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(Article begins on next page)

Asset Pricing in a Production Economy with Chew–Dekel Preferences *

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Abstract

In this paper we provide a thorough characterization of the asset returns implied by a simple general equilibrium production economy with Chew–Dekel risk preferences and convex capital adjustment costs. When households display levels of disappointment aversion consistent with the experimental evidence, a version of the model parameterized to match the volatility of output and consumption growth generates unconditional expected asset returns and price of risk in line with the historical data. For the model with Epstein–Zin preferences to generate similar statistics, the relative risk aversion coefficient needs to be about 55, two orders of magnitude higher than the available estimates. We argue that this is not surprising, given the limited risk imposed on agents by a reasonably calibrated stochastic growth model.

Key words. Disappointment Aversion, Epstein–Zin, Market Price of Risk, Equity Premium, Business Cycle.

JEL Codes: D81, E32, E43, E44, G12.

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1 Introduction

Production economies place greater demands on theoretical explanations of asset returns, since the quantities that lead to risk premia typically change as we modify preferences. In this paper we provide a thorough characterization of the asset returns implied by a simple variant of the neoclassical growth model, the workhorse of business cycle analysis since the seminal work of [Kydland and Prescott \(1982\)](#).

Our focus is on the role played by first-order risk aversion, modeled in the shape of [Gul \(1991\)](#)'s disappointment aversion (DA). While matching the standard deviations of consumption and output growth, a version of the model with mean-reverting technology shocks generates values for the unconditional first moments of risk-free rate and equity return, as well as the market price of risk, which are in line with the historical data. Most important, it does so for levels of disappointment aversion consistent with the experimental evidence.

The study of asset returns in the stochastic growth model goes back to [Naik \(1994\)](#), who investigated the impact of changes in aggregate risk on asset prices. Shortly thereafter, [Rouwenhorst \(1995\)](#) showed that, for the basic model employed in business cycle analysis,¹ generating plausible asset returns is virtually impossible. This is the case for two main reasons. First, and differently from what happens in an endowment economy, raising the coefficient of relative risk-aversion does not help increase the volatility of the stochastic discount factor. In fact, higher risk aversion implies lower elasticity of substitution, and therefore lower volatility of consumption growth. Second, the price of capital being constant at 1, the volatility of stock returns equals that of the marginal product of capital, which is quite limited in that framework.

[Tallarini \(2000\)](#) showed that the first of the two issues just outlined can be addressed by disentangling risk aversion from the elasticity of intertemporal substitution. By assuming Epstein-Zin preferences, he was able to raise risk aversion at arbitrarily high levels, while keeping the elasticity of substitution anchored at 1. Tallarini went on to show the existence of RRA coefficients such that the market price of risk is consistent with the empirical evidence. However, the price of capital being constant at 1, his model essentially generates no equity premium. This issue was dealt with successfully by [Jermann \(1998\)](#) and [Boldrin, Christiano, and Fisher \(2001\)](#), who assumed that the allocation of capital cannot adjust immediately or costlessly to productivity

¹We refer to the case of agents maximizing expected discounted utility under complete markets and no frictions in capital accumulation.

shocks.² Impediments to the smooth adjustment of capital imply that the price of capital is not anchored to 1 any longer. However, without limiting households' willingness to substitute consumption intertemporally, this innovation results mainly in a higher volatility of consumption growth. Both [Jermann \(1998\)](#) and [Boldrin, Christiano, and Fisher \(2001\)](#) lower the IES by introducing habit formation. The resulting increase in the curvature of the Bernoulli utility function, by increasing risk-aversion, also takes care of increasing the volatility of the stochastic discount factor.

Here we assume that agents' attitude towards risk is described by preferences belonging to the Chew–Dekel class. In particular, we focus on the case in which individuals display disappointment aversion. The more popular scenario in which agents compare atemporal lotteries by means of the expected utility criterion arises as a special case of Gul's representation, and gives rise to the model known as Epstein–Zin.

We assign values to parameters following the methodology typical of most modern macroeconomic studies. Whenever possible, we use direct empirical evidence. Alternatively, we choose them so that the model is consistent with certain low-frequency statistics for the US economy.

When setting the parameters governing risk aversion, we look for guidance in the fast-growing experimental literature. To our knowledge, the only attempt at estimating parameters for [Gul \(1991\)](#)'s representation is due to [Choi, Fisman, Gale, and Kariv \(2007\)](#). Under the assumption that the Bernoulli utility function belongs to the CRRA class, their median estimate for the function's curvature parameter is 0.481. This figure is remarkably close to the estimates obtained by several recent studies that assume the expected utility criterion.³ For this reason, we will set the RRA coefficient to 1/2 and explore our model's properties over the whole range of [Choi, Fisman, Gale, and Kariv \(2007\)](#)'s estimates of disappointment aversion.

Consistent with the seminal work of [Jermann \(1998\)](#) and [Boldrin, Christiano, and Fisher \(2001\)](#), we show that with a relatively low IES the model with mean-reverting technology shocks can generate a sizeable volatility of equity returns while matching the standard deviations of consumption and output growth. Most important, we also find that the precautionary motive implied by levels of disappointment aversion well

²In [Jermann \(1998\)](#), this feature is obtained by means of convex adjustment costs, while [Boldrin, Christiano, and Fisher \(2001\)](#) assume the existence of two production sectors, with limited inter-sectoral factor mobility.

³The estimates are 0.48 for [Chen and Plott \(1998\)](#), 0.3–0.5 for [Holt and Laury \(2002\)](#), 0.52 for [Goeree, Holt, and Pfaffey \(2002\)](#), and 0.45 for [Goeree and Holt \(2004\)](#).

within [Choi, Fisman, Gale, and Kariv \(2007\)](#)’s range yields unconditional expected risk-free rate and market price of risk in line with historical data. As is always the case for models that rely on low IES, the cost we pay is in terms of excessive volatility of the risk-free rate.

In [Section 6](#) we argue that our representative agent exhibits plausible levels of risk aversion over the whole support of the ergodic distribution. In the case of our DA benchmark parameterization, at the modal state of nature, an individual that expects to consume for 25,000 dollars per quarter faces a lottery over future quarterly consumption with a standard deviation of only 34 dollars. In order to avoid that lottery, she is willing to pay only 1.25 dollars (per quarter).

Essentially by construction, an agent endowed with Epstein–Zin preferences must display a similar willingness to pay in order to avoid the same risk. Without disappointment aversion, such willingness is obtained postulating a coefficient of relative risk-aversion of about 55. This is the case because an agent featuring second-order risk-aversion is almost indifferent to low risks (such as those implied by the stochastic growth model), for a large range of RRA coefficients.⁴

When productivity shocks are permanent, disappointment aversion is still crucial in generating a sensible level for the price of risk. However, the model is not nearly as successful in replicating the data on asset returns. In this scenario, the volatility of consumption growth is decreasing in the IES. In turn, this means that only by postulating relatively high levels of the elasticity, one can obtain a substantial volatility of capital gains (and therefore equity returns), while matching the volatility of consumption growth measured in the data. Unfortunately, our simulations show that even for a IES greater than 1 the volatility of equity returns falls about two orders of magnitude short of its empirical counterpart. A positive effect of assuming a relatively high elasticity is the drop in the volatility of the risk-free rate.

We conclude by briefly considering the issue of returns’ predictability. The model with disappointment aversion features acyclical price of risk and acyclical volatility of equity returns. It follows that, contrary to the empirical evidence, the price-dividend ratio does not predict the equity premium. In [Section 8](#) we show that when agents display Generalized Disappointment Aversion in the sense of [Routledge and Zin \(2004\)](#), the model is able to generate counter-cyclical price of risk and equity premium’s predictability.

In recent years, we have witnessed an increasing interest in the asset return prop-

⁴See [Epstein and Zin \(1990\)](#), [Epstein and Zin \(2001\)](#) and [Bonomo and Garcia \(1994\)](#).

erties of general equilibrium production economies. Uhlig (2004) carefully explores the interconnections between asset market observations, macroeconomic observations, and theoretical choices of key parameters in a fairly standard setup. Danthine and Donaldson (2002) and Guvenen (2008) focus on household heterogeneity, constructing environments in which consumption growth of stock market participants is more volatile because these agents provide insurance against aggregate shocks to non-participants. Papanikolaou (2008) studies the role of investment-specific technological change. The papers that are closer to ours are Croce (2008) and Kaltenbrunner and Lochstoer (2006), who conduct a comprehensive study of the asset pricing implications of the stochastic growth model when preferences are of the Epstein–Zin type. A further difference between their studies and ours is of methodological nature. Here we follow rigorously the calibration strategy used in macroeconomics, in that parameter values are set either using independent evidence or in such a way that the model’s equilibrium allocation is consistent with historical data on aggregate quantities, but not prices.

The remainder of the paper is organized as follows. The model is introduced in Section 2. Asset returns are defined in Section 3. In Section 4 we discuss our calibration strategy. Section 5 characterizes our main results, i.e. the properties of asset returns in the case of mean-reverting productivity shocks. The scenario with Epstein–Zin preferences is analyzed in Section 6. Section 7 contains our discussion of the case with permanent shocks. The predictability of asset returns is considered in Section 8. Finally, Section 9 concludes.

2 The Model

Ours is a simple version of the standard neoclassical growth model. Time is discrete and runs from $t = 0$ to infinity. The economy consists of a large number of identical and infinitely lived households that derive utility uniquely from consumption. Production takes place in firms, that finance the purchase of capital by selling claims to their cash flows to households. Our model differs from the standard framework in that labor supply is inelastic (it is assumed that each agent provides one unit of labor every period) and adjusting the capital stock is costly.

We denote by $S_t = \{s_v\}_{v=0}^t$ the history of the economy between dates 0 and t . In other words, S_t contains all payoff-relevant information as of time t . Once specified the other assumptions, we will be able to define its elements.

2.1 Preferences

We follow [Kreps and Porteus \(1978\)](#) and [Epstein and Zin \(1989\)](#) in constructing preference orderings over stochastic sequences of consumption by means of a Koopman's time aggregator and risk preferences in the Chew–Dekel class.⁵

In particular, we assume that after every history S_t , agents value stochastic sequences of consumption $\{c_v\}_{v=t}^{\infty}$ by means of the time aggregator

$$v(S_t) \equiv [c_t^\gamma + \beta \mu^\gamma(S_t)]^{1/\gamma}, \quad \gamma \leq 1, \beta > 0,$$

where $\mu(S_t)$ represents the certainty equivalent of the lottery over the streams of utility associated to all histories $S_t \cup \{s_v\}_{v=t+1}^{\infty}$, and β defines the relative importance of future versus current utility. The elasticity of intertemporal substitution is $1/(1 - \gamma)$. The nature of the certainty equivalent $\mu(\cdot)$ will depend on the preference relation over atemporal lotteries. This paper considers two popular formulations, known in the literature as *expected utility* and *disappointment aversion*.

As is well known, expected utility is the representation of a preference relation satisfying the axioms of monotonicity, completeness, transitivity, continuity, and independence. [Gul \(1991\)](#) showed that replacing the latter with a weaker requirement, which he called weak independence, leads an agent evaluating a lottery to attach a penalty in utility terms to events that she considers disappointing. These are all the events that fall below her certainty equivalent. The certainty equivalent $\mu(S_t)$ satisfies the following condition:

$$\begin{aligned} \mu^\eta(S_t) &= \sum_{S_{t+1}} \pi(S_{t+1}|S_t) v^\eta(S_{t+1}) - \theta \sum_{S_{t+1} \in \Delta_{t+1}} \pi(S_{t+1}|S_t) \{[\mu(S_t)]^\eta - v^\eta(S_{t+1})\}, \\ \Delta_{t+1} &= \{S_{t+1} : v(S_{t+1}) < \mu(S_t)\}, \quad \eta \leq 1, \theta \geq 0. \end{aligned}$$

With $\pi(S_{t+1}|S_t)$ we denote the probability of history S_{t+1} , conditional on S_t having occurred. The set Δ_{t+1} is the set of disappointing events. For $\theta = 0$ we retrieve the expected utility representation, with coefficient of relative risk aversion $1 - \eta$.

⁵By time-aggregator we simply mean a criterion for evaluating deterministic sequences of consumption. [Koopmans \(1960\)](#) characterizes the set of aggregators that satisfy the conditions of *history independence*, *future independence*, and *stationarity*. [Chew \(1989\)](#) and [Dekel \(1986\)](#) derive a class of risk preferences that include expected utility as a special case and lead to first-order conditions that are linear in probabilities. See [Backus, Routledge, and Zin \(2004\)](#) for a careful yet readable summary of this literature.

