A Model for the Analysis of the Interactions in a Digital Learning Environment During Mathematical Activities

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A model for the analysis of the interactions in a Digital Learning Environment during mathematical activities

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Abstract. Learning environments are the core of education and the recent digitalization of the learning processes brought the diffusion of Digital Learning Environments (DLE). In this paper we propose a definition and a conceptualization of DLEs and provide a model to analyze the interactions occurring among the members of a DLE during Mathematics activities. The conceptualization is rooted on results from the literature about digital education. The model is applied to analyze the activity of 299 8th grade students when tackling two tasks of algebraic modelling with automatic formative assessment in two different modalities: online and classroom-based. Two episodes have been selected and discussed, with the aim of identifying the interactions through which formative assessment strategies emerge. The results show that all the Black and Wiliam's strategies of formative assessment are developed during the activities through the interactions among the members of the learning community or with the technologies. This study can be a basis for extending the research in the learning analytics direction, to analyze the interactions during formative activities in large online courses.

Keywords: Automatic Formative Assessment, Digital Learning Environment, Interactions, Mathematics Education, Online Learning.

1 Introduction

The learning environment a key element of education. According to Wilson [1], a learning environment is "*a place where learning is fostered and supported*". It includes at least two elements: the learner, and a "*setting or space wherein the learner acts, using tools and devices, collecting and interpreting information, interacting perhaps with others, etc*." [1]. Accepting this definition, we agree that a learning environment is not limited to the physical place but includes also at least a learner. The traditional learning environment that everyone knows is the classroom, where the teacher teaches, students learn, individually or with their peers, using tools such as paper, pen, and a blackboard. The diffusion of technology transformed this traditional learning environment by adding digital tools, as tablets or computers, and the IWB (Interactive White Board). Besides equipping physical places with technologies, the technological revolution brought to the creation of a new learning environment, situated in a non-physical dimension: that of the Internet, accessible from everywhere via computers, tablets, or even smartphones. This is the essence of the "Digital Learning Environment" (DLE); besides the learner and a setting, which can be virtual, a device is needed to access the activities.

Today, due to the COVID-19 pandemic, online platforms have known increasing popularity, supporting smart-schooling, and class attendance from home [2, 3]. They have been invaluable to permit students from all social and cultural backgrounds to carry on their education. The interest in DLEs in the research has increased accordingly, making different theories and models come to life [4, 5].

This paper is an extension of the paper "Analyzing interactions in Automatic Formative Assessment activities for Mathematics in a Digital Learning Environment" [6] presented by the authors at the 13th International Conference on Computer Supported Education (CSEDU 2021), which intends to contribute to the discussion about the essence of DLEs providing a definition and a model for analyzing learning interactions in a DLE. In this revised and extended paper, the theoretical framework, which is the core of the paper, has been developed further, including a review of various studies on DLEs and discussing a proposal of definition. Particular characteristics of DLEs for Mathematics are considered, based on theories on formative assessment and Automatic Formative Assessment (AFA). Then, a model for the interactions among the members of a DLE is proposed, to highlight the interactions during AFA activities. Moreover, the results have been expanded, adding the analysis of a second AFA activity for grade 8 Mathematics in an online context, so that the model is applied in two different contexts (online and classroom-based teaching). The discussion about the kinds of interactions that can support formative assessment strategies is consequently enriched. The conclusions suggest how these findings could be used in learning analytics research.

2 Theoretical framework

2.1 Definition of Digital Learning Environment

The concept of "Digital Learning Environment" has a long history, and it has known several developments and many different names over the years. Suhonen [7] defined Digital Learning Environments as "*technical solutions for supporting learning, teaching and studying activities.*" Some years before, Abdelraheem [8] spoke about "Computerized Learning Environments" (CLEs), which are "*systems that provide rich databases, tools, and resources to support learning and information seeking and retrieval, as well as individual decision making.*" Abdelraheem's definition is more detailed than Suhonen's one, but the essence is similar: disseminating learning materials through the Internet. CLEs, or DLEs, emphasize empowerment through metaknowledge, which individuals invoke and refine while attempting to use their learning tasks. Other authors use the term "Online Learning Environments" as Khan [9], who defined them as "*hypermedia based instructional [systems], which utilizes the attributes and resources of the World Wide Web to create a meaningful learning environment where learning is fostered and supported*". Other scholars speak about "Virtual Learning Environments" referring to a particular type of Learning Environment where "*students interact primarily with other networked participants, and with widely disseminated information* *tools*”[10]. Here the interaction among learners is emphasized, and it is considered the most powerful key to learning.

The common factor among all these definitions is the use of the Internet and its tools to provide an environment where learning is supported, generally represented by a Learning Management System (LMS). An LMS, according to Watson and Watson [11], is “*the infrastructure that delivers and manages instructional content, identifies and assesses individual and organizational learning or training goals, tracks the progress towards meeting those goals, and collects and presents data for supervising the learning process of an organization as a whole*.” While similar environments are mainly used to support online educational processes, we are convinced and have proof of the fact that web-based platforms can also be successfully adopted in classroom-based settings: in our conception, DLEs should not only be confined to distance education [12–15].

