Lattice Study of the Jet Quenching Parameter

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We present a first-principle computation of the jet quenching parameter, which describes the momentum broadening of a high-energy parton moving through the deconfined state of QCD matter at high temperature. Following an idea originally proposed by Caron-Huot, we explain how one can evaluate the soft contribution to the collision kernel characterizing this real-time phenomenon, analyzing certain gauge-invariant operators in a dimensionally reduced effective theory (electrostatic QCD), which can be studied nonperturbatively via simulations on a Euclidean lattice. Our high-precision numerical computations at two different temperatures indicate that soft contributions to the jet quenching parameter are large. After discussing the systematic uncertainties involved, we present a quantitative estimate for the jet quenching parameter in the temperature range accessible at heavy-ion colliders, and compare it to results from phenomenological models as well as to strong-coupling computations based on the holographic correspondence.

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Introduction.—As first proposed by Bjorken [1], jet quenching provides a very important experimental signature for the creation of the quark-gluon plasma (QGP) in heavy-ion collisions [2]. When a hard parton propagates through the deconfined medium, multiple interactions with the QGP constituents induce energy loss and momentum broadening. This leads to suppression of back-to-back correlations among final-state hadrons, and of particle yields at large transverse momenta.

A first-principle, quantitative theoretical description of this phenomenon is, however, very challenging, given that both perturbative and nonperturbative dynamics is involved [3]. The momentum broadening of a parton can be described in terms of a phenomenological parameter $\hat{q}$, defined as the average increase in the squared transverse momentum component per unit length [4]. This quantity can be computed as the second moment of the differential transverse collision rate $C(p_\perp)$ describing the interaction between the hard parton (in the eikonal approximation) and the plasma constituents:

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L} = \int \frac{d^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp). \tag{1}$$

A leading-order (LO) perturbative evaluation of $\hat{q}$ was carried out in Ref. [5] and later extended to the next-to-leading order (NLO) in Ref. [6]. However, as is well known, perturbative computations in thermal QCD cannot be pushed to arbitrarily high order, due to the presence of infrared divergences which reveal the intrinsically non-perturbative nature of interactions for long-wavelength modes. In addition, at the temperatures probed at RHIC (and at LHC) the accuracy of LO or NLO perturbative computations may be questioned.

The gauge-string duality [7] provides a framework in which the phenomenon can be studied nonperturbatively, in the strong-coupling limit and in the large-N approximation. Following this approach, a holographic computation of $\hat{q}$ (for a light hard parton) was presented in Ref. [8]; for related studies, see also Refs. [9]. However, since the holographic dual of QCD is not known exactly, these types of computations are usually carried out for models (e.g., the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory), which are only expected to reproduce the features of QCD qualitatively or semiquantitatively.

Lattice simulations are the standard tool for nonperturbative, first-principle computations in QCD, but since they are based on the regularization of the theory in a Euclidean spacetime, they are not well suited for real-time phenomena, and typically require some analytical continuation. However, a closer inspection of the problem reveals that contributions from the parametrically different ("hard" $O(\pi T)$, "soft" $O(gT)$, and "ultrasoft" $O(g^2T/\pi)$) energy scales of the QGP can be disentangled from each other. In particular, following an idea first proposed in Ref. [6], and later discussed also in Refs. [10–14], it is possible to show that the contribution to jet quenching from soft modes can be *directly* evaluated in lattice simulations of a purely bosonic, dimensionally reduced effective theory (electrostatic QCD, or EQCD [15]). The latter, defined by the Lagrangian

$$\mathcal{L} = \frac{1}{4} F_{ij}^a F_{ij}^a + \mathrm{Tr}((D_i A_0)^2) + m_E^2 \mathrm{Tr}(A_0^2) + \lambda_3 (\mathrm{Tr}(A_0^2))^2, \tag{2}$$

is three-dimensional SU(3) Yang-Mills theory coupled to an adjoint scalar field A_0. The values of its parameters (the dimensionful, three-dimensional gauge coupling g_E and the coefficients for the quadratic and quartic terms in the scalar potential) are fixed by a *matching* procedure, to reproduce the physics of high-temperature QCD.