2.2 Production and Capital Accumulation

We assume that population grows at the constant rate $\varphi \geq 0$. For simplicity, we will express all variables in per-capita terms. Output y_t is produced according to

$$y_t = k_t^\alpha (z_t l_t)^{1-\alpha},$$

where k_t and $l_t \equiv 1$ are the capital and labor inputs, respectively, and $0 < \alpha < 1$. Labor augmenting technological progress at time t is given by

$$z_t = e^{\lambda t + \varepsilon_t},$$

where $\lambda > 0$ and $\varepsilon_t = \rho \varepsilon_{t-1} + \zeta_t$, $\zeta_t \sim N(0, \sigma^2)$, $\rho \in (0, 1)$. As commonly assumed in theoretical studies of the business cycle, deviations of productivity from the linear time-trend follow a first-order autoregressive process. The case of i.i.d. productivity growth will be considered in Section 7.

Investment at time t is subject to adjustment costs according to the function $g(k_t, k_{t+1})$, where $g(\cdot, \cdot)$ is twice differentiable, strictly increasing and strictly convex in k_{t+1} , and linearly homogenous in k_t . In the analysis that follows, we will assume that

$$g(k_t, k_{t+1}) \equiv \left| e^\varphi \frac{k_{t+1}}{k_t} - \psi \right|^\iota k_t, \quad \iota > 1, \psi > 0.$$

Finally, letting x denote investment (gross of depreciation but net of adjustment cost), the per-capita capital stock evolves according to

$$e^\varphi k_{t+1} = x_t + (1 - \delta)k_t$$

and the aggregate resource constraint is

$$y_t = c_t + x_t + g(k_t, k_{t+1}).$$

The timing and the resolution of uncertainty are as usual. It follows that the history S_t can be summarized by the pair (k_t, z_t) .

2.3 The Planner's Problem

Following much of the literature, we characterize the equilibrium allocation by solving the planner's problem. Expressed in terms of trend-stationary variables, the latter can be written in recursive form as

$$v(k, \varepsilon) = \max_{k'} \left\{ c^\gamma + \beta e^{[\varphi + \gamma \lambda]} \mu [v(k', \varepsilon')]^\gamma \right\}^{1/\gamma} \quad (1)$$

subject to

$$\begin{aligned} c + e^{(\lambda+\varphi)} k' &= k^\alpha (e^\varepsilon l)^{1-\alpha} + (1-\delta) k - \left| e^{(\lambda+\varphi)} \frac{k'}{k} - \psi \right|^\iota k, \\ \varepsilon' &= \rho\varepsilon + \zeta', \quad \zeta' \sim N(0, \sigma^2), \end{aligned}$$

where l is exogenously given and $\mu \equiv \mu[v(k', \varepsilon')]$ solves

$$\mu^\eta = \mathbb{E} \left\{ \frac{[1 + \theta I(k', \varepsilon', \mu)] v^\eta(\varepsilon', \zeta')}{1 + \theta \mathbb{E}[I(k', \varepsilon', \mu)]} \right\}, \quad I(k', \varepsilon', \mu) = \begin{cases} 1 & \text{if } v(k', \varepsilon') < \mu, \\ 0 & \text{otherwise.} \end{cases}$$

For $\theta = 0$, one recovers what is known as the *Epstein–Zin* model. Further, by setting $\gamma = \eta$, we obtain the classical case of expected discounted utility. Throughout this paper, we will impose $\psi \equiv e^{\lambda+\varphi}$. That is, we will assume that adjustment costs are strictly positive if and only if capital grows at a rate different from its balanced growth rate. As pointed out by [Abel \(2002\)](#), this hypothesis ensures that the balanced growth rate of the economy will be invariant to changes in the parameter ι .

3 On Asset Returns

3.1 Definitions

As in most of the asset pricing literature, our analysis will focus on the risk-free asset and on equity. In the data, we will identify the former with the 3-month Treasury Bill and the latter with a portfolio of stocks traded on the New York Stock Exchange. Details on data sources and elaboration are confined to [Appendix A](#). The theoretical counterpart of the 3-month T-Bill will be a 1-period lived asset that delivers one unit of consumption in all states of nature. Its conditional gross return, denoted as $R^f(k, z)$, satisfies⁶

$$\sum_i \pi(\varepsilon_i | \varepsilon) m(\varepsilon_i | k, \varepsilon) R^f(k, \varepsilon) = 1,$$

where the stochastic discount factor $m(\varepsilon_i | k, \varepsilon)$ is

$$m(\varepsilon_i | k, z) = \beta e^{\lambda(\gamma-1)} \frac{1 + \theta I(k', \varepsilon_i, \mu)}{1 + \theta \sum_i \pi(\varepsilon_i | \varepsilon) I(k', \varepsilon_i, \mu)} \left[\frac{c(k', \varepsilon_i)}{c(k, \varepsilon)} \right]^{\eta-1} \left[\frac{v(k', \varepsilon_i)/\mu(k', \varepsilon)}{c(k', \varepsilon_i)/c(k, \varepsilon)} \right]^{\eta-\gamma}, \quad (2)$$

and where $I(k', \varepsilon_i, \mu) = 1$ if $v(k', \varepsilon_i) \leq \mu$, and $I(k', \varepsilon_i, \mu) = 0$ otherwise.

⁶In the remainder of the paper, our notation will reflect the fact that our numerical procedure involves approximating the stochastic process for the productivity shock ε by means of a 6-state Markov chain.

Following [Cochrane \(1991\)](#), [Jermann \(1998\)](#), and [Boldrin, Christiano, and Fisher \(2001\)](#), we identify the gross return on equity (conditional on state i) with the marginal gross return on investment. That is, we posit that

$$R^e(k, \varepsilon; \varepsilon_i) = \frac{\alpha k'^{\alpha-1} (\varepsilon_i l')^{1-\alpha} + (1 - \delta) - \frac{\partial}{\partial k'} g[k', k''(k', \varepsilon_i)]}{1 + e^{-(\lambda+\varphi)} \frac{\partial}{\partial k'} g(k, k')}. \quad (3)$$

The conditional expected return is then computed as

$$R^e(k, \varepsilon) = \sum_i \pi(\varepsilon_i | \varepsilon) R^e(k, \varepsilon; \varepsilon_i).$$

In [Appendix B](#), we show that equation (3) is implied by a decentralization of the planner's problem where production is carried out by firms that maximize the sum of expected discounted dividends and finance investment by means of retained earnings only.

There are reasons why defining the right-hand side of (3) as the theoretical counterpart to the return on a stock portfolio is short of ideal. To start with, as shown in [Appendix B](#), it identifies aggregate dividends as the difference between aggregate consumption and labor income, i.e. the net resource flow from the corporate sector to the household sector. When consumption is greater than labor income, the net payout of the corporate sector is positive. This is the case in which capital income is greater than gross investment. However, when consumption is lower than labor income, the net payout of the corporate sector is negative. This is because capital income is not sufficient to finance gross investment. Households invest part of their labor income in the corporate sector. Therefore, $R^e(k, \varepsilon; \varepsilon_i)$ is the return to shareholders, once eventual new investments are taken into account. This implies that a more appropriate empirical counterpart of the payout to shareholders in this model consists of the gross flow of resources from the corporate sector to the household sector (dividends, shares and bonds repurchase, interest payments on bonds, interest payments on other loans, extinguishment of loans) minus the gross flow of resources from the household sector to the corporate sector (gross issue of equity and bonds plus new loans). A related issue is that in our decentralization firms are assumed to be completely equity-financed. This is obviously not the case for those corporations whose stocks form the market portfolios used in all empirical asset pricing studies.⁷

⁷These observations also constitute the premise of a recent paper by [Gomme, Ravikumar, and Rupert \(2007\)](#). In their work, the empirical counterpart of the return on equity generated by the model is not the return on an equity portfolio, but rather the return to a measure of aggregate business capital, which they construct using data from the National Income and Product Accounts.

3.2 The Role of Disappointment Aversion: Heuristics

In the Epstein–Zin model ($\theta = 0$), the intertemporal marginal rate of substitution depends on two risk factors, which we will label short- and long-run risk factors, respectively. The former is consumption growth and the latter is what the asset pricing literature often refers to as innovation to the wealth consumption ratio. As carefully laid out in [Croce \(2008\)](#) and [Kaltenbrunner and Lochstoer \(2006\)](#), when productivity shocks are mean reverting the two factors covary negatively. This implies that, for given allocation, the long-run factor contributes positively to the price of risk when the elasticity of substitution is lower than the inverse of the RRA coefficient (i.e. for $\eta > \gamma$), the less so as the IES increases (γ increases). For $\eta < \gamma$, the long-run factor ends up lowering the price of risk.

As one may expect after inspecting equation (2), these considerations are still valid when $\theta > 0$. What is the effect of adding disappointment aversion to the picture? Since in our model quantities are essentially invariant to θ , we can preview the answer to this question by considering a simple endowment economy. The intuition gained here will be useful in interpreting the findings illustrated in the next sections.

Think of introducing disappointment aversion in a setup à la [Mehra and Prescott \(1985\)](#), where detrended consumption takes only two values, indexed by $i = h, l$, respectively. In such scenario, no matter the current state of nature, the low state is disappointing, and the high is not. Denote with π_i the transition probability from state i to i and with q_{ij} the state-contingent prices that would obtain for $\theta = 0$. The state prices are

$$\begin{bmatrix} \frac{1+\theta}{1+\theta\pi_l}q_{ll} & \frac{1}{1+\theta\pi_l}q_{lh} \\ \frac{1+\theta}{1+\theta(1-\pi_h)}q_{hl} & \frac{1}{1+\theta(1-\pi_h)}q_{hh} \end{bmatrix}.$$

Introducing disappointment aversion means attaching weights greater than 1 to the Epstein–Zin prices contingent on reaching the low state in the future, and weights lower than 1 to the prices contingent on reaching the high state. This leads unequivocally to an increase in the volatility of the stochastic discount factor.

The conditional risk-free rates R_i^f satisfy

$$\begin{aligned} 1/R_l^f &= \frac{1-\pi_l}{1+\theta\pi_l}q_{lh} + \frac{\pi_l(1+\theta)}{1+\theta\pi_l}q_{ll}, \\ 1/R_h^f &= \frac{\pi_h}{1+\theta(1-\pi_h)}q_{hh} + \frac{(1-\pi_h)(1+\theta)}{1+\theta(1-\pi_h)}q_{hl}. \end{aligned}$$

Both expressions on the right-hand-side are larger, the greater is θ . The intuition is that the larger θ , the larger the disutility that agents derive from risk (the greater

the precautionary motive). In turn, this means that a lower risk-free rate is needed for the consumer to choose a given allocation.

4 Calibration

We borrow most of the parameter values from the real business cycle literature. The model period is one quarter. The income share of capital (α) is equal to 0.36. Following [Cooley and Prescott \(1995\)](#), we set λ and φ so that the yearly growth rates of population and per-capita output are 1.2% and 1.56%, respectively. Once again following [Cooley and Prescott \(1995\)](#), the autocorrelation coefficient ρ is set to 0.95. The parameter σ is chosen so that the model generates a standard deviation of output growth of about 1%.

The choice of the discount factor is crucial to the performance of asset pricing models. In business cycle studies, its value is chosen to generate sensible values for the capital-output ratio. Consider the deterministic version of our model. The optimality conditions imply two restrictions for $\beta^* \equiv \beta e^{\varphi + \lambda \gamma}$ and δ on the balanced growth path:

$$\begin{aligned}\delta &= \frac{y - c}{k} + 1 - e^{\lambda + \varphi}, \\ \alpha \frac{y}{k} &= \frac{e^{\lambda + \varphi}}{\beta^*} + \delta - 1.\end{aligned}$$

It is customary to assign a value to δ , and then choose β^* to match the capital-output ratio. Under certainty equivalence, this will also be the mean of the ergodic distribution of capital-output ratios in the stochastic model.

However, as pointed out by [Tallarini \(2000\)](#) and [Hansen, Sargent, and Tallarini \(1999\)](#), in general certainty equivalence will not hold in our model. Precautionary savings motives imply that the mean capital-output ratio will be larger than its value in deterministic steady-state. While the above conditions still offer some guidance, we will have to check that the mean conforms with the evidence.

