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HIghlights

- The management of elective and non-elective surgery is debated in the literature
- Literature reports conflicting results depending on the operative conditions
- We propose a hybrid and flexible model powered by online optimization algorithms
- Different stakeholder perspectives are taken into account defining a set of indices
- Dedicated, shared and hybrid policies are evaluated through a quantitative analysis

The management of non-elective patients: shared vs. dedicated policies

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Abstract

The approaches for the management of elective and non-elective surgery can be classified with respect to the choice of sharing or not the operating theater. The *dedicated operating room* policy consists in reserving, each day, one or more operating rooms to perform only non-elective surgeries. Conversely, the *shared operating room* policy allows to perform elective and non-elective surgeries in the same operating room session. Furthermore, hybrid policies are defined providing, each day, both dedicated and shared operating rooms. The issue of adopting one of these policies is debated in the literature and they all could be the best policy depending on the scenario and the operative conditions. In this paper we propose a hybrid and flexible model to deal with the surgery process scheduling of both elective and non-elective patients, in which new online and offline optimization algorithms are introduced, taking into account both patient- and facility-centered objectives. The aim of this paper is to provide a detailed comparison among different policies taking into account several scenarios and operative conditions in such a way to consider the characteristics of the operating theater and those of the patients it serves.

Keywords: Surgery process scheduling, Elective surgery, Non-elective surgery, optimization, discrete event simulation

1. Introduction

An elective surgery is a planned and non-emergency surgical procedure. It may be either medically required (e.g., cataract surgery), or optional (e.g., breast augmentation or implant) surgery. The patients requiring an elective surgery are inserted in a (usually long) waiting list and are scheduled through an ex-ante planning in such a way to ensure an efficient use of the resources in accordance with several priority rules.

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At the operational decision level, such a problem is called "surgery process scheduling" and is generally divided into two sub-problems referred to as "advanced scheduling" and "allocation scheduling". The former consists in selecting patients from the waiting list and assigning a specific surgery and operating room (OR) session to each patient over a certain planning horizon [1–9] trying also to take into account different stakeholder perspectives [10–14]. Given this advanced schedule, the latter deals with the sequencing of the surgical activities and the resource allocation for each OR session [15–18]. For a complete overview of the problems arising in the OR planning and scheduling, the reader can refer to the exhaustive reviews [19–21].

Conversely, due to the patient medical conditions, a non-elective surgery is an unpredictable surgery that should be performed within a time limit, which is shorter than that of an elective surgery. For this reason, non-elective patients cannot be inserted in the waiting list and scheduled through an ex-ante planning. Because of their unpredictability, the non-elective patients arrivals are therefore a further element of uncertainty, in addition to the stochasticity involving an elective surgery, whose most impactful component is its duration [22–24].

Non-elective surgeries deal with different time limits involving different goals: non-elective patients with a time limit of 30 minutes must be operated on as soon as possible while, when the time limit is equal to several hours, one can evaluate what is more beneficial between an immediate surgery or to postpone it waiting for the release of further ORs. An immediate surgery can determine a negative impact on the elective patient scheduling. To limit or to avoid such a negative impact, the surgery can be postponed increasing the risk of exceeding the time limit for the non-elective. Such a trade-off should be taken into account when scheduling a non-elective surgery.

In accordance with the analysis of 31 papers reported in the literature review [25], the policies for handling elective and non-elective patients are classified into dedicated, hybrid, and shared (or flexible). The Dedicated Operating Room (DOR) policy consists in reserving, each day, one or more OR sessions to perform only non-elective surgeries. Conversely, the Shared Operating Room (SOR) policy allows to perform elective and non-elective surgeries in the same OR sessions. Furthermore, a hybrid policy is a mix of the two previous policies providing both dedicated and shared ORs. The issue of adopting one of these policies is debated in the literature. In [26] and [27] the DOR and the SOR policies are respectively promoted and the improvement of the non-elective waiting times is proved in both papers. Same remarks are reported in [28] in which different hybrid policies are evaluated.

Since the conflicting conclusions reported in these papers could depend on the scenario and the operative conditions, a detailed comparison among the different policies is required. However, only two papers [27, 29] out of the 31 provide a (partial) comparison between different policies. The aim of this paper is to provide such a detailed comparison among different policies taking into account several scenarios and operative conditions in such a way to consider the characteristics of the operating theater and that of the patients it serves.

In our previous work [30], a hybrid simulation and optimization model for an elective patient flow has been proposed. Such a model deals with the surgery process scheduling including an online algorithm for the Real Time Management (RTM) of operating rooms: the RTM is the decision problem arising during the fulfillment of the surgery process scheduling, that is the problem of supervising the execution of such a schedule and, in case of delays, to take the more rational decision regarding the surgery cancellation or the overtime assignment [31].

Exploiting the same modeling framework depicted in [30], in this paper we propose an extended hybrid and flexible model to deal with the surgery process scheduling (including the RTM) of both elective and non-elective patients, in which new online and offline optimization algorithms are introduced in such a way to deal with the trade-off between the scheduling of elective and non-elective patients, taking into account both patient- and facility-centered objectives. The aim of this paper is therefore to exploit such a hybrid model to enable the analysis and the comparison of the DOR and the SOR policies. Further, we provide a tool capable to support the decision process in the surgery process scheduling. Indeed, the generality of the proposed model allows us to replicate and to compare a wide range of possible scenarios and policies, in which most of the case studies of the literature can be included.

The paper is organized as follows. The problem statement is presented in Section 2 while the online and offline algorithms for the optimization of the DOR and SOR policies are described in Section 3. The computational environment is defined in Section 4 describing scenarios, configurations and performance indices. Based on this environment, a comprehensive quantitative analysis is reported in 5: we evaluate the DOR, the SOR and the hybrid policies determining, for each policy, the best configuration with respect to the considered scenario; then, we use such configurations in order to compare the three policies in such a way to derive some insights in terms of supporting decision making; finally, the analysis also proves the effectiveness of the proposed approaches. Section 6 closes the paper.

2. Problem statement and literature review

An OR session (j, k) identifies a specific OR j of the set J of all the available ORs that have been assigned by the Master Surgical Schedule to one specialty in the k-th day of the planning time horizon, whose days are included into the set K. Given the set $S \subseteq J \times K$ of all the OR sessions assigned to a single specialty, the *elective surgical schedule* is represented by a set Λ of $n = |S|$ ordered sequences of elective patients belonging to the waiting list I. Such sequences are defined during the surgery process scheduling into two steps:

advanced scheduling: a set of patients $L \subseteq I$ is partitioned into n subsets L_{jk} corresponding to the patients that should be operated on within $(j, k) \in S$;

allocation scheduling: for each OR session $(j, k) \in S$, all the m_{jk} patients in L_{jk} are listed in an ordered sequence $\lambda_{jk} = (i_1, \ldots, i_{m_{jk}})$.

The former step is performed at the beginning of the planning horizon by the advanced scheduling, taking into account: (i) the estimation e_i of the surgery duration of the elective patients $i \in L_{jk}$ (called Estimated Operating Time – EOT) with respect to the duration d_{jk} of the OR session (j, k), and (ii) the waiting time t_i that should be less then or equal to the time limit $t_i^{\max}$, also called Maximum Time Before Treatment (MTBT). Such time limit is determined by the *diagnosis related group* of the patients, which indicates a general maximum time, and by the *urgency related group* assigned to the single patient, accordingly with his/her urgency [32]. In other words, the urgency related group allows to define a partition of the patients in the same diagnosis related group in order to prioritize their surgical operation.

The latter step is performed at the beginning of each OR session by the allocation scheduling, taking into account that the patients placed at the beginning of the sequence λ_{jk} will be exposed to less risk of cancellation than the patients at the end, due to delays or arrivals of non-elective patients. Hereafter, for simplicity the patients $i_1, \ldots, i_{m_{jk}} \in L_{jk}$ will be denoted with their position $1, \ldots, m_{jk}$ in the schedule.

The main objectives for scheduling the elective surgeries are the maximization of the OR utilization and the minimization of the cancellations, especially those concerning the exceeding of the MTBT [20, 32]. The conflict-proneness of these objectives is amplified by two uncertainty factors, which are the variability in the actual duration r_i of the surgery (Real Operating Time – ROT) and the inclusion of non-elective surgeries (only for shared ORs) [33].

To deal with such an uncertainty, the RTM of the ORs is performed during the execution of each OR session, taking decisions to achieve the objective fixed during the advanced and allocation scheduling with the information available at that time, that is delays in the ORs, elective and non-elective patients waiting for a surgery and the remaining amount of overtime. We observe that not always patients close to the MTBT are placed at the beginning of the OR sessions, but allocation scheduling rules are often adopted to maximize the OR utilization or to minimize the cancellations, although their impact has very assorted results in the literature [30, 33–38]. In such case the role of the RTM could be more considerable to avoid the exceeding of the MTBT [31].

Supposing that the first m patients of the sequence λ_{jk} have been already operated on at the instant τ, a delay occurs when

$$\rho_{jk}^{\tau} + \sum_{i=m+1}^{m_{jk}} e_i > d_{jk} \tag{1}$$

where $\rho_{jk}^{\tau} = \sum_{i=1}^{m} r_i$ is the time elapsed between the start of the OR session and the moment τ and $\sum_{i=m+1}^{m_{jk}} e_i$ is the total expected duration of the remaining surgeries. In that case the sequence λ_{jk} is not feasible without exceeding the session duration d_{jk}, then two different changes can be possible:

- re-definition of the sequence λ_{jk}: (i) changing the order of the patients in the last $m_{jk} - m$ positions, and/or (ii) canceling and postponing the last $m_{jk} - m$ patients of the sequence;

- assignment of a part ν_{jk} of the amount ν of available overtime shared with all the other OR sessions scheduled in the planning horizon.

