3 FORSCHUNGSBERICHTE UND -ÜBERSICHTEN

3.1 HELMHOLTZ IN NEO-KANTIANISM

HELMHOLTZ IN NEO-KANTIANISM

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1. WORKS BY HERMANN VON HELMHOLTZ

1847: *Über die Erhaltung der Kraft*, Berlin: Reimer; reprinted with additional notes in: Helmholtz 1882, 12-75; Eng. trans. in: Helmholtz 1971, 3-55. This essay includes a proof of an equivalent formulation for the conservation of force – namely, the principle that all forces can be calculated as functions of distances between pairs of points. Once the direction has been specified, all forces are supposed to be central forces. By 1882, however, Helmholtz admitted that the centrality of force is not necessary: it can only be assumed as an empirical generalization and such assumption can be called into question, if one considers more recent electromagnetic theories. Moreover, in an addition appeared in the 1882 edition, Helmholtz declared that, in 1847, he adhered to a Kantian conception of causality he was not willing to defend any more. By that time, Helmholtz tended to consider Kant's principles of experience as regulative ones: the law of causality is now understood as the requirement that the phenomena be related to one another in a lawful way.

1855: Über das Sehen des Menschen, Vortrag gehalten zu Königsberg am 27. Februar 1855, in: Helmholtz 1903, Bd. 1, 85-118. This lecture was given by Helmholtz when he was appointed professor of Physiology at the University of Königsberg. On that occasion, Helmholtz appealed to Kant in order to bridge the gap between philosophy and the sciences at a time when most scientists had become sceptical about philosophy. In Helmholtz's opinion, scepticism was a reaction to the philosophies of nature of Hegel and Schelling and their attempts to predict empirical results by means of pure thinking. Helmholtz put Kant's theory of perception in connection with physiological developments such as Johannes Müller's theory of specific sense energies: Kant's indication of subjective factors of representation is confirmed by Müller's proof that sensuous qualities depend not so much on the perceived object as on our nerves. This claim was vividly discussed by the neo-Kantians. Though none of them accepted the physiological interpretation of Kant's philosophy, Helmholtz's intent to go back to Kant to promote a science-oriented philosophy was appreciated especially by Alois Riehl. And Helmholtz's use of Kant's philosophy to study the foundations of the sciences can be considered one of the first developments of the neo-Kantian movement itself. Helmholtz considered the inquiry on the conditions of knowledge preliminary to any assumption regarding nature. He, therefore, distanced himself both from an idealist conception of nature and from a materialist one.

1862: Über das Verhältniss der Naturwissenschaften zur Gesammtheit der Wissenschaften, Akademische Festrede gehalten zu Heidelberg beim Antritt des Prorectorats, in: Helmholtz 1903, Bd. 1, 157-186. Helmholtz classifies the sciences according to their methods. Natural science presupposes the conscious formation of general categories and laws. By contrast, human sciences, such as jurisprudence, philology, and history, are characterized by their lack of clearly formulable principles. Their generalizations result from a kind of unconscious reasoning Helmholtz calls *artistic induction* in contrast with the logical induction of natural science.

1867: *Handbuch der physiologischen Optik*, 3 Bde., Leipzig: Voss. In this work Helmholtz rejects nativism (including Müller's theory of vision) and develops an empiricist physiology of vision, along with an empiricist explanation of spatial perception based on local signs and unconscious inferences (see also Helmholtz 1868a, 331-365). So it might seem that he calls into question also the Kantian theory of perception he endorsed in 1855. Still there are points of agreement with Kant, in several aspects. Firstly, inferences from our sensations to external

causes presuppose the principle of causality as an a priori law in Kant's sense. Owing to the principle of causality, the qualities of sensations can be deemed signs of something else. Secondly, Helmholtz traces back to Kant a fundamental distinction between subjective factors of experience and objective ones. The issue of philosophy then is to show how subjective impressions can be developed so that our knowledge acquires objectivity. This formulation of the problem of knowledge was adopted by Alois Riehl; whereas Marburg neo-Kantians considered the problem thereby formulated an unsolvable one.

1868a: Die neuere Fortschritte in der Theorie des Sehens, *Preussische Jahrbücher* 21, 149-170; reprint in: Helmholtz 1903, Bd. 1, 265-366.

1868b: Über die Tatsachen, die der Geometrie zugrunde liegen, *Abhandlungen der Königl. Gesellschaft der Wissenschaften zu Göttingen* 15; reprinted in: Helmholtz 1883, 618-639; Helmholtz 1921, 38-55. This paper includes an attempt to deduce Riemann's metric (1854) from the free mobility of rigid bodies (i.e., the fact that most solid bodies we experience remain unvaried in shape and size during displacements) and the remaining conditions of empirical measurements. Accordingly, Helmholtz defines space as a three-fold extended manifold of constant curvature. Helmholtz's inquiry differs from Riemann's one, firstly, because Riemann's *hypotheses* underlying geometry are replaced with *facts* to be induced by experience. A related problem is that Helmholtz overlooks the distinction between the finite level and the infinitesimal one: Riemann's metric is formulated at the infinitesimal level and cannot be derived from the free mobility of rigid bodies. Secondly, Riemann is not committed to the supposition that space is a manifold of constant curvature. His conjecture is that the curvature of space may be variable at the infinitesimal level, provided that the total curvature for intervals of a certain extension approximately equals zero. For a critical analysis of Helmholtz's attempt see also Torretti 1978, 155-71. Nevertheless, in philosophical discussions on that subject in the second half of the nineteenth century, Riemann's inquiry was usually read in connection with Helmholtz's one and the attempt to show how the form of space can be derived from empirical conditions was called "Riemann-Helmholtz theory".

1870: Über den Ursprung und die Bedeutung der geometrischen Axiome, Vortrag gehalten im Docentenverein zu Heidelberg, in: Helmholtz 1903, Bd. 2, 1-31; reprinted in: Helmholtz 1921, 1-24. This lecture includes a presentation of Helmholtz's inquiry on the foundations of geometry for a wide audience and his objections to Kant. There is, in fact, an important development in Helmholtz's view: around 1869, the Italian mathematician Eugenio Beltrami, with whom Helmholtz corresponded, had drawn his attention to the fact that manifolds of constant curvature might admit both Euclidean and non-Euclidean geometry. Therefore, in 1870, Helmholtz first presented a series of mental experiments showing how the form of intuition of space might be varied under suitably different empirical circumstances. The description of such circumstances should contradict Kant's assumption of an unchangeable form of intuition underlying any phenomenal changes and make the claim that geometrical axioms are synthetic a priori judgements untenable. Axioms can be either synthetic a posteriori, as Helmholtz claims, or analytic. Either way, they cannot be necessary. This lecture was hugely discussed in neo-Kantianism. And since the first discussions by Cohen and Riehl, two different strategies were developed to defend the apriority of geometrical axioms: on Cohen's view, critical philosophy is not committed to the necessity of some propositions; its issue, rather, is to prove that a connection of conditions is necessarily required for knowledge. On the other hand, Riehl restricted apriority to some fundamental concepts and emphasized necessity and universality as intrinsic properties of geometrical knowledge.

