Structural Vector Autoregressions

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Summary

Structural Vector Autoregressions (SVAR) represent a prominent class of time series models used for macroeconomic analysis. The model consists of a set of multivariate linear autoregressive equations characterizing the joint dynamics of economic variables. The residuals of these equations are combinations of the underlying structural economic shocks, assumed to be orthogonal to each other. Using a minimal set of restrictions these relations can be estimated, the so-called shock identification, and the variables can be expressed as linear functions of current and past structural shocks. The coefficients of these equations, called impulse response functions, represent the dynamic response of model variables to shocks. Several ways of identifying structural shocks have been proposed in the literature: short run restrictions, long run restrictions, sign restrictions, to mention a few.

SVAR models have been extensively employed to study the transmission mechanisms of macroeconomic shocks and test economic theories. Special attention has been paid to monetary and fiscal policy shocks, as well as other non-policy shocks like technology and financial shocks.

In recent years, many advances have been made both in terms of theory and empirical strategies. Several works have contributed to extend the standard model in order to incorporate new features like large information sets, nonlinearities, and time varying coefficients. New strategies to identify structural shocks have been designed and new methods to do inference have been introduced.

1 Macroeconomics and Reality

Understanding and measuring the effects of both policy actions and other non-policy shocks has been, and still is, the centerpiece of the design and the implementation of sound economic polices and the creation of economic theories to describe the functioning of modern economies. What are the sources of economic fluctuations? What are the transmission mechanisms of monetary and fiscal policy shocks? What are the effects of financial disruptions? This is just a sample of questions which have attracted the attention of macroeconomists for many decades, and the answer to these questions still represents, nowadays, the number one priority of the research agenda in macroeconomics.

The goal of this entry of the Encyclopedia is to present and discuss a class of models which have become very popular over the last four decades and which have been designed to address the type of questions above: *Structural Vector Autoregressions (SVAR).*

Structural Vector Autoregressions were introduced by Christopher Sims in a paper entitled ''Macroeconomics and Reality" appeared on Econometrica in January 1980. The paper establishes a before and after in the history of macroeconometric modeling and is at the root of any development in the field that has taken place since then.[1]

At the time the paper was published, there was a widespread dissatisfaction with the current state-of-the-art econometric techniques used to model macroeconomic

[1]See Canova (2007), Lütkepohl (2005), Hamilton (1994), Kilian and Lütkepohl (2017) for an extended textbook description of SVAR models, and Cooley and LeRoy (1985) for a critique.

dynamics. During the 70s, policy analysis was mainly performed with large-scale models which included hundreds of equations and variables. Several criticisms were raised to this class of models. First, no attention was payed on modeling agents' expectations. In particular the models were largely inconsistent with the new, at that time, and growing rational expectation paradigm. Second, model variables were ex-ante, arbitrarily and without any help of statistical models, categorized into exogenous and endogenous. Third, the models were full of arbitrary restrictions and assumptions about causal relationships among variables.

Sims (1980) introduces a new approach to model macroeconomic dynamics. The model consists of a set of linear multivariate autoregressive equations characterizing the joint dynamics of economic variables. The residuals of these equations are combinations of the underlying structural economic shocks, assumed to be orthogonal to each other. Using a minimal set of restrictions these relations can be estimated, the so-called shock identification, and the variables can be expressed as linear functions of current and past economic shocks. The coefficients of these equations are called impulse response functions and represent the reaction of model variables to structural economic shocks. Several ways of identifying structural shocks have been proposed in the literature: short run restrictions, long run restrictions, sign restrictions, just to mention a few of them.

Since the publication of Sims' paper, SVAR models have become the most prominent and popular tool for policy and macroeconomic analysis. Over the last three decades a myriad of papers has employed this class of models to study business cycles dynamics and the transmission mechanisms of both policy and non-policy shocks. Ramey (2016) provides an excellent and exhaustive review of methods and results concerning the identification of macroeconomic shocks. The study of the effects of monetary policy shocks has attracted a great deal of attention. Early contributions are Bernanke and Blinder (1992), Bernanke and Gertler (1995), Bernanke and Mihov (1998) Christiano, Eichenbaum and Evans (1996, 1999), Cochrane (1994), Leeper, Sims and Zha (1996), Leeper and Gordon (1992), Pagan and Robertson (1998), Rudebusch (1998), Sims and Zha (2006a), Strongin (1995). Important methodological advances were made in Romer and Romer (2004) and Uhlig (2005), and, more recently, Arias, Caldara and Rubio-Ramírez (2019), Gertler and Karadi (2015), Miranda-Agrippino and Ricco (2017), Jarocinsky and Karadi (2019), Caldara and Herbst (2019) .

Fiscal policy has also been a popular and widely studied topic. In particular, the response of private aggregate demand components to fiscal policy shocks has been the focus of many investigations. Blanchard and Perotti (2002) is the first paper using SVAR analysis to study the effects of government spending and tax shocks. The main finding is that government spending leads to a large increase in consumption. Similar results are obtained by Fatas and Mihov (2001), Galì, Lopez Salido, and Valles (2007), Mountford and Uhlig (2009), and Perotti (2005, 2007). On the contrary, Ramey and Shapiro (1998), find that consumption falls, implying a very smaller than one value for the government spending multiplier. Similar findings are obtained in Burnside, Eichenbaum, and Fisher (2004), Cavallo (2005), Edelberg, Eichenbaum, and Fisher

(1999), Eichenbaum and Fisher (2005) and Ramey (2011).

SVAR have also been a useful tool to test and assess competing economic theories. For instance, Galí (1999) employs SVAR analysis with long run restrictions à la Blanchard and Quah (1989) to investigate the effects of technology shocks on the economy. He finds that positive technology shocks reduce hours worked, an empirical finding which is at odds with RBC theory and in line with Neo-Keynesian models. Other works on the same topic are Shapiro and Watson (1988), King, Plosser and Watson (1991), Christiano, Eichenbaum and Vigfusson (2003, 2004), Francis and Ramey (2004), Uhlig (2004), Vigfusson (2004), Basu, Fernald and Kimball (2006), Beaudry and Portier (2006), Fisher (2006), Fernald (2007) and Francis, Owyang, Roush and Di Cecio (2010), Pagan and Pesaran (2008).

Over the last forty years, many advances have been made in terms of theory and empirical strategies, both within the frequentist and the Bayesian approach. Nonlinearities, time varying coefficients, large information sets are features which have been recently incorporated into SVAR models to study important features of modern economies. New strategies to identify structural shocks have been designed and new methods to do inference have been introduced.

This paper represents a short journey into this class of models. I will review the theoretical foundations of SVAR models and discuss several important applications. The remainder is organized as follows: Section 2 discusses the representations. Section 3 describes identification. Section 4 presents several extensions.

2 Representations

2.1 The Economy

I begin by discussing the class of economic models which is consistent with SVAR analysis. In the spirit of Frisch (1933) and Slutsky (1927), the macroeconomy is assumed to be the summation of agents' reactions to random economic disturbances of various types occurring at every point in time. Formally, let x_t be a n-dimensional stationary vector of time series with the following representation

$$x_t = F(L)u_t, \tag{1}$$

where u_t is a q-dimensional White Noise vector of orthonormal shocks and $F(L) = \sum_{k=0}^{\infty} F_k L^k$ is an $n \times q$ matrix of polynomials in the non-negative powers of the lag operator L. The vector u_t includes the structural economic shocks and the matrix $F(L)$ contains the impulse response functions, the object that captures agents' responses to economic shocks. I assume $q < n$, the number of shocks smaller than the number of variables.

Representation (1) is very general. A special case is when the representation is derived as the equilibrium solution of a Dynamic Stochastic General Equilibrium (DSGE) model. Consider the ABCD representation discussed in Fernández-Villaverde *et al.* (2007)

$$s_t = As_{t-1} + Bu_t \tag{2}$$

$$x_t = Cs_{t-1} + Du_t, \tag{3}$$

where s_t is an m-dimensional vector of state variables. Representation (1) can be derived as

$$x_t = [D + C(I - AL)^{-1}BL]u_t. \tag{4}$$

Given that x_t represents the whole economy, the number of variables n can be large. The typical situation is that the econometrician, in his empirical analysis, has to focus on a subset of variables. Let z_t be a s-dimensional subvector of x_t. Here we limit our attention to a subset of variables, but in principle z_t could contain combinations of the elements in x_t like principal components or averages. Moreover, we allow z_t to be driven only by a subset of shocks u_t^z of dimension q_z, with $q_z \leq q$. The structural economic representation for the subset of variables or linear combinations considered by the econometrician is given by

$$z_t = B(L)u_t^z \tag{5}$$

where $B(L) = \sum_{k=0}^{\infty} B_k L^k$ is the matrix of structural impulse response functions. Here I do not take any stand on the true underlying economic theory. Any model delivering representation (1) and (5), i.e. a linear (not necessarily square) MA, is compatible with the analysis discussed below.

2.2 Vector Autoregressions

Once the class of economic models under considerations has been clarified, let us focus on the Vector Autoregression representation of the vector z_t. By stationarity, the Wold representation of z_t exists, and is given by

$$z_t = C(L)\varepsilon_t \tag{6}$$

where $\varepsilon_t \sim WN(0, \Sigma)$ is the Wold shock, $C(L) = \sum_{k=0}^{\infty} C_k L^k$ represents the Wold impulse response functions and C_k, with $k = 1, 2, ...$, are matrices of coefficients. If there are no roots on the unit circle, then an infinite VAR representation exists, and can be well approximated with a finite-order VAR

$$A(L)z_t = \varepsilon_t \tag{7}$$

where $A(L) = I - A_1 L - ... - A_p L^p$ and A_j, $j = 1, ..., p$, are matrices of coefficients. Model (7) is a Vector Autoregression of order p, VAR(p).

The main goal of Structural Vector Autoregression (SVAR) analysis is to to recover the matrix of structural impulse response functions $B(L)$ (global identification) or some of the columns of $B(L)$ (partial identification) and the corresponding structural shocks u_t^z starting from equations (6) and (7). This, in a nutshell, is implemented in three steps. First, the matrix $A(L)$ and the innovation ε_t are estimated by least squares; second, $C(L)$ is obtained by inverting $A(L)$; third, the vector of structural shocks and structural impulse response functions are obtained as a linear combination of the vector of innovations and as combinations of the Wold impulse response functions respectively.

2.3 Estimation

Representation (7) (the VAR parameters A_i, $i = 1, ..., p$, and the covariance matrix Σ) can be consistently estimated with OLS equation by equation.[2] Using the estimates of the VAR parameters, the Wold impulse response functions can be obtained as follows. Consider the companion form representation

$$\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1} + \mathbf{e}_t$$

where $\mathbf{z}_t = [z_t' \; ... \; z_{t-p+1}' \;]'$, $\mathbf{A}\begin{pmatrix} & A \\ I_{s(p-1)} & 0_{s(p-1),s} \end{pmatrix}$, $A = [A_1 \; ... \; A_p]$ and $\mathbf{e}_t = [\varepsilon_t' \; \mathbf{0}']'$ and $\mathbf{0}$ is a $s(p-1) \times 1$ vector of zeros. The coefficients of the Wold representation are therefore

$$C_j = \left[\mathbf{A}^j\right]_{s,s} \tag{8}$$

where $\left[\mathbf{A}^j\right]_{s,s}$ represents the upper left $s \times s$ submatrix of $\mathbf{A}^j$. In this class of models, OLS represents also the conditional Maximum Likelihood Estimator (where the first p initial conditions, i.e. $\mathbf{z}_1$, are given). The OLS estimator of the VAR parameters in stationary models is biased but consistency and asymptotic Normality hold, see Hamilton (1994).[3]

[2]The variance covariance matrix can be estimated as $\hat{\Sigma} = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t'$ where T is number of datapoints used in the estimation.

[3]The OLS estimator is consistent even when the variables in the VAR are nonstationary, see Sims, Stock and Watson (1990). However in that case the long run impulse response functions are not estimated consistently as shown in Phillips (1998).

2.4 Lag selection

An important step in the empirical analysis is the selection of the order of the VAR, see Ivanov and Kilian (2005) for a review and a discussion. Typically, lag selection is based on information criteria like the Akaike Information Criteria (AIC), the Bayesian Information Criteria (AIC) or the Hannan and Quinn Criteria (HQC). The criteria select the number of lags which provides the best trade-off between model fit and parsimony. The information criteria take the form

$$IC(p) = \ln|\Sigma| + f(p)$$

where $f(p)$ is an increasing function of the number of lags p whose functional form varies depending on the specific criterion.[4] As p increases the first term of the right hand side of the above equation becomes smaller but the second increases. The optimal p is the one delivering the smallest $IC(p)$. BIC and HQC are consistent criteria, while AIC is not since tends to overestimate the true number of lags. However, the AIC might work better in small samples, and this is one of the reasons for his popularity.