Further issues occur when elective and non-elective patients are managed at the same time considering the DOR or the SOR policies. Since the DOR policy allows us to consider separately the two flows of patients, the elective patient flow is managed considering $S_E \subset S$, that is the set of OR sessions dedicated to the elective surgery, as we proposed in [30], while the non-elective surgery flow is simply managed in the remaining and dedicated OR sessions. In addition to the decisions regarding the management of the elective patients, the DOR policy imposes a further decision, that is how many OR sessions should be allocated for elective and non-elective surgery.

Table 1: Categorization of non-elective patients with respect to their time limit (extracted from [25]).

Category	Time Limit	Category	Time Limit
Trauma	30 minutes	*Emergent*	from 30 minutes to 24 hours
Urgent	from 4 to 24 hours	*Semi-urgent*	from 8 hours to 3 days
Add-on	24 hours	*Work-in*	from 24 hours to 3 days

Non-elective surgeries should be performed within a time limit that varies in accordance with their urgency, as reported in Table 1. Because the insertion of non-elective patients can have a negative impact on the elective patient scheduling, an appropriate handling of non-elective patients could significantly improve the performance of the two policies. Under the SOR policy, in [25] two classes of methods have been identified to deal with the *non-elective insertion*, that is the *slack management* and the *break-in-moment optimization*.

When elective and non-elective surgeries are performed in the same ORs, the schedule of the elective patients should take into account the possible insertion of non-elective patients during the execution of the OR sessions. If the whole session capacity is allocated to plan elective patients, such insertions will cause an overload that involves an higher demand of overtime, which generally is a scarce resource. Therefore slack management policies are introduced to avoid the increase of the cancellations [23, 39–41]. Different policies are obtained on the basis of two decisions, that is (i) the total amount b_{jk} of time reserved during the elective advanced scheduling and (ii) the distribution of the slacks within the schedule. Note that such decisions deal with the trade-off between cancellations and OR utilization, as well as having a different impact on the two flows of patients.

To the best of our knowledge, only [42] deals with the Break-In-Moment (BIM) optimization problem while the real time version of the problem is not yet studied. In [42], the allocation scheduling is performed focusing on the Break-In-Moment (BIM), that is each moment during the day in which one of the OR session is released by a patient, becoming available for the allocation to the next patient.

Let ι be the BIM in which the m-th patient leaves the OR session (j, k) and let Q^ι be the set of the non-elective patients waiting for an insertion at that instant. If $Q^\iota \neq \emptyset$, then the sequence λ_{jk} could be modified inserting one of such non-elective patients $i^{ne} \in Q^\iota$ at the position $m+1$ and shifting of one position the last $m_{jk} - m$ patients. Otherwise the sequence λ_{jk} could remain unchanged and the non-elective patients will wait the next BIMs for the insertion. The BIM optimization consists in determining the set $\Lambda_k = \{\lambda_{jk}\}_j$ that minimizes the time between two consecutive BIMs, called Break-In-Interval (BII), that involves a lower waiting time for the non-elective patient. The information available for the computation of the BIMs is the EOT of the patients $i \in L_{jk}$. All the notations introduced in this section are summarized in Table 2.

Table 2: Summary of the notation introduced in the problem statement.

Sets	
J: set of operating rooms	K: set of the days of the week
S: set of all OR sessions	S_E: set of OR sessions dedicated to elective patients
I: set of patients in the pre-admission waiting list	L: set of scheduled patients
L_{jk}: set of patients scheduled into the OR session (j,k)	λ_{jk}: sequence of the patients scheduled into (j,k)
Λ: set of all the sequences λ_{jk}	Λ_k: set of all the sequences λ_{jk} of the day k
Q^ι: set of non-elective patients waiting at the instant ι	
Indices and cardinalities	
j: index of the operating room	k: index of the day
i: elective patient	i^{ne}: non-elective patient
n: number of OR sessions	m_{jk}: number of patient scheduled into (j,k)
Times and durations	
τ: general instant during the OR session	ι: instant corresponding to a BIM
ρ^τ_{jk}: time elapsed since the beginning of (j,k)	d_{jk}: duration of (j,k)
t_i: waiting days to surgery of the i-th patient	$t_i^{\max}$: MTBT of patient i
e_i: EOT of patient i	r_i: ROT of patient i
b_{jk}: slack assigned to the OR session (j,k)	

This problem requires to be addressed during the allocation scheduling since the only way to change the BIMs is to determine an alternative surgery sequencing. Figure 1 reports an example of scheduling with three ORs planned for the day k, reserving slacks in two of them. We supposed to have OR sessions with the same duration and starting at the same time. Each gray rectangle represents the surgery of an elective patient that has been placed according to λ_{jk}. The length of the rectangle expresses the EOT of the corresponding patients, causing different BIMs corresponding to all the OR releases during the day k. Two consecutive BIMs have been indicated with a dashed vertical line: their distance of time determines one of the BIIs. From a real time management perspective, the uncertainty can change the BIMs determining the need of an online resequencing.

In addition, the non-elective insertion problem should consider also the real time decision of the OR sessions in which the surgery of the non-elective patient should be inserted: the decision should reach a good trade-off between the waiting time of the non-elective patients and the cancellation of the elective surgeries. The literature analysis reveals that such a decision is not considered, and for this reason we propose two online algorithms in Section 3.2.3.

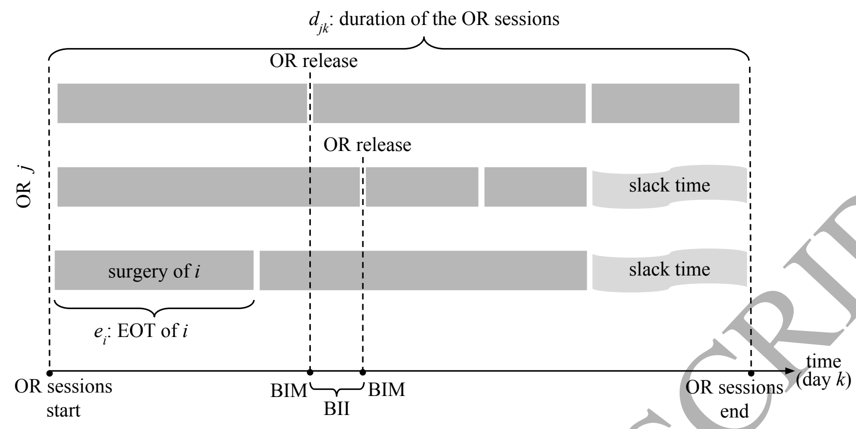


Figure 1: Slacks, BIMs and BIIs – example of configuration with three OR sessions.

3. Elective and non-elective optimization

The issues introduced in Section 2 can be grouped into two classes of optimization problems. The former is the class of *Elective-Oriented Optimization* (EOO) problems, involving all the phases of determining the elective surgery scheduling, that is selecting a set of patients from the waiting list, distributing and ordering them in all the available OR sessions and taking decisions regarding their cancellation or rescheduling during the real time management. The latter class of problems is called *Non-elective-Oriented Optimization* (NOO) problems, referring to the issues that arise when dealing with the arrival of non-elective patients, that is the choice of the moments and the OR sessions in which inserting such patients, but also all the decisions about the elective surgical schedule oriented to promote the insertion of non-elective patients (e.g., the BIM optimization). We observe that the latter class makes sense only when the SOR policy or hybrid policies are adopted, because of dedicating ORs to non-elective patients does not pose these issues: using the DOR policy, non-elective are basically inserted in specific ORs taking into account the urgency and the time of arrival.

3.1. Elective-Oriented Optimization

The EOO problems have been already studied in our previous works [30, 38]. These works determined the approaches providing the overall best performance to deal with a flow of only elective patients. In order to make the paper self-contained, we briefly present such approaches that are used in the quantitative analysis in Section 5.

Advanced scheduling: A metaheuristic divided into two phases:

1. a greedy algorithm selects patients i one by one from I in decreasing order of $w_i = \frac{t_i}{t_i^{\max}}$ and inserts them in L_{jk} if (j, k) is the first OR

session from Monday to Friday such that the available operating time is sufficient according with the EOT e_i and the duration d_{jk};

2. a local search algorithm starts from the schedule determined by the greedy algorithm and tries to improve it by exchanging pairs of scheduled patients belonging to two different OR sessions in such a way to release larger OR blocks in some OR sessions $(j, k) \in S$ and to allow us the insertion of further patients in L_{jk}.

Allocation scheduling: A LPT (Longest Processing Time) algorithm modified in such a way to give different priorities to the patients in λ_{jk} as follows:

1. we first schedule those patients close to their MTBT that can not be postponed to the next week avoiding to exceed such a time limit, that is patients $i \in L_{jk}$ such that $\tilde{w}_i = \frac{t_i+7-k}{t_i^{\max}} > 1$ in which $7-k$ are the days to the next week; such patients are scheduled in decreasing order of $\tilde{w}_i$ at the beginning of the OR session;
2. patients previously postponed with $\tilde{w}_i \leq 1$ are scheduled in decreasing order of the number of cancellations afterwards;
3. finally, all the remaining patients in L_{jk} are scheduled using the LPT rule with respect to e_i at the end of the OR session.

RTM – Sequencing check: The sequencing of the non operated patients of λ_{jk} is checked in such a way to ensure that (i) all the patients close to MTBT are scheduled prior to the other patients; (ii) if such patients run out the available operating time then a number of patients is selected in such a way to fill the available operating time following a rule similar to the Best Fit rule for the Bin Packing problem [43].