1876: The Origin and the Meaning of Geometrical Axioms I, *Mind* 1, 301-321. The English translation of Helmholtz 1870 caused a debate with the Dutch Kantian Jan Pieter Nicolaas Land. During that debate, Helmholtz partially reconsidered the Kantian theory of space.

1878a: The Origin and Meaning of Geometrical Axioms II, *Mind* 3, 212-225. The second part of Helmholtz (1876) includes Helmholtz's reply to Land (1877). It was republished in German as the third Appendix to Helmholtz (1878b). Here Helmholtz provides a physical interpretation of geometrical notions (e.g., congruence) and calls the study of such notions *physical geometry*. He then reformulates his objection to Kant as follows: physical geometry may not necessarily agree with pure geometry regarding the equality of some parts of space. If there

should be any disagreement, constraints imposed by pure geometry ought to be abandoned for the objectivity of empirical measurements.

1878b: Die Tatsachen in der Wahrnehmung, Rede, gehalten zur Stiftungsfeier der Friedrich-Wilhelm-Universität zu Berlin, in: Helmholtz 1921, 109-152. We already mentioned that Helmholtz rejects the priority of pure geometry over physical geometry. Nevertheless, he admits that neither his empiricist philosophy of geometry nor the defence of physical geometry excludes that space can be considered a general form of intuition in Kant's sense, provided that a choice among hypotheses regarding the specific axiomatic structure of space cannot be made independently of empirical investigation. This way of reconsidering the Kantian theory of space might have influenced Cassirer and was defended by him in Cassirer (1940). At the same time, Helmholtz argues for a naturalization of Kant's form of outer intuition: the localization of the objects in space also entails a construction of the concept of space, which differs from Kant's pure intuition because it depends on experience and therefore admits different specifications. Such naturalization was criticized by Marburg neo-Kantians, including Cassirer. A more favourable account is found in Riehl (1879). Other interpreters who emphasized this aspect were above all Johannes von Kries and Moritz Schlick.

1882: *Wissenschaftliche Abhandlungen*, Bd. 1, Leipzig: Barth.

1883: *Wissenschaftliche Abhandlungen*, Bd. 2, Leipzig: Barth.

1887: Zählen und Messen, erkenntnistheoretisch betrachtet, in: *Philosophische Aufsätze, Eduard Zeller zu seinem fünfzigjährigen Doctorjubiläum gewidmet*, Leipzig: Fues', 17-52; reprinted in: Helmholtz 1921, 70-97. This essay includes Helmholtz's inquiry on the foundations of arithmetic and his theory of measurement. It was first reviewed by Cohen in 1888 and received attention in those works by the neo-Kantians that were devoted to the philosophy of exact sciences. The neo-Kantians focused, in particular, on the psychological origin of the number series proposed by Helmholtz and presented by him as a natural basis for the use of number symbols. So the naturalized version of Kant's form of outer intuition was extended to that of inner intuition. Helmholtz identified the latter with the time sequence and used his interpretation of Kant to argue for an ordinal conception of number, on the one hand, and for a nominalist account of the theory of cardinal numbers, on the other. On such view, cardinal numbers should be treated as arbitrarily chosen signs. This claim, in particular, made Helmholtz's philosophy of arithmetic problematic, not only for the neo-Kantians, but also for Husserl and Frege.

1894: Heinrich Hertz. Vorwort zu dessen Prinzipien der Mechanik, Leipzig: Barth; reprinted in Helmholtz 1903, Bd. 2, 363-380. *Prinzipien der Mechanik* was Heinrich Hertz's main contributions to theoretical physics and appeared posthumous in 1894. Despite the fact that his approach to mechanics differs considerably from Helmholtz's approach, Hertz had been one of Helmholtz's favourite students in Berlin and Helmholtz supported his career until Hertz's death at the age of 37. Helmholtz himself died in the same year and one of his last writings was the preface to Hertz's book.

1903 [1884]: *Vorträge und Reden*, 2 Bde., 5. Aufl., Braunschweig: Vieweg.

1921: *Schriften zur Erkenntnistheorie*, herausgegeben und erläutert von Paul Hertz und Moritz Schlick, Berlin: Springer (*Epistemological Writings: The Paul Hertz/Moritz Schlick Centenary Edition of 1921*, trans. Malcom F. Lowe, Robert S. Cohen & Yehuda Elkana (eds.), Dordrecht: Reidel, 1977). The centenary edition of Helmholtz's epistemological writings includes a selection of papers that were representative of Helmholtz's inquiry on the foundations of geometry and its development from 1868 to 1887. References to the related subjects and helpful comments were provided by the German theoretical physicist and mathematician Paul Hertz and by Moritz Schlick, who was about to become one of the leading figures of the Vienna Circle. Schlick's reading of Helmholtz was opposed to that proposed by Marburg neo-Kantians, especially Cassirer: what Schlick called into question was not so much the naturalization of the forms of intuition, rather the possibility of interpreting Helmholtz's conception of space as a generalization of Kant's conception. Other comments show his more sympathetic attitude toward Riehl. And Schlick's own conception of space might have been influenced by Riehl in some aspects (see Heidelberger 2006).

1971: *Selected Writings*, edited with an introduction by Russell Kahl, Middletown: Wesleyan University Press.

1990: *Letters of Hermann von Helmholtz to his Wife 1847-1859*, edited by Richard L. Kremer, Stuttgart: Steiner. Helmholtz's private correspondence includes important information regarding his studies, interests, and researches. Several quotes from Helmholtz's letters to his wife and to his parents were first made available in Königsberger (1902-1903) and were used by Riehl in support of his reconstruction of Helmholtz's relationship with Kant.

1993: *Letters of Hermann von Helmholtz to his Parents: The Medical Education of a German Scientist 1837-1846*, edited by David Cahan, Stuttgart: Steiner.

2. WORKS BY THE NEO-KANTIANS

Bauch, Bruno 1907: Erfahrung und Geometrie in ihrem erkenntnistheoretischen Verhältnis, *Kant Studien* 12, 213-235. Bauch uses Poincaré's objections to Helmholtz's geometrical empiricism to better understand the relationship between geometry and experience. On the other hand, Bauch defends not so much Poincaré's geometrical conventionalism as a view that can be traced back to Riehl: while nineteenth-century inquiries on the foundations of geometry have proved the formal-logical possibility of a plurality of geometric systems, there is in fact only one geometry (i.e., Euclid's geometry) that can be related to human experience, and there can only be one geometry because it provides us with conditions of experience itself.

___ 1911: *Studien zur Philosophie der exakten Wissenschaften*, Heidelberg: Winter. Bauch's studies in the philosophy of exact sciences include an extended version of Bauch (1907). Other sections of the book are devoted to Helmholtz's theory of signs and to his relationship with Kant. Bauch interprets Helmholtz's conception of causality as a Kantian one, especially if one considers Helmholtz's formulation of the principle of causality as the condition for ordering the phenomena in a lawful way.

___ 1914: Über den Begriff des Naturgesetzes, *Kant Studien* 19, 303-337. Bauch refers to Helmholtz's insight into the conceptual character of natural laws in support of the Kantian view that laws are prescribed to nature by the categories of the understanding.

