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**Promoting formative assessment in a connected classroom environment:
design and implementation of digital resources**

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Abstract

This paper is based on a design-based research project investigating how to use digital resources to help activate formative assessment processes in the classroom. Performed as part of FaSMEd, a European Union project, our own project adopts a comprehensive theoretical framework, including the different functionalities of technology, formative assessment strategies, the agents involved, and teacher practices in classroom discussion management. Through this framework we analyze the design and implementation of specific digital worksheets that can be sent from teacher to students and vice versa, as well as displayed on the students' tablets, on teachers' computer and/or on interactive whiteboards by means of connected classroom technology. These digital resources are meant to help students share their results, opinions and reflections with their classmates and teachers during or at the end of mathematical activities. In this paper we focus on how to exploit digital worksheets supported by connected classroom technologies in order to help activate formative assessment strategies, especially during classwide activities. Our analysis reveals that formative assessment strategies emerge in the shape of typical patterns of their interaction when digital worksheets are implemented in the lessons. Hence we have outlined several criteria for the design and implementation of digital worksheets in support of FA processes.

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1. Introduction

Research in mathematics education has long been engaged in elaborating innovative instructional resources and studying their validity, often according to a design-based perspective (Cobb et al., 2003; DBRC, 2003) and focusing on “co-evolution of mathematics teaching resources and practices” (Ruthven, 2013, p. 1071).

FaSMEd (Improving Progress for Lower Achievers through Formative Assessment in Science and Mathematics Education) has adopted a design-based research approach in investigating the role of technologically enhanced formative assessment methods to support student learning. Formative assessment (FA) is conceived as a method of teaching where “evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited” (Black & Wiliam, 2009, p. 7).

Research into FA practices has particularly highlighted the role played by so-called connected classroom technologies (CCT) – i.e. networked systems of personal computers or handheld devices specifically designed to be used in classrooms for interactive teaching and learning (Irving, 2006). Within FaSMEd, we exploited CCT as communicative infrastructures (Hegedus & Moreno-Armella, 2009) and designed *digital resources* in order to help develop of formative assessment processes during classroom mathematics activities, especially discussions (Stein et al., 2008). The term ‘resource’ is used in the broad sense to include textbooks, curriculum materials, descriptions of mathematical tasks, devices, manipulatives, instructional software and including also particular pedagogical practices (Gueudet, Pepin & Trouche (2013); Remillard & Heck, 2014).

Thus adopting a design-based research perspective we analyze how CCT-supported digital worksheets can be designed and implemented in order to activate FA strategies, particularly during classwide activities.

2. Theoretical background

This section introduces the theoretical tools guiding: (a) the design of digital worksheets, where *design* includes the construction of worksheets and the planning of the methodology adopted for their implementation; and (b) the retrospective analyses of episodes from the design experiments.

2.1 Formative assessment and technology

There are five key strategies for FA practices in school setting, according to Wiliam and Thompson (2007): (a) *clarifying and sharing learning intentions and criteria for success*; (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding*; (c) *providing feedback that moves learners forward*; (d) *activating students as instructional resources for one another*; (e) *activating students as the owners of their own learning*. The teacher, student’s peers and the student him- or herself are the agents that activate these FA strategies.

Connected classroom technologies (CCT) support FA, as emphasized in previous research. CCT includes classroom response systems (Roschelle & Pea 2002), networked graphing calculators (Clark-Wilson, 2010) and participatory simulations (Ares, 2008). CCT have Specific features that make them effective tools for FA that help to: (1) monitor students’ progress, collecting the content of students’ interaction over longer timespans and over multiple sets of classroom participants (Roschelle & Pea 2002); (2) provide students with immediate private feedback, keeping them oriented on the path to deep conceptual understanding (Irving 2006); (3) encourage students to reflect and monitor their own progress (Roschelle & Pea 2002; Ares 2008).

1 We have investigated the use of technology to enhance FA practices by introducing a theoretical
2 model under the aegis of the FaSMEd project (Aldon et al., 2017), elaborated from Wiliam and
3 Thompson's FA model integrated with technology, This three-dimensional model takes into
4 account: (1) *the five FA key-strategies* described by Wiliam and Thompson (2007); (2) *the three*
5 *main agents* that intervene (teachers, students, peers), and (3) *the functionalities* through which
6 technology can support the three agents in developing the FA strategies.

7 This technology can support the three agents in developing the FA strategies through its three three
8 *functionalities*:

9 (1) *sending and displaying*, which fosters communication among the agents of FA processes (e.g.
10 sending and receiving messages and files, displaying and sharing screens or documents with the
11 whole class.;

12 (2) *processing and analyzing*, which supports the processing and the analysis of the data collected
13 during the lessons (e.g. through the sharing of the statistics of students' answers to polls or
14 questionnaires, the feedback given directly by the technology to the students while taking tests);

15 (3) *providing an interactive environment*, which creates environments where students can interact to
16 work individually or in groups on task or explore mathematical/scientific contents (e.g. through the
17 creation of interactive boards to be shared by teacher and students or through the use of specific
18 software that provide an environment where it is possible to explore mathematical problems
19 dynamically).

23 2.2 FA, Metacognition and Classroom Discussions

24 The perspective adopted on teaching-learning processes influences the ways in which FA may be
25 activated with the support of technology. Our study is based on a Vygotskyan perspective
26 (Vygotsky, 1978), where interaction with peers and experts plays a crucial role in students'
27 learning. In addition, FA must also focus on metacognitive factors (Schoenfeld, 1992). Hence we
28 designed activities to help students: (a) make their thinking visible (Collins, Brown & Newmann,
29 1989) through the sharing of their thinking processes with teachers and classmates through
30 argumentation; (b) develop their on-going reflections on the learning processes. In fact, students'
31 thinking can become visible during classroom discussions to help them become aware of facts and
32 acts that they had been unaware of (Mason, 1998). According to Mason, the teacher can play a role
33 in transforming the *awareness-in action* displayed by students' output into *explicit awareness* and
34 thus conducting them into "disciplined forms of thinking and perceiving" (p.258). To this end,
35 students should be involved in suitable mathematical tasks and effective classroom discussions.

