UNIVERSITÀ DEGLI STUDI DI TORINO

POLITECNICO DI TORINO

Department of Mathematics “Giuseppe Peano”

PhD in Pure and Applied Mathematics

XXXII Cycle

Dissertation

Automatic Formative Assessment in a Digital Learning Environment for learning and teaching Mathematics

Alice Barana

Tutor:
Marina Marchisio

Co-tutor:
Renato Grimaldi

CONTENTS

ACRONYMS

The following acronyms and abbreviations will be used throughout the thesis.

AAS	Automatic Assessment System
ACE	Advanced Computing Environment
AFA	Automatic Formative Assessment
AT	Activity Theory
BE	Behavioral Engagement+
CE	Cognitive Engagement
CPMP	Core-Plus Mathematics Project
DLE	Digital Learning Environment
EE	Emotional Engagement
FA	Formative Assessment
ICT	Information and Communications Technologies
INVALSI	Istituto Nazionale di Valutazione del Sistema di Istruzione (National Institute for the evaluation of the educational system)
IWB	Interactive White Board
LA	Learning Analytics
LMS	Learning Management System
OCSE	Organization for the Cooperation and Economic Development
PISA	Programme for International Student Assessment
PP&S	Problem Posing and Solving
SD	Standard Deviation
STEM	Science, Technology, Engineering and Mathematics
SY	School Year

PREFACE

This thesis work arises from the PhD research entitled "Development of digital methodologies for learning scientific disciplines in Virtual Learning Environments integrated with systems for advanced computing, automated assessment, and simulation." The research touched upon several research themes, such as using digital technologies in Mathematics Education at the secondary level, ranging from formative assessment to online learning, from modelling competence to engagement, moving through automatic assessment and learning analytics. It is the result of three years of work carried out within the DELTA (Digital Education for Learning and Teaching Advances) Research Group of the University of Turin, coordinated by prof. Marina Marchisio. The work started with studying theories and models of the literature about digital learning environments, automatic formative assessment, mathematical modelling, and engagement. This study led to the definition of Digital Learning Environments (DLE) and the development of models to create and analyze automatic assessment activities in a DLE. These kinds of activities, in particular, were experimented through two didactic experimentations, which were organized and developed within the Project "Educating City: teaching and learning processes in cross-media ecosystem" by the Delta Research Group in collaboration with the National Research Council. The Italian Ministry of Education funded the project in the frame of the National Technological Cluster "Technologies for Smart Communities", aimed at rethinking the learning processes by applying the newest advances in educational technologies. The research work was carried out between October 2016 and September 2019; the thesis writing was interrupted due to maternity leave and was concluded in Summer 2020. In this period, the world was upset by the COVID-19 pandemic, which provoked the schools' closure in the majority of countries for several weeks and even months. This shuttering event gave prompts to the research on new methodologies and instruments for smart schooling and online education. It compelled the educational world to thoroughly review teaching and learning methodologies. All the models and methodologies discussed within this thesis were conceived before the COVID-19 pandemic. However, they can – and have been – adapted and used to support online teaching and learning processes in crisis time.

We presented some of the results of this research at the following conferences:

1. 4th Conference on Smart Learning Environments and Regional Development (SLERD 2019). ASLERD. Roma, 22-24 May 2019.
2. 11th International Conference on Computer Supported Education (CSEDU 2019). INSTICC - University of Heraklion, 2-4 May 2019.
3. Eleventh Congress of the European Society for Research in Mathematics Education (CERME11). Utrecht University. Utrecht, 5-10 February 2019.
4. Technology Enhanced Assessment (TEA) Conference. Vrije Universiteit Amsterdam. Amsterdam, 10-11 December 2018.
5. I dati INVALSI: uno strumento per la ricerca, Bari, 26-28 October 2018.
6. I dati INVALSI: uno strumento per la ricerca. Firenze, 17-18 November 2017.
7. 2017 IEEE 41st Annual Computer Software and Applications Conference (COMPSAC). Politecnico di Torino, 4-8 July 2017.
8. HEAd'17 - 3nd International Conference on Higher Education Advances. Università Politecnica di Valencia, 21-23 June 2017

Moreover, some results of the research were published in the following papers, parts of which are reported in this thesis:

Accepted and in press

1. Barana, A., Marchisio, M. An interactive learning environment to empower engagement in Mathematics. Accepted for publication in the Journal IxD&A – Interaction Design and Architecture(s).
2. Barana, A., Marchisio, M., Miori, R. MATE-BOOSTER: Design of Tasks for Automatic Formative Assessment to Boost Mathematical Competence. Submitted for the inclusion in a volume edited by Springer including the extended selected papers from the Conference "Computer Supported Education" (CSEDU2019). (CSEDU2019).
3. Barana, A., Boffo, S., Gagliardi, S., Garuti, R., Marchisio, M. Un percorso digitale per l'apprendimento della matematica. Accepted for publication in the journal Scuola Democratica.
4. Barana, A., Boffo, S., Gagliardi, F., Marchisio, M. Problem Posing and Solving: a digital way to learn Mathematics. Accepted for publication in a Springer volume, Springer Short series.
5. Barana, A & Marchisio, M. Dalle formule ai modelli. Un percorso interattivo con le domande INVALSI. Presented at III Conference "I dati INVALSI: uno strumento per la ricerca" (Bari, 26-28 October 2018) and accepted for publication in a volume.

Published

Journal's article

6. Barana, A., Conte, A., Fissore, C., Marchisio, M., Rabellino, S. (2019). Learning Analytics to improve Formative Assessment strategies. Journal of e-Learning and Knowledge Society, 15(3), 75-88.
7. Barana, A., & Marchisio, M. (2016). Ten Good Reasons to Adopt an Automated Formative Assessment Model for Learning and Teaching Mathematics and Scientific Disciplines. Procedia - Social and Behavioral Sciences, 228, 608–613.

Book's chapters

8. Barana, A. & Marchisio, M. Le prove INVALSI per lo sviluppo di competenze matematiche e di problem solving (2020). In: P. Falzetti (ed.) Il dato nella didattica delle discipline. II seminario "I dati INVALSI: uno strumento per la ricerca" (pagg. 29-49). INVALSI per la ricerca, FrancoAngeli, Milano.
9. Barana, A., Boffo, S., Gagliardi, F., Garuti, R., Marchisio, M. (2020) Empowering Engagement in a technology enhanced learning environment. In: Rehm M., Saldien J., Manca S. (eds) Project and Design Literacy as Cornerstones of Smart Education (pagg. 75-77). Smart Innovation, Systems and Technologies, vol 158. Springer, Singapore.
10. Barana, A., Marchisio, M., Sacchet, M. Advantages of Using Automatic Formative Assessment for Learning Mathematics (2019). In: Draaijer S., Joosten-ten Brinke D., Ras E. (eds), Technology Enhanced Assessment. TEA 2018. Communications in Computer and Information Science, vol 1014. Springer, Cham.
11. Barana, A., Boffo, S., Gagliardi, F., Garuti, R., Marchisio, M., & Zich, R. (2018). Percorsi interattivi supportati dalle ICT per l'apprendimento della matematica attraverso il problem solving. In A. Raffone, La Città Educante: Metodologie e tecnologie a servizio delle Smart Communities (pagg. 115–128). Napoli: Liguori Editore.

Conference Proceedings

12. Barana, A., Fissore, C., Marchisio, M. (2020). From standardized assessment to automatic formative assessment for adaptive teaching. Proceedings of the 12th International Conference on Computer Supported Education, 1, 285-296.
13. Barana, A. & Marchisio, M. (2019). Strategies of formative assessment enacted through automatic assessment in blended modality. U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), Proceedings of Eleventh Congress of the European Society for Research in Mathematics Education, pp.

4041-4048. Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

14. Barana, A., Marchisio, M., Rabellino, S. (2019). Empowering Engagement through automatic formative assessment. Proceedings of 43rd IEEE International Computer Software and Applications Conference - COMPSAC 2019, 216-225.
15. Barana, A., Casasso, F., Marchisio, M. (2019). BYOD per imparare l'algebra in maniera interattiva. Atti di Didamatica 2019, 87-96.
16. Barana, A., Marchisio, M., & Miori, R. (2019). MATE-BOOSTER: Design of an e-Learning Course to Boost Mathematical Competence. Proceedings of the 11th International Conference on Computer Supported Education (CSEDU 2019), 1, 280–291.
17. Barana, A., Conte, A., Fioravera, M., Marchisio, M., & Rabellino, S. (2018). A Model of Formative Automatic Assessment and Interactive Feedback for STEM. Proceedings of 2018 IEEE 42nd Annual Computer Software and Applications Conference (COMPSAC), 1016–1025.
18. Barana, A., Fioravera, M., Marchisio, M., & Rabellino, S. (2017). Adaptive Teaching Supported by ICTs to Reduce the School Failure in the Project “Scuola Dei Compiti”. Proceedings of 2017 IEEE 41st Annual Computer Software and Applications Conference (COMPSAC), 432–437.
19. Barana, A., Fioravera, M., & Marchisio, M. (2017). Developing problem solving competences through the resolution of contextualized problems with an Advanced Computing Environment. Proceedings of the 3rd International Conference on Higher Education Advances, 1015–1023.

Abstracts in volume or journal

20. Marchisio, M., Barana, A., Fissore, C., Floris, F., Pulvirenti, M., Rabellino, S., Roman, F., Sacchet, M., Salusso, D. (2019). Learning Analytics per la valutazione formativa. Convegno internazionale SIRD SIe-L: Learning Analytics. Per un dialogo tra pratiche didattiche e ricerca educativa. E-book integrale degli interventi, 29-30.
21. Barana, A., Casasso, F., Marchisio, M. (2019). BYOD per imparare l'algebra in maniera interattiva. Mondo Digitale, 17(83).
22. Barana, A., Boffo, S., Gagliardi, F., Garuti, R., Marchisio, M., Zich, R. (2019). Interactive Pathway for an inclusive Mathematics Education. 1st International Conference of the Journal «Scuola Democratica»: Book of Abstracts, 373.

1 INTRODUCTION

The fast technological evolution in the last decade has brought about significant changes in many jobs and professions' content and methodology. This revolution requires a thorough review, not only of the school curricula but also of the didactic methodologies in the learning processes. According to a recent report (World Economic Forum, 2018a), the speed with which the education world will be able to evolve is one of the critical variables that will determine the labor world overview, therefore influencing the worker's position. Literacy in the STEM subjects, with Mathematics at their core, is becoming fundamental to keeping up with the technological development that affects workplaces (World Economic Forum, 2018b). Developing a Mathematical understanding is essential to comprehend and find innovative and competitive solutions to the rising issues that society's rapid evolution brings along (van der Wal et al., 2017).

For this reason, even the European Union, through the European Strategy 2020 and the Digital Agenda for Europe, set the objective of broadly introducing the ICT at a school level to increase the efficiency of didactics and to improve the learning process. Simultaneously, the acquisition of digital competences should be guaranteed; they are useful not only in a discipline aspect but also for an appropriate exercising of the citizenship rights in a more and more digital world. In Italy, there is a delay in this field. Above all, there is a significant delay regarding the spreading of the ICT, especially regarding the widespread distribution of hardware and software among the whole school system, among its divisions in grade and territory (Avvisati et al., 2013; Fondazione Giovanni Agnelli, 2010; Vivanet, 2013). On the other hand, many Italian students suffer from a delay in the learning of scientific subjects. It is shown in the learning tests periodically carried out by the INVALSI (Italian Institute for the evaluation of the education system) as part of the OCSE PISA (Organization for the Cooperation and Economic Development, Programme for International Student Assessment) ordered by the Ministry of Education in Italy (INVALSI, 2015). Finally, it is essential to point out the still existing delay in the spreading of the didactic methodologies aimed at using, in an appropriate way, the considerable potential of the ICT at an educational level.

In this context, this research thesis comes to life. In the following paragraphs, we will introduce the central problematics at the core of the research and the experimentation. A principal research question guides the study: "How an automatic assessment system can improve the teaching and learning of STEM disciplines, and in particular didactic methodologies as formative assessment?". The study about automatic assessment systems in Mathematics and scientific disciplines was the original issue that inspired the PhD research, which started addressing innovative methodologies for automatic assessment. We declined this principal research question in twelve specific questions through several experiments and analyses. They will be tackled in this thesis's following parts; they allowed us to draw results and conclusions.

Part of this chapter's paragraphs are taken from the following publications: (Barana, Conte, et al., 2018; Barana, Marchisio, & Sacchet, 2019)

1.1 PROBLEM SOLVING AND MODELLING

Problem posing and problem solving play a fundamental role in the process of teaching and learning Mathematics and in promoting a process of mathematization and modeling of reality (Baroni & Bonotto, 2015). As explained in many Italian national documents, Mathematics teaching should start with the use of language and mathematical reasoning as tools for interpreting reality and not only as baggage of notions (Anichini et al., 2003). Problem solving is a characteristic of mathematical practice. It has to be considered as an authentic and meaningful task, often linked to everyday life. It is not only a repetitive exercise to answer by simply recalling a definition or a rule (*Indicazioni Nazionali per Il Curricolo Della Scuola Dell'infanzia e Del Primo Ciclo d'istruzione*, 2012). The recent update of the European framework of the key competences for lifelong learning (European Parliament and Council, 2018) upholds the importance of the acquisition of a solid mathematical competence, which consists in the *"ability to develop and apply mathematical thinking and insight in order to solve a range of problems in everyday situations*. *Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge*. *The mathematical competence involves the ability and willingness to use mathematical modes of thought and presentation (formulas, models, constructs, graphs, charts)*". Therefore, it promotes didactic activities of inductive, experimental, and laboratory types that contribute to competence development.

Thus, all European citizens should acquire the ability to understand and use mathematical models to maintain an active role in the ever-changing society and current tumultuous times. Logic-mathematical skills are a critical prerequisite for access to numerous jobs, and they are verified in the selection process to the admission to study, training, specialization, or recruitment paths. A STEM (Science, Technology, Engineering, and Mathematics) degree offers good opportunities for employability in many sectors. The recruitment rate of graduated in STEM is ever-growing, notwithstanding the crisis that has hit the global economy for the last ten years. Moreover, the ability to analyze from a quantitative point of view data and situations improves the perception of reality, and it offers instruments to act within it, both in the working sphere and in daily life (OECD, 2016a).

The national and international learning surveys locate the ability to work with mathematical models at the highest levels of competence scale. In Italy, since 2010, INVALSI has been in charge of evaluating the Italian educational system and its capacity to train competent citizens. The standardized tests of Mathematics, which INVALSI administers to all Italian students at specific grades of education, verify that they can *"find solutions to various problems in daily situations, enacting mathematical thinking."* Similarly to OCSE PISA surveys, a scale of 5 competence levels has been developed to classify the students' results to computer-based tests; an analytic description has been elaborated, identifying what students can typically do at each level. The analytic description of the INVALSI levels of competence for grade 10 locates at level 4 out of 5 those students who can build a mathematical model with which to operate. At a level 3, students just manage to identify several forms of representation of the same mathematical object. Only those who achieve the 5th and highest level of competence demonstrate the ability to build a suitable mathematical model to solve problems in non-ordinary situations and correctly interpret information: it means that the percentage of students able to reach these objectives is relatively low (INVALSI, 2018). The qualitative description of the INVALSI levels of competence for Mathematics is consistent with the OCSE PISA's one, developed after the 2015 survey. It locates the 15-year students able to work with mathematical models in complex situations at a level 5 on a scale of 6 competence levels. In contrast, at level 4 out of 6, students can work only with explicit models and match different forms of representation (OECD, 2017). Table 1.1 shows a comparison of INVALSI and OCSE-PISA analytic descriptions of competence levels; sentences referring to modelling competence are highlighted.

There are many studies on the usefulness of INVALSI items and tests for the research in Mathematics education (Bolondi et al., 2018; Cascella et al., 2020). Answers to the INVALSI items can only partially measure competences' acquisition, since they can unlikely evidence aptitudinal and metacognitive aspects. On the other side, many of the questions proposed in the standardized tests, particularly those referring to "solving problems" dimensions, can constitute interesting contexts for the development of competences, if expanded and used in a formative way.

Table 1.1. Comparison between analytic descriptions of INVALSI levels for grade 10 and 2015 OCSE-PISA levels of competence. Sources: (INVALSI, 2018; OECD, 2017)

INVALSI (Grade 10)		**OCSE PISA (15-years old)**	
Levels	**Description**	**Levels**	**Description**

1	The student uses elementary knowledge and base abilities mainly acquired in previous school grades. **S/he answers to questions having simple formulations, referred to typical school situations** (e.g. concerning the identification and reading of data represented in different forms).	1	At Level 1, students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. **They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations**. They can perform actions that are almost always obvious and follow immediately from the given stimuli.
2	The student knows the principal base notions included in the national guidelines for grade 10 and carries out elementary procedures. **S/he solves problems in real-world contexts that require mathematical knowledge acquired in the previous school grades**. […]	2	Students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can **employ basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers. They are capable of making literal interpretations of the results.**
3	The student uses the base abilities acquired in grade 10 and connects the fundamental notions among them. S/he answers to questions that require simple reasonings starting from given information and data. S/he identifies the elements and properties of the principal mathematical objects […]. **S/he recognizes several representations of a mathematical object** (such as decimal numbers, fractions, and percentages).	3	Students can execute clearly described procedures, including those that require sequential decisions. **Their interpretations are sufficiently sound to be a base for building a simple model or for selecting and applying simple problem-solving strategies**. Students at this level can interpret and use representations based on different information sources and reason directly from them. […] Their solutions reflect that they have engaged in basic interpretation and reasoning.
		4	Students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. **They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations.** Students at this level can utilise their limited range of skills and can reason with some insight, in straightforward contexts. […]
4	Even in non-ordinary cases, the student knows the main mathematical objects […] tackled at grade 10 and efficiently uses the acquired knowledge. S/he interprets data linking facts and using several representations in different contexts**. S/he builds a mathematical model with which to operate, also using the symbolic language of Mathematics**. […]	5	**Students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models**. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insight pertaining to these situations. They begin to reflect on their work and can formulate and

			communicate their interpretations and reasoning.
5	The student confidently uses the conceptual and procedural aspects of the most important topics included in the national guidelines for grade 10**. S/he answers to questions that refer to non-ordinary and complex situations for which it is necessary to build an adequate model and to interpret information, activating strategies and reasonings**. **S/he uses different representations of the mathematical objects and confidently shifts from one to another. In the problem solving process s/he gathers even non-explicit relations among the available data.** […]	6	**Students can conceptualise, generalise and utilise information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts**. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. **These students can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for attacking novel situations.** Students at this level can reflect on their actions, and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situation.

1.2 ASSESSMENT AND LEARNING TECHNOLOGIES

Since the origins of humanity, various kinds of technologies have always supported learning. Nowadays, with electronic tablets replacing clay ones, we wonder about their effectiveness in improving teaching and learning quality. Evidence-based researches show that the use of a computer does not impact learning. The effect-size of computer-based instruction calculated by Hattie in his huge work of meta-analysis is 0.37, a not particularly high value, if compared with the 0.7 or 0.8 of the most effective methodologies (J. A. C. Hattie, 2009). Even lower is the effect-size of web-based learning, which halts at 0.18. However, Hattie noticed that when digital technologies were associated with different learning strategies, student-centered approaches, or teacher's training, the effect-size increased significantly. This means that digital technologies, as well as other instructional tools, are not effective per se, but they acquire relevance according to the way they are used. For instance, several findings in the research show that technologies can generate positive effects on students, both from a cognitive and from a metacognitive point of view, when they are used with appropriate methodologies (Drijvers et al., 2016).

One of the features of digital technology which makes them different from other kinds of technologies and is identified as a critical promoter of learning, especially in the research in Mathematics Education, is its interactive nature: stimulated by the student's action, a digital tool reacts and returns some information (E. Duval et al., 2017; Olive & Makar, 2010). Thus, with computers, it is possible to create materials that respond in several ways to the learner, giving prompts and feedback. When processed by the learner, this kind of feedback can be relevant for several issues, such as fostering deep reflection, putting the learner at the center of the learning process, and helping students become responsible for their learning (Heid, 1997). Under this perspective, the automatization of feedback can represent a considerable advantage. Digital learning materials with automatic assessment can be valuable resources since they are easily sharable by teachers, accessible by students, and offer interesting learning experiences to improve understanding. It is widely acknowledged that assessment has a great influence on learning, impacting on when and how students work and learn. In particular, formative assessment practices help develop understanding and motivation, encouraging positive attitudes toward learning (Black & Wiliam, 1998). Being responsive to the users' actions, digital technologies can make new room for formative assessment: with their capabilities of computing grades and offering feedback in real-time, they can return information to students and teachers relevant to support and enhance learning processes (Bennett, 2012). Moreover, web-based applications make it possible to interact among peers or with the instructor, even when they cannot meet physically. When these features are deployed, there is evidence that digital environments positively impact motivation and self-confidence, which are essential factors in promoting life-long learning.

The use of digital learning environments for assessment enables the generation and collection of data about learning agents (such as teachers, students, tutors), processes, and results. These data can be used to drive an adjust the learning path, make choices and decisions, and support learning in several ways. This is the study object of learning analytics, a recent research field, defined as "*the measurement, collection, analysis and reporting of data about learners and their contexts, for purposes of understanding and optimizing learning and the environments in which it occurs*" (*LAK 2011*, 2011).

Some issues can be raised on the students' access to technologies that potentially hinder the widespread adoption of digital tools in education. First of all, we must take into account the equity issue: digital tools should support every student's opportunity to learn fundamental Mathematics, as well as other subjects. Equity mainly involves socio-economic circumstances, but also differences in physical capabilities, gender, or teachers' development should not prevent students from getting quality instruction (Stacey & Wiliam, 2013). Equity in access to education is desirable and should be guaranteed to everyone. However, it is not always sufficient: every student should have the opportunity to be deeply engaged in the educational system as a key to achieving formative success (OECD, 2016b). Secondly, the complexity of technological tools should be suitable to the students' digital skills. Some findings suggest that "digital native" students do not easily transfer their skills in browsing social networks to fluent use of technologies in a learning context (ECDL Fundation, 2014). Lastly, while the students commonly appreciate the use of technology at school, there is still a small percentage of pupils that prefer a paper-and-pen approach to learning. The little appreciation of computer-aided instruction seems to be correlated to the dislike of computers in general (Mitra & Steffensmeier, 2000) or the little perceived usefulness of the learning activities with that particular instrument (Armstrong, 2011).

1.3 Online learning

The context of this research is mainly online learning, which, especially during the recent pandemic of COVID-19, has become a widespread issue. However, too often, it is confused with distance education. According to Borba et al. (2018), both are educational methods that occur partially or entirely at different times and/or spaces. However, in distance education, communication between the people involved is carried out through television and mailings, while online education occurs via the Internet and associated technologies. According to Kreber and Kanuka (2013), when we talk about online learning and teaching, the term "online" is used to describe learning activities that take place using computer networks or mobile devices to provide access to learning materials, activities, and support. Online courses currently use many asynchronous and synchronous Internet communication technologies and social software (such as Skype; blogs; Wikis; podcasts; instant messaging). Nevertheless, we believe that the best technology for online education is Learning Management Systems (LMS), such as Moodle, FirstClass, Blackboard, etc. An LMS is the infrastructure that delivers and manages instructional content, identifies and assesses individual and organizational learning or training goals, tracks the progress towards meeting those goals, and collects and presents data for supervising an organization's learning process as a whole (Watson & Watson, 2007). Online learning has become an essential vehicle for student learning (especially in higher education). However, especially in the context of Mathematics teaching, there is little research on its effectiveness in strengthening preservice teachers' subject-matter knowledge and pedagogical skills (Wachira & Keengwe, 2020). Many

Mathematics educators continue to prepare Mathematics teachers in traditional classrooms, and there is a tendency to move traditional educational practices into the online environment (Kreber & Kanuka, 2013).

Online teaching and learning cannot be a mere transfer of face-to-face lessons in synchronous online mode via web conference and a simple transmission of materials, tasks, and exercises. Interaction is a central element in online education: it can occur among participants or with the learning materials, and the technologies often mediate it. It can be synchronous or asynchronous, letting the students organize their study time according to their needs (Borba et al., 2018). We conceive online education as a form of education that mainly consists of resources and asynchronous activities that are always available, which students can carry out when they can and when they prefer. Online learning allows students to study from home, respecting their times, and independently organizing the study's schedule. Resources and activities can be multimedia and can result from the integration of different media to facilitate students' understanding and personalization based on each person's characteristics.

1.4 Problem Posing and Solving Project

In 2012 in Italy, a big project was born with the challenge of bringing innovation to the teaching and learning of Mathematics, aiming at integrating problem posing and solving, technologies, and online learning in the daily teaching practices. The Problem Posing and Solving (PP&S) Project was promoted by the Italian Ministry of Education, in collaboration with the University and the Polytechnic of Turin, the IIS Carlo Anti of Villafranca di Verona (VR) as pole school, as well as other promoters. The project fosters the training of Italian teachers of lower and upper secondary schools on innovative teaching methods, supported by the use of ICTs and the creation of a culture of Problem Posing and Solving (Brancaccio, Marchisio, Palumbo, et al., 2015; C. Demartini et al., 2013; Demartini et al., 2015).

The PP&S project makes available for free to all teachers in secondary schools in Italy an integrated Moodle platform (developed by the Department of Computer Science - ICT Services of the University of Turin) with an Advanced Computing Environment (ACE), an Automatic Assessment System (AAS) and a web conference system (Barana, Brancaccio, Conte, et al., 2019a, 2019b; Barana, Conte, et al., 2018). The Project proposes innovative learning methods such as problem posing and solving using an ACE, automatic formative assessment with immediate interactive feedback, and collaborative learning within a DLE. Each teacher participates in the project with one or more classes; for each class, the teachers have a DLE available to customize and to use as they prefer. The project was initially aimed only at teachers of the STEM disciplines. However, during the emergency from COVID-19, it was opened to teachers of all disciplines. It now involves 1861 teachers, more than 500 schools, 2079 classes, and about 40,000 students. A peculiar feature of the project is the continuous training of teachers on the proposed methodologies and technologies. All the teachers of the project become part of the community of teachers of the PP&S. They can collaborate (sharing ideas, experiences, or doubts through forums and sharing materials and activities), participate in training activities, self-train, and find many didactic materials ready to use.

1.5 Didactic experimentation

To study the problems discussed in this chapter, a didactic experimentation was designed and activated, with the idea of using interactive activities with automatic formative assessment in blended modality as part of the daily practice in 8th, 9th, and 10th-grade classes and evaluating the effectiveness of the innovative methodologies under different perspectives. The experimentation invested the whole 2017/2018 School Year (SY); it was preceded by a pilot experiment the previous SY, whose main results helped us orient and design the main experimentation. In particular, an interactive pathway aimed at developing Mathematics and modelling competences with special attention to the inclusion of students with special needs or challenging backgrounds. We designed the pathway according to problem posing and solving methodology and automatic formative assessment, and implemented it in a digital learning environment. The interactive pathway was experimented in the City of Turin in the 2017/2018 SY, involving six schools chosen in different city areas, characterized by different social and economic contexts. The digital materials were used during the Mathematics lessons in 13 classes (299 students), while other 11 classes (247 students) from the same schools composed the control group, going through the same topics in a traditional way. The digital materials were proposed in the classrooms via group-work supported by the Interactive White Board (IWB); automatically assessed assignments with interactive feedback were assigned as homework. Learning improvements were measured through pre and post-test. The effects of the methodologies on engagement and motivation were measured through questionnaires at the beginning and at the end of the experimentation. Some of the activities were held in the presence of the PhD student author of this thesis, and they were videotaped. Teachers were involved in the design and use of the materials through focus groups. The experimentation results have been analyzed under several perspectives: the growth of learning results, the enacted formative assessment processes, the development of mathematical and modelling competences, and the increasing of engagement. The inquiry has been conducted following the learning analytics' direction to study the learning processes and understand the learning methodologies' effectiveness.

The parents signed a consent form to use data of the students registered in the classroom and the platform for didactic and research purposes only. In reporting the results, we changed the names of the students to respect their anonymity.

1.6 PLAN OF THE THESIS

The thesis is structured as follows.

In the second chapter, we will present the theoretical framework for the research. It includes the discussion of the main results of the literature and original models developed during the research work. In particular, we will define and outline a model of Digital Learning Environment. We will discuss the main findings from the literature about formative assessment, automatic assessment, and feedback before introducing and discussing our model of Automatic Formative Assessment and its implications with Learning Analytics in a Digital Learning Environment. Moreover, we will present some theories about mathematical modelling which will be the experimentation's mathematical context. Lastly, we will discuss the construct of engagement and its relations with learning technologies.

The third chapter addresses the pilot experimentation: we will discuss its aims and the related research questions; we will present the methodology of experimentation and the instruments of assessment in detail; we will examine the interactive learning activities and discuss the results to answer the research questions.

The fourth chapter tackles the main experimentation design: the goal of the research and the research questions are presented, and the methodology of research is outlined, with particular reference to the instruments of evaluation (learning tests and questionnaires). We will show the interactive learning path based on Automatic Formative Assessment in a Digital Learning Environment and problem solving through meaningful activities.

In the fifth chapter, the results of the main experimentations are presented and discussed, under four main perspectives, which are: the growth in the learning results made possible by interactive learning path; the enactment of formative assessment strategies during the experimental activities, the development of mathematical and modelling competences through the problem solving and automatically assessed activities; and the increasing of the levels of engagement, especially for the initially less engaged students. The sixth chapter reports the conclusion of this research with implications for future work.

2 THEORETICAL FRAMEWORK

In this chapter, we will present theories and models on which we will frame our research. Many of those theories come from the literature and are grounded on findings and results of various studies; we have developed some models (the model of Digital Learning Environment, the model of Automatic Formative Assessment) within this thesis work. We will start discussing Activity Theory, a sociocultural learning theory on which we base much of our research. The analysis of the different names and definitions given to Digital Learning Environments (DLE) in literature will lead to our definition and characterization of DLE. In particular, the characteristics of interactive DLEs are discussed. In the second place, the most shared findings of the literature on formative assessment, online assessment, and feedback will be discussed to support the development of a model of Automatic Formative Assessment and interactive feedback, which will be fully described and validated with quantitative results. Then we will discuss the mutual relationship between Automatic Formative Assessment in a DLE and learning analytics. Moreover, the focus goes to mathematical modelling and how to learn it, discussing the main difficulties students may face when they start from the algebraic language and "translate" formulas into different representational registers. Lastly, we will present and examine a framework to analyze engagement and its relation to interactive technologies.

2.1 ACTIVITY THEORY

We begin the chapter introducing Activity Theory (AT), a sociocultural learning theory on which we will ground the theoretical framework developed in this thesis. AT is often used to frame studies about the use of digital technologies for learning.

Opposite to the traditional positions according to which activity must follow learning, AT assumes that learning emerges from human activity, it is rooted in activity, and is fundamental for the development of knowledge (Jonassen & Rohrer-Murphy, 1999).

An activity cannot be analyzed or understood outside the context in which it occurs. The historical context, the people engaged in, their aims and goal, the community in which they are immersed, and the rules that govern the situation must be considered when examining an activity.

When analyzing an activity, the smallest unit of analysis is an activity system represented in Figure 2.1. Its conceptualization changed over the decades: it originates from Vygotsky's idea of *mediated action*, which explains the semiotic process that enables human consciousness development through interaction with artifacts, tools, and social others in an environment and results in individuals to find new meanings in their world (Yamagata-Lynch, 2010). It is the so-called "first-generation Activity Theory", a name that considers the following developments in which the theory evolved. "Second-generation Activity Theory" refers to Leontiev's work, which developed the concept of mediated action and proposed a theory on *object-oriented activity.* It only considers the upper triangle of the schema in Figure 2.1 and views activity as intentional processes, driven by goals and motivation, in which subjects interact with the environment. Both subjects and objects are transformed during the experience (Yamagata-Lynch, 2010). Subjects can be either individuals or groups of actors engaged in the activity. The object of the activity is the physical or mental product that is pursued, and it represents the subject's intentions. The mediating artifacts can be material tools as well as abstract media used by the subject during the activity (D. H. Jonassen & Rohrer-Murphy, 1999). As an example, we can consider the activity of a student (the subject) who is learning (object). The mediating artifact could be a book, a digital technology, or a learning methodology.

In the "third-generation Activity Theory", Engeström added the lower part of the biggest triangle in Figure 2.1, underlying the contextualization of the activity into a social experience, in particular in a community, which is governed by rules that define the division of labor, which is how tasks are shared among the participants to the community (Yrjo Engeström, 1987). The outcome is the ultimate goal that motivates the activity. In the previous example of student learning, the community consists of classmates and teachers. The rules are those which define the classwork, and the division of labor corresponds to the support that the teacher or peers give to the learner. Moreover, Engeström (1987) theorizes that learning occurs through expansion when the interactions between the elements face some contradictions, and the systems modify themselves. In addition, more systems can interact to produce an outcome. For this reason, AT is a powerful tool to analyze both learning and working activities (Yrjo Engeström, 2000).

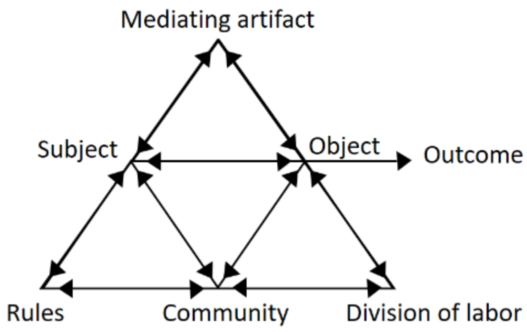


Figure 2.1 One activity system, the unit of analysis of activity in activity theory. Adapted from (Yrjo Engeström, 1987)

2.2 Digital Learning Environments

According to Wilson (1995), a learning environment is "*a place where learning is fostered and supported*". It includes at least two elements: the learner, and a "*setting or space wherein the learner acts, using tools and devices, collecting and interpreting information, interacting perhaps with others, etc*." (Wilson, 1995). The traditional learning environment that everyone knows is the classroom, where the teacher teaches, students learn, individually or with their peers, using tools such as paper, pen, and a blackboard. The diffusion of technology transformed this traditional learning environment by adding digital tools, as tablets or computers, and the IWB. Besides equipping physical places with technologies, the technological revolution brought to the creation of a new learning environment, situated in a non-physical dimension: that of the Internet, accessible from everywhere via computers, tablets, or even smartphones. This is the essence of the "Digital Learning Environment" (DLE); besides the learner and a setting, which can be virtual, a device is needed to access the activities.

The concept of "Digital Learning Environment" has a long history, and it has known several developments and many different names over the years.

Suhonen (2005) defined Digital Learning Environments as *"technical solutions for supporting learning, teaching and studying activities."* Some years before, Abdelraheem (2003) spoke about "Computerized Learning Environments" (CLEs), which are "*systems that provide rich databases, tools, and resources to support learning and information seeking and retrieval, as well as individual decision making*." Abdelraheem's definition is more detailed than Suhonen's one, but the essence is similar: disseminating learning materials through the Internet. CLEs, or DLEs, emphasize empowerment through metaknowledge, which individuals invoke and refine while attempting to use their learning tasks. Other authors use the term "Online Learning Environments" as Khan (1997) who defined them as "*hypermedia based instructional [systems], which utilizes the attributes and resources of the World Wide Web to create a meaningful learning environment where learning is fostered and supported*". Other scholars speak about "Virtual Learning Environments" referring to a particular type of Learning Environment where "*students interact primarily with other networked participants, and with widely disseminated information tools"* (Wilson, 1996). Here the interaction among learners is emphasized, and it is considered the most powerful key of learning.

The common factor among all these definitions is the use of the Internet and its tools to provide an environment where learning is supported, generally represented by a Learning Management System (LMS).

An LMS, according to Watson and Watson (2007), is "*the infrastructure that delivers and manages instructional content, identifies and assesses individual and organizational learning or training goals, tracks the progress towards meeting those goals, and collects and presents data for supervising the learning process of an organization as a whole*." While similar environments are mainly used to support online educational processes, we are convinced and have proof of the fact that web-based platforms can also be successfully adopted in classroom-based settings: in our conception, DLEs should not only be confined to distance education (Barana, Casasso, & Marchisio, 2019; Barana, Fioravera, Marchisio, et al., 2017; Barana & Marchisio, in pressa; Borba et al., 2018).

More recently, many authors have developed an interest in conceptualizing digital learning environments as ecosystems, borrowing the term from ecology (Guetl & Chang, 2008). According to Encyclopaedia Britannica (www.britannica.com), an ecosystem is *"a complex of living organisms, their physical environment, and all their interrelationships in a particular unit of space."* The natural ecosystem, constituted by a biological community in a physical environment, is the fundamental example; however, this definition can be applied to any domain, even artificial environments, by specifying the living community, environment, and space unit. According to Uden, Wangsa, and Damiani (2007), a digital ecosystem is *"a self-organizing digital infrastructure aimed at creating a digital environment for networked organizations that supports the cooperation, the knowledge sharing, the development of open and adaptive technologies and evolutionary business models"*. The digital components of digital infrastructure are software, applications, tools, knowledge, training modules, *"any useful idea, expressed by a language (formal or natural), digitalized and transported within the ecosystem, and which can be processed by humans or by computers"* (Uden et al., 2007). They define an e-learning ecosystem as "*all the components required to implement an e-learning solution*". The idea of ecosystem highlights the complexity of the relationships among the involved components, which evolve adapting to the variations of conditions; it is a more holistic approach than VLE or LMS. Following the same trend, García-Holgado and García-Peñalvo (García-Holgado & García-Peñalvo, 2018) define technological ecosystems as "*software solutions based on the integration of heterogeneous software components through information flows in order to provide a set of services that each component separately does not offer, as well as to improve the user experience.*" On this base, they define learning ecosystems as "*technological ecosystems focused on learning and knowledge management in different contexts such as educational institutions or companies*". This definition, though being more focused on user experience, underlines the interactions among the system's components, which are necessary for the proper functioning of the system itself.

There are several models of learning or e-learning ecosystems in the literature, which vary for the components included on the bases of the theoretical assumptions considered. In general, they contemplate individuals, computer-based agents, communities, and organizations in a network of relations and exchanges of data that supports the co-evolutions and adaptations of the components themselves (Guetl & Chang, 2008).

Following this trend, in this thesis, we chose to use the term "Digital Learning Environment" to indicate a learning ecosystem in which teaching, learning, and the development of competence are fostered in classroom-based, online or blended settings. It is composed of a human component, a technological component, and the interrelations between the two.

The human component consists of one or more learning communities whose members can be: teachers or tutors, students or learners, and their peers, the administrators of the online environment.

The technological component includes:

- a Learning Management System, together with software, other tools, and integrations which accomplish specific purposes of learning (such as web-conference tools, assessment tools, sector-specific software, and many others);

- activities and resources, static or interactive, which can be used in synchronous or asynchronous modality;
- technological devices through which the learning community has access to the online environment (such as smartphones, computers, tablets, Interactive White Board);
- systems and tools for collecting and recording data and tracking the community's activities related to learning (such as sensors, eye-trackers, video cameras.

The interrelations between the two components can be:

- the learning processes activated within the community and through the use of the technologies;
- pedagogies and methodologies through which the learning environment is designed.

Figure 2.2 shows a graphical representation of the components of a DLE. The community is in the middle in a human-centered approach to learning. In the ecology metaphor, it is the complex of living organisms, while the technological component surrounds the community, as the physical environment. The arrows linking the community and the technologies represent the learning processes as well as the pedagogies and methodologies used to design the learning materials and interpret data from the digital environment. They are double-ended to indicate the reciprocal relationships between the two components, which bring to the development of all the parts.

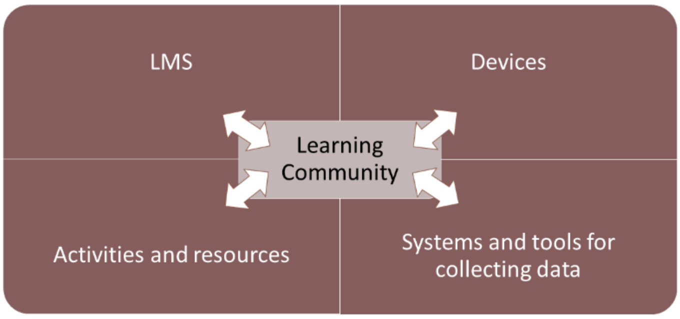


Figure 2.2 Schema of the components of a DLE.

Independently of the fact that the DLSs are based on a web-based platform, teaching and learning can occur in one of the following modalities:

- face to face, in the classroom or a computer lab, with students working autonomously or in groups through digital devices, or solving tasks displayed on the Interactive White Board with paper and pen or other tools;
- entirely online, using the DLE as the only learning environment in online courses or MOOCs;
- in a blended approach, using online activities to integrate classroom work, such as asking students to complete them as homework.

These three modalities can be adapted to different situations, grades, aims, and needs. For example, the face-to-face modality can be suitable with students of lowest grades and in scholastic situations where the classroom work is predominant. The blended approach can offer useful support to the face-to-face lessons at secondary school or university. Online courses are generally used for training and professional courses, university courses or learning in sparse communities, where face-to-face meetings are difficult to organize (Abdelraheem, 2003).

In this conceptualization, the DLE is not limited to the technological artifacts, even if they play a crucial role. The learning community takes a prominent place: it can include, according to the kind of DLE, students and peers, teachers and tutors (who are facilitators of learning activities), designers of educational materials, and administrators of the digital environment. There can also be more communities involved or a community of communities: it happens, as an example, in the Problem Posing and Solving Project where, hosted in an integrated LMS, there are many communities of students, one for each class participating to the project, and the community of all the teachers of the classes (C. G. Demartini et al., 2015). In this case, the communities of students are based on learning and teaching intentions, while the community of the teachers pursues the development of competence related to teaching with innovative technologies and didactic methodologies. The teachers' work in their students' communities allows them to practice the competences which are fostered through training courses in the teachers' community. In this case, the students' communities work in blended modality, and their online activities are mainly accomplished asynchronously, while the teachers participate in an online synchronous and asynchronous training (Barana, Fissore, Marchisio, et al., 2020; Brancaccio et al., 2014).

Activity Theory can be used to conceptualize and design a learning environment, especially when it involves using technologies (Yrjo Engeström, 2009; D. H. Jonassen & Rohrer-Murphy, 1999). In the AT framework, we can conceive a DLE as an activity system, or better, a system of interactive activity systems. The subject can be the teacher, the tutor, or the student. The activity's object can range from designing learning materials to supporting learning or learning itself, depending on the specific situation under analysis. According to the situation, the mediating artifacts are the digital tools or materials available in the learning environment and the chosen learning methodologies. The activity is immersed in and influenced by the learning community,thath lives in the learning environment, with its rules and methods forlabore divisior. When considering online learning, the rules are those which regulate computer-mediated collaborative learning, while in classroom settings, they are the school regulations and the norms of living together. Figure 2.3 shows two examples of activity systems through which we can schematize activities in a DLE.

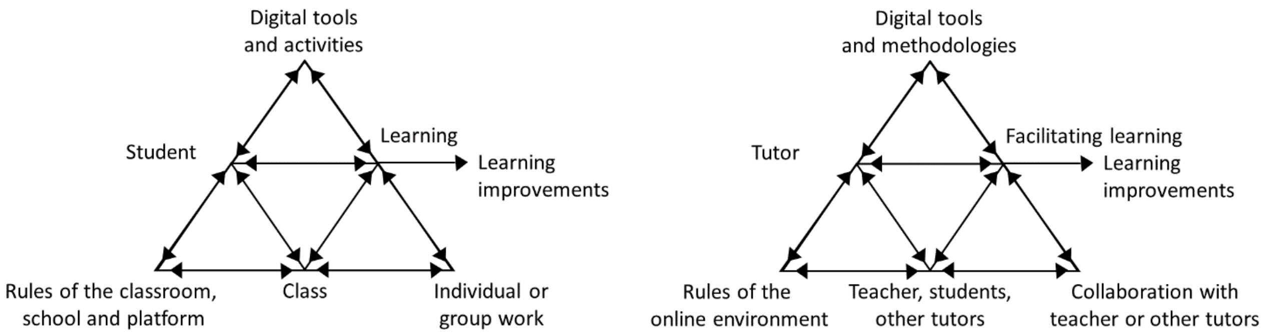


Figure 2.3 Two examples of activity systems that can constitute a DLE.

This definition of DLE does not disagree with the other definitions collected from the literature. However, it is more comprehensive: it is not limited to the web-based platform, which conveys the activities and is a relevant and essential part of a DLE; it also includes a "human" part.

The use of these technologies, such as web-based platforms, assessment tools, and other systems such as sensors or eye-trackers, allows for collecting, recording, and using learning data. These data can be elaborated within the DLE to provide information useful to make decisions and take action. In the following paragraphs, we will explain how these data can be used to improve learning, teaching, and the development of competences.

There is an in-depth discussion on the real effectiveness of DLEs (and their synonyms) that involves many researchers. For instance, in the paper "Media will never influence learning", Clark (1994) claims that the use of technologies, per se, is not more effective than traditional learning unless a learning theory supports it. The chosen learning theory should be coherent with the aims of the materials or the course and should guide the materials' design. Clark and Mayer (2008) analyzed the effect sizes gained in several studies that compare digital and traditional education. The average effect size is not much different from zero, meaning that digital tools are not better than paper and pen. However, they noticed that there are many cases where the effect size is considerably large: this means that digital technologies have great potential. When they are used following suitable principles, they can make a difference in education.

2.2.1 Interactive Learning Environments

Since the Eighties of the last century, many instructional designers have argued that the development of learning materials should be made on constructivist assumptions, according to which knowledge is situated, it being a product of the activity, context, and culture in which it is developed and used (Brown et al., 1989). Constructivist theories consider learning as a lifelong active process of knowledge building mediated by experiences and relations with the environment and the community (von Glasersfeld, 1989). Constructivist Learning Environments are defined as *"places where learners may work together and support each other as they use a variety of tools and information resources in their pursuit of learning goals and problem-solving activities"* (Wilson, 1995). Thus, students should be engaged through authentic and collaborative activities and real-world problems. We notice that Constructivist Learning Environments can be both physical and digital environments; technology is a mediating artifact, which can affect learning, but it is not an essential component of the learning environment. In Mathematics Education, this issue has been investigated by many researchers. According to A. Schoenfeld, Mathematical thinking should be a tool to interpret quantitative phenomena of the outside world and it should be developed at school through meaningful modelling activities (A. Schoenfeld, 1992)

One of the main implications of the constructivist idea of the learner creating his or her knowledge is the shift from a teacher-centered to a student-centered approach. If students become the protagonists, teachers need to leave the stage and move aside, changing their role from leaders to mentors. Consequently, their task moves from knowledge transmission to creating a suitable learning environment (Cornelius-White, 2007). The community in which the learner is integrated is a core element as well. The sharing of opinions opens the mind and favors the process of knowledge building. Thus, in a constructivist learning environment, activities that facilitate collaboration and require discussion and interaction among peers should take place (Lave, 1991).

From constructivism, the idea of Interactive Learning Environment originated; we consider "interactive" a learning environment where the student's action is encouraged and where what happens next depends on this action (Moreno & Mayer, 2007). In this perspective, the term "interactive" can be opposed to "transmissive". An interactive learning environment can be both physical – where the student learns through the interactions with peers – and virtual, populated not (or at least, not only) by "static" resources as texts and videos, but by activities that react to the user's action giving feedback.

Following this trend, Grabinger and Dunlap (1995) defined "Rich Environments for Active Learnings" (REALs) as comprehensive instructional systems that

- evolve from and are consistent with constructivist philosophies and theories;
- promote study and investigation within authentic (i.e., realistic, meaningful, relevant, complex, and information-rich) contexts;
- encourage the growth of student responsibility, initiative, decision-making, and intentional learning;
- cultivate an atmosphere of knowledge-building learning communities that utilize collaborative learning among students and teachers;
- utilize dynamic, interdisciplinary, generative learning activities that promote high-level thinking processes (i.e., analysis, synthesis, problem-solving, experimentation, creativity, and examination of topics from multiple perspectives) to help students integrate new knowledge with old knowledge and thereby create rich and complex knowledge structures; and,

- assess student progress in content and learning-to-learn through realistic tasks and performances.

REALs, or Interactive Learning Environments, can favor learning by acting in different directions, as the following ones: (Grabinger & Dunlap, 1995):

- promoting study in authentic contexts;
- encouraging the sense of responsibility, initiative, intentional learning of students;
- encouraging collaborations among students and teachers;
- using dynamic and interdisciplinary didactic activities that promote high-order thinking processes to help students to develop complex structures of knowledge;
- offering the chance of formative assessment and personalized feedback;
- assessing students' progress in authentic contexts using real-world tasks.

Technology can support the creation of interactive environments: it can provide computer-mediated communication, computer-supported collaborative work, case-based learning environments, computer-supported cognitive tools (D. Jonassen et al., 1995), as well as instruments for self and peer assessment (Kearns, 2012) and automatic evaluation (Barana et al., 2015).

The analysis of the implementation of digital constructivist learning environments has involved many authors in literature in the last thirty years. Several models have been designed to engage students of different school levels, in e-learning or blended modality, in learning several disciplines (Alonso et al., 2005; Czerkawski & Lyman, 2016; Lefoe, 1998; Sangsawang, 2015). Their results mainly deal with the relations between strategies, media, and tools used and processes activated. Constructivist instructional designers generally accept as a useful and well-established framework for building learning environments the seven learning goals devised by Cunningham, Duffy, and Knuth in 1993 and illustrated by Honebein (Honebein, 1996); they are:

1. to provide experience with the knowledge construction process;
2. to provide experience in and appreciation of multiple perspectives;
3. to embed learning in realistic and relevant contexts;
4. to encourage ownership and voice in the learning process;
5. to embed learning in social experience;
6. to encourage the use of multiple modes of representation; and
7. to encourage self-awareness in the knowledge construction process.

Part of paragraphs 2.2 and 2.2.1 is drawn, extended and adapted from (Barana, Marchisio, & Miori, 2019).

2.3 Formative Assessment, Automatic Assessment, Feedback

Formative assessment is one of the most important methodologies that should be included in an interactive DLE. In the following paragraphs, we will present its definition and characterization. Part of the following paragraphs (2.3.1-2.3.7) is drawn, extended and adapted from (Barana, Conte, et al., 2018; Barana, Marchisio, & Sacchet, 2019; Barana & Marchisio, 2019).

2.3.1 Formative Assessment

The term "formative evaluation" was coined by Michael Scriven in 1967 in opposition to "summative evaluation", to describe a practice aimed to collect information during a course in order to develop the curriculum (Scriven, 1967). The term was borrowed by Benjamin Bloom one year later to indicate a strategy for mastery learning, namely a set of diagnostic-progress tests that should assess the small units' achievement in which the program was divided (Bloom, 1968). According to Bloom, this strategy should motivate students to forge ahead with the learning path; each test should ensure that the set of learning tasks included in the unit is completely mastered before moving to the next one.

In 1989 D. Royce Sadler (Sadler, 1989) contributed to the definition between summative and formative assessment (FA) – a term that, nowadays, is preferred to evaluation (Wiliam, 2006) – theorizing that one key distinction lies in feedback. Sadler conceptualizes formative assessment as how learners use information from judgments about their work to improve their competence. According to Sadler, the distinction between formative and summative evaluation is not a matter of timing, but relies on purpose and effects.

Since the nineties up today, the concern about formative assessment has grown to cover one of the major educational research issues. The contributes of Paul Black and Dylan Wiliam stood out in the development of a theoretical framework for the formative assessment. The definition they gave, well-accepted in literature, is the following (Black & Wiliam, 2009):

"Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited."

This definition entails the collection of evidence, which can be gathered through tasks or questions, and the interpretation and use of the pieces of information gathered to act on learning. According to this definition, the mere collection of students' answers, without using them to make decisions to tailor their learning path, is not to be considered as a formative assessment. Wiliam made it clear in another paper in 2006 (Wiliam, 2006), where he stated that "*assessments are formative […] if and only if something is contingent on their outcome, and the information is actually used to alter what would have happened in the absence of the information*". To use the information gathered from assessment during teaching, it is essential to create *moments of contingence*. They are points in the instructional sequence where the instruction can change direction in light of evidence about the students' achievement. This allows teachers to adapt the instruction to meet better students' learning needs (Wiliam, 2006). These moments can be synchronous, including teachers' real-time adjustments during a classroom discussion after students' answers, or asynchronous, such as the use of evidence from students' homework to plan the next lesson. It seems that asynchronous moments of contingence are less effective, maybe due to the scarce experience of teachers with formative practices, or because of the use of inadequate tasks, or even due to the teachers' pressure to complete the program, to the disadvantage of students' understanding (Wiliam, 2006).

Black and Wiliam (2009) identified three agents that are principally activated during formative practices: the teacher, the student, and peers. All three agents can be the subject of the decision-making process which follows the collection of evidence and which is at the core of formative assessment. Depending on the learning environment and conditions, the three can be more or less involved in the assessment process stages. Black and Wiliam (2009) further developed a framework of formative assessment, individuating three different processes of instruction, that are the following:

- establishing where the learners are in their learning;
- establishing where they are going;
- establishing what needs to be done to get them there.

Moreover, the researchers theorized five key strategies enacted by the three agents during the three different processes of instruction:

- KS1. clarifying and sharing learning intentions and criteria for success;
- KS2. engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
- KS3. providing feedback that moves learners forward;
- KS4. activating students as instructional resources; and
- KS5. activating students as the owners of their own learning.

2.3.2 Feedback

The provision of feedback is only one of many strategies for formative assessment; nonetheless, it is probably the most distinctive and object of in-depth studies. The power of feedback emerges in Hattie's metanalysis: with an effect size of 0.73, it results in one of the most effective strategies for learning (J. A. C. Hattie, 2009). However, in the literature, results on feedback efficacy on learning are controversial (Azevedo & Bernard, 1995); for instance, one of the most surprising results that emerged from Kluger and DeNisi's review on feedback is that in more than one-third of the 607 analyzed cases (effect sizes), feedback interventions reduced performance (Kluger & DeNisi, 1996). This means that much attention should be paid to the feedback and task's design.

In the light of the results of their metanalysis, John Hattie and Helen Timperley conceptualized feedback as *"information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding"* (J. Hattie & Timperley, 2007). In that context, they provided a model for constructing effective feedback. The purpose of feedback is to reduce the discrepancy between current and desired understanding, and it can be fulfilled both by students and by teachers. Effective feedback must answer three main questions: *"Where am I going?", "How am I going?", "Where to next?".* In other words, they should indicate what are the learning goals (Feed Up); what progress is being made toward the goal (Feed Back); and what activities need to be undertaken to make better progress (Feed Forward). The three questions correspond to the three processes identified by Black and Wiliam in the model of formative assessment (establishing where the learners are in their learning, where they are going, and what needs to be done to get them there).

Feedback can work at four levels:

- task level, giving information about how well the task has been accomplished;
- process level, showing the main process needed to perform the task;
- self-regulation level, activating metacognitive process;
- self-level, adding personal evaluations and affects about the learner.

A major concern raised by many authors is that learners often do not go through feedbacks. If the learners do not process them, feedbacks lose all their potentialities (D. J. Nicol & Macfarlane-Dick, 2006; Timmers & Veldkamp, 2011). Sadler introduced the idea that feedback only works when it is used to alter the gap between current and reference performance (Sadler, 1989). If the information is not or cannot be processed by the learner to produce improvements, it will not affect learning. For feedback to be effective, students have to

(a) possess a concept of the standard (or goal, or reference level) being aimed for;
(b) compare the actual (or current) level of performance with the standard; and
(c) engage in appropriate action, which leads to some closure of the gap.

These conditions should be fulfilled when feedback is inserted in the context of formative assessment as theorized by Black and Wiliam.

Black and Wiliam argued that positive words of appreciation that concern the self-level can encourage the learner to process the whole feedback and use the information gained (Black & Wiliam, 2009). Since feedbacks at the self-level, as praises and rewards, are shown to have a minimal effect (J. A. C. Hattie, 2009), or even negative (Kluger & DeNisi, 1996), on learning, this is the reason why they are included in a framework of good practices.

In practice, how should good feedback be? The first feature that emerges from the literature is that feedbacks on the mere correctness of an answer, which only says yes or no, acting only at the task level, are not much effective: instead, they should provide more details about the correct solution (Chung et al., 2006; Shute, 2008). However, a simple explanation of the correct solution belongs to a transmissive educational style. It does not meet the interactive learning environment features, as it does not actively engage the student. "*A good feedback causes thinking*", Black and Wiliam affirm in their paper "In praise of educational research: formative assessment" (Black & Wiliam, 2003). In an interactive learning environment, feedback should activate students who have to do something with it, elaborate it, and discuss it to link it to prior knowledge (D. Nicol, 2019). In this perspective, feedback should be an interactive process, a formative interaction that involves the learner and the feedback's agent (teacher, peers, or students themselves) and which influences cognition (Black & Wiliam, 2009). Such interactive feedback can work at the process level helping students to understand how the task should be solved.

Feedback provides students with some external information; during this interactive process, this information should help students generate inner feedback, which can be used to understand and fill the gap between current and desired performance (D. Nicol, 2019). In other words, external feedbacks should be transformed into internal feedbacks to affect subsequent learning; internal feedbacks can activate self-regulation.

Self-regulation is such an important skill to be listed in the key competences for lifelong learning by the European Council (European Parliament and Council, 2018). According to Pintrich and Zusho, *"self-regulated learning is an active constructive process whereby learners set goals for their learning and monitor, regulate, and control their cognition, motivation, and behavior, guided and constrained by their goals and the contextual features of the environment"* (Pintrich & Zusho, 2007). Self-regulated learners are self-confident and resourceful when approaching a task. They are aware of whether they know something or not. They are systematic in their study and find a way to succeed in unfavorable conditions (Zimmerman, 1990). When engaged in learning activities, self-regulated learners generate internal feedback that monitors their choices, and they actively interpret external feedback. When they are well-thought, external feedback can facilitate the development of the processes of internal monitoring and self-regulation. David Nicol and Debra Macfarlane-Dick have identified seven key principles that feedback should satisfy to enhance self-regulation (D. J. Nicol & Macfarlane-Dick, 2006). Good feedback practice:

1. helps clarify what good performance is (goals, criteria, expected standards);
2. facilitates the development of self-assessment (reflection) in learning;
3. delivers high-quality information to students about their learning;
4. encourages teacher and peer dialogue around learning;
5. encourage positive behaviors, motivation, and self-esteem;
6. provides opportunities to close the gap between current and desired performance;
7. provides information to teachers that can be used to help shape the teaching.

