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Motivational Kantianism: Cassirer's late shift towards a regulative conception of the *a priori*

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ABSTRACT

Cassirer often pointed out that the physics of his time had progressively become a ‘physics of principles’ rather than a ‘physics of models'. Until the 1920s, Cassirer regarded these principles (the energy principle, the relativity principle etc.) as a constitutive but provisional form of the *a priori*, imposing specific limitations on the form of the allowable laws of nature. This paper argues that Cassirer shifted the role of the *a priori* to a deeper level in the 1930s. The *a priori* acquires a regulative meaning, motivating the search for the laws of nature without providing any particular insight into their form. The paper contends that, in this way, Cassirer embraced what might be called a ‘motivational Kantianism’. It can be argued whether this stance still deserves to be called a form of ‘neo-Kantianism’. However, depriving the *a priori* of any specific content allowed Cassirer to attribute a constitutive role to ‘statements of principle’ without granting them *a priori* status. This paper concludes that this attitude towards the role of ‘principles’ in physics sets Cassirer's philosophy apart from its positivist and post-positivist counterparts.

1. Introduction: from the physics of models to the physics of principles

The third ‘epistemological’ volume of Ernst Cassirer's *Philosophie der symbolischen Formen* appeared in print in 1929. In the last section of the book, dedicated to modern physics, Cassirer pointed out that since the nineteenth-century the latter had increasingly become a ‘physics of principles’ rather than a ‘physics of models.[1] The compatibility of candidate physical laws with abstract principles had progressively become more important than the construction of detailed intuitive pictures (*Bilder*) that behave according to such laws. Whereas the energy principle had dominated nineteenth-century physics (Cassirer, 1929, p. 541; tr. 1957, p. 464), Cassirer argued that by the turn of the century the least action principle had taken its place as a fundamental unifying principle that contained the former as a special case (Cassirer, 1929, p. 541; tr. 1957, p. 464). Just like the energy principle, it was initially established as a by-product of the laws of mechanics. However, the least action principle turned out to be particularly fruitful in its application to ‘extramechanical physics’; in particular, Cassirer pointed out, it turned out to be possible to derive the basic equations of electrodynamics via a variational approach (Cassirer, 1929, p. 541; tr. 1957, p. 464). The theory of relativity appeared to Cassirer as another instance of the ‘physics of principles’ (Cassirer, 1921, p. 16; tr. 1923, p. 359). Just as the energy principle does not imply the reduction of physics to mechanics, the relativity principle does not imply its reduction to electrodynamics. Rather, both principles serve as criteria for the admissibility of physical theories in general.

In this respect, Cassirer saw “a definite and unmistakable line” running “from the principle of the conservation of energy to the general principle of relativity” (Cassirer, 1929, p. 537; tr. 1957, p. 460). The highest unifying principle has changed over time. However, the general tendency to search for progressively more general principles has persisted (Cassirer, 1931, p. 126). Unlike the usual physical laws, these principles do not directly say anything about how any specific physical system behaves according to the known laws of nature. Instead, they impose general constraints on all *possible* laws; the latter cannot qualify as proper physical laws unless they satisfy these constraints (see Lange, 2009, 2016). In Cassirer's eyes, this ‘practice of principles’ (Seth, 2010)—the search for formal conditions that the laws of nature must satisfy—had an undeniable Kantian flavor. However, as the history of science has repeatedly shown, some of the constraining principles that Kant considered unshakable have been called into question. Cassirer's interest

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[1] The opposition of models and principles was a common trope. It was used, for instance, by Lorentz (1900) and Henry Poincaré (1904). Later, Einstein (1919) would classify relativity theory as a ‘theory of principles’; see Giovanelli (2020).

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in the 'physics of principles' can be seen as an attempt to preserve Kant's insight without embracing his hasty generalization. By the end of the 1920s, however, Cassirer's stance toward the status of such principles was somewhat ambiguous.

In his early epistemological works, Cassirer (1902, 1906b, 1910) regarded these constraining principles as instances of (1) 'invariants of experience' (see Cassirer, 1910, pp. 357ff.; tr. 1923, pp. 269ff.). Their 'resilience' despite the chaotic rise and fall of individual theories (Cassirer, 1906a, p. 17f. Cassirer, 1910, pp. 355ff.; tr. 1923, pp. 268ff.) cannot be a coincidence. The provisional hypothesis can be made that these principles remain 'invariant' because they are *constitutive* conditions *a priori* for the acceptability of *any* physical theories (Cassirer, 1910, p. 357; tr. 1923, p. 269). A theory that does not comply with such constraints would be rejected from the outset. However, some of these alleged 'invariants' may turn out not to be such. Thus, *stricto sensu*, only the last 'invariants' should be viewed as properly *a priori* (Cassirer, 1910, p. 357; tr. 1923, p. 269). Nevertheless, "we can never claim to grasp these invariants with our hands" (Cassirer, 1929, p. 552; tr. 1957, 476). We can only hope that it will always be possible to find 'better' constitutive principles in an infinite convergent process (Cassirer, 1910, p. 357; tr. 1923, p. 269). (2) By the 1920s, Cassirer's stance seems to have imperceptibly shifted (Cassirer, 1921). On some occasions, he suggested that only the general tendency to search for more general invariants should be regarded as *a priori* (Cassirer to Schlick, Oct. 23, 1920; ECN, Vol. 18, Doc. 88). Claims (1) and (2) were not clearly disentangled in Cassirer's early work, an ambiguity that left Cassirer open to attacks from his critics (Schlick, 1921, p. 102, 1929, p. 313). This paper claims that (3) Cassirer attempted to solve this ambiguity in the 1930s. In his last epistemological work, *Determinismus und Indeterminismus*, Cassirer (1936) made a twofold move: (3a) he attributed to 'statements of principle' an autonomous status as *constitutive* but not as *a priori* constraints imposed on the structure of the laws of nature; (3b) he attributed to the *a priori* a merely *regulative* meaning: the *a priori* motivates and guides the search for the laws of nature, without providing any particular insight into their structure.

It is often claimed that, whereas the young Hans Reichenbach (1920) promoted a constitutive but relativized *a priori* (Friedman, 2001), Cassirer and the Marburg School defended a regulative but absolute form of the *a priori* (Ryckman, 2005, pp. 245ff. Friedman, 2005, 2008). Others insist that Cassirer held a historicized *a priori* (Ferrari, 2009, 2012) or that both dynamical and absolute *a priori* elements coexist in Cassirer's neo-Kantianism (Heis, 2014). The root of these disagreements is ultimately a failure to fully appreciate the influence of Hermann Cohen's (1885) interpretation of Kant on Cassirer (Giovanelli, 2016).[2] This paper hopes to offer further evidence that the shift towards a regulative conception of the *a priori* happened only in Cassirer's later work. Indeed, in *Determinismus und Indeterminismus*, Cassirer (1936) seems to have settled for what might be called a *motivational Kantianism*.[3] The latter does not purport to search for the conditions without which physics would be impossible but only for the conditions without which physics would not be worth pursuing. One might wonder whether this stance still deserves to be regarded as a form of 'Kantianism' (Ferrari, 2009). However, the paper concludes that demoting the *a priori* to a 'regulative' function has interesting repercussions for Cassirer's philosophy of physics.

In particular, Cassirer was able to recognize the independent 'constitutive' role played by statements of principle without falling into the temptation of considering them *a priori*—even in a 'liberalized' sense of the expression. On the one hand, this move set Cassirer's late philosophy apart from the positivist epistemology of his time (Mormann, 2012), which did not recognize any role for principled thinking in physics (Stöltzner, 2009). On the other hand, it allowed Cassirer to outline a sort of *ante litteram* alternative to the post-positivist views of Quine (Quine & Ullian, 1970) and Kuhn (1962) without remaining entangled in the difficulties of the program of a relativized *a priori* (see Stump, 2015). In Cassirer's opinion, physicists' practice itself ultimately undermines both the positivist and the post-positivist accounts. Despite the superficially chaotic alternation of incompatible theories in the history of science, physicists trust that, at a deeper level, overarching physical principles can be found that preserve the key features of past theories and constrain the search for new ones (see Post, 1971). These principles are not the conditions without which scientific theories would be impossible, as Cassirer had initially surmised. However, the search for new theories would be reduced to mere guessing by trial and error without them.

