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(Article begins on next page)

# MOTIVIC GALOIS GROUPS OF 1-MOTIVES: A SURVEY

CRISTIANA BERTOLIN

ABSTRACT. We investigate the structure of the motivic Galois groups of 1-motives defined over a field of characteristic 0.

In this note we review the main results of [B03] and [B04].

Let k be a field of characteristic 0 and let  $\overline{k}$  be its algebraic closure. Let  $\mathcal{T}$  be a Tannakian category over k. The tensor product of  $\mathcal{T}$  allows us to define the notion of Hopf algebras in the category  $\operatorname{Ind}\mathcal{T}$  of Ind-objects of  $\mathcal{T}$ . The category of affine group  $\mathcal{T}$ -schemes is the opposite of the category of Hopf algebras in  $\operatorname{Ind}\mathcal{T}$ .

The fundamental group  $\pi(\mathcal{T})$  of  $\mathcal{T}$  is the affine group  $\mathcal{T}$ -scheme  $\operatorname{Sp}(\Lambda)$ , whose Hopf algebra  $\Lambda$  is endowed for each object X of  $\mathcal{T}$  with a morphism  $X \longrightarrow \Lambda \otimes X$ functorial in X, and is universal for these properties. Those morphisms  $\{X \longrightarrow \Lambda \otimes X\}_{X \in \mathcal{T}}$  define an action of the fundamental group  $\pi(\mathcal{T})$  on each object of  $\mathcal{T}$ . For each fibre functor  $\omega$  of  $\mathcal{T}$  over a k-scheme S,  $\omega\pi(\mathcal{T})$  is the affine group S-scheme  $\underline{\operatorname{Aut}}_{S}^{\otimes}(\omega)$  which represents the functor which associates to each S-scheme  $T, u: T \longrightarrow S$ , the group of automorphisms of  $\otimes$ -functors of the functor  $u^*\omega$ .

If  $\mathcal{T}(k)$  is a Tannakian category generated by motives defined over k (in an appropriate category of mixed realizations), the fundamental group  $\pi(\mathcal{T}(k))$  is called the *motivic Galois group*  $\mathcal{G}_{mot}(\mathcal{T}(k))$  of  $\mathcal{T}(k)$  and for each embedding  $\sigma: k \longrightarrow \mathbb{C}$ , the fibre functor  $\omega_{\sigma}$  "Hodge realization" furnishes the  $\mathbb{Q}$ -algebraic group

$$\omega_{\sigma}\mathcal{G}_{\mathrm{mot}}(\mathcal{T}) = \operatorname{Spec}\left(\omega_{\sigma}(\Lambda)\right) = \underline{\operatorname{Aut}}_{\mathbb{O}}^{\otimes}(\omega_{\sigma})$$

which is the Hodge realization of the motivic Galois group of  $\mathcal{T}(k)$ .

## EXAMPLES:

- (1) From the main theorem on neutral Tannakian categories, we know that the Tannakian category  $\operatorname{Vec}(k)$  of finite dimensional k-vector spaces is equivalent to the category of finite-dimensional k-representations of  $\operatorname{Spec}(k)$ . In this case, affine group  $\mathcal{T}$ -schemes are affine group k-schemes and  $\pi(\operatorname{Vec}(k))$  is  $\operatorname{Spec}(k)$ .
- (2) Let  $\mathcal{T} = \operatorname{Rep}_k(G)$  be the Tannakian category of k-representations of an affine group k-scheme G. The affine group  $\mathcal{T}$ -schemes are affine k-schemes endowed with an action of G and the fundamental group  $\pi(\mathcal{T})$  of  $\mathcal{T}$  is G endowed with its action on itself by inner automorphisms (see [D89] 6.3).
- (3) Let  $\mathcal{T}_0(k)$  be the Tannakian category of Artin motives over k, i.e. the Tannakian sub-category of the Tannakian category of mixed realizations for absolute Hodge cycles (see [J90] I 2.1) generated by pure realizations of 0-dimensional varieties over k. The motivic Galois group  $\mathcal{G}_{\text{mot}}(\mathcal{T}_0(k))$  of  $\mathcal{T}_0(k)$  is the affine group  $\mathbb{Q}$ -scheme  $\text{Gal}(\overline{k}/k)$  endowed with its action on itself by inner automorphisms. We denote it by  $\mathcal{GAL}(\overline{k}/k)$ . In particular,