More recently, many authors have developed an interest in conceptualizing digital learning environments as ecosystems, borrowing the term from ecology [2, 16–19]. According to Encyclopaedia Britannica (www.britannica.com), an ecosystem is “*a complex of living organisms, their physical environment, and all their interrelationships in a particular unit of space*.” The natural ecosystem, constituted by a biological community in a physical environment, is the fundamental example; however, this definition can be applied to any domain, even artificial environments, by specifying the living community, environment, and space unit.

There are several models of learning or e-learning ecosystems in the literature, which vary for the components included based on the theoretical assumptions considered. In general, they contemplate individuals, computer-based agents, communities, and organizations in a network of relations and exchanges of data that supports the co-evolutions and adaptations of the components themselves [17].

Following this trend, in this paper, we chose to use the term “Digital Learning Environment” to indicate a learning ecosystem in which teaching, learning, and the development of competence are fostered in classroom-based, online or blended settings. It is composed of a human component, a technological component, and the interrelations between the two.

The human component consists of one or more learning communities whose members can be: teachers or tutors, students or learners, and their peers, the administrators of the online environment.

The technological component includes:

- a Learning Management System, together with software, other tools, and integrations which accomplish specific purposes of learning (such as web-conference tools, assessment tools, sector-specific software, and many others);
- activities and resources, static or interactive, which can be used in synchronous or asynchronous modality;
- technological devices through which the learning community has access to the online environment (such as smartphones, computers, tablets, IWB);
- systems and tools for collecting and recording data and tracking the community's activities related to learning (such as sensors, eye-trackers, video cameras).

The interrelations between the two components can be:

- the interactions and learning processes activated within the community and through the use of the technologies, including dialogues between the members of the learning community, human-technology interactions, and so on;
- pedagogies and methodologies through which the learning environment is designed.

Figure 1 shows a graphical representation of the components of a DLE. The community is in the middle in a human-centered approach to learning. In the ecology metaphor, it is the complex of living organisms, while the technological component surrounds the community, as the physical environment. The arrows linking the community and the technologies represent the learning processes as well as the pedagogies and methodologies used to design the learning materials and interpret data from the digital environment. They are double-ended to indicate the reciprocal relationships between the two components, which bring to the development of all the parts.

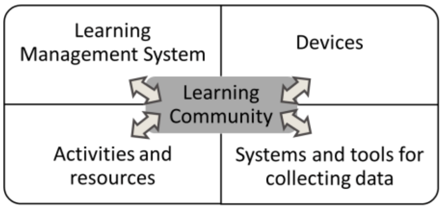

Fig. 1. Schema of the components of a DLE.

Independently of the fact that the DLSs are based on a web-based platform, teaching and learning can occur in one of the following modalities:

- face to face, in the classroom or a computer lab, with students working autonomously or in groups through digital devices, or solving tasks displayed on the IWB with paper and pen or other tools;
- entirely online, using the DLE as the only learning environment in online courses or MOOCs;
- in a blended approach, using online activities to integrate classroom work, such as asking students to complete them as homework.

These three modalities can be adapted to different situations, grades, aims, and needs. For example, the face-to-face modality can be suitable with students of the lowest grades and in scholastic situations where the classroom work is predominant [15]. The blended approach can offer useful support to the face-to-face lessons at secondary school or university [12, 20]. Online courses are generally used for training and professional courses, university courses, or learning in sparse communities, where face-to-face meetings are difficult to organize [8, 21, 22].

In this conceptualization, the DLE is not limited to technological artifacts, even if they play a crucial role. The learning community takes a prominent place: it can include, according to the kind of DLE, students and peers, teachers and tutors (who are facilitators of learning activities), designers of educational materials, and administrators of the digital environment. There can also be more communities involved or a community of communities: it happens, as an example, in the Italian National Problem Posing and Solving Project [23] where, hosted in an integrated LMS, there are many communities of students, one for each class participating to the project, and the community of all the classes' teachers [24]. In this case, the students' communities are based on learning and teaching intentions, while the teachers' community pursues the development of competence related to teaching with innovative technologies and didactic methodologies. The teachers' work in their students' communities allows them to practice the competences which are fostered through training courses in the teachers' community. The students' communities work in a blended modality, and their online activities are mainly accomplished asynchronously, while the teachers participate in online synchronous and asynchronous training [25, 26].

This definition of DLE does not disagree with the other definitions collected from the literature. However, it is more comprehensive: it is not limited to the web-based platform, which conveys the activities and is a relevant and essential part of a DLE; it also includes a “human” part.

The use of these technologies, such as web-based platforms, assessment tools, and other systems such as sensors or eye-trackers, allows for collecting, recording, and using learning data. These data can be elaborated within the DLE to provide information useful to make decisions and take action. In the following paragraphs, we will explain how these data can be used to improve learning, teaching, and the development of competences.

