Towards a Materialist Vision of 'Learning as Making': the Case of 3D Printing Pens in School Mathematics

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Towards a Materialist Vision of 'Learning as Making': the Case of 3D Printing Pens in School Mathematics

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Abstract
In this paper, we build on a previously developed notion of 'learning as Making' to examine mathematics thinking and learning in a highly transformative and technological Making environment: one that involves a handheld 3D printing technology which enables 3D models to be created instantly via one's moving hand. In particular, we present two examples of Maker-centred lessons for teaching and learning of primary mathematics. In these lessons, the students actively constructed artefacts with 3D Printing Pens while engaging in inquiry-based learning activities, where the target concepts were properties of prisms and cross-sections at the primary 5 (age 10–11) and primary 6 (age 11–12) levels respectively. We use diffractive analysis to capture the fine details in students' body-material interactions while engaging in the tasks with or without the 3D Printing Pens during the lessons. Through the lens of Making as a material act of creation and seeking to update Papert's constructionist view of learning, we propose to rethink Making in school mathematics according to a four-fold characterisation: *Making is co-constructing meanings*, *Making is mathematising*, *Making is assembling with technology* and *Making is inventing*. We discuss our contribution towards advancing a materialist perspective of learning mathematics and implications for a 'learning as Making' pedagogy and curriculum.

Keywords Constructionism · Gestures and diagramming · Making · Mathematics education · Materialism

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Introduction

Papert's (1980) success of Logo, an educational programming language which empowers students to construct digital artefacts, has pioneered the ways in which digital technologies could radically change how students think and learn in and out of school contexts. His theory of constructionism not only shares the constructivist view of learning as 'building knowledge structures', but also underpins the context whereby the learner is consciously constructing a public entity—a form of 'learning-by-Making' (Papert & Harel, 1991, p. 1). The value of Making has been argued by Sinclair, de Freitas and Ferrara (2013), who see experiences such as Logo's 'square-making […] as potential inventive moments in which the human-technology interaction gives rise to new ways of thinking and moving' (p. 242). While Papert's Logo was then a high-tech and active computational expressive medium, the emergence of multimodal technologies (touchscreen, 3D printing, etc.) has given rise to even more hands-on and direct modes of interactions and expressions of mathematical ideas (Hegedus & Tall, 2016). Of particular interest is 3D printing which is capable of transforming artefact creation and manipulation from 2D (digital) into 3D (physical). In the context of school mathematics learning, 3D printing stands in contrast with pre-made manipulatives and pre-designed digital representational tools for being a powerful and playful expressive medium, which draws upon the innate human desire to make things with our hands (Fleming, 2015). 3D printing is a form of Making which empowers students to create and innovate through 'hands-on production of artefacts that are technologically-enhanced' (Chu, Quek, Saenz, Bhangaonkar, & Okundaye, 2015, p. 330) as opposed to inhibit students as consumers or recipients of meanings as determined by others. It is also associated with a spatial and hands-on approach to learning which has implications for the growing global needs for expertise in the STEM disciplines (Wai, Lubinski, & Benbow, 2009).

Aligned with the prospect of Making and informed by previous empirical studies (e.g. Ng & Sinclair, 2018; Ng & Chan, 2019), we are interested in the potential transformations in thinking and learning of one specific form of 3D printing technology—a handheld 3D Printing Pen—which enables one to construct physical artefacts instantly via one's moving hands. One unique characteristic of the 3D Printing Pens is its diagrammatic nature: as the hand moves along with the Pen which the hand holds, a 3D diagram is created at once (Fig. 1). Ng and Sincalir (2018) showed that this diagrammatic nature mobilised some intriguing gestures diagram interplay during the Making process. Of particular significance is that the use of 3D Printing Pens offered new gestural forms of thinking: the students produced new gestures by which mathematical meanings of tangents and revolution about an axis emerged in the learning of calculus. Following the mathematician and philosopher Gilles Châtelet (2000), diagramming (the making of diagrams) and gesturing are inseparable, creative embodied acts that constitute new relationships between the mathematics and the material activity (of doing mathematics). For him, gestures and diagrams are sources of mathematical meaning, which presuppose each other. They are never complete and share similar mobility and potentiality: gestures give rise to the possibility of diagramming, while diagrams give rise to new possibilities for gesturing. Drawing on Châtelet, de Freitas and Sinclair (2014) posit that diagrams capture gestures 'mid-flight' (p. 64); they are provisionally emergent, affecting the individuation of mathematical concepts

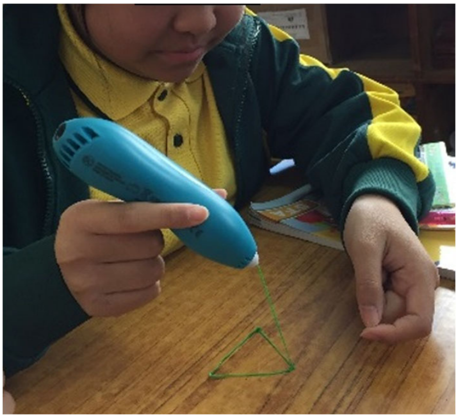

Fig. 1 Using a 3D Printing Pen to construct models in 3D

as 'kinematic capturing devices' (p. 65). This materialist vision is useful to identify the unique prospect of using 3D Printing Pens in mathematical activities due to its close resemblance to diagramming and gesturing, as in the embodied hand movements during the course of material creation, both of which engender new possibilities of encounter with mathematical concepts (see also Hall and Stevens, 2015 who called attention to the materiality of the task and tool environment from an interactional analysis perspective).

Châtelet (2000) claims that diagramming is a dynamic and material practice, which generates something ontologically new. His view advances a materialist conception of mathematics by seeking to move beyond the dichotomies of concrete/abstract and body/mind that pervade most mathematics education theories (e.g. Nemirovsky, Kelton, & Rhodehamel, 2013; Sinclair et al., 2013; de Freitas & Sinclair, 2014; Coles & Sinclair, 2018; Ng, Sinclair & Davis, 2018; Kelton & Ma, 2018). Mathematics is a product of the human activity, yet this does not mean that it is as simple as a product of human intentions and actions. The concept of assemblage has been taken up as the 'real unit' of study in new materialisms as a way of looking beyond the human by considering various kinds of non-human agencies (Deleuze & Parnet, 2007). In the context of understanding coordinated movements with motion sensor technology in mathematics learning, de Freitas, Ferrara, and Ferrari (2017) called attention to learning assemblages as 'provisional dynamic physical arrangements involving humans and other bodies *moving together* and *learning together*' (emphasis in original, p. 60). Assemblage refers to the entanglement of the components and its relational movement, rather than to the mere set of components. Therefore, an assemblage speaks directly to the provisional ways in which different bodies come together in mathematical activity and to how they constantly reconfigure activity, shedding light on its embodied and material dimensions. In this study, the hands-on production of artefacts with 3D Printing Pens is not only a form of Making but also a kind of *assembling* of learners, concepts and tools: moving and learning together with 3D Printing Pens through coordinated hand and eye movements.