RTM – Overtime allocation: If the overtime available is sufficient to avoid the cancellation of a patient, then the overtime is allocated if such a patient is close to the MTBT. Otherwise the overtime is allocated to the patient i, scheduled in the OR session (j, k), if and only if the following criterion is satisfied:

$$\left(1 + \frac{\sum_{h \leq k} n_h}{n} - \frac{\nu_k^\tau}{\nu}\right)\left(\frac{e_i + \rho_{jk}^\tau}{d_{jk}}\right) \leq 1, \tag{2}$$

where n_h is the number of OR sessions in the day h and ν_k^τ is the already allocated amount of overtime to the OR session (j, k) until the instant τ.

3.2. Non-Elective Oriented Optimization

As discussed in Section 2, the problem of inserting non-elective patients can be tackled with three methods, that is the slack management, the BIM optimization, and the *Non-Elective Real Time Insertion* (NERTI).

3.2.1. Slack management

Before scheduling the elective patients, there are two different choices regarding the slack management that should be taken. The first decision is in which OR session to provide a slack, which means to decide the number $n_s < n_k$ of ORs that will contain a slack during the day k. The second decision is about the fraction π of time to reserve in each of those OR sessions with respect to their total duration. The couple of parameters (n_s, π) will indicate that a slack of duration πd_{jk} has been reserved in each of the OR sessions (j, k), with $j = n_k - n_s + 1, \ldots, n_k$ and $k \in K$, while the OR session (j, k), with $j \leq n_k - n_s$ are entirely available during the advanced scheduling.

3.2.2. BIM optimization

With BIM (or BII) optimization we refer to the problem of sequencing the surgery of scheduled patients within the OR sessions of the day k in such a way to minimize the waiting time of possible non-elective patient arrivals. The problem has a strong stochastic component because of the unpredictability of the non-elective and their characteristics, that is the time of arrival, the surgery duration and the urgency (with the corresponding time limit). Although in literature such a problem is addressed before the beginning of the OR sessions, that is during the allocation scheduling, we also take into account the possibility of optimizing the BIIs configuration during the execution of the OR session, in such a way to exploit the updated information, that is the Real Operating Time (ROT) r_i of the patient i operated on (instead of the estimation e_i) and the insertion of non-elective surgery already performed.

We propose the *Break-In Layout Local Search* (BILLS), an algorithm inspired to that proposed in [42], but capable to deal with the elective patients close to their MTBT. The algorithm tries with a local search to improve an initial solution $\Lambda_k = \{\lambda_{jk}\}_j$ exchanging pairs of patients in the same sequence λ_{jk} in such a way to minimize an objective function accounting for the waiting time of the elective patients. The algorithm ends when there is no improvement of this function in the neighborhood. We propose two alternative objective functions to

$$z_1 = \max_{m \geq 1} (\iota_m - \iota_{m-1}) \tag{3}$$

$$z_2 = \frac{1}{d} \int_0^d \beta(t) dt \tag{4}$$

where ι_0 is the instant in which the OR sessions begin, ι_m are all the instants corresponding to all the other ordered BIMs ($m = 1, 2, \ldots$), d is the duration of the OR sessions of the day and $\beta : [0, d] \rightarrow [0, d]$ is the function which associates to each instant the time remaining to the release of the next OR. The former objective function represents the longest time interval between two consecutive BIMs. The latter is the average value of the estimated waiting time with respect to the overall duration of the OR sessions. Note that using deterministic surgery durations (EOTs), equation (4) is equal to

$$\frac{1}{d}\sum_{m\geq 1}\frac{\left(\iota_m-\iota_{m-1}\right)^2}{2}$$

therefore we can define the equivalent, but simpler, objective function

$$z_2' = \sum_{m\geq 1}\left(\iota_m-\iota_{m-1}\right)^2.$$

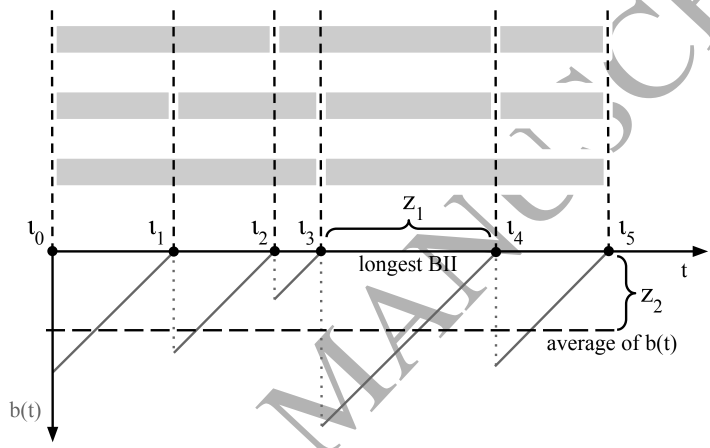


Figure 2: BIM optimization: computation of the objective functions z_1 and z_2 (we supposed that all the OR sessions begin at the same instant and that $d_{jk} = d$ for all (j, k)). In the figure, the decreasing function $b(t)$ from ι_i to ι_{i+1} $(i = 0, \dots, 4)$ measures the remaining time to ι_{i+1}, that is the waiting time of a non-elective patients arriving at $t \in (\iota_i, \iota_{i+1})$.

Figure 2 shows an example for the elective surgeries schedule and the corresponding values of the objective functions z_1 and z_2. In the lower part of the figure, the piece-wise linear function $b(t)$ has been obtained starting from the BIMs $\iota_0, \dots, \iota_5$. Note that $b(t)$ is equivalent to $\beta(t)$ when deterministic times are considered.

Since patients close to their MTBT are scheduled by the LPT modified algorithm (reported in Section 3.1) at the beginning of the session to avoid a cancellation, we impose that such patients can not be swapped during the local search.

We use two versions of this algorithm: an offline version will be used for the allocation scheduling at the beginning of each day while an online version will be used every time an operating room is released.

3.2.3. NERTI

The insertion of a non-elective patient within a certain OR session determines (i) the shift of the remaining elective surgeries and (ii) the variation of the BIIs configuration determining an effect to the other non-elective patients. Such modifications can have an impact that should be considered. With NERTI we refer to the problem of deciding when and in which OR session a non-elective patient could be inserted, which requires an online approach because such a patient could arrive in any instant during the day asking for a surgery within a short time limit. Such a decision could determine (i) a different need of overtime or the cancellation of the elective patients previously scheduled and (ii) longer waiting times of further more urgent non-elective patients (still not arrived). To deal with the two different impacts discussed above, we propose two algorithms: the Non-Elective Worst Fit (NEW-Fit) algorithm and the Non-Elective Insertion Criterion (NEIC).

Parameters. We take into account the arrivals of non-elective patients which must be treated within the end of the current day k taking into account different time limits $t_i^{\max} < 24$ hours, as reported in Table 1.

Let S_k be the set of the OR sessions planned on the day k. At the instant ι of the day k, let h be an operating room available after having operated on μ_{hk} patients $i_1, \dots, i_{\mu_{hk}}$. Let $i_{\mu_{jk}}$ be the patient that is still within the OR, with respect to the other OR sessions (j,k), $j \neq h$. Let L^{ι}_{hk} be the set of the waiting elective patient scheduled in $(h,k) \in S_k$, that are ordered in the last $m_{hk} - \mu_{hk}$ positions of the sequence λ_{hk} (i.e., $i_{\mu_{hk}+1}, \dots, i_{m_{hk}}$). Let Q^{ι} be the set of all the waiting non-elective patients at the instant ι. If at that moment the operating room j is available, then the next patient should be selected from $L^{\iota}_{hk} \cup Q^{\iota}$. Note that the problem arises only if $Q^{\iota} \neq \emptyset$. Let us introduce the parameter

$$\epsilon_{\mu_{jk}} = \begin{cases} \max\left(\sum_{i_1,\dots,i_{\mu_{jk}-1}} r_i + e_{i_{\mu_{jk}}} - \rho^{\iota}_{jk}, 0\right) & \text{if } j \neq h \\ 0 & \text{otherwise} \end{cases}$$

that is the estimated time for the next release of (j,k).

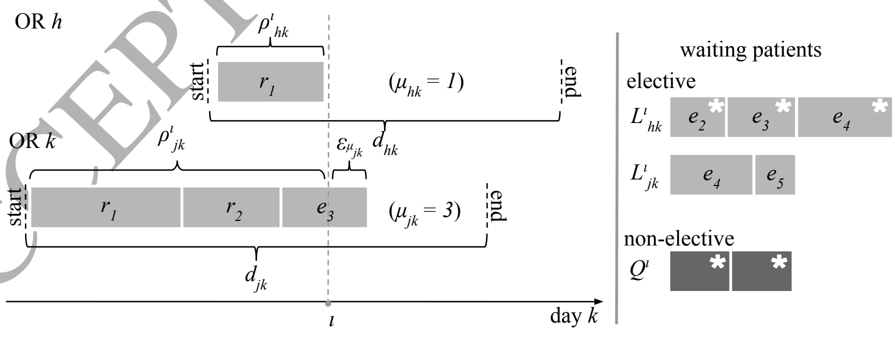


Figure 3: Parameters defined at the releasing of an operating room.

In Figure 3 an example of OR release at the instant ι is reported. In the OR session (h,k) the first surgery is concluded after r_1 minutes, that is the ROT of the first patient, then $\rho^{\iota}_{hk} = r_1$ and $\epsilon_{\mu_{hk}} = 0$. The time elapsed in the OR session (h, k) is equal to the sum of the ROTs of the operated patient plus the time elapsed from the entry of the current patient. The time required for the end of the surgery of such a patient is estimated by $\epsilon_{\mu_{jk}}$ computed using its EOT. The waiting patients that are candidates for the allocation of (h, k) are marked with an asterisk.

NEW-Fit. The algorithm provides an online greedy construction of an alternative schedule of the patients in which we try to insert the non-elective patients in Q^{ι}. On the basis of this auxiliary schedule, the NEW-Fit re-determines the sequence λ_{hk} establishing if to continue with the planned schedule or to insert a non-elective patient as next surgery in (h, k) in such a way to reduce the maximum exceeding time with respect to the duration of the sessions.