36 Planning and managing productive classroom discussions is indeed a demanding task for the
37 teacher. Stein et al. (2008) elaborated a pedagogical model according to which, "it is students' ideas
38 that provide the fodder for discussions, with students publicly serving as the primary evaluators of
39 them" (p.333). In particular, these authors identify five strictly interrelated practices:

40 (1) *anticipating* students' likely responses to cognitively demanding mathematical tasks;

41 (2) *monitoring* students' responses to the tasks in order to identify the mathematical learning
42 potential of particular strategies or representations used by them;

43 (3) *selecting* particular students to present their mathematical responses during the
44 discussion; introducing, if needed, particularly important strategies that no one in the class
45 has used (e.g. by sharing the work done by students from other classes);

46 (4) purposefully *sequencing* the students' responses to be shared and discussed to help
47 compare n related or contrasting strategies and make it more likely that important
48 mathematical ideas be discussed by the class;

49 (5) *helping the class make mathematical connections* among the responses of different
50 students in order to prompt them to reflect on other students' ideas while evaluating and
51 revising their own.

Within this model, teachers can organize discussions in order to make “students’ thinking public so it can be guided in mathematically sound directions, and encouraging students to construct and evaluate their own and each others’ mathematical ideas” (ibid., p.315).

3. Research methodology and questions

Based on this theoretical framework, we have followed a design-based approach (DBCR 2003) – i.e. one characterized by cycles of design, enactment, analysis and redesign. Indeed, our research project:

- (a) is meant to be useful for practitioners, carried out in classroom environments, and focused on “interactions that refine our understanding of the learning issues involved” (ibid. p. 5);
- (b) considers intervention itself as an important outcome of the research;
- (c) embodies “specific theoretical claims about teaching and learning”, reflects “a commitment to understanding the relationships among theory, designed artifacts, and practice” and aims to contribute “to theories of learning and teaching” (ibid. p. 6);
- (d) focuses on design experiments that “are implemented with a hypothesized learning process and the means of supporting it in mind in order to expose the details of that process to scrutiny” (prospective nature of design experiment, p. 10), and take the form of conjectures “about the means of supporting a particular form of learning that is to be tested”, conjectures that can evolve into “more specialized conjectures” during the research project (reflective nature of design experiments, p. 10);
- (e) is characterized by close collaboration between researchers and teachers, who aim to improve practices in view of contextual constraints and research aims, as in *Research for Innovation*, an Italian paradigm where teachers and researchers collaborate in mutual support during all the phases of the research process from planning to implementation and analysis (Arzarello and Bartolini Bussi 1998).

Our research project, through its design base, aims at studying how to design and implement specific digital resources that foster and activate FA strategies. The project involves the design and implementation of three different kinds of digital worksheets by different agents during classwide activities. We discuss the design of the three kinds of worksheets, but, for brevity, analyze the implementation of only two of them. In our analysis we do not go into detail about the process of design and redesign, but identify some criteria for the design of digital resources for FA resulting from this analysis. Our analysis of the design and implementation of digital worksheets strives to answer the following research question:

How can digital worksheets be designed and used to activate FA strategies during classroom discussions orchestrated by the teacher?

The following table summarizes the theoretical tools used to carry out the data analysis. In the second column, we specify how we use theoretical tools in order to answer this research question, referring to the theoretical tools introduced in Section 2.

Table 1: The theoretical tools for the design, implementation and analysis of the digital resources

The theoretical tools for the design, implementation and analysis of digital worksheets	
(1) <i>Key formative assessment strategies</i> (Wiliam & Thompson 2007)	(a) to help choose the design of the digital worksheets to help activate specific FA strategies;
	(b) to help choose the methodology for the implementation of the digital worksheets to help activate specific FA strategies;
	(c) to highlight when and how FA strategies are activated during classroom interaction in the retrospective analyses of design experiments.

<p>(2) Functionalities of technologies (three-dimensional FaSMEd framework, Aldon et al. 2017)</p>	<p>(a) to help choose the methodology of implementation of digital worksheets within the chosen digital environment;</p> <p>(b) to highlight, in the retrospective analysis, how the chosen digital technology supports the use of digital worksheets to activate FA processes.</p>
<p>(3) Teacher practices for discussion (Stein et al. 2008)</p>	<p>(a) to help identify the role played by the expert (teacher or researcher) in implementing digital worksheets during classroom activities in support of the process of making thinking visible (in connection with FA purposes);</p> <p>(b) to highlight, in the retrospective analysis of the design experiments, the role played by the expert (teacher or researcher) in meeting goals that had been set.</p>

The subsequent parts of the paper are organized according to Cobb et al.'s characterization of design experiments (Cobb et al. 2003): (a) preparing for a design experiment (Section 4); (b) conducting a design experiment (Section 5); (c) conducting retrospective analysis (Section 6).

4. Preparing for the design experiment: design of the *digital worksheets*

The digital resources consist in a set of digital worksheets, organized according to three different categories to be detailed below. The digital worksheets are used with the aid of IDM-TClass, a CCT software program that connects students' tablets with the teacher's laptop, enables the students to share their output and enables the teacher to collect the students' opinions and reflections easily, during or at the end of an activity by creating instant polls.

We created the digital worksheets in order to foster FA processes by adapting a set of activities on relations and functions. These activities were adapted from two main sources: ArAl project units (<http://www.progettoaral.it/>) and time-distance graph activities from the Mathematics Assessment Program, developed by the MARS Shell Centre team at the University of Nottingham (<http://map.mathshell.org/materials/lessons.php>).

