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Dynamical Pion Propagation in the Functional Approach to the Charge Response

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We calculate the charge response to an electromagnetic field for a homogeneous system of nucleons and pions in the functional framework. This requires dealing with the pion propagation in the medium within the random-phase-approximation scheme. A quenching of the response is obtained in line with the experimental findings. Remarkably the quenching takes place mainly in the isoscalar channel.

In recent years we have developed a theory of the nuclear response to an external electromagnetic field utilizing functional techniques.[1] Here we apply this approach to the charge response of a translationally invariant system of nucleons and pions interacting through pseudoscalar coupling. The path-integral formalism is of great convenience in dealing with the many-body problem since it provides unambiguous prescriptions to consistently select classes of diagrams contributing, in some approximation, to any given Green's function. As discussed in Ref. 1, for an appropriate treatment of the nuclear response, the functional scheme forcibly suggests a semiclassical expansion for the pionic field starting from a bosonic effective action S_{eff}^B, which is set up via a Gaussian integration over the fermionic fields in the path-integral representation of the generating functional:[2]

$$Z[j_\mu^{\text{ex}}] = \frac{1}{\mathcal{N}} \int \mathcal{D}[\psi,\bar{\psi},\phi,A_\mu] \exp\left[i\int dx\, j_\mu^{\text{ex}} A^\mu\right] \exp\left[i\int dx(\mathcal{L} - \tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} + j_\mu A^\mu + B_{\mu\nu}A^\mu A^\nu)\right]. \tag{1}$$

In Eq. (1),

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}m_\pi^2\phi^2 - ig\bar{\psi}\gamma_5\tau\psi\phi, \tag{2}$$

$$j_\mu = e\bar{\psi}\frac{1+\tau_3}{2}\gamma_\mu\psi + e(\phi\times\partial_\mu\phi)_3, \tag{3}$$

$$B_{\mu\nu} = \tfrac{1}{2}e^2(\phi_1^2+\phi_2^2)g_{\mu\nu}, \tag{4}$$

ψ and ϕ being the fermion and the pion fields, $\mathcal{N}$ the (infinite) normalization constant, and j_μ^{ex} the external source of the electromagnetic field. We also carried out the integration over the field A_μ to the order e^2, as required by linear-response theory.

Now, in the resulting functional integral

$$Z[a_\mu] = \frac{1}{\mathcal{N}} \int \mathcal{D}[\phi]\exp[iS_{\text{eff}}^B(\phi)], \tag{5}$$

S_{eff}^B turns out to have a quite complex structure and we refer to Ref. 1 for its explicit expression and more detailed discussions. In Eq. (5) we have set

$$a_\mu(x) = \int dy\, D_{\mu\nu}^0(x-y) j_{\text{ex}}^\nu(y), \tag{6}$$

$D_{\mu\nu}^0$ being the bare photon propagator.

The loop expansion for the polarization propagator $\Pi_{\mu\nu}$ is now obtained by expanding the effective action around the mean-field configuration. Thus, denoting by ϕ_0 the saddle point, one gets

$$Z_c[a_\mu] = -i\ln Z[a_\mu]$$

$$= S_{\text{eff}}^B(\phi_0) + \frac{i}{2}\text{tr}\ln\left[\frac{\delta^2 S_{\text{eff}}^B(\phi)}{\delta\phi^i(x)\delta\phi^j(y)}\right]_{\phi=\phi_0} + \cdots, \tag{7}$$

the first term corresponding to the semiclassical limit, and the second one accounting instead for the quadratic (one-loop) corrections. The loop expansion for the polarization propagator follows from the definition

$$\Pi_{\mu\nu}(x,y) = -\left.\frac{\delta^2 Z_c[a_\mu]}{\delta a_\mu(x)\delta a_\nu(y)}\right|_{a_\mu=0}. \tag{8}$$

In Ref. 1 we have shown that the stationary-phase approximation leads to the familiar Lindhard function. while the next order in the expansion (8) collects together six Feynman diagrams, displayed in Fig. 1 where all pionic lines *are meant to be dressed in ring approximation.* It ought to be recalled that the loop expansion should be performed *around the saddle point* ϕ_0 provid-
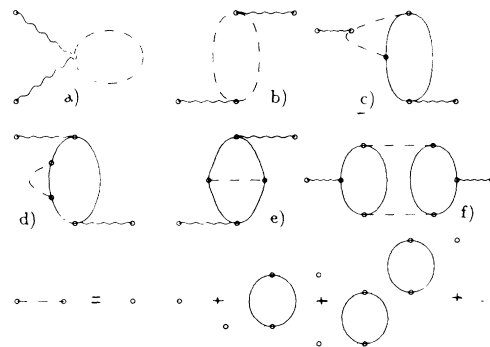