There is an in-depth discussion on the real effectiveness of DLEs (and their synonyms) that involves many researchers. For instance, in the paper “Media will never influence learning”, Clark [27] claims that the use of technologies, per se, is not more effective than traditional learning unless a learning theory supports it. The chosen learning theory should be coherent with the aims of the materials or the course and should guide the materials' design. Clark and Mayer [28] analyzed the effect sizes gained in several studies that compare digital and traditional education. The average effect size is not much different from zero, meaning that digital tools are not better than paper and pen. However, they noticed that there are many cases where the effect size is considerably large: this means that digital technologies have great potential. When they are used following suitable principles, they can make a difference in education.

2.2 Formative Assessment

Formative assessment is one of the key principles which, according to the majority of scholars, should be included in the design of a learning environment, being it physical or virtual [29, 30]. In this study, we refer to Black and Wiliam's definition and framework of formative assessment [31]. According to them, “*a practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they* *would have taken in the absence of the evidence that was elicited*". They identified three agents that are principally activated during formative practices: the teacher, the student, and peers. Moreover, they theorized five key strategies enacted by the three agents during the three different processes of instruction:

- KS1: clarifying and sharing learning intentions and criteria for success;
- KS2: engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
- KS3: providing feedback that moves learners forward;
- KS4: activating students as instructional resources; and
- KS5: activating students as the owners of their own learning.

2.3 DLEs for Mathematics

In this paper, we consider particular DLEs for working with Mathematics through suitable technologies and methodologies. The LMS that we use is based on a Moodle platform and it is integrated with an Advanced Computing Environment (ACE), which is a system for doing Mathematics through symbolic computations, geometric visualization, and embedding of interactive components [32], and with an Automatic Assessment System based on the ACE engine [33]. In particular, we chose Maple ACE and Moebius AAS. Through this system, we create interactive activities for Mathematics based on problem solving and Automatic Formative Assessment (AFA), which are the main methodologies used in the DLE, and that we have better defined and characterized in previous works [30, 34]. In detail, the characteristics of the Mathematics activities that we propose are the following:

- availability of the activities for a self-paced use, allowing multiple attempts;
- algorithm-based questions and answers, so that at each attempt different numbers, formulas, graphs, and texts are displayed, computed on the base of random parameters;
- open mathematical answers, accepted for its Mathematical equivalence to the correct one;
- immediate feedback, returned when the student is still focused on the task;
- interactive feedback, which provides a sample of a correct solving process for the task, which students can follow step-by-step;
- contextualization in real-life or other relevant contexts.

2.4 Modelling interactions in a Digital Learning Environment

The technological apparatus of a DLE, particularly when the LMS is integrated with tools for automatic assessment, has a mediating role in the learning processes. In particular, we can identify the following functions through which it can support the learning activities [34]:

- **creating and managing**: supporting the design, creation, editing, and managing of resources (e.g., interactive files, theoretical lessons, glossaries, videos), activities

(e.g., tests, chats for synchronous discussions, forums for asynchronous discussions, questionnaires, submission of tasks) and more generally of the learning environment by teachers, but also by students or peers;
- **delivering and displaying**: making the materials and activities available to the users;
- **collecting**: collecting all the quantitative and qualitative data concerning the actions of the students (such as movements and dialogues), the use of the materials (for example, if a resource has been viewed or not, how many times and how long), and the participation in the activities (such as given answers, forum interventions, number of tasks delivered, number of times a test has been performed, evaluations achieved);
- **analyzing and elaborating**: analyzing and elaborating all the data collected through the technologies related to teaching, learning, and the development of competences;
- **providing feedback**: giving the students feedback on the activity carried out and providing teachers, as well as students, with the elaboration of learning data.

To schematize these functions, we propose the diagram shown in Figure 2. The external cycle represents the five functions; the black dashed arrows represent how data are exchanged within the DLE through automatic processes. The technologies of a DLE, to accomplish one function, uses the data or the outputs resulting from the previous one: the learning materials, created through the LMS or other sector-specific software through the "creating and managing function", are displayed via devices through the "delivering and displaying function". Information about the students' activities is collected by the LMS, other software, or tools through the "collecting function" and it is analyzed by these systems, which may use mathematical engines, learning analytics techniques, algorithms of machine learning, or artificial intelligence, through the "analyzing and elaborating" function. The results of the analysis are feedback in the sense of Hattie's definition (i.e., "information provided by an agent regarding aspects of one's performance or understanding") [35]. They can be returned to students and teachers through the "providing feedback" function, and they can be used to create new activities or edit the existing ones. This circle represents a perfect adaptive system from the technological perspective [36–38].

In a human-centered approach, at the center of the DLE, there is the learning community, composed of students, teachers, and peers (who are the agents in the Black and Wiliam's theory of formative assessment [31]): they can interact with the DLE through its functions receiving and sending information. The blue dotted arrows represent the interactions between the community and the digital systems that occur through human actions, such as reading, receiving, inserting, providing, digiting. For example, the teacher, or designer, or tutor can create the digital activities through the "creating and managing" functions of the DLE; tasks are displayed ("delivering and displaying" function) and received, seen, or read by the students through some device. The students, individually or with their peers, can insert their answers or work. The technology collects them through the "collecting" function. The system analyzes the students' answers and provides feedback ("providing feedback" function) returned to the student. Simultaneously, the information about the students' activity is returned to the teacher through the "providing feedback" function; the teacher can use it to edit the existing task or create new ones. The continuous double-ended orange arrows represent the interactions among students, teachers, and peers, which in classroom-based settings can be verbal while in online settings can be mediated by the technology.