2.3.3 Automatic Formative Assessment in a Digital Learning Environment

The definition of FA that we have mentioned before can be adapted to consider the technologies' contribution. Pachler et al. (2010) define formative e-assessment as "*the use of ICT to support the iterative process of gathering and analyzing information about student learning by teachers as well as learners and of evaluating it in relation to prior achievement and attainment of intended, as well as unintended learning outcomes*." This definition highlights the role of ICT as a support for the process of formative assessment. It is open to several modalities of using the technologies (face to face, blended, and online). In the same vein, Gikandi, Morrow, and Devis (2011) define online formative assessment as "*the application of formative assessment within learning online and blended settings where the teacher and learners are separated by time and/or space and where a substantial proportion of learning/teaching activities are conducted through web-based ICT*." In this definition, formative assessment is intended as "*the iterative processes of establishing what, how much and how well students are learning in relation to the learning goals and expected outcomes in order to inform tailored formative feedback and support further learning, a pedagogical strategy that is more productive when the role is shared among the teacher, peers and the individual learner*". The latter definition excludes the cases in which online assessment tools are used in the classroom with the presence of the teacher. This not uncommon scenario allows enacting formative assessment and adaptive teaching strategies (Barana, Fioravera, Marchisio, et al., 2017).

Several forms of assessment can be included in these definitions, and the literature reports many good examples (e-portfolios, self-assessment, peer assessment, …) (Gikandi et al., 2011; Hallam & Glanville, 2007; van den Bogaard & Saunders-Smits, 2007). However, we mainly focus on automatic assessment, where the technology is used to analyze the students' answers and to return feedback. According to Kluger and DeNisi (1996), computerized feedback is more effective than human-delivered ones: this is one advantage of practicing formative assessment in e-learning. We define Automatic Formative Assessment (AFA) as the use of formative assessment in a Digital Learning Environment through the automatic elaboration of students' answers and provision of feedback, where formative assessment is intended as in the Black and Wiliam's definition.

Cusi, Morselli, and Sabena (2017) developed the FAsMEd (Formative Assessment in Mathematics Education) framework for explaining formative assessment with connected classroom technologies. Connected classroom technologies are networked systems of personal computers or handheld devices specifically designed to be used in classrooms for interactive teaching and learning. They identified three functionalities of the technologies, which can support the three agents (teacher, student, and peers) in developing the five key strategies for formative assessment: sending and delivering, processing and analyzing, and providing an interactive learning environment. They represented these three elements (agents, strategies, and functionalities) in a three-dimensional model that was used to analyze formative assessment strategies in a connected classroom. In this thesis, we try to develop a similar model, considering that we start from different assumptions. Firstly, our model should consider AFA activities, while Cusi, Morselli, and Sabena's model is more generally adaptable to formative assessment activities using technologies. Moreover, we need a framework that can explain the development of activities in face-to-face, blended and online settings, both individual and collaborative, while the FAsMEd model is conceived to analyze only face-to-face collaborative activities. Lastly, the technological apparatus is different for the two projects: we consider a DLE while they use connected classroom technology.

From the perspective of activity theory (Asghar, 2013), we can contextualize AFA in an activity system (Figure 2.4). In particular, we can consider the activity where the object is performing AFA and where the subjects are, in turn, the students, the teachers, and peers. The strategies of formative assessment identified by Black and Wiliam are mediating artifacts through which the action is completed. In this framework, the technologies are mediating artifacts as well. The outcome is the improvement in learning and, according to AT, it can be the result of the activities carried out by at least two activity systems. Rules, community, and division of labor are typical of the environment where the action occurs (a classroom, an online environment), which varies based on the modality of use of the technology (face-to-face, blended or online). When we also consider other more specific activities that occur during formative assessment, as the activity of enacting one of the key strategies of formative assessment, such as providing feedback that moves the learner forward. In this case, the strategy is the object of the action, the technology used is the mediating artifact and enacting formative assessment is the outcome. It is useful to analyze the formative assessment activities according to this model, as it helps to distinguish what causes learning. According to the AT, when the interactions between the elements face some contradictions, the systems modify themselves through expansion, and this provides learning (Yrjö Engeström, 2001).

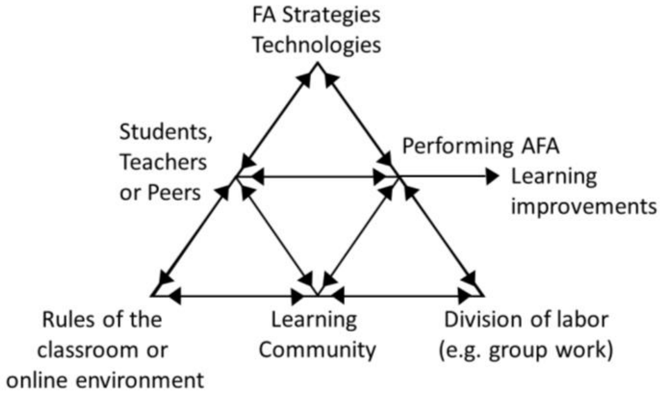


Figure 2.4 Activity system of formative assessment.

From this perspective, the technological apparatus of a DLE, particularly when the LMS is integrated with tools for automatic assessment, has a mediating role in the practice of formative assessment. In particular, we can identify the following functions through which it can support the activity:

- **creating and managing**: supporting the design, creation, editing, and managing of resources (e.g., interactive files, theoretical lessons, glossaries, videos), activities (e.g., tests, chats for synchronous discussions, forums for asynchronous discussions, questionnaires, submission of tasks) and more generally of the learning environment by teachers, but also by students or peers;
- **delivering and displaying**: making the materials and activities available to the users;
- **collecting**: collecting all the quantitative and qualitative data concerning the actions of the students (such as movements and dialogues), the use of the materials (for example, if a resource has been viewed or not, how many times and how long), and the participation in the activities (such as given answers, forum interventions, number of tasks delivered, number of times a test has been performed, evaluations achieved);
- **analyzing and elaborating**: analyzing and elaborating all the data collected through the technologies related to teaching, learning, and the development of competences;
- **providing feedback**: giving the students feedback on the activity carried out and providing teachers, as well as students, with the elaboration of learning data.

We can schematize these functions in a diagram, as in Figure 2.5. The external cycle represents the five functions; the black dashed arrows represent how data are exchanged within the DLE through automatic processes. The technologies of a DLE, to accomplish one function, uses the data or the outputs resulting from the previous one: the learning materials, created through the LMS or other sector-specific software through the "creating and managing function", are displayed via devices through the "delivering and displaying function". Information about the students' activities is collected by the LMS, other software, or tools through the "collecting function" and it is analyzed by these systems, which may use mathematical engines, learning analytics techniques, algorithms of machine learning, or artificial intelligence, through the "analyzing and elaborating" function. The results of the analysis are feedback in the sense of Hattie's definition (i.e., *information provided by an agent regarding aspects of one's performance or understanding*) (J. Hattie & Timperley, 2007). They can be returned to students and teachers through the "providing feedback" function, and they can be used to create new activities or edit the existing ones. This circle represents a perfect adaptive system from the technological perspective (Barana, Di Caro, Fioravera, Marchisio, et al., 2018; Di Caro et al., 2018; Marchisio et al., 2018).

In a human-centered approach, at the center of the DLE, there is the learning community, composed of students, teachers, and peers (who are the agents in the Black and Wiliam's theory of formative assessment): they can interact with the DLE through its functions receiving and sending information. The blue dotted arrows represent how data can be exchanged between the community and the digital systems through human actions, such as reading, receiving, inserting, providing, digiting. For example, the teacher, or designer, or tutor can create the digital activities through the "creating and managing" functions of the DLE; tasks are displayed ("delivering and displaying" function) and received, seen, or read by the students through some device. The students, individually or with their peers, can insert their answers or work. The technology collects them through the "collecting" function. The system analyzes the students' answers and provides feedback ("providing feedback" function) returned to the student. Simultaneously, the information about the students' activity is returned to the teacher through the "providing feedback" function; the teacher can use it to edit the existing task or create new ones. The continuous double-ended orange arrows represent the communications among students, teachers, and peers, which in classroom-based settings can be verbal while in online settings can be mediated by a device.

The diagram in Figure 2.5 helps us understand how data are shared among the components of a DLE, elaborated, and used; for this reason, it can be useful in the perspective of learning analytics.

In the diagram, one can follow the arrows along close paths and identify particular situations. For example, we can focus on the upper-left side of the circle as in Figure 2.6a and describe the process of content creation by a teacher or an instructor: she receives information about the activities completed by the students and uses them for the creation of new ones. If we focus on the right part of the circle as in Figure 2.6b we can examine the process of the fruition of static digital resources: the teacher makes them available to the students through the delivering function, and the students read/observe/study them. If we consider the lower-left part of the diagram as in Figure 2.6c, we have interactive activities: the student can insert answers in the system, they are automatically analyzed and feedback is returned to the student. Moreover, including or excluding interactions among peers, individual and collaborative activities are identified. Disregarding the arrows linking the teacher, the student, and the peers, we have a scenario of individual and self-paced online learning.

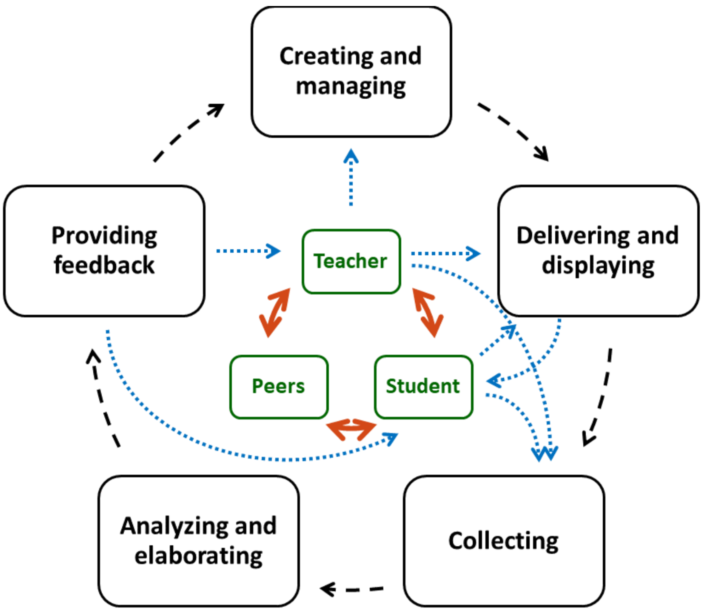


Figure 2.5 Diagram of the functions of the technologies in a DLE with AFA.

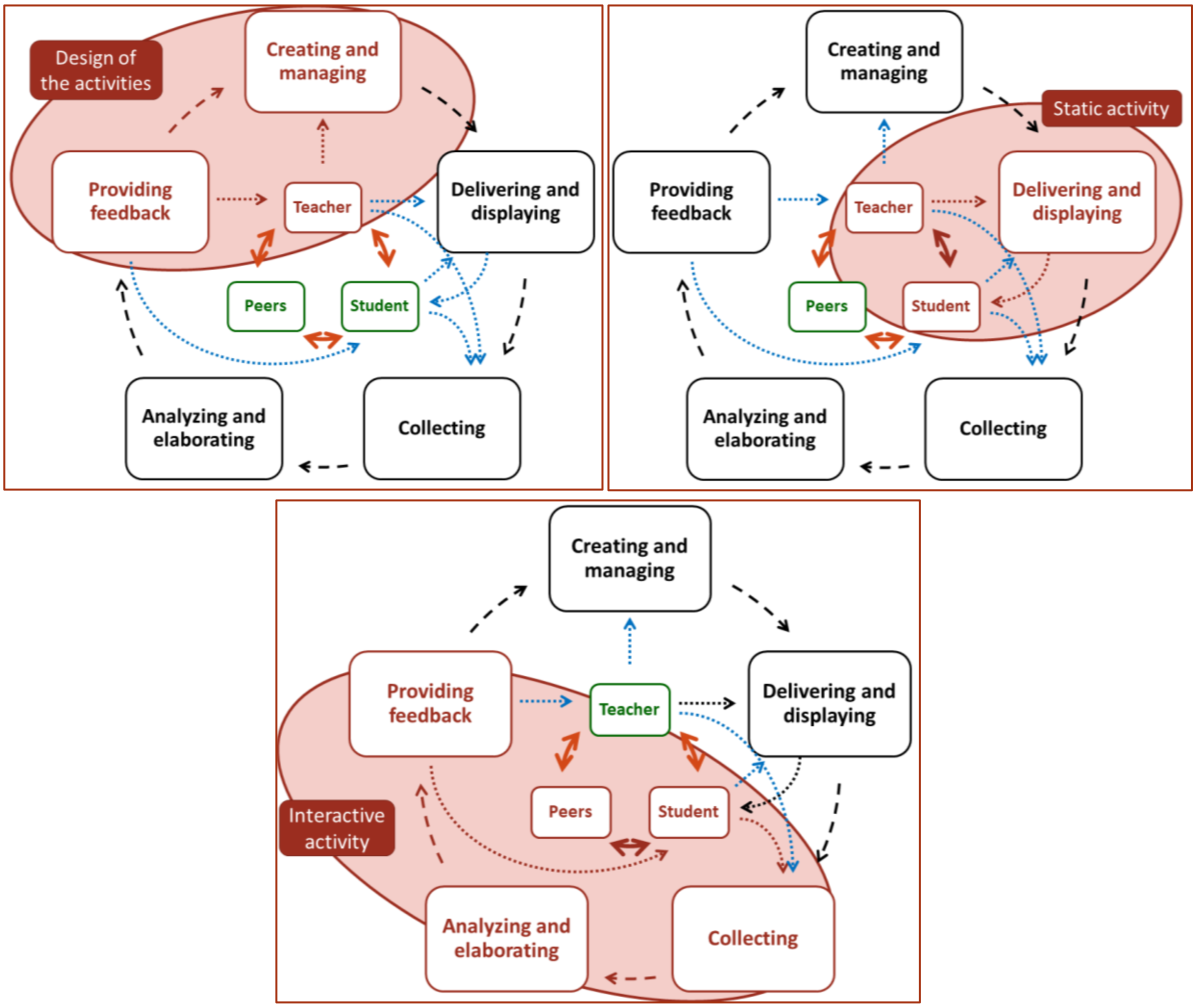


Figure 2.6 Paths identified in the model corresponding to design activities, static activities, and interactive activities.

Within this model, the formative assessment's key strategies can be located along the arrows, as shown in Figure 2.7. KS1 (clarifying and sharing learning intentions and criteria for success) can be enacted by the teachers when showing the materials to the students through the "delivering and displaying" function (or by the technologies when we are considering an online activity). The teacher can enact KS2 (engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding) through the "creating and managing" function when she creates learning materials or tasks with the automatic assessment. KS3 (providing feedback that moves learners forward) occurs through the automatic assessment system's action when it provides feedback to the student, after analyzing the student's answer. KS4 (activating students as instructional resources) can be located in the interactions between students and peers during the activities. Lastly, KS5 (activating students as the owners of their own learning) can be developed when students insert their answers in the AAS, which receives them through its "collecting" function. These are not the only occasions in which formative assessment strategies can be activated; as an example, during classroom activities, they can all be pursued through interactions between teachers and students.

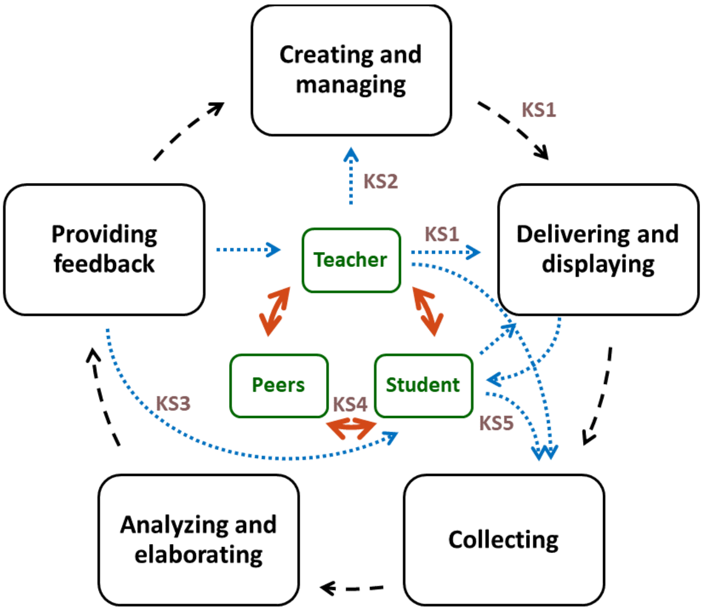


Figure 2.7 Key Strategies of formative assessment enacted in a DLE.

In the end, the model that we have analyzed in this section allows us to identify some outcomes that the adoption of a similar DLE with AFA, through the functions previously shown, makes it possible to achieve:

- to create an interactive learning environment: all the materials for learning and assessment can be collected in a single environment and be accessible at any time. They can activate the students who can be engaged in the navigation of the learning path, solve the tasks and receive feedback;
- to support collaborative learning, through specific activities, delivered to groups of students, which enhance the communication and sharing of materials, ideas, understanding;
- to promote formative assessment, by offering immediate feedback to students about their results, their knowledge and skills acquired, and their learning level. Feedback can also be returned to the teachers on the students' results and their activities, supporting decision-making.

The identification and classification of a DLE's functions can allow us to analyze the contribution of technology during the formative assessment process from a learning analytics perspective since they underline the role of data during the learning processes. This model helps us identify the functions and outcomes of technology in formative assessment processes. Individuating and separating functions and outcomes is necessary to have a clear frame and find causal connections, especially when analyzing large data quantities.

2.3.4 e-Assessment and Self-regulation

In the context of Technology Enhanced Learning, many systems have been studied to scaffold self-regulation and to guide students to develop autonomy in monitoring their processes. In particular, DLEs support metacognition and self-monitoring when they allow learners to go back through their actions and reflect upon their learning processes or strategies. Moreover, assessment can be managed to increase students' choices (e.g., through accessibility and availability), compare their work with exemplars or peers, and show the artifacts produced (e.g., portfolios). Technologies also offer personalization and adaptivity opportunities, which create a favorable condition for the regulation of their learning processes (Persico & Steffen, 2017).

Nicol and Milligan have declined the seven principles for good feedback practices (listed in paragraph 2.3.2) into strategies using virtual environments. From their analysis, it emerges that automatic assessment can be successful in accomplishing the seven key strategies. We will list some of the advice which mainly concerns online assessment.

1. To help clarify what good performance is, students can be given models of standards or other activities that encourage them to interact with and externalize criteria.
2. To facilitate self-assessment development, online objective tests can be useful for students to assess their understanding of a topic. In particular, multiple attempts help foster skills acquisition.
3. An online environment can increase the flexibility and range of feedback to deliver high-quality information to students about their learning;
4. One idea to encourage dialogue around learning in an online or blended environment is to distribute randomized questions with casual parameters and prompt discussion and collaboration. Since all assignments are different, discussions will focus on the solving processes, not on the results.
5. The online assessment is useful for encouraging positive behaviors and raising their motivation and self-esteem since students can privately assess their understanding without comparing performances with others. In particular, repeated attempts are highly motivational.
6. When students can resubmit their works, they have the opportunity to use feedback information to close the gap between current and desired performance.
7. In online assessment, all user data are collected and made available to students and teachers; the latter can thus use the information to shape the teaching.

2.3.5 Automatic Assessment and Mathematics

In Mathematics, AFA is widely used in online courses to keep the learner involved and increase motivation. Research centers and universities have developed systems that can process open-ended answers from a mathematical perspective and establish if they are equivalent to the correct solutions. This allows students to widen the possibilities of interaction with digital tools. Examples of similar Automatic Assessment Systems (AAS) are:

- CALM, developed by the Heriot-Watt University in the programming language of Pascal (Beevers et al., 1989);
- STACK, developed by the University of Edinburgh and relying on the Computer Algebra System (CAS) Maxima for its Mathematics capabilities (Sangwin, 2015);
- Moebius Assessment, developed by the University of Waterloo and running on the engine of the Advanced Computing Environment (ACE) Maple (Barana et al., 2015).

By exploiting programming languages or mathematical packages, these AASs allow instructors to build interactive worksheets based on algorithms where answers, feedback, and values are computed over random parameters and shown in different representational registers. The use of similar AASs becomes powerful when combined with the principles of formative assessment.

Research and experimentations have shown that digital materials that provide feedback through an AAS empowered by a mathematical engine can be useful under different perspectives, such as learning, problem solving, metacognition, adaptive teaching, and teacher’s practice.

First of all, they are effective in enhancing *learning*. Taking advantage of all the technology's potential, one can create questions that could not be replicated with paper and pen, but stimulate students’ cognitive processes. Stacey and Wiliam (Stacey & Wiliam, 2013) suggest new solutions for computer-based items, including the dynamic exploration of situations or solutions; Sangwin (Sangwin et al., 2010) shows examples of implicit feedback provided to the student instead of explicit solutions to enhance reflection and promote understanding. Questions can be enriched with dynamic images, animations, geometrical visualizations, symbolic manipulations, tables, and other features that can be created through the computing environments running behind the AAS, thus offering students experiences of mathematical construction and conceptual understanding (Paiva et al., 2015).

Secondly, they help *master problem solving procedures and strategies*. Beevers and Paterson discuss the use of step-by-step automatically graded resolutions of complex tasks that students cannot solve on their own (Beevers & Paterson, 2003). They state that the stepped approach guides learners towards breaking problems into smaller and manageable parts and helps them acquire control over the solving process. Students can thus develop procedural knowledge that, in Mathematics, is strictly connected to conceptual understanding (Bokhove & Drijvers, 2010). Several AASs support adaptive stepping capabilities; this area of research flows into studies on adaptive tutoring systems, which are at the most cutting-edge solutions in terms of e-assessment (Helder et al., 2017).

Third, automatic assessment proves to be useful at a *metacognitive level*. As seen before, Nicol and Milligan (D. Nicol & Milligan, 2006) have analyzed e-assessment in the light of seven principles of good feedback practices to promote self-regulation. They suggested several question formats or uses of the online assessment to help students become aware of their knowledge, draw abstract concepts out of the examples, reflect on their own mistakes, and set goals for their learning.

Another relevant opportunity offered by interactive materials with AFA is that of *facilitating adaptive teaching strategies*. Automatic formative feedback can work as an online tutor while students attempt the assignments at their own pace. The absence of restriction on the number of attempts allows weaker students to repeat the questions, while their most skillful classmates can put themselves to test with new ones (Barana, Fioravera, Marchisio, et al., 2017). Data collected by the platform can help teachers monitor learning and adjust instructional strategies accordingly. Moreover, digital interactive materials are incredibly helpful for students with learning disabilities: the organization and presentation of the worksheets, the multimodality, and the computing environments enhanced by technology can make up for their cognitive difficulties and motivate them to study.

Lastly, *teachers' work* can benefit from AFA, and not only for the time saved from manual correction. As mentioned above, data collected from the platform inform them about students' understanding. They can be useful to shape teaching and have a complete picture of the students' gains. Many systems also allow teachers to author their materials, adjust existing ones, and share their work with colleagues. This permits to overcome dissatisfaction with textbooks and contributes to professional communities' development and the growth of teachers' competences. Moreover, assignments prepared for one class can be easily reused and transferred to new courses in the following years; thus, the effort for producing online tests yields advantages and time saved in the future (Brancaccio, Marchisio, Palumbo, et al., 2015).

2.3.6 Task design for formative assessment

Designing tasks for formative assessment is not the same as designing tasks for summative assessment. In Mathematics Education, summative assessment design is generally affected by psychometric tradition, that requires that test items satisfy the following principles (Osterlind, 1998):

- unidimensionality: each item should be strictly linked to one trait or ability to be measured;
- local independence: the response of an item should be independent of the answer to any other item;
- item characteristic curve: low ability students should have low probability to answer correctly to an item;
- non-ambiguity: the question should be written so that students are led into the only correct answer.

In particular, tradition says that to build good assessment items, the following goals should be pursued (Osterlind, 1998):

1. there must be a high level of congruence between a particular item and the key objective of the whole test;
2. the key objective must be defined clearly and unambiguously;
3. the contribution to measurement errors must be minimized;
4. the format of the test items must be suitable to the goals of the test;
5. each item must meet specific technical assumptions;
6. the items should be well written and should follow prescribed editorial standards and style guidelines;
7. the items should satisfy legal and ethical questions.

As a result, items built according to this model are generally limited in the Mathematics that they can assess. The possible problems are reduced to those with one only solution, deducible through the text's data. If problems admit multiple solving strategies, the only information detected is the students' solution, thus removing the focus from the process, which is essential for assessing Mathematics understanding (van den Heuvel-Panhuizen & Becker, 2003).

If different summative assessment models are currently under research (Suurtamm et al., 2016), models for formative assessment should detach from these principles. Several studies are going into this direction; for example, Van den Heuvel-Panhuizen and Becker (2003) suggest that in formative assessment:

- problems should have multiple solutions, meaning both tasks with multiple possible paths to the single correct solution, and multiple correct results;
- tasks might be dependent, in the sense that information gained solving one task can be useful to solve a subsequent task or a second part of the same task;
- strategies should be the intended output: tasks should be designed in such a way to make the process visible, and the process itself should be considered more important than the answer.

2.3.7 A Digital Learning Environment for STEM

For the realization of online courses, the Delta Research Group decided to align with the University of Turin's choices and use Moodle as LMS (Barana et al., 2015). Moodle is an open source LMS, one of the world’s most popular one; it supports creating online courses, populated by resources and activities and accessible to students, teachers, and administrators. It is integrable with external systems according to one’s needs. To create an interactive DLE for STEM where it is possible to perform AFA, the Delta Research Group chose to integrate an Advanced Computing Environment and an Automatic Assessment System in the platform.

An ACE is a system able to perform numeric computations, symbolic calculus, geometric visualizations in two and three dimensions, and embedding of interactive components. Thus, it allows to enhance the capabilities of scientific objects representation, while maintaining the ease and immediacy of use of a common word processor (Barana, Fioravera, & Marchisio, 2017a). As an ACE, we chose Maple (www.maplesoft.com), which, through the MapleNet web tool, allows to make the interactive worksheets available into Moodle pages, so that students can use the interactive components to play with Mathematics without running the software on their computers (Barana, Conte, Fissore, et al., 2020). Moreover, the Maple engine runs behind the DigitalEd’s AAS Moebius Assessment (formerly known as Maple TA) (Moebius Assessment, 2019). Among all the existing AASs, Moebius Assessment was selected for its high suitability for STEM subjects, for its powerful capabilities for assessment, the robust mathematical engine running behind the system, and the possibility of integration within Moodle. In fact, in Moebius Assessment open mathematical answers can be implemented and graded through algorithms that verify if the student’s answer matches the correct one independently of the form. This allows us to go beyond the multiple-choice modality and to test different and complex cognitive processes. Moreover, it is possible to create variables based on algorithms, random parameters and mathematical formulas, graphics and even animated plots. Thus, questions appear different from student to student and at every attempt; data and solutions can be automatically computed in the algorithm within the question itself. In addition, adaptive assignments can be implemented, so that the next question – or question part – is proposed to the student according to the previous answer.

Several studies in the literature show how the adoption of Moebius Assessment (or Maple TA) improved students' results in different contexts. Pezzino (2018) shows that the introduction of automatic formative assessment with Maple TA enabled interactive learning and visualization and improved students' results in an Advanced Economics course. Rønning (2017) analyzed changes in the students' approach to problem solving after introducing Maple TA in Mathematics courses. At TU Delft, Moebius is successfully used to implement digital exams for Engineering courses (Keijzer-de Ruijter, 2019).

Figure 2.8 shows an example of a question implemented with Moebius Assessment. It asks students to identify a line in a sheaf of lines with a particular property. In the equation of the sheaf, there are random parameters that change at every attempt. The question is composed of two parts: firstly, students have to write the parameter that corresponds to the line, then the equation of the line. Thanks to the symbolic engine through which the question is graded, all the equivalent forms in which the equation can be written are graded as correct. Moreover, before submitting their answer, students can check its correctness through a graphic, automatically generated by the system, built using data from students' answer itself.

The Moodle platform integrated with the Maple ACE and the Moebius Assessment AAS is a complete and powerful LMS. It is the major technological component of the DLEs that we study, as it enables the enactment of interactive learning, automatic formative assessment, and collaborative learning in STEM subjects.

2.4 A MODEL FOR DESIGNING AUTOMATIC FORMATIVE ASSESSMENT ACTIVITIES IN MATHEMATICS

Our research group has studied the formative assessment's effectiveness in an interactive DLE, based on a Moodle platform integrated with Maple ACE and Moebius Assessment AAS (Barana & Marchisio, 2016b). Part of the following paragraphs (2.4, 2.4.1, 2.4.2, 2.4.3) is taken, expanded and adapted from (Barana, Conte, et al., 2018). Here we propose and discuss a model that can be used to design AFA activities in Mathematics. Activities with automatic assessment should have the following features:

- **Availability.** Assignments are always available to students, who can attempt them at their own pace, without limitation in data, time, and number of attempts.
- **Algorithm-based questions and answers**. In the questions, students find random values, parameters, or formulas that make questions, and the relative answers, randomly change at every attempt. This can be realized through the implementation of algorithms running behind the questions. By algorithmic variables, different representational registers (words, numbers, symbols, tables, graphics, schemas) can be shown in questions and feedback.
- **Open answers**. The multiple-choice modality is avoided whenever possible, to let room for open answers, where students are asked to respond in one of the different registers listed above.
- **Immediate feedback**. Results are computed in a few moments and they are shown to the students while they are still focused on the task. Assignments with no more than five questions are advised to increase the immediacy of feedback.
- **Interactive feedback**. Just after answering one question, the system can show if it is correct and go through a step-by-step guided resolution that interactively shows a possible process for solving the task. Students who fail to answer autonomously to the main question are asked questions about prerequisites or simpler tasks. At each step, if they give the wrong answer, the correct one is shown to be used in the following step. This schema is made possible by the "adaptive" modality available in Moebius Assessment. They can gradually acquire the background and the process that enables them to answer the initial problem. They earn partial credits for the correctness of their answers in the step-by-step process.

- **Contextualization**. Whenever possible, questions refer to real-world issues that can be relevant to students as well as for the discipline.

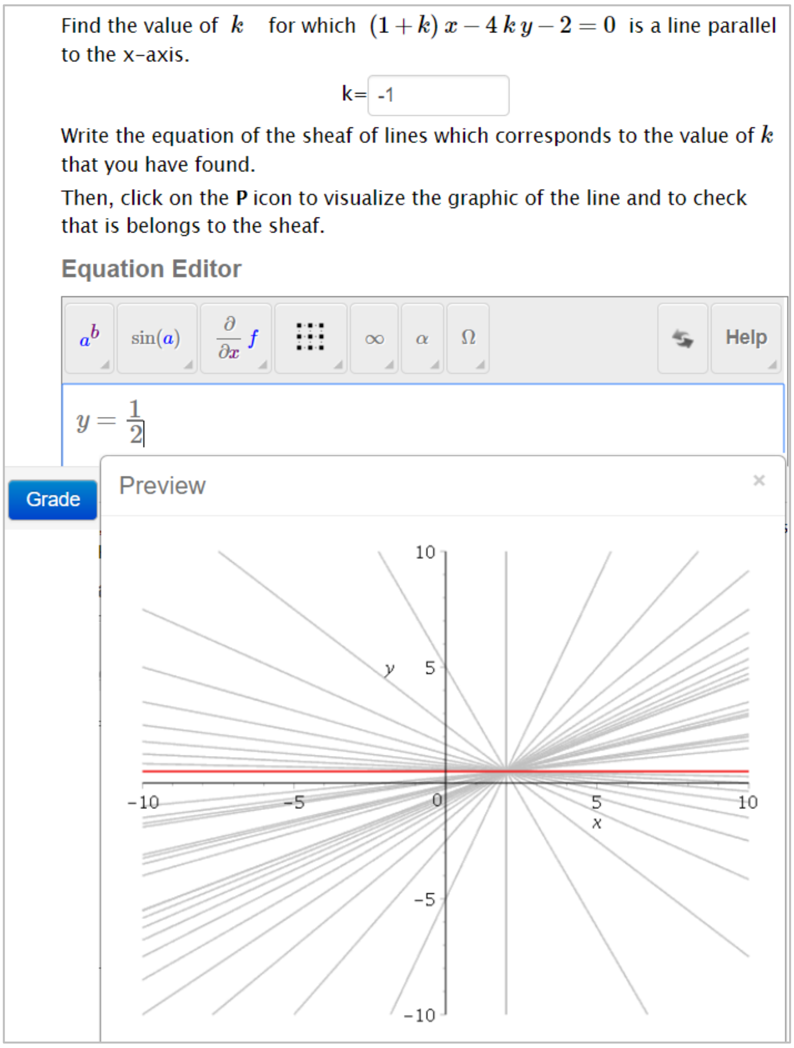


Figure 2.8 Example of a Moebius Assessment question. The system plots a graphic using data from the student's answer. (Barana, Conte, et al., 2018)

An example of a question built according to this model is shown in Figure 2.9. The first section shows the contextualized problem involving an ice-creamer and black cherry sauce; students are asked to solve a problem and insert the final solution in a response area. If they give the correct answer, they exit the question and move to the next one. Otherwise, if they provide an incorrect answer, before moving to the next questions, they must go through the other sections, one at a time, in a step by step process through the solution of the problem. This process is the interactive feedback: it activates students in understanding the solving process. Answers are open-ended, so students need to make all the needed computations to answer the questions. They can immediately see if their answers are correct, so they can easily identify their mistakes. Students have unlimited attempts available; an algorithm is running behind the question, which generates new data at every attempt. After following the interactive feedback, students can try the problem again and redo the solving process with different numbers.

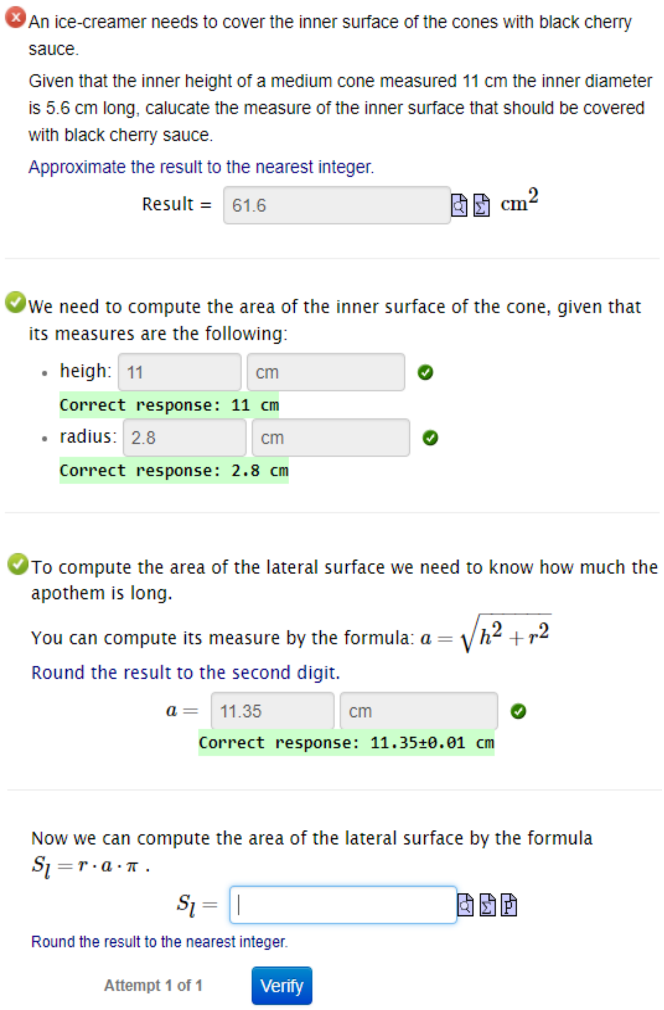


Figure 2.9. Example of a question with interactive feedback (Barana, Conte, et al., 2018).

Many of this model's features are typical of online assessment and used in many systems illustrated in the literature. The feedback's immediacy is one of the main advantages of using automatic assessment acknowledged by many authors (Kurvinen et al., 2014). Regarding the timing of the feedback, there is disagreement in the definitions of immediate and delayed feedback in the literature. While some authors define immediate feedback as the feedback provided after answering each item and delayed feedback like the one provided after a block of items, others include both cases in the "immediate feedback" and distinguish it from the "delayed feedback", which is provided hours or even days after ending the test (Shute, 2008). There is no doubt that immediate feedback is more useful than delayed feedback, intended as in the second definition; when considering the first definition's distinction, there is disagreement about the best one. For instance, Fabienne van der Kleij and her colleagues, in a study with 153 university students, found that immediate feedback was more useful than delayed feedback for learning (van der Kleij et al., 2012), while Gaona et al. (2018) argue that delayed feedback is preferable to improve learning. In her review of the literature about feedback, Shute (2008) argues that immediate feedback is more effective in the short term and for procedural knowledge, while delayed feedback may be superior for promoting the transfer of learning, especially concerning concept-formation tasks. Here we use the feedback after each item when coupled with interactive feedback, which often has a procedural aim, and feedback at the end of the quiz when items do not include interactive feedback. Feedback can also be delayed by multiple attempts before showing the correct answer or the interactive feedback.

Availability is another relevant advantage of computer-based education and a key difference compared with paper-and-pen instruction. It refers to the possibility of finding learning materials independently of time and space and the chance to repeat the activities and receive timely feedback. This aspect, coupled with suitable activities, could help students study and learn at their own pace, supported continuously by the technologies (Barana et al., 2016). Moreover, letting students repeat the assignments is an effective way to make them aware that the information from the feedback was useful to improve and make teachers and researchers sure that the feedback was well built (Boud, 2000).

In Mathematics Educations, there are several examples of the use of algorithms to randomize parameters and of CAS-based engines to assess correct answers (Cazes et al., 2007, p.; Gaona et al., 2018; Sangwin, 2015). Random parameters open two sceneries: on one side, each student has a different version of the same item; on the other, each student can repeat the tests finding different questions, which maintain the same structure but have new numbers, or new formulas, graphs or other elements each time. The algorithm can compute and generate even symbolic elements powered by a mathematical engine, thus allowing the creation of interesting items. Algorithmic questions have many advantages: the main ones are: restrictions to cheating, saving time in the creation of large question banks, inserting every kind of mathematical object (e.g., graphs, formulas, matrixes, diagrams) in a question, and the possibility to write codes even to grade students' answers (Barana & Marchisio, 2016b).

The possibility to write, edit, and adapt grading-codes to one's needs allows the assessment of open answers, through which open-ended tasks can be proposed, and question design becomes more flexible. Open-ended tasks are problems devoted to developing a mathematical idea that involves multiple methods, pathways, access points, and solutions (Edson, 2017). They are powerful for creating opportunities for student exploration, collaboration, and sophisticated mathematical reasoning (Chan & Clarke, 2017); they permit students to access problems from different perspectives and evoke meaningful mathematical discourse (Edson, 2017). Teaching through open-ended problems can foster students' higher-order thinking, reflection, and enjoyment, besides offering students a greater opportunity to exercise metacognitive skills related to tool selection and decision making (Boaler, 1998).

Contextualization of tasks in real-world settings contributes to creating meanings and a deeper understanding, as students can associate abstract concepts to real-life or concrete objects. It also works as a stimulus for motivation, since tasks can be closer to the students' lives and interests (Barana, Brancaccio, Conte, et al., 2019a; Samo et al., 2017). More details about problem solving and contextualization will be discussed later in this thesis (Chapter 2.6).

There has been a discussion in the literature aimed at comparing elaborated feedback and corrective feedback – those that just say if the answer is correct or not. Many studies show that the formers are more useful to improve learning (Shute, 2008; Timmers & Veldkamp, 2011; van der Kleij et al., 2012). Elaborated feedback can refer to explanations of the correct solution, links to further reading materials, cues, suggestions, or combinations of the previous (Shute, 2008). In Mathematics, elaborated feedbacks are often in the form of step-by-step resolutions of the task (Gaona et al., 2018). The great part of elaborated feedback models that the literature proposes is static: students have to read them carefully and compare them with their results. Some studies also show that, more often than expected, students do not read them at all, especially if they perceive the task as too complicated or they do not receive the feedback timely (Timmers & Veldkamp, 2011). To overcome this difficulty, Shute proposes to split feedback into manageable units, thus avoiding cognitive overload (Shute, 2008). This idea stands at the basis of the interactive feedback. It consists of a formative interaction between the students and the system, and actively engages students in resolving the task.

The model proposed is particularly relevant in making students process the feedback and use the information gained to improve their understanding. From a structural viewpoint, interactive feedback is part of the question itself, and the step-by-step process is shown immediately before moving on to the next question.

Consequently, it should help ensure that students go through the feedback after coming to know if their answer was correct. This way, feedback can be effective according to Sadler's model (Sadler, 1989): in fact,

- the interactive feedback offers a concept of the standard that students can actively possess;
- immediate feedback helps them compare the actual level of performance with the standard;
- when trying the assignment again, students find similar tasks with different values, so that they are engaged in an activity that makes them repeat the process until mastered.

Interactive feedback can help low-level students because if they are discouraged by a complex task, they are engaged in simpler and more manageable sub-questions and avoid cognitive overload. Since multiple attempts are allowed, students can try again the questions they failed: they will have to repeat the reasoning autonomously. Remembering the results by heart or developing a "trial and error" strategy will be useless. Values in questions randomly change at every attempt: students need to learn the process laying behind the question.

The interactive feedback recalls Beevers and Paterson's step-by-step approach to problem solving with automatic assessment (Beevers & Paterson, 2003). Here it is conceptualized in terms of feedback, highlighting the formative function that the sub-questions fulfill for a student who failed the main task. Moreover, students are rewarded with partial grading, which improves motivation (Beevers et al., 1999).

The step-by-step guided resolution can also be inserted after correct answers in multiple-choice questions when the probability of guessing the right option is high. In this case, if the student correctly answers because he knows the answer, subsequent questions will have the function of asking for a justification; otherwise, he will have the opportunity to see and enact a correct solving process.

This model for the design of AFA activities is in line with the abovementioned principles of the formative assessment task design:

- open answers and interactive feedback can be designed to focus on the process;
- the algorithmic power of the system enables different representations of the same concept to be provided to the students, or asked as an answer;
- multiple attempts with random parameters help students recognize and master solution paths.

Moreover, using this model, it is possible to design and build tasks that respect the constructivist principles for knowledge building, presenting the contents from multiple perspectives, activating students as owners of the learning process, and embedding learning in relevant contexts.

This kind of formative activities can be integrated into a DLE with other interactive resources and used in collaborative situations, or coupled with different activities of collective discussion and collaborative work.

Based on the studies of the literature summarized above, we can affirm that the structure of the assignments created according to this model is potentially highly motivating for students. They can attempt questions many times when they are ready and when they have time, and individually verify their understanding and compare with others their learning, not their results.

2.4.1 Analysis of the model for the design of AFA activities according to Hattie and Timperley's model of feedback

This paragraph will analyze the feedback information provided in activities with AFA designed according to our model, using Hattie and Timperley's model of feedback (J. Hattie & Timperley, 2007). Feedback developed according to Hattie and Timperley's theory should reduce the gap between actual and desired performance.

Interactive feedback answers the three questions *"Where am I going?", "How am I going?", "Where to next?".* In fact, interactive feedbacks show students the learning goals providing a sample of a good answer.

They can indicate the exact point where students stopped, if they made a mistake or did not have the background needed, thus giving information about their progress. Finally, they show the next step to drive students to the full accomplishment of the task.

Hattie and Timperley identified three levels of feedback:

- task-level: it is the information about correctness obtained immediately after submission. Depending on the kind of question and its settings, this information can be given through a green tick or red cross to indicate if students' response is correct or not, providing the correct answer that they have to compare with their one, and also in numeric form through the percentage of correct response of the whole test;
- process-level: interactive feedback, in addition to information about correctness, also provides a sample process for solving the task. It is interactive and integrated within the test at the end of each question and before submitting the test, to avoid students passing over it;
- self-regulation level: when engaged with the interactive feedback, students can locate the stage where they had difficulty, the exact error, or their missing prerequisite. They can learn how to self-assess and pinpoint their position along their learning path. The inclusion of the online assessment in a DLE such as Moodle allows students to picture their progress through tools such as progress bars or completion tracking.

Moreover, the feedback's text can focus on understanding, procedural, or metacognitive aspects to reinforce the feedback's action in one direction.

The fourth level of feedback will not be mentioned here. Unless the teacher adds standard comments in the feedback, such as "good" or "well done", the student will not receive personal words by the machine. It must be said that their function of driving students into reading the other information in the feedback is not essential anymore, as this function is covered by interactive feedback.

2.4.2 Analysis of the model according to Nicol and Macfarlane-Dick's model of feedback

In this paragraph we will analyze our model for designing AFA activities using Nicol and Macfarlane-Dick's model (D. J. Nicol & Macfarlane-Dick, 2006). We will start from the seven good feedback practices to promote self-regulation. In the following list, there are some ideas on how to implement them through the AFA model's features. Table 2.1 summarizes the analysis.

1. Guided resolutions in interactive feedbacks constitute interactive standards that can help clarify what good performance is.
2. Multiple attempts with immediate feedback can facilitate the development of self-assessment; the access to and the revision of the answers given in previous attempts can foster reflection on learning.
3. Through the interactive feedback, high-quality information can be delivered to students about their learning and how to close the gap to desired learning.
4. Algorithmic questions, which vary from student to student, can help encourage the dialogue around learning, since students, to discuss their work, need to share processes and not results.
5. The immediacy of feedback and the possibility of repeated attempts are useful to encourage positive motivational beliefs and self-esteem.
6. Feedback information can be used to close the gap between current and desired performance when students have the chance to resubmit the assignment; in particular, with algorithmic questions, students need to understand and use information regarding the process, not the result.

7. The AAS's gradebook collects all the students' results, attempts, and times; statistics on the class and the questions are automatically computed. This information can be used to help shape the teaching.

Table 2.1. Analysis of the model according to Nicol and Macfarlane-Dick's good feedback practices

Good feedback practices	Features	Functions
1) help clarify what good performance is	Interactive feedback	Offering an interactive standard
2) facilitate the development of self-assessment in learning	Immediate feedback	Facilitating self-evaluation
	Multiple attempts	Learning from mistakes
	Possibility to revise one's attempts	Promoting reflection on learning
3) deliver high-quality information to students about their learning;	Interactive feedback	Showing a process or reasoning; helping identify mistakes
4) encourage teacher and peer dialogue around learning;	Algorithm	Comparing solutions to the same problem with different data
5) encourage positive behaviors, motivation, and self-esteem;	Immediacy of feedback	Increasing motivation
	Multiple attempts	Promoting self-esteem
6) provide opportunities to close the gap between current and desired performance;	Multiple attempts	Using information to give evidence of understanding
	Algorithm	Focus on processes rather than results
7) provide information to teachers that can be used to help shape the teaching	Gradebook	Collecting students' attempts, answers, grades, and times.

2.4.3 Evaluation of the effectiveness of the model of AFA for learning STEM

the Delta Research Group of the University of Turin designed and conducted three experimentations to validate the model for designing AFA activities for learning STEM (Barana, Conte, et al., 2018). The three experiences differ in numbers, aims, and contexts.

First experimentation

Methodology

A module of Physics has been developed with AFA's aid during a Physics course for a class of 24 students of 11th grade. The class generally had little interest in scientific disciplines, as their curriculum's core subjects were Foreign Languages. The module lasted 12 hours and aimed at introducing students to problem solving; the contents concerned the Dynamics and Newton's law. 2-hour lessons were held weekly by a PhD student from the Department of Mathematics of the University of Turin, collaborating with the class Mathematics and Physics teacher. Students were asked to solve problems with randomly generated data and interactive feedback through weekly online assignments for homework. The online tests were developed through Moebius Assessment and made available to students in an e-learning Moodle platform managed by the ICT services of the Department of Computer Science of the University of Turin. All the questions were based on Physics problems, and they had interactive feedbacks, which showed a possible solving process for the students who gave the wrong answer.

At the end of the module, students had to answer a written test with 2 open theoretical questions and 4 problems to solve. Moreover, they had to fill in a questionnaire about the appreciation of the learning methodology. The final test has been developed and graded (on a scale from 1 to 100) in collaboration with the class teacher. Through an oral survey, the usual teacher methodology was investigated. It was mainly traditional, rooted in delivering knowledge rather than competence, without any integration with technologies. Students did not have much confidence with problem solving nor with an AAS.

Results

The first evidence collected during and after the experience was that the interactive feedback allowed students to master problem solving procedures autonomously. In this way, the time in class could be dedicated to a more fruitful clarification of doubts. Time optimization is necessary in a school system that asks teachers to develop competence, besides knowledge and abilities.

However, significant results emerged from the final questionnaire. The answers to the questions: *"in general, do you like Physics?"* and *"did you like the topic of this module?"*, expressed in a Likert scale from 1 to 5, register an increase from an average value of 2.46 (SD: 0.93) for the general appreciation of the subject to an average value of 3.23 (SD: 0.70) for the topic studied through the new methodology. The percentage increase is 31%.

Students liked the online tests and all the peculiar features of the model; in particular, they appreciated their availability, the immediate, multiple attempts, the guided solution of problems, and randomly generated data in problems. Moreover, they thought that these activities help them to understand better the topics studied. Details of students' answers are shown in Table 2.2. Notice that almost all answers are strongly positive.

Table 2.2. Students' answers to the final questionnaire.

To what extent did you appreciate the following features?	**1**	**2**	**3**	**4**	**5**	**Average**	**SD**
The online tests.	0%	0%	8%	38%	54%	**4.46**	**0.63**
The possibility to access the tests wherever and whenever you want.	0%	0%	0%	38%	46%	**4.61**	**0.49**
The possibility to make many attempts to the same test.	0%	0%	0%	23%	77%	**4.77**	**0.42**
Having different exercises at every attempt.	0%	0%	15%	38%	62%	**4.31**	**0.72**
Having the correct answer immediately.	0%	0%	0%	8%	15%	**4.92**	**0.27**
Having the possibility to solve problems through a guided resolution.	0%	0%	0%	46%	54%	**4.54**	**0.5**
Do you think that online activities helped you understand this topic?	0%	0%	15%	69%	92%	**4.0**	**0.55**

To evaluate the automatic assessment's impact on learning, we computed the correlation between the number of attempts students made to all assignments with their final test mark. The R squared value is 0.54: there is a high correlation between the two variables.

The results of this first experience show how using the AFA can be effective for learning. The number of students involved is low, so general statements cannot be inferred. Moreover, the sudden change of methodology and a PhD student's presence instead of their traditional teacher could have impacted the students. They could have had a particularly positive attitude because that module seemed to them unrelated to usual teaching. However, motivated by these encouraging results, we designed other two experimentations that could rely on more extensive data.

Second experimentation

Methodology

Some sets of assignments of Mathematics, built according to the model previously described, have been assigned to 155 students of 7 different classes of grade 8th, 9th, and 10th within 3 months. Table 2.3 shows the number of students and classes involved in the experimentation. The assignments were created by the researchers of the Delta Research Group and shared with the classes' Mathematics teachers. The assignments' topics were differentiated grade by grade, but homogeneous within each grade, and they were tailored to the teachers' programs. Thus, the teachers could use the materials as school homework for their students. The assignments were delivered through another Moodle platform managed by the University of Turin. The experimentation was designed to preserve the ecological validity; in particular, while in the three classes of 10th grade, the activities were considered supplementary by their teachers, in the 4 classes of 8th and 9th grade, the activities were naturally integrated within the curriculum.

Table 2.3. Numbers of the second experimentation.

Grade	Number of classes	Number of students
8th	2	36
9th	2	54
10th	3	65
Total	**7**	**155**

There were 10 assignments available within the platform for each class, with not more than 3 questions each. All questions were algorithmic so that the data in questions changed at every attempt

At the end of the experimentation, students were asked to fill a questionnaire for the appreciation and usefulness of the assignments for their study. Questions were mainly on a Likert scale from 1 to 5.

From the gradebook and usage data of the platform, we have extracted:

- the number of attempts made at each assignment by the whole class;
- for each student, the average grade to all the assignments attempted, calculated in percentage, considering for each assignment the grade of the student's first attempt;
- for each student, the average grade to all the assignments attempted, calculated in percentage, considering for each assignment the grade of the student's last attempt.

These data have been analyzed using SPSS 25.0 and cross-checked with the responses to the questionnaire. The aim was to check whether feedback from the automatic evaluation was used to improve their understanding.

Results

The ratios between the number of attempts and the number of students per assignment ranged from a minimum of 1 to a maximum of 6.9, with an average value of 1.92 (Standard Deviation: 0.92). The average value increases to 2.29 (SD: 1.03) if we consider only students of grade 8th and 9th, where the proposal of the activities was better integrated within the Mathematics lessons. These numbers show that there was a high trend to make more than one attempt to the assignments: according to the literature, letting students repeat the assignments is an effective way to make them aware that the information from the feedback was useful to improve, as well as to make teachers and researchers sure that the feedback was well built (Boud, 2000).

Moreover, from the analysis, it emerges that the feedbacks effectively made students improve their results. For each student, the average of their grades, considering only their first attempt to every assignment was compared with the grades' average considering only their last attempts through a pairwise Student's t-test.

Statistics of the test are shown in Table 2.4. The mean of initial grades was 50.30; it raised to 58.23 at the last attempt ($t = -8.05$). The p-value <0.001 shows that the increase is statistically significant. The normality of the two variables has been checked through a Shapiro-Wilk test ($p=0.645$ for the mean of initial grades, $p=0.203$ for the mean of most recent grades).

Table 2.4 Results of the pairwise Student's t-test.

	Mean	**N**	**SD**	**t (sign.)**
Mean of initial grades	50.30	155	16.67	-8.05 (<0.001)
Mean of most recent grades	58.23	155	17.63	

Students' answers to the questionnaire, shown in Table V, attest that they appreciated the use of automatic assessment. The model's peculiar features, such as the feedback's immediacy and the multiple attempts' availability, have been appreciated the most. We can argue that finding different values at every attempt was not much appreciated because it implicated students' greater effort.

Table 2.5 Students' answers to the questionnaire.

	Mean	**SD**	**Percentage of positive answers**
To what extent were the exercises useful to understand better the topics studied?	3.00	1.10	65.6%
To what extent the immediate grading after finishing an exercise was useful?	3.80	1.23	81.1%
To what extent did you appreciate having multiple attempts available?	3.94	0.98	91.8%
To what extent did you appreciate finding different values in the exercises?	3.06	1.04	72.1%

Compared to the first experimentation, the broader sample of students and the absence of a researcher in the classes caused data to vary. However, results confirm the good design of the model.

Third experimentation

Methodology

The first two experimentations used materials entirely created by researchers. The model of AFA has also been tested using assignments created directly by teachers for their students. The experimentation took place within a teacher training course on innovative didactic methodologies for Mathematics, including AFA (Barana, Fioravera, & Marchisio, 2017b). Sixteen teachers regularly adopted the automatic assessment in their 7th, 8th, and 9th-grade classes for a whole school year. Table 2.6 summarizes the participants' numbers. The interactive assignments were delivered through Moodle courses in an e-learning platform managed by the ICT services of the Department of Computer Science of the University of Turin.

Table 2.6 Numbers of the third experimentation.

Grade	**Number of classes**	**Number of students**
7^{th}	1	20
8^{th}	14	191
9^{th}	3	73
Total	**18**	**374**

The number of assignments created and used by each teacher widely varies from a minimum of 2 to a maximum of 42, with an average value of 12.61 (SD: 12.06).

At the end of the experimentation, a student questionnaire and a teacher questionnaire were delivered to investigate the appreciation and usefulness of the assignments for their study. Questions were on a Likert scale from 1 to 5.

Researchers have extracted the same statistics from the gradebook and the platform's usage data as those in the second experimentation. Data has been collected, cross-checked, and analyzed to check whether the AFA was useful to promote self-regulation.

Results

Of the 374 students involved in the experimentation, 325 of them effectively used the automatic assessment (87%). The number of attempts per assignment varies from a minimum of 1 to a maximum of 6, with an average value of 1.83 (SD: 0.87). It shows that the tendency to repeat the assignments is also confirmed for the tests created by teachers instead of researchers.

Moreover, students' average results increased from their first attempts to their last one, as well as in the previous experimentation. The pairwise Student's t-test supports this statement. As shown in Table 2.7, there is a significant difference between the two average values ($p<0.001$). It shows that the AFA model is effective for making students use the information gained in feedback to persevere and improve.

Table 2.7 Results of the pairwise Student's t-test.

	Mean	N	SD	T (sign.)
Mean of initial grades	53.47	325	21.87	-10.74 (<0.001)
Mean of most recent grades	63.01	325	23.11	

Two questions of the final students' questionnaire were considered for this research: "*Did you like doing the online tests?*" and "*Were the online tests useful?*" asked with a Likert scale from 1 to 5. The other questions are not considered here as they refer to other methodologies and materials introduced with the teacher training. From students' answers, it emerged that they appreciated this practice (mean: 3.40, SD: 1.02), and, most of all, they found it useful for studying (mean: 3.67, SD: 0.88). Figure 2.10 summarizes the students' answers.

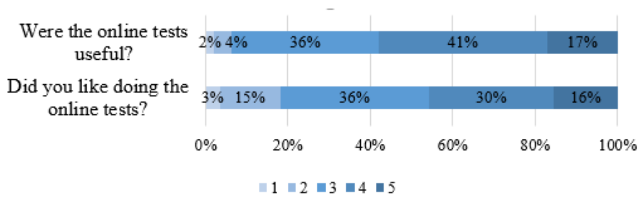


Figure 2.10 Students' answers about their appreciation of online tests (Barana et al., 2018)

At the end of the school year, teachers expressed their opinions through a questionnaire about the new methodologies' impact on their students. Answers about the AFA have been extracted for this research and analyzed. Again, the other questions will not be mentioned here as they concern other topics of the training. They were on a Likert scale from 1 to 5. The main question was the following: *"To what extent the use of automatic assessment had a positive impact on the following aspects?"*. The aspects and teachers' answers are listed in Figure 2.11. Teachers declared advantages in delivering exercises, controlling whether students accomplished them, and grading them, which are the immediate advantages of AFA for teaching. Moreover, they observed that students' motivation, self-confidence, and self-awareness have increased: automatic assessment did impact students' self-regulation.

