# Markets and Politics:
## Essays at the Intersection of Industrial Organization and Political Economy

**A Thesis submitted for the degree of Doctor of Philosophy**

**February 2025**

**Matteo Broso**

**Università di Torino**

**Vilfredo Pareto Doctorate in Economics**

UNIVERSITÀ DI TORINO

# Abstract

I study the interplay between markets and politics. Across three different chapters, I offer novel theoretical contributions to the fields of Industrial Organization and Political Economy.

In the first chapter, I study how political connections affect the optimal privatization policy in markets where public firms compete with private ones in quality and prices. In the second chapter (co-autored), we study how market concentration affects firms' lobbying. We provide a welfare analysis of mergers that takes into account mergers' political economy effects. In the third chapter (co-authored), we study how the extent of competitiveness of elections affect firms' campaign contributions.

These results highlight the importance of studying markets and politics as interconnected systems, where changes in one can influence the outcomes of the other.

**Ph.D. Candidate:** Matteo Broso

**Ph.D. Supervisors:** Andrea Gallice, Ignacio Monzón

**Ph.D. Cycle:** XXXVI

**Statement of Joint Work:** I declare that chapters 2 and 3 are co-authored with Tommaso Valletti and that I contributed to 50% of this work.

# Acknowledgments

I am grateful to my advisors, Andrea and Ignacio, for guiding me through this journey.

I extend my gratitude to my co-author Tommaso, for shaping my interests and curiosity.

I also thank all of my co-authors on the papers that are not part of this Thesis. I have learned something from each of them.

Finally, I thank all faculty and students at the Collegio Carlo Alberto, the University of Turin, the Polytechnic University of Turin, and Imperial College London with whom I interacted and exchanged ideas. This was the best part of the job.

I dedicate this work to the memory of my grandfather Claudio.
I wish he could read it because, somehow, he contributed to its writing.

# Table of contents

# Introduction

In this Thesis, I study the interplay between markets and politics. Each chapter explores this interaction in a different context: privatization policy (chapter 1), mergers and lobbying (chapter 2), and firms' campaign contributions (chapter 3).

In the first chapter, "*Privatization under Political Ties*", I study the impact of privatization in markets where a public firm competes with a private one in the provision of goods with different quality. Unlike previous literature on this topic, I assume that the public firm is politically connected. I show that public companies can gain market power and make profits due to their political connections, provided they are not too strong. Public firms may commit to pricing strategies that originate from political objectives but make them more profitable than private profit-seeking ones. This result challenges the conventional wisdom according to which political connections of public firms are detrimental to efficiency and profitability. When instead political connections are strong, public firms charge very low prices and then make negative profits to please voters, discouraging the entry of private competitors and resulting in high consumer surplus but low social welfare in equilibrium.

In the second chapter, "*Mergers, Lobbying, and Elections: Is there a 'Curse of Bigness'?*" (co-authored with Tommaso Valletti), we study how market concentration affects lobbying. We propose a political agency model with adverse selection and moral hazard. We show that mergers increase firms' incentives to lobby, potentially harming consumers through increased political power rather than market power. However, firms' increased political influence also enhances voters' ability to screen corrupt politicians. Our results challenge traditional antitrust policy focus solely on mergers' price effects (consumer welfare standard).

In the third chapter, "*Betting on the Right Horse: Corporate Campaign Contributions and Closeness of Elections*" (co-authored with Tommaso Valletti), we study how closeness of elections affects campaign contributions from corporations. We propose a model in which a corporation can offer contributions to office-seeking political candidates in exchange for favors. The model predicts that candidates in close elections may receive more funding because of both demand and supply effects. We use U.S. House election data from 1974 to 2014, and leverage the quasi-random variation in districts' competitiveness induced by incumbents' deaths, to provide empirical evidence supporting this claim. We find that corporate campaign contributions increase in districts following an incumbent's death relative to other districts in the same electoral cycle.

These results highlight the importance of studying markets and politics as interconnected systems, where changes in one can influence the outcomes of the other.

Table of Contents

# Chapter 1

Table of Contents

# Privatization under Political Ties*

Matteo Broso†

January 3, 2025

**Abstract**

I study a product differentiation model with endogenous entry where a politically connected public firm competes with a private one. Consumers are heterogenous in their willingness to pay. First, I argue that because of political ties, the public firm may mimic the preferences of the consumer with the median type. Then, I show that as privatization increases, the market outcome shifts from an inefficient public monopoly to a duopoly, where the public firm can even be more profitable than the private one, and welfare is higher. However, full privatization is not socially optimal as it implies excessive product differentiation.

**Keywords** Mixed Markets, Privatization, Product Differentiation.
**JEL Classification** L33, H44, D72.

*I thank Andrea Gallice and Ignacio Monzón for their help and support throughout this project. I also thank Vincenzo Denicolò for his advice during the initial phases of this research. I thank Rossella Argenziano, Federico Boffa, Angelica Bozzi, Carlo Cambini, Dino Gerardi, Luca Lambertini, Carol Propper, Enrico Mattia Salonia, Lorenzo Stanca, Tommaso Valletti, Davide Vannoni, Andre Veiga, and various seminar audiences for valuable comments. All remaining errors are my own.

†University of Turin and Collegio Carlo Alberto.

# 1 Introduction

State-Owned Enterprises (SOEs) compete with private ones in many markets. A recent IMF study reports that "*state-owned enterprises' assets are worth \$45 trillion, equivalent to half of global GDP*" (IMF [2021]).[1]

Oftentimes, SOEs compete with private firms in the production of goods of different qualities. Previous work refers to these markets as mixed oligopolies. Benassi et al. [2016] show evidence that SOEs usually produce goods of lower quality than their private competitors. However, that is not necessarily the case. Many public schools are of better quality than their private competitors both in the US and in Europe (Brunello and Rocco [2008]).

How does state ownership affect welfare and firms' profitability? On the one hand, political connections could negatively affect markets' allocative and firms' productive efficiency (Boycko et al. [1996]). On the other hand, in the interests of consumers and voters, SOEs may reduce markups and tackle externalities, thereby increasing consumer welfare (Willner [2001]). To answer this question, I focus on SOEs' incentives. While it is reasonable to assume that private firms maximize profits, the objective function of SOEs is not straightforward. Previous work assumes that SOEs maximize welfare. This assumption can be appropriate when benevolent technocrats run public firms. However, it does not capture the incentives of politicians who may control public firms. Moreover, the public authority may own only a share of the stocks of a company (partial privatization).

In this paper, I study a product differentiation model with endogenous entry where a politically connected public firm (a SOE) competes with a private one. Consumers are heterogeneous in their willingness to pay for quality (which is a proxy for income) and buy one unit of an indivisible good. Firms' marginal costs are increasing in quality. The timing of the game has four steps. First, firms choose whether to enter the market or not. Second, firms choose the quality of their goods. Third, firms choose prices. Finally, consumers buy one of the two goods.

I capture partial privatization by assuming that the public firm maximizes a convex combination between its own profits and the median consumer's utility. I provide a political economy micro-foundation of this objective function. Since consumers' preferences satisfy the single crossing property, the median voter

[1] See Smith and Trebilcock [2001], Heywood et al. [2021], and the references therein, for evidence about the presence of SOEs in modern economies.

theorem applies (Gans and Smart [1996]). Therefore, the public firm can maximize its consensus by simply aligning its preferences with those of the consumer with the median type (median consumer henceforth). This simple framework allows me to study the effect of a politically tied public firm's privatization on market structure, welfare, and profits.

I show that a Subgame Perfect Nash Equilibrium exists for any possible degree of privatization. The degree of privatization determines the market structure. When privatization is low, the public firm produces a high quality and sets prices below marginal costs. The private firm has no incentive to enter, so the market is a public monopoly.[2] In the public monopoly equilibrium, welfare can be negative because consumer surplus does not compensate for the losses of the public firm. Welfare increases with privatization.

When privatization is high enough, the market is a duopoly. There are two classes of duopoly equilibria, depending on which firm produces for the majority of consumers. In the first (second) class, privatization is low (high), and the public (private) firm produces for the majority of consumers. Each class features a equilibrium where the public firm produces the high quality good and another where the private firm produces the high quality good. Within each class, these two equilibria are payoffs- (and welfare-) equivalent.

The degree of privatization also determines market profits. If the degree of privatization is sufficiently low, the public firm makes negative profits. Surprisingly, if the degree of privatization is neither too high nor too low, the public firm makes higher profits than its private competitor. The intuition is simple. By committing to a political strategy, the public firm creates a lot of "local" monopoly power. The private firm is relegated to serving the very rich or very poor consumers and has little market power. The commitment device works as follows. The public firm produces an "intermediate" quality level, attracting the majority of consumers. The private firm differentiates its good for the minority of consumers whose willingness to pay is far from the median (either very high or very low). The private firm suffers from fierce competition. Its clients would only have to sacrifice a little utility if they had to buy the public firm's good, and so cannot be charged a too high markup. On the contrary, most of the public firm's customers would have to sacrifice a lot of utility to buy the private firm's good. Then, the public firm can charge a higher markup. To the best of my knowledge, this result is new to the literature on mixed oligopoly. If

[2] I allow the public firm to run a deficit. I introduce a strict budget constraint as an extension.

privatization is high enough, the private firm faces higher demand and makes more profits than the public firm.

The welfare-maximizing degree of privatization is interior. When the public firm only cares about the median consumer, the market is a public monopoly, and welfare is low. In the duopoly equilibria, if privatization is low, consumers enjoy low quality levels, and firms make small profits. On the contrary, when privatization is too high, excessive product differentiation causes high markups that harm consumer surplus. I show evidence of a trade-off between high quality levels and low markups. When privatization is high, no firm wants to invest in quality without making consumers pay for the cost of the investment. When privatization is low, the public firm would be willing to do so, but this would come at the cost of large losses.

The result on the social desirability of partial privatization is reminiscent of Matsumura [1998], although the intuition behind the two results is different. In Matsumura [1998], a welfare-maximizing public firm and a private one play a Cournot game under exogenous product differentiation and exogenous market structure. In my model, partial privatization is needed to induce the private firm's entry (which would not occur under zero privatization) and to limit the excessive product differentiation that characterizes the equilibrium under full privatization. Endogenous entry and product differentiation are two key determinants for the social desirability of partial privatization. These these forces are not at play in Matsumura [1998]. Moreover, in my setup, if the public firm maximizes welfare or consumer surplus, the equilibrium is First Best.[3] This is the case when the public firm cares about the median consumer's utility, wherein partial privatization is needed to restore efficiency.

In the Appendix, through some extensions, I explore the robustness of my results. First, I introduce a strict budget constraint on the public firm's maximization problem. The constraint can improve welfare, but partial privatization is still socially optimal. Second, I allow consumers not to buy any good (partial market coverage). In this case, I show that privatization decreases the share of buyers in equilibrium. Hence, privatization may raise inequality concerns, especially in the case of "essential" goods, like education or healthcare.

I contribute to the Industrial Organization literature on competition between

[3] In Matsumura [1998], zero privatization is socially optimal in a public monopoly or in a duopoly where the public firm cares about consumer surplus only. In Ishibashi and Kaneko [2008], instead, partial privatization is found to be socially desirable in a vertically differentiated duopoly where quality is not costly. In this paper, the cost of quality plays a crucial role, limiting the private firm's ability to compete with the public monopolist under zero privatization.

public and private firms. This literature usually employs the standard product differentiation model of Tirole [1988] in a duopoly where a welfare-maximizing firm competes with a private one.[4] Contributions differ from each other in the following dimensions. On the supply side, one could assume different cost structures and welfare functions for the public firm's objective function; on the demand side, full or partial market coverage and different distributions for consumers' willingness to pay for quality. With uniform distributions of consumers' types, Grilo [1994] considers a cost function increasing in quality and linear in quantity. She shows that the mixed duopoly with a welfare-maximizing public firm can replicate the First Best equilibrium outcome.[5] Delbono et al. [1991] extend this result to the partial market coverage case with a cost function quadratic in quality. Benassi et al. [2016], Benassi et al. [2017] and Laine and Ma [2017] relax the uniform distribution assumption for consumers' types and derive conditions for the social desirability of privatization. Some papers also relax the welfare-maximizing assumption. For instance, the public firm is subjected to price regulation in Estrin and De Meza [1995]. Klumpp and Su [2019] assume that public firms have a preference for re-distribution. In Benassi et al. [2016], the public firm cares about consumer surplus only. Inoue et al. [2009] assume that the public firm cares only about the welfare of its clients. Some papers consider the case of partial privatization (Matsumura [1998], Lu and Poddar [2007], Ishibashi and Kaneko [2008], among others). In these papers, partial privatization is captured by assuming that the public firm maximizes a convex combination between social welfare and its profits.

Unlike these papers, I assume that the public firm cares about the median consumer's utility, and I provide a micro-foundation for this objective function. Therefore, my main contribution is to add a Political Economy perspective to the Industrial Organization literature on mixed oligopoly. This new assumption changes the properties of the equilibrium in terms of market structure, firms' profitability, and aggregate welfare. Therefore, this paper reveals new economic forces associated with privatization under political ties, while it also demonstrates that assuming that public firms maximize welfare might lead to non robust policy implications.

[4]See Shaked and Sutton [1982], Wauthy [1996], Motta [1993], among others, for models of vertical product differentiation.

[5]Cremer and Thisse [1991] shows that the vertical differentiation model with quadratic costs for quality is equivalent to the Hotelling-type model with quadratic transportation costs. Cremer et al. [1991] prove that location choices in a mixed oligopoly are efficient when the public firm minimizes total transportation costs and the number of firms is 2 or greater or equal than 6.

The remainder of the paper is organized as follows. In Section 2, I present the model. In Section 3, I provide a micro-foundation of the public firm's objective function. In Section 4, I characterize the equilibrium; and in Section 5, I study its welfare properties. Finally, Section 6 concludes. Proofs and Extensions are relegated to the Appendix.

## 2 Model

I present a simple model of quality-price competition between a politically connected public firm and a private one. This model intends to capture how political connections affect the behavior of public firms and, in turn, market structure, profits, and welfare.

**Players** There are two firms. Firms are indexed by $i \in \{0, 1\}$, where 0 is the public firm, and 1 is the private one. Each firm produces a differentiated good. I refer to good $i$ as the good produced by firm $i$.

There is a continuum of consumers. Each consumer has a type $\theta$, which captures their marginal willingness to pay (WTP) for quality. Let $f(\theta)$ be the distribution of types: $\theta \sim f : [\theta_l, \theta_h] \rightarrow [0, 1]$, where $0 \leq \theta_l < \theta_h$. Let also $F(\theta)$ be the CDF of the types distribution. The type $\theta$ proxies consumers' income (Motta [1993], Grilo [1994]).

**Actions and Timing**

*Stage* (1) Firm $i$ chooses $a_i \in \{1, 0\}$, where $a_i = 1$ denotes the action of entering the market, and $a_i = 0$ the action of staying out. Firms that choose $a_i = 0$ do not move again. If $a_1 = a_0 = 0$, the game ends. If at least one firm enters the market, the game proceeds to the second stage.

*Stage* (2) Firms simultaneously choose the quality of their good $q_i \in [0, Q]$.[6]

*Stage* (3) Firms simultaneously choose the price of their good $p_i \in [0, \infty)$.

*Stage* (4) Each consumer chooses which good to buy.[7]

[6]The parameter $Q > 0$ represents the highest technologically feasible quality level.

[7]I assume that consumers cannot avoid consumption. This assumption is appropriate when demand is highly inelastic, as for healthcare or education (Grilo [1994]). If consumers could also choose not to buy any good and get a payoff of zero, full market coverage would hold in equilibrium if: (i) consumers' utility from consumption of good $i$ is: $u(q_i, p_i \mid \theta) = y + \theta q_i - p_i$, with $y$ high enough; (ii) $\theta_h$ is high enough. Any of these assumptions would not change players' maximization problems. I study the case of partial market coverage in Appendix C.2.

There is perfect and symmetric information. At the beginning of each stage, players observe the outcome of the previous stages.

**Payoffs** The payoff of a consumer of type $\theta$ when they buy good $i$ is:

$$u\left(q_i, p_i \mid \theta\right) = \theta q_i - p_i \ . \tag{1}$$

Firm $i$'s profit function is:

$$\pi_i\left(x_i, q_i, p_i, a_i\right) = a_i\left[x_i p_i - C\left(q_i, x_i\right) - \Phi\right] \ , \tag{2}$$

where $x_i$ is the output produced by firm $i$, $C(q_i, x_i)$ is the cost of producing $x_i$ units of quality $q_i$, and $\Phi \geq 0$ is the fixed cost of entry. Let $C(q_i, x_i)$ be increasing in both arguments. Moreover, I assume that the marginal cost of output is increasing in quality, i.e.,

$$\frac{\partial^2 C(x_i, q_i)}{\partial x_i \partial q_i} > 0 \ . \tag{3}$$

Since consumers have unitary demand, then $x_i \in [0, 1]$.

Firm 1 maximizes profits. Firm 0's payoff is a convex combination between its profits and the utility of the consumer with the median type $\overline{u}$:

$$V_0\left(\pi_0, \overline{u}\right) = \lambda \pi_0 + (1-\lambda)\,\overline{u} \ . \tag{4}$$

The parameter $\lambda \in [0, 1]$ captures the degree of partial privatization (Matsumura [1998]). In *Stage* (4), the median consumer chooses which good to buy. Then,

$$\overline{u} = \max\left\{\overline{\theta} q_1 - p_1, \overline{\theta} q_0 - p_0\right\} \ , \tag{5}$$

where $\overline{\theta}$ is defined by:

$$\int_{\theta_l}^{\overline{\theta}} dF(\theta) = \frac{1}{2} = \int_{\overline{\theta}}^{\theta_h} dF(\theta) \ . \tag{6}$$

I now provide a micro-foundation for (4).

# 3 Micro-Foundation of Firm 0's Objective Function

A large literature in Political Economy describes how the amount and/or the quality of public provision of private goods can be seen as the outcome of majority voting. Epple and Romano [1996a] study how public service quality is determined through majority voting when there are private alternatives. In their model, the median-income voter is pivotal if and only if, in equilibrium, no consumers choose the private alternative. They also show that a coalition of the rich and poor can prefer reduced public provision while the middle class prefers stronger public intervention. In my model, the median consumer is pivotal even if there are private alternatives. In a different paper, the same authors show that a mixed public-private system can be preferred by a majority of voters to a government-only or a market-only provision system (Epple and Romano [1996b]). In a recent paper, Lülfesmann and Myers [2011] find a similar result in a model where citizens fund public provision through taxes and vote over the tax rate.[8]

How can Political Economy insights be integrated into a mixed duopoly model from the IO tradition such as the one presented in the previous section? I now show that this can be done by invoking the median voter theorem.

Suppose that consumers are asked to vote between two alternative pairs of prices and qualities: $\delta = \left(q^{\delta}, p^{\delta}\right)$ and $\delta' = \left(q^{\delta'}, p^{\delta'}\right)$. Call $\Delta$ the space of all possible bundles. Because of the single-crossing property of preferences, the bundle $\delta$ can be preferred to $\delta'$ by a majority of consumers if and only if the median consumer shares this opinion. For this reason, the public firm may mimic the preferences of the median consumer when it is politically concerned.

In the product differentiation model, the single-crossing property has the following interpretation. Suppose that the bundle $\delta$ comes with a higher quality and a higher price than $\delta'$, i.e., $q^{\delta} > q^{\delta'}$ and $p^{\delta} > p^{\delta'}$. Whenever a consumer with type $\theta$ prefers $\delta$ over $\delta'$, then all consumers with types $\theta' > \theta$ also prefer $\delta$ over $\delta'$. This property is sufficient to prove the following result.

[8]For similar papers, see Glomm and Ravikumar [1998], Gouveia [1997], Dotti [2019], and the references therein.

**Proposition 1.** *(Gans and Smart [1996])*
*Suppose consumers are asked to vote between two alternative bundles: $\delta, \delta' \in \Delta$. A majority of consumers prefers $\delta$ over $\delta'$ if and only if:*

$$u\left(\delta \mid \overline{\theta}\right) > u\left(\delta' \mid \overline{\theta}\right) . \tag{7}$$

*Proof.* See Appendix A.1 □

Proposition 1 follows from Theorems 1 and 2 in Gans and Smart [1996]. Because of the median consumers' pivotality, the public firm can maximize its consensus by simply miming their preferences.

This result has implications for welfare and re-distribution. In particular, it implies that welfare-maximizing policies may not be politically feasible, as there is no guarantee that these would be supported by the median consumer.[9] Moreover, changes in the distribution $f$, by changing the position of the median consumer $\overline{\theta}$, affect the behavior of the public firm. As the density below the left tail of the distribution of types increases (the distribution shifts from red to blue in Figure 1), the median consumer's position $\overline{\theta}$ shifts to the left. Then, the public firm will mimic the preferences of a poorer consumer.[10] On the contrary, if the public firm cares only about aggregate surplus (or consumer surplus), it does not care about how surplus is distributed among consumers.

How does Proposition 1 extend to the case of partial privatization? Suppose that the government controls a share $\lambda$ of the public firm's stocks while the rest are owned by private investors. Private investors seek to maximize profits. By Proposition 1, the government cares about the median consumer. Thus, the median consumer's utility enters the firm's objective function with a weight of $\lambda$, and profits enter with a weight of $1 - \lambda$. Previous papers in the mixed oligopoly literature have adopted this approach, altough they typically assume that the government cares about welfare (Matsumura [1998]).

I now proceed to solve for the equilibria of the game described in Section 2. This allows me to study how privatization affects the market outcome when the public firm cares about the median voter's utility.

[9]As an example, in Section 5, I show that - given a pair of qualities $q_0, q_1$ - the consumer surplus maximizing price is not the one preferred by the median consumer.
[10]As we discussed in the previous Section, $\theta$ is a proxy for income.

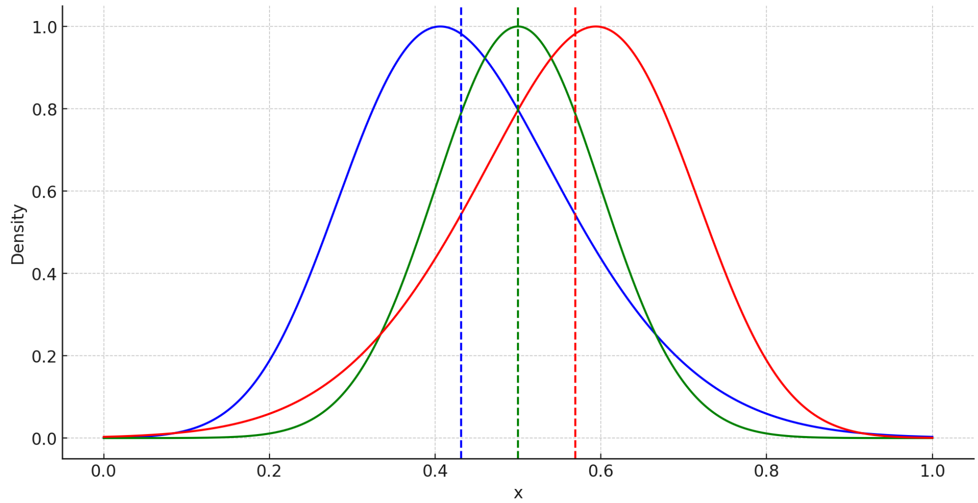



Figure 1: **The Position of the Median Consumer for Different Distributions of Types**. This plot corresponds to the specific case of bell-shaped distributions and $\theta \in [0,1]$. Each vertical line denotes the median of the distribution of the same color.

# 4 Market Equilibrium

The solution concept is Subgame Perfect Nash Equilibrium (equilibrium henceforth). To obtain a closed-form solution for equilibrium qualities and prices, I posit the following assumptions.

**Assumption 1.** *(Uniform Distribution of Types).*
*Consumers' types are uniformly distributed, i.e.,* $f(\theta) = \frac{1}{\theta_h - \theta_l}$, *with* $\theta_l = \theta_h - 1$, *and* $\theta_h > 1$.

**Assumption 2.** *(Quadratic Costs of Quality).*
*Firms' cost function is quadratic in quality and linear in quantity, i.e.,* $C(q_i, x_i) = \frac{x_i q_i^2}{\alpha}$, *with* $\alpha > 0$.

Assumption 1 and Assumption 2 are standard in the product differentiation literature.[11]

I now proceed by Backward Induction. Throughout the paper, starred variables denote equilibrium values.

[11]Some recent papers (Benassi et al. [2016], Benassi et al. [2017], Laine and Ma [2017]) relax the uniform-distribution assumption in mixed oligopoly models where a welfare-maximizing firm competes with a private one.

## 4.1 Demand Functions

Suppose $a_0 = a_1 = 1$. In *Stage* (4), each consumer chooses which good to buy. Let $j \in \{0, 1\}$ and $j \neq i$. Suppose $q_i > q_j$ and $p_i > p_j$. The indifferent consumer $\hat{\theta}(p_0, p_1, q_0, q_1)$ is:

$$\hat{\theta}(p_i, p_j, q_i, q_j) = \frac{p_i - p_j}{q_i - q_j} \,. \tag{8}$$

By the single crossing property of preferences, all consumers with $\theta > \hat{\theta}(p_i, p_j, q_i, q_j)$ buy good $i$.[12] If $(q_i - q_j)(p_i - p_j) < 0$, then one firm is a monopolist.[13]

Demand functions are as follows. The demand for firm $i$ is:

$$x_i\left(\hat{\theta}(p_i, p_j, q_i, q_j), p_i, p_j, q_i, q_j\right) =$$

$$\begin{cases} \theta_h - \hat{\theta}(p_i, p_j, q_i, q_j) & \text{if } p_i > p_j,\, q_i > q_j, \text{ and } \hat{\theta}(p_i, p_j, q_i, q_j) \in [\theta_h - 1, \theta_h] \\ \hat{\theta}(p_i, p_j, q_i, q_j) - (\theta_h - 1) & \text{if } p_i < p_j,\, q_i < q_j, \text{ and } \hat{\theta}(p_i, p_j, q_i, q_j) \in [\theta_h - 1, \theta_h] \\ 0 & \text{if } p_i > p_j,\, q_i > q_j \text{ and } \hat{\theta}(p_i, p_j, q_i, q_j) > \theta_h; \\ & \text{or } p_i < p_j,\, q_i < q_j \text{ and } \hat{\theta}(p_i, p_j, q_i, q_j) < \theta_h - 1\,; \\ & \text{or } q_i \leq q_j \text{ and } p_i > p_j; \\ & \text{or } p_i = p_j \text{ and } q_i < q_j \\ 1 & \text{if } p_i > p_j,\, q_i > q_j \text{ and } \hat{\theta}(p_i, p_j, q_i, q_j) < \theta_h - 1; \\ & \text{or } p_i < p_j,\, q_i < q_j \text{ and } \hat{\theta}(p_i, p_j, q_i, q_j) > \theta_h; \\ & \text{or } q_i \geq q_j \text{ and } p_i < p_j; \\ & \text{or } p_i = p_j \text{ and } q_i > q_j \\ \frac{1}{2} & \text{if } p_i = p_j \text{ and } q_i = q_j \end{cases} \,. \tag{9}$$

The demand for firm $j$ is:

$$x_j\left(\hat{\theta}(p_i, p_j, q_i, q_j), p_i, p_j, q_i, q_j\right) = 1 - x_i\left(\hat{\theta}(p_i, p_j, q_i, q_j), p_i, p_j, q_i, q_j\right) \,. \tag{10}$$

Firms' demand functions are not continuous on $\Delta$. To see this, fix $q_i = q_j$. Then,

$$\begin{aligned} x_i(p_j, p_j, q_j, q_j) &= \frac{1}{2} \\ \lim_{p_i \to p_{j-}} x_i(p_i, p_j, q_j, q_j) &= 1 \\ \lim_{p_i \to p_{j+}} x_i(p_i, p_j, q_j, q_j) &= 0 \,. \end{aligned} \tag{11}$$

[12] See Appendix A.1 for a formal illustration of the single crossing property.
[13] Assume $q_i > q_j$ but $p_i < p_j$. In this case, all consumers buy good $i$.

Firm $i$'s demand function jumps discontinuously from one to one half as soon as $p_i$ "hits" $p_j$ from below, and it jumps again to zero as soon as $p_i$ "crosses" $p_j$. Therefore, if $q_i = q_j$, firms' demand functions are discontinuous at $p_i = p_j$.[14] Payoff functions are also not continuous on $\Delta$.

I now solve for a NE in prices and discuss its properties.

## 4.2 Price Stage

There are several challenges to the characterization of a NE in prices. First, firms' payoff functions are discontinuous (see Section 4.1). Therefore, the standard Glicksberg [1952]'s result about the existence of a NE does not go through.

Second, because of the max operator in (5), firm 0's payoff is not quasi-concave and not single peaked. Firm 0 needs to anticipate the action of the median consumer, generating a discontinuity in its best response function. From condition (5), the median consumer buys good 0 if and only if:

$$p_0 \leq p_1 - \overline{\theta}(q_1 - q_0) \,. \tag{12}$$

For a given $p_1$, by moving $p_0$, firm 0 can determine whether to serve the median consumer or not. As a result, to identify a NE, I need to characterize the set of all $p_1$ such that serving the median consumer is optimal for firm 0. This set depends on the quality gap between firms, i.e., $q_1 - q_0$.

Third, firms' payoffs also depend on the market structure. Let $v_i\left(p_i, p_j, q_i, q_j\right)$ be the payoff function of firm $i$. Let us define the following prices:

$$\begin{aligned} \underline{p}_i\left(p_j, q_i, q_j\right) &= \inf\left\{p : x_j\left(p, p_j, q_i, q_j\right) > 0\right\} \;; \\ \overline{p}_i\left(p_j, q_i, q_j\right) &= \sup\left\{p : x_i\left(p, p_j, q_i, q_j\right) > 0\right\} \;. \end{aligned} \tag{13}$$

Given $\left(p_j, q_i, q_j\right)$, firm $i$ (firm $j$) has a positive market share if and only if $p_i$ is lower than $\overline{p}_i\left(p_j, q_i, q_j\right)$ (higher than $\underline{p}_i\left(p_j, q_i, q_j\right)$. It follows that the set of $p_i$ such that, given $\left(p_j, q_i, q_j\right)$, both firms have positive market shares is:

$$D_i\left(p_j, q_i, q_j\right) = \left(\underline{p}_i\left(p_j, q_i, q_j\right), \overline{p}_i\left(p_j, q_i, q_j\right)\right) \,. \tag{14}$$

[14] The discontinuity of the demand functions does not depend on the tie-breaking rule. I assume that if $q_i = q_j$ and $p_i = p_j$, half of the consumers break the tie in favor of good $i$, and the other half break it in favor of good $j$. Demand functions are discontinuous under any possible tie-breaking rule. Moreover, the tie-breaking rule does not affect any of the results. In the same way, it is not important to specify which half of the consumers break the tie in favor of which firm.

Let me also define the following complement sets:

$$\begin{aligned} \underline{D}_i\left(p_j, q_i, q_j\right) &= \left[0, \underline{p}_i\left(p_j, q_i, q_j\right)\right] \\ \overline{D}_i\left(p_j, q_i, q_j\right) &= \left[\overline{p}_i\left(p_j, q_i, q_j\right), \infty\right) \ . \end{aligned} \tag{15}$$

For a given $p_j$, by moving $p_i$, firm $i$ can end up in three different market structures: $i$'s monopoly, $j$'s monopoly, and duopoly. Necessarily, any duopoly NE must be such that deviations towards monopoly are unattractive. Therefore, to construct the NE, FOCs are neither necessary nor sufficient. Suppose $p_j^*$ is a candidate NE price of firm $j$. I need to compare the payoffs from three different strategies of firm $i$:

$$\begin{aligned} p_i^{'} &\in \arg \max_{p_i \in D_i\left(p_j^*, q_i, q_j\right)} v_i\left(p_i, p_j^*, q_i, q_j\right) \\ p_i^{''} &\in \arg \max_{p_i \in \underline{D}_i\left(p_j^*, q_i, q_j\right)} v_i\left(p_i, p_j^*, q_i, q_j\right) \\ p_i^{'''} &\in \arg \max_{p_i \in \overline{D}_i\left(p_j^*, q_i, q_j\right)} v_i\left(p_i, p_j^*, q_i, q_j\right) \ . \end{aligned} \tag{16}$$

FOCs only generate $p_i^{'}$.

In the following Lemma, I tackle these challenges and show the main results of this Section.

**Lemma 1.** *(Subgame NE Prices)*
*Suppose $a_1 = a_0 = 1$. Consider the subgame induced in Stage (3) by a pair of qualities $(q_0, q_1)$.*

*(i) If $q_0 \neq q_1$, there exists $\hat{p}_1(q_0, q_1)$ such that firm 0 finds it optimal to serve the median consumer if and only if:*

$$p_1 \geq \hat{p}_1(q_0, q_1) \ . \tag{17}$$

*(ii) There exists a NE.*

*(iii) If in the NE both firms have positive market shares with probability 1, the NE is unique.*

*(iv) Suppose $q_0 = q_1$. If $\lambda \geq \frac{1}{2}$, in the unique NE, both firms price at marginal costs. If $\lambda < \frac{1}{2}$, firm 0 is a monopolist.*

*(v) Let $\lambda \geq \frac{1}{2}$. In the NE, firm 0 has a positive market share. There exist $\left(q_1^a, q_1^b\right)$ such that firm 1 has a positive market share if and only if:*

$$q_0 \leq q_1 \leq q_1^a \quad or \quad q_0 \geq q_1 \geq q_1^b. \tag{18}$$

*Proof.* All the proofs are in Appendix A.2. See Appendix A.2.1 for the proof of (i); Appendix A.2.2 for the proof of (ii); Appendix A.2.4 for the proof of (iv); and Appendix A.2.5 for the proof of (v). □

Lemma 1 is an useful result because it shows the existence and the properties of a mapping from qualities to NE prices.

Let us start from (i). Firm 0 wants to serve the median consumer if and only if $p_1$ is high enough. When $p_1$ is low, it is more likely that the median consumer is better off buying good 1, thereby increasing firm 0's payoff.[15] To obtain $\hat{p}_1(q_0, q_1)$, I compare the maximum payoff that firm 0 can obtain by serving the median consumer and by not serving them. The comparative statics of (i) is as follows. The floor $\hat{p}_1(q_0, q_1)$ is increasing in $\lambda$. Intuitively, as the degree of privatization increases, the set of $p_1$ such that firm 0 wants to attract the median consumer shrinks. The impact of $\theta_h$ on $\hat{p}_1(q_0, q_1)$ depends on the ordering of

[15] In Appendix A.2.1, I show the expression of $\hat{p}_1(q_0, q_1)$.

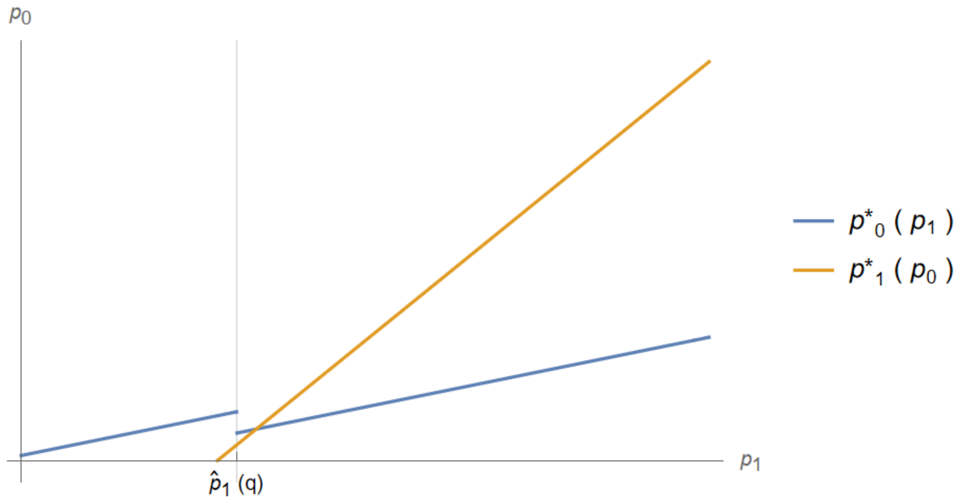



Figure 2: **(Duopoly) Best Responses in the Price Stage**. This plot corresponds to the case where neither of the two firms wants to "push" its competitor out of the market (see Appendix **??**).

qualities, i.e.,

$$\frac{\partial \hat{p}_1 (q_0, q_1)}{\partial \theta_h} > 0 \Leftrightarrow q_1 > q_0 \,. \tag{19}$$

When $\theta_h$ increases, the median consumer's WTP increases. Then, it is more likely that they will buy the high quality good.