2. The role of 'statements of principles' in physics

According to Toni Cassirer's recollections, even before leaving Germany in 1933 Cassirer had planned to write a book on the new quantum theory (Cassirer Bondy, 1981, p. 189). In the 1920s, Cassirer had participated in the philosophical controversies around the alleged 'relativism' implied by Einstein's theory. In the 1930s, Cassirer aimed to participate in the debate concerning the alleged 'indeterminism' introduced by quantum mechanics, a debate in which both physicists (Planck, 1932; Schrödinger, 1932) and a growing number of philosophers were engaged (Frank, 1932; Hermann, 1935; Meyerson, 1933; Popper, 1935; Schlick, 1931, 1932, 1936). The book was finished during his first year at Gothenburg. In February 1936, Cassirer requested the Göteborgs Högskolas Årsskrift–the Gothenburg university's yearbook–to publish the manuscript (Hansson & Nordin, 2006, p. 71), which was finished in April 1936. It was published the following year (Cassirer, 1936).

Cassirer opens the monograph by insisting on the consistency of his current philosophical stance with his previous work. When his book on relativity was published, Cassirer (1921) recollects, sympathetic critics "supplemented their agreement with the question whether I as a 'neo--Kantian' was *permitted* to draw such conclusions" (Cassirer, 1936, p. VIII; tr. 1956, p. xxii). However, Cassirer replies that the objection was based on a misguided representation of neo-Kantianism. Cassirer insists that the neo-Kantianphilosophers' relation to Kant was no different than the relation of a modern physicist to Galileo, Newton, Maxwell, or Helmholtz. As Paul Natorp (1912) had already insisted, it was never "the intention of the 'Marburg School' to hold fast unconditionally to Kant's teaching or to wish to do so" (Cassirer, 1936, p. VII; tr. 1956, p. xxii). For this reason, Cassirer emphasizes that his ties to "the founders of the Marburg School are not loosened and [his] debt of gratitude to them is not diminished", although his results differ from those set forth by Cohen and Natorp (Cassirer, 1936, p. VIIIf.; tr. 1956, p. xxii). An evaluation of the continuity and discontinuity of Cassirer's late work with his early neo-Kantianism would require a separate investigation (Mormann, 2015, 2021). However, it is clear that Cassirer continued to consider the goal of theoretical philosophy in typical 'Marburg' fashion. Philosophy must "interrogate the scientific 'fact' of physics, in order to obtain information concerning methodological principles [*Prinzipien*] underlying it" (Cassirer, 1936, p. 37f.; tr. 1956, p. 29; translation modified).

Cassirer argues, referring explicitly to his 1910 monograph (Cassirer, 1910), that in all of the major changes in the history of physics there are features that seem to be "invariant in the face of this shift" (Cassirer, 1936, p. 149; tr. 1956, p. 120): "These principles," like the principles of conservation of energy and momentum, "seem to be part of the basic scaffolding of every exact description of nature; they belong among the

[2] Cassirer often emphasized that it was one of the fundamental merits of Cohen Cohen's (1885) *Kants Theorie der Erfahrung* to have shown that the core of Kant's first *Critique* was the 'synthetic principles' (see, e.g. Cassirer, 1918, p. 187). The synthetic principles (*Grundsätze*) are said to be constitutive in that they impose specific formal constraints on possible candidate laws of nature. The search for the actual empirical laws of nature is guided by the principle (*Prinzip*) of the 'formal finality of nature'-the expectation that the empirical laws are organized in a progressively more coherent system. The latter principle is merely *regulative:* it provides a guide for the search for empirical laws without imposing any specific constraints on them (Cohen, 1885).

[3] I model this expression on Fine's (1986) 'motivational realism'

invariants which are independent of any special form adopted by the system of physical description" (Cassirer, 1936, p. 145; tr. 1956, p. 117). However, Cassirer seems to have abandoned his plan to classify each of these 'invariants' as *a priori*, even provisionally. On the contrary, in *Determinismus und Indeterminismus*, Cassirer for the first time explicitly gives a special role to such 'principles' in his account of the conceptual structure of physical theories, a role that is explicitly distinguished from that of *a priori* statements.

According to Cassirer, modern disputes concerning the foundation of physics depended on the fact that physicists were not always aware of the clear distinction between "*different types of physical statements*" Lr (Cassirer, 1936, p. 241; tr. 1956, p. 30).[4] Thus, Cassirer introduces a sort of 'type theory' of physical statements, a nested 'hierarchical structure' in which the higher level transcends the peculiarities of the lower ones, such that the former would still have held even if the latter had been different. Cassirer's classification, pared to the bone, is as follows:

- *statements of measurement* (*Massaussagen*): at the lower level are those statements that refer to any property that a system may have that is amenable to being measured (at least in principle) by an experimental apparatus, and thus to be designated by numbers. For example, in pre-general relativistic physics, the position and time coordinates of particles can be measured using rods and clocks, whereas the strengths of various fields at each point (e.g., the electromagnetic field) can be measured by charged test particles.
- *statements of law* (*Gesetzes-Aussagen*): at the next level are what we would call laws of nature, "functional equations by means of which we combine the different classes of measured quantities" (Cassirer, 1936, p. 53; tr. 1956, p. 42), usually differential equations (e.g., Maxwell's equations) that impose a restriction on the dynamical variables.
- *statements of principle* (*Prinzipien-Aussagen*): finally, Cassirer introduced 'principles' as a separate category of statements. They also appear in the form of 'functional equations', which, however, are not used as single equations but as constraints on the formulations of *all* possible lower-level law-like statements. The latter do not qualify as proper statements of law unless they satisfy such requirements.

The relationship between statements of principle and statements of law is analogous to that between the latter and statements of measurement. In each case, there is a *jump*, or, according to one of Cassirer's favorite turns of phrase, there is a μετάβασις είς ἄλλο γένος, a change into another kind of genus. For this reason, variations at the lower level do not imply changes at the higher level. Statements of laws are insufficient to fully determine statements of measurement because they do not entail the initial and boundary conditions. Likewise, statements of principle are insufficient to determine first-order statements of law; they merely impose constraints, which provide criteria for distinguishing what can be taken as 'laws of physics'. Nevertheless, despite this hierarchical structure, Cassirer insists that all three levels mutually condition and support each other (Ryckman, 2015). The structure of physics should not be thought of as a pyramid but as a well-rounded sphere (Cassirer, 1936, p. 45; tr. 1956, p. 35).[5]

At first sight, Cassirer's classification of statements seems to come out of the blue. However, it is merely a more perspicuous ordering of Cassirer's scattered reflections from the previous decades (see e.g., Cassirer, 1910, pp. 351ff.; tr. 1923, pp. 264ff.). Indeed, in *Determinismus und Indeterminismus* he uses the same examples of 'principles' that he used in the past. He mentions Leibniz's use of the continuity principle to exclude Descartes' rules of collision from the outset (Cassirer, 1936, p. 197f.; tr. 1956, p. 158f.); the 'virtual principle', ie., the principle of virtual work (Cassirer, 1936, p. 201; tr. 1956, p. 161), which mediates between statics and dynamics; and the energy principle, which completes the transition from mechanics to general physics (Cassirer, 1936, p. 58; tr. 1956, p. 45). However, possibly because he treated these episodes in his previous works, in *Determinismus und Indeterminismus* Cassirer resorts to the history of the principle of least action, as "a paradigm in which the character of *statements of principle*, statements of the third order[, can] be exhibited and studied" (Cassirer, 1936, p. 64; tr. 1956, p. 50). In the form given to it by Helmholtz, Cassirer argues that the least action principle represents "the typical form of physical principles and the epistemological problem latent in it" (Cassirer, 1936, p. 60; tr. 1956, p. 47).[6]