Cassirer, Ernst 1907: Kant und die moderne Mathematik, *Kant Studien* 12, 1-49. Cassirer adopts an ordinal conception of number, though not so much in the version of Helmholtz as in that of Dedekind. Cassirer's critical remark against Helmholtz is made explicit in Cassirer (1910). Note, however, that, already in 1907, Cassirer rejects the identification of Kant's form of inner intuition with the time sequence and interprets it, rather, in Hamilton's sense as the general form of a progression whatsoever.

___ 1910: *Substanzbegriff und Funktionsbegriff: Untersuchungen über die Grundfragen der Erkenntniskritik*, Berlin: B. Cassirer; Hamburger Ausgabe, Bd. 6. Important sections of Cassirer's work are devoted to Helmholtz's nominalist account of the cardinal theory of numbers and to his theory of signs. Cassirer substitutes "symbols" for "signs" to clearly distinguish between sense data associated with numerals and their intended meaning. His objection to Helmholtz is that such meaning presupposes not so much a psychological explanation as a logical development of the theory of numbers, and therefore an ordinal conception of numbers in Dedekind's sense (see Dedekind 1888). Cassirer's related objection to Helmholtz's theory of signs in the philosophy of perception regards the empiricist formulation of the problem of knowledge: the problem of motivating the belief in the objectivity of the knowledge that has been first acquired by subjective data is an unsolvable one, because the objectivity of knowledge already presupposes general, logical conditions of experience.

___ 1918: Hermann Cohen und the Erneuerung der Kantischen Philosophie, *Kant Studien* 17, 252–273 (Eng. trans. Lydia Patton, Hermann Cohen and the Renewal of Kantian Philosophy, *Angelaki* 10.1 (2005), 95–108). Cassirer contrasts Cohen's theory of apriority with the physiological interpretation of the a priori proposed by Helmholtz in 1855.

___ 1920-1921: *Die philosophischen Probleme der Relativitätstheorie*, in: *Vorlesungen und Vorträge zu philosophischen Problemen der Wissenschaften 1907-1945*, hrsg. v. Jörg Fingerhut, Gerald Hartung u. Rüdiger Kramme, Hamburg: Meiner, 2010, 29-116. Cassirer's

lectures on the philosophical questions regarding Einstein's general relativity include several remarks on Helmholtz's theory of measurement. Despite Helmholtz's commitment to classical mechanics (see especially Helmholtz 1847), Cassirer interprets Helmholtz as one of the forerunners of a functionalist conception of physical objects that had been developed in late nineteenth-century energetics.

___ 1921: *Zur Einstein'schen Relativitätstheorie: Erkenntnistheoretische Betrachtungen*, Berlin: Bruno Cassirer; Hamburger Ausgabe, Bd. 10. Cassirer emphasizes the importance of the philosophical background of Einstein's general relativity for his reflexion on the conditions of measurement. In particular, Cassirer analyzes Helmholtz's notion of rigid body and the related debate on the status of geometrical axioms. Despite the compelling objections to Helmholtz's geometrical empiricism that had been developed by Poincaré, it was in the context of that debate that the priority of Euclid's geometry was first called into question, so that a more general system of hypotheses could be taken into consideration. And Einstein himself was involved in that debate (see Einstein 1921).

___ 1923: *Substance and Function and Einstein's Theory of Relativity*, authorized translation by William C. and Marie C. Swabey, Chicago: Open Court.

___ 1927a: Erkenntnistheorie nebst den Grenzfragen der Logik und Denkpsychologie, *Jahrbücher der Philosophie* 3, 31-92; Hamburger Ausgabe, Bd. 17, 13-81. A series of Cassirer's writings dated 1927-1929 give evidence of his interest in Helmholtz's conception of sign during the development of the third part of the philosophy of symbolic forms. At the same time, Cassirer emphasizes the role of the mathematical concept of function for the coordination between symbols and phenomena. He distances himself from Helmholtz because Cassirer sharply distinguishes such coordination, which is an ideal one, from actually given, psychic processes (see also Cassirer 1927b, 297-301).

___ 1927b: Das Symbolproblem und seine Stellung im System der Philosophie, *Zeitschrift für Ästhetik und allgemeine Kunstwissenschaft* 21, 295-322; Hamburger Ausgabe, Bd. 17, 253-282.

___ 1929a: Hermann von Helmholtz, in: *Encyclopedia Britannica: A New Survey of Universal Knowledge*; Hamburger Ausgabe, Bd. 17, 308-210. This entry includes Cassirer's account of the physiological interpretation of the a priori by Helmholtz (1855) and Lange (1866).

___ 1929b: Formen und Formwandlungen des philosophischen Wahrheitsbegriffs, in: Hamburgische Universität. Reden, gehalten bei der Feier des Rektoratswechsels am 7. November 1929, 17-36; Hamburger Ausgabe, Bd. 17, 342-356. In this talk, Cassirer defends his functionalist conception of truth. The talk includes Cassirer's critical remarks on Helmholtz's philosophy of arithmetic (see also Cassirer 1910).

___ 1929c: *Philosophie der symbolischen Formen*, 3. Teil: *Phänomenologie der Erkenntnis*, Berlin: B. Cassirer; Hamburger Ausgabe, Bd. 13. The third volume of *Philosophie der symbolischen Formen* includes a detailed account of Helmholtz's theory of signs, along with Cassirer's critical remark: while Helmholtz's theory of signs enables him to emphasize the role of the understanding in the interpretation of sense data, he is unclear about their symbolic function (i.e., that which provides us with their meaning). This is because Helmholtz reduces the connections signs stand for to causal connections: this particular way of interpreting sense data is influenced by classical mechanics and certainly is not the only one possible in physics as well as in other fields of knowledge.

___ 1936-1937: *Ziele und Wege der Wirklichkeitserkenntnis*, in: *Nachgelassene Manuskripte und Texte*, hrsg. v. K.C. Köhnke u. J.M. Krois, Bd. 2, Hamburg: Meiner, 1999. This manuscript was part of Cassirer's project to deepen some of the subjects treated in the fourth volume of the *Erkenntnisproblem* by writing another book on "the goals and ways of knowing reality". Since Cassirer had emigrated during Nazism (first to England and then to Sweden) and did not succeed to find an editor at that time, both manuscripts were left in Sweden when he moved to the United States. They first appeared posthumous, *Erkenntnisproblem* in 1950 (in English translation) and *Ziele und Wege* in 1999. A section of the latter is devoted to Helmholtz's theory of signs and his controversy with Hering.

___ 1940: *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit,* 4. Bd.: *Von Hegels Tod bis zur Gegenwart: (1832-1932)*, Stuttgart: Kohlhammer, 1957; Hamburger

Ausgabe, Bd. 5. Cassirer interprets Helmholtz's conception of space as a generalization of Kant's form of outer intuition. It follows from Cohen's interpretation of the Kantian philosophy of space Cassirer adheres to that the form of outer intuition alone does not suffice to determine the geometrical properties of space. What is made clear by Helmholtz's analysis of the relationship between space and geometry is that, once general properties of space (e.g., continuity, homogeneity, etc.) have been formulated, there might still be a choice among hypotheses regarding its specific axiomatic structure.