We designed a set of different digital worksheets for each activity, to be sent by the teacher to the students' tablets, to be displayed on the IWB and/or displayed through the data projector. We designed three main types of digital worksheets:

- (1) *problem worksheets*: worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
- (2) *helping worksheets*, aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
- (3) *poll worksheets*: worksheets prompting a poll among proposed options.

Our specific design choices concern: (a) the type of file to be shared; (b) the task proposed to the students; (c) the way in which each worksheet is implemented. The next paragraphs presents these choices. .

4.1 Design of the *problem worksheets*

A problem worksheet contains a task to be tackled by the students, usually organized in groups of two or three. The teacher sends the problem worksheet from her or his computer to the students' tablets (*sending* functionality of the technology). The task contains an open question and an explicit requirement to justify the eventual answer. This is in line with our choice to encourage students to make their thinking visible (Collins, Brown, & Newman 1989). Students are strongly encouraged to

1 create proper sentences rather than just sketch answers or provide “closed answers” without
2 justifying them. Students write on the doc files they received, which are then sent to the teacher and
3 may be displayed on the interactive board. Doc files accommodate answers that are easily read by
4 the teacher, selected, and displayed to the other students to be commented on. This is why we chose
5 them. While the students are taking on the problem, the teacher may *monitor* the group work
6 through IDM-TClass software, or through appearing at the group desks.

7 In this way, a preliminary *monitoring* practice aids in the subsequent *selection* practice. Once a
8 written solution has been produced, each group sends the document containing the solution to the
9 teacher’s laptop. In this way, the teacher can immediately read the answers and *select* some of them
10 to start the discussion. After all the groups have sent their answers, the first step of the FA
11 classroom discussion takes place: the teacher shows the chosen written output to the whole class (or
12 shows all of them, following a chosen order), using the *display* functionality of the technology. In
13 order to organize the discussion, the teacher can make strategic choices for selecting and
14 sequencing the students’ output. The *selection* of the written answers is guided by the following
15 aims:
16

- 17 • to bring typical mistakes so that students can discuss them and that students who wrote them
18 can receive feedback both from their peers and teachers (*FA Strategies C, D and E*);
- 19 • to highlight more or less efficient ways of processing tasks (with a focus on problem solving
20 and not only on the final product), thus sharing criteria for success (*FA Strategy A*);
- 21 • to contrast different justifications or identify similar ones. This aim is in line with the
22 decision to propose tasks that require not only an answer, but also its justification.
23

24 The *selection* of solutions may be made on the basis of the final answer, or of the provided
25 justification, or both. Each written solution could be analyzed according to the following questions:
26 “What do you think of this answer?” “Do you find it correct?” “Do you find it clear?” and “Do you
27 find it complete?”
28

29 In the first cycle of design experiments, the selected answers were displayed and discussed one by
30 one. In later cycles, thanks to the continuous dialectic between experimentation and analysis of the
31 results, we gradually moved towards a more planned *sequencing* modality, in which some selected
32 answers were grouped within the same file, to be displayed on the screen simultaneously. Hence the
33 teacher could introduce comparisons and contrasts among answers by asking specific questions,
34 such as “What do they have in common and how are they different?” “Why did I [the teacher]
35 choose to group them together?” The students could also be asked to identify the criteria underlying
36 the grouping presented by the teacher.
37

41 **4.2 Design of the *helping worksheets***

42 *Helping worksheets* are doc files drafted to be sent to those students who have trouble with specific
43 tasks within the problem worksheets (“*sending*” functionality of technology).

44 They were initially designed to be sent only to some students or groups of students when:

- 45 a) they ask for help;
- 46 b) the teacher realizes that they are blocked because of the problems they are having;
- 47 c) the answers sent to the teacher highlight mistakes or problems.
48

49 Usually, from one to three helping worksheets were elaborated for each problem worksheet.
50 Helping worksheets were designed with attention towards a pedagogical analysis of the related
51 problem worksheet. When we worked on the helping worksheets, we avoided “suggesting an
52 answer,” which may relegate students to a procedural role. Rather, we involved the students in
53 questions so that we could respect their active role. In fact, the helping worksheets contain the
54 original problem plus an additional question or task whose answer is helpful for solving the original
55 problem, such as:
56

- 57 - an additional question focusing on a specific part of the text of the problem, which contains
58 information to be considered carefully;
59

- a table or another kind of representation to be completed, so that students may have a clearer situation of the problem and a tool that could support their reasoning;
- a reminder of a previous concept that could be used in the present task;
- gradual questions, leading students to focus on a specific strategy to identify the correct answer.

After the first cycle of experiments, two additional uses of helping worksheets emerged:

- d) When a group sends its answer to the teacher, one helping worksheet is sent to the group as a tool to check the correctness of the answer (activation of FA strategy C) – i.e. a tool to support the students in building meta-cognitive competencies regarding the control aspect of the problem-solving (Schoenfeld 1992);
- e) Helping worksheets are displayed to the whole class during classroom discussions (“displaying” functionality of technology) and students are asked to reflect on the “sense of the help”– i.e. on why the additional question/table/reminder was given as a help to solve the original problem. This is a specific form of practice in “helping the class make mathematical connections” (Stein et al. 2008), which may be connected with FA strategies. In fact, the discussions are planned by the teacher to elicit students’ understanding at a metacognitive level (FA Strategy B). During these, the students who did not receive the worksheets have the chance to reflect on the role played by the help provided, becoming instructional resources for their classmates (FA Strategy D). Meanwhile the students who received help can discuss how they used it, making their reasoning explicit (Strategy E).