Our goal in this paper is to examine what learning entails as students engage in one specific form of Making—with 3D Printing Pens—from a materialist perspective. We are not much interested in engaging in the question, 'What have the students learned', but rather in exploring the new ontology of mathematics learning which emerges in and from Making. We present two examples of Maker-centred lessons for teaching and learning primary mathematics respectively, from which we explore the implications of tools and Making in student learning. Before this, we discuss, in the next section, in more depth how we see mathematical learning in this highly transformative and technological Making environment. In particular, we discuss the entanglements of human, mathematical concepts, and technology in learning, and gestures and diagramming as a form of inventiveness in mathematical practices.

Theoretical Framework

Inclusive Materialism and Learning Assemblages

In the chapter of their book entitled, 'When does a body become a body', de Freitas and Sinclair (2014) offer a theoretical approach of unbinding the body from its skin to shed new lights on the ontologies of body and of mathematics. They redefine the boundaries of the body by considering the interactionist point of view that materials are not inert but are constantly interacting with each other and with the human body. The interactionist approach to the body reflects a more social perspective of meaning-making with respect to the conceptualist tradition, which ultimately demotes activity to simulation rather than full-body Making. To challenge the binary of human and tools, de Freitas and Sinclair refer back to an image from Merleau-Ponty (1945), that of a man walking in a dark room with a stick: does the man feel his hand touching the stick or does the man feel the end of the stick touching the contours of the dark room? This image implicates that in an interactionist approach we might avoid to associate agency with human will and intention, and, instead, reconsider the ways in which the human and non-human form bond thus reconfigure the world (in other words, agency is distributed across surfaces and bodies and tools). Accordingly, it shows that within human-material interactions, boundaries are re-created and *assemblages* emerge as 'the minimum real unit' in human activities (Deleuze & Parnet, 2007, p. 51). Following assemblage theory and various new materialisms and new empiricisms in the social sciences (e.g. Bennett, 2010; de Freitas, 2012), assemblages are therefore the fundamental 'real unit' of study.

Drawing on assemblage theory and inclusive materialism, de Freitas et al. (2017) propose the term 'learning assemblage' to refer to the mobile human–non-human arrangements and reconfigurations of the world in learning situations. In learning assemblages, materials are not merely passive but actively involved in the assembling of meaning and in the Making event, through relational movement and collaborative activity. Therefore, assemblages possess emergent properties in contrast to the conceptualist idea that materials have confined properties of their own. In a learning assemblage, learners' material encounters, state of emotions and the surrounding contexts constantly form new relationships in the assemblage and contribute to reshape the body of mathematics. This perspective suggests that bodies are provisional relationships between moving parts and embodiment is a compromised intra-action between human and material, a move beyond the embodied cognition tradition and literature, and enactivist perspectives (Varela, Thompson, & Rosch, 1991; Lakoff & Núñez, 2000;

Radford, Edwards, & Arzarello, 2009; Maheux & Proulx, 2015), in which the centre of the activity is still given to the learner, but towards distributing agency across the learning situation (Rotman, 2008; de Freitas & Sinclair, 2013).

In this paper, we use the perspective of learning assemblage to investigate the kind of learning as Making that occurs as students engage and assemble with 3D Printing Pens in mathematical activities. In particular, assemblage theory and inclusive materialism help us analyse this data less in terms of tool use and the affordances for the human, and more in terms of the potential energy and force of the technology, insofar as it partakes of the Making.

Gestures, Diagramming and Inventiveness in Mathematical Practice

Diagramming and gesturing are important mathematical acts in emerging assemblages. De Freitas and Sinclair (2014) refer to them as a kind of 'boundary-drawing apparatus' (p. 82), devices that reconfigure the world rather than representing it or coding it. When a cut or divide is enacted, boundaries are created which both 'conjoin and separate the 'real' from the mathematical, the matter from the meaning' (p. 51). Therefore, diagrams and gestures are not *only* iconic representations of the 'real', they also affect the individuation of the 'real', simultaneously engendering its meaning. Likewise, diagramming and gesturing are not merely acts of meaning-making but of boundary-making which is crucial in performing the separateness of mathematical meaning (Châtelet, 2000). These movements are particularly relevant in Making processes such as those associated with the use of 3D Printing Pens, where gesturing and diagramming specifically affect the individuation of geometrical objects in Making. Located in the physical world, these embodied acts can potentially evoke mathematical meanings within body-material assemblages: 'Does mathematics really just stand there, silently waiting for the breakthrough insight or shift in attention? Or might it somehow be much more implicated in the moving hands […]? If so, what do we mean when we say that the actions are concrete and the mathematical expression abstract?' (de Freitas & Sinclair, 2014, p. 30).

In studying the historical interplay between gesturing and diagramming, Châtelet (2000) suggests that this dance is the source for mathematical inventiveness. He proposes the notion of the virtual as that which allows us to think of the mathematical and the physical together, as bond, challenging visions that deny mobility in mathematics and that typically associate mathematical thinking to the mind. Briefly speaking, the virtual is the indeterminacy, mobility and potentiality which is latent in the matter and which is actualised through activity, primarily by gestures and diagrams, in provisional configurations. In so doing, Châtelet does not see the mathematical as abstract and the physical as concrete. Instead, he reconceives mathematics as partaking of the virtual; mathematical entities are physico-mathematical entities with both virtual and actual dimensions, material objects on which mathematicians perform thought experiments. The virtual of mathematics is mobilised and actualised through these experiments. Drawing on this perspective, in the context of learning as Making, we might reconceive mathematics as material Making by learners with technology. We can thus investigate whether Making creates new mathematical spaces of encounter with the virtual for learners.