___ 1944: The Concept of Group and the Theory of Perception, *Philosophy and Phenomenological Research* 5.1, 1-35. Here and in Cassirer (1945), Cassirer refers to Helmholtz's theory of perception in support of the view that there are possibly progressive generalizations of the form of space. This way of proceeding at least implicitly entails a group-theoretical reasoning. There is in fact no evidence that Helmholtz himself was committed to the group-theoretical classification of geometries that was first developed by Klein (1872). Nevertheless, implicitly group-theoretical considerations in Helmholtz (1870) had been pointed out by Klein (1898).

___ 1945: *The Concept of Group*, in: *Vorlesungen und Vorträge zu philosophischen Problemen der Wissenschaften 1907-1945*, 181-201.

___ 2011: *Nachgelassene Manuskripte und Texte*, Bd. 4: *Symbolische Prägnanz, Ausdruckssphänomen und "Wiener Kreis"*, hrsg. von C. Möckel, Hamburg: Meiner. Some of the texts by Cassirer made available in this edition give further evidence of his interest in Helmholtz's conception of sign during the development of Cassirer's philosophy of symbolic forms.

Cohen, Hermann 1883: *Das Princip der Infinitesimal-Methode und seine Geschichte: Ein Kapitel zur Grundlegung der Erkenntniskritik*, Berlin: Dümmler; Neudruck in: *Werke*, herausgegeben vom Hermann-Cohen-Archiv unter der Leitung von Helmut Holzhey, Hildesheim: Olms, Bd. 5, 1984. Cohen maintains that Helmholtz's inquiry on the foundations of geometry makes it clear that there is a connection between intuition and understanding. On the other hand, Cohen points out that such connection cannot be used as an argument against Kant. The inseparability of both kinds of activities follows, in fact, from the structure of Kant's *Critique of Pure Reason*. Helmholtz's objections to Kant seem to depend on the fact that Kant's Transcendental Aesthetic is considered independently of the Analytic of Principles.

___ 1885: *Kants Theorie der Erfahrung*, 2. Aufl., Berlin: Dümmler, 1885; Neudruck der 3. Aufl. von 1918 in: *Werke*, Bd. 1.1, 1987. In the second edition of *Kants Theorie der Erfahrung* Cohen defends the apriority of geometrical axioms against the so-called „Riemann-Helmholtz's theory". Cohen interprets "axioms of intuition" in the sense of Kant's Analytic of Principles, as the conditions of experience that make geometry applicable to the empirical manifold. But Cohen is not committed to the assumption of a geometric structure of space independently of empirical science. Therefore he does not call into question mathematical freedom in the development of non-Euclidean geometry. In this connection, the formulation of geometrical axioms or definitions is not an issue of transcendental philosophy, which is concerned not so much with pure mathematics per se as with the relationship between pure and applied mathematics.

___ 1888: Jubiläums-Betrachtungen. Rezension von: Philosophische Aufsätze. Eduard Zeller zu seinem fünfzigjährigen Doctorjubiläum gewidmet (1887), *Philosophische Monatshefte* 24, 257-291; 2. Aufl. in: *Schriften zur Philosophie und Zeitgeschichte*, hrsg. von Ernst Cassirer u. Albert Görland, Berlin: Akademie-Verlag, 1928, Bd. 1. Cohen's „Jubiläums-Betrachtungen" begin with a review of Helmholtz (1887). Cohen appreciates the structure of Helmholtz's inquiry – namely, the use of the laws of arithmetic analyzed in the first part of the paper to develop the theory of measurement presented in the second. Nevertheless, Cohen makes several critical remarks. The first regards Helmholtz's naturalization of the forms of intuition, which for Cohen makes them unable to provide foundations of mathematics. Helmholtz seems to presuppose the mathematical meaning of the concept of number from the outset: the psychological explanation of its origin would entail a *petitio principii*. Secondly, Cohen points out Helmholtz's confusion between numbers and numerals. This is another consequence of the said unclearness about the mathematical meaning of the concepts under consideration.

___ 1896: Einleitung mit kritischem Nachtrag zur „Geschichte des Materialismus“ von F.A. Lange, 5. Aufl. von 1896; Neudruck der 3. Aufl. von 1914 in: *Werke*, Bd. 5, 1984. Cohen refers to Helmholtz in defence of the analytic method in geometry against the preference for synthetic method Kant inherited from Newton. On Cohen's view, the importance of analytic method in nineteenth-century geometry speaks against Kant's assumption of pure intuition as an autonomous source of mathematical knowledge.

___ 1902: *Logik der reinen Erkenntnis*, Berlin: B. Cassirer; Neudruck der 2. Aufl. von 1914, in: *Werke*, Bd. 6, 1977. Cohen's logic of pure knowledge is the first part of his system of philosophy and includes Cohen's analysis of the relationship between logic and the exact sciences. In this connection, Cohen resumes his epistemological studies, including his discussions on the concept of number and on the origin of geometrical axioms.

Cohn, Jonas 1908: *Voraussetzungen und Ziele des Erkennens: Untersuchungen über die Grundfragen der Logik*, Leipzig: Engelmann. Cohn's theory of knowledge rests upon a dualistic principle called *utraquismus*, according to which there are two fundamental elements of knowledge – namely, a logical, formal element and a nonlogical, material one. He applies this principle to a number of issues in the philosophy of mathematics of his time, including the debates on the status of natural numbers and the foundations of geometry Helmholtz was involved with.

Hönigswald, Richard 1908: Über den Unterschied und die Beziehungen der logischen und der erkenntnistheoretischen Elemente in dem kritischen Problem der Geometrie, in: *Bericht über den III. Internationalen Kongress für Philosophie, 1. bis 5. September 1908*, hrsg. v. Theodor Elsenhans, Heidelberg: Winter, 1909, 887-893. In this talk, Hönigswald maintains that, though all geometries that are logically possible are independent of experience, there is only one geometry that makes experience first possible – namely, Euclid's geometry. To support this view, Hönigswald refers to Riehl's conception of critical philosophy as theory of experience – namely, in this case, theory of the geometrical presuppositions of experience. A similar argument had been developed also by Bauch in 1907. And it is reported in the proceedings of the Congress that Bauch himself took part to the discussion that followed the talk to express his agreement with Hönigswald.

___ 1912: *Zum Streit über die Grundlagen der Mathematik*, Heidelberg: Winter. Hönigswald defends the apriority of mathematics, including geometry, and distinguishes the concepts of space and time from space-time measurements in physics. His book includes one of the first philosophical discussions of the principle of relativity (see also Natorp 1910, 392-404).

Natorp, Paul 1901: Zu den logischen Grundlagen der neueren Mathematik, *Archiv für systematische Philosophie* 7, 177-209, 372-384. The goal of this article is to prove the singularity of three-dimensional Euclidean space by deducing its defining properties a priori. The a priori argument for assuming three-dimensionality (whose most detailed version is found in Natorp 1910, 303-09), in particular, should directly contradict Helmholtz's attempt to explain the same property empirically.

