Intermittency in Integrable Turbulence

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We examine the statistical properties of nonlinear random waves that are ruled by the one-dimensional defocusing and integrable nonlinear Schrödinger equation. Using fast detection techniques in an optical fiber experiment, we observe that the probability density function of light fluctuations is characterized by tails that are lower than those predicted by a Gaussian distribution. Moreover, by applying a bandpass frequency optical filter, we reveal the phenomenon of intermittency; i.e., small scales are characterized by large heavy-tailed deviations from Gaussian statistics, while the large ones are almost Gaussian. These phenomena are very well described by numerical simulations of the one-dimensional nonlinear Schrödinger equation.

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The way through which nonlinearity and randomness interplay to influence the propagation of waves is of major interest in various fields of investigation, such as, e.g., hydrodynamics, wave turbulence, optics, or condensed-matter physics. If the spatiotemporal dynamics of the wave system is predominantly influenced by some disorder found inside the nonlinear medium, localized eigenmodes may emerge through the process of Anderson localization [1]. Conversely, if randomness mostly arises from the initial field, linear and nonlinear effects occurring inside the propagation medium interplay to modify the statistical properties of the incoherent wave launched as initial condition. This process, extensively studied during the past years, may result in the formation of extreme or rogue waves [2–5].

In this context, the nonlinear Schrödinger equation (NLSE) plays a special role because it provides a universal model that rules the dynamics of many nonlinear wave systems. In the fields of oceanography and laser filamentation, numerical simulations of the NLSE with random initial conditions have been made in $2 + 1$ dimensions to study the emergence of rogue waves [6–8]. In the context of nonlinear fiber optics, the propagation of incoherent waves is often treated in $1 + 1$ dimensions from generalized NLSEs including high-order linear and nonlinear terms [2,9–11]. Wave turbulence (WT) theory provides an appropriate framework for the statistical treatment of the interaction of random nonlinear waves that are described by these *nonintegrable* equations [12].

If third-order nonlinearity and second-order (linear) dispersion dominate other physical effects in a one-dimensional medium, wave propagation is ruled by the 1D NLSE, which is an *integrable* equation. In the focusing regime, the 1D NLSE possesses soliton and breather solutions that have been recently observed in various physical systems [13–15]. Considering a random input field, these breather solutions may exist embedded in random waves, thus behaving as prototypes of rogue waves that modify the statistical properties of the random input field [16,17]. In the defocusing regime, the 1D NLSE has dark or gray soliton solutions. These solutions have also been experimentally observed [18,19], but the influence of these coherent structures on the statistics of systems of nonlinear random waves is not known.

Some interesting approaches based on WT theory for studying the probability density function (PDF) of the 1D NLSE in the weakly nonlinear regime have been made [20]; however, as discussed in Ref. [21], the WT theory does not provide an appropriate framework for investigating the statistical phenomena described by integrable wave equations, especially in a strongly nonlinear regime. The theoretical analysis of these phenomena enters within the framework of an emerging field of research introduced by Zakharov under the appellation of "integrable turbulence" [21–23]. Although the inverse scattering theory (IST) provides natural tools to tackle problems related to integrable turbulence, only a few experiments made with one-dimensional incoherent waves have been analyzed using IST [24–28]. Currently, the accurate description of the statistical properties of incoherent waves described by an integrable system is still a complex and open problem.

In addition to the determination of the PDF of global fields, separating large scales from small scales is known to provide rich statistical information about nonlinear systems of random waves. In this respect, the phenomenon of intermittency is defined in the general context of turbulence as a departure from the Gaussian statistics that grows increasingly from large scales to small scales [29]. The intermittency phenomenon is usually evidenced by using spectral filtering methods and by showing the existence of deviations from Gaussianity through the measurement of the kurtosis of fluctuations that are found at the output of some frequency filter. Spectral fluctuations can be examined at the output of an ideal one-mode spectral filter passing only a single Fourier component [12,30]. Time fluctuations at the output of ideal high-pass or bandpass frequency filters can also be considered [29,31]. PDFs of second-order differences of the wave height have also been measured in wave turbulence [32]. Using this kind of technique, the phenomenon of intermittency has been initially reported in fully developed turbulence [29], but it is also known to occur in wave turbulence [30,32,33], solar wind [34], or in the Faraday experiment [35].