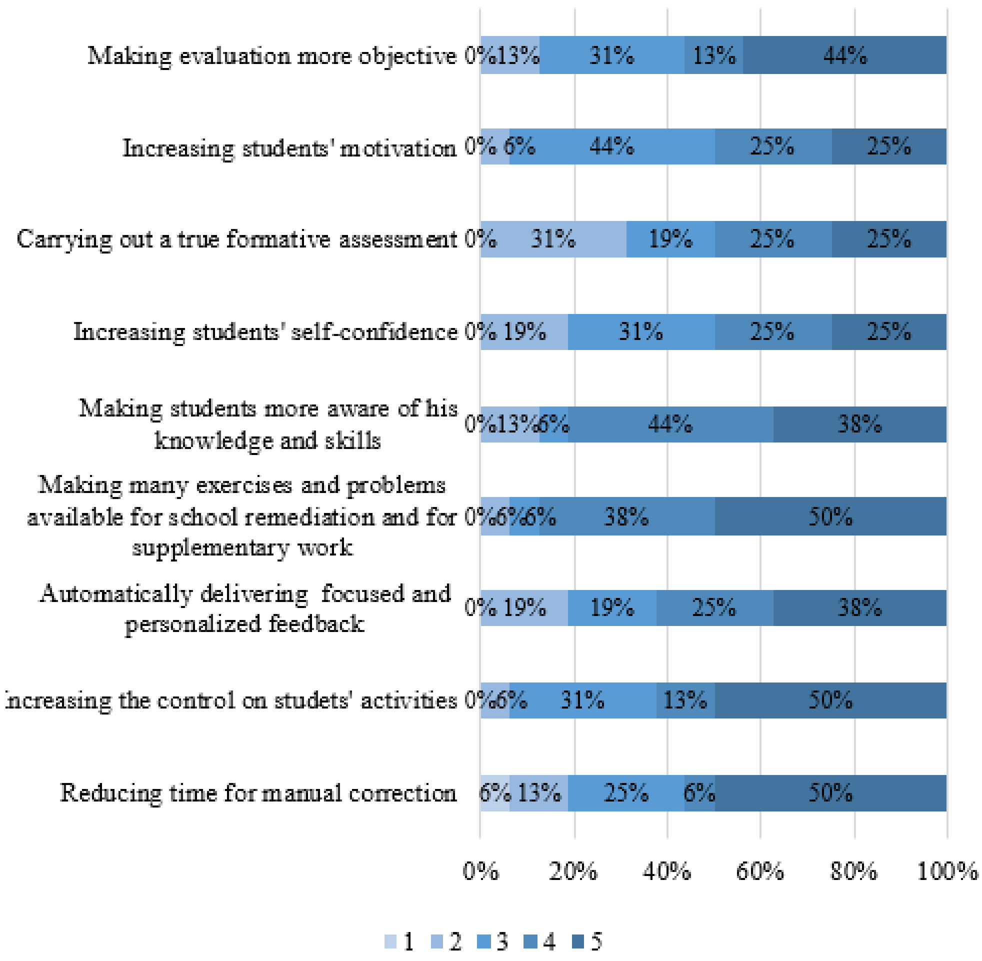


Figure 2.11 Teachers' answers about the aspects of teaching and learning on which the automatic formative assessment had a positive impact (Barana et al., 2018).

The second experience showed that the automatic assessment model is useful for making students use the feedback's information. The third one shows that the practice is replicable by teachers after suitable training. As Black and Wiliam (1998) noticed, to introduce formative assessment in a classroom, a radical change of the pedagogy is necessary, and for teachers, it requires undergoing a training path with continuous support from experts.

Table 2.8 shows a summary sheet with data and results of the three experimentations.

Discussion

Data analysis shows that students have appreciated this AFA model and that it was found useful for understanding. The use of interactive feedback effectively ensures that students use the information to improve their performance, which is a significant problem raised in the literature. According to teachers, assignments effectively foster self-assessment, self-confidence, and awareness in one's capabilities, which are essential steps for the development of self-regulation.

The design of questions and feedbacks according to the model requires that the instructors carefully consider the questions' formulation and the processes involved during the resolution. This reflection over assessment entails the professional development of the teachers when they build activities for their students. It happened in the third experimentation presented above: with appropriate teacher training, also teachers were led to create meaningful questions for their classes, joining the technical structure to interesting content and making appropriate use.

Table 2.8 Summary of data about the 3 experimentations.

		1st Experimentation	2nd Experimentation	3rd Experimentation
Data	**Number of students**	24	155	374
	Number of classes	1	7	18
	Grade	11	8, 9, 10	7, 8, 9
	Subject	Physics	Mathematics	Mathematics
Experimentation	**Modality**	Lessons held by a PhD student in collaboration with the class teacher.	Lessons held by the teacher in collaboration with a PhD student	Lessons held by the class teacher
	Number of assignments	5	10	12,61 (mean)
	Author of assignments	Delta Research Group	Delta Research Group	Teachers
	Time	4 weeks	3 months	1 school year
Evaluation	**Modality**	Post-test; platform data; student questionnaire	platform data; student questionnaire	platform data; student questionnaire; teacher questionnaire
Results	**Number of attempts per assignment**	2.46	1.92	1.83
	Increment in the assignments' grades after repeated attempts		7.97/100	9.54/100

During the experiences, some criticisms by students and teachers have been collected. They mainly concern technical details about the use of an AAS. In particular, some students complained that "*it was difficult to type the correct answers into the computer to make them accepted*." It happened mainly in the third experience, probably because teachers, less friendly with the AAS, did not give students enough information about the correct syntax or made some mistakes in writing grading codes. These problems can be overcome with experience, both by teachers and by students. Many times even when instructions are evidenced near the response areas, students skip them without reading them. Moreover, reading and following instructions are essential matters that students should learn, if possible, before graduating and entering the working world and adult society. Digital technologies, with their inner "rigidity" in accepting inputs to elaborate outputs, can have a fundamental role in this aspect of education.

Other students complained that tests are difficult to attempt using a smartphone, as mathematical language was quite difficult to write and some interactive components could not be activated on smartphones' operating systems. This is because not all students have a computer or a tablet with internet connection when studying, especially younger students living in low social contexts, who would benefit from these methodologies. In recent times, having a computer has become common, even in families with economic problems; thus, the number of students who are cut off from the possibility of accessing web-based materials from home will soon be zeroed.

From the third experimentation data, we can notice that 13% of students did not use the automatic assessment. Only a small percentage of these students filled the final questionnaire. According to their teachers, they were not those without an internet connection at home, but those students who never do their homework, never get prepared for class tests, those who remain uninterested by the school. If a little motivated student tries to do the online exercises, then the AFA will have a chance to show him that it can be useful to improve or that he will get less bored this way than with paper and pen homework. Nevertheless, if students do not even try to open one, it is not the formative assessment that failed, but the instructional system allows leaving one student behind.

2.5 Learning Analytics and Automatic Formative Assessment

Big data and algorithms are the keywords of modern society: nowadays, even the most traditional workplaces, such as mechanic's or carpenter's workshops, require data analysis expertise to perform market surveys and decide how to manage a business (World Economic Forum, 2018a). Education is not left out of this panorama: the increasing adoption of learning technologies enables the production of data, which can be used to understand, guide, and optimize learning processes. Here the field of Learning Analytics (LA) comes to life. The call of paper of the First Learning Analytics and Knowledge Conference (*LAK 2011*, 2011) introduced the definition of learning analytics later adopted by the Society for Learning Analytics Research (SoLAR): "*the measurement, collection, analysis and reporting of data about learners and their contexts, for purposes of understanding and optimizing learning and the environments in which it occurs*." Unlike the general use of statistics to provide evidence of learning methodologies' effectiveness, learning analytics should support active monitoring and dynamic decision-making during learning processes (de Waal, 2017). The data gathered and elaborated should inform not only teachers and researchers but also students about their achievements, thus letting them keep control of their learning path.

Learning analytics are based on assessment (Knight & Buckingham Shum, 2017), often the main source of data in a digital environment; assessment can be seen as summative or formative. Learning analytics is not the mere introduction of algorithms into teaching: data collection and analysis must be driven by a theoretical framework rooted in pedagogy (Friend Wise & Williamson Schaffer, 2015). The theory has a key role in guiding the researcher in choosing the variables that should be included in a model, in focusing on some results and drawing relevant conclusions out of large datasets.

2.5.1 From Automatic Formative Assessment to Learning Analytics

The Delta Research Group has used AFA activities designed through the AFA model in a DLE in different ways and contexts. For example, at lower secondary school level in a face to face modality (Barana, Boffo, Gagliardi, Garuti, et al., 2018), at lower and upper secondary school level in a blended modality (Barana, Fioravera, Marchisio, et al., 2017; Anna Brancaccio, Marchisio, Palumbo, et al., 2015), in online modality at upper secondary school level (Barana, Marchisio, & Rabellino, 2019; Barana & Marchisio, 2016a) or in a university context (Bruschi et al., 2018; Marchisio et al., 2019). Through these projects and experimentations, we began to study how learning analytics can help us understand how formative processes occur in a DLE (Barana, Conte, Fissore, et al., 2019). The model of AFA in a DLE that we previously described (paragraph 2.3.3) has a relevant role in outlining how data are produced, used, and exchanged, so we will use it as a starting point to study how learning analytics can be applied to AFA.

Through the “collecting” function of these technologies, it is possible to collect many different types of data about the activities carried out by students: data on the use of the online environment (such as number and time of logins), qualitative data concerning the use of materials (such as the completion of activities) and specific quantitative data for each type of activity. Evaluation data, elaborated through the “analyzing and elaborating” function, is automatically saved in the AAS gradebook, also integrated within the grader report. All these data can describe the activity carried out by the student. The possibility of keeping these data in memory can offer an overview of the student's learning path over time. These data can be made available to students and teachers through the “providing feedback” function via different tools. For example, progress bars provide students with visual information about their completed activities; the grader report allows teachers to see the activities carried out by the students, their progress and thus highlighting the students at risk. Data can be combined and analyzed with various Learning Analytics techniques (such as dashboards, recommender systems, predictive analytics, and alerts/warnings/interventions) to address concerns related to a broad range of teaching and learning areas. These areas include retention and student success; improvement of learning design, units, courses, and teaching practice; the development of personalized learning pathways; and student support (West et al., 2018). For LA to help improve formative assessment, it is essential to refer to a clear pedagogical framework for the interpretation of the data and to be able to use them for future actions. In our case, we used the framework described in the previous sections for formative assessment in a DLE.

Section 2.5 and its subsections are taken and adapted from (Barana, Conte, Fissore, et al., 2019).

2.5.2 From Learning Analytics to Automatic Formative Assessment

This section focuses on how the extensive data collected in a DLE can be useful to "go back" to Black and Wiliam’s FA strategies and support their implementation. Considering the reference to the LA definition of Solar (*LAK 2011*, 2011), we show some examples of collecting and analyzing different types of data relating to students and their activities, support formative assessment strategies, and optimize the DLE and learning. The examples described below reflect the theoretical framework of AT. Technologies are the tool that mediates the action of the subject (student, teacher, or peers) towards the object (implementing or improving the FA strategy).

Clarifying and sharing learning intentions and criteria for success

For this strategy, the data on the use of materials and interactive activities by the students can be analyzed and related to their assessment data to evaluate the materials and activities' effectiveness. In this way, it is possible to improve the teaching materials and increase the internal coherence of the platform's contents. Usually, the content is organized in learning objects, a collection of content items, practice items, and assessment items that are combined based on a single learning objective. An analysis of this type has been carried out on the Realignment Course in Mathematics of Orient@mente (Barana, Bogino, et al., 2018). Orient@mente is a platform of self-paced open online courses aimed to guide students in choosing a scientific university program at our University (Barana et al., 2017). The course lessons have many activities, such as online readable books, interactive activities of exploration or simulation, pages with theory applications and curiosities, automatically assessed online tests, exercises with their solutions. The assessment data have been related to the completion data of the various resources (viewed/not viewed) to understand if the student had completed the other activities or used the other resources before trying the tests. Our analysis showed that the students who used the activities before the tests did better than those who completed the tests based only on their knowledge. This result shows that the materials made available were effective and consistent with the tests. Different results would have been a clue of the need for a redesign of the course contents, to make materials more effective, or the tests more coherent with the learning activities.

Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding

For this formative assessment strategy, it is possible to use the gradebook to view assessment data organized by test, by student, or by question item, and the gradebook statistics. In this way, it is possible to analyze indexes such as the discrimination index of the items, the percentage of correct answers, and students' common mistakes. The teacher can identify the topics that are not clear, improve the existing learning objects, create new ones, or prepare classroom activities to clarify the unclear points. An example of this FA strategy was carried out within MATE-BOOSTER, a project conceived to strengthen students' mathematical competences attending the first year of a technical upper secondary school through an online course (Barana et al., in press; Barana, Marchisio, & Miori, 2019). The development of an online course was preceded by the analysis of the learning needs. It was carried out through an entry test to assess the initial competence and a questionnaire to understand students’ motivations. The entry test results aggregated by content areas showed the most difficult topics; moreover, the questions with low discrimination indexes identified common misunderstandings and improvement areas. Researchers and teachers listed the course's learning outcomes in light of the entry test and questionnaire results. The teaching materials' design was made considering the students' frequent mistakes, which emerged from the entry test and the teachers’ experience.

Providing feedback that moves learners forward

The data collected in the gradebook can be used to provide more detailed feedback that moves learners forward. In particular, the percentage of correct answers to a question in subsequent attempts with interactive feedback. In this way, it is possible to evaluate interactive feedback effectiveness, improve feedback itself, and provide useful activities for learning. In (Barana, Conte, et al., 2018), and in chapter 2.4 of this thesis, some examples of this strategy are presented. The results showed a high trend to make more than one attempt on the assignments developed according to the AFA model and containing interactive feedback. This means that letting students repeat the assignments is an effective way to make them aware that the information from the feedback was useful to improve their performance and make teachers and researchers sure that the feedback was well built. From the analysis, it emerged that the feedback effectively made students improve their results. In fact, for each student, the average of their grades considering only their first attempt on every assignment was compared with the average of the grades considering only their last attempts through a pairwise Student’s t-test. It resulted that the activities were useful for making students use the information obtained through the feedback to persevere and improve.

Activating students as instructional resources for one another

To support the activation of this strategy, it is possible to consult the grader report to analyze the students' relationships and interactions. In this way, it is possible to study the effectiveness of collaborative activities to support learning, and eventually improve them.

An example of this strategy was used in the Digital Math Training project (Barana & Marchisio, 2016a) and presented in (Barana & Marchisio, 2017). We analyzed the resolutions of the same problem by two groups of students, one in the context of individual work during a competition, and the other in online collaborative work in the Project’s platform. In the second group, the students could discuss their resolution through an asynchronous forum. The forum's interventions were analyzed, and the scores of the two groups of students were compared. The results showed that the second group of students performed better than the first group. Moreover, the analyses of the forum’s intervention confirmed that the collaborative activities supported learning and the development of mathematical, problem solving, computer, digital and collaborative work skills.

Activating students as the owners of their own learning

For this FA strategy, it is possible to use the data of the interactive activities and the questionnaires in the grader report to study the relationship between students' performance and engagement. The objective is to evaluate the effects of interactive activities on engagement, one of the most powerful driving forces that push students forward into a learning experience. Some examples were presented in (Barana, Marchisio, & Rabellino, 2019), in (Barana, Boffo, Gagliardi, et al., 2020) and in (Barana & Marchisio, in pressa), as well as in chapter 5.4 of this thesis. These studies are focused on experimentation where interactive technologies were used to improve students' engagement in Mathematics at grade 8. All students involved in the project with their teachers had access to an online platform populated with interactive worksheets with real-life mathematical problems coupled with automatically assessed quizzes for the whole school year. According to the results of initial and final questionnaires, the level of engagement notably increased in students that initially showed low levels of engagement. We believe that the nature of interaction enabled by the interactive files and by automatic assessment elicited engagement. This kind of interaction supported the exploration and the understanding of complex concepts, facilitated teachers' explanations in the classroom, and allowed students to self-correct and understand mistakes. Increasing students' engagement in such environments is a great goal. The online activities managed to catch students' attention thanks to the use of the computer and the interactive feedback, which opens a dialogue between students and the system and encourages them to understand solving processes.

2.5.3 Challenges

Being a new approach to formative assessment, LA techniques are not free from risks and challenges. Firstly, creating tasks and activities in a DLE to be used with formative purposes requires technical skills, knowledge of the tools, and a pedagogical preparation in the strategies and models of formative assessment. Otherwise, there is the risk of merely replicating traditional instruction with digital tools without reaping the benefits gained from a correct, informed, and conscious use of these technologies. This can be tackled through a specific training dedicated to the teachers or the instructors who will author the learning activities. Our research group has designed and experimented a teacher training model on these themes. It involves face-to-face and online training sessions through which many secondary school teachers became skilled in adopting AFA in a DLE (Barana, Fioravera, & Marchisio, 2017b; Anna Brancaccio, Marchisio, Meneghini, et al., 2015). The teacher training is flanked sharing the produced materials in a virtual community of practice. There, the trainees' contribution and the control of tutors from the University assure that high-quality materials are proposed to students.

But this is only a part of the risk mitigation. As Black and Wiliam stress (Black & Wiliam, 2009), it is not the mere use of proper tasks at the appropriate time that makes assessment formative: data from the assessment need to be used to make decisions in the instruction process. Here the learning analytics techniques can facilitate the visualization and analysis of learning data. However, it is not easy, especially for school teachers, to do the analyses and use the results just in time to influence the next steps in instruction. Several reasons can undermine the dynamism of the decision-making process that takes place in a classroom: for instance, sometimes teachers need the help of researchers to complete the analyses; gathering the data and starting the analyses requires some time; the results could not immediately be available (de Waal, 2017). To tackle these difficulties, it is possible to act on the automatization of the analyses processes and the improvement of the visualization of the results directly into the LMS. Teachers and instructors need to be trained to read these results and use them in their daily practices. Lastly, the contribution of Artificial Intelligence will give new and important prompts to the research about AFA and Learning Analytics.

2.6 MODELLING

This chapter will give some insights into the theories of mathematical models and modelling, a key competence for developing mathematical thinking, around which a substantial part of the topics covered in the upper secondary school rotates.

2.6.1 Working with mathematical models

In Mathematics, the term "modelling" refers to describing a real situation in mathematical terms and vice versa, which is used to solve real-life problems (Kaiser, 2014). As highlighted by the most recent national and international learning surveys, it is one of the most complex processes in Mathematics learning and it causes not few difficulties to the students. This is due to several factors that are analyzed in the vast literature of this field. Without any doubt, the main one is the cognitive impact that the modelling processes require. Infact, it is closely connected with other skills in the mathematical sphere necessary to carry out the process, such as the abilities to read and communicate in mathematical language, operate with mathematical symbols, interpret the results (Blum & Ferri, 2009). Moreover, metacognitive competences are needed to monitor the modelling process and identify instruments and results (Greefrath & Vorhölter, 2016). Of fundamental importance for the modelling activity is the knowledge of the different representation forms of the mathematical objects, indispensable in expressing a mathematical model correctly and working with it to solve a problem. In this sense, the modelling process is mediated by the tools – technological or cognitive – chosen and used (Lesh & Lehrer, 2003). Since different media emphasize different aspects of the conceptual system they are intended to describe, the solving processes for modelling activities often involve shifting back and forth among a variety of representations, each one holding part of the meaning associated with the system (Lesh & Doerr, 2003).

Working with mathematical models often involves linking together different aspects of the discipline, which are usually treated separately. For instance, functions are one of the most common forms in which mathematical models are expressed at secondary school. Functions, their different characteristics and representation forms (such as formulas, graphs, tables, verbal descriptions, and diagrams) are often presented in a fragmented way since lower secondary school. At the end of secondary school, the study of Mathematics shifts towards calculus, and students are expected to be able to flexibly work with functions and their representations. However, they can have difficulties in overcoming this fragmented view (Best & Bikner-Ahsbahs, 2017). These obstacles in understanding the concept of a function and its characteristics are observed even at the university level. They can be attributed to an underlying difficulty in understanding the dependence relation between two variables and their joint variation (Carlson et al., 2002).

Furthermore, students often have trouble transferring the knowledge developed during the resolution of a problem to a new problematic situation or applying again a previously analyzed model. This difficulty comes from an intrinsic property of learning, which is particularly significant for learning modelling: as several studies in the context of situated cognition point out, learning is always dependent on the specific learning context. Hence, a simple transfer from one situation to another cannot be expected (Brown et al., 1989). Modelling activities should always originate from a specific context, so this difficulty is natural and comprehensible (Blum & Ferri, 2009). One possible approach to help students overcome this difficulty is working on the generalization of a model and identifying similar cases where it can possibly be applied to deeper understand its properties and potentialities (English & Sriraman, 2010).

Modelling activities are interdisciplinary by their nature; this makes them not easy to design since the teacher must have extensive knowledge of the reality or connections with other disciplines. The collaboration with other colleagues who are experts in different fields would favor this kind of activities. However, too often, the school offers little chances for discussion among teachers of different disciplines.

Notwithstanding these difficulties, working with mathematical models is useful to favor the research of problem solving strategies, develop a numerical view of the reality, get students used to organizing data, analyze information with a scientific approach, and individuate key variables, processes, and transformations. As Blum and Ferri (2009) state, even "*modelling can be learnt*"; the decisive factor for successful teaching seems to be "quality teaching".

2.6.2 Algebra as a language for modelling

Algebra has a central role in every field of Mathematics; its concepts, principles, and techniques are essential to operate in all the other branches of the discipline. For this reason, algebra has been at the core of secondary school Mathematics for many years, and it has been a common belief that a good mastery of algebraic techniques is required to understand advanced mathematics (Huntley et al., 2000). Therefore, the teaching of algebra has always focused on technicisms and pure syntactic rules, as a set of procedures disconnected both from other mathematical knowledge and from students' real worlds, with the risk of losing the sense that the symbols convey (Arcavi, 1994).

In this panorama, Italy is not an exception. Traditionally, algebra is introduced at the end of lower secondary school (grade 8), when often entire months are dedicated to operating with monomial and polynomials, solving pages and pages of technical exercises which many textbooks propose. However, in the National Indications for the first cycle of instruction of 2012, there is just one brief reference to algebraic computations among the learning objectives at the end of lower secondary school: "*interpreting, building and transforming formulas which contain letters in order to express relations and properties in a general form*" (*Indicazioni Nazionali per Il Curricolo Della Scuola Dell'infanzia e Del Primo Ciclo d'istruzione*, 2012). This objective is included in the "relation and functions" content stream and not in the "numbers" one, which includes the other objectives linked to computation techniques, such as fractions, powers, and properties of the operations. Thus, it is suggested to introduce formulas and letters not to solve numeric expressions, but to generalize and build mathematical models. Actually, the suggestion to avoid presenting algebra as a set of rules to be learned by heart is not a novelty of 2012. It can be found in the Ministry Programs of 1979, where algebra of polynomials is not mentioned. However, instead, they say: *"algebraic manipulations detached from concrete references must not have a prevalent role in teaching as well as in assessment"* (Decreto Ministeriale 9 Febbraio 1979: Programmi, Orari Di Insegnamento e Prove Di Esame per La Scuola Media Statale, 1979).

These indications are in line with the trends and findings of the literature in Mathematics Education. In fact, since the Seventies, the algebraic content strand considered for school teaching has been widened to include the concept of function (Cusi et al., 2011), driven by the conviction that modelling provides both a critical reason and an important strategy for studying algebra (Yerushalmy, 2000). The shift towards a functional approach to teaching and learning algebra brought about a greater attention to algebraic language as a representational tool for modelling (Malara, 1996). Under this new light, the traditional studied objects such as symbols, formulas, and equations assume new meanings, and the interactive technologies help explore and convey them (Yerushalmy, 2000).

In this approach, representations play a crucial role. The Encyclopedia of Mathematics Education (Goldin, 2014) defines mathematical representations as "*visible or tangible productions – such as diagrams, number lines, graphs, arrangements of concrete objects or manipulatives, physical models, mathematical expressions, formulas and equations, or depictions on the screen of a computer or calculator – that encode, stand for, or embody mathematical ideas or relationships*." Duval (1999) extensively studied the role of representations in Mathematics education, starting from the assumption that mathematical objects are inaccessible to human knowledge, and students can only identify them through semiotic representations. This can hinder them from understanding mathematics when they cannot associate a mathematical object to its several representations. Mathematical reasoning occurs through the coordination of registers via two main processes, namely treatment, and conversion. A treatment is a transformation from a representation into another within the same semiotic register; a conversion is a transformation between two representations in different semiotic registers (Bolondi et al., 2019). While syntactic rules guide treatments, conversions can be more difficult for students because they require to associate different apparently unrelated representations to the same mathematical concept (R. Duval, 1993). For this reason, according to Duval (1993), the conversion is the key cognitive function that guarantees the conceptual acquisition of mathematical objects.

Under this perspective, a functional approach to algebra should help students focus on conversions and reason through different media, linking different registers to underline different aspects of the mathematical model intended to describe. However, several studies show that this approach is not so rooted in daily teaching practices. Pozio and Bolondi (2019) analyzed the answers given by all the Italian students of grade 8 in 2018 to an item of the INVALSI tests asking to express algebraically a geometrical model, namely the perimeter of a trapezoid in which one side’s length is given through a parameter. The results show the students' great difficulties in formalizing algebraically geometrical concepts and investigating the possible causes, among which teaching practices have a prominent role. This study is an example of how Italian students do not master the conversion processes in grade 8. In another study based on data from INVALSI tests, Bolondi, Ferretti, and Santi (2019) focus on an item of the 2018 tests for grade 10 involving algebraic manipulation, that is, treatments within the same semiotic register. They analyze the students’ answers, 30% only of which were correct. They conclude that in the absence of a meaningful personal activity, the students interpret the semiotic representations as unrelated objects, thus losing the connection with the original mathematical concept, even when they can rely on precise transformation rules.

Another study from an English context (Huntley et al., 2000), the effects on the growth of student understanding, skill, and problem-solving ability in algebra of two high-school Mathematics curriculum - The Core-Plus Mathematics Project (CPMP) and a traditional curriculum – are compared. The CPMP was based on a real-life problem solving approach, letting room explore concepts rather than teacher explanations and student practice of routine symbol-manipulation skills, as it was a common habit in the traditional curriculum. From the study results, it emerges that the abilities in algebraic manipulation are weakly correlated to problem solving skills, and the authors argue that it is possible to develop advanced problem solving skills without innumerable repetitive algebraic manipulation exercises. Moreover, managing conversions between different representation registers resulted in a key for success in problem solving. In the CPMP, the sense of connection among representations (such as graphs and expressions) for Mathematical concepts (such as that of a linear function) was enabled through a variety of experiences with representational activities and applied-problem situations (Huntley et al., 2000). Similarly, Yerushalmy (2000) noted that, working on long-term vertical paths with algebraic modelling problems that aim to coordinate different forms of representation, students manage to integrate them and better understand the concepts. In this case, technology plays a leading role, mainly helping students connect the graphic and symbolic registers. Through her results, Yerushalmy (2000) rejected the hypothesis that using graphing technologies or computer algebra systems might prevent students from developing skills such as drawing graphs or manipulating expressions autonomously. She observed that the students who successfully used graphing technologies to solve the tasks completed the tasks just as well, even without the software. On the other hand, low achievers who had difficulties solving the tasks with paper and pen had the same problems even with the software's help. As Huntley et al. (2000) observed, graphing technologies and computers make modeling and functions attractive and accessible for students independently from their prior achievements, aptitudes, and interests. The use of calculating tools also offers students a variety of powerful new learning and problem-solving strategies. The need for students to acquire a high degree of skill in symbol manipulation diminishes.

Geometry can be a relevant context to work with algebraic formulas: several scholars suggest to use algebraic thinking in geometry to favor the understanding of variables as quantities to be related rather than symbols to manipulate (Dindyal, 2004). Activities of this kind include tasks that ask to compute the perimeter, area, or volume of a figure whose measures are given through variables. Varying the variable's value to enlarge or reduce the size of the figure can help move from symbolic manipulation to functional covariation. Dindyal (2004) found that the students' main difficulties in facing this kind of problem are the understanding of geometric properties and algebraic rules. The previously cited study by Pozio and Bolondi (2019) added that, besides misconception, formal and procedural errors in both the geometric and algebraic sphere, other problems could derive from the inability to coordinate different forms of representation. Its roots can be found in the usual didactic practices, which tend to focus mainly on manipulative mechanisms in the construction of concepts, giving little relevance to functional and semantic aspects.

Malara, Cusi, and Navarra (2011) suggested using natural language as a didactic mediator for learning algebra. Analogously to every language, algebra has a syntactic structure and a semantic aspect; therefore, it should be taught as a new language: starting from its meanings and setting them gradually in the syntactic structure. They called this process "*algebraic babbling*" (Cusi et al., 2011). The process of translation (from algebra to natural language and vice versa; from one representation to another) is central. Natural language sets up the bases for producing and interpreting representations written in algebraic language. Through natural language students can illustrate the systems of relations (additive and multiplicative ones at the beginning) among elements in a problem situation inducing a translation of the process itself into a mathematical sentence. In this way, in the resolution of a problem, attention is shifted from the arithmetic goal of solving it to produce a result, to the algebraic one of representing it, its variables and relations. Under this perspective, algebra becomes a language for modelling.

Many authors have recently suggested starting the learning of algebra early, using the approaches that we presented above, since primary school (Cusi et al., 2011; Kaput, 2000; Kieran et al., 2016). In order to improve the teaching and learning of algebra, Kaput (2000) suggested to begin early, integrate the learning of algebra with that of other disciplines (that is, to apply it), include different forms of algebraic thinking, grounded on the students' cognitive and linguistic abilities, and encourage active learning. He distinguishes five different forms of algebraic thinking:

a. generalization and formalization of patterns and constraints;
b. syntactically guided manipulation of (opaque) formalisms;
c. study of structures abstracted from computations and relations;
d. study of functions, relations, and joint variation;
e. cluster of modeling and phenomena-controlling languages.

Using these interacting forms of algebraic thinking, it is possible to move from the abstract and formal algebraic rules to the language for modelling.

2.6.3 The first mathematical models: linear models

Linear functions are the first mathematical models that students face in their Mathematics studies. They emerge within algebra, since they involve the simplest operations among numbers and variables, and they offer many prompts for reflecting on and understanding mathematical models. The first hurdle that students have to overcome when dealing with linear models is the concept of variable, which is often not well defined in school Mathematics. It can create confusion among the terms variable, unknow, parameter, and their relations with numbers and constants (A. H. Schoenfeld & Arcavi, 1988). The second hurdle in understanding linear functions is the dependence between x and y, the variables through which they are usually expressed. The concept of joint variation is one of the most problematic at school teaching. It seems that many difficulties with Mathematics, even at the university level, can be attributed to an underlying misunderstanding of this concept (Carlson et al., 2002). It is a recurrent concept in secondary school Mathematics since several functions are studied with their properties and representation forms; linear functions are the first example through which this concept is approached. The third point that needs attention is the relationship between "*m*", the slope, and "*c*", the intercept, in the standard equation $y =$

$mx + c$, which determine the trend of the line. A study by Bardini and Stacey (2006), which focuses on the understanding of *m* and *c* in linear functions, shows that, as expected, the slope is a more complex concept than the intercept. However, students tend to omit *c* as it is not part of the function, maybe due to the little attention dedicated to the intercept compared to the slope in the classroom activities. Conversions among different semiotic registers (numeric, symbolic, graphic, and real-world context) seem to influence the students' interpretation and understanding of these elements (Bardini & Stacey, 2006).

A good understanding of a linear function's slope as "rate of change" is essential to develop the following important mathematical concepts, such as the derivative and differential equations (Rasmussen & King, 2000). An interesting vertical study by Gambini, Banchelli, and Nolli (2020) in the Italian context tries to analyze how the understanding (and misunderstanding) of the concept of slope changes from grade 8 to 14 (from lower secondary school to university), using data from INVALSI tests and university entry tests. From the results, they observe that students have trouble integrating symbolic thinking and meaning. By grade 8, mainly reasoning numerically, they should have acquired the concept of variation associated with the slope of a linear structure. By grade 10, students should have associated the symbolic and graphic aspects, but it seems that they have abandoned the quantitative reasoning, which helps confer meaning to the involved objects. This split between the different interpretations of the slope continues and is consolidated in grades 13 and 14 when the comprehension of the derivative concept would require integrating the different aspects, joining the mathematical formalism to the numeric, symbolic, and graphic aspects. What too often remains is the algebraic computations, disconnected by their meaning (Gambini et al., 2020).

2.6.4 Modelling and Problem solving

As seen above, mathematical modelling refers to the process of describing a real situation in mathematical terms and vice versa and using it to solve real-life problems (Greefrath & Vorhölter, 2016). Therefore, mathematical modelling always originates from a real-life problem, which is then described by a mathematical model and solved using it.

In model-eliciting activities, students must develop, extend, and revise a model (e.g., interpretation, description, explanation) that is useful for accomplishing some specified purpose. What needs to be produced is a model to make sense of the situation for which students' currently available interpretations of the givens and goals lack enough detail, elaboration, precision, and development. The interpretation of the given and goals is considered a significant challenge, making the selection and application of procedures a cyclical process integrated into the interpretation phases of problem solving (Zawojewski & Lesh, 2003).

Problem solving

The term "problem-solving" in Mathematics education refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development (*Executive Summary Principles and Standards for School Mathematics*, 2000). Problem solving is one of the objectives of Mathematics learning, and it includes the ability to understand the problem, devise a mathematical model, develop the solving process and interpret the obtained solution (Samo et al., 2017). Mathematical problems are central in mathematical practice to develop the discipline and foster student learning (Pólya, 1945). Mathematics should not be considered different from the possible daily activities of the students. A real situation in which students might find themselves can be used to propose a problem to bring together the teaching and learning practices of school Mathematics and the wealth of experiences that students develop outside of the school (D'Amore & Pinilla, 2006). According to Samo, Bana and Kartasasmita (2017) "*contextual teaching and learning is a learning which links the material with the real-world context of students' everyday life either within family, community, environment, and the world of work so that students are able to make connections between knowledge possessed by its application in everyday life*". Problem solving can stimulate interest and motivation towards the study of Mathematics by creating a bridge between school and extracurricular Mathematics, bringing out realistic considerations and developing modeling skills (Baroni & Bonotto, 2015).

Furthermore, by solving mathematical problems students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the Mathematics classroom (*Executive Summary Principles and Standards for School Mathematics*, 2000). To gain experience in problem solving, students should solve various types of problems regularly and over a prolonged period. Non-routine problems and open problems should be used to offer students a wide range of possibilities for choosing and making decisions. Students' resolution of a problem can be used to evaluate progress in the problem solving skill, using a rubric score scale (Leong & Janjaruporn, 2015).

The analysis of how mathematical problems are formulated and the process involved in solving problems generates important information to structure learning environments to guide students' construction of mathematical concepts (Liljedahl et al., 2016).

Problem Posing

Problem posing is an important component of the Mathematics curriculum, and it is considered an essential part of mathematical doing (*Executive Summary Principles and Standards for School Mathematics*, 2000). It involves creating new problems and questions aimed at exploring a given situation and the reformulation of a problem during the process of solving it (Lavy & Shriki, 2007). Kilpatrick (1987) marked a historic milestone in research related to problem posing: he pointed out that problem formulating should be viewed not only as a goal of instruction but also as a means of instruction and that the students should have opportunities to live the experience of discovering and posing their problems. Kilpatrick also pointed out how other people have posed all problems that students solve. However, in the real-life, many problems are created or discovered by the solver, who gives the problem an initial formulation (Liljedahl et al., 2016). According to Lavy & Shriki (2007), providing students with opportunities to pose their own problems can foster more diverse and flexible thinking, enhance students' problem solving skills, broaden their perception of Mathematics and enrich and consolidate basic concepts.

Stoyanova and Ellerton (1996, p. 519) considered mathematical problem posing as "*the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems*." They identified three categories of problem-posing situations: free, semi-structured, or structured. In free situations, students pose problems without restrictions; in semi-structured situations, students explore a given open situation, and they have to complete it; in structured situations, students pose problems by reformulating already solved problems or by varying the conditions of given problems (Singer et al., 2015).

In conclusion, problem posing is an opportunity for interpretation and critical analysis of reality since

a. the students have to discern significant data from immaterial data;
b. they must discover the relations between the data;
c. they must decide whether the information in their possession is sufficient to solve the problem; and
d. they have to investigate if the numerical data involved is numerically and contextually coherent.

Technologies for Problem Posing and Solving

The use of digital technologies in problem solving activities makes teachers and learners become active participants in the learning process generated within the mathematical working space. These instruments offer a precious diversity of ways to represent and explore the tasks (Santos-Trigo et al., 2016). Kuzniak et al. (2013) argue that digital technology offers teachers and learners an opportunity to extend and deepen ways of reasoning about the mathematical strategies involved in solving problems. Moreover, representing and exploring mathematical tasks mediated by digital technologies bring in new challenges for teachers. These challenges include appropriating the instruments provided by these technologies to identify and analyze what changes to mathematical contents and teaching practices their use fosters. It is not the tool itself that produces this transformation: it is the process of appropriation led by the teacher and the students that eventually transforms the digital artifact into a mathematical instrument (Moreno-Armella & Santos-Trigo, 2016).

Among the classic examples of technologies that can be successfully used for problem solving, we can list: electronic spreadsheets, graphic calculators such as Graphing Calculator 4.0(Avitzur, 2011), online computational engines such as Wolfram Alpha (Dimiceli et al., 2010), dynamic geometry systems such as Geogebra or Cabri Geometry (Hohenwarter et al., 2008), Advanced Computing Environments but also Learning Management Systems or Automatic Assessment Systems. Technology can foster conjecturing, justifying and generalizing by enabling fast, accurate computation, collection and analysis of data, and exploration of multiple representational registers (Goos et al., 2003). The increasing availability and power of electronic devices such as computers and graphic calculators offer students new opportunities to communicate and analyze their mathematical thinking, since the objects generated on the screen can act as a common referent for discussion (*Executive Summary Principles and Standards for School Mathematics*, 2000). They can be tools that free students from the calculations, allowing them to focus on more interesting activities, such as exploring, conjecturing, searching, and concluding, leading to a deeper understanding of mathematical concepts. Graphic calculators can be used as exploratory tools but also to verify the solution to a problem (Mesa, 1997). Dynamic geometry systems provide learners with a set of affordances to represent and dynamically explore mathematical problems. By moving objects within the dynamic model, students can identify and examine mathematical relations that emerge (Moreno-Armella & Santos-Trigo, 2016). According to Caprotti et al. (2006), technologies are also used to enhance teaching materials in Mathematics by adding java illustrations or animated graphics and allowing students to experiment with mathematical statements. ACEs such as Maple and Mathematica can easily produce sophisticated animations, and the materials can be used without needing to install the software (Caprotti et al., 2006). Moreover, a DLE encourages social interactions in problem solving: through discussions in a community of practice, the construction of knowledge and awareness of learning are fostered (Barana, Brancaccio, Esposito, Fioravera, et al., 2018). Lastly, AASs can enhance problem solving skills through interactive feedbacks and reflection on one's mistakes (Barana, Marchisio, & Sacchet, 2019).

Technologies play a fundamental role not only in problem solving but also in problem posing. According to Singer et al. (Singer et al., 2015), technology can facilitate and foster skills in formulating problems. The authors explained that most of the studies had been conducted on developing conjectures in dynamic (or partially dynamic) geometry environments and on mathematical problem posing in web-based learning environments. Singer et al. presented modern digital technology tools such as electronic spreadsheets, computer-based Graphing Calculator, Maple, and Wolfram Alpha in facilitating and advancing skills of preservice teachers in mathematical problem posing. Their research shows that the teachers' success with technology-enabled problem posing requires practical experience with mathematical modelling and problem solving as well as theoretical preparation in pedagogical issues. This is important for the development of skills in formulating new problems and modifying the existing ones. They also explained that using technology in problem posing encourages open-ended classroom pedagogy, fosters mathematical reasoning and thinking skills of preservice teachers, and has excellent potential to make students better problem solvers.

The interventions of a teacher in a problem solving activity using the technologies can have different purposes: to help use the tools, to support the students in understanding the mathematical concepts (for example, solving an apparent contradiction between the solutions returned by an Equation Editor, graphic methods and spreadsheets), to encourage students to conjecture and justify (for example by presenting their results to their peers) and to support a peer discussion in developing a deep understanding (Goos et al., 2003).

Part of section 2.6.4 is taken, expanded and adapted from (Barana, Brancaccio, Conte, et al., 2019a, 2019b).

2.7 ENGAGEMENT

Engagement is one of the most powerful driving forces that moves learners forward in a learning experience. When students are engaged in a task, they tend to keep focused, to enact in-depth learning strategies and self-regulation, to achieve good results, and even to get satisfaction and pleasure for their activity. More generally, encouraging and controlling engagement is an effective communication technique: the evolution of audience attention is a key factor that has to be considered when scheduling the duration of lessons, presentations, and performances. In education, increasing engagement can be useful because engaged learners may enter a virtuous circle: when they obtain good results because of their work, their self-efficacy beliefs are intensified; this keeps them engaged and makes them continue succeeding. On the other side, disengaged students who have difficulties achieving good results may be trapped in a negative rather than a positive loop, which hinders them from success (Ng et al., 2018). Students coming from poor families or low socio-economic status might find it harder to be engaged in learning activities than students coming from medium-high social classes. This fact is due to the little support that the former may find in their family or the greater difference they may perceive between their school and home environments (Appleton et al., 2006). These might be the root cause for drop-outs and early school leaving. That is why supporting didactic projects aimed at enhancing learning engagement, especially for students with challenging backgrounds, is often a key strategy pursued by policymakers and institutions interested in improving education quality over their territory.

In Mathematics, which is often considered a "hard science", engagement is related to developing strong aspirations for carrying on advanced studies in this field (Ng et al., 2018). Since the development of Mathematical understanding is a crucial access key for the workplace in modern society, teachers and educators from the very early school years should pay much attention to student engagement in Mathematics.

Even more than for traditional schooling, engagement is a key point, especially for online learning. In particular, it is a strong predictor of MOOC retention (Xiong et al., 2015). Studying solutions aimed at keeping users engaged is crucial to increase the completion rates. They are often very low due to the enrolled students' weak motivations to complete the courses (Jordan, 2014). Technologies often support the engagement process: learning materials provided through gamification, simulation, or interaction seem to be more effective than static resources in maintaining the users involved.

Technology also supports the teaching process: in virtual environments, unlike in traditional classrooms, the large amount of data registered and made available by the systems can provide useful information that helps researchers understand the processes activated during learning situations, evaluate the effectiveness of teaching strategies, and support decision-making.

Part of section 2.7 and its subsections are taken, adapted and extended from (Barana, Boffo, Gagliardi, et al., 2020; Barana, Marchisio, & Rabellino, 2019).

2.7.1 What is "engagement"?

Engagement is highly studied in the educational research field, and it is possible to find many different definitions and characterizations of this construct in literature. Some authors associate engagement with the level of attention (J. A. C. Hattie, 2009) or with motivation (Skinner & Pitzer, 2012). Others conceptualize it in terms of visible students' behavior, which should reflect the way they engage with learning materials (Chi & Wylie, 2014), in terms of intensity and quality of students' involvement in learning activities (Yang, 2011), or in terms of effort and investment students expend in the learning task (Marks, 2000). In all these researches, active participation is a central theme for understanding engagement. In this paper, we accept the definition given by Ng, Barlett, and Elliott (2018), who refer to engagement as "*students' dynamic participation and co-participation in recognition of opportunity and purpose in completing a specific learning* *task*.” The peculiarity of this definition is the characterization of engagement as an interactive and purposive process. It allows to examine how it may change over time and vary according to situations and contexts. When students participate eagerly in a specific learning activity, they deploy appropriate strategies, regulate processes, and monitor their actions. They feel happy, spend time and effort on the task, and show high focus and concentration levels. However, these conditions may fluctuate: sometimes, students can fail to plan their actions, feel worried or not so willing to making efforts, or they can become distracted. Thus, engagement is not mere personal property, but a set of actions undertaken by a person in a specific context where interactions with other people, artifacts, and tools occur. Engagement has a fluctuant nature. This means that it depends on specific situations, and it may change over time; it has a focal object; it is situational and malleable: changing task design, support or rules project can modify it (Ng et al., 2018). The malleable nature of engagement means that it is possible to create repeated episodes eliciting engagement and so, in time, contribute to these students establishing stable positive beliefs and behaviors (Pierce et al., 2007).

Despite the number of definitions, research on engagement agrees that it is a multidimensional construct (Fredricks & McColskey, 2012). In line with the main trend, we recognize the three main components of student engagement identified by Fredricks, Blumenfeld, and Paris (2004): behavioral, emotional, and cognitive.

Behavioral engagement (BE) draws on the idea of participation and it includes behaviors such as effort, perseverance, attention, concentration, and completion of work (Fredricks et al., 2004). It also concerns positive conduct, such as following rules and participating in social or school-related activities (Ng et al., 2018). When the focus is on homework, effort expenditure, and timely completion are indicators of behavioral engagement. However, strict adherence to norms is not a good indicator for high-order thinking, enjoyment, and interest: students could just keep quiet and pretend to pay attention, their level of interest being indeed very low (Dagley, 2004).

Emotional engagement (EE) is understood as students’ affective reactions in a classroom, which can vary from interest to boredom, from happiness to sadness, from satisfaction to anxiety. Interest and value for learning are important indicators of emotional engagement (Fredricks et al., 2004). It is linked to several outcomes such as learning achievements, liking school subjects and positive attitudes towards school (Ng et al., 2018).

Cognitive engagement (CE) is the mental investment people make in learning; it involves using deep strategies, self-regulation, openness to problem solving, and positive coping in the face of failure (Fredricks et al., 2004). A high level of cognitive engagement can be detected when students enter into an interactive dialogue to generate new knowledge (Chi & Wylie, 2014). Students with high cognitive engagement levels are less likely to give up their learning and more likely to keep engaged with school (Ng et al., 2018).

In analyzing the effects of engagement during learning activities, a distinction should be made amongst these components, labeled as “indicators” of engagement, and the so-called “facilitators” of engagement, which can be cognitive and social (Skinner & Pitzer, 2012). Among cognitive facilitators, the most acknowledged are:

- self-efficacy, the child’s perceived ability to successfully complete a task within a specific domain or setting. Students who have developed a high sense of self-efficacy are more confident in their capacities and are more likely to get involved in tasks;
- achievement goals, that are students’ perceived purposes to learn. There are two main types of achievement goals: mastery and performance goals. Mastery goals represent a focus on learning for the sake of improvement and understanding, whereas performance goals reflect students’ attention to achievement; the former are more desirable than the latter, as they are linked to higher outcomes;

- autonomy, which refers to the choices that students can make freely during the learning process, and it is at the core of the self-determinant theory for promoting engagement (Skinner & Pitzer, 2012);
- interest, which involves both cognitive and emotional components and promotes and supports learning motivations.

Social facilitators refer to social conditions, interactions, and relationships that promote engagement; especially for adolescents, the influence that peers, teachers, family, and the environment have on them is critical for their choices, behaviors, and emotions (Kindermann, 2007).

Low levels of engagement and low achievements are often related. In a study conducted by the OECD after PISA 2012 survey, low performers in Mathematics, who scored less than 2 on a 6-level scale, are less interested in Mathematics than better-performing students. They report low levels of perseverance, that is associated with work ethic. It seems that they do not devote enough time to homework. Their effort in school-related activities is not very productive, as it does not result in significant outcomes. Moreover, they tend to skip classes and school days and show little sense of belonging at school (OECD, 2016b). Students with a disadvantaged background are more likely low performers than top ones: socio-economic status greatly influences school achievements, attitudes, and beliefs towards Mathematics (OECD, 2013).

2.7.2 Engagement and learning technologies

Students engagement in technology-enhanced learning environments includes any interaction of the learner with instructors, peers, or learning content through digital technology. The interactions can happen face to face or remotely, and the courses involved may be entirely online, blended, or face to face (Henrie et al., 2015). The potential of technologies can open up new ways to research engagement, contributing both to the measurement of the indicators and the creation of facilitators of engagement.

When the focus is on detecting engagement, technologies offer many sources of data such as logs, registration of dialogues, and answers that can be usefully integrated into the research (Yang, 2011). Many authors agree that the mere number of logs is not a reliable indicator of behavioral engagement if considered alone: the amount of time and actions spent on activities may vary largely among students according to their cognitive needs or external factors. However, the data provided by automated systems can be combined in order to generate meaningful information about user experience (Henrie et al., 2015).

On the other hand, digital technologies can contribute to the creation of cognitive and social facilitators of engagement for several reasons: they enhance the possibilities to activate learning by doing or active learning strategies, which enable students to intellectually engage in the task (Gossen et al., 2018; Prince, 2004); they increase the chances of interactions among peers and with the instructor (Yang, 2011); they can facilitate self-regulation and adaptive learning through formative assessment (Barana, Fioravera, Marchisio, et al., 2017; D. Nicol & Milligan, 2006); asynchronous activity enable learners to study at their own pace and to reflect on the learning process (Barana et al., 2017, 2017). In Mathematics and other STEM, real-world problem solving activities help create a connection between the subject and the environment, making Mathematics interesting and relevant. Software for advanced computation and data analysis makes students analyze real and complex data and solve the problems (Barana, Fioravera, & Marchisio, 2017a; Pierce et al., 2007).

AFA activities developed according to our model should be cognitive facilitators of engagement. When students fail one answer, the interactive feedback activates them through the solving process, making it possible to identify the mistake's exact source, thus acting on self-efficacy. Students can try the question again and find different parameters, so they have to repeat the solving process autonomously. Multiple attempts before showing the correct solution act on autonomy as well. The immediate feedback helps them focus on mistakes as a source of knowledge and to set mastery goals instead of performance goals. Adaptivity allows the creation of personalized learning paths to tailor the learning experience to the students' needs. Lastly, real-world settings act on the student's interest in the subject, connecting abstract Mathematics to the real world (Barana, Marchisio, & Miori, 2019; Anna Brancaccio, Marchisio, Palumbo, et al., 2015). Similar activities can also be used to support collaborative classwork, becoming social facilitators of engagement: well-organized peer discussions foster students' participation in learning activities and contribute to creating a comfortable learning environment, which is fundamental for school well-being (Kindermann, 2007).

2.7.3 Reluctant learners and reluctant students

Every teacher, especially at the secondary level, has met at least one "reluctant learner" in their career. Reluctant learners are those students who achieve low academic results as a consequence of their poor motivation, self-esteem, and low efficacy. They are usually disengaged from school, and they do not easily get involved in learning activities; they often end up leaving school without any formal qualification. Since Mathematics is considered one of the most challenging disciplines, developing negative aptitudes towards it is somewhat more common than other subjects (Sanacore, 2008). For formative assessment, digital learning environments are generally considered successful in engaging reluctant learners in Mathematics. The presence of digital and dynamic elements can offer them new ways of engaging with mathematical thinking and facilitating understanding (Calder & Campbell, 2016).

In this framework, we propose a distinction between the reluctant learners, who are dissatisfied with school, or with Mathematics, in general, and reluctant users, who cannot be engaged in learning technologies. In particular, we define "reluctant users" as those students who make too few attempts to the AFA activities in a course to benefit from their effects on learning, metacognition, and self-regulation. They are counterposed to the "active users", who can be actively involved in the online activities. Activity and reluctance here concern only the use of the automatically graded assignments; an a priori relation with learning is not assumed; however, it will be discussed through the following analyses.

3 PILOT EXPERIMENTATION

The use of AFA with interactive feedback in Digital Learning Environments is not much common at school: it is a novelty coming from the research world. It requires that teachers and the whole school system make a significant investment to integrate this methodology into the daily teaching practices. Educational politics need evidence that it works before introducing it in teacher training programs. Therefore, it is clear how studies on AFA's effectiveness for learning under different perspectives are essential. The Delta Research Group started a research project to analyze how the adoption of AFA in a DLE affects learning and teaching processes.

The project started with a pilot experimentation, aimed at testing interactive activities with AFA with a problem solving approach on a limited sample to identify the most suitable conditions for the design of a more robust experimentation. In particular, we were interested in investigating the following research questions:

(pRQ1) What is the most suitable target for an experimentation aimed at analyzing how the adoption of interactive activities with AFA in a DLE affects teaching and learning processes?

(pRQ2) What kind of activities can be proposed to the students?

(pRQ3) What are the main effects on learning of the adoption of interactive activities with AFA in a DLE?

The experimentation was developed by the Delta Research Group of the University of Turin, in collaboration with the National Research Council. It was conducted within a wider research project entitled: "Educating City: teaching and learning processes in cross-media ecosystem" funded by the Italian Ministry of Education in the frame of the National Technological Cluster "Technologies for Smart Communities", to rethink the learning processes through the application of the newest advances in educational technologies.

In the following chapters, we will give details about the methodology of research, the experimented activities, the instrument of evaluation of the experimentation, and the results, discussing their implications for further research. Table 3.1 shows a sheet which schematizes the pilot experimentation's main points.

3.1 METHODOLOGY

The pilot experiment took place in 2016/2017 SY and involved 258 students from 11 classes of three different lower and upper secondary schools of Turin (Italy), in particular:

- 4 classes of the third year of lower secondary school (grade 8);
- 3 classes of the first year of upper secondary school (grade 9);
- 4 classes of the second year of upper secondary school (grade 10).

The classes of upper secondary school (grades 9 and 10) belonged to a prestigious school, a Scientific Lyceum, attended mostly by students of a middle-high social class, and with a cultural family background relatively high. The classes of grade 8 belonged to 2 lower secondary schools from the same area of the city attended by students of similar social conditions, mainly lower classes.

The experimentation has undergone an assessment analysis that involved the learning processes through a systematic description of the facts (what is observed, stated in a report) and the processes' success by implementing an assessment model of a counterfactual type (Martini, 2006). This assessing approach aims to establish a causal link between what is observed before carrying out an intervention and the intervention itself. In the specific case, the intervention under assessment has regarded the didactic methodology's success or failure compared to the results with the original objectives. In particular, and to evaluate the so-called net effects (the effective added value of the used methodology), a comparative assessment of the counterfactual type has been implemented. This type of assessment required identifying two groups: a treated group, composed of classes that undertook the experimentation, and a control group, composed of reference classes that were not part of the experimental activities, but only to the experimentation's assessment. For selecting the control group, the classes with more similar characteristics to the treated ones have been taken into account (Cerulli, 2015). Thus, eight classes were identified and composed the treated group, while three classes (one for each grade) the control group. Table 3.2 shows the number of students in each group. This technique was integrated with qualitative methods through questionnaires, focus groups, and observations collected during the experimentation, aimed at studying several aspects of the effectiveness of the didactic methodologies proposed, as the activation of learning processes and changes in student engagement.

Table 3.1 Pilot experimentation's sheet, which outlines the main points.

	PILOT EXPERIMENTATION
DURATION	15 hours of experimental activities in March-June 2017
GOALS	Testing AFA interactive activities with a problem solving approach on a limited sample to identify the most suitable conditions for the design of a more robust experimentation.
RESEARCH QUESTIONS	1. What is the most suitable target for an experimentation aimed at analyzing how the adoption of interactive activities with AFA in a DLE affects teaching and learning processes? 2. What kind of activities can be proposed to the students? 3. What are the main effects on learning of the adoption of interactive activities with AFA in a DLE?
TARGET	Students of grade 8, 9, and 10 of Turin.
LEARNING OBJECTIVES	• Equations and statistics (grades 8 and 9); • quadratic functions, linear systems, and second-grade equations (grade 10).
DIDACTIC METHODOLOGIES	• AFA in a DLE; • problem-solving; • collaborative learning.
FORMAT OF THE EXPERIMENTAL ACTIVITIES	1. stimulus questions with AFA in small groups; 2. problem solving activities in small groups; 3. individual online homework with AFA
SETTING	• Grades 8 and 10: in the classroom, tasks displayed at the IWB, students working in groups with paper and pen; • Grade 9: in the computer lab, students working in groups with the ACE • Online activities: Moodle platform integrated with an ACE and an AAS.
LMS	https://cittaeducante.i-learn.unito.it/
RESEARCH METHODS	Counterfactual method, comparing the effects on learning of a treated and a control group, integrated with qualitative analyses of data from observations, questionnaires and focus groups.
PARTICIPANTS	• TREATED GROUP: 179 students (8 classes) from 3 different schools of Turin • CONTROL GROUP: 79 students (3 classes) from 3 different schools of Turin
ASSESSMENT INSTRUMENTS	• Initial test (treated and control group; 10 INVALSI questions in 30 minutes, paper-based) • Final test (treated and control group; 10 INVALSI questions in 30 minutes, paper-based) • Final questionnaire (treated group only) • Focus groups with teachers of the treated classes.

Table 3.2 Numbers of participants to the pilot experimentation.

Grade	Classes			Students		
	Control	**Treated**	**Total**	**Control**	**Treated**	**Total**
8	1	3	**4**	23	51	**74**
9	1	2	**3**	29	57	**86**
10	1	3	**4**	27	71	**98**
Total	**3**	**8**	**11**	**79**	**179**	**258**

Ten teachers were involved teachers, one for each class, except for the control class for grade 10, which shared the teacher with another 10th-grade class in the treated group. Some of the teachers had previously attended specific training activities on the methodologies proposed.

To fulfill the conducted analysis, all the instruments and the didactic technologies and methodologies were shared with the teachers through a focus group technique. This way, the intention was not only to identify the time and method for using and adopting the new technologies and instruments produced as part of the project but also to acquire information regarding the dynamic triggered by the experimentation in the classes, even to outline the strengths and weaknesses of the experience and the possible future variations. The choice of the focus group technique was also due to the need to acquire information from the experimentation respecting each teacher's total freedom of expression. In this way, we avoided them feeling forced to provide answers that do not correspond to their own opinion, which would have originated non-foreseeable situations at the stage of defining the assessment design.

The parents signed a consent form to use data of the students registered in the classroom and the platform for didactic and research purposes only.