3. The principle of least action as the prototype of a statement of principle

Cassirer presents a brief history of the principle of least action up to Lagrange.[7] Since Lagrange, Cassirer points out, "[the principle of least action] has dominated all the subsequent development of mechanics" (Cassirer, 1936, p. 61; tr. 1956, p. 48). Nevertheless, in the form given by Lagrange, the principle of least action depended in its formulation on the law of conservation of energy, which ought to be postulated separately. It was Hamilton (1834, 1835) who generalized the principle of least action to the case in which the conservation of energy does not hold. Among the possible paths between two fixed points, Hamilton considered those that have the same time rather than the same energy. "In Hamilton's formula", it is "the excess of kinetic over potential energy that provides the determining quantity", that is, the so-called 'Lagrangian' $L = T - U$, the difference between the kinetic (T) and the potential energy (U) (Cassirer, 1936, p. 65; tr. 1956, p. 51). Hamilton's principle asserts that the actual motion realized in nature is the particular path for which the time-integral of L assumes its smallest value, or better, a stationary value $\delta \int_{t_1}^{t_2} L dt = 0$. "In the form of the 'Hamilton's principle'" the principle of least action has become the "fundamental theorem of mechanics" (Cassirer, 1936, p. 61; tr. 1956, p. 48). In particular, it even remains valid for nonconservative systems, that is, for systems that include forces that cannot be derived from potential functions (e.g., friction, galvanic resistance etc.). In these cases, the work W made by such forces in displacing the system could be taken into account by adding an external term to the action integral. Hamilton's principle would then read: $\int_{t_1}^{t_2} (\delta L + \delta W) dt = 0$.

Up to this point, debatable details aside, Cassirer's historical reconstruction is rather conventional. What is characteristic of Cassirer's historiographical approach is the central role that, following Planck (1915), he attributes to Helmholtz's work (Mayerhofer, 1994). By the end of the 1880s, Helmholtz (1884, 1887) had abandoned the conservation of energy as the main guide in physics, and elevated the principle of least actionto be the fundamental "heuristic principle" in the quest for the laws of new classes of phenomena (Helmholtz, 1887a, 1887b, 143). With Helmholtz, Cassirer writes, "[t]he principle entered an entirely new

[4] The word 'type' is used in analogy to Russell's *type theory*; (Russell, 1903, Appendix B. The Doctrine of Types).

[5] The image of an 'arch' as a self-supporting structure might have been more effective. The keystones, voussoirs and footers all support each other while belonging to different 'types'; on Cassirer's holism, see (Richardson, 2015; Cassirer, 1936, p. 45; tr. 1956, p. 35).

[6] It is interesting to compare Cassirer's stance towards the principle of least actionwith the rather dismissive attitude held by the members of the Vienna Circle at the same time (Frank, 1932; Hahn, 1933). On this point, see Stöltzner, 2003. In Cassirer's judgment, the empiricist tradition is incapable of appreciating the hierarchical structure of physical theories insofar as it puts all physical statements at the same level (Petzoldt, 1891; Mach, 1904, ch. 3). The same criticism might be raised against logical empiricists. However, in my view, Cassirer did not consider the principle of least action a form of constitutive, relativized *a priori* (cf. Stöltzner, 2009). On the relationship between Cassirer and the Vienna Circle in the 1930s, see Mormann (2012).

[7] Cassirer's historical reconstruction is based on Helmholtz (1887a, 1887b), Planck (1915) and Kneser (1928).

phase of its development" (Cassirer, 1936, p. 61; tr. 1956, p. 49). Helmholtz (1887a, 1887b) realized that the sharp distinction between kinetic energy T and potential energy V—where the first depends on square velocity and the second on positions alone—is not a necessary condition for the application of the principle of least action. There are cases in which the potential energy may be a function of velocity as well as position (as in electrodynamics), and the kinetic energy may depend on position and non-squared velocities (as in thermodynamics). In such cases, kinetic and potential energy cannot be clearly separated.

As Cassirer puts it, "Helmholtz designates the magnitude, whose time integral represents Hamilton's action, as the 'kinetic potential'" H (Cassirer, 1936, p. 65; tr. 1956, p. 51). Helmholtz introduced H as the fundamental quantity (which can be any function whatsoever of coordinates and velocities) without specifying the individual parts of the 'kinetic potential' as kinetic or potential energy. Only in hindsight can H be split into two separate terms: the translational energy which depends on velocity, and the internal energy of the system at rest. Helmholtz showed that the kinetic potential thus modified still maintained the stationary property of its time-integral, in the form $\int_{t_1}^{t_2}(\delta H+\delta W)dt=0$. Thus, Cassirer points out that with Helmholtz's work "the range of validity of the principle has grown far beyond the limits of mechanics, and it is to be regarded as highly probable that it embraces electrodynamics and thermodynamics[8] as well" (Cassirer, 1936, p. 62; tr. 1956, p. 49). The form of the function that determines the kinetic potential depends on the particular nature of the system under investigation. As soon as we have determined H experimentally for a particular field of investigation, the principle of least action allows us to derive the equation of the motion of the system as a solution of the Lagrange equations.

In this way, Cassirer points out, "Helmholtz freed the principle from its restriction to the mechanics of ponderable bodies and proclaimed it a *physical postulate of completely universal validity*" (Cassirer, 1936, p. 62; tr. 1956, p. 49; my emphasis). Initially, the principle of least action was considered to be a consequence of the laws of mechanics. Thus, it enjoyed equal rank and regard with numerous other mechanical 'principles' (e.g., d'Alembert's principle, the principle of virtual displacements etc.). In this form, the principle of least action was nothing more than a "mathematical curiosity", an interesting but ultimately "dispensable appendix" to Newton's laws of motion (Planck, 1915, p. 699). Helmholtz showed that one could take the opposite path. He assumed the universal validity of the principle itself as his starting point and used it as a potent selection principle to find new laws. The laws of mechanics only happen to satisfy the principle alongside the laws of electrodynamics, the thermodynamics of reversible processes, and so on. In this way, Helmholtz introduced the peculiar *Umkehrung* that characterizes the role of principles in physics. At first, statements of principle seem simply to be a way to encode certain regularities of the known statements of law; at a certain point, however, they are turned into constraints on possible laws and in many cases serve as a guide in the formulation of new laws[9]:

> "Here in fact we find a basic methodological characteristic common to all genuine statements of principle. *Principles do not stand on the same level as laws*, for the latter are statements concerning specific concrete phenomena. Principles are not themselves laws, but rules for seeking and finding laws. This heuristic point of view applies to all principles. They set out from the presupposition of certain common determinations valid for all natural phenomena and ask whether in the specialized disciplines one finds something corresponding to these determinations, and how this 'something' is to be defined in particular cases. […] Principles are invariably such bold anticipations that justify themselves in what they accomplish by way of construction and inner organization of our total knowledge. *They refer not directly to phenomena but to the form of the laws according to which we order these phenomena*. A genuine principle, therefore, is not equivalent to a natural law. It is rather the birthplace of natural laws, a matrix as it were, out of which new natural laws may be born again and again" (Cassirer, 1936, p. 66; tr. 1956, pp. 52–53).

Thus, the principles are justified not only insofar as they successfully express common features of available dynamical laws but insofar as they serve as criteria for their selection. For this reason, when reflecting on the epistemological *status* of principles and studying their history, one is confronted with a continuous fluctuation between empiricism and rationalism, between a bottom-up *a posteriori* approach and a top-down *a priori* approach.