Lattice formulation.—The lattice regularization of the theory described by the Lagrangian density (2) is straightforward (see Ref. [16] and references therein). We chose parameter values corresponding to QCD with $n_f = 2$ light quark flavors, at two temperatures $T \simeq 398$ MeV and 2 GeV, respectively, equal to about twice and ten times the deconfinement temperature.

The contribution to jet quenching from soft QGP modes can be extracted from the two-point correlation function of long light-cone Wilson lines (stretching, for example, along the $x_3 - t =$ const direction at fixed x_1 and x_2, and separated by a distance r along the x_1 direction), which can be made gauge-invariant by "closing the loop" with two transverse parallel transporters, and taking the trace over color indices [10]. In the lattice regularization of EQCD, each lightlike side of this loop becomes a "staircase" path, built from products of unitary parallel transporters $U_3(x)$ over one lattice spacing unit a to the x_3 direction, and matrices $H(x) = \exp[-ag_E^2 A_0(x)]$, which are EQCD analogues of the parallel transporters along paths of length a in the *real-time* direction. Note that $H(x)$ is Hermitian (rather than unitary). Suppressing the coordinate dependence, the lattice operator corresponding to a single light-cone Wilson line is then: $L_3 = \prod(U_3 H)$. Denoting the path-ordered product of gauge links $U_1(x)$ along the transverse direction as L_1, the original light-cone Wilson loop can be represented in lattice EQCD as a "decorated" Wilson loop (see Fig. 1) defined as

$$W(\ell, r) = \mathrm{Tr}(L_3 L_1 L_3^{-1} L_1^\dagger), \tag{3}$$

where ℓ denotes the length of the loop along the x_3 direction. $W(\ell, r)$ enjoys well-defined renormalization

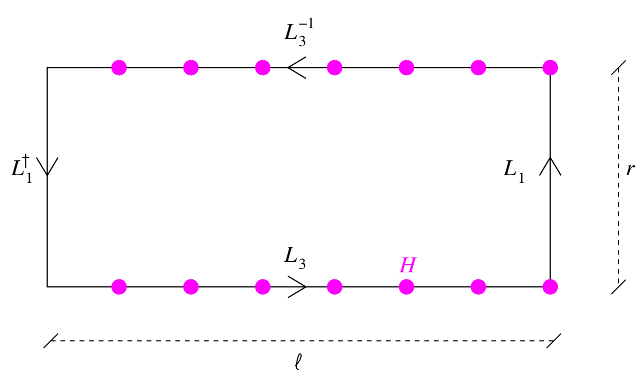

FIG. 1 (color online). The "decorated" Wilson loop $W(\ell, r)$ describing a two-point correlation function of light-cone Wilson lines involves Hermitian parallel transporters $H(x)$ along the real-time direction.

properties [17]. We compute the $\langle W(\ell, r)\rangle$ expectation values with a multilevel algorithm [18] and extract the coordinate-space expression for the differential transverse collision kernel,

$$V(r) = -\lim_{\ell\to\infty} \frac{1}{\ell} \ln\langle W(\ell, r)\rangle, \tag{4}$$

which equals the transverse Fourier transform of $-C(p_\perp)$ (up to an additive term, which is inessential to our discussion). Note that the presence of "real time" in $W(\ell, r)$ implies that this operator is actually very different from a usual spatial Wilson loop. In particular, perturbation theory predicts a partial cancellation between the gauge and scalar propagators [5,6,12,19]. In turn, this implies that $V(r)$ starts from zero and is a decreasing function at very small r, while at distances larger than the inverse of the Debye mass m_D it includes a linear term of positive slope $7g_E^4 \mathcal{C}_f \mathcal{C}_a/(64\pi)$, with $\mathcal{C}_f = 4/3$, $\mathcal{C}_a = 3$ [6,12].

Results.—Using Eq. (4), we extracted the differential transverse collision kernel V as a function of the separation r between the lightlike Wilson lines, at the two temperatures that we investigated. Our simulation results exhibit good scaling properties: data obtained from lattices of different spacing a (recall that, in three-dimensional SU(3) lattice gauge theory, $\beta = 6/(ag_E^2)$) fall within a narrow band. This indicates that discretization effects are under control, and that our data allow us to obtain a reliable extrapolation to the continuum limit.