#### CRISTIANA BERTOLIN

for any fibre functor  $\omega$  over Spec ( $\mathbb{Q}$ ) of  $\mathcal{T}_0(k)$ , the affine group scheme  $\omega(\mathcal{GAL}(\overline{k}/k)) = \underline{\operatorname{Aut}}_{\operatorname{Spec}(\mathbb{Q})}^{\otimes}(\omega) \text{ is canonically isomorphic to } \operatorname{Gal}(\overline{k}/k).$ 

- (4) The motivic Galois group  $\mathcal{G}_{mot}(\mathbb{Z}(0))$  of the unit object  $\mathbb{Z}(0)$  of  $\mathcal{T}_0(k)$  is the affine group  $\langle \mathbb{Z}(0) \rangle^{\otimes}$ -scheme Sp( $\mathbb{Z}(0)$ ). For each fibre functor "Hodge realization"  $\omega_{\sigma}$ , we have that  $\omega_{\sigma}(\mathcal{G}_{\text{mot}}(\mathbb{Z}(0))) := \operatorname{Spec}(\omega_{\sigma}(\mathbb{Z}(0))) = \operatorname{Spec}(\mathbb{Q}),$ which is the Mumford-Tate group of  $T_{\sigma}(\mathbb{Z}(0))$ .
- (5) Let  $\langle \mathbb{Z}(1) \rangle^{\otimes}$  be the Tannakian category over  $\mathbb{Q}$  defined by the k-torus  $\mathbb{Z}(1)$ . The motivic Galois group  $\mathcal{G}_{mot}(\mathbb{Z}(1))$  of the torus  $\mathbb{Z}(1)$  is the affine group  $(\mathbb{Z}(1))^{\otimes}$ -scheme  $\mathbb{G}_m$  defined by the  $\mathbb{Q}$ -scheme  $\mathbb{G}_{m/\mathbb{Q}}$ . For each fibre functor "Hodge realization"  $\omega_{\sigma}$ , we have that  $\omega_{\sigma}(\mathbb{G}_m) = \mathbb{G}_{m/\mathbb{Q}}$ , which is the Mumford-Tate group of  $T_{\sigma}(\mathbb{Z}(1))$ .
- (6) If k is algebraically closed, the motivic Galois group of motives of CM-type over k is the Serre group (cf. [M94] 4.8).
- (7) The Tannakian category  $\mathcal{T}_1(k)$  of 1-motives over k is the Tannakian subcategory of the Tannakian category of mixed realizations (for absolute Hodge cycles) generated by mixed realizations of 1-motives over k. Recall that a 1-motive  $M = [X \xrightarrow{u} G]$  over k consists of
  - a group scheme X over k, which is locally for the étale topology, a constant group scheme defined by a finitely generated free Z-module,
  - a semi-abelian variety G defined over k, i.e. an extention of an abelian • variety A by a torus Y(1), which cocharacter group Y,
  - a morphism  $u: X \longrightarrow G$  of group schemes over k.

1-motives are mixed motives of level  $\leq 1$ : the weight filtration W<sub>\*</sub> on M is  $W_i(M) = M$  for each  $i \ge 0$ ,  $W_{-1}(M) = G$ ,  $W_{-2}(M) = Y(1)$ ,  $W_j(M) = 0$ for each  $j \le -3$ . If  $\operatorname{Gr}_n^W = W_n/W_{n-1}$ , we have the quotients  $\operatorname{Gr}_0^W(M) = X$ ,  $\operatorname{Gr}_{-1}^W(M) = A$  and  $\operatorname{Gr}_{-2}^W(M) = Y(1)$ . We will denote by  $W_{-1}\mathcal{T}_1(k)$ (resp.  $\operatorname{Gr}_0^W \mathcal{T}_1(k), \ldots$ ) the Tannakian sub-category of  $\mathcal{T}_1(k)$  generated by all  $W_{-1}M$  (resp.  $\operatorname{Gr}_0^W M$ , ...) with M a 1-motive. With this notation we can easily compute the following motivic Galois groups