In this Letter, we present an optical fiber experiment accurately ruled by the one-dimensional integrable and *defocusing* NLSE. The experiment has been designed to investigate a situation of integrable turbulence and it has been dimensioned in order to capture the entire dynamics of a partially coherent optical wave having initially a Gaussian statistics. This is usually never achieved in optics where the spectrum of the incoherent wave is often much broader than the detector bandwidth [2]. Using fast detection techniques, we explore the changes in the global statistics of the incoherent wave occurring in the nonlinear regime and we observe that the PDF of intensity of light fluctuations is characterized by tails that are lower than those predicted by a Gaussian distribution. Implementing an optical filtering technique, we also examine the statistics of intensity of light fluctuations on different scales and we observe that the PDFs show heavy tails that strongly depend on the scales. This reveals an unexpected phenomenon of intermittency that is similar to the one reported in several other wave systems, though they are fundamentally far from being described by an integrable wave equation [30,32–35].

Our experimental setup is schematically shown in Fig. 1. It includes a continuous partially coherent light source that brings together the abilities of (i) delivering a beam that is confined in a unique transverse spatial mode, (ii) having a narrow bandwidth ($\Delta\nu_0 \sim 14$ GHz) and a high power (up to 10 Watt), while (iii) still presenting a stationary Gaussian statistics over approximately 7 decades (see black curve in Fig. 2). The partially coherent light is launched inside a 1.5-km-long polarization-maintaining (PM) single-mode optical fiber after being transmitted by a polarizing cube. The light wave acquires a linear polarization direction that is maintained all along the PM fiber. Hence, our experiment deals with the nonlinear propagation along the z direction of a *scalar* incoherent optical field $A(z,t)$. The optical power P_0 launched inside the PM fiber is ~205 mW. As the losses of the PM fiber at 1064 nm are approximately of 1 dB/km, the optical power decreases from ~205 to ~145 mW over the propagation distance $L = 1.5$ km. Rigorously speaking, dissipation cannot be fully ignored in our experiment. However, as will be discussed in more detail from numerical simulations, it does not play a significant role in the statistical features reported in this Letter.

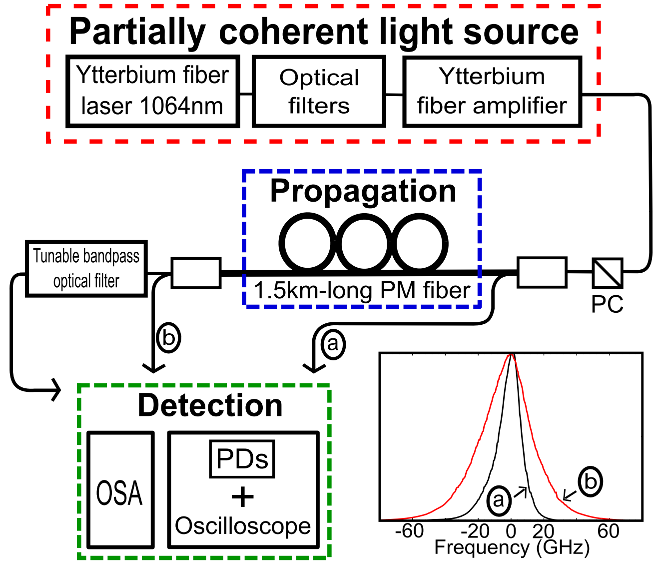

FIG. 1 (color online). Experimental setup. Linearly polarized partially coherent light having a Gaussian statistics is launched inside a polarization maintaining (PM) single-mode fiber through a polarizing cube (PC). Mean optical power spectra are measured by using an optical spectrum analyzer (OSA). The light spectrum broadens from 14 to 32 GHz from nonlinear propagation. Time detection of light power fluctuations $P(t)$ is made by using fast photodiodes (PDs) and a 36-GHz oscilloscope. A tunable optical bandpass filter with a narrow bandwidth of 6 GHz is used to analyze fluctuations of output light in sliced parts of the spectrum.

