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A Note on the Maximum Value of the Kakwani Index

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Abstract The Kakwani index computes the departure from proportionality of a progressive income tax by measuring the difference between the concentration coefficient for tax liabilities and the Gini coefficient for pre-tax incomes. In case of maximum progression, that is a situation in which only one taxpayer faces the overall tax burden, the index reaches its theoretical maximum value, given by 1 minus the Gini coefficient for pre-tax incomes. We argue that this phenomenum can happen in one special case that is not satisfied in real-world personal income taxes. As a matter of fact, the overall tax revenue of a real-world personal income tax cannot be eventually paid only by the richest taxpayer. Therefore, the maximum concentration coefficient for taxes cannot be equal to 1, and, consequently, the maximum value of the Kakwani index cannot be 1 minus the Gini coefficient for pre-tax incomes, as generally

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described in the related literature. According to different hypotheses, we give evidence of this phenomenon by employing the Italian personal income tax.

Keywords Kakwani index · Redistributive effect · Personal income tax · Microsimulation models

JEL Codes C81 · H23 · H24

1 Introduction

In their seminal papers, Jakobsson (1976), Fellman (1976), Kakwani (1977) and Reynolds and Smolensky (1977) show how the degree of progression and the redistributive effect of a tax can be quantified. In particular, Kakwani (1977) proposes his famous index able to compute the departure from proportionality of a progressive income tax. This index measures the difference between the concentration coefficient for the tax liability distribution and the the Gini coefficient for the pre-tax income one.

All the related tax literature (e.g., Lambert 2001) states that its maximum value is one minus the Gini coefficient for the pre-tax income distribution, and its minimum value -1 minus the same Gini coefficient. We argue that these phenomena can happen in one special case that is not satisfied in real-world personal income taxes. As a consequence, the maximum (minimum) value of the Kakwani index is lower (greater) than its theoretical one. Focusing on the maximum value, we give evidence of its magnitude by two examples regarding the Italian personal income taxation.

The structure of the paper is as follows. Section 2 presents the basic inequality indexes. Section 3 focuses on the highest admittable value of the Kakwani index. Section 4 briefly introduces the data and the microsimulation model employed in this work (Subsection 4.1), and subsequently reports the results (Subsection 4.2). Section 5 concludes.

2 Basic notation

A population of N income earners, with $i = 1, \ldots, N$, is considered. We denote by $X = (x_1, \ldots, x_N)$ the gross income distribution ordered in non decreasing order. Similarly, we call $T = (t_1, \ldots, t_N)$ the tax liability distribution and $Z = (z_1, \ldots, z_N)$ the post-tax income one. To evaluate the inequality within these distributions, we employ the Gini (1914) coefficient $G_\epsilon = 2{\mu_\epsilon}^{-1} cov\big(\epsilon, F(\epsilon)\big)$ and the corresponding concentration one $C_{\epsilon|\eta} = 2{\mu_\epsilon}^{-1} cov\big(\epsilon, F(\eta)\big)$, where $\epsilon, \eta = (X, T, Z)$, $C_{\eta|\eta} = G_\eta = G_\epsilon = C_{\epsilon|\epsilon}$, μ_ϵ is the average value of the considered distribution, cov represents the covariance, and $F(\epsilon)$ is the cumulative distribution function (Kakwani 1980; Jenkins 1988). As it is well known, Gini and concentration coefficients range between zero and $\frac{N-1}{N}$, $1 = \lim_{N\to\infty} \frac{N-1}{N}$ in case of large samples.

Following the existing literature (e.g., Lambert 2001), the redistributive effect RE can be measured by $RE = G_X - G_Z = RS - RR$ where $RS =$

$G_X - C_{Z|X}$ is the Reynolds-Smolensky index and $RR = G_Z - C_{Z|X}$ is the Atkinson-Plotnick-Kakwani index (Atkinson 1980; Plotnick 1981; Kakwani 1984). Similarly, the degree of tax progressivity can be computed by the Kakwani index $K = C_{T|X} - G_X$, linked to RS by the overall average tax rate $\theta = \frac{\sum_{i=1}^{N} t_i}{\sum_{i=1}^{N} x_i}$: $RS = \frac{\theta}{1-\theta}K$.

3 The maximum value

For large samples, all the tax literature states that the maximum value of the Kakwani index is $K^{MAX} = 1 - G_X$ and its minimum value is $K^{MIN} = -1 - G_X$. These extreme bounds are possible under the condition that the highest admittable value for $C_{T|X}^{MAX}$ is 1 and the corresponding minimum value $C_{T|X}^{MIN}$ is equal to -1. It has to be noted that the above mentioned extreme values for K can be verified in one special case: the overall tax revenue (hereafter Υ) is lower than the top (bottom) gross income x_N (x_1) observed in the income distribution.[1] This is not what researchers observe in real-world taxation, since in general $\Upsilon > x_N$. As a consequence, the highest value of the tax liability concentration $C_{T|X}^{MAX}$ is necessarily lower than $\frac{N-1}{N}$ (1 for large samples) and the corresponding lowest value $C_{T|X}^{MIN}$ is greater than $\frac{1-N}{N}$ (-1 for large samples). In turn this implies that K^{MAX} (K^{MIN}) depends on the distribution of X and the overall amount of the tax revenue to be collected Υ. In particular, ceteris paribus, K^{MAX} (K^{MIN}) increases (decreases) as the overall amount of the tax revenue Υ decreases.