In line with Châtelet's study, Rotman (2008) also provocatively proposes to consider how the advent of new digital technologies might lead to new kinds of gestural and diagrammatic inventions. Rather than focusing on how technology might change the logical necessity of mathematics, he shifts attention to how it changes the mathematics, entailing a move away from symbolic formal language and towards new ways of seeing, touching, hearing and in our case moving and Making. Similarly, Sinclair et al. (2013) offered a new vision of inventiveness in the mathematics classroom, one that captures the temporal and dynamic moment when the new or the original comes into the world at hand in unscripted and unexpected ways. Therefore, inventiveness is seen 'as *an action taken* that emerges in context, without being exhausted by it' (Sinclair et al., 2013). Where these authors contend that attention be shifted away from the doer(s) to the doing, we propose to move away from the Maker(s) and towards the Making. In learning as Making, we will then analyse how the drawing hand and the gestures can together occasion new ways of thinking and moving for learners. We will end up in stating Making as a material process not only of mathematical creation (in the physical sense), but of the mathematical invention (in the physico-mathematical sense *à la* Châtelet).

The Study

Participants and Context

The mathematics lessons exemplified in this paper were part of a design-based study, in which the first author collaborated with two classroom teachers in integrating cycles of lesson planning and implementation 'as part of a complex, evolving design process attempting to positively influence and effect change in a learning context through the building of a design intervention' (Bannan, Cook, & Pachler, 2016, p. 940). The intervention and design element was the teaching and learning of mathematics with 3D Printing Pens. The study took place at two public primary schools in Hong Kong with an average socioeconomic status. The participants consisted of two teachers, one from each primary school, and primary 5 students ($n = 25$) and primary 6 students ($n = 28$) who were enrolled in their respective mathematics classes. The teachers and students provided consent to participate, upon invitation by the researcher, on the basis that the study posed no conflict of interests and harm but would be beneficial to their teaching and learning experience. The participants had never used the 3D Printing Pens before this study.

Lessons with 3D Printing Pens

We briefly describe the lessons that integrated Making with 3D Printing Pens in two primary mathematics classrooms in the design-based study. We chose the topics, properties of prisms and pyramids in the primary 5 (age 10–11) level and cross-sections of 3D solids in the primary 6 (age 11–12) level from the local mathematics curriculum (Hong Kong Curriculum Development Council [HKCDC], 2015), as they were deemed complementary to teach mathematics with 3D Printing Pens based on previous empirical research (Ng & Sinclair, 2018).

Specifically, the former topic was complementary because of the ease for primary mathematics students to construct 3D models without necessarily using any pre-made manipulatives (for example, nets of solids) nor working with the constraint of tools that render 3D into a 2D representation (paper-and-pencil and computer screen). The latter topic was chosen because it was hoped that the diagrammatic and manipulative nature of 3D Printing Pens, coupled with a hands-on, investigatory approach to learning, would support students' spatial reasoning and visualisation of cross-sections. Generally speaking, local teachers have found it challenging to approach this topic pedagogically. Though local curriculum documents have suggested the use of teaching aids in the form of digital tools (such as dynamic geometry environments) or physical manipulatives (such as transparent liquid containers in the shape of various polyhedra), teachers have found that these tools were only helpful insofar as to show the cross-sections visually but did little to support conceptual learning through explorations and reasoning about mathematical relations.

In both lessons, the classroom teachers began with a brief introduction to the learning objective of the lessons. Then, they introduced the relevant terminologies in the lessons by drawing on everyday examples such as cutting an orange to illustrate cross-sections and showing a box of chocolate bar to review the meaning of triangular prisms. The main student-centred, inquiry-based activity followed, which invited students to construct some 3D models actively with the 3D Printing Pens. In both lessons, every two students were given one 3D Printing Pen for completing the construction tasks as given by the teacher, and each student was given ample time to take turns in using the 3D Printing Pens to construct the artefacts as planned in the lessons.

In the properties of prisms lesson, the students actively constructed different forms (triangular, rectangular and pentagonal) of prisms with 3D Printing Pens (Fig. 2a and b). The students were given minimal guidance on how to construct them; the rationale was that the construction process should remain open-ended in order to encourage different construction strategies and different sizes of solids to be constructed. After the construction task, the classroom teacher and students together completed a chart about the learning targets: the number of lateral faces, bases and a total number of faces of prisms. Referring to this chart, the teacher eventually led a class discussion on generalising these properties for an *n*-sided polygon.

In the cross-section lesson, the students were tasked with constructing the outline of various cross-sections of 3D solids with the use of 3D Printing Pens. Having just learned the meaning of cross-sections, the students used the 3D Printing Pens to anticipate and trace around the outline of a cut through two given solids (a cylinder and a square pyramid). Figure 2 c and d show two kinds of constructions, i.e. both cross-sections of a cylinder that the students would complete during the activity. A worksheet was given to each student, which facilitated them to complete a total of seven 'cross-sections' with various cuts (horizontal to the base, vertical to the base and oblique) through the given cylinders and square pyramids. The first part of the worksheet asked students to guess what shape the cross-sections would take on before they began their constructions. The students then used the 3D Printing Pens to construct and then detach the artefacts from the

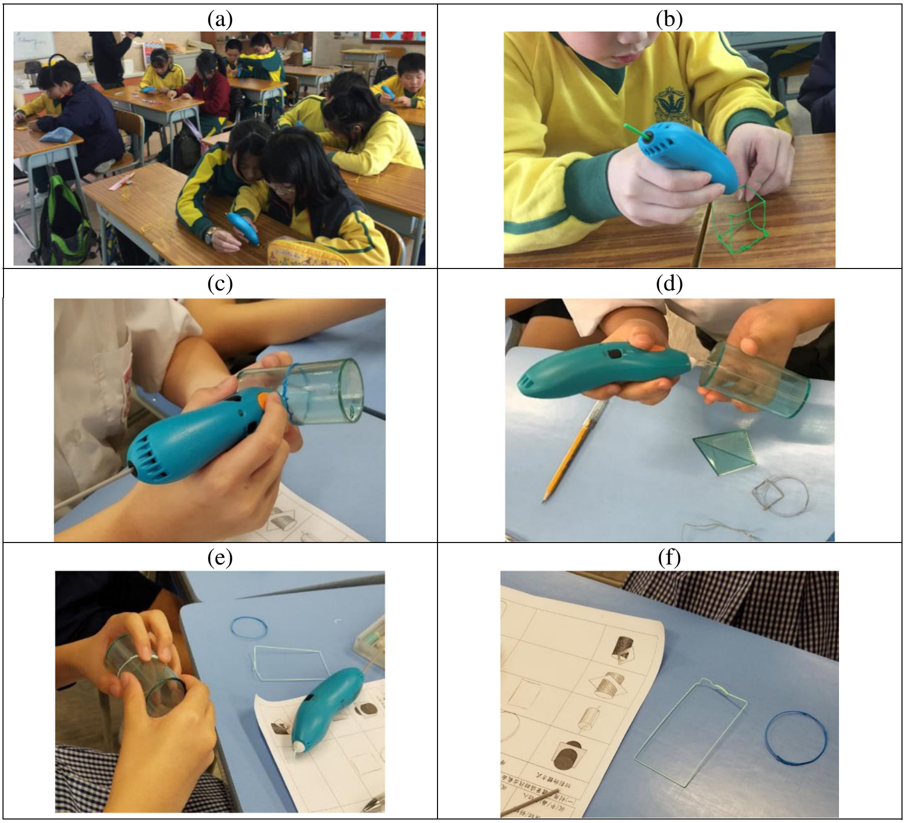

Fig. 2 **a** and **b** Snapshots of the 'properties of prisms and pyramids' lesson. **c**–**f** Snapshots of students' active Making of artefacts during the 'cross-section' lesson

solids physically, upon which they would draw the shape of their artefacts on the worksheet (Fig. 2e and f).