We assume that the annual depreciation rate is 6%. The measurement of the capital-output ratio depends crucially on the definition of capital. In business cycle studies, it ranges from 2.5 to 3.32.⁸ We pick β^* so that, along the balanced-growth

⁸According to [Cooley and Prescott \(1995\)](#), 3.32 is the value that obtains once housing capital is accounted for. [Cooley and Soares \(1999\)](#) and [Gomme and Rupert \(2007\)](#) report that the capital output ratio was a lot higher in the 1930s, a period of abnormally low output. However, their estimates for the post-WWII period fall in the range mentioned above.

path of the deterministic model, the capital–output ratio is 3. The annual investment to capital ratio is 8.76%, slightly higher than the US historical average of 7.6% reported by [Cooley and Prescott \(1995\)](#).

We are left with four choices: the preference parameters η , θ , and γ , and the elasticity parameter in the capital adjustment cost function, ι . Recall that η and θ control the agent’s attitude towards risk in atemporal lotteries. To our knowledge, the only attempt at jointly estimating the two parameters was carried out by [Choi, Fisman, Gale, and Kariv \(2007\)](#). The second and third quartile of their estimates of η are 0.481 and 0.88, respectively.⁹ Since asset returns turn out to be virtually invariant to changes in η over $[0,1]$ for a wide range of the other parameters, we set $\eta = 0.5$. Conversely, asset returns are highly sensitive to both θ and γ . This leads us to explore the model’s implications for wide ranges of their values. We will consider $\theta \in [0, 0.4]$, the upper bound being close to the third quartile of [Choi, Fisman, Gale, and Kariv \(2007\)](#)’s estimates distribution. The range of γ , for which the empirical literature offers little guidance, will be dictated by the model.

Since quantity dynamics is essentially invariant with respect to both θ and η , the volatility of consumption growth is pinned down by γ and ι . In the comparative statics exercises to follow, as we increase γ , we will vary ι in such a way that the ratio between the standard deviations of consumption growth and output growth is always 0.5. We will also vary β to keep β^* constant. The calibration is summarized in Table 1.

α	δ	λ	φ	β^*	η	ρ	σ
0.36	0.015	0.00387	0.00298	0.9919	0.5	0.95	0.0164

Table 1: Calibration.

The results reported in the remainder of the paper are the output of an algorithm that involves i) obtaining a numerical approximation to the optimal policy for consumption of the optimization program (1) and ii) applying standard Monte Carlo simulation methods in order to approximate the stationary distribution of capital implied by that policy.¹⁰

⁹See Table 1 and Figure 6 of their paper.

¹⁰A detailed description of the algorithm, which uses finite element methods, is available from the authors upon request. Recently, [van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez \(2008\)](#) argued that high–order perturbation methods also yield accurate predictions for asset returns under Epstein–Zin preferences. It remains to be seen whether such methodology can be extended to models with preferences, such as disappointment aversion, which depend on time–varying reference points.

5 Main Results

The purpose of this section is twofold. First, we want to understand the effects on asset prices generated by varying the preference parameters θ and γ . Second, we want to ask whether there exist pairs (θ, γ) that lead to sensible implications for the first two moments of risk-free rate and equity return, as well as for the market price of risk. At the end of the section, we will also consider the roles played by β^* and ρ , which govern discounting and the persistence of productivity shock, respectively.

5.1 Dynamics of Consumption and Investment

Qualitatively, the business-cycle dynamics implied by the model is that of the prototypical stochastic growth model with mean-reverting productivity shocks (see [Rouwenhorst \(1995\)](#) and [King and Rebelo \(1999\)](#)).

Figure 1 reports the impulse responses for selected variables that obtain under our benchmark parameterization, i.e. for $\theta = 0.25$, $\gamma = -28$ (IES=0.0345), and $\iota = 1.23$. The rationale behind these choices will be clear shortly. The plots were obtained by averaging out the values generated by simulating the model for 500,000 runs of 150 periods each. In period 1 of each run, detrended capital is set equal to the mean of its unconditional ergodic equilibrium distribution, while the shock is set to the highest value in its grid. The variables are expressed as percentage deviations from their unconditional means.

In response to a positive productivity shock, both consumption and investment jump. Investment increases in the expectation that productivity will stay high in the near future. The ensuing rise in the capital stock allows the agent to smooth consumption over time. The dynamics of consumption depends on preference parameters, namely the discount factor and the intertemporal elasticity of substitution, on the capital adjustment cost, and on the persistence of the productivity shock. In particular, the initial response is going to be larger the lower the discount factor, the higher the IES, the greater the marginal capital adjustment cost, and the lower the persistence of the shock.

The bottom-right panel of Figure 1 reproduces the response of the price of capital, i.e. the cost of diverting the marginal unit from current consumption to capital accumulation. In the absence of investment adjustment costs, the price would be

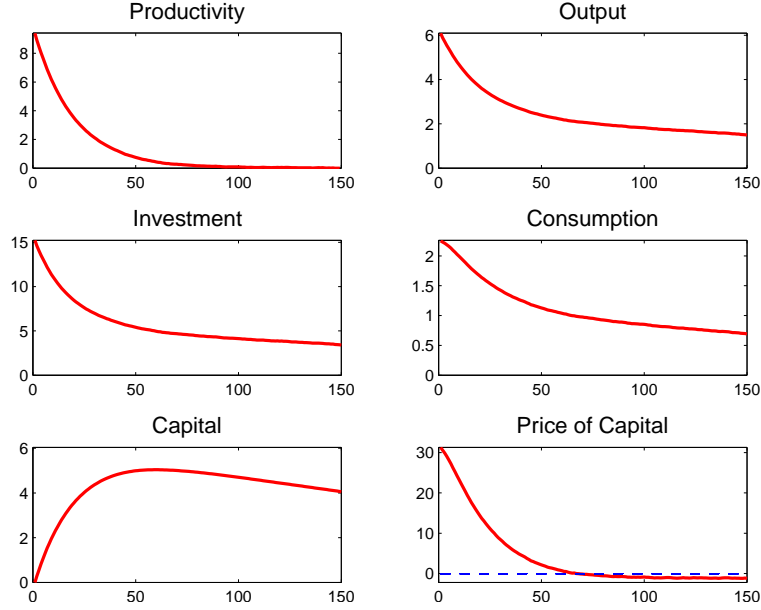


Figure 1: Impulse Responses of Quantities – Percentage Deviations from Trend.

constant at 1. In our case, it jumps to reflect the sudden increase in investment, and then decreases over time, as the progressive decline in the marginal product of capital calls for a declining pattern of investment.

5.2 Asset Returns

Table 2 reproduces two sets of statistics for asset returns in the US. Details about the calculation of the post-WWII figures can be found in Appendix A. The data for the longer sample is drawn from Bansal and Yaron (2004).¹¹ Here we are interested

	$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e - r^f)$	Sharpe Ratio
1947–2005	1.008%	1.668%	7.572%	15.16%	0.499
1929–1998	0.86%	0.97%	6.33%	19.42%	0.326

Table 2: Unconditional Moments at Annual Frequency – Data.

in assessing our model’s ability to generate unconditional moments close to their empirical counterparts and a market price of risk higher than the Hansen–Jagannathan bound.

¹¹Table 11 in the Appendix compares our unconditional moments with those reported by other authors. Notice that our equity premium and Sharpe Ratio are higher than reported by other studies. The main reason is that we include the 1990s in our sample.

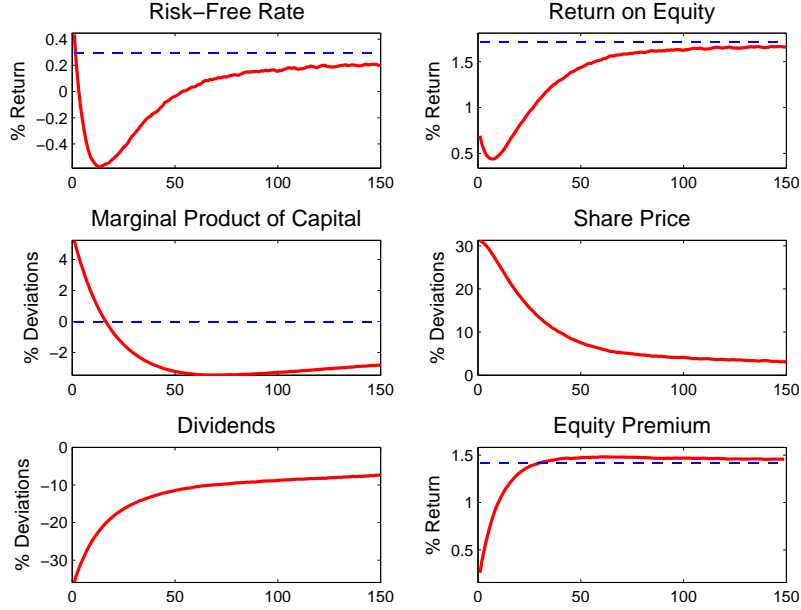


Figure 2: Impulse Responses of Financial Variables.

Figure 2 illustrates the impulse responses of asset returns. Not surprisingly, the dynamics of the risk-free rate is dictated by the pattern of expected consumption growth depicted in Figure 1. It is above average on impact, and then drops. The share price is the product of the price of capital and the capital stock, which is a slow-moving variable. It jumps to reflect the sudden increase in investment, but then regresses to its unconditional mean as expected productivity growth and investment drop towards their trend levels. Dividends, defined as the difference between consumption and labor income, drop below trend to allow for the increase in investment. The expected return on equity falls as a result of the drop in dividends and of the expected decline in the price of capital.

Now consider Figure 3. It depicts the unconditional moments of asset returns that obtain for $\theta \in [0, 0.4]$ and $\gamma \in [-34, -19]$ ($\text{IES} \in [0.0286, 0.05]$). The figure is constructed by varying the parameters ι and β in such a way that the standard deviation of consumption growth and the effective discount rate β^* are constant. This implies that for all allocations described, the volatility of consumption growth is 0.5% per quarter. A larger IES means that agents are less eager to smooth the effect of temporary productivity shocks. In turn, this implies that the volatility of consumption growth increases. To bring it back to its target value, we need to make investment

more appealing at the margin. This is accomplished by lowering the marginal capital adjustment cost (i.e. by raising ι). The right panel in the bottom row depicts the values of ι that keep the volatility of consumption growth constant as we increase the intertemporal elasticity of substitution.

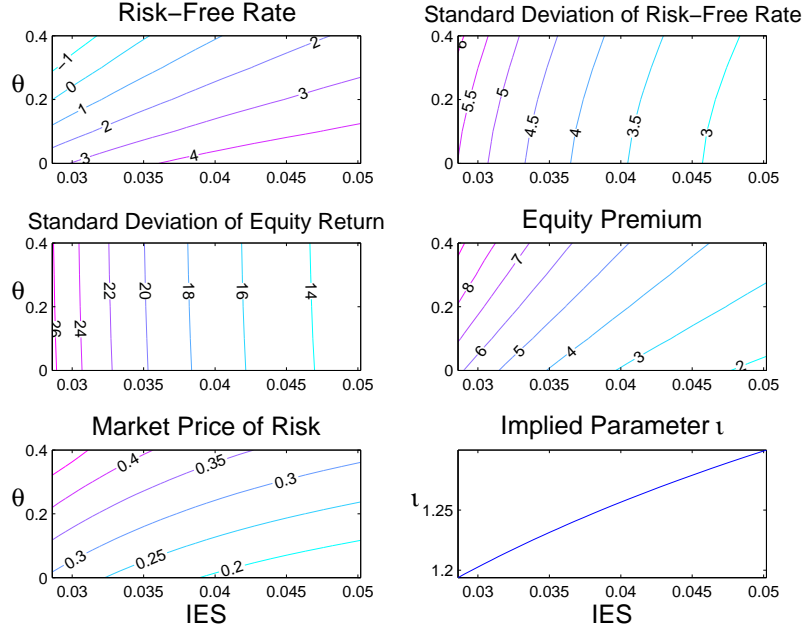


Figure 3: Comparative Statics. Statistics are Annualized.

The model delivers statistics of the right order of magnitude for all variables of interest. Given the wide range of estimates for asset returns obtained in the literature, we decide against calibrating the parameters to minimize a measure of the distance between target moments and their model counterparts. By inspection of Figure 3, the reader can single out the parameterization that she/he prefers.

$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e)$	MPR
1.182%	4.457%	6.838%	20.309%	0.34

Table 3: Unconditional Moments at Annual Frequency – Model.

Table 3 reports the results that obtain for our benchmark parameterization. It shows that the model can generate a low expected risk-free rate and sizeable equity premium and price of risk, at the cost of a somewhat excessive volatility of both

returns. Because of the precautionary motive, in our experiments the mean capital–output ratio is always larger than the steady–state value in the deterministic version of the model. For our benchmark parameterization, it equals 3.33, roughly the value considered by [Cooley and Prescott \(1995\)](#).