Since the jet quenching parameter is the second moment of $C(p_\perp)$, the soft contribution to $\hat{q}$ is encoded in the curvature of $V(r)$. Following a procedure similar to the one used in Ref. [11], we fit V/g_E^2 to a functional form including linear, quadratic, and quadratic-times-logarithmic terms in rg_E^2, i.e., $c_1 rg_E^2 + c_2(rg_E^2)^2 + c_3(rg_E^2)^2 \ln(rg_E^2)$, and estimate the contribution to $\hat{q}$ from soft modes as $\{4c_2 + 2c_3[2 + \gamma + 4\ln(r_0 g_E^2/\sqrt{2})]\}g_E^6$, where γ is the Euler-Mascheroni constant and r_0 denotes Sommer's scale [20], with $r_0 g_E^2 \simeq 2.2$ [21]. At the two temperatures considered, we find that the total contribution to $\hat{q}$ from soft modes is significantly larger than the perturbative NLO expectation,

$$\hat{q}^{\mathrm{NLO}} = g^4 T^2 m_D \mathcal{C}_f \mathcal{C}_a \frac{3\pi^2 + 10 - 4\ln 2}{32\pi^2}, \tag{5}$$

(where, at this order, $m_D = m_E + O(g^2 T)$, with $m_E = gT\sqrt{(\mathcal{C}_a + n_f t_f)/3}$ and $t_f = 1/2$). The fact that contributions beyond NLO (which are intrinsically nonperturbative) are large is not surprising: at those temperatures also the Debye mass m_D receives a numerically large nonperturbative correction [22], which dominates over the leading perturbative term (m_E). In fact, it is interesting to note that, when plotted in terms of the nonperturbatively evaluated Debye mass, our lattice results for $V(r)$ become compatible with the theoretical NLO curve [6,12], as seen in Fig. 2 (which also shows that this rescaling in terms of the Debye mass makes

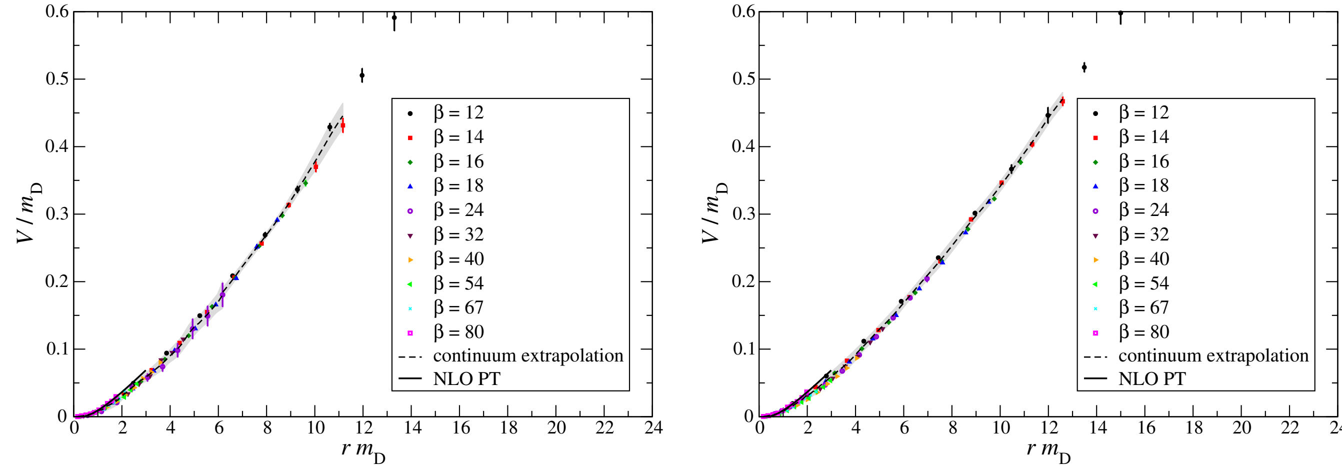

FIG. 2 (color online). The coordinate-space collision kernel V, extracted from expectation values of $W(\ell, r)$ computed nonperturbatively in our EQCD simulations. Symbols of different colors correspond to different lattice spacings, while the dashed black line (and the gray band) indicate the continuum limit (and the associated uncertainty). The lhs panel shows results for $T \simeq 398$ MeV, the one on the rhs for $T \simeq 2$ GeV. Both V and r are shown in the appropriate units of the Debye screening mass m_D, as evaluated nonperturbatively in Ref. [22]. The NLO perturbative prediction [6,12] is also shown (solid black line).