Method

Having collaborated with the classroom teachers on planning the target lessons, the first author was present during these lessons to observe and interact with students while collecting video data. Simultaneously, she was also participating in the same space and time as the study's subjects, forming a research assemblage of the researcher, subjects, data and method during the intervention. Also, as researchers partaking in this study, we both assemble with the data and the theoretical perspectives that we are investigating. Following Mazzei (2014), we offer diffractive analyses to explore the learning assemblages that emerged in activities with 3D Printing Pens. Diffractive analyses stem from the work of Barad (2008) who explored new ontologies in quantum physics by studying the relations between matter and meaning. Such analyses involve a diffractive apparatus, which produces effects or diffracts meanings as entangled with the apparatus. Important for this study is that unlike reflections, which like in a mirror 'reflects the themes of mirroring and sameness' from a distance, diffractions 'are attuned to differences' (Barad, 2008, p. 72) and focus on fine details. Hence, a diffractive analysis 'is respectful of the entanglement of ideas and other materials in ways that reflexive methodologies are not' (Barad, 2008, pp. 29–30). Sinclair (2017) infers that the work of Barad may offer a model to design and conduct research experiments in an educational context. In her case, the apparatus was a multitouch iPad application for young children to learn number concepts, which helped her show 'how number is created and re-created through the children's gesture and touch in this experiment' (p. 118).

In this study, the 3D Printing Pen is a diffractive apparatus, which produces effects that help us see how mathematical meanings are entangled with the physical/Pen. A diffractive analysis is suitable, as we aim to explore new ontologies of learning/Making rather than to study the 'effect' of using 3D Printing Pens on students' mathematical learning. Our materialist frame positions us differently from 'most educational technology experiments that separate the students from the apparatus and study students' acquisition of concepts (which are considered to be timeless and given)' (Sinclair, 2017, p. 118). Of particular importance for this study is that diffraction focuses on the details, however fine, that emerge in a phenomenon, and therefore helping us to put the focus on the temporal and dynamic nature of the mathematical activity and to speak of change and difference in the activity. As we reviewed and selected the video data for analysis, we did not focus on what the students have learned or how the tool has mediated a particular concept; instead, we focused on how the 3D Printing Pen was involved in producing new ideas, paying particular attention to the specific material configurations at play when the 3D Printing Pens were in use and recognising that these configurations (ways of holding the pen, speed of drawing, etc.) actually matter. The aim is to not only analyse how and what the 3D Printing Pen produces, but rather how it can be productive of new possible realities of mathematics learning.

Diffractive Analyses

In this section, we provide our diffractive reading of the video data in selected moments of students' Making in both lessons. Upon our diffractive reading of each lesson, we use a materialist perspective to propose some characteristics of Making as being significant and highlighted in the analyses.

Properties of Prisms

In the properties of the prism lesson, the students showed their excitement and eagerness to try out the 3D Printing Pens for the first time through their verbal expressions and body language. The configurations of the classroom and task were arranged that had every two students who sat beside each other to use one 3D Printing Pen to construct different forms of prisms. The first task was of constructing two triangular prisms, one by each student. Students were quick to begin Making by constructing artefacts on the surface of the table; however, the task of constructing the mesh of 3D solids became difficult as the students were encountered with the third dimension of 3D solids. One important characteristic of the Making task was that the 3D Printing Pen afforded the students to construct in the third dimension by extruding plastic 'in the air'. Yet, the hardening of the plastic took some time, and this could make those plastics that were hanging ‘in the air’ to fall due to the force of gravity. As such, two students often worked as a unit in crafting their 3D models, especially when constructing in the third dimension. In particular, the students without the 3D Printing Pen would use their hands to hold parts of the 3D models for support while the other students completed the construction. As seen in Fig. 3, both hands of this student (on the left) were actively involved in the construction even though he was not the one holding the 3D Printing Pen. Beyond moving the 3D Printing Pen from one point to another in space, the Making process called upon other engagements with the human body: the students coordinated their eyes and hands with each other while gripping and holding certain parts of the 3D models. Furthermore, the surface of the table and forces of gravity was at play in the construction as shown in Fig. 3. These bodily and material experiences were important in the learning/Making process inasmuch as the process was shaped and, at the same time, constrained by these experiences.

Though the context was to construct specific prisms, the task was open-ended in the sense that students would construct prisms in different sizes and in different ways. Most students would begin by constructing the prism’s ‘base’, such as a triangle, rectangle or pentagon, on and attached to the surface of the table. Having established a ‘base’ (literally and in a mathematical sense), they then extruded plastic from the vertices of the ‘base’ while moving the 3D Printing Pen above and away from the table, thereby forming plastic pillars that stood vertically (from the table) in the third dimension. In the case of triangular prisms, for instance, many students constructed a triangular base initially, followed by three vertical pillars ‘in the air’ (Fig. 4a). The last step of this construction was to make a triangular shape ‘in the air’, by moving the 3D Printing Pen from the top of one pillar to another in a triangular path. The students coordinated movements of their eyes, their hands and the 3D Printing Pens slowly while their partners assisted them in the process. However, we did observe a different process of Making which was worth describing because of differences in how the triangular prism was realised. These students would first construct a rectangular shape (Fig. 4b), followed by two pairs of oblique pillars, which met at one point ‘in the air’ while the other ends held onto the vertices of the rectangular face. In doing so, two upright triangles had been formed. The last step was to construct a horizontal pillar which joined the upright triangles at their vertices which acted as anchors of the pillar.