Our optical fiber experiment has been dimensioned in order to keep the bandwidth of light fluctuations below the bandwidth of the detection setup. The stochastic dynamics

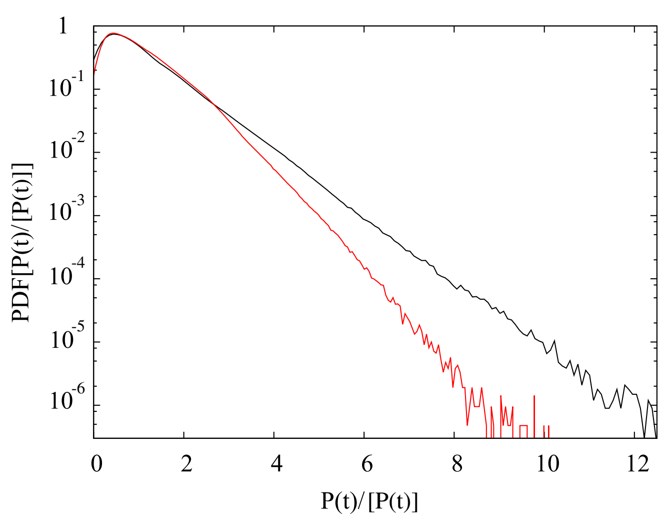

FIG. 2 (color online). Experiments. Global PDFs measured at the input (black line) and at the output (red line) ends of the PM fiber without using the optical bandpass filter.

of light power fluctuations $P(t)$ is simultaneously captured at the input and output ends of the PM fiber by using two fast photodiodes connected to a fast oscillocope having a bandwidth of 36 GHz. This kind of fast detection technique has been recently implemented to evidence a laminar-turbulent transition in a Raman fiber laser [36]. The electrical bandwidth of the two photodiodes is 38 GHz. For each of its two channels, the oscilloscope records in a single shot an ensemble of 40×10^6 points at a sampling rate of 80 GSa/s (one point every 12.5 ps) in a total duration of 500 μs. Despite the fact that the detection bandwidth is just a little bit wider than the FWHM of the output spectrum (∼32 GHz), the statistical analysis made from the recorded time series is weakly influenced by the finite detection bandwidth; see the Supplemental Material [37] for numerical simulations showing the influence of the detection bandwidth.

Figure 2 compares the PDFs for the power fluctuations of the partially coherent light at the input and at the output of the PM fiber. The PDF for intensity fluctuations of light at the input of the fiber is exponentially decaying (exponential distribution), which means that the statistics of the input field is actually Gaussian. As a result of nonlinear propagation in the defocusing medium, the PDF of the output light intensity shows tails lower than those defined by the exponential distribution [37]. This result could possibly be associated with the presence of coherent structures such as dark solitons or dispersive shock waves embedded inside the nonlinear random wave system. However, an accurate answer could only be given using more sophisticated tools based on the IST. We mention that our results are consistent with the one obtained in Ref. [24], where a different experimental setup was adopted. Optical experiments reported in Ref. [24] are not placed in the time domain but in the spatial domain. They examine the diffraction of an intense beam of spatially incoherent light in a medium with a thermal defocusing nonlinear response. Although experiments of Ref. [24] evidence statistical features similar to those plotted in Fig. 2, the physics of the two wave systems cannot be compared in a straightforward way. Experiments of diffraction in a nonlinear medium indeed involve a spatially localized field propagating over a zero background of light intensity, whereas our experiment involves a continuous wave propagating over a nonzero dc background. As pointed out in Ref. [21], the theoretical problem of nonlinear statistical changes observed in our experiment should be addressed by IST tools developed for quasiperiodic functions rather than by usual IST tools related to decaying fields [25–27].

The output end of the nonlinear fiber can be connected to a tunable bandpass optical filter (see Fig. 1) having a 6-GHz bandwidth that is much narrower than the bandwidth of light fluctuations (∼32 GHz). Observing the fluctuations of stochastic light at the output of this tunable bandpass filter reveals the occurrence of an intermittent behavior. Detuning the central frequency of the optical filter from the center of the spectrum to its wings and measuring the PDFs of the power fluctuations $P(t)$ at the output of the optical filter, we observe a continuous distortion of the PDFs; see Fig. 3. Increasing the frequency detuning, the PDF moves from the exponential distribution to distributions having tails that become increasingly larger. In other words, the statistics of the light field is Gaussian at small frequency scales, and deviations from Gaussian statistics become larger and larger when considering fluctuations at larger and larger frequency scales.