An alternative strategy to select the number of lags is represented by the sequential testing procedure, either general-to-specific or specific-to-general. The former involves, starting from a maximal number of lags, a likelihood ratio tests of the null hypothesis that the coefficients in A_p are equal to 0. This procedure iterates backwards until the null is rejected. The latter involves a sequence LM test of the null

[4] Let T the sample size and let s the dmension of z_t. For the AIC, $f(p) = \frac{2}{T}s^2p$. For the BIC, $f(p) = \frac{\ln T}{T}s^2p$. For the HQC, $f(p) = \frac{2\ln\ln T}{T}s^2p$.

hypothesis of no serial correlation of the VAR residuals starting from a minimum number of lags. The procedure stops when the null is not rejected.

A third alternative is to choose the number of lags so to minimize the mean squared prediction error in out-of-sample forecasts.

2.5 Inference

There are several methods available to construct confidence bands for structural impulse response functions. One alternative is to use confidence intervals based on the asymptotic distribution of impulse response functions (Lütkepohl, 1990). The problem is that in small samples the distribution of the impulse response functions can be considerably different. A popular alternative, which does not rely on asymptotic theory, is represented by bootstrap techniques.[5] The idea behind these techniques is to characterize the distribution of the impulse responses by resampling the sample of VAR residuals. The standard bootstrap methodology was originally proposed by Runkle (1987) and relies on the following steps: (i) a VAR(p) is estimated by ordinary least squares using actual data; (ii) a new sequence of residuals of length T (the sample size) is re-sampled with replacement from the sample of estimated residuals; (iii) using the re-sampled residuals, the estimated coefficients and p initial observations, a new vector of time series is generated using the VAR equations; (iv) with the new data, the impulse response functions are estimated. Steps (ii)-(iv) are repeated a large

[5]Bayesian Monte Carlo Integration, Sims and Zha (1999) represents a third alternative within a Bayesian approach.

number of times and the bands are constructed by taking the percentiles of interest across all the realizations of impulse responses. Kilian (1998) proposes a bootstrap after bootstrap approach to correct for the OLS bias in the impulse response functions which considerably improves the accuracy of the confidence bands.

A potential weakness of the standard bootstrapping approach is that the errors have to be i.i.d. (although not necessarily normally distributed). An alternative procedure that doesn't rely on such a strong assumption is wild bootstrapping, see Goncalves and Kilian (2004). With this method the sample is computed as in the bootstrapping procedure above, but the residual is drawn differently. Each residual is multiplied by a scalar value from a $N(0, 1)$. The paper shows that this approach is preferable whenever there is conditional heteroskedasticity of any form.

In recent years, several papers have studied several interesting features related to inference and provided solutions to different problems. Kilian and Chang (2000) shows that confidence bands constructed with standard methods, especially asymptotic intervals and standard bootstrapping, have very low accuracy especially after horizon 16 (with quarterly data). The simulation is based on an estimated VAR model including standard US variables. Motivated by these results, several papers have developed methods based on on local-to-unity asymptotic theory to improve the reliability of confidence bands, see Wright (2000), Gospodinov (2004) and Pesanvento and Rossi (2006).

3 Identification

In this section I present and discuss several approaches that have been employed in the literature to obtain the structural shocks u_t^z and the structural impulse response functions $B(L)$ starting from the Wold innovations and impulse response functions.

3.1 Main assumptions and concepts

Estimating u_t^z and $B(L)$ is known as *VAR identification*. I make two assumptions: (i)$q_z = s$, i.e. the number of shocks is equal to the number of variables; (ii) representation (5) is invertible. The first assumption defines the typical case studied in the literature where the number of shocks and variables coincide. The second assumption is need for the structural analysis to be successful. The two assumptions imply that the structural shocks and impulse response functions are related to the reduced form model as follows:

$$\begin{aligned} B(L) &= C(L)B_0 \\ u_t &= B_0^{-1}\varepsilon_t \end{aligned}$$

where the matrix B_0 has to satisfy the condition $B_0B_0' = \Sigma$.

Given that the likelihood function of the model is invariant with respect to B_0, identification in the SVAR context amounts to fixing the elements of matrix B_0 under the covariance restrictions above. The restriction $B_0B_0' = \Sigma$ provides $s(s+1)/2$ restrictions on the elements of B_0. Therefore, there are other $s(s-1)/2$ free elements which have to be fixed. Typically, these remaining parameters are obtained using restrictions derived from economic theory. Below we will see several identification schemes, i.e. several ways of pinning down the matrix B_0.

As mentioned before, in many situations the researcher is only interested in estimating the effects of a single shock or a subset of shocks. This is the case of *partial identification*, and requires fixing only on a column or subset of columns of B_0 corresponding to the shocks of interests. In the case of identification of a single shock only $(s-1)$ restrictions have to be imposed.

Once the structural impulse response functions, $B(L)$, are available, the variance of the series can be decomposed in order to understand the relative importance of the identified shocks, the so-called *variance decomposition analysis*. The variance of variable j is given by

$$\mathrm{Var}(z_{jt}) = \sum_{i=1}^{q^z} \sum_{k=1}^{\infty} (B_k^{ji})^2$$

where B_k^{ji} refers to element j, i of B_k. The contribution of shock l to the variance of z_{jt} is given by $\sum_{k=1}^{\infty} (B_k^{jl})^2$ and the proportion of variance atributable to the shock is

$$\frac{\sum_{k=1}^{\infty} (B_k^{jl})^2}{\sum_{i=1}^{q^z} \sum_{k=1}^{\infty} (B_k^{jl})^2}.$$

The quantity

$$\frac{\sum_{k=1}^{K} (B_k^{jl})^2}{\sum_{i=1}^{q^z} \sum_{k=1}^{K} (B_k^{jl})^2}$$

represents the proportion of the forecast error variance of the K-step ahead forecast of z_{jt} attributable to shock l.

Another useful tool for the analysis is represented by the historical decomposition analysis. The idea is to generate counterfactual histories for the series of the model in absence of the estimated structural shock of interest, or in presence exclusively of the estimated shock. In practice, for the former, the counterfactual histories are simply generated using the VAR equations with the estimated coefficients, B_0, and the vector of structural shocks with the shock of interest replaced by zero. In the latter, the counterfactual series are generated as above but with the remaining structural shocks equal to zero.

3.2 A general approach

I first discuss a general approach to implement restrictions on B_0. Let S be the Cholesky factor of Σ (the variance covariance matrix of ε_t), i.e. the unique lower triangular matrix such that $SS' = \Sigma$. Then consider the representation

$$\begin{aligned} z_t &= C(L)SS^{-1}\epsilon_t \\ z_t &= D(L)\eta_t \end{aligned}$$

with $D(L) = C(L)S$ and $\eta_t = S^{-1}\epsilon_t$. The representation is called Cholesky representation, and the shocks have the property of being orthonormal, $E(\eta_t\eta_t') = I$.[6]

[6]Another way of obtaining an orthonormal representation is by means of the spectral decomposition. Let V be the vector of eigenvectors of Σ and Λ a diagonal matrix with the eigenvalues of Σ on the main diagonal. Then an orthogonal system is obtained as

$$\begin{aligned} z_t &= C(L)V\Lambda^{1/2}(V\Lambda^{1/2})^{-1}\epsilon_t \\ z_t &= G(L)\xi_t \end{aligned}$$

where $G(L) = C(L)V\Lambda^{1/2}$ and $\xi_t = (V\Lambda^{1/2})^{-1}\epsilon_t$

Now let H be an orthogonal matrix, i.e $HH' = I$, with the property that the representation

$$\begin{aligned} z_t &= D(L)HH'\eta_t \\ z_t &= B(L)u_t^z \end{aligned}$$

is the structural representation. The matrix H is the identifying matrix, i.e. the matrix such that $B_0 = SH$. Notice that, given the uniqueness of S, identification ultimately reduces to pinning down the orthogonal matrix H.

Orthogonal matrices can be found as rotation matrices using trigonometric functions, or Givens rotations. Consider for instance the case $s = 3$. An orthogonal matrix is given by

$$H = \begin{pmatrix} cos(\theta_1) & sin(\theta_1) & 0 \\ -sin(\theta_1) & cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cos(\theta_2) & 0 & sin(\theta_2) \\ 0 & 1 & 0 \\ -sin(\theta_2) & 0 & cos(\theta_2) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\theta_3) & sin(\theta_3) \\ 0 & -sin(\theta_3) & cos(\theta_3) \end{pmatrix}$$

In general the orthogonal matrix can be written as the product of $s(s-1)/2$ matrices

$$H = \prod_{i=1}^{s-1} \prod_{j=i+1}^{s} \tilde{H}^{ij}$$

where $\tilde{H}^{ij}$ is a matrix with sine and cosine on the i-th and j-th row and column, ones on the remaining diagonal elements and zeros in the remaining off-diagonal elements.

Orthogonal matrices can also be randomly drawn. Typically the elements of H are drawn in such a way that the columns of H represent the coordinates of orthogonal uniformly distributed points on the s-dimensional hypersphere. A common technique to draw H is represented by the QR decomposition. Let P be a $s \times s$ matrix whose elements are independent draws from a standardized normal. Then the decomposition decomposes the matrix P into two matrices, Q and R, such that $P = QR$, where Q is an orthogonal matrix and R an upper triangular matrix. Which of the two methods should be used to generate orthogonal matrices depends to a large extent on the type of application and identification scheme used.

Now let us consider the case of partial identification where only one shock in u_t^z, say u_{jt}^z, and the related impulse response functions have to be obtained. In this case only one column of the matrix $B(L)$, say $b(L)$, has to be obtained. This amounts to finding one column of the matrix H, say h, satisfying the unit norm condition and the identifying restrictions. The column of impulse response functions corresponding to the shock of interest is found as $b(L) = C(L)Sh$ and the structural shock is obtained as impulse response functions will be found as $u_{jt}^z = h'S^{-1}\varepsilon_t$. In the above three-variable example, the first column of H is given by

$$h = \begin{pmatrix} cos(\theta_1)cos(\theta_2) \\ -sin(\theta_1)cos(\theta_2) \\ -sin(\theta_2) \end{pmatrix}$$

so that only two restrictions are needed in order to pin down the two unknown parameters θ_1 and θ_2. In general $s-1$ restrictions are required in order to identify only one shock.

3.3 Short-run recursive

The short-run recursive approach represents one of the most popular identification schemes, and has been extensively used in the literature to identify both policy and non-policy shocks.[7] The main idea is to impose a recursive structure that relates the innovations and the structural shocks, i.e. the matrix of impact effects of structural shocks on model variables. From a technical point of view the identification amounts to considering the Cholesky shocks as the structural shocks, i.e. $H = I$. This identification scheme has a few properties. First, different ordering of the variables in z_t will produce different representations. Second, a recursive structure also emerges in the contemporaneous relationships among variables in the SVAR representation so that variable j depends on the contemporaneous value of the $j-1$ ordered before but not on the value of the $s-j$ variables ordered after. Third, the effects of a given shock j are invariant with respect to the ordering of the $j-1$ variables ordered before variables j and the $s-j$ variables ordered after.

Such a scheme has been employed for identifying monetary policy shocks, fiscal policy shocks, oil price shocks and uncertainty shocks, among others. Blanchard and Perotti (2002) identifies the government spending shock by assuming that the shock

[7] A partial list of papers using short-run restrictions, although not necessarily within a recursive approach, includes Bernanke (1986), Bernanke and Blinder (1992), Bernanke and Mihov (1998), Blanchard and Perotti (2002), Blanchard and Watson (1986), Bloom (2009), Cochrane (1994), Cushman and Zha (1997), Davis and Kilian (2011), Fernández-Villaverde et al. (2015), Inoue, Kilian and Kiraz (2009), Kilian (2009), Kilian and Vega (2011), Monacelli, Perotti and Trigari (2010), Sims (1992).

is the only one which affects government spending on impact. The restriction is implemented by ordering government spending first in a VAR. The first shock of the Cholesky representation of the model is the government spending shock.[8]

Kilian (2009) estimates a VAR with three variables, oil production, an indicator of world demand and oil price and imposes a Cholesky identification scheme to identify several types of oil shocks: oil price shock (first), demand-driven shock (second) and precautionary shock (third). The main finding is that oil price fluctuations are mostly driven by world demand and precautionary demand shocks.[9]

The monetary policy shock, in the standard monetary VAR including inflation, GDP growth and the interest rate, is identified by the restrictions that the monetary policy shock has no effects contemporaneously on both inflation and GDP growth, see Christiano Eichenbaum and Evans (1996).[10] In practice, the interest rate is ordered

[8]Many other papers have adopted VAR techniques to identify fiscal policy shocks, see, among others, Burnside, Eichenbaum and Fisher (2004), Cavallo (2005), Caldara and Kamps (2006), Edelberg, Eichenbaum and Fisher (1999), Fatás and Mihov (2001), Galí, López-Salido, and Vallés (2007), Mertens and Ravn (2012,2013), Mountford and Uhlig (2009), Pappa (2009), Perotti (2005), Ramey (2011), Romer and Romer (2010), Rotemberg and Woodford (1992).