In terms of hand movements, the students’ hands were mobilised via constructing the mesh of prisms. Some of these movements were continuous and two-dimensional, as in constructing a triangle on the surface of the table continuously without stoppage of

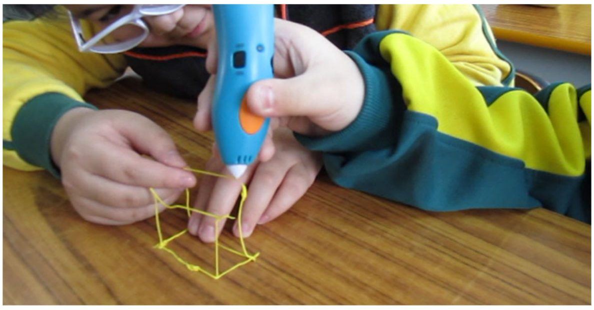

Fig. 3 Two students working together to construct a rectangular prism

(a) (b)

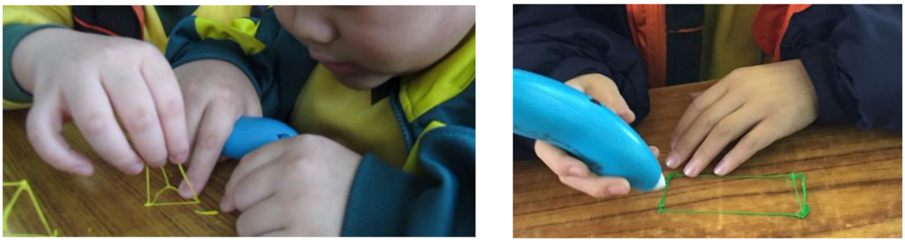

Fig. 4 **a** and **b** Different ways to construct a triangular prism

plastic using the 3D Printing Pen. Other movements were one dimensional, as in constructing vertical pillars ‘in the air’. Linguistically, the students talked about their constructions as initially Making ‘triangles’, ‘rectangles’ or ‘pentagons’, after which they would construct ‘lines’ that stood upright or at an angle. The significance was that they were speaking of 2D shapes (‘triangles’, ‘rectangles’, etc.) and 1D ‘lines’ by realising prisms as a composition of 2D (shapes) and 1D (lines) parts. Non-linguistically, the students’ constructions called upon the hand—along with the 3D Printing Pen which it held—to observe perpendicularity and parallelism between lines, between lines and planes and between planes. The symmetry of 2D shapes and 3D objects also emerged in these constructions. In the episode below, we further illustrate some intriguing interplay between a student’s hands and the mathematics communicated by the student in the properties of the prism lesson. The student has just finished constructing a triangular prism when the researcher approached to interact with him.

Episode 1

00:01	Researcher:	How did you draw [the triangular prism]? What did you draw first?	
00:06	Student:	In the beginning? The rectangle.	(Fig. 5a)
00:10		Then draw an oblique line and wait for it to harden, and draw a line to here.	(Fig. 5b)
00:18		This side is done by the same method, then drag from here to here to finish this.	(Fig. 5c)
		<2 min and 3 s later>	
02:21	Researcher:	Ok, one more question <the researcher took the triangular prism away>. How many vertices are there? We have a vertex when two or more lines meet. How many vertices?	
02:35	Student:	<Silence for 2 s while moving his finger above the table> Six.	
02:38		There is a triangle on the top left, so there is one vertex, on the right side there is another vertex, and for the rectangle…it has two vertices on the left, and two on the right.	(Fig. 5d)

In this episode, the student described how he (re-)constructed his triangular prism. He began by using his right index finger to imagine just a rectangle on the surface of the table (Fig. 5a). Then, he moved along two oblique lines thereby creating an upright triangle through the movement of his fingers (Fig. 5b). He repeated the same movement on the other side of the rectangle (Fig. 5c), and finally, he (re-)constructed a line ‘in the air’ by moving his finger ‘from here to here’ (Fig. 5d). Two minutes later, the student was encountered by the question: ‘How many vertices are there’, as posed by the researcher. Since the artefact was

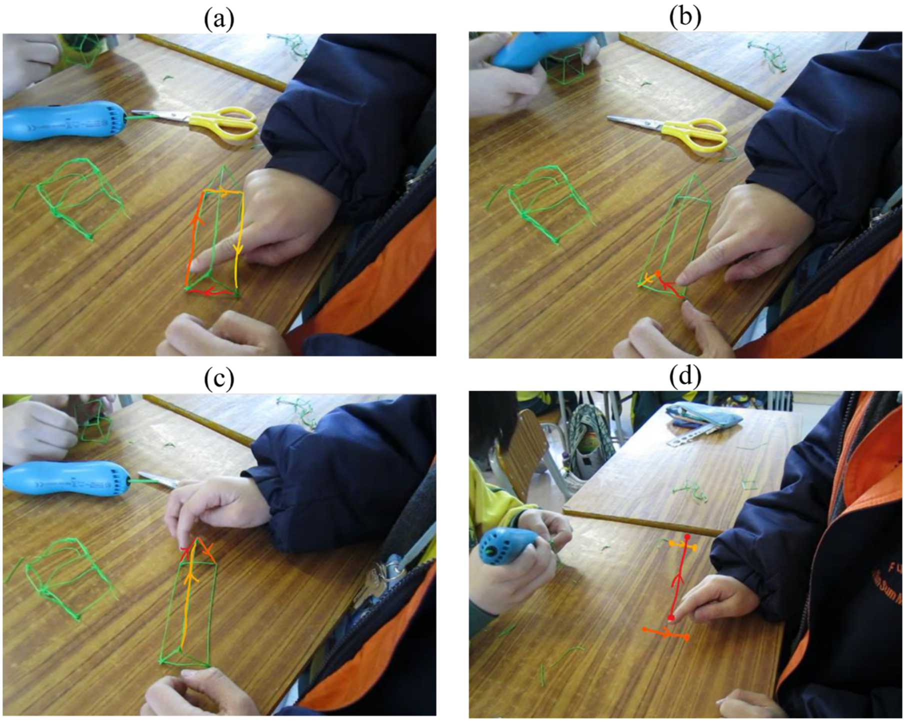

Fig. 5 **a**–**d** Re-constructing the triangular prism

taken out of sight by the researcher, the student again (re-)constructed the triangular prism as emerging from his moving hand. In particular, he actualised the vertices one by one with his right index finger as if they were 'real'. This time, he mentioned that there were two 'triangles', where one vertex sat on the 'top left' and another one 'on the other side'. Interestingly, he was bringing forth that triangular prism in a different way this time: namely, two triangular faces emerged out of words and gestures. This speaks directly to the mobility and indeterminacy of prisms encapsulated by the use of the 3D Printing Pen. In fact, making a prism might, for example mean starting from the rectangular base and add one triangle on the one side and another triangle on the other side before completing or the other way around, immediately drawing two facing triangles, perpendicular to the surface of the table, and then individuate the rectangle at the bases. The student here changes from one configuration to the other, through provisional ways of bringing the prism into being that is also potential ways of making it with the Pen. Diffracting here means to be sensitive to, and notice, these differences. From this analysis, we could see how the 3D models were more than simply mediators of learning, but rather, the very movement that constituted students' mathematical thinking. Therefore, Making did not only yield a *product* that was physical and sharable, but it was also a material *process* of thinking mathematically that outlined differences in how to make a triangular prism.