3.1 When re-ranking of post-tax incomes occurs

The empirical occurrence of the maximum theoretical value of $C_{T|X}^{MAX}$ is even more unlikely if the condition of marginal tax rates not exceeding 100% holds.[2]

Suppose initially that only the richest taxpayer has to face a positive tax liability. Until the tax revenue Υ is lower than or at most equal $x_N - x_{N-1}$, $C_{T|X} = \frac{N-1}{N}$, and the maximum Kakwani index is $K^{MAX} = \frac{N-1}{N} - G_X$; moreover, $G_T^{MAX} = C_{T|X}^{MAX}$, and $RR^{MAX} = 0$. If $x_N - x_{N-1} < \Upsilon \leq x_N$, K^{MAX} is still $\frac{N-1}{N} - G_X$; in this circumstance not only $RR^{MAX} > 0$ but it also monotonically increases with Υ. For all possible values $x_N < \Upsilon \leq \sum_{i=1}^{N-1} x_i$, two or more taxpayers are needed for Υ to be paid, so that $K^{MAX} < \frac{N-1}{N} - G_X$ monotonically decreases with Υ (also RE^{MAX} decreases with Υ, whilst RS^{MAX} increases): in these cases an unwanted and disproportionate re-ranking of post-tax income values is likely to occur, leading to situations

[1] For simplicity here we focus on positive values for X and T as well as Z.

[2] It has to be noted that this condition is invariably assumed in both theoretical and empirical analysis on redistribution in order to avoid the re-ranking (see for example Kakwani and Lambert (1998) and Pellegrino and Vernizzi (2013)).

in which $RE^{MAX} < 0$ even if $RS^{MAX} > 0$. Finally, K^{MAX} becomes zero when $\Upsilon = \sum_{i=1}^{N} x_i$; in this extreme case also RR^{MAX} would be zero, whilst $RE^{MAX} = RS^{MAX}$ would be equal to G_X.

3.2 When re-ranking of post-tax incomes is avoided

Whilst focusing on the highest value for the Kakwani index, in order to avoid the unpleasant outcomes underlined in Subsection 3.1, a different strategy has to be employed: the tax structure associated to K^{MAX} needs to avoid post-tax incomes re-ranking.

Following the methodology described in Mantovani (2017), this distribution can be obtained by imposing a deduction D, equal for all taxpayers, to their pre-tax income x_i whilst applying a 100% statutory marginal tax rate and avoiding the negative income tax (Keen et al. 2000).

Given a pre-tax income distribution (and its G_X value) and a specific amount for Υ, RE^{MAX} can be obtained through the minimum value for G_Z^{MIN}. Focusing on a tax, and avoiding the negative income taxation, $z_i \leq x_1$ for all incomes. G_Z^{MIN} can then be obtained by levelling top pre-tax incomes in order to obtain $Z = (x_1, x_2, \ldots, x_n, D, D, \ldots, D)$. This is possible by applying $T = (0, 0, \ldots, 0, x_{k+1} - D, x_{k+2} - D, \ldots, x_N - D)$, with $\sum_{n=k+1}^{N} x_n - D = \Upsilon$. Let $Z_{inf} = (x_1, x_2, \ldots, x_n)$ and $Z_{sup} = (D, D, \ldots, D)$.

For a given Υ and excluding negative taxes, three situations are possible: a) were a redistributive transfer from Z_{inf} to Z_{sup} considered, G_Z would increase and RE decrease; b) either a redistributive transfer within Z_{inf} or an egalitarian redistributive transfer from Z_{sup} towards Z_{inf} is not possible to occur, because some pre-tax incomes within Z_{inf} would increase; c) a redistributive transfer taking place within Z_{sup} would determine an increase of G_Z. As a consequence, the maximum value of the Kakwani index compatible with the maximisation of RE^{MAX} is obtainable by considering $t_i = x_i - D$ if $x_i > D$ and $t_i = 0$ otherwise, with $\sum_{i=1}^{N} t_i = \Upsilon$.

4 An application to a real-world tax

4.1 The data and the microsimulation model

We make use of a static microsimulation model concerning the Italian personal income tax (Pellegrino 2007) updated to the 2014 fiscal year (Pellegrino et al. 2017). Results of the model are very close to the Department of Finance (2016) official statistics. Moreover, inequality indexes both for taxpayers and equivalent households are also very close to the ones evaluated by the Department of Finance official microsimulation model (Di Nicola et al. 2015).