Algorithm 1: Non-Elective Worst Fit

Input : δ;
begin
  $p^s \leftarrow i_{\mu_{hk}+1}$; `/* next elective patient` p^e `in` L^{τ}_{hk} `*/`
  $p^u \leftarrow \arg\min_{i \in Q^{\iota}} (t_i^{\max} - t_i)$; $Q' \leftarrow Q^{\iota}$;
  foreach *OR session* (j, k) **do** $L'_j \leftarrow L^{\iota}_{jk}$; $\epsilon'_{\mu_{jk}} \leftarrow \epsilon_{\mu_{jk}}$;
  $S = (i_1, \ldots, i_{\mu_{hk}}, i_{\mu_{hk}+1}, \ldots, i_{m_{hk}})$; flag $\leftarrow$ **false**; stop $\leftarrow$ **false**;
  while $Q' \neq \emptyset$ ***and not*** *stop* **do**
    $p^{ne} \leftarrow \arg\min_{i \in Q'} (t_i^{\max} - t_i)$; $x^\star \leftarrow +\infty$; $j^\star \leftarrow -1$;
    foreach $(j, k) \in S_k$ **do**
      $x = \rho^{\iota}_{jk} + \epsilon_{\mu_{jk}} + \sum_{i \in L'_j} e_i - d_{jk}$;
      if $x < x^\star$ *and* $\epsilon'_{\mu_{jk}} \leq \delta(t^{\max}_{p^{ne}} - t_{p^{ne}})$ **then** $x^\star \leftarrow x$; $j^\star \leftarrow j$;
    if $j^\star = -1$ **then** $S = (i_1, \ldots, i_{\mu_{hk}}, p^u, i_{\mu_{hk}+1}, \ldots, i_{m_{hk}})$; stop $\leftarrow$ **true**;
    if $j^\star = h$ ***and*** *flag* = ***false*** **then** $p^s \leftarrow p^{ne}$; flag $\leftarrow$ **true**;
    $L'_{j^\star} \leftarrow L'_{j^\star} \cup \{p^{ne}\}$; $Q' \leftarrow Q' \setminus \{p^{ne}\}$; $\epsilon'_{\mu_{j^\star k}} \leftarrow \epsilon'_{\mu_{j^\star k}} + e_{p^{ne}}$;
  if *flag* = ***true*** **then** $S = (i_1, \ldots, i_{\mu_{hk}}, p^s, i_{\mu_{hk}+1}, \ldots, i_{m_{hk}})$;
Output: S;

The pseudo-code reported in Algorithm 1 describes the algorithm NEW-Fit, having the parameter $\delta \in (0, 1]$ which is used to define, for each non-elective patient i, an *early deadline* $\delta t_i^{\max}$ until which the insertion can be planned. The early deadline is introduced in such a way to deal with the uncertainty of the surgery duration. When δ is close to 0, NEW-Fit reduces the risk of exceeding the non-elective time limit. On the contrary, when δ is close to 1, the number of feasible insertions increases and better global solutions can be computed. In Section 5, we study the performance of the algorithm varying the value of δ.

After the initialization of the auxiliary data structures, the algorithm starts a loop to determine the auxiliary schedule. At each iteration, the current non-elective patient p^{ne} is scheduled on one of the OR sessions (j, k) such that the condition of the early deadline in correspondence of the instant of insertion

$$\epsilon'_{\mu_{jk}} \leq \delta(t^{\max}_{p^{ne}} - t_{p^{ne}}) \tag{5}$$

is satisfied, where $\epsilon'_{\mu_{jk}}$ is equal to $\epsilon_{\mu_{jk}}$ plus the sum of the EOTs of the non-elective patients planned in (j, k) in the previous iterations. The algorithm selects the OR session that minimizes the difference between the estimated total duration of the operated and non-operated patients in L_{jk}

$$\rho^{\iota}_{jk} + \epsilon_{\mu_{jk}} + \sum_{i \in L'_j} e_i \tag{6}$$

and its duration d_{jk}. Such a rule corresponds to insert the patient p^{ne} in the OR session with the maximum unused OR time in such a way to minimize the overtime demand, when d_{jk} is greater than (6). The aim is to balance the workload among the OR sessions. At a certain iteration, if the condition (5) is not satisfied for any OR session of the day, it means that we are not able to plan all the non-elective patients before their early deadlines, then the NEW-Fit terminates inserting the most urgent non-elective patient p^u as next operation within the sequence λ_{hk}, that is at the $(\mu_{hk} + 1)$-th position. When all the insertions are feasible within the time limits and at least one non-elective patient has been inserted in the OR session (h, k), the NEW-Fit returns adding at the $(\mu_{hk}+1)$-th position of the sequence λ_{hk} the one with the shortest early deadline. Otherwise, the sequence λ_{hk} remains unchanged and the elective patient $i_{\mu_{hk}+1}$ will be the next to be operated on in (h, k).

NEIC. This algorithm establishes the best BIM for inserting a non-elective patient on the basis of the number of BIMs available up to the end of the OR session. The idea is to schedule a non-elective patient only when a sufficient number of BIMs is available in the next minutes, in such a way to guarantee the insertion of further and more urgent arriving non-elective patients. Let δ be the same parameter used in NEW-Fit. Let $i \in Q^{\iota}$ be the non-elective patient with the minimum value of $t_i^{\max} - t_i$. On the basis of the EOTs of the elective schedule, let $\bar{\iota}$ be the time estimated for the next OR release, which is the first BIM after the time t. Finally, let $\eta(t_0, T)$ be the number of BIMs within a certain interval of time (t_0, T). Then the patient i is inserted in the released OR (h, k) if and only if at least one of the following conditions is satisfied

$$t_i^{\max} - t_i \leq T \tag{7}$$

$$\delta(t_i^{\max} - t_i) < \bar{\iota} \tag{8}$$

$$\frac{\eta(t, t + \delta(t_i^{\max} - t_i))}{\delta(t_i^{\max} - t_i)} \leq \frac{\eta(t, t + \delta T)}{\delta T} \tag{9}$$

otherwise the schedule remains unchanged.

Let i^+ be a possible further patient that arrives right after the entry of a patient in the OR session (h, k), which we have to allocate within the shortest time limit T, that is the worst case for our online problem. The condition (7) is satisfied if the patient i is closer to the time limit than i^+, while the condition (8) is satisfied if patient i can not wait until the next OR release without exceeding the early deadline. In both cases, it is not convenient to optimize the waiting time of further non-elective patients, because of the short time limit of an already waiting patient. The condition (9) is satisfied when the frequency of BIMs in the next $\delta(t_i^{max} - t_i)$ minutes, that is the early deadline of i, is lower than the frequency of BIMs in the next δT minutes, that is the early deadline of i^+. In this case it is better to insert i even if $\delta(t_i^{\max} - t_i) > \delta T$ because there are more frequent BIMs in the next minutes than hereafter.

4. Setting up the computational environment

We performed a quantitative analysis in order to assess the impact of the different policies (the DOR, the SOR and the hybrid policies) and the optimization approaches when they are used separately or jointly.

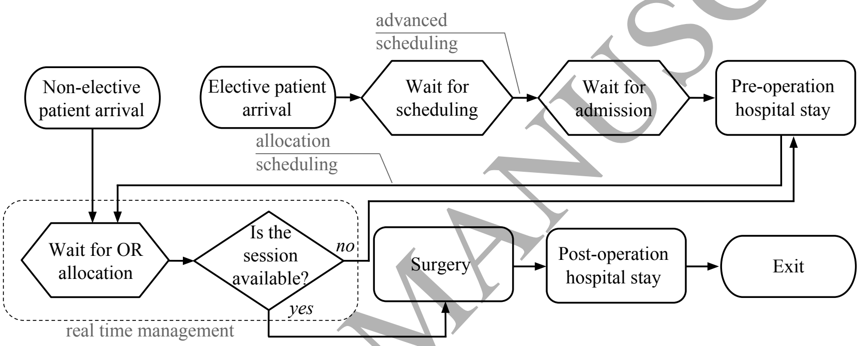


Figure 4: The surgical CP and the optimization problems.

We implemented a Discrete Event Simulation (DES) model that generates single and heterogeneous items belonging to both elective or non-elective patient flows, and makes them move across different activities depending on the respective surgical clinical pathways, which are described in Figure 4. We provided the possibility of adopting both DOR and SOR policy or combining them getting hybrid policies. Further, we embedded our offline and online optimization approaches introduced in Section 3 in such a way to evaluate their impact week by week, that is how the previous decisions (e.g., determining less or more cancellations) can impact on the current decisions.

The choice of using DES is because of its suitability to analyze a discrete and stochastic workflow and its capability to represent single entities allowing the application of our algorithms. The resulting hybrid model has been implemented using AnyLogic 7.1: its Enterprise Library is exploited for the implementation of the DES simulation framework whilst the optimization modules are implemented from scratch in Java, which is the native programming language of AnyLogic.

4.1. Scenarios

We introduce four scenarios S1–S4 in such a way to provide more accurate insights from our quantitative analysis. Such scenarios are obtained by varying the number of OR sessions per day and the distribution of the surgery durations to represent different settings of the operating theater and different characteristics of the patient population, respectively. We fixed the arrival rate of the patients in such a way to match the surgery time per week with the total number of arriving patients per week multiplied for the average surgery duration. Note that the total duration of the OR sessions is just sufficient to operate all the patients if their durations were deterministic and if it were possible to predict the non-elective arrivals and to perform the advanced scheduling with the 100% of the OR utilization. This choice allows us to have four scenarios in which the capacity is adequate to the need of interventions, but extra time is necessary to deal with uncertainty: we provided 30 minutes of overtime per OR session.