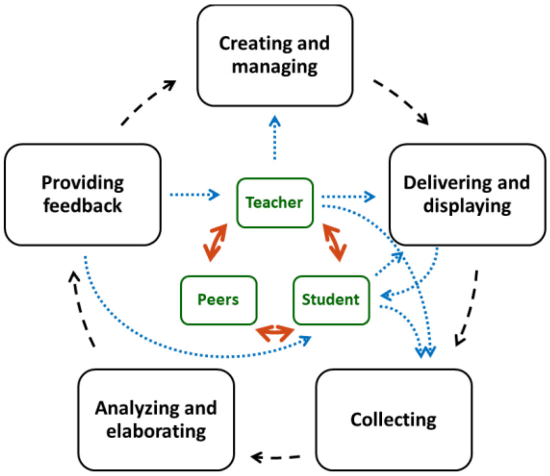

Fig. 2. Diagram of the interactions among the components of a DLE through the functions of the technology [6].

The diagram in Figure 2 is a proposal of schematization of the interactions among the components of a DLE. It helps us understand how data are shared among the components of a DLE, elaborated, and used; for this reason, it can be useful in the perspective of learning analytics.

In the diagram, one can follow the arrows along close paths and identify particular situations. For example, we can focus on the upper-left side of the circle as in Figure 3 and describe the process of content creation by a teacher or an instructor: she receives information about the activities completed by the students and uses them for the creation of new ones. If we focus on the right part of the circle as in Figure 4, we can examine the process of the fruition of static digital resources: the teacher makes them available to the students through the delivering function, and the students read/observe/study them. If we consider the lower-left part of the diagram as in Figure 5, we have interactive activities: the student can insert answers in the system, they are automatically analyzed and feedback is returned to the student. Moreover, including or excluding interactions among peers, individual, and collaborative activities are identified. Disregarding the arrows linking the teacher, the student, and the peers, we have a scenario of individual and self-paced online learning.

In the end, the model that we have analyzed in this section allows us to identify some outcomes that the adoption of a similar DLE with AFA, through the functions previously shown, makes it possible to achieve:

- **to create an interactive learning environment**: all the materials for learning and assessment can be collected in a single environment and be accessible at any time.

They can activate the students who can be engaged in the navigation of the learning path, solve the tasks and receive feedback;

- **to support collaborative learning**, through specific activities, delivered to groups of students, which enhance the communication and sharing of materials, ideas, understanding;
- **to promote formative assessment**, by offering immediate feedback to students about their results, their knowledge and skills acquired, and their learning level. Feedback can also be returned to the teachers on the students' results and their activities, supporting decision-making.

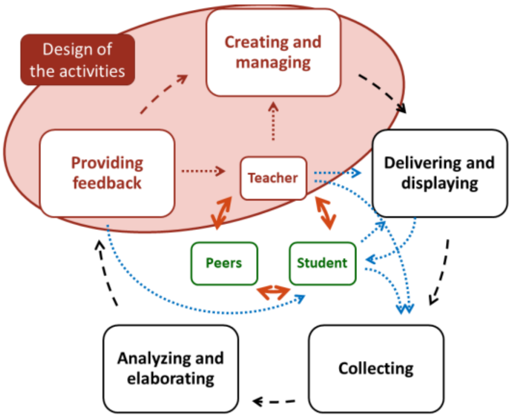

Fig. 3. Paths identified in the model corresponding to design activities

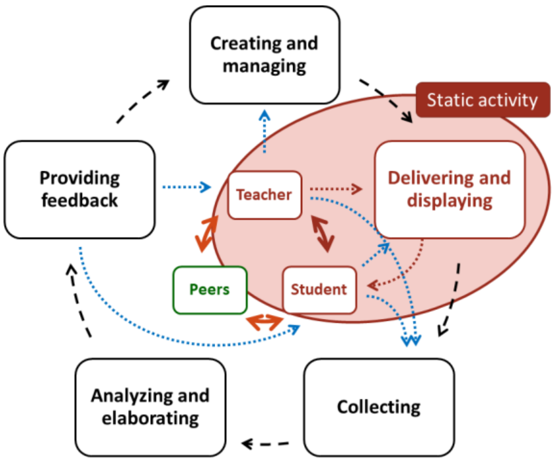

Fig. 4. Paths identified in the model corresponding to static activities.

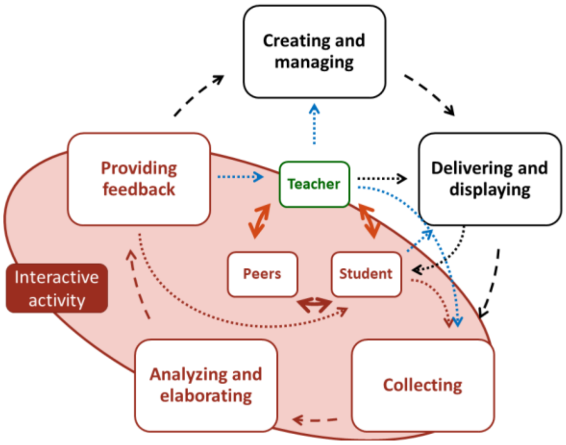

Fig. 5. Paths identified in the model corresponding to interactive activities.