Figure 3 shows that the price of risk rises with the level of disappointment aversion and declines with γ . The impact of θ on the volatility of the stochastic discount factor was already discussed. Now consider the comparative statics with respect to γ . Recall that (i) since β^* is held constant, $\beta e^{\lambda(\gamma-1)}$ is also constant and that (ii) the parameter ι varies with γ to keep the volatility of consumption growth constant. Then the result follows from the fact that, for given consumption growth process, a larger γ (higher IES) reduces the impact of long–run risk on the volatility of the stochastic discount factor.

Since $\beta e^{\lambda(\gamma-1)}$ is constant, the comparative statics of the risk–free rate is uniquely driven by changes in the precautionary motive. Thanks to the mechanism illustrated in Section 3.2, a rise in θ increases the precautionary motive, leading to a lower unconditional expected risk–free rate. A rise in γ has the opposite effect.

Consider now the standard deviation of the risk–free rate. Given the persistence built in the process for productivity growth, it is in good states that productivity growth is expected to be above average. In turn, this means that in such scenarios most of the conditional probability mass is over states characterized by positive consumption growth. A larger γ will increase state prices (reduce the conditional risk free rate). The converse will occur in bad states. As a result, the volatility of the risk–free rate drops with γ .

The right panel at the top of Figure 3 shows that the volatility of the risk–free rate slowly increases with θ . The intuition behind this result is not transparent. The simple example considered in Section 3.2 hints that a larger θ leads to a greater precautionary motive in all states of nature. Even in such simple environment, however, it is not possible to point out conditions under which such effect is greater conditional on high or low consumption growth.

By definition (see equation (3)), the standard deviation of equity return depends on the volatility of marginal product of capital and capital gains. Since the latter is by far the largest of the two, it is safe to disregard the former. As quantities are essentially invariant with respect to θ , the same must be true for the volatility of capital gains. This is confirmed in Figure 3. Now consider the effect of increasing the IES. Recall that in our exercise, ι increases with γ to keep the volatility of consumption

growth constant. In turn, this means that the marginal capital adjustment cost drops with γ . This leads to a lower volatility in the price of capital, i.e. to a lower volatility in capital gains.

The Hansen–Jagannathan decomposition expresses the expected unconditional equity premium as the product of market price of risk, the standard deviation of the premium, and the correlation between premium and stochastic discount factor:

$$E(r_{t+1}^e - r_{t+1}^f) = -\frac{\text{Std}(m_{t+1})}{E(m_{t+1})} \times \text{Std}(r_{t+1}^e - r_{t+1}^f) \times \text{Corr}(m_{t+1}, r_{t+1}^e - r_{t+1}^f).$$

Then, it is not surprising that the expected equity premium increases with θ and decreases with γ . A larger θ leaves risk unchanged, but leads to a higher price of risk. A rise in γ prompts a drop in both risk and its price.

In summary, here is what we learned in this section. As in [Jermann \(1998\)](#), the combination of relatively low IES and relatively high marginal capital adjustment cost allows the model to attain a high volatility of the equity return while matching the volatility of consumption growth. For values of θ consistent with the experimental evidence, the model also generates data-conforming outcomes for the price of risk and for the first moments of risk-free rate and equity return.

The values for β^* and γ used in the simulations of which in [Figure 3](#) imply that $\beta > 1$. In the past, many scholars felt uneasy with this assumption, on the grounds that households with $\beta > 1$, when faced with a constant stream of consumption, prefer future consumption to current consumption. This implication is inconsistent with the observation that interest rates tend to be positive. Consistent with what argued by [Kocherlakota \(1990\)](#) in the case of endowment economies, our paper shows that equilibria with positive interest rates may exist in growing economies, in spite of the fact that $\beta > 1$. This finding led Kocherlakota himself to conclude that “*many researchers are turning their attention to simulating artificial economies using ‘reasonable’ values of preference parameters. It is essential that this criterion of reasonableness not be used to preclude β being larger than one.*” To this, it is worth adding that several econometric studies, among which [Hansen and Singleton \(1982\)](#), estimated β to be significantly greater than 1.

We conclude this section by briefly explaining why our setup does not deliver when $\beta \leq 1$. Imposing that β be no larger than 1 while matching the capital-output ratio leads to values of the IES much higher than our model needs in order to generate sensible asset pricing moments, and also much higher than found in the empirical

literature.¹² Given our values of β^* and δ , requiring $\beta \leq 1$ implies $\gamma \geq -2.8472$, or $IES \geq 0.26$. Staring at Figure 3 reveals that for such parameter values our model yields counterfactual values for the risk-free rate, the market price of risk, and the equity premium. The reason is that (varying ι as described above) the precautionary motive, the volatility of the stochastic discount factor, and the volatility of the price of capital are all decreasing in γ .

5.3 Assessment of γ and ι

Our conclusion that a rather low IES is needed in order to generate a reasonable volatility of equity return is consistent with what found by the early literature on habit preferences (see Jermann (1998) and Boldrin, Christiano, and Fisher (2001)) and by more recent contributions which adopt Epstein–Zin preferences, such as van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2008). The latter estimate the IES to be 0.06. Although very far from the unitary value that we are used to see assumed in macroeconomic models, an intertemporal elasticity of substitution of $1/29$ falls in the confidence interval of several econometric studies, among which Hall (1988). Other studies, such as Croce (2008) and Kaltenbrunner and Lochstoer (2006), focus on scenarios in which the elasticity of substitution is greater than 1. This allows them to lower the volatility of the risk-free rate, at the price of an equity premium which is off by at least one order of magnitude.

In our framework, capital adjustment costs are needed to produce variation in the price of capital, a necessary condition to generate a substantial volatility of share prices. Essentially all studies in the literature on asset pricing in production economy assume the existence of such costs or other rigidities that impede the smooth adjustment of capital to shocks. The question is whether the model produces overidentifying restrictions that can be used to assess the values we assigned to ι . The restriction we will be considering is for the elasticity of the investment rate with respect to Tobin's q :

$$\frac{d \log(i/k)}{d \log(q)} = \frac{d(i/k)}{dP_{k'}} \frac{P'_k}{i/k} = \frac{1}{i/k} \frac{1 + \chi\left(\frac{i}{k}\right) \iota \left| \frac{i}{k} + (1 - \delta) - \psi \right|^{\iota-1}}{\iota(\iota - 1) \left| \frac{i}{k} + (1 - \delta) - \psi \right|^{\iota-2}},$$

where $\chi(i/k) = 1$ if $(i/k) > \psi - (1 - \delta)$, and $\chi(i/k) = -1$ otherwise. In the stationary distribution of our benchmark parameterization, the average point elasticity is 0.92. How realistic is that?

It is well-known that all attempts to estimate the elasticity using aggregate data

¹²See Section 5.3.

have given disappointing outcomes. Typically, the estimates are not significantly different from zero, and the variation in aggregate q accounts for a negligible fraction of the variation in investment rates.¹³ In the last ten years or so, several authors have studied the relation between investment and Tobin’s q using firm-level data. In particular, we refer to the work of Eberly (1997) with the Global Vantage dataset, and to those of Barnett and Sakellaris (1998) and Abel and Eberly (2002) with Compustat data. One of the main lessons learned from these studies is that the relationship between investment and q is highly non-linear. A corollary is that information on the cross-sectional distribution of q can be used to improve the predictive power of investment equations. For example, when they estimate aggregate elasticity by computing the increase in total investment implied by a 1% increase in q for all firms, Barnett and Sakellaris (1998) obtain a point estimate of 0.84. When controlling for higher moments of the distribution of q , Eberly (1997) finds that regressing $\log(\sum_i I_i/K_i)$ on $\sum_i q_i$ generates a point estimate of 0.62, with a R^2 of 0.08. Regressing $\sum_i \omega_i \log(I_i/K_i)$ over $\sum_i \omega_i q_i$ (with $\omega_i = K_i/\sum_i K_i$) yields a coefficient estimate of 0.72, with R^2 of 0.38. Interestingly, Abel and Eberly (2002) also estimate that adjustment costs amount to 1.1% of the cost of investment in manufacturing, and to 9.7% in non-manufacturing sectors. In the simulations of our model’s ergodic distribution, the mean value is about 1.45%.

While the findings we have just summarized cannot be used for direct falsification of our model, they suggest that its implications for the magnitude of adjustment costs and for the elasticity of investment to Tobin’s q are hardly out of line.

5.4 The Role of Discounting and Persistence

Consider raising the effective discount factor β^* to 0.9948, which implies a capital-output ratio of 3.3 on the balanced-growth path of the deterministic version of our model. Figure 4 shows that such change enhances the model’s ability to match asset pricing data.

As an example, Table 4 shows the statistics that obtain for $\theta = 0.25$, $\gamma = -21$ (IES=0.0455), and $\iota = 1.26$. The model now generates lower volatilities of both returns, while matching the post-WWII expected risk-free rate and producing sizeable

¹³We refer to studies, such as Von Furstenburg (1977), where the investment to capital ratio is gross aggregate investment divided by an estimate of the aggregate capital stock, and q is the ratio of market capitalization to the capital stock.

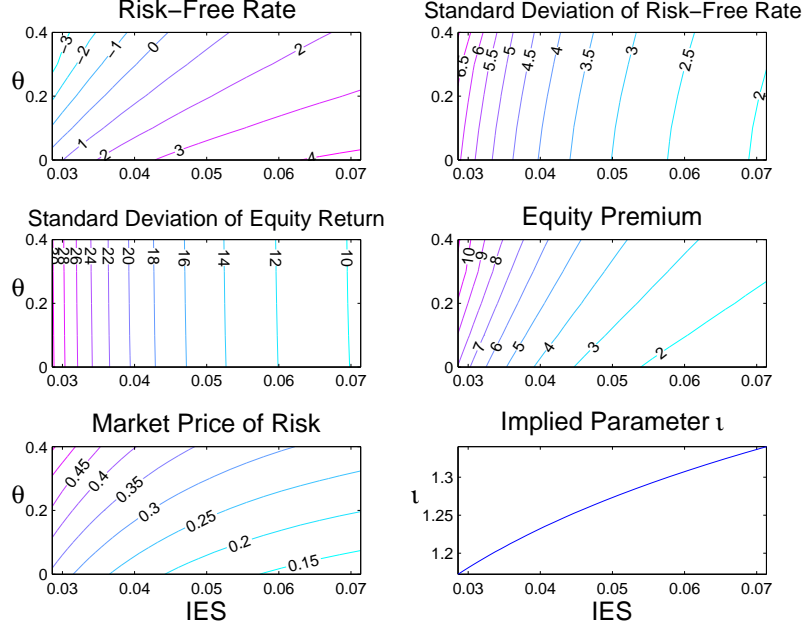


Figure 4: Comparative Statics with $\beta^* = 0.9948$.

expected equity premium and market price of risk. The price of these achievements is that the mean capital-output ratio rises to about 3.6, higher than the values considered acceptable in the business cycle literature.

$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e)$	MPR
1.159%	3.55%	5.439%	16.723%	0.301

Table 4: Unconditional Moments at Annual Frequency – Model with $\beta^* = 0.9948$.

An increase in β^* fosters the desire to smooth the effects of productivity shocks on consumption. In turn, this implies a drop in the volatility of consumption growth. To keep the latter constant, ι must decrease (the marginal capital adjustment cost must increase) to make investment less appealing at the margin. This is why the volatility of equity returns increases.

Let's now turn to the market price of risk. Notice that $\beta^{\lambda(\gamma-1)} = \beta^* e^{-(\lambda+\varphi)}$. This means that, given the allocation, for each γ a rise in β^* leads to a higher volatility of the stochastic discount factor and a lower unconditional expected risk-free rate.

For given allocation, a rise in β^* also implies a less volatile risk-free rate. However, as already argued, the lower covariance (in absolute value) between short- and long-run risk factors means that state prices drop in bad states (when the risk-free rate is

above average) and rise in good states (when the risk-free rate is below average). In turn, this leads to a more volatile risk-free rate. In the case of our parameterization, the latter effect prevails.

Finally, Figure 5 illustrates how our comparative statics exercise changes when $\rho = 0.96$. A rise in the autocorrelation coefficient implies that the absolute value of the covariance between short- and long-term risk factors drops. In turn, this means that, given the range of values assigned to γ , the volatility of the stochastic discount factor and the precautionary motive decline.

