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Telling the story of a diagram: affective and aesthetic mathematical experiences

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Abstract

In this paper, we are interested in a dynamic vision of diagrams, focusing on their evocative power, their force in mathematical activity and their potential for interaction with mathematical objects. We are inspired by the work of C. S. Pierce to see diagrams as inscriptions that organise space and articulate relations and that of the philosopher of mathematics G. Châtelet that emphasises diagrams as the very means of ontogenesis. In particular, diagrams are sophisticated mathematical devices for thinking about and communicating mathematics. They have stories and purposes. Focus is on the use of diagrams in the mathematics classroom to tell stories in order to trigger narrative modes of thinking (a là Bruner) as part of the mathematical work. Through some examples from the written productions of students at different ages, we examine how the task of telling a story about a given diagram gives rise to multiple views of the diagram. We use this analysis to begin characterising ways in which the mathematical relationships embedded in a diagram acquire a character that is both affective and aesthetic and to suggest implications of such use of diagrams in and for mathematics education.

Keywords Diagrams · Story · Mathematics · Affect · Aesthetics

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From diagrams to stories

Diagrams in mathematics education are often treated as an accessory to mathematical problem solving, a tool that can support understanding while dealing with a situation that can be modelled through an organised set of arrows, lines, or circles. They may also be used as part of tasks that prompt exploration of geometrical properties (in paper and pencil or in digital environments), or may appear as sketches in the practice of teachers, students and mathematicians, complementing symbols and words, and in textbooks to present definitions or exemplify properties (e.g., Arcavi, 2003; Inglis & Mejía-Ramos, 2009; Dimmel & Herbst, 2015; De Toffoli, 2023).

In this article, we discuss data from teaching experiments conducted in several primary grades and focus on the use of specific diagrams in the mathematics classroom. As we will detail in the following sections, students were asked to tell stories about given diagrams or to think of a title for them. These types of tasks involve very different interactions with the diagram from those mentioned above, which are common practice in the mathematics classroom and see the diagram primarily as an aid. Indeed, in these tasks we see the potential of using mathematical diagrams as a means of expression, an open window into students' mathematical experiences.

Our primary goal in this article is to explore innovative and relevant ways in which diagrams can be used in the classroom to support creativity and mathematical sense making. In doing so, we investigate what a pedagogy of the diagram in the mathematics classroom might look like.

In other works, we have focused on the study of diagrams produced by students in the classroom to talk about the agency of the diagram in mathematical activity (Ferrara & Ferrari, 2017) or to see diagrams as the site of the tension between abstract and concrete in mathematics (Ferrara & Ferrari, 2022).

In this article, we want to further develop this line of research by studying how dealing with diagrams is not only about *seeing* relationships, but about *creating* new forms of relationships, which we aim to begin characterising. Therefore, a second goal is to discuss the kinds of relationships that could emerge in creating stories on diagrams.

We will investigate both issues through presenting case studies, which help us delve into the complex nature of students' experiences with the diagram.

Diagrams in mathematics education

The literature in mathematics education shows the potential of representations, more generally, for the teaching and learning of mathematics (e.g., Duval, 2006; Finesilver, 2022). For Duval (2006), semiotic representations are a gateway to mathematical objects and an essential condition to the development of mathematical thought. Signs, or semiotic systems of representation, play an important part in mathematics to work on mathematical objects and with them. This aspect is crucial for mathematics education, especially if we think of learning mathematics as a material and discursive practice that develops around mathematical objects. Both discursive and visual representations, for example, are associated with the treatment of a geometrical figure. Duval speaks of four different *representation registers* in mathematics to underline that mathematical activity consists of semiotic systems that permit transformations of representations. As Finesilver (2022) observes, “a great deal of research has contrasted mathematical representations which are considered to be ‘formal’, ‘abstract’, or ‘standard’ with those considered informal, concrete, or nonstandard (chosen terminology varies) as though these were distinct binaries” (p. 272). But Duval already saw as deceptive the opposition between comprehension as conceptual and semiotic representations as external to the mind. The very distinction between abstract and concrete in mathematics seems to be challenged by a more modern view of representations as dynamically partaking in thinking processes. The specific focus on diagrams in this article helps us to account for the relevance of semiotic registers in mathematics while attending to the material nature of mathematical activity and the body-diagram interaction in mathematics.

The term *diagram* in mathematics education is often used with different conceptions and meanings, many times referring simply to pictorial schemas or to informal, *naïve* representations. However, it has an extraordinary evocative power, covering a whole galaxy of interconnected ideas such as graph, schema, form and arrow, which seem to recall the potential to interact with mathematical objects.

Bender and Marrinan (2010) have argued that diagrams function both as representations *and* objects situated in the world of the observer. They claim that the diagram—which they define as a typically reductive rendering, usually drawn, with few if no colours, notated, coded and labelled or referenced through explanatory captions—has played a crucial role historically in concretising processes and in visualising the temporality of action and motion. In this article, we use the term *diagram* to refer to any *inscription that organises space and articulates relations*, drawing on the philosopher and mathematician Charles S. Pei- rce. For Peirce, “the diagram is a skeleton-like sketch of its object in terms of relations between its parts, but what makes it apt to reason with, to experiment on, respectively, is the fact that it is constructed from rational relations” (Stjernfelt, 2000, p. 363). Therefore, a diagram is not just an image, it must be interpreted. It requires the subject to actively participate in giving meaning to its articulation and to experiment on it. It becomes a thing in motion, a site of activity. Unlike a photograph or a painting, a mathematical diagram fails to position a viewer in one designated or legitimate location from which to observe, and thus it invites a more active and yet virtual engagement. It is this kind of engagement that interests us in mathematical activity.