Figure 2 plots firms' best responses for a generic quality pair $q_0 < q_1$. Unsurprisingly, prices are strategic complements. Firm 0's best response function is discontinuous at $\hat{p}_1 (q_0, q_1)$. Firm 1's best response can be found by a simple FOC. There are three cases: either best responses intersect at some $p_1 < \hat{p}_1 (q_0, q_1)$, or they interesct at some $p_1 > \hat{p}_1 (q_0, q_1)$, or there is no intersection.

Lemma 1 ((ii)) ensures the existence of a NE. The proof of existence follows from Reny [1999]. The game satisfies the following properties. The first property is reciprocal upper semi-continuity: when the payoff of one player "jumps" up, the payoff of the other player "jumps" down (Reny [1999], Simon [1987]). The second property is payoff-security.[16] The intersection of these two properties, by Corollary 5.2 of Theorem 3.1 in Reny [1999], guarantees the existence of a (mixed strategy) NE. Since firm 0's payoff is not quasi-concave in its own price, and its best response function is not continuous, the NE does not need to be in pure strategies. In (iii), I show that duopoly NE can be unique because of a local form of strict concavity of firm 1's payoff.

In (iv), I characterize the NE in the case of homogeneous goods ($q_0 = q_1$).

[16]See Appendix A.3.1, Reny [1999] and Edwards and Routledge [2023].

When $\lambda \geq \frac{1}{2}$, Bertrand undercutting drives prices down to marginal costs. When $\lambda < \frac{1}{2}$, to please the median consumer, firm 0 undercuts firm 1's price below marginal costs, thereby becoming a monopolist.

In (v), I provide a full characterization of the NE for $\lambda \geq \frac{1}{2}$.[17] To obtain NE prices, I proceed as follows. First, I assume that firms are in a duopoly, and I intersect best response functions. Then, I check that the intersection of best responses implies that firms are indeed in a duopoly. Finally, I check that firms do not want to "push" their competitor out of the market.[18]

Then, there exists a mapping from a given quality pair to a pair of NE prices. The mapping is unique when $(q_0, q_1)$ maps to a duopoly NE. There are three classes of (duopoly) subgame price NE. In the first class, the median consumer buys good 0 and $p_1 > \hat{p}_1(q_0, q_1)$. In the second class, the median consumer buys good 1 and $p_1 < \hat{p}_1(q_0, q_1)$. In the third class, $p_1 = \hat{p}_1(q_0, q_1)$, and firm 0 plays a mixed strategy. So, the median consumer buys either good with positive probability.

Holding qualities fixed, the NE where the median consumer buys good 0 is characterized by lower prices. I refer to this NE as the *low* prices NE ($p^L$). The NE such that the median consumer buys good 1 is the *high* prices NE ($p^H$). I refer to the NE in mixed strategies as the *mixed* prices NE ($p^M$). With a slight abuse of notation, I refer to $p^K(q_0, q_1)$, or simply $p^K$, as the region of the space $(q_0, q_1)$ where the $K = L, M, H$ price NE exists. Figure 3 shows these three regions, and it can be illustrated as follows. First, let us define two thresholds, $\underline{q}_0$ and $\overline{q}_0$, such that $\underline{q}_0 \leq \overline{q}_0$.[19] Consider a generic $q_0$. There are three possible cases:

*Case* $(a)$ $q_0 < \underline{q}_0$.

*Case* $(b)$ $q_0 > \overline{q}_0$.

*Case* $(c)$ $q_0 \in \left[\underline{q}_0, \overline{q}_0\right]$.

Let's start with *Case* $(a)$ (low $q_0$). When $q_1$ is very low compared to $q_0$, the median consumer buys good 0. As $q_1$ increases and approaches $q_0$, we reach the $p^H$ region, where the median consumer buys good 1. If we keep increasing $q_1$, at

[17]When $\lambda < \frac{1}{2}$, it is complicated to solve for the values of $p_1$ such that firm 0 wants to become a monopolist. Then, it does not seem easy to provide a full characterization of the NE. See the Appendix for details.

[18]For some $(q_0, q_1)$, the NE is such that firm 0 is a monopolist. I discuss this case in detail in Appendix A.2.5.2.

[19]The expressions for the different thresholds are shown in the Appendix.

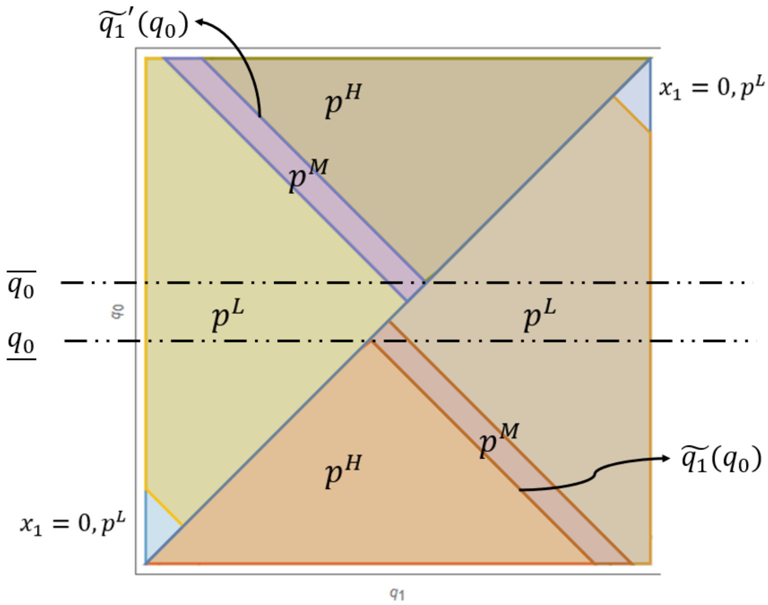



Figure 3: **Existence of Price NE in the** $(q_1, q_0)$ **Space**. Firm 1 is out of the market in the top-right and bottom-left corners.

some point we will hit another threshold, $\tilde{q}_1(q_0)$, such that for all $q_1 > \tilde{q}_1(q_0)$, the price NE is either $p^M$ or $p^L$. The median consumer buys good 0 with some positive probability, as the price of good 1 is high due to the high quality. A similar reasoning applies to Case (b) (high $q_0$). In this case, the median consumer buys good 1 with probability one as long as $q_1$ is neither too high nor too low ($q_1 \in \left[\tilde{q}_1{}'(q_0), q_0\right]$). Interestingly, in Case (c), the median consumer never buys good 1 with probability one, no matter the value of $q_1$. In this case, the quality of firm 0 is never too low to not be attractive or too high to be too costly.

I now go back to the quality and entry stages.

### 4.3 Equilibrium

I posit an additional assumption.

**Assumption 3.** *Let* $\phi = 0$ *and* $Q \in \left[\frac{\theta_h}{\alpha}, \infty\right)$.

Assumption 3 has two implications. First, $\phi = 0$ ensures that firms enter the market if and only if they expect a positive payoff.[20] Second, I assume that $Q$ is high enough to never be binding in a duopoly equilibrium.

[20] I assume that firms enter the market when indifferent between entering or not.

The main challenge to the characterization of an equilibrium is as follows. Given the mapping from qualities to NE prices described by Lemma 1, firms have a "technology" to move across the different price regions. Across regions, firms' expected payoffs (although continuous at the "borders") are different, and, therefore, many cases must be taken into account.[21] As a result, firms' payoff functions are not quasi-concave, and best responses are not continuous (and potentially are not even functions).

In any equilibrium, the following conditions must be satisfied. Let

$$v_i\left(q_i, q_j, p^k\left(q_i, q_j\right)\right) \tag{20}$$

be the payoff function of firm $i \in \{0, 1\}$ inside the $p^k$ region, with $k \in \{L, M, H\}$. Let $\Psi_i^k(q_j)$ be the set of $q_i$ such that, given $q_j$, the point $(q_i, q_j)$ belongs to the $p^k$ region. Suppose $(q_i^*, q_j^*)$ are some candidate equilibrium qualities. A necessary condition for the pair $(q_i^*, q_j^*)$ to be an equilibrium is:

$$q_i^* \in \arg \max_{q_i \in \Psi_i^k\left(q_j^*\right)} v_i\left(q_i, q_j, p^k\left(q_i, q_j\right)\right) . \tag{21}$$

By combining FOCs and SOCs, I check these necessary conditions (21). However, firm $i$ knows that, by increasing (or decreasing) further its quality, it may end up in another price region $p^h$, with $h \in \{P, M, L\}$, and $h \neq k$. Put it differently, for a given $q_j$, different $q_i$ map to different induced price NE. Then, I need to check:

$$\begin{aligned} & v_i\left(q_i^*, q_j^*, p^k\left(q_i^*, q_j^*\right)\right) \geq v_i\left(q_i^h, q_j^*, p^h\left(q_i^h, q_j^*\right)\right) \\ & \text{where:} \\ & q_i^h = \arg \max_{q_i \in \Psi_i^h\left(q_j^*\right)} v_i\left(q_i, q_j^*, p^h\left(q_i, q_j^*\right)\right) . \end{aligned} \tag{22}$$

Finally,

$$v_i\left(q_i^*, q_j^*, p^k\left(q_i^*, q_j^*\right)\right) > 0 , \tag{23}$$

guarantees that firms want to enter the market in *Stage* (1).

In the following Proposition, I show the existence of equilibria and how partial privatization affects the market structure.

[21] See Figure 3 for an illustration of these regions.

**Proposition 2.** *(Market Structure in Equilibrium)*
*For each $(q_0, q_1)$, consider the induced price NE obtained in Lemma 1. Consider Stage (2) and Stage (1).*

*(i) For all $\lambda \in [0, 1]$, there exist an equilibrium.*

*(ii) If $\lambda = 0$, in the unique equilibrium, firm 0 is a monopolist ($a_1^* = 0$).*

*(iii) There exists $\overline{\lambda} \in (0, \frac{1}{2}]$ such that firm 1 enters the market in equilibrium ($a_1^* = 1$) if and only if $\lambda \geq \overline{\lambda}$.*

*(iv) For all $\lambda \geq \frac{1}{2}$, there always exist (at least) two duopoly equilibria. In the first equilibrium, $q_0 < q_1$. In the second, $q_0 > q_1$. The two equilibria are payoffs- (and welfare-) equivalent.*

*Proof.* All the proofs are in Appendix A.3. See Appendix A.3.1 for the proof of (i); Appendix A.3.2 for the proof of (ii); Appendix A.3.3 for the proof of (iii); and Appendix A.3.4 for the proof of (iv). □

Proposition 2 describes the equilibria of the model.

In (i), I show that since *Stage* (2) is a continuous game, at least one equilibrium exists (Glicksberg [1952]).

In (ii), I show that when $\lambda = 0$, the market is a public monopoly. Firm 0 only cares about the median consumer, and it has a dominant strategy: $q_0 = Q$ and $p_0 = 0$. Given this strategy, firm 1 does not want to enter the market. By entering the market, it can only make negative profits. Therefore, staying out is optimal.

For $\lambda \in \left(0, \frac{1}{2}\right)$, it does not seem easy to fully characterize the set of equilibria. However, in (iii), I show how partial privatization affects the market structure in equilibrium. If $\lambda$ is low, NE prices are low. Anticipating this, firm 1 does not enter the market. This result strictly depends on the assumption that firm 0 does not have a budget constraint.[22] However, it may explain the persistence of public monopolies to the entry of private firms. Public firms may credibly commit to charging low prices (below marginal costs) because of their political ties, thereby discouraging the entry of private firms. Proposition 2 shows that natural (public) monopolies may not only stem from technology but also from political ties.

[22]I relax this assumption in Appendix C.1.

The proof of (iii) follows two steps. First, I show that if firm 1 wants to enter at some $\lambda = \overline{\lambda}$, it also wants to enter for any $\lambda > \overline{\lambda}$, as its value function in the quality stage is non-decreasing in $\lambda$ (because NE prices are non-decreasing in $\lambda$, too). Second, I show that if $\lambda = \frac{1}{2}$, firm 1 wants to enter the market. Therefore, it must exists some $\bar{\lambda} \in \left(0, \frac{1}{2}\right]$ such that firm 1 wants to enter only when $\lambda \geq \bar{\lambda}$.

When $\lambda$ is high enough ($\lambda \geq \overline{\lambda}$), the market is a duopoly ((iv)).[23] I provide a full characterization of the different equilibria in Appendix A.4. For all $\lambda \geq \frac{1}{2}$, there always exists a pair of equilibria. In the first equilibrium of the pair, the public firm produces the low quality, and the private firm produces the high quality. In the second one, the opposite is true. The two equilibria are payoffs- (and welfare-) equivalent. The symmetric nature of equilibria does not depend on the public firm's political connections, as it would hold true even if the public firm maximizes welfare, but rather on firms' ability to adjust markups given qualities, and on Assumptions 1 and 2.[24]

For all $\lambda \geq \frac{1}{2}$, I can always obtain a closed-form solution for equilibrium quality pairs. We can distinguish between two classes of equilibrium quality pairs, depending on whether the median consumer buys good 0 or good 1. When privatization is low ($\lambda \leq \lambda_{p_2}$), the median consumer buys good 0. The public firm wants to attract the median consumer. When privatization is high ($\lambda \geq \lambda_{p_1}$), there exist equilibria where the median consumer buys good 1. The public firm lets the median consumer buy from its competitor, as it cannot offer them a better product without sacrificing too much profits. Since $\overline{\lambda} < \lambda_{p_1} < \lambda_{p_2}$, when $\lambda < \lambda_{p_1}$ ($\lambda > \lambda_{p_2}$), the median consumer buys good 0 (good 1) in any equilibrium, while there are equilibria of both classes when $\lambda \in \left[\lambda_{p_1}, \lambda_{p_2}\right]$.

I provide details about equilibria characterization in the Appendix. In Table A.3.4, I show the numeric values for the thresholds $\lambda_{p_1}, \lambda_{p_2}$.[25] In Appendix A.4, I detail all the different equilibria and the respective existence regions. I also discuss under what conditions equilibria are unique.

I now illustrate the main welfare properties of equilibrium.

[23] In the proof of (iv), I report a characterization of the set of equilibria for specific $\theta_h = 2$ and $\alpha = \frac{1}{2}$. This is without loss of generality because the results on equilibria' existence and their welfare properties do not depend on $\theta_h$ and $\alpha$. In *Stage* (2), $\theta_h$ and $\alpha$ are just multiplicative factors of firms' payoff functions. Therefore, varying $\theta_h$ and $\alpha$ does not alter firms' strategic behavior in the choice of quality. As in Motta [1993] and Lambertini [2006], the relation between equilibrium qualities and $\theta_h$, $\alpha$ is linear.

[24] Chang et al. [2018] show that when prices are regulated, a welfare-maximizing public firm may produce a higher quality than the private competitor. Laine and Ma [2017] show that for non-uniform types distributions and different cost functions, when the public firm maximizes welfare, different quality orderings can appear in equilibrium.

[25] These thresholds are numbers and not functions of parameters.

# 5 Welfare Analysis

With a slight abuse of notation, let me define $p_L$ ($p_H$) and $q_L$ ($q_H$) as the price and quality of the lower (higher) quality good. Given (8), consumer surplus can be written as:

$$CS\,(p_H, p_L, q_H, q_L) = \int_{\theta_h - 1}^{\hat{\theta}(p_H, p_L, q_H, q_L)} \theta q_L - p_L, d\theta + \int_{\hat{\theta}(p_H, p_L, q_H, q_L)}^{\theta_h} \theta q_H - p_H, d\theta \ . \tag{24}$$

Aggregate welfare is:

$$W\,(CS, \pi_H, \pi_L) = CS + \pi_L + \pi_H \ , \tag{25}$$

where $\pi_L$ ($p_H$) are the profits of the firm producing the low quality (high quality) good. Clearly, $H, L \in \{0, 1\}$.

I now define the First Best qualities and prices as the solution of the following problem:

$$\begin{aligned} &\max_{q_H, q_L, p_H, p_L} \ W\,(CS, \pi_H, \pi_L) \\ &\textit{subject to} \\ &\quad q_H \geq q_L \geq 0 \\ &\quad p_H \geq p_L \geq 0 \end{aligned} \tag{26}$$

A vector of qualities and prices solves problem (26) if and only if it satisfies allocative efficiency (Grilo [1994], Laine and Ma [2017]). Allocative efficiency requires that the indifferent consumer (8) is the one whose switch from the low quality good to the high quality one equalizes social costs and social benefits:

$$\hat{\theta} = \frac{\alpha(q_H^2 - q_L^2)}{(q_H - q_L)} \ . \tag{27}$$

A sufficient (but not necessary) condition for (27) is that prices equal marginal costs.

The mixed oligopoly literature has widely relied on the assumption that public firms are benevolent welfare-maximizing agents. In this model, if the public firm maximizes welfare, equilibrium qualities and prices are First Best. This result can be found in Matsumura and Matsushima [2004]. In a horizontal differentiation model with quadratic transportation costs and a welfare-maximizing public firm, Matsumura and Matsushima [2004] show that the socially optimal level of privatization is zero. Because of quadratic costs (Assumption 2), their model is isomorphic to mine (Anderson et al. [1997], Cremer and Thisse [1991]). Similar results are in Grilo [1994] and in Delbono et al. [1991].

What if the public firm maximizes consumer surplus? Benassi et al. [2016] study this question under the assumptions of zero production costs and exogenous quality levels, but they allow for a generic distribution of consumers' types.[26] They do not focus on welfare but rather on the existence of a NE in prices. Their main finding is that not every distribution of types supports the existence of a price equilibrium. The more convex the distribution of types, the more likely that an equilibrium exists.

In my model, types are uniformly distributed but quality is costly. If the public firm maximizes consumer surplus, equilibrium quality levels are First Best. We can see this as follows. Suppose both firms entered the market. The public firm maximizes (24). The private firm maximizes profits. In the price stage, qualities are given. The best response of firm 1 is analogous to the one obtained in the previous Section. The public firm sets a price such that all consumers buy the high quality good. That is, for all $q_0, q_1$, then $p_0$ is such that $\hat{\theta}(p_0, p_1, q_0, q_1) = \theta_l$. Let us go back to the quality stage. In this stage, the private firm has a dominant strategy to set $q_1 = Q$.[27] The public firm's best response is also $q_0 = q_1 = Q$. Prices collapse to marginal costs. Then, (27) is satisfied, and equilibrium qualities and prices solve (26).[28]

If the public firm maximizes consumer surplus or total welfare, equilibrium is First Best, and total welfare is maximum. Under the former assumption, all surplus in equilibrium is internalized by consumers (as prices at marginal cost imply zero profits). Under the latter, surplus is shared between consumers and producers. Welfare implications change substantially when the public firm cares about the median consumer's utility.

[26]In Footnote 11, they argue that the median voter theorem could be invoked when the distributions of consumers' types is asymmetric. They claim that the median consumer's utility coincides with consumer surplus under symmetric distributions. However, consumer surplus coincides with the average surplus, which is different from the surplus of the consumer with the average (and median) type. The two coincide if and only if surplus (and not types) is uniformly distributed in the population.

[27]Fix $q_0$. All $q_1 \leq q_0$ cannot be optimal as they imply zero profits. For any $q_1 > q_0$, firm 1's profits are strictly increasing in $q_1$. Then $q_1 = Q$ is optimal given any possible $q_0$.

[28]Anticipating the equilibrium of the quality stage, the private firm would be indifferent between entering the market or not. As in the previous Section, I assume that indifference is broken in favor of entry. Nonetheless, results are robust to any change in this tie-breaking rule such that entry happens with strictly positive probability.

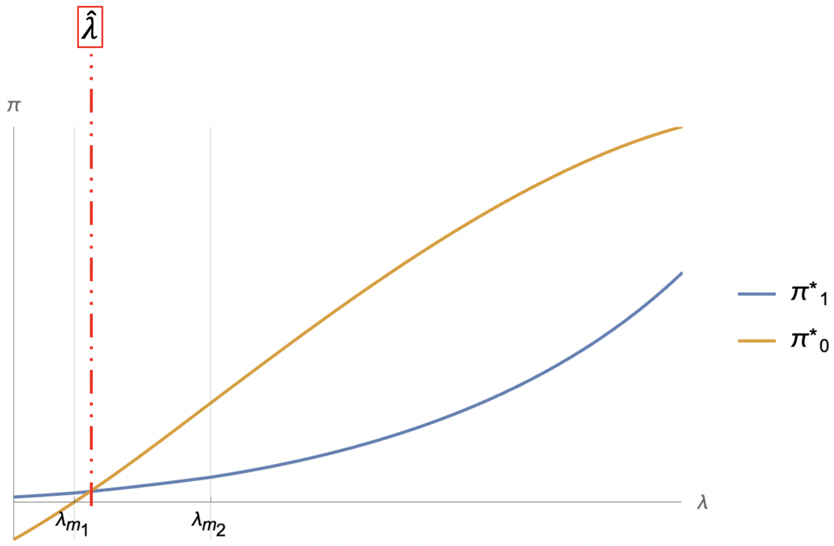



Figure 4: **Profits in Equilibrium** for $\lambda \in \left[\frac{1}{2}, \lambda_{p_2}\right]$. The figure shows profits in the equilibria where the median consumer buys good 0. This equilibria exists if $\lambda \leq \lambda_{p_2}$, and it is unique if $\lambda < \lambda_{p_1}$. See Appendix B for profits in the equilibria where the median consumer buys good 1 ($\lambda > \lambda_{p_1}$). See Appendix A.4 for a detailed characterization of all equilibria.

**Proposition 3.** *(Profits and Welfare in Equilibrium)*
*Consider the equilibria described in Proposition 2.*

*(i) In the public monopoly equilibrium, social welfare is monotonically increasing in* $\lambda$.

*(ii) The socially optimal level of privatization* $\lambda^*$ *is interior:* $\lambda^* \in (0, 1)$.

*(iii) There exists* $\hat{\lambda} \in (0, 1)$ *such that:*

$$\begin{aligned} \lambda < \hat{\lambda} &\Rightarrow \pi_0^* < \pi_1^* \, ; \\ \lambda \geq \hat{\lambda} &\Rightarrow \pi_0^* \geq \pi_1^* \, . \end{aligned} \tag{28}$$

*Proof.* See Appendix A.5. □

Proposition 3 contains the main results of this paper. Zero privatization is not socially optimal. If $\lambda = 0$, the market is a public monopoly where firm 0 plays the maximum quality level ($q_0 = Q$) and sells its product for free ($p_0 = 0$), realizing negative profits that are not compensated by consumer surplus. Then,

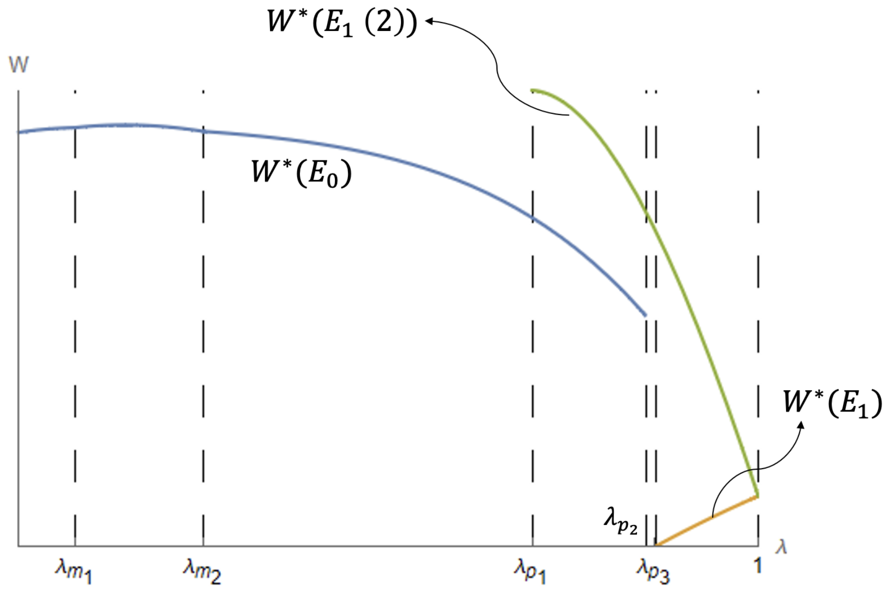



Figure 5: **Welfare in Equilibrium** for $\lambda \geq \frac{1}{2}$. The figure shows profits in the equilibria where the median consumer buys good 0 (denoted by $W^*(E_0)$); as well as in the two equilibria where the median consumer buys good 1 (denoted by $W^*(E_1)$ and $W^*(E_1(2))$). The former equilibria exist if $\lambda \leq \lambda_{p_2}$, and it is unique if $\lambda < \lambda_{p_1}$. The latter equilibria exist if $\lambda \geq \lambda_{p_3}$ (for $E_1$) or if $\lambda \geq \lambda_{p_1}$ (for $E_1(2)$). As one can see, welfare is maximized at $\lambda = \lambda_{p_1}$ See Appendix A.4 for a detaided characterization of all equilibria.

a marginal increase in the level of privatization is optimal. As emphasized above, this would not be the case if the public firm maximizes welfare or consumer surplus. Therefore, the social desirability of privatization is driven by the public firm's political ties.

In the same way, it can be shown that full privatization ($\lambda = 1$) is not socially optimal. Along the equilibria described in Proposition 2, welfare is maximized at $\lambda = \lambda_{p_1}$, where the median consumer buys good 1 (Figure 5). Under full privatization, there exists a unique equilibrium where welfare is lower than under partial privatization ($\lambda = \lambda_{p_1}$). This proves that there exists some $\lambda^* \in (0, 1)$ that maximizes welfare. Possibly, there exist different equilibria where welfare is higher ($\lambda^* \neq \lambda_{p_1}$), but necessarily, they do not occur under zero or full privatization ($\lambda^* \in (0, 1)$).

Under efficient partial privatization ($\lambda = \lambda_{p_1}$), firms produce relatively similar quality levels. Therefore, they charge low (but positive) markups. The model displays a trade-off between high qualities and low markups. When $\lambda$ is low, qualities and markups are low. Then, increasing $\lambda$ can be welfare enhancing. However, if $\lambda$ is too high, firms have incentives to invest in qualities, but they ask for too high markups.

Partial privatization is socially optimal. This result is not new to the literature on mixed oligopoly, albeit it is new to the literature on mixed oligopoly under endogenous product differentiation. Matsumura [1998] shows that partial privatization can be optimal in a Cournot mixed duopoly where the public firm maximizes welfare. There are some key differences between my model and Matsumura [1998]. First, Matsumura [1998] assumes that firms choose quantity taking product differentiation as given, whereas, in my model, the need to reduce excessive product differentiation is the key driver for the social optimality of partial privatization rather than full privatization. Second, in Matsumura [1998], the social optimality of partial privatization depends on the assumption that the market is a duopoly. If the public firm is a monopolist, then zero privatization is socially optimal. In my model, the market structure is endogenous, and privatization is needed precisely to avoid an inefficient public monopoly. Finally, in Matsumura [1998], if the public firm maximizes consumer surplus subject to a non-negativity constraint on its profits, again zero privatization is socially optimal. As we have seen above, this would not be the case in my model.

The following table summarizes how different assumptions on the public firm's objective function affect the equilibrium market structure, product differentiation, and optimal privatization policy.

| | Public firm maximizes | | |
|---|---|---|---|
| | Consumer surplus | Total welfare | Median voter's utility |
| Market structure (under zero privatization) | Duopoly | Duopoly | Public monopoly |
| Optimal privatization policy | Zero privatization (all surplus in equilibrium goes to consumers) | Zero privatization (surplus in equilibrium is shared between firms and consumers) | Partial privatization |
| Product differentiation (under zero privatization) | $q_0 = q_1 = Q$ | $q_0 \lesseqgtr q_1 < Q$ | $q_0 = Q$ |

Table 1: Market structure, optimal privatization, and product differentiation under different assumptions for the public firm's objective function.

I now focus on the most novel result of my paper: how privatization shapes market profits. Figure 4 shows profits in the equilibrium where the median consumer buys good 0.[29] When $\lambda$ is low, the public firm makes negative profits and is generally less profitable than the private one, as one would expect. However, when $\lambda$ is sufficiently high (but not so high that the median consumer switches to firm 1), the public firm becomes more profitable than the private one. The firm least concerned with profits ends up being the most profitable in equilibrium. This is an example of commitment power. To the best of my knowledge, this result is new to the literature on mixed oligopoly.

The intuition behind it is simple. Firm 0 wants to attract the median consumer. However, because of partial privatization, it cannot produce a very high quality product at a very low price. Then, it produces an "intermediate" quality/price ratio. This product appeals to most consumers in equilibrium. The private firm is forced to produce for the minority of rich or poor consumers and to sell its good at an even lower markup. If firm 1 raised the markup, it would lose too many customers because the quality/price ratio of firm 0 is relatively close to their preferences. On the contrary, as firm 0 serves the majority of the

[29]See Appendix B for an analogous picture for the remaining equilibria. Let us recall that the equilibrium in Figure 4 is unique (among the class described in Appendix A.2.3) whenever $\lambda < \lambda_{p_1}$. Appendix B also shows that firm 0's profits are non-monotone in $\lambda$.

market, most of its customers find the good of firm 1 too far from their WTP. These buyers would sacrifice a lot of utility if they had to change provider, and then can be charged a higher markup. This is a specific feature of product differentiation models, where firms' "local" monopoly power depends on how far the competitor's product is from their customers' tastes.[30]

The empirical evidence about politically connected public firms' profitability is still controversial. For instance, Agrawal and Knoeber [2001], Menozzi et al. [2012], among others, find negative effects of political ties on firms' profitability. Faccio [2006] and Goldman et al. [2009], among others, have opposite results.[31]

Proposition 3 suggests a potential explanation for this empirical puzzle. For political ties to be profitable, they must be not too tight. Provided that this is the case, politically connected firms can credibly commit to market strategies that, albeit originally designed to attract the median consumer without sacrificing too much profit, are very profitable. Political connections can create market power for public firms.

In Appendix C, I offer two extensions to the baseline model. First, I study the case of a public firm that is constrained to price at marginal costs. I show that this constraint does not make zero privatization socially optimal. Second, I study the case of partial market coverage. In this extension, consumers can also choose not to buy any of the two goods. In this case, privatization increases the share of consumers who do not have access to any good in equilibrium. Therefore, while potentially increasing efficiency, privatization can raise inequality concerns, especially for "essential" goods, such as education or healthcare.

# 6 Concluding Remarks

In this paper, I propose a model of quality competition between a public and a private firm. Previous papers in the literature assume that public firms maximize welfare. I contribute to this literature by providing a new definition for the public firm using the median voter theorem. In particular, given that consumers' preferences satisfy the single crossing property, the policy preferred by the median-type consumer is always a majority voting equilibrium, so that the public firm can mimic their preferences to maximize consensus. I argue that

[30] This commitment device requires the firm 0 to serve the majority of consumers. In fact, in any equilibrium such that the median consumer buys good 1, $\pi_1^* > \pi_0^*$.

[31] In a recent paper, Akcigit et al. [2023] show that private firms gain revenues and market shares from political connections, whereas these are harmful to innovation and productivity.

this definition might be adopted when public firms have political ties.

Under this new assumption, I study the impact of partial privatization on market structure, profits, and welfare. As privatization increases, the market structure changes from public monopoly to duopoly, where the public firm can even be more profitable than the private one. I also show that the socially optimal degree of privatization is interior. These results are new to the mixed oligopoly literature and strictly depend on my new assumption of political ties. Assuming that public firms maximize welfare might then lead to non-robust implications for privatization policy.

My paper has several limitations, as it is based on restrictive and unrealistic assumptions such as a uniform distribution of types, a fixed number of firms, and symmetric cost functions. Future research may explore the robustness of the findings by relaxing these assumptions.

# References


A. Agrawal and C. R. Knoeber. Do some outside directors play a political role? *The Journal of Law and Economics*, 44(1):179–198, 2001.

U. Akcigit, S. Baslandze, and F. Lotti. Connecting to power: political connections, innovation, and firm dynamics. *Econometrica*, 91(2):529–564, 2023.

S. P. Anderson, J. K. Goeree, and R. Ramer. Location, location, location. *Journal of Economic Theory*, 77(1):102–127, 1997.

C. Benassi, M. Castellani, and M. Mussoni. Price equilibrium and willingness to pay in a vertically differentiated mixed duopoly. *Journal of Economic Behavior & Organization*, 125:86–96, 2016.

C. Benassi, A. Chirco, and C. Colombo. Mixed spatial duopoly, consumers' distribution and efficiency. *Economics Letters*, 156:74–77, 2017.

M. Boycko, A. Shleifer, and R. W. Vishny. A theory of privatisation. *The Economic Journal*, 106(435):309–319, 1996.

G. Brunello and L. Rocco. Educational standards in private and public schools. *The Economic Journal*, 118(533):1866–1887, nov 2008.

C.-W. Chang, D. Wu, and Y.-S. Lin. Price control and privatization in a mixed duopoly with a public social enterprise. *Journal of Economics*, 124:57–73, 2018.

H. Cremer and J.-F. Thisse. Location models of horizontal differentiation: a special case of vertical differentiation models. *The Journal of Industrial Economics*, pages 383–390, 1991.

H. Cremer, M. Marchand, and J.-F. Thisse. Mixed oligopoly with differentiated products. *International Journal of Industrial Organization*, 9(1):43–53, 1991.

F. Delbono, V. Denicolo, and C. Scarpa. Quality choice in a vertically differentiated mixed duopoly. Technical report, Quaderni-Working Paper DSE, 1991.

V. Dotti. The political economy of public education. *Research in Economics*, 73 (1):35–52, 2019.

R. Edwards and R. Routledge. Existence and uniqueness of nash equilibrium in discontinuous bertrand games: a complete characterization. *International Journal of Game Theory*, 52(2):569–586, 2023.

D. Epple and R. E. Romano. Ends against the middle: Determining public service provision when there are private alternatives. *Journal of Public Economics*, 62(3):297–325, 1996a.

D. Epple and R. E. Romano. Public provision of private goods. *Journal of Political Economy*, 104(1):57–84, 1996b.

S. Estrin and D. De Meza. Unnatural monopoly. *Journal of Public Economics*, 57 (3):471–488, 1995.

M. Faccio. Politically connected firms. *American Economic Review*, 96(1):369–386, 2006.

J. S. Gans and M. Smart. Majority voting with single-crossing preferences. *Journal of Public Economics*, 59(2):219–237, 1996.

I. L. Glicksberg. A further generalization of the kakutani fixed theorem, with application to nash equilibrium points. *Proceedings of the American Mathematical Society*, 3(1):170–174, 1952.

G. Glomm and B. Ravikumar. Opting out of publicly provided services: A majority voting result. *Social Choice and Welfare*, 15(2):187–199, 1998.

E. Goldman, J. Rocholl, and J. So. Do politically connected boards affect firm value? *The Review of Financial Studies*, 22(6):2331–2360, 2009.