Table 3: Parameters characterizing the four scenarios.

Varying parameters

scenario	**capacity**		**EOT distribution**			**patients**	
id	OR sessions	ν	e_0	μ_{EOT}	σ_{EOT}	arrival rate	initial
S_1	10 per day	25 hours	60 min	150 min	60 min	160 per week	400
S_2	5 per day	12.5 hours	60 min	150 min	60 min	80 per week	200
S_3	10 per day	25 hours	30 min	90 min	15 min	266 per week	400
S_4	5 per day	12.5 hours	30 min	90 min	15 min	133 per week	200

Common parameters to all scenarios $S_1 - S_4$

		patient distribution			
		elective (85% of the total)		**non-elective** (15% of the total)	
parameter	**value**	**urgency** (freq.)	**MTBT**	**type** (freq.)	**time limit**
d_{jk}	480 min	A (3%)	8 days	trauma (20%)	30 min
$e_{\max}$	480 min	B (5%)	15 days	emergent (40%)	90 min
q	15 min	C (7%)	30 days	urgent (30%)	3 hours
σ_{ROT}	30 min	D (10%)	60 days	add-on (10%)	24 hours
$r_{\min}$	15 min	E (15%)	90 days		
$r_{\max}$	480 min	F (25%)	120 days		
		G (35%)	180 days		

The main parameters used in the four scenarios $S_1 - S_4$ are summarized in Table 3, in which we adopted the same terminology introduced in Table 1 for the definition of the time limit for non-elective patients, that is *trauma*, *emergent*, *urgent*, *add-on*. According to [44, 45], the EOT of each patient is obtained generating a value with a 3-parameters Lognormal distribution of minimum value e_0, average μ_{EOT} and standard deviation σ_{EOT}, truncated to the maximum value $e_{\max}$. Such values are then approximated to the nearest multiple of a discretization constant q, which models the estimate made by the physician during the pre-operative visit. Once the EOT e_i has been determined, the ROT is generated with a Gaussian distribution with average e_i and standard deviation σ_{ROT}, truncated to the minimum and maximum values $r_{\min}$ and $r_{\max}$. We observe that if m_{jk} patients are scheduled in the OR session (j, k), then the total duration of their surgeries is a random variable with average equal to the sum of the EOTs and standard deviation $\sqrt{m_{jk}}\sigma_{\text{ROT}}$. For instance, if 4 patients with EOT of 75, 105, 120 and 180 minutes are scheduled in an OR session of duration 480 minutes and we fix $\sigma_{\text{ROT}} = 30$ minutes, then the total surgery duration will have average 480 minutes and standard deviation 60 minutes, which means that the probability of exceeding more than 30 and 60 minutes the closing time is 31% and 16%, respectively.

Finally, we remark that our model allows to modify the settings of all the parameters reported in Table 3 in order to represent a large variety of operative conditions.

4.2. Configurations

In order to provide a term of comparison in our quantitative analysis, we introduce a baseline configuration valid for the DOR, the SOR and the hybrid policies, in which:

- advanced scheduling is performed using the same greedy algorithm, that is executing only the first step of the metaheuristic previously introduced;
- allocation scheduling is performed by ordering the elective patients in decreasing order of w_i;
- resequencing is never performed;
- the overtime is subdivided a priori among assigning the amount $\frac{\nu}{n}$ to each OR session, which is always allocated to patients in need until exhaustion;
- all non-elective patients are inserted as soon as possible in the first dedicated or shared (in accordance with the policy used) OR session, giving the priority to the patient closest to the time limit.

We remark that the baseline configuration for the DOR policy has an additional parameter representing the number of daily ORs dedicated to non-elective patients.

Table 4: Optimization modules available for the three different phases of the surgery process scheduling: the first column denotes the optimization module and its parameter(s).

mod.(par.)	**description**	**type**		advanced	allocation	
		EOO	NOO	sched.	sched.	RTM
$\mathcal{A}$	Greedy + Local Search	√		√		
	LPT modified	√			√	
	Best Fit resequencing	√				√
	Overtime criterion	√				√
$\mathcal{B}(n_s, \pi)$	Slack		√	√		
$\mathcal{C}(z)$	offline BILLS		√		√	
$\mathcal{D}(z)$	online BILLS		√		√	√
$\mathcal{E}(\delta)$	NEW-Fit		√			√
$\mathcal{F}(\delta)$	NEIC		√			√

Starting from the baseline configuration, further configurations can be obtained enabling the optimization modules introduced in Section 3. Table 4 reports all the EOO and the NOO modules we will consider in the quantitative analysis reported in Section 5, specifying the problem in which they are included and the parameters required ($z = z_1$ or z_2 is the objective function used in the BILLS algorithm). Since the aim of this study focuses on the impact of the DOR, the SOR and the NOO optimization approaches, we will study only the overall impact of the EOO approaches. We refer to [30] for a complete analysis, on the basis of which we define an unique best EOO module (the configuration giving the best overall performance) that will be used in our quantitative analysis.

Finally, we remark that in Section 5, we report only the results of several representative configurations with the aim of giving a general idea of the analysis that our model allows us to do. This choice is determined by the high numbers of possible configurations: as a matter of fact, limiting both the parameters π and δ to 4 different values, there are 104 possible configurations for the DOR, $11,160$ for the SOR and $145,080$ for a hybrid policy.

4.3. Performance indices

We define a set indices in such a way to evaluate the performance of each representative configurations from both the patient and the facility point of view. Table 5 reports the typical criteria adopted in the literature to evaluate an operating room planning and scheduling solution: the w and the f indices are a reformulation of the *need adjusted waiting days* proposed in [46] while the remaining ones are reported in [19].

The strong trade-off among the facility- and the patient-centered indices does not allow us to state what configurations are better than the others, because it depends on the particular scenario and the individual objectives of hospital managers. In order to provide a concise analysis, we define an objective function Z that allow us to determine uniquely what is the more rational configuration, that is

$$\max \quad Z = 3f_{\mathrm{E}} + (1-c) + 4f_{\mathrm{NE}} + 2u_{\mathrm{OR}}. \tag{10}$$

We derived the equation (10) in such a way to balance the contribution of the performance indices related to different stakeholders. We included four performance indices from Table 5: f_{E} and c to consider the the elective patients point of view, f_{NE} to take into account the non-elective patients point of view and u_{OR} for the facility-centered aspect. The coefficients have been fixed in order to assign the 40% of the weight to both elective and non-elective patients and the remaining 20% for the efficiency point of view. We observe that $Z \in [0, 10]$ is equal to 10 when the OR sessions are fully utilized, there are not cancellations and all (elective and non-elective) patients are operated within their time limits. The objective function Z can be redefined changing the weights and/or involving other indices in such a way to account for the different perspectives of the stakeholders.

Table 5: Patient-centered and facility-centered indices.

Index	Definition
	Facility-centered
u_{OR}	OR utilization
u_{over}	overtime utilization
p	number of surgeries performed
	Elective patient-centered
c	fraction of cancellations
t	average waiting time spent by elective patients in the waiting list
f_{E}	fraction of elective patients operated within the MTBT
w_{E}	average value of elective patient's $w_i = t_i/t_i^{\max}$ at the time of their surgery
	Non-elective patient-centered
f_{NE}	fraction of all non-elective patients operated within the time limit
$f_{\mathrm{tr,em,ur,ad}}$	fraction of patients operated within the time limit in the classes "trauma" "emergent", "urgent" and "add-on"
w_{NE}	average value of all non-elective patient's w_i at the time of their surgery
$w_{\mathrm{tr,em,ur,ad}}$	average value of patient's w_i at the time of their surgery in the classes "trauma", "emergent", "urgent" and "add-on"

5. Quantitative analysis

In this section we report the results obtained by the quantitative analysis described in Section 4. In Section 5.1 we discuss the analysis for the DOR policies, which are more straightforward than the SOR ones because of the reduced number of possible configurations. Then, the analysis of the SOR policies is reported and discussed in Section 5.2 with a particular attention to the evaluation of the several NOO algorithms introduced in Section 3.2. Starting from the best configuration for the DOR policies, we provided the analysis for the hybrid policies in Section 5.3. Finally, we compare the performance of all the best configurations of the different policies in Section 5.4.

All the results reported in this section are the average value of the performance indices over 30 different simulation runs for each scenario and configuration. Each run starts from a different seed in such a way to obtain an independent and identical distributed replication. A time horizon of two years has been fixed: after a warm up period of one year, the steady state results are collected over the second year. This allows us also to appreciate the impact of decisions over time and not only over the single planning horizon of one week. Such parameters are those already used in [30] in which the patient pathway has been validated.

For each policy, we focus on the results of the scenario S_1 showing the impact of the EOO and, after, the effect of each single NOO module on the performance.

On the basis of the best values of Z, we also evaluate the impact of enabling all the best NOO modules at the same time. Because of the huge number of configurations, we will show only the best configuration for the scenarios $S_2 - S_4$ to remark that different scenarios could require a different approach.

The average execution time for a single simulation running over the whole time horizon ranged between 7 and 348 seconds, depending on the fixed scenario and configuration. The scenario S3 required the longest computational times, because of the higher amount of patients. For the same reason, S2 had the best performance in terms of execution time: all configurations required on average less than 23 seconds for each complete run. In general, the greater impact is given by the optimization modules $\mathcal{A}$ and $\mathcal{D}$, because of the use of a local search algorithm in both of them. However, the times required are satisfactory for our aims.