[9]Other papers studying the effects of oil prices are Bernanke, Gerter and Watson (1997), Blanchard and Galí (2007), Kilian and Murphy (2014), Baumeister and Kilian (2011), Kilian and Vigfusson (2011a, 2011b).

[10]The literature on VAR and monetary policy shocks identified using short run zero restrictions is huge: Bernanke and Blinder (1992), Bernanke and Gertler (1995), Bernanke and Mihov (1998) Christiano, Eichenbaum and Evans (1996, 1999), Cochrane (1994), Cochrane and Piazzesi (2002), D'Amico and M. Farka (2011), Faust, Swanson, and Wright (2004), Gürkaynak, Sack and Swanson (2005), Hanson (2004), Leeper, Sims and Zha (1996), Leeper and Gordon (1992), Leeper and Zha

last and the policy shock is the last shock of the Cholesky representation. When the model includes fast moving variables, like financial variables or other interest rates, these variables are typically ordered after the interest rate, and the Cholesky scheme still can be used to estimate the policy shock as the shock whose position corresponds to the position of the federal funds rate in the vector of variables. The drawback in this case is that the recursive identification imposes that the interest rate does not react contemporaneously to the variables ordered after and in some cases such an assumption might be controversial.

Here, as an illustration, I estimate the monetary policy shock in a VAR which includes monthly US data for the (log) CPI, (log) GDP, the one-year rate, and the excess bond premium (as in Gilchrist and Zakrajsek, 2012). The span is 1979:7-2012:6. Figure 1 below plots the responses of the four variables to the third Cholesky shock, the monetary policy shock. The shock increases the interest rate and significantly reduces industrial production. Prices increases in line with the very well known price puzzle, see Sims (1992), which is solved (prices reduce) by including an index of commodity prices.

3.4 Long-run recursive

Since the seminal paper Blanchard and Quah (1989), identification by means of long-run restrictions has become a very popular approach.[11] The idea is to identify eco-

(2003), Rudebusch (1998), Sims and Zha (2006), Strongin (1995).

[11]See, for example, Canova, Lopez-Salido and Michelacci (2010), Enders and Lee (1997), Galí (1992, 1999).

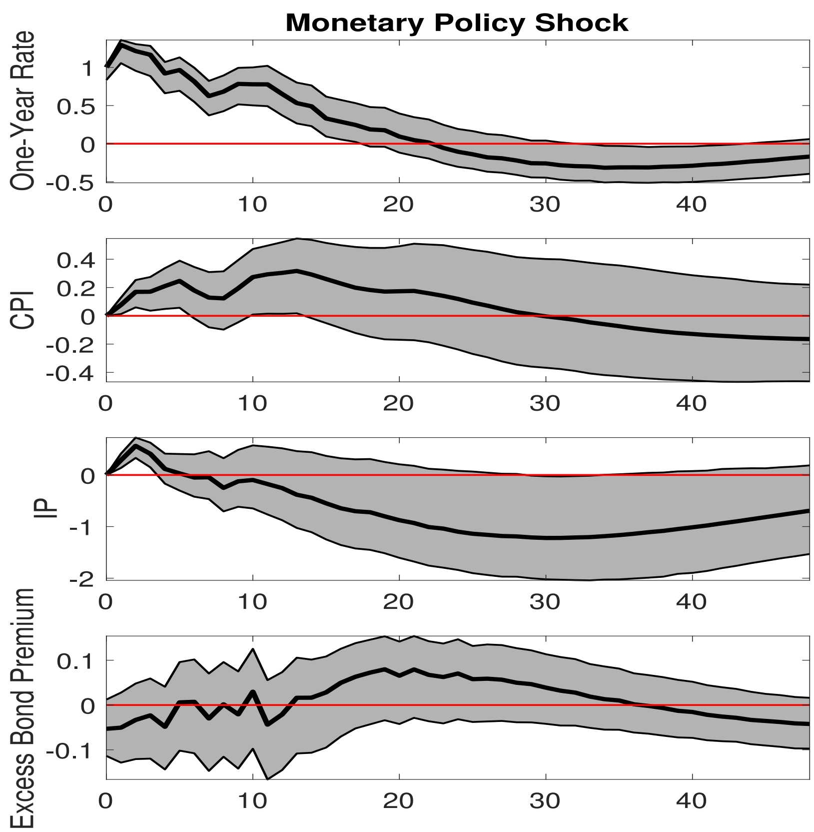

Figure 1: Impulse response functions obtained using a Cholesky identification. Solid lines – point estimates; grey areas – 90% confidence bands.

nomic shocks by restricting some of the elements of $B(1) = \sum_{j=0}^{\infty} B_j$. Blanchard and Quah (1989) estimates a bivariate VAR with GDP growth and the unemployment rate and identify two shocks: aggregate supply and aggregate demand shocks. The restriction imposed is that only aggregate supply drives (log) GDP in the long run.

In practice the restriction is implemented as follows. Let $S = \text{chol}(C(1)\Sigma C(1)')$. Then $B_0 = C(1)^{-1}S$. This creates a lower triangular structure in $B(1) = C(1)B_0$ where the first shock of the resulting representation is the aggregate supply shock, the second is the aggregate demand shock.

Long-run restrictions have been extensively used for the identification of technology shocks, to assess the importance of these shocks in shaping business cycles fluctuations.[12] In the seminal paper Galí (1999), the technology shock is identified by assuming that is the only shock driving labor productivity in the long run. In the baseline specification the vector z_t includes labor productivity growth and the growth rates of employment. The technology shock is the first shock of the representation obtained using the procedure described above. The main result is that employment reduces after a positive technology shock. Notice that the same identification can be applied in larger systems. However, notice that the remaining $s-1$ shocks do not have a structural interpretation, and they can be combined to identify other shocks leaving unchanged the effects of the technology shock.

[12]See, for example, Basu, Fernald and Kimball (2006), Beaudry and Portier (2006), Christiano and Eichenbaum and Vigfusson (2003, 2004), Fernald (2007), Fisher (2006), Francis and Ramey (2004), Francis, Owyang, Roush and Di Cecio (2010), King, Plosser, Stock and Watson (1991), Shapiro and Watson (1988), Uhlig (2004), Vigfusson (2004).

When the growth rate of employment is replaced by the growth rate of per-capita hours worked the results are unchanged: a positive technology shock significantly reduces the labor input. The result however is not robust across specifications of hours worked. Indeed when hours are specified in per-capita terms and in levels a technological improvement significantly increases hours worked, see Christiano and Eichenbaum and Vigfusson (2003). The results are reported in Figure 2. The left column reports the results when hours enter the VAR in growth rates while the right column the results for hours in levels.

A potential drawback of long run restrictions, is that the estimates of the impulse response functions might be distorted when the variables in the VAR display low-frequency comovements, see Faust and Leeper (1997), An example is precisely the case of labor productivity growth and (log) per-capita hours worked. Indeed Fernald (2007) shows that when low frequency comovements between labor productivity growth and per-capita hours are removed, then a positive technology shocks significantly reduces hours even when specified in levels. However, Gospodinov, Maynard and Pesavento (2011) shows that, if a long run comovement is actually in the data generating process, then removing it can produce a bias in the impulse response functions of the detrended variables.

3.5 Sign restrictions

Sign restrictions were introduced in the seminal paper Uhlig (2005). Since then, they have become a popular tool for identifying structural shocks in VAR models. Rubio-

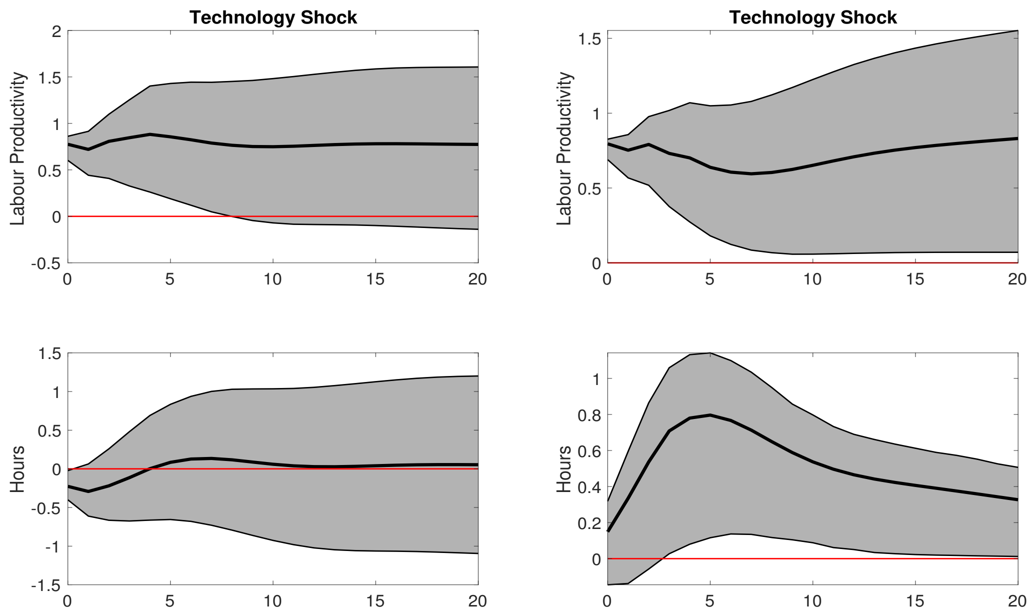

Figure 2: Impulse response functions of labor productivity and hours worked. Left column – hours in growth rates. Right column – hours in levels. Solid lines – point estimates; grey areas – 90% confidence bands.

Ramírez, Waggoner and Zha (2010) presents a formal and exhaustive discussion of this approach.[13] In this section I first discuss some technical details of the approach with some examples and then I will provide a discussion at the end of the subsection.

The main idea is to restrict the sign of the effect of the shock, instead of imposing quantitative restrictions like the zero restrictions discussed before. Consider the following example. Consider a VAR for inflation and GDP growth. Assume the two structural shocks driving the two variables are a supply and a demand shock. A positive demand shock is assumed to increase both inflation and GDP growth on impact. A positive supply shock is assumed to reduce inflation and increase GDP growth on impact. How can these restrictions be implemented? The impact effects are given by SH, where

$$H = \begin{pmatrix} cos(\theta_1) & sin(\theta_1) \\ -sin(\theta_1) & cos(\theta_1) \end{pmatrix}.$$

Assume, with no loss of generality, that inflation is ordered first and GDP growth second, the demand shock is the first one and the supply shock the second one. The

[13]This approached has been extensively used to identify a wide range of economic shocks. See, for example: Baumeister and Peersman (2010), Canova and De Nicolò (2002), Canova and Paustian (2011), Canova and Pappa (2007), Chari, Kehoe and McGrattan (2008), Dedola and Neri (2007), Faust (1998), Foroni, Furlanetto and Lepetit (2018), Fujita (2011), Furlanetto, Ravazzolo and Sarferaz (2018), Inoue and Kilian (2011), Kilian and Murphy (2011, 2014), Pappa (2009), Scholl and Uhlig (2008), Weale and Wieladek (2016).

four restrictions that have to hold are

$$
\begin{aligned}
S_{11}\cos(\theta_1) &> 0 \\
S_{21}\cos(\theta_1) - S_{22}\sin(\theta_1) &> 0 \\
S_{11}\sin(\theta_1) &< 0 \\
S_{22}\sin(\theta_1) - S_{22}\cos(\theta_1) &> 0.
\end{aligned}
$$

The first two restrictions concern the demand shock while the remaining two define the supply shock. Notice that within this approach we might be confronted with two very different situations. First, there is a set of values of θ_1 satisfying the restrictions (this is why this type of identification is also known as set identification). The impulse response functions associated to the different values of θ_1 can be similar or different. In this sense there exists identification uncertainty which contributes, together with the estimation uncertainty, to the total uncertainty surrounding the impulse response functions. Increasing the number of restrictions typically helps in reducing identification uncertainty. On the contrary it could be the case that there is no value of θ_1 satisfying the restrictions. In this case some of the restrictions should be relaxed in order to identify the shocks.

Sign restrictions are typically implemented in a Bayesian VAR framework, as in Uhlig (2005) since a distribution for θ has to be specified. However, very recently, Granziera, Moon and Schorfheide (2018) proposes an approach to construct confidence bands that are valid from a frequentist perspective. Giacomini and Kitagawa (2018) develops a multiple-prior Bayesian approach which reconciles the discrepancies between the Bayesian and frequentist approaches in set identified models.

Sign restrictions have received a lot of attention and several criticisms were raised. For instance, Canova and Paustian (2011) shows that sign restrictions can be successfully used only when the shock is important and has large effects. Fry and Pagan (2011) points out that the median response will not correspond to any specific draw of the impulse response functions but it will be a mix of several models. For this reason, one should be careful in interpreting the median impulse response functions. They suggest to take the draw which is the closest to the median response.

Baumeister and Hamilton (2015) shows that the typical uniform prior on H implies nonuniform distributions for some of the structural parameters. This means that the prior, unlike typically thought, is actually informative about the parameter space. To address this problem, the paper proposes a general framework for Bayesian inference in structural VARs, designed to optimize the prior information.