As input data, it employs those provided by the Bank of Italy (2015) in its Survey on Household Income and Wealth published in 2015 with regard to the 2014 fiscal year. The survey contains information on income and wealth of 8,156 households and 19,366 individuals, and it is representative of the Italian population, composed of about 24.7 million households and 60.8 million individuals.

According to the microsimulation model, the 2014 overall amount of pre-tax incomes $\sum_{i=1}^{N} x_i$ is 807.85 billion euros, whilst the overall tax revenue $\sum_{i=1}^{N} t_i$ is 151.67 billion euros. As a consequence, $\theta = 0.18774$.

The Gini coefficient for the gross income distribution G_X is 0.45253, whilst that for the net income distribution G_Z is 0.40248, and the one for the tax liability distribution is $G_T = 0.68484$. The overall redistributive effect RE is then 0.05005. The concentration coefficient for the net income distribution $C_{Z|X}$ is 0.40160, whilst the one on the net tax liability distribution $C_{T|X}$ is 0.67289; therefore, the Reynolds-Smolensky RS index is equal to 0.05093 and the Kakwani index K is 0.22035. Finally, the Atkinson-Plotnick-Kakwani index RR is equal to 0.00088 (Table 1, column Present tax).

4.2 Results

Having ranked pre-tax values in non decreasing order and considered sample weights, the top 1.46 million taxpayers (3.7% of all) earn a pre-tax income equal to $\sum_{i=1}^{N} t_i = \Upsilon$. Supposing all these taxpayers face a tax liability equal to their pre-tax income, and the remaining ones a zero tax liability (Table 1, column Maximum with re-ranking), $G_T^{MAX} = C_{T|X}^{MAX} = 0.97379$ and $K^{MAX} = 0.52116$. Note that the empirical maximum value of the Kakwani index is lower than its theoretical one $(1 - G_X = 0.54747)$.[3] RE^{MAX} would be remarkably lower than the one observed according to the present tax structure (0.04734) since this hypothetical tax liability distribution would generate a huge re-ranking of post-tax incomes ($RR^{MAX} = 0.07312$, 83 times greater than the one observed today). As a consequence, RS^{MAX} would be 2.36 times the one registered according to the present tax structure (0.12046) even if RE^{MAX} is lower.

From the empirical point of view it can be interesting to determine an hypothetical tax liability distribution able both to guarantee the total tax revenue observed according to the present tax structure and no re-ranking of post-tax incomes (see Subsection 3.2).

In order to obtain a total tax revenue equal to 151.67 billion euros, the value of D in the 2014 Italian case should be 29,763.83 euros (Table 1, column Maximum without re-ranking). This tax liability distribution would be able to maximise RE^{MAX}, whilst guaranteeing no re-ranking of both tax liability and post-tax income distributions. In particular, $G_T^{MAX} = C_{T|X}^{MAX} = 0.94395$, $K^{MAX} = 0.49142$, $RE^{MAX} = RS^{MAX} = 0.11358$ and $RR^{MAX} = 0$.

[3] A similar discussion would refer to the minimum value of the index, here omitted.

Table 1 Inequality indexes

Index	Present tax	Maximum with re-ranking	Maximum without re-ranking
G_X	0.45253	0.45253	0.45253
G_T	0.68484	0.97369	0.94395
$C_{T\|X}$	0.67289	0.97369	0.94395
G_Z	0.40248	0.40519	0.33895
$C_{Z\|X}$	0.40160	0.33207	0.33895
RE	0.05005	0.04734	0.11358
RS	0.05093	0.12046	0.11358
K	0.22035	0.52116	0.49142
RR	0.00088	0.07312	0.00000
θ	0.18774	0.18774	0.18774
$\sum_{i=1}^{N} x_i$	807.85	807.85	807.85
$\sum_{i=1}^{N} t_i$	151.67	151.67	151.67

5 Conclusions

In this paper we stress that, even if desired, the overall tax revenue of a personal income tax cannot be concentrated only on the richest income earner, simply because the overall tax revenue is remarkably greater than the top gross income observed in real-world income distributions.

From this simple observation follows that the maximum concentration coefficient for taxes cannot be 1, and, consequently, the maximum value of the Kakwani index cannot be equal to 1 minus the Gini coefficient for pre-tax incomes as generally described in the related literature. We give evidence of this phenomenon by illustrating empirical examples when considering a real-world tax.

6 Compliance with Ethical Standards

The authors Daniela Mantovani, Simone Pellegrino and Achille Vernizzi declare that they have no relevant or material financial interests that relate to the research described in this manuscript. Moreover, the authors declare that they have no conflict of interest.

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