FIG. 1. The Feynman diagrams contributing to the nuclear response function at the first order in the loop expansion. Also displayed is the dynamical pion propagator.

ed it is unique, which, in general, is not true: In the present case another stationary point has been found,[3] connected with collective nuclear states outside the response region. Hopefully this does not affect too much the nuclear response in the quasielastic peak region. Here we ignore this difficulty and confine our task to evaluating the contributions displayed in Fig. 1. Of these, the first two are irrelevant to our purposes as they describe the photon self-energy and the hadronic components of the photon field, whereas the third represents a meson-exchange-current (MEC) contribution to the charge response, which is found to be quite small in the particle-hole (p-h) sector.[4,5] We therefore need to calculate the last three terms only: Yet, an acceptable degree of numerical accuracy can be achieved only at the price of a very large amount of computational time, even when use is made of the analytical results of Ref. 6 which allow us to reduce the problem to three-dimensional numerical integrations.

Further problems arise from the ultraviolet behavior of the integrands appearing, for example, in Fig. 1(d). Since in the present approach we neglect the frequency dependence of the *free* pion propagator, in first order no renormalization is required. In second order, however, divergencies do arise, cured by the usual phenomenological πNN form factor:

$$\Gamma_\pi(p) = \Lambda^2/(\Lambda^2 + \mathbf{p}^2).   \tag{9}$$

In the latter we take $\Lambda = 1.2$ GeV/c, which sets the scale of the problem as far as the size of the constituents of the system is concerned.

Moreover, to the pion exchange among the constituents, we heuristically added the Landau-Migdal parameter g' with a momentum dependence which should allow its use far from the Fermi surface:

$$g'(q) = 1 + (g'_0 - 1)[q_c^2/(q_c^2 + q^2)]^2   \tag{10}$$

($q_c = 900$ MeV/c). The parametrization (10) cancels exactly the high-momentum components of the one-pion exchange potential in the nonrelativistic limit and is equivalent to the one employed by Brown et al.[7] and Osaka and Toki[8] in dealing with short-range correlations. These are crucial for damping the otherwise wild spin-isospin density oscillations induced by the pion during its propagation.

We evaluated the energy behavior of the charge response at $q = 300$, 330, 370, and 410 MeV/c, using $k_F = 1.2$ fm^{-1}, which corresponds to the average density of a medium-size nucleus. Our results are displayed in Figs. 2(a)–2(d) for $g'_0 = 0.5$, together with the experimental data measured at Saclay[9] on ^{40}Ca. We account for the electromagnetic nucleonic size by means of the Sachs form factor $G_E(q_\mu^2)$. In a fully relativistic treatment, however, this prescription does not hold[10] and relativistic corrections are expected[4] to play some role al-
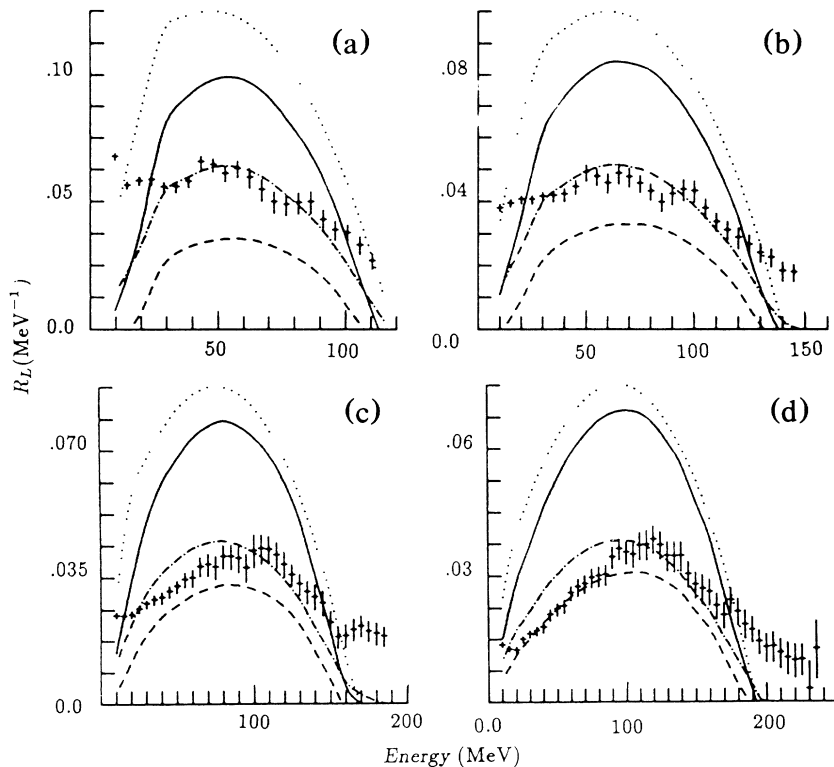