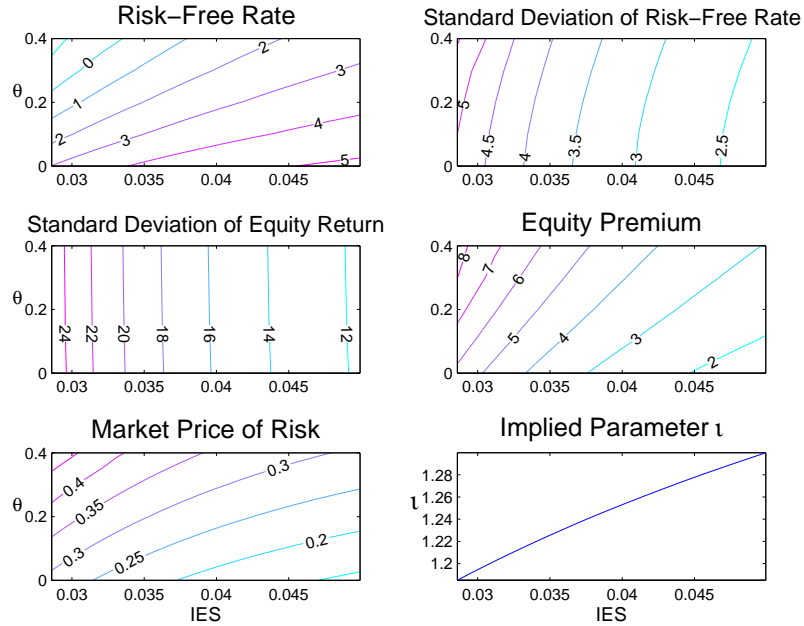


Figure 5: Comparative Statics with $\rho = 0.96$.

The discussion of the case in which productivity shocks are permanent ($\rho = 1$) is postponed to Section 7.

6 Epstein–Zin Preferences

Here we are interested in exploring the potential of Epstein–Zin preferences. We set $\theta = 0$ and assess the comparative statics with respect to γ and η . Our results are illustrated in Figure 6. The stochastic discount factor is now

$$m(\varepsilon_i|k, \varepsilon) = \beta e^{\lambda(\gamma-1)} \left[\frac{c(k', \varepsilon_i)}{c(k, \varepsilon)} \right]^{\eta-1} \left[\frac{v(k', \varepsilon_i)/\mu(k', \varepsilon)}{c(k', \varepsilon_i)/c(k, \varepsilon)} \right]^{\eta-\gamma},$$

where

$$\mu(k', \varepsilon) = \left[\sum_i \pi(\varepsilon_i | \varepsilon) v^\eta(k', \varepsilon_i) \right]^{1/\eta}.$$

As it was the case in the exercises presented in Section 5, we adjust β and ι in such a way that $\beta^* = 0.9919$ and consumption growth is half as volatile as output growth. The main conclusion is that, if one is willing to accept a relatively high RRA coefficient, the model with Epstein–Zin preferences can generate results that are very similar to those of the DA model.

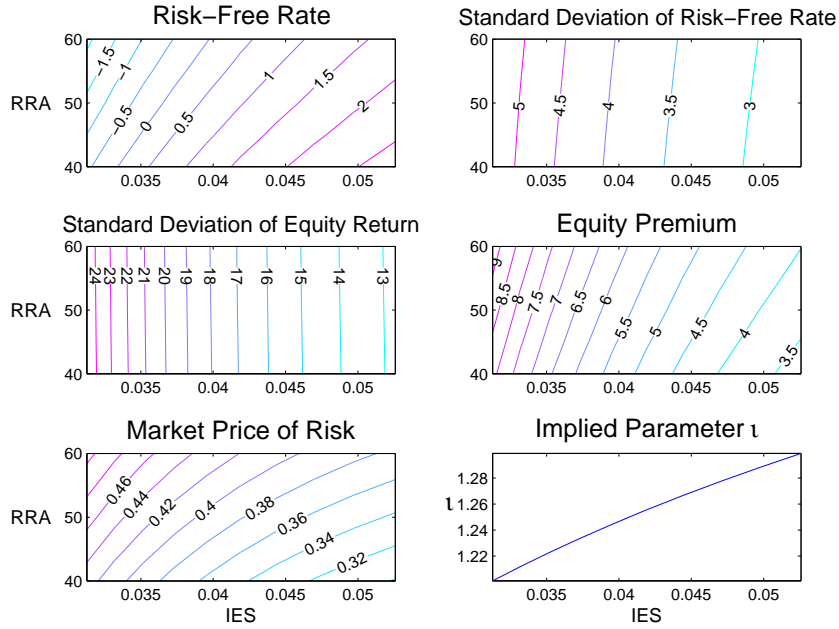


Figure 6: Asset Pricing with Epstein–Zin Preferences ($\theta = 0$).

For example, Table 5 shows that for $RRA = 55$ and $IES = 0.0455$, the EZ model matches the post–WWII averages of risk-free rate and equity return volatility while generating sizeable market price of risk and expected equity premium. While higher than its empirical counterpart, the volatility of the risk-free rate is close to the value generated by the DA model.

In all the allocations considered here, $\eta < \gamma$. This means that, differently from Section 5, the introduction of long-run risk decreases the volatility of the stochastic discount factor. The qualitative impact of varying η is essentially the same as that induced by changes in θ in the DA model. For the sake of brevity, we do not comment further on it.

$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e)$	MPR
1.146%	3.352%	4.848%	15.323%	0.384

Table 5: Unconditional Moments at Annual Frequency ($\theta = 0$, $\eta = -54$, $\gamma = -21$).

6.1 Evaluating the Implied Risk Aversion

Starting with [Mehra and Prescott \(1985\)](#), the debate on what is a reasonable range for relative risk aversion has been central to the asset pricing literature. Much of this debate was flawed by the failure to acknowledge two facts. First, the attitude towards risk in dynamic models does not depend only on the parameters governing aversion to risk in atemporal bets. In fact, in general the price of risk will still be positive for $\eta = 1$ (i.e. in the case of risk neutrality with respect to atemporal bets). Second, and most important, relative risk-aversion is a local concept, i.e. it depends on the amount of risk an individual is faced with. Regardless of her risk preferences, an agent's relative risk aversion is defined as the fraction of her wealth she would pay in order to avoid a multiplicative atemporal bet. The RRA is nothing else but the measure that obtains when the variance of the payoffs' distribution goes to zero, in the case of expected utility.¹⁴

Given state variables (k, ε) , and regardless of the shape of her risk preferences, our agent faces a lottery over continuation utilities $\{v[k'(k, \varepsilon), \varepsilon_i], \pi(\varepsilon_i|\varepsilon)\}_{i=1}^n$, where n is the number of values attainable by the productivity shock. By definition, relative risk-aversion is the value $P(k, \varepsilon)$ such that

$$[1 - P(k, \varepsilon)] \sum_i \pi(\varepsilon_i|\varepsilon) v[k'(k, \varepsilon), \varepsilon_i] = \mu[v(k', \varepsilon')].$$

That is, relative risk-aversion is measured by the discount over the fair value of the lottery that the agent would accept in order to avoid the risk implied by the lottery itself.

For the DA and EZ models, Table 6 reports (i) the coefficient of variation of the equilibrium bet over continuation utilities (i.e. the standard deviation as percentage of the mean), and (ii) the value of $P(k, \varepsilon)$. The figures refer to the benchmark parameterizations described in Sections 5 and 6, respectively. The levels of detrended per-capita capital are set equal to the minimum, average, and maximum levels in the ergodic distribution, respectively. The realization of the shock (ε) is the third highest (out of six).

¹⁴For an illustration, see the simple calculations shown in Appendix D.

One may argue that the informativeness of these measures is limited by the fact that they are expressed as fractions of utils. Notice however that, for any (k, ε) , $v(k, \varepsilon)$ can be implemented by a constant sequence of detrended consumption values $c = v(k, \varepsilon)[1 - \beta e^{\varphi + \lambda \gamma}]^{1/\gamma}$. This means that the lottery over continuation utilities v_i is equivalent to a lottery over constant detrended consumption levels $v_i[1 - \beta e^{\varphi + \lambda \gamma}]^{1/\gamma}$. In turn, this implies that the coefficient of variation of the lottery over continuation utilities is also the coefficient of variation of the bet over consumption sequences, and $P(k, v)$ is also the fraction of the mean of contingent consumption levels that the agent would give up in order to avoid the lottery.

	DA		EZ	
k	C.V.	R.A.	C.V.	R.A.
min.	0.2123	0.0096	0.2112	0.0131
mean	0.1354	0.0062	0.1519	0.0067
max.	0.0976	0.0042	0.1168	0.0040

Table 6: Risk and Risk Aversion – In Percentage Points.

It appears that the risk generated by the stochastic growth model is smaller than the risks implied by many decisions in people’s lives. A further observation, perhaps not surprising, is that the two models imply very similar levels of risk-aversion. The crucial question is whether such level is reasonable.

At the mean capital stock, a DA agent who consumes for 25,000 dollars (units of consumption good) per quarter faces a lottery over future constant consumption sequences whose standard deviation is about 33.85 dollars. In order to avoid this risk, the agent is willing to give up 1.25 dollars. Is it too much? Too little? Based on personal introspection, [Cochrane \(1997\)](#) does not find it unreasonable for a household earning \$50,000 per year to be willing to pay 25 cents in order to avoid a bet involving the gain or loss of \$10 with even probability. This leads us to conjecture that, most likely, he would not find the behavior of our agent to be unreasonable either.

In the Introduction we have summarized the conclusions of the experimental literature so far. Estimations of the expected utility model consistently find evidence of a relative risk aversion coefficient around $1/2$, a lot lower than required by the EZ model to generate sensible asset return statistics. The same literature provides evidence in support of disappointment aversion parameters in the range $[0, 0.5]$. It follows that, for parameterizations delivering similar levels of risk aversion in the stochastic growth

model, the two risk preferences imply very different attitudes towards risk in experiments. This is consistent with the observation that most risk-aversion estimates are obtained confronting subjects with gambles, either real or hypothetical, whose risk is much larger (most often by orders of magnitude) than the risk implied by the stochastic growth model.

As pointed out by many, among which [Epstein and Zin \(1990\)](#), [Kandel and Stambaugh \(1991\)](#), and [Rabin \(2000\)](#), relative risk aversion grows faster with risk under expected utility than under disappointment aversion. Since under expected utility a bet's standard deviation has only a second-order effect on relative risk-aversion, individuals are essentially risk-neutral when faced with low risk, for a very large range of RRA coefficients. The implication is that RRA coefficients that generate realistic levels of risk aversion for small risk, also generate implausibly large risk aversion with respect to larger risks. With reference to our model, this means that while the relative risk aversion of an EZ agent with $\eta = -55$ is plausible at the risk levels implied by our model's equilibrium, the same agent will display unreasonable risk aversion when facing considerably greater risks.

Consider a simple atemporal bet that pays $1 - \kappa$ or $1 + \kappa$ with equal probability. For a few values of $\kappa \in (0, 1)$ (which also equals the bet's coefficient of variation), [Figure 7](#) plots the loci of pairs $(\theta, 1 - \eta)$ that satisfy

$$\left[\frac{1 + \theta}{2 + \theta} \sqrt{1 - \kappa} + \frac{1}{2 + \theta} \sqrt{1 + \kappa} \right]^2 = \left[\frac{1}{2} (1 - \kappa)^\eta + \frac{1}{2} (1 + \kappa)^\eta \right]^{1/\eta}.$$

The left-hand side is the certainty equivalent of the bet for a DA agent with disappointment aversion parameter θ , endowed with a Bernoulli utility function with a 0.5 RRA coefficient. The right-hand side is the certainty equivalent for an EU agent with relative risk-aversion coefficient $1 - \eta$. Notice that the slope of the loci is decreasing in κ . The larger the risk, the higher the value of θ that (under DA) produces the same relative risk aversion generated by a given η (under EU).

The second column of [Table 6](#) indicates that at the mean of the distribution of capital, our calibration of the DA model generates a coefficient of variation of about 0.15%. In the case of the simple atemporal lotteries considered here, a disappointment averse agent with $\theta = 0.4$, when faced with that risk, displays the same attitude towards risk as an EU agent with a RRA coefficient of about 230. As the risk increases, the RRA coefficient of the *equivalent* EU agent drops. For a coefficient of variation of 20%, such agent is close to being risk-neutral.