3.2 Experimented activities

Interactive Mathematics paths were created and made available to the students and teachers that belonged to the treated group. These paths were developed using a problem posing and solving approach. They are constituted of interactive materials with AFA and interactive feedback. They were conceived to be conducted in a blended modality and dealt with selected topics agreed with the classes' teachers. The interactive materials were developed and distributed in a DLE whose LMS was based on a Moodle platform specifically designed for the project (http://cittaeducante.i-learn.unito.it/). The platform was integrated with the ACE Maple and the AAS Moebius Assessment. Interactive worksheets allowed students to explore the solutions to the problems and share the developed models, check their understanding through the computer's immediate feedback, and learn through step-by-step adaptive processes (Barana, Boffo, Gagliardi, Garuti, et al., 2018; Barana et al., In press).

The topics chosen for the experimentation were linear equations and statistics for grades 8 and 9, quadratic functions, second-grade equations, and linear systems for grade 10. For each topic, the following activities were proposed:

1. stimulus questions with automatic assessment, in small groups, to promote the interaction and the debate among peers. The stimulus questions have been designed to introduce the fundamental concepts of the topic and to function as a pretext for promoting further classroom discussions;
2. an open problem to be solved in the classroom in small groups. The group's solutions and solving processes were afterward presented to the whole class and discussed with the teacher;

3. problems proposed with automatic assessment and interactive feedback to be carried out in the platform, substituting the classic homework. The activities were designed starting from INVALSI questions, mainly related to the dimension "solving problems", expanded and adapted to the automatic assessment.

The classes of the treated group worked on experimental activities for about 15 hours in Spring 2017. Almost all the activities took place with the PhD student author of this thesis as a support for the teachers, for the management of the group work and the laboratory activity, and the use of the DLE. In this way, she could collect observations about the students' answers and the classes' attitudes. During the same experimentation period, the control classes dealt with the same topics, although using traditional methodologies.

In the Scientific Lyceum case, the activities took place in a computer laboratory; students in pairs could answer the stimulus questions using the AAS and solve the problems using the ACE. The teacher had the guiding and supporting role and encouraged and guided the discussions that followed the class' activities. On the other hand, in the 8th-grade classes, lessons were all carried out with the assistance of the IWB. Students organized in groups of 3-4 answered the stimulus questions; their answers were collected by the teacher, assessed through the AAS displayed to the class through the IWB and discussed with the students. In particular, all groups were requested to make their arguments clear to compare different solving processes that lead to a correct answer. This strategy allowed students to highlight their mistakes and get used to verbally describing their reasoning processes. The resolution of the problems was carried out similarly, with students working in small groups. Some of the problems were more focused on a solving strategy; others were more open to diverse approaches. The different solving processes were then shared and the possible solutions collectively discussed with the assistance of the ACE's interactive instruments. As for the homework, the interactive feedback limited the requests of explanations regarding the possible solution approaches. Therefore, in the classroom, it was not necessary to go over the exercises; the students' only doubts mainly regarded technical problems such as the answers' input method.

In the platform, a course was created for each class. It was available to students and teachers of that class and to the researchers involved in the experimentation. In the course, for each topic, the following activities were uploaded:

- the stimulus questions which, even that they were used in the classroom activities, could be repeated at home;
- the contextualized problem with a proposed resolution, a generalization, and an explanation of the mathematical content laying behind the problem;
- the automatically graded exercises that students could use at home.

Moreover, the courses included a forum for asynchronous discussion, and the students' questionnaires (Figure 3.1).

3.3 Evaluation of the experimentation

The improvement of the students' mathematical competences was measured by implementing two tests, one at the beginning and one at the end, both for the treated group and for the control group. Each test consisted of 10 items to be solved in 30 minutes. The tests were delivered in paper-and-pen modality to all the students both before and after the experimentation period. The paper-and-pen modality was preferred to the computer-based one not to penalize the control group which was not used to work in a computerized environment. The tests were differentiated according to the grade (grade 8 and 9 shared the same tests since their learning objectives in Mathematics are similar). The questions were borrowed from INVALSI tests of previous years; they belonged to the dimension "solving problems", since this is the process of major interest for this thesis's purposes. The initial tests included questions of all content areas (3 of the area "numbers", 3 of the area "relations and functions", 2 of "space and shape" and 2 of "data and prediction"). The final test included only questions about the topics covered by the interactive paths (linear equations and statistics for grades 8 and 9; quadratic functions, second-grade equations and linear systems for grade 10).

The students' appreciation of the materials used and of the methodologies adopted was detected through a questionnaire administered online to the treated group at the end of the experience, the last school week in June. The questionnaire was composed of 17 items, listed in Table 3.3, with answer on Likert scale from 1 to 5 (1 = not at all, 5 = very much); they involved three main themes: the general appreciation of the project, the classroom activities and the online activities (in particular, the AFA).

To collect the teachers' opinions about the experience, the focus group techniques was used at the end of the school year. During the focus group, the results of the students' questionnaire were the object of reflection for a complete view of the effectiveness of the interactive path.

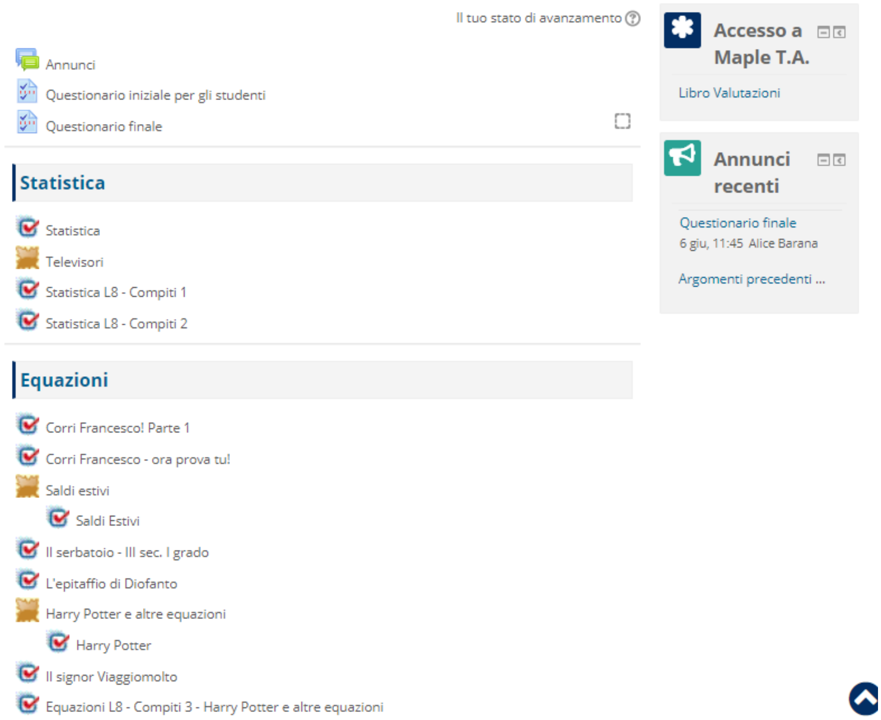


Figure 3.1 Online course for students (Barana & Marchisio, 2020).

Table 3.3 Items of the questionnaire about the appreciation of the interactive activities.

General appreciation of the project	**Answer**				
Were the activities interesting?	1	2	3	4	5
How useful were the activities to better understand the topics dealt with in Mathematics?	1	2	3	4	5
Were the activities enjoyable?	1	2	3	4	5
To what extent were the activities similar to your typical lessons of Mathematics?	1	2	3	4	5
Appreciation of classroom activities					
To what extent the proposed problems were interesting?	1	2	3	4	5

How useful was solving problems to understand better the topics dealt with in Mathematics?	1	2	3	4	5
To what extent were the classroom activities enjoyable?	1	2	3	4	5
To what extent did you appreciate to work together with your classmates in solving problems?	1	2	3	4	5
To what extent did you appreciate working together with your teacher and/or with the University's researchers to solve problems?	1	2	3	4	5
Appreciation of online activities					
To what extent has to solve the online exercises been useful to understand better the topics dealt with?	1	2	3	4	5
To what extent solving the online exercises was enjoyable?	1	2	3	4	5
To what extent was using the platform difficult?	1	2	3	4	5
To what extent was to have the automatic assessment available after each of your answers useful?	1	2	3	4	5
To what extent have you appreciated being able to do the exercises more than once?	1	2	3	4	5
To what extent have you appreciated finding different values in the exercises at every attempt?	1	2	3	4	5
To what extent have you appreciated being able to find interactive material always available on the platform?	1	2	3	4	5
To what extent would you like to have similar interactive exercises for each topic?	1	2	3	4	5

3.4 EXAMPLES OF INTERACTIVE ACTIVITIES

To clarify and discuss how the activities were designed and created, we propose and discuss an example related to the activities of Statistics, on which all the classes of grade 8 and 9 worked. In the two grades, the materials were used differently, adapting their presentation to the classes and schools. In the following paragraphs, the experimentation's typical lessons will be illustrated, mainly focusing on the classes of grade 8 (Barana & Marchisio, 2020).

3.4.1 Stimulus questions

The statistics module begins with three stimulus questions implemented through the AAS: the first one deals with the graphic representation of some data and their mean through a scatterplot (Figure 3.2); the second one is related to the interpretation of data on a table and their weighted mean (Figure 3.3); the third one is related to a graphic representation of the mode (Figure 3.4). The three questions are drawn from INVALSI tests and adapted to automatic assessment: the answer's format, originally multiple-choice, was made open, and a blank space was added to let students describe their reasoning. The stimulus questions were used with the classes of grade 8 during a classroom lesson lasting 90 minutes. Each question was displayed through the IWB. In groups of three, students were asked to find the solution and write it in their notebook, also describing their solving process. Then the teacher asked each group to share their result with the other groups. The most frequent solution was inserted as an answer to the question in the platform and automatically graded. Once the correct answer was agreed, one spokesperson from each group was called to illustrate the reasoning to the rest of the class to compare different resolutions and find mistakes in the incorrect ones. The teachers then used the three questions to explain, starting from the examples, the graphic and tabular representations of data, the concepts of mean and mode.

3.4.2 Contextualized problems

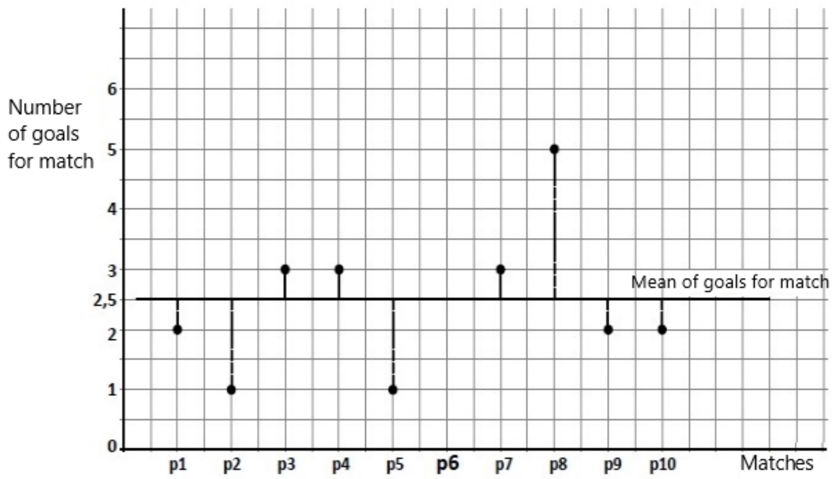
After the stimulus questions' activity, a wider problem was proposed to the students concerning absolute and relative frequencies, data representations in tabular and graphic form, the weighted mean (Figure 3.5). This problem had been created in collaboration with the teachers specifically for this lesson. At grade 8, the students in small groups solved the problem in the classroom. We followed the same schema applied for the stimulus questions, checking step-by-step the students' resolutions. At grade 9, students solved the problem independently in the computer lab using the ACE. The teacher marked time and managed discussions about the most problematic steps. In both cases, at the end of the resolution, the fundamental concepts behind the problem were reviewed and made available in the platform within the interactive file.

3.4.3 Online activities

Figure 3.6 shows an example of a Statistics exercise, automatically graded, available through the platform. The exercise is algorithmic: numbers vary at every student's attempt. It is drawn from an INVALSI item but expanded and adapted to the AFA. We will give more details about transforming items from standardized to formative assessment afterward (section 4.3.3).

The following graph shows the number of goals for match that San Lorenzo team scored during the last championship.

The continuous line corresponds to the mean of the goals scored for match in the 10 matches of the championship.



How many goals did the San Lorenzo team scored in the sixth match?

Number

Explain your reasoning.

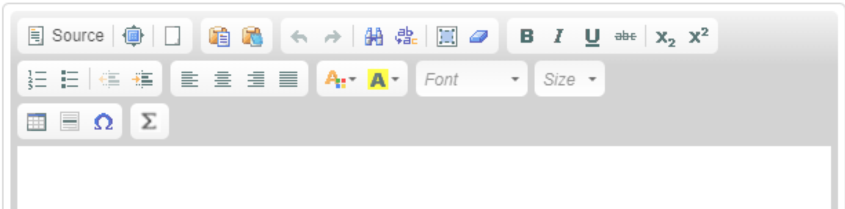
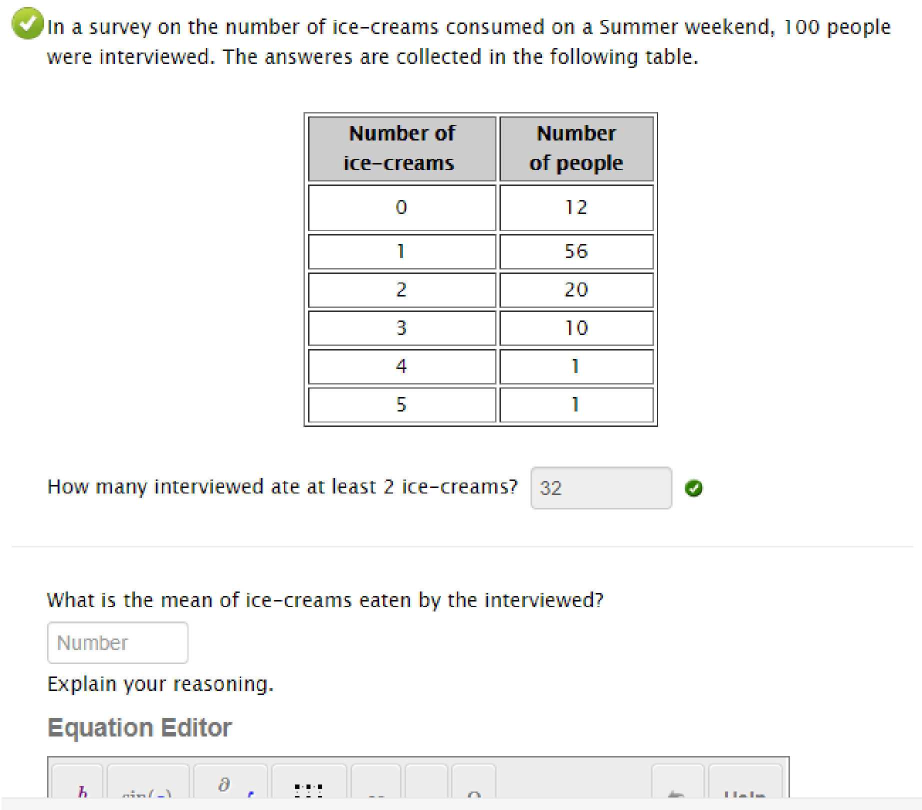


Figure 3.2 First stimulus question of Statistics: the mean. The text has been translated into English for the comprehension of the thesis.



In a survey on the number of ice-creams consumed on a Summer weekend, 100 people were interviewed. The answeres are collected in the following table.

Number of ice-creams	Number of people
0	12
1	56
2	20
3	10
4	1
5	1

How many interviewed ate at least 2 ice-creams? 32

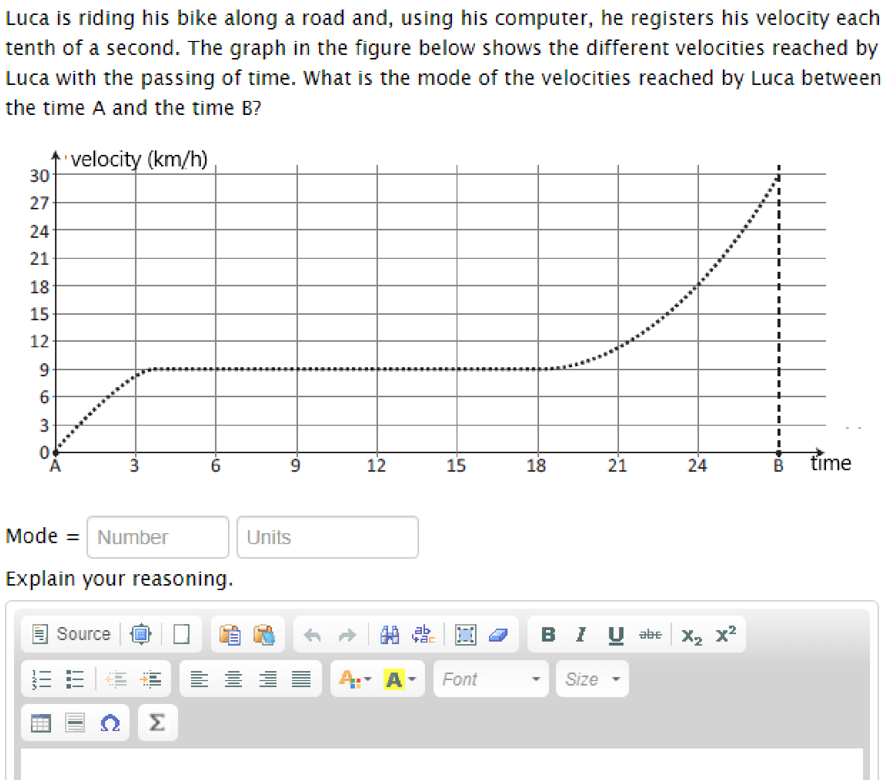
What is the mean of ice-creams eaten by the interviewed?

Number

Explain your reasoning.

Equation Editor

Figure 3.3 Second stimulus question of Statistics: the weighted mean. The text has been translated into English for the comprehension of the thesis.



Luca is riding his bike along a road and, using his computer, he registers his velocity each tenth of a second. The graph in the figure below shows the different velocities reached by Luca with the passing of time. What is the mode of the velocities reached by Luca between the time A and the time B?

Mode = Number Units

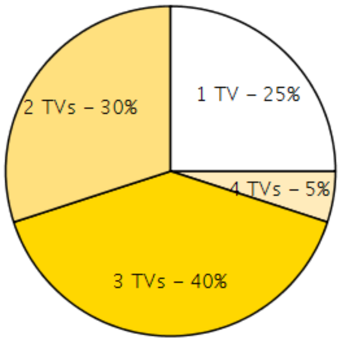
Explain your reasoning.

Figure 3.4 Second stimulus question of Statistics: the mode. The text has been translated into English for the comprehension of the thesis.

TVs

A survey carried out on a group o 500 people on the number of TVs at home produced the following graph:

Graph



Compute the average number of TVs for each person and, after examining what changes when working with different data, draw a generalization.

Resolution

- Build a table which represents the previous graph, inserting the number of people (that is the absolute frequency) instead of the percentages.
- Compute the average number of TV for person.
- 10 people with only one TV and 20 people with 2 TVs buy a new one. How does the percentage distribution vary and how does the mean vary?

Figure 3.5 Problem of Statistics: TVs. The text has been translated into English.

3.5 RESULTS OF THE PILOT EXPERIMENTATION

In line with the adopted counterfactual approach, the analysis focused on the comparison of the evaluations obtained by the two groups of students (treatment group and control group) in the initial and final tests. Some students were absent from school the days of the tests' administration, and they could not do it in another moment; in the analyses, only the results of the students who completed both the tests were considered (N=234, that is the 90% of the students). Results, given on a scale from 0 to 100, are shown in Table 3.4. In lower secondary schools, students of the treated group obtained better results than those of the control group, whereas this is not statistically evident in the high school classes. However, the ANOVA test conducted on the results of the tests, grade by grade, considering as dependent variable the difference between the grade of the final test and that of the initial test, given in a 100-points scale, and as an independent variable the treatment modality (treated group and control group).

In grade 8, the treated group performed slightly better than the control group, but the difference is not statistically significant. In grade 9, the difference is significantly higher for the control group than the treated group. However, a great part of the difference is due to lower performance in the initial test by the control group. Moreover, the final test scores are above 90%, and the standard deviation is very little for both groups. It means that the test was not correctly calibrated for the students of the school, so it did not allow us to measure differences among students and between the groups. A similar discourse can be repeated for grade 10: here, there is no difference between the performances of the two groups, and the results of the final test are very high.

A survey was carried out on a sample of 2400 women aged between 25 and 50 years old, in order to know their opinion about a monthly magazine which deals with health. The following results have been obtained:

	Employed	Unemployed
Positive opinion	636	651
Negative opinion	377	736

What is the percentage of women who expressed a positive opinion?

Round the result to the second digit. E.g.: 25.19

Number

What is the percentage of unemployed women who took part in the survey?

Round the result to the second digit. E.g.: 25.19

Number

Select the graphs which correctly express the situation.

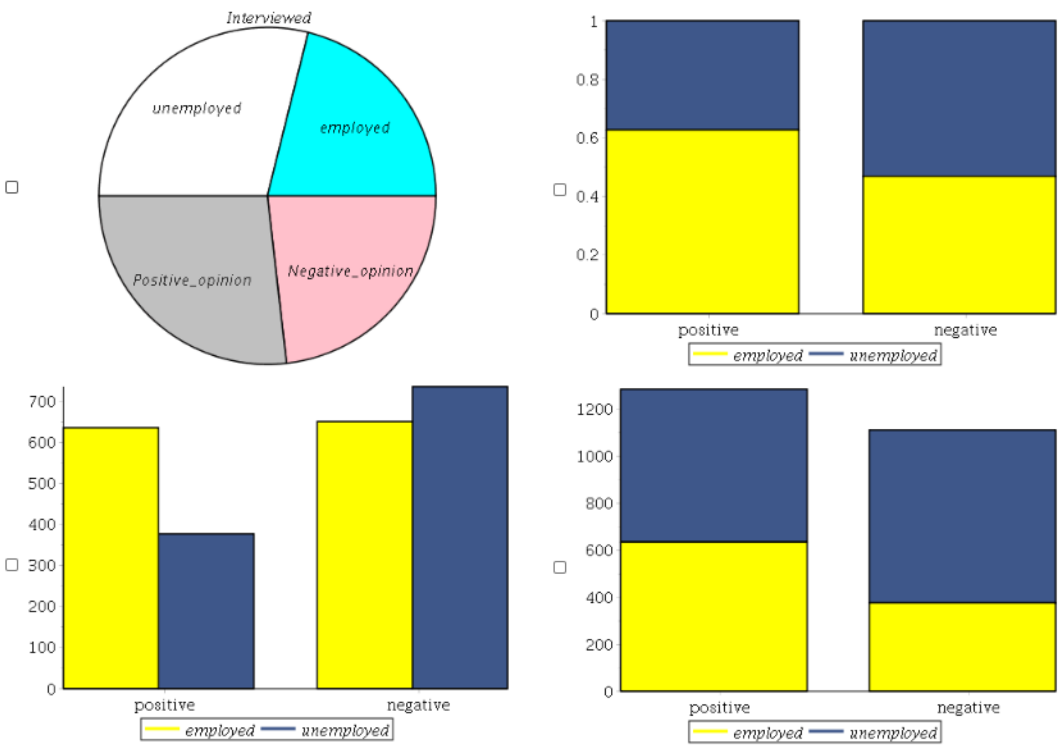


Figure 3.6 Example of online activities with AFA. The text has been translated into English for the comprehension of the thesis.

Together with the counterfactual assessment analysis, the collection of opinions from the treated group students through a questionnaire was used. The questionnaire was administered during the last week of the school year, after the final test. 141 students (out of the 179 of the treated group) answered the questions, which is 79% of the participants. In particular, only 40% of students of grade 10 answered, while almost all the students of grade 8 and 9 filled the questionnaire. The low percentage of answers from grade 10 can be due to a scarce control of the teachers on the students who, at the end of the school year, are often absent from school or give less importance to the final scholastic requests.

Table 3.4 Results of the tests of the pilot experimentation.

Grade	Group		Initial test	Final test	Difference	N	F	Sign.
8	**Control**	Mean	**45.23**	**63.49**	**16.23**	**19**	**0.665**	**0.421**
		SD	18.86	26.80	21.16			
	Treated	Mean	**45.14**	**67.10**	**20.49**	**45**		
		SD	20.71	22.38	18.39			
9	**Control**	Mean	**65.33**	**96.28**	**30.80**	**28**	**19.40**	**0.000**
		SD	11.56	4.97	11.62			
	Treated	Mean	**75.10**	**92.78**	**17.78**	**55**		
		SD	15.71	9.39	13.26			
10	**Control**	Mean	**80.19**	**92.75**	**11.47**	**23**	**0.038**	**0.846**
		SD	14.23	10.47	15.32			
	Treated	Mean	**77.51**	**90.87**	**11.93**	**64**		
		SD	14.45	9.73	12.49			

At first glance, the results of the questionnaire (Table 3.5) show the positive appreciation from a great part of the students regarding the activities, in particular from the students of the third year of lower secondary school. A large majority regarded problem solving activities for better understanding the mathematics. The classroom activities' organization was different from the usual lessons, especially for students of upper secondary school. The most appreciated aspect was working in groups, maybe because it is not a regular practice for Mathematics lessons. They did not find much difficulty in using the platform. They particularly appreciated the immediate feedback and the materials' availability: these two aspects registered the best scores in the whole questionnaire. The algorithmic questions did not register a great appreciation, perhaps because it is an advantage perceived by the teacher rather than the student.

The qualitative observations made during the classroom activities were also collected and considered for the project's whole evaluation. In particular, it was noticed that activities had the most significant impact in 8^{th}-grade classes regarding students' interest and engagement. The division in groups worked very well, the classroom discussions were managed with attention, and every student could participate. In particular, the automatic assessment of the groups' answers, displayed through the IWB, created moments of suspense which contributed to increasing the class attention. The level of attention remained elevated when the groups told each other how they reasoned to solve the problem: different strategies always emerged, not necessarily wrong. The two teachers of grade 9 were quite used to organizing lessons in the computer lab and using the automatic assessment; so, there were no problems in the management of the activities with their classes. However, those students perceived the most significant difference with their usual lessons: they were probably not accustomed to problem solving and formative assessment. The classes of grade 10 were those in which working was more complicated than the others. Here, the teachers told the students that the project's activities were eligible. Moreover, the activities were used to review previously studied topics, unlike grades 8 and 9, where the activities were used to introduce new topics. Thus, students were less engaged during the activities; this also explains why they were reluctant to fill the final questionnaire.

The teachers' opinions involved in the experimentation were collected using the focus group technique at the end of the experimentation. They agreed on what was observed during the classroom activities: a high level of students' engagement and more careful attention than when moving to more theoretical and abstract lessons. For this reason, several teachers appreciated the contextualization of the problems and the fact that they allowed students to solve them in different ways, even via non-standardized rules and implementing non-formalized strategies, often in ways that differ from what teachers expected. Working in groups was proven to be a useful method to encourage students to get involved in developing the activities. The teachers observed that the well-structured and seriously developed group activities helped optimize the lessons and avoid wasting time.

Table 3.5 Results of the questionnaire of the final experimentation.

		Grade 8	Grade 9	Grade 10	Total
	N (percentage of answers)	**46 (90%)**	**56 (98%)**	**39 (40%)**	**141 (79%)**
Were the activities interesting?	Mean	**3.41**	**3.43**	**3.36**	**3.40**
	SD	0.75	0.92	0.93	0.83
How useful were the activities to better understand the topics dealt with in Mathematics?	Mean	**3.59**	**2.91**	**2.56**	**3.04**
	SD	0.90	1.05	1.05	1.08
Were the activities enjoyable?	Mean	**2.93**	**2.98**	**2.79**	**2.91**
	SD	1.04	1.00	1.10	1.04
To what extent were the activities similar to your typical lessons of Mathematics?	Mean	**2.93**	**2.38**	**2.59**	**2.62**
	SD	0.93	1.00	1.02	1.01
To what extent the proposed problems were interesting?	Mean	**3.20**	**3.14**	**3.03**	**3.13**
	SD	0.81	0.80	0.84	0.81
How useful was solving problems to understand better the topics dealt with in Mathematics?	Mean	**3.61**	**3.23**	**2.56**	**3.13**
	SD	0.80	1.11	1.07	1.09
To what extent were the classroom activities enjoyable?	Mean	**3.22**	**2.89**	**2.97**	**3.02**
	St. Dev.	1.07	0.99	0.99	1.01
To what extent did you appreciate to work together with your classmates in solving problems?	Mean	**3.78**	**3.57**	**3.38**	**3.59**
	SD	1.11	1.17	1.12	1.14
To what extent did you appreciate working together with your teacher and/or with the University's researchers to solve problems?	Mean	**3.33**	**3.48**	**3.31**	**3.38**
	SD	0.92	1.04	1.00	0.99
To what extent has to solve the online exercises been useful to understand better the topics dealt with?	Mean	**3.24**	**3.13**	**2.51**	**2.99**
	SD	1.10	1.10	1.02	1.11
To what extent solving the online exercises was enjoyable?	Mean	**2.83**	**2.95**	**2.69**	**2.84**
	SD	1.14	0.86	1.08	1.02
To what extent was using the platform difficult?	Mean	**2.28**	**2.05**	**2.69**	**2.30**
	SD	1.22	1.14	1.37	1.25
To what extent was to have the immediate feedback available after each of your answers useful?	Mean	**3.74**	**3.68**	**4.05**	**3.80**
	SD	1.27	1.32	1.05	1.24
To what extent have you appreciated being able to do the exercises more than once?	Mean	**3.64**	**4.04**	**3.77**	**3.84**
	SD	1.27	0.95	0.99	1.08
To what extent have you appreciated finding different values in the exercises at every attempt?	Mean	**2.98**	**2.96**	**3.15**	**3.02**
	SD	1.09	1.11	0.96	1.06
To what extent have you appreciated being able to find interactive material always available on the platform?	Mean	**3.41**	**3.46**	**3.03**	**3.33**
	SD	1.00	0.91	0.83	0.94
To what extent would you like to have similar interactive exercises for each topic?	Mean	**3.52**	**3.45**	**2.79**	**3.29**
	SD	1.21	1.17	1.11	1.20

The organization allowed all students to participate and keep engaged in the activities; we can find confirmation of this teachers' point of view in the students' answers to the questionnaire. This was made possible due to an external person's intervention in the organization and management of the activities. As for the request to reuse the didactic material prepared by external people, teachers have presented no objection and even appreciated the work and contribution. As regards online activities, more contrasted opinions emerged. Some teachers highlighted that the students' interest during the activity in class was reduced during the homework, where they seemed to have had more difficulties in getting students to access the platform. Often students have several extracurricular activities that take away their time for working at home. Some difficulties were also observed regarding the syntax in the open answers of the online texts. In particular, in the case of grade 8 classes, teachers reported fewer difficulties in getting students to cooperate at this point. Students used the platform autonomously; they could choose when and how to make use of the materials and activities always available on the platform.

The experimented activities were also analyzed using Black and Wiliam's formative assessment model to show that the activities complied with the five key strategies and pursued formative interventions.

1. *Clarifying and sharing learning intentions and criteria for success*: the classes' participation to the experimentation was initially discussed by the teachers directly with their students, then repeated at the beginning of the classroom activities, thus sharing the objectives of learning.
2. *Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding*: each group's request to explain the solving strategy used with questions and problems allowed the creation of effective discussions to highlight students' understanding.
3. P*roviding feedback that moves learners forward*: the interactive feedback in automatically assessed questions was designed to improve learning.
4. *Activating students as instructional resources*: students were activated as learning resources for each other during group working.
5. *Activating students as the owners of their own learning*: the online activities that students had to do autonomously under the guidance of the ICTs favored the students' activation as responsible for their learning

3.5.1 Discussion and conclusions

This experimentation's quantitative results need to be considered cautiously since the methodological frame adopted presents several limitations: firstly, the number of students involved in the experimentation was small, the control group was more restricted than the treated group, and the school time evaluated was also restrained. Moreover, many of the students, especially of grades 9 and 10, belonged to a middle-high social class (including students of the control groups) and had a high-standard school curriculum. These types of students are less susceptible to learning positive effects regarding the introduction of didactic methodologies based on ICT, as shown by several studies carried out on learning through the use of ICT (Giusti et al., 2015; Gui et al., 2014; Thiessen & Looker, 2007). Lastly, the commitment of the teachers and the students involved in this experimentation was limited by the curricular obligations that, especially towards the end of the school year, had a priority over other educational initiatives.

However, the qualitative results highlight a considerable potential of the didactic methodologies used, particularly problem solving and the use of a DLE with AFA activities. The latter was generally appreciated by teachers and students, who, through the questionnaire, were asked to express their opinion about the main features of the model of AFA (in particular, availability, immediate feedback, algorithmic questions and contextualization). The interactive feedback deserves more in-depth analysis through more precise questions in other questionnaires matched with the experimentation of appositely designed educational activities.

The results of this experience allowed us to identify key strengths and weaknesses to design and organize a bigger experimentation, more reliable from a methodological point of view, which would involve a higher number of participants and a longer time dedicated to the activities. First of all, we noticed that students and teachers of grade 8 were the most receptive to the kind of activities proposed, both in the classroom and online. Classes of lower secondary schools are generally more heterogeneous from the learning perspective, allowing us to observe a broader spectrum of effects on students' learning and behavior. As mentioned before, the social factor is also important to be considered in an experiment's design, since it seemed to influence the students' engagement with and appreciation of the interactive activities. Thus, we can answer to the (pRQ1) "what is the most suitable target for an experimentation aimed at analyzing how the adoption of interactive activities with AFA in a DLE affects teaching and learning processes?". We can say that the most suitable target is constituted by students of grade 8 coming from lower social classes.

The interactive activities with problem solving and AFA were appreciated both by teachers and students. The better success of the activities at grade 8 and 9 than grade 10 suggests that the experimental activities should be naturally integrated into the usual teacher's program, involving new topics just when the teacher intends to introduce them, rather than being a one-time event. Thus we can answer to the (pRQ2) "what kind of activities can be proposed to the students?", affirming that the interactive activities with problem solving and AFA can be proposed to the students; they should be integrated into the usual lessons.

On the base of the results of the learning tests, of the questionnaires, and of the focus group with the teachers we can answer to (pRQ3) "what are the main effects on learning of the adoption of interactive activities with AFA in a DLE?". We can say that the adoption of interactive activities with AFA in a DLE was useful to increase students' interest and engagement, support problem solving, and understand concepts. The improvement of learning results is more uncertain but promising: a more carefully designed experimentation is needed to shed light on this aspect.

Lastly, teachers' level of confidence about the didactic methodologies used and their ability to use the DLE autonomously seemed to influence the results. This opens a new issue: the need for teacher training, even on a larger scale, so that similar projects could not be isolated experiences of learning for students, but systematically enter the classrooms becoming part of the typical approach to Mathematics.

4 MAIN EXPERIMENTATION: DESIGN

Based on the pilot experimentation results, a larger one was designed and implemented during the 2017/2018 school year. In this chapter, the experimentation design will be presented and discussed; in particular, we will focus on the research questions, the methodology of research, and the instruments of evaluation of the experimentation. Table 4.1 outlines the key points of the main experimentation.

Table 4.1 Main experimentation's sheet, which outlines the key points.

	MAIN EXPERIMENTATION
DURATION	40 hours of experimental activities in November-June 2018
GOALS	• Studying the effectiveness of interactive materials built according to the AFA model based on a problem solving approach, in a DLE in blended settings, from the learning perspective; • investigating how formative assessment processes could be enacted in a DLE using automatic assessment and interactive activities; • understanding if and how this AFA activities in a DLE can activate cognitive processes in the field of Mathematics, with particular reference to modelling processes; • studying if the use of interactive activities with AFA might improve the students' engagement levels.
TARGET	Students of grade 8 of Turin.
LEARNING OBJECTIVES	Formulas and functions
DIDACTIC METHODOLOGIES	• AFA in a DLE; • problem-solving; • collaborative learning.
FORMAT OF THE EXPERIMENTAL ACTIVITIES	• CLASSROOM ACTIVITIES: problem solving activities with AFA in small groups • ONLINE ACTIVITIES: individual online homework with AFA
SETTING	• CLASSROOM ACTIVITIES: in the classroom, tasks displayed at the IWB, students working in groups with paper and pen; • ONLINE ACTIVITIES: Moodle platform integrated with an ACE and an AAS.
LMS	https://cittaeducante.i-learn.unito.it/
RESEARCH METHODS	Counterfactual method, comparing the effects on learning of a treated and a control group, integrated with qualitative analyses of data from observations, questionnaires and focus groups.
PARTICIPANTS	• TREATED GROUP: 299 students (13 classes) from 6 different schools of Turin • CONTROL GROUP: 248 students (11 classes) from 6 different schools of Turin
ASSESSMENT INSTRUMENTS	• Initial test (treated and control group; 19 items from INVALSI questions in 30 minutes, paper-based); • Intermediate test (treated and control group; 25 items from INVALSI questions in 45 minutes, paper-based); • Final test (treated and control group; 11 items from INVALSI questions in 30 minutes, paper-based); • Initial questionnaire (treated and control group); • Final questionnaire (treated group only); • Platform's gradebook and logs reports; • Focus groups with teachers of the treated classes.

4.1 RESEARCH QUESTIONS

The main research question which guides the whole study is the following: "How an automatic assessment system can improve the teaching and learning of STEM disciplines, and in particular didactic methodologies as formative assessment?". Now we will decline this big question in more precise and specific research questions, following the issues emerged in the theoretical framework to design an experimentation and drive the analyses.

The first goal of this experimentation is the study of the effectiveness of interactive materials built according to the AFA model based on a problem solving approach, in a Digital Learning Environment in blended and online settings, from the learning perspective, in order to understand the advantages of using these kinds of online activities for improving learning results. Interesting insights can be drawn from comparing a treated group, composed of classes that use the interactive materials, with a control group of classes that received traditional instruction. Also, the comparison between students who actively used the technologies and those who did not use them much should lead to interesting considerations. Moreover, we are interested in understanding the extent to which the reluctant users of the online activities are reluctant learners and what reasons motivate their scarce use of AFA activities.

Secondly, we will investigate how formative assessment processes could be enacted in a DLE using automatic assessment and interactive activities to verify that the model for the design AFA activities previously presented can support formative processes. In particular, we will consider the main models of the literature (Sadler's model of feedback; Black and Wiliam's strategies of FA) as reference for describing good feedback and formative assessment practices to which we aspire.

In third place, we aim at understanding if and how this kind of digital activity can activate cognitive processes in the field of Mathematics. We choose mathematical models and modelling as a context for research. As we said before, they represent very useful competences not only for the learning of Mathematics at school and university level, in the world of work and daily life. In particular, we will study whether algebraic thinking problems in a geometric context can be a valid starting point to introduce functions. Then we will investigate how the use of different registers of representation can help understand the concepts underlying mathematical models and modelling. Lastly, we will focus on AFA's interactive activities and whether they can support learning and modelling competence development.

In the end, we will focus on student engagement and try to understand if the use of interactive activities with AFA might be useful to improve the engagement level of students who, at the beginning of a learning path, show a weak engagement in Mathematics or come from challenging backgrounds.

If the initial target of this research were Italian students of grades 8, 9 and 10, the pilot experimentation results suggested refining the target, focusing on 8^{th}-grade students only. The reasons for this choice are various. First of all, through the pilot experimentation, we could notice that 8^{th}-grade classes were more heterogeneous than the upper grades, thence facilitate the assessment and observations. Moreover, they can be considered a high-risk category since they are in a delicate transition period from lower to upper secondary school when students, in Italy, begin to decide their future studies. In addition, lower secondary schools are often let out of innovation and teacher training projects and are often considered a "weak ring" of the Italian school system (Cornoldi & Israel, 2015). Data tell us that in this period, the "early school leaving" phenomenon has its origin; this phenomenon is correlated with the familiar economic situation, well-being, and educational level (MIUR - Gestione Patrimonio Informativo e Statistica, 2019). This target could really benefit from interactive learning technologies that allow the personalization of learning and improve engagement. Since engagement, as seen before, is strictly connected to the quality of learning, acting on it could have a dramatic impact on their future life, especially for demotivated and disinterested students or those coming from challenging backgrounds. The last reason belongs to the sphere of Mathematics Education: since we are interested in studying the development of modeling competence, we can consider that at the end of lower secondary school, with the introduction of symbolic calculus, students have all the necessary instruments to establish relations among the various forms of semiotic representation of mathematical models: verbal descriptions, numerical tables, formulas, and graphs. We hypothesize that working at this level with elementary mathematical models, for instance, those determined by linear and non-linear functions, through activities that promote the flexible use of these tools and the link between concepts belonging to different disciplinary areas, can help prevent conceptual difficulties in the future.

These considerations led us to the formulation of the following research questions:

(RQ1) Are classroom and online activities with automatic formative assessment in a digital learning environment effective in improving students' learning results?

(RQ2) How does belonging to a lower social class affect the impact of the interactive activities with AFA on learning?

(RQ3) Are mathematical knowledge, procedural understanding, and self-assessment skills influenced by the regular adoption of the automatic formative assessment for learning Mathematics?

(RQ4) Who are the AFA's reluctant users and are there any differences between active and reluctant users in learning achievements?

(RQ5) Can the interactive feedback be effective according to Sadler's model (Sadler, 1989)?

(RQ6) Can the blended use of the automatic assessment support formative assessment's strategies?

(RQ7) Could an activity of algebraic thinking in a geometric context help students develop multiple approaches to a problem?

(RQ8) In what ways allowing multiple representations can support the understanding of algebraic formulas?

(RQ9) What are the main problems faced by the students in using geometric thinking in geometry?

(RQ10) How is it possible to introduce the difference between linear and non-linear models in grade 8 through interactive activities?

(RQ11) What are the effects of interactive activities with AFA to approach the linear model's concepts, with particular reference to the intercept and the slope?

(RQ12) Could interactive materials with automatic formative assessment and interactive feedback in blended and online situations improve the engagement level of students who, at the beginning of the learning path, show a weak engagement in Mathematics or come from challenging backgrounds?

4.2 METHODOLOGY

The main experimentation aimed to produce quantitative evidence that the methodologies object of study can be effective under different points of view. For this reason, it involved a larger number of students: 547 students belonging to 24 8th-grade classes from 6 different lower secondary schools from different areas of Turin (Figure 4.1). In particular, about 50% of the selected classes belonged to schools attended by students from lower social classes and with a presence of second-generation migrants close to 40%; the remaining 50% of the sample belonged to a middle-high social class and wealthier families.

The design was based on a counterfactual approach, comparing the effects of the innovative methodologies on a treated group of students with those of a control group, as for the pilot experimentation (Angrist & Pischke, 2008). For the validity of the study, it is essential to identify a valid control group. The correct way would consist of identifying individuals in the control group with the most similar characteristics that those in the treated group. In the pilot experimentation, the choice of the control group having similar observable characteristics to the treated group proved to be inadequate. In fact, the experimentation showed that non-observable characteristics, such as a high personal motivation for learning or the teachers' involvement, also had an impact upon the control group. These elements did not make it possible to confirm if the observed differences in the learning results of the treated classes related to the control group, with a statistic certainty, are due to the didactic methodologies carried out or to other reasons. Thus, during the main experimentation, a different approach was adopted for the selection of the control classes. It was based on the casual selection in each of the six schools of the group to be compared to the treated classes. The result was that 11 out of 24 classes belonged to the control group. This procedure, based on randomization, and the larger number of participants than the previous stage, permitted to organize two potentially equivalent groups, at least under a statistic aspect, for each of the observable and non-observable characteristics. This approach is seemingly more solid than that of the pilot experimentation; therefore, the comparison of the learning results between treated and control group is deemed more trustworthy. Table 4.2 shows the number of classes and students involved in the experimentation. 23 teachers were involved, one for each class, except for one teacher who had two classes of the same school in the treated group.

The classes in the treated group had access to online courses with digital interactive materials with AFA to revise and learn mathematical contents relevant for the competences to be acquired by the end of the 8^{th} grade. The materials were prepared by experts from the University of Turin, in collaboration with Rossella Garuti, INVALSI expert, after agreeing on the topics with their Mathematics teachers; the online course was then shared with teachers, who were trained on the use of the platform. The DLE's technological apparatus was the same as the pilot experimentation (https://cittaeducante.i-learn.unito.it/): a Moodle platform integrated with Maple ACE and Moebius Assessment AAS. On the LMS, an online course containing all the interactive activities aimed to share the materials with the teachers and keep in touch with them through forums, synchronous web-meetings, and questionnaires. The didactic materials were duplicated into 13 identical courses, one for each treated class so that teachers can easily control their own students' progress and give them personalized support and advice. The course homepage is shown in Figure 4.2.

Table 4.2 Numbers of students and classes participating in the experimentation.

Schools	**Classes**			**Students**		
	Control	**Treated**	**Total**	**Control**	**Treated**	**Total**
B. Chiara	1	2	**3**	13	39	**52**
E. Fermi	0	2	**2**	0	46	**46**
G. Marconi	2	2	**4**	47	47	**94**
Via Revel	5	4	**9**	117	100	**217**
U. Saba	2	2	**4**	47	43	**90**
A. Manzoni	1	1	**2**	24	24	**48**
Total	**11**	**13**	**24**	**248**	**299**	**547**

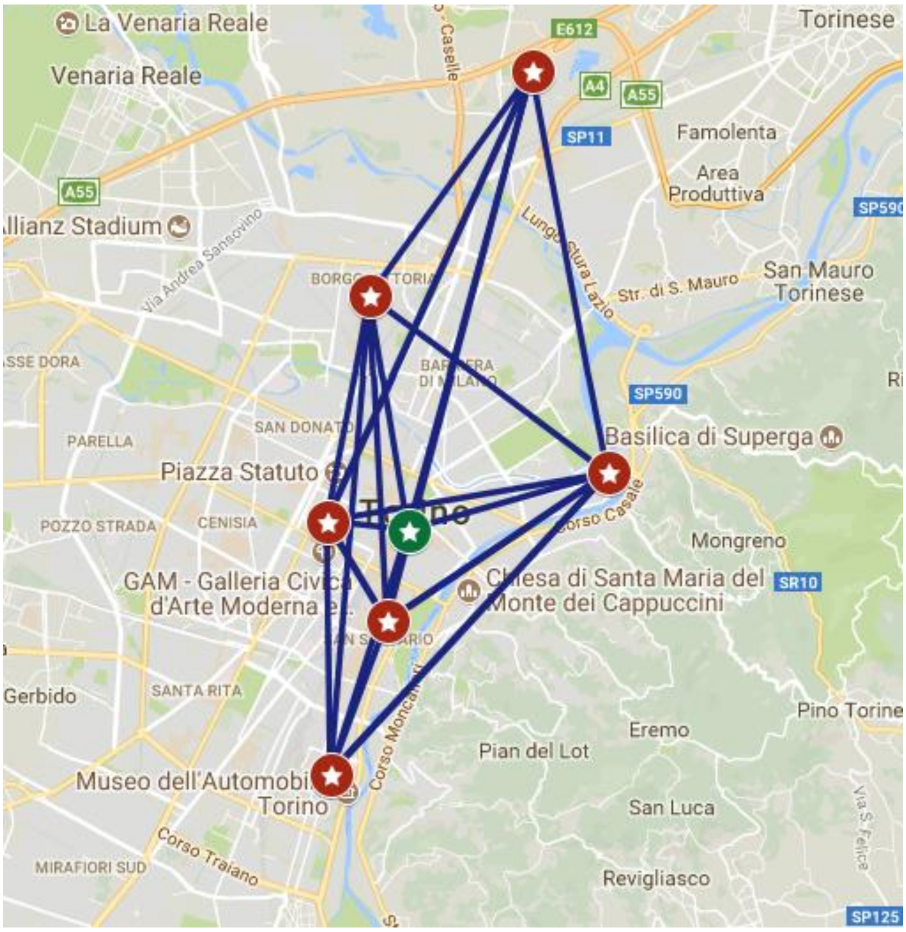


Figure 4.1 Map of the schools involved in the main experimentation. The green star is the Department of Mathematics of the University of Turin.

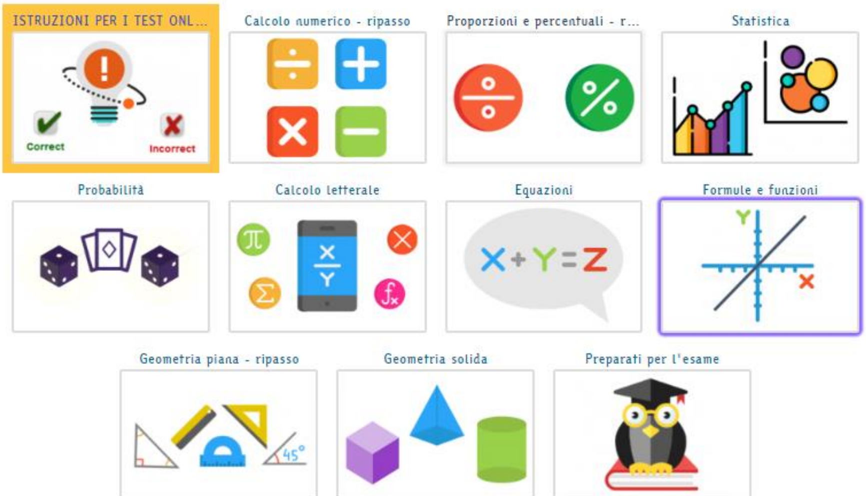


Figure 4.2 Homepage of the online course for the treated classes. Each box corresponds to a different topic.

The teachers could use interactive materials in two modalities:

- in the classroom, with the support of the IWB, where tasks were displayed. Students, in small groups of 3 or 4, were asked to solve one task. The teacher collected all the answers; the class agreed on one of them to be checked using the AAS. After verifying whether it was correct, all the groups, in turn, had to show their solving process to the others. The interactive worksheets displayed at the IWB supported the collective discussion and gave prompts for deeper reflection;

- online, as homework, using the online assessment and the interactive feedback to check understanding. Students could autonomously navigate within the platform and make one or more attempts to the assignments.

Moreover, a particularly relevant topic in Mathematics Education for grade 8, that is, formulas, symbolic manipulation, and modeling, was chosen as the didactic goal for the experimentation. A series of lessons around it were held in collaboration with the PhD student author of this thesis. The main experimentation duration was more significant than that of the pilot one: the students worked on the interactive materials for about 40 hours during the whole school year, from December to June, 10 of which were held under the supervision of the PhD student.

The teachers were periodically updated on the experimentation's events through individual and collective meetings to share the methodologies adopted and the results obtained, and gain their perceptions of the classes' reactions. Moreover, a final focus group was organized at the end of the experimentation.

The experimentation was evaluated through several instruments of quantitative and qualitative analysis:

- an initial, an intermediate, and a final learning test, performed both by the test group and by the control group;
- an initial questionnaire about students' engagement, filled by both groups;
- a final questionnaire on the students' engagement relative to the interactive activities, proposed to the treated group only;
- observations during the face-to-face lessons, collected through the students' videotapes while solving the problems and through their worksheets;
- extraction of data from the platform about its usage: logs to the activities, assessment data.

Students' socio-economic factor was determined using data from INVALSI surveys; the sample has been split into two categories: students from low social class and students from medium or high social classes. The division broadly coincides with schools' division: four out of the six schools were mostly attended by students with low socioeconomic status (B. Chiara, E. Fermi, A. Manzoni, and U. Saba). The two schools attended by students from a medium-high class (Via Revel and G. Marconi) participated in the project with more classes.

Table 4.3 shows the project's time-table.

Table 4.3 Project's timetable.

Time	Activity
November 2017	Project's presentation and selection of the participants
	Initial test
December 2017	Enrollment of the students on the platform
	Students' initial questionnaire
	Start of the online activities
February 2017	Intermediate test
	Start of the classroom activities with a PhD student
March – May 2017	Classroom activities with a PhD student
June 2017	Final test
	Students' final questionnaire
	Final focus group with teachers

The parents signed a consent form for the use of data of the students registered in the classroom and the platform for didactic and research purposes only.

4.2.1 Learning tests

All the students of both treated and control classes took a test at the beginning of the school year, an intermediate test halfway through, and a final one in June. All the tests were created using questions from INVALSI surveys for grades 8 and 10 of the previous years, with little arrangements. This allowed us to have an initial idea of the items' difficulty level, based on the national sample results, and to compare it to the participants' results. The tests were administered in the paper-and-pen modality not to create disadvantages for the control classes, which were not accustomed to the computerized assessment environment.

The initial test aimed to gain a picture of students' abilities and competence in Mathematics; it was composed of 19 items with different levels of difficulty and varied topics. The test was of an acceptable level of reliability (Cronbach's alpha: 0.773).

The intermediate test was mainly focused on symbolic manipulation and modeling. It was administered before the beginning of the classroom activities about that particular module to measure students' prior modeling skills. It was composed of 25 items with different levels of difficulty, all on the same topic; its Cronbach's alpha is 0.800, revealing high internal reliability.

The final test aimed to assess high-level mathematical competences that students should develop during the lower secondary school. It was composed of 11 items, all of them rather difficult. Its reliability level is acceptable, considering that the number of items was low (Cronbach's alpha: 0.652). During the final test, students were also asked to fill a self-assessment form after solving the exercises: for each question, they were asked to indicate if they thought that their answer was correct, incorrect or whether they did not know it.

4.2.2 Students' questionnaires

Part of this section is taken and adapted from (Barana, Marchisio, & Rabellino, 2019).

All the students filled an initial questionnaire in November 2017. It aimed at investigating the initial level of students' engagement toward school and, more specifically, toward Mathematics. The questionnaire was administered online through the platform; it was composed of 35 Likert-scale questions inspired to PISA 2012 student questionnaire (OECD, 2014). Table 4.4 shows the items of the questionnaire. The emotional engagement subscale includes items aimed at investigating how students are interested in and value Mathematics. The behavioral engagement subscale includes items on students' effort and completion of work, perseverance, and participation in school and social-related activities. Items of the cognitive engagement subscales are related to the perceived control of success, self-regulation and openness to problem solving. All items were on a 4-points Likert scale (strongly disagree/disagree/agree/strongly agree) except for the items from IQ21 to IQ25 on perseverance and items from IQ31 to IQ35 on openness to problem solving, where, through a 5-point Likert scale, a neutral position was allowed.

At the end of the school year, the students were asked to fill a second online questionnaire to evaluate the impact of the project's activities on engagement. This questionnaire was composed of three main subscales as well, corresponding to EE, CE, and BE. The emotional engagement subscale items focused on the affective reactions to the classroom and online activities, such as interest, enjoyment, and value for the learning activities. The cognitive engagement subscale items were related to cognitive and metacognitive processes enacted by the learning materials, such as understanding, the use of in-depth strategies for problem solving and self-regulation. The behavioral engagement subscale included items on attention, persistence, effort, and completion of work. Part of the questions are related to the classroom activities; other items are related to the platform's individual use in homework activities, with particular reference to automatic assessment. Table 4.5 reports the 37 items; they are all 5-point Likert scale, where 1 is the lowest and 5 the highest.

Table 4.4 Items of the initial questionnaire.

Subscale	Code	Items
Emotional Engagement	IQ1	I like lectures about Mathematics.
	IQ2	I can't wait for Mathematics lessons
	IQ3	I do Mathematics because I like it
	IQ4	I am interested in the things that I learn in Mathematics
	IQ5	Making an effort in mathematics is worthy because it will help me in the job that I want to do later on
	IQ6	Mathematics is an important subject for me because I need it for what I want to study later on
Behavioral Engagement	IQ7	I finish my homework in time for mathematics class
	IQ8	I work hard on my Mathematics homework
	IQ9	I am prepared for my Mathematics exams
	IQ10	I study hard for mathematics quizzes
	IQ11	I keep studying until I understand Mathematics material
	IQ12	I pay attention in Mathematics class
	IQ13	I avoid distractions when I am studying mathematics
	IQ14	I keep my Mathematics work well organized
	IQ15	I talk about mathematics problems with my friends
	IQ16	I help my friends with Mathematics
	IQ17	I do Mathematics as an extracurricular activity
	IQ18	I do Mathematics more than 2 hours a day outside of school
	IQ19	I play chess
	IQ20	I program computers
	IQ21	When confronted with a problem, I give up easily
	IQ22	I put off difficult problems
	IQ23	I remain interested in the tasks that I start
	IQ24	I continue working on tasks until everything is perfect
	IQ25	When confronted with a problem, I do more than what is expected from me
Cognitive Engagement	IQ26	If I put enough effort, I can succeed in Mathematics
	IQ27	It is completely my choice whether or not I do well in Mathematics
	IQ28	Family demands or other problems prevent me from spending a lot of time for my Mathematics work
	IQ29	If lessons were different, I would try harder in Mathematics
	IQ30	Whether I study or not, I am bad at Mathematics
	IQ31	I can handle a lot of information
	IQ32	I am quick at understanding things
	IQ33	I seek explanations for things
	IQ34	I can easily link facts together
	IQ35	I like to solve complex problems

Table 4.5 Items of the final questionnaire.