FIG. 2. The electromagnetic response function at (a) $q = 300$, (b) $q = 330$, (c) $q = 370$, and (d) $q = 410$ MeV/c: The dotted line is the free Lindhard function, the dashed line is the total isoscalar contribution, the dot-dashed line is the total isovector contribution, and the solid line is the total. Experimental data are also displayed for reference.

ready at $q = 2$ fm^{-1}.

The reduced strength of the separated longitudinal response, as compared with naive nuclear models, poses a challenging problem: Several interpretations of this puzzle have been offered[11-18] thus far. In our calculation the depletion of the longitudinal response arises almost entirely from the quenching of the isoscalar channel, while the isovector response is affected very little by the pionic correlations (being roughly half the free total one). This important result follows from the different combinations of the contributions associated with diagrams (d), (e), and (f) of Fig. 1 (we call them, for short, self-energy, exchange, and correlation, respectively) entering the $\tau = 1$ and $\tau = 0$ responses. We illustrate the intertwining among these different contributions in Fig. 3, where they are displayed at $q = 300$ MeV/c both in the isoscalar and in the isovector channel. The self-energy term is obviously the same in both channels, while the exchange term has different sign and magnitude as a result of isospin algebra. Furthermore, while the strong *negative* contribution to the isoscalar response of the correlation term is most significant, this is quite smaller in the isovector channel. The emphasis on isoscalar correlations seems to outline the importance of a quasideuteron component in the nuclear response to both real[19] and virtual photons in the intermediate-energy domain.

The origin of the quenching is found in certain negative Goldstone diagrams, associated with the Feynman correlation graph, which give intrinsically *different* contributions to the longitudinal and to the transverse response.[20] In the latter case a substantial component of the tail of the response is provided by the 2p-2h Goldstone term, already considered in Ref. 21. By contrast, in the charge longitudinal channel, where the experimental tail seems to be less pronounced than in the transverse one, the positive 2p-2h tail of the self-energy term is slightly overcome by the negative one of the correlation graph. The balance between these two contributions to the charge response is, however, delicate and our one-loop approximation fails to deal with it properly, leading, as a matter of fact, to a small violation of unitarity.

Whether the two-loop terms can restore unitarity at large energies is a question whose answer would require an even larger computational effort than the present one. It should also be recalled that the MEC, neglected here in spite of being first order in the loop expansion, may also lead to an appreciable cross section above the quasielastic peak (QEP). Moreover, the true π emission, neglected here due to the use of static free pion propagators, should also contribute to the experimental tail outside the 1p-1h continuum.

Concerning the Coulomb sum rule, we recall the findings of Ref. 4 that, in first-order pion exchange and without short-range correlations ($g' = 0$), the charge response is "depleted" by the pion at low frequencies, but "enhanced" at large ones: As a consequence, the
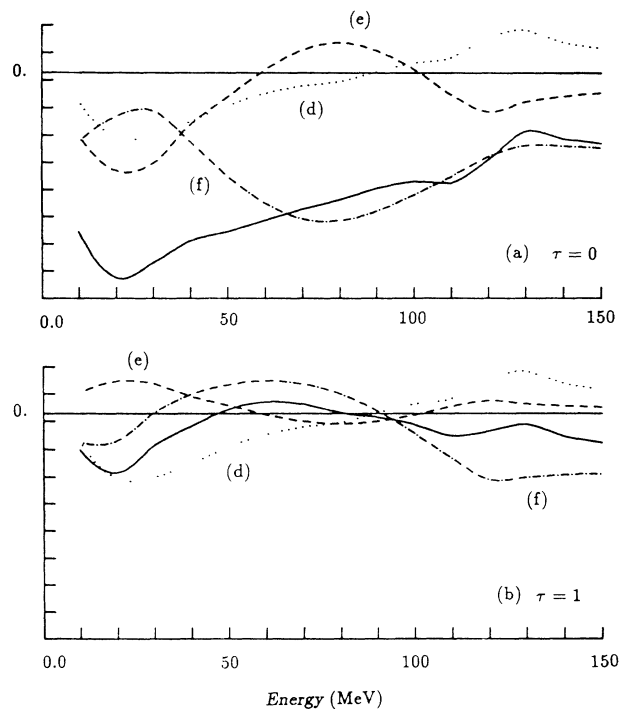

FIG. 3. Separated one-loop contributions in the (a) isoscalar and (b) isovector channels at $q = 300$ MeV/c: The self-energy (dotted line), exchange (dashed line), and correlation (dot-dashed line) contributions are shown. In both channels the solid line is the total.