Arias, Caldara and Rubio-Ramírez (2019) extends the sign restrictions approach by imposing restrictions also on the parameters of the implied VAR representation. For instance, in the context of a monetary policy shock, they impose the restrictions that the parameters of the monetary policy rule associated to inflation and output cannot be negative. By adding these restrictions they overturn the main result of Uhlig (2005) that monetary policy is neutral.

3.6 Narrative approach

The approach consists of two steps. In a first step, narrative evidence obtained from historical records is used to construct a proxy measure of the shock of interest. In a second step, the variable is used in a statistical model in order to derive the impulse response functions of the variables of interest to the shock. The second step typically involves the estimation of a SVAR, a VARX or simply a linear regression with controls. The approach has been used for identifying several types of shocks: oil shocks (Hamilton, 1985), government spending shocks (Ramey, 2011), monetary policy shocks (Romer and Romer, 2004), and tax shocks (Romer and Romer, 2010, Mertens and Ravn, 2012, 2013).[14]

Let us consider a simple example. Suppose that N_t is the measure for the structural shock of interest constructed using narrative evidence. In the second step a VAR for $(N_t \ z_t')'$ is estimated and the impulse response functions are derived as the impulse response functions of the first shock of the Cholesky decomposition.

An alternative is to estimate the VARX model

$$A(L)z_t = P(L)N_t + \varepsilon_t \tag{9}$$

where $P(L) = P_0 + P_1 L + ... + P_r L^r$ and obtain the impulse response functions to N_t as $A(L)^{-1}P(L)$. The two alternative coincide as long as N_t is truly exogenous, no other structural shock have any effect on the variable, and N_t has no serial correlation.

[14] Other papers using the narrative approach are Alloza (2017), Coibion et al. (2017), Favero and Giavazzi (2012), Owyang, Ramey and Zubairy (2013), Ramey and Zubairy (2018), Tenreyro and Thwaites (2016).

A third alternative is to estimate the impulse response functions for variable z_{it} at horizon j as the coefficient β_i^j in the following linear regressions

$$z_{it+j} = \alpha_i + \beta_i^j N_t + X_t' \gamma_i^j + e_t \tag{10}$$

where X_t is a vector of controls and γ_i^j a vector of coefficients.

Here, as an illustration, I study the effects of fiscal policy shocks on consumption using the war news approach proposed by Ramey (2011). Ramey (2011) constructs a news variable reporting changes in government spending, as a percentage of GDP, driven by war episodes. I use both a VAR model and a linear regression.

I consider four types of consumption: total, durable, non-durable and services. For each type of consumption, I estimate a separate VAR(4) which includes, in this order, the war news, the log of real government expenditure per capita, the log of real GDP per-capita, the 3-month T-bill rate, the average marginal income tax rate of Barro-Redlick and the consumption measure of interest. The government spending shock is the first shock of the Cholesky decomposition. The VAR also includes a quadratic trend. The left column of Figure 3 plots the responses of the four types of consumption. Except for services, consumption significantly reduces, consistently with Ramey (2011).

Direct projections can be obtained by regressing the consumption measure of interest on a constant, the news variable and the first lag of the same set of variables included in the baseline VAR as controls.

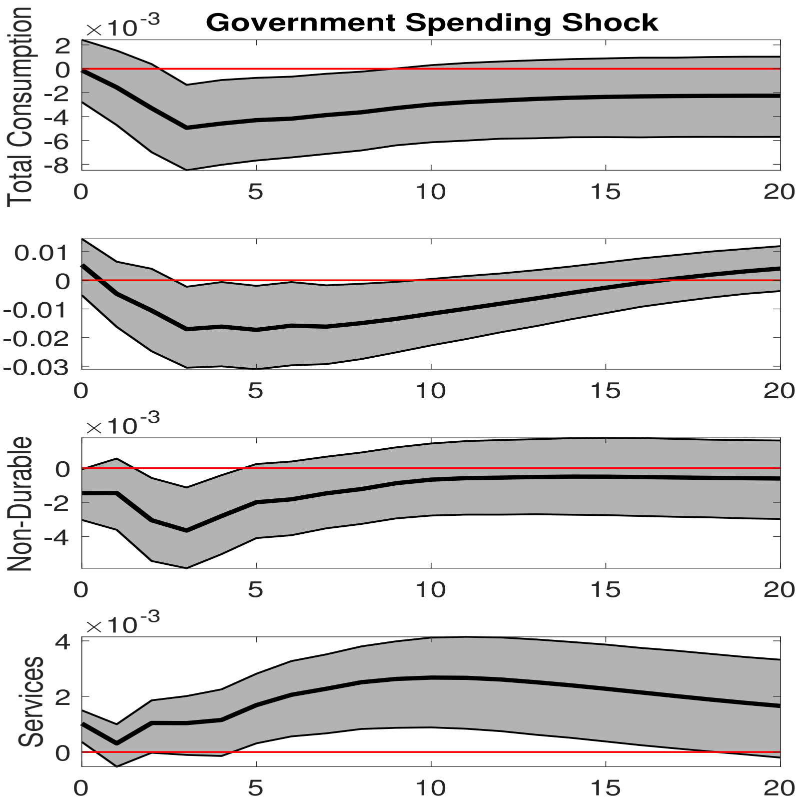

Figure 3: Impulse response functions of a government spending shock using the war news variable of Ramey (2011). Left column – VAR; right column – linear projections. Solid lines – point estimates; grey areas – 90% confidence bands.

3.7 Proxy-External Instruments approach

Another recent approach which is becoming more and more popular is the proxy approach or external instrument approach developed by Stock and Watson (2012) and Mertens and Ravn (2013). The approach has been used in many applications, among others by Gertler and Karadi (2015), to study the effects of monetary policy shocks.[15] The main idea is that in many situations the shock itself is not available to the econometrician but a proxy is. In what follows I describe how to use this proxy to estimate the effects of the shock.

Let u^z_{1t} be the structural shock of interest whose effects have to be estimated. Let u^z_{-1t} be the vector of remaining shocks in u^z_t. Let b_0 denote the first column in matrix B_0, the vector representing the impact effects of u^z_{1t} on z_t. Let g_t be the external instrument (the proxy) and assume that the following two conditions hold:

1. g_t is correlated with u^z_{1t}: $E(g_t u^z_{1t}) = \phi$ (relevance condition);

2. g_t is orthogonal to the other shocks in u^z_{-1t}: $E(g_t u^z_{-1t}) = 0$ (contemporaneous exogeneity condition).

Notice that $E(\varepsilon_t g_t) = \phi b_0$, i.e. the covariance between the innovations and the external instrument is proportional to the impact effects of u^z_{1t}. To estimate the effects of the shock one can proceed as follows:

i Estimate, by ordinary least squares, the VAR(p) process and obtain the vector

[15] Other examples: Caldara and Kamps (2017), Carriero et al. (2015), Drautzburg, Fernández-Villaverde and Guerrón-Quintana (2017), Mertens and Ravn (2014), Stock and Watson (2018).

of residuals $\hat{\varepsilon}_t$.

ii Regress each element of the vector of innovations $\hat{\varepsilon}_{it}$ on g_t. The OLS estimator is a consistent estimator of $\frac{\phi m_i}{\sigma_g^2}$.

iii Normalize some of the effects, say the impact on the jth variable $b_{0,j}$, and obtain an estimate of $\frac{b}{b_{0,j}}$.

Notice that the normalization will also rescale the standard deviation of the structural shocks, which will be now equal to $b_{0,j}$. Olea, Stock and Watson (2018) discuss how to do inference in this framework, in particular how to cope with the potential problem represented by the weak correlation between the instrument and the structural shock.

Gertler and Karadi (2015) applies this procedure to estimate the effects of monetary policy shocks. Here we replicate their exercise. The VAR includes: the one-year rate, the logarithm of CPI, the logarithm of industrial production and the excess bond premium of Gilchrist and Zakrajšek (2012). Therefore, z_t includes the policy variable ordered first and then a set of macroeconomic and financial variables. The data span 1979:7 to 2012:6 and $p = 12$. The external instrument g_t is the change in the 3 months ahead federal funds future rate in a 30 minute window after the FOMC announcement in a given month. This instrument is chosen as it is the one that performs the best, in terms of F-test, in the two stages regression described above.

Figure 4 displays the estimated impulse response functions. Consistently with the existing literature, a contractionary shock decreases significantly and persistently both inflation and output. Furthermore, it increases significantly the excess bond premium on impact and for the first 7 months after the shock.

A recent paper, Miranda-Agrippino and Ricco (2017), shows that the instrument used in Gertler and Karadi (2015) is not exogneous. More specifically the paper shows that the instrument is driven, to some extent, by the Fed's expectations about future economic conditions which are ultimately influenced by nonpolicy shocks. The authors regress the instrument on the Greenbook Forecasts, the component attributable to Fed's expectations, and they use the residual as a cleaned and exogenous version of the instrument. The authors show that with the new variable the results are much more robust and consistently point to a contactionary effect on prices and output of the policy shock.

3.8 Mixed restrictions approach

The restrictions discussed above can be used simultaneously yielding mixed identification schemes.[16] Let me start considering a simple example. Suppose $s = 3$. The goal is to identify a single shock using the restriction that such a shock has a zero effect on the first variable on impact, a positive effect on the second variable and negative on the third variable on impact. Given that we are interested in a single shock we can focus on a single column of H. With no loss of generality suppose the shock is the first. The first column of the product of the three orthogonal matrices,

[16] See, for example, Galí (1992), Mountford and Uhlig (2009), Binning (2013), Arias, Rubio-Ramírez, Waggoner (2018). Two recent papers prposing a new identifiction approach based on sign restrictions are Ben Zeev (2018) and Antolín-Díaz and Rubio-Ramírez (2018). The authors propose combining narrative identification and sign restrictions for SVARs.

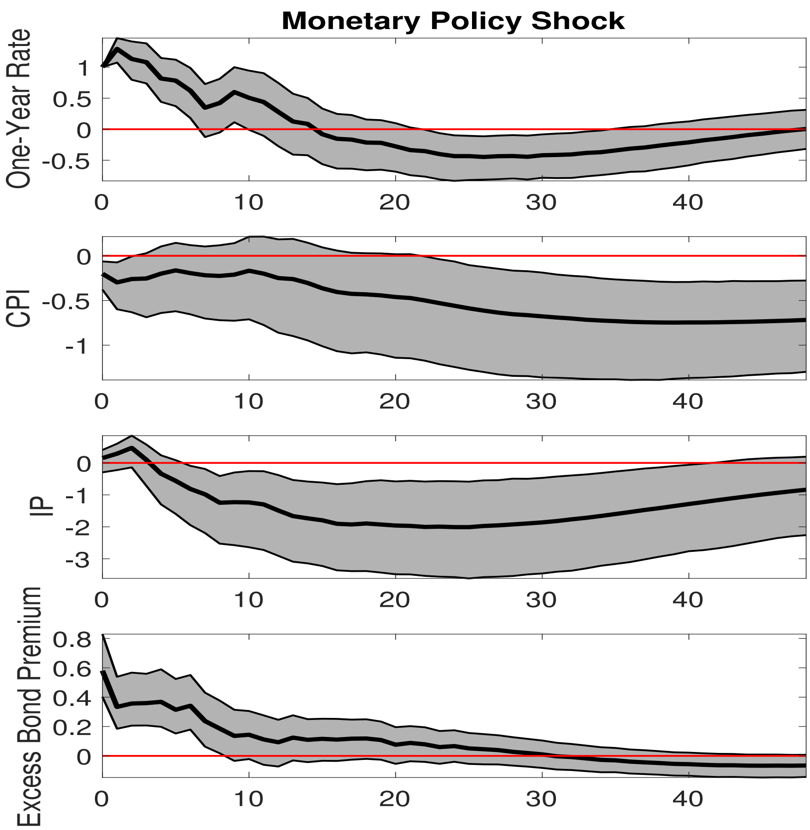

Figure 4: Impulse response functions of a monetary policy shock using Gertler and Karadi (2015) identification. Solid lines – point estimates; gray areas – 90% confidence bands.

as seen before, is

$$h = \begin{pmatrix} \cos(\theta_1)\cos(\theta_2) \\ -\sin(\theta_1)\cos(\theta_2) \\ -\sin(\theta_2) \end{pmatrix}.$$

The impact effect of the first shock on the three variables, i.e. the first column of B_0, again denoted by b_0, is given by the product

$$b_0 = \begin{pmatrix} S_{11} & 0 & 0 \\ S_{21} & S_{22} & 0 \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} \cos(\theta_1)\cos(\theta_2) \\ -\sin(\theta_1)\cos(\theta_2) \\ -\sin(\theta_2) \end{pmatrix}.$$

To implement the first restriction we can simply set $\theta_1 = \pi/2$, i.e. $\cos(\theta_1) = 0$. This implies that

$$b_0 = \begin{pmatrix} S_{11} & 0 & 0 \\ S_{21} & S_{22} & 0 \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} 0 \\ -\cos(\theta_2) \\ -\sin(\theta_2) \end{pmatrix}.$$

The second restriction implies that

$$-S_{22}\cos(\theta_2) > 0$$

and the third

$$-S_{32}\cos(\theta_2) - S_{33}\sin(\theta_2) < 0.$$

All the values of θ_2 satisfying the two restrictions yield impulse response functions consistent with the identification scheme.