4.3 Design of the *poll worksheets*

Instant polls are used with the support of the *processing and analyzing* functionality of the technology, which enables teachers to create instant polls, submit them to the students, gather the answers and show the results (both individual answers and clustered ones) immediately. According to our theoretical assumptions, polls are designed around not only mathematical but also metacognitive and affective focuses, such as:

- (a) a mathematical task, as in the example we are presenting below;
- (b) a meta-cognitive reflection with the aim of clarifying and sharing learning intentions and criteria for success (*FA Strategy A*); for example, students can be asked to reflect on the completeness of a set of written justifications produced by their classmates (“Look at these answers to question 2. Which is the most complete?”): they can be asked to look at those written or by students from other classes or by fictitious students, in order to introduce a particularly important strategy that no one in the class has used, as suggested by Stein et al. (2008);
- (c) a meta-cognitive reflection with the aim of making explicit the difficulties they meet when facing specific kind of tasks (“What was the most difficult task today?”; “What are the aspects that it is more difficult to highlight when you have to interpret a graph?”);
- (d) a reflection on affective aspects; for example, students can be asked to express how they felt when facing a specific kind of task (“How did you feel when you had to work on worksheet 1?”) or when a particular methodology were adopted during the lessons (“How did you feel when your written answer was displayed on the IWB and discussed by the teacher and your classmates?”).

In answering the poll, students have to choose one answer among a set of possible ones. The IDM-TClass software collects all the students’ choices and processes them, displaying an analytical record (collection of each answer) as well as a summarized overview (bar chart). Using the software, the teacher can choose to provide or not an immediate automatic correction of students’ answers (right/wrong). The software also enables teachers to set the time given to students before completing the poll. During the activities focused on polls, we decided *not* to provide the immediate automatic correction to students, so that they could be engaged in subsequent classroom discussion based on the results of the poll. In fact, polls were conceived as an alternative way of prompting

1 focused discussions to foster sharing of results and comparison (*FA Strategy B*). Comparison was
2 meant to focus on the justification of the answers specifically. In our view, this choice has many
3 positive aspects: (a) it prompts the students to focus on the justification of the answers more than on
4 the identification of the correct answer, in tune with Stein et al.'s practice in *helping the class make*
5 *mathematical connections*; (b) it pushes the students to motivate their answers; (c) it prevents the
6 students from feeling labeled as right/wrong, thus contributing to a positive affective climate with
7 respect to the activity.

8 After the first cycle of design experiments, polls were also used to assess whether the students had
9 reached understanding on some key-features of the lesson during or at the end of classroom
10 discussions. Thus polls were a precious tool serving teachers to be better equipped in the design of
11 the subsequent learning steps.
12
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14 **5. Conducting the design experiment**

15
16 The experimental part was conducted in 36 4th, 5th, 6th and 7th-grade classes from three different
17 clusters over two consecutive school years (2014-15 and 2015-16) in northwestern Italy. Each
18 school was provided with tablets for the students and computers for the teachers, linked to an IWB
19 or data projector. Students were asked to work in pairs or in small groups on the same tablet in
20 order to foster collaboration and sharing of ideas.
21
22

23 At least one of the authors was present in the classroom as participant observer during the design
24 experiments. We chose to make researchers play an active role in the implementation as well as
25 design of classwide activities (as seen in the examples below). In fact, our design experiments were
26 mainly aimed at highlighting how experts could use our digital resources effectively to support FA
27 processes in the classroom. Researchers closely involved in design experiments were able to
28 develop the necessary changes in each repeated design cycle while they observed the designs being
29 implemented in classrooms. In this way, their active participation both supported the identification
30 of criteria for the design of resources for FA strategies and enabled them to highlight organized and
31 effective ways to implement these resources. Masters-level students (future primary or lower
32 secondary school teachers) also helped in filming and observing the lessons, as part of their master
33 thesis projects. The primary and middle-school students were told what the researcher(s) and master
34 students were doing and welcomed them as additional resources for their learning.
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36
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38 All the lessons were filmed, fields notes were taken, and students' output (doc files) were collected,
39 to become part of a large corpus of data (about 450 hours of class sessions in collaboration with 20
40 teachers). In addition, teachers were interviewed every two or three lessons and, after each lesson,
41 were asked to write a report on how effective the lesson was in activating FA processes and how
42 effective the technological support was. Then all the members of the research team performed a
43 cross analysis of this data.
44
45

46 The section below is a report of the analysis of sequences from classroom discussions, illustrating
47 "selected aspects of the envisioned learning and of the means of supporting it as paradigm cases of
48 a broader class of phenomena" (Cobb et al. 2003, p.10). Any generalizations made are meant to be
49 analytical rather than statistical. The researchers, actively involved in classroom activities, were
50 able to identify examples rich in terms of the dynamics of FA strategies and the different agents
51 involved.
52
53

54 A preliminary selection of class sessions was made on the basis of researchers' direct observations.
55 Each researcher selected a series of class sessions based on the following criteria:
56

- 57 - The session was in line with the design (the worksheets were used according to the design;
58 the students filled out their tasks in the expected time; and the discussions on the worksheets
59 were organized according to the planned focus);
60
61
62
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- The researcher and the teacher could manage the technology without technical problems.

After the preliminary selection, the class discussion episodes were transcribed and analyzed separately by the three researchers. Each researcher coded the transcript separately in terms of FA strategies. Afterwards, problematic codes were discussed together so that researchers could come to an agreement.

In the following section we present and analyze paradigmatic examples through the theoretical tools introduced in Table 1. In this paper, we have limited ourselves to two examples: the use of a problem worksheet and the use of a poll worksheet. The episodes we chose are rich in terms of activated FA strategies, and paradigmatic, because the same FA strategies were also observed in other episodes.

6. Retrospective analysis: examples from the classroom

The two examples relate to an activity on time-distance graphs (below) adapted from the task sequence “Interpreting time-distance graphs”, from the Mathematics Assessment Program (<http://map.mathshell.org/materials/lessons.php>). We created a set of 19 digital worksheets from the original source based on paper-and-pencil materials for grade 8 so that we could use them with students from grade 5 to 7. The sequence starts with a short text about the walk of a student, Tommaso, from his home to the bus stop. This text is accompanied by a time-distance graph:

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.