Our approach mainly aligns with Châtelet’s (2000) perspective that considers mathematical objects as physico-mathematical entities, which function in light of a tension between the real and the virtual, the experimental and the abstract, the physical and the mathematical. In other words, mathematical concepts are “material objects on and with which mathematicians perform thought experiments. These thought experiments are not the disembodied mental ruminations with which we typically associate mathematical thinking but, rather, gestural choreographies and exploratory diagramming” (Sinclair & de Freitas, 2014, p. 562).

According to Châtelet, diagrams created by mathematicians shed light on how mathematics evolves and how new concepts or strategies are developed. Diagrams speak directly to the transformative character of mathematics. In the hands of mathematicians, diagrams are able to open up new mathematical worlds and new stories about mathematics.

This ontological relevance of diagrams in mathematics requires a didactic focus on the use of diagrams in the mathematics classroom and the mathematical stories we can trigger and tell with them.

Diagrams and their stories

Rotman (2012) points out that the Bourbaki programme in the twentieth century has excluded diagrams from the picture in its set-theoretical framing of mathematical objects, privileging the static formalism over the dynamic becoming of concepts. From Châtelet's perspective of diagrams as operating in the space between the body and the written, "the exclusion of diagrams both protects the purity of mathematical objects from any kind of physical or corporeal contamination and cannot but be silent regarding mathematics' becoming" (Rotman, 2012, p. 251). Briefly speaking, set theory mandates an interior epistemology of mathematics. In contrast, the more recent field of category theory mandates an exterior epistemology, which understands objects relationally, not as fixed, isolated and self-contained entities. Categories deliver a dynamic logic using (schemes of) arrows to understand and practice mathematics as *diagrammatic thought*, and the fact that they "deploy diagrams in a substantive way differentiates them from set theory, but conveys no hint of a deeper sense of diagrams, namely, the pivotal role they play in mathematical ontogenesis" (Rotman, 2012, p. 255). Paton (2025) also calls to attention the power of category theory and its diagrams to describe objects in relation and therefore how they act in context. Thinking through the lens of category theory, the author experiments with diagrams to reimagine equality in mathematics education and thus reframe both knowledge and assessment. Category theory provides multiple ways of thinking through the complexity of relations and aligns with entangled ontologies. Infinity categories even suggest that in mathematics education we look for the relations of the relations, telling us that "all differences matter, and all details make a difference" (Paton, 2025, p. 371).

We return here once again to Châtelet for pointing out the force of diagrams in exactly being the very means of ontogenesis, a main element of the becoming of objects and relations in mathematics. Because working with diagrams necessarily involves the body, the body–diagram interaction becomes the focus of mathematical activity.

In particular, we see diagrams as sophisticated mathematical devices for thinking about and communicating mathematics. A diagram is in fact a kind of geometric artifact, which has a history and a purpose. It is geometric insofar as it captures spatial relations. It was certainly constructed with a purpose (e.g., to model a situation or solve a problem), and the way its construction may have occurred can be told, in written or oral form. Therefore, its "history" is the story or narrative of the diagram's construction. But the result of this process of "story telling," a narrative, could be different if produced by different storytellers. At the same time, any dia- gram can trigger some narrative when one is asked what mathematical story the diagram tells. The story, in the sense just discussed, introduces temporality (time) into the diagram, helping to realise how its construction entails a series of steps that are specific to the particular constraining relationships embodied in the diagram. It positions the storyteller through point of view and voice and may also reflect a tie to the diagram, a certain way of looking at and, perhaps, perceiving it, as well as a way of situating it in a more or less familiar context.

This is relevant to the questioning of the kinds of stories students might have about their school mathematics, which is often felt as too abstract, too difficult, too distant. Narrative modes of thinking in relation to mathematical understanding are discussed in mathematics education literature. Healy and Sinclair (2007), for example, see them as playing “a part in the claiming of mathematical territories as our own, in navigating mathematical landscapes and in conversing with the mathematical beings that inhabit them” (p. 3).

Drawing on Bruner’s (1996) vision of narrative as “a mode of thinking” (p. 119) with specific characteristics, these researchers first identify narrative thinking in forms of mathematical activity undertaken by mathematicians, then examine the kinds of stories generated by students in the mathematics classroom, in contexts where interaction with computational objects occurs to make sense of mathematical concepts. They distinguish narrative thinking from paradigmatic thinking. While the second is an explicit reasoning about the world of facts tending towards abstraction (more resonant to most conceptions of mathematics), the first may linguistically express different kinds of ingredients, like “possibility, wishes, emotion, judgements or statements that may be contrary to the facts in hand” (Diezmann & English, 2001, p. 6).

Interestingly, on the one side, paradigmatic modes and narrative modes seem to appear together, in tension, for example in mathematicians’ thinking. On the other side, the more personal, private and interpretative, but less formal character of the latter seems very interesting to better understand the nature of the mathematical experiences we give our learners.

