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A target-based foundation for the “hard-easy effect” bias

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Abstract

The “hard-easy effect” is a well-known cognitive bias on self-confidence calibration that refers to a tendency to overestimate the probability of success in hard-perceived tasks, and to underestimate it in easy-perceived tasks. This paper provides a target-based foundation for this effect, and predicts its occurrence in the expected utility framework when utility functions are S-shaped and asymmetrically tailed. First, we introduce a definition of hard-perceived and easy-perceived task based on the mismatch between an uncertain target to meet and a suitably symmetric reference point. Second, switching from a target-based language to a utility-based language, we show how this maps to an equivalence between the hard-perceived target/gain seeking and the easy-perceived target/loss aversion. Third, we characterize the agent’s miscalibration in self-confidence. Finally, we derive sufficient conditions for the “hard-easy effect” and the “reversed hard-easy effect” to hold.

Keywords: Expected utility, Hard-easy effect bias, Endowment effect bias, Sunk cost effect bias, Benchmarking procedure, Loss-gain asymmetry, van Zwet skewness conditions.

JEL classification: C91; D81

1. Introduction

Since the 1970s an extensive experimental research has amassed evidence of systematic cognitive biases in individuals’ misperception on own performance. In this paper we focus on the well-known self-misconfidence bias called “hard-easy effect”, that refers to a tendency to overestimate the probability of success in hard-perceived tasks, and to underestimate it in easy-perceived tasks. The phenomenon was originally pinpointed by Lichtenstein and Fischhoff (1977) and subsequently ample experimental literature has confirmed this human behavior; see Peon et al. (2014) and references therein. In recent years, many authors have attributed findings of misconfidence to psychological biases occurring in specific economic environments; see e.g. Burks et al. (2013), Ludwig et al. (2011), Moore and Healy (2008), and references therein.

This paper tackles the problem from a different perspective. We demonstrate sufficient conditions for an Expected Utility (EU) agent endowed with an asymmetric tailed S-shaped utility function to act according to the "hard-easy effect" or its reversed effect. Our findings are achieved along the following steps.

First, we present a characterization of hard-perceived and easy-perceived targets on the basis of the target distribution asymmetry.

Second, we illustrate the link between the perceived-task difficulty and the perceived-gain-loss asymmetry. Following the seminal ideas of Borch (1968) and Berhold (1973), we normalize the agent utility function and interpret it as the cumulative distribution function (c.d.f.) of the uncertain target to meet. Then we extend the local definition of loss aversion given by Kahneman and Tversky (1979, p. 279) to formalize the sentiment "losses loom larger than gains" over the wealth domain. In the same way we formalize the patterns "gains loom larger than losses" and "gains loom equal to losses". Then we prove in Theorem 2 the equivalence of these patterns with the perception of easy, hard and well-calibrated tasks, respectively.

Our results match the empirical evidence: underperforming an easy-perceived task (negatively) looms more than outperforming it; and vice versa, outperforming a hard-perceived task (positively) looms larger than underperforming it. That behavior can be explained by the "*endowment effect*" (Kahneman et al., 1991) based on the fact that people ascribe more value to an object when they feel own than when they don't.

Third, we present the main contribution of the paper. We provide sufficient conditions for an EU agent to act according to the "hard-easy effect" or "reversed hard-easy effect"; see Theorem 3. They involve both the perception of task-difficulty and the "quality" of the lottery that the agent has at hand to successfully meeting the desired target. As a by-product we identify situations in enterprise risk management (ERM) where misconfidence in judgments emerges.

The remainder of the paper is organized as follows. In Section 2, we introduce the definitions of hard-perceived and easy-perceived targets. In Section 3, we show the equivalence between the perceived loss-gain asymmetry and perceived task difficulty augmented by the so-called endowment effect. Sufficient conditions for the "hard-easy effect" and its reversed effect are set out in Section 4. Section 5 concludes the article. An Appendix collects the proofs.

2. The task difficulty calibration: the van Zwet (1979) conditions

Ample experimental psychology literature has investigated how an agent calibrates the subjective perception of task difficulty; see Lichtenstein et al. (1982), Yates (1990) among others.