___ 1910: *Die logischen Grundlagen der exakten Wissenschaften*, Leipzig: Teubner.

Nelson, Leonard 1905-1906: Bemerkungen über die nicht-euklidische Geometrie und den Ursprung der mathematischen Gewißheit, *Abhandlungen der Friesschen Schule*, neue Folge 1.2-3, 373-430; reprinted in: *Gesammelte Schriften*, 3. Bd.: *Die kritische Methode in ihrer Bedeutung für die Wissenschaft*, hrsg. v. Paul Bernays, Willi Eichler, Arnold Gysin, Gustav Heckmann, Grete Henry-Hermann, Fritz von Hippel, Stephan Körner, Werner Kroebel, Gerhard Weisser, Hamburg: Meiner, 3-52. In this paper, as well as in Nelson (1906) and (1914), the founder of the neo-Friesian School, Leonard Nelson, defends the understanding of geometrical axioms as synthetic a priori judgement and maintains that mathematical knowledge is grounded in pure intuition, though in a way that makes this compatible with the use of axiomatic method: only the fundamental elements are univocally and immediately determined; the formulation of the first premises is not. This is because the foundations of mathematics lie outside the domain of language. And since they provide us with certain knowledge, they cannot be derived from experience either. In order to reply to Helmholtz's objections to Kant, Nelson points out Helmholtz's false inference from the nonlogical origin of geometrical axioms to their empirical origin: Helmholtz overlooks that the notion of pure

intuition was meant by Kant precisely to indicate a source of mathematical knowledge that could be neither logical nor empirical.

___ 1906: Kant und die nicht-Euklidische Geometrie, *Das Weltall* 6.10-12, 147-155, 174-182, 187-193; reprinted in: *Gesammelte Schriften*, Bd. 3, 53-94.

___ 1914: Des fondements de la géométrie, Vortrag, gehalten in Paris am 8. April 1914 bei der Gründung der Société internationale de philosophie mathématique, in: *Die Reformation der Philosophie durch die Kritik der Vernunft*, Leipzig: Der Neue Geist-Verlag, 1918, 87-118; reprinted in: *Gesammelte Schriften*, Bd. 3, 129-156.

Rickert, Heinrich 1929 [1896]: *Die Grenzen der naturwissenschaftlichen Begriffsbildung: Eine logische Einleitung in die historischen Wissenschaften*, 5. Aufl. Tübingen: Mohr; Neudruck mit einer Einleitung herausgegeben von Rainer A. Bast, Hildesheim: Olms, 2007. Rickert mentions Heinrich Hertz's *Prinzipien der Mechanik* as an example of natural-scientific reduction of the phenomena to lawful connections and refers to Helmholtz's preface to Hertz's book in support of the view that concrete aspects of natural phenomena cannot be accounted for in terms of general laws. Note, however, that Helmholtz's remark does not entail a lack of explanatory power of natural science per se. He only points out the limits of Hertz's attempt to generalize mechanics to all known natural phenomena by replacing action at distance theories with a field theory: such a generalization might be achievable in principle; but its application to entire branches of physics could not be accomplished without great effort.

Riehl, Alois 1872a: Zur Aprioritätslehre, Rezension von: H. Cohen, Kants Theorie der Erfahrung (1871), *Philosophische Monatshefte* 8, 212-215. Riehl points out the relevance of Cohen's theory of apriority for the contemporary debate on the foundations of geometry, though Cohen himself did not explicitly discuss Helmholtz's geometrical empiricism until the second edition of his work.

___ 1872b: *Über Begriff und Form der Philosophie*, Berlin: Duncker. By 1872, Riehl had distanced himself from Herbart's realism he first adhered to and adopted the thing-in-itself realism that followed from his interpretation of Kant. Riehl was also influenced by Helmholtz's conception of sensations as signs of external objects. Therefore Riehl rejected Herbart's project of a mathematical psychology and defended Helmholtz's empiricist approach.

___ 1876: *Der philosophische Kriticismus und seine Bedeutung für die positive Wissenschaft*, 1. Bd.: *Geschichte und Methode des philosophischen Kriticismus*, Leipzig: Engelmann. Riehl's major work on critical philosophy shows Helmholtz's influence in several aspects. First of all, he adopts Helmholtz's distinction between subjective factors of experience and objective ones. The goal of the theory of knowledge then is to motivate our belief in the objectivity of knowledge, despite its subjective factors. Secondly, especially in 1879, Riehl's attempt is to show that the Kantian theory of space is compatible with Helmholtz's analysis of spatial perceptions. Therefore Riehl interprets Kant's distinction between form and matter of the phenomena as follows: only the general form of space is a priori; whereas the localization of particular objects in space also requires sensations.

___ 1879: *Der philosophische Kriticismus*, 2. Bd.: *Die sinnlichen und logischen Grundlagen der Erkenntnis*. The second volume of Riehl's work includes a detailed analysis of Helmholtz's physiology of vision. And Riehl admits the empirical origin of properties of space such as three-dimensionality. Therefore he rejects Kant's assumption of pure intuition as the source of mathematical knowledge. Nevertheless, he argues for the assumption of intuitive foundations determining the Euclidean character of geometric space. On his view, the logical possibility of developing non-Euclidean geometry already presupposes the concept of a three-dimensional Euclidean space. For the same reason, Euclid's hypothesis should be preferred also in the choice of hypotheses regarding physical space. Note that Riehl was one of the fist to distinguish between an intuitive, a geometric, and a physical meaning of space and, correspondingly, of the external world (see also Riehl 1887, chap. 4).

___ 1887: *Der philosophische Kriticismus*, 3. Bd.: *Zur Wissenschaftstheorie und Metaphysik*.

___ 1904a: Helmholtz in seinem Verhältnis zu Kant, *Kant Studien* 9, 260-285. This essay includes a reconstruction of Helmholtz's relationship with Kant and its development, along with a synthesis of Riehl's interpretation of the Kantian theory of space. Here Riehl's critical remark

against Helmholtz is made more explicit than in 1879. Riehl's point is that Helmholtz's association between Kant's philosophy and nativism in 1855 was misguided and led him to a misconception of the notion of "a priori", which is confused by Helmholtz with that of "transcendental" (e.g., in Helmholtz's use of the non-Kantian expression "transcendental intuition"). A priori knowledge (e.g., geometry) is the object of transcendental knowledge, whose goal for Riehl is to show that some knowledge is objective, despite its being a priori.

___ 1904b: Anfänge des Kritizismus. – Methodologisches aus Kant, *Kant Studien* 9, 492-517. Despite the said critical remark, Riehl's appreciation for Helmholtz's contribution to the theory of knowledge is expressed by him in this paper, which is the revised version of the introduction to the second edition of Riehl's *Kritizismus*, as well as in Riehl (1921, 1922).

___ 1921: Helmholtz als Erkenntnistheoretiker, *Die Naturwissenschaften* 35: *Dem Andenken an Helmholtz zur Jahrhundertfeier seines Geburtstages*, 702-708.