At first, Cassirer points out that principles "are only valid as hypotheses. They cannot stipulate dogmatically from the beginning a particular result of investigation" (Cassirer, 1936, p. 68; tr. 1956, p. 54). Confidence in the principles arises from the fact that they work across very different fields of application. In this sense, they are often discovered *a posteriori* by a second level induction on the character of well-established statements of law. At a certain point, physicists become extremely leery of any proposed physical theory that violates one of these principles. Thus, they start assuming that statements of principle are more fundamental than the laws they constrain: if the available laws do not satisfy the principle, then they surmise that there are other laws to be discovered. Thus, the temptation to consider the principle a condition *sine qua non* of a good physical theory in general becomes hard to resist. For this reason, Cassirer suggests that it is indeed a "*historical* criterion" for identifying principles that "the attempt [be] made repeatedly to ascribe to them the highest form of 'universality'; that is, to identify them in some way with the general causal principle"dass immer wieder in der Geschichte des philosophischen und naturwissenschaftlichen Erkennens der Versuch hervortritt, ihnen die höchste Form der 'Universalität' zuzusprechen, d. h. sie in irgendeiner Weise mit dem allgemeinen Kausalsatz selbst zu identifizieren oder aus ihm unmittelbar abzuleiten (Cassirer, 1936, p. 69; tr. 1956, p. 55), or, more generally, as principles *a priori*. This was indeed the destiny of the energy principle or of the least action principle. The "higher we ascend in the hierarchy of scientific propositions, the harder it becomes to distinguish between these propositions and the summit of the hierarchy" (Cassirer, 1936, p. 69; tr. 1956, p. 55), *a priori* statements.

However, this historical criterion for identifying principles cannot be taken at face value. Cassirer conceded that "the difference between statements of principle, no matter how general", and the *a priori* principles themselves "cannot be wiped out" (Cassirer, 1936, p. 71; tr. 1956, p. 57). Even the energy principle, Cassirer now argues, "which has so well justified itself in its universality", is not a necessary condition of the possibility of science, "whose negation would by no means be tantamount to an abrogation of the causal principle as such" (Cassirer, 1936, p. 71f.; tr. 1956, p. 57). After repeated failures to find laws that conform to the principle, physicists often entertain the hypothesis that the failure is not due to a lack of effort or imagination. Under the pressure of empirical evidence or in virtue of theoretical considerations, physicists might concede that the principles themselves must be abandoned or modified.

Helmholtz initially assumed that forces that depend only on distances are a necessary condition for defining energy (Helmholtz, 1847). Consequently, he rejected Weber's (1848) velocity-dependent forces in electrodynamics. Following Clausius's criticisms, Helmholtz abandoned his original formulation of the energy principleand ultimately embraced the least action principle as a supreme unifying principle that was more general and more restrictive than the energy principle (Bevilacqua,

[8] See Bierhalter (1993).

[9] Cassirer is aware that the generality of the principle of least action seems to be the consequence of the ambiguity of the definition of 'action', which is different in every domain in which the principle is applied (Cassirer, 1936, p. 65; tr. 1956, p. 52). This "iridescent indeterminateness" (Cassirer, 1936, p. 64; tr. 1956, p. 51) seems to reduce the principle to an empty definition. However, Cassirer pointed out that the problem is analogous to the definition of 'mechanical equivalent' in applying the energy principle (Cassirer, 1936, p. 65; tr. 1956, p. 52). The choice of a certain definition is ultimately legitimized by its capacity to recover already known laws and to lead to new ones.

1994). At the turn of the century, it appeared that the application of the principle of least action fails in cases where internal and external energy cannot be distinguished, in particular when electromagnetic radiation is present (Mosengeil, 1906). In these cases, the internal energy of a moving system is different from that of a system at rest. As Planck (1907) realized, the introduction of the relativity principle solved this problem by allowing physicists to determine the energy of radiation of the moving system. Thus, by Planck (1915) declared the least action principle the crown jewel of physics for governing all of the reversible processes of nature. In the same year, it was also shown (Einstein, 1916; Hilbert, 1915) that the fundamental equation of the "general theory of relativity [could be] set out from the Hamiltonian principle" (Cassirer, 1936, p. 63; tr. 1956, p. 48f.).

4. *A priori* statements: Cassirer's 'motivational Kantianism' and the regulative conception of the *a priori*

To a certain extent, Cassirer's 'phylogenesis' of the role of principles in physics seems to mirror the 'ontogenesis' of the role of principles in Cassirer's own thought. The young Cassirer (1902) seemed to have identified the energy principle, the continuity principle etc., more or less explicitly with constitutive *a priori* principles in a proto-Kantian sense. Later, he did concede that the identification of such 'invariants' with *a priori* statements could be at most provisional (Cassirer, 1906b, 1910). However, even this compromise turned out to be ultimately untenable. In *Determinismus und Indeterminismus*, Cassirer includes an analysis of domains of physics that were absent from his earlier work. In particular, the entropy principle in thermodynamics, despite its undeniable generality and heuristic power, does not have the aprioristic flavor of its companion, the energy principle (Cassirer, 1936, pp. 94ff.; tr. 1956, pp. 75ff.). Most of all, Cassirer reconstructs the history of the old quantum theory as the passage from 'the quantum law', Planck's quantum of action, to 'quantum principles', such as Ehrenfest's 'adiabatic hypothesis' and Bohr's 'correspondence principle' (Cassirer, 1936, p. 139; tr. 1956, p. 112). The old quantum theory appears to Cassirer as the transformation of the quantum hypothesis from a 'statement of law' into a 'statement of principle'. Ultimately, Cassirer seems to consider even quantum mechanics in its final form as a 'theory of principles' in which the 'uncertainty relations' play the role of an 'uncertainty principle'[10]. The renunciation of intuitive models in favor of abstract principles has become even more apparent than in the past. However, none of these 'statements of principle' can plausibly be considered *a priori*, even provisionally, in the sense of serving as a condition without which physical science *in general* would be impossible.

Unsurprisingly, in *Determinismus und Indeterminismus*, Cassirer shifted the position of the *statements a priori* to a deeper level. One can reach this level even "from the level of the universal principles, only by means of a jump" similar to that which characterized the passage between the previous levels of statements (Cassirer, 1936, p. 72; tr. 1956, p. 57). Cassirer recognized that at this point a fundamental problem emerges: "what new insight does it add to what we have already learned from the foregoing epistemological analysis?" (Cassirer, 1936, p. 75; tr. 1956, p. 60). Cassirer conceded that one could legitimately wonder about the role played by this further level of statements, "after we have traversed the earlier levels, after we have progressed from statements of the results of measurements to those of laws, and from these to statements of principle" (Cassirer, 1936, p. 75; tr. 1956, p. 60). Cassirer's response to these rhetorical questions is, by his own admission, somewhat baffling:

> "I would like to give an answer to this question, which at first sight will perhaps seem paradoxical. There is in fact *nothing* left over, nothing new in principle to be added to the description of the process of cognition and of the epistemological structure of science. […] What the [*a priori*] signifies […] is not a new insight concerning the content, but solely concerning the method. It does not add a single factor homogeneous with the foregoing, which could be placed alongside it as a material supplement. With regard to content it does not go beyond what has already been observed; it only confirms it and confers upon it as it were the epistemological *imprimatur*. […] [It] specifies fundamentally nothing more than that the process, which we have sought to describe in detail, is possible without limitation. It does not maintain that the process of translating data of observation into exact statements of the results of measurements, or the process of gathering together the results of measurements into functional equations by means of general principles, is ever complete. What it demands and what it axiomatically presupposes, is only this: that the completion can and must be sought, that the phenomena of nature are not such as to elude or to withstand in principle the possibility of being ordered by the process we have described" (Cassirer, 1936, p. 75; tr. 1956, p. 60).