Intuitively a task is hard-perceived if the target to meet is most likely higher than the objective reference point; and vice versa if the target is lower; see Roy et al. (2013, p. 201) for a discussion about the link between the target distribution asymmetry and the perceived task difficulty.

Clearly, a fundamental question is how to set the subjective and objective reference points. During the past two decades, that question has become a major research topic in the experimental and theoretical economics area, see, e.g., Heath et al. (1999), Hoffmann et al. (2013), and references therein. In experimental research the most common measure used is the *median*; see Heath et al. (1999). Given a random target *T*, the value *m* such that $P(T < m) < 0.5$ and $P(T \geq m) \geq 0.5$ is called a median of *T*. The median thus defined always exists, and is unique. Alternative definitions are possible, but they yield the same answer when the distribution of *T* is continuous and unimodal. Hereafter we consider only targets with unimodal distributions.

In the next section we argue that the subjective reference point for a (strictly) unimodal target is better captured by its mode than by its median.

2.1. The perceived central value: the role of the mode

In real world situations targets are very often uncertain. This may occur when the agents are asked to outdo the competitors' future performances. Or simply, the target may be vague at the time of decision, as is the case for a risky investment plan in view of achieving some target wealth at a future time. That is the case for an investor trying to forecast her future needs in order to pick the best retirement planning. The different the agent risk attitude, the different the perceived benchmark she feels she is asked to meet. Now we show a way to extract risk preference information from the target perception.

As it was seminally discussed by Borch (1968) in the context of ruin probabilities, the agent utility function *u* is assumed bounded, increasing (but not constant) and continuous. Without loss of generality, by a positive affine transformation the cardinal utility *u* can be normalized so that $\inf u(x) = 0$ and $\sup u(x) = 1$. Then *u* satisfies all the properties characterizing a c.d.f. As Berhold (1973, p. 825) states "there are advantages to having the utility function represented by a distribution", because that permits the use of the known properties of distribution functions to find analytic results. In fact, we have:

$$u(x) = P(x \geq T) = F(x) \tag{1}$$

where *F* is the c.d.f. of the uncertain target *T*. This somewhat surprising equivalence states that the c.d.f. *F* of the target *T* *just* coincides with the normalized utility function *u*. To put it differently, Equation (1) tells us that we can think of the agent cardinal utility $u(x)$ as the probability that the uncertain target *T* is not greater than *x*. For example, we can interpret $u(1)$ as the probability that the agent target is not greater than 1 euro, rather than as the cardinal utility of 1 euro, and vice versa.

In this paper, the target *T* is assumed to be a unimodal variable about the mode *M*, i.e. $F(x)$ on the support (a,b) is convex for $x \in (a,M)$ and concave for $x \in (M,b)$. (If *T* is not strictly unimodal, let *M* be the midpoint of the modal set.) By equivalence (1) it follows that the utility function *u* is S-shaped with the concavity switching point in correspondence of the mode *M.* As a consequence *the mode M* is the subjectively perceived "watershed" value that divides the domain of losses from that of gains.

Now we are ready to introduce the definition of hard-perceived and easy-perceived tasks by reformulating the van Zwet (1979) conditions for unimodal distributions. These are referred to the median *m*, that by virtue of its definition, objectively splits the values of a r.v. *X* into two equally likely sets of gains and losses. As we will see, it is the inherent tension between the "subjective" watershed *M* and the "objective" watershed *m* that underlies the hard-easy effect.

Definition 1. Let a unimodal continuous target *T* with median *m* taken as reference point and continuous c.d.f. *F.* Then *T* is called:

a) a *hard-perceived target* with respect to *m* if $P(T \le m - x) \ge P(T \ge m + x)$ for any $x \ge 0$;

b) an *easy-perceived target* with respect to *m* if $P(T \le m - x) \le P(T \ge m + x)$ for any $x \ge 0$;

c) a *symmetric target with* respect to *m* if $P(T \le m - x) = P(T \ge m + x)$ for any $x \ge 0$.

Most of the unimodal continuous asymmetric distributions satisfy conditions a) and b); whereas condition c) is the usual definition of a symmetric distribution.