___ 1922: Hermann von Helmholtz, in: *Führende Denker und Forscher*, Leipzig: Quelle & Meyer, 223-240.

___ 1924: *Der philosophische Kritizismus*, Bd. 1, 3. Aufl., Leipzig: Kröner.

___ 1925: *Der philosophische Kritizismus*, Bd. 2, 2. Aufl., Leipzig: Kröner. The second edition of this volume includes Riehl's additional comments on the relationship between space and geometry after Einstein's general relativity. Riehl uses his distinction between intuitive and physical space to defend his view that the Newtonian conception of space reflects the characteristics of the form of our spatial intuition.

___ 1926: *Der philosophische Kritizismus*, Bd. 3, 2. Aufl., Leipzig: Kröner.

3. SECONDARY AND OTHER RELEVANT LITERATURE

Biagioli, Francesca (forthcoming): Hermann Cohen and Alois Riehl on Geometrical Empiricism, forthcoming in *HOPOS: The Journal of the International Society for the History of Philosophy of Science*.

Boi, Luciano 1995: *Le problème mathématique de l'espace: Une quête de l'intelligible*, Berlin : Springer. The author analyzes the development of the mathematical concept of space in the nineteenth century and discusses the related philosophical problems. Helmholtz's relationship with Kant and the neo-Kantian is analyzed in chap. 7.5.

Cahan, David (ed.) 1993: *Hermann von Helmholtz and the Foundations of Nineteenth-Century Science*, Berkeley: University of California Press.

Coffa, Alberto J.:1991: *The Semantic Tradition from Kant to Carnap: To the Vienna Station*, Cambridge: Cambridge University Press. Chap. 3 is devoted to the debate on Helmholtz's inquiry on the foundations of geometry and its discussions by Cohen and Riehl.

Darrigol, Olivier 1994: Helmholtz's Electrodynamics and the Comprehensibility of Nature, in: Krüger 1994a, 216-244. The author studies the connections between Helmholtz's reflection on the comprehensibility of nature and his contributions to physics.

___ 2003: Number and Measure: Hermann von Helmholtz at the Crossroads of Mathematics, Physics, and Psychology, *Studies in History and Philosophy of Science* 34, 515-573. The article offers a detailed analysis of Helmholtz's theory of measurement.

Dedekind, Richard 1888: *Was sind und was sollen die Zahlen?*, Braunschweig: Vieweg.

DiSalle, Robert 1993: Helmholtz's Empiricist Philosophy of Mathematics: Between Laws of Perception and Laws of Nature, in: Cahan 1993, 498-521.

___ 2006: *Kant, Helmholtz, and the Meaning of Empiricism*, in: Friedman & Nordmann 2006, 123-139.

Dosch, Hans Günter 1997: The Concept of Sign and Symbol in the Work of Hermann Helmholtz and Heinrich Hertz, in: N. Janz (ed.), Ernst Cassirer 1945-1995: Science et culture, Actes du colloque des 2-3 juin 1995 à l'Université de Lausanne, *Étude des Lettres* 1.2, 47-61.

Einstein, Albert 1921: *Geometrie und Erfahrung, erweiterte Fassung des Festvortrages gehalten an der Preussischen Akademie der Wissenschaften zu Berlin am 27. Januar 1921*, Berlin: Springer.

Erdmann, Benno 1877: *Die Axiome der Geometrie: Eine philosophische Untersuchung der Riemann-Helmholtz'schen Raumtheorie*, Leipzig: Voss. This book includes one of the first

philosophical discussions of the so-called „Riemann-Helmholtz's theory of space". It was praised by Helmholtz himself as a good account of the nineteenth-century inquiries on the foundations of geometry in philosophical terms and played an important role in the philosophical reception of Helmholtz.

Ferrari, Massimo 1994: Cassirer, Schlick und die Relativitätstheorie. Ein Beitrag zur Analyse des Verhältnisses von Neukantianismus und Neupositivismus, in: E.W. Orth & H. Holzhey (eds.), *Neukantianismus, Perspektiven und Probleme*, Würzburg: Königshausen & Neumann, 418-441 (originally published as: Cassirer, Schlick e l'interpretazione "kantiana" della teoria della relatività, *Rivista di filosofia* 82 (1991), 243-278).

___ 1997: *Introduzione a Il neocriticismo*, Roma: Laterza.

___ 2003: *Categorie e a priori*, Bologna: Il Mulino. The author interprets Helmholtz's psychological understanding of the a priori as one of the sources of the dynamical conception of the a priori developed by Marburg neo-Kantians.

___ 2009: Le forme della conoscenza scientifica: Cohen e Helmholtz, in: *Unità della ragione e modi dell'esperienza: Hermann Cohen e il neokantismo. Atti del Convegno internazionale di studi, Salerno 21-23 maggio*, a cura di G.P. Cammarota, Soveria Mannelli: Rubettino, 77-96. The author analyzes Cohen's review of Helmholtz 1887 (see Cohen 1888) and connects it to later discussions of Helmholtz's philosophy of arithmetic by Husserl, Frege, and Cassirer.

Friedman, Michael 1997: Helmholtz's *Zeichentheorie* and Schlick's *Allgemeine Erkenntnislehre*: Early Logical Empiricism and its Nineteenth-Century Background, *Philosophical Topics* 25, 19-50. The author contrasts his reading of Helmholtz's theory of signs with that of Schlick and interprets Helmholtz's conception of space as follows: the localization of the objects in space provides us with a construction of the concept of space. This can be interpreted as a generalization of Kant's conception (see also Friedman 1999, 45; 2000, 199-206; 2001, 29-30; Ryckman 2005, chap. 3; DiSalle, 2006, 138). The problem with Schlick's reading lies, in particular, in his separation between sensibility and understanding, which, on the contrary, for Helmholtz are strictly related to each other (see also Pulte 2006, 197-200).

___ 1999: *Reconsidering Logical Positivism*, Cambridge: Cambridge University Press.

___ 2000: Geometry, Construction and Intuition in Kant and his Successors, in: Sher, Gila & Tieszen, Richard (eds.), *Beyond Logic and Intuition: Essays in Honor of Charles Parsons*, Cambridge: Cambridge University Press, 186–218.

—— 2001: *Dynamics of Reason: The 1999 Kant Lectures at Stanford University*, Chicago: University of Chicago Press.

___ 2002: Geometry as a Branch of Physics: Background and Context for Einstein's "Geometry and Experience", in: D.B. Malament (ed.), *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics to Honor Howard Stein*, Chicago: Open Court, 193-229. The author reconstructs Einstein's reflection on the conditions of measurement and its philosophical background, including Helmholtz's use of the notion of rigid body.

Friedman, Michael & Nordmann, Alfred (eds.) 2006: *The Kantian Legacy in Nineteenth-Century Science*, Cambridge (Mass.): The MIT Press.

Hagner, Michael & Wahrig-Schmidt, Bettina (eds.) 1992: *Johannes Müller und die Philosophie*, Berlin: Akademie Verlag.

Hatfield, Gary 1990: *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz*, Cambridge (Mass.): The MIT Press. The author analyzes the conception of the relationship between mind, space, and geometry from Kant to Helmholtz. The development of Helmholtz's view on that subject is reconstructed and discussed in detailed in chap. 5.