In my view, it is only at this point in his intellectual biography that Cassirer embraces a 'regulative' conception of the *a priori*; however, his notion of the regulative *a priori* differs markedly from that used by scholars today. For the young Cassirer, as for every good neo-Kantian, the historical *fact* of physics had to be turned into the *problem* of how it is possible (see, e.g., Gawronsky, 1910, pp. 1ff.). The Cassirer of the 1930s seems to have come to the conclusion that this problem is unsolvable. He saw no other alternative than to give a new twist to Goethe's old trick. He transformed the *problem* into the *postulate* that physics *must* be possible.[11]

This postulate is *a priori* because it cannot be derived from experience; however, it is only a "regulative principle and nothing else" (Cassirer, 1936, p. 79; tr. 1956, p. 63) since it does not have any content and thus does not impose any specific 'constraint' on lower-level statements. It only expresses a "universal trust [*Vertrauen*]" (Cassirer, 1936, p. 79; tr. 1956, p. 63) on the part of the scientists that "the procedure from experimental findings and their exact formulation to ever stricter statements of law and ever more general statements of principle" will continue without end (Cassirer, 1936, p. 79; tr. 1956, p. 65). This requirement is not objectively necessary for the constitution of individual physical theories. However, it is subjectively necessary as an expression of an 'interest of reason', as a stimulus for scientific research (Cassirer, 1936, p. 100; tr. 1956, p. 80) that makes it seem worth pursuing. In this way, Cassirer settled for what might be called a 'motivational Kantianism'[12] that gives meaning to the work of physicists but is powerless to justify their theories.

Cassirer seems to be clutching at straws by constantly moving the goalpost of what counts as 'Kantianism' in the first place (Frank, 1938; Weizsäcker, 1937). However, Cassirer's point is possibly more subtle than it first appears. To appreciate it, one should consider the passages where he compares the 'critical' with the 'empiricist' concept of experience. According to Cassirer, the inadequacy of naive empiricism lies not in the fact that it regards experience as the ultimate source of scientific credibility but in the fact that "it does not push its *analysis* of experience far enough" (Cassirer, 1936, p. 68; tr. 1956, p. 54). Recognizing that experience is the only tribunal of scientific knowledge "by no means implies that within empirical knowledge itself all logical differences are to be leveled and obliterated" (Cassirer, 1936, p. 68; tr. 1956, p. 54).

- Dogmatic empiricism of the Machian persuasion interprets even more abstract principles, such as the energy principleand the least action

[10] This latter case is less straightforward, but I plan to discuss this issue elsewhere. See, e.g., Cassirer, 1936, p. 241; tr. 1956, p. 193); on Cassirer's interpretation of quantum mechanics, see Ryckman, 2018, 2021.

[11] One of Cassirer's favorite quotes from Goethe states: "The highest art in intellectual life and worldly life consists *in turning the problem into a postulate*" (Goethe to Zelter, Aug. 9, 1828; Goethe & Friedrich Zelter, 1832, p. 61) See Giovanelli, 2015.

[12] See fn. 3

principle, as statements that are not qualitatively different from simpler empirical laws, which in turn are only catalogs of empirical data (Cassirer, 1936, p. 86; tr. 1956, p. 68). For this reason, from this point of view, "[e]xperience constantly alters its form as the circle of the given widens" (Cassirer, 1936, p. 91; tr. 1956, p. 73); it is a process of accumulation of statements of the same kind, in which new "elements are introduced from without" and "added singly one to the other, piece by piece" (Cassirer, 1936, p. 91; tr. 1956, p. 73). Thus, physics appears to be constantly changing; theories are short-lived and volatile, and laws that were previously believed to be correct can be overturned at any time by new discoveries.

- Critical empiricism concedes that *all* physical statements are always open to empirical revision. However, it does not conclude that they are all on the same level. Just as laws are not merely records of measurements, principles cannot be reduced to a mere 'economical' collection of individual laws. One can move from laws to principles "only by means of a 'jump'" (Cassirer, 1936, p. 68; tr. 1956, p. 54) that requires "a considerable *expenditure* of intellectual energy" (Cassirer, 1936, p. 68; tr. 1956, p. 54). In Cassirer's view, the history of science repeatedly shows that such 'jumps' cannot be fully justified theoretically or inductively (Cassirer, 1936, p. 58; tr. 1956, pp. 46ff.). They involve the physicists' intuitive capacity to single out relevant features of the available theories and to turn them into mathematically formulated selection criteria for new theories.

As one might expect, according to Cassirer, it was Kant's merit to have pointed out that scientific experience, contrary to the claims of British empiricism, has an articulate and stable 'structure' that is defined "not through its material but through its form" (Cassirer, 1936, p. 91; tr. 1956, p. 73). Cassirer of course acknowledged that Kant's analysis of this structure "was too closely bound to a specific form of science": Euclidean geometry and Newtonian physics (Cassirer, 1936, p. 92; tr. 1956, p. 74). In this sense, the critical project in its original form has failed. When pressured by experience, physics has been forced to demolish that structure piece by piece. Nevertheless, Cassirer pointed out that scientific experience "did not simultaneously give up its general structure" (Cassirer, 1936, p. 93; tr. 1956, p. 74). Instead, it only reveals that this structure is based not on a fixed set of assumptions but on the internal logic according to which, historically, physics has moved from one theory to another: "This process has its rules, and these rules provide the presuppositions and foundation for what we may call the 'form of experience'" (Cassirer, 1936, p. 93; tr. 1956, p. 74).

At first glance, if one considers its results, the history of physics appears to be a "curious phantasmagoria" of theories, a "sequence of pictures which move past us in unending change and kaleidoscopic brilliance" (Cassirer, 1936, p. 186; tr. 1956, p. 149). However, from a point of view of method, a "comparison of the various empirical theories shows us that their succession is not characterized by lack of law or by arbitrariness" (Cassirer, 1936, p. 93; tr. 1956, p. 75). The evolution of physics takes the form of a process of progressive integration of measurements under laws, and of a great number of laws under progressively fewer and more general principles. By the turn of the century, it could be shown that the laws of all reversible processes, whether mechanical or electromagnetic, can be derived from a single principle, the principle of least action. The entropy principle did introduce a "foreigner and intruder" (Cassirer, 1936, p. 95; tr. 1956, p. 76) into the system of classical mechanics and electrodynamics. However, Cassirer argues that, as often happened in the past, pressure from the side of experience was matched by a counter-pressure from the side of theory. The gap between reversible and irreversible processes could be bridged again by interpreting entropy as probability (Cassirer, 1936, pp. 94ff.; tr. 1956, pp. 75ff.). Thus, contrary to first appearances, the history of physics is not "guided by mood or caprice but by a definite rule"; it is a movement that progresses steadily in the same direction. It is "in this *direction* and in it alone" that we can recognize the structure of scientific experience (Cassirer, 1936, p. 187; tr. 1956, p. 150).

Cassirer's 'historical continuism' appears rather anodyne if one considers its content. However, it is remarkable if one considers its justification. For Cassirer, the continuity of the history of science is the result neither of empirical generalization nor of rational deduction. It is not even a necessary condition of the possibility of science. The legitimacy of historical continuism ultimately resolves in its effectiveness as a 'maxim of scientific inquiry' (Cassirer, 1936, p. 100; tr. 1956, p. 80). From a logical point of view, the dogmatic empiricist's stance is unobjectionable: new theories do not need to show any kinship with the old ones; physicists are spoiled for choice and can only arrive at new theories by trial and error. However, the critical empiricist can retort that, from a historical point of view, physics has progressed by building on past knowledge[13]; physicists have always been able to condense the wisdom of past theories in the form of overarching principles that restrict the range of possible new theories. Resorting to Goethe's stratagem, Cassirer turned the historiographical *problem* of the continuity of the evolution of science into a heuristic *postulate* promoting the growth of science (see Cassirer, 1906b, p. 19). In other words, Cassirer does not simply claim that the development of science *happened to be* continuous as a matter of historical fact but also declares that it *must* be continuous if the search for new theories is not to be reduced to a hit-or-miss groping in the dark.