In this article, we are interested in pursuing exactly this kind of investigation, by profiting from the power of diagrams. The interactive nature of diagrams opens up the possibility of a different, dynamic way of using them in mathematics teaching and learning than using them simply as visual representations that present information in a spatial layout (see Diezmann & English, 2001).

In addition, diagrams are often studied in terms of if and how they can help students in problem-solving situations, even non-routine ones, and whether their presence positively influences the students’ performance (e.g., Pantziara et al., 2009). While this strand of research demonstrates once again the power and significance of diagrams, there is little investigation concerning the kind of tasks that can be associated to diagrams to think of them as tools for creating new, non-conventional mathematical activities. Instead, depending on the task at hand, we can mobilise the mathematics of a diagram along different lines of thought.

Therefore, we also want to see how non-routine tasks about *telling stories* for a given diagram could elicit a mathematical experience that is infused with aesthetic and affective elements, which make the mathematics in the diagram more personal and familiar. (This approach is opposite in direction from the one taken by Healy and Sinclair (2007), who analysed the stories that emerged out of specific diagrams.)

Aesthetics and affect in mathematics education

Aesthetics and affect are two dimensions that are of interest for mathematics education researchers. In this section, we consider only a few arguments about them, which we see as relevant to our discussion in this article.

Aesthetics in mathematics can refer to a sense of beauty or a judgement of value that is assigned to a particular mathematical formula, object, solution, and so on. Since ancient Greece, the affinities between aesthetics and mathematics have been highlighted, especially in the work of Aristotle. In the book *Metaphysics*, he suggests that beauty can be described through characteristics such as order, symmetry and definiteness. In doing so, Aristotle establishes a relationship between mathematics and beauty, which would hold until the eighteenth century: "mathematics themselves are beautiful; furthermore, they provide a model for beauty" (Jullien, 2012, p. 93).

According to Jullien (2012), since then, "the influence of mathematics on speculations concerning beauty, and more generally on the analysis of aesthetic properties, has already been subjected to serious scrutiny. Yet, […] none of these studies highlight the fact that mathematics have in return been a subject of aesthetic inquiry" (p. 95). Scholars like Wells (1990) have discussed aesthetics in mathematics as dependent upon historical and cultural contexts, while mathematicians like Hardy (1940) had highlighted rather objective criteria that make a mathematical object beautiful *per se*. This contraposition of approaches reveals the intricated nature of the relationships between mathematics and aesthetics. [1] Interestingly for our discourse, Sinclair (2004) identifies three distinct roles that aesthetics play in mathematical inquiry:

- *evaluative*, which concerns judgments about beauty and elegance of entities such as proofs and theorems;
- *generative*, which is related to the affective response elicited, which might guide the action of the mathematician;
- *motivational*, which might attract mathematicians (or not) to particular lines of inquiry or mathematical fields.

Sinclair (2011) also acknowledges that these roles might not be valid for those who contrast the epistemic with the aesthetic sharply. This discussion is beyond the objective of this article, but the fact that mathematicians are moved by aesthetics in various ways opens room for considering the possibilities of students' aesthetic experiences in learning mathematics. In mathematics education, this is often analysed in relation to activities mixing art and mathematics, but aesthetics does not pertain necessarily to such kinds of activities nor to a specific mathematical object or concept. Rather, according to Sinclair (2008), "interaction of the emotion with understanding and mathematical meaning gives rise to the aesthetic artifact" (p. 34). Therefore, we can study aesthetic experiences entangled with affective forces that move the activity and transform the learner's encounter with mathematics. We do not intend to reduce all experiences of mathematics to the same emotional note. Rather, we are more interested in attending to the nuanced or tonal differences between one experience and another, highlighting how the specific mathematics at play is fuelling affective forces.

The positioning of the student towards mathematics surely influences the experience itself, in the same way as the kind of environment, tasks and materials matter. Nevertheless, we assume that aesthetics and affect always play a role in mathematical activity and are

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[1] We introduce the term *intricated* to suggest that the intricacy comes from the subjectivity of the perceiver rather than using the term *intricate* to refer to something that is objectively intricate.

generative of forms of visibility, sensibility and expression. Drawing on this, we propose here to discuss ways in which diagrams can elicit aesthetic and affective engagements with mathematics that allow the students to be positioned at the centre of the mathematical activity.

A diagram and a mathematical task

In this section, the focus is on a specific diagram that we offered, in the context of a teacher training program, to a large group of primary and secondary mathematics teachers, with the aim of reflecting on the importance of using diagrams and representations in mathematics as a didactic resource.

Other diagrams were also offered in the program, but we will not consider them all here. The diagrams were different from each other, but they all consisted of relatively simple geometric figures and incorporated various types of relationships.

We also present some tasks that were discussed with teachers before they assigned them to their students. We are in fact particularly interested in the extent to which we can design tasks that involve diagrams in a non-routine manner and in investigating how they allow exploration of new worlds and enrich the mathematical possibilities/experiences of learners.

The diagram

The diagram, which is the focus of this article, is presented in Fig. 1. In itself, the diagram incorporates mathematical relationships. From one point of view, one can see several geometric figures: a small square on the inside, and five non-standard hexagons (or L shapes), adjacent and similar to each other, which make a large square. From another viewpoint, six squares of different sizes can be seen overlapping. In this case, one can even see the side of each fol- lowing square halved (or doubled) in length, and that all the squares have a common vertex.