Taking a materialist perspective, we claim the following about Making in the above analyses:

Making Is Co-Constructing Meaning

We see in these moments of Making that the artefacts were constantly being co-constructed: by the surface of which it was constructed on, the forces of gravity, the 3D Printing Pens and both students' eyes and hands. The final constructions of prisms retained that same movement by the eyes, the hand and the 3D Printing Pens, like the student who explained his construction of a triangular prism in Episode 1. If the materials and task had been configured differently, the physical construction as well as the meaning of prisms would have changed. Hence, the materials, task and the students who engaged in Making played a fundamental role in co-constructing mathematical meanings.

Making Is Mathematising

The students' hands were constantly entangled with the artefact and mathematical concepts in the learning assemblage. These material entanglements configure what prisms are rather than simply represent them. In Châtelet's term, the student from Episode 1 was awakening the virtual and bringing the triangular prism into being through (re-)constructing it (actualising its particular view) with his gestures. Within his hand movements, this student showed how the concepts of 'lines', 'rectangles', 'triangles' and 'prisms' came back to life. Importantly, he also showed how the artefact he made encapsulated the mobility and indeterminacy of 'prisms' as unfolded in the Making process. On the other hand, the freedom of where and how to begin Making seems to be a relevant quality in terms of mobilising the diagrams: the students are doing thought experiments together with the hand experiments of creation of a figure, through which they encounter and actualise the virtual of prisms.

Cross-Sections of Solids

In the cross-section lesson, the main student activity was that of exploring various cross-sections of 3D solids by constructing them physically with 3D Printing Pens. This task called upon students first to imagine a cut of two given solids, a cylinder and a pyramid. Then, they would trace, with their 3D Printing Pens, the outline of the anticipated cross-sections and detach them from the solids to observe their properties (shapes). The students initially did struggle with the tasks of extruding plastic onto the surface of the solids as well as to detach them from the solids without breaking them apart. As they continued to engage in the task, they became more adept to working with their hands, the 3D Printing Pens and the given solids for completing the task. Yet, each cross-section construction demanded a different material experience for the students. For example, in constructing a circular cross-section of a cylinder, as in Fig. 2c, it demanded some forces be exerted upon the surface of the cylinder by the student's hand holding the 3D Printing Pen while it moved along the surface of the cylinder. Conversely, the curved surface of the cylinder also exerted force upon the 3D Printing Pen and provided certain feedback to the student's embodied experience. These sorts of explorations brought forth different ways of 'assembling' the meaning of a circle: some students moved the 3D Printing Pen around the cylinders by 360° thereby forming a circle, and others rotated the cylinder by 360° while holding the 3D Printing Pen still to the same effect. In both cases, these movements make the circle emerge as a cross-section in the learning event.

Where there is an oblique cut to the cylinder and the cross-section is elliptical, the construction of artefact called upon a different material encounter for the students. Most students used a combination of moving the 3D Printing Pen and rotating the cylinder in their constructions (Fig. 6a–c). Moreover, the configuration of holding the 3D Printing Pen on one hand and the cylinder on the other made it difficult for the students to complete the construction 'in one go'. Therefore, they would divide the construction into two or more segments, where each segment would be joined by another using 3D Printing Pens immediately before the plastic parts would harden after extrusion. Moreover, as shown in Fig. 6a–c, the movement of the 3D Printing Pen was in such a way that took a longer time and distance to travel around the cylinder compared to when constructing a circular cross-section. In other words, *time* and *distance* were made manifest in the students' assembling of artefacts.

When realising the cross-section of a cylinder perpendicular to the base, the construction required making four right-angled turns, two at the top circular face and two more at the bottom face which lied on the same vertical plane, with the 3D Printing Pen (Fig. 7a). These actions of moving along the four straight paths and making four right-angled turns with the 3D Printing Pen was similar to when dealing with cross-sections of a square pyramid. For example, the students constructed artefacts by moving the 3D Printing Pens along with four straight paths and making different amount of turns with the Pen in the case of working with horizontal and vertical cuts of a square-based pyramid (Fig. 7b and c). Thus, within these three different Making processes, *angles* and *turns* were made manifest, where the meanings of rectangles, squares and trapezoids were also co-implicated. Though the students were constructing rectangles and squares in the first two examples, the material encounters of the two were different, both in terms of the surface which the 3D Printing Pen was in contact with (curved versus flat), and in the way the other hand took a grip of the solid. For instance, the student in Fig. 7 a held objects (the 3D Printing Pen and the solid) in both his hands, while the student in Fig. 7 b rested the solid on the table while tracing around it. As can be seen, the meanings of rectangles and squares were assembled differently with 3D Printing Pens and the given solids.

The most challenging cross-section task in the lesson involved an oblique cut of a cylinder which did not yield an ellipse or a rectangle, but instead, an irregular shape whose outline was made up of a straight line and a curve. This task was different from all other ones in this lesson in the sense that the 3D Printing Pen needed to travel along a combination of straight and curved paths on the surface of a solid (Fig. 8a). It could be said that the curved path was not obvious for the students, which made it more