The identification and classification of a DLE's functions can allow us to identify the interactions in a DLE, to analyze their nature and the contribution of technology that mediates them. The information gained is useful from a learning analytics perspective since it allows us to identify the role of data during the learning processes. Moreover, this model helps us identify the functions and outcomes of technology in learning processes. Individuating and separating functions and outcomes is necessary to have a clear frame and find causal connections, especially when analyzing large data quantities.

3 Methodology

In this study, we aim at showing how the diagram of the interactions among the components of a DLE can be used to model learning processes, and in particular to understand how formative assessment can be enacted in a DLE for Mathematics.

To this purpose, we analyzed two AFA activities, both concerning symbolic computations for students of grade 8, experimented in two different contexts: an online context and a classroom-based one. Both tasks ask students to formulate, represent, and compare different functions derived from several geometrical figures. The first one is shown in Figure 6. Firstly, students are asked to write a formula for the area of a geometrical figure whose lengths are given through a variable and write the formula in the blank space. The geometrical figure is not standard, but students can decompose it in simpler parts, such as rectangles or squares; they can use several decompositions to reach different forms of the same formula. Thanks to the Maple engine, the system can recognize the formula's correctness independently of its form, so every formula obtained through different reasoning is considered correct. Students have three attempts to provide the formula: they can self-correct mistakes and deepen their reasoning if a red cross appears. After three attempts, either correct or not, a second section appears, showing a table that students have to fill in with the values of the figure's area when the variable assumes specific values. In this part, students have to substitute in the formula different

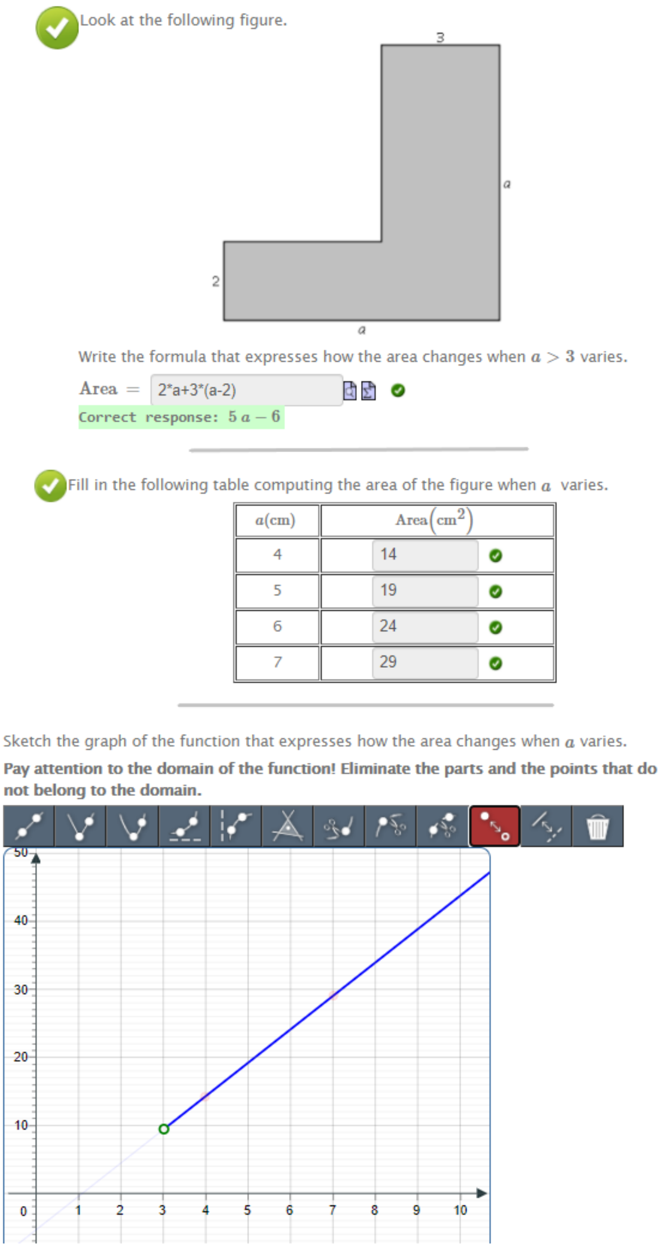

Fig. 6. First activity on symbolic computations

values of the variable; the purpose is to increase the awareness that variables are symbols that stand for numbers and that a formula represents a number, which has a particular meaning in a precise context. The table is a bridge to the last part of the question, where students are asked to sketch the graph of the function, using an interactive response area of Moebius Assessment that accepts answers within a fixed tolerance, without manual intervention. A second task repeats similar questions with the perimeter of the same figure. The activity is algorithmic: at every attempt the numbers, and consequently the figure and the function, change. This activity was tested in a bigger project involving 299 students of 13 classes of grade 8; they were asked to complete it from home individually.