Mixed restrictions have been largely used in the literature since the seminal paper Galí (1992), which mixes short and long-run restrictions. More recently, mixed restrictions have been employed to identify news shocks.[17] Beaudry and Portier (2006, 2014) uses short and long-run restrictions to identify news shocks. TFP is driven by two types of technology shocks: surprise and news. The two shocks are the only shocks driving TFP in the long run but news shocks do not have a contemporaneous effect on TFP, while surprise shocks do. Barsky and Sims (2011), in the spirit of Uhlig (2004), identifies the news shock by assuming that the shock has no contemporaneous effect on TFP but has a maximal effect on the forecast error variance of the TFP up to a 40-quarter horizon. The approach is also employed in Kurmann and Otrok (2013) to study the dynamics of the slope of the term structure. Forni, Gambetti and Sala (2014) uses a similar approach but instead of maximizing the forecast error variance over the whole horizons, they maximize the effect of the shock in the long run. The rationale behind all these identification schemes is that the news shock is an important driver of TFP in the long run but, by definition, does not have immediate effect on productivity.

Here, as an example I identify the news shock assuming that has zero contemporaneous effect on total factor productivity on impact and has a maximal effect on total factor productivity after 60 quarters. Implementation is relatively easy. Let me first use a three-variable example in order to better understand the mechanics. Again suppose, with no loss of generality that the TFP is ordered first. Consider again the

[17] Basu, Fernald and Kimball (2006), Barsky and Sims (2011), Beaudry and Lucke (2009), Beaudry and Portier (2006), Dupaigne and Portier (2006), Feve, Matheron and Sahuc (2009), Forni, Gambetti and Sala (2014), Jaimovich and Rebelo (2009), Schmitt-Grohé and Uribe (2012), Sims (2011).

rotation vector

$$h = \begin{pmatrix} 0 \\ -\cos(\theta_2) \\ -\sin(\theta_2) \end{pmatrix}$$

Given that the first element of the vector $b_0 = Sh$ is zero, the restriction that the news shock has no effect of TFP on impact is satisfied. Now, let $\tilde{D}_{60} = \sum_{j=0}^{60} D_j$ the effects on the level of three variables after 60 quarters in the Cholesky representation. To maximize the effects after 60 periods one can simply maximize with respect to θ_2 the coefficient $-\tilde{D}_{60}^{12}\cos(\theta_2) - \tilde{D}_{60}^{13}\sin(\theta_2)$, where $\tilde{D}_{60}^{ij}$ is the element i, j of $\tilde{D}_{60}$.

I estimate a VAR model for TFP, stock prices, consumption, hours worked GDP, investment, current and expected consumer sentiment.[18] The news shock is identified as described earlier: imposing a zero impact effect and maximal effect after 60 quarters on TFP. Figure 5 display the results. TFP increases slowly to its new long-run level. Consumption increases on impact but investment reduces, which implies a small effect on GDP. Hours significantly reduce on impact.

3.9 Identification as a statistical device

Sometimes identification is adopted as a simple statistical device. In this strand of literature, rather than identifying one single type of economic shock, a convolution of shocks with desired characteristics is isolated. Examples of this approach are: (a) Generalized IRF, for example, Pesaran and Shin (1998). (b) Business cycle shocks, combination of shocks that explain most of the variance of GDP at business cycle

[18] I use the variables in levels to avoid cointegration problems.

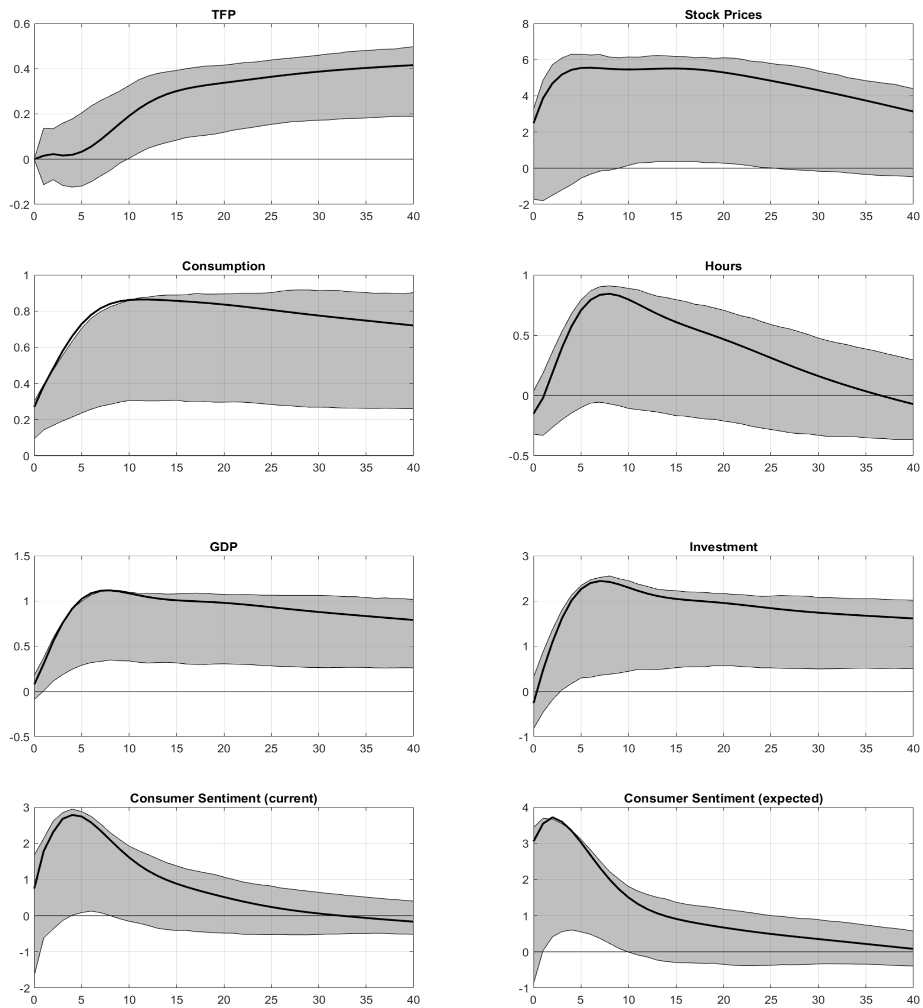

Figure 5: Impulse response functions to a news shock. Solid lines – point estimates; gray areas – 90% confidence bands.

frequencies, e.g. Giannone, Lenza and Reichlin (2008), Angeletos, Collard and Dellas (2018). (c) Real shocks, combinations of shocks that explain most of the variance of real variables, e.g. Giannone, Reichlin and Sala (2005). (d) Medium run shocks, combinations of shocks that explain most of the variance at medium horizons, e.g. Uhlig (2005).

4 Extensions

In this section I will discuss a few recent extensions of SVAR models.

4.1 Large-N VARs

A potential shortcoming of VAR models is represented by limited number of varables that can be handled. Bernanke Boivin and Eliasz (2005) proposes a model that circunvents such a limitation, Factor Augmented VAR (FAVAR) models. The idea is to augmenent the VAR with a set of unobserved factors. The the factors are the objects that convey all of the relevant information about the dynamics of the economy. When the data are generated by a DSGE model, the factors span the space spanned by the state variables of the model. The factors can be consistently estimated with the principal components of a large dataset, see Stock and Watson (2002). Thus, the factors add to the model the relevant information coming from many economic series, and this can solve the deficient information problem.

FAVAR models have been extensively used over the last years precisely to cope with the problem of narrow information sets and it has been shown that they can solve many existing puzzles.[19] The factors can be consistently estimated with the principal component estimator and the number of factor to be included can be established by repeating the same test (Granger causality or orthogonality depending on the goal of the application) with the vector of variables augmented by the factors.

An alternative to FAVAR models to solve the problem of deficient information is represented by large Bayesian VAR, see Banbura, Giannone and Reichlin (2010). The Bayesian approach allows to handle large dataset and include hundreds of variables in the model. The curse of dimensionality problem is solved by setting the degree of prior tightness in relation to the model dimension. A potential drawback is that in large BVAR the number of restrictions to identify economic shocks increases substantially. Large BVAR have been extensively used for both forecasting and structural analysis, see, among others, Ellahie and Ricco (2017).

There are class of models that can handle high dimensional datasets. For instance, Dynamic Factor Models have been extensively used for structural analysis in recent years, see Forni et al (2009). Panel VAR, see Canova and Ciccarelli (2013) for a survey, or Global VAR, see Pesaran, Schuermann and Weiner (2004), represent alternative approaches. PVAR and GVAR are typically employed when the goal is to have a multicountry or multisector models.

[19]See, for example, Bianchi, Mumtaz and Surico (2009), Boivin, Giannoni and Mihov (2009), Ludvigson and NG (2009), Mumtaz and Surico (2008) and Moench (2008), among others. Bernanke and Boivin (2003) investigates the mapping between large-N models and DSGE models.

4.2 Time-Varying Coefficients VAR

In standard SVAR models the dynamics and the propagation mechanisms of structural shocks are constant over time. In recent years, however, many papers have documented several structural changes that industrialized economies have experienced over the last decades. For instance the literature has shown how the real economy and inflation are more stable since the mid 80, the phenomenon called the Great Moderation, see McConnel and Perez Quiros (2000). Another notable example is the change in the conduct of monetary policy since Volcker's chairmanship.

The evidence implies that economic dynamics have evolved over time and the effects of shocks might have changed. For this reason the macroeconometric literature has developed over the last ten years Time-Varying Coefficients VARs. The model, developed by Cogley and Sargent (2002), and extended by Primiceri (2005) and Del Negro and Primiceri (2015), is a generalization of the standard SVAR in the sense that the coefficients are assumed to be varying over time:

$$A_t(L)z_t = \varepsilon_t \tag{11}$$

where $A_t(L) = I - A_{1t} - ... - A_{pt}$ is a polynomial matrix of time-varying coefficients. A second generalization, due to Primiceri (2005), has been to introduce, in addition to variation in the VAR coefficients, stochastic volatility of the reduced form residuals, so that $\varepsilon_t \sim N(0, \Sigma_t)$.

The resulting impulse response functions, are also time-varying so that the underlying MA representation has time-varying coefficients.

$$z_t = B_t(L)u_t^z \tag{12}$$

Representation (11) is typically estimated using MCMC methods and $B_t(L)$ is derived from the estimated time-varying VAR coefficients. To estimate the model some stochastic process for the elements af $A_t(L)$ and Σ_t is assumed.

Cogley and Sargent (2002) uses this model to characterize changes in the dynamics of the US economy. What they find is that the mean of the volatility and the persistence of inflation has substantially declined after mid 80s. They argue that loose monetary policy, i.e. not enough aggressive against inflationary pressures, was the main cause of these bad ouctomes in terms of inflation.[20]

4.3 Nonlinear VAR

Many papers have studied nonlinearities in the transmission mechanisms of both policy and non policy shocks, see among others Morley and Piger (2012), Abadir, Caggiano, and Talmain (2013), Morley, Piger, and Tien (2013). A great deal of attention has been paid on investigating whether policy shocks have state-dependent effects. For instance do the effects of fiscal policy expansions or contractions dependent on the state of the economy?[21] From a methodological perspective, the Smooth

[20]The paper sparked an interesting debate about the causes of the Great Moderation, see, among others, Benati (2008), Cogley and Sargent (2005), Primiceri (2005), Galí and Gambetti (2009) and Canova and Gambetti (2009).

[21]A partial list of papers includes Auerbach and Gorodnichenko (2012), Bachmann and Sims (2012), Berger and Vavra (2014), and Caggiano, Castelnuovo, Colombo, and Nodari (2015),

Transition VAR (STVAR) represents a popular tool employed to investigate nonlinear dynamics (see Terasvirta, Tjostheim, and Granger, 2010). In this model the transmission of shocks depend on an underlying state variable which reflects the state of the economy. The idea is that the effects of a certain shock can be different depending of the level of this state variable. The typical example is recession versus expansion.[22] The model is the following

$$z_t = (1 - I_t)\tilde{A}^1(L)z_{t-1} + I_t\tilde{A}^2(L)z_{t-1} + \varepsilon_t \tag{13}$$

where $\tilde{A}^1(L) = \tilde{A}^1_1 + ...\tilde{A}^1_p L^p$ $\tilde{A}^2(L) = \tilde{A}^2_1 + ...\tilde{A}^2_p L^p$, $I_t = \frac{exp(-\gamma x_t)}{1+exp(-\gamma x_t)}$ is the logistic function governing the transition from one regime to the other and x_t is the underlying state variable, and λ governs the speed of transition from one regime to the other. The implied MA representation is given by

$$z_t = (1 - I_t)B_1(L)u^z_t + I_t B_2(L)u^z_t \tag{14}$$

The model can be estimated using MCMC methods or, when x_t is exogenous with respect to the structural shocks, by nonlinear least squares.