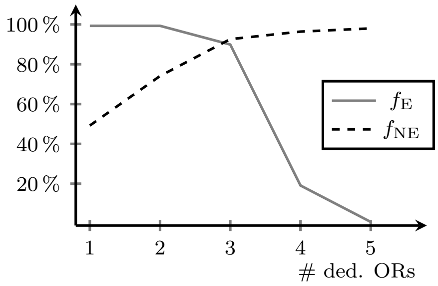
5.1. DOR

We simulated different configurations of the DOR, which are obtained varying the number of dedicated OR sessions over the total number of 10, using or not the EOO modules and adopting or not a policy $\mathcal{G}$ for the immediate insertion of trauma patients: while as default they can access only to the dedicated ORs, adopting this policy they are allowed in any OR that is released first. The reason of such a policy is the need of an immediate intervention for the patients of this type. The main results about the scenario S_1 are reported in Table 6, in which several baseline configurations are obtained varying the number of dedicated ORs.

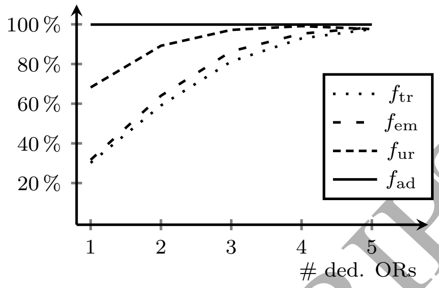
Table 6: DOR – Scenario S_1 with 10 ORs – main performance indices.

conf. id	# ded. ORs	enabled modules	**Performance indices** u_{OR}	u_{over}	p	c	t	f_{E}	w_{E}	f_{NE}	w_{NE}	Z
A1	1		78.0%	9.3%	6.5k	6.9%	58	99.0%	0.52	47.1%	2.01	7.344
B1	1	$\mathcal{G}$	78.6%	9.3%	6.5k	6.9%	56	99.0%	0.50	49.3%	1.89	7.446
C1	1	$\mathcal{A}, \mathcal{G}$	86.6%	57.8%	7.2k	0.0%	11	99.3%	0.13	49.2%	1.88	7.681
A2	2		76.4%	9.0%	6.0k	7.1%	92	80.1%	0.82	72.2%	1.01	7.747
B2	2	$\mathcal{G}$	76.8%	9.0%	6.0k	7.1%	91	81.2%	0.81	73.7%	0.92	7.849
C2	2	$\mathcal{A}, \mathcal{G}$	85.6%	57.0%	6.8k	0.0%	39	99.3%	0.36	74.1%	0.90	8.656
A3	3		71.6%	8.7%	5.4k	7.3%	132	22.5%	1.22	87.1%	0.43	6.520
B3	3	$\mathcal{G}$	71.8%	8.6%	5.4k	7.2%	131	23.7%	1.21	87.8%	0.42	6.586
C3	3	$\mathcal{A}, \mathcal{G}$	80.5%	53.2%	6.1k	0.0%	85	89.9%	0.76	92.6%	0.35	8.984
A4	4		64.3%	7.8%	4.6k	7.4%	171	1.6%	1.68	93.4%	0.22	5.993
B4	4	$\mathcal{G}$	64.4%	7.8%	4.7k	7.2%	170	1.6%	1.67	93.5%	0.21	6.007
C4	4	$\mathcal{A}, \mathcal{G}$	71.1%	38.8%	5.1k	0.0%	136	19.1%	1.27	96.4%	0.12	6.852
A5	5		56.0%	6.7%	3.9k	7.3%	201	0.1%	2.16	95.5%	0.15	5.873
B5	5	$\mathcal{G}$	56.0%	6.8%	3.9k	7.2%	204	0.1%	2.18	95.6%	0.14	5.875
C5	5	$\mathcal{A}, \mathcal{G}$	61.7%	31.8%	4.3k	0.0%	183	0.8%	1.86	98.8%	0.04	6.211

As expected, increasing the number of ORs dedicated to non-elective patients, their waiting times decrease allowing the respect of the time limits. However this causes a worsening of the elective patient performance, which have less available resources, but also a lower OR utilization.



(a) elective vs. non-elective



(b) non-elective classes

Figure 5: DOR – Scenario 1 – Percentage of patients treated in time for various numbers of dedicated ORs, with $\mathcal{A}$ and $\mathcal{G}$ enabled.

Figure 5(a) remarks the strong trade-off between the percentage of elective and non-elective patients operated on time, while Figure 5(b) focuses on the different type of non-elective patients and shows that the most urgents have a higher risk of exceeding the time limits when the number of dedicated ORs is not sufficient.

Regardless of the number of dedicated ORs, the EOO is able to better exploit the overtime than the baseline configurations. Then the OR utilization is significantly improved and cancellations are almost totally annulled. This fact is also due to the lower uncertainty that the DOR policy has because of the insertion of non-elective patients does not affect on the risk of elective patients cancellation, as in the SOR.

When the module $\mathcal{G}$ is enabled, a slight improvement of the non-elective waiting times has been observed, but an even greater contribution is given by the EOO. In Table 7 can be seen that this fact is more evident for trauma and emergent patients. In particular, $w_{\mathrm{tr}} = 1.24$ in the baseline configuration, that means that the average waiting time is 7 minutes over the time limit, but enabling the modules $\mathcal{A}$ and $\mathcal{G}$ such exceeding is less than one minute and there is 3.6% more trauma patients operated on time. Therefore, the proposed EOO approaches have also a positive impact on the non-elective patients, although they are designed for an elective patient flow.

Table 7: DOR – Scenario S_1 – Focus on non-elective patient-centered indices, 3 dedicated ORs

config. id	# ded. ORs	enabled modules	**Performance indices**									
			f_{NE}	f_{tr}	f_{em}	f_{ur}	f_{ad}	w_{NE}	w_{tr}	w_{em}	w_{ur}	w_{ad}
A3	3		87.1%	77.7%	82.4%	95.4%	100%	0.43	1.24	0.34	0.17	0.03
B3	3	$\mathcal{G}$	87.8%	78.5%	83.4%	95.7%	100%	0.42	1.20	0.31	0.16	0.03
C3	3	$\mathcal{A}$, $\mathcal{G}$	89.9%	81.3%	86.4%	97.2%	100%	0.35	1.01	0.27	0.13	0.02

Although the baseline configuration A2 is better than the baseline configuration A3, when the optimization modules are enabled configuration B3 has a greater value of Z than B2. Then the number of dedicated ORs that maximizes the performance strictly depends on the optimization approaches that are used.

Table 8: DOR – Scenarios $S_1 - S_4$ – Best configurations.

scen. id	config. id	# ded. ORs	enabled modules	Performance indices									
				u_{OR}	u_{over}	p	c	t	f_{E}	w_{E}	f_{NE}	w_{NE}	Z
S_1	C3	3	$\mathcal{A}$, $\mathcal{G}$	80.5%	53.2%	6.1k	0.0%	85	92.6%	0.76	89.9%	0.35	8.984
S_2	C1	1	$\mathcal{A}$, $\mathcal{G}$	81.9%	52.7%	3.3k	0.0%	46	99.3%	0.42	64.6%	1.30	8.202
S_3	C2	2	$\mathcal{A}$, $\mathcal{G}$	83.6%	60.6%	10.8k	0.1%	55	99.3%	0.48	71.4%	1.01	8.505
S_4	C1	1	$\mathcal{A}$, $\mathcal{G}$	81.3%	69.1%	5.4k	0.3%	57	99.2%	0.50	60.5%	1.43	8.018

All the previous remarks for the scenario S_1 are confirmed also for the other scenarios, whose best configurations are listed in Table 8. We observe that, in scenario S_1, the best configuration C3 provides the 30% of the ORs to the non-elective patients, that are the 15% of the total, because the unpredictability of such patients requires a higher amount of resource to deal with the time limits. Differently, the scenario S_3 maximizes the objective function with the configuration C2, which provides the 20% of the ORs to the non-elective. This result indicates that the need of dedicated ORs depends also on the surgery duration distribution, that is the only difference between the two scenarios.

5.2. SOR

Starting from the unique baseline configuration D1 defined for the SOR, Table 9 reports the results of different configurations obtained enabling and combining the optimization modules to maximize the objective function Z for the scenario S_1.

The configuration E1 corresponds to the configurations C1–C5 of the DOR, because of the use of the EOO and the implication of the module $\mathcal{G}$ in any SOR policy. All the performance indices are improved by the module $\mathcal{A}$: OR utilization and waiting times of both elective and non-elective patients are better than those of the DOR. The higher OR utilization is due to the advanced scheduling that plans elective patients in all the OR session and, as opposed to the DOR, never an OR slot is unused because of the lack of non-elective patients to operate on, this allows us to operate on more elective patients per week and to have shorter waiting times. On the other hand, non-elective patients do not need the release of a specific dedicated OR to be inserted, then their waiting times are lower than those of the DOR up to 3 dedicated ORs. The high utilization of the overtime suggests that the online approach included in the EOO avoids a high number of cancellations, nevertheless the value of c is high because of the uncertainty determined by the insertion of non-elective surgery in almost filled OR sessions. Figure 6 shows that the used EOO approaches avoid the lengthening of the waiting list. Because of the general improvement given by the EOO optimization, the module $\mathcal{A}$ has been always enabled in the further configurations involving NOO approaches.