- $\mathcal{G}_{mot}(\operatorname{Gr}_{0}^{W}\mathcal{T}_{1}(k)) = \mathcal{GAL}(\overline{k}/k),$   $\mathcal{G}_{mot}(\operatorname{Gr}_{-2}^{W}\mathcal{T}_{1}(k)) = \mathcal{GAL}(\overline{k}/k) \times \mathbb{G}_{m}.$   $\mathcal{G}_{mot}(\operatorname{Gr}_{0}^{W}\mathcal{T}_{1}(\overline{k})) = \mathcal{G}_{mot}(\mathcal{T}_{0}(\overline{k})) = \operatorname{Sp}(\mathbb{Z}(0))$   $\mathcal{G}_{mot}(\operatorname{Gr}_{-2}^{W}\mathcal{T}_{1}(\overline{k})) = \mathbb{G}_{m}$

# 1. MOTIVIC GALOIS THEORY

For each Tannakian sub-category  $\mathcal{T}'$  of  $\mathcal{T}$ , let  $H_{\mathcal{T}}(\mathcal{T}')$  be the kernel of the faithfully flat morphism of group  $\mathcal{T}$ -schemes  $I: \pi(\mathcal{T}) \longrightarrow i\pi(\mathcal{T}')$  corresponding to the inclusion functor  $i: \mathcal{T}' \longrightarrow \mathcal{T}$ . In particular we have the short exact sequence of group  $\pi(\mathcal{T})$ -schemes

$$0 \longrightarrow H_{\mathcal{T}}(\mathcal{T}') \longrightarrow \pi(\mathcal{T}) \longrightarrow i\pi(\mathcal{T}') \longrightarrow 0.$$

In [D89] 6.6, Deligne proves that the Tannakian category  $\mathcal{T}'$  is equivalent, as tensor category, to the sub-category of  $\mathcal{T}$  generated by those objects on which the action of  $\pi(\mathcal{T})$  induces a trivial action of  $H_{\mathcal{T}}(\mathcal{T}')$ . In particular, this implies that the fundamental group  $\pi(\mathcal{T}')$  of  $\mathcal{T}'$  is isomorphic to the group  $\mathcal{T}$ -scheme  $\pi(\mathcal{T})/H_{\mathcal{T}}(\mathcal{T}')$ . The group  $\mathcal{T}$ -scheme  $H_{\mathcal{T}}(\mathcal{T}')$  characterizes the Tannakian sub-category  $\mathcal{T}'$ . In fact

we have a clear dictionary between Tannakian sub-categories of  $\mathcal{T}$  and normal affine group sub- $\mathcal{T}$ -schemes of the fundamental group  $\pi(\mathcal{T})$  of  $\mathcal{T}$ :

**Theorem 1.0.1.** There is bijection between the Tannakian sub-categories of  $\mathcal{T}$  and the normal affine group sub- $\mathcal{T}$ -schemes of  $\pi(\mathcal{T})$ , which associates

- to each Tannakian sub-category  $\mathcal{T}'$  of  $\mathcal{T}$ , the kernel  $H_{\mathcal{T}}(\mathcal{T}')$  of the morphism of  $\mathcal{T}$ -schemes  $I : \pi(\mathcal{T}) \longrightarrow i\pi(\mathcal{T}')$  corresponding to the inclusion  $i : \mathcal{T}' \longrightarrow \mathcal{T}$ ;

- to each normal affine group sub- $\mathcal{T}$ -scheme H of  $\pi(\mathcal{T})$ , the Tannakian subcategory  $\mathcal{T}(H)$  of objects of  $\mathcal{T}$  on which the action of  $\pi(\mathcal{T})$  induces a trivial action of H.