Auerbach and Gorodnichenko (2012) use a version of this model to study the effects of government spending in recessions and booms. Using the same identification scheme as in Blanchard and Perotti (2002), the authors find that government spending

Caggiano, Castelnuovo, and Groshenny (2014).

[22]Barnichon and Matthes (2018) propose an alternative method to directly estimate the nonlinear moving average representation where the impulse response functions are parametrized by Gaussian basis functions. They find a high extent of nonlinearity in the responses, especially in terms of sign of the monetary policy shock.

is much more effective in periods of recession, the government spending multiplier being substantially larger than one. Similar results are found in Caggiano et al. (2015). On the contrary Ramey and Zubairy (2018), with a longer sample and a different version of the model, do not find significant differences between recessions and booms, the multiplier being always smaller than one.

A prominent alterative is represented by regime-switching VAR, see Krolzig (1997), Hamilton (1985, 1994) and Sims and Zha (2006b). The idea behing these models is similar to the one discussed above. There is a state variable which governs the model dynamics but the state is now unobserved and assumed to be generated by a discrete-time Markov chain. This class of models has been extensively used in recent to study several important questions see, among others, Sims and Zha (2006b), Barnett, Groen and Mumtaz (2010), Nason and Tallman (2015) and Hubrick and Tetlow (2015).

4.4 DSGE Models and VARs

This subsection focuses on the relation between DSGE models and VARs, see Giacomini (2013) and Pagan and Robinson (2016). More specifically, we review the conditions under which a DSGE model admits a VAR representation, and the conditions under which SVAR models can be employed to estimate the effects of DSGE shocks, see Ravenna (2007). Consider the following (log) linear solution of a DSGE model

$$s_t = As_{t-1} + Bu_t \tag{15}$$

$$z_t = Cs_{t-1} + Du_t \tag{16}$$

where s_t is an r-dimensional vector of stationary state variables, z_t again is the vector considered by the econometrician, $q \leq r \leq n$, A, B, C and D are conformable matrices of parameters and B has a left inverse B^{-1} such that $B^{-1}B = I_q$. Also

$$u_t = B^{-1}s_t - B^{-1}As_{t-1}.$$

Substituting in z_t we have

$$z_t \quad = \quad \left[DB^{-1} - (DB^{-1}A - C)L\right] s_t$$

In the square case $q = s$ we have

$$z_t = DB^{-1}\left[I - (A - BD^{-1}C)L\right] s_t.$$

The shocks can be obtained as a square summable combination of the present and past of z_t if and only if the eigenvalues of $(A - BD^{-1}C)$ are strictly less than one in modulus, the so called "poor man's condition" discussed in Fernández-Villaverde et al (2007). when this condition holds a VAR representation in the structural shocks exists

$$z_t = \sum_{j=0}^{\infty}(A - BD^{-1}C)^j BD^{-1}z_{t-j} + Du_t.$$

In many cases, however, the condition is too restrictive since the researcher might be interested in investigating the effects of a single shock. The theoretical conditions under which sufficiency holds have been for the first time discussed in Sims and Zha (2006a). Let us consider the projection of u^z_{it} onto the entries of ε_t

$$u^z_{it} = M\varepsilon_t + e_{it} \tag{17}$$

where ε_t is the innovation of the VAR for z_t and the fraction of unexplained variance in the above regression (recall $\sigma^2_{u_i^z} = 1$)

$$\delta_i = \sigma^2_{e_i}. \tag{18}$$

When $\delta_i = 0$ then partial informational sufficiency holds for shock i. The idea is that in this case the structural shock is an exact linear combination of the innovations. On the contrary, a large value of δ_i means that the structural shock cannot be obtained from the innovations and therefore from a VAR. Notice that, in many cases δ_i might be nonzero but small. In these cases information sufficiency is only *approximate* but still, as shown in Forni et al. (2019), the shocks can be estimated with very good approximation.

Notice that the Sims and Zha (2006a) statistic can be used for DSGE validation (see Canova and Paustian, 2011). For a given DSGE and parametrization δ_i can be computed to understand whether a given VAR specification can be used to compare the empirical impulse response functions to the theoretical DSGE impulse response functions.

References

[1] Abadir, K. M., G. Caggiano, and G. Talmain (2013), Nelson-Plosser revisited: The ACF approach, Journal of Econometrics, 175, issue 1, p. 22-34.

[2] Alloza, M. (2017), Is fiscal policy more effective in uncertain times or during recessions?, Working Papers 1730, Banco de España.

[3] Angeletos, G. M., F. Collard and H. Dellas (2018), Business Cycle Anatomy, NBER Working Papers 24875, National Bureau of Economic Research.

[4] Antolín-Díaz, J., and J. F. Rubio-Ramirez (2018), Narrative Sign Restrictions for SVARs, American Economic Review, 108 (10): 2802-29.

[5] Arias, J.E., D. Caldara and J.F Rubio-Ramírez (2019), The Systematic Component of Monetary Policy in SVARs: An Agnostic Identification Procedure, Journal of Monetary Economics, Elsevier, vol. 101(C), pages 1-13.

[6] Arias, J.E., J.F. Rubio-Ramírez and D.F. Waggoner (2018), Inference Based on Structural Vector Autoregressions Identified With Sign and Zero Restrictions: Theory and Applications. Econometrica, 86: 685-720.

[7] Auerbach, Alan J., and Y. Gorodnichenko (2012), Measuring the Output Responses to Fiscal Policy, American Economic Journal: Economic Policy, 4 (2): 1-27.

[8] Bachmann, R., and E. R. Sims (2012), Confidence and the transmission of government spending shocks, Journal of Monetary Economics, Elsevier, vol. 59(3), pages 235-249.

[9] Banbura, M., D. Giannone, and L. Reichlin (2010), Large Bayesian Vector Autoregressions, Journal of Applied Econometrics, 25, 71-92.

[10] Barnett, A., Groen, J. J. and Mumtaz, H. (2010), Time-varying inflation expectations and economic fluctuations in the United Kingdom: a structural VAR analysis, Bank of England working papers 392, Bank of England.

[11] Barnichon, R., and C. Matthes (2018), Functional Approximations of Impulse Responses (FAIR), Journal of Monetary Economics, 99:41-55.

[12] Barsky, R.B., and E.R. Sims (2011), News Shocks and Business Cycles, forthcoming: Journal of Monetary Economics.

[13] Basu, S., J. Fernald and M.S. Kimball (2006), Are Technology Improvements Contractionary?, American Economic Review, American Economic Association, vol. 96(5), pages 1418-1448, December.

[14] Baumeister, C., and J.D Hamilton (2015), Sign Restrictions, Structural Vector Autoregressions, and Useful Prior Information, Econometrica, 83: 1963-1999.

[15] Baumeister, C., and L. Kilian (2011), Real-Time Forecasts of the Real Price of Oil, Journal of Business & Economic Statistics 30(2):326-336.

[16] Baumeister, C., and G. Peersman (2010), Sources of the Volatility Puzzle in the Crude Oil Market, mimeo, Ghent University.

[17] Beaudry, P., and B. Lucke (2009), Letting Different Views about Business Cycles Compete, NBER Working Paper No. 14950, May 2009.

[18] Beaudry, P., and F. Portier (2006), Stock Prices, News, and Economic Fluctuations, American Economic Review, American Economic Association, vol. 96(4), pages 1293-1307, September.

[19] Beaudry, P., and F. Portier (2014), "News-Driven Business Cycles: Insights and Challenges." Journal of Economic Literature, 52 (4): 993-1074.

[20] Benati, L. (2008), The "Great Moderation" in the United Kingdom, Journal of Money, Credit and Banking, 40, 121-147.

[21] Ben Zeev, N. (2018).' What can we learn about news shocks from the late 1990s and early 2000s boom-bust period? Journal of Economic Dynamics and Control, 87(C): 94-105.

[22] Berger, D., and J. Vavra (2014), Measuring How Fiscal Shocks Affect Durable Spending in Recessions and Expansions, American Economic Review, 104 (5): 112-15.

[23] Bernanke, B.S. (1986), Alternative Explanations of the Money-Income Correlation, Carnegie-Rochester Conference Series on Public Policy, 25, 49-99.

[24] Bernanke, B.S., and A. Blinder (1992), The Federal Funds Rate and the Channels of Monetary Transmission, American Economic Review, 82, 901-921.

[25] Bernanke, B.S., and J. Boivin (2003), Monetary Policy in a Data-Rich Environment, Journal of Monetary Economics, 50, 525-546.

[26] Bernanke, B.S., J. Boivin, and P. Eliasz (2005), Measuring Monetary Policy: A Factor Augmented Vector Autoregressive (FAVAR) Approach, Quarterly Journal of Economics, 120, 387-422.

[27] Bernanke, B.S., M. Gertler, (1995), Inside the Black Box: The Credit Channel of Monetary Policy Transmission, Journal of Economic Perspectives, Vol. 9, No. 4, pages 27 - 48.

[28] Bernanke, B.S., M. Gertler, and M. Watson (1997), Systematic Monetary Policy and the Effects of Oil Price Shocks, Brookings Papers on Economic Activity, 1, 91-142.

[29] Bernanke, B.S., I. Mihov (1998), Measuring Monetary Policy, The Quarterly Journal of Economics, Oxford University Press, vol. 113(3), pages 869-902.

[30] Bianchi, F., H. Mumtaz and P. Surico (2009), The great moderation of the term structure of UK interest rates, Journal of Monetary Economics, 56, issue 6, p. 856-871.

[31] Binning, A. (2013), Underidentified SVAR models: A framework for combining short and long-run restrictions with sign-restrictions, Working Paper 2013/14, Norges Bank.

[32] Blanchard, O.J., and and J. Galí (2007), The Macroeconomic Effects of Oil Price Shocks: Why are the 2000s so different from the 1970s?, NBER Chapters, in: International Dimensions of Monetary Policy, pages 373-421 National Bureau of Economic Research, Inc.

[33] Blanchard, O.J., and R. Perotti (2002), An Empirical Investigation of the Dynamic Effects of Shocks to Government Spending and Taxes on Output, Quarterly Journal of Economics, 117, 1329-1368.

[34] Blanchard, O.J. and D. Quah (1989), The Dynamic Effects of Aggregate Demand and Supply Disturbances, American Economic Review, 79, 654-673.

[35] Blanchard, O.J., and M.W. Watson (1986), Are Business Cycles All Alike? in: R.J. Gordon (ed.), The American Business Cycle, NBER and Chicago Press, 123-179.

[36] Bloom, N. (2009), The Impact of Uncertainty Shocks, Econometrica 77, pp. 623-685.

[37] Boivin, J., M.P. Giannoni and I. Mihov (2009), Sticky Prices and Monetary Policy: Evidence from Disaggregated US Data, American Economic Review, 99 (1): 350-84.

[38] Burnside, C., M. Eichenbaum and J. Fisher (2004), Fiscal shocks and their consequences, Journal of Economic Theory, Elsevier, vol. 115(1), pages 89-117, March.

[39] Caggiano, G., E. Castelnuovo, V. Colombo, and G. Nodari (2015), Estimating Fiscal Multipliers: News From A Non?linear World, Economic Journal, Royal Economic Society, vol. 0(584), pages 746-776, May.

[40] Caggiano, G., E. Castelnuovo, and N. Groshenny (2014), Uncertainty shocks and unemployment dynamics in U.S. recessions, Journal of Monetary Economics, Elsevier, vol. 67(C), pages 78-92.

[41] Caldara, D., and C. Kamps (2017), The Analytics of SVARs: A Unified Framework to Measure Fiscal Multipliers, The Review of Economic Studies, Volume 84, Issue 3, July 2017, Pages 1015?1040.

[42] Caldara, D., and E. Herbst (2019), Monetary Policy, Real Activity, and Credit Spreads: Evidence from Bayesian Proxy SVARs, American Economic Journal: Macroeconomics, 11 (1): 157-92.

[43] Canova, F. (2007), Methods for Applied Macroeconomic Research, Princeton University Press: Princeton.

[44] Canova, F., and M. Ciccarelli (2013), Panel Vector Autoregressive Models: A Survey, ECB Working Paper No. 1507.

[45] Canova, F. and G. De Nicolò (2002), Monetary Disturbances Matter for Business Fluctuations in the G-7, Journal of Monetary Economics, 49, 1131-1159.

[46] Canova, F., and L. Gambetti (2009), "Structural Changes in the U.S. Economy: Is There a Role for Monetary Policy?," Journal of Economic Dynamics and Control, 33, 477-490.

[47] Canova, F., D. López-Salido and C. Michelacci (2010), The Effects of Technology Shocks on Hours and Output: A Robustness Analysis, Journal of Applied Econometrics, 25,755-773.