For example Gamma distributions, that include Exponential and Chi-square distributions as special cases, and Beta distributions satisfy condition b), see Abadir (2005). More generally, it can be easily proven that any Pearson distribution of Type I to XII satisfies inequality b)[1]. Note that distributions enjoying condition a) relate to those enjoying condition b). In fact, if a random variable *X* satisfies condition a), then its symmetric reversal $Y = -(X - m) + m = 2m - X$ satisfies condition b), and vice versa; this will be discussed in Sec. 4.

Clearly, there exist asymmetric distributions that satisfy neither condition a) nor b). In such a case the agent perceives the target as neither hard nor easy, so the "hard-easy effect" does not apply.

A simple necessary condition for testing the perceived difficulty is offered by the so-called van Zwet mean-median-mode inequalities; see van Zwet (1979).

[1]A sufficient condition for inequality b) is that the probability density *f* of *T* satisfies for some $\zeta > 0$, $f(m-x) + f(m+x) \ge 0$ for $0 \le x \le \zeta$ and $f(m-x) + f(m+x) \le 0$ for $\zeta < x < \infty$, see Equation (2.1) van Zwet (1979). The probability density *f* of any Pearson distribution of Type I to XII satisfies the above condition; see also Sato (1997).

Theorem 1. Let a unimodal continuous target *T* with mean μ, median *m* and mode *M*. Then:

a) if T is a *hard-perceived target,* then $\mu \le m \le M$;

b) if *T* is an *easy-perceived target*, then $M \le m \le \mu$;

c) if T is a *symmetric-perceived target, then T* is symmetric and $M = m = \mu$.

Proof. The definition of hard-perceived and easy-perceived task is equivalent with the van Zwet (1979) conditions. Mean-median-mode inequalities come from van Zwet (1979)'s Theorem.□

The alphabetical/counter-alphabetical order among mean, median and mode offers a quick-and-dirty test for task difficulty; however, it provides only a necessary but not sufficient condition. For a number of counter-examples, see Abadir (2005).

The intuition underlying Theorem 1 follows. If the target is hard-perceived, then the agent sets her subjective reference point, i.e. the mode *M*, higher than the objective one, i.e. the median *m*; and vice versa if the target is easy-perceived. If the target is symmetric-perceived, the subjective and objective reference point coincides.

3. The perceived loss-gain asymmetry in "the large"

An S-shaped utility function may be asymmetrically tailed. As mentioned by Kahneman and Tversky (1979, p. 279) the tail asymmetry captures the attitude that "losses loom larger than gains", called "loss aversion". That expresses the common sentiment that the disutility of losses exceeds the utility of commensurate gains. Although this pattern is one of the most robust findings in behavioral economics, recent studies have documented reversed preferences called "gain seeking" or "reversed-loss aversion"; see Moore and Cain (2007), and Sacchi and Stanca (2014).

In the following we introduce a definition of the asymmetric loss-gain perception captured by the following statements: a) "*gains loom larger than losses*"; b) "*losses loom larger than gains*"; and c) "*gains loom equal to losses*". These sentiments are assumed to hold at any level of the agent wealth.

Definition 2. Let *u* be an increasing cardinal utility function with $0 \le u(x) \le 1$. Given a reference point *m*,

a) if $u(m+x)-u(m) \ge u(m)-u(m-x)$ for all $x \ge 0$, then *u* exhibits *gain seeking* around *m*;

b) if $u(m+x)-u(m) \le u(m)-u(m-x)$ for all $x \ge 0$, then *u* exhibits *loss aversion* around *m*;

c) if $u(m-x)+u(m+x)=1$ for all *x,* then *u* exhibits *gain-loss neutrality* around *m.*

It is worthwhile pinpointing the link between Definition 2.b) and the notion of loss aversion introduced by Tversky and Kahneman (1992, p. 303), i.e., $u'(x) < u'(-x)$ for all $x \geq 0$. The former provides a definition "in the large", based on the *absolute* change in utility that holds for any *m* of the wealth domain; the latter applies "in the small" and is based on the *marginal* change in utility at $m = 0$. Clearly, if *u* is differentiable at $m = 0$, the definition of loss aversion "in the large" implies that "in the small" given by Tversky and Kahneman (1992, p. 303).