Subscale	Code	Items
Emotional Engagement	FQ1	The classroom activities were interesting.
	FQ2	The classroom activities were enjoyable
	FQ3	I think that I will remember the things I learned during the classroom activities in the next months
	FQ4	I think that the classroom activities will be useful for my future studies
	FQ5	Using the platform during the classroom activities was useful
	FQ6	Using the platform made Mathematics lessons more interesting

Subscale	Code	Items
	FQ7	Having the materials available at every moment is useful
	FQ8	The proposed problems are interesting
	FQ9	The tests are useful to practice
	FQ10	It is useful to visualize the correct answer after submitting a response.
	FQ11	The online assignments are a valid help for studying
	FQ12	Online assignments made me appreciate the topics studied more
Cognitive Engagement	FQ13	The classroom activities were comprehensible
	FQ14	During the classroom activities, I could learn new things
	FQ15	The classroom activities were useful to understand some Mathematical topics better
	FQ16	The classroom activities were useful to make connections among different areas of Mathematics
	FQ17	The classroom activities were useful to see Mathematics in a different way
	FQ18	The possibility to see the materials used in the classroom again from home is useful
	FQ19	Using the platform in the classroom helped me understand the topics covered
	FQ20	Using the platform from home helped me identify the topics on which we worked in class
	FQ21	The online tests helped me better understand the topics studied
	FQ22	The online tests helped me understand if I understood the topics studied
	FQ23	The immediate feedback helped me understand how the task should be solved
	FQ24	Problems with step-by-step resolution helped me understand the solving process
	FQ25	Online assignments helped me autonomously solve Mathematics exercises
	FQ26	Online assignments helped me become more confident about my capabilities
	FQ27	Online assignments helped me acknowledge my preparation
Behavioral Engagement	FQ28	I paid attention during the classroom activities
	FQ29	I completed the homework assigned after the activities through the platform
	FQ30	This year I was very involved in my work with Mathematics
	FQ31	I paid attention to the lessons when we used the platform
	FQ32	I used the platform to review the topics on which we worked
	FQ33	I used the platform to do my homework
	FQ34	I used the platform to prepare myself for the final exam
	FQ35	I used the platform to prepare myself for the INVALSI tests
	FQ36	I used the platform to study Mathematics
	FQ37	When I gave an incorrect answer, I used to try the exercise again

Moreover, the final questionnaire also included the following open questions, conceived to analyze the students' perceptions about the experience. Table 4.6 lists them.

Table 4.6 Open items in the final questionnaire.

Subscale	Code	Items
Classroom activities	FQ38	What could you learn during the classroom activities of the project?
	FQ39	What do you think these activities were useful for?
	FQ40	What aspect of the classroom activities did you appreciate most?
	FQ41	Is there any aspect of the classroom activities that could be improved?
	FQ42	Other free comments about the classroom activities.
Online activities	FQ43	Explain why, in your opinion, the online activities were (or were not) useful, and you liked (or did not like) them.

Lastly, a question for investigating the students' preference for online or paper-based homework was inserted in the questionnaire; it had a dichotomous answer (computer-based or paper-based). It is shown in Table 4.7.

Table 4.7 Dichotomous item in the final questionnaire.

Subscale	Code	Items
Online activities	FQ44	Do you prefer doing computer-based or paper-based homework?

4.3 Design of the didactic materials: the interactive path on formulas and functions

The interactive materials created for the experimentation were designed according to the principles of task design for formative assessment and the AFA model presented in the theoretical framework, using a functional approach to algebra and a modelling perspective. The ideas for the problems were mainly from items of the INVALSI tests. A consistent amount of INVALSI questions is available on the Gestinv database (www.gestinv.it) and on the INVALSI website (www.invalsi.it). Questions of INVALSI tests are designed with summative purposes according to the principles of the psychometric tradition. However, there are excellent reasons that this source is incredibly valuable for this kind of course. On the one hand, these questions reveal the specific topics in which students of grade 8 are particularly weak, their frequent mistakes and misunderstandings, so they provide precious information about the concepts and skills on which, in general, it is necessary to work. On the other hand, although conceived for a standardized evaluation, they offer rich prompts for classroom tasks and formative assessment; moreover, the items' statistical properties are available, showing the difficulties that students at each stage usually face with Mathematics.

The items were expanded in order to explicitly touch upon the key points and help students clarify how the questions should be solved; they were also adapted to the technologies used in order to take full advantage of the potential of the system for the exploration of Mathematical concepts and for the automatic feedback on students' answers. In the following paragraphs, we will provide further details on the design of tasks for automatic assessment through some examples.

The core of the didactic experimentation is an interactive path on formulas and functions. The interactive materials were used in the classroom activities held under the supervision of the PhD student author of this thesis. They were available to the students, who could access them from home and complete the assigned homework (Barana & Marchisio, in pressb).

4.3.1 Part 1: algebraic computations

The path originates from some activities on algebraic computations and manipulation of algebraic formulas, seen in a geometry context. Symbolic computation is a core topic in the Mathematics curriculum of grade 8: it is the basis for working with functions, which will be one of the main objectives of Mathematics education at the secondary level. Writing a formula to calculate the area, the perimeter or the volume of a figure whose measures are expressed through letters and numbers is a pretext to analyze, on a geometrical, numerical, graphic, and algebraic perspective, linear, quadratic, and cubic functions. Algebraic thinking in geometry is proposed by several scholars to favor the understanding of variables as quantities to be related rather than symbols to manipulate (Dindyal, 2004). For the development of a set of tasks on symbolic computation, we took inspiration from an item of the INVALSI tests for grade 8 of 2009, shown in Figure 4.3, which asks to compute the area of a geometrical figure, a rectangle trapezoid, whose measures are given in function of a variable. In order to provide the correct answer, as a first step students need to find a way to calculate the area of a trapezoid. They can recall the formula or decompose it in simpler shapes, with the given figure's help. As a second step, they need to use algebraic rules to add and multiply algebraic expressions. Synthetizing the obtained formula in a compact form is not requested, being the answer open. The item requires both geometrical and algebraic reasoning in order to be solved. The link between geometry and algebra is what makes this question interesting. Using measures of geometrical figures to visualize algebraic operations is very useful in conferring concreteness to abstract computations. The question was difficult, with 27% of correct answers: it is probably due to the students' low familiarity with connections between different parts of Mathematics; nevertheless, the processes that this item involves are very interesting from a didactic point of view.

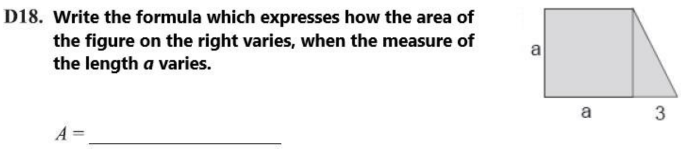


Figure 4.3 Item from INVALSI tests involving algebraic reasoning on a geometric figure. The text has been translated into English. Source: Gestinv 2.0.

Formulas and Areas

We started from these considerations to build a set of tasks with automatic assessment and formative purposes, aimed at making algebra less abstract and at helping students visualize algebraic computations. During the classroom activities, they were used as a starting point of the learning path to introduce algebra with a functional approach and use it as a language for modelling. The first activity asks students to write the formula that expresses the area of a trapezoid (Figure 4.4). The symbolic register is immediately required, associated with the geometric representation. An interactive file built using Maple ACE integrated within the platform, part of which is shown in Figure 4.5, served as a support for a classroom discussion about the possible correct formulas and as a guide for discovering the function obtained when the parameter varies. The increasing of the trapezoid measures and, consequently, of its area, is associated with the movement of a point along a curve of the cartesian plane. It corresponds to the numerical results previously computed and collected in a table. Here the numeric, geometric, and graphic registers are associated and jointly vary. The activity was repeated modifying the initial figure and fixing the lenght of one size (the triangle's base, like in the original INVALSI item).

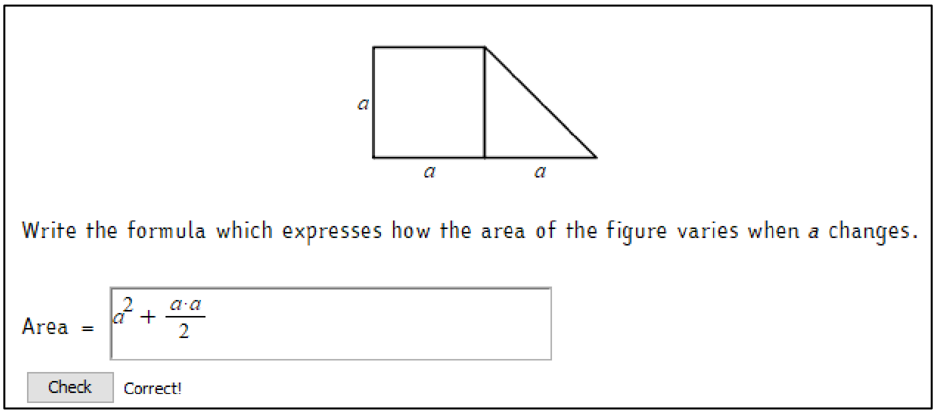


Figure 4.4 First part of the interactive activity "Formulas and Areas". The students have to find a formula to express the trapezoid area, write it in the box, and check its correctness. The activity, originally in Italian, has been translated into English for the comprehension of the thesis.

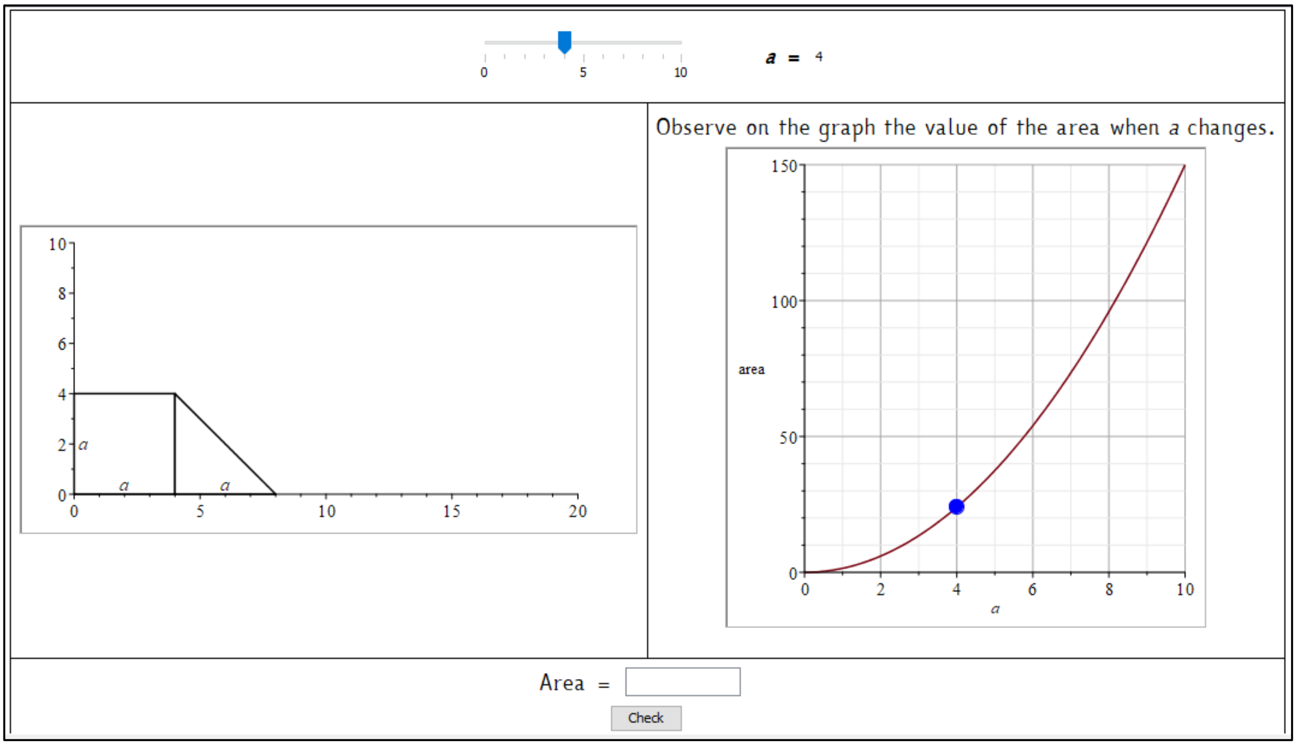


Figure 4.5 Part of the interactive activity about the area of the trapezoid. Students can move the slider choosing a value for the variable a, and observe the figure increasing and the point moving along the graph of the function. The activity, originally in Italian, has been translated into English for the comprehension of the thesis

Formulas and Volumes

Using a similar stimulus, we designed an activity on the same schema of the previous one, aimed at observing some differences in the models generated by different algebraic formulas. The first task asks students to write a formula for a cube volume whose sides are of length *a*. Then, students are asked to complete a table which associates different values of *a* to the volume of the cube; the table is used to build the graph of the function $Volume = a^3$. Extending the graph to negative values for *a*, the cubic's negative part is visualized, too. The activity is repeated for a parallelepiped built on the base of the cube, but "blocking" one side to the fixed measure $b = 1$ and letting the other two vary in order to study the function $Volume = a^2$, then for a parallelepiped with one only side varying and two sides fixed to the length 1, to study the function $Volume = a$. The three curves (a cubic, a parabola, and a line) are thus compared under a geometric, symbolic, numeric, and graphic perspective. Figure 4.6 shows part of the interactive worksheet.

Other Strange Figures – Online activities

In the wake of the previous interactive activities, other algebraic manipulation tasks in a geometric context were designed and developed with the automatic assessment. They ask students to formulate, represent, and compare different functions derived from several geometrical figures in 2 or 3 dimensions to repeat and consolidate the processes with different data and situations. One of them is shown in Figure 4.7. The geometrical figure is not standard, but students can calculate its area decomposing it in simpler parts, such as rectangles or squares; they can use several decompositions to reach different forms of the same formula. Firstly, students are asked to compute the area and write the formula in the blank space. Thanks to the Maple engine, the system can recognize the formula's correctness independently of its form, so every formula obtained through different reasoning is considered correct. Students have three attempts to provide the formula: they can self-correct mistakes and deepen their reasoning if a red cross appears. After three attempts, either correct or not, a second section appears, showing a table that students have to fill in with the values of the figure's area when the variable assumes specific values.

In this part, students have to substitute in the formula different values of the variable; the purpose is to increase the awareness that variables are symbols that stand for numbers and that a formula represents a number, which has a particular meaning in a precise context. The numbers in the left column are in progressive order to help students grasp the linear relationship between the variable and the area. The table is a bridge to the last part of the question, where students are asked to sketch the graph of the function, using an interactive response area of Moebius Assessment that accepts answers within a fixed tolerance, without manual intervention. Students can draw the points found in the last part in the cartesian plan to sketch the line.

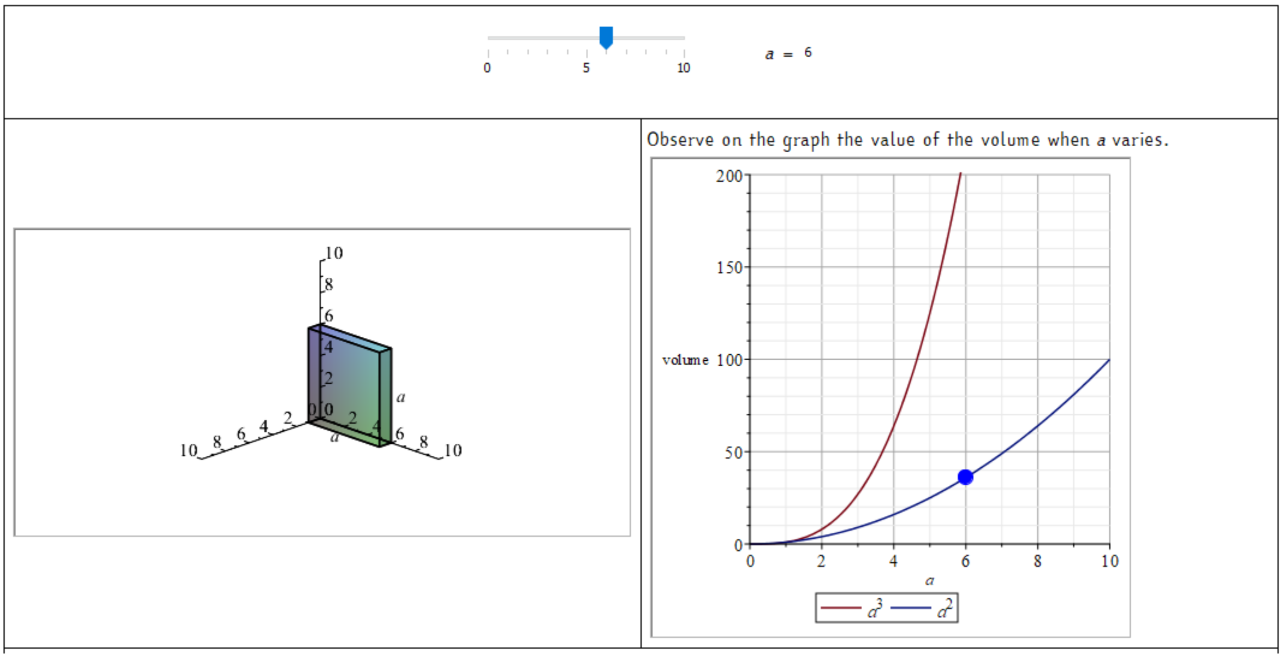
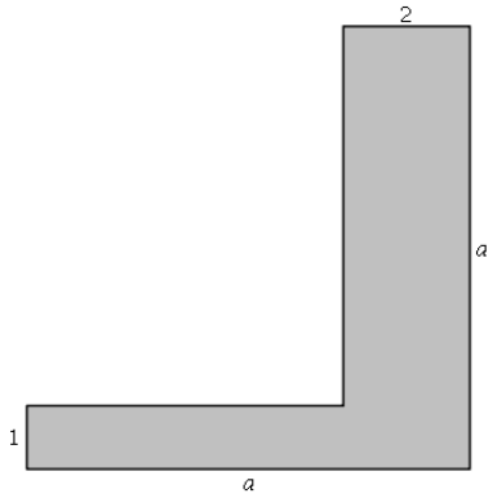


Figure 4.6 Part of the interactive worksheet about the volume of the parallelepipeds. Students can move the slider choosing a value for the variable a, and observe the figure increasing and the point moving along the graph of the function. The graph is confronted with that of the cubic given by the volume of the cube. The activity, originally in Italian, has been translated into English for the comprehension of the thesis.

This question allows students to explore an algebraic formula from several points of view: geometrical, symbolic, numeric, functional, and graphic. The second and third parts help them deepen their familiarity with this kind of formula, make the algebraic computation more concrete and raise the awareness that in Mathematics, concepts from different areas are often connected. The integration of different semiotic registers in the resolution of these tasks is intended to make students confident with treatments and conversions, as well as to introduce them to a new way of reasoning when dealing with mathematical models. This question is algorithmic, in the sense that the numbers of the upper and left side of the figure are randomly chosen and change at every attempt; the picture, the formula, the numbers, and the graph in the following sections change accordingly. There are other versions of the same task involving different figures so that students can be activated to comprehend the meaning of variables and formulas.

The question, adapted in this way, follows the principles for the formative assessment task design: in particular, there are more strategies to find the correct solution, namely, the different ways of decomposing the figure to compute the area; moreover, the requests are dependent on each other; the system provides students with the correct answer to be used in the following parts if they fail all the available attempts.

Look at the following figure.



Write the formula that expresses how the area changes when $a > 2$ varies.

Area $=$ 2*a+a-2

Correct response: $3a - 2$

Fill in the following table computing the area of the figure when a varies.

a(cm)	Area$\left(\text{cm}^2\right)$
3	7
4	10
5	13
6	16

Sketch the graph of the function that expresses how the area changes when a varies.

Pay attention to the domain of the function! Eliminate the parts and the points that do not belong to the domain.

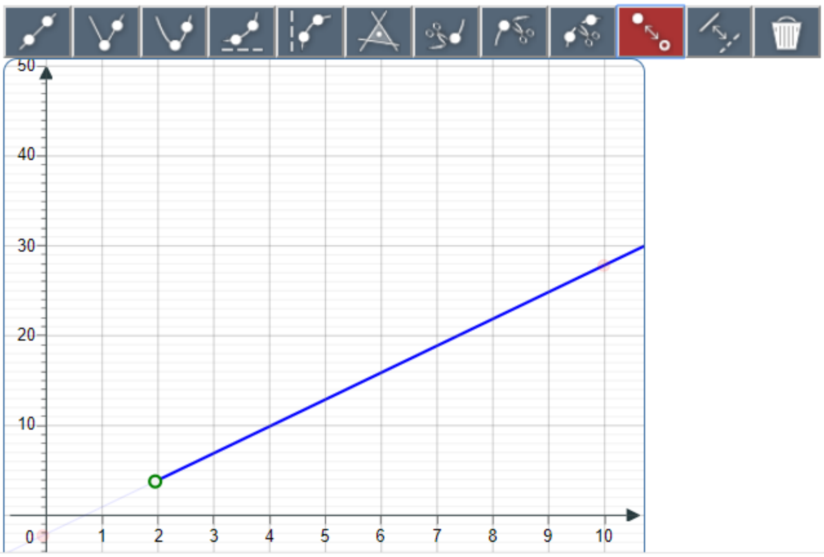


Figure 4.7 Task with AFA on algebraic computations. The activity, originally in Italian, has been translated into English for the comprehension of the thesis (Barana et al., in press).

4.3.2 Part 2: linear models

After the first activities on the on algebraic computations that introduced functions and models, the learning path continues with problems contextualized in real-world situations. Students have to detect variables and variations, formulate and represent mathematical models, with particular reference to linear models. If, in the previous activities, building formulas was facilitated by the use of formulas studied in geometry, in this second phase, much attention is paid to the algebraic formulation of verbally described models and their various representation forms. In these problems, more precise questions are oriented to the analysis of linear models with all their properties, such as the slope interpreted as velocity, the intersection with the cartesian axes in the starting point of the motion or of the action, the intersection of two models as concurrence in time and space, the relation between the model's algebraic and graphic elements.

The Holiday Book

As an example, we show a task based on the INVALSI question in Figure 4.8, proposed to grade 10 in 2012 with 51% of correct answer. The original item asks, given a real-world situation where a girl reads a book at different speeds during different periods of time, to find the graph that correctly describes the situation. The question's format is multiple-choice; students have to qualitatively analyze the different graphs and associate different speeds with different lines. They need to clearly understand that, if the number of pages the girl reads per day is constant, the reading trend is linear, that horizontal traits represent periods with no reading, and that higher slopes correspond to higher speeds.

We expanded the item in a wider interactive task, shown in Figure 4.9, which allows students to study every linear function aspect. The problem leads to exploring three kinds of linear functions: one passing through the origin, one intersecting the x-axis, and one intersecting the y-axis. Students are asked to explore the numerical representation of the problem filling in the tables in the interactive file, which are initially empty. In the box below, graphs are interactively generated with points and lines using data from the tables. The tables and the interactive graphs help students reason and visualize the trend of the reading of a book by the three friends. The activity actively engages the learners asking them to insert the graph of one's reading, envisaging the speed they would read the book with; thus, it opens up to explorations, comparisons, and discussions. A set of questions complete the activity, focused on the graphs' analysis, leading to writing the formulas through which it is possible to express the mathematical models.

On the base of this task, other activities with AFA were designed. The one shown in Figure 4.10 is expanded, moving backward, from the graph to the situation, proposing a punctual analysis and interpretation of the graph. In the first section, students have to read, from a graph showing a similar reading trend, the number of pages that a student reads in three different periods of time at different speeds. The following section asks students to determine how many pages the student reads per day in the three different periods: they find the reading speeds in this way. The last section asks students to choose the period in which Marta was faster in reading the book, thus interpreting the numbers inserted above in the original situation.

The question is algorithmic: the initial graph and the following values change randomly at every attempt. The task aims to help students explore a graph of a function and interpret it in light of a real-world situation; they are asked to explicitly find the graph's properties so that they are facilitated in connecting them with the properties of the function and reading it within the context. The requests are dependent on each other; students can use the previous results, automatically checked, in the following sections, so that their mistakes are immediately corrected and not dragged through the problem's resolution.

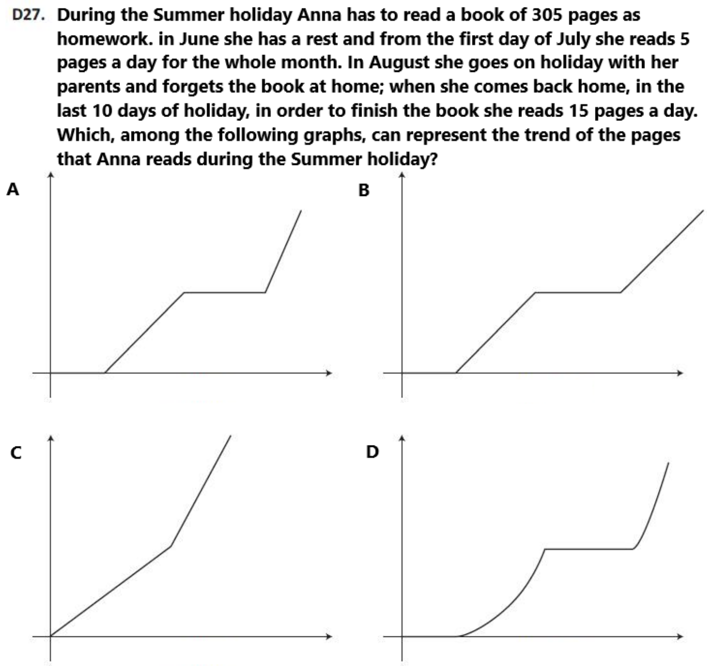


Figure 4.8 Item from INVALSI tests on graphs of functions. The text has been translated into English. Source: Gestinv 2.0.

Space Launch

Another example of activity on linear models is shown in Figure 4.11. It asks students to solve a problem with linear models, open to different approaches. Students who give the incorrect answer to the main task are engaged with exploring the situation through a table to complete and a graphic to draw interactively. This question was used online with some classes and in the classroom with others. On the platform, students were guided through the solving process and could repeat the problem with different data; in the classroom, only the main task was initially displayed, and the different solutions made by the groups of students were shared and discussed, with the support of the automatic assessment.

4.3.3 Discussion on the design of the tasks

The tasks that populate the online course, which include the ones shown above, follow the model for designing AFA activities and exploit the system's potential to engage students in interactive activities. In fact:

1. questions, collected and made available through assignments, are always accessible to students, who can make multiple attempts in order to repeat reasonings and to reinforce concepts;
2. questions are algorithm-based: in fact, numbers change at every attempt, and figures, formulas, answers change accordingly. In this way, in multiple attempts, students will find different questions with the same scheme, that will help them master the solving process, since simply remembering the correct answer is not useful, as it varies every time;
3. questions are, as much as possible, open-ended: students have to write formulas instead of selecting the correct one, and this enables high cognitive processes. Maple engine assures that the correct formulas are considered correct independently of the form;
4. feedback is immediate so that students can immediately acknowledge their mistakes and even self-correct them. In questions that include more than one section, this is very useful as students can identify their mistakes during the solving process and use the correct answers in the following steps;
5. whenever it makes sense, questions are contextualized in the reality or show connections with other disciplines or other areas of Mathematics; this helps students to associate abstract concepts to concrete ideas;

6. Feedback is interactive because it involves students in a step-by-step active resolution that is more efficient than just reading a correct solving process.

The Holiday Book

For the Summer Holidays the Italian teacher has asked students to read the book. The book is 180 pages long.

- Valentina starts reading it on the first day of Holidays and she reads 15 pages a day.
- Marco has already started to read the book and he has already read 30 pages, so he starts from page 31 on the first day of Holidays. He reads 10 pages a day.
- Luca rests for the first 4 days, then he starts reading the book and he reads 20 pages a day.

Who will be the first to finish the book?

Fill in the following table writing, in the "Pages" column, the book's point that the three friends reach at the end of each day.
Use the "You" table to indicate how you would read this book.
Then observe the graphs below and answer the following questions.

VALENTINA		MARCO		LUCA		YOU	
Days	Pages	Days	Pages	Days	Pages	Days	Pages
0	0	0	30	0	0	0	0
1	15	1	40	1	0	1	30
2	30	2	50	2	0	2	60
3	45	3	60	3	0	3	90
4	60	4	70	4	0	4	120
5	75	5	80	5	30	5	150
Draw the points		Draw the points		Draw the points		Draw the points	
Draw the graph		Draw the graph		Draw the graph		Draw the graph	
Clear		Clear		Clear		Clear	

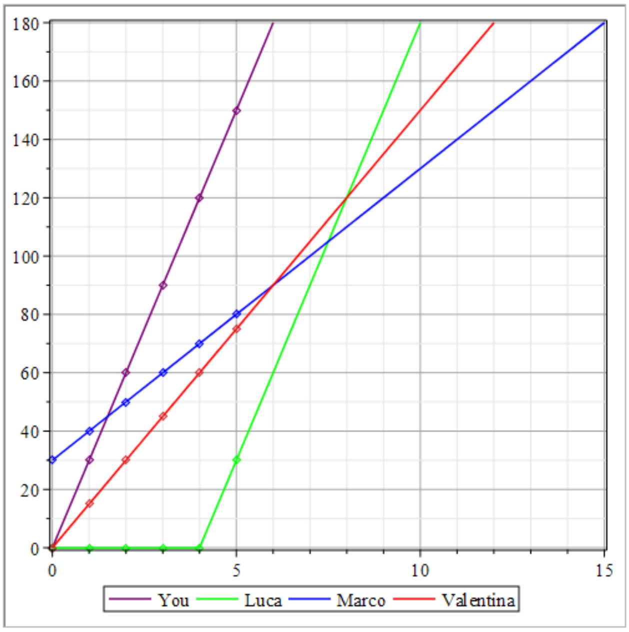
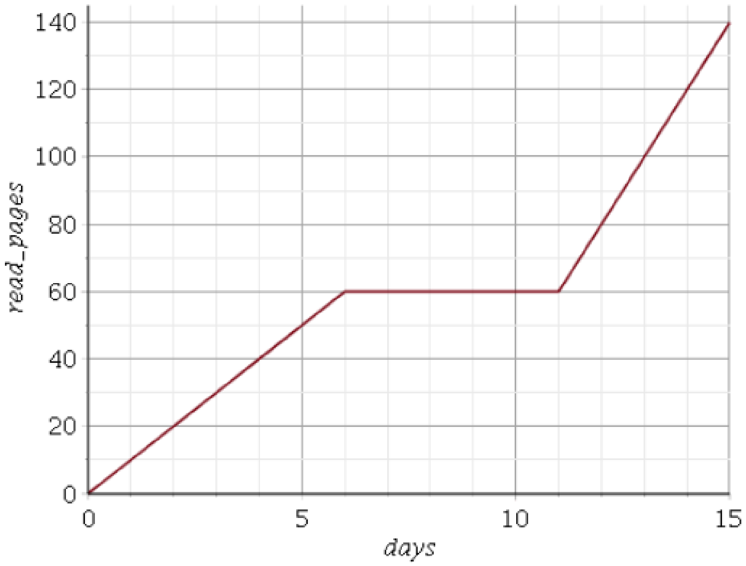


Figure 4.9 Part of the interactive activity "The Holiday Book" on linear models. The activity, originally in Italian, has been translated into English for the comprehension of the thesis.

Holiday reading

During the Summer Holiday, the Italian teacher asked students to read a book 145 pages long.

Marta began to read on the first day of July. The following graph shows the trend of her reading in the days of July.



How many pages did she read during the first 6 days? 60

How many pages did she read during the next 5 days? 0

How many pages did she read during the last 4 days? 80

How many pages did she read each day during the first 6 days?

Answer: 10 pages each day.

How many pages did she read each day during the next 5 days?

Answer: 0 pages each day.

How many pages did she read each day during the last 4 days?

Answer: 20 pages each day.

was Marta faster during the first 6 days or during the last 4 days?

- During the first 6
- During the last 4

Figure 4.10 Question with AFA on graphs of functions. The question, originally in Italian, has been translated into English for the comprehension of the thesis (Barana et al., in press).

Space Launch

An interplanetary rocket leaves the Earth atmosphere at 16 at the speed of 7000 km/h and it follows a linear trajectory with constant speed. The rocket must hook a spaceship which follows the same trajectory at the constant speed of 5000 km/h, which left the atmosphere 2 hours before, that is at 14.

At what time will the roket hook the spaceship? Number

At what distance from the atmosphere will they be? Number Units

Attempt 1 of 2 Verify

Complete the following table.

Time (h)	Distance from the atmosphere (km)	
	Spaceship	Rocket
14	0	0
15	5000	0
16	10000	0
17	15000	7000
18	20000	14000
19	25000	21000
20	30000	28000

Sketch the graphic representing the distances covered by rocket and spaceship.

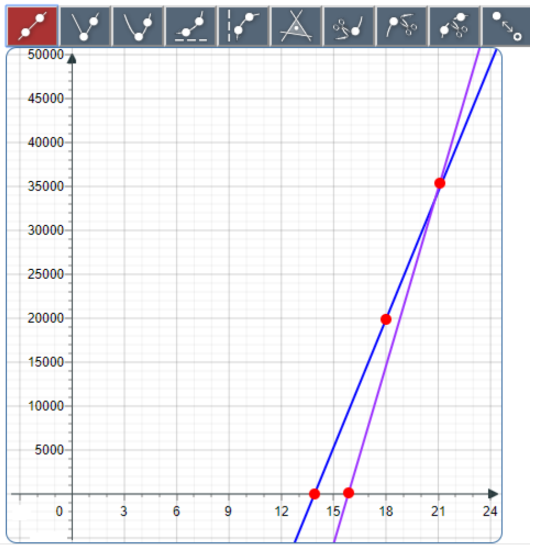


Figure 4.11 The activity "Space Launch" (Barana & Marchisio, 2019).

The AFA model to shape questions allowed us to follow the principles of formative assessment design (section 2.3.6) (van den Heuvel-Panhuizen & Becker, 2003). In particular, the step-by-step capabilities allowed us to build well-structured questions containing multiple requests. Since students can immediately visualize the correctness of their answer and use the information in the next steps, it is possible to create inter-dependent tasks within a question. Through the step-by-step capability of the system, different forms of questions were created: questions with interactive feedback, where students have a chance to understand why their answer was correct or incorrect following an interactive process; step-by-step processes which guide students in the resolution of one problem; and exploration of mathematical objects that, at each step, provide further details and new viewpoints that are useful for a complete analysis of a concept. These forms of questions help students make the process visible: they are guided in making reasonings explicit and go through good examples of correct justifications interactively, increasing their autonomy. This way, the process becomes an output; to get the maximum score, students have to complete the full reasoning, not only to provide the correct answer. These questions also increase argumentation competence, one of the greatest difficulties for students at the secondary level. Questions that involve different possible strategies to get the solution can be carefully studied and adapted to the automatic assessment. It is possible to write algorithms in order to accept answers in different forms. Moreover, one can prepare interactive feedback that shows students several approaches to the correct answer or different representations of the same problem, as shown in the first example (Barana et al., in press; Barana, Fissore, & Marchisio, 2020).

The tasks designed for the experimentation are created according to constructivist directions. They respect the seven goals for building constructivist learning environments theorized by Cunningham, Duffy, and Knuth (Honebein, 1996).

1. Real-world problems offer students a learning environment to create mathematical knowledge starting from a specific case. Exploring tasks let students build and associate meanings to mathematical concepts. Questions with step-by-step, guided solutions help them manage a complex resolution following their ideas. Thus, students get to experience the very knowledge construction process.
2. Questions, as shown in the first example, often show students different approaches to resolving a problem, and highlight how it is possible to express the same Mathematical object in different registers (through words, formulas, graphs, geometrical figures). Moreover, in the classroom

activities, questions can be discussed with peers, so students have the opportunity to come to terms with different opinions and ways of understanding. These features can provide learners with experience in and appreciation of multiple perspectives.

3. A great part of the activities is contextualized in real-world situations, interesting and challenging for students, or in other disciplines, or other areas of Mathematics. In this way, learning is embedded in realistic and relevant contexts.
4. Step-by-step processes and interactive feedback actively involve students in the correction of their mistakes and exploring solving strategies. Students are not merely reading static texts, but they must follow the reasoning and demonstrate their understanding. They are at the center of their learning. Moreover, the questions and their sections do not flow automatically in front of students' eyes: they have to autonomously get into each one and browse questions with a click, thus enhancing their commitment. In this way, ownership and voice in the learning process can be encouraged.
5. Students' work, their problems and successes are not isolated: a great part of the activities are conceived to be carried out in the classroom in small groups, always followed by collective discussions. Students can share their results or their problems with other learners through the forum to embed learning in social experience even at home.
6. Questions often present the same concept with different registers (such as in words, symbolic, graphic, tabular). The shift of register is aimed to facilitate understanding. This approach aims at encouraging students to the use of multiple modes of representation.
7. Immediate feedback facilitates students' acknowledgment of their preparation; moreover, automatically graded open answers and interactive feedback ask students to explain processes, not only to give results. Hence, the assessment activities pursue the goal to encourage self-awareness in the knowledge construction process.

Thus, this DLE provides a suitable learning environment where students can reinforce their knowledge with a constructivist approach. AFA activities make Città Educante an interactive learning environment to acquire and reinforce mathematical knowledge and competence.

5 MAIN EXPERIMENTATION: RESULTS

In this chapter, we will present and discuss the main experimentation results from several perspectives, using the data collected as previously explained in the chapter on the research methodology and analyzing them according to the models and theories discussed in the theoretical framework. The four dimensions which will be the object of this analysis are: the advantages of the use of a DLE from a general learning perspective (including learning general learning results and the various factors which influenced them); the enactment of formative assessment strategies; the development of modelling competences; and the development of engagement. Through these analyses, we will obtain a complete overview of the effectiveness of interactive activities with AFA in a digital learning environment at the school level.

5.1 ADVANTAGES OF THE USE OF THE DLE FOR LEARNING

In this chapter, we will examine the results of the main experimentation from a general learning perspective to understand which advantages the use of the DLE used in the experimentation, with the activities of problem solving and AFA, can bring to learning processes. Firstly, we will focus on the experiment's net results by comparing the outcomes of the learning tests of the treated and the control group to understand if the experimental activities were effective in improving learning results. We will then investigate if the social environment affects our experimentation results, focusing mainly on students from lower social classes. Lastly, we will focus on the online activities to identify the advantages that the use of AFA for individual study might bring to learning; a profile of students who made little individual use of the AFA will be conducted.

In particular, the following research questions, considered in the context of the experimentation, will be discussed:

> (RQ1) Are classroom and online activities with automatic formative assessment in a digital learning environment effective in improving students' learning results?
>
> (RQ2) How does belonging to a lower social class affect the impact of the interactive activities with AFA on learning?
>
> (RQ3) Are mathematical knowledge, procedural understanding, and self-assessment skills influenced by the regular adoption of the automatic formative assessment for learning Mathematics?
>
> (RQ4) Who are the AFA's reluctant users and are there any differences between active and reluctant users in learning achievements?

In order to discuss the answers to the research questions, the following primary sources of data were collected and cross-checked:

- the percentage of correct answers of the initial, intermediate, and final tests returns the information about the learning achievements made by the two groups of students (treated and control group);
- answers from the self-assessment form in the final test express students' self-assessment skills;
- students' answers to selected questions of the initial questionnaire indicate their motivations to study Mathematics – and to schoolwork in general – before the experimentation;
- the answers of the treated group to selected questions of the final questionnaire shows students' perceptions of the usefulness of the online interactive materials;
- the answers of the treated group to the open questions of the final questionnaire, to detect if the strategies of formative assessment were implemented in the classroom and online activities;
- data about the social condition of students used to split the whole sample into two sub-groups: medium-high social classes and lower social classes;

- data about the platform usage have been extracted, in particular, the number of attempts made and the grades obtained, to analyze the relationship between learning achievements and frequency of usage.

Data have been analyzed using SPSS 25; the main results and conclusions will be reported in the following paragraphs. For reasons of clarity, we reported in Table 5.1 the list of variables used in the analysis of chapter 5.1.

Table 5.1 List of variables used in the analyses of chapter 5.1.

Variable	**Type**	**Values**
IT	Cardinal	Percentage of correct answers to the initial test
INTT	Cardinal	Percentage of correct answers to the intermediate test
FT	Cardinal	Percentage of correct answers to the final test
DIFF_FT_IT	Cardinal	Difference between the percentages of correct answer to the final test and the initial test (FT – IT)
GROUP	Dichotomous	Control Group or Treated Group
DIFF_FT_INTT	Cardinal	Difference between the percentages of correct answer to the final test and the intermediate test (FT – INTT)
DIFF_INTT_IT	Cardinal	Difference between the percentages of correct answer to the intermediate test and the initial test (INTT – IT)
USERS	Dichotomous	Active Users or Reluctant Users

5.1.1 Effectiveness of the experimentation from a learning perspective

Firstly, we will start the analysis of the experimentation's results by studying the net effects on learning achievements of the experimental activities, answering the research question:

(RQ1) Are classroom and online activities with automatic formative assessment in a digital learning environment effective in improving students' learning results?

For that purpose, we compared the results of the learning tests carried out by the treated group to the ones carried out by the control group. The data considered for this analysis are the percentage of correct answers of the initial, intermediate and final test, which we registered under the cardinal variables IT, INTT, and FT; they assume values from 0 to 100 depending on the percentage of correct answer.

As a first step, we computed the difference between the final and initial test results, thus defining a new cardinal variable: DIFF_FT_IT. This variable assumes positive values for students who improved their results at the end of the school year and negative values for those who had a lower performance in the final test than in the initial one. The sample was then restricted to the only students who took both the initial and the final test, being present at school on the days the tests were administered: it is the 83% of the whole sample (80% of the control group and 85% of the treated group). This variable is normally distributed for the two groups (Shapiro-Wilk test: 0.991 and 0.992, sig.: 0.263 and 0.236 for control and treated group, respectively). We performed the variance analysis (ANOVA) on the dependent variable DIFF_FT_IT, using the dichotomous variable GROUP, expressing the belonging to the treated or control group as the independent variable. The results are shown in Table 5.2: students of the treated group started from a lower level (39.87 out of 100) than students of the control group (45.83 out of 100), but at the end of the experimentation, their results improved their score significantly more than students of the control group (the mean is 45.77, with an increase of 7.20 out of 100, while the latter obtained a mean of 45.83, with a decrease of 0.58 out of 100). The ANOVA test showed that the experimentation modality influenced the increase in the results between initial and final tests in a statistically significant way ($F = 19.23$, $p < 0.001$).

Table 5.2 Results of the initial and final tests for the treated and the control group.

		Control group	Treated group	Total
Initial test (IT)	N	233	280	513
	Mean	**45.83**	**39.87**	**42.58**
	SD	21.28	19.00	20.22
Final test (FT)	N	207	267	474
	Mean	**45.77**	**47.24**	**46.60**
	SD	20.48	19.60	19.98
Difference (FT - IT)	N	197	255	452
	Mean	**-0.58**	**7.20**	**3.81**
	SD	20.30	17.35	19.07

These data allow us to compute the effect size of the experimental activities, which measure the intervention's efficacy. We use Cohen's *d* index (Cohen, 1969), which is defined as:

$$d = \frac{M_1 - M_2}{S}$$

Where M_1 and M_2 are the means of the two groups and S is an esteem of the standard deviation of the population from which the samples are extracted; in particular,

$$S = \sqrt{\frac{(n_1 - 1) \cdot {S_1}^2 + (n_2 - 1) \cdot {S_2}^2}{(n_1 - 1) + (n_2 - 1)}}$$

where n_1 and n_2 are the numerosity of the two groups and S_1 and S_2 are the standard deviations. We chose this index as the whole sample is numerous (>20), and the standard deviations are not so different (Pellegrini et al., 2018). Using the results in Table 5.2, Cohen's d is 0.42, a medium value for an effect size (Pellegrini et al., 2018).

If we consider the intermediate test results, we find out that they are more similar to those of the initial test than the final one. Similarly to the final test, it was exclusively focused on formulas and functions, and it was administered before the beginning of classroom activities on that topic. Therefore, it can represent the students' starting point about this particular area. We need to underline that the intermediate test was easier than the final one, so comparisons between these tests' results are meaningless in absolute terms. The difference between the percentages of correct answer to the final test and the intermediate one (cardinal variable DIFF_FT_INTT) has a negative mean, and the difference between results of the intermediate test and those of the initial one (DIFF_INTT_IT) has a much higher mean than the previously studied DIFF_FT_IT. Data are shown in Table 5.3.

We computed an ANOVA test on the dependent variables DIFF_FT_INTT and DIFF_INTT_IT, considering the dichotomous variable GROUP as the independent variable. We found that the treated group decreased its results much less than the control group in the final test compared to the intermediate one: the difference is statistically significant ($F = 12.69$, $p < 0.001$), while there was no evident difference in the improvements from initial to intermediate tests between the two groups ($F = 1.624$, $p = 0.20$).

To provide more easily interpretable results, we used the covariance analysis to compute the adjusted mean of the final test results, considering the initial test results as the covariate. Then we repeated the test using the results of the intermediate test as the covariate. The results are shown in Table 5.4. Values are statistically significant ($p = 0.002$ and $p = 0.003$ respectively). These means can be interpreted as the grades that students would have obtained in the final test once we eliminate the variance provided by their initial level, expressed though their grades in the initial (or intermediate) tests. The effects are more evident when we refer to the initial test than to the intermediate one.

Table 5.3 Results of the initial, intermediate and final test. Values are expressed in a scale from 1 to 100, corresponding to the percentage of correct answer.

		Control group	Treated group	Total
Initial test (IT)	N	233	280	513
	Mean	**45.83**	**39.87**	**42.58**
	SD	21.28	19.00	20.22
Intermediate test (INTT)	N	233	279	512
	Mean	**61.06**	**57.12**	**58.91**
	SD	18.06	16.98	17.57
Final test (FT)	N	207	267	474
	Mean	**45.77**	**47.24**	**46.60**
	SD	20.48	19.60	19.98
Difference (FT - INTT)	N	201	255	456
	Mean	**-14.92**	**-9.50**	**-11.89**
	SD	16.30	15.97	16.32
Difference (INTT - IT)	N	222	264	486
	Mean	**15.03**	**16.93**	**16.06**
	SD	18.04	14.88	16.41

Table 5.4. Adjusted means of the final test, for the initial and intermediate test.

Covariate	Group	N	Mean	Standard error
Initial test	**Control group**	197	43.99	1.19
	Test group	255	49.01	1.04
Intermediate test	**Control group**	201	44.38	1.08
	Test group	255	48.78	0.96

In conclusion, we can positively answer the research question (RQ1) “Are classroom and online activities with automatic formative assessment in a digital learning environment effective in improving students’ learning results?”. We can affirm that students from the treated group increased their learning achievements, significantly more than students from the control group, with a medium effect of the experimental intervention.

5.1.2 Social factor and learning achievement

From the pilot experimentation results, we argued that the socio-economic situation could have influenced the experimentation’s results. In the context of the main experimentation, we aim to find an answer to the research question:

(RQ2) Does belonging to a lower social class affect the impact of the interactive activities with AFA on learning?

Thus, we restricted the analysis to schools located in socially disadvantaged milieux of Turin, attended by students from low socio-economic classes (identified through INVALSI data and self-evaluation school reports). The sample, restricted in this way, is composed of 236 students, 86 in the control group, and 152 in the treated group. As before, we considered the results of the initial and final learning tests, in particular the cardinal variables IT, FT, and DIFF_VF_VI. We performed the ANOVA test on the increments in the students’ grade in the final test as compared to the initial one (DIFF_VF_VI): results are shown in Table 5.5. As expected, the quantitative effects of the experimental activities become more evident: while the control group decreased its mean grade of almost 8 points out of 100, the treated group increased its mean grade of almost the same extent. The difference between the two groups is statistically significant ($F = 27.58$, $p < 0.001$).

Table 5.5 Results of the learning tests restricted to the students belonging to lower social classes.

		Control group	Treated group	Total
Initial test (IT)	N	78	140	218
	Mean	**42.17**	**34.74**	**37.39**
	SD	20.06	17.44	18.71
Final test (FT)	N	76	136	212
	Mean	**35.39**	**41.90**	**39.57**
	SD	20.19	20.24	20.42
Difference (FT - IT)	N	71	130	201
	Mean	**-7.90**	**7.07**	**1.78**
	SD	21.98	17.71	20.57

The experimentation's effect size restricted to the students from lower social classes nearly doubles: Cohens' *d*, calculated with these data, is $d = 0.77$, which is a medium-high value (Pellegrini et al., 2018).

Through these results, we can positively answer the research question (RQ2) "does belonging to a lower social class affect the impact of the interactive activities with AFA on learning?": in fact, it seems that belonging to a lower social class affects the impact of the experimentation on learning; in particular, the effects of interactive activities nearly doubled.

One could argue that these results are due to the Hawthorne effect, that is, the set of modifications of a phenomenon or of a behavior which occurs due to the presence of observers, but which do not last in the time (Cook, 1962). If the Hawthorne effect could have influenced a little part of the experimentation results, we need to consider that:

a) the project lasted the whole school year, while the Hawthorne effect would have had a more limited effect;
b) only a few hours were held in the presence of the researchers, while the teachers adopted the methodologies and used the interactive materials for the whole school year;
c) the students had little perception that they were the treated group of an experimentation, they just knew to be part of a project with the University for learning Mathematics;
d) the initial and final tests were considered by teachers and students as regular class tests;
e) also the students from higher social classes would have suffered from this effect.

Thus, we think that the main part of these results is not compromised by the Hawthorn effect.

This analysis confirms that interactive activities that make extensive use of innovative technologies are more successful with students of lower social classes. Observations made during the classroom activities confirm that the level of attention and students' engagement was higher in the schools situated in more disadvantaged areas than those attended by wealthier families. Our feeling was that students, in the former case, enjoyed the attention that we had towards them: taking part in special activities, specially dedicated to them, carried out by people from the university, probably made them feel special. We suppose that many of them do not often have the chance to participate in similar activities. Many of their families do not have cultural end economic tools to support other extracurricular activities, unlike their peers of higher social classes, for whom we were just one of their many chances of learning. This aspect will be analyzed deeper in the following part of this thesis.

These results also open to further discussion, that partially goes beyond the scope of this thesis. We can notice that in the treated group, the improvements made by the students from lower social class in the final test (7.07) is very similar to that of the whole group (7.20). It means that the social background did not limit the possibility of using the interactive path and the technologies to improve learning. On the other hand, in the control group, the decreasing in the results to the final test of students from lower social classes (-7.90) is noticeably worse than the average value of the whole group (-0.58), and as a consequence, of the students from higher social classes. This suggests that, especially for students from lower social classes, the use of traditional learning methodologies is not the best strategy to improve their results.

5.1.3 Focus on online activities: active users and reluctant users

As previously explained, the experimental activities involved two important and interlaced parts: the classroom activities and online activities (done as homework). All the students in the treated group participated in the classroom activities for a simple reason: they were present at school; we do not have a track of how much students paid attention and actively worked during the classroom activities. Usually, teachers logged in the platform with their credentials and shared the activities through the IWB, while students could work with paper and pen. On the other hand, the online activities were available on the DLE, and students were asked to work on them from home. Sometimes, teachers used the online activities from the school lab, where available; in this case, the students' individual work is registered, and we can measure the number of online activities completed by each of them. This is an interesting variable for evaluating the experimentation: we are interested in studying how much the individual use of the AFA affects the learning achievements. We cannot expect astonishing effects, since those students who never logged in the platform in the whole school year could benefit from classroom activities' effects, and it is not possible to separate the effects of these two kinds of activities. However, we want to examine the added value that AFA's individual use confers to the experimental activities. In other words, we try to answer the following research questions:

(RQ3) are mathematical knowledge, procedural understanding, and self-assessment skills influenced by the regular adoption of the automatic formative assessment for learning Mathematics?

(RQ4) Who are the AFA's reluctant users and are there any differences between active and reluctant users in learning achievements?

To this purpose, we restricted our analysis to the treated group only and considered the following sources of data:

- the percentage of correct answers of the initial, intermediate, and final tests;
- answers from the self-assessment form in the final test express students' self-assessment skills. For each item, one point was assigned if the student's impression of giving the correct or incorrect answer matched the item's evaluation, while no points were assigned in case it did not match the evaluation or in case students were unsure of their answers. The sum of the points earned in the whole test was then scaled from 0 to 10;
- students' answers to the initial questionnaire, which indicate their motivations to study Mathematics – and to schoolwork in general – before the experimentation;
- the answers of the treated group to the final questionnaire – and in particular to the set of questions about the online assignments – from which we can gather students' satisfaction with the online interactive materials;
- data about the platform usage, in particular, the number of attempts made and the grades obtained, to analyze the relationship between learning achievements and frequency of usage.

As in the previous section, only the fully completed attempts were considered "attempts", meaning that students requested and obtained feedback and grades. The number of attempts was used to define the two sub-groups of students in the treated group. We defined "active users" as those who made at least five graded attempts for the course assignments. The students who made less than five attempts were defined "reluctant users". The latter represents about 30% of the students in the treated group. As active and reluctant users were defined based on the number of accesses to online materials, a parallel definition is not provided for students belonging to the control group, who did not have access to online activities.

To better understand the relationship between active users and reluctant users in the treated group concerning learning gains, the ANOVA test was performed on their results in the initial, intermediate and final tests and in the final test's self-assessment. The results are summarized in Table 5.6. It emerges that active users performed a little, but not significantly better in the initial and intermediate tests than reluctant users. However, active users performed significantly better in the final test and the final test's self-assessment. Given the similarity in the initial results, it is not possible to conclude that reluctant users were initially weaker than their active classmates; however, the regular use of the AFA led to higher learning achievements and more effective self-assessment skills.

An analysis of the platform usage leads to affirming that the questions implemented with automatic assessment according to the abovementioned educational models, were effective under several points of view. Students' number of attempts to any test is, on average, 15.81; this value grows when considering only active users (mean: 20.76). Active students attempted the same assignments an average rate of 1.75 times, which means that they have generally made more than one attempt at the online quizzes.

Table 5.6 Results of the ANOVA test conducted on the learning test results for active and reluctant users.

		Reluctant users	**Active users**	**p-value**
Initial test	N	90	190	0.060
	Mean	**36.94**	**42.51**	
	SD	19.26	19.12	
Intermediate test	N	93	186	0.091
	Mean	**55.41**	**59.85**	
	SD	17.07	16.37	
Final test	N	85	182	0.049
	Mean	**43.16**	**49.13**	
	SD	18.20	20.21	
Self-assessment	N	85	182	0.011
	Mean	**3.64**	**4.47**	
	SD	2.09	2.13	

The feedback gained with the automatic assessment has been used to improve the performance in subsequent attempts, where tasks were repeated with different numerical values, functions, and geometrical figures, which varied algorithmically. This statement can be statistically shown using the platform's assignments results, comparing the average score of students' first assignment attempts with the average score of their last attempt. Active students' results grew from an average value of 51.97 at their first attempts to 60.25 at their last attempts (values are expressed in percentage of correct answer); a pairwise t-test shows that the difference is statistically significant ($p<0.001$). It is interesting to notice that the average grade of the first attempts made by reluctant users (49.76) is similar to that of active students: the ANOVA test shows no statistically significant difference between the two means ($p=0.524$). This fact confirms what emerged from the initial and intermediate tests: reluctant users were not initially weaker than active users in terms of learning. However, in the platform, like in the learning tests, reluctant users did not improve their results: the average of their last attempts increased only of 2.15 points out of 100, rising to 51.91, while the average active users' grade increased by 8.28 points out of 100, rising to 60.25. The difference in the improvements between the two groups can be seen as a consequence of the time spent using the online assignments: reluctant users' average rate of attempts per assignment is 1.22. Table 5.7 summarizes the statistics on platform usage for active and reluctant users.

Table 5.7 Statistics on the platform usage for active and reluctant users

		Reluctant users	**Active users**	**p-value**
Number of attempts	Mean	**1.48**	**20.76**	<0.001
	SD	1.52	15.63	
Rate of attempts per test	Mean	**1.22**	**1.75**	0.001
	SD	0.43	0.90	
Mean of the first attempts	Mean	**49.76**	**51.97**	0.524
	SD	24.90	18.49	
Mean of the last attempts	Mean	**51.91**	**60.25**	0.016
	SD	22.82	18.80	

The improvements registered through the platform suggest that the automatic assessment format helped students develop problem solving strategies, acquire procedural understanding, and identify and correct their mistakes. To support this hypothesis, we selected some meaningful questions from the final questionnaire, which could help us highlight the main advantages of the regular use of online activities. Active users affirmed that it was useful to visualize the feedback immediately after answering. It helped them understand how to solve a task. Step-by-step guided resolutions were useful to understand the solving process. Their answers are compared with the reluctant users' ones, which are lower, as expected. Interestingly, ANOVA results show that, between the two groups, there is not any statistically significant difference neither in the appreciation of AFA as a support to study, nor in the effectiveness of AFA to raise the awareness of one's capabilities. The main differences are on the perceived usefulness of the system to help them learn a method to solve problems. In other words, reluctant users acknowledged the potential of the automatic assessment; nevertheless, the effectiveness of the system to learn problem solving strategies could be appreciated only after multiple attempts and after an adequate time spent on online homework. The data are summarized in Table 5.8.

Table 5.8 Active and reluctant users' answers to questions about automatic assessment in the final questionnaire. Answers are in a Likert scale from 1 (lowest value) to 5 (highest value).

		Active users	**Reluctant users**	**p-value**
FQ10 It is useful to visualize the correct answer after submitting a response.	Mean	4.29	3.93	0.047
	SD	0.89	0.91	
FQ23 The immediate feedback helped me understand how the task should be solved	Mean	4.16	3.77	0.042
	SD	0.95	0.97	
FQ24 Problems with step-by-step resolution helped me understand the solving process.	Mean	3.91	3.47	0.025
	SD	0.96	1.01	
FQ25 Online assignments helped me autonomously solve Mathematics exercises	Mean	3.34	2.83	0.015
	SD	1.00	1.08	
FQ11 Online tests are valid support for studying.	Mean	3.62	3.43	0.362
	SD	0.99	1.07	
FQ27 Online assignments helped me acknowledge my preparation	Mean	3.69	3.43	0.20
	SD	0.99	1.00	

Students' appreciation of the automatic assessment is confirmed by their answers to the open question FQ43 "Explain why, in your opinion, the online activities were (or were not) useful and you liked (or did not like) them". Examples of comments repeated in the answers collected are the following:

- "I appreciated the fact that the exercises were on the computer, which made them more interesting. In my opinion, giving an evaluation to each exercise pushed me to put more effort into my homework and to improve my results."

- “They are useful because if you give an incorrect answer, you can try it again, while you cannot do the same on the book.”
- “The online exercises are useful because you can access them whenever you need. Also, the problems that gave the correct solution immediately after your answer were excellent since you could understand the solving process and try them again with different data.”
- “In my opinion, they are useful because they help you understand the topic, and the presence of figures and graphs helped you better understand the task.”