Coulomb sum rule $S(q)$, which is only affected by ground-state correlations, coincides with that of the free Fermi gas. Here the dynamical random-phase-approximation (RPA) propagation of the pion entails the existence of a highly correlated ground state: hence, the depletion of the charge response at all energies and, in turn, of $S(q)$.

Because of the present limited knowledge of the underlying physics, we have introduced here two phenomenological scales: one fixing the size of the constituents of the system, Λ, and one setting the range of the NN correlations induced by the repulsive core, q_c. Our results are quite stable against moderate variations of Λ, whereas they do depend upon g'_0 and, more strongly so, upon q_c. Specifically, the trend emerging from our calculations points to an increased depletion of the isoscalar channel when the high-momentum components of the interaction are increased as well: This damping can be achieved either through a lower value for g'_0 or via an enhancement of the scale set by q_c.

We notice that the quenching we obtain in the charge response is induced by the dynamical RPA propagation of the pion, which should not be confused with the conventional RPA correlations pertaining to this channel, which are due to the standard scalar and isoscalar Landau-Migdal parameters f and f'. Their role is illustrated in Fig. 4, where the longitudinal response calculat-

ed at $q = 300$ MeV/c within the RPA scheme,

$$R_L(q,\omega) = G_E^2(q_\mu^2) \left[-\frac{3\pi^2 Z}{2k_F^3} \right] \text{Im} \left[\frac{\Pi_{\tau=0}^*}{1 - Cf\Pi_{\tau=0}^*} + \frac{\Pi_{\tau=1}^*}{1 - Cf'\Pi_{\tau=1}^*} \right], \tag{11}$$

is shown. We have set $f' = 1.2$ and $f = -0.7$. This last choice stems from the necessity of accounting for the attraction developed by the density-dependent, isoscalar force on the surface of the nucleus.[17] Obviously this can only be done empirically in a Fermi-gas framework, and the value -0.7 is just for orientation.

The impact of the RPA correlations is felt through a reduction and a reshaping of the response, which tends to improve the overall agreement with experimental data.

In conclusion, the present work indicates that the nuclear charge response is much quenched in the isoscalar channel with respect to the Fermi-gas one and this outcome is to a large extent due to the pion. It is remarkable that the inclusion of retardation effects on the pion-carried interaction inside nuclei is effective in pulling strength out of the QEP domain. We also notice how the functional approach collects together *at the same order in the loop expansion* exactly those diagrams needed for this purpose. However, our approach is still insufficient to explain the trend of the experimental data which show that strength is still missing in the charge response at momenta as large as 3 fm^{-1}, where the effects associated with the pion are no longer operative in our approach.

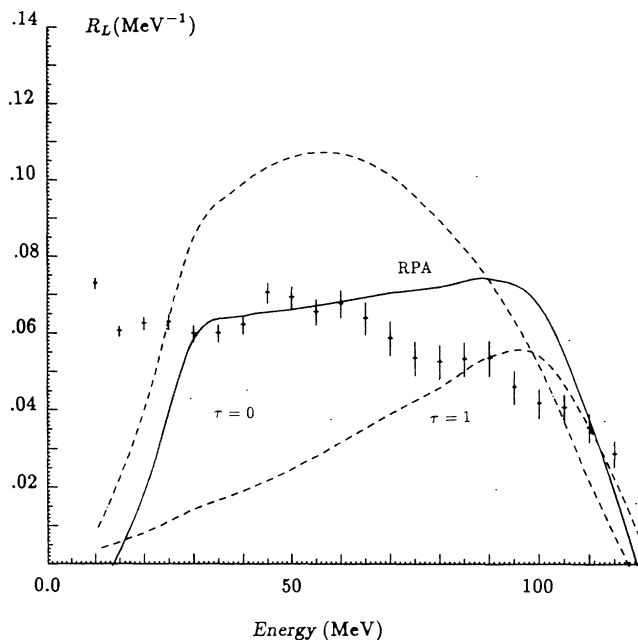It therefore appears that a yet unsolved and challenging problem remains to be tackled.

We wish to thank Professor M. Ericson for fruitful discussions.

FIG. 4. Longitudinal electromagnetic response function for $q = 300$ MeV/c and $f = -0.7$, $f' = 1.2$: The free Lindhard function (sparse dots), the total without RPA correlations (dot-dashed), the separated isoscalar (dots) and isovector (dashed) contributions, and the total response in RPA (solid).

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