M. Gouveia. Majority rule and the public provision of a private good. *Public choice*, 93(3):221–244, 1997.

I. Grilo. Mixed duopoly under vertical differentiation. *Annales d'Economie et de Statistique*, pages 91–112, 1994.

J. S. Heywood, S. Wang, and G. Ye. Partial privatization upstream with spatial price discrimination downstream. *Review of Industrial Organization*, 59(1):57–78, 2021.

IMF. State-owned enterprises in the time of covid-19. *IMF Blog*, 2021.

T. Inoue, Y. Kamijo, and Y. Tomaru. Interregional mixed duopoly. *Regional Science and Urban Economics*, 39(2):233–242, 2009.

K. Ishibashi and T. Kaneko. Partial privatization in mixed duopoly with price and quality competition. *Journal of Economics*, 95(3):213–231, 2008.

T. Klumpp and X. Su. Price–quality competition in a mixed duopoly. *Journal of Public Economic Theory*, 21(3):400–432, 2019.

L. T. Laine and C.-t. A. Ma. Quality and competition between public and private firms. *Journal of Economic Behavior & Organization*, 140:336–353, 2017.

L. Lambertini. *The Economics of Vertically Differentiated Markets*. Edward Elgar Publishing, 2006.

Y. Lu and S. Poddar. Firm ownership, product differentiation and welfare. *The Manchester School*, 75(2):210–217, 2007.

C. Lülfesmann and G. M. Myers. Two-tier public provision: comparing public systems. *Journal of Public Economics*, 95(11-12):1263–1271, 2011.

T. Matsumura. Partial privatization in mixed duopoly. *Journal of Public Economics*, 70(3):473–483, 1998.

T. Matsumura and N. Matsushima. Endogenous cost differentials between public and private enterprises: a mixed duopoly approach. *Economica*, 71(284): 671–688, 2004.

A. Menozzi, M. Gutiérrez Urtiaga, and D. Vannoni. Board composition, political connections, and performance in state-owned enterprises. *Industrial and Corporate Change*, 21(3):671–698, 2012.

M. Motta. Endogenous quality choice: price vs. quantity competition. *The Journal of Industrial Economics*, pages 113–131, 1993.

P. J. Reny. On the existence of pure and mixed strategy nash equilibria in discontinuous games. *Econometrica*, 67(5):1029–1056, 1999.

A. Shaked and J. Sutton. Relaxing price competition through product differentiation. *The Review of Economic Studies*, pages 3–13, 1982.

L. K. Simon. Games with discontinuous payoffs. *The Review of Economic Studies*, 54(4):569–597, 1987.

D. A. C. Smith and M. J. Trebilcock. State-owned enterprises in less developed countries: Privatization and alternative reform strategies. *European Journal of Law and Economics*, 12(3):217–252, 2001.

J. Tirole. *The Theory of Industrial Organization*. MIT press, 1988.

X. Wauthy. Quality choice in models of vertical differentiation. *The Journal of Industrial Economics*, pages 345–353, 1996.

J. Willner. Ownership, efficiency, and political interference. *European Journal of Political Economy*, 17(4):723–748, 2001.

# Appendix

## A Proofs and Derivations

The solution of the model can be replicated by downloading the *Mathematica* Replication Package.

### A.1 Proof of Proposition 1

Take $\delta, \delta' \in \Delta$ such that $\delta = \left(q^{\delta}, p^{\delta}\right)$, $\delta' = \left(q^{\delta'}, p^{\delta'}\right)$ and $q^{\delta} > q^{\delta'}$, $p^{\delta} > p^{\delta'}$. Consumers' preferences satisfy the single crossing property:

$$u\left(\delta \mid \theta\right) > u\left(\delta' \mid \theta\right) \Rightarrow u\left(\delta \mid \theta^{'}\right) > u\left(\delta' \mid \theta^{'}\right) , \tag{29}$$

for all $\theta' \geq \theta$. Then, Theorems 1 and 2 in Gans and Smart [1996] prove the result.

### A.2 Proof of Lemma 1

#### A.2.1 Proof of (i)

I compare the maximum payoff that firm 0 can achieve by serving the median consumer and by not serving them. There are two possible cases: $q_0 < q_1$ and $q_0 > q_1$.

- $q_0 < q_1$. Firm 0's payoff is:

$$\begin{aligned} &V_0\left(p_0, p_1, q_0, q_1\right) = \\ &\lambda\left(p_0 - \alpha q_0^2\right)\left(\frac{p_1 - p_0}{q_1 - q_0} + 1 - \theta_h\right) + \\ &(1-\lambda)\max\left\{\frac{1}{2}(2\theta_h - 1)q_0 - p_0, \frac{1}{2}(2\theta_h - 1)q_1 - p_1\right\} . \end{aligned} \tag{30}$$

  If the median consumer buys from the public firm (condition (12) is satisfied):

$$\max\left\{\frac{1}{2}(2\theta_h - 1)q_0 - p_0, \frac{1}{2}(2\theta_h - 1)q_1 - p_1\right\} = \frac{1}{2}(2\theta_h - 1)q_0 - p_0 , \tag{31}$$

firm 0's payoff is:

$$\lambda\left(p_0-\alpha q_0^2\right)\left(-\theta_h+\frac{p_1-p_0}{q_1-q_0}+1\right)+(1-\lambda)\left(\frac{1}{2}(2\theta_h-1)q_0-p_0\right)\ . \tag{32}$$

Otherwise, firm 0's payoff rewrites as:

$$\lambda\left(p_0-\alpha q_0^2\right)\left(-\theta_h+\frac{p_1-p_0}{q_1-q_0}+1\right)+(1-\lambda)\left(\frac{1}{2}(2\theta_h-1)q_1-p_1\right)\ . \tag{33}$$

I now compare the maxima of (32) and (33). To find the maximizers, FOCs are sufficient by quasi-concavity.[32] The FOC of (32) requires:

$$\begin{aligned}&\frac{\lambda(p_1-2p_0+q_0(\theta_h+\alpha q_0-2))-q_1((\theta_h-2)\lambda+1)+q_0}{q_1-q_0}=0\Rightarrow\\ &p_0=\frac{\lambda(p_1+q_0(\theta_h+\alpha q_0-2))-q_1((\theta_h-2)\lambda+1)+q_0}{2\lambda}=\underline{p}_0^{L^*}(p_1,q_0,q_1)\end{aligned} \tag{34}$$

The FOC of (33) requires:

$$\begin{aligned}&\frac{\lambda(p_1-2p_0-\theta_h q_1+q_1+q_0(\theta_h+\alpha q_0-1))}{q_1-q_0}=0\Rightarrow\\ &p_0=\frac{1}{2}(p_1-\theta_h q_1+q_1+q_0(\theta_h+\alpha q_0-1))=\underline{p}_0^{H^*}(p_1,q_0,q_1)\ .\end{aligned} \tag{35}$$

Note that $\underline{p}_0^{H^*}(p_1,q_0,q_1)\geq\underline{p}_0^{L^*}(p_1,q_0,q_1)$. Then, the superscripts $H,L$ refer, respectively, to *high* and *low*. Moreover,

$$\lim_{\lambda\to 1}\underline{p}_0^{L^*}(p_1,q_0,q_1)=\underline{p}_0^{H^*}(p_1,q_0,q_1)\ . \tag{36}$$

When the public firm does not serve the median consumer, it just maximizes profits.

I now plug (34) into (32) to obtain:

$$\frac{1}{4}\left(2(\lambda-1)(2p_1-2\theta_h q_1+q_1)+\frac{\lambda(p_1-\theta_h q_1+q_1+q_0(\theta_h+\alpha(-q_0)-1))^2}{q_1-q_0}\right)\ , \tag{37}$$

[32] The payoff function (30) is not quasi-concave in $p_0$. However, both (32) and (33) are quasi-concave in $p_0$.

and (35) into (33) to obtain:

$$\frac{1}{4}\left(2(\lambda-1)(2p_1-2\theta_h q_1+q_1)+\frac{\lambda(p_1-\theta_h q_1+q_1+q_0(\theta_h+\alpha(-q_0)-1))^2}{q_1-q_0}\right) . \tag{38}$$

Since (37) is higher than (38) if and only if:

$$p_1 \geq \frac{q_1(2\theta_h\lambda+\lambda-1)+\lambda q_0(-2\theta_h+2\alpha q_0-1)+q_0}{2\lambda} = \hat{p}_1^1(q_0,q_1) , \tag{39}$$

the best response of firm 0 is:[33]

$$\underline{p}_0^*(p_1,q_0,q_1) = \begin{cases} \underline{p}_0^{L^*}(p_1,q_0,q_1) & \text{if } p_1 > \hat{p}_1^1(q_0,q_1) \\ \underline{p}_0^{H^*}(p_1,q_0,q_1) & \text{if } p_1 < \hat{p}_1^1(q_0,q_1) \\ \left\{\underline{p}_0^{L^*}(p_1,q_0,q_1), \underline{p}_0^{H^*}(p_1,q_0,q_1)\right\} & \text{if } p_1 = \hat{p}_1^1(q_0,q_1) . \end{cases} \tag{40}$$

It is interesting to note that:

$$\begin{aligned} \frac{\partial \hat{p}_1^1(q_0,q_1)}{\partial \lambda} &> 0 \\ \frac{\partial \hat{p}_1^1(q_0,q_1)}{\partial \theta_h} &> 0 \\ \frac{\partial \hat{p}_1^1(q_0,q_1)}{\partial \alpha} &> 0 . \end{aligned} \tag{41}$$

- $q_0 > q_1$. I adopt the same approach as the previous case. Let:

$$\frac{(2\theta_h-3)\lambda q_1+q_1+q_0(\lambda(-2\theta_h+2\alpha q_0+3)-1)}{2\lambda} = \hat{p}_1^2(q_0,q_1) . \tag{42}$$

[33] Condition (39) implies condition (12). The function (40) is the best response of firm 0 only for those $p_1$ such that firm 0 does not want to push firm 1 out of the market and become a monopolist. There exists some $\tilde{p_1}(q_0,q_1)$ such that for all $p_1 \geq \tilde{p_1}(q_0,q_1)$, firm 0 wants to become a monopolist. Necessarily, $\tilde{p_1}(q_0,q_1) > \hat{p}_1^1(q_0,q_1)$. There exists an analogous threshold ($\tilde{p_0}(q_0,q_1)$) for firm 1's best response. In the remainder of this Section, I derive firms' optimal monopoly pricing.

The best response of firm 0 is:

$$\overline{p}_0^*(p_1, q_0, q_1) = \begin{cases} \frac{\lambda(p_1+q_0(\theta_h+\alpha q_0+1))-((\theta_h+1)\lambda q_1)+q_1-q_0}{2\lambda} = \overline{p}_0^{L^*}(p_1, q_0, q_1) & \text{if } p_1 > \hat{p}_1^2(q_0, q_1) \\ \frac{1}{2}(p_1 - \theta_h q_1 + q_0(\theta_h + \alpha q_0)) = \overline{p}_0^{H^*}(p_1, q_0, q_1) & \text{if } p_1 < \hat{p}_1^2(q_0, q_1) \\ \left\{\overline{p}_0^{L^*}(p_1, q_0, q_1), \overline{p}_0^{H^*}(p_1, q_0, q_1)\right\} & \text{if } p_1 = \hat{p}_1^2(q_0, q_1). \end{cases} \tag{43}$$

The comparative statics of $\hat{p}_1^2(q_0, q_1)$ is as follows.

$$\begin{aligned} \frac{\partial \hat{p}_1^2(q_0, q_1)}{\lambda} &> 0 \\ \frac{\partial \hat{p}_1^2(q_0, q_1)}{\alpha} &> 0 \\ \frac{\partial \hat{p}_1^2(q_0, q_1)}{\theta_h} &< 0 \end{aligned} \tag{44}$$

Therefore, $\hat{p}(q_0, q_1)$ is as follows.

$$\hat{p}_1(q_0, q_1) = \begin{cases} \max\left\{\hat{p}_1^1(q_0, q_1), 0\right\} & \text{if } q_0 < q_1 \\ \max\left\{\hat{p}_1^2(q_0, q_1), 0\right\} & \text{if } q_0 > q_1 \end{cases} \tag{45}$$

### A.2.2 Proof of (ii)

The proof follows from Reny [1999]. The price game satisfies the following properties. The first property is reciprocal upper semi-continuity. Payoffs are discontinuous. However, when the payoff of one firm "jumps up," the payoff of the other firm "jumps down." In addition, the sum of the payoffs is continuous on $\Delta$. Second, the game satisfies payoff security: Given any pair $(p_i, p_j)$, firm $i$ always has a strategy $p_i'$ that secures a payoff of at least $v_i(p_i, p_j, q_i, q_j)$ even if its opponent slightly deviates from $p_j$ to some $p_j'$.[34] By Corollary 5.2 of Theorem 3.1 in Reny [1999], there exists a (mixed strategy) NE. The NE is not (necessarily) in pure strategies because firm 0's payoff is not quasi-concave.

[34] Edwards and Routledge [2023] shows that any Bertrand game is payoff-secure when marginal costs (with respect to quantity) are constant.

### A.2.3 Proof of (iii)

The function $\pi_1(p_1, p_0, q_1, q_0)$ is strictly concave in $p_1$ on $D_1(p_0, q_1, q_0)$. However, $\pi_1(p_1, p_0, q_1, q_0)$ is not strictly concave in $p_1$ on $[0, \infty)$, although it is quasi-concave. Therefore, for all $p_0$ such that some $p_1 \in D_1(p_0, q_1, q_0)$ is a best response for firm 1, the best response of firm 1 to $p_0$ is unique and pure. Then, in any duopoly NE, firm 1 plays a pure strategy.[35]

Let us now consider firm 0. Firm 0's payoff is neither strictly concave in $p_0$ on $D_0(p_1, q_0, q_1)$, nor quasi-concave in $p_0$ on $[0, \infty)$. Since firm 1 plays a pure strategy, there are two cases: $p_1 \neq \hat{p}_1(q_0, q_1)$ and $p_1 = \hat{p}_1(q_0, q_1)$.

- $p_1 \neq \hat{p}_1(q_0, q_1)$. By concavity of (32) and (33), if some $p_0 \in D_0(p_1, q_0, q_1)$ is optimal, the best response of firm 0 is a pure strategy. Therefore, there is a unique intersection of best responses, and the NE is unique.

- $p_1 = \hat{p}_1(q_0, q_1)$. By (40), firm 0 has two best responses. As I show in Appendix A.2.5, there exists a unique mixed strategy for firm 0 such that $p_1 \neq \hat{p}_1(q_0, q_1)$ is optimal for firm 1.

### A.2.4 Proof of (iv)

By the standard Bertrand's undercutting argument, if $\lambda \geq \frac{1}{2}$, the unique duopoly NE is such that prices are equal to marginal costs. However, if $\lambda < \frac{1}{2}$, firm 0 still has an incentive to undercut its price below marginal costs to increase the median consumer's utility.

### A.2.5 Proof of (v)

When $\lambda \geq \frac{1}{2}$, I can fully characterize the NE.[36] I distinguish between two types of NE: duopoly NE (Appendix A.2.5.1) and public monopoly NE (Appendix

[35] Extending this argument to a possible randomization of firm 0 is straightforward. Let $\eta_0$ be a mixed strategy of firm 0, with support $S_{\eta_0}$. The expected payoff of firm 1 is $\mathbb{E}_{\eta_0}[\pi_1(p_1, p_0, q_1, q_0)]$. Suppose, toward a contradiction, that firm 1's best response to $\eta_0$ is some mixed strategy $\eta_1$, with support $S_{\eta_1}$. By definition of NE, any $p_1 \in S_{\eta_1}$ must yield firm 1 the same payoff. Then, $S_{\eta_1} \subseteq D_1(p_0, q_1, q_0)$. Now, consider two cases: either firm 0 randomizes over finitely many actions, or it randomizes over infinitely many actions. If $|S_{\eta_0}| < \infty$, $\mathbb{E}_{\eta_0}[\pi_1(p_1, p_0, q_1, q_0)]$ is strictly concave in $p_1$ on $D_1(p_0, q_1, q_0)$ because it is the sum of finitely many strictly concave functions. If $|S_{\eta_0}| = \infty$, concavity is preserved under the expectation sign as $\pi_1(p_1, p_0, q_1, q_0)$ is bounded above. In both cases, strict concavity guarantees single peakedeness.

[36] When $\lambda < \frac{1}{2}$, it seems complicated to characterize the NE. In this case, the optimal monopoly price of firm 0 is $p_0 = 0$. Given some candidate duopoly NE prices, it is not easy to determine under what conditions on $p_1$, deviating to $p_0 = 0$ is optimal for firm 0. See also Appendix **??**. However, as I show in (iii), when it exists a duopoly price NE, the NE is described by the same expressions that characterize the duopoly NE for $\lambda \geq \frac{1}{2}$.

A.2.5.2).

To characterize duopoly NE, I proceed as follows. First, I assume that firms are in a duopoly, and I intersect firms' (duopoly) best response functions. Second, I check that the intersection of best responses implies that firms are indeed in a duopoly. Finally, I check that firms do not want to deviate towards their optimal monopoly price.

To characterize monopoly price NE, I solve firm 0's optimal monopoly problem, and I check that neither firm 1 wants to enter nor firm 0 wants to increase its price to give its opponent a positive market share.

#### A.2.5.1 Duopoly NE

Step (1) I start by obtaining candidate duopoly NE. There are three possible cases: $q_0 < q_1$, $q_1 > q_0$, and $q_0 = q_1$.

- $q_0 < q_1$. Firm 0's (duopoly) best response function is (40). The payoff of firm 1 is:

$$\pi_1(p_0, p_1, q_0, q_1) = \left(p_1 - \alpha q_1^2\right)\left(\theta_h - \frac{p_1 - p_0}{q_1 - q_0}\right) . \tag{46}$$

The FOC of (46) (that is necessary for an interior maximizer) requires:

$$\begin{aligned} &\theta_h + \frac{-2p_1 + p_0 + \alpha q_1^2}{q_1 - q_0} = 0 \Rightarrow \\ &p_0 = \frac{1}{2}\left(p_0 + \alpha q_1^2 + \theta_h q_1 - \theta_h q_0\right) = \overline{p}_1^*(p_0, q_0, q_1) \ . \end{aligned} \tag{47}$$

Assuming that firm 1 does not want to deviate towards monopoly, $\overline{p}_1^*(p_0, q_0, q_1)$ is its best response.

There are two intersections of best responses.

$$\begin{aligned} p_0 &= \frac{1}{3}\left(\alpha q_1^2 - (\theta_h - 2)q_1 + q_0(\theta_h + 2\alpha q_0 - 2)\right) = \underline{p}_0^L(q_0, q_1) \\ p_1 &= \frac{1}{3}\left(2\alpha q_1^2 + (\theta_h + 1)q_1 + q_0(-\theta_h + \alpha q_0 - 1)\right) = \overline{p}_1^L(q_0, q_1) \end{aligned} \tag{48}$$

$$\begin{aligned} p_0 &= \frac{1}{3}\left(\alpha q_1^2 - (\theta_h - 2)q_1 + q_0(\theta_h + 2\alpha q_0 - 2)\right) = \underline{p}_0^H(q_0, q_1) \\ p_1 &= \frac{1}{3}\left(2\alpha q_1^2 + (\theta_h + 1)q_1 + q_0(-\theta_h + \alpha q_0 - 1)\right) = \overline{p}_1^H(q_0, q_1) \end{aligned} \tag{49}$$

The pair (48) is a NE only if (12) holds and $\overline{p}_1^L(q_0, q_1) \geq \hat{p}_1^1(q_0, q_1)$, which requires:

$$\left(0 \le \lambda \le \frac{1}{4\theta_h - 1}\right) \text{ or }$$
$$\left(\lambda(q_1+q_0)(-4\theta_h\lambda + \lambda + 4\alpha\lambda(q_1+q_0)+1) \le 0 \text{ and } \frac{1}{4\theta_h - 1} < \lambda < 1\right). \tag{50}$$

The pair (49) is a NE only if (12) does not hold and $\overline{p}_1^H < \hat{p}_1^1(q_0, q_1)$, i.e.,

$$\frac{3}{4\theta_h + 1} \le \lambda \le 1 \text{ and } \alpha \le \frac{4\theta_h\lambda + \lambda - 3}{4\lambda q_1 + 4\lambda q_0}. \tag{51}$$

Note that (50) and (51) are never jointly satisfied. However, there are some quality pairs $(q_0, q_1)$ such that neither (50) nor (51) is satisfied. When that is the case, there exists a NE in mixed strategies.

Let us assume that firm 1 plays $p_1 = \hat{p}_1^1(q_0, q_1)$. Firm 0 is indifferent between $\overline{p}_1^{H^*}\left(\hat{p}_1^1(q_0, q_1), q_0, q_1\right)$ and $\overline{p}_1^{L^*}\left(\hat{p}_1^1(q_0, q_1), q_0, q_1\right)$. Assume then that firm 0 randomizes between $\overline{p}_1^{H^*}\left(\hat{p}_1^1(q_0, q_1), q_0, q_1\right)$ and $\overline{p}_1^{L^*}\left(\hat{p}_1^1(q_0, q_1), q_0, q_1\right)$ with probabilities $h_0$ and $1 - h_0$, for some $h_0 \in [0, 1]$. Let $\sigma_0$ denote this mixed strategy. For all $h_0$, $\sigma_0$ is optimal for firm 0. The expected value of $\sigma_0$ is:

$$\begin{aligned}
&\mathbb{E}[\sigma_0] = \\
&h_0\left[\overline{p}_0^{L^*}\left(\hat{p}_1^1(q_0, q_1), q_0, q_1\right)\right] + (1 - h_0)\left[\underline{p}_0^{H^*}\left(\hat{p}_1^1(q_0, q_1), q_0, q_1\right)\right] = \\
&h_0\left(\frac{(5\lambda - 3)q_1 + q_0(\lambda(4\alpha q_0 - 5) + 3)}{4\lambda}\right) + \\
&(1 - h_0)\left(\frac{(3\lambda - 1)q_1 + \lambda q_0(4\alpha q_0 - 3) + q_0}{4\lambda}\right) \\
&= -\frac{(2h_0 + 1)(q_1 - q_0)}{4\lambda} + \frac{1}{4}(2h_0 + 3)(q_1 - q_0) + \alpha q_0^2.
\end{aligned} \tag{52}$$

For $\hat{p}_1^1(q_0, q_1)$ to be optimal for firm 1 against $\sigma_0$, it must be that

$$h_0 = \frac{4\theta_h\lambda + \lambda - 4\alpha\lambda(q_1 + q_0) - 3}{2(\lambda - 1)} = h_0^*. \tag{53}$$

Let $\sigma_0^*$ denote the NE mixed strategy. Note that:

$$
\begin{aligned}
&h_0^* \in (0,1) \Leftrightarrow \\
&\left(\frac{1}{4\theta_h - 1} < \lambda \leq \frac{3}{4\theta_h + 1} \text{ and } 0 < \alpha < \frac{4\theta_h\lambda - \lambda - 1}{4\lambda q_1 + 4\lambda q_0}\right) \\
&\text{or} \left(\frac{3}{4\theta_h + 1} < \lambda < 1 \text{ and } \frac{4\theta_h\lambda + \lambda - 3}{4\lambda q_1 + 4\lambda q_0} < \alpha < \frac{4\theta_h\lambda - \lambda - 1}{4\lambda q_1 + 4\lambda q_0}\right).
\end{aligned} \tag{54}
$$

In the mixed NE:

$$
\begin{aligned}
&\mathbb{E}\left[\sigma_0^*\right] = \frac{-\alpha\lambda q_1^2 + q_1(\theta_h\lambda + \lambda - 1) + \lambda q_0(-\theta_h + 2\alpha q_0 - 1) + q_0}{\lambda} = \underline{p}_0^M(q_0, q_1) \\
&p_1 = \hat{p}_1^1(q_0, q_1) = \overline{p}_1^M(q_0, q_1) \ .
\end{aligned} \tag{55}
$$

By comparing (50), (51) and (54), one can see that for any $(q_1, q_0)$ it is impossible for two or more of these conditions to be simultaneously true. However, at least one of them is always true.

- $q_0 > q_1$. I adopt the same approach of the latter case. Some computations are omitted in the interest of brevity. The (duopoly) best response of firm 0 is (43). The (duopoly) best response of firm 1 is:

$$
\underline{p}_1^*(p_0, q_0, q_1) = \frac{1}{2}\left(p_0 + \alpha q_1^2 + (\theta_h - 1)q_1 - \theta_h q_0 + q_0\right) \ . \tag{56}
$$

There are two intersections of best responses.

$$
\begin{aligned}
p_0 &= \frac{\alpha\lambda q_1^2 + q_1(2 - (\theta_h + 3)\lambda) + q_0(\lambda(\theta_h + 2\alpha q_0 + 3) - 2)}{3\lambda} = \overline{p}_0^L(q_0, q_1) \\
p_1 &= \frac{2\alpha\lambda q_1^2 + (\theta_h - 3)\lambda q_1 + q_1 + q_0(\lambda(-\theta_h + \alpha q_0 + 3) - 1)}{3\lambda} = \underline{p}_1^L(q_0, q_1)
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
p_0 &= \frac{1}{3}\left(\alpha q_1^2 - (\theta_h + 1)q_1 + q_0(\theta_h + 2\alpha q_0 + 1)\right) = \overline{p}_0^H(q_0, q_1) \\
p_1 &= \frac{1}{3}\left(2\alpha q_1^2 + (\theta_h - 2)q_1 + q_0(-\theta_h + \alpha q_0 + 2)\right) = \underline{p}_1^H(q_0, q_1)
\end{aligned} \tag{58}
$$

There also exists a mixed strategy NE, where firm 1 plays $\hat{p}_1^2(q_0, q_1)$ and firm 0 randomizes between its two best responses with probabilities

$$h_0 = \frac{\lambda(-4\theta_h + 4\alpha(q_1 + q_0) + 5) - 3}{2(\lambda - 1)} = h_0^{'} \tag{59}$$

and $1 - h_0^{'}$. Let $\sigma_S^{**}$ be the NE mixed strategy of firm 0. Expected prices are as follows.

$$\begin{aligned} \mathbb{E}\left[\sigma_S^{**}\right] &= \frac{-\alpha\lambda q_1^2 + (\theta_h - 2)\lambda q_1 + q_1 + q_0(\lambda(-\theta_h + 2\alpha q_0 + 2) - 1)}{\lambda} = \overline{p}_0^M(q_0, q_1) \\ p_1 &= \hat{p}_1^2(q_0, q_1) = \underline{p}_1^M(q_0, q_1) \end{aligned} \tag{60}$$

(57) is a NE only if:

$$\alpha \leq \frac{4\theta_h\lambda - 3\lambda + 1}{4\lambda q_1 + 4\lambda q_0}\ . \tag{61}$$

(58) is a NE if and only if:

$$\alpha \geq \frac{4\theta_h\lambda - 5\lambda + 3}{4\lambda q_1 + 4\lambda q_0}\ . \tag{62}$$

(60) is a NE if and only if:

$$\frac{4\theta_h\lambda - 3\lambda + 1}{4\lambda q_1 + 4\lambda q_0} < \alpha < \frac{4\theta_h\lambda - 5\lambda + 3}{4\lambda q_1 + 4\lambda q_0}\ . \tag{63}$$

Thus, for any $q_0 < q_1$, there exists a unique NE.

For further reference, define:

$$p^L(q_0, q_1) = \begin{cases} \underline{p}_0^L(q_0, q_1), \overline{p}_1^L(q_0, q_1) & \text{if } q_0 < q_1 \\ \overline{p}_0^L(q_0, q_1), \underline{p}_1^L(q_0, q_1) & \text{if } q_0 > q_1 \end{cases} \tag{64}$$

$$p^H(q_0, q_1) = \begin{cases} \underline{p}_0^H(q_0, q_1), \overline{p}_1^H(q_0, q_1) & \text{if } q_0 < q_1 \\ \overline{p}_0^H(q_0, q_1), \underline{p}_1^H(q_0, q_1) & \text{if } q_0 > q_1 \end{cases} \tag{65}$$

$$p^M(q_0, q_1) = \begin{cases} \underline{p}_0^M(q_0, q_1), \overline{p}_1^M(q_0, q_1) & \text{if } q_0 < q_1 \\ \overline{p}_0^M(q_0, q_1), \underline{p}_1^M(q_0, q_1) & \text{if } q_0 > q_1 \end{cases} \tag{66}$$

- $q_0 = q_1$. See Appendix A.2.4.

In the remainder of the paper, I refer to $p^k(q_0, q_1)$ as the region of existence of the $k = L, M, H$ price NE. With a slight abuse of notation, I often use the notation $p^k$, omitting the arguments $(q_0, q_1)$.

Finally, it is important to note that in the $p^L$ region, the median consumer always buys good 0. In the $p^H$ region, the median consumer always buys good 1. In the $p^M$ region, the median consumer buys from firm 0 with a certain probability and from firm 1 with the remaining probability.[37]

Step (2) Firm 0 has a positive market share in all the different NE. However, this is not necessarily the case for firm 1. In the $p^L$ NE, firm 1 is out of the market whenever:

$$\begin{aligned} q_0 < q_1 \text{ and } q_1 < \frac{\theta\lambda + 2\lambda + \alpha\lambda(-q_0) - 1}{\alpha\lambda} = q_1^a \text{ or} \\ q_0 > q_1 \text{ and } q_1 > \frac{\theta\lambda - 3\lambda + \alpha\lambda(-q_0) + 1}{\alpha\lambda} = q_1^b \; . \end{aligned} \tag{68}$$

Note that $q_1^a$ is increasing in $\lambda$, while $q_1^b$ decreases in $\lambda$. Both firms are always active in the $p^H$ and $p^M$ regions. Whenever (68) is satisfied, the NE market structure is a public monopoly. I characterize public monopoly NE in the Appendix A.2.5.2.

Step (3) Take a pair of candidate NE prices $(p_0^*, p_1^*)$ that, for $(q_0, q_1)$, it implies

$$\begin{aligned} x_1\left(p_0^*, p_1^*, q_0, q_1\right) > 0 \; , \\ x_0\left(p_0^*, p_1^*, q_0, q_1\right) > 0 \; . \end{aligned} \tag{69}$$

Let us consider the maximization problem of firm $i$, given $p_j^*$. By FOC, firm $i$ strictly prefers $p_i^*$ to any $p_i'$ such that $p_i' \neq p_i^*$ and $p_i' \in D_i\left(p_j^*, q_i, q_j\right)$. The payoff function of firm $i$ is increasing in $p_i$ for all $p_i \in \underline{D}_i\left(p_j^*, q_i, q_j\right)$, and decreasing in $p_i$ for all $p_i \in \overline{D}_i\left(p_j^*, q_i, q_j\right)$. Hence, if $\underline{p}_i\left(p_j^*, q_i, q_j\right)$ and $\overline{p}_i\left(p_j^*, q_i, q_j\right)$ are not profitable, any other monopoly price is not profitable

[37] To see this, let us consider the following example. Let $q_0 < q_1$. Then,

$$\begin{aligned} x_0\left(\underline{p}_0^{L^*}\left(\hat{p}_1\left(q_0, q_1\right), q_0, q_1\right), \hat{p}_1\left(q_0, q_1\right), q_0, q_1\right) \geq \frac{1}{2} \\ x_0\left(\underline{p}_0^{H^*}\left(\hat{p}_1\left(q_0, q_1\right), q_0, q_1\right), \hat{p}_1\left(q_0, q_1\right), q_0, q_1\right) \leq \frac{1}{2} \; . \end{aligned} \tag{67}$$

An analogous result applies if $q_0 > q_1$.

as well.[38]

**A.2.5.2 Public Monopoly NE** Consider a generic pair of candidate NE prices from the $p_L$ region, i.e., (64). Call these $\left(p_0^*, p_1^*\right)$. When (68) holds, firm 1 is out of the market. In this case, the NE is characterized as follows. Firm 1 still plays $p_1 = p_1^*$ and firm 0 plays $p_0 = \underline{p}_0\left(p_1^*, q_1, q_0\right)$, i.e., the highest price that keeps firm 1 out of the market.

We can see this as follows. Let us start from firm 0. It is straightforward to see that firm 0 does not want to deviate to any $p_0 \in D_0\left(p_1^*, q_0, q_1\right)$. In particular, any $p_0 \in D_0\left(p_1^*, q_0, q_1\right)$ such that the median consumer buys from from 0 is dominated by the corner solution $\underline{p}_0\left(p_1^*, q_1, q_0\right)$. Any $p_0 \in D_0\left(p_1^*, q_0, q_1\right)$ such that the median consumer buys from from 1 cannot be optimal since $p_1^* > \hat{p}_1\left(q_0, q_1\right)$. In the same way, all $p_0 \in \overline{D}_0\left(p_1^*, q_0, q_1\right)$ are strictly dominated.

What about firm 1? Given any pair of prices in the $p^L$ region, $\pi_1\left(p_0^*, p_1^*, q_1, q_0\right) > 0$. So, if $x_1\left(p_0^*, p_1^*, q_1, q_0\right) < 0$, then $p_1^* < \alpha q_1^2$. Since firm 1 could retrieve market shares only by decreasing its price (any deviation to some $p_1 > p_1^*$ cannot be profitable because it does not increase the market share), but this is not profitable because it would imply negative profits, $p_1^*$ is optimal.

While a duopoly price NE is always unique, the same is not necessarily true for a public monopoly NE.[39]

## A.3 Proof of Proposition 2

In all the following proofs, I assume that firms have entered the market in *Stage* (1). Then, I find a NE of *Stage* (2), and I check that firms have indeed an incentive to enter the market in *Stage* (1).

[38] The best response functions in Appendix (ii) are well-defined at $p_i = \underline{p}_i\left(p_j, q_i, q_j\right)$ and $p_i = \overline{p}_i\left(p_j, q_i, q_j\right)$. If $\lambda < \frac{1}{2}$, firm 0's payoff is strictly decreasing in $p_0$ for all $p_0 \leq \underline{p}_0\left(p_1^*, q_0, q_1\right)$, and I cannot exclude that deviations to $p_0 = 0$ are un-profitable.

[39] In particular, firm 1's best response to $\underline{p}_0\left(p_1^*, q_0, q_1\right)$ is potentially not unique (any price such that firm 1 is out of the market generates a zero profit for firm 1 but a different payoff for firm 0). This is potentially problematic in the quality stage, where a mapping from qualities to NE prices is necessary to construct payoff functions. Therefore, along an equilibrium path, the same quality pair could lead to different NE in prices, and, therefore, to different expected payoffs. Nonetheless, given candidate equilibrium qualities (see Proposition 2), firms cannot reach the region of existence of the public monopoly NE. That is, given any candidate equilibrium quality of firm $j$ ($q_j^*$), firm $i$ can only deviate towards some $q_i$ such that the pair $\left(q_i, q_j^*\right)$ sustains a (unique) duopoly price NE. This allows me to appropriately check firms' incentives to deviate from candidate equilibrium quality pairs.

### A.3.1 Proof of (i)

Given *Stage* (3) NE prices, each firm's payoff function in *Stage* (2) is continuous in its own action and in the action of its opponent. Given NE prices $p^*\left(q_i, q_j\right)$:

$$\begin{aligned}
&\lim_{q_i \to q_{j-}} \pi_i\left(q_i, q_j, p^*\left(q_i, q_j\right)\right) = \\
&\lim_{q_i \to q_{j+}} \pi_i\left(q_i, q_j, p^*\left(q_i, q_j\right)\right) = \\
&\pi_i\left(q_j, q_j, p^*\left(q_j, q_j\right)\right) = 0\,.
\end{aligned} \tag{70}$$

Hence, in *Stage* (2), there exists at least one (mixed strategy) NE (Glicksberg [1952]).[40] In *Stage* (1), firms only choose whether to enter or not, depending on whether their expected payoff is greater than zero or not.