Heavy-tailed deviations from Gaussian statistics qualitatively similar to those presented in Fig. 3 have already been evidenced in several optical wave systems [9,10,39,40]. So far, these observations have been ascribed to extreme-type statistical phenomena having physical origins that specifically depend on the particular optical system under consideration. For instance, the noise transfer between the pump and the signal waves induces heavy-tailed deviations from Gaussianity in Raman fiber amplifiers [9,10], whereas similar statistical features arise from turbulentlike wave-mixing processes in Raman fiber lasers [39,40]. We stress here that the deviations from Gaussianity reported in Fig. 3 can be related, in a more general way, to several observations of the intermittency phenomenon previously made in wave turbulence [30,32,33], solar wind [34], or in the Faraday experiment [35]. However, it must be emphasized that all previous works have investigated wave systems that are far from being integrable. On the other hand, our results show that the intermittency features

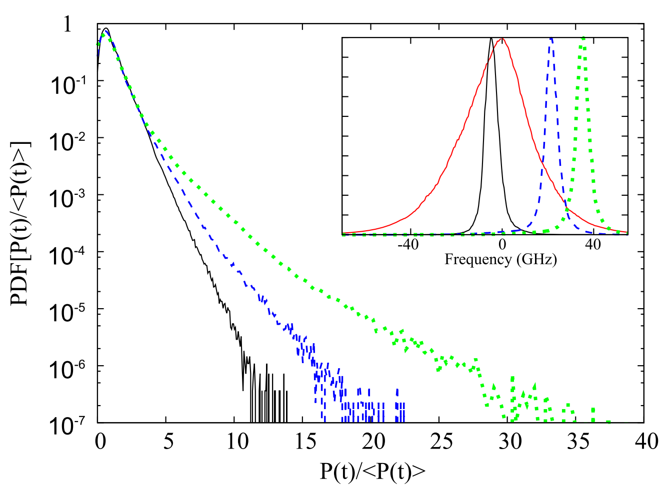

FIG. 3 (color online). Experiments. PDFs measured at the output of the optical bandpass filter by detuning its central frequency by ∼ −5 GHz (black full line), ∼21 GHz (blue dashed line), and ∼35 GHz (green dotted line) from the center of the output spectrum. Inset: Corresponding power spectra measured at the output of the tunable bandpass optical filter. The ∼32-GHz-wide power spectrum plotted by the red line is measured without bandpass filter at the output of the PM fiber.

can be sustained by an underlying dynamics that is essentially of an integrable nature.

As a matter of fact, statistical features shown in Figs. 2 and 3 dominantly arise from the interplay between third-order (Kerr) nonlinearity and second-order group velocity dispersion. They can be described by the following 1D NLSE:

$$i\frac{\partial A(z,t)}{\partial z} = -\frac{\beta_2}{2}\frac{\partial^2 A(z,t)}{\partial t^2} + \gamma|A(z,t)|^2 A(z,t), \qquad (1)$$

where $\beta_2 = +20\ \mathrm{ps^2\,km^{-1}}$ is the second-order group velocity dispersion parameter and $\gamma = 6.2\ \mathrm{W^{-1}\,km^{-1}}$ is the nonlinear Kerr coefficient of the fiber [41]. As the bandwidth of light spectrum remains relatively narrow, dispersive effects of order higher than two influence light propagation in a negligible way. We have carefully checked that the amount of Stokes light arising from stimulated Brillouin and Raman scattering remains at a very negligible level. Although dissipation plays a perturbative role, Eq. (1) describes our incoherent light wave system in an accurate way.

We have conducted a series of numerical simulations of Eq. (1) with random initial conditions. The field $A(z=0,t)$ used as the initial condition is made from a discrete sum of uncorrelated Fourier modes with random phases and amplitudes [12]. It is prepared in such a way that the PDF for the real and imaginary parts of each individual Fourier mode is Gaussian. Experimental results plotted in Figs. 2 and 3 can be reproduced with a very good accuracy from precise numerical simulations taking into account the exact shape of the input spectrum, the fiber losses, the finite bandwidth of the tunable optical filter, and the finite detection bandwidth (see Supplemental Material [37]). In the experiment, the nonlinear Kerr length $L_{\mathrm{NL}} = 1/(\gamma P_0)$ is 0.8 km and the linear dispersive length $L_D = 1/[\beta_2(2\pi\Delta\nu_0)^2]$ is 6.5 km [41]. With a propagation distance of $L = 1.5$ km, the experiment captures a transient evolution in which the nonlinear Kerr effect strongly dominates the linear dispersive effect.

To investigate the asymptotic evolution of the wave system, we have performed numerical simulations of Eq. (1) at a propagation distance that is 1 order of magnitude greater than the propagation distance reached in the experiment. After a transient evolution of a few kilometers, the wave system reaches a statistically stationary state in which the mean optical power spectrum and the PDFs no longer change with propagation distance. After a propagation distance $z = 15$ km, they assume the shapes shown in Figs. 4(a)—4(c). The features evidenced in Fig. 4 do not qualitatively depend on the exact numerical values of the parameters, as long as the power of the initial condition is sufficient for the nonlinear interaction regime to be explored.