The second activity (Figure 7) is similar; it involves a different figure and students are asked to find as many formulas as they can to express its area. In the first section, they have 3 attempts to write a formula for the area. In the second one, they are asked to fill other 4 response areas with different formulas expressing the same area. The intent is to make them explore the symbolic manipulation of an algebraic formulas through the geometric context, to confer a more concrete meaning to the technical operations. In the last part, students have to substitute the variable with a given value and compute the area. This activity was tested in a classroom-based setting in another experiment involving 97 students of 4 different classes of grade 8. In the classrooms there were the teacher and 2 researchers of the group; the students worked in pairs using a computer or a tablet. The work and discussions of some pairs of students were recorded through a video camera.

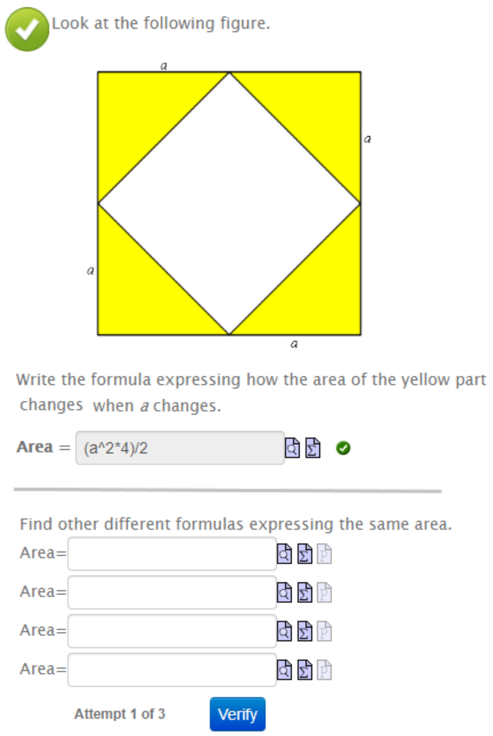

Fig. 7. Second activity on symbolic computations [6].

In both cases, data from the platform were analyzed through the diagram of the interactions in a DLE presented in the previous section. For the second activity, the video recordings were analyzed as well.

4 RESULTS

To analyze the interactions and the development of formative assessment strategies during individual automatic assessment activities in online modality (first activity), we used data from the AAS gradebook, where all the students' results are collected.

We analyzed the attempts made by all the students to the questions. We noticed that some students made more than one attempt at this assignment; we analyzed Erica's answers, who made 3 attempts, obtaining 64%, 83%, and 100%, respectively. In her first attempt, which lasted 19 minutes, she answered correctly to the formula's request, although she wrote the formula in a different form, without the simplifications; she obtained the simplified form as feedback. The attempt is shown in Figure 8. Then, she correctly used the formula to fill the table with the values of the area for the given values of the variable, but she failed the choice of the graph, choosing the graph of the function f(a)=5a instead of f(a)=5a-4. Then she moved on to the second question, related to the perimeter. The figure was similar to the previous one, with just a numerical value changing. At this point, she gave an incorrect answer to the first part, asking the formula for the perimeter. Erica received the correct formula as feedback and correctly filled the table with the perimeter's values for the given values of a, then she correctly sketched the graph of the function. The system allows one to sketch a line clicking on two passing points on a cartesian plane: she chose as points the first two points of the table. After that, she submitted the assignment. She obtained the percentage of correct answers (64%) as final feedback, together with all her answers paired with the correct ones, which she had already seen after each step during the test.

After 5 minutes, Erica started a new attempt, which lasted 11 minutes: the previous reasonings helped her accelerate the procedure. In the second attempt, she found a new figure, similar to the previous ones but having different numbers. She correctly answered the first two items; then, she failed the choice of the graph. In the second question, related to the perimeter, she correctly inserted the formula. We can notice that she did not write 4a as the answer, still not noticing the invariance for all the figures of the same kind. Instead, she added all the measures of the sizes expressed through the parameter a. Then she correctly answered the following parts, except for a number in the table, which probably was a distraction mistake.

She ran the third attempt just 2 minutes after finishing the second one, and it lasted 7 minutes, further reducing the duration. She answered correctly to all the items; in particular, she inserted 4a as perimeter, meaning that she understood the invariance of the perimeter for the class of figures.

Here, the human component of DLE is Erica, who is the only member of the learning community; the technological components are the LMS integrated with the AAS, the digital activity, and the computer used to access it.

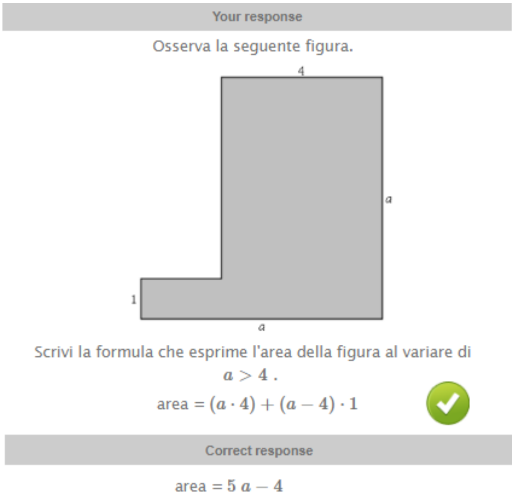

Fig. 8. Erica's answers to the first part of the first question in her first attempt. The question asks to write a formula for the area of the given figure. Erica’s answer on the left matches the correct one, on the right.