### A.3.2 Proof of (ii)

Suppose, towards a contradiction, that $a_1^* = a_0^* = 1$ and $\lambda = 0$.

Firm 0's payoff is:

$$V_0\left(p_0, p_1, q_0, q_1\right) = \max\{\overline{\theta} q_0 - p_0, \overline{\theta} q_1 - p_1\}\,. \tag{71}$$

Firm 0 has a dominant strategy: $p_0^* = 0$. The best response of firm 1 is:

$$p_1^*(p_0^* = 0) = \begin{cases} \frac{1}{2}\left(\alpha q_1^2 + \theta_h q_1 - \theta_h q_0\right) & \text{if } q_0 < q_1 \\ \frac{1}{2}\left(\alpha q_1^2 + (\theta_h - 1) q_1 - \theta_h q_0 + q_0\right) & \text{if } q_0 \geq q_1 \end{cases} \tag{72}$$

Given subgame NE prices, both firms have positive market shares if and only if:

$$q_1 > q_0 \text{ and } x_1\left(0, p_1^*(0), q_0, q_1\right) \geq \theta_h \Leftrightarrow \frac{\theta_h q_1 - \alpha q_1^2}{\theta_h} < q_0 < q_1\,. \tag{73}$$

Let us go back to *Stage* (2). Firm 0's payoff is:

$$V_0\left(p_0, p_1, q_0, q_1\right) = \max\{\overline{\theta} q_0, \overline{\theta} q_1 - p_1^*\left(q_0, q_1\right)\}\,. \tag{74}$$

Note that:

$$\frac{\partial p_1^*\left(q_0, q_1\right)}{\partial q_0} < 0\,. \tag{75}$$

[40] As in Appendix A.2.2, the NE does not need to be in pure strategies because of the lack of quasi-concavity of payoffs.

Then, the dominant strategy of firm 0 is $q_0^* = Q$, which yields the median consumer the highest possible payoff. By Assumption 3, for any $q_1$, firm 1 makes negative profits, which contradicts the definition of equilibrium (by choosing $a_1 = 1$, firm 1 can secure a payoff of zero).[41]

Given firm 0's dominant strategy, firm 1 cannot make non-negative profits. Then, it cannot exist a SPNE where $a_1^* = 1$.

#### A.3.3 Proof of (iii)

I proceed in two steps.

Step (1) In this step, I show that if firm 1 wants to enter the market when $\lambda = \bar{\lambda}$, then it also wants to enter whenever $\lambda > \bar{\lambda}$.

In any public monopoly equilibrium:

$$q_0 = q_0^M \in \arg\max_{q\in[0,Q]} V_0\left(q, p_0(q) \mid a_1 = 0\right) , \tag{76}$$

where

$$p_0(q) = \arg\max_{p_0\in[0,\infty)} V_0\left(p_0(q), q \mid a_1 = 0\right) . \tag{77}$$

As in any monopoly problem, it does not matter whether firm 0 chooses first $q_0^M$ or $p_0$. Now, let us define firm 1's optimal entry quality:

$$q_1^E \in \arg\max_{q\in[0,Q]} \pi_1\left(q, q_0^M, p_1^*\left(q, q_0^M\right), p_0^*\left(q, q_0^M\right)\right) , \tag{78}$$

where $p_i^*\left(q, q_0^M\right)$ is a mapping from the pair of qualities $\left(q, q_0^M\right)$ to some NE price of firm $i$. By Lemma 1, such mapping exists and it is unique whenever $\pi_1\left(q, q_0^M, p_1^*\left(q, q_0^M\right), p_0^*\left(q, q_0^M\right)\right) > 0$.[42] By assumption, if $\lambda = \overline{\lambda}$, then

$$\pi_1\left(q_1^E, q_0^M, p_1^*\left(q_1^E, q_0^M\right), p_0^*\left(q_1^E, q_0^M\right)\right) > 0 . \tag{79}$$

Since the function $\pi_1\left(q, q_0^M, p_1^*\left(q, q_0^M\right), p_0^*\left(q, q_0^M\right)\right)$ is continuous and non-decreasing in $\lambda$, then, for all $\lambda > \overline{\lambda}$:

$$\pi_1\left(q_1^E, q_0^M, p_1^*\left(q_1^E, q_0^M\right), p_0^*\left(q_1^E, q_0^M\right)\right) > 0 . \tag{80}$$

[41] Assumption 3 is not consequential for this result, as it would hold true whenever $Q \leq \frac{\theta_h}{4\alpha}$. I assume $Q \geq \frac{\theta_h}{\alpha}$ so that $q_i \leq Q$ is never binding in a duopoly.

[42] The mapping is potentially not unique only when qualities induce a public monopoly price NE. Clearly, in this case, entry is not profitable for firm 1.

This is guaranteed by subgame NE prices being continuous and non-decreasing in $\lambda$.[43]

Step (2) In this step, I show that $\bar{\lambda}$ exists and $\bar{\lambda} \in \left(0, \frac{1}{2}\right]$.

By (ii), $\overline{\lambda} > 0$. That is, under zero privatization, firm 1 does not want to enter.

To prove the result, it is sufficient to note that, if $\lambda = \frac{1}{2}$,

$$\pi_1\left(q_1^E, q_0^M, p_1^*\left(q_1^E, q_0^M\right), p_0^*\left(q_1^E, q_0^M\right)\right) > 0 \,. \tag{81}$$

Firm 1 wants to enter the market when $\lambda = \frac{1}{2}$. Therefore, it must exists $\overline{\lambda}$ such that the equilibrium market structure switches from public monopoly to duopoly as soon as $\lambda$ is bigger than $\overline{\lambda}$.

### A.3.4 Proof of (iv)

In this proof, I construct and list all duopoly equilibria.

For the sake of the readability of expressions, I adopt the following normalization: $\alpha = \frac{1}{2}$, $\theta_h = 2$. This simplifies the algebra but it does not affect any of the results. In this stage, $\alpha$ and $\theta_h$ are just multiplicative factors of firms' payoff functions, so that the relation between equilibrium qualities and $\alpha$, $\theta_h$ is linear (Motta [1993], Lambertini [2006]).

Subgame prices simplify as follows.

$$\begin{aligned}
\underline{p}_0^L(q_0, q_1) &= \frac{\lambda(q_1+4)q_1 - 4q_1 + 2\lambda(q_0-2)q_0 + 4q_0}{6\lambda} \\
\overline{p}_0^L(q_0, q_1) &= \frac{\lambda(q_1-10)q_1 + 4q_1 + 2\lambda q_0(q_0+5) - 4q_0}{6\lambda} \\
\underline{p}_1^L(q_0, q_1) &= \frac{2\lambda(q_1-1)q_1 + 2q_1 + \lambda q_0(q_0+2) - 2q_0}{6\lambda} \\
\overline{p}_1^L(q_0, q_1) &= \frac{2\lambda(q_1+4)q_1 - 2q_1 + \lambda(q_0-8)q_0 + 2q_0}{6\lambda}
\end{aligned} \tag{82}$$

[43] See subgame NE prices obtained in Lemma 1. The function $\pi_1\left(q, q_0^M, p_1^*\left(q, q_0^M\right), p_0^*\left(q, q_0^M\right)\right)$ has the following properties. First, it is not quasi-concave because by moving $q$, firm 1 can end up in different price regions. Second, at the "borders" of these regions, and everywhere else, is continuous. Third, "inside" of any region, it is non-decreasing in $\lambda$, which guarantees the required result.

| $\lambda_i$ | Value |
|---|---|
| $\lambda_{m_1}$ | $\frac{7}{13}$ |
| $\lambda_{m_2}$ | $\frac{5}{8}$ |
| $\lambda_{p_1}$ | $\approx 0.847$ |
| $\lambda_{p_2}$ | $\approx 0.924$ |
| $\lambda_{p_3}$ | $\approx 0.93$ |

Table 2: Different Thresholds for $\lambda$

$$\begin{aligned}
\underline{p}_0^H(q_0, q_1) &= \frac{1}{6}\left(q_1^2 + 2q_0^2\right) \\
\overline{p}_0^H(q_0, q_1) &= \frac{1}{6}((q_1 - 6)q_1 + 2q_0(q_0 + 3)) \\
\underline{p}_1^H(q_0, q_1) &= \frac{1}{6}\left(2q_1^2 + q_0^2\right) \\
\overline{p}_1^H(q_0, q_1) &= \frac{q_1^2}{3} + q_1 + \frac{1}{6}(q_0 - 6)q_0
\end{aligned} \tag{83}$$

$$\begin{aligned}
\underline{p}_0^M(q_0, q_1) &= \frac{q_0 - q_1}{\lambda} - \frac{1}{2}(q_1 - 6)q_1 + (q_0 - 3)q_0 \\
\overline{p}_0^M(q_0, q_1) &= q_0^2 + \frac{q_1 - q_0}{\lambda} - \frac{q_1^2}{2} \\
\overline{p}_1^M(q_0, q_1) &= \hat{p}_1^1(q_0, q_1) = \frac{5\lambda q_1 - q_1 + \lambda q_0^2 - 5\lambda q_0 + q_0}{2\lambda} \\
\underline{p}_1^M(q_0, q_1) &= \hat{p}_1^2(q_0, q_1) = \frac{\lambda q_0^2 - \lambda q_0 - q_0 + \lambda q_1 + q_1}{2\lambda}
\end{aligned} \tag{84}$$

Table A.3.4 defines some thresholds that will be useful for the remainder of the proof. Note that these values are not functions of $\theta_h$ and $\alpha$.

In some equilibria, the median consumer buys good 0. In others, they buy good 1. Moreover, in some equilibria, both firms play a pure strategy in *Stage* (2). In others, firm 0 plays a mixed strategy. For each type of equilibrium, for the sake of brevity, I show the proof only for the first equilibrium (usually the one where $q_0 < q_1$).[44]

**A.3.4.1 Pure strategies equilibria where the median consumer buys from the public firm** I am looking for a NE in qualities. Take a candidate equilibrium quality pair $\left(q_i^*, q_j^*\right)$.[45] The following conditions must be satisfied:

[44] The other proofs are analogous and available upon request.

[45] Throughout this section, sometimes I will refer to NE qualities as "an equilibrium." This is because firms will always have an incentive to enter the market given any candidate NE quality pair. Therefore, any NE in qualities is also an equilibrium of the whole game.

1. The induced price NE $p^L\left(q_i^*, q_j^*\right)$ exists, or, equivalently, $\left(q_i^*, q_j^*\right)$ lies in the $p^L$ region;
2. Given $q_j^*$,
   (a) $q_i^*$ must maximize the payoff of firm $i$ inside the $p^L\left(q_i, q_j^*\right)$;
   (b) firm $i$ has no incentive to deviate to any other price region.[46]

Given prices $p^L(q_0, q_1)$, I now write down firms' payoffs. $\underline{V}_0^L\left(q_0, q_1, p^L(q_0, q_1)\right)$ (respectively, $\overline{V}_0^L\left(q_0, q_1, p^L(q_0, q_1)\right)$) be the *Stage* (2) payoff function of firm 0 when $q_0 \leq q_1$ (respectively, $q_0 > q_1$). Analogously, I define $\underline{\pi}_1^L\left(q_0, q_1, p^L(q_0, q_1)\right)$ and $\overline{\pi}_1^L\left(q_0, q_1, p^L(q_0, q_1)\right)$.

$$V_0\left(q_0, q_1, p^L(q_0, q_1)\right) = \begin{cases} \underline{V}_0^L\left(q_0, q_1, p^L(q_0, q_1)\right) & \text{if } q_0 \leq q_1 \\ \overline{V}_0^L\left(q_0, q_1, p^L(q_0, q_1)\right) & \text{if } q_0 > q_1 \end{cases} =$$
$$= \begin{cases} \frac{\left(\lambda^2\left(\left(q_1^2-70\right)q_0-(q_1-10)q_0^2+q_1(q_1+4)^2-q_0^3\right)-2\lambda(4q_1(q_1+4)+q_0(5q_0-43))+16(q_1-q_0)\right)}{36\lambda} & \text{if } q_0 \leq q_1 \\ \frac{q_0\left(16-\lambda\left(\lambda\left(q_1^2-46\right)+26\right)\right)-q_1(\lambda(q_1-10)+4)^2+\lambda q_0^2(\lambda(q_1-2)-10)+\lambda^2 q_0^3}{36\lambda} & \text{if } q_0 > q_1 \end{cases} \tag{85}$$

$$\pi_1\left(q_0, q_1, p^L(q_0, q_1)\right) = \begin{cases} \overline{\pi}_1^L\left(q_0, q_1, p^L(q_0, q_1)\right) = \frac{(q_1-q_0)(\lambda(q_1+q_0-8)+2)^2}{36\lambda^2} & \text{if } q_0 \leq q_1 \\ \overline{\pi}_1^L\left(q_0, q_1, p^L(q_0, q_1)\right) = \frac{(q_0-q_1)(\lambda(q_1+q_0+2)-2)^2}{36\lambda^2} & \text{if } q_0 > q_1 \end{cases} \tag{86}$$

Let $\underline{q}_i^L(q_j)$ (respectively, $\overline{q}_i^L(q_j)$) denote the (interior) optimal quality of firm $i$ when $q_i \leq q_j$ (respectively, $q_i > q_j$). To characterize these strategies, I combine FOCs and SOCs. FOCs read as follows.

$$\frac{\partial \underline{V}_0^L\left(q_0, q_1, p^L(q_0, q_1)\right)}{\partial q_0} = \frac{\lambda^2((q_1+q_0)(q_1-3q_0)+20q_0-70)+\lambda(86-20q_0)-16}{36\lambda} = 0 \tag{87}$$

$$\frac{\partial \overline{V}_0^L\left(q_0, q_1, p^L(q_0, q_1)\right)}{\partial q_0} = \frac{\lambda\left(\lambda\left(3q_0^2+2q_0(q_1-2)-q_1^2+46\right)-20q_0-26\right)+16}{36\lambda} = 0 \tag{88}$$

[46] I present a formal illustration for these conditions in the main text.

$$\frac{\partial \underline{\pi}_1^L\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_1} = \frac{(\lambda(q_0 - 3q_1 - 2) + 2)(\lambda(q_0 + q_1 + 2) - 2)}{36\lambda^2} = 0 \tag{89}$$

$$\frac{\partial \overline{\pi}_1^L\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_1} = \frac{(\lambda(3q_1 - q_0 - 8) + 2)(\lambda(q_1 + q_0 - 8) + 2)}{36\lambda^2} = 0 \tag{90}$$

Combining FOCs and SOCs, I obtain:

$$\begin{aligned}
&\frac{\partial \underline{V}_0^L\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_0} = 0 \text{ and} \\
&\frac{\partial^2 \underline{V}_0^L\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_0^2} < 0 \Leftrightarrow \\
&q_0 = \frac{-(\lambda(q_1 - 10)) + \sqrt{2\lambda(-55\lambda + 2q_1(\lambda(q_1 - 5) + 5) + 29) + 52} - 10}{3\lambda} = \underline{q}_0^L(q_1)
\end{aligned} \tag{91}$$

$$\begin{aligned}
\frac{\partial \overline{\pi}_1^L\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_1} &= 0 \text{ and} \\
\frac{\partial^2 \overline{\pi}_1^L\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_1^2} &< 0 \Leftrightarrow \\
q_1 = \frac{\lambda(q_0 + 8) - 2}{3\lambda} &= \overline{q}_1^L(q_0)
\end{aligned} \tag{92}$$

$$\begin{aligned}
&\frac{\partial \overline{V}_0^L\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_0} = 0 \text{ and} \\
&\frac{\partial^2 \overline{V}_0\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_0^2} < 0 \Leftrightarrow \\
&q_0 = -\frac{\lambda(q_1 - 2) + \sqrt{2\lambda(-67\lambda + 2q_1(\lambda(q_1 - 1) - 5) + 59) + 52} - 10}{3\lambda} = \overline{q}_0^L(q_1)
\end{aligned} \tag{93}$$

$$\begin{aligned}
\frac{\partial \underline{\pi}_1^L\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_1} &= 0 \text{ and} \\
\frac{\partial^2 \underline{\pi}_1\left(q_0, q_1, p^L\left(q_0, q_1\right)\right)}{\partial q_1^2} &< 0 \Leftrightarrow \\
q_1 = \frac{\lambda(q_0 - 2) + 2}{3\lambda} &= \underline{q}_1^L(q_0)
\end{aligned} \tag{94}$$

I omit the SOCs for the sake of brevity. Each FOC has two solutions, but only one is a maximum. Note that both firms play the same strategy when $\lambda \to 1$.

The intersection of $\underline{q}_0^L(q_1)$ and $\overline{q}_1^L(q_0)$ is:

$$q_0^* = \frac{37\lambda + 9\sqrt{\lambda(34-39\lambda)+9} - 43}{16\lambda} < \frac{55\lambda + 3\sqrt{\lambda(34-39\lambda)+9} - 25}{16\lambda} = q_1^* \,. \quad (95)$$

The intersection of $\underline{q}_1^L(q_0)$ and $\overline{q}_0^L(q_1)$ is:

$$q_1^{**} = \frac{-7\lambda - 3\sqrt{\lambda(34-39\lambda)+9} + 25}{16\lambda} < \frac{11\lambda - 9\sqrt{\lambda(34-39\lambda)+9} + 43}{16\lambda} = q_0^{**} \,. \quad (96)$$

Define the vectors: $q^* = (q_0^*, q_1^*)$ and $q^{**} = (q_0^{**}, q_1^{**})$. For these to be an equilibrium quality pair, it must be that they are not subject to profitable unilateral deviations.

First of all, note that:

$$\begin{aligned} q_0^* &< q_1^* \\ q_0^{**} &> q_1^{**} \,. \end{aligned} \quad (97)$$

Let us start with $q^*$ (the proof for $q^{**}$ is analogous). I need to check that given $q_j^*$, firm $i \neq j$ does not want to deviate to some $q_i \neq q_i^*$. By Lemma 1, there exists a bi-jective mapping from a pair $(q_0, q_1)$ to some duopoly price NE. Thus, I distinguish between different types of deviations, depending on whether the pair $(q_j^*, q_i)$ is such that the induced price NE is $p^L\left(q_j^*, q_i\right)$, $p^H\left(q_j^*, q_i\right)$ or $p^M\left(q_j^*, q_i\right)$. Payoffs along (95) and (96) are symmetric and they are given by:

$$\begin{aligned} &V_0\left(q^*, p^L(q^*)\right) = \\ &\frac{1}{256\lambda^2}\left[93\sqrt{\lambda(34-39\lambda)+9} + \lambda\left(-108\sqrt{\lambda(34-39\lambda)+9}\right.\right. \\ &\left.\left.+\lambda\left(323\lambda + 39\sqrt{\lambda(34-39\lambda)+9} - 561\right) + 693\right) - 407\right] \end{aligned} \quad (98)$$

$$\begin{aligned} &\pi_1\left(q^*, p^L(q^*)\right) = \\ &-\frac{3\left(-3\lambda + \sqrt{\lambda(34-39\lambda)+9} - 3\right)^3}{512\lambda^3}. \end{aligned} \quad (99)$$

*Deviations of type* (1) Given $q_j^*$, firm $i$ deviates to some $q_i$ such that the induced price NE is $p^L\left(q_i, q_j^*\right)$.

Consider firm $i = 0$. Any deviation $q_0 < q_0^*$ is not profitable by FOC (87).[47] Then, consider some deviation $q_0 > q_1^*$ (Motta [1993] and Lambertini [2006], among others, call these "leapfrogging deviations"). Among these deviations, $\overline{q}_0^L\left(q_1^L\right)$ yields the highest possible payoff. Note that:

$$\begin{aligned}
&\underline{V}_0^L\left(\underline{q}_0^L(q_1), q_1, p^L\left(\underline{q}_0^L(q_1), q_1\right)\right) \geq \\
&\overline{V}_0^L\left(\overline{q}_0^L(q_1), q_1, p^L\left(\overline{q}_0^L(q_1), q_1\right)\right) \Leftrightarrow \\
&q_1 \geq \frac{3}{2};
\end{aligned} \tag{100}$$

$$\begin{aligned}
&\underline{\pi}_1^L\left(\underline{q}_1^L(q_0), q_0, p^L\left(\underline{q}_1^L(q_0), q_0\right)\right) \geq \\
&\overline{\pi}_1^L\left(\overline{q}_1^L(q_0), q_0, p^L\left(\overline{q}_1^L(q_0), q_0\right)\right) \Leftrightarrow \\
&q_0 \geq \frac{3}{2}.
\end{aligned} \tag{101}$$

That is, firm $i$ wants to produce a higher quality than $q_j$ if and only if $q_j$ is low enough. Since $q_1^* > \frac{3}{2}$ for $\lambda \geq \frac{1}{2}$, firm 0 does never want to leapfrog the quality of its opponent.

Let us now consider firm 1. Any $q_1 > q_0^*$ is not optimal by FOC. Since $q_0^* > \frac{3}{2}$ for $\lambda \in \left[\frac{7}{13}, \frac{5}{8}\right]$, firm 1 has an incentive to deviate to $\underline{q}_1^L\left(q_0^*\right) < q_0^*$ in this range. Let us define $\frac{7}{13} = \lambda_{m_1}$ and $\frac{5}{8} = \lambda_{m_2}$. I conclude that the pair (95) is "safe" from profitable unilateral deviation of *Deviations of type* (1) when $\lambda \in \left[\frac{1}{2}, \lambda_{m_1}\right] \bigcup [\lambda_{m_2}, 1]$.

*Deviations of type* (2) Given $q_j^*$, firm $i$ deviates to some $q_i$ such that the induced price NE is $p^H\left(q_i, q_j^*\right)$.

As in the previous point, I adopt the following approach. I compare (98) and (99) to the payoff that firm $i$ obtains by playing its optimal quality, given $q_j^*$, in the region where subgame prices are $p^H\left(q_i, q_j^*\right)$. I define the

[47] It can be easily shown that firms' payoffs inside this region are strictly concave. Therefore, FOCs are sufficient.

following payoffs.

$$V_0^H\left(q_0, q_1, p^H(q_0, q_1)\right) =$$
$$= \frac{1}{36}\left[\lambda\left(q_1^3 + q_1^2(q_0+12) - q_1\left(q_0^2+18\right)\right.\right.$$
$$\left.\left. - q_0((q_0-6)q_0+36)) + 6q_1(3-2q_1) - 6(q_0-6)q_0\right)\right]$$
$$\text{if } q_0 \leq q_1; \tag{102}$$

and

$$\frac{1}{36}\left(-\lambda\left(q_1^2-36\right)q_0 - q_1(90\lambda + q_1(\lambda(q_1-24)+12)-54) + \right.$$
$$\left. q_0^2(\lambda(q_1-6)-6) + \lambda q_0^3\right)$$
$$\text{if } q_0 > q_1$$

$$\pi_1^H\left(q_0, q_1, p^H(q_0, q_1)\right) = \begin{cases} \frac{1}{36}(q_1-q_0)(q_1+q_0-6)^2 & \text{if } q_0 \leq q_1 \\ \frac{1}{36}(q_0-q_1)(q_1+q_0)^2 & \text{if } q_0 > q_1 \end{cases} \tag{103}$$

FOCs and SOCs produce the following (interior) optimal strategies:

$$\begin{aligned} \underline{q}_0^L(q_1) &= \frac{6\lambda - \lambda q_1 + 2\sqrt{\lambda(q_1+3)(\lambda(q_1-6)+3)+9} - 6}{3\lambda} \text{ if } q_0 \leq q_1; \\ \overline{q}_0^L(q_1) &= -\frac{\lambda(q_1-6) + 2\sqrt{\lambda(q_1-6)(\lambda(q_1+3)-3)+9} - 6}{3\lambda} \text{ if } q_0 > q_1; \end{aligned} \tag{104}$$

$$\begin{aligned} \underline{q}_1^H(q_0) &= \frac{q_0}{3} \text{ if } q_0 \leq q_1; \\ \overline{q}_1^H(q_0) &= \frac{q_0+6}{3} \text{ if } q_0 > q_1; \end{aligned} \tag{105}$$

Whenever these strategies are feasible (i.e., they induce $p^H$ as a NE in prices), they are not attractive.

However, firms might still want to reach the $p^H(q_0, q_1)$ region, even if their (interior) optimal strategy in the region is not achievable. In other words, without an interior maximizer, I need to consider corner solutions (see Figure 6). There are three possible cases.

Case (1) $q_1 = q_0$. This deviation implies zero profits for both firms, and it is not profitable.

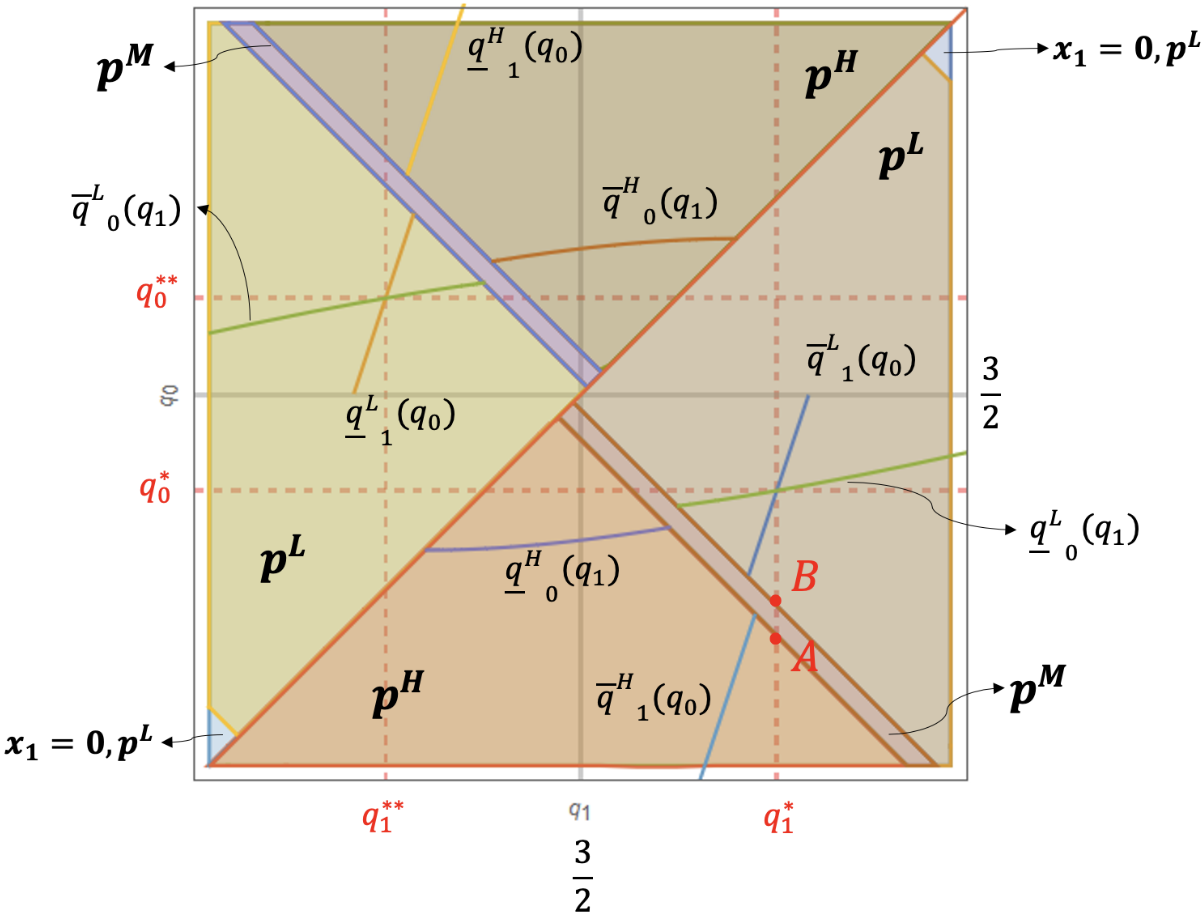



Figure 6: **Optimal Qualities** when $\lambda \geq \lambda_{m_2}$. In the $p^L$ region, I only plot the strategy yielding higher payoffs. Recall that, when $q_i, q_j$ induces $p^L$, $q_i > q_j$ is optimal for firm $i$ if and only if $q_j \leq \frac{3}{2}$.

Cbse (2) Given $q_j^*$, firm $i \neq j$ deviates to some $q_i$ such that $\overline{p}_1^H\left(q_i, q_j^*\right) = \hat{p}_1^1\left(q_i, q_j^*\right)$ or $\underline{p}_1^H\left(q_i, q_j^*\right) = \hat{p}_1^2\left(q_i, q_j^*\right)$. As an example of this kind of deviations, see the candidate equilibrium $\left(q_0^*, q_1^*\right)$ in Figure 6. Consider firm 0. By decreasing its quality, firm 0 can deviate to point $A$, which belongs to the $p^H$ region.[48]

It can be shown that these deviations solve the equation:

$$\begin{aligned}
\overline{p}_1^H(q_0, q_1) = \hat{p}_1^1(q_0, q_1) &\Rightarrow \frac{(q_1 - q_0)(\lambda(2q_1 + 2q_0 - 9) + 3)}{6\lambda} = 0 \text{ if } q_0 \leq q_1 \\
\underline{p}_1^H(q_0, q_1) = \hat{p}_1^2(q_0, q_1) &\Rightarrow \frac{(q_1 - q_0)(\lambda(2q_1 + 2q_0 - 3) - 3)}{6\lambda} = 0 \text{ if } q_0 > q_1
\end{aligned} \tag{106}$$

Take $i = 0$ and then $q_1 = q_1^*$. Equation (106) reduces to :

$$\begin{aligned}
&\frac{\left(3\sqrt{\lambda(34-39\lambda)+9} + \lambda(55-16q_0) - 25\right)\left(3\sqrt{\lambda(34-39\lambda)+9} + \lambda(16q_0 - 17) - 1\right)}{768\lambda^2} = 0 \\
&\text{if } q_0 < q_1; \\
&\text{and} \\
&\frac{1}{384\lambda^2}\Bigg[-111\sqrt{\lambda(34-39\lambda)+9} \\
&+ \lambda\left(677\lambda + 129\sqrt{\lambda(34-39\lambda)+9} - 64q_0(\lambda(2q_0-3)-3) - 1582\right) + 653\Bigg] = 0 \\
&\text{if } q_0 > q_1 \,.
\end{aligned} \tag{107}$$

There are two solutions.

$$\begin{aligned}
&q_0 = \frac{17\lambda - 3\sqrt{\lambda(34-39\lambda)+9} + 1}{16\lambda} = q_0^{d_1} < q_1^* \\
&q_0 = \frac{1}{16\lambda^2}\Bigg[\sqrt{2}\Bigg[\lambda^2\Bigg(-111\sqrt{\lambda(34-39\lambda)+9} + \\
&\lambda\left(749\lambda + 129\sqrt{\lambda(34-39\lambda)+9} - 1438\right) + 725\Bigg)\Bigg]^{\frac{1}{2}} + 12\lambda(\lambda+1)\Bigg] = q_0^{d_2} > q_1^* \\
&\Leftrightarrow \lambda < \frac{161}{275}
\end{aligned} \tag{108}$$

[48]Clearly, analogous deviations are available from the other equilibria and for firm 1 too.

Plugging the above solutions into (102) reveals that deviating to $q_0^{d_1}$ is never profitable. However, if firm 0 deviates to $q_0^{d_2}$, its payoff is:

$$\frac{1}{256\lambda^2}\Bigg(57\sqrt{\lambda(34-39\lambda)+9} \\ +\lambda\left(-204\sqrt{\lambda(34-39\lambda)+9}+\lambda\left(511\lambda+171\sqrt{\lambda(34-39\lambda)+9}-1205\right)+1017\right)-275\Bigg). \tag{109}$$

I obtain (109) by substituting $q_0^{d_2}$ and $q_1^*$ into (102). This reveals that (109) is higher than (98) for $\lambda > 0.953$. Note that this deviation is no more attractive if $\lambda = 1$ because $q_0^{d_2} = q_0^*$.

Let us now consider firm $i = 1$. Equation (106) has a unique feasible solution.

$$\overline{p}_1^H\left(q_1, q_0^*\right) = \hat{p}_1^1\left(q_1, q_0^*\right) \Rightarrow q_1 = \frac{-9\sqrt{-39\lambda^2+34\lambda+9}+35\lambda+19}{16\lambda} = q_1^{d_1} > q_0^*. \tag{110}$$

Substituting (110) and $q_0^*$ into (103), I obtain the deviation payoff:

$$-\frac{(\lambda+1)^2\left(\lambda+9\sqrt{\lambda(34-39\lambda)+9}-31\right)}{128\lambda^3}, \tag{111}$$

which is higher than (99) if $\lambda > 0.924 = \lambda_{p_2}$.

Ccse (3) Given $q_j^*$, firm $i \neq j$ deviates to some $q_i$ such that $q_i \to Q$ or $q_i = 0$. It is relatively easy to show that these "corner" deviations are not profitable.

*Deviations of type* (3) Given $q_j^*$, firm $i$ deviates to some $q_i$ such that the induced price NE is $p^M\left(q_i, q_j^*\right)$.

I now write down firms' payoff functions along this subgame price NE.

$$V_0^M\left(q_0, q_1, p^M\left(q_0, q_1\right)\right) = \begin{cases} \frac{(3-5\lambda)^2 q_1 + q_0(\lambda(-49\lambda+8(\lambda-1)q_0+54)-9)}{16\lambda} & \text{if } q_0 < q_1 \\ \frac{q_0(\lambda(\lambda+8(\lambda-1)q_0-6)+9)-(3-5\lambda)^2 q_1}{16\lambda} & \text{if } q_0 > q_1 \end{cases} \tag{112}$$

$$\pi_1^M\left(q_0, q_1, p^M\left(q_0, q_1\right)\right) = \begin{cases} \frac{(q_1-q_0)(\lambda(q_1+q_0-5)+1)^2}{4\lambda^2} & \text{if } q_0 < q_1 \\ \frac{(q_0-q_1)(\lambda(q_1+q_0-1)-1)^2}{4\lambda^2} & \text{if } q_0 > q_1 \end{cases} \tag{113}$$

The mixed strategies price NE exists if and only if:

$$\begin{aligned}
&q_0 \leq q_1 \text{ and } \overline{p}_1^H(q_0, q_1) < \hat{p}_1^1 < \overline{p}_1^L(q_0, q_1) \Rightarrow \\
&\frac{9\lambda - 2\lambda q_0 - 3}{2\lambda} < q_1 < \frac{7\lambda - 2\lambda q_0 - 1}{2\lambda} \\
&q_0 > q_1 \text{ and } \underline{p}_{-1}^H < \hat{p}_1^2(q_0, q_1) < \underline{p}_{-1}^L(q_0, q_1) \Rightarrow \\
&\frac{5\lambda - 2\lambda q_0 + 1}{2\lambda} < q_1 < \frac{3\lambda - 2\lambda q_0 + 3}{2\lambda}
\end{aligned} \tag{114}$$

I now show that by fixing $q_j$, the payoff function of firm $i$ is continuous. Let us start with firm 1. Suppose $q_0 \leq q_1$. Then,

$$\begin{aligned}
&\lim_{q_1 \to \frac{9\lambda - 2\lambda q_0 - 3}{2\lambda}} \left\{ \pi_1^H \left( q_0, q_1, p^H(q_0, q_1) \right) \right\} = \\
&\lim_{q_1 \to \frac{9\lambda - 2\lambda q_0 - 3}{2\lambda}} \left\{ \frac{1}{36}(q_1 - q_0)(q_1 + q_0 - 6)^2 \right\} = \\
&\lim_{q_1 \to \frac{9\lambda - 2\lambda q_0 - 3}{2\lambda}} \left\{ \pi_1^M \left( q_0, q_1, p^M(q_0, q_1) \right) \right\} = \\
&\lim_{q_1 \to \frac{9\lambda - 2\lambda q_0 - 3}{2\lambda}} \left\{ \frac{(q_1 - q_0)(\lambda(q_1 + q_0 - 5) + 1)^2}{4\lambda^2} \right\} = \\
&-\frac{(\lambda + 1)^2(\lambda(4q_0 - 9) + 3)}{32\lambda^3}.
\end{aligned} \tag{115}$$

$$\begin{aligned}
&\lim_{q_1 \to \frac{7\lambda - 2\lambda q_0 - 1}{2\lambda}} \left\{ \pi_1^L \left( q_0, q_1, p^L(q_0, q_1) \right) \right\} = \\
&\lim_{q_1 \to \frac{7\lambda - 2\lambda q_0 - 1}{2\lambda}} \left\{ \frac{(q_1 - q_0)(\lambda(q_1 + q_0 - 8) + 2)^2}{36\lambda^2} \right\} = \\
&\lim_{q_1 \to \frac{7\lambda - 2\lambda q_0 - 1}{2\lambda}} \left\{ \pi_1^M \left( q_0, q_1, p^M(q_0, q_1) \right) \right\} = \\
&\lim_{q_1 \to \frac{7\lambda - 2\lambda q_0 - 1}{2\lambda}} \left\{ \frac{(q_1 - q_0)(\lambda(q_1 + q_0 - 5) + 1)^2}{4\lambda^2} \right\} = \\
&-\frac{(1 - 3\lambda)^2(\lambda(4q_0 - 7) + 1)}{32\lambda^3}.
\end{aligned} \tag{116}$$

The same is true for the case $q_1 < q_0$.