The diagram caught our attention because, although it is extremely simple (in a sense, it is just a set of squares), it has the potential to evoke interesting mathematical concepts including (but not limited to) recursive processes and proportional reasoning.

A feature of the diagram is the alternation between neighbouring figures (overlapping or not), which is made evident by the presence of different colours but can also be sensed when the diagram is in grey tones.

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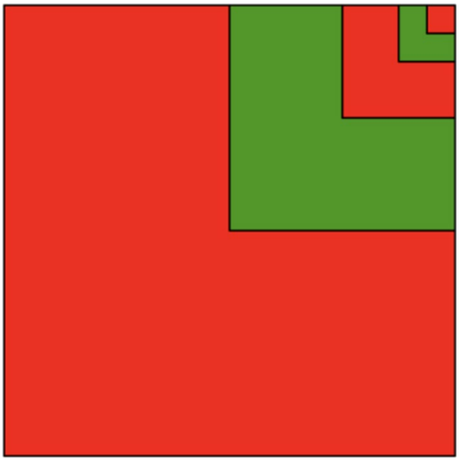

Fig. 1 The selected diagram

The task

In pursuing our original idea of using the diagram as a didactic resource, we discussed with teachers partaking in the program some ways of (inter)acting with the diagram that require more than just recovering information from its spatial layout. We are not so much interested in the diagram as a representation as we are in the diagram as a powerful tool to explore and mobilise mathematical relationships (and, therefore, the activity itself).
We assume that the task plays an essential role to prime the mathematical activity in this direction but maintain that the potential of diagrams in mathematical sense making has so far remained poorly understood. We have chosen to work with tasks that can help students create their own mathematical stories from working with diagrams: paying attention to the use of paradigmatic and narrative thinking (language) in accounting for a diagram reveals how students are engaging with the mathematics in the diagram. Regarding this engagement, there is a particular semiotics that is encoded into a diagram and reading the diagram requires a familiarity with it. A different kind of engagement can be given by intervening in the diagram, which may involve introducing new lines, codes, or movements. Some tasks we designed with the above commitment are as follows (a diagram is always provided at the beginning of the task):

- What can be the title of this diagram?
- Tell the story of this diagram.
- Tell a mathematical story for making this diagram.
- Explain how and why the following words [some specific words are provided] can be used to describe this diagram.
- What do you make of this diagram?

These tasks can all be seen as non-routine mathematics problems that call for the use of diagrams as problem-framing and problem-solving tools. In particular, some of them might destabilise students accustomed only to standard mathematical tasks. For example, what does "the title" of a diagram mean? *What is it*? It could be something that describes and summarises the diagram or something that the diagram reminds of, that is evoked by the diagram. However, this openness is relevant when we think that, in mathematics, questions can have more than one (correct) answer and multiple paths to solution. In short, with these kinds of tasks, diagrams become for the students' tools *to think with* (Turkle, 2011), on which to act and with which to interact.

At the same time, involving narrative thinking, the task allows for a different encounter with the diagram. It invites some freedom to look at the diagram or express how we are affected by it, so it also allows the diagram to affect us and be affected by us. Drawing attention to this dual relationship between us and the diagram helps account for the materiality of mathematical activity and the affective and aesthetic forces at play.

The following section presents the methodological choices of our study and some examples of work with the diagram illustrating types of student engagement at different ages. The students are all specifically tasked in this case with telling the story of the diagram.

The shaping of storylines by diagram

In this section, we offer an investigation of the way in which the students in this study have interacted with the diagram. In particular, we look at diagrams as partaking in the mathematical activity, agents that make mathematical relationships emerge from paper. We will see that aesthetic and affective forces are also circulating in the activity and operate at the heart of the act of mathematical sense making.

Methodological choices

In each class involved, the mathematics teacher (who participated in our training program as trainee or trainer) assigned the task to individual students. Each student was handed the diagram on a sheet of paper and asked to write the story on that sheet (if they wished, they could add a title). In some cases, the diagram was presented with a grid around it to provide students with a resource they could use as needed to explore mathematical relationships. Each teacher decided how much time to devote to the task in each class according to contextual constraints, approximately one to two hours.

The data was produced through carrying out the activity in school, with teachers who collected the materials and shared them with the researchers via an online shared folder. We selected four examples from these materials. One of the aims of the selection process was to have diverse data in its range that could account for the manifold ways in which students interacted with the diagram. Therefore, we selected data ranging from non-mathematical stories to stories about growth (both an everyday and a mathematical experience) to stories that play on specific numbers or mathematical objects.

The selection provides the reader with a glimpse into the wide diversity of responses that the task allowed, without exhausting them all. Our aim is not to categorise the possible stories; rather, we want to exploit them to bring out the richness of the mathematical activity with the diagram, which is why we speak of characterising. To this aim, the study follows a qualitative analysis, under a descriptive approach. Likewise, the data corresponding to the students' written productions have been looked at in terms of the work on and around the diagram. We see the diagram and its associated story as the unit of analysis for our investigation and adopt a method of writing to understand data (St. Pierre & Pillow, 2000). In doing so, we describe and examine ways in which the diagram partakes in the activity as a tool to think with, which mobilises relationships. We also seek to shed light on how narrative thinking triggers a more personal engagement with the mathematics of the diagram, enabling aesthetic or affective encounters with it. In short, we show how the diagram affects and is affected by the student and use the story as a narration of the personal engagement.