The interactions among these components can be schematized with repeated cycles of AFA, as shown in yellow in Figure 9. The assignment she opened was displayed through the “delivering and displaying” function of the technology. The AAS accepted the answers she inserted through the “collecting” function. The mathematical engine of the AAS processed the answers through its “analyzing and elaborating” function. Feedback was provided (“providing feedback” function) to the student, who could enter a new cycle, moving on to the following item, or running a new attempt. Erica was activated as the owner of her learning (KS5) every time she ran a new attempt through the “delivering and displaying” function and inserted her answers through the “collecting” function. The KS3 (“providing feedback which moves the learner forward”) was activated every time she received feedback.

We analyzed the videos realized during the second activity in the classrooms, to understand how the interactions among the components of the DLE changed and how the formative assessment strategies took place during a group activity. We choose some episodes which we considered most significant. Here, the learning community includes a class of students and a teacher; the digital activities are created in a LMS integrated with an AAS, and the devices used to access them are an Interactive White Board (IWB) and computers.

The first episode involves the teacher (T) who illustrates the task to the students (S). The teacher was at the IWB and was pointing at the figure shown.

T: Look at this figure. Write the formula which expresses how the area of this figure varies when *a* varies. That is, [pointing at one of the sizes of the yellow triangles] how long is this side?

S: *a*.

T: Well, you have to calculate the area of this figure using *a*. Those sides measure *a*. What does it mean? What is *a*?

S: A variable.

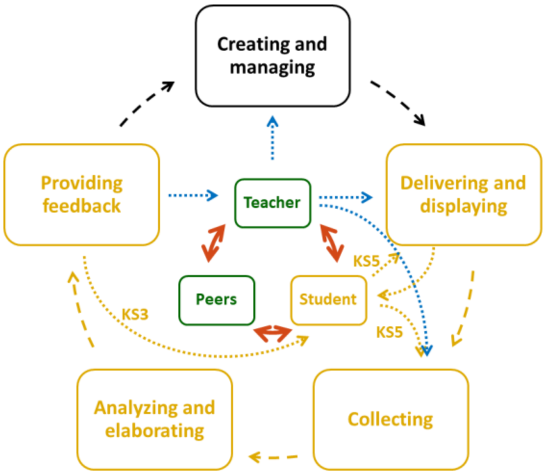

Fig. 9. Diagram of the strategies of formative assessment enacted during individual online activities.

In this excerpt, the teacher introduced the activity and explained to the students what their task was. The explanation took the form of a dialogue, as he engaged the students with questions to make sure that they were following the discourse. The teacher exploited the "delivering and displaying" function of the technology to display the task and, in particular, the figure; then, she interacted with the students. If we consider the diagram, we are in the right part; the parts of the model involved in this excerpt are shown in yellow in Figure 10. While explaining the tasks, she developed the KS1 "clarifying and sharing learning intentions and criteria for success". The KS2 "engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding" was accomplished during the phase of the creation of this activity by the researchers (that we can include in the "Teacher" subject of our analysis) through the "creating and managing" function of the technologies; it is also activated when the teacher asks questions to the class aimed at making students reason in the correct direction.

The second episode involves Marco (M) and Giulia (G), two students of medium level who were trying to solve the first part of the activity, working together. In the beginning, they observed the figure displayed on the screen of their computer and tried to understand the task.

M: We have to compute the area, but we don't have any data!

G: But we have *a*.

M: But *a* is not a number!

G: Ok, but we can compute the area using *a*.

M: Teacher, how can we compute the area without numbers? Can we use *a*?
T: Yes, it is like a generic number.
G: We have to write a formula using *a*, isn't it?
T: That's right.

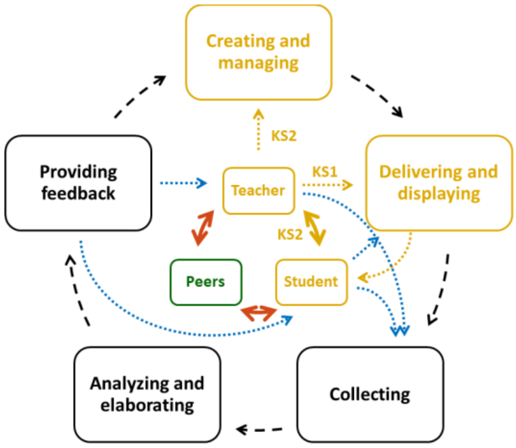

Fig. 10. Diagram of the formative assessment strategies enacted in the first episode of activity 2 through the interactions in the DLE [6].