the data sets at the two different temperatures compatible with each other: this suggests that, essentially, the temperature dependence of V is inherited from m_D). Physically reasonable values of the coupling $g^2 \sim 2.6$ [23] for RHIC temperatures (at which the LO contribution to the jet quenching parameter is known to be subdominant [6]) then lead us to estimate $\hat{q} \simeq 6\ \mathrm{GeV}^2/\mathrm{fm}$. This value is in the same ballpark as those obtained from holography [8,9], as well as from certain phenomenological models [24] (although the latter are somewhat dependent on the details of the approach that is used [25], and more recent studies tend to favor smaller values [26]).

Conclusions.—In this Letter, we presented a nonperturbative investigation of the momentum broadening of a hard parton (specifically, a light quark) in the quark-gluon plasma. Although Monte Carlo simulations on a Euclidean lattice are generally ill-suited for studying phenomena involving real-time dynamics (because they require some analytical continuation), in the present work, following an idea originally proposed in Ref. [6], we extracted the contribution to the jet quenching parameter from soft QGP modes, with momenta $O(gT)$, in a high-precision numerical study of a dimensionally reduced, Euclidean and purely bosonic effective theory (EQCD). Recent works discussing related ideas include Refs. [10–14], while a different approach to study jet quenching on the lattice was suggested in Ref. [27].

Our nonperturbative estimate of the soft contribution to $\hat{q}$ in the dimensionally reduced effective theory is obtained from the numerical evaluation of a lattice operator describing (a gauge-invariant version of) the two-point correlator of light-cone Wilson lines. Our results give direct access to $V(r)$, which is related to the Fourier transform of the collision kernel $C(p_\perp)$. While, by construction, our effective theory approach misses the contribution to $\hat{q}$ from hard thermal modes with momenta $O(\pi T)$ (which, however, can be reliably estimated perturbatively, and is numerically subdominant), we remark that it does so in a well-defined way, consistent with the modern theoretical approach to thermal QCD [15,23]. Although the separation between hard, soft and ultrasoft scales may be partially blurred at temperatures accessible to present-technology experiments, in this work we presented a first concrete attempt to provide a quantitative, first-principle estimate for $\hat{q}$ that is free from uncontrolled approximations. We found that soft contributions to $\hat{q}$ are significantly larger than the perturbative prediction up to NLO. However, this mismatch can be accommodated, by expressing the results in terms of the nonperturbatively evaluated Debye mass. Our final result for $\hat{q}$ at RHIC temperatures is about $6\ \mathrm{GeV}^2/\mathrm{fm}$ (with total uncertainty around 15%–20%). While this estimate should be taken *cum grano salis*, given that the very definition of $\hat{q}$ is affected by intrinsic ambiguities (see Ref. [11] for a discussion), we stress that all sources of uncertainty related to the lattice regularization are under control, and systematically improvable. In particular, our simulations were carried out with large and fine lattices, so finite-volume and finite-cutoff effects are small. Finally, a recent classical lattice gauge theory study [13] finds conclusions that are compatible with ours.

To extend this study, the determination of $V(r)$ could be refined using an improved formulation of the lattice action, for which a multilevel algorithm has been recently proposed [28]. We also plan to repeat the present calculation at several different temperatures T, in order to study the dependence of $\hat{q}$ on T. As the temperature is increased, the QGP should interpolate between a regime dominated by nonperturbative physics, and one in which it becomes weakly coupled. It would also be interesting to study the dependence of $\hat{q}$ on the number of color charges N, in particular in the large-N limit [29], which provides insight into many properties of real-world QCD (see Refs. [30]) and plays a crucial role in all holographic computations. Lattice studies in both four [31] and three [32] spacetime dimensions show that static equilibrium quantities characterizing the QGP have very mild (nontrivial) dependence on N. It would be interesting to see if this also holds for quantities involved in real-time dynamics.

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