If the median *m* of the target *T* is chosen as reference point, the notions of task difficulty and risk attitude overlap, as shown below.

Theorem 2. *Let T a hard/easy target according to Definition 1, with a continuous c.d.f. u and a median m. If the agent is endowed with an S-shaped utility function u, then*

a) u exhibits gain seeking around the median m if and only if the agent judges T a hard-perceived target around the median m;

b) u exhibits loss aversion around the median m if and only if the agent judges T an easy-perceived target around the median m;

c) u exhibits equal loss-gain weighting around the median m if and only if the agent judges T a symmetric-perceived target around the median m.

Proof. The proof is a direct consequence of the equivalence (1) between the c.d.f. *F* of the perceived target and the agent cardinal utility *u*. Since the reference point is the median *m*, substituting $u(m) = 0.5$ in the inequalities a), b) in Definition 2, the correspondent inequalities a), b) in Definition 1 come out, and vice versa. □

The equivalence between the gain seeking/hard-perceived and the loss aversion/easy-perceived task has an intuitive explanation that matches the experimental evidence. That may be augmented by the so-called *endowment effect* (also called deprivation or divestiture aversion).

Dealing with easy-perceived tasks the agent tends to feel the potential gains as virtually own. As a consequence, for the endowment effect, no-gains are felt as a deprivation. So the agent exhibits loss aversion when facing an easy-perceived task; see LiCalzi (1999). Vice versa, dealing with hard-perceived tasks the agent does not ascribe the possible gains as his own and no endowment effect appears. However, no-gains give no-deprivation, but albeit gains are strongly sought. So, in the presence of hard-perceived tasks, the agent becomes a gain seeker.

4. Misconfidence in hard and easy tasks

According to Moore and Healy (2008) people may exhibit confidence miscalibration in three different ways: (1) in estimating their own performance (misconfidence); (2) in estimating their own performance relative to others (overplacement or 'better-than-average' effect); (3) and having an excessive precision to estimate future uncertainty (overprecision). In the paper at hand, we focus on the cognitive bias (1): the self-misconfidence.

Self-misconfidence has been long recognized since the pioneering works of Lichtenstein and Fischhoff (1977). Evidences document the presence of the “hard-easy task effect” that overconfidence is much prevalent if the commitments are difficult-perceived, whereas underconfidence prevails if they are easy-perceived.

We build on the target-based interpretation and provide foundations for this behavior. Consequently, we demonstrate sufficient conditions for a rational agent abiding EU paradigms and endowed with an S-shaped utility to act according to the “hard-easy” and the “reversed hard-easy” effect.

Before introducing the formal definition of misconfidence, let us gain an intuitive insight on this concept. The misconfidence is defined as the gap between the self-confidence and the confidence of a well-calibrated agent, i.e. an individual who feels “gains loom equal to losses”. To formalize this ideal equidistance, we postulate for each target T a corresponding well-calibrated target T_0 with the same median m and a distribution obtained by suitably symmetrizing the distribution of T. Its construction is the following. First, we demonstrate that for each hard-perceived (respectively, easy-perceived) target, there is an opposite-perceived target. The proof is in the Appendix.

Proposition 1. Let T a hard-perceived/easy-perceived target with respect to m, we call Y its *opposite-perceived* variable of T, if $Y-m=-(T-m)$. Clearly, T and Y are symmetric with respect to m. If T is a hard-perceived/easy-perceived target, then its opposite-perceived variable Y is an easy/hard-perceived one, and vice versa.

Second, we construct the symmetric benchmark T_0. Let $\overline{T}$ and $\underline{T}$ two opposite-perceived targets with c.d.f. $\overline{u}$ and $\underline{u}$, respectively. Let $u_0(x)=\frac{\underline{u}(x)+\overline{u}(x)}{2}$ for any x. Since $\underline{u}(m-x)=1-\overline{u}(m+x)$, it follows immediately that $u_0(m-x)+u_0(m+x)=1$ for any x. So u_0 is the c.d.f. of a *symmetric-perceived target* T_0. We single out this choice for T_0 because it is the only symmetric target for which

$$P(m-x\le T\le m+x)=P(m-x\le T_0\le m+x)$$

holds for any $x\ge 0$.