Let us consider firm 0 and $q_0 \leq q_1$. Let us also note that $q_0 = \frac{9\lambda - 2\lambda q_1 - 3}{2\lambda}$ and

$q_0 = \frac{7\lambda - 2\lambda q_1 - 1}{2\lambda}$ bound the existence of the mixed strategies price NE. Then,

$$\lim_{q_0 \to \frac{9\lambda-2\lambda q_1-3}{2\lambda}} \left\{ V_0^M \left( q_0, q_1, p^M(q_0, q_1) \right) \right\} =$$
$$\lim_{q_0 \to \frac{9\lambda-2\lambda q_1-3}{2\lambda}} \left\{ \frac{(3-5\lambda)^2 q_1 + q_0(\lambda(-49\lambda + 8(\lambda-1)q_0 + 54) - 9)}{16\lambda} \right\} =$$
$$\lim_{q_0 \to \frac{9\lambda-2\lambda q_1-3}{2\lambda}} \left\{ V_0^H \left( q_0, q_1, p^H(q_0, q_1) \right) \right\} =$$
$$\lim_{q_0 \to \frac{9\lambda-2\lambda q_1-3}{2\lambda}} \Bigg\{ \frac{1}{36}\lambda \left( q_1^3 + q_1^2(q_0+12) - q_1\left(q_0^2+18\right) - q_0((q_0-6)q_0+36) \right) + 6q_1(3-2q_1) - 6(q_0-6)q_0 \Bigg\}$$
$$= \frac{\lambda\left(3\lambda(31-39\lambda) + 16(\lambda-1)\lambda q_1^2 + 4(\lambda(\lambda+6)-3)q_1 + 9\right) - 9}{32\lambda^2} \tag{117}$$

and

$$\lim_{q_0 \to \frac{7\lambda-2\lambda q_1-1}{2\lambda}} \left\{ V_0^M \left( q_0, q_1, p^M(q_0, q_1) \right) \right\} =$$
$$\lim_{q_0 \to \frac{7\lambda-2\lambda q_1-1}{2\lambda}} \left\{ \frac{(3-5\lambda)^2 q_1 + q_0(\lambda(-49\lambda + 8(\lambda-1)q_0 + 54) - 9)}{16\lambda} \right\} =$$
$$\lim_{q_0 \to \frac{7\lambda-2\lambda q_1-1}{2\lambda}} \left\{ V_0^L \left( q_0, q_1, p^L(q_0, q_1) \right) \right\} =$$
$$\lim_{q_0 \to \frac{7\lambda-2\lambda q_1-1}{2\lambda}} \Bigg\{ \frac{1}{36\lambda}\Big[ \lambda^2 \left( \left( q_1^2 - 70 \right) q_0 - (q_1-10)q_0^2 + q_1(q_1+4)^2 - q_0^3 \right)$$
$$- 2\lambda(4q_1(q_1+4) + q_0(5q_0-43)) + 16(q_1-q_0) \Big] \Bigg\}$$
$$\frac{\lambda\left(7\lambda(25-21\lambda) + 16(\lambda-1)\lambda q_1^2 + 4(\lambda(9\lambda-10)+5)q_1 - 57\right) + 5}{32\lambda^2} \tag{118}$$

Note again that an analogous condition holds for the case of $q_1 < q_0$. Hence, payoff functions are continuous, and I do not need to deal with corner solutions in this region. So I can focus on "interior" deviations. Put it differently, the payoff of firm 0 at points $A$ and $B$ in Figure 6 is the same, no matter if $q_0$ approaches $A$ or $B$ from above or from below. The same is true for both firms and for all $(q_0, q_1)$.

Take firm 1.

Suppose that firm 1 deviates to some $q_1$ such that $q_1 > q_0^*$ and $\left(q_1, q_0^*\right)$ is such that the induced price NE is $p^M$. Then, if it exists an optimal deviation inside this region, it must be characterized by the following equation.

$$\frac{\partial \pi_1^M\left(q_0, q_1, p^M\left(q_0, q_1\right)\right)}{\partial q_1} = \frac{\partial}{\partial q_1}\left[\frac{(\lambda(3q_1 - q_0 - 5) + 1)(\lambda(q_1 + q_0 - 5) + 1)}{4\lambda^2}\right]$$
$$\Rightarrow q_1 = \frac{5\lambda + \lambda q_0 - 1}{3\lambda}. \tag{119}$$

Since $q_0 = q_0^*$, (119) simplifies to:

$$q_1 = \frac{117\lambda + 9\sqrt{\lambda(34 - 39\lambda) + 9} - 59}{48\lambda} = q_1^{d_{m1}}. \tag{120}$$

The pair $\left(q_1^{d_{m1}}, q_0^*\right)$ satisfies (114) if and only if $\frac{119}{141} \leq \lambda \leq \frac{1}{132}\left(3\sqrt{345} + 61\right)$. The payoff from deviation is:

$$\pi_1^M\left(q_0^*, q_1^{d_{m1}}, p^M(q_0^*, q_1^{d_{m1}})\right) = \frac{\left(3\lambda - 9\sqrt{\lambda(34 - 39\lambda) + 9} + 35\right)^3}{13824\lambda^3}, \tag{121}$$

which is lower than (99) for any $\frac{119}{141} \leq \lambda \leq \frac{1}{132}\left(3\sqrt{345} + 61\right)$.

Suppose that firm 1 deviates to some $q_1$ such that $q_1 < q_0^*$ and $\left(q_1, q_0^*\right)$ is such that the induced price NE is $p^M$. Then,

$$\frac{\partial \pi_1^M\left(q_0, q_1, p^M\left(q_0, q_1\right)\right)}{\partial q_1} = \frac{\partial}{\partial q_1}\left[\frac{(q_0 - q_1)(\lambda(q_1 + q_0 - 1) - 1)^2}{4\lambda^2}\right]$$
$$\Rightarrow q_1 = \frac{\lambda + \lambda q_0 + 1}{3\lambda}. \tag{122}$$

However, the intersection of $q_0^*$ and $q_1 = \frac{\lambda + \lambda q_0 + 1}{3\lambda}$ does not satisfy (114).[49]

Let us now consider firm 0.

Suppose that firm 0 deviates to some $q_0$ such that $q_0 < q_1^*$ and $\left(q_0, q_1^*\right)$ is

[49] In particular, by decreasing or increasing $q_1$, firm 1 can never reach the region of existence of the mixed strategies subgame price NE where $q_1 < q_0$. See Figure 6.

such that the induced price NE is $p^M$. Then, if it exists an optimal interior deviation, it is given by:

$$q_0^{d_{m1}} = \frac{49\lambda^2 - 54\lambda + 9}{16\lambda^2 - 16\lambda} . \tag{123}$$

I obtain (123) by taking the FOC of (112). The first deviation (123) is feasible (it satisfies (114)) if and only if $0.859 \leq \lambda \leq 0.870$, but in this case it is not profitable.

Suppose that firm 0 deviates to some $q_0$ such that $q_0 > q_1^*$ and $\left(q_0, q_1^*\right)$ is such that the induced price NE is $p^M$. In this case, the FOC of (112) and condition (114) are never jointly satisfied given $q_1 = q_1^*$. Therefore, none of the two firms is attracted by qualities that lie inside the $p^M$ region.

Since the pair $q_0^*, q_1^*$ is not subject to profitable unilateral deviations and firms get positive NE payoffs, the following profile of strategies:

$$\begin{aligned}
&\textit{Stage (1)}: a_i^* = 1 \text{ for } i \in \{0,1\}\\
&\textit{Stage (2)}: q_0^*, q_1^*\\
&\textit{Stage (3)}: \underline{p}_0^L\left(q_0^*, q_1^*\right), \overline{p}_1^L\left(q_0^*, q_1^*\right)\\
&\textit{Stage (4)}: \text{All consumers with } \theta \leq \hat{\theta}(\cdot) \text{ buy good 0 and}\\
&\text{all consumers with } \theta > \hat{\theta}(\cdot) \text{ buy good 1}
\end{aligned} \tag{124}$$

is an equilibrium for any $\lambda \in \left\{\left[\frac{1}{2}, \lambda_{m_1}\right] \bigcup \left[\lambda_{m_2}, \lambda_{p_2}\right], 1\right\}$.

Note that an analogous proof applies to the following profile of strategies.

$$\begin{aligned}
&\textit{Stage (1)}: a_i^* = 1 \text{ for } i \in \{0,1\}\\
&\textit{Stage (2)}: q_0^{**}, q_1^{**}\\
&\textit{Stage (3)}: \overline{p}_0^L\left(q_0^{**}, q_1^*\right), \underline{p}_{-1}^L\left(q_0^{**}, q_1^{***}\right)\\
&\textit{Stage (4)}: \text{All consumers with } \theta \geq \hat{\theta}(\cdot) \text{ buy good 0 and}\\
&\text{all consumers with } \theta < \hat{\theta}(\cdot) \text{ buy good 1}
\end{aligned} \tag{125}$$

It is trivial to prove that (124) and (125) are payoffs- (and, therefore, welfare-) equivalent.

**A.3.4.2 Mixed strategies equilibria where the private firm plays a mixed strategy and the median consumer buys from the public firm** In this Section, I show that if $\lambda \in (\lambda_{m_1}, \lambda_{m_2})$, there exist a mixed strategy NE where firm 0 plays $q_0 = \frac{3}{2}$ and firm 1 mixes between its two best responses.

Suppose $q_0 = \frac{3}{2}$. Firm 1's best responses are:

$$\underline{q}_1^L\left(\frac{3}{2}\right) = -\frac{\lambda - 4}{6\lambda} \tag{126}$$

and

$$\overline{q}_1^L\left(\frac{3}{2}\right) = \frac{19}{6} - \frac{2}{3\lambda} \tag{127}$$

Given $q_0 = \frac{3}{2}$, the strategies (126) and (127) are such that the price NE $p^L$ exists. These strategies yield firm 1 the following payoff:

$$\frac{(5\lambda - 2)^3}{243\lambda^3}. \tag{128}$$

It is relatively easy to show that, given $q_0 = \frac{3}{2}$, firm 1 never wants to deviate to some $q_1$ such that another price NE exists.

Therefore, let us suppose that firm 1 plays (127) with probability $k_1 \in (0, 1)$ and (126) with probability $1 - k_1$. Let us denote with $\omega_1$ this mixed strategy. For any $k_1$, $\omega_1$ is a best response of firm 1 to $q_0 = \frac{3}{2}$.

For this to be a NE, it must be that firm 0's best response against $\omega_1$ is $q_0 = \frac{3}{2}$. Let us define $S(\omega_1)$ as the support of $\omega_1$. Firm 0's expected payoff given some $q_0$ such that $p^L(q_0, q_1)$ exists is:

$$\mathbb{E}[V_0(\omega_1, q_0)] =$$

$$\begin{cases} k_1 \underline{V}_0^L\left(q_0, \underline{q}_1^L\left(\frac{3}{2}\right), p^L\left(\underline{q}_1^L\left(\frac{3}{2}\right)\right)\right) + (1-k_1)\overline{V}_0^L\left(q_0, \overline{q}_1^L\left(\frac{3}{2}\right), p^L\left(\overline{q}_1^L\left(\frac{3}{2}\right)\right)\right) & \text{if } \underline{q}_1^L\left(\frac{3}{2}\right) \le q_0 \le \overline{q}_1^L\left(\frac{3}{2}\right) \\ k_1 \underline{V}_0^L\left(q_0, \underline{q}_1^L\left(\frac{3}{2}\right), p^L\left(\underline{q}_1^L\left(\frac{3}{2}\right)\right)\right) + (1-k_1)\underline{V}_0^L\left(q_0, \overline{q}_1^L\left(\frac{3}{2}\right), p^L\left(\overline{q}_1^L\left(\frac{3}{2}\right)\right)\right) & \text{if } q_0 < \underline{q}_1^L\left(\frac{3}{2}\right) \le \overline{q}_1^L\left(\frac{3}{2}\right) \\ k_1 \overline{V}_0^L\left(q_0, \underline{q}_1^L\left(\frac{3}{2}\right), p^L\left(\underline{q}_1^L\left(\frac{3}{2}\right)\right)\right) + (1-k_1)\overline{V}_0^L\left(q_0, \overline{q}_1^L\left(\frac{3}{2}\right), p^L\left(\overline{q}_1^L\left(\frac{3}{2}\right)\right)\right) & \text{if } q_0 > \overline{q}_1^L\left(\frac{3}{2}\right) > \underline{q}_1^L\left(\frac{3}{2}\right) \end{cases} \tag{129}$$

which, if $\underline{q}_1^L\left(\frac{3}{2}\right) < q_0 < \overline{q}_1^L\left(\frac{3}{2}\right)$, reduces to:

$$\frac{1}{7776\lambda^2}\Bigg[\lambda\Bigg( - 6k_1(2q_0 - 3)(\lambda(\lambda(36(q_0 - 3)q_0 + 1745) - 1936) + 560)+ \\ \lambda^2(6q_0(6q_0(6q_0 - 13) + 1655) + 3721) - 12\lambda(8q_0(21q_0 + 58) + 1525) + 336(10q_0 + 43)\Bigg) - 3136\Bigg] \tag{130}$$

Note that if $\lambda_{m_1} \leq \lambda \leq \lambda_{m_2}$ and $k_1 = \frac{1}{2}$, (130) is maximized at $q_0 = \frac{3}{2}$. Moreover, any $q_0$ such that

$$\begin{aligned} & q_0 \leq \underline{q}_1^L\left(\frac{3}{2}\right) < \overline{q}_1^L\left(\frac{3}{2}\right) \text{ or} \\ & \text{if } q_0 \geq \overline{q}_1^L\left(\frac{3}{2}\right) > \underline{q}_1^L\left(\frac{3}{2}\right) \end{aligned} \tag{131}$$

it is strictly dominated by $q_0 = \frac{3}{2}$. Firm 0's expected payoff is then:

$$\begin{aligned} & \underline{V}_0^L\left(q_0, \overline{q}_1^L\left(\frac{3}{2}\right), p^L\left(q_0, \overline{q}_1^L\left(\frac{3}{2}\right)\right)\right) = \\ & \overline{V}_0^L\left(q_0, \underline{q}_1^L\left(\frac{3}{2}\right), p^L\left(q_0, \underline{q}_1^L\left(\frac{3}{2}\right)\right)\right) = \\ & \frac{\lambda(\lambda(4573\lambda - 7797) + 4872) - 784}{1944\lambda^2} \end{aligned} \tag{132}$$

It is relatively easy to check that given $\omega_1$, any $q_0$ that induces $p^H$ or $p^M$ as a price NE is strictly worse than $q_0 = \frac{3}{2}$.[50]

Finally, note that expected payoffs are positive so that firms have an incentive to enter the market in *Stage* (1). Thus, let us denote with $\omega_1^*$ the equilibrium mixed strategy of firm 1. I can state the following. If $\lambda \in (\lambda_{m_1}, \lambda_{m_2})$, the following strategies profile is an equilibrium:

$$\begin{aligned} & \textit{Stage } (1)\text{: } a_i^* = 1 \text{ for } i \in \{0,1\} \\ & \textit{Stage } (2)\text{: } \frac{3}{2}, \omega_1^* \\ & \textit{Stage } (3)\text{: } p^L\left(\frac{3}{2}, \omega_1^*\right) \\ & \textit{Stage } (4)\text{: If } q_1 \geq \frac{3}{2} \text{ all consumers with } \theta \leq \hat{\theta}(\cdot) \\ & \text{buy good 0 and all consumers with } \theta > \hat{\theta}(\cdot) \text{ buy good 1.} \\ & \text{If } q_1 < \frac{3}{2}, \text{ all consumers with } \theta \geq \hat{\theta}(\cdot) \text{ buy good 0} \\ & \text{and all consumers with } \theta < \hat{\theta}(\cdot) \text{ buy good 1.} \end{aligned} \tag{133}$$

By definition of mixed strategy equilibrium, payoffs and welfare are symmetric, no matter what strategy does firm 1 play. When $\lambda = \lambda_{m_1}$ or $\lambda = \lambda_{m_2}$, the

[50] In particular, one can check that given any strategy in the support of $\omega_1$, any $q_0$ that induces $p^M$ or $p^H$ is strictly dominated by $q_0 = \frac{3}{2}$.

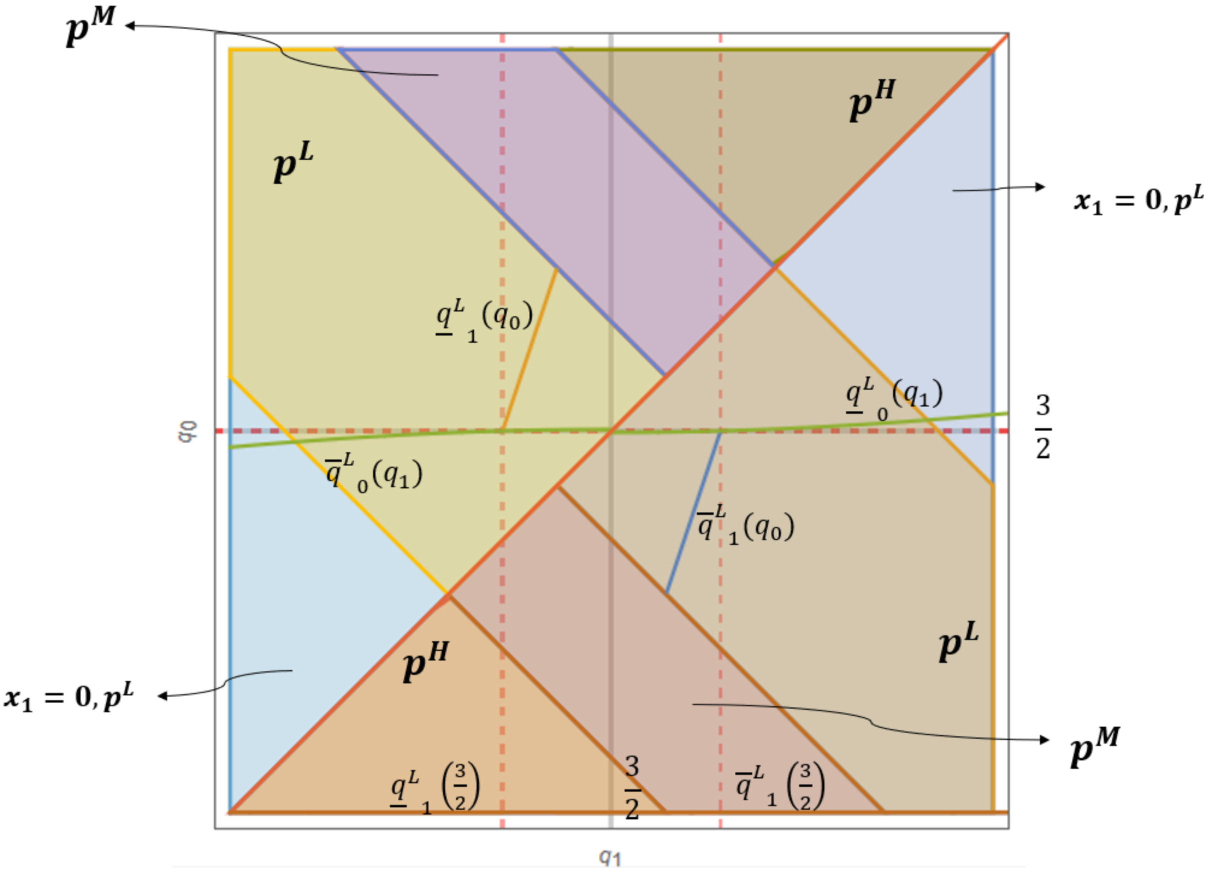



Figure 7: **Mixed Strategy Equilibria** when $\lambda = \lambda_{m_1}$. In the $p^L$ region, I only plot the strategy yielding higher payoffs. Recall that, when $q_i, q_j$ induces $p^L$, $q_i > q_j$ is optimal for firm $i$ if and only if $q_j \leq \frac{3}{2}$.

(expected value of) equilibrium strategies, payoffs and welfare is the same as in the equilibria (124), (125).

Note that (133) may be an equilibrium also for other values of $\lambda$. In Figure 7, I show the strategies in the support of $\omega_1^*$ when $\lambda = \lambda_{m_1}$. Let $E_0$ denote the (collection of) equilibria where the median consumer buys good 0. Then, $E_0$ exists when $\lambda \in \left[\frac{1}{2}, \lambda_{p_2}\right]$.

#### A.3.4.3 Pure strategies equilibria where the median consumer buys from the private firm

Suppose we are in the NE $p^H$ region. The intersection of optimal qualities (104) and (105) are:

$$q_1 = \frac{3\left(13\lambda + \sqrt{\lambda(18 - 23\lambda) + 9} - 3\right)}{16\lambda} > q_0 = \frac{3\left(7\lambda + 3\sqrt{\lambda(18 - 23\lambda) + 9} - 9\right)}{16\lambda}; \tag{134}$$

and

$$
\begin{aligned}
&q_0 = \\
&\frac{1}{16\lambda}\Bigg[29\lambda + \sqrt{\lambda(18-23\lambda)+9} \\
&- 2\sqrt{2}\sqrt{5\sqrt{\lambda(18-23\lambda)+9} + \lambda\left(-287\lambda + 5\sqrt{\lambda(18-23\lambda)+9} + 226\right) + 113} + 29\Bigg] \\
&> q_1 = \frac{9\lambda - 3\sqrt{\lambda(18-23\lambda)+9} + 9}{16\lambda}\,,
\end{aligned}
\tag{135}
$$

However, given (134) and (135), the conditions such that the price NE $p^H(q_0, q_1)$ exists are not satisfied. In particular, the median consumer would buy from firm 0. Hence, I look for corner solutions. Consider $q_0, q_1$ such that:

$$
\begin{aligned}
&\begin{cases} \overline{p}_1^H(q_0, q_1) = \hat{p}_1^1(q_0, q_1) \\ q_1 = \overline{q}_1^H(q_0) \end{cases} \quad \Leftrightarrow \\
&q_1 = \frac{3}{8}\left(7 - \frac{1}{\lambda}\right) = q_1^{***} \\
&q_0 = \frac{3}{8}\left(5 - \frac{3}{\lambda}\right) = q_0^{***}
\end{aligned}
\tag{136}
$$

Payoffs along (136) are:

$$
\begin{aligned}
V_0^H\left(q_0^{***}, q_1^{***}, p^H\left(q_0^{***}, q_1^{***}\right)\right) &= \frac{3(\lambda+1)(\lambda+3)(5\lambda-3)}{128\lambda^2} \\
\pi_1^H\left(q_0^{***}, q_1^{***}, p^H\left(q_0^{***}, q_1^{***}\right)\right) &= \frac{3(\lambda+1)^3}{64\lambda^3}\,.
\end{aligned}
\tag{137}
$$

It can be shown that none of the two firms has a profitable deviation inside the $p^H(q_0, q_1)$ region. Then, to check that (136) is an equilibrium, I need to check for possibly profitable deviations toward qualities such the induced NE prices are either $p^L(q_0, q_1)$ or $p^M(q_0, q_1)$. Firm 0 can deviate to $\underline{q}_0^L\left(q_1^{***}\right)$, gaining a payoff of

$$\frac{1}{31104\lambda^2}\left[721\sqrt{\lambda(1762-2159\lambda)+721}+\right.$$
$$\lambda\left(1762\sqrt{\lambda(1762-2159\lambda)+721}+\right. \tag{138}$$
$$\left.\left.\lambda\left(-8828\lambda-2159\sqrt{\lambda(1762-2159\lambda)+721}+107214\right)-73248\right)-19306\right],$$

which is higher than the public firm's payoff in (137) for $\lambda < 0.93 = \lambda_{p_3}$. If $\lambda \geq 0.93$, firm 0 has no profitable unilateral deviation given $q_1^{***}$. Analogously, it can be shown that if $\lambda \geq 0.93$, it exists a NE:

$$\begin{cases} \underline{p}_1^H(q_0, q_1) = \hat{p}_1^2(q_0, q_1) \\ q_1 = \underline{q}_1^H(q_0) \end{cases} \Leftrightarrow$$
$$q_1 = \frac{3(\lambda+1)}{8\lambda} = q_1^{****} \tag{139}$$
$$q_0 = \frac{9(\lambda+1)}{8\lambda} = q_0^{****}.$$

Payoffs are still (137). If $\lambda < 0.93$, in fact, firm 0 can deviate to $\overline{q}_0^L\left(q_1^{****}\right)$, gaining again (138). There are two other equilibria that are obtained as corner solutions.

$$\begin{cases} \overline{p}_1^H(q_0, q_1) = \hat{p}_1^1(q_0, q_1) \\ q_0 = \underline{q}_0^H(q_1) \end{cases} \Leftrightarrow$$
$$q_1 = -\frac{6}{\lambda+1} - \frac{15}{8\lambda} + \frac{57}{8} = q_1^{*****} \tag{140}$$
$$q_0 = \frac{3}{8}\left(\frac{16}{\lambda+1} + \frac{1}{\lambda} - 7\right) = q_0^{*****}.$$

The NE (140) exists if and only if $\lambda \geq 0.847 = \lambda_{p_1}$ because otherwise firm 1 has an incentive to deviate toward $\overline{q}_1^L(q_0^{*****})$. In the same way, if $\lambda \geq 0.847$, it exists the NE:

$$\begin{cases} \underline{p}_1^H(q_0, q_1) = \hat{p}_1^2(q_0, q_1) \\ q_0 = \overline{q}_0^H(q_1) \end{cases} \Leftrightarrow$$
$$q_1 = \frac{6}{\lambda+1} + \frac{15}{8\lambda} - \frac{33}{8} = q_1^{******} \tag{141}$$
$$q_0 = -\frac{6}{\lambda+1} - \frac{3}{8\lambda} + \frac{45}{8} = q_0^{******}.$$

Payoffs along (140) and (141) are symmetric and are given by:

$$\begin{aligned}
&V_0^H\left(q_0^{*****},q_1^{*****},p^H\left(q_0^{*****},q_1^{*****}\right)\right)=\\
&V_0^H\left(q_0^{******},q_1^{******},p^H\left(q_0^{******},q_1^{******}\right)\right)=\\
&\frac{3(\lambda(\lambda(\lambda(\lambda(965\lambda-1237)-150)+558)-15)-57)}{128\lambda^2(\lambda+1)^2};\\
&\pi_1^H\left(q_0^{*****},q_1^{*****},p^H\left(q_0^{*****},q_1^{*****}\right)\right)=\\
&\pi_1^H\left(q_0^{******},q_1^{******},p^H\left(q_0^{******},q_1^{******}\right)\right)=\\
&\frac{3(\lambda+1)(\lambda(13\lambda-6)-3)}{64\lambda^3}.
\end{aligned} \tag{142}$$

Therefore, it is possible to state the following results. If $\lambda \geq \lambda_{p_3}$, the following profile of strategies are equilibria.

$$\begin{aligned}
&\textit{Stage (1)}: a_i^*=1 \text{ for } i\in\{0,1\}\\
&\textit{Stage (2)}: q_0^{***},q_1^{***}\\
&\textit{Stage (3)}: p^H\left(q_0^{***},q_1^{***}\right)\\
&\textit{Stage (4)}: \text{All consumers with } \theta\leq\hat{\theta}(\cdot)\\
&\text{buy good 0 and all consumers with } \theta>\hat{\theta}(\cdot) \text{ buy good 1.}
\end{aligned} \tag{143}$$

$$\begin{aligned}
&\textit{Stage (1)}: a_i^*=1 \text{ for } i\in\{0,1\}\\
&\textit{Stage (2)}: q_0^{****},q_1^{****}\\
&\textit{Stage (3)}: p^H\left(q_0^{****},q_1^{****}\right)\\
&\textit{Stage (4)}: \text{All consumers with } \theta\geq\hat{\theta}(\cdot)\\
&\text{buy good 0 and all consumers with } \theta<\hat{\theta}(\cdot) \text{ buy good 1.}
\end{aligned} \tag{144}$$

It is trivial to prove that (143) and (144) are payoffs- (and, therefore, welfare-) equivalent.

If $\lambda \geq \lambda_{p_1}$, the following profile of strategies are equilibria.

$$\begin{aligned}
&\textit{Stage (1)}: a_i^*=1 \text{ for } i\in\{0,1\}\\
&\textit{Stage (2)}: q_0^{*****},q_1^{*****}\\
&\textit{Stage (3)}: p^H\left(q_0^{*****},q_1^{*****}\right)\\
&\textit{Stage (4)}: \text{All consumers with } \theta\leq\hat{\theta}(\cdot)\\
&\text{buy good 0 and all consumers with } \theta>\hat{\theta}(\cdot) \text{ buy good 1.}
\end{aligned} \tag{145}$$

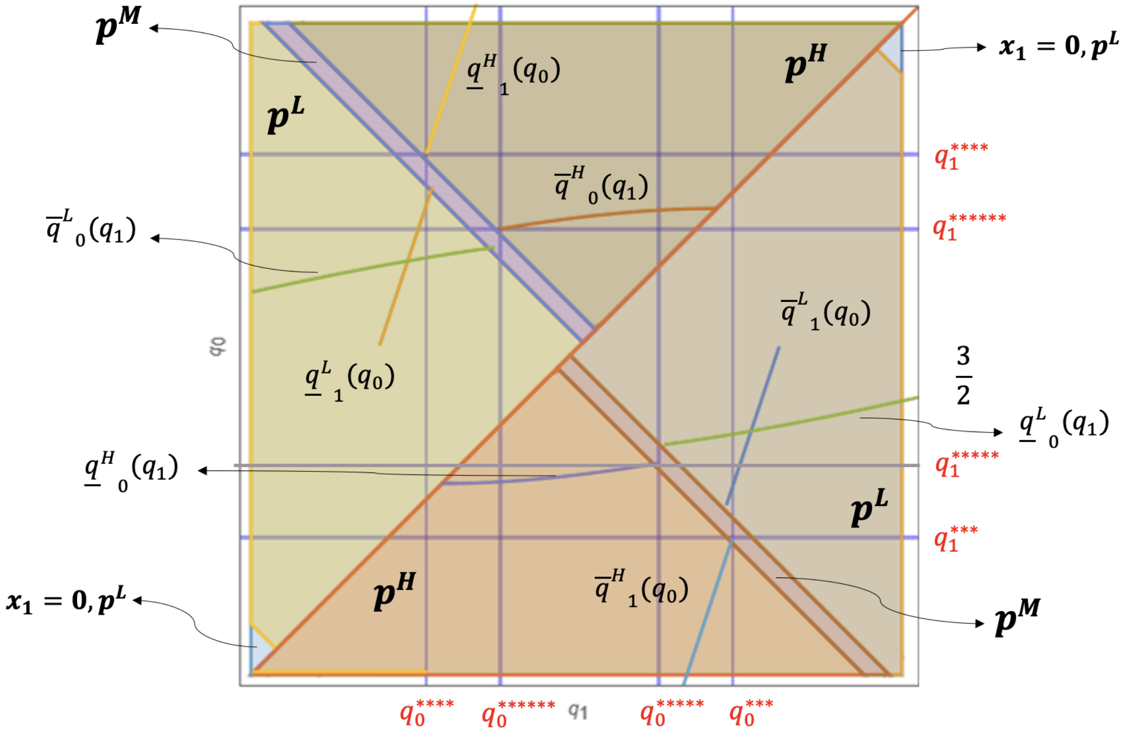



Figure 8: **"Corner" Equilibria** when $\lambda \geq \lambda_{m_2}$. In the $p^L$ region, I only plot the strategy yielding higher payoffs for each firm. Recall that, when $q_i, q_j$ induces $p^L$, $q_i > q_j$ is optimal for firm $i$ if and only if $q_j \leq \frac{3}{2}$.

$$\begin{aligned}
&\textit{Stage (1)}: a_i^* = 1 \text{ for } i \in \{0,1\} \\
&\textit{Stage (2)}: q_0^{******}, q_1^{******} \\
&\textit{Stage (3)}: p^H\left(q_0^{******}, q_1^{******}\right) \\
&\textit{Stage (4)}: \text{All consumers with } \theta \geq \hat{\theta}(\cdot) \\
&\text{buy good 0 and all consumers with } \theta < \hat{\theta}(\cdot) \text{ buy good 1.}
\end{aligned} \tag{146}$$

It is trivial to prove that (145) and (146) are payoffs- (and, therefore, welfare-) equivalent.

Throughout the paper, I refer to $E_1$ (respectively, $E_1(2)$) as the (collection of) equilibria where the median consumer buys good 1, i.e., (143), and (144) (respectively, (145), and (146)). In Table 3, I recap the duopoly equilibria and clarify the notation.

| / | Median Consumer Buys: | Existence | Strategies | Quality |
|---|---|---|---|---|
| (124) | Good 0, $E_0$ | $\lambda \in \left[\frac{1}{2}, \lambda_{m_1}\right] \bigcup \left[\lambda_{m_2}, \lambda_{p_2}\right]$ | Pure | $q_0 < q_1$ |
| (125) | Good 0, $E_0$ | $\lambda \in \left[\frac{1}{2}, \lambda_{m_1}\right] \bigcup \left[\lambda_{m_2}, \lambda_{p_2}\right]$ | Pure | $q_0 > q_1$ |
| (133) | Good 0, $E_0$ | $\lambda \in (\lambda_{m_1}, \lambda_{m_2})$ | 1: Mixed. 0: Pure. | $\mathbb{E}[q_1] = q_0$ |
| (143) | Good 1, $E_1$ | $\lambda \geq \lambda_{p_3}$ | Pure | $q_0 < q_1$ |
| (144) | Good 1, $E_1$ | $\lambda \geq \lambda_{p_3}$ | Pure | $q_0 > q_1$ |
| (145) | Good 1, $E_1(2)$ | $\lambda \geq \lambda_{p_1}$ | Pure | $q_0 < q_1$ |
| (146) | Good 1, $E_1(2)$ | $\lambda \geq \lambda_{p_1}$ | Pure | $q_0 > q_1$ |

Table 3: Characterization of Duopoly Equilibria for $\lambda \in \left[\frac{1}{2}, 1\right)$.