Since using the automatic assessment provides such good effects on learning, it seems a pity that there were students who missed this opportunity and barely opened the online assignments. Data collected during the experimentation help understand who the reluctant users are. As emerged above, reluctant users do not coincide with reluctant learners, in fact, their initial learning achievements were not significantly far from the active users’ ones. We supposed that students who made little use of the online platform had a preference for a paper-and-pen learning approach. To test this hypothesis, we analyzed the students’ answers to the question FQ44: “do you prefer doing computer-based or paper-based homework?” to which students could choose one of the two options. It emerged that 60% of active users opted for “computer-based homework”, while 45% of reluctant users made the same choice (Cramer’s V: 0.117, p-value: 0.11). This means that, among reluctant users, there is a slight preference for a paper-and-pen learning style, but it does not explain the difference in the platform’s usage. At least, the 45% of reluctant users who stated that they preferred using the computer for doing their homework actually did not do it.

We investigated reluctant users’ general approach to learning through their answers to meaningful selected questions of the initial questionnaire, provided on a Likert scale from 1 (strongly disagree) to 4 (strongly agree) and displayed in Table 5.9.

Table 5.9 Answers of reluctant and active users to some initial questionnaire questions, given on a Likert scale from 1 (strongly disagree) to 4 (strongly agree).

		Reluctant users	**Active users**	**Cramer’s V (p-value)**
IQ2 I can’t wait for Mathematics lessons	Mean	2.20	2.45	0.151 (0.116)
	SD	0.73	0.81	
IQ3 I do Mathematics because I like it	Mean	**2.58**	**2.74**	**0.22 (0.006)**
	SD	0.83	0.88	
IQ5 Making an effort in mathematics is worthy because it will help me in the job that I want to do later on	Mean	3.46	3.51	0.14 (0.167)
	SD	0.53	0.66	
IQ6 Mathematics is an important subject for me because I need it for what I want to study later on	Mean	3.43	3.46	0.15 (0.093)
	SD	0.61	0.75	
	SD	0.68	0.66	
IQ7 I finish my homework in time for mathematics class	Mean	2.94	2.94	0.073 (0.716)
	SD	0.97	1.03	
IQ8 I work hard on my Mathematics homework	Mean	2.69	2.94	0.148 (0.129)
	SD	0.79	0.69	
IQ16 I help my friends with Mathematics	Mean	**1.89**	**2.29**	**0.244 (0.002)**
	SD	0.89	0.86	
IQ21 When confronted with a problem, I give up easily	Mean	2.52	2.32	0.107 (0.565)
	SD	1.17	1.14	
IQ22 I put off difficult problems	Mean	2.45	2.20	0.145 (0.246)
	SD	1.25	1.04	
IQ26 If I put enough effort, I can succeed in Mathematics	Mean	3.42	3.42	0.086 (0.595)
	SD	0.68	0.66	

It emerges that they are as aware as active users of the importance of studying Mathematics for their educational and professional future; however, they do not appreciate the subject per se as much as their classmates (Cramer’s V: 0.22, p=0.006). Some differences emerge in how they appreciate the way Mathematics is taught at school: reluctant users are slightly less willing to attend Mathematics lessons. They are aware that good results depend on their dedication to their classmates' same extent, and they arrive at school with their homework done. However, they work less hard on their homework, and they usually do not help their friends with homework as much as active students (Cramer’s V: 0.24, p=0.002). There are no statistically significant differences in questions asking about their attitude towards problem solving, even though reluctant users seem to be a little less persistent in problem solving.

One could argue that students living in poorer families or disadvantaged contexts could have less chance to access the technologies and have more probability of falling into the “reluctant users” category. We tested this hypothesis through Chi-squared and Cramer’s V test, in order to check the dependence of the variable “users” (having values active or reluctant) from the sociocultural factor (lower social class and medium-high social class). It emerged that the percentage of reluctant users is similar and even lower in schools located in areas with low socioeconomic conditions (29%) than in wealthy areas (33.1%). Chi-squared is 0.343 (sign.: 0.558), and the Cramer’s V is 0,036 (sign.: 0.058). These results mean that it is not possible to hypothesize a relation between the two variables. Therefore, we can affirm that sociocultural factors did not affect students’ usage of the automatic assessment.

Not surprisingly, the chances are that their school teachers strongly influence students’ reluctance (or willingness) towards the use of learning technologies: in this case, Cramer’s V is high ($V = 0.52$, p-value < 0.001). Examining the different percentages of reluctant users for teacher, it emerged that the teachers who participated more eagerly in the projects and who managed to integrate the methodologies proposed into their daily practices had very low percentages of reluctant users (less than 20%). Most of these teachers were previously trained on the autonomous use of the technologies involved in the experimentation and created interactive digital learning materials with AFA for their students. On the other hand, teachers who spent less time in the classroom to the interactive activities registered the highest percentages of reluctant users. This confirms what was observed in the pilot experimentation. It suggests that teachers’ confidence in the experimented methodologies and how they propose them to their students in daily practices greatly influence the students’ adoption of learning technologies for self-study.

Summing it up, the picture of the reluctant users emerging from data is that of students who are not lacking talents or economic conditions for accessing the platform or performing well; they are conscious that school is essential even though they do not like Mathematics. This lack of inner motivation could be linked to the scarce effort they put into studying. An essential element of the distinction of reluctant users from the general reluctant learners is their acknowledgment that school success depends on their effort in studying. It is interesting that they hardly help their friends with Mathematics. Associated with the fact that they complete their homework without working hard on it, it suggests that they are mainly the students who are usually helped by others or even copy their homework, which becomes nearly impossible with an online system. The teachers’ opinions collected through a focus group at the end of the experimentation confirmed this insight: they affirmed that the weakest students with learning disorders or little self-awareness really appreciated online work; it was useful for skilled students to improve even more; however, the most undisciplined and lazy students did not seize the opportunities given by digital tools: they worked very little on the platform, just like they did with paper-and-pen homework.

In conclusion, this analysis helped us answer the research questions: (RQ3) “are mathematical knowledge, procedural understanding and self-assessment skills influenced by the regular adoption of the automatic formative assessment for learning Mathematics at the secondary school level?” and (RQ4) “who are the reluctant users of the AFA and are there any differences between active and reluctant users in the learning achievements?”.

Regarding (RQ3), the immediate feedback, given at the right time and in an interactive way, helps students master a process for solving mathematical tasks and finding problem solving strategies. When students repeated the exercises with similar tasks but different data, they improved their grades, as shown in the analysis before (Table 5.7). This finding is slightly in disagreement with many findings in the literature, which show that students pay little attention to feedback information and consequently do not use it to improve (Timmers & Veldkamp, 2011). In this study, the critical strength was the feedback's interactivity, which helped students process the information. Improvements in understanding also emerged in the previous paragraphs from comparing initial and final tests between the treated and control groups. The gains involve not only mathematical knowledge but also metacognitive factors, self-assessment skills above all. This fact emerged clearly from the performance in the self-evaluation of the final test of students who regularly used the automatic assessment compared with that of students who rarely worked on the platform. Hence, we have positively answered the research question (RQ3): mathematical knowledge, procedural understanding, and self-assessment skills are positively influenced by the regular adoption of the automatic formative assessment for learning Mathematics at the secondary school level.

Regarding (RQ4), through an analysis of the data usage, we have selected a group of students who made few or no attempts with the online assignments, and we defined them as “reluctant users”, opposed to the “active users”. Data show that students that did not take advantage of this chance did not improve in learning as much as their classmates. Their profile can be inferred from their answers to the initial questionnaire, thus answering the research questions. Reluctant users are students who put little effort in school in general, and who dislike Mathematics in particular. Economic disadvantages or learning disorders did not prevent students from working in the online courses, as reluctant users’ social distribution and initial results show. However, their teacher’s didactic practices and confidence in the experimental methodologies can influence their online work commitment. Some of them prefer a paper-based approach to learning, but it is not why they did not use the technologies.

Part of section 5.1.3 has been taken, expanded and adapted from (Barana, Marchisio, & Sacchet, 2019).

5.2 Enactment of Automatic Formative Assessment in the experimental activities

This section's focus is to show how automatic assessment, implemented in a blended modality through classroom activities and online work in a DLE, allows the enactment of formative assessment. In particular, we will refer to Sadler’s and Black and Wiliam’s formative assessment models, aimed to enhance learning and self-regulation (Barana & Marchisio, 2019). We will use the model of AFA in a DLE that we have conceived and presented in the theoretical framework of this thesis (section 2.3.3). Part of the following text is taken, adapted and expanded from (Barana & Marchisio, 2019) In particular, we investigate the following research questions:

(RQ5) Can the interactive feedback be effective, according to Sadler’s model (Sadler, 1989)?

(RQ6) Can the blended use of the automatic assessment support formative assessment strategies?

To this purpose, we considered the following sources of data:

- data about the platform usage, in particular, the number of attempts made and the grades obtained to the automatically assessed online tests, to understand if the interactive feedback helped students improve their performances;
- the videotapes of the classroom activities held in the presence of a PhD student;
- the answers of the treated group to the open questions of the final questionnaire, to detect if the strategies of formative assessment were implemented in the classroom and online activities;

The first point deserves some additional clarifications. The number of attempts to all the available tests – included repeated attempts to the same tests – was drawn from the platform for each student. We considered only the fully complete attempts, where the students requested and obtained feedback and grade. We excluded from the analysis the incomplete attempts, where the students just answered a few questions and left the AAS without grading the assignment nor viewing the feedback. We also excluded the cases where the students opened the assignments to look at the questions and did not answer them. This choice is in line with the models of formative assessment adopted in this thesis. Without automatic grading and interactive feedback, the activity would not have a formative value.

In order to evaluate whether the interactive feedback was effective according to Sadler's model (RQ5), for each student, we computed the average number of attempts per assignment: it ranges from 1 to 12, with an average value of 1.70, which expresses students' tendency to repeat the questions. Then, we compared the average grade each student earned in their first assignment attempt with the average grade she earned in her last attempt through a pairwise Student's t-test. We remember that, in questions with the interactive feedback, students earn the full grade if they answer correctly to the initial question. If they fail it, students are led to the interactive feedback, through which they can earn partial grades, up to a maximum of 80% of the question's full grade. Thus, repeating the tasks, students have the chance to improve their scores.

We found that the mean of the initial grades was 51.55/100 (SD: 19.63), while the mean of final grades was 59.02/100 (SD: 19.87); the increase is statistically significant ($p < 0.001$). These results show that students did use the information provided in the feedback to improve their results in subsequent attempts. Therefore, the interactive feedback was effective according to Sadler's model, in fact:

a) the interactive feedback shows an example of correct resolution to the students, which is a concept of the standard being aimed for;
b) answering and grading the sub-questions in the interactive feedback, students can compare the actual level of performance with the standard;
c) repeating the tasks, students engage in an appropriate action which leads to some closure of the gap.

Thus, we have positively answered to (RQ5).

To clarify if the interactive activities with AFA could support the implementation of Black and Wiliam's formative assessment strategies, we will use our model of AFA in a DLE (elaborated in section 2.3.3) to analyze some episodes of classroom activities and the students' behavior in online activities. For the classroom activities, we will use the videotapes realized during the activities. They were carefully seen and analyzed in order to find relevant episodes. Then, they were transcribed and coded. For the online activities, we will analyze the students' answers gathered in the AAS's gradebook.

Here we present an episode that occurred during the first classroom activity at the "Manzoni" school, dealing with algebraic computations through the interactive activity "Formulas and Areas". The components of the DLE present in this excerpt are the following:

- the human component is the class community, constituted by the students working in peers and the teacher; there is also the presence of a researcher, who supported the teacher in managing the activities and observed the students' work;
- the technological component is constituted by the LMS integrated with the ACE and the AAS; the interactive activity "Formulas and Areas" available through the LMS thanks to the integration with the ACE, and displayed through the IWB to the class; the discussions among the groups were videotaped using a video camera.

The interactions between the technological apparatus and the community are the learning processes and the didactic methodologies. They will be explained in the following lines.

Excerpt 1

The teacher is at the IWB. He is pointing at the figure of a trapezoid (as the one in Figure 4.4).

TEACHER	Look at this figure. Write the formula which expresses how the area of this figure varies when a varies. That is, [pointing at the vertical side of the trapezoid] how long is this side?
STUDENTS	*a*
TEACHER	[Pointing at the base of the square] How about this?
STUDENTS	*a*
TEACHER	[Pointing at the base of the triangle] How about this?
STUDENTS	*a*
TEACHER	Well, you have to calculate the area of this figure using *a*. Those sides measure *a.* What does it mean? What is *a*?
STUDENTS	It is a variable.

In this excerpt, the teacher introduces the activity and explains to the students what their task is. The explanation takes the form of a dialogue, as he engages the students with questions to make sure that they follow the discourse. The teacher exploits the "delivering and displaying" function of the technology to display the task and, in particular, the figure; then, he interacts with the students. If we consider the diagram with the functions of a DLE (Figure 2.5), we are in the right part; the parts of the model involved in this excerpt are shown in yellow in Figure 5.1. While explaining the tasks, he develops the KS1 "clarifying and sharing learning intentions and criteria for success". The KS2 "engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding" was accomplished during the phase of the creation of this activity by the researchers (that we can include in the "Teacher" subject of our analysis) through the "creating and managing" function of the technologies.

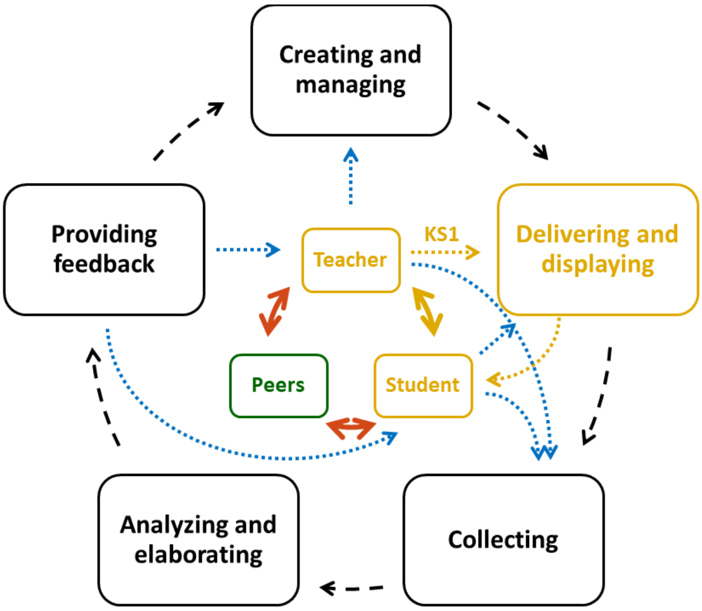


Figure 5.1 Diagram of the DLE functions and the formative assessment strategies enacted in excerpt 1.

Excerpt 2

In groups of 3, the students solve the tasks in their notebook with paper and pen. They have reported the figure, and they are trying to understand how to answer the teacher's question. In the following lines, we can see a discussion happening among the members of one group.

CLAUDIA	So, we have to write the formula.

ALBERTO	The formula is easy! But then can we transform *a* into whatever number?
STEFANO	Yes!
ALBERTO	Ok, then let's say that *a* is 5, is it ok?
STEFANO	Yes
ALBERTO	[writing] *a* = 5. Then, [pointing at the figure's sides on his notebook] if this is 5, this is 5, this is 5, this [pointing at the oblique side] is easy to compute, using the Pythagorean theorem…
STEFANO	[pointing at the trapezoid] What figure is this?
ALBERTO	It is a square plus a triangle.
STEFANO	No, this whole [pointing at the whole figure].
ALBERTO	It is a trapezoid. Do you remember a formula for the trapezoid?
STEFANO	Yes, I remember it, but the point is another. We have to say a formula… we have *a*…
ALBERTO	We have to say how the formula changes when *a* changes.
STEFANO	So we have to write a formula.
ALBERTO	For example, if *a* is 5…
STEFANO	No, we have to write a formula.
ALBERTO	No, we have to say, if *a* is 5, we have an area, if *a* is 10, it changes, that is "when a varies."
STEFANO	No, we have to find a formula that fits the case *a* = 5 but also *a* = 10.
ALBERTO	Yes, but the formula for the trapezoid works for both the numbers.
STEFANO	Yes, we have to… personalize it… using a.
ALBERTO	What do you mean? It means that you change the number.
STEFANO	*a*. Write a formula using *a*.
ALBERTO	*a* times… But can't we use the formula for the trapezoid? Tell me the formula!
STEFANO	Long base plus short base times height divided by two.
ALBERTO	Let's write it [writing the formula on his notebook].
CLAUDIA	Then, the long base is 10.
ALBERTO	Wait, if we change the number *a*, nothing changes, only the result changes! But the formula remains the same.
STEFANO	But the formula! We have to write the formula using *a*!
CLAUDIA	So, the long base is *a* + *a*...

This excerpt shows how the three students collaborate to understand the given task; what hinders them is the relation between numbers and variables, a key point of the activity. Alberto has many ideas, not all correct. Stefano tries to focus his attention on the task's main problem, and Claudia is more pragmatic and summons the attention on answering the task. This discussion is wholly face-to-face, without the use of the technologies and their functionalities. In the diagram, the episode involves the student, the peers, and the arrow linking them, as shown in Figure 5.2, highlighted in yellow. It is an example of how the KS4 (activating students as instructional resources one another) was pursued.

Excerpt 3

The last excerpt is related to an episode that occurred some minutes later, after that the students were engaged in the research of other different formulas expressing the same area. The researcher, who is supporting the teacher in managing the activities and summoning the conclusions reached by the groups, is at the IWB. She uses the interactive file, in particular the response areas in which to insert the formulas.

RESEARCHER	Well, who can tell me the formula for the area of the figure?

STEFANO	Two times *a* plus *a*, times *a*, divided by two.
RESEARCHER	[After writing the formula in the response area] What did you think when you wrote this formula? How did you see this figure?
ALBERTO	A trapezoid.
RESEARCHER	[Pointing at the sides of the figure] A trapezoid. In fact, what is 2*a*?
ALBERTO	The long base, then *a* is the short base, times *a* is the height.
RESEARCHER	[After checking the correctness of the formula through automatic assessment] It's right.
ALBERTO AND STEFANO	Oh yeah!
RESEARCHER	Do you agree, you guys? [directed to the rest of the class]
STUDENTS	Yes.
RESEARCHER	Ok. Who can tell me another formula?

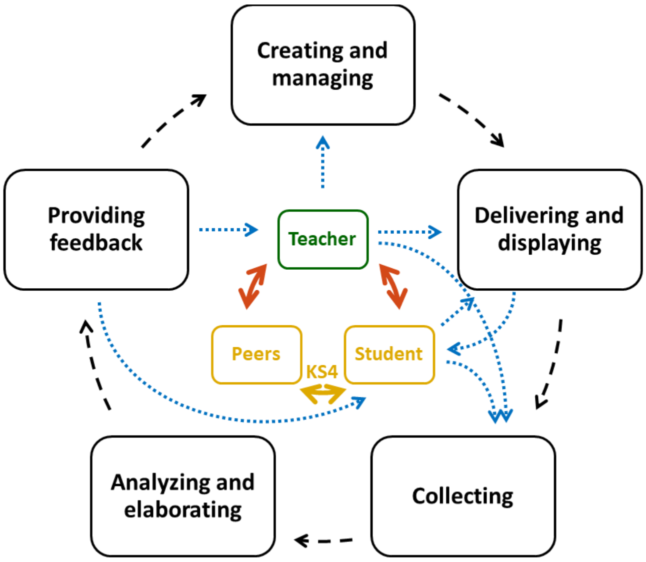


Figure 5.2 Diagram of the DLE functions and the formative assessment strategies enacted in the excerpt 2.

After that, the other groups of students proposed other formulas, obtained decomposing the trapezoid in different ways. The researcher asked students to explain how they obtained the formulas, checked their correctness through the automatic assessment, and guided students in simplifying the formulas to trace them all back to the same simplified form.

In this episode, the "collecting" function of the technology is used to insert the formula in the interactive worksheet. Formulas can be written through a simple equation editor, which makes them simple to be read. The system processes the answer through the "analyzing and elaborating" function, and its correctness is communicated to the class through the "providing feedback" function. The KS3 (providing feedback that moves learners forward) is enacted firstly through the automatic feedback displayed through the IWB, then through the discussion between the teacher and the students. Moreover, the researcher develops the KS5 (activating students as the owners of their learning) by asking them to explain their answers. This strategy fosters the development of argumentation competences. Figure 5.3 shows a schematization of the strategies enacted during this excerpt.

We suppose that formative assessment strategies are enacted differently during the online activities compared to the classroom ones. We consider the case of students working independently on their online homework at home. In this case, the DLE is restricted: the human component is represented by the student working autonomously in asynchronous online modality; the technological component is composed of the LMS integrated with the AAS, automatically graded assignments as learning activities, a computer or a tablet to access them. The relationships between the two components are the learning processes that we will analyze in the following lines.

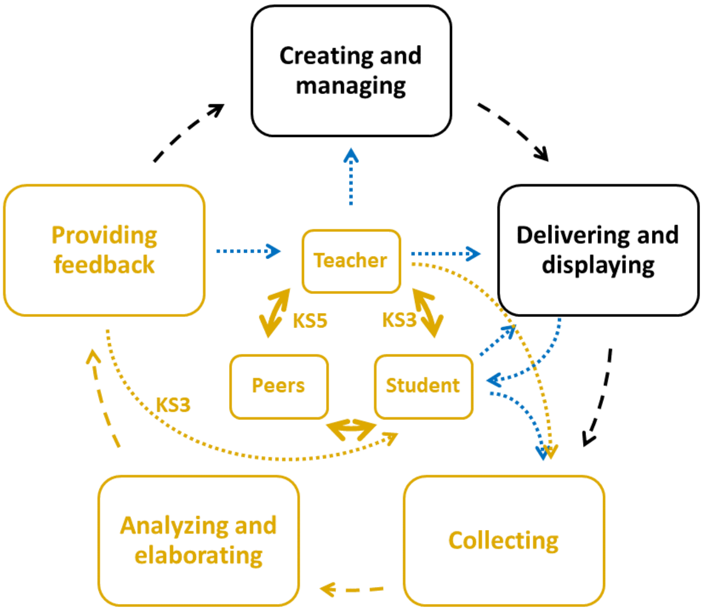


Figure 5.3 Diagram of the DLE functions and the strategies of formative assessment enacted during the excerpt 3.

To analyze the development of formative assessment strategies during individual automatic assessment activities in online modality, we used data from the AAS gradebook, where all the students' results are collected. We consider the online activities "Other Strange Figures" shown in figure 4.6: they are additional exercises related to the part of the interactive path involving algebraic computations. The assignment is composed of 2 questions related to the same figure, the first one involving the area and the second one the figure's perimeter. Both questions are composed of 3 parts: the first one asks a formula for the area (or the perimeter), the second one shows a table to be filled with numerical values, obtained assigning different values to the variable, and the third one asks to sketch the graph of the function or to choose it among four options. The students have feedback about their answers' correctness and the comparison with the correct one after each part. The question is algorithmic: at every attempt, students find the same figure but with different numerical values for the sides, so that they have to repeat the reasonings.

We analyzed the attempts made by all the students to the questions. We noticed that some students made more than one attempt at this assignment; we analyzed Erica's answers, who made 3 attempts, obtaining 64%, 83%, and 100%, respectively. In her first attempt, which lasted 19 minutes, she answered correctly to the formula's request, although she wrote the formula in a different form, without the simplifications; she obtained the simplified form as feedback. The attempt is shown in Figure 5.4. Then, she correctly used the formula to fill the table with the values of the area for the given values of the variable, but she failed the choice of the graph, choosing the graph of the function $f(a) = 5a$ instead of that of $f(a) = 5a - 4$. Then she moved on to the second question, related to the perimeter. The figure was similar to the previous one, with just a numerical value changing. At this point, she gave an incorrect answer to the first item, asking the formula for the perimeter, as shown in Figure 5.5. Erica received the correct formula as feedback and correctly filled the table with the perimeter's values for the given values of a, then she correctly sketched the graph of the function. The system allows one to sketch a line clicking on two passing points on a cartesian plane: she chose as points the first two points of the table. After that, she submitted the assignment. She obtained the percentage of correct answers (64%) as final feedback, together with all her answers paired with the correct ones, which she had already seen after each step during the test.

After 5 minutes, Erica started a new attempt, which lasted 11 minutes: the previous reasonings helped her accelerate the procedure. In the second attempt, she found a new figure, similar to the previous ones but having different numbers. She correctly answered the first two items; then, she failed the choice of the graph. In the second question, related to the perimeter, she correctly inserted the formula. We can notice that she did not write $4a$ as the answer, still not noticing the invariance for all the figures of the same kind. Instead, she added all the measures of the sizes expressed through the parameter a. Then she correctly answered the following parts, except for a number in the table, which probably was a distraction mistake.

She run the third attempt just 2 minutes after finishing the second one, and it lasted 7 minutes, further reducing the duration. She answered correctly to all the items; in particular, she inserted $4a$ as perimeter, meaning that she understood the invariance of the perimeter for the class of figures.

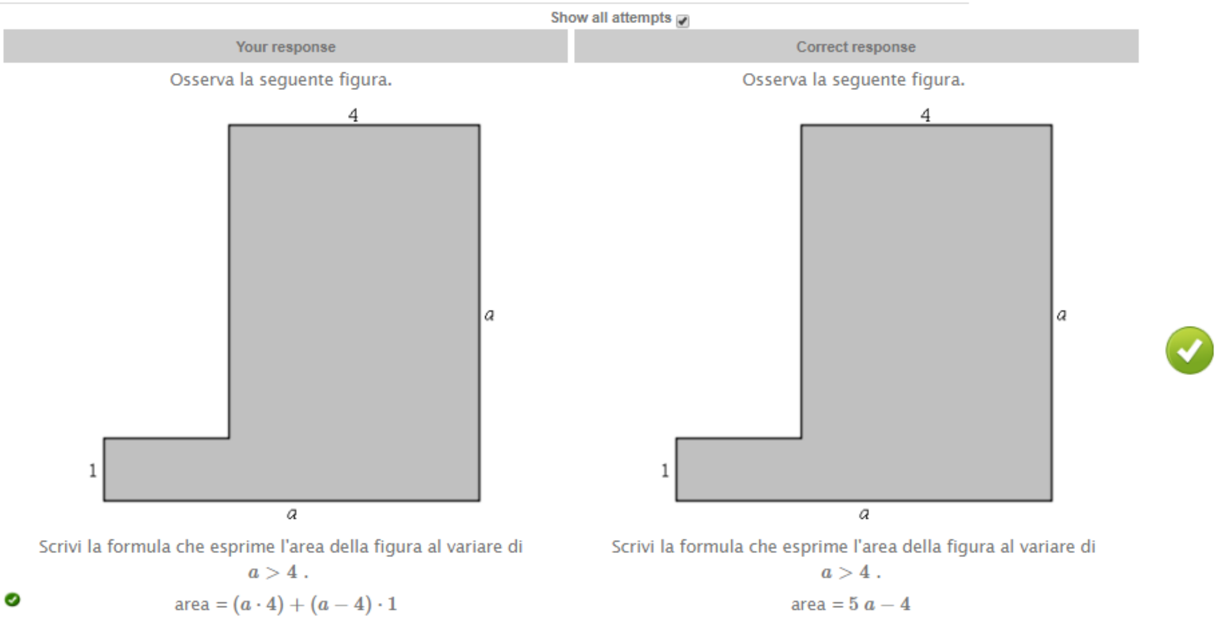


Figure 5.4 Erica's answers to the first part of the first question in her first attempt. The question asks to write a formula for the area of the given figure. Erica's answer on the left matches the correct one, on the right.

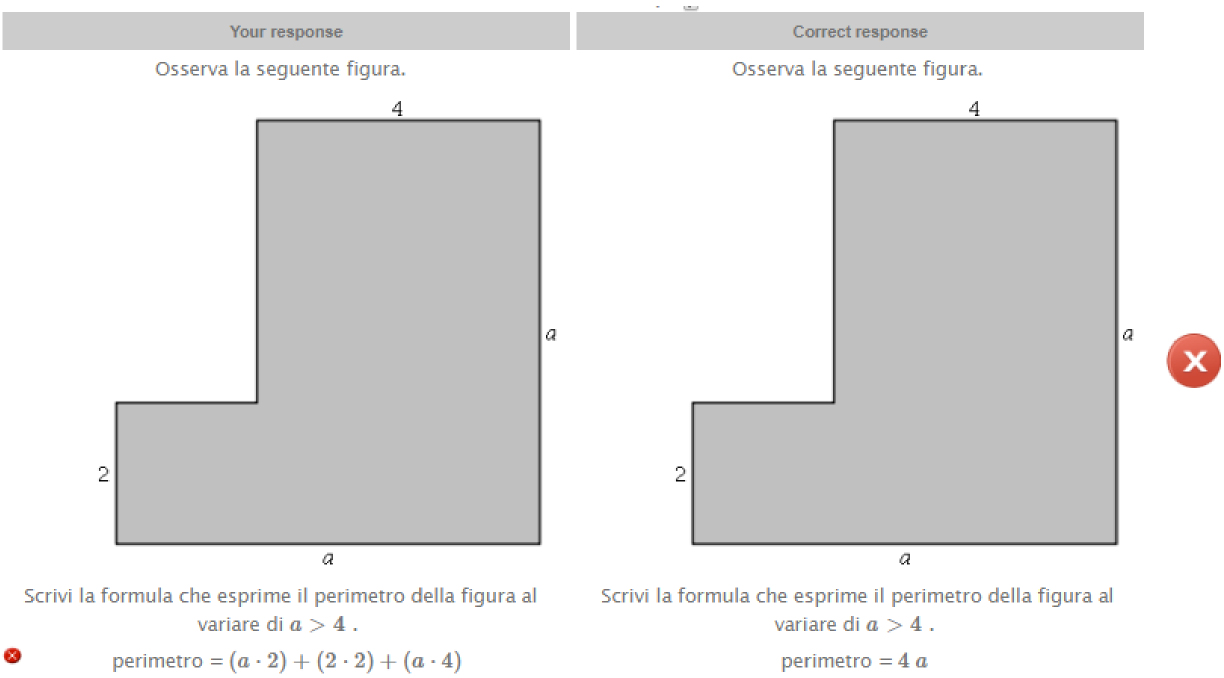


Figure 5.5 Erica's answers to the first part of the second question at her first attempt. The question asks to write a formula for the perimeter of the given figure. Erica's answer on the left is compared to the correct one, on the right.

These episodes can be schematized with repeated cycles of AFA, as shown in yellow in Figure 5.6. Erica is the only member of the community involved. The assignment she opened was displayed through the "delivering and displaying" function of the technology. The AAS accepted the answers she inserted through the "collecting" function. The mathematical engine of the AAS processed the answers through its "analyzing and elaborating" function. Feedback was provided ("providing feedback" function) to the student, who could enter a new cycle, moving on to the following item, or running a new attempt. Erica was activated as the owner of her learning (KS5) every time she ran a new attempt through the "delivering and displaying" function and inserted her answers through the "collecting" function. The KS3 ("providing feedback which moves the learner forward") was activated every time she received feedback.

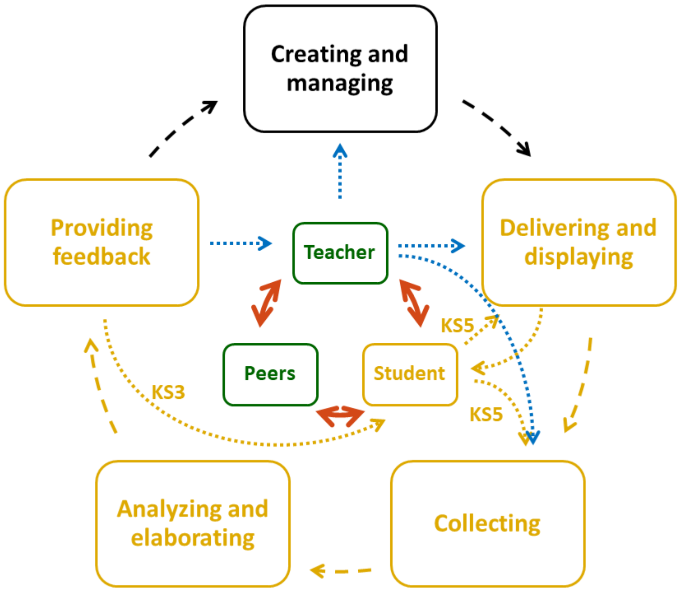


Figure 5.6 Diagram of the strategies of formative assessment enacted during online activities.

To confirm our analysis and complete it with the students' point of view, we considered the answers to the final questionnaire's open questions. We searched for evidence that the five key strategies were applied. In particular, we considered the open answers to the questions FQ40 "What aspect of the classroom activities did you appreciate most?" and FQ43 "Explain why, in your opinion, the online activities were (or were not) useful and you liked (or did not like) them." They evidenced how the use of automatic assessment in the classroom and online supported the enactment of formative assessment strategies. Some comments, such as the following, show how online assignments could clarify and share criteria for success:

- "In my opinion, online exercises are useful to understand better the process of building a formula."
- "The platform was very useful to understand better both the current topic and the resolution of problems."
- "I think that the online tests were useful because they showed me the many possibilities to solve one problem."

Tasks in the online tests were effective to elicit evidence of students' understanding, as other students report:

- "Online tests were useful because only by doing them, I was able to acknowledge whether I had understood a topic or not."
- "Online tests were useful to identify the points where I should improve."

Other comments prove that automatic and interactive feedback supported learning improvements:

- "I could have immediate access to the result to understand where I made a mistake, and I had the chance to make another attempt with different data to drill."

- “With the book, you can do the assignments, but you cannot acknowledge if your resolution is correct, while the platform always shows you both the correct answers and the solving processes.”
- “If you don’t understand a topic, with guided exercises, you can gradually learn how to solve them autonomously.”
- “In the classroom, it often happens that I think I have understood a topic, but the next day I can’t understand it anymore, while with the platform, I can work whenever I need to.”

Groupwork was very appreciated, and it allowed students to be activated as learning resources one for another, as their comments show:

- “Classroom lessons were useful because, besides solving problems, we had to interact with each other and this allowed us to understand the tasks better.”
- “I appreciated reasoning together on the things that we were not able to do.”
- “I appreciated sharing ideas with classmates and helping each other.”

The assignments were also effective to activate students as owners of their own learning, as they report:

- “Technology encourages youngsters to work hard with homework; therefore, they can better understand Mathematics.”
- “During classwork or homework, I often felt willing and happy to solve the exercises, and they were useful to understand the lesson.”
- “I appreciated the online tests because they made me reasoning and work hard; sometimes, I also had fun when doing Mathematics.”

In conclusion, we can answer to (RQ6) “can the blended use of the automatic assessment support formative assessment’s strategies?”. In fact, through the analyses of the DLE's functions, we could notice on what occasion, and through what components of the DLE, the formative assessment strategies were activated. The students’ reports confirmed that the automatic assessment's blended use made it possible to activate all the formative assessment agents (students, peers, and the teacher) and all the five key strategies identified by Black and Wiliam. In particular, KS1 was developed in classroom activities using the “delivering and displaying” functions of the technologies, when the teacher or the researcher showed the task to the students. Moreover, from the students’ comments, we can affirm that this strategy was also activated in online activities since the interactive feedback showed them a possible solving process for the problems. KS2 was activated in the phase of creating the learning materials to be used in the classroom or online through the “creating and managing” function, and by the teacher during the classroom activities, when displaying the tasks through the “delivering and displaying” function. For KS3, the feedback to the students was provided both in the classroom and online activities through the “providing feedback” function, and in the dialogues between students and teachers. KS4 was enacted in the classroom activities, through the group work and the discussions among peers. Lastly, KS5 was enacted in the classroom with the dialogues between teacher and students, and online when students chose to open a new assignment through the “delivering and displaying” function, and when they had to insert an answer through the “collecting” function. More generally, the students affirmed that both classroom and online activities engaged them, made them work hard, and even appreciate doing Mathematics.

AFA provides enhancements compared to paper-and-pen work and traditional book exercises: students can identify their mistakes and make more attempts to improve their understanding. They can be actively engaged in mathematical work and even have fun.

5.3 MODELLING AND DEVELOPMENT OF MATHEMATICAL COMPETENCE

In this section, we will analyze the results of the main experimentation under the perspective of the development of modelling competence, studying in particular how students worked on the activities of the interactive path on formulas and functions and their effects on learning.

The research questions that will guide our analysis are the following:

> (RQ7) Could an activity of algebraic thinking in a geometric context help students develop multiple approaches to a problem?
>
> (RQ8) In what ways allowing multiple representations can support the understanding of algebraic formulas?
>
> (RQ9) What are the main problems faced by the students in using geometric thinking in geometry?
>
> (RQ10) How is it possible to introduce the difference between linear and non-linear models in grade 8 through interactive activities?
>
> (RQ11) What are the effects of interactive activities with AFA to approach the linear model's concepts, with particular reference to the intercept and the slope?

In order to answer these questions, we used the following sources of data:

- observation and notes, taken during the classroom activities;
- videotapes of the classroom activities;
- students' answers to specific items of the final test.

Using the data collected during the activities, some meaningful episodes were selected, and the related videotapes were analyzed using the theoretical framework previously presented (section Modelling2.6). In order to confirm the qualitative findings, some items of the final test - those mainly related to the algebraic and modelling skills under analysis - were considered. For these questions, the treated group results were compared with those of the control group and with those of the national sample that faced similar questions during the INVALSI tests.

5.3.1 Algebraic computations

We start our analysis discussing the research questions related to algebraic computations, which are the following:

(RQ7) Could an activity of algebraic thinking in a geometry context help students develop multiple approaches to a problem?

(RQ8) In what ways allowing multiple representations can support the understanding of algebraic formulas?

(RQ9) What are the main problems faced by the students in using geometric thinking in geometry?

In particular, we refer to the first part of the interactive path, where algebraic formulas and computations were the main topics. In all the classes, these activities were carried out in the classroom through paper and pen, displaying the tasks through the IWB and using the interactive materials to support the discussion. All the materials and additional online activities with automatic assessment were available on the platform and left for homework. The three activities presented in section 4.3.1 – Part 1: algebraic computations (Formulas and Areas, Formulas and Volumes, Other Strange Figures) were used in the classroom. The following schema was used: at the beginning, the main task's figure was shown through the IWB, and students were asked to find a formula to express its area, volume or perimeter (according to the activity). The groups of students were encouraged to find as many formulas as they could. The teacher and the researcher passed through the groups to monitor their work and answer their doubts, while a university student was videotaping some chosen groups' work. When all the groups had found some formulas, all the formulas were shared through the IWB, asking each group to explain how they found them. The equivalence of the different formulas under a geometrical and algebraic perspective was discussed. After this more “algebraic” part, the activity shifted toward a functional interpretation of the formulas. Firstly, the students were asked to choose a unit, assign progressive values to the variable, and draw one figure for each value, computing the related area. In this way, they could observe the figure growing together with its area. Then the values of the variable and of the corresponding areas were collected in a table; these values were used to build the graph of the function. The groups worked autonomously, each student working on his or her notebook. In the end, the function was checked through the interactive worksheet at the IWB and collectively discussed.

The three activities are examples of mathematical modelling problems; we focus on the first one, “Formulas and Areas” (Figure 4.4). The context is not from the real-world because adding a real contextualization would be a forcing, a further mask which adds an irrelevant difficulty to the problem. However, it allowed multiple interpretations to the students, without suggesting any particular solving strategy. In all the classes, at least 7 or 8 formulas emerged, such as:

- $\frac{(a+a+a)\cdot a}{2}$, seeing the figure as a trapezoid;
- $a^2 + \frac{a\cdot a}{2}$, interpreting it as the composition of a square and a triangle;
- $3 \cdot \frac{a\cdot a}{2}$, splitting the square into two triangles through the diagonal, and interpreting the trapezoid as composed by three identical triangles;
- $2a^2 - \frac{a\cdot a}{2}$, seeing a rectangle with base $2a$, minus the upper right triangle;
- $2a^2 - \frac{1}{4}(2a^2)$, seeing a rectangle to which a quarter of a triangle was removed;
- $\frac{2a\cdot a}{4} \cdot 3$, dividing the rectangle into 4 parts to find a triangle and multiplying it by 3.

These are only some examples, as the students gave rein to their imagination and decomposed the figure in many different ways, each one leading to a new formula. Figure 5.7 shows, as an example, the notebook of a group of students full of different formulas.

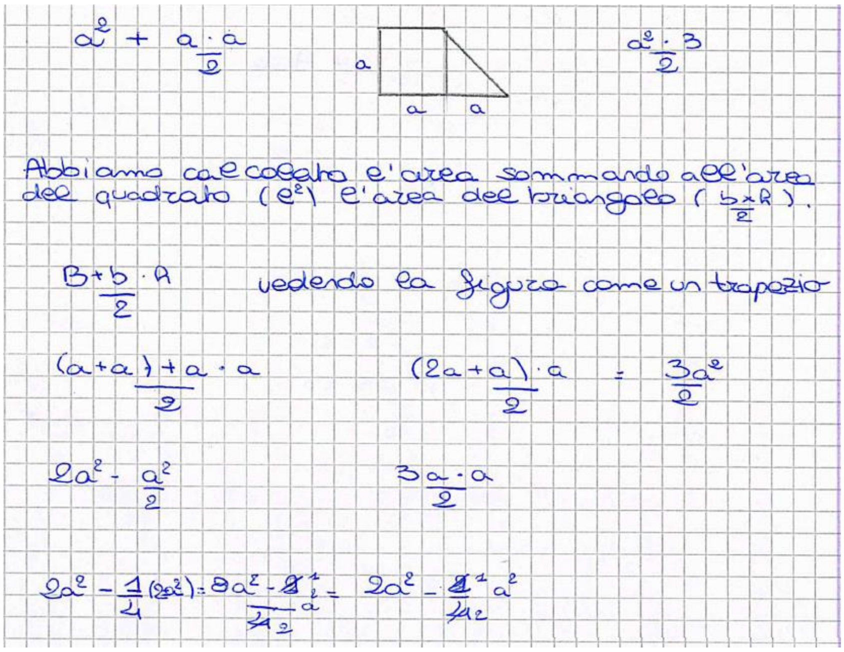


Figure 5.7 Notes of a group of students working on the formulas for computing the trapezoid area. Some sentences next to the formulas describe the reasonings made to write those formulas.

In all the classes, the teacher and the researcher spent some time making students notice that they could solve the problem in many different correct ways and that everyone could choose the strategy they preferred. All the formulas were reconducted to the simplified form $\frac{3}{2}a^2$, not to do exercises of symbolic manipulation, but rather to show that all the formulas were equivalent from an algebraic perspective, not only from a geometric perspective. In this way, algebraic treatments were supported by a geometrical interpretation to confer them a meaning. The algebraic equivalence of the different formulas expressing the same area was not taken for granted by most of the students; they needed to prove it to be sure. Moreover, when they were completing the tables with some values of the variable and the corresponding areas, the teacher and the researcher asked students which of the many found formulas they were using to compute the area. There were many different answers. Some students used a geometric approach and chose the first formula they found, reflecting their preferred geometric reasoning (often expressing the trapezoid area or the composition square plus triangle). In this way, each time, they followed a classic procedure to compute the area. Others preferred an algebraic approach and chose the simplified formula so that computations were easier. Figure 5.8 shows two examples of students' computation of the trapezoid area for several values of the variable. On the left, the student chose a geometric approach and used the classic formula for the trapezoid area to calculate the area. On the right, the student followed an algebraic reasoning and used the simplified formula. Evidencing the different choices, all of them correct, some more convenient for the computations, was another way to stress that problems can be solved in many ways, some more convenient than others. Everyone is allowed to choose the most preferred strategy.

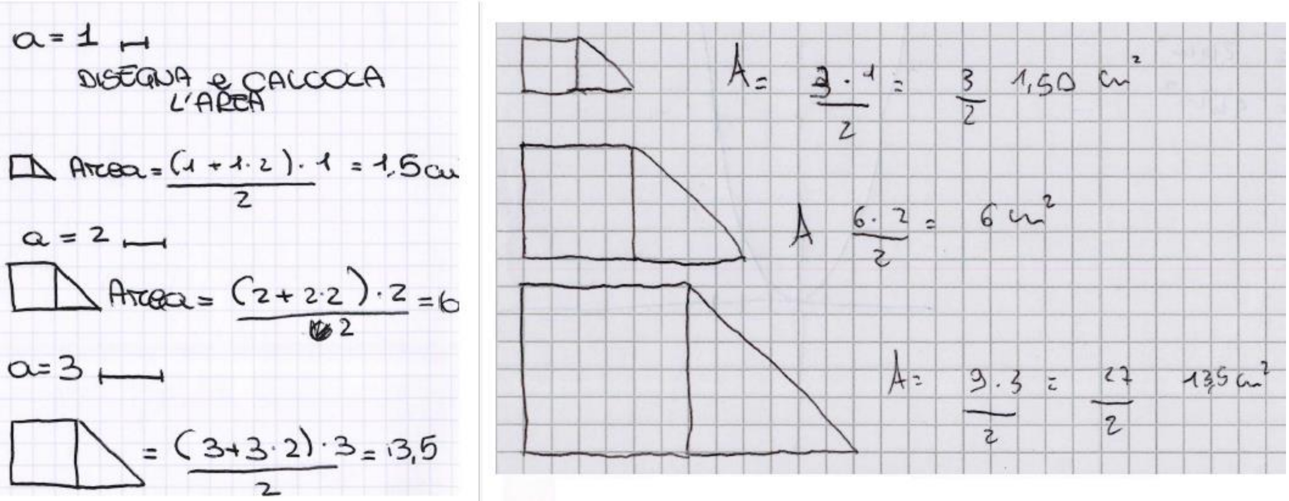


Figure 5.8 Two students' computations of the area of the trapezoid. On the left, the student used a geometric approach and computed the area using the classic formula for the trapezoid area. On the right, the student used an algebraic approach and used the simplified formula to compute the area.

In a class of Via Revel school, attended mainly by high-achiever students, while all the groups were finding formulas for the area of the trapezoid and I was going around the desk to observe their work, one of the students called the researcher. The following conversation took place.

Excerpt 1

JUAN	I am solving the task differently. I am trying to solve it using functions.
RESEARCHER	Good! How are you doing?
JUAN	I tried to compute the area for some cases, and I'm plotting them in a graph.
RESEARCHER	Very good! Go on.

After some minutes, the researcher came back to Juan and asked if he managed to find the function.

RESEARCHER	How are you doing with your function?
JUAN	I am doing it, but I do not think it is correct, because the points are not aligned. I think it's wrong, isn't it?

In this excerpt, Juan understood the role of *a* as a variable, and he was trying to represent the area graphically without passing through the symbolic representation. The class had already studied the functions and their representation in the Cartesian plane. They were used to deal with linear functions, so the fact that the points were not aligned made Juan think that his function was incorrect.

Students also found the same formulas reasoning in different ways. The geometry context helped them understand that they were all equivalent. To this extent, we report an excerpt that happened at Manzoni school during the discussion on the students' formulas.

Excerpt 2

SARA	We found: $a^2 + \frac{a \cdot a}{2}$
RESEARCHER	Good! How did you see the figure to write this formula?
SARA	We divided the figure into a square, whose area is a^2, and a triangle, and we computed base times height divided by two, that is $\frac{a \cdot a}{2}$
RESEARCHER	Very good! Did anyone find another formula?
MARTA	Yes, we did! Ours is similar, but we thought it differently.
RESEARCHER	What did you think?
MARTA	We thought that the figure is a square plus half the square, so we did $a^2 + \frac{a^2}{2}$.
RESEARCHER	Is it the same formula?
MARTA	Yes, because $a \cdot a$ is equal to a^2.
TEACHER	Correct! [pointing at the figure at the IWB] You see, guys, computing the area of the triangle, that is a times a divided by two, is the same that dividing by two the area of the square, that is squared a divided by two.

In the previous excerpt, we could notice how the students interpreted the problem in different ways, though finding the same results. Moreover, the correspondence with geometry helped the students understanding mathematical equivalences and treatments, even those that often we take for granted.

In a class of B. Chiara school, a school mainly attended by students from lower social classes, the following discussion was registered during the activity about “Formulas and Areas”.

Excerpt 3

SANDRO	We found: $2a^2 - \frac{1}{4}$
RESEARCHER	Ok. How did you reason?
SANDRO	We considered the rectangle that has as a base the longer base of the trapezoid and as a height its height, and we removed a quarter, which is the upper right triangle.
RESEARCHER	Ok. So you considered the whole rectangle and said that the trapezoid is three-quarters of the rectangle. Is it correct?
SANDRO	Yes, that’s right.
RESEARCHER	Are you sure that your formula is correct, written like that?
SANDRO	Yes… mmmm… I’m not so sure.
RESEARCHER	And you guys? Who thinks that the formula is correct?
CLASS	[Silence, and whispers of students discussing in groups]
PAOLO	No, in my opinion, the formula cannot be correct because it cannot be $\frac{3}{2}a^2$, which is the correct formula.
TEACHER	Why not?

PAOLO	Because you cannot do $2a^2 - \frac{1}{4}$, you cannot subtract it, because you can only subtract similar monomials.
TEACHER	Ok, Paolo made a good point. Do you agree with him?
CLASS	[Whispers of students who do not seem much convinced with Paolo's idea]
CHIARA	We could do some tries and try with a number.
TEACHER	Yes, Chiara's idea is good. We can try with $a = 2$ and see if the result is the same. If $a = 2$, what is $\frac{3}{2}a^2$?
CLASS	[After some calculations with paper and pen] It's 6.
TEACHER	Ok. What about $2a^2 - \frac{1}{4}$?
CLASS	[After some calculations with paper, pen, and calculator] It's 7.75.
TEACHER	Ok, so it is not the same. Let's try with 2 and 3.
CLASS	[After working for some minutes] No, they are not the same!
MATTIA	You do not subtract enough things each time!
TEACHER	Why not? What are you subtracting?
MATTIA	A quarter.
TEACHER	And what should you subtract?
MATTIA	One quarter… of the rectangle!
TEACHER	Yes. And how do you write it? A quarter OF the rectangle? How do you express the reference to the rectangle?
ANNALISA	Ah, we have to write $2a^2 - \frac{1}{4}(2a^2)$!

At that point, they were quite convinced of why the functions were not the same. To reinforce the concept, when they drew the function of the area, the teacher asked them to add also the function $2a^2 - \frac{1}{4}$ on the graph. Observing the different trends of the two functions, the students understood the concept.

In the last episode, we can see how the discussion involved several registers of representation and conversions. The original idea for the function comes from geometrical reasoning. Paolo's doubts were of algebraic nature, as he considered that the formula does not match the "correct" one, which had previously been discussed. Reasoning using a numerical approach helped students confer meaning to the formulas, thus understanding what the problem was with the incorrect one. The association with a graphic representation helped them clarify the difference between the correct and incorrect functions, have an organic picture of the concept, and understand it. Towards the end of the discussion, the translation from natural language and algebraic language was used to focus on a missing part of the sentence, which is the reference to the rectangle. In this episode, the students were engaged in multiple conversions of the same concept expressed in different registers: natural language, geometric, algebraic, numeric, and graphic. The orchestration of the different representations was successful in improving the understanding of a non-trivial concept.

Through the observations during classroom activities and the videotapes analysis, we collected the most frequent mistakes made by the students when solving the tasks of algebraic modelling in geometry context. Here we present some examples showing the most typical mistakes.

In the following excerpt, occurred during group work, Mariana and Federico discuss about the area of the trapezoid.

Excerpt 4

MARIANA	I have an idea! We can consider this rectangle, you see, and choose only one quarter, then multiply it by 3.

FEDERICO	I can't follow you. Write here.
MARIANA	Well, we can take this rectangle, its area is $2a \cdot a$. Then we divide it by one quarter, so we have only one triangle.
FEDERICO	[Writing] so, $\frac{2a \cdot a}{\frac{1}{4}}$.
MARIANA	Yes, now we can simplify it, and we have $\frac{a \cdot a}{2}$.
FEDERICO	[Writing the simplifications] Yes.
MARIANA	And now we can multiply it by 3, so we have three triangles that are our figure!
FEDERICO	Well done, you are a genius!

In this excerpt, Mariana had a brilliant idea, but she committed an error in formalizing it: choosing one quarter became "dividing by one quarter." Federico did not notice the mistake but continued to follow her reasoning. A second mistake in the simplification restored the correct result. This kind of error derives from misconceptions in working with fractions, which are very common but relate to the basic arithmetic area.

The following excerpt refers to an episode that occurred during the collective discussion of the formulas for the trapezoid area.

Excerpt 5

ALEX	We wrote $a + a$, that is a^2, then…
TEACHER	Are you sure?
ALEX	Yes, $a + a$… oh no, that's $2a$!
TEACHER	Look at the figure. What is $a + a$?
ALEX	It's the long base of the trapezoid
TEACHER	And what is a^2?
ALEX	It's the area of the square
TEACHER	Could they be the same thing?

Here Alex made a very typical mistake of algebraic nature, confusing the position of power and coefficient, constant multiplications with powers. The geometric context helped him understand the mistake: the addition is associated with a linear measure, while the square is associated with an area, they have two different natures and can't be equivalent.

Excerpt 6

The class was discussing the formulas resulted from the second part of the activity "Formulas and Functions," where, in the trapezoid, the horizontal base of the triangle is fixed $b = 1$.

MARCO	We wrote: squared a plus square root of squared a plus squared b, which is c.
TEACHER	What do you mean?
MARCO	We computed the area of the square, that is a^2, then the area of the triangle, using the Pythagorean Theorem, so $\sqrt{a^2 + b^2} = c$.
TEACHER	And what do you find doing $\sqrt{a^2 + b^2}$
MARCO	The hypothenuse.
TEACHER	And how do you compute the area of a triangle?
MARCO	It's base times height divided by two.
TEACHER	So, where is the hypothenuse?

In this episode, Marco and his group tried to compute the area of a triangle using the Pythagorean Theorem. This mistake, related to the problem's geometric interpretation, probably derives from the habit of using the

Pythagorean Theorem when dealing with triangles. It causes an immediate reaction in the student's mind: every time there is a triangle in a problem, they try to use the Pythagorean Theorem, without thinking about whether it makes sense. Reasoning on the formulas' semantic aspect and on the computations' meaning, students can identify the mistake and correct it.

Excerpt 7

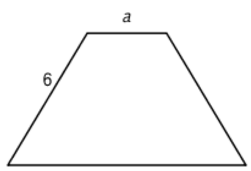
The students of a class in Fermi school were working in groups on the task "Other Strange Figures". After some difficulties, the class managed to write the formula $3a + 2(a - 3)$, and they were collectively discussing it with the teacher.

TEACHER	Ok, we found the formula $3a + 2(a - 3)$. It is correct. Can we carry out the multiplication?
CLASS	[Silence, thinking about the teacher's request]
ANDREI	No, we cannot do the multiplication.
TEACHER	Why not?
ANDREI	Because we cannot say how much it is.

The students were destabilized because the result was not a number, as they were used in geometry tasks. After a lengthy discussion, the teacher managed to convince the class that, even if they were solving a geometry task, they could do algebraic simplifications of the formulas. The habit of studying separately the various Mathematics branches blocked the students who were in difficulty finding and manipulating a formula when dealing with measures and areas.

The final test included an item of algebraic manipulation in a geometrical context, where students were asked to compute the perimeter of a trapezoid whose sizes were given through variables and constants. The text of the item is shown in Figure 5.9. Among students of the treated group, the item registered a high percentage of correct answer (67%) clearly higher than the national sample of grade 8 during the 2017 INVALSI tests, which only achieved the 27% of correct answer when they faced a similar question. The control group's score was a bit lower, even if not significantly (63%); we need to remember that the control group achieved higher results in the initial tests, showing an initially higher mathematical competence.

In the isosceles trapezoid in the following figure the longer base is three times the shorter base.



Write a formula to express the perimeter *p* of the trapezoid in function of *a*.

Answer: $p =$ ______

Figure 5.9 Item of the final test dealing with algebraic modelling in geometry.

Synthetizing the results presented in this section, we can answer the research questions initially proposed. We can positively answer to (RQ7) "could an activity of algebraic thinking in a geometry context help students develop multiple approaches to a problem?". In fact, we saw that the students interpreted the figure of "Formulas and Areas" in several different ways: even when they proposed the same formula, often there were different reasonings in background. Also, the different choices of the formula used to substitute the values and compute the figure's areas show the heterogeneity of the approaches used to tackle the problem. The modelling process itself was undertaken in different ways: the case of Juan, who successfully tried to solve the problem using a graph rather than a formula, shows that the problem was not limited to algebraic solutions. Thence we can affirm that the tasks of the first part of the interactive path concerning algebraic computations were true problems of algebraic modelling, since they were open to multiple interpretations and approaches.

Regarding the (RQ8) "in what ways allowing multiple representations can support the understanding of algebraic formulas?", we saw that the association of algebraic formulas with their correspondent geometric measure supported the understanding of concept and misconception. As an example, we can cite excerpt 2, where students could understand why $\frac{a \cdot a}{2}$ is equal to $\frac{a^2}{2}$, and excerpt 5, where the teacher used geometry to explain why $a + a$ is not a^2. Moreover, the exploration of the formula in four different representations (geometric, algebraic, numeric and graphic) was very useful to help clarify a difficult concept, as why $2a^2 - \frac{1}{4}$ is not the three-quarters of the area of a rectangle (excerpt 3). The translation in natural language added a fifth layer through which to interpret the formula. It was not one particular reasoning that made the students understand the concept, but the orchestration of different representations was essential for understanding.

Concerning (RQ9) "what are the main problems faced by the students in using geometric thinking in geometry?", we found:

- fundamental misconceptions from arithmetic concepts, as Mariana and Federico's problems with fractions; algebraic errors, such as $a + a = a^2$;
- errors rooted in the comprehension of geometry, such as Marco's recalling of the Pythagorean Theorem to compute the triangle area;
- problems coming from the well-established practice of studying the various Mathematics branches separately, which lead students to stop reasoning algebraically when dealing with geometry.

At the beginning of the interactive learning path on formulas and functions, the effects of a mnemonic method of teaching and learning Mathematics, based on routine exercises rather than on the semantic aspects, are still visible.

5.3.2 Linear and non-linear models

The activities about algebraic computations were conceived to act as a bridge between algebraic manipulation and modelling. In this section, we will continue the analysis of this part of the interactive path under the modelling perspective, to answer the following research question:

(RQ10) how is it possible to introduce the difference between linear and non-linear models in grade 8 through interactive activities?