Heidelberger, Michael 1994: Helmholtz' Erkenntnis- und Wissenschaftstheorie im Kontext der Philosophie und Naturwissenschaft des 19. Jahrhunderts, in: Krüger 1994a, 168-185. The article offers a reconstruction of Helmholtz's epistemological views in the context of nineteenth-century philosophy and science.

___ 2006: Kantianism and Realism: Alois Riehl (and Moritz Schlick), in: Friedman & Nordmann 2006, 227-248. Here and in Heidelberger (2007), the author analyzes the relationship between

Riehl and Schlick and maintains that Schlick was influenced by Riehl, in particular, in the development of his method of coincidence.

___ 2007: From Neo-Kantianism to Critical Realism: Space and the Mind-Body Problem in Riehl and Schlick, *Perspectives on Science* 15.1, 26-48.

Hendricks, Vincent F., Klaus Frovin Jørgensen, Jesper Lützen, & Stig Andur Pedersen (eds.) 2006: *Interactions: Mathematics, Physics and Philosophy, 1860-1930*, Dordrecht: Springer.

Hentschel, Klaus 1990: *Interpretationen und Fehlinterpretationen der speziellen und der allgemeinen Relativitätstheorie durch Zeitgenossen Albert Einsteins*, Basel: Birkhäuser. The book offers a thorough analysis of the early philosophical interpretations (and misinterpretations) of special and general relativity. A detailed account of neo-Kantian interpretations and their connection with the debate on the philosophical consequences of non-Euclidean geometry is found in chap. 4.1.

Holzhey, Helmut & Wolfgang Röd 2004: *Die Philosophie des ausgehenden 19. und des 20. Jahrhunderts*, 12. Bd.: *Neukantianismus, Idealismus, Realismus, Phänomenologie*, München: Beck. This book includes a detailed account of the life and work of the neo-Kantians, along with an overview of the historical development of the neo-Kantian movement and its context, including the philosophical reception of Helmholtz.

Hyder, David 2006: Kant, Helmholtz and the Determinacy of Physical Theory, in: Hendricks et al. 2006, 1-43. The author emphasizes the line of transcendental argument that we find in Helmholtz (1847) and interprets the structure of Helmholtz's first geometrical papers as a development of the same line of argument: until 1869, Helmholtz believed that Euclidean geometry was entailed in his measurement postulates. For a detailed analysis of Helmholtz's inquiry on the foundations of geometry from 1868 to 1878, see Hyder 2009, chap. 6.

___ 2009: *The Determinate World: Kant and Helmholtz on the Physical Meaning of Geometry*, Berlin: De Gruyter.

___ 2010: German Scientific Epistemology and Early Analytic Philosophy, in: *The Oxford Handbook of the History of Analytic Philosophy*, edited by M. Beaney, Oxford: Oxford University Press.

Jammer, Max 1993 [1957]: *Concepts of Space: The History of Theories of Space in Physics*, third, enlarged edition, New York: Dover Publications. The book offers an overview of different concepts of space from antiquity to recent developments in the philosophy of physical space.

Janz, Nathalie 2001 : *Globus symbolicus: Ernst Cassirer, un épistémologue de la troisième voie ?*, Paris : Kimé. The book offers a detailed analysis of Cassirer's epistemology, with special focus on his conception of symbolism and its sources, including Helmholtz.

Kant, Immanuel 1786: *Metaphysische Anfangsgründe der Naturwissenschaft*, Riga: Hartknoch; Akademie-Ausgabe 4, 465-565.

___ 1787: *Kritik der reinen Vernunft*, 2 Aufl., Riga: Hartknoch; Akademie-Ausgabe 3.

Klein, Felix 1872: *Vergleichende Betrachtungen über neuere geometrische Forschungen*, Erlangen: Duchert.

___ 1898: Gutachten, betreffend den dritten Band der Theorie der Transformationsgruppen von S. Lie anlässlich der ersten Vertheilung des Lobatschewsky-Preises, *Mathematische Annalen* 50, 583-600.

Köhnke, Klaus-Christian 1986: *Entstehung und Aufstieg des Neukantianismus: Die deutsche Universitätsphilosophie zwischen Idealismus und Positivismus*, Frankfurt am Main: Suhrkamp. This book offers a detailed account of the development of neo-Kantianism, its origins, and historical context. The author interprets Helmholtz and Lange as the proponents of a psychological direction of neo-Kantianism and analyzes the reception of this direction by the next generation of neo-Kantians, especially Cohen and Riehl.

Königsberger, Leo 1902-1903: *Hermann von Helmholtz*, 3 Bde., Braunschweig: Vieweg. This work offers a detailed account of Helmholtz's life and work. Here unpublished manuscripts by Helmholtz, as well as numerous quotations from private letters and reports of discussions, were first made available.

Krause, Albrecht 1878: *Kant und Helmholtz über den Ursprung und die Bedeutung der Raumanschauung und der geometrischen Axiome*, Schauenburg: Lahr. Krause infers the apriority of geometrical axioms from that of space. His interpretation of the Kantian theory of

space motivated Helmholtz to make it clear that the general form of space might have a transcendental function in Kant's sense, provided that its specific axiomatic structure cannot be determined a priori.

Kries, Johannes von 1935: Helmholtz als Physiolog, *Die Naturwissenschaften* 9, 673-693.

Krüger, Lorenz (ed.) 1994a: *Universalgenie Helmholtz: Rückblick nach 100 Jahren*, Berlin: Akademie Verlag.

___ 1994b: Helmholtz über die Begreiflichkeit der Natur, in: Krüger 1994a, 201-215. The author contrasts his reading of Helmholtz's epistemology with that of Cassirer (1940). Krüger maintains that Cassirer tends to overlook the realist aspect of Helmholtz's epistemology by interpreting his research program in phenomenalist terms. At the same time, he agrees with Cassirer that such program ought to be put in connection with early modern rationalist philosophy of nature and with Kant.

Land, Jan Pieter Nicolaas 1877: Kant's Space and Modern Mathematics, *Mind* 2, 38-46.

Lange, Friedrich Albert 1866: *Geschichte des Materialismus und Kritik seiner Bedeutung in der Gegenwart*, Iserlohn: Baedeker. Lange, as Helmholtz (1855), proposed a physiological interpretation of Kant's philosophy by assuming that the a priori is rooted in the psycho-physical organization of the human mind.

Lenoir, Timothy 1992: Helmholtz, Müller und die Erziehung der Sinne, in: Hagner & Wahrig-Schmidt 1992, 207-223. The author analyzes Helmholtz's relationship with Müller from 1855 to 1863. During that period, Helmholtz develops a pragmatic approach to physiology of vision and distances himself from Müller.

___ 2006: Operationalizing Kant: Manifolds, Models, and Mathematics in Helmholtz's Theory of Perception, in: Friedman & Nordmann 2006, 141-210. The author interprets Helmholtz's theory of perception as an operationalist development of Kant's transcendental philosophy.

Neuber, Matthias 2012: *Die Grenzen des Revisionismus: Schlick, Cassirer und das „Raumproblem"*, Vienna: Springer. The book offers a deep analysis of the epistemologies of Cassirer and Schlick. Their different ways to treat the relationship between space and geometry reflect a differentiation in the reception of Helmholtz (see also Ferrari 1994 [1991]; Ryckman 1991).