## A.4 Discussion on Equilibria Characterization

Let us recall that:[51]

$$\frac{1}{2} < \lambda_{m_1} < \lambda_{m_2} < \lambda_{p_1} < \lambda_{p_2} < \lambda_{p_3} < 1 \ . \tag{147}$$

When $\lambda \leq \lambda_{p_2}$, there are two pairs of (payoffs-equivalent) equilibrium qualities such that the median consumer buys good 0. These equilibrium qualities are $q_0^* < q_1^*$ and $q_1^{**} > q_0^{**}$. Fix any of these quality pair. When $\lambda$ is low, firm 0 produces an "intermediate" quality, neither too high nor too low. Combined with a low price in *Stage* (3), this strategy appeals to most consumers. Firm 1 produces a very high (or very low) quality, serving consumers with very high (or very low) WTP. When $\lambda > \lambda_{p_2}$, firm 1 wants to deviate to some $q_1$ inside the $p^H$ region, and these pairs stop to be an equilibrium. Along the deviation path to the $p^H$ region, firm 1 faces a trade-off between a higher markup and higher product differentiation, two key drivers of profits. On the one hand, the region $p^H$ would always tempt firm 1 because of the high markups. However, deviation towards that region would imply lower product differentiation, thereby decreasing profits.[52] The degree of product differentiation along this possibly profitable deviation increases in $\lambda$. Therefore, the deviation becomes profitable for firm 1 only when $\lambda$ is high enough. Firm 0 is attracted by the $p^H$ region only if $\lambda$ is high enough for the increase in public profits to compensate for the loss in the median consumer's payoff.

In the region $\lambda \leq \lambda_{p_2}$, there exists an interval $(\lambda_{m_1}, \lambda_{m_2})$ where the equilibrium is in mixed strategies. The lack of pure strategies equilibria is due to the discontinuity of best response functions. Inside the $p^L$ region, each firm $i$ wants to produce a higher quality than $q_j$ if and only if $q_j$ is below a certain threshold $\hat{q}$. If $\lambda = \lambda_{m_1}$, then $q_0^* = \hat{q}$ and $q_0^{**} = \hat{q}$. So, a marginal increase in $\lambda$ makes firm 1

[51] See Table A.3.4 for the expression of these thresholds.

[52] See Figure 7.

want to leapfrog the quality of its opponent. Take, for example, the pair $(q_0^*, q_1^*)$. As soon as $\lambda$ "hits" the threshold $\lambda_{m_1}$, then $q_0^* > \hat{q}$ and firm 1's best response "jumps." This deviation is profitable as long as $q_0^* > \hat{q} \Leftrightarrow \lambda < \lambda_{m_2}$.[53] In the region $(\lambda_{m_1}, \lambda_{m_2})$, I show that there exists an equilibrium where firm 0 plays $\hat{q}$ and firm 1 randomizes between its two best responses. Note that the strategies in the mixed strategy equilibrium approach those of the pure strategy equilibria when $\lambda \to \lambda_{m_1}$ and $\lambda \to \lambda_{m_2}$ (see Figure 10).

When $\lambda$ is high enough ($\lambda \geq \lambda_{p_1}$), there exists at least two equilibria where the median consumer buys good 1. In particular, there are two pairs of equilibrium qualities if $\lambda \in [\lambda_{p_1}, 1]$, ($q_0^{****}, q_1^{****}$ and $q_0^{******}, q_1^{******}$), and two more ($q_0^{***}, q_1^{***}$ and $q_0^{*****}, q_1^{*****}$) if $\lambda \in [\lambda_{p_3}, 1]$. To obtain these equilibria, I intersect the optimal quality of firm $i \in \{0, 1\}$ with the highest $q_j$ such that the $p^H$ price equilibrium exists. Therefore, I refer to these as "Corner" equilibria.[54] In Figure 10, I show equilibrium qualities. See Table 3 for a classification of the different equilibria.

When $\lambda = 1$, there exists a unique equilibrium. All equilibrium quality pairs in Table 3 coincide. The market is a standard private duopoly.

**Equilibrium Uniqueness** Equilibrium uniqueness seems too much to be expected from this setting. However, I can give some insights into the problem of equilibrium selection. First, I can claim that the equilibria in Proposition 2 are unique in the class of equilibria where both firms play pure strategies and at least one firm plays an "interior" strategy (a strategy resulting from its FOC). Second, I focus on mixed strategy equilibria only in the region where it does not exist a pure strategy equilibrium because of the discontinuity of best responses. Note that there might possibly exist other equilibria in mixed strategies, which, however, are complicated to construct.

### A.5 Proof of Proposition 3

The proof is straightforward and obtained by substituting equilibrium qualities and prices in the profits and welfare functions.

# B Additional Plots

[53] The same is true for the pair $(q_0^{**}, q_1^{**})$.
[54] The intersection of FOCs inside the $p^H$ region is not feasible.

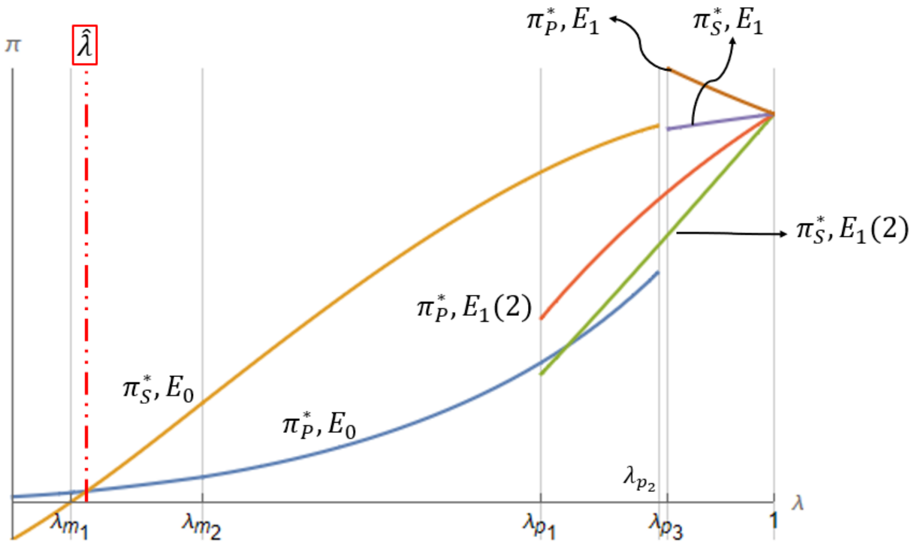



Figure 9: **Equilibrium Profits for** $\lambda \geq \frac{1}{2}$. See Table 3 for the classification of SPNE.

# C Extensions

Without loss of generality, I adopt the normalization of the previous Section: $\theta_h = 2, \alpha = \frac{1}{2}$.

## C.1 Marginal Cost Pricing

Consider a "modified" version of the game in Section 2. Firm 0 now does not get to choose $p_0$, because it commits to $p_0 = \alpha q_0^2$. Then, the game runs as in the previous sections. I have the following result.

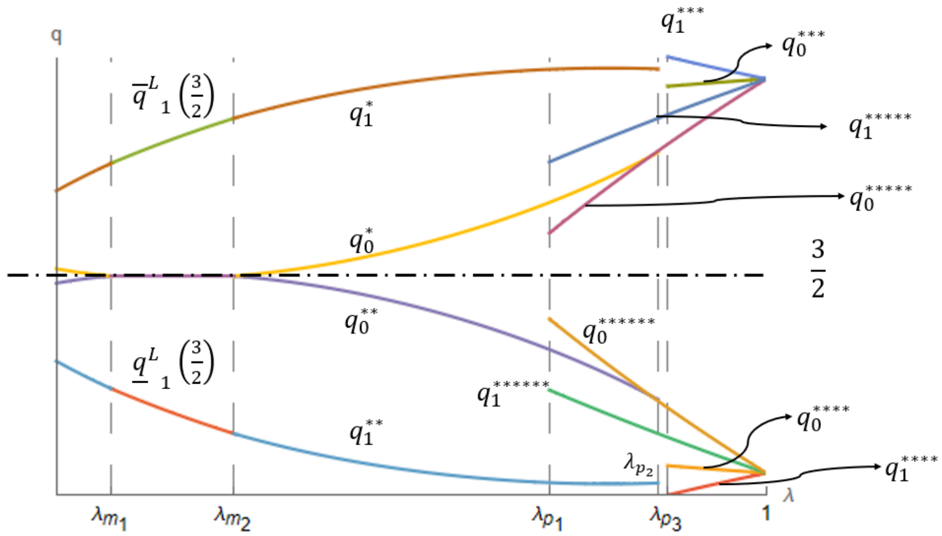



Figure 10: **Equilibrium Qualities for** $\lambda \geq \frac{1}{2}$. See Table 3 for the classification of equilibria, and Table A.3.4 for the expressions of the different thresholds for $\lambda$. The plot is obtained fixing some $\theta_h, \alpha$, which imply $\hat{q} = \frac{3}{2}$.

**Proposition 4.** *(Strict Budget Constraint)*
*There exist two (duopoly) payoffs-equivalent equilibria. In the first equilibrium, $q_0 < q_1$. In the second, $q_0 > q_1$. Welfare in the un-constrained equilibrium described in Proposition 2 (if evaluated at $\lambda = \lambda^*$) is higher than in these constrained equilibria.*

*Proof.* In *Stage* (4), the game runs as in the previous Section. In *Stage* (3), firm 0 commits to $p_0 = \frac{1}{2}q_0^2$. When $q_0 < q_1$, the profit function of firm 1 is maximized if $p_1 = \frac{1}{4}(q_1(q_1+4)+(q_0-4)q_0) = \overline{p}_1^{c_*}(q_0, q_1)$. If $q_1 < q_0$, the optimal price of firm 1 is: $p_1 = \frac{1}{4}(q_1(q_1+2)+(q_0-2)q_0) = \underline{p}_1^{c_*}(q_0, q_1)$.

Let us now consider *Stage* (2). If firm 0 serves the median consumer, its payoff is: $(1-\lambda)\left(q_0 - \frac{1}{2}q_0^2\right)$. This payoff is maximized at $q_0 = \frac{3}{2}$. If evaluated at $q_0 = \frac{3}{2}$, it reduces to: $\frac{1}{8}(9)(1-\lambda)$. If firm 0 does not serve the median consumer, its payoff is: $\frac{1}{4}(\lambda-1)((q_1-2)q_1+(q_0-4)q_0)$, which is maximized at $q_0 = 2$, yielding a payoff of: $\frac{1}{4}(\lambda-1)((q_1-2)q_1-4)$. The profit of firm 1 is: $\frac{1}{16}(q_1-q_0)(q_1+q_0-4)^2$, which is maximized at $\frac{q_0+4}{3} = \overline{q}_1^{c_*}(q_0)$. The intersection of best responses is given by $\frac{11}{6} = q_1^{c_*}$ ; $\frac{3}{2} = q_0^{c_*}$. These qualities are an equilibrium because neither firm has an incentive to deviate. Analogously, it can be shown that there exists an equilibrium where $q_0 = \frac{3}{2} = q_0^{c_{**}}$ and $q_1 = \frac{9}{6} = q_1^{c_{**}}$. The welfare comparison is straightforward. □

Public firms may be constrained to make non-negative profits, and they may be legally obligated to price their goods at marginal costs. Proposition 4 shows that this commitment is not necessarily welfare improving, but rather, it depends on the degree of partial privatization.

## C.2 Partial Market Coverage

In this Section, consumers are also allowed not to buy any good. The setup is standard for product differentiation models with partial market coverage. Unfortunately, in this case, it does not seem easy to solve for an equilibrium quality pair. However, I have the following result for the price stage.

**Proposition 5.** *(Partial Market Coverage)*
*Consider the subgame induced in the price stage by a pair of qualities $(q_0, q_1)$. The share of consumers that do not buy any good is non-decreasing in $\lambda$.*

*Proof.* Without loss of generality, suppose $q_i > q_j$. There are now two indifferent consumers: $\hat{\theta}_1 = \frac{p_j}{q_j}$; and $\hat{\theta}_2 = \frac{p_i - p_j}{q_i - q_j}$. Hence, under duopoly, the demand for firm $j$ is: $\hat{\theta}_2 - \hat{\theta}_1$. The demand for firm $i$ is: $\theta_h - \hat{\theta}_2$. If consumers do not buy any good, they get a zero payoff.

Let us consider the price stage. As in Appendix A.2.2, the existence of a NE is guaranteed by the results in Reny [1999]. Firm 0's payoff is:

$$V_0(\pi_0, q_1, q_0, p_1, p_0) = \lambda \pi_0 + (1 - \lambda) \max\{0, \overline{\theta} q_1 - p_1, \overline{\theta} q_0 - p_0\} . \tag{148}$$

The FOC of (148) w.r.t. $p_0$ is non-decreasing in $\lambda$. Then, the equilibrium price of firm 0 is non-decreasing in $\lambda$. By strategic complementarity of prices, the price of firm 1 is also non-decreasing in $\lambda$. Hence, for any quality ordering, in a price equilibrium, $\hat{\theta}_1 = \frac{p_j}{q_j}$ is also non-decreasing in $\lambda$. □

When $\lambda$ increases, firm 0 charges higher prices. Because of the strategic complementarity of prices, the NE price of firm 1 is also non-decreasing in $\lambda$. Then, the share of consumers who do not buy any good increases accordingly. Proposition 5 shows that privatization may raise inequality concerns, especially in the case of "essential" goods, like education or healthcare.

# Chapter 2

Table of Contents

# Mergers, Lobbying, and Elections: Is there a "Curse of Bigness"?*

Matteo Broso† and Tommaso Valletti‡

January 3, 2025

**Abstract**

We study the impact of market concentration on elections and lobbying in a political agency model with adverse selection and moral hazard. Two incumbent firms can lobby a politician (P) to prevent a pro-competitive reform. P's type determines whether they care about bribes or not. A representative voter tries to infer P's type monitoring the policymaking process. We investigate the welfare implications of a merger between the two firms. In equilibrium, the merger increases firms' incentives to lobby and their ability to influence politics. This additional political power reduces the chances that the pro-competitive reform is approved, hurting consumers; but it allows the voter to defeat a corruptible P with higher probability. Thus, it improves the voter's screening and mitigates adverse selection. We discuss how this new trade-off interacts with traditional competition considerations in the merger's assessment.

**Keywords** Lobbying, Political Agency, Mergers and Acquisitions, Antitrust
**JEL Classification** P16, L41, D72, G34

*We thank Benjamin Blumenthal, Steven Callander, Carlo Cambini, Enrique Carreras, Markus Dertwinkel-Kalt, Andrea Gallice, Filippo Lancieri, Andrea Matranga, Ignacio Monzón, Amedeo Piolatto, Federico Trombetta, Stephane Wolton, as well as seminar participants in Amsterdam (EARIE), Athens, Barcelona, Bergen, Bologna, Boston (IIOC), Brussels, Durham, Mannheim and Turin for useful comments and suggestions. Tommaso Valletti acknowledges support from the Leverhulme Trust. The authors have no conflict of interest to declare.

†University of Turin and Collegio Carlo Alberto.
‡Imperial College London and CEPR.

# 1 Introduction

Market concentration entails market power costs and potential efficiency benefits. Following consolidation, like after a merger or an acquisition, firms can become more efficient. Due to synergies, economies of scale, or economies of scope, they might produce at lower costs. Cost efficiencies may then translate into lower consumer prices. However, consolidation can harm consumers as market power may reduce output and increase prices. This trade-off is first pointed out in Williamson [1968], and lies at the core of merger control.

The *consumer welfare standard*, which is the current dominant perspective in antitrust enforcement, follows this logic and posits that mergers should be approved if and only if efficiencies are large enough to compensate for the increase in consumer prices due to higher market power, holding market structure constant.

This approach has been criticized for being too narrow by the Neo-Brandeisian antitrust movement (Wörsdörfer [2023]). Following the legacy of Associate Justice of the US Supreme Court Louis Brandeis, critics are concerned about the wider risks that market concentration poses for political freedom and democracy.[1] In this view, antitrust should not only contrast market power but also prevent excessively large corporations from influencing democratic institutions to the detriment of citizens. This is Brandeis' "Curse of Bigness", according to which industrial concentration not only affects consumers directly through market power but also indirectly through politics. Empirical research supports this concern, finding that market concentration increases firms' lobbying expenditures in the US (Cowgill et al. [2024], Moshary and Slattery [2024]). Recent empirical evidence also shows that lobbying expenditures and market concentration have both been increasing over time and across industries.[2]

From a welfare perspective, however, it is not obvious that size is "bad" *per se*. Economists champion efficiencies, and it is not straightforward why size should result in additional political influence and consumer harm, as well as how antitrust should react to the political consequences of market power.

In this paper, we study how market concentration induced by a merger affects

[1]See here for an overview of the Neo-Brandeisians' critiques to the consumer welfare standard. The debate around the consumer welfare standard goes back at least to Bork [1978], and has been revamped by Khan [2016], Wu [2018], Shapiro [2019], and Tirole [2022], among others.

[2]See here, here, and here for evidence about firms' lobbying activity in the US and the EU. The literature on industrial concentration trends, and its causes and consequences, is an active area of ongoing research. See, e.g., Grullon et al. [2019], Philippon [2019], De Loecker et al. [2020], Benkard et al. [2021], Kwon et al. [2024].

firms' political power and how this impacts a merger's welfare assessment. We build a political agency model with regulation, mergers, and lobbying. As the market becomes more concentrated, firms have more political power, and lobbying activity increases. Our model reveals a new trade-off associated with market concentration. On the one hand, this additional political power is bad as it is used to lobby politicians to implement policies that protect large incumbents in the market, hurting consumers. On the other hand, policies are observed and this allows voters to screen politicians, thereby mitigating problems of adverse selection of policymakers. Our model captures how this trade-off interacts with traditional price and efficiency effects in a merger's assessment.

We consider an industry producing three differentiated goods. Goods 1 and 2 are produced by two incumbent firms with market power. Good 3 is produced by a competitive fringe.

An incumbent politician decides whether to approve a pro-competitive market reform or not. The reform determines the extent of the competitive pressure faced by the two incumbent firms: if approved, it makes the fringe more aggressive. The reform can be seen as the elimination of a barrier to entry that protects incumbent firms' profits by keeping good 3's price artificially high. The reform benefits the consumer, and it is bad for the two incumbent firms.

The two incumbent firms can lobby the politician to persuade them not to implement the reform (*quid pro quo lobbying*, Grossman and Helpman [1994]). The politician can be of two types: Good or Bad. If the politician is Good, they are un-corruptible. They do not care about bribes and always implement the reform in the consumer's interest. If the politician is Bad, they care about bribes and re-election. As it is usual in political agency models, the Bad politician faces a tension between behaving well to try to win re-election and accepting bribes and giving up the office with some probability.

A representative voter chooses whether to re-elect the politician or not. The objective of the voter is to elect a Good politician. However, they do not observe the politician's type. They try to infer the type by imperfectly monitoring the political process. Imperfect monitoring creates an incentive for moral hazard. Imperfect information creates a problem of adverse selection of politicians.

We show that this game may have two types of equilibria: Pooling and Separating. In a Pooling equilibrium, both types of P implement the pro-competitive reform. However, this equilibrium is inefficient from the voter's perspective as it prevents learning. With some probability, the voter re-elects a corruptible incumbent. Adverse selection bites. In a Separating equilibrium, the Bad politician does not implement the pro-competitive reform in exchange for bribes. The consumer suffers from higher prices, but the voter is able to infer the politician's type. Adverse selection is mitigated.

Before playing the lobbying game, the two firms may merge. The merger has efficiency benefits, as it reduces firms' marginal costs, but also market power costs. Additionally, if the merger takes place, firms internalize higher benefits from protection and, therefore, have more incentives to lobby. It may be the case that firms have incentives to lobby *only* if the merger is approved. Then, the merger can change the nature of the political equilibrium, from Pooling to Separating.

Within this setting, we then address our main policy question: Which mergers are welfare improving? Should antitrust authorities account for the political architecture of market power? To put it differently, should a merger's assessment consider only market effects of market power, following the consumer welfare standard, or should it extend to non-market effects as well? To answer this question, we introduce an additional player: an antitrust authority that decides whether to approve the merger or not before firms play the lobbying game. The authority cares about consumers and voters, and its optimal behavior depends on the nature of the political equilibrium. We show that three possible cases arise.

First, suppose that the equilibrium is always Pooling, no matter if the merger is allowed or not. All politicians behave in the same way and approve the pro-competitive reform. There is no screening of P. With some probability, the voter re-elects the Bad politician, but the authority cannot do anything to prevent this. Then, the authority approves only mergers that are efficient enough to decrease prices. Mergers have no political effects, and the consumer welfare standard is optimal.

Second, suppose instead that the equilibrium is always Separating, no matter if the merger is allowed or not. As in the previous case, the merger has no political effects, and the voter's payoff does not depend on the merger's approval. In either case, the voter successfully screens P. However, with some probability, nature draws a Bad politician, and the reform is not implemented. Then, a higher level of efficiencies is required (in expectations) for the merger to be pro-competitive compared to the previous case. For this reason, the consumer welfare standard is too lenient.

Finally, suppose that the equilibrium is Separating if and only if the merger is approved. It is the merger that gives the two firms the ability to influence P. Then, the merger can increase prices not only because it increases market power, but also because it increases political power, which causes the failure of the pro-competitive reform. However, the voter would like the merger to be approved, as this allows them to defeat a corrupt politician. The authority's optimal merger policy depends on the voter's benefits from screening politicians' types.

In this third case, if the value of screening is low, the separating equilibrium is very inefficient as it induces the Bad politician to accept bribes in exchange for limiting competition, thereby hurting the consumer. Then, the merger would need to be very efficient to be approved. The authority may reject pro-competitive mergers (mergers that decrease prices) because in the resulting Pooling equilibrium, there would never be lobbying, the pro-competitive reform would always be approved, and prices would drop even further.

The higher the value of screening, the greater the price increase that the authority can tolerate to make the voter learn the politician's type. If the value of screening is high enough, the authority may even approve anti-competitive mergers (mergers that increase prices) to make the voter learn the politician's type and defeat a corrupt one. Committing to the consumer welfare standard can now result in both type I and type II errors.[3]

In the Appendix, we test the robustness of our results. We explore different bargaining settings, we explore a dynamic model with endogenous bribes and office values for politicians, and extend our findings to a broad class of supermodular market games.

However, our model relies on some key assumptions which are worth discussing. First, we assume lobbying is a *quid-pro-quo* exchange, where politicians grant favors to firms in return for private benefits, such as monetary bribes, non-monetary perks, or job offers after their term. In this regard, we do not consider the case of informational lobbying, whereby politicians gain information from

[3]In the baseline model, we assume that the value of screening is exogenous. In the Appendix, we endogenize it in a two-period model. In this dynamic model, the voter wants to avoid appointing Bad politicians because, in the last period, they always fail to pass the pro-competitive reform in exchange for bribes. In this model, one may want to approve anti-competitive mergers if and only if Bad politicians could harm consumers beyond the market's partial equilibrium (e.g., corruption, extraction of public resources, etc.). In the absence of these out-of-market externalities, the consumer welfare standard is always too lenient.

interacting with firms.[4]

Second, we adopt a partial equilibrium approach. While we acknowledge that voting and competition are complex and general equilibrium phenomena, we believe that our simple model of a single industry can still shed light on the incentives of politicians, voters, and firms in settings where large corporations can use influence to obtain favors from politics. For example, telecommunications is an industry where there have been several mergers among large operators with market power. As documented by Faccio and Zingales [2022], telecoms is also an industry where lobbying is common, as are political connections. In line with one of our key modeling assumptions, they show that when incumbent firms are politically connected, regulation does not favor competition, and prices are higher. This example fits with our model, where incumbent firms lobby politicians to block pro-competitive reforms. We show that firms have stronger incentives to lobby after a merger.

We contribute to two main streams of literature. The first is the literature at the intersection of industrial organization and the political economy of regulated markets. In a recent theoretical paper, Callander et al. [2022] show that politicians have an incentive to protect firms from competition and raise barriers. However, their incentive is not perfectly aligned with the incumbent firms. Politicians strategically do not protect firms too much; otherwise, firms would stop demanding protection. The same authors show that a liberalization policy can be effective or not depending on the *ex-ante* market structure (Callander et al. [2023]). Recent empirical work by Akcigit et al. [2023] shows that, as firms increase in size, they tend to rely more on non-market strategies, such as political connections, to maintain a dominant position. Akcigit and Ates [2023] document the lack of "creative destruction" in the modern US economy, pointing to regulation and weak antitrust enforcement among the different potential explanations. Kang [2016] structurally estimates a rent-seeking lobbying game, where interest groups lobby politicians to obtain favorable regulations. She finds that lobbying has a small effect on a policy's probability of approval. However, since policies are, on average, very valuable for firms, average private returns from lobbying expenditures are substantial.

Our two most closely related papers are Cowgill et al. [2024] and Moshary and Slattery [2024], which focus empirically on the impact of mergers on lob-

[4]For a model of informational lobbying, see Schnakenberg [2017], and Besanko and Spulber [1993] for an application to antitrust. Bombardini and Trebbi [2020] review the empirical literature, distinguishing between *quid-pro-quo* and informational lobbying.

bying, finding that mergers increase lobbying activity. Our main contribution is to provide the first (to the best of our knowledge) welfare analysis of mergers that takes into account the political economy effects of market concentration.

The second is the literature about political agency with adverse selection and moral hazard. Seminal papers in this literature are Fearon [1999], Coate and Morris [1995], Maskin and Tirole [2004], and Banks and Sundaram [1998].[5] The key insight of this literature is that voters and politicians can be seen as principals and agents, respectively. Then, imperfect information generates agency problems such as adverse selection of politicians and moral hazard. We contribute to this literature by linking electoral accountability with market concentration.

The rest of the paper is organized as follows. In Section 2, we illustrate our model setup. In Section 3, we characterize the equilibrium of the model. Mergers and their welfare properties are considered in Section 4. Finally, Section 5 concludes. Proofs and extensions are relegated to the Appendix.

## 2 Model

We present a simple model of mergers and lobbying. In the Appendix, we extend the model in several directions to test the robustness of the results and explore further implications.

The model intends to capture market and non-market aspects of competition, and for this reason, it has two main blocks that interact with each other: a political economy block and a competition block. In the first one, two firms lobby a politician who can make a decision that impacts their profitability via a market mechanism, and a representative voter appoints the politician. In the second one, these firms compete against each other in the market, as affected by the politician's decision. Firms may merge or not before playing the lobbying game and the competition game. Our goal is to perform a welfare assessment of the merger decision by considering both channels. With this in mind, we now detail every component of the model.

**Players** We consider a market with three differentiated goods $i \in \{1, 2, 3\}$. Goods $1, 2$ are produced by two incumbent firms with market power. These

[5] For more recent contributions, see Wolton [2019], Duggan and Martinelli [2020], Blumenthal [2023], Sasso and Morelli [2021], among many others. For political agency papers related to the topic of money in politics and *quid-pro-quo* lobbying, see Bils et al. [2021] and Daley and Snowberg [2011]. See also Schnakenberg and Turner [2023] for a recent survey of this literature.

are the firms that can eventually lobby. A competitive fringe sells good 3, and they do not lobby.[6] A representative consumer buys the three goods. At the beginning of the game, the two firms $(1,2)$ can merge or not. We say that $m = 1$ if the merger occurs, and $m = 0$ otherwise. The merger is exogenous but always profitable for the two firms. In Section 4, we allow a benevolent antitrust authority to decide upon the merger approval.

There is an incumbent politician (P). P can implement a pro-competitive reform that decreases the price of the fringe good $p_3$, for instance, by making entry into the market by the competitive fringe easier. Incumbent firms can lobby P trying to avoid that. P has a type $\theta_P \in \{\text{Good}, \text{Bad}\}$. If P is Good, they are not interested in lobbying money, such as bribes. If P is Bad, they care about bribes. A representative voter chooses whether to re-elect P or replace them with a challenger. The challenger type is denoted by $\theta_C \in \{\text{Good}, \text{Bad}\}$.

**Policy** The price of the fringe good $p_3(a)$ depends on a pro-competitive reform $a$. We say that the reform is implemented by P if $a = 1$ and it is not implemented if $a = 0$. We assume that $p_3(1) < p_3(0)$. Without loss of generality, let $p_3(1) = 0$ and $p_3(0) = \eta$, with $\eta > 0$.[7]

There are various interpretations for $a$. For instance, firms $1,2$ may be two domestic firms competing against a fringe of foreign goods whose prices depend on the level of import tariffs. Alternatively, firms $1,2$ may describe two private firms (e.g., hospitals, schools, or pharmacies) competing with a fringe of public providers whose number and/or quality is affected by political decisions.[8] As a further example, consider the case of entry regulation. Politicians can restrict entry in many markets (e.g., retail, or ride-sharing), thereby (indirectly) controlling prices. All these examples share a crucial feature of our parsimonious model. Politicians can take actions that influence the degree of competition firms face in the market and, therefore, their market rents, and firms can lobby politicians to shape those rents. In the context of this tension, we will study

[6]This assumption is based on the idea that lobbying from the fringe might require coordination or the payment of a fixed cost that is possibly too high for very small players. If the fringe captures competition from imports, foreign firms may not have access to domestic politicians.

[7]For the sake of simplicity, we assume that the price of the fringe good is a function of $a$ only. This assumption is not consequential for our results, as we would obtain analogous results in a model where good 3 is also sold by a firm with market power. Our results depend on two assumptions: (i) $p_3$ depends on $a$; (ii) the seller of good 3 cannot lobby. See Appendix B, for a generalization of our main results to a more general supermodular game.

[8]This example may be particularly relevant given the recent wave of mergers between US hospitals. See Gowrisankaran et al. [2015] and Prager and Schmitt [2021], among others.

how a merger between the two firms affects lobbying and, in turn, welfare.

We discuss the details of the lobbying and merger processes in the following paragraphs.

**Timing and Actions** The timing of the game runs as follows.

Stage 1 Nature draws $\theta_P, \theta_C$.

Stage 2 P chooses whether to implement the reform ($a = 1$) or not ($a = 0$) by committing to a mechanism $a(l_1, l_2)$.

Stage 3 Firms observe the mechanism and choose whether to lobby ($l_i = 1$) or not ($l_i = 0$). The reform $a$ is implemented.

Stage 4 Firms observe $p_3(a)$ and simultaneously set prices $p_1, p_2$. The consumer observes prices and chooses a consumption plan $d_1, d_2, d_3$.

Stage 5 The voter chooses whether to re-elect P ($r = 1$) or not ($r = 0$). Payoffs are realized.[9]

In the baseline model, we make some assumptions to keep the model as simple as possible to convey our intuition. We assume that lobbying is binary and that P can commit to a TIOLI offer for the two firms. In the Appendix, we generalize the model to different bargaining and competition structures.[10]

**Payoffs** The representative consumer has a quadratic utility function *à la* Singh and Vives [1984]:

$$u(d_1, d_2, d_3) = d_1 + d_2 + d_3 - \frac{1}{2}\left(d_1^2 + d_2^2 + d_3^2\right) - \gamma\left(d_1 d_2 + d_2 d_3 + d_1 d_3\right) . \quad (2.1)$$

The parameter $\gamma \in (0, 1)$ captures the degree of product differentiation.[11] If $\gamma = 0$, the three products are independent, and the two incumbent firms act as separate monopolists. If $\gamma = 1$, goods are perfect substitutes, and Bertrand

[9]At the end of Stage 5, $\theta^*(r)$ is revealed to the voter.

[10]In our reduced-form approach, we assume that either the politician (in the baseline model) or the two firms (in the Appendix) have commitment power. While this assumption simplifies the analysis, it could be relaxed in a dynamic model where reputation, for example, might serve as a commitment device.

[11]Assume that the consumer has a quasi-linear utility function, where (2.1) is the non-linear part. Then, the demand for goods 1, 2, 3 does not depend on income, and a partial equilibrium analysis is justified (Singh and Vives [1984], Motta [2004], Choné and Linnemer [2020]). In Appendix B, we show that our results generalize to a whole class of games with strategic complements.

competition brings profits down to zero. In either limiting case, firms would never lobby as the reform $a$ does not impact their profits. Hence, we study the case when $0 < \gamma < 1$, that is, products are imperfect substitutes (Amir et al. [2017]).

The voter cares about electing a Good politician. Let $\theta^*(r)$ be the type of politician winning the election. Then,

$$\theta^*(r) = \begin{cases} \theta_P & \text{if } r = 1 \\ \theta_C & \text{if } r = 0 \,. \end{cases} \tag{2.2}$$

The voter's payoff is:

$$v(r) = \begin{cases} -\phi & \text{if } \theta^*(r) = \text{Bad} \\ 0 & \text{if } \theta^*(r) = \text{Good} \,, \end{cases} \tag{2.3}$$

where $\phi > 0$ captures the benefits associated with screening. A possible interpretation for $\phi$ is as follows. Suppose that a Bad politician is re-elected. While in office, a corruptible politician may adopt policies that appeal to specific interest groups (perhaps in exchange for bribes) rather than voters. In this context, $\phi$ represents the cost of such actions to the voters in present-value terms.[12]

The profits of firms $i \in \{1, 2\}$ are:

$$\pi_i(p_i, d_i, l_i) = d_i\left[p_i - c\left(m\mu + (1 - m)\right)\right] - tl_i \,, \tag{2.4}$$

where $d_i$ is the quantity sold by firm $i$, $c > 0$ is the (symmetric) marginal cost, $\mu \leq 1$ captures the efficiencies activated by the merger, and $t > 0$ is the value of the bribe.

The parameter $t$ captures how costly it is for firms to engage in lobbying. A possible interpretation is that $t$ is a proxy for the stringency of the lobbying legislation (Schnakenberg and Turner [2019]).[13]

[12]The assumption of exogenous $\phi$ is the simplest way of capturing the value of screening by the voter. However, the same intuition could arise endogenously following standard dynamic political agency models in finite horizon where Bad politicians always extract public resources for themselves during the last period of office. In Appendix C, we consider this case explicitly.

[13]For instance, in the US, the lobbying legislation is more permissive than in the EU. Our interpretation suggests that the parameter $t$ would be lower in the US than in the EU, as firms would have to pay lower costs to lobby, other things equal. The assumption of exogenous lobbying costs is convenient for illustrating the intuition behind our results. However, this assumption is not consequential for our results. In Appendix C, we allow for endogenous transfers.

We assume that the merger can generate merger-specific efficiencies from the joint production of goods $1, 2$. We model this by assuming that if the merger goes ahead ($m = 1$), marginal costs are $\mu c$, where $\mu \leq 1$. If $m = 1$, the merged entity chooses prices $p_1, p_2$ and lobbying efforts $l_1, l_2$ to maximize the joint sum of profits from selling the two goods. Notice that this framework allows for a meaningful welfare assessment, as a merger entails a trade-off: market power always increases, but there may be a countervailing force if efficiencies are large enough (Williamson [1968]). In Section 3, we solve the model taking $m$ as given. In Section 4, we perform a welfare assessment of the merger.

The Good politician P is a behavioral type. They always implement the reform: $a = 1$. One interpretation of this assumption is that they are ethical agents who care about decreasing prices, as they maximize the welfare of consumers that are also voters. The Bad P's payoff is:

$$U_{\text{Bad}}(r, l_1, l_2) = rV + (l_1 + l_2)\,t\ , \tag{2.5}$$

where $V > 0$ captures the value of the office, if re-elected, and $t\,(l_1 + l_2)$ is the total amount of bribes received from firms. In our model, *quid-pro-quo* lobbying takes the forms of bribes. In reality, there are various possible interpretations for $t$: revolving doors, bribes, campaign contributions, informational benefits, etc. In the remainder, we simply refer to bribes as a shorthand for all these alternatives but we do not take a position on the legality of lobbying or bribery.[14]

**Information Structure** The voter does not observe politicians' types $\theta_P, \theta_C$. Let $q \in [0, 1]$ be the voter's prior belief that $j =$ Good, where $j \in \{\theta_P, \theta_C\}$. We assume that $m$ is common knowledge: everyone knows whether the merger occurred or not.