Now we are ready to introduce the definition of self-misconfidence in meeting uncertain hard-perceived and easy-perceived targets.

Definition 3. Let T a hard-perceived/easy-perceived target according to Definition 1, with c.d.f. u and median m. Given a lottery X:

a) if $P(X>T)\ge P(X>T_0)$ then the agent endowed with the utility function u exhibits *overconfidence on X;*

b) if $P(X > T) \le P(X > T_0)$ then the agent endowed with the utility function u exhibits *underconfidence on X*,

where T_0 is the symmetric-perceived target with median m.

4.1 Confidence miscalibration: the benchmarking procedure

To circumvent the problem of eliciting beliefs about the probability of meeting an uncertain target, we reword it in the target-based language of the benchmarking procedure. Benchmarking is a prescriptive target-based model that satisfies von Neumann and Morgenstern (1947)'s and Savage (1954)'s axiomatization through a probabilistic and intuitive interpretation of the expected utility of a lottery *X;* see Castagnoli and LiCalzi (1996), Bordley and LiCalzi (2000). They show that the expected utility of a lottery *X* can be read as the probability that *X* outperforms a stochastically *independent* target *T* with c.d.f. *u*

$$P(X \ge T) = \mathrm{E}(u(X)). \tag{2}$$

This interpretation highlights the benchmark-procedure advantage: the subjective beliefs about the probability of successfully meeting the target constitute all that is needed to inform the subjective expected utility of the lottery at hand. The main result of this paper gives sufficient conditions for the "hard-easy" effect bias to occur.

Theorem 3. (The "hard-easy effect" bias) *Suppose that agent has a hard-perceived target T with c.d.f. u and median m. Let X be a lottery independent of T. Then the agent exhibits*

a) overconfidence if the support of X belongs to $[m, +\infty)$*, and*

b) underconfidence if the support of X belongs to $(-\infty, m]$*;*

where m is the objective reference point. Correspondingly, if T is easy-perceived, then the agent exhibits underconfidence for lotteries over $[m, +\infty)$, and *overconfidence for lotteries over* $(-\infty, m]$.

Proof. See the Appendix.

Theorem 3 gives a normative guideline to highlight circumstances where confidence is misaligned. When the target is hard-perceived, the agent is overconfident for lotteries with payoffs higher than the median (gains), and underconfident over lotteries delivering payoffs lower than the median (losses). When the target is easy-perceived, this pattern reverses. The "hard-easy" effect summarizes the two results over gains. The Theorem predicts a symmetric "easy-hard" effect over the domain of losses.

Shifting from the target-language versus the loss-gain one, we reword our findings in the card game context. A player will be o*verconfident* if:

- she is a gain-seeker holding "trump cards"; or if

- she is a loss averter holding “bad cards”.

Vice versa, she will be *underconfident* if:

- she is a gain-seeker holding “bad cards”; or if
- she is a loss-averter holding “trump cards”.

These conclusions reflect well-known biases in cognitive psychology. Contrary to a common belief, it is not only risk-seeking players that may fall into the trap of overconfidence, but loss adverse players as well. An explanation of overconfidence in loss adverse agents is offered by the *“sunk cost effect*” also called the “Concorde effect” and the "commitment bias". That cognitive bias describes the phenomenon where people do not want to admit a failure in running a bad business. The probability of meeting the targets is overestimated and people go on "throwing good money after bad". A behavioral explanation for sunk cost effects was originally proposed by Thaler (1980) in relation with the presence of loss aversion and the endowment effect. However the sunk cost bias is still a questionable “fallacy” in psychology. Kelly (2004) legitimates the tendency of “honoring” sunk costs as rational driver. McAfee and al. (2010) argue that agents may rationally engage in sunk costs because of reputational concerns, or financial and time constraints. Our findings support this documented human behavior with sound normative motivations.

Remark 1. Theorem 3 finds applications in business environments. According to the enterprise risk management (ERM) directives, managers should prudentially manage the risks and strategically seize the opportunities. To achieve this double objective it is essential to have an accurate grasp of the probability of succeeding the commitments (see Conine, 2014). Theorem 3 sheds light on circumstances where this judgment is normatively biased. Summing up, the decision maker will be prone to o*verconfidence* if:

- she is a risk-seeker and she is handling a very good business;
- she is a loss averter and she is handling a very bad business.