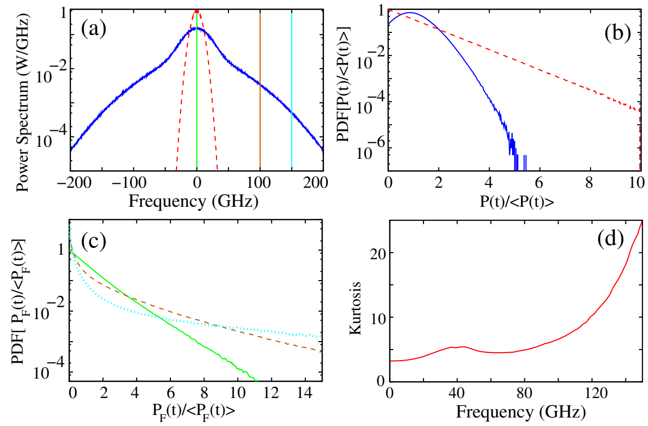

FIG. 4 (color online). Numerical simulations of Eq. (1) between $z = 0$ km and $z = 15$ km with $P_0 = 205$ mW, $\Delta\nu_0 = 14$ GHz, $\beta_2 = +20\ \mathrm{ps^2\,km^{-1}}$, $\gamma = 6.2\ \mathrm{W^{-1}\,km^{-1}}$. (a) Input (red dashed line) and output (blue full line) mean power spectra of the partially coherent wave. (b) Input (red dashed line) and output (blue full line) PDFs for $P(t) = |A(z,t)|^2$. (c) PDFs for the power fluctuations $P_F(t)$ found at the output of an ideal bandpass filter centered at $\nu_0 = 0$ GHz (green full line), $\nu_0 = 100$ GHz (brown dashed line), $\nu_0 = 150$ GHz (cyan dotted line). Vertical colored lines in (a) indicate the corresponding central frequencies. (d) Kurtosis κ of fluctuations at the output of the bandpass frequency filter.

The field $A(z=L,t) = \int d\nu \tilde{A}(z=L,\nu)e^{i2\pi\nu t}$ computed at $z = L$ has been filtered by an ideal bandpass frequency filter centered at the frequency ν_0. Figure 4(c) shows the PDFs for the power fluctuations $P_F(t) = |S_{\nu_0}^{\Delta\nu}(t)|^2$ of the filtered field $S_{\nu_0}^{\Delta\nu}(t) = \int_{\nu_0-\Delta\nu}^{\nu_0+\Delta\nu} d\nu \tilde{A}(z=L,\nu)e^{i2\pi\nu t}$ for a bandwidth $2\Delta\nu = 6$ GHz, as in the experiment. As shown in Fig. 4(c), the PDF computed in the center of the spectrum follows the exponential distribution, whereas the PDFs computed in the wings of the spectrum show tails that become larger and larger as ν_0 increases (see Supplemental Material [37] for an additional description of the dynamics found at the output of the bandpass filter). Considering the real part $R(t) = \mathrm{Re}[S_{\nu_0}^{\Delta\nu}(t)]$ of the filtered field, the intermittency phenomenon can be quantified in a more accurate way through the measurement of the kurtosis $\kappa = \langle R(t)^4\rangle/\langle R(t)^2\rangle^2$ of $R(t)$. As shown in Fig. 4(d), the deviations from ~3 of the value of κ for frequencies greater than ~100 GHz clearly indicate an intermittent behavior. The features evidenced in Figs. 4(c) and 4(d) do not depend qualitatively on the exact nature of the spectral filtering process: similar deviations from Gaussianity are also observed at the output of ideal high-pass or one-mode frequency filters.

In summary, we have performed a nonlinear fiber optic experiment in which light propagation is accurately ruled by the one-dimensional defocusing and integrable NLSE. We have examined the changes in the statistics of a partially coherent wave that has initially a Gaussian statistics. Global changes in the statistics of the light field have been evidenced and the occurrence of PDFs having tails lower than those defined by the Gaussian distribution has been reported. Using bandpass spectral filters, we have evidenced an intermittency phenomenon in a wave system that is close to integrability. From our results, we hope to stimulate other works providing a better understanding of the mechanisms of wave intermittency. Moreover, because of the universality of the NLSE, we expect that similar results could be obtained in different fields where the dynamics is close to be integrable.

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