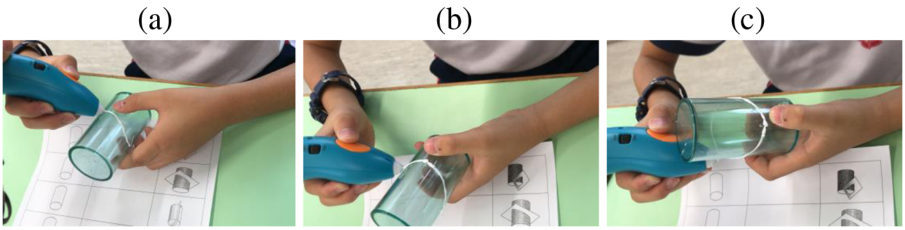

Fig. 6 a–c Oblique cuts to the cylinder

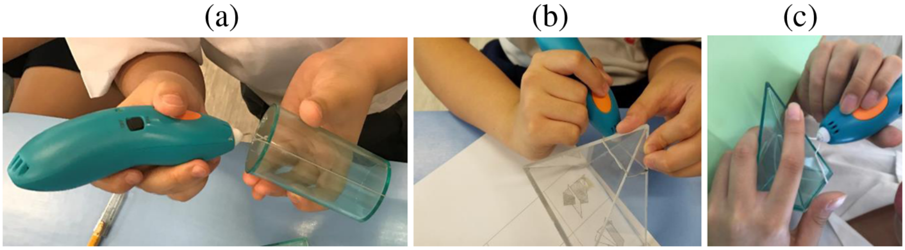

Fig. 7 Students actively Making artefacts on the surface of the given solids, i.e., a cylinder (**a**) and a square-based pyramid (**b** and **c**)

challenging than other constructions. In addition, the students had not realised that it was possible for the shape of cross-sections to be irregular, and they ended up constructing artefacts that looked somewhat like triangles (Fig. 8b) after engaging in the task. Interestingly, some students elected not to use the 3D Printing Pens but to produce gestures that enacted the construction virtually. They did so by moving their fingers slowly along an imagined path on the surfaces of the solids. For example, a student moved his finger across the circular face (Fig. 8c and d) and then onto the curved surface (Fig. 8e) before eventually describing the cross-section as triangular.

We end our analysis with an episode during the cross-section lesson, in which a student has just used the 3D Printing Pen to construct the outline of a (trapezoidal) cross-section, which was vertical to the base of a square pyramid. He has also just detached the artefact from the solid, which now sat on the table. The researcher approached to interact with this student and the following conversation unfolded:

Episode 2

23:08	Researcher:	How do you think you can make an even bigger trapezoid?	
23:10	Student:	Bigger trapezoid? Using [the 3D Printing Pen]?	
23:15	Researcher:	Yes.	
		<Student constructed an artefact on the surface of the solid>	
23:36	Researcher:	Why did you draw this way? How was it different from the one you drew before?	
23:40	Student:	It's bigger.	
24:44	Researcher	How did you make it bigger? What was different about the way you drew before?	
23:48	Student:	I made it taller.	
23:51	Researcher:	Taller… Good. Why are you sure that it is a trapezoid [even without detaching it from the solid]?	
23:55	Student:	Because… Because from the way I did it before, I made a trapezoid. So I used the previous method but made it higher. If I did it in this way, it must be a trapezoid.	(Fig. 9a)

In Episode 2, the student used the 3D Printing Pen to construct a second 'trapezoid' which was now attached to the solid, whereas the first one had been detached from the solid (Fig. 9b). This construction was initiated by the researcher's prompt to make a 'bigger trapezoid'. The student elected to use a 3D Printing Pen to show how he could do it and explained that this 'bigger' one was different from the first because he 'made

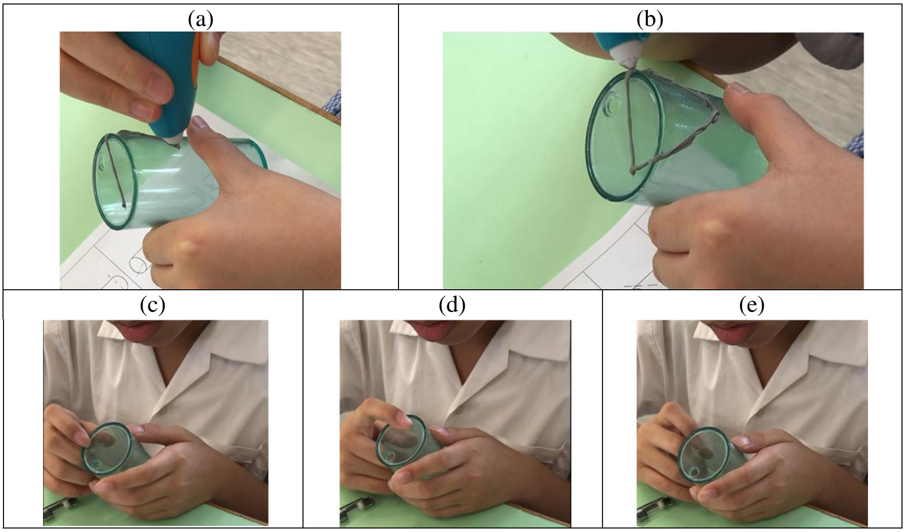

Fig. 8 **a** and **b** Constructing the outline of a cross-section with an oblique cut to the cylinder. c–e A student electing to move his index finger along with the surface of the cylinder as a way to visualise a cross-section with an oblique cut to the cylinder

it taller'. Then, the researcher asked him to justify why he was sure that the artefact was trapezoidal. In response, the student referred to his previous construction: 'Because from the way I did it before, I made a trapezoid. So I used the previous method but made it higher'. He mobilised his previously made 'trapezoid' by making present the height as a changing entity, thereby actualising the new trapezoid as the taller figure (Fig. 9a). Since the two constructions were similar in the way the hand moved in relation to the solid, it could be precisely why this student was so sure in his response that 'If I did it in this way, it must be a trapezoid'. Our diffractive analysis points out the way that the difference in size for the trapezoidal section is actualised as a difference in height. Through the moving hands and the created diagrams, the mobility which characterises the cross-sections perpendicular to the base of the solid is brought forth as identification between a family of figures and a family of cuts, which differ from each other in height and location respectively.

From a materialist perspective, we claim two more characteristics of Making in the above analyses:

Making Is Assembling with Technology

We see that each artefact was assembled differently by the students' hands, the surface of the solids and the 3D Printing Pens, and a 'Making assemblage' was formed each time the students used the 3D Printing Pens. Besides, the concept of time, distance, angles and turns entered the 'Making assemblage' which gave rise to the meanings of circles, ellipses, squares, rectangles, trapezoids, etc. In terms of the students' 'struggles' with the most challenging task, the materialist perspective focuses on the evolving relationships in the 'Making assemblage' of students, technology and mathematics.