The selected examples are presented as a series in the following. We briefly introduce and comment on a written protocol, which is included as a figure. The figure shows both the diagram and the work done on and with it, including a text in Italian. The translation of the text is presented in italics (any marks or sketches in the original story are indicated in parentheses). In addition, we use italics in the comments to refer to words explicitly used by the students in the written protocol.

A big square

The first example is shown in Fig. 2 and comes from the work of a grade 3 student. The diagram has been coloured and called "quartatone." This an incorrect name in Italian, a modification of the word "quadratone," which means "big square" (this is the chosen title). The addition of drawings of planets and stars, of the sun, the moon and the earth, and the blue night sky and bold, not quite straight lines enriches the diagram.

We interpret the added drawings as a way of making sense of the mathematics in the diagram. They give a spatial character to the very short story told by this young student, while also showing how affect and aesthetic are at play. The blue sky where the planets and stars live occupies the farthest space, the one enclosed by the largest hexagon, as if to remind us of the universe. A perception of depth emerges from the adjacent hexagons whose contours have been emphasised but not coloured. In the space in between is a kind of spaceship moving towards space.

The story seems to be one of a space travel that leads far away. For this student, "far away" may have to do with a space where he feels *accepted*. We cannot be sure about the few words he uses to close his story, but we know from the teacher that he has some learning problems and find it fascinating how this kind of work led him to write: *Now I like to do new things because I feel accepted*. Such feeling seems to be more important than the rest in this case, so much that it is expressed before other thoughts. We also see it emerge from the precise way the diagram acquires aesthetic features that are captured by the drawn elements and colours more than by other written words.

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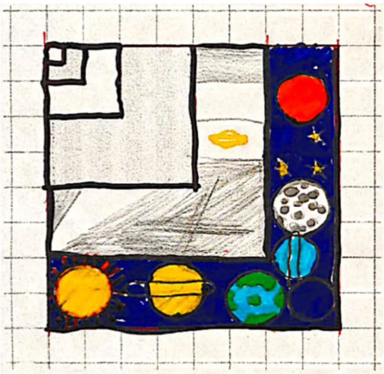
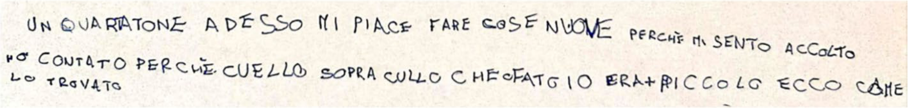




UN QUARTATONE ADESSO MI PIACE FARE COSE NUOVE PERCHÉ MI SENTO ACCOLTO
HO CONTATO PERCHÈ CUELLO SOPRA CULLO CHE OFATGIO ERA+PICCOLO ECCO CAME
LO TROVATO

Translation. *A big square. Now I like doing new things because I feel accepted. I counted because the one above, the one I made was smaller, this is how I found it.*

Fig. 2 The first protocol: A quartatone

The life of a square

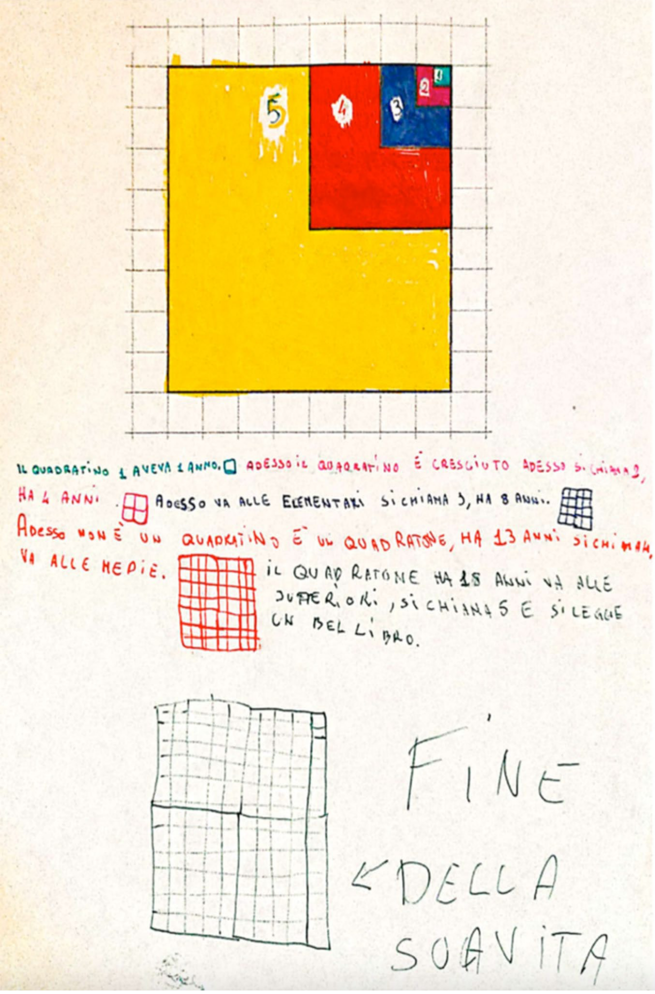
The second example comes from another grade 3 student and is presented in Fig. 3. The diagram has been coloured again, some numbers have been added to it, and drawings of squares of different sizes appear in the story (no title has been given in this case). In the original story, the words "quadratino" and "quadratone" refer to a small and a big square respectively.