For this purpose, we start considering some episodes that occurred during the classroom activities. Through the dialogue with Juan presented in the excerpt 1 in the previous section, we can realize that grade 8 students, when dealing with graphs, mainly know linear functions and linear models; they have little confidence with other non-linear models. Like Juan, working on the graph of the function representing the trapezoid area, many other students were perplexed when they noticed that the points they drew were not aligned. Like Juan, some of them thought they had made a mistake somewhere. Many tried to force the points into a line, tracing a sort of regression line, or connecting only two of the points. Most of them traced a piecewise function connecting the points two-by-two. Someone even changed the unit in the y-axis for each point so that the distance between two subsequent points was the same, and they resulted aligned. Only in the classes where the parabola had been studied during the previous year, the idea of a non-linear curve was natural. We report, as an example, a discussion occurred in a class of Via Revel school. The teacher had interrupted the group work because all the students were confused because the curve was not straight. She used the interactive worksheet displayed through the IWB to make students reason about it.

Excerpt 8

TEACHER	So, the graph is correct. Why is it not a straight line?
CLASS	[Silence, and whispers of students discussing in groups]
TEACHER	What happens when the value of a increases?
CLASS	The area increases.
TEACHER	Ok. But how does it increase?
MAURO	It is directly proportional to a.
TEACHER	Does everyone agree with Mauro?
CLASS	Yes!
TEACHER	Are you sure? What are the characteristics of two directly proportional quantities?
ANDREA	They increase by the same quantities.
TEACHER	That's right. Does the area increase by the same quantity for each value of a?
CLASS	No.
TEACHER	Why?
CLASS	[Whispers of students thinking to various possibilities]
TEACHER	What is the expression of the direct proportionality? Give me some examples.
FRANCESCO	It does not have a constant.
TEACHER	Ok, but does our expression have a constant?
FRANCESCO	Mmm no, it doesn't have it either.
TEACHER	What is the role of the constant?
CLAUDIA	If there is not a constant, the graph passes through the origin.
TEACHER	So, our curve does not have a constant, so it passes through the origin, is it correct? We can try to add a constant [adding and changing some constant values to the function's expression through the interactive worksheet and observing how the graph changes]. You can see how the graph varies if we add a constant?
CLASS	Yes.
TEACHER	But the curve is still not a straight line. Do you remember other properties of the direct proportionality?
ELENA	It should be $y = \frac{3}{2}x$.
TEACHER	[displaying the graph of $y = \frac{3}{2}x$ at the IWB] Yes, in $y = \frac{3}{2}x$, the two variables are directly proportional. What is the difference between this and our expression?
ELENA	The power! Yes, the power makes the area increase so much!

The interactive worksheet displayed at the IWB was used to change the formula and see how the graph varies, adding different constants and changing the power. The immediate and interactive visualization supported the discussion and understanding.

Similar discussions occurred in many classes: through this activity and the following ones, the students started to distinguish the constant rate of change, typical of a linear function, and a non-constant rate of change typical of quadratic and cubic functions.

In the activity "Formulas and Volumes" (Figure 4.6), the students encountered a cubic graph. To study the complete graph of a cubic and of a parabola, the teacher asked to draw the negative part of the two curves, pretending that a could assume negative values. At the Bernardo Chiara school, during the group work, after completing the table with the values of a^3 and drawing the positive part of the cubic, one student called the researcher.

Excerpt 9

FILIPPO	I have understood how the negative part is!
RESEARCHER	Good! How is it?
FILIPPO	It is here [pointing at the third quadrant of the cartesian plane], but it goes rapidly down in a dive! [miming the trend of the curve with his hand].
	It's like this part [pointing at the positive part] but upside down, like that [rotating the sheet of 180°, so that the positive branch became the negative one]
RESEARCHER	That's right. How did you understand it?
FILIPPO	Because the values are like these ones [pointing at the positive part], but they are negative. They are rapidly decreasing: here there is -8, but when a is -10, the volume is -1000!

The activity "Formulas and Volumes" offered the students the possibility to analyze different non-linear functions and compare their rate changes. Working with symbolic, numeric, and graphic registers helped Filippo understand, even before drawing the points on the plane, how the cubic trend was. He realized that the values are negative and that they change at a very high rate, compared with the line and the parabola's known trends, the latter seen during the previous activity on formulas and areas.

The following activity, "Other Strange Figures" (as that in Figure 4.7, but with other algorithmic values, which changed at every attempt), offered the chance to analyze a linear function deeply and distinguish it from the quadratic and cubic models previously explored. In a class of B. Chiara school, after finding the correct formula for the area of the figure, which was $Area = 5a + 8$, the students were filling in the interactive table displayed at the IWB and discussing it altogether.

Excerpt 10

GABRIELE	I have noticed something! The area always increases by 5!
TEACHER	Good! Why?
MARGHERITA	Because it is 5a.
TEACHER	And if it was 3a?
YASSIN	It would increase by 3.
TEACHER	[pointing at the table] If we draw all these points on a cartesian plane, what would we find?
GABRIELE	A straight line.

The teacher proceeded to the next part of the automatically assessed activity and drew the line's graph by clicking on two of its points. The students could observe the points of the table aligned on a line and make it clear the trend of the function, which, at every step, increases by 5. The teacher then moved the line showing what happens when changing the parameters to link the formula and the line's rate of change.

The final test results shed light on the competence acquired by the students of the treated group through these activities compared to the control group. Specifically, we can notice significant differences between the two groups in the answers to the single questions. Concerning the distinction between linear and non-linear models, we report an item of the final test that asks to identify the graph of an exponential model verbally described in a real context between four proposed graphs. The text of the question is shown in Figure 5.10. It is an item of the INVALSI tests of grade 10 administered in 2013. It had 30.7% of correct answers, so it is rather difficult. In the control group, the item had a slightly higher percentage of correct answer (33.3%). In comparison, in the treated group, it registered 52.4% of correct answer, which is an outstanding result, noticeably higher than the national sample of grade 10 and the control group. Data are reported in Table 5.10. The Chi-squared test, run on the treated and control group, is 26.609 (p-value: <0.001), and the Cramer's V test, run on the treated and control group as well, is 0.237 (p-value: <0.001). It means that belonging to the treated or control group explains 24% of the variance. Since the control group did not report lower results than the treated group in the initial test, the difference can be attributed to the didactic approach used in the experimented activities.

Table 5.10 Results of an item of the final test related to the difference between linear and non-linear models

Groups	**Correct answers (percentage)**	**Incorrect answers (percentage)**
National sample (grade 10)	30.7%	62.9%
Control group (grade 8)	33.3%	66.7%
Treated group (grade 8)	52.4%	47.6%

The following table reports the number N of million of bacteria of the population with the passing of days:

number of passed days	0	1	2	3	4	5	...
number N of bacteria (in millions)	1000	1100	1210	1331	...	...	...

a. Which of the following graphs can represent the trend of the number N of bacteria when the time *t* varies, in at least 20 days?

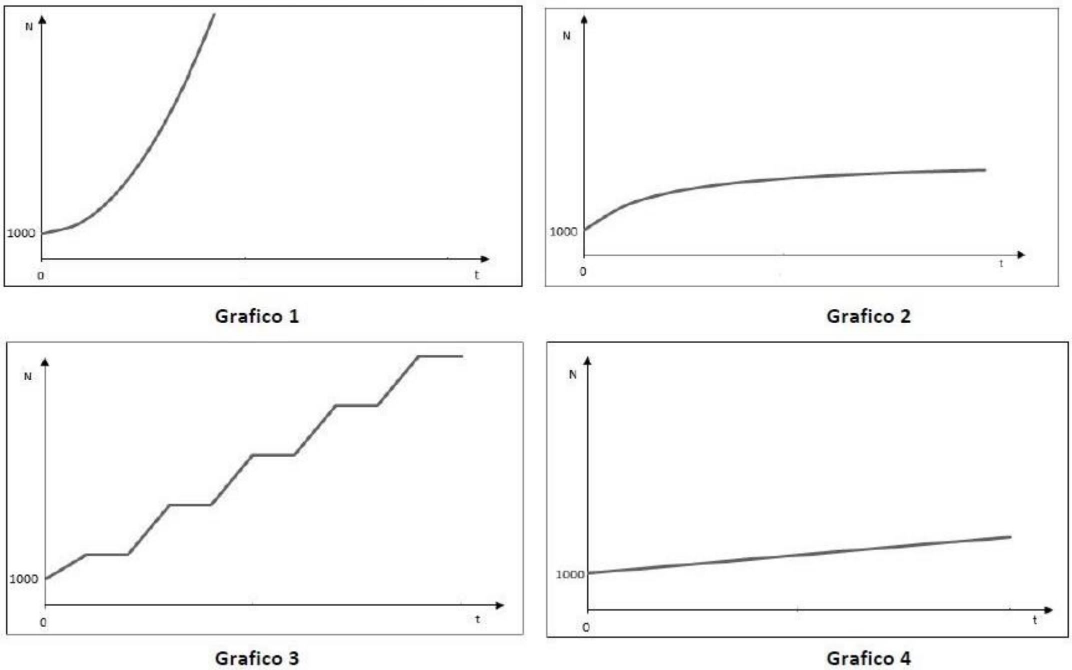


Figure 5.10 Item in the final test about the difference between linear and non-linear models.

In conclusion, we can answer to the (RQ10) "how is it possible to introduce the difference between linear and non-linear models at grade 8 through interactive activities?": we have shown that interactive activities of geometry modelling in a DLE successfully helped students recognize linear and non-linear models. The linear, quadratic, and cubic functions written to compute geometric figures' areas and volumes made students notice the difference between a linear and a non-linear trend. The possibility of modifying the function and immediately visualizing the graph allowed them to identify the role of powers and coefficients in the functions' graphs. The parallel visualization of the figure growing in size and the point moving along the curve (such as in the activity shown in Figure 4.5) allowed them to link the function's trend to a concrete quantity. The interplay among different kinds of representations contributed to the understanding of this concept. The results of the final test, particularly those of the item related to the recognition of a non-linear trend, showed that the concept was mastered much more by the students after solving these activities than by those who studied the trend of the functions using traditional approaches.

5.3.3 From linear models to the slope

Through the activities proposed in the second part of the interactive path, the students could explore several examples of linear models in real-world problematic contexts. In this section we will try to answer the last research question of this thesis:

(RQ11) What are the effects of interactive activities with AFA to approach the linear model's concepts, with particular reference to the intercept and the slope?

We will mainly refer to the activity "The Holiday Book" (section 4.3.2; activity shown in Figure 4.8). The students could examine the linear model identified by the reading of a book at a constant speed. Through the interactive worksheets, the students could compare and contrast different graphs corresponding to the reading with different speeds and how the graphs change when the book is started before or after the beginning of the holidays, writing the corresponding formulas. This activity was carried out in the classroom similarly to what happened for the previous ones: the students worked on one task at the time in small groups, with paper and pen. Each step was discussed with the teacher and the researcher through the IWB, using the interactive worksheets and the automatically assessed questions to support the discussion. The activity started with a verbal description of the real context. Students had to translate it in a numeric register filling in some tables. The students moved from the tables to the graphic register, and drew the points and the lines on a cartesian plane. As the last step, they had to deduce the algebraic formulas for the models.

In the following excerpt, we present part of the discussion, which followed the graphs' drawing, under the input of imagining how students themselves would read the book and add the trend of their reading to the other graphs. The discussion occurred at Saba school, mainly attended by students from lower social classes.

Excerpt 11

RESEARCHER	Well, how would you read this book?
LUIGI	I would read one page a year.
RESEARCHER	One page a year? Let's say one page a day. How would Antonio's graph be if he reads one page a day?
CAMILLA	Very little inclined.
RESEARCHER	Yes, how many days does he need to read the whole book?
CLASS	180 days.
RESEARCHER	180 days. Look how little its values increase from the horizontal axis. [She displays the line through the interactive worksheet at the IWB]. Ok. Is there someone who reads the book a little faster?
SIMONE	I would read 20 pages a day.
RESEARCHER	So, 20 the first day, 40 the second… [filling in the table at the IWB and displaying the line]. This is the graph. How much time will he take to complete the book?
CLASS	9 days.
RESEARCHER	Ok. Anyone else?
CECILIA	I would rest for two days, then start the book and read 20 pages a day.

RESEARCHER	[Filling in the table] Two days of rest, so we start from 0 and have 0 for the first two days. Then we reach 20 at the end of the third day, 40, 60, … This is the graph [displaying the graph of the function]. Cecilia, are you faster than Simone?
CECILIA	Yes, I am the fastest one.
RESEARCHER	Are you sure? Indeed you are the first one to end the book.
CECILIA	Yes, I meant that I finish the book before everyone else.
RESEARCHER	Yes, you finish the book one day before Valentina, but what about Simone? Who reads faster?
GIANLUCA	They are the same.
RESEARCHER	How do you understand it from the graph?
GIANLUCA	Because they are parallel lines.
RESEARCHER	Exactly. The lines are parallel. Even if the book was very much longer, Cecilia would never reach Simone. They increase by the same number of pages each day, but Cecilia started later. Ok, is there anyone else who reads even faster than Cecilia?
MATTIAS	If I work hard, I think I could read even 35 pages a day.
RESEARCHER	[After filling in the table and displaying the graph]. Ok, you can see that Mattias is faster than the others. How long does he take?
MATTIAS	Less than 5 days.
RESEARCHER	Ok. Would anyone read even faster?
BIAGIO	One time I read a whole book in a day.
RESEARCHER	Ok. Let's say that he rests four days and then reads the whole book in one day. How would his graph be?
ALESSIA	[miming an L] Horizontal until 4, and then vertical.
MATTIAS	Parallel to the y-axis.
BIAGIO	Yes, it's like that [miming a vertical line with his hand]
CAMILLA	No, it's not vertical!
ANDREA	She's right. It cannot be vertical. The fourth and fifth points should be connected.
CAMILLA	Yes, you have to connect the fourth day [pointing at the point (4,0) on the plane] to 180 [pointing at (5,180)]
ANDREA	Yes, on the fifth day, he reads 180 pages.
ALESSIA	But it's more or less vertical.
ANDREA	Almost, but it's not vertical.
RESEARCHER	[displaying the graph at the IWB] It's very, very steep. At the end of the fourth day, he was at 0 pages, but at the end of the fifth day, he was at 180. If we imagine that he reads the same amount of pages each hour, we have a very steep line.
SAMUELE	So, if he takes one second, it would be vertical?
RESEARCHER	How should he read the book to obtain a vertical graph?
LUIGI	He should have already read the book.
RESEARCHER	But if he had already read the book, he would start from 180, not from 0.
GIANLUCA	He should take one second.
SIMONE	Yes, but the graph would be inclined of the space of one second.
RESEARCHER	Exactly, there should be a little time which makes the line to be inclined.
SAMUELE	That's right. If there is a bit of time, there is a bit of inclination.
CECILIA	There should not be any time at all.

CAMILLA Right, the time should be zero.

Similar discussions took place in all the classes when solving this problem. In all the classes, all the students actively participated with interest in the discussion. The problem was comprehensible for everyone. It allowed to present the underlying mathematical concepts, such as the slope and the impossibility to have a vertical line as the graph of a function, in a natural and meaningful way. The discussion about how they would have read the book actively engaged even the less interested students, such as Luigi, who usually disturbed his classmates. Thanks to the well-designed contextualization, even Luigi's provocative answer could become a very interesting prompt for mathematical discussion: lines with a low slope. As a result, Luigi kept concentrated until the end of the discussion, when he proposed a new intuition, this time incorrect. The worksheet's interactivity allowed students to observe the different functions obtained by reading the book at different speeds: graphs could be rapidly created using the numeric values in the tables with the numeric values, supporting the discussion in real-time.

Similar tasks were repeated in the classroom during the following lessons and as online homework, to facilitate students to generalize the acquired knowledge and transfer it to new cases. The interactivity and the automatic assessment helped students explore the other problematic situations and check their understanding step-by-step. At home, the interactive feedback in activities such as "Space Launch" (Fig 4.11) allowed students to explore the situations using different registers (verbal, numeric, symbolic, and graphic), thus gaining a full understanding of linear models.

Analyzing some final test questions, we can investigate if the classroom discussions and the online activities helped students understand linear functions and develop modelling competences. Figure 5.11 shows an item of model identification, where the text of the problem verbally illustrates a situation of linear modelling (a typical situation of fixed and variable costs).

In order to attend a gym, this year Paolo has to pay a fixed cost of 60 euros and 5 euros for each entry. Which of the following graphs describes the cost C (in euros) of the gym in function of the number n of entries?

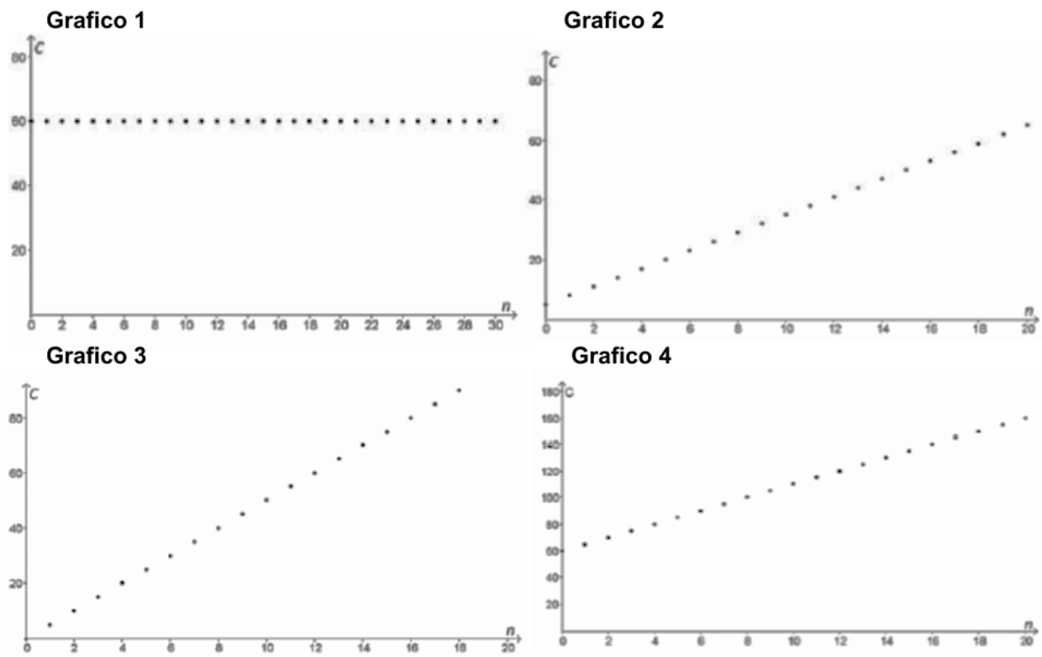


Figure 5.11 Item of the final test on linear models.

The task consists of choosing the graph that corresponds to the verbal description of the function in a real context among 4 options. The incorrect choices are designed so that, to solve the task, students have to consider the intercept and distinguish the model from a constant function. This item was included in the 2014 INVALSI national test for grade 10 and registered 61% of correct answers. In the experimentation, 81% of the treated group answered correctly to this item: it is a very high percentage for grade 8. It means that the interactive activities were effective in learning how to individuate a graph of a linear function. Also a high percentage of the control group answered correctly to this question (79%). This percentage is not statistically different from that achieved by the treated group.

Another final test item aimed to test students' understanding of the slope concept, linking a verbal description of a real-life piecewise linear model to the corresponding graph. Figure 5.12 shows the item. It took inspiration from a similar item published by INALSI as an example for the grade 13 test. It is noticeable that 50.2% of the students of the treated group answered correctly. Compared to the previous item, which mainly dealt with the intercept, this one is more difficult: it confirms results from the literature that affirm that the slope is a more complex object that the intercept (Bardini & Stacey, 2006). Also in this case, the control group performed similarly, with 48.8% of correct answers.

In the last two items considered here, the control group achieved slightly lower results than the treated one. Even if we know that the control group performed better than the treated group in the initial test, we cannot affirm that the treated group students learned how to distinguish a linear model graphically better than the control group. However, we can say that they achieved higher results than the national sample of grade 10 about this competence.

In a city the taxi fare for a journey consists of a fixed cost of 4,00€ and a variable cost which depends on the number of kilometers covered.

Fixed cost	4,00 €
Variable cost for km for the first 10 km	1,50 €
Variable cost for each km after the first 10	0,90 €

In the following figure there are the graphs of 4 taxi fare. Which one corresponds to the taxi fare described above?

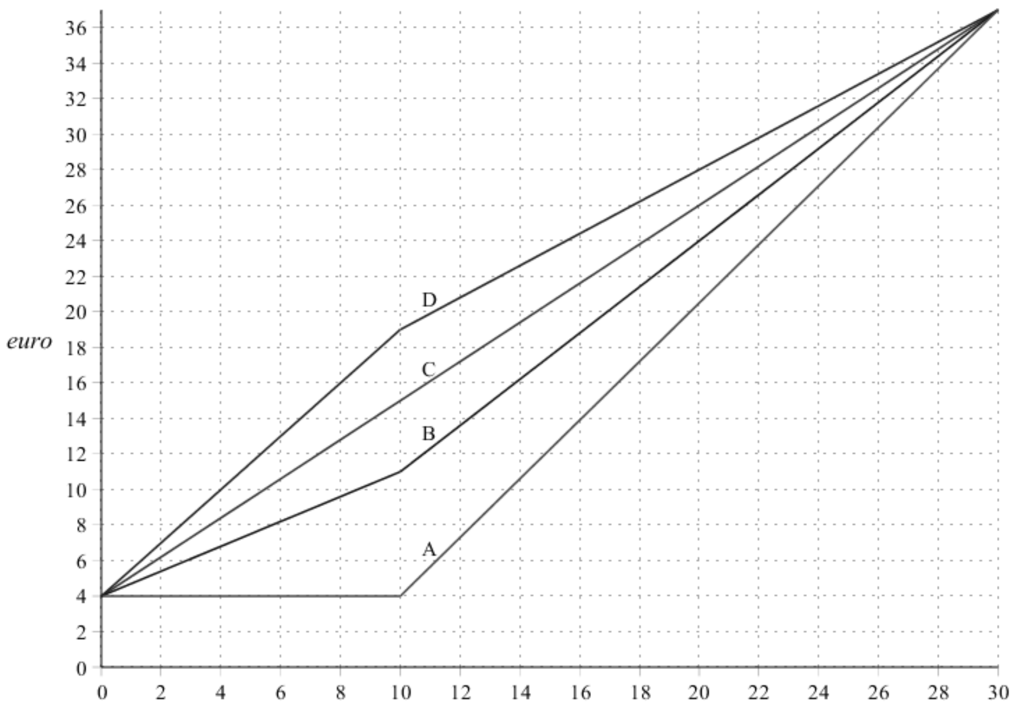


Figure 5.12 Item of the final test on the slope of linear models.

In conclusion, we can answer to the (RQ11) “what are the effects of interactive activities with AFA to approach the linear model's concepts, with particular reference to the intercept and the slope?”. The linear models' activities made students reason on all the linear functions’ characteristics (slope, intercept, intersection between two models). Through a classroom discussion during the activity “The Holiday Book”, we saw that in the classroom students appreciated the interactive part, which asked to imagine how they would read the book and insert the trend of their reading with the other graphs. The students had fun proposing their data to obtain graphs with particular characteristics; the interactivity of the file displayed through the IWB was useful to rapidly explore different speeds, examining the corresponding lines' slope. When increasing the reading speed, the students noticed how the curve identified by the points becomes steeper and steeper. However, it is not possible to obtain a vertical line, which corresponds to null reading time. In all the classes, this activity helped activate all the students, even the less engaged ones, in interesting discussions. The students could repeat the same reasonings and consolidate their comprehension through other problems about linear models in different contexts. The final test results show that these concepts were mainly understood by students, who could integrate different registers of representation to identify linear models.

5.4 Engagement

In this paragraph, we focus on the engagement that the classroom and online activities engender in students of the treated group, trying to answer the following research question:

(RQ12) Could interactive materials with automatic formative assessment and interactive feedback in blended and online situations improve the engagement level of students who, at the beginning of the learning path, show a weak engagement in Mathematics or come from challenging backgrounds?

To investigate this issue, we used data from the initial and final questionnaires, answered by students of the treated group. We did not consider the control group since we focused on the engagement generated during the interactive activities that were the object of this experimentation.

Part of this section and its subsections is taken, expanded and adapted from (Barana, Marchisio, & Rabellino, 2019).

To evaluate the impact of the learning activities on student’s engagement, an initial profile of all the students has been depicted through their answers to the initial questions. We reversed the answers to IQ21, IQ22, IQ28, IQ29, and IQ30 so that higher answers correspond to higher attitudes. Factor analysis was used to confirm the items' classifications into three main sets, corresponding to emotional, behavioral, and cognitive components of initial student engagement. Three continual and categorical variables were defined as indicators of the initial level of engagement in the three components.

Similarly, we split the final questionnaire questions into three subscales related to emotional, cognitive, and behavioral engagement. Three continuous variables were created as indicators of the final level of emotional, cognitive, and behavioral engagement. As additional behavioral indicators, the number of logs to any course activity, the number of submissions of automatically assessed assignments, and the average submission rate per assignment were collected and taken into consideration. The reason for considering such variables is to double-check data from the self-assessment questionnaires, which were filled after the activities, with data automatically collected by the use of technology during the activities, when the engagement process took place. Technical data can reflect the effective participation in online activities and are not affected by personal and subjective evaluations. However, the number of logs and visualizations can vary from student to student according to their needs. Scholars disagree on which is the most appropriate measure to understand how engagement changes (Henrie et al., 2015).

The final level of engagement in each subscale was compared with the initial one; analyses were conducted using SPSS 25. The reliability of all the questionnaires and the subscales were checked through Cronbach Alpha.

The variables used in the following analysis are listed in Table 5.11.

Table 5.11 List of variables used in section 5.4.

Variable	**Type**	**Values**
EE_i	Cardinal (standardized)	Initial EE value (from the initial questionnaire)
CE_i	Cardinal (standardized)	Initial CE value (from the initial questionnaire)
BE_i	Cardinal (standardized)	Initial BE value (from the initial questionnaire)
EE_cat	Categorical	Low/Moderate-high/High level of initial EE
BE_cat	Categorical	Low/Moderate-high/High level of initial BE
CE_cat	Categorical	Low/Moderate-high/High level of initial CE
EE_f	Cardinal (standardized)	Final EE value (from the final questionnaire)
CE_f	Cardinal (standardized)	Final CE value (from the final questionnaire)
BE_f	Cardinal (standardized)	Final BE value (from the final questionnaire)
EE_diff	Cardinal	EE_f – EE_i
CE_diff	Cardinal	CE_f – CE_i
BE_diff	Cardinal	BE_f – BE_i
LOGS	Cardinal	Student's number of logs in the platform
NUMBER_ASSIGNMENT	Cardinal	Total number of attempts to the assignments by one student (counting repeated attempts)
RATE_SUBMISSION	Cardinal	NUMBER_ASSIGNMENT divided by the number of attempted assignment

5.4.1 Analyses and results

The initial questionnaire was answered by 278 out of 299 students (93%). Students who did not answer the questionnaire were excluded from the sample. The reliability of the survey was checked through Cronbach Alpha, which was sufficiently high (0.82). An initial factor analysis led to the elimination of 5 variables: IQ7, IQ19, IQ20, IQ28, and IQ29. As for IQ19 and IQ20 (related to playing chess and programming computers), they are probably not everyday actions for 8^{th}-grade students, so they did not contribute effectively to the detection of Mathematics behavior. IQ28 and IQ29 ("Family demands or other problems prevent me from spending a lot of time for my Mathematics work", and "If lessons were different, I would try harder in Mathematics") probably concerned external factors compromising students' success more than interior control of their actions. Although the teacher and the family environment's effect may be important factors for learning and developing competences, they are usually not so influential as to impede school work and the achievement of success (OECD, 2016b). Regarding completing homework before classes (IQ7), we previously noticed that mere compliance with rules does not necessarily imply engagement. In fact, the homework can be finished just to avoid punishments – in grade 8, many teachers are usually rigorous in demanding that homework is done on time. However, this does not necessarily mean that there is an effort behind the homework: it could also be copied from classmates. Factor analysis also showed a distinction between two groups of emotional subscale items: IQ5 and IQ6 seemed to conceal a different nature compared to the first four items. IQ5 and IQ6 refer more to the extrinsic motivation to study Mathematics than intrinsic involvement with the subject. Therefore, we only considered items from IQ1 to IQ4 in the emotional engagement subscale. The Cronbach Alpha computed on the remaining 30 items increased to 0.85.

Through factor analysis, three standardized variables were created as linear functions of students' answers to the remaining items: emotional engagement (EE_i); behavioral engagement (BE_i), composed by the remaining items on Mathematics work ethic, Mathematics behavior, and perseverance; cognitive engagement (CE_i), to which the remaining items on perceived control of success and openness to problem solving contributed.

Three categorical variables were built on the basis of the sum of students' answers to questions in the subscales. The variables had values 1, 2, or 3, meaning low, moderate-high, and high attitude. In particular, for emotional engagement (EE_cat), the value of the sum of the answers to the four items could range from 4 to 16. A low level was defined for values equal to or below 8; a medium-high level was defined for values ranging from 9 to 12, and high level for values higher than 12. For cognitive engagement (CE_cat) the cut-off values were 21 and 29 in a range from 8 to 37; for behavioral engagement (BE_cat) the cut-off values were 37 and 53 in a range from 16 to 69. Table 5.12 shows students' distribution in the three levels of initial emotional, cognitive, and behavioral attitudes.

Table 5.12 Percentage of students in the three levels of each subscale of initial engagement.

Initial engagement level	EE	CE	BE
Low level	8.4%	20.9%	15.8%
Moderate-high level	24.3%	52.1%	72.6%
High level	68.3%	27.0%	11.6%

The final questionnaire was answered by 85% of students; they had previously completed the initial questionnaire. Cronbach Alpha was very high (0.93), showing that the items were sufficiently reliable.

We wanted to exclude the hypothesis that students who did not answer to the final questionnaire were concentrated in the lowest initial engagement levels. Thus, we run a Chi-squared test to verify the incidence of having or not having answered the final questionnaire on the distributions of the initial levels of emotional, cognitive, and behavioral engagement. None of the tests gave significant results ($p=0.56$ for EE; $p=0.78$ for CE; $p=0.35$ for BE), meaning that answerers and non-answerers were equally distributed in terms of initial engagement.

As a preliminary analysis, ANOVA was conducted over the students' answers on all the single final questionnaire items of the three subscales, considering independent variables the corresponding level of initial engagement, emotional, cognitive, or behavioral. We found that the initial engagement level explains only the items related to emotional and behavioral engagement concerning classroom activities (such as FQ1, FQ2, and FQ28). For all the items related to cognitive engagement and all the items of the other subscales explicitly involving the platform's use, the initial level of engagement does not explain students' answers ($p>0.1$). For some items, the trend was even decreasing; that is, students with low engagement levels showed a higher interest in this kind of activity than their classmates. Table 5.13 reports some examples of students' answers to some items of the cognitive subscales, analyzed for levels of the related engagement.

These results are interesting: students in the highest initial engagement levels tended to give higher - or similar - answers than students from the lowest levels. However, it seems that the use of the interactive materials designed for the project had some impact on the levels of engagement, particularly on students from initial lower levels. These results are worth investigating.

Factor analysis on the final questionnaire items left the original schema unchanged: thus, three standardized variables, expressing the emotional, cognitive, and behavioral engagement, were created as output of the factor analysis (EE_f, CE_f, and BE_f); they embed the students' answers to the items of the final questionnaire. Moreover, ANOVA analysis was conducted over these variables, considering the corresponding initial engagement levels (EE_cat, CE_cat, and BE_cat) as independent variables. Results showed a slight dependence of emotional and behavioral engagement on the relative initial levels; the final cognitive engagement follows the same trend of the initial one, but results are not significant. It seems that the initially most engaged students remain the most engaged at the end of the path, but we do not have information on how engagement changed with the project's activities.

Table 5.13 Examples of students' answers to the final questionnaire for levels of respective engagement.

Initial EE/CE/BE level	**Means of students' answers**				
	FQ11 The online assignments are a valid help for studying	**FQ18** The possibility to see the materials used in the classroom again from home is useful	**FQ20** Using the platform from home helped me identify the topics on which we worked in class	**FQ23** The immediate feedback helped me understand how the task should be solved	**FQ36** I used the platform to study Mathematics
Low level	3.62	3.74	3.59	4.07	3.30
Moderate-high level	3.31	3.70	3.53	3.96	3.15
High level	3.66	3.56	3.66	4.30	3.11
p-value	0.23	0.31	0.75	0.16	0.84

In order to investigate whether any effect occurred on changes in the factors of student engagement, the difference between EE_f and EE_i was computed; it expresses the variation in the levels of EE with online activities during the project. The same was done for CE and BE. We thus defined three new variables: EE_diff, CE_diff, and BE_diff. The statistics of these variables are collected in Table 5.14.

Table 5.14. Means and Standard Deviations of the three variables expressing variation in the engagement levels.

Variable	Mean	Standard Deviation
EE_diff	-0.061	1.186
CE_diff	-0.028	1.345
BE_diff	-0.046	1.045

Through ANOVA, the dependence of EE_diff from the initial categorical level of emotional engagement (EE_cat) was tested. Results are reported in Table 5.15: students with initial low EE levels improved their level by 1.302, which is more than one standard deviation. The difference decreases as the initial engagement level increases. ANOVA test shows significant relations among the variables ($p<0.001$); Eta test shows that this relation is moderate, explaining the 16% of the variance (Squared eta: 0.157, $p<0.001$). We restricted the sample to students with initially low levels of emotional engagement and calculated the effect size of the interactive activities on their EE levels using Hedges' *g*, which is more appropriate for small samples (Pellegrini et al., 2018). The formula is:

$$g = d \cdot \left(1 - \frac{3}{4\cdot(n_1+n_2-2)-1}\right),$$

Where *d* is Cohen's *d*. In this case, we have $g = 1.53$, which is a very high value: it means that the classroom and online activities dramatically increased the emotional engagement level for students who, in the beginning, showed a low level of EE.

A similar analysis was conducted for cognitive and behavioral engagement; Results are reported in Table 5.15. By analyzing, through ANOVA, the dependence of CE_diff on the CE_cat., we can notice that students with initially low levels of CE improved their level by 1.264, which is almost one standard deviation. The difference decreases as the initial engagement level increases, reaching -1.122 for initially highly engaged students. ANOVA test shows significant relations among the variables (p<0.001); the Eta test shows that this relationship is strong, explaining 37% of the variance (Squared eta: 0.368, p<0.001). Considering only students having low initial levels of cognitive engagement, the effect size is *g*=1.26, which is a very high value as well. It shows that students who at the beginning where little cognitively engaged with Mathematics perceived a noticeable increase in their CE level through the project's activities.

As for behavioral engagement, students with initially low levels improved their level by 0.942, which is almost one standard deviation. The difference has a decreasing trend, reaching -0.939 for initially highly engaged students. ANOVA test shows significant relations among the variables (p<0.001); the Eta test shows that this relationship is strong, explaining the 20% of the variance (Squared eta: 0.204, p<0.001). Considering only students having low initial levels of behavioral engagement, the effect size is *g*=1.16, which is again a high value. It means that the experimental activities had a strong impact on the BE's levels of initially little engaged students.

Table 5.15 Differences between initial and final levels of engagement, per level of initial engagement.

Initial level of EE/CE/BE	EE_diff	CE_diff	BE_diff
Low level	1.302	1.264	0.942
Moderate-high level	0.351	0.164	-0.486
High level	-0.317	-1.122	-0.939
p-value	<0.001	<0.001	<0.001

The technologies can contribute to the measurement of engagement providing important data on the students' real use of the platform, which can be integrated into the research (Yang, 2011). We decided to consider as additional behavioral variables the number of logs registered in the platform (LOGS), the number of submitted assignments on the platform (NUMBER_ASSIGNMENT), and the rate of submission per assignment (RATE_SUBMISSION). Data from the informatics systems offer the advantage that they were collected for the whole 100% of students, so there are no missing data. However, in this case they reflect only the individual work in online activities and not the participation in classroom activities. In fact, in the classroom, the students mainly worked on paper, and only the teacher was logged in the platform, displaying the activities through the IWB. Table 5.16 shows the means and standard deviations of the three variables.

Table 5.16 Data from the platform's usage.

Variable	Mean	Standard Deviation
Number of logs	96.51	72.48
Number of assignment submissions	18.65	17.15
Rate of submission per assignment	1.66	0.87

From the literature, we already know that logs are related to behavioral engagement, but they can be influenced by other factors (Henrie et al., 2015). As a matter of fact, in our analysis, 28% of variances of the number of logs and 26% of variances of the number of submissions is explained by the class teacher: probably the way teachers asked students to do the online activities and the way they checked the homework impacted on students' work. These variables turn out to be weakly associated with the initial level of behavioral engagement, as shown in Table 5.17: students with a low level of BE tended to work less on the platform than their classmates. For the number of submissions, the relation is statistically significative; for the number of logs, instead, there is not statistical evidence. Both variables are correlated with the variable that measures the final level of behavioral engagement built using data from questionnaires (R-squared are respectively 0.30 and 0.28, $p<0.001$).

The situation changes when considering the average rate of resubmission per assignment. In our interactive path, assignments have unlimited attempts, numbers and situations change at every attempt, and mistakes are explained through interactive feedback. Thus, when students try questions again, it means that they are eager to solve the problem autonomously, that they understood the solution, and want to challenge themselves once again. Thus, the task managed to engage students. The variable RATE_SUBMISSION seems not to be related to the initial BE level, as shown in Table 5.17 ($p=0.21$). Even students with initially low levels of BE could be engaged in activities with AFA. This variable is not correlated with the final level of students' BE (BE_f) (R-squared: 0.075, $p=0.37$). This means that it measures something different, namely the rate of the engagement generated by the interactive activities and their feedback, while the variable expressing BE is linked more to attention in classwork and mere completion of homework.

Table 5.17 Average data from the platform per level of initial behavioral engagement.

Initial BE level	**LOGS**	**NUMBER_ASSIGNMENT**	**RATE_SUBMISSION**
Low level	78.52	14.52	1.75
Moderate high level	97.33	18.45	1.60
High level	116.06	25.56	1.89
p-value	0.079	0.020	0.210

Lastly, we focused on the students' socio-economic status to verify that online activities experimented were useful also for students with challenging backgrounds. Through ANOVA tests, we noticed that the two groups registered similar values in the difference between final and initial EE and BE ($p=0.99$ and $p=0.57$, respectively). From a cognitive point of view, engagement level grew significantly more in students from a lower social class than in those from a higher one ($p=0.01$); the same trend was registered in the submission rate per assignment ($p=0.027$). The results are displayed in Table 5.18. Since the socio-cultural origin is a strong predictor of scholastic success (OECD, 2013), acting on students' engagement level from disadvantaged backgrounds is extremely important for promoting school success and preventing drop-out rates.

Table 5.18 Impact of engagement on students of different socio-economic status.

Socio-economic status	**EE_diff**	**CE_diff**	**BE_diff**	**RATE_SUBMISSION**
Low	-0.063	0.319	-0.065	1.80
Medium-high	-0.061	-0.245	0.035	1.54
p-value	0.993	0.013	0.573	0.027

5.4.2 Discussion and conclusions

Through the data analysis above, we positively answered to the research question (RQ12) "could interactive materials with AFA and interactive feedback in blended and online situations improve the engagement level of students who, at the beginning of the learning path, show a weak engagement in Mathematics or come from challenging backgrounds?". We compared the initial and final levels of emotional, behavioral, and cognitive engagement in students who participated in a didactic experimentation for Mathematics using interactive materials with AFA. We found that the most significant increase in the engagement level was observed in the students who started the path from the lowest engagement levels. Moreover, belonging to a lower social class did not influence the increase in the engagement levels, except for cognitive engagement, for which it is related to the most significant increases.

The theoretical framework on engagement helped us understand what engagement is, distinguish its components, and find useful indicators to build questionnaires to measure the students' one. Moreover, a clear understanding of the nature of engagement and its components allowed us to build meaningful learning activities that could promote its development. It also provided us with a good frame to analyze the answers and interpret the results, thus understanding which factor is mainly involved and how didactic activities can act.

We observed that cognitive engagement is the subscale where the effect is the most evident. CE is linked to self-regulation and persistence with school work. Cognitively engaged students are less likely to give up their learning and more likely to keep engaged with school (Ng et al., 2018). We found that the students who had the most significant increases in the CE levels are the initially less engaged ones and those coming from the lowest social classes. This result suggests that the methodologies used in this experimentation can be useful to prevent early school leaving and reduce the drop-out rates. A crucial role in this process seems to be played by the AFA, and in particular, by the interactive feedback designed in the online tasks, as the answers to the questionnaire's items show. The interactive feedback offers the possibility to understand mistakes and be guided in a possible solving path, being actively involved. The results related to the rate of submission to the assignments show that the chance to repeat the reasoning with different numbers was indeed taken by the students, especially those with initially low levels of engagement or coming to lower social classes, in order to improve their results: it means that the interactive feedback really engaged them and that they were activated as the owner of their learning. Thus, the digital learning materials acted as cognitive facilitators of engagement. We argue that other features of the tasks, as the real-world settings, could contribute to capturing students' interest and attention. Even the mere use of technology as school activity could facilitate engagement development: it is still an unusual practice in the majority of 8^{th}-grade classes in Italy. Classroom activities were managed through group working: the peer collaboration and the collective discussions are social facilitators of engagement, and they contributed to the creation of an active learning community.

These activities were not occasional practices, but they were regularly repeated over the school year: we can suppose that the effects on students' engagement could become stable and influence students' attitudes and beliefs towards Mathematics. Related research shows that, in Mathematics, engagement is linked to the development of aspirations for challenging Mathematics: similar educational models might even contribute to the students' choice to undertake scientific careers (Ng et al., 2018).

The effect of these activities on students from low social classes is of considerable importance. According to our results, they do not increase the social inequalities; rather, they overturn them, offering disadvantaged students tools to reengage with school. During the classroom activities, we could perceive a higher level of attention in schools located in Turin's disadvantaged areas than in schools located in the city center. Despite the potentially lower availability of digital technologies in their homes, students from low social classes used the online platform to the same extent, or even more, than students coming from wealthier families. It is shown that school disengagement is related to disaffection, disruptive behaviors, bullying, and early school leaving, and these problems are more common in disadvantaged areas (Skinner & Pitzer, 2012). Developing interest for and understanding of Mathematics in poorly educated families may improve young people's capacity to engage in society and in the workplace actively, thus opening the possibilities of social mobility.

We also showed that the teachers had a significant impact on the amount of students' online work, particularly on the number of logs and submissions. The teacher's role is always fundamental in learning processes at the school level, especially when the technologies are involved. How they are confident with innovative methodologies and convinced of their effectiveness influences the students' activities. Teacher training is, therefore, essential for the success of similar projects. The teachers involved in this experimentation soon realized that the adopted methodologies lead to important learning results, so they were motivated to change their teaching practices by adopting these methodologies. Most of them enrolled in a teacher training course the following year, aimed at making them autonomous in the preparation of interactive learning materials with AFA.

6 CONCLUSIONS

6.1 SUMMARY OF THE OBTAINED RESULTS

Summing it up, in this thesis, we have illustrated and discussed a possible theoretical framework for the creation and analysis of formative assessment activities using a DLE, connecting frameworks on Activity Theory, digital and interactive learning environments, and formative assessment. In particular, we defined Digital Learning Environment as a learning ecosystem in which teaching, learning, and the development of competence are fostered, in classroom-based, online, or blended settings; it is composed of a human component, a technological component, and the interrelations between the two. Then we studied the processes of Automatic Formative Assessment in a DLE, outlining the main functions of the technologies which support the processes (creating and managing; delivering and displaying; collecting; analyzing and elaborating; providing feedback) and synthesizing them on a diagram (Figure 2.5). Starting from models from the literature of effective feedback for closing the gap between actual and desired performance and for promoting self-regulation in a context of traditional learning, we proposed a model for the design of AFA activities, mainly based on the following features: availability, algorithm-based questions and answers, open-ended answers, immediate feedback, interactive feedback, and real-world contextualization. In particular, interactive feedback can be useful for ensuring that students use the information to improve their performance, which is a significant problem raised in the literature. When formative assessment is paired with technologies, applying learning analytics techniques can enhance the FA's potential. The examples discussed show how the data coming from using a DLE, and the evaluation data could be used to improve the enactment of formative assessment strategies, strengthen and evaluate their action.

Using these models, we proposed some interactive activities to 179 students of grades 8, 9, and 10 in a pilot experimentation, while other 79 students from the same schools composed the control group. We could identify the best target for a more significant and better-designed experimentation through this pilot experimentation: 8^{th}-grade students, who better reacted to the interactive activities in a DLE. Moreover, we observed that teachers and students appreciated the interactive activities designed and created due to their potential for favoring the learning process, motivation, and self-assessment skills. The methodologies with which the activities were carried out were deemed valid and useful to stimulate cognitive and metacognitive processes.

Based on these results, we designed a more robust experimentation, aimed at studying the effectiveness of interactive activities with AFA with a problem solving approach in a digital learning environment at the school level under several perspectives. The experimentation involved 24 classes of grade 8 from 6 lower secondary schools, for a total of 547 students, 299 composed the treated group, and the other 248 the control group. Many interactive learning materials with AFA were available in a DLE based on a Moodle platform; students and teachers could use them both in the classroom and from their home. In particular, an interactive path on formulas and functions constituted the core of the experimentation. The didactic activities' effectiveness was evaluated through learning tests and questionnaires administered at the beginning and the end of the learning path.

We had a complete overview of the effectiveness of interactive activities with AFA in a digital learning environment at school level through the conducted analyses. In the first place, we could observe a clear improvement of the learning results of the treated group (+7.20/100), while the control group did not increase their results (-0.58/100); the effect size is moderate (0.41). These results considerably improve when considering only students from lower social classes (treated group: +7.07/100; control group: -7.90/100; effect size: 0.77). In both cases, the difference between the improvements of treated and control groups is statistically significant.

Secondly, the results of the above-reported experimentation show that the blended use of the online assessment can effectively activate all the formative assessment agents (students, peers, and the teacher) and all the five key strategies identified by Black and Wiliam (2009). Students received feedback from discussing and sharing ideas with their peers in the classroom, while on the platform, the interactive feedback offered guided support to understand concepts and processes.

Thirdly, we qualitatively analyzed some episodes that occurred during classroom activities and integrated the results with quantitative analyses of the answers to some final learning test questions. The goal was to evidence that the interactive path helped students develop multiple approaches to modelling problems, understand algebraic formulas, and approach the concept of linear functions, in particular in understanding the difference between linear and non-linear models and the concepts of intercept and slope. The use of geometry to introduce mathematical models and functions and the use of conversions among different registers of representation were valuable strategies to accomplish these goals. Specifically, we observed that students from the treated group, after solving the activities of the interactive path, obtained a neatly higher percentage of correct answer compared to the control group to a question of identification of a non-linear model (treated group: 52.4%; control group: 33.3%). All Italian students of grade 10 who faced the same question in their INVASI tests obtained 30.7% of correct answers. Similarly, other questions of the final test dealing with algebraic modelling and linear models evidence similar differences between the treated group and the national sample at the INVALSI tests. The proposal of activities of this kind could have long-term results, as the ability to integrate different kinds of reasoning while dealing with mathematical models and with the concept of the slope; it is crucially important since studies show that the risk of losing this ability is common (Gambini et al., 2020).

Lastly, we showed that the technologically enhanced learning activities experimented in the project successfully contributed to increasing students' engagement level that initially showed little involvement in Mathematics. In particular, we found that the initially most engaged students in terms of emotional, cognitive, and behavioral engagement remained the most engaged until the end of the project. However, the initially least engaged students are those on which the activities had the maximum impact. Cognitive engagement is the subscale where the effect is the most evident: it is a positive result because CE is linked to self-regulation and persistence with school work, and cognitively engaged students are likely to remain engaged in school activities. Therefore, the methodologies adopted in designing interactive activities can help support the reengagement and prevent early school leaving.

6.2 CHALLENGES AND NEW DIRECTIONS

The results of this research open to several challenges. First of all, we can consider that the AFA activities included in the interactive path were mainly developed by the PhD student author of this thesis in collaboration with her colleagues of the Delta Research Group and with other experts (first of all, Rossella Garuti, INVALSI expert). This resulted in top-quality activities, both from a didactic and from a technical point of view; however, they do not always reflect didactic activities elaborated by teachers themselves, whose results could have different impacts on students' learning. The creation of tasks and activities in a DLE to be used with formative purposes requires technical skills and knowledge of the tools, as well as a pedagogical preparation in the strategies and models of formative assessment. Otherwise, there is the risk of merely replicating traditional instruction with digital tools without reaping the benefits gained from a correct, informed, and conscious use of these technologies. This can be tackled through a specific training dedicated to the teachers or the instructors to author the learning activities. The Delta Research Group has designed and experimented a model of teacher training that involves face-to-face and online training sessions through which many secondary school teachers became skilled in the adoption of AFA through a DLE (Barana, Fioravera, & Marchisio, 2017b). The teacher training is flanked sharing the produced materials in a virtual community of practice, where the trainees' contribution and the control of tutors from the University assures that high-quality materials are proposed to students.

Nevertheless, this is only a part of the risk mitigation. As Black and Wiliam (2009) stress, it is not the mere use of proper tasks at the appropriate time that makes assessment formative: data from the assessment need to be used to make decisions in the instruction process. Here the learning analytics techniques can facilitate the visualization and analysis of learning data. However, it is not easy, especially for school teachers, to do the analyses and use the results in time to influence the next steps in instruction. Sometimes, they need the help of researchers to complete the analyses. Moreover, gathering the data and starting the analyse requires time, and the results are not immediately available. Thus, the dynamism of the decision-making process in a classroom can be undermined (de Waal, 2017). To tackle these difficulties, it is possible to act on the automatization of the analyses processes and the improvement of the visualization of the results directly into the DLE; teachers and instructors need to be trained to read these results and use them in their daily practices.

To this end, new prompts to this research could be given by applying Artificial Intelligence algorithms to the automatic assessment, which could help enhance the grading performance of open questions and increase the adaptivity of assignments, thus improving the formative assessment experience to teachers and students.

Another challenge arising from this work is managing to engage all the students in online activities. We noticed a high number of reluctant users of the AFA activities; some of them never logged into the platform. From their answers to the initial questionnaire, we argued that they are the students who often do not do their homework and copy it from their friends. Their teachers also testified that they were not those without an internet connection at home, but those students who never do their homework. If a little motivated student tries to do the online exercises, then the automatic formative assessment will have a chance to show him that it can be useful to improve or that he will get less bored this way than with paper and pen homework. Nevertheless, if students do not even try to open one, it is not the formative assessment that failed, but the instructional system allows leaving one student behind.

The experience of Educating City can be replicated in other contexts to improve the quality of Mathematics teaching and learning. The didactic methodologies used in Città Educante project have been proposed to enhance the teaching and learning of Mathematics of all Italian teachers through the Problem Posing and Solving (PP&S) Project, supported by the Ministry of Education to renew Mathematics teachers' practices (Brancaccio, Marchisio, Palumbo, et al., 2015). The materials used during the project were made freely available to all the Italian teachers enrolled in the PP&S Project; they can use, edit, and adapt them to their needs. Specific training on these methodologies is offered to the participants. In March 2020, when all Italian schools were closed due to the COVID-19 pandemic, the PP&S Project platform was open to all Italian teachers of every subject, even non-scientific. Thus, AFA interactive materials could help many Italian students keep on learning in an emergency situation.

This experimentation's results gave prompts for the activation of other research projects, aimed at studying the impact of these methodologies on students of different levels and backgrounds, as well as for the application of this AFA model to other subjects, even outside the STEM area, such as Latin and Foreign Languages. Interdisciplinary research collaborations are essential to developing innovative materials for learning and new insights for the research. Algorithms and grading codes can be implemented not only with numbers and formulas but also with words and phrases, for the creation of interesting and meaningful questions (Barana, Floris, Marchisio, et al., 2019; Marello et al., 2019).

In the field of Mathematics Education, in the 2018/2019 SY other minor experimentations were activated, focused on particular aspects that emerged during this study. In particular, a study was undertaken, focused on introducing algebraic computations using algebraic modeling activities in a geometric context, expanding those proposed in the "Formulas and Functions" interactive path (Barana, Casasso, & Marchisio, 2019).

Another study was conducted to introduce linear equations starting from the graph of linear functions seen in real-world contexts. These studies were carried out with smaller numbers of students, more suitable for qualitative analyses. With few classes, we could experiment the use of the DLE and the AAS during the classroom activities directly by the groups of students through tablets and computers. It was not possible during the main experimentation object of this thesis due to the large number of classes involved and the schools' scarce technological equipment. Thus, we could study the students' learning processes when dealing with automatic assessment activities: the interest was to observe how interactive feedback and multiple attempts impacted their mathematical reasoning. Moreover, it would be interesting to keep track of the study path of students who took part in the main experimentation in order to study the long-term effects of the interactive activities on their understanding of Mathematics.

These studies will be objects of further research to analyze AFA's effectiveness in a digital learning environment in other contexts, such as other levels of instruction, different topics, online or blended learning. The contribution from a more in-depth knowledge of Learning Analytics and Artificial Intelligence will be precious for the development of this research. Indeed, it may be significant to carry out new research on LA's use and improve formative assessment strategies and learning processes.

Lastly, the results of this experimentation gave prompts to the research on adaptive assessment about which the Department of Mathematics of the University of Turin is very active (Barana, Di Caro, Fioravera, Floris, et al., 2018; Barana, Di Caro, Fioravera, Marchisio, et al., 2018; Di Caro et al., 2018; Marchisio et al., 2018). This research's main output is the design of an automatic system that provides students with questions according to their competence level, choosing them among a database of shared items appositely clustered. We hope to broaden the set of good experiences and examples of learning through technologies through the cooperation of research, training, and practice to offer more and more students effective opportunities for education.

6.3 Further reflections and conclusions

This thesis's results inspire some reflections at, at least, three different levels: at a didactic level, at a research level, and at a political level.

6.3.1 Didactic level

Interactive and adaptive activities

This thesis's results show that this conception of automatic assessment provides enhancements compared to paper-and-pen work and traditional book exercises. Students can identify their mistakes and make more attempts to improve their understanding; they can be actively engaged in mathematical work and even have fun, although items are not game-based.

The tasks that composed the interactive path on formulas and functions mainly come from items of INVALSI tests and were initially conceived for standardized assessment; they were expanded and adapted to AFA. Students appreciated these tasks, and they became a chance to learn. The experimentation results highlighted how the formative use of the questions from standardized tests, matched with interactive tools with automatic assessment, effectively developed modeling competences. This learning path aimed to reinforce the cognitive roots of concepts that become fundamental for learning Mathematics in secondary school, on which problems of understanding are evidenced even at the university level. The competences on which this learning path works are relevant to offer students tools to interpret the reality around them to act with awareness in a society pervaded by data and complex models.

AFA, and in particular the interactive feedback designed in the online tasks, seems to have played a vital role in the engagement process. The interactive feedback offers the possibility to understand mistakes and be guided in a possible solution, being actively involved. Those students indeed took the chance to repeat the reasoning with different numbers in order to improve their results. We argue that other features of the tasks, as the real-world settings, and the use of the technology as school activity, could capture students' interest and attention. During the classroom activities, we also observed that the use of the digital materials displayed through the IWB supported peer collaboration and collective discussions, contributing to creating an active learning community.

The students' point of view

In the analyses, we did not consider students' emotions and reactions during the learning path. Without any claim of scientific evidence, we report some comments made by the students in the final questionnaire's open answers that we consider meaningful to show their appreciation of the activities.

In particular, to the question "what could you learn during the classroom activities of the project?" some of the meaningful answers were the following:

- "I have understood that it is better to learn mathematics through logic, without all the formulae and that that it is not impossible";
- "I have learned that it is possible to learn even using technology";
- "I have learned to think more, and they have made Mathematics look easy to me";
- "During these activities, I learned to read more carefully the given tasks, and above all to discuss the problems that I faced within Mathematics and Geometry";
- "A lot of things, such as the formulas that I couldn't do before";
- "To think with my own brain";
- "I learned that there are many ways to solve a task";
- "That sometimes Algebra can also be fun";
- "I understood that geometry could be transferred to the graphs".

To the question, "What do you think these meetings were useful for?" some of the significant answers were the following:

- "for improving myself and not getting stuck at the first doubt";
- "for thinking as a group where everyone has to express themselves before answering, and for thinking before answering";
- "I believe learning in ways different from the traditional ones can also be fun";
- "To see Mathematics differently";
- "It made us understand that sometimes you do not need to remember the rules by heart, but you can get it also with the logic";
- "I am happy not to have had class";
- "Usually, Mathematics is rather boring, but if it's done in such a way, it's not boring anymore!".

These comments support many points that we showed through the analyses in the previous chapters. The activities were useful for connecting different representational registers and offering students a new perspective of mathematical reasoning; they were engaging and inspirational and made them better understand different topics. Many students had fun during the lessons so that some even thought that they were not actual Mathematics lessons, given the new approach used. This highlights the need for a new approach to Mathematics teaching practices.

Many of the treated group classes had an outstanding performance in the INVALSI tests that year, after following the interactive learning path. We collected the results from the INVALSI tests of the classes involved in the experimentation; we intend to analyze them in-depth in the future: nevertheless, we can affirm that, in most schools, the classes of the treated group performed better than the school average.

Inclusion

One significant result that this work highlighted is the effect of interactive activities with AFA on students coming from low social classes. As we observed above, during the classroom activities, we could perceive a higher level of attention in schools located in disadvantaged areas of Turin than in schools located in the city center attended by wealthy families. The behavioral and emotional engagement level increased at the same extent for the two groups of students, while the cognitive engagement level increased significantly more in students from lower socio-economic conditions. Developing an interest in and understanding Mathematics in poorly educated families may improve young people's ability to actively engage in society and in the workplace.

Moreover, we saw that, if restricting the experimentation results to the students from lower social classes, the learning activities' effects nearly doubled, and the effect size was large. In particular, in the treated group, students from lower and medium-high social classes had a similar improvement in their results. On the other hand, in the control group, students from lower social classes decreased their results significantly more than the medium-high social class. These results suggest that AFA in a DLE had a positive impact on students from disadvantaged backgrounds and, at the same time, the traditional methodologies had a negative impact on them.