Vice versa, she will move to *underconfidence* if:

- she is a risk-seeker and she is handling a very bad risky project;
- she is loss averter and she is handling a very good risky project.

Overconfidence in evaluating risks may end up in not defensible losses; on the other hand, underconfidence may lead to miss out on opportunities for gains because of a small risk of failure. Recognizing the influence of these cognitive biases on confidence miscalibrations can induce managers to be more mindful in decision making.

5. Conclusion

The cognitive bias “hard-easy effect” is a widely acknowledged phenomenon in human behavior documented by an ample literature on the experimental psychological economics. In this paper we study this bias and its reversed version from a normative perspective. We show that these effects are compatible with EU prescriptions for a rational agent endowed with an asymmetrically tailed S-shaped cardinal utility function.

The key reading of our findings is based on the equivalence between the asymmetry in task-difficulty and the asymmetry in loss-gain perception.

Sufficient conditions for acting according to the "hard-easy effect" and the "reversed hard-easy effect" biases are set out. More specifically, these effects arise if the agent feels to face a hard or an easy task and the lottery at her disposal promises extremely good or extremely bad odds. As by-product we derive a normative guide for ERM to detect situations where confidence miscalibrations arise. Recognizing these cognitive biases, and being mindful of to be normatively influenced by them, gives the managers a better framework for decision making.

Our findings suggest a novel normative reading of other cognitive biases in ERM, but that is left to future research.

Appendix

Proof of Proposition 1.

We will prove that if T is a hard-perceived/easy-perceived task with respect to m its opposite-perceived variable Y is an easy-perceived/hard-perceived task.

Let $\overline{T}$ a hard-perceived target with c.d.f. $\overline{u}$. Construct the variable Y with c.d.f. $\underline{u}$ such that $Y - m = -\left(\overline{T} - m\right)$. It follows that

$$\underline{u}\left(m+x\right) = 1 - \overline{u}\left(m-x\right) \text{ for all } x \geq 0 . \tag{3}$$

In fact

$$\underline{u}\left(m+x\right) = P\left(Y \leq m+x\right) = P\left(Y-m \leq x\right) = P\left(-\left(\overline{T}-m\right) \leq x\right) = P\left(\overline{T} \geq m-x\right) = 1 - \overline{u}\left(m-x\right)$$

And analogously

$$\underline{u}\left(m-x\right) = 1 - \overline{u}\left(m+x\right) \text{ for all } x \geq 0 . \tag{4}$$

Summing up (3) and (4), we get

$$\underline{u}\left(m+x\right) + \underline{u}\left(m-x\right) = 1 - \left[\overline{u}\left(m-x\right) + \overline{u}\left(m+x\right) - 1\right]$$

Since $\overline{T}$ is a hard-perceived target, then $\left[\overline{u}\left(m-x\right) + \overline{u}\left(m+x\right) - 1\right] \geq 0$, so

$$\underline{u}\left(m+x\right) + \underline{u}\left(m-x\right) \leq 1 \text{ for all } x \geq 0 ,$$

Then by Definition 1.b) Y is an easy-perceived target.

The two opposite-perceived targets have the same median *m*. In fact, if $x = 0$, it follows $\underline{u}\left(m\right) = 1 - \overline{u}\left(m\right) = 0.5$. If T is an easy-perceived target, the relations are reversed and the opposite-perceived variable Y can be proved to be hard-perceived. □

Proof of Theorem 3.

Let $\overline{T}$ a hard-perceived target with c.d.f. $\overline{u}$ and $\underline{T}$ its opposite-perceived variable with c.d.f. $\underline{u}$, such that $\underline{T}-m=-\left(\overline{T}-m\right)$. We will prove that the relation between $\overline{u}$ and $\underline{u}$ switches at *m*. Specifically, $\overline{u}\left(m-x\right)\leq\underline{u}\left(m-x\right)$ and $\underline{u}\left(m+x\right)\leq\overline{u}\left(m+x\right)$for any $x\geq 0$.