We conclude that the problem with Epstein-Zin preferences in asset pricing models

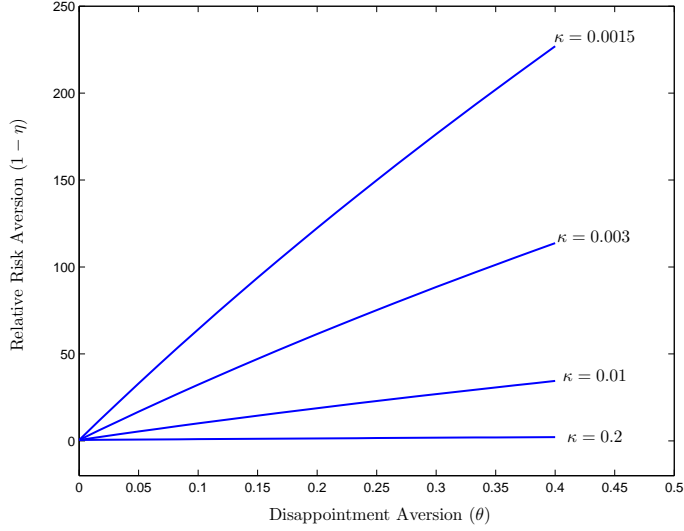


Figure 7: Comparison between Risk Preferences.

does not lay in the excessive risk-aversion they supposedly generate. When measured at the equilibrium bets implied by the stochastic growth model, agents' relative risk aversion seems plausible. Furthermore, a model with disappointment averse agents generates the same level of relative risk aversion with parameters in line with the experimental evidence. Our conclusion, consistent with those reached by [Epstein and Zin \(1990, 2001\)](#) and [Bonomo and Garcia \(1994\)](#) for endowment economies, is that for preference relations displaying second-order risk aversion, the challenge comes from the universality requirement. Relative risk aversion grows too fast with risk, to be consistent with both asset returns and experimental evidence.

7 Permanent Productivity Shocks

In this section we consider the case in which $\rho = 1$, or $\log(z_{t+1}/z_t) = \lambda + \zeta_t$, with $\zeta_t \sim N(0, \sigma^2)$.¹⁵ As is well known,¹⁶ the quantity dynamics implied by this version of the model are quite different from those induced by mean-reverting shocks. [Figure 8](#) shows the response of the model to a positive, permanent productivity shock. The figure displays the average values that obtain from 500,000 150-period long simulations. As in [Section 4](#), we picked the variance of the innovation (σ^2) to match the volatility of output growth. The other parameters are $\beta^* = 0.99468$, $\theta = 0.25$, $\eta = -9$, $\gamma = 0.5$, $\iota = 1.85$. The rationale behind these choices will be clear shortly.

¹⁵The planner's problem is shown in [Appendix C](#).

¹⁶See [Rouwenhorst \(1995\)](#) and [King and Rebelo \(1999\)](#)

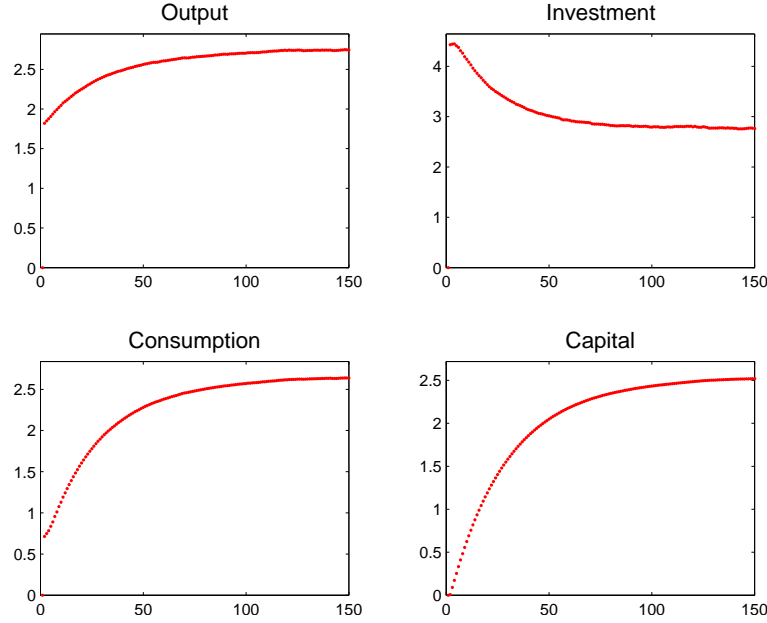


Figure 8: Impulse Responses – Percentage Deviations from the Initial Trend.

Permanently higher productivity means that output will also be permanently higher. The household wishes to take advantage of this by increasing consumption. At the same time, given that the marginal product of capital is permanently higher, she also wants to build up the capital stock. Eventually, all quantities will converge to the new balanced-growth path. However, neither consumption nor capital or output will adjust instantaneously. The length of the adjustment process will depend on parameter values, and in particular on the intertemporal elasticity of substitution.

Everything else equal, the initial response of consumption will be higher, the lower the IES. This also hints that a decrease in the elasticity increases the volatility of consumption growth. This is exactly the opposite of what happens in the case of transitory shocks. To keep the volatility of consumption growth constant as we lower the IES, the marginal capital adjustment cost must decrease (ι must increase). In turn, this will lower the volatility of capital gains (and equity returns). It follows that, when looking to increase the volatility of equity returns, one needs to explore scenarios with relatively high elasticity of substitution. This is why, to give the model the best chance, we assumed the IES to be 2. As elsewhere in this paper, ι was chosen so that the volatility of consumption growth is 0.5.

In spite of these choices, Table 7 reveals that the volatility of equity returns is roughly two order of magnitudes lower than in the data. Furthermore, in spite of the fact that β^* was set at the highest level among those that do not generate grossly counterfactual capital–output ratios, and the RRA coefficient was assigned a value higher than most studies find acceptable, the unconditional risk–free rate is roughly four times its post–WWII empirical counterpart.

$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e)$	MPR
4.542%	0.210%	0.13%	0.148%	0.471

Table 7: Unconditional Moments at Annual Frequency – Model with Unit Root in TFP.

In the case of Epstein–Zin preferences, Croce (2008) and Kaltenbrunner and Lochstoer (2006) find that when productivity growth is i.i.d., long–run risk adds to the volatility of the stochastic discount factor for $\gamma > \eta$, and the more so the larger is γ . As expected, the same holds in our framework.

When turning to the effects of increasing disappointment aversion, we find that the signs of our comparative statics are the same as in the scenario with mean–reverting shocks. Disappointment aversion has little or no impact on the volatility of either equity or risk–free returns. However, as hinted by the right panel of Figure 9, it has a positive and sizeable impact on the price of risk.

For $\eta = 0.5$ and $\theta = 0$, our setup collapses to the case of a discounted utility maximizer with an elasticity of substitution of 2 (RRA coefficient of 0.5). As is well known from Rouwenhorst (1995), this model produces a very low price of risk. Allowing for a level of disappointment aversion consistent with experimental evidence is enough for the model to generate a price of risk which is quite close to what required by the Hansen–Jagannathan bounds. The left panel suggests that increasing θ also lowers the expected risk–free rate, but by far less than needed to match the data.

8 Predictability of Asset Returns

Every student of finance soon learns that stock prices, dividends, and returns are linked by a simple identity. For any stock i , $R_{t+1}^i \equiv (P_{t+1}^i + D_{t+1}^i)/P_t^i$. It follows that if the price of an asset is high today, agents must be expecting that either next period’s price will be high, or the dividends will be high, or the rate of return will be low. Or a combination of these events. In turn, this means that, if stock prices are

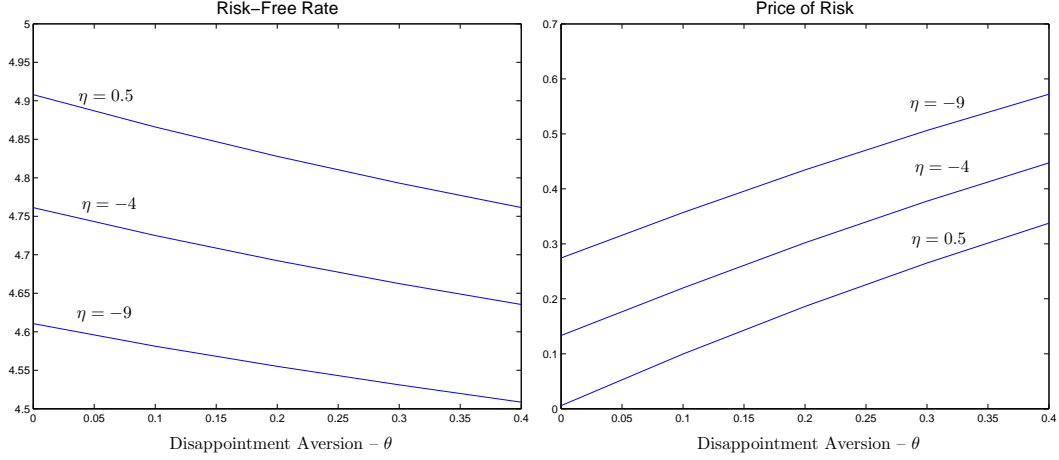


Figure 9: Comparative Statics.

not explosive, a high price today must be associated with either low returns or high dividends in the future, or both. [Campbell and Shiller \(1988\)](#) formalized this simple argument by log-linearizing the above identity and iterating forward, to obtain

$$\log(P_t^i/D_t^i) = \frac{b}{1-\nu} + E_t \sum_{s=0}^{\infty} \nu^s [\log(D_{t+1+s}^i/D_t^i) - \log(r_{t+1+s}^i)],$$

where b and $\nu < 1$ are linearization constants. Several authors, among which [Campbell and Shiller \(1988\)](#) and [Fama and French \(1988a,b\)](#), showed that real stock returns, in particular at long horizons, are forecasted by the price-dividend ratio, while dividend growth is not. The leftmost among the columns labeled “Data” in [Table 8](#) documents the results of regressing cumulative stock returns on the current price-dividend ratio. As expected, the regression coefficients associated with the cumulative returns are negative. Furthermore, their absolute values and the R^2 statistics are increasing with the horizon. The figures reported in the column labeled “Model” are computed by running the same regressions on simulated data. The pattern is very similar to the one just discussed. Stock returns are forecastable, and increasingly so as the horizon increases.

The relation between current price-dividend ratio and future stock returns may be due to the predictability of the risk-free rate, or of the equity premium, or both. The evidence, summarized in the rightmost column of [Table 8](#), suggests that it is equity premia that are predictable, and increasingly so as the time horizon widens. This confirms the finding of many others before us, among which [Campbell \(1999\)](#). Unfortunately, the model’s population coefficients and R^2 reported in [Table 8](#) show that our setup is not consistent with this feature of the data. The expected equity

Table 8: Long-Horizon Regressions of Equity Returns and Equity Premia.

Dependent Variables	$r_{t,t+s}^e$				$r_{t,t+s}^e - r_{t,t+s}^f$			
	<i>Model</i>		<i>Data</i>		<i>Model</i>		<i>Data</i>	
Horizon (s)	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2
1	-0.075	0.06	-0.115 (-2.18)	0.08	-0.0067	0.00	-0.117 (-2.27)	0.08
2	-0.157	0.10	-0.22 (-3.04)	0.14	-0.0138	0.00	-0.219 (-3.13)	0.15
3	-0.235	0.13	-0.284 (-3.46)	0.18	-0.0215	0.00	-0.28 (-3.60)	0.19
5	-0.391	0.17	-0.509 (-4.65)	0.29	-0.0367	0.00	-0.484 (-4.78)	0.31
7	-0.547	0.20	-0.778 (-5.35)	0.36	-0.0513	0.00	-0.712 (-5.29)	0.36

Note: t statistics in parenthesis. Specification: $x_{t,t+s} = a + b \log(P/D)_t + \epsilon_t$

Data sources: See Appendix A.

premium is essentially acyclical, and the predictability of stock returns is entirely due to time-variation in the risk-free rate. These findings are not surprising. First, the model generates a much higher volatility of the risk-free rate than in the data. Second, and most important, both the price of risk and the volatility of stock returns are acyclical by construction. This is evidenced in the first two columns of Table 9, which list both statistics by state of nature (1 denoting the lowest level of the detrended productivity shock and 6 the highest).