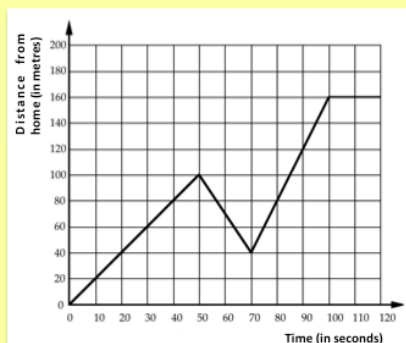


Fig.1: The time-distance graph of Tommaso's walk

The students' interpretations of this graph were guided through the questions they were asked in the problem, helping, and poll worksheets. These were gauged at developing students' awareness-in-action (Mason, 1998). In this specific case, this awareness was related to the interpretation of the ascending, descending and horizontal lines of the graph. Students were also asked to focus on the reasons supporting the correct interpretation of a time-distance graph. This was meant to get them to consolidate their competencies in justifying their answers and, in this way, to develop what Mason calls “disciplined forms of thinking and perceiving.”

After the interpretation of the given time-distance graph according to the given story, the activity developed through the matching among different graphs and the corresponding stories and the construction of graphs associated to specific stories. Since the students were encountering time-distance graphs for the first time in this activity, we designed an introductory activity based on the use of a motion sensor, in which students could explore the construction of a graph in practice after a motion experience along a straight line.

We stress that teachers used the designed sequence of worksheet flexibly. In primary school classes (grades 4-5) the teachers always decided to propose the worksheets in the order in which they were conceived (from 1 to 5). In lower secondary school (especially in grade 7), after a first cycle of

1 experiments, the teachers chose to change the sequence, proposing at first Worksheet 5 (which
2 requires a global interpretation of the graph), and later Worksheets 1-4.

3 **6.1 Excerpts from a discussion focused on a problem worksheet**

4
5 The episode refers to Worksheet 5, which requires a global interpretation of the graph: “*Every*
6 *morning Tommaso walks along a straight road from home to a bus stop, a distance of 160 meters.*
7 *The graph shows his journey on one particular day. Describe how Tommaso walked on the road*
8 *from his home to the bus stop. What could have happened to him?”*
9

10 The episode comes from a 7th-grade class, where the teacher decided to start the task sequence with
11 problem Worksheet 5. Here the teacher and students are exploiting the *sending and displaying*
12 *functionality of technology*. In the first lesson, the students received the problem worksheet on their
13 tablets, tackled the task and sent their written answers to the teacher. After the lesson, the teacher
14 and the researcher selected some written solutions to be displayed during the second lesson. The
15 *selection* was made in order to spark a discussion on two crucial issues: how to interpret a change of
16 direction in the graph and how to derive further information from the reading of the graph. The two
17 written solutions that were chosen illustrate the process of *sequencing* (our translation):
18

19 *Answer 1. We think that Tommaso had some problem: for instance, some men at work that made*
20 *him go back and take another road, and after go on normally and stop.*
21

22 *Answer 2. Tommaso leaves home and goes on for 100 meters. After having gone 100 meters he*
23 *goes back for 60 meters, probably because he got lost, and he gets closer to his house.*
24 *Afterwards, he changes direction and he gets closer to the bus stop, walking for 140*
25 *meters.*
26
27

28 Answer 1 does not contain any numerical data and interprets the change of direction in the graph
29 (from ascending to descending) as a change of road on the walk (“take another road”). Answer 2
30 contains numerical data (inferred from the reading and interpretation of the graph) to describe the
31 walk. One of these pieces of data is not correct since Tommaso does not walk for 140 but 120
32 meters. In this answer, the change of direction in the graph is interpreted as a change in the
33 direction that Tommaso is walking on the same road (towards home or towards the bus stop). In
34 other words, in Answer 1, Tommaso is said to have *changed roads*. Meanwhile, in Answer 2
35 Tommaso is said to have *gone back* on the same road. Comparing the two answers may therefore
36 spark a discussion on the meaning of the *change of direction* in the graph and on the way of
37 deriving information from the interpretation of the graph. Summing up, the teacher and the
38 researcher had two goals when they set up the discussion. The first, task-level goal consisted in
39 clarifying that there is only one possible interpretation: *Tommaso changes his direction, going back*
40 *towards home*. This is so because the text says explicitly that Tommaso is walking down a straight
41 road. The second, metacognitive goal consisted in emphasizing that close reading of a text is an
42 efficient problem solving strategy.
43

44 In the first part of the discussion, one author of Answer 2, Rob recognized that Tommaso walked
45 for 120 meters and so changed his last words immediately. When he had the chance to look back at
46 his group’s solution, Rob activated himself as owner of his own learning (FA *Strategy E*). Other
47 students noted that, in both answers, Tommaso is said to return closer to home, but only Answer 1
48 mentions the final part of the walk as well. This is when Tommaso does not walk anymore,
49 corresponding to the horizontal part in the graph. Afterwards, the discussion focuses on the
50 decreasing part of the graph, which the students interpret in terms of returning closer to home:
51

52 74. Chiara: Anyway, I think he (*Tommaso*) didn’t really take another road. He could have... I
53 don’t know... forgotten about something.... He lost his pencil case in the
54 middle of the street. He only turned back. There, he changed his direction.
55 That’s why there is a peak, but he didn’t necessarily change roads... take
56 another road because maybe it (*the graph*) would have reported more distance.
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Chiara *activates herself as a resource for the other students* (FA Strategy 4), because she points out something to be fixed in Answer 1. Chiara is efficient in explaining that Tommaso did not change roads, but only changed directions on the same road. In the last part of her sentence, she points out that, if Tommaso had changed roads, the distance would have increased.