## 2. The case of motives of level $\leq 1$

In order to study the category  $\mathcal{T}_1(k)$  of motives of level  $\leq 1$ , in [B04] we have applied the above theorem to some sub-categories of  $\mathcal{T}_1(k)$ . The weight filtration  $W_*$  of 1-motives induces an increasing filtration  $W_*$  of 3 steps on the motivic Galois group  $\mathcal{G}_{\text{mot}}(\mathcal{T}_1(k))$  which we describe through the action of  $\mathcal{G}_{\text{mot}}(\mathcal{T}_1(k))$  on the generators of  $\mathcal{T}_1(k)$ : For each 1-motive M over k, we have that

- $W_0(\mathcal{G}_{\mathrm{mot}}(\mathcal{T}_1(k))) = \mathcal{G}_{\mathrm{mot}}(\mathcal{T}_1(k))$
- $W_{-1}(\mathcal{G}_{\mathrm{mot}}(\mathcal{T}_1(k))) = \{g \in \mathcal{G}_{\mathrm{mot}}(\mathcal{T}_1(k)) \mid (g id)M \subseteq W_{-1}(M), (M) \in W_{-1}(M)\}$
- $(g id)W_{-1}(M) \subseteq W_{-2}(M), (g id)W_{-2}(M) = 0\},$
- $W_{-2}(\mathcal{G}_{\text{mot}}(\mathcal{T}_1(k))) = \{g \in \mathcal{G}_{\text{mot}}(\mathcal{T}_1(k)) \mid (g id)M \subseteq W_{-2}(M), (g id)W_{-1}(M) = 0\},\$
- $W_{-3}(\mathcal{G}_{\text{mot}}(\mathcal{T}_1(k))) = 0.$

According to the motivic analogue of [Br83] §2.2,  $\operatorname{Gr}_{0}^{W}(\mathcal{G}_{\mathrm{mot}}(\mathcal{T}_{1}(k)))$  is a reductive group sub- $\mathcal{T}_{1}(k)$ -scheme of  $\mathcal{G}_{\mathrm{mot}}(\mathcal{T}_{1}(k))$  and  $W_{-1}(\mathcal{G}_{\mathrm{mot}}(\mathcal{T}_{1}(k)))$  is the unipotent radical of  $\mathcal{G}_{\mathrm{mot}}(\mathcal{T}_{1}(k))$ . Each of these 3 steps  $W_{-i}(\mathcal{G}_{\mathrm{mot}}(\mathcal{T}_{1}(k)))$  (i = 0, 1, 2) can be reconstructed as intersection of group sub- $\mathcal{T}_{1}(k)$ -schemes of  $\mathcal{G}_{\mathrm{mot}}(\mathcal{T}_{1}(k))$  associated to Tannakian sub-categories of  $\mathcal{T}_{1}(k)$  through the bijection 1.0.1:

Lemma 2.0.2. (1) 
$$W_{-1}(\mathcal{G}_{\text{mot}}(\mathcal{T}_{1}(k))) = \bigcap_{i=-1,-2} H_{\mathcal{T}_{1}(k)}(\text{Gr}_{i}^{W}\mathcal{T}_{1}(k)),$$
  
(2)  $W_{-2}(\mathcal{G}_{\text{mot}}(\mathcal{T}_{1}(k))) = H_{\mathcal{T}_{1}(k)}(W_{-1}\mathcal{T}_{1}(k)) = H_{\mathcal{T}_{1}(k)}(W_{0}/W_{-2}\mathcal{T}_{1}(k)).$ 

The explicit computation of these group sub- $\mathcal{T}_1(k)$ -schemes involved in the above lemma will provide four exact short sequences of group sub- $\mathcal{T}_1(k)$ -schemes of  $\mathcal{G}_{\text{mot}}(\mathcal{T}_1(k))$ :

**Theorem 2.0.3.** We have the following diagram of affine group  $T_1(k)$ -schemes

where all horizontal short sequences are exact and where the vertical arrows on the left are inclusions and those on the right are surjections.