With probability $x \in (0, 1]$, the voter observes the action $a$. There are two cases. The first case is $x = 1$. In this case, the voter and the consumer can be thought of as the same player. This player's payoff would be given by the sum of (2.1) and (2.3). In fact, if $x = 1$, the voter and the consumer have the same information.

The second case arises when $x < 1$. In this case, there is imperfect monitoring. Voters cannot always hold politicians accountable upon observing market

[14]In our *quid-pro-quo* setting, when an agreement is reached, the bribe $t$ is transferred one-for-one from firms to politicians. If this transfer provides a non-monetary benefit of value $t$ for politicians, the cost to firms may be $\alpha t$, where $\alpha < 1$ or $\alpha \geq 1$. For simplicity and without loss of generality, we assume $\alpha = 1$.

outcomes (Blumenthal [2023]). Formally, the voter and the consumer are two different players. There are at least two possible reasons. First, consumers and voters may posses different information. Suppose that $x$ captures the industry's relevance from the voter's perspective. For instance, the voter may not be directly interested in the consumption of goods $1, 2, 3$, but still be interested in learning $\theta_P$. When $x$ is low, market issues are not very salient for the voter, and politicians can exploit this lack of monitoring to their advantage. Second, voters and consumers may coincide, but prices might not be perfectly informative. Imagine that prices are informative only with probability $x$. With probability $1 - x$, the consumer/voter thinks that prices are pure noise.[15] Also in this case, P can use ambiguity at their advantage.

In general, $x \in (0, 1]$ generates a moral hazard problem from the P's side. The extent of moral hazard determines how likely the politician can accept bribes and still get re-elected. Only in the limiting case when $x = 0$, the voter does never observe P's action, and the Bad politician does not trade off re-election and bribes.

We now present the joint political-market equilibrium of our model for a given market structure $m$. Then, we study how a merger affects the nature of the such equilibrium.

## 3 Results

Our solution concept is Perfect Bayesian Equilibrium (equilibrium henceforth) in pure strategies. A strategies–belief pair is an equilibrium if and only if:

1. Each player's strategy maximizes their expected payoff given all the other players' strategies and beliefs;

2. For any observation of P's action, the voter's belief is updated via Bayes' Rule, and their action is optimal given the equilibrium beliefs.

[15]Indeed, in many real-world examples, prices can be "high" not because of a lack of competition but because of macroeconomic shocks or external factors. Therefore, if consumers (voters) observe high prices, they may be uncertain about the causes behind such prices. For instance, during the 2022-2023 inflation wave, there was an intense debate on the causes of such inflation, with some experts attributing them mainly to "external factors," such as the War in Ukraine or the Covid 19 Pandemic; and others pointing to companies' opportunistic behaviors ("greedflation"). This imperfect information makes prices not (fully) informative and hence generates imperfect monitoring from the point of view of consumers and a problem of moral hazard from the politicians' perspectives. Despite being formally more complicated, a model that features this intuition would be qualitatively equivalent to the one we propose here.

In the following Subsections, we take the merger $m$ to be exogenous, and we study the impact of the merger on prices, lobbying, and elections. In Section 4, we study how these effects interplay with each other in the welfare analysis of the merger.

### 3.1 Preliminary Results

Let us start from the election stage (Stage 5). Let $\hat{q}$ denote the voter's updated belief that $\theta_P$ = Good. Let us assume that, in case of indifference, the voter chooses to re-elect the incumbent.[16] The voter's optimal re-election rule is:

$$r^* = \begin{cases} 1 & \text{if } \hat{q} \geq q \\ 0 & \text{otherwise .} \end{cases} \tag{3.1}$$

If the Bad P chooses $a = 0$ and the voter observes it, then $\hat{q} = 0$. Therefore, they are re-elected with probability $1 - x$. If they choose $a = 1$, they are re-elected with probability 1. P then faces a trade-off between behaving well today and ensuring re-election tomorrow, or accepting bribes and giving up the office (with some probability).

Let us now consider the market stage. In Stage 4, firms set prices anticipating the consumer's demand functions. The representative consumer's demands for the three goods $i, j, k \in \{1, 2, 3\}$, $i \neq j \neq k$ are:

$$d_i = \frac{1 - \gamma - p_i(\gamma + 1) + \gamma(p_j + p_k)}{(1 - \gamma)(2\gamma + 1)} \ . \tag{3.2}$$

We obtain (3.2) by simply maximizing (2.1) given prices. As $\gamma > 0$, the demand for good $i$ increases in the price of goods $j, k$.

In Appendix A.1, we obtain subgame equilibrium prices as a function of $p_3$ and market structure, with and without the merger $m$. We omit computations from the main text, and all the expressions are reported in the Appendix. The important results from Appendix A.1 are as follows. First, for any market structure $m$, equilibrium prices increase in $p_3$ (and thus decrease in $a$). This result stems from prices' strategic complementarity. Second, equilibrium profits of firms 1, 2 decrease in $a$, as they face a stronger fringe. Finally, we compare

[16] Our findings remain qualitatively unchanged under any change of the tie-breaking rule where there is a strictly positive probability that the incumbent is re-elected (for a similar discussion, see Blumenthal [2023]).

prices with and without the merger. Unsurprisingly, the merger decreases prices if and only if efficiencies are high enough, that is, if $\mu$ is low enough. If the reform is not implemented ($a = 0$), higher efficiency (lower $\mu$) is required for the merger to be pro-competitive. This result will play a key role in the merger's welfare assessment. When the merger induces P to adopt the non-competitive policy ($a = 0$), a higher level of efficiency must be achieved by the merger to be pro-competitive. This is an instance when the merger can increase prices not only through market power but also through political power.

## 3.2 Lobbying Equilibrium

We now consider the lobbying game (Stage 2 and Stage 3). For all $m$, there are two possible types of equilibria: Pooling and Separating. In a Pooling equilibrium, both types of P implement the reform. The consumer enjoys lower prices, but the voter does not learn P's type and, with some probability, re-elects the Bad P. In a Separating equilibrium, the Bad P does not implement the reform and both firms lobby. With probability $x$ the voter observes the action, and learns P's type.

We first show how a Separating equilibrium looks like, and then we discuss conditions under which such an equilibrium exists. In a Separating equilibrium, the Bad P commits to the mechanism $a^*(1,1) = 0$ and both firms choose $l_i^* = 1$. Suppose that P does not offer protection, then no firm wants to lobby. In the same way, if neither firm lobbies, P ensures re-election by pooling. Then, any (possibly) optimal contract implies protection in exchange for some lobbying.

We are left to consider the alternative mechanism $a^*(1,0) = 0$. We show that this cannot be optimal for P. In fact, P knows that if $l_i = 1$ is incentive compatible (IC) for firm $i$, $l_j = 1$ is also IC for firm $j$. Therefore, the contract $a^*(1,0) = 0$ is strictly dominated. If P can "sell" protection ($a = 0$) for $2t$, there is no reason to give out a 50% discount and accept a price of $t$. This result depends on the fact that P has the power of commitment to the offer. In Appendix C, we show that if firms can commit to the offer, there is an equilibrium where only one firm lobbies.[17]

When is a Separating equilibrium possible? Let $p^m(a)$ be the vector of equilibrium prices as a function of $a$ and $m$. For $a = 0$ to be individually rational

[17]The mechanism $a^*(1,1) = 0$ also solves the coordination problem for firms as they both know they need to lobby to induce $a = 0$. When firms offer the contract and $m = 0$, there is a coordination problem arising from the multiplicity of equilibria.

(IR) for the Bad P, it must be that:

$$(1 - x) V + 2t \geq V \; . \tag{3.3}$$

P knows they can ensure re-election by pooling ($a = 1$), which yields a payoff of $V$. By separating, P gets bribes ($2t$) but wins the office only with probability $1 - x$. P trades off tomorrow's re-election chances with today's bribes.

Paying bribes must be incentive-compatible (IC) for firms.[18] Let

$$\Delta\pi(m) = \begin{cases} \pi_i\left(p^{m=0}(0)\right) - \pi_i\left(p^{m=0}(1)\right) & \text{if } m = 0 \\ \frac{1}{2}\left[\sum_i \pi_i\left(p^{m=1}(0)\right) - \sum_i \pi_i\left(p^{m=1}(1)\right)\right] & \text{if } m = 1 \end{cases} \tag{3.4}$$

be the return for firms from protection as a function of $m$. We derive the expressions of $\Delta\pi(m)$ in Appendix A.2. Firms' IC is:

$$t \leq \Delta\pi(m) \; . \tag{3.5}$$

Combining P's IR and firms' IC, we obtain the following result.

**Proposition 1.** *In a Pooling equilibrium, both P's types implement the pro-competitive reform ($a^* = 1$). In a Separating equilibrium, a Bad P does not implement the reform ($a^* = 0$), and both firms lobby ($l_1^* = l_2^* = 1$). Moreover,*

*(i) For all $m \in \{0, 1\}$, the equilibrium is Separating if and only if*

$$\frac{xV}{2} \leq t \leq \Delta\pi(m) \; .$$

*In a Separating equilibrium, the voter learns P's type with probability $x$. Otherwise, the equilibrium is Pooling, and the voter does never learn P's type.*

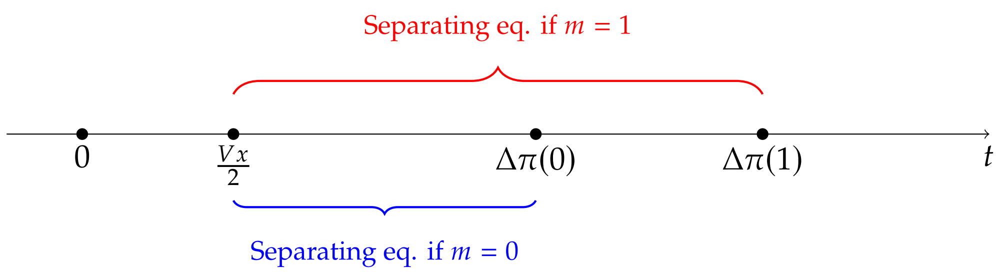
*(ii) The merger increases firms' incentives to lobby:*

$$\Delta\pi(1) \geq \Delta\pi(0) \, .$$

Figure 3.1 shows the findings of Proposition 1. First, a Separating equilibrium

[18] In Appendix C.2, we consider a two-period model and we show that firms do not have incentives to give up bribing in the first period to induce the re-election of a Bad P in the second period. As the discount factor is always smaller or equal than 1, firms' returns from bribing tomorrow cannot be higher than returns from bribing today. For this to be the case, it must be that the size of market rents for the firms that P controls via the policy $a$ increases over time.

Figure 3.1: Existence of Separating Equilibrium




exists if and only if the cost of lobbying $t$ is neither too high nor too low.[19] If $t$ is too low, bribes are not sufficiently valuable, and the Bad P prefers to ensure re-election by pooling. If $t$ is too high, lobbying is not worth it for firms.

Second, a merger increases firms' incentives to lobby as it increases the returns from protection. The intuition is simple. Lobbying to fend off competition from the fringe is valuable, as it can increase rents to firms 1, 2. Without a merger, however, these rents are partly dissipated by the rivalry between 1, 2. With a merger, this dissipation is muted, and therefore merging firms are willing to pay more for protection. In Appendix B, we generalize this intuition to a more general setting.[20]

The increase in lobbying after a merger is not driven by the public good nature of lobbying, as in Moshary and Slattery [2024], or by economies of scale. In our model, by muting price competition, the merger creates additional rents that trigger firms' incentives to lobby. This result follows a similar intuition to the one in Cowgill et al. [2024]. As a consequence, even if firms maximize the total sum of profits in the lobbying game, as would happen if firms exert influence via industry trade associations, our result would still go through. In our model, the merger increases firms' incentives to lobby because prices (and not lobbying efforts) are set non-cooperatively.

We now discuss the comparative statics of the upper and lower bounds in Figure 3.1. The lower bound $\frac{Vx}{2}$ does not depend on market structure. The merger does not impact the politician's incentives to accept the bribes. This lower bound increases as $x$ and $V$ increase. When $x$ is low, moral hazard bites:

[19] In Appendix C, we show the conditions for the existence of a Separating equilibrium when $t$ is endogenous.

[20] In particular, we show that Proposition 1-(ii) follows from the supermodularity of the Bertrand game. Therefore, it does not require any specific assumption on the demand or the profit functions, and it easily extends to any supermodular game with spillovers.

the Bad P knows that by separating, they can enjoy bribes and still be re-elected with a relatively high probability. Therefore, the interval for $t$ that supports a Separating equilibrium decreases in $x$. On the contrary, as $V$ increases, holding the office is more valuable, and the Bad P has a higher incentive to ensure re-election by pooling.

From the firm's perspective, the merger increases the incentives to lobby. The upper bound $\Delta\pi(1)$ decreases in $\mu$. The more efficient the merger, the higher the incentives to lobby, as the the returns internalized from protection are higher. In the same way, for all $m$, $\Delta\pi(m)$ is also increasing in $\eta$. The higher the threat from the fringe, the higher the incentives to lobby.

**Equilibrium Uniqueness** The equilibrium described in Proposition 1 is unique among the class of equilibria in pure strategies. Uniqueness is guaranteed by our assumption on the tie-breaking rule for the voter in Stage 5 and by the following refinement. Whenever there are multiple equilibria in the lobbying game (Stage 2-Stage 3), we select the voter-preferred equilibrium.

The only potential multiplicity of equilibria arises when $t$ equals either the lower or the upper bounds of the interval supporting separation in Proposition 1. For a given $m$, inside the interval, the Separating equilibrium is unique as the equilibrium mechanism is dominant for the Bad P. Outside the interval, Separating is either not IC for firms or not IR for P. Hence, the unique equilibrium is Pooling. At the boundaries of the interval, either the two firms or the politician are indifferent between their actions. Given our refinement, we always select the Separating equilibrium over a possible Pooling one.[21]

# 4 Merger Assessment

We now provide a welfare assessment of the merger. That is, we compare players' welfare if $m = 1$ and $m = 0$. Our model captures the market and non-market effects of the merger, via prices and lobbying respectively. As shown above, a merger increases firms' lobbying activity and their ability to influence policymaking. On the one hand, this political power is bad for the consumer as it further increases prices on top of the usual market power effect. On the other hand, the additional political power is good for the voter when it allows them

[21]In this way, we also rule out possible equilibria in mixed strategies. In the market game (Stage 4), the equilibrium is unique by the strict concavity of payoffs.

to learn about P's type. We now discuss how this new trade-off interplays with the traditional price effects in the merger's assessment.

To this end, we introduce an additional player: a benevolent antitrust authority. At the beginning of the game (before Stage 1), the competition authority decides whether to approve the merger ($m = 1$) or not ($m = 0$) by maximizing the sum of consumers' and voters' payoffs. Therefore, from now on, the market structure $m$ is endogenous. The two firms notify a merger application, and the authority decides whether to approve the merger or not.[22] Let us recall that if $x = 1$, we can think of the voter and the consumer as a unique player. If $x < 1$, the two players are distinct as they possess different information.[23] This objective function of the antitrust authority is based on the impact on citizens (consumers and voters) and does not take into account firms or the politician. In this regard, our authority's mandate goes beyond what competition authorities usually do. Their remit is typically to maximize the welfare of the consumers in the market affected by the transaction, which goes under the name of *consumer welfare standard*, as we anticipated in the Introduction. It is important to remark that we are not claiming that competition authorities *do* or *should* start to consider the direct impact of mergers on elections and lobbying. Our results only illustrate the broader welfare implications of merger policy and allow us to contribute to the debate on what antitrust authority *could* do. At the end of this Section, we provide a possible way to interpret our findings in practice.[24]

Notice that we also assume that firms cannot lobby the antitrust authority. Specifically, firms $1, 2$ cannot lobby to get the merger approved. This simplifying assumption is based on the empirical findings of Cowgill et al. [2024]. They find that merging firms in the US do spend considerable lobbying money but mostly in the Congress. They also do not find anticipatory effects before the approval date of a merger, suggesting that the main targets of lobbying money are not antitrust enforcers.[25] Moreover, the objective of our model is to analyze

[22]The merger is always profitable. Hence, the merger application is optimal for the two firms.

[23]In a more realistic setting, there are multiple consumers and voters. If the sets of consumers and voters are disjoint or perfectly overlapped, a welfare function that sums voters' and consumers' payoffs implies no bias due to the double-counting of players. However, if only some consumers were also voters, this welfare measure would double-count the overlapping players. However, our results are qualitatively robust to players' re-weighting in the welfare function.

[24]See Motta [2004] for a discussion about antitrust authorities' mandates and, in particular, for an introduction to the debate around the consumer welfare standard. Currently, in both the US and in the EU, competition authorities follow, albeit with some differences, a consumer welfare standard.

[25]This is possibly due to the fact that most mergers fall below screening thresholds for merger notification and, therefore, are not vetted by the antitrust enforcers.

how market concentration shapes firms' ability to influence politics and how this affects optimal merger control. To this end, the benevolent competition authority is introduced as a rhetorical device to conduct a welfare analysis rather than an actual player of the game. We do not model the political pressure faced by competition authorities in reality, as this is beyond the scope of our paper. However, this is a natural avenue for further research on the broad topic of the political economy of competition policy.[26]

We start by writing down the authority's payoff across the different equilibria. We suppose that the authority does not observe $\theta_P$. Thus, they do not know what type of P will be at play, but they know what kind of equilibrium will occur and, therefore, how different types of P would behave. We assume that the authority shares the prior beliefs of the voter about P's type. Let $W(m)$ denote the authority's payoff as a function of market structure. Let also $d^*(p^m(p_3))$ be the vector of demands. In a Pooling equilibrium,

$$W_{Pool}(m) = u(d^*(p^m(0))) + (1-q)\left(-\phi\right) \ . \tag{4.1}$$

In a Pooling equilibrium, the consumer always enjoys lower prices, but the voter always re-elects P. Then, with probability $1-q$, the voter re-elects a Bad P and pays the penalty $-\phi$.

In a Separating equilibrium,

$$\begin{aligned} &W_{Sep}(m) = q\left[u(d^*(p^m(0)))\right] + \\ &(1-q)\left[u(d^*(p^m(\eta))) + (1-x)\left(-\phi\right) + x(1-q)\left(-\phi\right)\right] \ . \end{aligned} \tag{4.2}$$

With probability $q$, P is Good, and the consumer enjoys lower prices. It does not matter if the voter observes $a$ or not; in either case, the voter re-elects the incumbent and pays no penalty. With probability $1-q$, P is Bad, and the reform is not implemented. In this case, with probability $x$, the voter observes $a^* = 0$, and the Bad P is defeated. The newly elected politician is Bad with probability $1-q$. With probability $1-x$, the action is not observed, and the Bad P is re-elected. We show the expressions of (4.1) and (4.2) in Appendix A.3.

Lobbying comes at the cost of corruption and high prices today, but it increases the probability that Bad politicians are defeated tomorrow. In particular, in a Pooling equilibrium, the voter appoints a Bad politician with probability

[26]See Motta and Ruta [2012] for a political economy model of mergers affecting international markets, in which firms can lobby and governments can influence antitrust authorities.

$1-q$. In a Separating equilibrium, a Bad P is appointed with probability

$$(1-q)\left[(1-x)+x(1-q)\right] < 1-q\ . \tag{4.3}$$

Therefore, the voter always prefers a Separating equilibrium over a Pooling equilibrium. The opposite is true for the consumer. The authority needs to trade off these two effects as well as to anticipate the effect of their decision on the likelihood that lobbying will occur in equilibrium.

Let us define two thresholds for $\mu$, corresponding to the different levels of efficiencies required for prices to decrease after a merger, and as a function of the policy $a$:

$$\begin{aligned} &\overline{\mu}(0) : \mu \leq \overline{\mu}(0) \Rightarrow p^{m=1}(0) < p^{m=0}(0) \\ &\overline{\mu}(\eta) : \mu \leq \overline{\mu}(\eta) \Rightarrow p^{m=1}(\eta) < p^{m=0}(\eta)\,. \end{aligned} \tag{4.4}$$

Intuitively, when the pro-competitive reform is not approved, the merger must be more efficient to decrease prices, that is, $\overline{\mu}(\eta) < \overline{\mu}(0)$.[27] A traditional competition authority, assuming that politicians are not corrupt and considering only price effects, would approve the merger if and only if $\mu \leq \overline{\mu}(0)$. We refer to this approach as the consumer welfare standard.[28]

To describe the decision of the authority and how that compares with the consumer welfare standard, we distinguish three cases corresponding to the different regions in Figure 3.1.

(*a*) $t \in \left[0, \frac{Vx}{2}\right)$ or $t > \Delta\pi(1)$. For any $m$, the equilibrium is always Pooling. Political effects do not matter: there is no lobbying, and the Bad P is always re-elected with probability $1-q$. All P types approve the reform: $a^* = 1$. Thus, the authority approves the merger if and only if $\mu \leq \overline{\mu}(0)$. In this case, the consumer welfare standard is optimal.

[27] See Appendix A.1 for the expressions of (4.4).

[28] Competition authorities typically take merger control decisions without considering dynamic effects on market structure, such as merger waves. For example, in a situation when two mergers occur consecutively in the same market and are notified at close but different dates, the European Commission must adopt the so-called priority rule. This means the Commission must vet the first notified transaction as if the second transaction would *not* occur (even if they know that it will), freezing market structure at the date of the first notification. Similarly in our model, a traditional competition authority focused on the consumer welfare standard would ignore the dynamic effects on market structure (i.e., the block of pro-competitive reforms) caused by the increased political power firms gain through mergers. See Nocke and Whinston [2010] for a dynamic model of merger review and a discussion on the optimality of a static consumer welfare standard approach.

(b) $t \in \left[\frac{Vx}{2}, \Delta\pi(0)\right]$. For any $m$, the equilibrium is always Separating. As in the previous case, the probability of election of a Bad politician does not depend on $m$. The authority approves the merger if and only if $\mu$ is low enough for prices to decrease. However, with probability $1-q$, a Bad P is in office, and the reform is not implemented ($a^* = 0$). The optimal standard to be adopted would be $\overline{\mu}(\eta)$. Therefore, the authority's optimal merger policy is stricter than a consumer welfare standard that ignores lobbying that does happen. The critical threshold, $\hat{\mu}$, is such that $\hat{\mu} \in [\overline{\mu}(\eta), \overline{\mu}(0)]$.[29] Clearly, $\hat{\mu} \to \overline{\mu}(\eta)$ if $q \to 0$ and $\hat{\mu} \to \overline{\mu}(0)$ if $q \to 1$.

(c) $t \in (\Delta\pi(0), \Delta\pi(1)]$. In this case, there is a Separating equilibrium if the merger is approved ($m = 1$) and a Pooling equilibrium if the merger is not approved ($m = 0$). If $m = 0$, the consumer enjoys lower prices (the reform is always implemented), but the voter does not learn about P's type. If $m = 1$, with probability $1-q$, the consumer faces higher prices in the market, but the voter learns P's type (with probability $x$), thereby defeating the corrupt politician. The authority faces a trade-off between having the Good reform today and being able to screen P types tomorrow. This new trade-off augments the problem of the authority, otherwise only trying to balance market power and efficiencies. The authority approves the merger if and only if $\mu \leq \tilde{\mu}$, but potentially $\tilde{\mu} < \overline{\mu}(0)$ or $\tilde{\mu} > \overline{\mu}(0)$. In particular, $\tilde{\mu} > \overline{\mu}(0)$ when $\phi$ is high enough; and $\tilde{\mu} < \overline{\mu}(0)$ when $\phi$ is low enough.[30] In this case, the consumer welfare standard can be either too strict or too weak, depending on the benefits of separation $\phi$.

When $\phi$ is low, the merger must be very efficient ($\tilde{\mu} < \overline{\mu}(0)$) to be approved. The intuition is simple. Suppose that the authority commits to approve the merger whenever $\mu \leq \overline{\mu}(0)$. With probability $1-q$, P is Bad, and the merger gives firms enough power to influence P's behavior. As a result, if $\mu \in (\tilde{\mu}, \overline{\mu}(0))$, prices increase (in expectations) after the merger because of firms' political power rather than market power. The voter's benefits from separating P types do not offset the loss in consumer utility. It can also be the case that $\tilde{\mu} < \overline{\mu}(\eta)$.[31] Efficiencies must be high enough not only

[29] To see this, suppose $\hat{\mu} > \overline{\mu}(0)$. Then, if $\mu \in (\overline{\mu}(0), \hat{\mu})$, the merger increases prices with probability 1 but it is accepted, which cannot be optimal for the authority. Analogously, say $\hat{\mu} < \overline{\mu}(0)$. Then, if $\mu \in (\hat{\mu}, \overline{\mu}(0))$, the merger decreases prices with probability 1, but it is rejected, which cannot be optimal for the authority.

[30] We show this formally in Appendix A.3 and A.4.

[31] See Appendix A.3 and A.4.

to decrease prices, but also to compensate for the fact that, if the merger were not allowed, the consumer would always enjoy the pro-competitive reform, no matter the type of P.

As $\phi$ increases, welfare in the Separating equilibrium increases. The authority starts to trade off more of today consumer surplus to allow the voter to defeat the corrupt politician tomorrow. When $\phi$ is high enough, the authority can even allow mergers that increase prices ($\tilde{\mu} > \overline{\mu}(0)$), as the increase in the voter's payoff fully compensates for the decrease in the consumer's utility.[32]

The following Proposition summarizes the results of this Section.

**Proposition 2.** *The authority allows the merger ($m^* = 1$) if and only if $\mu \leq \mu^*$, where*

$$\mu^* = \begin{cases} \overline{\mu}(\eta) & \text{if } t \in \left[0, \frac{Vx}{2}\right) \text{ or } t > \Delta\pi(1) \\ \hat{\mu} & \text{if } t \in \left[\frac{Vx}{2}, \Delta\pi(0)\right] \\ \tilde{\mu} & \text{if } t \in (\Delta\pi(0), \Delta\pi(1)], \end{cases}$$

$\hat{\mu} \in [\overline{\mu}(\eta), \overline{\mu}(0)]$, *and* $\tilde{\mu} \gtreqless \overline{\mu}(0)$ *depending on* $\phi$.

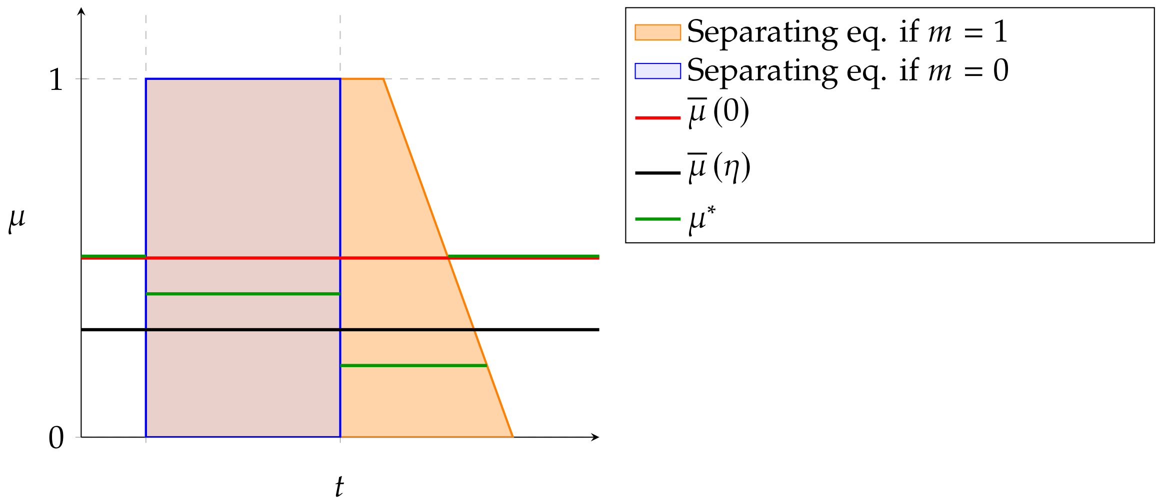
Proposition 2 captures the key insights of our model.[33] We summarize the results with the help of Figure 4.1, which draws the efficiency thresholds in various regions as a function of the level of bribes and $\phi$. Figure 4.1-panel a) corresponds to the case of low $\phi$. Figure 4.1-panel b) corresponds to the case of high $\phi$. The optimal merger policy coincides with the consumer welfare standard when the equilibrium does not exhibit lobbying, that is, when the equilibrium is Pooling for all $m$ (white region).

The optimal merger policy is stricter than the consumer welfare standard when the equilibrium exhibits lobbying, but the merger does not induce the voter's learning, that is, when the equilibrium is Separating for all $m$ (blue region). In the same way, the optimal policy is stricter than the consumer welfare standard when the merger induces a Separating equilibrium, but the benefits from the voter's learning do not offset the increase in prices (orange region in Figure 4.1-panel a)). When, instead, the merger can induce learning, and benefits from the voter's learning are high enough (orange region in Figure

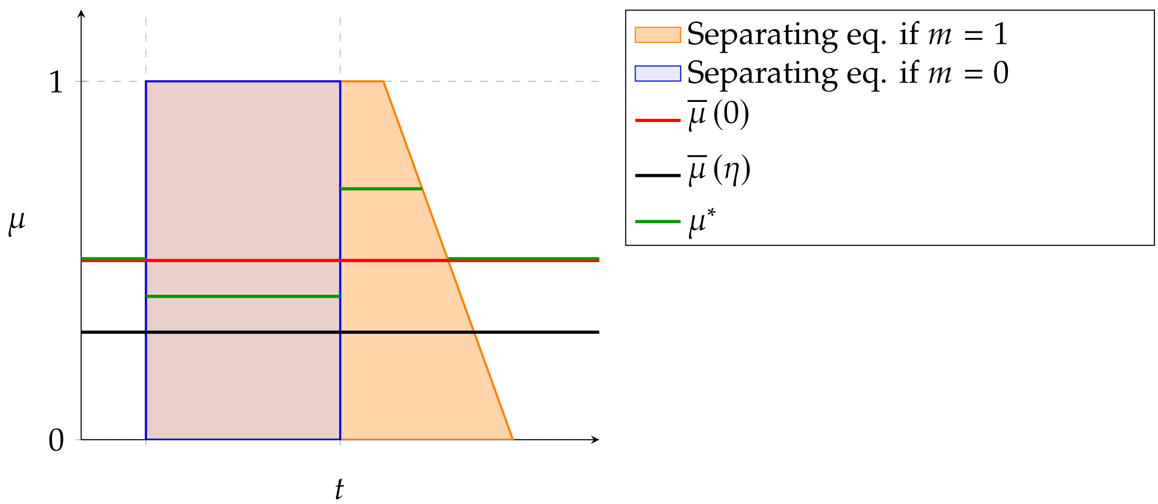
[32] At the end of this Section and in Appendix C.2, we discuss conditions under which this can be the case in practice.

[33] The expressions for the different thresholds in Proposition 2 are reported in Appendix A.3.

Figure 4.1: Panel a). Merger policy when $\phi$ is low




Panel b). Merger policy when $\phi$ is high



4.1-panel b)), the optimal merger policy can be more lenient than what the consumer welfare standard would suggest.

**Discussion** We have shown that the traditional consumer welfare standard can be sub-optimal when political effects enter the picture. We now briefly recap the intuition behind this result and later discuss its implications for competition policy in practice.

In our model, a politician decides whether to approve a pro-competitive reform or not. The reform benefits consumers, but it is detrimental to firms' profits. Firms can pay a bribe to the politician to prevent the approval of the reform. Before playing the lobbying game, the two firms can merge. The merger increases incentives to lobby. If the two firms do not merge, preventing the pro-competitive reform is less profitable, as some of the rents are dissipated by price competition. Then, if the merger is allowed, lobbying activity increases. Because of firms' additional political power, the pro-competitive reform is less likely to be approved, but the voter's ability to screen politicians improves. Bad politicians accept firms' bribes and, therefore, are defeated with a higher probability. This trade-off complements the conventional wisdom according to which only price effects matter for merger control.

The traditional consumer welfare standard is too weak whenever the voter's benefits from screening politicians do not offset the loss in consumer surplus. We distinguish two cases. First, it can be the case that the equilibrium exhibits lobbying regardless of the merger approval. In this case, the consumer welfare standard fails because - with some probability - a corrupt politician is in office, and the pro-competitive reform does not pass. When the reform is not approved, the optimal merger policy is stricter than the consumer welfare standard, as a higher level of efficiencies is required for prices to decrease. In this first case, the failure of the consumer welfare standard is not related to firms' additional political power, but rather to the assumption that politicians always behave in the consumers' interests. A stricter antitrust standard is needed in this case to compensate for the probability that Bad politicians do not approve market reforms.

The second case is more interesting. This arises when the equilibrium exhibits lobbying only if the merger is approved. The consumer welfare standard falls short as the merger allows firms to influence the Bad politicians and induce them to block the pro-competitive reform. It may also be optimal to reject mergers that are pro-competitive, that is, mergers that are efficient enough to decrease prices regardless of the approval of the reform (Figure 4.1).[34] This happens because, if the merger were rejected, (expected) prices would be even lower as the pro-consumer reform would always be approved (and firms would still be competing against each other). In a sense, a very strict standard is optimal in this case (that is, very high efficiencies are required for a merger approval) because the antitrust enforcer anticipates that it is precisely the merger approval that will trigger lobbying and then block pro-competitive reforms.

As the benefits from screening politicians increase, the welfare performance of the lobbying equilibrium improves. The merger makes firms lobby in equilibrium. Then, the voter can defeat corrupt politicians with a higher probability. This effect is positive for welfare. When the benefits of screening are large enough, one might accept a bigger increase in prices after the merger to let voters remove Bad politicians. Potentially, the consumer welfare standard can be too strict.

When, in practice, can this be the case? One may want to approve anti-competitive mergers and bear the cost of bribery today in order to avoid re-electing Bad politicians tomorrow. Three conditions then need to be satisfied. First, the voters' time discount factor should not be too low. Second, there must be welfare gains from avoiding regulations detrimental to consumers/voters in *other* parts or sectors of the economy. Suppose instead that the only regulation that corrupt politicians can implement to the detriment of consumers/voters is the one limiting the extent of competition within the narrowly defined market (i.e., the policy $a$). Then, it can never be optimal to suffer the policy for sure today in order to avoid it happening with some probability tomorrow.[35] Third, legislating tomorrow on these other regulations should not be too appealing for Bad politicians. Otherwise, they may prefer staying in power today, and the Separating equilibrium would disappear. If instead there are no out-of-market externalities, then the consumer welfare standard is always too lenient in our model.

**Implications for Lobbying Regulation** Our model shows that the stringencies of the lobbying and merger regulations can be complements or substitutes from a social welfare perspective, depending on the extent of the benefits from screening. Suppose that the lobbying legislation is strict; that is, the cost of lobbying is high enough that firms lobby if and only if the merger is approved.

[34] Our results go through even if one identifies the consumer welfare standard with $\mu < \bar{\mu}(\eta)$.
[35] We show this formally in Section C.2 in a two-period model.

When benefits from screening are low, merger policy and lobbying regulation are complements. The optimal merger policy should be strict as the lobbying equilibrium is inefficient for consumers/voters. The additional political power firms unlock due to the merger is used to corrupt the incumbent politician and prevent the approval of the pro-competitive reform. The improved screening of politicians does not compensate for this negative effect. In contrast, when screening is efficient, lobbying is good for welfare and the merger policy can be weak to allow mergers that induce the voter's learning, which has a high social value: The optimal merger policy and lobbying regulation are substitutes.