After a brief discussion aimed at clarifying Chiara's intervention (lines 75-93), the teacher wanted to make clear that Chiara's interpretation is the only possible one since the text reports that Tommaso moves along a straight line. The following sequence illustrates the discussion on this crucial issue:

94 Teacher: But look, is the graph the only thing that gives us information? Was there only the graph in the worksheet?

95 Paul: There was the text too.

96 Teacher: So, there was the text too. Let's read the text again? Go on, Rob.

97 Rob (*reading*): Every morning Tommaso walks along a straight road from home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.

98 Teacher: Do we have any other information?

99 Cate: Ah, but Tommaso walks down a straight road.

100 Guglielmo: Yeah, really.

101 Teacher: And so?

102 Mark: Then, yes, he had to change roads... then..

103 Teacher: He changed roads?

104 Guglielmo: No!

105 Teacher: Chiara?

106 Chiara: I wanted to say that he did not change roads because, beside the fact the road is straight, if the road was straight and then let's say that there was a little road there, the motion sensor would not have caught him...

107 Teacher: Yes, let's imagine we're observing him, not that there was a motion sensor.... we measure the time periods and we take a look at his distance from home. But the text gives you some more information: he was walking down a straight road. So I really know he was walking down a straight road. This thing, the fact that he was walking down a straight road: does the graph tell me this or not?

108 Rob: No.

109 Teacher: Because what's the only thing the graph tells me?

110 Rob: Distance and time.

On line 94, the teacher emphasizes that, in order to accomplish the task, it is important to take into account both the graph and the text in the problem worksheet. She activates two FA strategies: *Strategy A* (*Clarifying and sharing learning intentions and criteria for success*) and *Strategy C* (*Providing feedback that moves learners forward*).

Starting from line 98, the teacher involves the students in close reading of the text. In this way, she pursues a double goal: working on the task and promoting close reading of the text as an efficient strategy for solving any problem.

6.2 Excerpts from a discussion focused on poll worksheets

Figure 2 presents an example of a poll worksheet (Worksheet 3 in the designed sequence). After having reflected on what happened in the periods of time 50s-70s (Question 1) and 100s-120s (Question 2), students are now asked to determine when Tommaso reaches the bus stop (Question

3). Here the focus is therefore on the interpretation of a point in a time-distance graph as a bearer of two kinds of information: the distance from home and the time spent. Students have to identify the point (100,160) as the one on which they have to focus in order to find the answer.

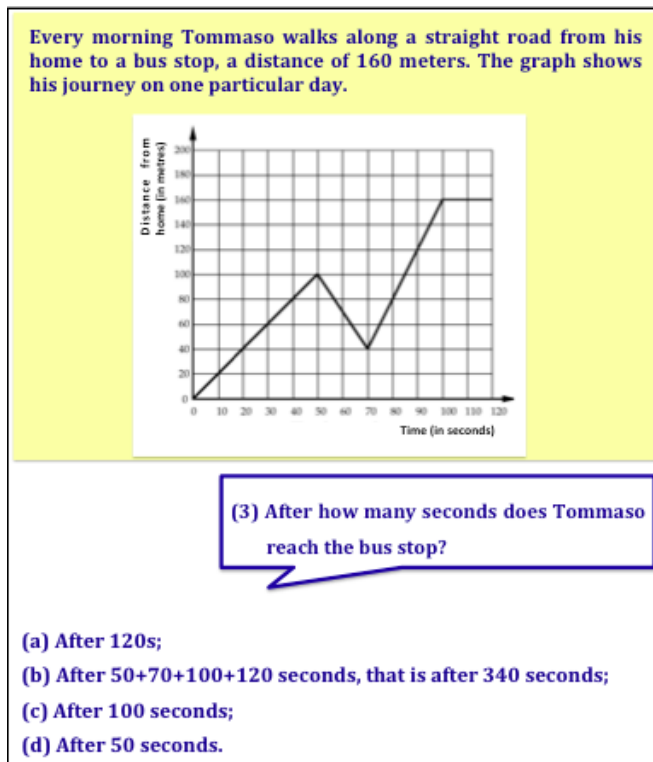


Fig.2: An example of poll worksheet (Worksheet 3 in the designed sequence)

In designing Worksheet 3, we chose these four options in order to highlight possible inappropriate approaches and misconceptions to students: for example, adding the numbers of seconds that correspond to the right end of each part of the graph, as in Answer B; and thinking that the arrival at the bus stop corresponds to the last point of the graph, as in Answer A.

Below, we have analyzed a sequence from a discussion on the results of the poll, carried out in a 5th-grade class. After having received the poll worksheet, students had 5 minutes to answer, organized in pairs or groups of three. The discussion started showing the global bar chart diagram and the list of students' answers together with their timing on the IWB: 18% of the couples answered A ("after 120s") and 81% provided the correct answer, C ("after 100s").

At the beginning, the discussion focused on the result of the poll. Several students declare that they thought C was the right answer. Initially the teacher and the researcher asked the students who chose Answer A to explain why they chose it. In commenting the results of the poll, students were invited to reflect on their own reasoning, thus being given the chance to highlight possible reasons for making mistakes. Students were therefore activated as owners of their own learning (*FA Strategy E*). Some of those who gave answer A said that they had not read the question carefully and had not realized that the question asks the exact moment when Tommaso reaches the bus stop. Others declare that they were deceived by the fact that, on the horizontal axis, the last value represented is exactly 120 seconds.

The discussion was then focused on students' justifications of the choice of the correct answer (in this case, C) and on the comparison of the ways in which students justified this choice. The teacher noticed that the group Rodolfo-Marianna-Valeria went through a lot of discussion before answering because they were undecided between B and C. Since Marianna and Valeria wanted to choose answer B and Rodolfo convinced them to choose C, Rodolfo was asked to explain how he managed to convince his classmates. Rodolfo approached the IWB and provides his justification:

130 Rodolfo: So... he starts out from 50s (*with his finger, he traces the segment from point (0,0) to point (50,100)*). Then we add 20s (*with his finger, he traces the segment from*

1 point (50,100) to point (70, 40) – that is, 50 plus 20 equals 70. Then he moves and
2 you arrive at 100 (with his finger, he traces the segment from point (70,40) to
3 point (100, 160)). So, we add other 30s. 70 plus 30 is 100. And then he stops

4 131 Researcher: So you split the time ... You say, “he takes 50s to finish the first part.” (She
5 points, with two fingers, at the two ends of the segment (0,0) and (50,0)). To
6 complete the second (part) (she points, with two fingers, at the two ends of the
7 segment (50.0) and (70.0)), it takes him...