The price of risk is acyclical under disappointment aversion because so is the probability mass over the disappointing states. Recently, [Routledge and Zin \(2004\)](#) were able to overcome this issue in a Mehra-Prescott economy by replacing [Gul \(1991\)](#)'s weak independence axiom with a less demanding version of it, which they labeled δ -weak independence. In the remainder of this section, we will investigate whether their result extends to our production economy.¹⁷

In [Routledge and Zin \(2004\)](#)'s utility representation, which goes under the name of Generalized Disappointment Aversion (GDA), the disappointment threshold is a linear function of the certainty equivalent. More formally, the certainty equivalent

¹⁷[Kuehn \(2007\)](#) proposes an alternative way of generating predictability in a production economy. His main assumption is that building new capital is a multi-period endeavour and firms must commit to it. Even conditional on negative productivity shocks, investment projects already started must be completed on schedule. This limits consumers' ability to smooth negative shocks, but not positive shocks.

$\mu(S_t)$ satisfies¹⁸

$$\mu^\eta(S_t) = \sum_{S_{t+1}} \pi(S_{t+1}|S_t) v^\eta(S_{t+1}) - \theta \sum_{S_{t+1} \in \Delta_{t+1}} \pi(S_{t+1}|S_t) \{[\xi \mu(S_t)]^\eta - v^\eta(S_{t+1})\},$$

$$\Delta_{t+1} = \{S_{t+1} : v(S_{t+1}) < \xi \mu(S_t)\}, \quad \eta \leq 1, \theta \geq 0, \xi \geq 0.$$

The stochastic discount factor writes as

$$m(\varepsilon_i|k, z) = \beta e^{\lambda(\gamma-1)} \frac{1 + \theta I(k', \varepsilon_i, \mu)}{1 + \theta \xi^\eta \sum_i \pi(\varepsilon_i|\varepsilon) I(k', \varepsilon_i, \mu)} \left[\frac{c(k', \varepsilon_i)}{c(k, \varepsilon)} \right]^{\eta-1} \left[\frac{v(k', \varepsilon_i)/\mu(k', \varepsilon)}{c(k', \varepsilon_i)/c(k, \varepsilon)} \right]^{\eta-\gamma},$$

where $I(k', \varepsilon_i, \mu) = 1$ if $v(k', \varepsilon_i) \leq \xi \mu$, and $I(k', \varepsilon_i, \mu) = 0$ otherwise.

The fourth column of Table 9 reports the conditional price of risk for a parameterization (labeled GDA (1)) that coincides with our benchmark, with the exception of $\theta = 2$ and $\xi = 1.005$. Increasing ξ beyond 1 has the effect of increasing the probability mass over disappointing states, conditional on high current states. This lowers the price of risk in those states. However, it also lowers the average price of risk and increases the expected unconditional risk-free rate. Raising θ (perhaps beyond sensible levels) takes care of this problem.

	DA		GDA (1)		GDA (2)	
Shock	MPR	$Std(R^e - R^f)$	MPR	$Std(R^e - R^f)$	MPR	$Std(R^e - R^f)$
1	.209	8.62%	.503	8.72%	.493	14.56%
2	.236	17.40%	.464	17.67%	.413	31.05%
3	.306	27.59%	.410	26.99%	.363	20.76%
4	.471	18.10%	.354	17.88%	.197	9.47%
5	.260	6.60%	.157	6.95%	.104	4.48%
6	.220	3.84%	.068	2.97%	.057	2.71%

Table 9: Conditional Volatilities of Market Price of Risk and Equity Premium.

Running predictability regressions with simulated data produced by this parameterization shows some success. Still, the extent of the predictability of the equity premium falls short of that implied by the data. The results of regressing future cumulative equity premia on current price-dividend ratios, which we report in Table 10, make it clear. The fifth column of Table 9 hints that, at least in part, this negative result may be due to the acyclicity of equity premium's volatility. As expected, changes in risk preference parameters have no sizeable effect on the latter. This is why the figures in columns 3 and 5 of Table 9 are essentially the same.

¹⁸In the case of $\xi > 1$, one needs to multiply the right-hand side by the constant $[1 - \theta(\xi^\eta - 1)]^{-1}$ and impose $\theta(\xi^\eta - 1) < 1$ in order to preserve monotonicity and $\mu(x) = x$ for x constant. See Section 2.3 of [Routledge and Zin \(2004\)](#).

Notice that the volatility of equity returns is mostly driven by the returns that realize when the system changes productivity state. When productivity is relatively low, capital is declining. The stock return implied by an increase in productivity will be higher, the lower the capital stock. Conversely, when productivity is relatively high, capital is increasing. The stock return implied by a decrease in productivity will be lower, the higher the capital stock. It follows that a simple way to make equity premia volatility countercyclical is to modify the shock's probability distribution in order to induce a stationary distribution of capital skewed to the right (more probability mass to the right of the mode). The result of doing so is illustrated in the last two columns of Table 9.¹⁹ With the exception of the lowest level of productivity, for which the volatility of stock returns is limited by the impossibility of a further drop, volatility is countercyclical. As documented in columns 6 and 7 of Table 10, this leads to an increase in the R^2 for the predictability regression. However, such statistics still fall short of what found in the data.

	Data		GDA (1)		GDA (2)		GDA (2)*	
Horizon (s)	<i>Slope</i>	R^2	<i>Slope</i>	R^2	<i>Slope</i>	R^2	<i>Slope</i>	R^2
1	-.117	.08	-.0396	.01	-.0700	.01	-0.1191	.042
2	-.219	.15	-.0792	.02	-.1378	.02	-0.2123	.074
3	-.280	.19	-.1190	.03	-.2051	.03	-0.2951	.104
5	-.484	.31	-.1976	.05	-.3391	.06	-0.4222	.151
7	-.712	.36	-.2760	.07	-.4742	.08	-0.5090	.180

Note: Specification: $(r_{t,t+s}^e - r_{t,t+s}^f) = a + b \log(P/D)_t + \epsilon_t$.

Table 10: Predictability of the Equity Premium: GDA.

Predictability regressions suffer from a severe small-sample problem, which may bias both the coefficients and the R^2 . This suggests that evaluating the model using its population values may be unfair to it. Therefore we have also run separate regressions on 200 236-period long simulations. The last two columns of Table 10 (those labeled GDA (2)*) report the means across all regressions for the two statistics.²⁰ Indeed, both the regression coefficient and the R^2 coefficient are considerably higher than their population counterparts.

¹⁹The parameterization labeled GDA (2) has $\theta = 2.2$ and $\xi = 1.00315$. The most relevant unconditional statistics are $E(r^f) = 1.164\%$, $\sigma(r^f) = 5.182\%$, $E(r^e - r^f) = 6.838\%$, $\sigma(r^e - r^f) = 22.49\%$, and $\text{MPR} = 0.373$.

²⁰We thank an anonymous referee for pointing out the issue to us and suggesting this further test.

9 Conclusion

This paper shows that when agents display levels of disappointment aversion consistent with the experimental evidence, an otherwise standard general equilibrium production economy with mean-reverting productivity shocks is able to produce sensible implications for the price of risk, the unconditional expected risk-free rate, and first and second unconditional moments of the equity return. Capital adjustment costs and low elasticity of substitution generate a sizeable volatility of equity return while matching the relative volatility of consumption and output growth. Disappointment aversion is responsible for generating a price of risk and unconditional risk-free rate consistent with the historical data.

Without disappointment aversion (the Epstein–Zin case), the model generates similar implications for a RRA coefficient of about 55, a figure two orders of magnitude larger than commonly found by the experimental literature and about one order of magnitude higher than the highest values assumed in most economics literature. Given the relatively small risk imposed on agents by the stochastic growth model, it is hardly surprising that risk preferences displaying second-order risk aversion require a substantial curvature in order to generate seemingly reasonable levels of risk aversion. The issue is that when agents face risk orders of magnitude higher, as those implied by many decisions in people’s lives, that curvature implies pathologically high levels of relative risk aversion.

Essentially by construction, our model features acyclical price of risk and acyclical volatility of equity return. This directly implies that the equity premium is not predictable. We show that under Generalized Disappointment Aversion, there exist parameterizations that yield a countercyclical price of risk. When we also allow for counter-cyclical volatility, the model generates predictability to extents similar to those suggested by the data.

The main shortcoming of the model appears to be an exceedingly high risk-free volatility. We show that allowing for permanent productivity shocks takes care of the problem, but at the price of essentially no volatility of equity returns. In turn, this leaves no hope of generating a sizeable equity premium.

We conclude that understanding how to amend the model with mean-reverting shocks in order to reduce the volatility of the risk-free rate should be at the top of the research agenda. In the present framework, the excessive volatility is the result of the perfect rigidity of bond supply and the high rigidity of bond demand, implied by the

low intertemporal elasticity of substitution and the fast-rising investment adjustment costs. Allowing for a bond supply schedule with non-zero elasticity seems to us the most promising direction. This could be accomplished by assuming some form of heterogeneity across households, or by introducing a government entity that finances a deficit by issuing securities to the public.

A Data

Our business cycle data is drawn from FRED (Federal Reserve Economic Data) at <http://research.stlouisfed.org/fred2/>. Output is variable GDPC96 (Real Gross Domestic Output). Consumption is the sum of variables PCNDGC96 and PCESVC96 (Real Personal Consumption Expenditures on Nondurable Goods and on Services, respectively). Investment is variable FPIC96 (Real Fixed Private Investment). All series are quarterly from 1947:1 to 2005:4, expressed in billions of chained 2000 dollars, and seasonally adjusted.

Our data on asset returns is drawn from CRSP (Center for Research in Security Prices at the University of Chicago Booth School of Business). Nominal Equity Returns correspond to the NYSE variable VWRETD (Value-Weighted Return on the NYSE Index, including Dividends). The nominal risk-free returns correspond to the yields to maturity on 90-day T-Bills (based upon the average between bid and ask prices, and drawn from the Fama T-Bill Term Structure Supplemental Files). All series are quarterly from 1947:1 to 2005:4. (Gross) real returns were computed as gross nominal returns divided by the gross inflation rate. The inflation rate is based upon the CPI from FRED, variable CPIAUCNS (CPI for all urban consumers, all items). This variable is monthly and not seasonally adjusted – quarterly observations correspond to the value of the index in the last month of each quarter.

We computed price-dividend ratios as follows. The variable VWRETD provides $(P_t + D_t)/P_{t-1} - 1$, where P_t is the value-weighted index and D_t are dividends at time t . From the CRSP dataset we also obtained the NYSE variable VWINDX, which provides P_t (the value of the index relative to a base year). Price-Dividend ratios are thus the inverse of $(1 + \text{VWRETD}_t) \times \text{VWINDX}_{t-1} / \text{VWINDX}_t - 1$.

We ran the return predictability regressions on an annual basis. We computed annual dividends by summing up quarterly dividends. Annual price-dividend ratios are the NYSE value-weighted index for the last quarter divided by annual dividends. The real returns between years t and $t + k$ were computed by summing up all real quarterly returns between the two dates.

Table 11 compares our estimates for the first two moments of risk-free rate and equity premium with those reported by other papers in the literature on asset pricing in general equilibrium production economies. As mentioned in Section 5.2, our value for the equity premium is higher than most others, because our sample includes the 90's.

	$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e - r^f)$	$Std(r^e)$
This paper	1.01	1.67	7.57	15.16	15.39
Tallarini (2000)	0.92	1.60	7.88	15.64	—
Jermann (1998)	0.80	5.67	6.18	—	16.54
Boldrin, Christiano, and Fisher (2001)	1.19	5.27	6.63	—	19.40
Danthine and Donaldson (2002)	0.80	5.67	6.18	16.67	16.54
Guvenen (2008) – long	1.91	5.44	6.17	19.40	—
Guvenen (2008) – short	1.33	2.70	7.21	17.00	—

Table 11: Moments Estimates (Annualized).

B Decentralization

Here we describe our decentralization of the planner's problem illustrated in the main body of the paper. For the sake of clarity, we will consider the case in which agents maximize expected discounted utility ($\theta = 0$ and $\gamma = \eta$). The generalization to the cases of $\gamma \neq \eta$ and $\theta > 0$ is straightforward and is omitted for the sake of brevity.

Our decentralization is essentially the same as in Cochrane (1991), Jermann (1998), and Boldrin, Christiano, and Fisher (2001). Because of our assumptions, there is no loss of generality in assuming that production is carried out by one representative firm, owned by the households. The firm maximizes the expected discounted sum of dividends, does not issue new shares, and finances investment exclusively through retained earnings.