5. Conclusion

Cassirer left Sweden in May of 1941 to become Visiting Professor of Philosophy at Yale (Hansson & Nordin, 2006). He left behind the manuscript of the fourth and final volume of the series *Das Erkenntnisproblem*, which was published posthumously in English (Cassirer, 1950). The chapter 'The Goal and Methods of Theoretical Physics'—which was likely written around the time of *Determinismus und Indeterminismus*—offers a good overview of Cassirer's take on the role of principles in physics. Cassirer observed a tendency in nineteenth- and twentieth-century physics to rely on progressively more abstract mathematical principles that distill the common structure of different, possibly incompatible theories. What changes in the succession of these theories are the various models (e.g., the different models of the atom or the wave and the particle pictures of light etc.). For this reason, models that were initially introduced as 'images' of physical systems have been progressively recognized as 'symbols' that are implicitly defined within the structure of the theory as a whole:

> "The old physics was for the most part a physics of symbols in this sense: it tried to represent the nature of every object or event investigated in a corresponding mechanical model, whose single items passed as replicas of the details and properties of the object. Modern physics has increasingly renounced this procedure, and *from a physics of literal pictures has become a physics of principles*. Its development in the nineteenth century was signaled by the discovery and precise formulation of several principles, such as Carnot's principle, the principle of the conservation of energy, the principle of least action, and so on. A principle is neither a mere synopsis of facts nor solely an *epitomization of single laws*. It contains in its meaning the claim of 'always and universally,' which experience as such is never warranted in making. Instead of deriving a principle directly from experience we use it as a *criterion of experience*. Principles constitute the fixed points of the compass that are required for successful orientation in the world of phenomena. They are not so much assertions about empirical facts as maxims by which we interpret these facts in order to bring them together into a complete and coherent whole" (Cassirer, 1932–1940, pp. 127f.; tr. 1956, pp. 110f.; my emphasis).

This passage effectively summarizes a theme that passes throughout the entire body of Cassirer's work, until it becomes explicit in *Determinismus und Indeterminismus*. In physics, there are general rules with which all individual laws of nature seem to comply. As Cassirer points out in the

[13] The spirit of Cassirer's heuristics seems broadly comparable to that outlined by Post (1971). See also, Rovelli (1997, 2009, 2015).

cited passage, these principles are not simply the *epitomization* of the actual laws of nature but serve as *criteria* for selecting possible laws of nature. In Cassirer's reading, it was Kant's merit to have fully grasped the philosophical importance of the search for criteria, separating the wheat from the chaff: the law-like statements that can be taken as 'laws of physics'[14] from those that do not. However, as Cassirer conceded, Kant mistakenly believed that a set of selection criteria could be fixed once and for all. Cassirer's historical-critical analysis of the role of 'principles' in physics can ultimately be considered his life-long attempt to avoid the shortcomings of Kant's original program by preserving its key insight.

In the early years, Cassirer seemed to have considered each of these constraints as a possible, although provisional, candidate for a *constitutive*, *a priori* condition of the acceptability of physical laws in general. As I have tried to show, in his later years Cassirer shifted the role of statements *a priori* to a deeper level. The *a priori* no longer provides a specific criterion for deciding what counts as a law of nature; it only provides a regulative maxim for the search for new laws. The role of the *a priori* was moved, so to speak, from the 'context of justification' to the 'context of discovery'. In this way, Cassirer seems to have settled for what I have called a 'motivational Kantianism' in which the *a priori* merely gives meaning to the search for theories without attempting to justify their legitimacy. Physics would not be worth pursuing without a deep-seated belief that it is *always* possible to compress measurements into laws, and laws into principles. Paraphrasing Kant, the question was no longer 'What can we know?', but only 'What may we hope?' (A805/B833).

Cassirer's 'motivational Kantianism' is, *qua* 'Kantianism', somewhat disappointing. However, by relegating the *a priori* to a *regulative* role, Cassirer was able to maintain the *constitutive*, constraining function of 'principles' in physics[15] without being forced to consider them conditions of the possibility of science in general. In this way, he managed to escape the logical positivist dualism between the physical-empirical and the mathematical-tautological (Stöltzner, 2003) without running aground in the sands of the synthetic *a priori*. At the same time, he was able to outline *ante litteram* a conception of the structure and evolution of physics that offered an alternative to that which would dominate post-positivist philosophy of science, without remaining entangled in the incommensurability issues that plague the program of a relativized *a priori* (Friedman, 2001; see; Richardson, 2009). Contrary to Quine's holistic empiricism (Quine & Ullian, 1970), Cassirer sees the key feature of critical empiricism as lying in the acknowledgment of different levels of physical statements, which is deeper than the mere distinction between the periphery and the center of the 'web of belief'. Contrary to Kuhn's (1962) historiographical rupturism, Cassirer insists on the continuity of the historical development of physics. It is physicists' 'practice of principles' by itself that justifies this stance. Historically, the passage from one theory to another has not been made not by irrational leaps but by the search for more encompassing principles that preserve the success of previous theories and constrain the search for new ones (see Post, 1971). These two aspects seem to complement each other. The vertical discontinuity in the synchronic structure of each physical theory (Cassirer's 'type theory') allows for the horizontal continuity in their diachronic succession (Cassirer's 'historical continuism'). As the late Cassirer conceded, physics would still be possible without such a multilayered structure; however, research in physics would be demoted to a 'see what sticks' strategy.

Abbreviations

ECN Ernst Cassirer (1995–2009). Nachgelassene Manuskripte und Texte. Ed. by John Michael Krois. 18 vols. Hamburg: Meiner, 1995–2009

ECW Ernst Cassirer (1998–2009). Gesammelte Werke. Hamburger Ausgabe. Ed. by Birgit Recki. 26 vols. Hamburg: Meiner, 1998–2009

[14] See fn. 2

[15] The importance of the 'meta-character' of some statements in physics has recently been emphasized by Lange (2009, 2016).

References

Bevilacqua, F. (1994). Theoretical and mathematical interpretations of energy conservation. The Helmholtz-Clausius debate on central forces 1852-54. In L. Krüger (Ed.)*, 1994. Universalgenie Helmholtz. Rückblick nach 100 Jahren* (pp. 89–106). Berlin: Akademie Verlag.

Bierhalter, G. (1993). Helmholtz's mechanical foundation of thermodynamics. In D. Cahan (Ed.), *1993. Hermann von Helmholtz and the foundations of nineteenth-century science* (pp. 432–496). Berkeley: University of California Press.

Cassirer, E. (1902). *Leibniz' system in seinen wissenschaftlichen Grundlagen*. Marburg: Elwert, 1902. Repr. in ECW Vol. 1.

Cassirer, E. (1906a). *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit* (Vol. 1). Berlin: Bruno Cassirer, 1906. Repr. in ECW, Vol. 2.

Cassirer, E. (1906b). *Der kritische Idealismus und die Philosophie des 'gesunden Menschenverstandes'*. Giessen: Töpelmann, 1906.

Cassirer, E. (1910). *Substanzbegriff und Funktionsbegriff. Untersuchungen über die Grundfragen der Erkenntniskritik*. Berlin: Bruno Cassirer, 1910. Repr. in ECW, Vol. 6.

Cassirer, E. (1918). *Kants Leben und Lehre*. Berlin: B. Cassirer, 1918. Repr. in ECW, Vol. 8.

Cassirer, E. (1921). *Zur Einstein'schen Relativitätstheorie. Erkenntnistheoretische Betrachtungen*. Berlin: B. Cassirer. Repr. in ECW, Vol. 10.

Cassirer, E. (1923). *Substance and function, and Einstein's theory of relativity*. In Chicago, London: Open Court, 1923.

Cassirer, E. (1929). *Philosophie der symbolischen formen. Dritter Teil: Phänomenologie der Erkenntnis*. Berlin: B. Cassirer, 1929. Repr. in ECW, Vol. 13.

Cassirer, E. (1931). Die Einheit der Wissenschaft, 1931. In *Two lectures given on October 21 and October 28, 1931 for the program Hochsculfunk aired by the radio station Deutsche Welle*. Pub. in ECN, Vol. 8, pp. 117–134.

Cassirer, E. (1932–1940). *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit. Von Hegels Tod bis zur Gegenwart (1832–1932)*. In (Vol. 4). Berlin: B. Cassirer, 1932–1940 German manuscript 1940.