#### CRISTIANA BERTOLIN

#### 3. The case of a 1-motive

Let  $M = [X \xrightarrow{u} G]$  be a 1-motive defined over k. The motivic Galois group  $\mathcal{G}_{\text{mot}}(M)$  of M is the fundamental group of the Tannakian sub-category  $\langle M \rangle^{\otimes}$  of  $\mathcal{T}_1(k)$  generated by M i.e. the affine group  $\langle M \rangle^{\otimes}$ -scheme Sp( $\Lambda$ ), where  $\Lambda$  is the Hopf algebra of  $\langle M \rangle^{\otimes}$  universal for the following property: for each object X of  $\langle M \rangle^{\otimes}$ , there is a morphism  $\lambda_X : X^{\vee} \otimes X \longrightarrow \Lambda$  functorial in X. The morphisms  $\{\lambda_X\}$ , which can be rewritten in the form  $X \longrightarrow X \otimes \Lambda$ , define an action of the group  $\mathcal{G}_{\text{mot}}(M)$  on each object X of  $\langle M \rangle^{\otimes}$ , and in particular on itself. The main result of [B03] is that

**Theorem 3.0.4.** The unipotent radical  $W_{-1}(\text{Lie }\mathcal{G}_{\text{mot}}(M))$  of the Lie algebra of  $\mathcal{G}_{\text{mot}}(M)$  is the semi-abelian variety defined by the adjoint action of the graded  $\operatorname{Gr}^W_*(W_{-1}\text{Lie }\mathcal{G}_{\text{mot}}(M))$  on itself.

The idea of the proof is as followed: Before recall that according to [D75] (10.2.14), to have M is equivalent to have the 7-uplet  $(X, Y^{\vee}, A, A^*, v, v^*, \psi)$  where

- X and Y<sup>∨</sup> are two group k-schemes, which are locally for the étale topology, constant group schemes defined by a finitely generated free Z-module;
- A and A\* are two abelian varieties defined over k, dual to each other;
  v : X → A and v\* : Y<sup>∨</sup> → A\* are two morphisms of group k-schemes;
- $\psi$  is a trivialization of the pull-back  $(v, v^*)^* \mathcal{P}_A$  by  $(v, v^*)$  of the Poincaré biextension  $\mathcal{P}_A$  of  $(A, A^*)$ .

Observe that the 4-uplet  $(X, Y^{\vee}, A, A^*)$  corresponds to the pure part of the 1motive, i.e. it defines the pure motives underlying M, and the 3-uplet  $(v, v^*, \psi)$ represents the "mixity" of M.

Consider the motive  $E = W_{-1}(\underline{\operatorname{End}}(\operatorname{Gr}^W_* M))$ : it is a split 1-motive of weight -1 and -2 obtained from the endomorphisms of the graded  $\operatorname{Gr}^W_* M$  of M. The composition of endomorphisms endowed E with a Lie algebra structure (E, [, ]), whose crochet [, ] corresponds to a  $\Sigma - X^{\vee} \otimes Y(1)$ -torsor  $\mathcal{B}$  living over  $A \otimes X^{\vee} + A^* \otimes Y$ . The action of E on the motive  $\operatorname{Gr}^W_*(M)$  is described by a morphism

$$E \otimes \operatorname{Gr}^W_*(M) \longrightarrow \operatorname{Gr}^W_*(M)$$

which endowed the motive  $\operatorname{Gr}^W_*(M)$  with a structure of (E, [, ])-module.