[48] Canova, F., and E. Pappa (2007), Price Dispersions in Monetary Unions: The Role of Fiscal Shocks, Economic Journal, 117, 713-737.

[49] Canova, F., and M. Paustian (2011), Measurement with Some Theory: Using Sign Restrictions to Evaluate Business Cycle Models, forthcoming: Journal of Monetary Economics.

[50] Carriero, A., H. Mumtaz, K. Theodoridis and A. Theophilopoulou (2015), The Impact of Uncertainty Shocks under Measurement Error: A Proxy SVAR Approach, Journal of Money, Credit and Banking, vol 24, issue 6.

[51] Cavallo, M. (2005), Government employment and the dynamic effects of fiscal policy shocks, Working Paper Series 2005-16, Federal Reserve Bank of San Francisco.

[52] Chari, V.V., P. Kehoe and E. McGrattan (2008), Are structural VARs with long-run restrictions useful in developing business cycle theory?, Journal of Monetary Economics, Elsevier, vol. 55(8), pages 1337-1352, November.

[53] Christiano, L., M. Eichenbaum, and C. Evans (1996), The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds, The Review of Economics and Statistics, MIT Press, vol. 78(1), pages 16-34, February.

[54] Christiano, L., M. Eichenbaum, and C. Evans (1999), Monetary Policy Shocks: What Have We Learned and to What End?, in: J.B Taylor and M. Woodford (eds.), Handbook of Macroeconomics, 1, Elsevier: North Holland, 65-148.

[55] Christiano , M. Eichenbaum and R. Vigfusson (2003), What Happens After a Technology Shock?, NBER Working Papers 9819, National Bureau of Economic Research, Inc.

[56] Christiano , M. Eichenbaum and R. Vigfusson (2004), The Response of Hours to a Technology Shock: Evidence Based on Direct Measures of Technology, Journal of the European Economic Association, MIT Press, vol. 2(2-3), pages 381-395, 04/05.

[57] Cochrane, J.H. (1994), Permanent and Transitory Components of GNP and Stock Prices, Quarterly Journal of Economics, 109, 241-265.

[58] Cochrane, J.H., and M. Piazzesi (2002), The Fed and Interest Rates: A High-Frequency Identification,American Economic Review, 92, 90-95.

[59] Cogley, T., and T. J. Sargent (2002), Evolving Post-World War II U.S. Inflation Dynamics, NBER Macroeconomics Annual, 16:331-388.

[60] Cogley, T., and T. J. Sargent (2005), Drifts and volatilities: monetary policies and outcomes in the post WWII US, Review of Economic Dynamics, Volume 8, Issue 2, 2005, Pages 262-302.

[61] Coibion, O., Y. Gorodnichenko, L. Kueng, and J. Silvia (2017), Innocent Bystanders? Monetary policy and inequality, Journal of Monetary Economics, Elsevier, vol. 88(C), pages 70-89.

[62] Cooley, T.F., and S. LeRoy (1985), Atheoretical Macroeconometrics: A Critique, Journal of Monetary Economics, 16, 283-308.

[63] Cushman, D.O., and T. Zha (1997), Identifying Monetary Policy in a Small Open Economy under Flexible Exchange Rates, Journal of Monetary Economics, 39, 433-448.

[64] Davis, L.W., and L. Kilian (2011), Estimating the Effect of a Gasoline Tax on Carbon Emissions, forthcoming: Journal of Applied Econometrics.

[65] Dedola, L. and S. Neri (2007), "What Does a Technology Shock Do? A VAR Analysis with Model-Based Sign Restrictions" Journal of Monetary Economics, 54, 512-549.

[66] Del Negro, M, and G.E. Primiceri (2015), Time Varying Structural Vector Autoregressions and Monetary Policy: A Corrigendum, The Review of Economic Studies, Volume 82, Issue 4, October 2015, Pages 1342-1345.

[67] Drautzburg, T., J. Fernández-Villaverde and P. Guerrón-Quintana (2017), Political Distribution Risk and Aggregate Fluctuations, NBER Working Papers 23647, National Bureau of Economic Research, Inc.

[68] Dupaigne, M., and F. Portier (2006), "News" Shocks in International Business Cycles, 2006 Meeting Papers 473, Society for Economic Dynamics.

[69] D'Amico, S., and M. Farka (2011), The Fed and the Stock Market: An Identification Based on Intraday Futures Data, Journal of Business and Economic Statistics, 29, 126-137.

[70] Edelberg, W., M. Eichenbaum and J. Fisher (1999), Understanding the Effects of a Shock to Government Purchases, Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 2(1), pages 166-206, January.

[71] Eichenbaum, M., and J. Fisher (2005), Fiscal Policy in the Aftermath of 9/11, Journal of Money, Credit and Banking, 37, issue 1, p. 1-22.

[72] Ellahie, A., and G. Ricco (2017), Government purchases reloaded: Informational insufficiency and heterogeneity in fiscal VARs, Journal of Monetary Economics, 90, issue C, p. 13-27.

[73] Enders, W., and B. Lee (1997), Accounting for real and nominal exchange rate movements in the post-Bretton Woods period, Journal of International Money and Finance, Elsevier, vol. 16(2), pages 233-254, April.

[74] Fatás, A., and I. Mihov (2001), The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence, CEPR Discussion Papers 2760, C.E.P.R. Discussion Papers.

[75] Faust, J. (1998), The Robustness of Identified VAR Conclusions about Money, Carnegie- Rochester Conference Series on Public Policy, 49, 207-244.

[76] Faust, J., and E. Leeper (1997), When Do Long-Run Identifying Restrictions Give Reliable Results? Journal of Business Economics and Statistics, 15, 345-53.

[77] Faust, J., E. Swanson, and J.H. Wright (2004), Identifying VARs Based on High Frequency Futures Data, Journal of Monetary Economics, 51, 1107-1131.

[78] Favero, C., and F. Giavazzi (2012), Measuring Tax Multipliers: The Narrative Method in Fiscal VARs, American Economic Journal: Economic Policy, 4 (2): 69-94.

[79] Fernald, J. (2007), Trend breaks, long-run restrictions, and contractionary technology improvements, Journal of Monetary Economics, Elsevier, vol. 54(8), pages 2467-2485, November.

[80] Fernández-Villaverde, J., P. Guerrón-Quintana, K. Kuester, J. Rubio-Ramírez (2015), Fiscal Volatility Shocks and Economic Activity, American Economic Review 2015, 105(11): 3352-3384.

[81] Fernández-Villaverde, J., J.F. Rubio-Ramírez, T.J. Sargent, and M.W. Watson (2007), ABCs (and Ds) of Understanding VARs, American Economic Review, 97, 1021-1026.

[82] Feve, P., Matheron, J. and J. G. Sahuc (2009), On the dynamic implications of news shocks, Economics Letters, 102, issue 2, p. 96-98.

[83] Fisher, J. (2006), The Dynamic Effects of Neutral and Investment-Specific Technology Shocks, Journal of Political Economy, University of Chicago Press, vol. 114(3), pages 413-451, June.

[84] Forni, M., L. Gambetti, and L. Sala (2014), No News in Business Cycles, Economic Journal, Royal Economic Society, vol. 124(581):1168-1191, December.

[85] Forni, M., L. Gambetti, and L. Sala (2019) Structural VARs and Non-invertible Macroeconomic Models, Journal of Applied Econometrics, 34(2): 221-246, 2019

[86] Forni, M., D. Giannone, M. Lippi, and L. Reichlin (2009), Opening the Black Box: Structural Factor Models with Large Cross-Sections, Econometric Theory, 25, 1319-1347.

[87] Foroni, C., F. Furlanetto and A. Lepetit (2018), Labor Supply Factors and Economic Fluctuations, forthcoming: International Economic Review.

[88] Francis, N., M. Owyang, J.E. Roush, and R. DiCecio (2010), A Flexible Finite-Horizon Alternative to Long-run Restrictions with an Application to Technology Shocks, mimeo, Federal Reserve Bank of St. Louis.

[89] Francis, N., and V. Ramey (2004), The Source of Historical Economic Fluctuations: An Analysis using Long-Run Restrictions, NBER Working Papers 10631, National Bureau of Economic Research, Inc.

[90] Frisch, R. (1933), Propagation problems and impulse problems in dynamic economics, Economic essays in honour of Gustav Cassel (Allen & Unwin, London) 171-205.

[91] Fry, R. and A. Pagan (2011), Sign Restrictions in Structural Vector Autoregressions: A Critical Review, Journal of Economic Literature, Vol. 49, No. 4 (DECEMBER 2011), pp. 938-960.

[92] Fujita, S. (2011), Dynamics of Worker Flows and Vacancies: Evidence from the Sign Restriction Approach, Journal of Applied Econometrics, 26, 89-121.

[93] Furlanetto, F., F. Ravazzolo and S. Sarferaz (2018), Identification of financial factors in economic fluctuations, forthcoming: Economic Journal.

[94] Galí, J. (1992), How Well Does the IS-LM Model Fit Postwar U.S. Data? Quarterly Journal of Economics, 107, 709-738.

[95] Galí, J. (1999), Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?, American Economic Review, 89, 249-271.

[96] Galí, J., and L. Gambetti (2009), On the Sources of the Great Moderation, American Economic Journal: Macoeconomics, 1(1):26-57.

[97] Galí, J., López-Salido, J.D., and J. Vallés (2007), Understanding the Effects of Government Spending on Consumption, Journal of the European Economic Association, MIT Press, vol. 5(1), pages 227-270, March.

[98] Gertler, M., and P. Karadi (2015), Monetary Policy Surprises, Credit Costs, and Economic Activity, American Economic Journal: Macroeconomics, 2015, 7(1): 44-76.

[99] Giacomini, R., (2013), The relationship between VAR and DSGE models, in Advances in Econometrics, vol. 31.

[100] Giacomini, R., and T. Kitagawa (2018), Robust Bayesian inference for set-identified models, cemmap Working Paper, CWP61/18.

[101] Giannone, D., M. Lenza and L. Reichlin (2008), Business Cycles in the Euro Area, NBER Working Papers 14529, National Bureau of Economic Research, Inc.

[102] Giannone, D., L. Reichlin and L. Sala (2005), Monetary Policy in Real Time, p. 161-224, NBER Macroeconomics Annual 2004, Volume 19, National Bureau of Economic Research, Inc.

[103] Gilchrist, S., and E. Zakrajsek (2012), Credit Spreads and Business Cycle Fluctuations, American Economic Review, American Economic Association, vol. 102(4), pages 1692-1720, June.

[104] Goncalves, S. and L. Kilian (2004), Bootstrapping autoregressions with conditional heteroskedasticity of unknown form, Journal of Econometrics, Elsevier, vol. 123(1), pages 89-120, November.

[105] Gospodinov, N. (2004), Asymptotic Confidence Intervals for Impulse Responses of Near-integrated Processes, Econometrics Journal 7(2): 505-527.

[106] Gospodinov, N., A.Maynard and E. Pesavento (2011), Sensitivity of Impulse Responses to Small Low Frequency Co-movements: Reconciling the Evidence on the Effects of Technology Shocks, Journal of Business & Economic Statistics, 29:4, 455-467.

[107] Granziera, E., H.R. Moon and F. Schorfheide, 2018, Inference for VARs identified with sign restrictions, Quantitative Economics, Econometric Society, vol. 9(3), pages 1087-1121, November.

[108] Gürkaynak, R.S., B. Sack and E. Swanson (2005), Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and

Statements, International Journal of Central Banking, International Journal of Central Banking, 1(1).

[109] Hamilton, J. D. (1985), Historical Causes of Postwar Oil Shocks and Recessions, Energy journal, 6, pp. 97-116.

[110] Hamilton, J. D. (1994), Time series analysis, Princeton University Press.

[111] Hanson, M.S. (2004), The "Price Puzzle" Reconsidered, Journal of Monetary Economics, 51, 1385-1413.

[112] Hubrich, K., and R. J. Tetlow. F. (2015). Financial Stress and Economic Dynamics: The Transmission of Crises, Journal of Monetary Economics, 70, pp.100-115.

[113] Inoue, A., and L. Kilian (2011), Inference on Impulse Response Functions in Structural VAR Models, mimeo, University of Michigan.

[114] Inoue, A., L. Kilian, and F.B. Kiraz (2009), Do Actions Speak Louder Than Words? Household Expectations of Inflation Based on Micro Consumption Data, Journal of Money, Credit, and Banking, 41, 1331-1363.

[115] Ivanov, V. and L. Kilian (2005), A Practitioner's Guide to Lag Order Selection For VAR Impulse Response Analysis, Studies in Nonlinear Dynamics & Econometrics 9(1): Article 2.

[116] Jaimovich, N., and S. Rebelo (2009), Can News about the Future Drive the Business Cycle?, American Economic Review, 99 (4): 1097-1118.

[117] Jarocinski, M., and P. Karadi (2019), Deconstructing Monetary Policy Surprises - The Role of Information Shocks, American Economic Journal: Macroeconomics, forthcoming.