Cassirer, E. (1936). Determinismus und Indeterminismus in der modernen Physik. In *Historische und systematische Studien zum Kausalproblem*. -. Göteborgs Högskolas Årsskrift 42. Repr. in ECW, Vol. 19.

Cassirer, E. (1950). *The problem of knowledge philosophy, science, and history since Hegel*. New Haven: Yale University Press, 1950.

Cassirer, E. (1956). *Determinism and indeterminism in modern physics. Historical and systematic studies of the problem of causality*. New Haven: Yale University Press, 1956.

Cassirer, E. (1957). *The philosophy of symbolic forms. Volume Three: The Phenomenology of Knowledge*. New York: Yale University Press, 1957.

Cassirer Bondy, T. (1981). *Mein Leben mit Ernst Cassirer*. Hildesheim: Gerstenberg, 1981.

Cohen, H. (1885). *Kants theorie der Erfahrung* (2nd ed.). Berlin: Dümmler, 1885. Repr. in Cohen, 1977–, Vol. 1/I.

Cohen, H. (1977). In J. H. Holzhey (Ed.), *Werke* (Vol. 15). Hildesheim: Olms, 1977–.

Einstein, A. (1916). Hamiltonsches Prinzip und allgemeine Relativitätstheorie. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, 1111–1116. Repr. in Einstein 1987–, Vol. 6, Doc. 41.

Einstein, A. (1919). *What is the theory of relativity?* The Times. Nov. 28, 1919. Repr. in Einstein, 1987–, Vol. 7, Doc. 25.

Einstein, A. (1987–). In J. Stachel (Ed.)*, 16 Vols.. The collected papers of Albert Einstein*. Princeton: Princeton University Press.

Ferrari, M. (2009). Is Cassirer a neo-Kantian methodologically speaking? In R. A. Makkreel, & S. Luft (Eds.), *Neo-Kantianism in contemporary philosophy* (pp. 293–312). Bloomington: Indiana University Press, 2009.

Ferrari, M. (2012). Between Cassirer and Kuhn. Some Remarks on Friedman's Relativized A Priori. *Studies in History and Philosophy of Science Part A, 18*, 18–26.

Fine, A. (1986). *The Shaky Game. Einstein, Realism, and the Quantum Theory*. Chicago: University of Chicago Press.

Frank, P. (1932). *Das Kausalgesetz und seine Grenzen*. Wien: Springer, 1932.

Frank, P. (1938). Discussion of E. Cassirer: *Determinismus und Indeterminismus in der modernen Physik. Theoria, 4*, 70–80.

Friedman, M. (2001). *Dynamics of reason. The 1999 Kant lectures at*. Stanford, Calif: Stanford University. CSLI Publ., 2001.

Friedman, M. (2005). Ernst Cassirer and the philosophy of science. In G. Gutting (Ed.), *Continental philosophy of science* (pp. 71–83). Oxford: Oxford University Press, 2005.

Friedman, M. (2008). Ernst Cassirer and Thomas Kuhn. The neo-Kantian tradition in history and philosophy of science. *Philosophical Forum, 39*, 239–252.

Gawronsky, D. (1910). *Das Urteil der Realität und seine mathematischen Voraussetzungen*. Marburg: Dissertation, 1910.

Giovanelli, M. (2015). 'Das Problem in ein Postulat verwandeln': Cassirer und Einsteins Unterscheidung von konstruktiven und Prinzipien-Theorien. In M. Neuber (Ed.), *Husserl, Cassirer, Schlick - 'Wissenschaftliche Philosophie' im Spannungsfeld von Phänomenologie, Neukantianismus und logischem Empirismus* (pp. 150–175). Vienna: Springer, 2015.

Giovanelli, M. (2016). 'Zwei Bedeutungen des Apriori'. Hermann Cohens Unterscheidung zwischen metaphysischem und transzendentalem *a priori* und die Vorgeschichte des Relativierten *a priori*. In C. Damböck (Ed.), *Philosophie und Wissenschaft bei Hermann Cohen/Philosophy and Science in Hermann Cohen* (pp. 177–203). Dordrecht: Springer, 2016.

Giovanelli, M. (2020). Like thermodynamics before Boltzmann'. On Einstein's distinction between constructive and principle theories. *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics, 71*, 118–157.

von Goethe, J. W., & Friedrich Zelter, C. (1832). *Briefwechsel zwischen Goethe und Zelter in den Jahren 1799 bis 1832*. In L. Geiger (Ed.). Leipzig: Reclam, 1832.

Hahn, H. (1933). *Logik, Mathematik und Naturerkennen*. Wien: Gerold, 1933.

Hamilton, W. R. (1834). On a general method in dynamics; by which the study of the motions of all free systems of attracting or repelling points is reduced to the search and differentiation of one central relation, or characteristic function. *Philosophical Transactions of the Royal Society of London, 124*, 247–308.

Hamilton, W. R. (1835). Second essay on a general method in dynamics. *Philosophical Transactions of the Royal Society of London, 125*, 95–144.

Hansson, J., & Nordin, S. (2006). *Ernst Cassirer. The Swedish years*. Bern: Lang, 2006.

Heis, J. (2014). Realism, functions, and the a priori. Ernst Cassirer's philosophy of science. *Studies in History and Philosophy of Science, 48*, 10–19.

Helmholtz, H. von (1847). *Über die Erhaltung der Kraft. Eine physikalische Abhandlung*. Berlin: Reimer, 1847.

Helmholtz, H. von (1884). Principien der Statik monocyclischer Systeme. *Journal fur die reine und angewandte Mathematik, 97*, 111–140; 317–336.

Helmholtz, H. von (1887a). Zur Geschichte des Prinzips der kleinsten Aktion. *Sitzungsberichte der Berliner Akademie*.

Helmholtz, H. von (1887b). Ueber die physikalische Bedeutung des Princips der kleinsten Wirkung. *Journal für die Reine und Angewandte Mathematik, 100*, 137–166.

Hermann, G. (1935). Die naturphilosophischen Grundlagen der Quantenmechanik. *Abhandlungen der Fries'schen Schule, 6*, 69–152.

Hilbert, D. (1915). Grundlagen der Physik. Erste Mitteilung, vorgelegt in der Sitzung vom 20. November 1915. *Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse, I*, 395–407. Repr. in Hilbert, 2009, pp. 28–46.

Hilbert, D. (2009). *David Hilbert's lectures on the foundations of physics 1915-1927*. In T. Sauer, & U. Majer (Eds.), Berlin/Heidelberg: Springer, 2009.

Kneser, A. (1928). *Das Prinzip der kleinsten Wirkung von Leibniz bis zur Gegenwart*. Leipzig/Berlin: Teubner, 1928.

Kuhn, T. (1962). *The structure of scientific revolutions*. Chicago: University of Chicago Press, 1962.

Lange, M. (2009). *Laws and lawmakers. Science, metaphysics, and the laws of nature*. Oxford/New York: Oxford University Press, 2009.

Lange, M. (2016). *Because without cause. Non-causal explanations in science and mathematics*. New York: Oxford University Press, 2016.

Lorentz, H. A. (1900). Elektromagnetische Theorien physikalischer Erscheinungen. Rektoratsrege, gehalten zur Feier des 325. Jahrestages der Universität Leyden am 8. Februar 1900. *Physikalische Zeitschrift, 1*, 498–501; 514–519.

Mach, E. (1904). *Die Mechanik in ihrer Entwickelung. Historisch-Kritisch dargestellt*. Leipzig: Brockhaus, 1904.

Mayerhofer, B. (1994). Das Prinzip der kleinsten Wirkung bei Hermann von Helmholtz. *Centaurus, 37*, 304–320.

Meyerson, É. (1933). *Réel et déterminisme dans la physique quantique. With a forew. by Louis de Broglie*. Paris: Hermann, 1933.

Mormann, T. (2012). A virtual debate in exil. Cassirer and the Vienna Circle after 1933. In M. Friedman (Ed.)*, 2012. Rudolf Carnap and the legacy of logical empiricism* (pp. 149–167). Dordrecht: Springer.