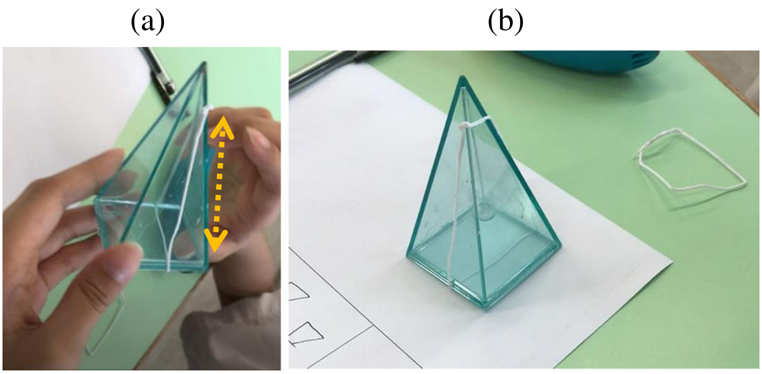

Fig. 9 **a** A student's gestures to highlight the height of his artefact. **b** Two artefacts in the shape of trapezoids were constructed at the end of the episode

From this perspective, we say that these relationships were developing in such a way that compromised the students' realisation of mathematics. This was especially since the idea of an irregular shape of cross-section had not yet entered the students' 'Making assemblage'. In other words, mathematics was not some abstract concepts to be conceived or acquired 'in the head' but it emerged as an *assemblage with technology* from the students' drawing and gesturing hands.

Making Is Inventing

We see two moments of inventiveness in this analysis. First, we consider the student's gestures as shown in Fig. 8 c–e as acts of inventiveness. His creative act was in introducing a new way of thinking about cross-section, which involves touching and moving one's finger along the surface of the solid. Through these gestures, he was also engaging in a thought experiment about the cutting of a cylinder, where mathematics was *invented* and something new, unplanned and unscripted was created at the moment. On the other hand, the student in Episode 2 was inventing another way of thinking about cross-sections: he was relating one cross-section to another in terms of the location of the cut and of the movement along with the surface of the solids from one location to the other. Therefore, the student had not only created an artefact, but he was also mobilising the cross-section to visualise new potential cross-sections of the same kind, in that the second section must be taller than the first. The emergence of this mobility, especially out of gestures, fundamentally changed the nature and meaning of cross-sections as ever changing with respect to the cut.

Discussion and Conclusion

In this paper, we pursued an exploratory approach to understand 'What learning entails' in contrast to 'What students learn' in a technologically enhanced, Maker-centred environment from a materialist perspective. Our diffractive analyses helped us explore a new ontology of learning *as* it emerges in Making with a four-fold characterisations: (1) the co-construction of meanings by the students' coordinated bodily movements with the material world; (2) the interplay between the students' hands and the emerging 3D models—between the bodily and the diagrammatic—as a form of mathematising; (3) the Making assemblage of human, mathematics and technology, which constantly reconfigures and reorients thinking, and where 3D Printing Pens play a central active role in affording this material act of creation; (4) the inventive nature of Making as students mobilise artefacts to think with, and as they perform creative and unscripted ways of thinking mathematically what was not present or available before.

This study contributes towards the changing nature of knowledge, learning and pedagogy in the digital age. In particular, having advanced materialist perspectives of learning mathematics as a form of Making, we believe that this exploration helps us to better understand the complex nature of students' mathematics learning by attending to the fine details and emerging relationships in their bodily material experience while Making. On the other hand, materialism provides us with the language to describe Making beyond distinguishing between categories of concrete and abstract (physical and mathematical) and body and mind (Making and learning). Revisiting Papert and Harel's (1991) constructionism, his notion of 'learning-by-Making' is dualistic in the sense Making is a process of creating artefacts that affects what goes on psychologically, such as building mental schemas 'in the head'. From this perspective, learning and Making are conceived as separate and sequential processes, as illustrated by the phrase, 'learning-*by*-Making'. From our investigations of what learning entails *as* Making, we have shown that learning/Making was essentially all entangled with the students' hands, their eyes, the 3D Printing Pens and the material surrounding. Not only was learning inseparable from Making, but so were the hand's movements, the technology and material agencies in the creative assembling of physical artefacts and mathematical ideas. This way of conceiving mathematics and mathematical activity as assembling in the Making also goes beyond socio-cultural assumptions that still set the learner at the centre of the activity and the materials as inert, looking at them as mediators of knowledge. In our view, materials actively partake of mathematics and mathematical thinking, and speaking of learning as Making is a way to claim this in the specific context of Making with the 3D Printing Pens.

In reflecting upon the implications for a 'learning as Making' pedagogy, we concur with Papert that Making empowers students as producers as opposed to consumers of knowledge. A 'learning as Making' pedagogy is one which presents as hands-on and goal-oriented, Making activities with the use of a 'playful' and flexible technological media. Through Making, students engage in problem-solving, inquiry-based learning and *invention* of new ways to think mathematically. This pedagogy is mindful of that Making does not produce artefacts that represent some abstract mathematical concepts nor mediate students' learning, but it is a practice of *mathematising*—a material act of creation by which the virtual is actualised. As such, Making is both the means and goals of learning, and the idea of 'learning objectives' which implies learning is acquiring timeless and given mathematical concepts would give way to 'Making objectives'. Besides, a 'learning as Making' pedagogy is considerate of the *co-constructing meanings* and *assembling with technology* by attending to the fine details of students' body-material intra-actions and designing tasks that foster these intra-actions. In terms of the gesture-diagram interplay, we see the potential for 'learning as Making' pedagogy to offer new modes of interactions to think, encounter, touch and move mathematics with students' hands and eyes.

With reference to the geometry curriculum that prevails in current mathematics education, Gerofsky (2018) encourages us to look beyond 'geometries of straight lines and rectangular grid' (p. 2) that often treats the sense of shape and space as subordinate to the learning of other school mathematics domains, such as algebra. She proposes a transdisciplinary approach with a new consideration of 'geometries to think and act with' (p. 3) which would not only potentially reconfigure education but help to solve profound problems in our societies. Our notion of 'learning as Making' fits well with this transdisciplinary approach especially in the context of STEM education, because of the three-dimensional nature of artefacts that the 3D Printing Pens create, making them applicable to real-world problems and projects and thereby helping to shape the learning experience of STEM/STEAM (English, 2016; Ng, 2017). Therefore, Making invites students to see geometries (and mathematics) as concepts 'to think and act with', and 'to inquire and invent with'. A 'learning as Making' curriculum is not just about supporting students' technical literacy at an early age but to enculture a 'Maker mindset' in which students are free to move and invent the mathematics they are Making. In closing, our four-fold characterisations of the proposed lens of 'learning as Making' pedagogy contribute towards refining what it means to learn abstract mathematical concepts in technological, hands-on, and innovation-oriented environments that bring forth the issues of temporality, mobility and virtuality at the heart of mathematics.

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