Abdous and Theodorescu (1998, equation (2), p. 357) set the equivalence between the van Zwet conditions (1979, (1.2), p. 1) and the first stochastic order between the two tails of *T* around *m,* it holds

$\underline{u}\left(m+x\right)+\underline{u}\left(m-x\right)\leq 1$ is equivalent to $\left(\underline{T}-m\right)^{+}\succ_{st}\left(\underline{T}-m\right)^{-}$ and

$\overline{u}\left(m+x\right)+\overline{u}\left(m-x\right)\geq 1$ is equivalent to $\left(\overline{T}-m\right)^{-}\succ_{st}\left(\overline{T}-m\right)^{+}$,

where $X^{\pm}$ denote the positive (negative) part of X.

Since $\underline{T}-m=-\left(\overline{T}-m\right)$ we have $\left(\underline{T}-m\right)^{+}=\left(-\left(\overline{T}-m\right)\right)^{+}=\left(\overline{T}-m\right)^{-}$, then

$$\left(\overline{T}-m\right)^{-}\succ_{st}\left(\underline{T}-m\right)^{-},\text{ so }\overline{u}\left(m-x\right)\leq\underline{u}\left(m-x\right)\text{ for }x\geq 0$$

And since $\left(\overline{T}-m\right)^{-}=\left(-\left(\underline{T}-m\right)\right)^{-}=\left(\underline{T}-m\right)^{+}$, we have

$$\left(\underline{T}-m\right)^{+}\succ_{st}\left(\overline{T}-m\right)^{+},\text{ so }\underline{u}\left(m+x\right)\leq\overline{u}\left(m+x\right)\text{ for }x\geq 0$$

The above can be rewritten as $\underline{u}\left(s\right)\leq\overline{u}\left(s\right)$ for $s\leq m$ and $\overline{u}\left(s\right)\geq\underline{u}\left(s\right)$ for $s\geq m$.

Let $u_0\left(m+x\right)=\dfrac{\underline{u}\left(m+x\right)+\overline{u}\left(m+x\right)}{2}$ for any *x*. By construction, it holds

$\overline{u}\left(m-x\right)\leq u_0\left(m-x\right)\leq\underline{u}\left(m-x\right)$ and $\overline{u}\left(m+x\right)\geq u_0\left(m+x\right)\geq\underline{u}\left(m+x\right)$ for $x\geq 0$.

Let now quantify the probability that the lottery *X* outperforms the target *T.* Consider the lottery *X* such that:

a) the *outcomes of X belong on* $\left[m,+\infty\right)$. Since $\overline{u}\left(s\right)=u_0\left(s\right)\leq\underline{u}\left(s\right)=0$ for all $s<m$, following relation holds

$$\mathrm{E}\left(\underline{u}\left(X\right)\right)<\mathrm{E}\left(u_0\left(X\right)\right)<\mathrm{E}\left(\overline{u}\left(X\right)\right).$$

Then

$$P\left(X\geq\underline{T}\right)<P\left(X\geq T_0\right)<P\left(X\geq\overline{T}\right).$$

So if the lottery *X* promises high stakes, all above or equal to the external reference point *m*, then the agent is prone to underconfidence in easy-perceived tasks, and to overconfidence in hard-perceived tasks. That risk attitude follows the “hard-easy effect”.

b) The *outcomes of X belong on* $\left(-\infty,m\right]$. Since $\overline{u}\left(s\right)=u_0\left(s\right)\leq\underline{u}\left(s\right)=0$ for all $s>m$, following relation holds

$$\mathrm{E}\left(\bar{u}\left(X\right)\right) < \mathrm{E}\left(u_0\left(X\right)\right) < \mathrm{E}\left(\underline{u}\left(X\right)\right).$$

Then

$$P\left(X \geq \bar{T}\right) < P\left(X \geq T_0\right) < P\left(X \geq \underline{T}\right).$$

So if the lottery X promises bad outcomes, all below or equal to the external reference point m, then the agent is prone to underconfidence in hard-perceived tasks, and to overconfidence in easy-perceived tasks. That risk attitude follows the "reversed hard-easy effect".□

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