Denote by  $b = (b_1, b_2)$  the k-rational point  $b = (b_1, b_2)$  of the abelian variety  $A \otimes X^{\vee} + A^* \otimes Y$  defining the morphisms  $v: X \longrightarrow A$  and  $v^*: Y^{\vee} \longrightarrow A^*$ . Let B be the smallest abelian sub-variety of  $X^{\vee} \otimes A + A^* \otimes Y$  containing this point  $b = (b_1, b_2)$ . The restriction  $i^*\mathcal{B}$  of the  $\Sigma - X^{\vee} \otimes Y(1)$ -torsor  $\mathcal{B}$  by the inclusion  $i: B \longrightarrow X^{\vee} \otimes A \times A^* \otimes Y$  is a  $\Sigma - X^{\vee} \otimes Y(1)$ -torsor over B. Denote by  $Z_1$  the smallest  $\operatorname{Gal}(\overline{k}/k)$ -module of  $X^{\vee} \otimes Y$  such that the torus  $Z_1(1)$ , that it defines, contains the image of the restriction  $[, ]: B \otimes B \longrightarrow X^{\vee} \otimes Y(1)$  of the Lie crochet to  $B \otimes B$ . The direct image  $p_*i^*\mathcal{B}$  of the  $\Sigma - X^{\vee} \otimes Y(1)$ -torsor  $i^*\mathcal{B}$  by the projection  $p: X^{\vee} \otimes Y(1) \longrightarrow (X^{\vee} \otimes Y/Z_1)(1)$  is a trivial  $\Sigma - (X^{\vee} \otimes Y/Z_1)(1)$ -torsor over B. We denote by  $\pi: p_*i^*\mathcal{B} \longrightarrow (X^{\vee} \otimes Y/Z_1)(1)$  the canonical projection. The morphism  $u: X \longrightarrow G$  defines a point  $\tilde{b}$  in the fibre of  $\mathcal{B}$  over b. We denote again by  $\tilde{b}$  the points of  $i^*\mathcal{B}$  and of  $p_*i^*\mathcal{B}$  over the point b of B. Let Z be the smallest sub-Gal $(\overline{k}/k)$ -module of  $X^{\vee} \otimes Y$ , containing  $Z_1$  and such that the subtorus  $(Z/Z_1)(1)$  of  $(X^{\vee} \otimes Y/Z_1)(1)$  contains  $\pi(\tilde{b})$ . If we put  $Z_2 = Z/Z_1$ , we have that  $Z(1) = Z_1(1) \times Z_2(1)$ .

With these notations, the unipotent radical  $W_{-1}(\text{Lie}\,\mathcal{G}_{\text{mot}}(M))$  of the Lie algebra of  $\mathcal{G}_{\text{mot}}(M)$  is the extension of the abelian variety B by the torus Z(1) defined by the adjoint action of (B + Z(1), [, ]) on itself. Since in the construction of B and Z(1) are involved only the parameters  $v, v^*$  and u, the computation of the unipotent radical  $W_{-1}(\text{Lie}\,\mathcal{G}_{\text{mot}}(M))$  of the Lie algebra of  $\mathcal{G}_{\text{mot}}(M)$  depends only on the 3-uplet  $(v, v^*, \psi)$ , i.e. on the "mixity" of the 1-motive M.

EXAMPLES:

- (1) Let M be the split 1-motive  $\mathbb{Z} \oplus A \oplus \mathbb{G}_m$ . In this case all is trivial:  $W_{-1}(\text{Lie}\,\mathcal{G}_{\text{mot}}(M)) = B = Z(1) = 0.$
- (2) Let  $M = [\mathbb{Z} \xrightarrow{u} \mathcal{E}]$  be a 1-motive over k defined by u(1) = P with P a non-torsion k-rational point of the elliptic curve  $\mathcal{E}$ . We have that the torus Z(1) is trivial and the unipotent radical  $W_{-1}(\text{Lie }\mathcal{G}_{\text{mot}}(M))$  is the elliptic curve  $B = \mathcal{E}$ .
- (3) Let  $M = [\mathbb{Z} \xrightarrow{u} \mathbb{G}_m^3 \times A]$  be a 1-motive over k defined by  $u(1) = (q_1, q_2, 1, 0)$ with  $q_1, q_2$  two elements of  $\mathbb{G}_m(k) - \mu_\infty$  multiplicatively independents  $(\mu_\infty \text{ is the group of roots of the unity in } \overline{k})$ . In this example the abelian variety B is trivial and the unipotent radical  $W_{-1}(\text{Lie } \mathcal{G}_{\text{mot}}(M))$  is the torus  $Z(1) = \mathbb{G}_m^2$ .

With the above notations we have also that

**Proposition 3.0.5.** The derived group of the unipotent radical  $W_{-1}(\text{Lie }\mathcal{G}_{\text{mot}}(M))$ of the Lie algebra of  $\mathcal{G}_{\text{mot}}(M)$  is the torus  $Z_1(1)$ .

# Proposition 3.0.6.

 $\dim \operatorname{Lie} \mathcal{G}_{\mathrm{mot}}(M) = \dim B + \dim Z(1) + \dim \operatorname{Lie} \mathcal{G}_{\mathrm{mot}}(\operatorname{Gr}^W_* M).$ 

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