Table 9: SOR – Scenario S_1 – Performance indices.

conf. id	enabled modules	**Performance indices** u_{OR}	u_{over}	p	c	t	f_{E}	w_{E}	f_{NE}	w_{NE}	Z
D1		89.0%	12.2%	6.8k	21.4%	40	98.8%	0.37	91.0%	0.35	9.167
E1	$\mathcal{A}$	93.1%	99.9%	7.1k	11.5%	8	99.3%	0.11	92.4%	0.32	9.423
F1	$\mathcal{A}, \mathcal{B}(5, 0.3)$	89.6%	99.7%	6.8k	25.2%	48	99.1%	0.43	92.9%	0.35	9.208
F2	$\mathcal{A}, \mathcal{B}(5, 0.4)$	88.6%	93.1%	6.7k	21.0%	54	99.1%	0.48	92.8%	0.33	9.247
F3	$\mathcal{A}, \mathcal{B}(5, 0.5)$	85.6%	80.1%	6.4k	13.7%	73	99.2%	0.64	93.8%	0.29	9.300
F4	$\mathcal{A}, \mathcal{B}(5, 0.6)$	79.6%	63.6%	5.9k	10.2%	103	61.2%	0.92	94.9%	0.23	8.120
F5	$\mathcal{A}, \mathcal{B}(10, 0.15)$	93.3%	90.0%	7.1k	4.5%	8	99.3%	0.11	93.0%	0.31	9.520
F6	$\mathcal{A}, \mathcal{B}(10, 0.2)$	93.1%	84.5%	7.1k	2.7%	17	99.3%	0.17	92.8%	0.32	9.528
F7	$\mathcal{A}, \mathcal{B}(10, 0.25)$	87.9%	63.7%	6.7k	0.6%	52	99.3%	0.46	93.2%	0.30	9.459
F8	$\mathcal{A}, \mathcal{B}(10, 0.3)$	84.6%	51.6%	6.4k	0.2%	69	99.2%	0.61	93.6%	0.28	9.410
G1	$\mathcal{A}, \mathcal{C}(z_1)$	92.9%	99.6%	7.1k	11.0%	8	99.3%	0.11	93.2%	0.29	9.454
G2	$\mathcal{A}, \mathcal{C}(z_2)$	93.0%	99.2%	7.1k	10.5%	8	99.3%	0.11	94.2%	0.25	9.503
G3	$\mathcal{A}, \mathcal{D}(z_1)$	92.6%	99.6%	7.1k	10.7%	8	99.3%	0.11	93.6%	0.28	9.469
G4	$\mathcal{A}, \mathcal{D}(z_2)$	92.7%	99.3%	7.0k	10.3%	8	99.3%	0.11	94.2%	0.25	9.499
H1	$\mathcal{A}, \mathcal{E}(0.25)$	92.6%	99.9%	7.1k	10.1%	8	99.3%	0.11	87.7%	0.46	9.240
H2	$\mathcal{A}, \mathcal{E}(0.5)$	92.9%	100%	7.1k	10.0%	8	99.3%	0.11	86.2%	0.51	9.183
H3	$\mathcal{A}, \mathcal{E}(0.75)$	92.5%	99.9%	7.1k	9.5%	7	99.3%	0.11	85.6%	0.53	9.158
H4	$\mathcal{A}, \mathcal{E}(1)$	92.6%	99.8%	7.1k	9.6%	8	99.3%	0.11	84.4%	0.55	9.112
I1	$\mathcal{A}, \mathcal{F}(0.25)$	93.1%	100%	7.1k	11.8%	8	99.3%	0.11	92.5%	0.33	9.424
I2	$\mathcal{A}, \mathcal{F}(0.5)$	92.9%	99.9%	7.1k	11.6%	8	99.3%	0.11	91.8%	0.35	9.393
I3	$\mathcal{A}, \mathcal{F}(0.75)$	92.5%	99.8%	7.1k	11.4%	8	99.3%	0.11	91.6%	0.37	9.380
I4	$\mathcal{A}, \mathcal{F}(1)$	93.1%	100%	7.1k	11.3%	8	99.3%	0.11	90.9%	0.40	9.365
J1	$\mathcal{A}, \mathcal{B}(10, 0.2), \mathcal{E}(1)$	92.9%	82.8%	7.1k	2.5%	17	99.3%	0.18	92.9%	0.32	9.528
J2	$\mathcal{A}, \mathcal{C}(z_2), \mathcal{F}(0.25)$	92.7%	99.2%	7.1k	10.4%	8	99.3%	0.11	94.3%	0.25	9.501
K1	$\mathcal{A}, \mathcal{B}(10, 0.2), \mathcal{C}(z_2), \mathcal{F}(0.25)$	92.8%	83.4%	7.1k	2.3%	18	99.3%	0.19	94.8%	0.24	9.605

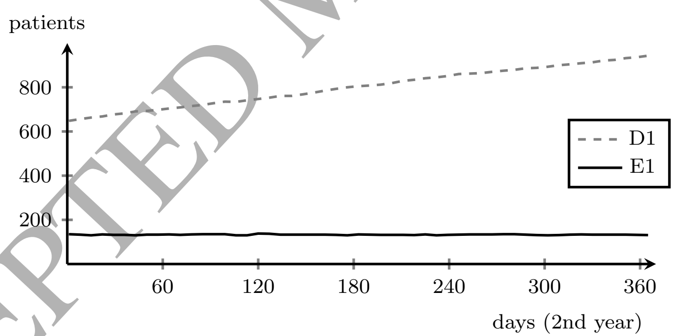
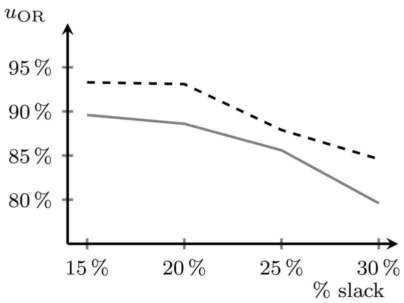


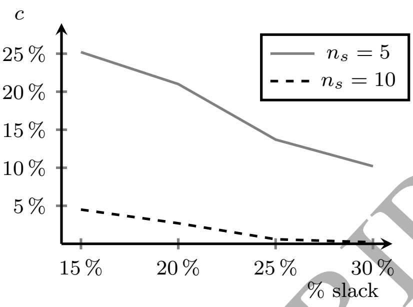
Figure 6: SOR – Scenario S_1 – Length of the elective waiting list with and without EOO.

Configurations F1–F8 concern the slack management and have been obtained fixing the number n_{s} equal to 5 (half of daily ORs) and 10 (all daily ORs), and ranging the parameter π in such a way to reserve a percentage between 15% and 30% of the total time with a 5% step, that is π ranges between 0.3 and 0.6 when $n_s = 5$ and between 0.15 and 0.3 when $n_s = 10$.

As shown in Figure 7, at the increasing of π both the OR utilization and the number of cancellations decreases, because there is less probability to exceed



(a) OR utilization



(b) cancellations

Figure 7: SOR – Scenario S_1 – OR utilization and fraction of cancellation, varying the fraction of slack on the total surgery time.

the total duration of the OR sessions that involves a lower request of overtime, which is proved by a lower value of u_{over}. Conversely, the waiting times of the elective patients raise when π increases, causing a significantly lowering of f_{E} for the configuration F4. In all the other configurations involving slacks, more than 99% of elective patients are operated within the MTBT, but the growth of the waiting list can be unmanageable on a longer period. Furthermore, slacks lead to a slight improvement of non-elective patients performance.

The effectiveness of BIM optimization has been tested in configurations G1–G4, using the two different objective functions z_1 and z_2 in the BILLS algorithm, for both the offline and the online version. A first difference with the configuration E1 is the higher percentage of non-elective patients operated on within the time limits, that is more remarkable for the trauma patients using the objective function z_2, as can be seen in Table 10. However, it seems that the online version of the algorithm does not provide a further improvement respect to the offline version. Furthermore, the NOO modules $\mathcal{C}$ and $\mathcal{D}$ slightly impact also on the trade-off between OR utilization and cancellations, fostering the improvement of the latter at the expense of the former.

Table 10: SOR – Scenario S_1 – Impact of BIM optimization on the non-elective patients.

config. id	enabled modules	**Performance indices**									
		f_{NE}	f_{tr}	f_{em}	f_{ur}	f_{ad}	w_{NE}	w_{tr}	w_{em}	w_{ur}	w_{ad}
E1	$\mathcal{A}$	92.4%	68.9%	96.8%	99.8%	100%	0.32	0.95	0.24	0.12	0.02
G1	$\mathcal{A}, \mathcal{C}(z_1)$	93.2%	71.2%	97.7%	99.7%	100%	0.29	0.84	0.21	0.10	0.02
G2	$\mathcal{A}, \mathcal{C}(z_2)$	94.2%	74.9%	98.3%	99.8%	100%	0.25	0.72	0.18	0.09	0.02
G3	$\mathcal{A}, \mathcal{D}(z_1)$	93.6%	72.9%	97.8%	99.8%	100%	0.28	0.81	0.21	0.10	0.02
G4	$\mathcal{A}, \mathcal{D}(z_2)$	94.2%	74.9%	98.3%	99.8%	100%	0.25	0.74	0.18	0.09	0.02

The NEW-Fit algorithm has been used in configurations H1–H4 varying the value of the parameter δ between 0.25 and 1.00 with step 0.25. We observe that when $\delta = 1$ the early deadline for the non-elective patients is fixed equal to the time limit, while decreasing δ to 0 the algorithm uses an even more restrictive early deadline. As expected, enabling $\mathcal{E}$ the number of cancellations decreases because the non-elective patients are inserted in such a way to balance the workload among the OR sessions. In proportion to the value of δ, this causes higher non-elective patients waiting times that induce to a higher number of patients exceeding the time limit due to the uncertainty of the surgery durations of the previous patients. The reason of the limited impact of the NEW-Fit algorithm is probably due to the workload of the chosen scenario, whose baseline configuration D1 shows high performance from the elective patients point of view ($f_{\mathrm{E}} = 98.8\%$). As a matter of fact, the effectiveness of the the NEW-Fit algorithm has been proved in other more overloaded scenarios, as reported in [31]. Furthermore we analyzed the impact of the NEW-Fit algorithm jointly to the insertion of slacks in the schedule. Configuration J1 gives the best value of Z varying δ in $(0, 1]$, that is the combination of the configurations F6 and H4. The results are very similar to that computed for the configuration F6, but it is interesting that the negative impact of the NEW-Fit algorithm on the non-elective patients can be canceled using slacks.