We see that, in the diagram, the five adjacent figures have been numbered and filled with a specific colour, following an ascending order from the smallest to the largest figure. In the story, subsequent sentences follow the same order and are written using a different colour and referring to a specific number. In this case, the colour used in the text is the same as that used to colour the corresponding figure (with the exception of the fifth figure for which yellow was not used in the text).

The story reveals that the five figures are seen as squares and refer to different stages in the life of a single square, growing from 1 to 18 years old. The story unfolds through the changes a *small square* undergoes, in age, size and type of school (elementary to high school). The mathematics in the diagram gives rise to the temporality of the story, with a unit square growing to four pasted copies of itself to form a new, bigger, 4-year-old square, then to four more pasted copies of this new square forming a bigger, 8-year-old square, and so on. The square changes *names* throughout the story and becomes a *big square* with the transition to *middle school*.

The change in size of the different squares in the diagram is seen as the change under- gone by the individual character in the story, personified by the smallest square. Aesthetic and affective elements emerge from the way the story interweaves aspects of real life (going to school and changing age) and mathematical growth and regularity (the growing rate of the little square, in the sketches).

For this student, the diagram seems to personify a life that could be his own, where the experience of growth is strictly associated to changes in height (size), birthdays and corresponding changes of school.



IL QUADRATINO 1 AVEVA 1 ANNO. ADESSO IL QUADRATINO È CRESCIUTO ADESSO SI CHIAMA 2, HA 4 ANNI. ADESSO VA ALLE ELEMENTARI SI CHIAMA 3, HA 8 ANNI. ADESSO NON È UN QUADRATINO È UN QUADRATONE, HA 13 ANNI SI CHIAMA 4, VA ALLE MEDIE. IL QUADRATONE HA 18 ANNI VA ALLE SUPERIORI, SI CHIAMA 5 E SI LEGGE UN BEL LIBRO.

FINE
DELLA
SUA VITA

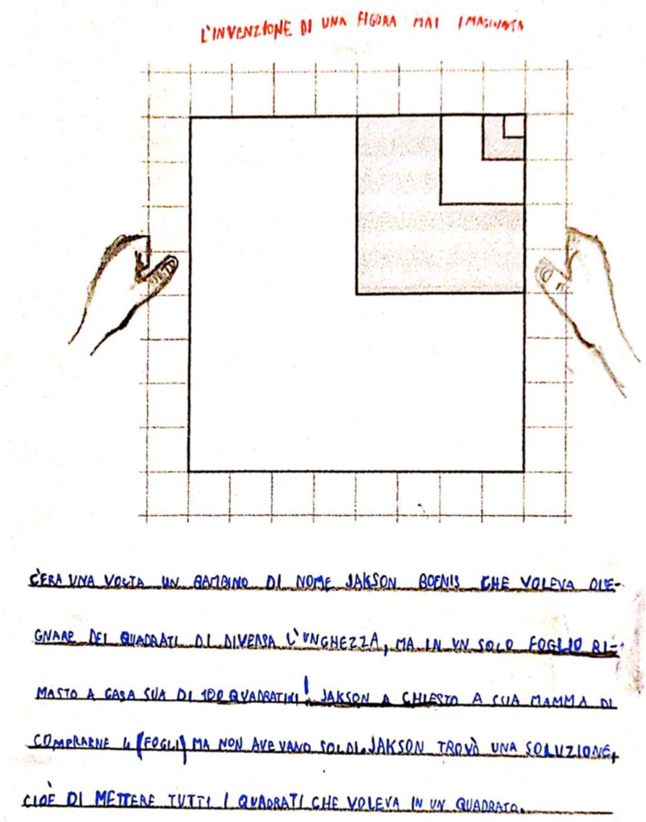
Translation. *The small square 1 was 1 year old (sketch of a little square resembling the little square in the diagram). Now the small square has grown up, now it is called 2, it is 4 years old (sketch of a square made of 4 copies of the small square). Now it goes to elementary school, its name is 3, it is 8 years old (sketch of a square made of four copies of the previous one, then 16 small squares). Now it is no longer a small square, it is a big square, it is 13 years old, its name is 4, it goes to middle school (sketch of a square made of four copies of the previous one, then 64 small squares).The big square is 18 years old, it goes to high school, its name is 5 and it is reading a good book. End of its life (sketch of a square bigger than the previous one).*

Fig. 3 The second written protocol: A diagram that personifies a real life

A figure never imagined

Figure 4 shows the third example that again comes from the same class as above. The diagram was not coloured by this student, but two hands were drawn on the left and right sides of the grid. The word "quadratini," which appears in the Italian text, is associated with small squares.

The hands added to the sides of the grid seem to suggest from the beginning a specific point of view taken when looking at the diagram. The story unveils interesting elements of this perspective. The student talks about *a single sheet of paper made up of 100 small squares*: reference is made to the entire outer square and the use of the grid (supported by the hands) to see that it "covers" 100 small squares. These squares are the grid squares, not copies of the smallest square in the diagram. The narrative develops around them.