[118] Kilian, L. (1998), Small-Sample Confidence Intervals For Impulse Response Functions, The Review of Economics and Statistics, MIT Press, vol. 80(2), pages 218-230, May.

[119] Kilian, L. (2009), "Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market," American Economic Review, 99, 1053-1069.

[120] Kilian, L. and P. Chang (2000), How accurate are confidence intervals for impulse responses in large VAR models?, Economics Letters, Elsevier, vol. 69(3), pages 299-307, December.

[121] Kilian, L., and H. Lütkephol (2017), Structural Vector Autoregressive Analysis, Cambridge University Press.

[122] Kilian, L., and D.P. Murphy (2011), Why Agnostic Sign Restrictions Are Not Enough: Understanding the Dynamics of Oil Market VAR Models, forthcoming: Journal of the European Economic Association.

[123] Kilian, L., and D.P. Murphy (2014), The Role of Inventories and Speculative Trading in the Global Market for Crude Oil, Journal of Applied Econometrics, John Wiley and Sons, Ltd., vol. 29(3), pages 454-478, April.

[124] Kilian, L., and C. Vega (2011), Do Energy Prices Respond to U.S. Macroeconomic News? A Test of the Hypothesis of Predetermined Energy Prices, Review of Economics and Statistics, 93, 660-671.

[125] Kilian, L., and R.J. Vigfusson (2011a), Are the Responses of the U.S. Economy Asymmetric in Energy Price Increases and Decreases?, mimeo, University of Michigan.

[126] Kilian, L., and R.J. Vigfusson (2011b), Nonlinearities in the Oil Price-Output Relationship, forthcoming: Macroeconomic Dynamics.

[127] King, R.G., C.I. Plosser, J.H. Stock, and M.W. Watson (1991), Stochastic Trends and Economic Fluctuations, American Economic Review, 81, 819-840.

[128] Krolzig, H.M (1997), Markov-Switching Vector Autoregressions : Modelling, Statistical Inference, and Application to Business Cycle Analysis, Berlin: Springer.

[129] Kurmann, A., and C. Otrok (2013), News Shocks and the Slope of the Term Structure of Interest Rates, American Economic Review, 103 (6): 2612-32.

[130] Leeper, E.M. and D.B. Gordon (1992), 'In Search of the Liquidity Effect', Journal of Monetary Economics, 29(3):341-369.

[131] Leeper, E.M., C.A. Sims and T. Zha (1996), What Does Monetary Policy Do? Brookings Papers on Economic Activity, 2, 1-63.

[132] Leeper, E.M., and T. Zha (2003), Modest policy interventions, Journal of Monetary Economics, Elsevier, vol. 50(8), pages 1673-1700, November.

[133] Ludvigson, S.C., and S. Ng (2009), Macro Factors in Bond Risk Premia, The Review of Financial Studies, Volume 22, Issue 12, December 2009, Pages 5027-5067.

[134] Lütkepohl, H. (1990), Asymptotic Distributions of Impulse Response Functions and Forecast Error Variance Decompositions of Vector Autoregressive Models, The Review of Economics and Statistics, 72, issue 1, p. 116-25.

[135] Lütkepohl, H. (2005), New Introduction to Multiple Time Series Analysis. Springer: New York.

[136] McConnell, M.M., and G Perez-Quiros (2000), Output fluctuations in the United States: what has changed since the early 1980s? American Economic Review 90 (5), 1464-76.

[137] Mertens, K., and M. Ravn (2012), Empirical Evidence on the Aggregate Effects of Anticipated and Unanticipated US Tax Policy Shocks, American Economic Journal: Economic Policy, 2012, 4(2): 145-181.

[138] Mertens, K., and M. Ravn (2013), The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States, American Economic Review, 2013, 103(4): 1212-1247.

[139] Miranda-Agrippino and Ricco (2017), The Transmission of Monetary Policy Shocks, Bank of England, Staff Working Paper n. 657.

[140] Moench, E., (2008), Forecasting the yield curve in a data-rich environment: A no-arbitrage factor-augmented VAR approach, Journal of Econometrics, 146, issue 1, p. 26-43.

[141] Monacelli, T., R. Perotti and A. Trigari (2010), Unemployment fiscal multipliers, Journal of Monetary Economics 57 (2010) 531-553.

[142] Morley, J., and J. Piger (2012), The Asymmetric Business Cycle, The Review of Economics and Statistics, 94, issue 1, p. 208-221.

[143] Morley, J., J. Piger and P. Tien (2013), Reproducing business cycle features: are nonlinear dynamics a proxy for multivariate information?, Studies in Nonlinear Dynamics & Econometrics, De Gruyter, vol. 17(5), pages 483-498, December.

[144] Mountford, A., and H. Uhlig (2009), What are the Effects of Fiscal Policy Shocks? Journal of Applied Econometrics, 24, 960-992.

[145] Mumtaz, H., and P. Surico (2008), Evolving international inflation dynamics: evidence from a time-varying dynamic factor model, Bank of England working papers 341, Bank of England.

[146] Nason, J. M., and E. Tallman (2015), Business Cycles And Financial Crises: The Roles Of Credit Supply And Demand Shocks, Macroeconomic Dynamics, Cambridge University Press, vol. 19(04), pages 836-882, June.

[147] Olea, M.,L, J., Stock, and M. Watson (2018), Inference in Structural VARs with External Instruments, mimeo.

[148] Owyang, M., Ramey, V., Zubairy, S. (2013), Are government spending multipliers greater during periods of slack? Evidence from 20th century historical data, American Economic Review Papers and Proceedings, vol. 103(3), pp. 129-34.

[149] Pagan, A.R. and H.M. Pesaran (2008), Econometric analysis of structural systems with permanent and transitory shocks, Journal of Economic Dynamics and Control, Elsevier, vol. 32(10), pages 3376-3395.

[150] Pagan, A.R., and J.C. Robertson (1998), Structural Models of the Liquidity Effect, Review of Economics and Statistics, 80, 202-217.

[151] Pagan, A.R., and T. Robinson (2016), Investigating the Relationship Between DSGE and SVAR Models, NCER Working Paper Series 112, National Centre for Econometric Research.

[152] Pappa, E. (2009), The Effects of Fiscal Shocks on Employment and Real Wages, International Economic Review, 50, 217-244.

[153] Perotti, R. (2005), Estimating the effects of fiscal policy in OECD countries, Proceedings, Federal Reserve Bank of San Francisco.

[154] Perotti, R. (2007), In Search of the Transmission Mechanism of Fiscal Policy, NBER Macroeconomics Annual 2007, Chicago University Press: 2007, Vol. 22, pp. 169-226

[155] Pesaran M.H., T. Schuermann and S.M. Weiner (2004), Modeling Regional Interdependencies Using a Global Error-Correcting Macroeconometric Model, Journal of Business and Economic Statistics, 22:129-162.

[156] Pesaran, M.H., and Y. Shin (1998), Generalized impulse response analysis in linear multivariate models, Economics Letters, 58, issue 1, p. 17-29.

[157] Pesavento, E., and B. Rossi (2006), Small-sample confidence intervals for multivariate impulse response functions at long horizons, Journal of Applied Econometrics, John Wiley & Sons, Ltd., vol. 21(8), pages 1135-1155.

[158] Phillips P.C., (1998), Impulse response and forecast error variance asymptotics in nonstationary VARs, Journal of Econometrics, 83(1-2):21-56.

[159] Primiceri, G.E. (2005), Time Varying Structural Vector Autoregressions and Monetary Policy, Review of Economic Studies, 72, 821-852.

[160] Ramey, V.A. (2011), Identifying Government Spending Shocks: It's all in the Timing, The Quarterly Journal of Economics, Oxford University Press, vol. 126(1), pages 1-50.

[161] Ramey, V.A. (2016), Macroeconomic Shocks and Their Propagation, Handbook of Macroeconomics.

[162] Ramey, V.A., and M.D. Shapiro (1998), Costly capital reallocation and the effects of government spending, Carnegie-Rochester Conference Series on Public Policy, Elsevier, vol. 48(1), pages 145-194, June.

[163] Ramey, V.A., and S. Zubairy (2018), Government Spending Multipliers in Good Times and in Bad: Evidence from US Historical Data, Journal of Political Economy, University of Chicago Press, vol. 126(2), pages 850-901.

[164] Ravenna, F. (2007), Vector Autoregressions and Reduced Form Representation of Dynamic Stochastic General Equilibrium Models, Journal of Monetary Economics 54:7.

[165] Romer, C., and D. Romer (2004), A New Measure of Monetary Shocks: Derivation and Implications, American Economic Review, American Economic Association, vol. 94(4), pages 1055-1084, September.

[166] Romer, C., and D. Romer (2010), The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks, American Economic Review, American Economic Association, vol. 100(3), pages 763-801, June.

[167] Rotemberg, J., M. Woodford (1992), Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity, Journal of Political Economy, University of Chicago Press, vol. 100(6), pages 1153-1207, December.

[168] Rubio-Ramírez, J.F., Waggoner, D., and T. Zha (2010), Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference, Review of Economic Studies, 77, 665-696.

[169] Rudebusch, G. (1998), Do Measures of Monetary Policy in a VAR Make Sense? International Economic Review, 39, 907-931.

[170] Runkle, D. (1987), Vector Autoregressions and Reality, Journal of Business & Economic Statistics, 5(4), 437-442.

[171] Schmitt-Grohé, S., and M. Uribe (2012), What's News in Business Cycles, Econometrica, Econometric Society, vol. 80(6), pages 2733-2764, November.

[172] Scholl, A. and H. Uhlig (2008), New Evidence on the Puzzles: Results from Agnostic Identification on Monetary Policy and Exchange Rates, Journal of International Economics, 76, 1-13.

[173] Shapiro, M.D., and M.W. Watson (1988), Sources of Business Cycle Fluctuations, in S. Fischer (ed.), NBER Macroeconomics Annual 1988, Cambridge, MA: MIT Press, 111-148.

[174] Sims, C.A. (1980), Macroeconomics and Reality, Econometrica, 48, 1-48.

[175] Sims, C.A. (1992), Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy, European Economic Review, 36, 975-1011.

[176] Sims, C. , Stock, J.H., and M.W. Watson (1990), Inference in Linear Time Series Models with Some Unit Roots, Econometrica, 58(1):113-44.

[177] Sims, C.A., and T. Zha (1999), Error Bands for Impulse Responses, Econometrica, 67: 1113-1155.

[178] Sims, C. A., and T. Zha, (2006a). Does monetary policy generate recessions?, Macroeconomic Dynamics, 10(02), pp. 231-272.

[179] Sims, C.A., and T. Zha (2006b), Were There Regime Switches in U.S. Monetary Policy?, American Economic Review, 96, 54-81.

[180] Sims, E.R. (2011), Permanent and Transitory Technology Shocks and the Behavior of Hours: A Challenge for DSGE Models, manuscript.

[181] Stock, J.H., Watson, M.W., (2002), Forecasting Using Principal Components from a Large Number of Predictors, Journal of the American Statistical Association 97:1167-1179.

[182] Stock, J.H., and M.W. Watson (2012), Disentangling the Channels of the 2007-09 Recession, Brookings Papers on Economic Activity, 2012, vol. 43, issue 1 (Spring), 81-156.

[183] Stock, J.H., and M.W. Watson (2018), Identification and Estimation of Dynamic Causal Effects in Macroeconomics Using External Instruments, The Economic Journal, vol 128(610), pages 917-948.

[184] Slutzky, E. (1927), The Summation of Random Causes as the Source of Cyclic Processes, Problems of Economic Conditions, ed. by The Conjuncture Institute, Moskva (Moscow), Vol. 3, No. 1, 1927.

[185] Strongin, S. (1995), The Identification of Monetary Policy Disturbances: Explaining the Liquidity Puzzle, Journal of Monetary Economics, 35, 463-498.

[186] Tenreyro, S., and G. Thwaites (2016), Pushing on a String: US Monetary Policy Is Less Powerful in Recessions, American Economic Journal: Macroeconomics 2016, 8(4): 43-74.

[187] Teräsvirta, T., D. Tjostheim, and C. W. J. Granger (2010), Modelling Nonlinear Economic Time Series, Oxford University Press.

[188] Uhlig, H. (2004), Do Technology Shocks Lead to a Fall in Total Hours Worked?, Journal of the European Economic Association, MIT Press, vol. 2(2-3), pages 361-371, 04/05.

[189] Uhlig, H. (2005), What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure, Journal of Monetary Economics, 52, 381-419.

[190] Vigfusson, R. (2004), The delayed response to a technology shock: a flexible price explanation, International Finance Discussion Papers 810, Board of Governors of the Federal Reserve System (U.S.).

[191] Weale, M., and T. Wieladek (2016), What are the macroeconomic effects of asset purchases?, Journal of Monetary Economics, Elsevier, vol. 79(C), pages 81-93.

[192] Wright, J. (2000), Confidence Intervals for Univariate Impulse Responses with a Near Unit Root, Journal of Business and Economic Statistics 18(3): 368-373.