At every t , and for given capital K_t , the firm's value P_t is

$$\begin{aligned}
P_t &\equiv \max_{\{K_{t+v}\}_{v=1}^{\infty}} E_t \sum_{v=0}^{\infty} \frac{\lambda_{t+v}}{\lambda_t} D_{t+v}, \\
\text{s.t. } D_{t+v} &= K_{t+v}^{\alpha} (z_{t+v} L_{t+v})^{1-\alpha} - w_{t+v} L_{t+v} - X_{t+v}, \\
X_{t+v} &= K_{t+v+1} - (1 - \delta) K_{t+v} + g(K_{t+v}, K_{t+v+1}), \\
L_{t+v} &= e^{\varphi v} L, \quad L \text{ given.}
\end{aligned}$$

where $g(K_{t+v}, K_{t+v+1}) \equiv \left| \frac{K_{t+v+1}}{K_{t+v}} - \psi \right|^{\ell}$ and λ_{t+v}/λ_t is the owners' marginal rate of substitution between consumption at the dates t and $t + v$. Denote $g_i(\cdot, \cdot)$ as the partial derivatives with respect to argument i . Necessary condition for the optimum is that, at every t :

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\alpha K_{t+1}^{\alpha-1} (z_{t+1} L_{t+1})^{1-\alpha} + (1 - \delta) - g_1(K_{t+1}, K_{t+2})}{1 + g_2(K_t, K_{t+1})} \right] = 1.$$

Multiplying numerator and denominator by K_{t+1} , one obtains

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\alpha Y_{t+1} + (1 - \delta)K_{t+1} - g_1(K_{t+1}, K_{t+2})K_{t+1}}{[1 + g_2(K_t, K_{t+1})]K_{t+1}} \right] = 1.$$

Now notice that

$$g_1(K_{t+1}, K_{t+2})K_{t+1} = g(K_{t+1}, K_{t+2}) - g_2(K_{t+1}, K_{t+2})K_{t+2}$$

and

$$\alpha Y_{t+1} + (1 - \delta)K_{t+1} = D_{t+1} + K_{t+2} + g(K_{t+1}, K_{t+2}).$$

The two imply

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{D_{t+1} + [1 + g_2(K_{t+1}, K_{t+2})]K_{t+2}}{[1 + g_2(K_t, K_{t+1})]K_{t+1}} \right] = 1. \quad (4)$$

For given shareholding a_t , the household's optimization problem at time t writes as

$$\begin{aligned} \max_{\{a_{t+v}\}_{v=1}^{\infty}} E_t \sum_{v=0}^{\infty} [\beta e^{\varphi}]^v c_{t+v}^{\gamma}, \\ \text{s.t. } w_{t+v} l_{t+v} + a_{t+v}(P_{t+v} + D_{t+v}) = c_{t+v} + a_{t+v+1} P_{t+v}, \\ l_{t+v} = e^{\varphi v} l, \quad l \text{ given.} \end{aligned}$$

where c_{t+v} is per-capita consumption and l_{t+v} is the inelastic supply of labor. Necessary condition for the optimum is that, at every t :

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_{t+1} + D_{t+1}}{P_t} \right] = 1, \quad (5)$$

where $\lambda_{t+v} \equiv [\beta e^{\varphi}]^v \gamma c_{t+v}^{\gamma-1}$. Finally, (4) and (5) imply that the conditional gross return to the firm's owners from time t to time $t + 1$ is

$$R_{t,t+1}^e \equiv \frac{D_{t+1} + P_{t+1}}{P_t} = \frac{D_{t+1} + [1 + g_2(K_{t+1}, K_{t+2})]K_{t+2}}{[1 + g_2(K_t, K_{t+1})]K_{t+1}}.$$

Notice that the latter expression equals the right-hand side of equation (3) in Section 2.

C I.I.D. TFP Growth

In the scenario discussed in Section 7, productivity growth follows a random walk, that is $\log \left(\frac{z_{t+1}}{z_t} \right) = \lambda + \zeta_{t+1}$, with $\zeta_t \sim N(0, \sigma^2)$. In this case, the planner's problem

can be made stationary by dividing all time- t variables by z_t . The Bellman equation of the stationary model is

$$\begin{aligned} v(\hat{k}) &= \max_{\hat{k}'} \left\{ \hat{c}^\gamma + \beta e^{[\varphi+\gamma\lambda]} \mu^\gamma \left[e^{\zeta'} v(\hat{k}') \right] \right\}^{1/\gamma} \\ \text{s.t. } \hat{c} + e^{(\lambda+\varphi)} \hat{k}' e^{\zeta'} &= \hat{k}^\alpha l^{1-\alpha} + (1-\delta) \hat{k} - \left| e^{(\lambda+\varphi)} \frac{\hat{k}'}{\hat{k}} e^{\zeta'} - \psi \right|^\iota \hat{k}, \\ \zeta' &\sim N(0, \sigma^2). \end{aligned}$$

Notice that \hat{k}' is a random variable. In order to implement the above optimization program on a computer, define $\tilde{k}' = \frac{k'}{ze^\lambda}$ and rewrite it as

$$\begin{aligned} v(\hat{k}) &= \max_{\tilde{k}'} \left\{ \hat{c}^\gamma + \beta e^{[\varphi+\gamma\lambda]} \mu^\gamma \left[e^{\zeta'} v(e^{-\zeta'} \tilde{k}') \right] \right\}^{1/\gamma} \\ \text{s.t. } \hat{c} + e^{(\lambda+\varphi)} \tilde{k}' &= \hat{k}^\alpha l^{1-\alpha} + (1-\delta) \hat{k} - \left| e^{(\lambda+\varphi)} \frac{\tilde{k}'}{\hat{k}} - \psi \right|^\iota \hat{k}, \\ \zeta' &\sim N(0, \sigma^2), \\ \hat{k}' &= \tilde{k}' e^{-\zeta'}. \end{aligned}$$

The conditional stochastic discount factor is

$$m(\zeta_i | \hat{k}) = \beta e^{\lambda(\gamma-1)} \frac{1 + \theta I(\tilde{k}', \zeta_i, \mu)}{1 + \theta \sum_i \pi(\zeta_i) I(\tilde{k}', \zeta_i, \mu)} \left[\frac{\partial'(\zeta_i)}{\hat{c}} \right]^{\gamma-1} e^{\zeta_i(\eta-1)} \left[\frac{v(e^{-\zeta_i} \tilde{k}')}{\mu(\tilde{k})} \right]^{\eta-\gamma},$$

where $I(\tilde{k}', \zeta_i, \mu) = 1$ if $e^{\zeta_i} v(e^{-\zeta_i} \tilde{k}') \leq \xi \mu$, and $I(\tilde{k}', \zeta_i, \mu) = 0$ otherwise.

D Disappointment Aversion – A Primer

In most applied work in economics, agents choose among risky outcomes (lotteries over simple events) after evaluating each of them according to the expected utility criterion. According to this paradigm, the certainty equivalent μ of a lottery over a finite set of payoffs $\{x_1, x_2, \dots, x_N\}$ satisfies

$$u(\mu) = \sum_{i=1}^N \pi_i u(x_i),$$

where $\pi_i \geq 0$, $\sum_{i=1}^N \pi_i = 1$, is the probability of event i and u is an increasing and continuous function. The preference relation over lotteries represented by this utility specification is known to satisfy the axioms of monotonicity, completeness, transitivity, continuity, and independence. The latter has repeatedly come under attack, as experimental studies have found an increasing number of instances in which

individuals' decision making appears to violate it. Perhaps the most famous of these violations is that known as the Allais Paradox.

Decision theorists have therefore sought to identify preference relations that satisfy a weaker version of the independence axiom. Here we consider the work of Faruk Gul. Gul (1991) defines as *disappointing* those outcomes that lie below a lottery's certainty equivalent. His *weak independence axiom* requires independence only for lotteries that are disappointment-comparable. Gul shows that the preference relation satisfying this axiom, along with those named above, can be represented by the certainty equivalent μ that solves

$$u(\mu) = \sum_{i=1}^N \pi_i u(x_i) - \theta \sum_{x_i \leq \mu} \pi_i [u(\mu) - u(x_i)].$$

Outcomes below the certainty equivalent receive a greater weight in the computation of overall utility. Such weight depends positively on the parameter θ and on the distance from the certainty equivalent itself. Notice that μ appears on both sides of the above condition, and therefore must be determined together with the set of disappointing states.

It is sometimes handy to express μ as the following weighted average:

$$u(\mu) = \sum_{i=1}^N \tilde{\pi}_i u(x_i), \quad \tilde{\pi}_i = \pi_i \frac{1 + \theta \times I(x_i, \mu)}{1 + \theta \sum_{(x_i \leq \mu)} \pi_i},$$

where $\sum_i \tilde{\pi}_i = 1$ and $I(x_i, \mu)$ is an indicator function that takes value 1 if $x_i \leq \mu$ and 0 otherwise. This formulation makes it clearer that under DA the attitude towards risk not only depends on the curvature of u , but also on the value of θ . For the sake of illustration, consider the case of $N = 2$ and $\pi = 1/2$. In Figure 10 we have pictured the qualitative behavior of indifference curves, in the cases of expected utility ($\theta = 0$) and disappointment aversion ($\theta > 0$). It is important to notice that, contrary to the expected utility case, under disappointment aversion the indifference curve is not differentiable at the certainty equivalent. The kink reflects what Segal and Spigal (1990) call first-order risk aversion, as opposed to second-order risk aversion, which characterizes expected utility.

The difference between first- and second-order risk aversion can be appreciated by computing the effect of θ on relative risk aversion, i.e. on the amount P (the risk premium) that an agent is willing to pay in order to avoid a given bet.²¹ Consider the following example. For some $\kappa \geq 0$, let an agent endowed with wealth ω_0 consider an

²¹Here we focus on multiplicative bets, but similar arguments can be made for additive bets. For an accessible review of the definition and measurement of risk aversion, see Chapter 2 of Gollier (2001).

investment opportunity that pays $w_0(1 + \kappa)$ and $w_0(1 - \kappa)$ with equal probabilities. With disappointment aversion, the risk-premium associated with κ , $P(\kappa)$, solves

$$\frac{1 + \theta}{2 + \theta} u[w_0(1 - \kappa)] + \frac{1}{2 + \theta} u[w_0(1 + \kappa)] = u[w_0(1 - P(\kappa))], \quad \kappa > 0$$

and satisfies $P(0) = 0$. In a neighborhood of $\kappa = 0$, we have that

$$P(\kappa) = P(0) + \frac{dP}{d\kappa} d\kappa + \frac{1}{2} \frac{d^2 P}{d\kappa^2} d\kappa^2 = \frac{\theta}{2 + \theta} d\kappa - \frac{1}{2} \frac{u''(w_0)w_0}{u'(w_0)} \left[\frac{4(1 + \theta)}{(2 + \theta)^2} \right] dk^2.$$

If the utility function is isoelastic with $\frac{u''(w_0)w_0}{u'(w_0)} = \eta - 1$, we can write

$$P(\kappa) = \frac{\theta}{2 + \theta} d\kappa + \frac{1}{2} (1 - \eta) \left[\frac{4(1 + \theta)}{(2 + \theta)^2} \right] dk^2,$$

For $\theta > 0$, an increase in κ , i.e. an increase in risk, has a first-order effect on the risk premium. This is why in this case we talk of first-order risk aversion. In the case of expected utility ($\theta = 0$) instead, risk has only a second-order effect, as we obtain the familiar expression $P(\kappa) = [(1 - \eta)/2] dk^2$. This is why $|\frac{u''(w_0)w_0}{u'(w_0)}| = 1 - \eta$ is known as the de Finetti–Arrow–Pratt coefficient of relative risk-aversion.

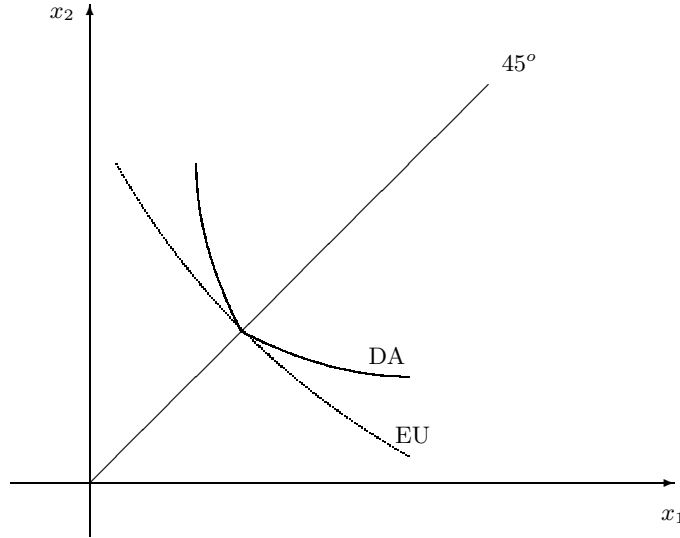


Figure 10: Indifference Curves.

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