Nordmann, Alfred 2006: Critical Realism, Critical Idealism, and Critical Common-Sensism: The School and World Philosophies of Riehl, Cohen, and Peirce, in: Friedman & Nordmann 2006, 249-274. The author shows how a selective reception of Helmholtz, along with scientists such as Hertz, Mayer, and Ostwald, in neo-Kantianism contributed to the development of Riehl's critical realism, on the one side, and Cohen's critical idealism, on the other.

Parrini, Paolo 1979: *Fisica e geometria dall'Ottocento a oggi*, Torino, Loescher. The book offers an overview of the debate on the foundations of geometry in the nineteenth century, along with selected texts by Helmholtz, Poincaré, Schlick, and Cassirer, among others.

Patton, Lydia 2008: Hermann von Helmholtz, *Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/hermann-helmholtz/

___ 2009: Signs, Toy Models, and the A Priori: From Helmholtz to Wittgenstein, *Studies in the History and Philosophy of Science* 40.3, 281–289. The author partially defends Helmholtz's empiricist account of the a priori against the Marburg neo-Kantians by using pragmatics to account for the role of a priori reasoning in translating between frameworks.

Pettoello, Renato 1998: De Herbart à Kant. Quelques considérations sur le réalisme de Alois Riehl, *Revue de métaphysique et de morale* 102.3, 347-366. The author analyzes the development of Riehl's philosophy from his reception of Herbart to the thing-in-itself realism Riehl proposed after 1872.

Poincaré, Henri 1902 : *La Science et l'Hypothèse*, Paris : Flammarion. Poincaré's rejection of Helmholtz's geometrical empiricism was seminal in later neo-Kantian studies on space and geometry: Poincaré made it clear that geometrical knowledge is grounded not so much in some facts as in idealized constructions (e.g., rigid figures) providing us with general rules such as free mobility. On the other hand, neither Bauch nor Cassirer adopted Poincaré's view that the choice among hypotheses regarding space is a matter of convention. For the neo-Kantians habit, convenience, and the usual criteria for a conventional choice do not suffice to

explain the relationship between geometry and experience: if there has to be such a relationship, there must be some epistemic reason for the use of geometry in physics.

Poggi, Stefano 1972: Neokantismo e fondazione della psicologia: J.B. Mayer, F.A. Lange, H. Cohen e P. Natorp, *Rivista di filosofia* 63.1, 37-58.

___ 1977: *I sistemi dell'esperienza: Psicologia, logica e teoria della conoscenza da Kant a Wundt*, Bologna: Il Mulino. The author explores the interactions between psychology, logic, and theory of knowledge from Kant to Wundt. A section of the book is devoted to Helmholtz's psychology and epistemology and gives information on its reception in neo-Kantianism.

___ 1980: *Le origini della psicologia scientifica*, Torino: Loescher. The concluding part of Poggi's book on the origins of scientific psychology is devoted to the Helmholtz-Hering controversy and to the psychological foundation of physiology by Helmholtz and Wundt.

Pulte, Helmut 2006: The Space between Helmholtz and Einstein: Moritz Schlick on Spatial Intuition and the Foundations of Geometry, in: Hendricks et al. 2006, 185-206. The author reconsiders the aspects of Helmholtz's conception of space that have been overlooked by Schlick – namely, above all, Helmholtz's commitment to the idea of constant curvature of space and the inductive character of his approach to the foundations of geometry.

Riemann, Bernhard 1867 [1854]: Über die Hypothesen, welche der Geometrie zu Grunde liegen, *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen* 13, 133-152.

Ryckman, Thomas A. 1991: Conditio sine qua non? *Zuordnung* in the Early Epistemologies of Cassirer and Schlick, *Synthese* 88.1, 57-95. The author reconsiders Cassirer's reception of Helmholtz's theory of signs. Ryckman's analysis shows that Helmholtz was one of Cassirer's sources for the conception of a univocal coordination of the symbols of a theory to the objects of experience as a condition of objective knowledge. On the other hand, the same sources, especially Helmholtz, were seminal for the development of the empiricist epistemology that Schlick contrasts with Cassirer's neo-Kantianism. For a detailed analysis of Schlick's reception of Helmholtz, see also Ryckman (2005), chap. 3.

___ 2005: *The Reign of Relativity: Philosophy in Physics 1915-1925*, New York: Oxford University Press.

Schiemann, Gregor 1994: Die Hypothetisierung des Mechanismus bei Hermann von Helmholtz: Ein Beitrag zum Wandel der Wissenschafts- und Naturauffassung im 19. Jahrhundert, in: Krüger 1994a, 149-167. Here and in Schiemann (2009), the author emphasizes the originality of Helmholtz's conception of science and contrasts it with classical mechanism, on the one hand, and with Kantianism, on the other. The author points out, in particular, Helmholtz's commitment to empirical and therefore hypothetical explanations.

___ 2009: *Hermann von Helmholtz's Mechanism: The Loss of Certainty. A Study on the Transition from Classical to Modern Philosophy of Nature*, translated by Cynthia Klohr, Dordrecht: Springer (originally published as *Wahrheitsgewissheitsverlust: Hermann von Helmholtz' Mechanismus im Anbruch der Moderne. Eine Studie zum Übergang von klassischer zu moderner Naturphilosophie*, Darmstadt: Wissenschaftliche Buchgesellschaft, 1997).

Schlick, Moritz 1921: Kritizistische oder empiristische Deutung der neuen Physik? Bemerkungen zu Ernst Cassirers Buch „Zur Einstein'schen Relativitätstheorie", *Kant Studien* 26, 96-111.

Siegel, Carl 1932: *Alois Riehl: Ein Beitrag zur Geschichte des Neukantianismus*, Festschrift der Universität Graz, Graz: Leuschner & Lubensky. The book offers a general account of Riehl's philosophy and gives evidence of his commitment to Helmholtz's epistemology.

Torretti, Roberto 1978: *Philosophy of Geometry from Riemann to Poincaré*, Dordrecht: Reidel.

Turner, Steven R. 1993: Consensus and Controversy: Helmholtz on the Visual Perception of Space, in: Cahan 1993, 154-204. This essay offers a reconstruction of the development of Helmholtz's approach to the physiology of vision between 1860 and 1865. The author explores, in particular, the role of Helmholtz's controversy with Ewald Hering in the development of Helmholtz's views. A detailed study of the same controversy is found in Turner (1994).

___ 1994: *In the Eye's Mind: Vision and the Helmholtz-Hering Controversy*, Princeton: Princeton University Press.

Wahsner, Renate 1994: Apriorische Funktion und aposteriorische Herkunft: Hermann von Helmholtz' Untersuchungen zum Erfahrungsstatus der Geometrie, in: Krüger 1994a, 245-259.

Willey, Thomas E. 1978: *Back to Kant: The Revival of Kantianism in German Social and Historical Thought, 1860-1914*, Detroit: Wayne State University Press. The book gives an account of the "back to Kant" movement in context and offers a detailed analysis of the development of social and historical thought in neo-Kantianism.