8
9 132 Rodolfo: 20

10 133 Researcher: And we get to 70... To complete the third (she points, with two fingers, at the
11 two ends of the segment (70,0)-(100,0)) it takes him...

12
13 134 Rodolfo: 30

14 135 Researcher: ...and I get to 100... And why are you really sure that... (indicating point
15 (100,160))?

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17 136 Rodolfo: Because then he stops and doesn't walk anymore

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19 137 Researcher: So, you're saying: “if he stops after this, it means that he has reached the bus
20 stop for sure.”

21
22 This brief sequence highlights how Rodolfo activated himself as the owner of his own learning
23 (*Strategy E*). In fact, when he spoke (line 130, in particular), he clearly presented the reasoning he
24 constructed to identify the correct answer. This way of reasoning represents a possible approach to
25 facing this task – i.e. determining the time spent by Tommaso to reach the bus stop as a sum of the
26 times corresponding to the different sections into which the graph could be subdivided (the time
27 spent by Tommaso to reach a distance of 100m from home, the time spent to go back to a distance
28 of 40m from home and the time spent to reach the bus stop starting from this point). The researcher
29 guided Rodolfo in making his reasoning more explicit (lines 131-133-135-136), so that Rodolfo
30 could be also activated as an instructional resource for his classmates (*Strategy D*).

31
32 The researcher repeated Rodolfo's claim (line 136) in order to focus the students' attention on a
33 possible source of misconception: the idea that Tommaso reached the bus stop because there is a
34 horizontal line that means that he is not moving. In the subsequent part of the discussion, this
35 misconception became more evident when another student, Elsa, raised her hand to suggest a
36 different way of approaching this question. She declared that the solution “100s” could have been
37 found as the correct answer not by “adding the times”, but simply by looking at the point (100,160):
38 since this is the point corresponding to the moment when Tommaso stops. It is also the moment in
39 which Tommaso reaches the bus stop.

40
41 The researcher makes students observe that both Elsa and Rodolfo have identified (100,160) as the
42 point that represents when Tommaso stops, and that, for this reason, they have considered that this
43 point also represents when Tommaso arrives at the bus stop. Then she proposes a challenge to the
44 students:
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46
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48 148 Researcher: But then, let me ask everybody something. Because, in another class, this what
49 came out, “‘After, he stopped’ means he arrived at the bus stop.” But there was a
50 child who said “what if, instead, he stopped to tie his shoes? Who can tell me that
51 he was exactly at the bus stop?”

52
53 149 Sabrina: Over there (pointing at the graph on the IWB).

54
55 150 Researcher: You come too to show it to us.

56
57 *Sabrina goes up to the IWB.*

58 151 Sabrina: Because, there, he is at 160m (with her finger, she traces the segment from the
59 point (120,160) to the point (0, 160)) and, above (pointing to the text of the
60 problem), it tells you that the path between his home and the bus stop is 160m.
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152 Researcher: Did you hear what she said?

153 Chorus: Yes.

154 Teacher: He did not stop before, Sabrina says, to tie his shoes, but he arrived at 160m. And the text says “every morning ... a distance of 160m”. So, she says, so he stopped at the bus stop! ... Are you with her?

155 Chorus: Yes!

The researcher challenged the class (line 148) in order to activate the students *as owners of their own learning (Strategy E)*. This challenge involving an answer proposed by a student from another class focused the discussion on one specific aspect, putting into practice one of the suggestions of Stein (2008). This challenge is connected with Rodolfo and Elsa’s previous remarks (“Tommaso reached the bus stop because there is a horizontal line, which means that he is not moving.”) in order to give the students feedback (FA Strategy C): seeing a horizontal line is not enough, the most important thing is that 100s correspond to the moment when Tommaso is at 160m from home.

Sabrina (line 151) showed that she understood the problem that the researcher wanted to highlight. She showed what part of the graph she focused on and referred to the sentence in the text that was a key to the problem. She was therefore activated as an *instructional resource for her classmates (Strategy D)*.

7 Conclusion

This paper relies on our analysis of the design and implementation of specific categories of digital resources tailored for FA: problem worksheets, helping worksheets, and poll worksheets. In Section 4, we illustrated how our assumptions and theoretical tools guided our specific choices of the design of three categories of digital resources.

Our fine-tuned data analysis gave us new insights in terms of principles for the design and use of digital resources to support FA practices: they are detailed in Table 2 for each category of worksheets.

Table 2: Guidelines emerging from data analysis

Guidelines for the design and implementation of the digital resources	
PROBLEM WORKSHEETS	
Characteristics of the digital resources	- They consist in doc files that contain open questions.
Use of digital resources during classwide activities to help activate formative assessment processes	- They are sent to the students who work in pairs or small groups (<i>sending</i>).
	- The teacher collects the students’ answers, selects and groups some of them to be displayed on the IWB (<i>sending and displaying</i>).
	- The <i>selection and sequencing</i> of the answers to be displayed determine the focus and the planning of the subsequent classroom discussion (<i>FA Strategy B</i>). <i>Specific questions</i> can be posed to students, according to the teacher’s aims: (a) to focus on students’ mistakes; (b) to focus on the strategies adopted to process the task (metacognitive level); (c) to make students compare their answers and their justifications to these answers.
	- The different ways in which <i>the teacher displays (displaying)</i> students answers lead to <i>different ways of focusing the discussion</i> : a) showing the answers one by one could enable students to reflect on their mistakes (<i>FA Strategy E</i>) or provide feedback to the authors of the