As we noticed above, the percentage of reluctant users of the DLE in students from lower social classes is similar to that of medium-high social classes. Moreover, students from lower socio-economic conditions submitted automatically assessed assignments with a higher rate than those from higher social classes. It means that, despite the potentially lower availability of digital technologies in their homes, students from low social classes used the online platform to the same extent, or even more, than students coming from wealthier families.

From the discussions with the teachers both in the pilot and in the main experimentation, it turned out that students who mainly benefitted from AFA were those with learning disorders. This is in line with most of the literature on this subject (Giusti et al., 2015). In other words, the most important effects of the experimentation are focused on those students who at the beginning of the learning path had learning difficulties, less motivation for studying or came from disadvantaged backgrounds. Therefore, a positive aspect of using said didactic methodologies could be inferred regarding aspects such as inclusion and improvement of chances. This seems to confirm that a vision of using ICT in the didactic field and its effects on the learning process as a benefit erga omnes has not unique feedback regarding the experimentation.

The role of the teachers

From the results of the experimentations conducted in this research, it is clear that teachers play a pivotal role: they have a tremendous impact on the effectiveness of the learning methodologies and technologies used. In fact, we saw that the number of reluctant users in the treated group's classes strongly depended on teachers; similarly, the number of logs was highly influenced by them. This means that the positive effects are not only a product of the mere implementation of didactic ICT-based methods, but also a product of the fundamental role of teachers who are the executors of said methods in a focused and aware way. The role of teachers and their characteristics, including the non-observable ones (the capacity to motivate, digital competences, and, above all, the decisive opening to didactic innovations), are fundamental for maximizing the opportunities offered by this type of science learning methods. Only in this way can AFA in a DLE effectively bring to the education field the benefits from learning a new way of thinking that is composed of different learning styles, which allows, at the same time, a greater individualization and more teamwork. These results support an important claim: even though going towards adaptive models where the technological components, driven by data and artificial intelligence, have a prominent role in choosing the direction of the learning path, the technology could not substitute the teacher at the school level. The teacher plays a fundamental role in proposing the technologies and making them use by the students.

Since the teachers' confidence in these new methodologies and their autonomous use affected the results of the experimentation, the necessity of teacher training is evidenced, even at a larger scale. Teacher training projects would prevent similar experimentations from remaining limited occasions of innovative learning for students, and help AFA in a DLE systematically enter the classes and become the typical approach to Mathematics.

In this project, the online course materials were all designed and developed by university researchers, who are experts both in the platform's technical use and in didactic methodologies for teaching and learning Mathematics. The teachers who participated in the experimentation, satisfied with this kind of technology in their classes, soon realized that similar strategies lead to positive learning results, so they were motivated to change their teaching practices by adopting these methodologies. Most of them enrolled in a teacher training course the following year to make them autonomous in authoring digital materials of this kind. For a similar training to be effective, teachers should have constant support from expert tutors and inner motivation to change their teaching methods.

Moreover, using AFA is formative per se: the design of questions and feedback according to the model requires that the instructors carefully consider how questions are asked and the processes involved during the resolution. This reflection over assessment entails the teachers' professional development when they build activities for their students. With appropriate teacher training, the teachers can create meaningful questions for their classes, too, joining the technical structure to interesting content and making an appropriate use.

6.3.2 Research level

All these reflections acquire a new meaning if considered under the research point of view. The AFA in a DLE leads to the generation of many data that can help understand and improve teaching and learning processes. To collect, analyze, cross-check, and use data stored in the DLE, knowledge and research in the Learning Analytics field are required. LA is a multidisciplinary field that encompasses advanced Mathematics, Data Science, and Computer Science, as well as Pedagogy: collaborations among experts in different disciplines, starting from advanced Mathematics, should be fostered for making effective use of the data from AFA.

As we said before, the results of this thesis gave prompts to other interdisciplinary studies. On the one hand, the Delta Research Group undertook several research projects to apply this AFA model to the learning of other disciplines, even outside the STEM area, such as Latin and Foreign Languages. On the other hand, research on adaptive assessment has involved Mathematics and Computer Sciences researchers and professors to design an automatic system that provides students with questions according to their competence level, choosing them among a database of shared items appositely clustered. Interdisciplinary research collaborations are essential to developing innovative materials for learning and new insights for the research.

6.3.3 Political level

At a third level, these results should inspire educational politics and policies. From this study, policymakers should gather suggestions on strategic investments, in terms of resources and money, and on the emerging priorities. For instance, to apply these models, there is a need for an infrastructure that should reach every student and every teacher. This thesis's results, the interactive path, and the methodologies studied can be shared throughout Italy and abroad. To help institutions and teachers make use of the interactive activities and the AFA, educational projects should be designed and fulfilled, encompassing training and technical support, the availability of suitable technologies, researchers, and trainers, thus building a complete DLE.

As an example, the Delta Research Group started a project with the Clark Memorial International High School in Japan, involving a net of 57 Japanese schools over the territory and more than 10,000 students. It aims at adopting the problem posing and solving and AFA methodologies for learning and teaching Mathematics at the secondary level. The project includes creating a shared DLE with teacher training, sharing of materials and educational activities for the students. Similar projects and experiences could be replicated in many countries and adapted to different needs and cultures.

As we said before, this research was completed in Autumn 2019 before the COVID-19 pandemic shook the entire world, especially in the educational environment. However, the reformulation and development of these methodologies and models to be adapted to a fully online setting can offer prompts for this inevitable digital revolution.

REFERENCES

Abdelraheem, A. Y. (2003). Computerized Learning Environments: Problems, Design Challenges and Future Promises. *The Journal of Interactive Online Learning*, *2*(2), 1–9.

Alonso, F., Lopez, G., Manrique, D., & Vines, J. M. (2005). An instructional model for web-based e-learning education with a blended learning process approach. *British Journal of Educational Technology*, *36*(2), 217–235. https://doi.org/10.1111/j.1467-8535.2005.00454.x

Angrist, J. D., & Pischke, J.-S. (2008). *Mostly harmless econometrics: An empiricist's companion*. Princeton university press.

Appleton, J. J., Christenson, S. L., Kim, D., & Reschly, A. L. (2006). Measuring cognitive and psychological engagement: Validation of the Student Engagement Instrument. *Journal of School Psychology*, *44*(5), 427–445. https://doi.org/10.1016/j.jsp.2006.04.002

Arcavi, A. (1994). Symbol Sense: Informal Sense-making in Formal Mathematics. *For the Learning of Mathematic*, *14*(3), 24–35.

Armstrong, D. A. (2011). Students' perceptions of online learning and instructional tools: A qualitative study of undergraduate students use of online tools. *The Turkish Online Journal of Educational Technology*, *10*(3), 222–226.

Asghar, M. (2013). Exploring Formative Assessment Using Cultural Historical Activity Theory. *Turkish Online Journal of Qualitative Inquiry*, *4*(2), 18–32.

Avitzur, R. (2011). *Graphing calculator (Version 4.0)*. Berkeley.

Avvisati, F., Hennessyi, S., Kozma, R. B., & Vincent-Lancrin, S. (2013). Review of the Italian Strategy for Digital Schools. *OECD Education Working Papers*, *90*. http://dx.doi.org/10.1787/5k487ntdbr44-en

Azevedo, R., & Bernard, R. (1995). A meta-analysis of the effects of feedback in computer-based instruction. *Journal of Educational Computing Research*, *13*(2), 111–127.

Barana, A., Boffo, S., Gagliardi, F., Garuti, R., & Marchisio, M. (2020). Empowering Engagement in a Technology-Enhanced Learning Environment. In M. Rehm, J. Saldien, & S. Manca (Eds.), *Project and Design Literacy as Cornerstones of Smart Education* (Vol. 158, pp. 75–77). Springer. https://doi.org/10.1007/978-981-13-9652-6_7

Barana, A., Boffo, S., Gagliardi, F., Garuti, R., Marchisio, M., & Zich, R. (2018). Percorsi interattivi supportati dalle ICT per l'apprendimento della matematica attraverso il problem solving. In A. Raffone, *La Città Educante: Metodologie e tecnologie a servizio delle Smart Communities* (pp. 115–128). Liguori Editore.

Barana, A., Boffo, S., Gagliardi, F., & Marchisio, M. (In press). *Problem Posing & Solving: A Digital Way to Learn Mathematics*.

Barana, A., Bogino, A., Fioravera, M., Floris, F., & Marchisio, M. (2018). Realignment Course in Mathematics: Design of an online valuable experience for students. *Proceedings of the 4th International Conference on Higher Education Advances (HEAd'18)*, 1465–1473. https://doi.org/10.4995/HEAD18.2018.8226

Barana, A., Bogino, A., Fioravera, M., Floris, F., Marchisio, M., Operti, L., & Rabellino, S. (2017). Self-Paced Approach in Synergistic Model for Supporting and Testing Students. *Proceedings of 2017 IEEE 41st Annual Computer Software and Applications Conference (COMPSAC)*, 407–412. https://doi.org/10.1109/COMPSAC.2017.211

Barana, A., Bogino, A., Fioravera, M., Marchisio, M., & Rabellino, S. (2016). Digital Support for University Guidance and Improvement of Study Results. *Procedia - Social and Behavioral Sciences*, *228*, 547–552. https://doi.org/10.1016/j.sbspro.2016.07.084

Barana, A., Bogino, A., Fioravera, M., Marchisio, M., & Rabellino, S. (2017). Open Platform of self-paced MOOCs for the continual improvement of Academic Guidance and Knowledge Strengthening in Tertiary Education. *Journal of E-Learning and Knowledge Society*, *13*(3), 109–119. https://doi.org/10.20368/1971-8829/1383

Barana, A., Brancaccio, A., Conte, A., Fissore, C., Floris, F., Marchisio, M., & Pardini, C. (2019a, June 26). Immersive teacher training experience on the methodology of problem posing and solving in Mathematics. *5th International Conference on Higher Education Advances (HEAd'19)*. Fifth International Conference on Higher Education Advances. https://doi.org/10.4995/HEAD19.2019.9489

Barana, A., Brancaccio, A., Conte, A., Fissore, C., Floris, F., Marchisio, M., & Pardini, C. (2019b). The Role of an Advanced Computing Environment in Teaching and Learning Mathematics through Problem Posing and Solving. *Proceedings of the 15th International Scientific Conference ELearning and Software for Education*, *2*, 11–18. https://doi.org/10.12753/2066-026X-19-070

Barana, A., Brancaccio, A., Esposito, M., Fioravera, M., Fissore, C., Marchisio, M., Pardini, C., & Rabellino, S. (2018). Online Asynchronous Collaboration for Enhancing Teacher Professional Knowledges and Competences. *The 14th International Scientific Conference ELearning and Software for Education*, 167–175. https://doi.org/10.12753/2066-026x-18-023

Barana, A., Casasso, F., & Marchisio, M. (2019). BYOD per imparare l'algebra in maniera interattiva. *Atti di Didamatica 2019*, 87–96.

Barana, A., Conte, A., Fioravera, M., Marchisio, M., & Rabellino, S. (2018). A Model of Formative Automatic Assessment and Interactive Feedback for STEM. *Proceedings of 2018 IEEE 42nd Annual Computer Software and Applications Conference (COMPSAC)*, 1016–1025. https://doi.org/10.1109/COMPSAC.2018.00178

Barana, A., Conte, A., Fissore, C., Floris, F., Marchisio, M., & Sacchet, M. (2020). The Creation of Animated Graphs to Develop Computational Thinking and Support STEM Education. In J. Gerhard & I. Kotsireas (Eds.), *Maple in Mathematics Education and Research* (pp. 189–204). Springer. https://doi.org/10.1007/978-3-030-41258-6_14

Barana, A., Conte, A., Fissore, C., Marchisio, M., & Rabellino, S. (2019). Learning Analytics to improve Formative Assessment strategies. *Journal of E-Learning and Knowledge Society*, *15*(3), 75–88. https://doi.org/10.20368/1971-8829/1135057

Barana, A., Di Caro, L., Fioravera, M., Floris, F., Marchisio, M., & Rabellino, S. (2018). Sharing system of learning resources for adaptive strategies of scholastic remedial intervention. *Proceedings of the 4th International Conference on Higher Education Advances (HEAd'18)*, 1495–1503. https://doi.org/10.4995/HEAD18.2018.8232

Barana, A., Di Caro, L., Fioravera, M., Marchisio, M., & Rabellino, S. (2018). Ontology Development for Competence Assessment in Virtual Communities of Practice. In C. Penstein Rosé, R. Martínez-Maldonado, H. U. Hoppe, R. Luckin, M. Mavrikis, K. Porayska-Pomsta, B. McLaren, & B. du Boulay (Eds.), *Artificial Intelligence in Education* (Vol. 10948, pp. 94–98). Springer. https://doi.org/10.1007/978-3-319-93846-2_18

Barana, A., Fioravera, M., & Marchisio, M. (2017a). Developing problem solving competences through the resolution of contextualized problems with an Advanced Computing Environment. *Proceedings of the 3rd International Conference on Higher Education Advances*, 1015–1023. https://doi.org/10.4995/HEAD17.2017.5505

Barana, A., Fioravera, M., & Marchisio, M. (2017b). Teacher training: A model for introducing innovative digital methodologies for learning Mathematics. *Proceedings of the 3rd International Conference on Higher Education Advances*, 608–616. https://doi.org/10.4995/HEAD17.2017.5303

Barana, A., Fioravera, M., Marchisio, M., & Rabellino, S. (2017). Adaptive Teaching Supported by ICTs to Reduce the School Failure in the Project "Scuola Dei Compiti". *Proceedings of 2017 IEEE 41st Annual Computer Software and Applications Conference (COMPSAC)*, 432–437. https://doi.org/10.1109/COMPSAC.2017.44

Barana, A., Fissore, C., & Marchisio, M. (2020). From Standardized Assessment to Automatic Formative Assessment for Adaptive Teaching. *Proceedings of the 12th International Conference on Computer Supported Education CSEDU*, *1*, 285–296.

Barana, A., Fissore, C., Marchisio, M., & Pulvirenti, M. (2020). Teacher Training for the development of Computational Thinking and Problem Posing & Solving skills with technologies. *Prooceding of ELearning Sustainment for Never-Ending Learning. Proceedings of the 16th International Scientific Conference ELearning and Software for Education*, *2*, 136–144.

Barana, A., Floris, F., Marchisio, M., Marello, C., Pulvirenti, M., Rabellino, S., & Sacchet, M. (2019). Adapting STEM Automated Assessment System to Enhance Language Skills. *Proceedings of the 15th International Scientific Conference ELearning and Software for Education*, *2*, 403–410. https://doi.org/10.12753/2066-026X-19-126

Barana, A., & Marchisio, M. (in pressa). *An interactive learning environment to empower engagement in Mathematics*.

Barana, A., & Marchisio, M. (in pressb). *Dalle formule ai modelli. Un percorso interattivo con le domande INVALSI*.

Barana, A., & Marchisio, M. (2016a). Dall'esperienza di Digital Mate Training all'attività di Alternanza Scuola Lavoro. *Mondo Digitale*, *15*(64), 63–82.

Barana, A., & Marchisio, M. (2016b). Ten Good Reasons to Adopt an Automated Formative Assessment Model for Learning and Teaching Mathematics and Scientific Disciplines. *Procedia - Social and Behavioral Sciences*, *228*, 608–613. https://doi.org/10.1016/j.sbspro.2016.07.093

Barana, A., & Marchisio, M. (2020). Le prove INVALSI per lo sviluppo di competenze matematiche e di problem solving. In P. Falzetti (Ed.), *Il Dato nella Didattica delle Discipline. II Seminario "I dati INVALSI: uno strumento per la ricerca"* (pp. 29–49). FrancoAngeli.

Barana, A., & Marchisio, M. (2019). Strategies of formative assessment enacted through automatic assessment in blended modality. *Eleventh Congress of the European Society for Research in Mathematics Education*, 4041–4048.

Barana, A., & Marchisio, M. (2017). Sviluppare competenze di problem solving e di collaborative working nell'alternanza scuola-lavoro attraverso il Digital Mate Training. *Atti di Didamatica 2017*, 1–10.

Barana, A., Marchisio, M., & Miori, R. (in press). *MATE-BOOSTER: Design of Tasks for Automatic Formative Assessment to Boost Mathematical Competence*.

Barana, A., Marchisio, M., & Miori, R. (2019). MATE-BOOSTER: Design of an e-Learning Course to Boost Mathematical Competence. *Proceedings of the 11th International Conference on Computer Supported Education (CSEDU 2019)*, *1*, 280–291.

Barana, A., Marchisio, M., & Rabellino, S. (2015). Automated Assessment in Mathematics. *Proceedings of 2015 IEEE 39th Annual Computer Software and Applications Conference*, 670–671. https://doi.org/10.1109/COMPSAC.2015.105

Barana, A., Marchisio, M., & Rabellino, S. (2019). Empowering Engagement through Automatic Formative Assessment. *2019 IEEE 43rd Annual Computer Software and Applications Conference (COMPSAC)*, 216–225. https://doi.org/10.1109/COMPSAC.2019.00040

Barana, A., Marchisio, M., & Sacchet, M. (2019). Advantages of Using Automatic Formative Assessment for Learning Mathematics. In S. Draaijer, D. Joosten-ten Brinke, & E. Ras (Eds.), *Technology Enhanced Assessment* (Vol. 1014, pp. 180–198). Springer. https://doi.org/10.1007/978-3-030-25264-9_12

Bardini, C., & Stacey, K. (2006). Students' conceptions of m and c: How to tune a linear function. In J. Novotná, H. Moraová, M. Krátká, & M. Stehlíková (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 113–120). PME.

Baroni, M., & Bonotto, C. (2015). *Problem posing e problem solving nella scuola dell'obbligo*. 62.

Beevers, C. E., Cherry, B. S. G., Clark, D. E. R., Foster, M. G., McGuire, G. R., & Renshaw, J. H. (1989). Software tools for computer-aided learning in mathematics. *International Journal of Mathematical Education in Science and Technology*, *20*(4), 561–569. https://doi.org/10.1080/0020739890200410

Beevers, C. E., & Paterson, J. S. (2003). Automatic assessment of problem solving skills in mathematics. *Active Learning in Higher Education*, *4*(2), 127–144. https://doi.org/10.1177/1469787403004002002

Beevers, C. E., Wild, D. G., McGuine, G. R., Fiddes, D. J., & Youngson, M. A. (1999). Issues of partial credit in mathematical assessment by computer. *Research in Learning Technology*, *7*(1), 26–32. https://doi.org/10.3402/rlt.v7i1.11236

Bennett, R. E. (2012). Inexorable and Inevitable: The Continuing Story of Technology and Assessment. *The Journal of Technology, Learning, and Assessment*, *1*(1), 1–22. https://doi.org/10.1002/9780470712993.ch11

Best, M., & Bikner-Ahsbahs, A. (2017). The function concept at the transition to upper secondary school level: Tasks for a situation of change. *ZDM*, *49*(6), 865–880. https://doi.org/10.1007/s11858-017-0880-6

Black, P., & Wiliam, D. (1998). Assessment and Classroom Learning. *Assessment in Education: Principles, Policy & Practice*, *5*(1), 7–74. https://doi.org/10.1080/0969595980050102

Black, P., & Wiliam, D. (2003). 'In praise of educational research': Formative assessment. *British Educational Research Journal*, *29*(5), 623–637. https://doi.org/10.1080/0141192032000133721

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability*, *21*(1), 5–31. https://doi.org/10.1007/s11092-008-9068-5

Bloom, B. S. (1968). Learning for Mastery. *Evaluation Comment*, *1*(2).

Blum, W., & Ferri, R. B. (2009). Mathematical Modelling: Can It Be Taught And Learnt? *Journal of Mathematical Modelling and Application*, *1*(1), 45–58.

Boaler, J. (1998). Open and Closed Mathematics: Student Experiences and Understandings. *Journal for Research in Mathematics Education*, *29*(1), 41–62. https://doi.org/10.2307/749717

Bokhove, C., & Drijvers, P. (2010). Digital Tools for Algebra Education: Criteria and Evaluation. *International Journal of Computers for Mathematical Learning*, *15*(1), 45–62. https://doi.org/10.1007/s10758-010-9162-x

Bolondi, G., Branchetti, L., & Giberti, C. (2018). A quantitative methodology for analyzing the impact of the formulation of a mathematical item on students learning assessment. *Studies in Educational Evaluation*, *58*, 37–50. https://doi.org/10.1016/j.stueduc.2018.05.002

Bolondi, G., Ferretti, F., & Santi, G. (2019). National standardized tests database implemented as a research methodology in mathematics education. The case of algebraic powers. *Eleventh Congress of the European Society for Research in Mathematics Education*.

Borba, M. C., Chiari, A. S. de S., & de Almeida, H. R. F. L. (2018). Interactions in virtual learning environments: New roles for digital technology. *Educational Studies in Mathematics*, *98*(3), 269–286. https://doi.org/10.1007/s10649-018-9812-9

Boud, D. (2000). Sustainable assessment: Rethinking assessment for the learning society. *Studies in Continuing Education*, *22*(2), 151–167.

Brancaccio, A., Demartini, C. G., Marchisio, M., Pardini, C., & Patrucco, A. (2014). The PP&S computer science project in school. *Mondo Digitale*, *13*(51), 565–574.

Brancaccio, Anna, Marchisio, M., Meneghini, C., & Pardini, C. (2015). Matematica e Scienze più SMART per l'Insegnamento e l'Apprendimento. *Mondo Digitale*, *14*(58), 1–8.

Brancaccio, Anna, Marchisio, M., Palumbo, C., Pardini, C., Patrucco, A., & Zich, R. (2015). Problem Posing and Solving: Strategic Italian Key Action to Enhance Teaching and Learning Mathematics and Informatics in the High School. *Proceedings of 2015 IEEE 39th Annual Computer Software and Applications Conference*, 845–850. https://doi.org/10.1109/COMPSAC.2015.126

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated Cognition and the Culture of Learning. *Educational Researcher*, *18*(1), 32–42.

Bruschi, B., Cantino, V., Cavallo Perin, R., Culasso, F., Giors, B., Marchisio, M., Marello, C., Milani, M., Operti, L., Parola, A., Rabellino, S., Sacchet, M., & Scomparin, L. (2018). Start@unito: A Supporting Model for High School Students Enrolling to University. *Proceedings 15th International Conference CELDA 2018: Cognition and Exploratory Learning in Digital Age*, 307–312. http://www.iadisportal.org/digital-library/startunito-a-supporting-model-for-high-school-students-enrolling-to-university

Calder, N., & Campbell, A. (2016). Using Mathematical Apps with Reluctant Learners. *Digital Experiences in Mathematics Education*, *2*(1), 50–69. https://doi.org/10.1007/s40751-016-0011-y

Caprotti, O., Seppl, M., & Xambo, S. (2006). Using Web Technologies to Teach Mathematics. *Society for Information Technology & Teacher Education International Conference*, 2679–2684.

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying Covariational Reasoning While Modeling Dynamic Events: A Framework and a Study. *Journal for Research in Mathematics Education*, *33*(5), 352–378. https://doi.org/10.2307/4149958

Cascella, C., Giberti, C., & Bolondi, G. (2020). An analysis of Differential Item Functioning on INVALSI tests, designed to explore gender gap in mathematical tasks. *Studies in Educational Evaluation*, *64*.

Cazes, C., Gueudet, G., Hersant, M., & Vandebrouck, F. (2007). Using E-Exercise Bases in Mathematics: Case Studies at University. *International Journal of Computers for Mathematical Learning*, *11*(3), 327–350. https://doi.org/10.1007/s10758-006-0005-8

Cerulli, G. (2015). *Econometric evaluation of socio-economic programs* (Vol. 49). Springer.

Chan, M. C. E., & Clarke, D. (2017). Structured affordances in the use of open-ended tasks to facilitate collaborative problem solving. *ZDM*, *49*(6), 951–963. https://doi.org/10.1007/s11858-017-0876-2

Chi, M. T. H., & Wylie, R. (2014). The ICAP Framework: Linking Cognitive Engagement to Active Learning Outcomes. *Educational Psychologist*, *49*(4), 219–243. https://doi.org/10.1080/00461520.2014.965823

Chung, G. K. W. K., Shel, T., & Kaiser, W. J. (2006). An Exploratory Study of a Novel Online Formative Assessment and Instructional Tool to Promote Students' Circuit Problem Solving. *The Journal of Technology, Learning, and Assessment*, *5*(6), 4–26.

Clark, R. C., & Mayer, R. (2008). *E-Learning and The Science of Instruction* (Second Edition). Pfeiffer.

Clark, R. E. (1994). Media will never influence learning. *Educational Technology Research and Development*, *42*(2), 21–29. https://doi.org/10.1007/BF02299088

Cohen, J. (1969). *Statistical power analysis for the behavioral sciences*. Academic Press.

Cook, D. L. (1962). The Hawthorne Effect in Educational Research. *The Phi Delta Kappan*, *44*(2).

Cornelius-White, J. (2007). Learner-Centered Teacher-Student Relationships Are Effective: A Meta-Analysis. *Review of Educational Research*, *77*(1), 113–143. https://doi.org/10.3102/003465430298563

Cornoldi, C., & Israel, G. (2015). *Abolire la scuola media?* Il Mulino.

Cusi, A., Malara, N. A., & Navarra, G. (2011). Early Algebra: Theoretical Issues and Educational Strategies for Bringing the Teachers to Promote a Linguistic and Metacognitive approach to it. In J. Kai & E. Knuth, *Early Algebraization: Cognitive, Curricular, and Instructional Perspectives* (pp. 483–510). Springer.

Cusi, A., Morselli, F., & Sabena, C. (2017). Promoting formative assessment in a connected classroom environment: Design and implementation of digital resources. *ZDM*, *49*(5), 755–767. https://doi.org/10.1007/s11858-017-0878-0

Czerkawski, B. C., & Lyman, E. W. (2016). An Instructional Design Framework for Fostering Student Engagement in Online Learning Environments. *TechTrends*, *60*(6), 532–539. https://doi.org/10.1007/s11528-016-0110-z

Dagley, V. (2004). Making the invisible visible: A methodological and a substantive issue. *Educational Action Research*, *12*(4), 613–630. https://doi.org/10.1080/09650790400200260

D'Amore, B., & Pinilla, M. I. F. (2006). Che problema i problemi! *L'insegnamento della matematica e delle scienze integrate*, *6*(29), 645–664.

de Waal, P. (2017). Learning analytics for continuous learning-processes im- provement through dynamic data-informed decisions. *Formazione & Insegnamento*, *15*(2), 43–51. https://doi.org/107346/-fei-XV-02-17_04

Demartini, C. G., Bizzarri, G., Cabrini, M., Di Luca, M., Franza, G., Maggi, P., Marchisio, M., Morello, L., & Tani, C. (2015). Problem posing (& solving) in the second grade higher secondary school. *Mondo Digitale*, *14*(58), 418–422.

Demartini, C., Lamberti, F., Marchisio, M., Pardini, C., & Patrucco, A. (2013). *The "CSCT Living Lab" for Computer Science and Computational Thinking*.

Di Caro, L., Rabellino, S., Fioravera, M., & Marchisio, M. (2018). A model for enriching automatic assessment resources with free-text annotations. *Proceedings of the 15th International Conference on Cognition and Exploratory Learning in the Digital Age, CELDA 2018*, 186–194.

Dimiceli, V. A., Lang, A. S. I. D., & Locke, L. A. (2010). Teaching calculus with Wolfram|Alpha. *International Journal of Mathematical Education in Science and Technology*, *41*(8), 1061–1071.

Dindyal, J. (2004). Algebraic Thinking in Geometry at High School Level: Students' Use of Variables and Unknowns. In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010: Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 183–190). MERGA Inc.

Drijvers, P., Ball, L., Barzel, B., Heid, M. K., Cao, Y., & Maschietto, M. (2016). *Uses of digital technology in lower secondary mathematics education: A concise topical survey*. Springer.

Duval, E., Sharples, M., & Sutherland. (2017). Research Themes in Technology Enhanced Learning. In Erik Duval, M. Sharples, & R. Sutherland, *Technology enhanced learning: Research themes* (pp. 1–10). Springer.

Duval, R. (1993). Registres de représentations sémiotique et fonctionnement cognitif de la pensée. *Annales de Didactique et de Sciences Cognitives*, *5*, 37–65.

Duval, R. (1999). Representation, Vision and Visualization: Cognitive Functions in Mathematical Thinking. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 3–26.

ECDL Fundation. (2014). *The Fallacy of the 'Digital Native': Why Young People Need to Develop their Digital Skills*.

Edson, A. J. (2017). Learner-controlled scaffolding linked to open-ended problems in a digital learning environment. *ZDM*, *49*(5), 735–753. https://doi.org/10.1007/s11858-017-0873-5

Engeström, Yrjo. (1987). *Learning by Expanding: An Activity-Theoretical Approach to Developmental Research* (2nd ed.). Cambridge University Press. https://doi.org/10.1017/CBO9781139814744

Engeström, Yrjo. (2000). Activity theory as a framework for analyzing and redesigning work. *Ergonomics*, *43*(7), 960–974. https://doi.org/10.1080/001401300409143

Engeström, Yrjö. (2001). Expansive Learning at Work: Toward an activity theoretical reconceptualization. *Journal of Education and Work*, *14*(1), 133–156. https://doi.org/10.1080/13639080020028747

Engeström, Yrjo. (2009). From Learning Environments and Implementation to Activity Systems and Expansive Learning. *An International Journal of Human Activity Theory*, *2*, 17–33. https://doi.org/10.1017/CBO9781316225363

English, L. D., & Sriraman, B. (2010). Problem Solving for the 21st Century. In L. D. English & B. Sriraman, *Theories of Mathematics Education* (pp. 263–290). Springer.

European Parliament and Council. (2018). Council Recommendation of 22 May 2018 on key competences for lifelong learning. *Official Journal of the European Union*, 1–13.

Fondazione Giovanni Agnelli. (2010). *Rapporto sulla scuola in Italia*. Laterza.

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School Engagement: Potential of the Concept, State of the Evidence. *Review of Educational Research*, *74*(1), 59–109. https://doi.org/10.3102/00346543074001059

Fredricks, J. A., & McColskey, W. (2012). The Measurement of Student Engagement: A Comparative Analysis of Various Methods and Student Self-report Instruments. In S. L. Christenson, A. L. Reschly, & C. Wylie (Eds.), *Handbook of Research on Student Engagement* (pp. 763–782). Springer. https://doi.org/10.1007/978-1-4614-2018-7_37

Friend Wise, A., & Williamson Schaffer, D. (2015). Why theory matters more than ever in the age of big data. *Journal of Learning Analytics*, *2*(2), 5–13. https://doi.org/10.18608/jla.2015.22.2

Gambini, A., Banchelli, S., & Nolli, N. (2020). Analisi verticale del concetto di pendenza: Dalla scuola secondaria di primo grado all'università. In P. Falzetti (Ed.), *Il Dato nella Didattica delle Discipline. II Seminario "I dati INVALSI: uno strumento per la ricerca"* (pp. 184–200). FrancoAngeli.

Gaona, J., Reguant, M., Valdivia, I., Vásquez, M., & Sancho-Vinuesa, T. (2018). Feedback by automatic assessment systems used in mathematics homework in the engineering field. *Computer Applications in Engineering Education*, *26*(4), 994–1007. https://doi.org/10.1002/cae.21950

García-Holgado, A., & García-Peñalvo, F. J. (2018). Human Interaction in Learning Ecosystems Based on Open Source Solutions. In P. Zaphiris & A. Ioannou (Eds.), *Learning and Collaboration Technologies. Design, Development and Technological Innovation. LCT 2018* (Vol. 10924, pp. 218–232). Springer. https://doi.org/10.1007/978-3-319-91743-6_17

Gikandi, J. W., Morrow, D., & Davis, N. E. (2011). Online formative assessment in higher education: A review of the literature. *Computers & Education*, *57*(4), 2333–2351. https://doi.org/10.1016/j.compedu.2011.06.004

Giusti, S., Gui, M., Micheli, M., & Parma, A. (2015). *Gli effetti degli investimenti in tecnologie digitali nelle scuole del Mezzogiorno* (Vol. 33). Collana Materiali Uval.

Goldin, G. A. (2014). Mathematical Representations. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 409–413). Springer.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. *The Journal of Mathematical Behavior*, *22*(1), 73–89. https://doi.org/10.1016/S0732-3123(03)00005-1

Gossen, F., Kuhn, D., Margaria, T., & Lamprecht, A.-L. (2018). Computational Thinking: Learning by Doing with the Cinco Adventure Game Tool. *Proceedings of 2018 IEEE 42nd Annual Computer Software and Applications Conference (COMPSAC)*, 990–999. https://doi.org/10.1109/COMPSAC.2018.00175

Grabinger, R. S., & Dunlap, J. C. (1995). Rich environments for active learning: A definition. *ALT-J*, *3*(2), 5–34. https://doi.org/10.1080/0968776950030202

Greefrath, G., & Vorhölter, K. (2016). *Teaching and Learning Mathematical Modelling: Approaches and Developments from German Speaking Countries*. Springer.

Guetl, C., & Chang, V. (2008). Ecosystem-based Theoretical Models for Learning in Environments of the 21st Century. *International Journal of Emerging Technologies in Learning (IJET)*, *3*(1), 50–60. https://doi.org/10.3991/ijet.v3i1.742

Gui, M., Micheli, M., & Fiore, B. (2014). Is the internet creating a learning gap among students? Evidence from the Italian PISA data. *Italiana Journal of Sociology of Education*, *6*(1), 1–24.

Hallam, G., & Glanville, C. (2007). Blending Process with Product: Using Assessment to Drive Learning Through the Creation of an Online Journal. In S. Frankland (Ed.), *Enhancing teaching and learning through assessment: Deriving an appropriat model* (1. ed, pp. 242–255). Springer.

Hattie, J. A. C. (2009). *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement*. Routledge.

Hattie, J., & Timperley, H. (2007). The Power of Feedback. *Review of Educational Research*, *77*(1), 81–112. https://doi.org/10.3102/003465430298487

Heid, M. K. (1997). The Technological Revolution and the Reform of School Mathematics. *American Journal of Education*, *106*(1), 5–61. https://doi.org/10.1086/444175

Helder, E., Sosnovsky, S., & Dimitrova, V. (2017). Adaptive Intelligent Learning Environments. In Erik Duval, M. Sharples, & R. Sutherland, *Technology enhanced learning: Research themes* (pp. 109–114). Springer.

Henrie, C. R., Halverson, L. R., & Graham, C. R. (2015). Measuring student engagement in technology-mediated learning: A review. *Computers & Education*, *90*, 36–53. https://doi.org/10.1016/j.compedu.2015.09.005

Hohenwarter, M., Hohenwarter, J., Kreis, Y., & Lavicza, Z. (2008). Teaching and Learning Calculus with Free Dynamic Ma- thematics Software GeoGebra. *11th International Congress on Mathematics Education*, 1–10.

Honebein, P. C. (1996). Seven Goals for the Design of Constructivist Learning Environments. In B. Wilson, *Constructivist Learning Environments* (pp. 11–24). Educational Technology Publications.

Huntley, M. A., Rasmussen, C. L., Villarubi, R. S., Sangtong, J., & Fey, J. T. (2000). Effects of Standards-Based Mathematics Education: A Study of the Core-plus Mathematics Project Algebra and Functions Strand. *Journal for Research in Mathematics Education*, *31*(3), 328. https://doi.org/10.2307/749810

INVALSI. (2015). *Indagine OCSE- PISA (2015): I risultati degli studenti italiani in scienze, matematica e letteratura*. INVALSI. https://www.invalsi.it/invalsi/ri/pisa2015/doc/rapporto_PISA_2015.pdf

INVALSI. (2018). *I livelli INVALSI in Matematica—II secondaria di II grado. Descrizione analitica*. https://invalsi-areaprove.cineca.it/docs/2018/Descrittori%20analitici%20dei%20livelli%20INVALSI%20-%20MATEMATICA.pdf

Jonassen, D., Davidson, M., Collins, M., Campbell, J., & Haag, B. B. (1995). Constructivism and computer-mediated communication in distance education. *American Journal of Distance Education*, *9*(2), 7–26. https://doi.org/10.1080/08923649509526885

Jonassen, D. H., & Rohrer-Murphy, L. (1999). Activity theory as a framework for designing constructivist learning environments. *Educational Technology Research and Development*, *47*(1), 61–79. https://doi.org/10.1007/BF02299477

Jordan, K. (2014). Initial trends in enrolment and completion of massive open online courses. *The International Review of Research in Open and Distributed Learning*, *15*(1), 133–160. https://doi.org/10.19173/irrodl.v15i1.1651

Kaiser, G. (2014). Mathematical Modelling and Applications in Education. In Stephen Lerman, *Encyclopedia of Mathematics Education* (pp. 396–404). Springer.

Kaput, J. J. (2000). *Teaching and Learning a New Algebra with Understanding.*

Kearns, L. R. (2012). Student Assessment in Online Learning: Challenges and Effective Practices. *MERLOT Journal of Online Learning and Teaching*, *8*(3), 198–208.

Keijzer-de Ruijter, M. (2019). Digital Exams in Engineering Education. In S. Draaijer & D. Joosten-ten Brinke (Eds.), *Technology Enhanced Assessment. TEA 2018.* (Vol. 1014). Springer. https://doi.org/10.1007/978-3-030-25264-9_10

Khan, B. H. (1997). Web-Based Instruction: What Is It and Why Is It? In B. H. Khan, *Web-Based Instruction, Educational Technology Publications* (pp. 5–18). Englewood Cliffs.

Kieran, C., Pang, J., Schifter, D., & Swee Fong, N. (2016). *Early algebra*. Springer.

Kilpatrick, J. (1987). Problem formulating: Where do good problem come from? In *Cognitive science and mathematics education* (A. H. Schoenfeld, pp. 123–147). Erlbaum.

Kindermann, T. A. (2007). Effects of Naturally Existing Peer Groups on Changes in Academic Engagement in a Cohort of Sixth Graders. *Child Development*, *78*(4), 1186–1203. https://doi.org/10.1111/j.1467-8624.2007.01060.x

Kluger, A. N., & DeNisi, A. (1996). The effects of feedback interventions on performance: A historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological Bulletin*, *119*(2), 254–284. https://doi.org/10.1037/0033-2909.119.2.254

Knight, S., & Buckingham Shum, S. (2017). Theory and Learning Analytics. In C. Lang, G. Siemens, A. Wise, & D. Gasevic (Eds.), *Handbook of Learning Analytics* (pp. 17–22). Society for Learning Analytics Research (SoLAR). https://doi.org/10.18608/hla17

Kreber, C., & Kanuka, H. (2013). The Scholarship of Teaching and Learning and the Online Classroom. *Canadian Journal of University Continuing Education*, *32*(2). https://doi.org/10.21225/D5P30B

Kurvinen, E., Lindén, R., Rajala, T., Kaila, E., Laakso, M.-J., & Salakoski, T. (2014). Automatic assessment and immediate feedback in first grade mathematics. *Proceedings of the 14th Koli Calling International Conference on Computing Education Research - Koli Calling '14*, 15–23. https://doi.org/10.1145/2674683.2674685

Kuzniak, A., Parzysz, B., & Vivier, L. (2013). Trajectory of a problem: A study in Teacher Training. *The Mathematics Enthusiast*, *10*(1 & 2), 407–440.

LAK 2011. (2011). https://tekri.athabascau.ca/analytics/

Lave, J. (1991). Situating learning in communities of practice. In L. B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), *Perspectives on socially shared cognition.* (pp. 63–82). American Psychological Association. https://doi.org/10.1037/10096-003

Lavy, I., & Shriki, A. (2007). Problem posing as a means for developing mathematical knowledge of prospective teachers. *31st Conference of the International Group for the Psychology of Mathematics Education*, *3*, 129–136.

Lefoe, G. (1998). Creating Constructivist Learning Environments on the Web: The Challenge in Higher Education. *ASCILITE Conference*, 453–464.

Leong, Y. H., & Janjaruporn, R. (2015). Teaching of Problem Solving in School Mathematics Classrooms. In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education* (pp. 645–648). Springer. https://doi.org/10.1007/978-3-319-12688-3_79

Lesh, R., & Doerr, H. M. (2003). Foundation of a Models and Modeling Perspectives on Mathematics Teaching, Learning and Problem Solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching* (pp. 3–33). Routledge. https://doi.org/10.4324/9781410607713

Lesh, R., & Lehrer, R. (2003). Models and Modeling Perspectives on the Development of Students and Teachers. *Mathematical Thinking and Learning*, *5*(2 & 3), 109–129.

Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). *Problem solving in mathematics education*. Springer.

Malara, N. A. (1996). Il pensiero algebrico: Come promuoverlo sin dalla scuola dell'obbligo limitandone le difficoltà? *L'educazione matematica*, *17*(1), 80–99.

Marchisio, M., Di Caro, L., Fioravera, M., & Rabellino, S. (2018). Towards Adaptive Systems for Automatic Formative Assessment in Virtual Learning Communities. *Proceedings of 2018 IEEE 42nd Annual Computer Software and Applications Conference (COMPSAC)*, 1000–1005. https://doi.org/10.1109/COMPSAC.2018.00176

Marchisio, M., Operti, L., Rabellino, S., & Sacchet, M. (2019). Start@unito: Open Online Courses for Improving Access and for Enhancing Success in Higher Education: *Proceedings of the 11th International Conference on Computer Supported Education*, 639–646. https://doi.org/10.5220/0007732006390646

Marello, C., Marchisio, M., Pulvirenti, M., & Fissore, C. (2019). Automatic assessment to enhance online dictionaries consultation skills. *Proceedings of 16th International Conference on Cognition and Exploratory Learning in the Digital Age*, 331–338.

Marks, H. M. (2000). Student Engagement in Instructional Activity: Patterns in the Elementary, Middle, and High School Years. *American Educational Research Journal*, *37*(1), 153–184.

Martini, A. (2006). Metodo sperimentale, approccio controfattuale e valutazione degli effetti delle politiche pubbliche. *Rassegna Italiana Di Valutazione*, *34*, 61–75.

Mesa, V. (1997). *The use of graphing calculators in solving problems on functions* (E. Pehkonen, Ed.; Vol. 3, pp. 240–247). Program Committee.

Decreto Ministeriale 9 febbraio 1979: Programmi, orari di insegnamento e prove di esame per la scuola media statale, no. 50 (1979).

Mitra, A., & Steffensmeier, T. (2000). Changes in Student Attitudes and Student Computer Use in a Computer-Enriched Environment. *Journal of Research on Computing in Education*, *32*(3), 417–433. https://doi.org/10.1080/08886504.2000.10782289

Indicazioni Nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione, (2012) (testimony of MIUR).

MIUR - Gestione Patrimonio Informativo e Statistica. (2019). *La dispersione scolastica nell'anno scolastico 2016/2017 e nel passaggio all'anno scolastico 2017/2018*. https://www.miur.gov.it/documents/20182/2155736/La+dispersione+scolastica+nell%27a.s.2016-17+e+nel+passaggio+all%27a.s.2017-18.pdf/1e374ddd-29ac-11e2-dede-4710d6613062?version=1.0&t=1563371652741

Moebius Assessment. (2019, January 2). Https://Www.Digitaled.Com/Products/Assessment/. https://www.digitaled.com/products/assessment/

Moreno, R., & Mayer, R. (2007). Interactive Multimodal Learning Environments. *Educational Psychology Review*, *19*(3), 309–326. https://doi.org/10.1007/s10648-007-9047-2

Moreno-Armella, L., & Santos-Trigo, M. (2016). The use of digital technologies in mathematical practices: Reconciling traditional and emerging approaches. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 595–616). Taylor & Francis.

Executive Summary Principles and standards for school mathematics, (2000) (testimony of National Council of Teachers of Mathematics).

Ng, C., Bartlett, B., & Elliott, S. N. (2018). *Empowering engagement: Creating learning opportunities for students from challenging backgrounds*. Springer.

Nicol, D. (2019). Reconceptualising feedback as an internal not an external process. *Italian Journal of Educational Research*, *12*(Special issue: May), 71–83.

Nicol, D. J., & Macfarlane-Dick, D. (2006). Formative assessment and self-regulated learning: A model and seven principles of good feedback practice. *Studies in Higher Education*, *31*(2), 199–218.

Nicol, D., & Milligan, C. (2006). Rethinking technology-supported assessment practices in relation to the seven principles of good feedback practice. In C. Bryan & K. Clegg, *Innovative Assessment in Higher Education* (pp. 1–14). Taylor and Francis Group Ltd.

OECD. (2013). *PISA 2012 results: Ready to learn* (Vol. 3). OECD.

OECD. (2014). *PISA 2012 Technical Report*. PISA, OECD Publishing. https://www.oecd.org/pisa/pisaproducts/PISA-2012-technical-report-final.pdf

OECD. (2016a). *Equations and Inequalities. Making Mathematics accessible to all*. OECD Publishing. http://dx.doi.org/10.1787/9789264258495-en.

OECD. (2016b). *Low-performing students: Why they fall behind and how to help them succeed*. OECD Publishing.

OECD. (2017). *PISA 2015 Assessment and Analytical Framework: Science, Reading, Mathematic, Financial Literacy and Collaborative Problem Solving* (revised edition). PISA, OECD Publishing. https://doi.org/10.1787/19963777

Olive, J., & Makar, K. (2010). Mathematical Knowledge and Practices Resulting from Access to Digital Technologies. In C. Hoyles & J.-B. Lagrange (Eds.), *Mathematics education and technology: Rethinking the terrain: The 17th ICMI study* (Vol. 13, pp. 133–178). Springer.

Osterlind, S. J. (1998). *Constructing test items* (Vol. 25). Springer. https://doi.org/10.1007/978-94-009-1071-3

Pachler, N., Daly, C., Mor, Y., & Mellar, H. (2010). Formative e-assessment: Practitioner cases. *Computers & Education*, *54*(3), 715–721. https://doi.org/10.1016/j.compedu.2009.09.032

Paiva, R. C., Ferreira, M. S., Mendes, A. G., & Eusébio, A. M. J. (2015). Interactive and Multimedia Contents Associated with a System for Computer-Aided Assessment. *Journal of Educational Computing Research*, *52*(2), 224–256. https://doi.org/10.1177/0735633115571305

Pellegrini, M., Vivanet, G., & Trinchero, R. (2018). Gli indici di effect size nella ricerca educativa. Analisi comparativa e significatività pratica. *Educational, Cultural and Psychological Studies*, *18*, 275–309. http://dx.doi.org/10.7358/ecps-2018-018-pel1

Persico, D., & Steffen, K. (2017). Self-regulated learning in technology enhanced learning environments. In Erik Duval, M. Sharples, & R. Sutherland, *Technology enhanced learning: Research themes* (pp. 115–124). Springer.

Pezzino, M. (2018). Online assessment, adaptive feedback and the importance of visual learning for students. The advantages, with a few caveats, of using MapleTA. *International Review of Economics Education*, *28*, 11–28. https://doi.org/10.1016/j.iree.2018.03.002

Pierce, R., Stacey, K., & Barkatsas, A. (2007). A scale for monitoring students' attitudes to learning mathematics with technology. *Computers & Education*, *48*(2), 285–300. https://doi.org/10.1016/j.compedu.2005.01.006

Pintrich, P. R., & Zusho, A. (2007). Student motivation and self-regulated learning in the college classroom. In R. P. Perry & J. C. Smart, *The Scholarship of Teaching and Learning in Higher Education: An Evidence-Based Perspective*. Springer.

Pólya, G. (1945). *How to solve it*. Princeton NJ: Princeton University.

Pozio, S., & Bolondi, G. (2019). Difficulties in formulating a geometric situation algebraically: Hints from a large-scale assessment. *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education*.

Prince, M. (2004). Does Active Learning Work? A Review of the Research. *Journal of Engineering Education*, *93*(3), 223–231. https://doi.org/10.1002/j.2168-9830.2004.tb00809.x

Rasmussen, C. L., & King, K. D. (2000). Locating starting points in differential equations: A realistic mathematics education approach. *International Journal of Mathematical Education in Science and Technology*, *31*(2), 161–172. https://doi.org/10.1080/002073900287219

Rønning, F. (2017). Influence of computer-aided assessment on ways of working with mathematics. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, *36*(2), 94–107. https://doi.org/10.1093/teamat/hrx001

Sadler, D. R. (1989). Formative assessment and the design of instructional systems. *Instructional Science*, *18*(2), 119–144.

Samo, D. D., Darhim, D., & Kartasasmita, B. (2017). Culture-Based Contextual Learning to Increase Problem-Solving Ability of First Year University Student. *Journal on Mathematics Education*, *9*(1). https://doi.org/10.22342/jme.9.1.4125.81-94

Sanacore, J. (2008). Turning Reluctant Learners into Inspired Learners. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *82*(1), 40–44. https://doi.org/10.3200/TCHS.82.1.40-44

Sangsawang, T. (2015). Instructional Design Framework for Educational Media. *Procedia - Social and Behavioral Sciences*, *176*, 65–80. https://doi.org/10.1016/j.sbspro.2015.01.445

Sangwin, C. (2015). Computer Aided Assessment of Mathematics Using STACK. In S. J. Cho (Ed.), *Selected Regular Lectures from the 12th International Congress on Mathematical Education* (pp. 695–713). Springer. https://doi.org/10.1007/978-3-319-17187-6_39

Sangwin, C., Makar, K., Cazes, C., Lee, A., & Wong, K. L. (2010). Micro-level Automatic Assessment Sup-ported by Digital Technologies. In C. Hoyles & J.-B. Lagrange (Eds.), *Mathematics education and technology: Rethinking the terrain: The 17th ICMI study* (Vol. 13, pp. 227–250). Springer.

Santos-Trigo, M., Moreno-Armella, L., & Camacho-Machín, M. (2016). Problem solving and the use of digital technologies within the Mathematical Working Space framework. *ZDM*, *48*(6), 827–842. https://doi.org/10.1007/s11858-016-0757-0

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grows, *Handbook of Research on Mathematics Teaching and Learning* (pp. 334–370). Macmillian.

Schoenfeld, A. H., & Arcavi, A. (1988). On the Meaning of Variable. *The Mathematics Teacher*, *81*(6), 420–427.

Scriven, M. (1967). *The methodology of evaluation*. Lafayette, Ind: Purdue University.

Shute, V. J. (2008). Focus on Formative Feedback. *Review of Educational Research*, *78*(1), 153–189. https://doi.org/10.3102/0034654307313795

Singer, F. M., Ellerton, N. F., & Cai, J. (Eds.). (2015). *Mathematical problem posing: From research to effective practice*. Springer.

Skinner, E. A., & Pitzer, J. R. (2012). Developmental Dynamics of Student Engagement, Coping, and Everyday Resilience. In S. L. Christenson, A. L. Reschly, & C. Wylie (Eds.), *Handbook of Research on Student Engagement* (pp. 21–44). Springer. https://doi.org/10.1007/978-1-4614-2018-7_2

Stacey, K., & Wiliam, D. (2013). Technology and Assessment in Mathematics. In M. A. Clements (Ed.), *Third International Handbook of Mathematics Education* (Vol. 27, pp. 721–751). Springer.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. C. Clarkson (Ed.), *Technology in mathematics education* (pp. 518–525). Mathematics Education Research Group of Australasia.

Suhonen, J. (2005). *A formative development method for digital learning environments in sparse learning communities* [University of Joensuu]. http://epublications.uef.fi/pub/urn_isbn_952-458-663-0/index_en.html

Suurtamm, C., Thompson, D. R., Kim, R. Y., Moreno, L. D., Sayac, N., Schukajlow, S., Silver, E., Ufer, S., & Vos, P. (2016). *Assessment in Mathematics Education*. Springer. https://doi.org/10.1007/978-3-319-32394-7_1

Thiessen, V., & Looker, D. (2007). Digital Divides and Capital Conversion:the Optimal Use of Information and Communication Technology for Youth Reading Achievement. *Information, Communication & Society*, *10*(2), 159–180. https://doi.org/10.1080/13691180701307370

Timmers, C., & Veldkamp, B. (2011). Attention paid to feedback provided by a computer-based assessment for learning on information literacy. *Computers & Education*, *56*(3), 923–930. https://doi.org/10.1016/j.compedu.2010.11.007

Uden, L., Wangsa, I. T., & Damiani, E. (2007). The future of E-learning: E-learning ecosystem. *Inaugural IEEE International Conference on Digital Ecosystems and Technologies*, 113–117.

Matematica 2001, (2001) (testimony of Unione Matematica Italiana).

van den Bogaard, M. E. D., & Saunders-Smits, G. N. (2007). *Peer & Self Evaluations as Means to Improve the Assessment of Project Based Learning*. *Session S1G*, 12–18.

van den Heuvel-Panhuizen, M., & Becker, J. (2003). Towards a Didactic Model for Assessment Design in Mathematics Education. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second International Handbook of Mathematics Education* (pp. 689–716). Springer. https://doi.org/10.1007/978-94-010-0273-8_23

van der Kleij, F. M., Eggen, T. J. H. M., Timmers, C. F., & Veldkamp, B. P. (2012). Effects of feedback in a computer-based assessment for learning. *Computers & Education*, *58*(1), 263–272. https://doi.org/10.1016/j.compedu.2011.07.020

van der Wal, N. J., Bakker, A., & Drijvers, P. (2017). Which Techno-mathematical Literacies Are Essential for Future Engineers? *International Journal of Science and Mathematics Education*, *15*(S1), 87–104. https://doi.org/10.1007/s10763-017-9810-x

Vivanet, G. (2013). Le ICT nella scuola italiana. Sintesi dei dati in un quadro comparativo europeo. *Form@re - Open Journal per La Formazione in Rete*, *4*(13), 47–56.

von Glasersfeld, E. (1989). Constructivism in Education. In T. Husen & T. N. Postlethwaite, (Eds.), *The International Encyclopedia of Education: Vol. Supplement Vol.1.* (pp. 162–163). Pergamon Press.

Wachira, P., & Keengwe, J. (2020). *Handbook of Research on Online Pedagogical Models for Mathematics Teacher*. Information Science Reference.

Watson, W. R., & Watson, S. L. (2007). An Argument for Clarity: What are Learning Management Systems, What are They Not, and What Should They Become? *TechTrends*, *51*(2), 28–34. https://doi.org/10.1007/s11528-007-0023-y

West, D., Luzeckyj, A., Searle, B., Toohey, D., & Price, R. (2018). *The Use of Learning Analytics to Support Improvements in Teaching Practice*. Innovative Research Universities.

Wiliam, D. (2006). Formative Assessment: Getting the Focus Right. *Educational Assessment*, *11*(3–4), 283–289. https://doi.org/10.1080/10627197.2006.9652993

Wilson, B. G. (1995). Metaphors for instruction: Why we talk about learning environments. *Educational Technology*, *35*(5), 25–30.

Wilson, B. G. (1996). *Constructivist learning environments: Case studies in instructional design*. Educational Technology Pubns.

World Economic Forum. (2018a). *Eight Futures of Work. Scenarios and their Implications*. https://www.weforum.org/whitepapers/eight-futures-of-work-scenarios-and-their-implications

World Economic Forum. (2018b). *The Future of Jobs Report*. https://www.weforum.org/reports/the-future-of-jobs-report-2018

Xiong, Y., Li, H., Kornhaber, M. L., Suen, H. K., Pursel, B., & Goins, D. D. (2015). Examining the Relations among Student Motivation, Engagement, and Retention in a MOOC: A Structural Equation Modeling Approach. *Global Education Review*, *2*(3), 23–33.

Yamagata-Lynch, L. C. (2010). Understanding Cultural Historical Activity Theory. In L. C. Yamagata-Lynch, *Activity Systems Analysis Methods* (pp. 13–26). Springer. https://doi.org/10.1007/978-1-4419-6321-5_2

Yang, Y.-F. (2011). Engaging students in an online situated language learning environment. *Computer Assisted Language Learning*, *24*(2), 181–198. https://doi.org/10.1080/09588221.2010.538700

Yerushalmy, M. (2000). Problem solving strategies and mathematical resources: A longitudinal view on problem solving in a function based approach to algebra. *Educational Studies in Mathematics*, *43*, 125–147.

Zawojewski, J. S., & Lesh, R. (2003). A Models and Modeling Perspective on Problem Solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching* (pp. 317–336). Routledge. https://doi.org/10.4324/9781410607713

Zimmerman, B. J. (1990). Self-Regulated Learning and Academic Achievement: An Overview. *Educational Psychologist*, *25*(1), 3–17. https://doi.org/10.1207/s15326985ep2501_2

ACKNOWLEDGEMENTS

At the end of this work, I desire to spend a few words to thank those who helped to make it possible.

First of all, I would like to thank my supervisors, prof. Marina Marchisio and prof. Renato Grimaldi, for the invaluable scientific advice they provided, guiding me during this 3-year path.

I also need to mention the referees, prof. Giorgio Bolondi and prof. Sorel Reisman, whose precious suggestions improved the quality of this thesis. You have my deepest gratitude.

This work would not be possible without all the members of the working group of the Educating City project, namely Stefano Boffo, Francesco Gagliardi, Rossella Garuti, and Rodolfo Zich, who collaborated in ideating, designing, and realizing the didactic experimentations at the heart of this study.

Moreover, I feel grateful to the numerous teachers and students who believed in this project and took part in the experimentations.

A precious contribution to my thesis work came from the Problem Posing and Solving Project, within which part of the research has been carried out. In particular, prof. Anna Brancaccio and prof. Massimo Esposito of the Ministry of Education, and all the rest of the working group: prof. Claudio Demartini, prof. Marina Marchisio, prof. Claudio Pardini, prof. Rodolfo Zich. Then, all the teachers of the PP&S Project, who experimented the materials and methodologies presented here, giving prompts and suggestions.

I am deeply grateful to the whole Delta Research Group and all the present and past colleagues who contributed to the research with help, discussions, and hints. In particular, to prof. Alberto Conte, for inspiring our research themes. Then, to Sergio Rabellino and ICT services of the Department of Computer Sciences of the University of Turin, for the management of the platforms where I could work.

Last, I would like to thank all my family and my friends for their warm support during these last years. My special thanks go to Daniele, my rock in good and hard times, and to the little Vittoria, who was always with me during this thesis's writing and filled with joy the last months.