The two students started reasoning together on the figure trying a way to compute the area. After about 15 minutes, they came up with a quite complex formula, built subtracting the area of the inner white square to that of the external square. They used the Pythagorean theorem to compute the length of the white square's side. They inserted the formula in the response area and the system returned a green tick with positive feedback. They passed to the following part, which asked them to find other 4 formulas for the same area. For the first two formulas, they reasoned algebraically, manipulating the original formula. For the other two, they reasoned geometrically, developing new ways to compute the area. The peer discussion allowed them to correct mistakes before entering the formulas in the response areas, so their answers were marked as correct at their first attempt.

In this episode, the students look at the task displayed on the screen through the "delivering and displaying" function, then interact among them discussing the task. They also interact with the teacher asking questions about their doubts. Then they insert their answers in the system, which collects them through the "collecting" function, analyzes them, and returns feedback. They repeat the same cycle several times. The students activate KS4 "activating students as instructional resources" when discussing in pair. KS5 "activating students as the owners of their own learning" is enacted when they insert their answers in the AAS, and KS3 is developed when they receive feedback from the AAS, but also by the teacher. The yellow parts in Figure 11 schematize the interactions that occurred in this episode and the formative assessment strategies developed.

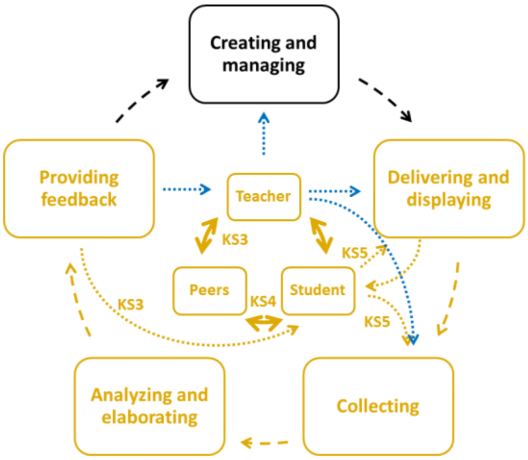

Fig. 11. Diagram of the formative assessment strategies enacted in the second episode of activity 2 through the interactions in the DLE [6].

5 DISCUSSION AND CONCLUSIONS

The episodes presented in the previous section help clarify how the interactions among the members of a DLE occur during AFA Mathematics activities in online and classroom-based settings. In the first case, the computerized interactive feedback has a key role in providing feedback and in engaging the student, who is working individually. The design of the activity enables KS3 and KS5, which keep Erica engaged with the task until its full comprehension, demonstrated by the repeated attempts and the correctness of the last one. In the second case, the main feedback is provided by the social interactions within the learning community and especially among peers; in fact, Marco and Giulia reasoned more time on the tasks and they tended to answer correctly at the first attempt. We would like to underline that also these interactions, which are mainly dialogues among the learning community, are part of the Digital Learning Environment as we have conceptualized it. Similar activities can lead to a deep understanding of fundamental Mathematics concepts; the technologies and methodologies used – in particular, an AAS based on a mathematical engine and AFA – supported the design and implementation of interesting activities for the development of mathematical competences.

The diagram used for the analyses helped clarify what functions of the technologies and through which kinds of interactions the formative assessment strategies are elicited in different situations. In particular, we can see that all the Black and Wiliam's strategies of formative assessment can be enacted through AFA activities, and all of them are identified and located along the arrows of our diagram, that is during the interactions among the human components of the DLE or between human and technological components. Thus, we can include a fourth agent in Black and Wiliam's framework: in the DLEs that we consider, the technology is also an agent of the formative assessment strategies, especially for providing feedback and engaging students (KS3 and KS5).

Through the analysis of the interactions among the members of these DLEs, we can also point out that the three outcomes mentioned in our framework are achieved. In particular:

- the analyzed learning environments are interactive, since students are actively engaged in the activities, they are stimulated to reflect and have the opportunity to achieve important understanding;
- the formative assessment is promoted by the activities, as all the 5 key strategies are enacted;
- collaboration among students is supported, especially in the classroom-based setting where students are asked to work together.

The diagram used in the analyses allows us to conceptualize the DLE as an ecosystem: we can see that the human and technological components are strictly related, and the interrelations among them cause the development of the learning community, in terms of learning processes, knowledge, and competences gained; but also an improvement of the learning activities on the base of the results obtained.

The analyses conducted in this study have a qualitative nature: they are aimed at showing how the schema of the interactions among the components of a DLE can be used to model formative assessment practices, especially when the AFA is adopted. However, they can be a starting point for extending the research about learning analytics for formative assessment. This model can be used to create a taxonomy of the interactions occurring in a DLE, identifying which support formative assessment or other learning processes. Since interactions in a DLE can be described using log data, this model can also be used with extensive learning data to identify the formative assessment strategies or other learning processes occurring in large online courses. This would allow us to identify the learning activities which are better related to the development of knowledge, abilities, and competences or the elicitation of interactions and engagement. The results of similar analyses could help adjust and improve the digital materials in online courses. Using other technologies and different learning methodologies to build suitable activities, this model of analysis could also be adapted to other disciplines.

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