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	<p>answers that is displayed (<i>FA Strategies C and D</i>);</p> <p>b) showing a group of answers could lead students to reflect at a metacognitive level;</p> <p>c) the answers could also be displayed in a disorganized way for the students to group them according to criteria they or the teacher choose. The discussions carried out in (b) and (c) makes students highlight similarities/differences among answers and reflect on them, becoming instructional resources for their classmates (<i>FA Strategy D</i>). In this way, they could also interiorize effective strategies to be used when they will face similar tasks and identify general criteria to plan and assess their answers (<i>FA Strategy A</i>).</p>
HELPING WORKSHEETS	
Characteristics of the digital resource	<p>- They consist in doc files, aimed at providing students with a support to face the tasks within problem worksheets.</p> <p>- The “help” provided to students could consist of: (a) additional questions; (b) a sequence of questions; (c) reminders; (d) tools to support students’ reasoning (tables, graphs, symbolic or numerical expressions, etc).</p>
Use of digital resources during classwide activities to help activate formative assessment processes	<p>- The teacher can send the digital resource (<i>sending</i>) to:</p> <p>(a) specific groups of students to support those who face difficulties;</p> <p>(b) all the groups of students to make them check the correctness of the answers.</p> <p>In this way the teacher activates <i>FA Strategy C</i>, providing feedback to students with the main aim of activating them as owners of their own learning (<i>FA Strategy E</i>).</p> <p>- The helping worksheets could also be displayed (<i>displaying</i>) during classroom discussions to make students reflect on the reasons why a specific “help” was given:</p> <p>(a) the students who did not receive the worksheets have the chance to reflect on the role played by the help provided, becoming instructional resources for their classmates (<i>FA Strategy D</i>);</p> <p>(b) the students who received help can discuss how they used it, making their reasoning explicit (<i>FA Strategy E</i>).</p> <p>Thanks to this focused discussion, all the students receive feedback from the teacher and their classmates (<i>FA Strategy C</i>) and have the chance to clarify the learning intentions associated with the worksheet (<i>FA Strategy A</i>).</p>
POLL WORKSHEETS	
Characteristics of the digital resource	<p>- They consist in doc files to be displayed on the IWB (<i>displaying</i>). They contain a multiple-choice question and sets of possible answers. Students are asked to choose one of them. The answers are gathered and processed by the technology (<i>processing and analyzing</i>).</p> <p>- The focus of the polls could vary according to the corresponding aims: (a) focus on a mathematical task; (b) focus on the effectiveness of strategies or on the completeness of the answers (meta-cognitive reflections); (c) focus on students’ difficulties; (d) focus on affective aspects.</p> <p>When the poll worksheets are redesigned, the identification of set of possible answers in (a) and (b), could be informed by the answers previously collected through problem worksheets.</p> <p>- If the feedback is not directly provided by the digital technology (our choice), students are guided to focus mainly on the reasons underlying</p>

	their choice of specific answers.
Use of digital resources during classwide activities to help activate formative assessment processes	- The poll worksheets could be used to assess whether the students understood some key-features of the lesson (<i>processing and analyzing</i>), making both them and the teacher become aware of where they are in their learning.
	- The poll worksheets could be used to activate a focused discussion on the answers given by students, starting from the displaying of the results of the poll (<i>processing and analyzing</i> , and <i>displaying</i>): (a) at the beginning of the discussion the students who gave the incorrect answers are asked to explain their choice, reflecting on the mistakes they made (<i>FA Strategy E</i>); (b) then, students who gave the correct answer are asked to justify their choice. In this way, the strategies are shared and discussed within the class (<i>FA Strategies E, D, C</i>); (c) all the students could be asked to focus on comparing different ways of processing the task, with the aim of making them expand the “repertoire” of possible strategies to adopt when facing similar problems; (d) when groups of students say they had problems agreeing on the answer, they could be asked to share their doubts and problems with all their classmates, so that potential misconceptions can surface and, consequently, feedback can be given to students at different levels (<i>FA Strategy C</i>).

Through our overall data analysis, we gained evidence that the use of the digital worksheets within a CCT environment was efficient in promoting FA strategies in classwide mathematics activities, in particular in supporting the activation of all the five FA strategies envisaged and in fostering the involvement of the three main agents (the teacher, the peers, the learner). This answers our research question concerning the role of digital worksheets in fostering formative assessment.

Further research is needed, especially about two important points. First, the potentials and limits of the different ways of displaying the students’ answers from problem worksheets. When is one way better than the other? Second, the displaying of the helping worksheets provided a suitable way of fostering students’ reasoning at a meta-level; yet, deeper research is still needed about the actual support that helping worksheets may give during the group-work phase. As a matter of fact, helping worksheets are prepared in advance and may not meet specific students’ needs during the activity. On the one hand, they are digital resources in the teacher’s hands to help students, but students are the ones who are responsible for the answers. On the other hand, the number of helping worksheets that a teacher can manage is limited, and so they may not be tailored to difficulties that come up. On a more general plane, the efficiency of the digital worksheets relies on two connected aspects: 1) digital worksheets are inserted in a teaching sequence, which encompasses the alternation and integration of a variety of worksheets; 2) the teacher plays a crucial role in dealing with the resource, managing the whole teaching sequence and orchestrating the discussions.

As regards (1), on-going data analysis suggests a fruitful synergy in the coordinated and complementary use of different types of worksheets in a sequence of lessons or even within the same lesson.

As regards (2), we are aware that the presence of the researcher during our design experiments was a key component in the development of the sessions. This is a typical feature of design-based research. The researcher may fade away in further experiments, once the teacher becomes expert in the developed methodology, and fully aware of the theoretical assumptions of the project. A further development is therefore the promotion of teacher education focused on teachers’ mastering digital resources (Ruthven, 2013) in their ever evolving forms.

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