An analogous analysis has been provided for the NEIC algorithm in the configurations I1–I4. All the performance indices are very close to those of the configuration E1, except the waiting times of the non-elective patients that increases slightly. However the NEIC algorithm is implemented to preserve the BIM optimization when the non-elective patients are inserted, therefore in configuration J2 we tested the impact when the BIM are optimized, but without better results.

Finally, the configuration K1 is the configuration that maximizes Z and, compared to E1, improves all the performance indices except a very slight loss of the OR utilization, with the advantage of using 16.5% less overtime. Table 11 lists the best configurations of all scenarios $S_1 - S_4$. We observe that the EOO and the same configuration of slacks are always enabled, while the BILLS algorithm is used in the offline or online version using the objective function z_2. Both NEW-Fit and NEIC contribute in the best configuration with a slight improvement in only one of the four scenarios. The most interesting thing is that the four not very different scenarios provide four different best configurations, which remarks the usefulness of a decision support system that is specifically adapted to the operative environment.

Table 11: SOR – Scenario $S_1 - S_4$ – Best configurations.

scen. id	conf. id	enabled modules	Performance indices u_{OR}	u_{over}	p	c	t	f_{E}	w_{E}	f_{NE}	w_{NE}	Z
S_1	K1	$\mathcal{A}, \mathcal{B}(10, 0.2), \mathcal{C}(z_2), \mathcal{E}(1)$	92.8%	83.0%	7.0k	2.3%	10	99.3%	0.13	94.6%	0.24	9.595
S_2	K2	$\mathcal{A}, \mathcal{B}(5, 0.2), \mathcal{C}(z_2)$	91.8%	81.2%	3.5k	2.8%	14	99.3%	0.16	90.1%	0.39	9.391
S_3	K3	$\mathcal{A}, \mathcal{B}(10, 0.2), \mathcal{D}(z_2), \mathcal{F}(1)$	94.7%	79.6%	11.8k	1.1%	14	99.3%	0.15	98.4%	0.13	9.793
S_4	K4	$\mathcal{A}, \mathcal{B}(5, 0.2), \mathcal{D}(z_2)$	94.3%	79.6%	5.9k	2.0%	17	99.2%	0.18	95.6%	0.21	9.668

5.3. Hybrid policies

A hybrid policy is a mix of dedicated and shared policies. In our settings, a number of ORs are reserved for the non-elective patients (as in the DOR) while the remaining ORs are used to operate on both elective and non-elective patients (as in SOR). The elective patients are scheduled into the shared ORs. When a non-elective patient arrives, his/her surgery is scheduled into a dedicated OR, if available at that time; on the contrary, the surgery is scheduled into the first released (dedicated or shared) OR. If two or more non-elective patients are waiting for the insertion, the priority is given to the patient that is closer to his/her time limit. We observe that the module $\mathcal{G}$ for the immediate insertion of trauma patients is already included in this hybrid policy.

Table 12: Hybrid policy – Scenario S_1 – Performance indices.

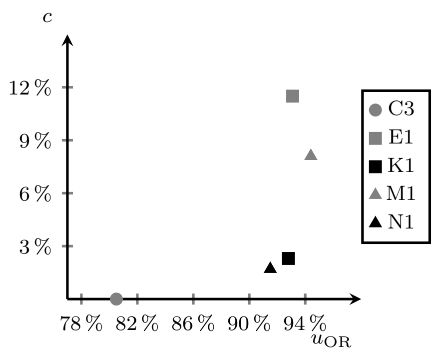
conf. id	# ded. ORs	enabled modules	Performance indices u_{OR}	u_{over}	p	c	t	f_{E}	w_{E}	f_{NE}	w_{NE}	Z
L3	3		72.6%	8.9%	5.4k	9.3%	131	23.5%	1.22	95.7%	0.15	6.893
L2	2		79.7%	10.1%	6.0k	11.6%	90	82.6%	0.81	95.4%	0.17	8.769
L1	1		85.2%	11.0%	6.5k	15.8%	58	98.8%	0.52	94.0%	0.24	9.270
M1	1	$\mathcal{A}$	94.4%	97.8%	7.2k	8.1%	8	99.2%	0.12	96.2%	0.17	9.633
N1	1	$\mathcal{A}, \mathcal{B}(10, 0.2), \mathcal{C}(2)$	91.5%	78.1%	6.9k	1.7%	26	99.3%	0.25	96.3%	0.16	9.659

Table 12 reports the results for the scenario S_1. We started considering 3 dedicated ORs (configuration L3), which corresponds to the best configuration of the DOR, and we decreased this number (configuration L2 and L1) in order to improve the elective patients performance. In addition the OR utilization has been improved using less dedicated ORs. As expected, allowing the access of non-elective patients to resources that the DOR dedicated to elective patients, the distribution assignment of the operating rooms to the two patient flows needs to be changed to have a fair balance. Configuration M1 is obtained from configuration L1 enabling the EOO modules that, also in this case, provide a very significant and general improvement. Finally, configuration N1 is the best one using also the NOO approaches. Similar results are obtained for the other scenarios $S_2 - S_4$.

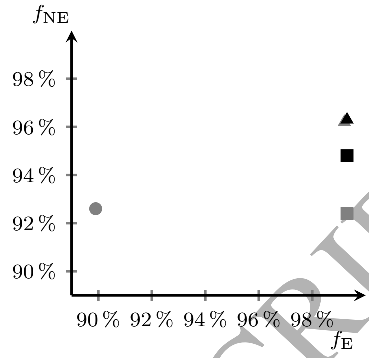
5.4. Comparing policies

We evaluate the impact of the best configurations for the DOR, the SOR and the hybrid policy comparing their results on the four indices involved in the objective function Z. Such a comparison is summarized in Figure 8: a facility-centered index is compared with an elective one in 8(a) while an elective patient-centered index is compared with a non-elective one in 8(b). We plotted the configurations using only the EOO approaches for the SOR and the hybrid policy, in order to appreciate the impact of the NOO. Finally, note that the results for the configurations M1 and N1 are overlapping in Figure 8(b).

In Figure 8(a) the trade-off between OR utilization and cancellations is evident. We remark that all the configurations represent a different compromise between the two indices, except the configuration E1 that is dominated by K1



(a) u_{OR} vs. c



(b) f_{E} vs. f_{NE}

Figure 8: Scenario S_1 – Comparing DOR, SOR and hybrid on the main performance indices.

and M1. Further, the best compromise seems to be provided by K1 and N1 configurations. On the other side, considering the trade-off between the percentage of elective and non-elective patients operated within their MTBT, Figure 8(b) shows that hybrid configurations (M1 and N1) dominate the DOR and the SOR configurations. Globally, the configuration N1 seems the more rational one in accordance with the coefficients adopted in (10).

6. Conclusions

In the literature, the problem of sharing operating rooms between elective and non-elective patients counts a number of different approaches whose results are usually conflicting. Such approaches are applied and tested to different operative conditions making their comparison very difficult. In order to determine the best approach under certain operative conditions an ad hoc study is therefore necessary. In this paper, we fill this research gap providing a hybrid model capable to represent a large range of operative and decision-making conditions to study and evaluate the impact of such approaches.

Our hybrid model uses discrete event simulation to represent the general surgical pathways of the two patient flows (elective and non-elective) under the dedicated and shared policies, and their hybrid versions. A set of optimization approaches are embedded within the model. We consider the approaches proposed in the literature to deal with the sharing of the operating rooms (the reservation of slacks and the break-in-moment optimization). Further, we introduce three new online algorithms for the real time management of the non-elective patients, that is the Break-In Layout Local Search, the Non-Elective Worst Fit and the Non-Elective Insertion Criterion. In particular, the last two algorithms deal with the Non-Elective Real Time Insertion problem, which suffers from a lack of studies in the literature.

Because of the capability of the model to represent a high number of scenarios and configurations, we have to restrict the quantitative analysis to four representative scenarios, choosing conditions such that the workload caused by the elective patients is proportional to the available resources. For each of the four scenarios, we show the performance of both the dedicated and shared operating room policies. Further, for the latter we observe the impact of the optimization modules when they are enabled separately and, on the basis of their results, we combine them to find the best configuration with respect to both the (elective and non-elective) patient and facility points of view. Furthermore, the impact of a hybrid policy is evaluated.

From the management policy point of view, the results confirm the strong trade-off between the OR utilization and number of cancellations, which is widely discussed in literature. While the dedicated operating room policy allows us to have a very low probability of elective-patients cancellations, the shared operating room policy is able to increase the use of the resources and, consequently, to reduce the length of the waiting list. However, a better trade-off between the performance of the elective and non-elective patients is given by the shared operating room policy. We also show that hybrid policies could provide a further performance improvement.

In summary, our analysis suggests the use of a hybrid policy to manage elective and non-elective patients even if shared and dedicated policies can be a good compromise in certain operative conditions. However, to account for the different perspectives of the stakeholders, it is always recommended to provide an ad hoc analysis.

From an algorithmic point of view, we prove the effectiveness of the elective-oriented optimization approaches: they are able to manage the elective patient flow and, counter-intuitively, also some non-elective performance indices take advantage from them. This result suggests that an appropriate management of the elective patient flow is a necessary condition to have a positive impact when dealing also with the non-elective patient flow.

Regarding the non-elective-oriented optimization algorithms, our analysis suggests that an appropriate slack management can overcome the limitation of a dedicated policy, at parity of overall operating time, causing less cancellations but lowering the OR utilization, with a slight improvement of the non-elective patients performance. The Break-In Layout Local Search algorithm has a positive impact on non-elective patients without deteriorating the performance of the elective patients. Finally, the impact of the the Non-Elective Worst Fit and the Non-Elective Insertion Criterion seems limited in the operative conditions represented by the four scenarios. However, the effectiveness of the Non-Elective Worst Fit is proved in [31] in more crowded surgical pathways.

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