Mormann, T. (2015). Mathematics to quantum mechanics. On the conceptual unity of Cassirer's philosophy of science (1907-1937). In J. T. Friedman, & S. Luft (Eds.)*, 2015. The Philosophy of Ernst Cassirer. A novel assessment* (pp. 31–63). Berlin/Boston: De Gruyter.

Mormann, T. (2021). Two kindred neo-Kantian philosophies of science. Pap's the a priori in physical theory and Cassirer's 'Determinism and indeterminism in modern physics. *Journal of transcendental philosophy, 2*, 47–69.

Mosengeil, K.von (1906). *Theorie der stationären Strahlung in einem gleichförmig bewegten Hohlraum*. PhD thesis. Friedrich-Wilhelms-Universität zu Berlin, 1906.

Natorp, P. (1912). Kant und die Marburger Schule. *Kant-Studien, 17*, 193–221.

Petzoldt, J. (1891). *Maxima, Minima und Ökonomie*. Schnuphase: Altenburg, 1891.

Planck, M. (1907). Zur Dynamik bewegter Systeme. *Gesammtsitzung, 13*. Juni 1907. Sitzungsberichte der Preußischen Akademie der Wissenschaften, 542–570. Repr. in Planck, 1958, Vol. 2, Doc. 64.

Planck, M. (1915). Das Prinzip der kleinsten Wirkung. Ed. by Paul Hinneberg. Leipzig: Teubner, 1915, Part 3, Sec. 1. In *Die Kultur der Gegenwart. Ihre Entwicklung und ihre Ziele* (Vol. 3, pp. 692–702). Emil Warburg. Physik.

Planck, M. (1932). *Der Kausalbegriff in der Physik*. Leipzig: Barth, 1932.

Planck, M. (1958). Physikalische Abhandlungen und Vorträge. In *Verband Deutscher Physikalischer Gesellschaften and Max-Planck-Gesellschaft zur Förderung der Wissenschaften, 3 vols*Braunschweig: Vieweg, 1958.

Poincaré, H. (1904). L'état actuel et l'avenir de la physique mathématique." Conférence lue le 24 Septembre 1904 au Congrés d'art et de Science de Saint-Louis. *Bulletin des Sciences Mathematiques, 28*, 302–324. repr. in: La valeur de la science. Paris: Flammarion, 1905.

Popper, K. R. (1935). *Logik der Forschung. Zur Erkenntnistheorie der modernen Naturwissenschaft*. Berlin: Springer, 1935.

Post, H. R. (1971). Correspondence, invariance and heuristics: In praise of conservative induction. *Studies In History and Philosophy of Science Part A, 2*, 213–255.

Quine, W. Van O., & Ullian, J. S. (1970). *The web of belief*. New York: Random House, 1970.

Reichenbach, H. (1920). *Relativitätstheorie und Erkenntnis apriori* (pp. 191–332). Berlin: Springer, 1920. Repr. in Reichenbach, 1977, Vol. 3.

Reichenbach, H. (1977). *Gesammelte Werke in 9 Bänden*. In A. Kamlah, & M. Reichenbach (Eds.). Braunschweig/Wiesbaden: Vieweg, 1977.

Richardson, A. W. (2009). Ernst Cassirer and Michael Friedman. Kantian or Hegelian dynamics of reason? In M. Dickson, & M. Domski (Eds.), *Discourse on a new method. Reinvigorating the marriage of history and philosophy of science* (pp. 279–293). Chicago: Open Court, 2009.

Richardson, A. W. (2015). Holism and the constitution of 'Experience in its Entirety' Cassirer contra Quine on the lessons of Duhem. In J. T. Friedman, & S. Luft (Eds.)*, 2015. The philosophy of Ernst Cassirer. A novel assessment* (pp. 103–122). Berlin/Boston: De Gruyter.

Rovelli, C. (1997). Halfway through the woods. In J. Earman, & J. D. Norton (Eds.), *The cosmos of science. Essays of exploration* (pp. 180–223). Pittsburgh: University of Pittsburgh Press, 1997.

Rovelli, C. (2009). Quantum spacetime. What do we know? In C. Callender, & N. Huggett (Eds.), *Physics Meets Philosophy at the Planck Scale: Contemporary Theories in Quantum Gravity* (pp. 101–122). Cambridge: Cambridge University Press.

Rovelli, C. (2015). Aristotle's physics. A physicist's look. *Journal of the American Philosophical Association, 1*, 23–40.

Russell, B. (1903). *The principles of mathematics*. Cambridge: Cambridge University Press, 1903.

Ryckman, T. (2005). *The reign of relativity. Philosophy in physics 1915-1925*. Oxford/New York: Oxford University Press, 2005.

Ryckman, T. (2015). A retrospective view of 'Determinism and indeterminism in modern physics'. In J. T. Friedman, & S. Luft (Eds.), *The philosophy of Ernst Cassirer. A novel assessment* (pp. 65–102). Berlin/Boston: De Gruyter, 2015.

Ryckman, T. (2018). Cassirer and Dirac on the symbolic method in quantum mechanics. A confluence of opposites. *Journal for the history of analytical philosophy, 6*, 213–243.

Ryckman, T. (2021). Quantum mechanics as the ultimate mode of symbol formation. In J. T. Friedman, & S. Luft (Eds.), *The philosophy of Ernst Cassirer. A novel assessment* (pp. 89–108). Berlin/Boston: De Gruyter, 2021.

Schlick, M. (1921). Kritizistische oder empiristische Deutung der neuen Physik? Bemerkungen zu Ernst Cassirers Buch: *Zur Einsteinschen Relativitätstheorie*. *Kant-Studien, 26*, 96–111. Repr. in Schlick 2006, Vol. 1/5, pp. 223–250.

Schlick, M. (1929). Erkenntnistheorie und moderne Physik. *Scientia, 45*. Repr. in Schlick 2006, Vol. 1/6, pp. 161–174.

Schlick, M. (1931). Die Kausalität in der gegenwärtigen Physik. *Naturwissenschaften, 19*, 145–162. Repr. in Schlick 2006, I/6, pp. 237–292.

Schlick, M. (1932). Causality in Everyday Life and in Recent Science. *University of California Publications in Philosophy, 15*, 99–125. Repr. in Schlick 2006, I/6, pp. 419–445.

Schlick, M. (1936). Quantentheorie und Erkennbarkeit der Natur. *Erkenntnis, 6*, 317–326. Repr. in Schlick 2006, Vol. 1/6, pp. 807–822.

Schlick, M. (2006). In F. Stadler, & H. Jürgen Wendel (Eds.), *Gesamtausgabe*. Berlin: Springer, 2006.

Schrödinger, E. (1932). *Über Indeterminismus in der Physik. Ist die Naturwissenschaft milieubedingt? Zwei Vorträge zur Kritik der naturwissenschaftlichen Erkenntnis*. Leipzig: Barth, 1932.

Seth, S. (2010). *Crafting the quantum. Arnold Sommerfeld and the practice of theory, 1890–1926*. Cambridge: MIT Press, 2010.

Stöltzner, M. (2003). The principle of least action as the logical empiricist's shibboleth. *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics, 34*, 285–318.

Stöltzner, M. (2009). Can the principle of least action be considered a relativized a priori? In M. Bitbol, P. Kerszberg, & J. Petitot (Eds.), *Constituting objectivity. Transcendental perspectives on modern physics* (pp. 215–228). Dordrecht: Springer, 2009.

Stump, D. (2015). *Conceptual change and the philosophy of science. Alternative interpretations of the a priori*. New York: Routledge, 2015.

Weber, W. E. (1848). Elektrodynamische Maassbestimmungen. *Annalen der Physik, 73*, 193–240.

Weizsäcker, C. F.von (1937). Review of Cassirer, 'Determinismus und Indeterminismus' [Cassirer, 1936]. *Physikalische Zeitschrift, 38*, 860–861.