L'INVENZIONE DI UNA FIGORA MAI IMAGINATA

C'ERA UNA VOLTA UN BAMBINO DI NOME JAKSON BOENIS CHE VOLEVA DIEGNARE DEI QUADRATI DI DIVERSA L'UNGHEZZA, MA IN UN SOLO FOGLIO RIMASTO A CASA SUA DI 100 QUADRATINI! JAKSON A CHIESTO A SUA MAMMA DI COMPRARNE 4 (FOGLI) MA NON AVEVANO SOLDI. JAKSON TROVÒ UNA SOLUZIONE, CIOÈ DI METTERE TUTTI I QUADRATI CHE VOLEVA IN UN QUADRATO.

Translation. *The invention of a figure never imagined. Once upon a time there was a child named Jakson Boenis who wanted to draw squares of different lengths, but on a single sheet of paper left at home, made up of 100 small squares! Jakson asked his mother to buy 4 (sheets of paper) but they had no money. Jakson found a solution, which was to put all the squares he wanted into one square.*

Fig. 4 The third protocol: The invention of a figure never imagined

The side of the external square is 10 units long, where the unit is one grid square. This relates to the *squares of different lengths*, which Jakson, the character in the story, had to draw *on a single sheet of paper*. The focus is again on a group of squares: beyond the outer square, the four squares that are inside it (*to put all the squares he wanted into one square*). The mathematical relationships in the diagram are interpreted here as the solution to a real (albeit narrated) problem: Jakson wanted to draw the four squares separately on *4 sheets of paper*, but the lack of *money* to buy these did not allow him to do so. Then, the diagram is the result of the act of drawing the four squares all inside the outer square so as to use only one sheet of paper.

The story, told in the style of a tale (*Once upon a time*), seems to imaginatively bring out this act from the written words and the point of view assumed by the student, underlying the affective engagement with the diagram, which displays *a figure never imagined*. Aesthetically, we see how beautifully realised are the two hands, with the trait of the pencil very attentive to specific details, like nails and finger creases.

The big square family

The fourth example is shown in Fig. 5 and comes from the work of a grade 4 student. The written protocol is quite long: the story is written below the drawings and signs presented in Fig. 5 (upper left and upper right of the protocol, respectively; in this case, the original text is in the footnote, not in the figure).[2]

To let the reader better understand the protocol, we need to translate also the written words appearing above the squares drawn on the upper left, as well as the words connected with an arrow on the upper right. Concerning the first, we find (reading from left to right) the following four words: "Great-grandparents," "Grandparents," "Parents" and "Children." Concerning the second, the arrow expresses a relationship between members from previous groups (the relationship is given by "contains" written above the arrow). So, we can read (from top to bottom): "1 great-grandparent contains 4 grandparents," "1 grandparent contains 4 parents," and "1 parent contains 4 children."

This example illustrates how the mathematical relationships in the diagram are thought of as family relationships.

Affective forces are at play here in the way the story is told (we know from the teacher that this student lives in a rather large family), and the diagram is mobilised to give rise to new sketches and signs. While the size of the family and the regularity of the 1–4 relationships may not be typical of a real family, other types of relationships are nothing but real. We also see how the regularity of the square is associated with the regularity of the 1–4 relationship which is maintained and is at the same time mathematical (contains, which refers to how we can always divide each square into 4 smaller squares) and real (as each member of the family is associated with 4 younger members to look after). The age of the members of the family is associated with the size of the squares (each a different size based on age), therefore to their area, showing from where the 1–4 ratio comes.

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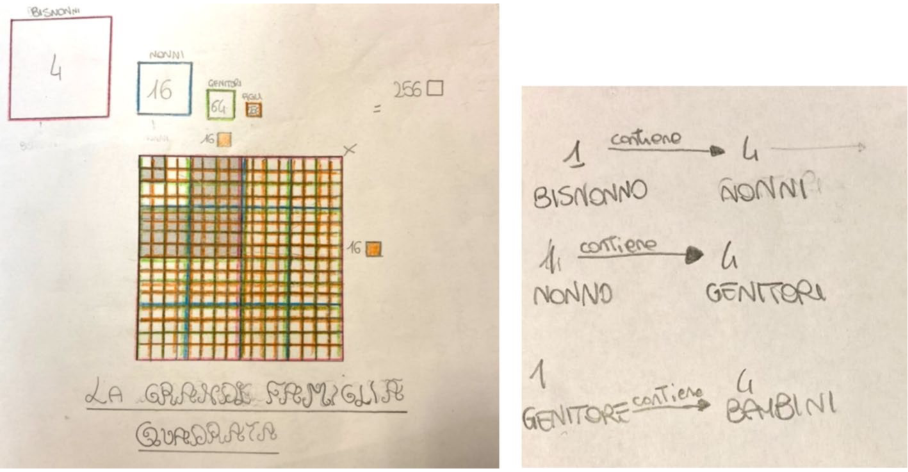
[2] Original text. "La grande famiglia quadrata. C'era una volta una famiglia molto molto numerosa. Era formata da bisnonni, nonni, genitori e bambini. Si chiamavano famiglia Quadrata, perché erano tutti quadrati, ognuno in base all'età di grandezze diverse. La cosa strana di quella famiglia era il modo di invecchiare perché quando si diventava sempre più grandi aumentava la grandezza della propria area: l'area aumentava perché ognuno, tranne i bambini, dovevano accudire i propri figli di qualsiasi era la loro età. Infatti i genitori che, in quella famiglia erano 64, dovevano accudire i propri figli cioè i bambini, ogni genitore aveva quattro bambini da contenere nella propria area, i nonni avevano l'area più grossa dei genitori, perché li dovevano contenere, ogni nonno doveva accudire quattro genitori che a loro volta in totale contenevano 16 bambini; i bisnonni erano i quadrati con la maggiore area perché dovevano contenere i nonni i genitori e i bambini, e i bisnonni erano solo quattro e ognuno conteneva altrettanti nonni. Una cosa che non accadeva mai erano i confronti tra bambini o tra genitori, tra nonni, tra bisnonni. Questo non succedeva perché, per esempio, tutti i bambini erano equiestesi e isoperimetrici, perché erano di diverse età ma erano pur sempre bambini! Così era per i genitori, per i nonni, e anche per i bisnonni, di cui però non c'era motivo di preoccuparsi dei confronti perché erano troppo impegnati a tenere e a far giocare 256 bambini in totale!"

The issue of care as a common task of the older members of the family, except for the children, seems to characterise the story. Aesthetic forces are also at play, for example, in the way in which colour is used to differentiate squares that speak to different groups of people both outside of and inside the diagram.



Translation. *The big square family. Once upon a time there was a very, very large family. It was made up of great-grandparents, grandparents, parents and children. They were called the Square family, because they were all squares, each a different size based on age. The strange thing about that family was the way they grew older because as they got older the surface area also grew: the surface area increased because everyone, except the children, had to look after their children of any age.*

In fact, the parents, who were 64 in that family, had to look after their children, that is, the kids, each parent had four children to contain in their own area, the grandparents had an area larger than the parents, because they had to contain them, each grandparent had to look after four parents who in turn contained 16 children in total; the great-grandparents were the squares with the largest area because they had to contain the grandparents, parents and children, and the great-grandparents were only four and each contained as many grandparents.

One thing that never happened were comparisons between children or between parents, between grandparents, between great-grandparents. This didn't happen because, for example, all the children were equally extended and isoperimetric, because they were of different ages, but they were still children! This was the case for parents, grandparents, and even great-grandparents, but there was no reason to worry about comparisons because they were too busy caring for and playing with 256 children in total!

Fig. 5 The fourth protocol: The big square family

Conclusion

In this article, we have discussed some stories originated from working with a specific diagram, which was proposed to several primary school classes, with the task of telling a story for the diagram. The diagram and its associated story have been seen as our unit of analysis, with the story used as a narration of the personal engagement with the diagram, and the diagram interpreted as a tool *to think with* rather than an aid to mathematical thinking. In this way, the qualitative descriptive analysis has drawn attention to the written productions of the students with specific focus on the work around and on the diagram. One of our goals was to gain insight into the kinds of relationships that could emerge in creating stories on diagrams. The selected examples have helped us present a variety of experiences that are elicited by the narrative thinking associated with the diagram. While recognising the mathematical relationships that are at play in the story, the analysis has also sought to characterise these experiences in terms of the aesthetic and affective engagement of the students with the diagram. In particular, we saw that some students connected real-life experience (such as that of growing up) to the regularity of the squares in the diagram, or the proportionality of these figures to hierarchical and affective relationships within a special family.

In telling the story, the diagram was mobilised by the students through inserting new lines, drawings or signs that were directly related to the story while remaining coherent to the mathematics in the diagram.

In the analysis, we recognised the generative and motivational roles of the aesthetics, which Sinclair (2004, 2011) found for mathematicians and which for our students created productive engagement with the diagram, diversifying feelings of belonging (Turkle, 2011). Indeed, we have seen that stories are *generative* of affective responses and infused with *motivation* towards specific lines of narration and investigation together.

Summarising, we propose thinking of diagrams as particular tools with some potential to evoke and trigger particular kinds of relationships, which we can characterise as aesthetic, affective *and* mathematical. The work with the diagram surfaces affective relation- ships, both in terms of the characters involved in the story and in the ways in which the work with the diagram makes the students feel engaged and accepted.

We also aimed to investigate what a pedagogy of the diagram in the mathematics classroom might look like. We are aware that we cannot account for all aspects of the experience by solely looking at the writing, but we are also aware of the quality of these kinds of experiences. The feedback we received from teachers who participated in the training program was enthusiastic. One common observation was that this work with diagrams gave all the students the possibility to be part of the mathematical discourse developed in the classroom. All students had (and felt they had) something to say about the diagram, resulting in a range of solutions and approaches to the task. Therefore, we believe that this perspective may open up new lines of activity in the mathematics classroom that is inclusive at all levels.

In conclusion, we strive to pursue a commitment towards creating *a pedagogy of the diagram* that is not only limited to avoiding possible misconceptions linked to stereotyped figures, nor solely to analysing the pitfalls in the use of representations. Rather, we aim at eliciting sensitivity towards the use of diagrams in the mathematics classroom as powerful tools for engaging students in mathematical sense making. The idea of pedagogy of the diagram does not focus on problem-solving tasks that have unknown but definite solutions. Instead, we suggest a pedagogy where non-routine, open-ended tasks can motivate the integration of the task with life experience at large, making mathematics a more personal and enjoyable experience.

We do not expect this article to be exhaustive with a study restricted to young children. We invite other researchers to build on our work to enrich understanding of the pedagogy of the diagram by exploring it in other contexts—for example with older students or mathematicians—or in other cultures.

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