**Implications for Competition Policy** Should antitrust authorities consider non-market dimensions too in their analysis? A simple answer is that they do have the statutory powers, in a merger investigation, to request internal documents and reports from the merging parties. These could well include information referring to lobbying activities. If evidence of lobbying efforts is found in this discovery phase, then the competition authority could use it in its decision, and subject the transaction to a close scrutiny.

However, we do not think this is feasible in practice in many instances. Tirole [2022] warns about the risks associated with mission creep. He argues that expanding authorities' mandates to social and political goals, albeit possibly desirable, can be challenging from an organizational point of view. Mission creep can increase political interference, compromise authorities' independence, and decrease accountability. Expanding agencies' goals can decrease focus, making it difficult to measure and assess their performance. Our model is silent on these dimensions, as we assume the antitrust authority to be benevolent and not subject to agency problems. While this is a natural avenue for future research, we can however offer some implications for competition policy.

Policy has several dimensions of uncertainty that are not captured by our model but are very relevant in reality. Two important dimensions, in particular, are related to the burden of proof and the standard of proof. The standard of proof is the degree to which a party must prove its case to succeed. The burden of proof, or the onus, is the requirement to satisfy that standard. Our results can then be re-interpreted from this perspective. In the current system, the burden of proof for efficiencies falls on to the firms, as they have far better information than the enforcer about technology. This is very reasonable. It is when it comes to the standard of proof that our considerations kick in. A strict optimal merger policy can be interpreted as a situation where the standard of proof required for efficiencies is very high. The strictest case arises when there is a structural presumption that a merger is bad, and this cannot be rebutted by any efficiency claim. Antitrust authorities can retain a consumer-centric mandate, even in a world with political dimensions. However, a strict merger policy might be needed to account for the fact that mergers can decrease consumer welfare not only through market power effects but also through political ones. Our model then provides conditions that support the strengthening of a system of (rebuttable) structural presumptions in merger assessment. This is in line with the spirit of the recent 2023 revision of the US merger guidelines.[36]

# 5 Concluding Remarks

Zingales [2017] calls our attention to the risk of a "Medici vicious circle", in which economic and political power reinforce each other. Large firms can influence the rules of the game they play in the market. Society can anticipate this process by designing institutions and empowering them with legal tools: how should they account for the linkage between economic and political power?

In this paper, we consider how antitrust enforcement should react to the presence of political power, by joining a simple model of mergers from industrial organization with a political economy model of firms lobbying for regulation. Our model highlights a new trade-off associated with mergers. Mergers increase firms' political influence. On the one hand, this additional political power is bad for consumers as it reduces the level of competition politicians implement in the market. On the other hand, political power allows voters to screen politicians and punish corrupt ones. We also investigate how this trade-off interacts with traditional competition considerations in a merger's assessment. This allows us to contribute to the debate on the appropriateness of the consumer welfare standard in antitrust, and we discuss how our results largely lend support to a system of rebuttable structural presumptions.

Our findings come from a very stylized model, though they are robust to changes in consumers' utility function and in the bargaining model between firms and politicians. Further research could extend our framework to different models of lobbying and voting. Exploring these connections will provide further insights for a deeper analysis of the relationship between market power and political power.

[36] Source: Department of Justice. See Lancieri and Valletti [2024].

## References

U. Akcigit and S. T. Ates. What happened to US business dynamism? *Journal of Political Economy*, 131(8):2059–2124, 2023.

U. Akcigit, S. Baslandze, and F. Lotti. Connecting to power: political connections, innovation, and firm dynamics. *Econometrica*, 91(2):529–564, 2023.

R. Amir, P. Erickson, and J. Jin. On the microeconomic foundations of linear demand for differentiated products. *Journal of Economic Theory*, 169:641–665, 2017.

J. S. Banks and R. K. Sundaram. Optimal retention in agency problems. *Journal of Economic Theory*, 82(2):293–323, 1998.

C. L. Benkard, A. Yurukoglu, and A. L. Zhang. Concentration in product markets. *Working Paper*, 2021.

D. Besanko and D. F. Spulber. Contested mergers and equilibrium antitrust policy. *The Journal of Law, Economics, and Organization*, 9(1):1–29, 1993.

P. Bils, J. Duggan, and G. Judd. Lobbying and policy extremism in repeated elections. *Journal of Economic Theory*, 193:105223, 2021.

B. Blumenthal. Political agency and implementation subsidies with imperfect monitoring. *The Journal of Law, Economics, and Organization*, 39(3):775–800, 2023.

M. Bombardini and F. Trebbi. Empirical models of lobbying. *Annual Review of Economics*, 12(1):391–413, 2020.

R. H. Bork. *The antitrust paradox: A policy at war with itself*. New York: Basic Books, 1978.

S. Callander, D. Foarta, and T. Sugaya. Market competition and political influence: An integrated approach. *Econometrica*, 90(6):2723–2753, 2022.

S. Callander, D. Foarta, and T. Sugaya. The dynamics of a policy outcome: Market response and bureaucratic enforcement of a policy change. *American Journal of Political Science*, 2023.

P. Choné and L. Linnemer. Linear demand systems for differentiated goods: Overview and user's guide. *International Journal of Industrial Organization*, 73: 102663, 2020.

S. Coate and S. Morris. On the form of transfers to special interests. *Journal of Political Economy*, 103(6):1210–1235, 1995.

B. Cowgill, A. Prat, and T. Valletti. Political power and market power. *NBER Working Paper*, 2024.

B. Daley and E. Snowberg. Even if it is not bribery: the case for campaign finance reform. *The Journal of Law, Economics, & Organization*, 27(2):324–349, 2011.

J. De Loecker, J. Eeckhout, and G. Unger. The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, 135(2):561–644, 2020.

J. Duggan and C. Martinelli. Electoral accountability and responsive democracy. *The Economic Journal*, 130(627):675–715, 2020.

M. Faccio and L. Zingales. Political determinants of competition in the mobile telecommunication industry. *The Review of Financial Studies*, 35(4):1983–2018, 2022.

J. D. Fearon. *Electoral accountability and the control of politicians: selecting good types versus sanctioning poor performance*. Democracy, Accountability, and Representation, Cambridge University Press, 1999.

G. Gowrisankaran, A. Nevo, and R. Town. Mergers when prices are negotiated: Evidence from the hospital industry. *American Economic Review*, 105(1):172–203, 2015.

G. M. Grossman and E. Helpman. Protection for sale. *The American Economic Review*, 84(4):833–850, 1994.

G. Grullon, Y. Larkin, and R. Michaely. Are US industries becoming more concentrated? *Review of Finance*, 23(4):697–743, 2019.

K. Kang. Policy influence and private returns from lobbying in the energy sector. *The Review of Economic Studies*, 83(1):269–305, 2016.

L. M. Khan. Amazon's antitrust paradox. *Yale Law Journal*, 126:710–805, 2016.

S. Y. Kwon, Y. Ma, and K. Zimmermann. 100 years of rising corporate concentration. *American Economic Review*, forthcoming, 2024.

F. Lancieri and T. Valletti. Towards an effective merger review policy: A defence of rebuttable structural presumptions. *Oxford Review of Economic Policy*, forthcoming, 2024.

J. Levin. Supermodular games. *Lectures Notes, Department of Economics, Stanford University*, 2003.

E. Maskin and J. Tirole. The politician and the judge: Accountability in government. *American Economic Review*, 94(4):1034–1054, 2004.

P. Milgrom and C. Shannon. Monotone comparative statics. *Econometrica*, 62(1): 157–180, 1994.

S. Moshary and C. Slattery. Market structure and political influence in the auto retail industry. *Working Paper*, 2024.

M. Motta. *Competition policy: theory and practice*. Cambridge University Press, 2004.

M. Motta and M. Ruta. A political economy model of merger policy in international markets. *Economica*, 79(313):115–136, 2012.

V. Nocke and M. D. Whinston. Dynamic merger review. *Journal of Political Economy*, 118(6):1200–1251, 2010.

T. Philippon. *The great reversal: How America gave up on free markets*. Harvard University Press, 2019.

E. Prager and M. Schmitt. Employer consolidation and wages: Evidence from hospitals. *American Economic Review*, 111:397–417, 2021.

G. Sasso and M. Morelli. Bureaucrats under populism. *Journal of Public Economics*, 202:104497, 2021.

K. Schnakenberg and I. R. Turner. Formal theories of special interest influence. *Working Paper*, 2023.

K. E. Schnakenberg. Informational lobbying and legislative voting. *American Journal of Political Science*, 61(1):129–145, 2017.

K. E. Schnakenberg and I. R. Turner. Signaling with reform: How the threat of corruption prevents informed policy-making. *American Political Science Review*, 113(3):762–777, 2019.

C. Shapiro. Protecting competition in the american economy: Merger control, tech titans, labor markets. *Journal of Economic Perspectives*, 33(3):69–93, 2019.

N. Singh and X. Vives. Price and quantity competition in a differentiated duopoly. *The RAND Journal of Economics*, 15(4):546–554, 1984.

J. Tirole. Socially responsible agencies. *Competition Law & Policy Debate*, 7(4), 2022.

D. M. Topkis. *Supermodularity and complementarity*. Princeton University Press, 1998.

O. E. Williamson. Economies as an antitrust defense: The welfare tradeoffs. *The American Economic Review*, 58(1):18–36, 1968.

S. Wolton. Are biased media bad for democracy? *American Journal of Political Science*, 63(3):548–562, 2019.

M. Wörsdörfer. Louis D. Brandeis and the New Brandeis movement: Parallels and differences. *The Antitrust Bulletin*, 68(3):440–459, 2023.

T. Wu. *The curse of bigness*. New York, Columbia Global Reports, 2018.

L. Zingales. Towards a political theory of the firm. *Journal of Economic Perspectives*, 31(3):113–130, 2017.

# Appendix

## A Proofs and Derivations of the Baseline Model

The solution of the model can be replicated by downloading the attached Mathematica code.

### A.1 Equilibrium Prices

Suppose the two firms have not merged ($m = 0$). Profits are as follows:

$$\pi_i\left(p_i, p_j, p_3\right) = \frac{(p_i - c)(1 - \gamma - p_i(\gamma + 1) + (\gamma(p_j + p_3)))}{(1-\gamma)(2\gamma + 1)} . \tag{A.1}$$

The FOCs imply:

$$\begin{aligned} \frac{\partial \pi_i\left(p_i, p_j, p_3\right)}{\partial p_i} &= \frac{1 + c - 2p_i + \gamma(c - 2p_i + p_j + p_3 - 1)}{1 - 2\gamma^2 + \gamma} = 0 \Rightarrow \\ p_i\left(p_j, p_3\right) &= \frac{\gamma(c + p_j + p_3 - 1) + c + 1}{2(\gamma + 1)} . \end{aligned} \tag{A.2}$$

Intersecting firms' best response functions, we get:

$$p_i^{m=0}(p_3) = \frac{\gamma\left(c + p_3 - 1\right) + c + 1}{\gamma + 2} . \tag{A.3}$$

It is easy to see that (A.3) is increasing in $p_3\,(a)$. Substituting (A.3) into demand functions and profits, we get:

$$\begin{aligned} d_i\left(p_i^{m=0}(p_3), p_j^{m=0}(p_3), p_3\right) &= \frac{(\gamma + 1)(1 - c - \gamma + \gamma p_3)}{(1-\gamma)(\gamma + 2)(2\gamma + 1)} \\ \pi_i\left(p_i^{m=0}(p_3), p_j^{m=0}(p_3), p_3\right) &= \frac{(\gamma + 1)(1 - c - \gamma + \gamma p_3)^2}{(1-\gamma)(\gamma + 2)^2(2\gamma + 1)} , \end{aligned} \tag{A.4}$$

with

$$\frac{\partial d_i\left(p_i^{m=0}(p_3), p_j^{m=0}(p_3), p_3\right)}{\partial p_3} < 0\,,$$
$$\frac{\partial \pi_i\left(p_i^{m=0}(p_3), p_j^{m=0}(p_3), p_3\right)}{\partial p_3} < 0\,. \tag{A.5}$$

Therefore, profits of firm $i$ increase in $p_3$ via two channels. First, an increase in the price of good 3 increases the demand of good $i$. Second, the higher the price of good 3, the milder the competition, and the higher the price that firm $i$ can set.

If the merger is approved ($m = 1$), the merged firm solves:

$$\max_{p_1, p_2 \geq 0} \pi_1(p_1, p_2, p_3) + \pi_2(p_2, p_1, p_3)\ . \tag{A.6}$$

By FOCs, optimal prices are:

$$p_i^{m=1}(p_3) = \frac{1}{2}(c\mu + \gamma(p_3 - 1) + 1)\,, \tag{A.7}$$

with

$$\begin{aligned} &x_i\left(p_i^{m=1}(p_3), p_j^{m=1}(p_3), p_3\right) = \frac{c\mu + \gamma - \gamma p_3 - 1}{4\gamma^2 - 2\gamma - 2}\,,\\ &\pi_1\left(p_1^{m=1}(p_3), p_2^{m=1}(p_3), p_3\right) + \pi_2\left(p_2^{m=1}(p_3), p_1^{m=1}(p_3), p_3\right) =\\ &\frac{(c\mu + \gamma + \gamma(-p_3) - 1)^2}{2(1-\gamma)(2\gamma + 1)}\,. \end{aligned} \tag{A.8}$$

The comparative statics of (A.8) is analogous to (A.5).

We now compare prices (A.3), (A.7). The merger decreases prices if and only efficiencies are high enough:

$$p_i^{m=1}(p_3) < p_i^{m=0}(p_3) \Leftrightarrow \mu < \frac{2c(\gamma+1) - \gamma(1 - \gamma + \gamma p_3)}{c(\gamma+2)} := \overline{\mu}(p_3)\ . \tag{A.9}$$

## A.2 Returns from Protection

If $m = 0$, firms' return from protection is:

$$\begin{aligned}&\Delta\pi_i(0) = \\ &\pi_i\left(p_i^{m=0}(\eta), p_j^{m=0}(\eta), \eta\right) - \pi_i\left(p_i^{m=0}(0), p_j^{m=0}(0), 0\right) = \\ &\frac{\gamma(\gamma+1)\eta(2(c-1)-\gamma(\eta-2))}{(1-\gamma)(\gamma+2)^2(2\gamma+1)}.\end{aligned} \tag{A.10}$$

If $m = 1$, firms' return from protection is:

$$\begin{aligned}&\Delta\pi_i(1) = \\ &\pi_i\left(p_i^{m=1}(\eta), p_j^{m=1}(\eta), \eta\right) - \pi_i\left(p_i^{m=1}(0), p_j^{m=1}(0), 0\right) = \\ &\frac{\gamma\eta(\gamma(\eta-2)-2c\mu+2)}{4(1-2\gamma^2+\gamma)}.\end{aligned} \tag{A.11}$$

It is easy to see that $\Delta\pi(1) > \Delta\pi(0) \geq 0$ always.

## A.3 Merger Policy

We start by writing down the authority's payoff across the different equilibria. Let $m = 0$. In a Pooling equilibrium :

$$\begin{aligned}&W_{Pool}(0) = \\ &\frac{1}{4}\left(c^2-2c-4(1-q)\phi+3\right)+ \\ &+\frac{\gamma\left(6\gamma^3+c^2\left(2\gamma^2+7\gamma+7\right)\gamma+7\gamma^2+c\left(-4\gamma^3-6\gamma^2+2\gamma+8\right)-5\gamma-8\right)}{4(1-\gamma)(\gamma+2)^2(2\gamma+1)}.\end{aligned} \tag{A.12}$$

Let $m = 1$. In a Pooling equilibrium :

$$\begin{aligned}&W_{Pool}(1) = \\ &\frac{1}{4}\left(c^2\mu^2-2c\mu+4(q-1)\phi+3\right)-\frac{\gamma\left(\gamma\left(-2c^2\mu^2+4c\mu-3\right)+c^2\mu^2-4c\mu+3\right)}{4(1-\gamma)(2\gamma+1)}.\end{aligned} \tag{A.13}$$

Let $m = 0$. In a Separating equilibrium :

$$W_{Sep}(0) = \\ A\eta^2 + B\eta + C \ , \tag{A.14}$$

where

$$C = \\ \frac{2c^2(\gamma+1)^2 + 4c(\gamma-1)(\gamma+1)^2 + (\gamma-1)\left(-7\gamma^2 - 14\gamma + (\gamma+2)^2(2\gamma+1)q - 6\right)}{2(1-\gamma)(\gamma+2)^2(2\gamma+1)} + \\ + \frac{q}{2} - (q-1)\phi(qx-1) \ ; \\ A = \\ \frac{\left(4 - \gamma\left(3\gamma^2 + \gamma - 8\right)\right)(1-q)}{2(1-\gamma)(\gamma+2)^2(2\gamma+1)} \ ; \\ B = \\ \frac{\eta\left(4c\gamma(\gamma+1)^2(q-1) - 2(\gamma-1)(5\gamma(\gamma+2)+4)(q-1)\right)}{2(1-\gamma)(\gamma+2)^2(2\gamma+1)} \ . \tag{A.15}$$

Let $m = 1$. In a Separating equilibrium :

$$W_{Sep}(1) = \\ D\mu^2 + E\mu + F \ , \tag{A.16}$$

where

$$F = \\ \frac{\gamma(\gamma(q+2) - 2q - 1)}{4(1-\gamma)(2\gamma+1)} + \frac{(1-\eta)(1-q)\left(3\gamma^2(\eta-1) - 2\gamma\eta - 2\eta + 2\right)}{4(1-\gamma)(2\gamma+1)} + \\ + \frac{1}{4}\left(\mu^2 - 2\mu - 4(1-q)\phi(1-qx) + 2q + 1\right) \ ; \\ D = \\ \frac{\left(2\gamma^2 - \gamma - (1-c)(1+c)\right)}{4(1-\gamma)(2\gamma+1)} ; \\ E = \\ - \frac{\left(4\gamma^2 + 2c\gamma\eta - 2(c+1)\gamma - 2c\gamma\eta q - 2(1-c)\right)}{4(1-\gamma)(2\gamma+1)} \ . \tag{A.17}$$

We now obtain the critical thresholds $\tilde{\mu}, \overline{\mu}(0), \hat{\mu}, \overline{\mu}(\eta)$. These thresholds are obtained from finding the admissable roots of a quadratic equation in $\mu$. Specifically, for all $h, l \in \{Sep, Pool\}$, we solve

$$W_k(1) - W_l(0) = W\mu^2 + S\mu + T \ , \tag{A.18}$$

where $W, S, T$ are constant in $\mu$ and $W > 0$ always. Findings are reported next.

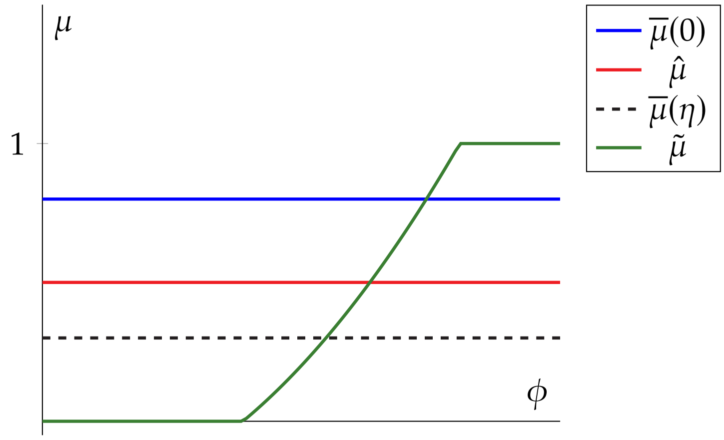
$$\begin{aligned}
&\mu \leq \hat{\mu} \Rightarrow W_{Sep}(1) \geq W_{Sep}(0) \ \text{, where} \\
&\hat{\mu} = \\
&\frac{(\gamma+2)(1-\gamma+\gamma\eta(1-q)) - \sqrt{G\eta^2 + H\eta + I}}{c(\gamma+2)} \ \text{and} \\
&G = \gamma^2(1-q)\left(4(\gamma+1)^2 - (\gamma+2)^2 q\right) \\
&H = 8\gamma(\gamma+1)^2(1-q)(1-c-\gamma) \\
&I = 4(\gamma+1)^2(c+\gamma-1)^2 \ ;
\end{aligned} \tag{A.19}$$

$$\begin{aligned}
&\mu \leq \tilde{\mu} \Rightarrow W_{Sep}(1) \geq W_{Pool}(0) \ \text{, where} \\
&\tilde{\mu} = \\
&\frac{(\gamma+2)(1-\gamma+\gamma\eta(1-q)) - \sqrt{L\gamma^4 + M\gamma^3 + N\gamma^2 + Q\gamma + R}}{c(\gamma+2)} \ \text{and} \\
&L = (1-q)(8qx\phi - \eta(8-\eta(q-4))) + 4 \\
&M = 8c - 2(1-q)\left(\eta(\eta(2q-7)+14) - 14qx\phi\right) \\
&N = 4c(c+2) - 2(1-q)\left(\eta(\eta(2q-3)+6) - 6qx\phi\right) - 8 \\
&Q = 8((c-1)c - 2(1-q)((\eta-2)\eta + 2qx\phi)) \\
&R = 4(c-2)c - 8(1-q)((\eta-2)\eta + 2qx\phi) + 4 \ ;
\end{aligned} \tag{A.20}$$

$$\begin{aligned}
&\mu \leq \overline{\mu}(0) \Rightarrow W_{Pool}(1) \geq W_{Pool}(0) \ \text{, where} \\
&\overline{\mu}(0) = \frac{2c(\gamma+1) + (\gamma-1)\gamma}{c(\gamma+2)} \ .
\end{aligned} \tag{A.21}$$

Additional details are available in the attached Mathematica code. In Figure A.1, we plot the different thresholds as a function of $\phi$.

Figure A.1: Thresholds $\tilde{\mu}, \overline{\mu}(0), \hat{\mu}, \overline{\mu}(\eta)$ as a function of $\phi$




## A.4 Proof that $\mu^* \gtreqless \bar{\mu}(0)$ (when $\phi$ is high) and $\mu^* < \bar{\mu}(\eta)$ (when $\phi$ is low)

The authority approves the merger if and only if:

$$\begin{aligned}
&W_{Sep}(1) - W_{Pool}(0) \geq 0 = \\
&q\left[u\left(d^*\left(p^{m=1}(0)\right)\right)\right] + \\
&(1-q)\left[u\left(d^*\left(p^{m=1}(\eta)\right)\right) + (1-x)(-\phi) + x(1-q)(-\phi)\right] + \\
&-u\left(d^*\left(p^{m=0}(0)\right)\right) - (1-q)(-\phi) \geq 0\,.
\end{aligned} \tag{A.22}$$

Let us define $s(\phi)$ as the expected benefit for the voter in the Separating equilibrium :

$$s(\phi) = -\phi \Delta P(\theta^* = \text{Bad})\ , \tag{A.23}$$

where

$$\begin{aligned}
&\Delta P(\theta^* = \text{Bad}) = \\
&[(1-q)(x(1-q) + (1-x))] - (1-q) = \\
&-(1-q)qx \leq 0
\end{aligned} \tag{A.24}$$

is the difference in the probability of election of a Bad P between the two equilibria. Since $s(\phi) \geq 0$ always, the voter prefers the Separating equilibrium. Moreover, $s' > 0$ and $s(0) = 0$.

Now, we rewrite (A.22) as follows:

$$W_{Sep}(1) - W_{Pool}(0) \geq 0 = \\ \left[qu\left(d^*\left(p^{m=1}(0)\right)\right) + (1-q)\,u\left(d^*\left(p^{m=1}(\eta)\right)\right) - u\left(d^*\left(p^{m=0}(0)\right)\right)\right] + s(\phi) \geq 0 \tag{A.25}$$

(A.25) allows us to isolate the effects of the merger on the consumer's and the voter's payoffs.

Let us assume $\phi = 0$ and consider the consumer's perspective. If the merger is approved, the reform is implemented with probability $q$, while if the merger is not approved, the reform is implemented with probability 1. Therefore, a very high level of efficiencies (lower $\mu$) is needed for the merger to be pro-competitive. In particular, if $\phi = 0$, $\tilde{\mu} < \overline{\mu}(0)$ necessarily.

We can see this by contradiction. Assume $\phi = 0$ and $\tilde{\mu} \geq \overline{\mu}(0)$. For the merger to be pro-competitive, it must decrease prices. Then,

$$\begin{aligned} &\mu \in [\overline{\mu}(0), \tilde{\mu}) \Rightarrow \\ &q p_i^{m=1}(0) + (1-q)\, p_i^{m=1}(\eta) \leq p_i^{m=0}(0) \Rightarrow \\ &p_i^{m=1}(0) < p_i^{m=0}(0). \end{aligned} \tag{A.26}$$

which contradicts (4.4). In words, if $\mu \in (\overline{\mu}(0), \tilde{\mu})$ and the merger is pro-competitive when the reform is implemented with probability $< 1$, then it must also be pro-competitive when the reform is implemented with probability 1, which we know it is not the case. Finally, since (A.25) is increasing in $\phi$, then also $\tilde{\mu}$ increases in $\phi$.[37]

By the same logic, one can see that $\tilde{\mu}$ can be lower than $\overline{\mu}(\eta)$. To see this, consider the extreme case than $q = 0$ (all politicians are Bad) and $\phi = 0$. The authority approves the merger if and only if

$$\begin{aligned} &u\left(d^*\left(p^{m=1}(\eta)\right)\right) > u\left(d^*\left(p^{m=0}(0)\right)\right) \Rightarrow \\ &p^{m=1}(\eta) < p^{m=0}(0) \ . \end{aligned} \tag{A.27}$$

Recall that

$$\mu \leq \overline{\mu}(\eta) \Rightarrow p^{m=1}(\eta) < p^{m=0}(\eta) \ . \tag{A.28}$$

[37]Suppose $\phi \rightarrow \infty$, then even if the merger does not bring efficiencies ($\mu = 1$), it is implemented. By monotonicity, it exists some $\tilde{\phi} < \infty$ such that $\tilde{\mu} = \overline{\mu}(0)$.

Since

$$p^{m=0}(0) < p^{m=0}(\eta) \ , \tag{A.29}$$

a higher level of efficiencies is required for (A.27) to hold:

$$\tilde{\mu} < \overline{\mu}(\eta) \ . \tag{A.30}$$

We show next some numerical examples of the relationship between $\tilde{\mu}$, $\overline{\mu}(\eta)$ $\overline{\mu}(0)$ , $\hat{\mu}$, and we comment on some limiting cases.

Table A.1 shows that as we increase $\phi$, $\tilde{\mu}$ becomes larger than $\overline{\mu}(0)$.

Table A.2 shows that when $q = 1$ (all politicians are Good), the optimal merger policy always coincides with the consumer welfare standard $\overline{\mu}(0)$.

Table A.3 shows that when $q = 0$ (all politicians are Bad), $\hat{\mu} = \overline{\mu}(\eta)$. In this case, $\tilde{\mu} = 0$. No merger is approved when allowing the merger creates a Separating equilibrium, as this Separating equilibrium is very inefficient from the voter's perspective.

| $\phi$ | $\tilde{\mu}$ | $\overline{\mu}(0)$ | $\hat{\mu}$ | $\overline{\mu}(\eta)$ |
|---|---|---|---|---|
| 0.3 | 0 | 0.366667 | 0.222818 | 0.116667 |
| 0.5 | 0 | 0.366667 | 0.222818 | 0.116667 |
| 0.8 | 0.134601 | 0.366667 | 0.222818 | 0.116667 |
| 1 | 0.951899 | 0.366667 | 0.222818 | 0.116667 |

Table A.1: Optimal merger policy when $\eta = 0.3; \gamma = 0.5; x = 0.5; q = 0.5; c = 0.12$

| $\phi$ | $\tilde{\mu}$ | $\overline{\mu}(0)$ | $\hat{\mu}$ | $\overline{\mu}(\eta)$ |
|---|---|---|---|---|
| 0.3 | 0.366667 | 0.366667 | 0.366667 | 0.116667 |
| 0.5 | 0.366667 | 0.366667 | 0.366667 | 0.116667 |
| 0.8 | 0.366667 | 0.366667 | 0.366667 | 0.116667 |
| 1 | 0.366667 | 0.366667 | 0.366667 | 0.116667 |

Table A.2: Optimal merger policy when $\eta = 0.3; \gamma = 0.5; x = 0.5; q = 1; c = 0.12$

| $\phi$ | $\tilde{\mu}$ | $\overline{\mu}(0)$ | $\hat{\mu}$ | $\overline{\mu}(\eta)$ |
|---|---|---|---|---|
| 0.3 | 0 | 0.366667 | 0.116667 | 0.116667 |
| 0.5 | 0 | 0.366667 | 0.116667 | 0.116667 |
| 0.8 | 0 | 0.366667 | 0.116667 | 0.116667 |
| 1 | 0 | 0.366667 | 0.116667 | 0.116667 |

Table A.3: Optimal merger policy when $\eta = 0.3; \gamma = 0.5; x = 0.5; q = 0; c = 0.12$

# B Generalization of Proposition 1

In this Section, we show that Proposition 1 follows from the supermodularity of the Bertrand game, and so it easily extends to any supermodular game. The concepts used in this Section are standard and come from Milgrom and Shannon [1994], Topkis [1998], and Levin [2003].

Let us define the following game. Let $i \in \{1,2\}$ index the two firms. Each firm's action is $a_i \in A_i$, where $A_i$ is the action space, which we assume to be a compact set. Let $\pi_i\left(a_i, a_j, a\right)$ be the payoff of firm $i$, where $a \in \{0,1\}$ is the politician's action, which we interpret as in the baseline model, so that $\pi_i\left(a_i, a_j, a\right)$ is decreasing in $a$. Assume P chooses $a$ before firms choose their actions. Then, we can interpret $a$ as a parameter. We assume that $\pi_i\left(a_i, a_j, a\right)$ is continuous.

If $m = 1$, firms' choose $a_i, a_j$ cooperatively. If $m = 0$, they choose $a_i, a_j$ simultaneously and independently. Our aim is to show that each firm's marginal benefit from reducing $a$ increases after a merger ($m = 1$).

Before introducing the notion of supermodularity, we define two additional elements. First, for expositional convenience, let us introduce the following notation: $\bar{a} = -a$, so that $\pi_i\left(a_i, a_j, \bar{a}\right)$ increases in $\bar{a}$. Second, we introduce the property of Increasing Differences (ID). A function $f(x,t)$ satisfies ID in $(x,t)$ if and only if, for all $x \in X$, $t \in T$, where $X, T$ are two (partially) ordered sets, such that $x' \geq x, t' \geq t$, the following property holds:

$$f(x', t') - f(x, t') \geq f(x', t) - f(x, t) \ . \tag{B.1}$$

Condition (B.1) implies that the marginal value from increasing $x$ is higher when $t$ is higher.

We assume that the game described above is a supermodular game with positive spillovers indexed in $\bar{a}$ (Levin [2003]), that is,

$(a)$ For all $i, j$, the function $\pi_i\left(a_i, a_j, \bar{a}\right)$ satisfies ID in $\left(a_i, a_j\right)$;

$(b)$ For all $i$, the function $\pi_i\left(a_i, a_j, \bar{a}\right)$ satisfies ID in $(a_i, \bar{a})$;

$(c)$ For all $i, j$, the function $\pi_i\left(a_i, a_j, \bar{a}\right)$ increases in $a_j$.

Let us define $\left(a_i^{m=1}, a_j^{m=1}\right)$ as the optimal actions for the two firms if $m = 1$.[38] If $m = 0$, by the supermodularity of the game, there is at least one Nash

[38]These maximizers exist by the continuity of payoff functions and the compactness of action spaces.

Equilibrium in pure strategies. Let $\left(a_i^{m=0}, a_j^{m=0}\right)$ be the Nash Equilibrium actions of the game if $m = 0$.[39]

By $(c)$:

$$\left(a_i^{m=1}, a_j^{m=1}\right) \geq \left(a_i^{m=0}, a_j^{m=0}\right) , \tag{B.2}$$

which implies:

$$\forall \bar{a}, \pi_i \left(a_i^{m=1}, a_j^{m=1}, \bar{a} = 0\right) - \pi_i \left(a_i^{m=0}, a_j^{m=0}, \bar{a} = 0\right) \geq 0. \tag{B.3}$$

Because of pair-wise ID between any pair of its arguments, the function $\pi_i$ is supermodular.[40] Therefore, increasing both actions $a_i, a_j$ has an higher marginal return when $\bar{a}$ is higher:

$$\begin{aligned} &\pi_i \left(a_i^{m=1}, a_j^{m=1}, \bar{a} = 0\right) - \pi_i \left(a_i^{m=0}, a_j^{m=0}, \bar{a} = 0\right) \geq \\ &\pi_i \left(a_i^{m=1}, a_j^{m=1}, \bar{a} = -1\right) - \pi_i \left(a_i^{m=0}, a_j^{m=0}, \bar{a} = -1\right) \end{aligned} \tag{B.4}$$

and by the symmetry of ID:

$$\begin{aligned} &\pi_i \left(a_i^{m=1}, a_j^{m=1}, \bar{a} = 0\right) - \pi_i \left(a_i^{m=1}, a_j^{m=1}, \bar{a} = -1\right) \geq \\ &\pi_i \left(a_i^{m=0}, a_j^{m=0}, \bar{a} = 0\right) - \pi_i \left(a_i^{m=0}, a_j^{m=0}, \bar{a} = -1\right) \geq 0 \end{aligned} . \tag{B.5}$$

Intuitively, because of a merger, firms set actions cooperatively and take positive spillovers into account. Then, they choose higher actions. If the marginal value of increasing actions $a_i, a_j$ increases as a $\bar{a}$ increases ($a$ decreases), then, by symmetry, the marginal value of increasing $\bar{a}$ (decreasing $a$) increases when actions are higher.

Note that the Bertrand game is a supermodular game. The Cournot duopoly is a supermodular game if one player's strategy set is given the reverse of its usual order (Levin [2003]). For more examples, see Levin [2003]. Note also that this proof does not require any assumption on the degree, concavity, or differentiability of demand and profit functions. If $a_i, a_j$ are interpreted as prices, then (B.5) implies

$$\forall i \ , a_i^{m=1}(\bar{a} = 0) - a_i^{m=0}(\bar{a} = 0) > a_i^{m=1}(\bar{a} = -1) - a_i^{m=0}(\bar{a} = -1) \, , \tag{B.6}$$

[39] In case of a multiplicity of equilibria, we select, without loss of generality, the Pareto undominated equilibrium, which exists by $(c)$ (Levin [2003]).

[40] To apply the properties of supermodular functions, we need to impose the assumption that the set $A_i \times A_j \times \{0, 1\}$ is a lattice.

which indicates that the welfare analysis performed in Section 4 would easily extend to this more general model.[41]

If the sign of (B.5) is inverted, then a merger decreases firms' incentives to lobby. We have shown that this can never be the case if the market interaction is a game with spillovers and supermodular payoffs.

# C Extensions

In the main text, we presented a simple model highlighting the political effects of mergers and how these effects interplay with market power considerations in a merger's assessment. We now extend the model to discuss the robustness of our findings. In particular, we first consider an alternative bargaining structure where firms offer a mechanism to P. Then, we consider an alternative model setup with two periods and endogenous $t$, $\phi$, and $V$.

## C.1 Firms Offer the Mechanism

Let us consider a modified version of the timing introduced in Section 2. In Stage 2, both firms submit a TIOLI offer to P: $l_i(a) \in \{0,1\}$. If $m = 1$, the two (merged) firms coordinate their offers. If $m = 0$, the two firms submit their offers simultaneously and independently. In Stage 3, P observes both offers and decides on the implementation of the reform.

How does this alternative bargaining structure affect the nature of the lobbying equilibrium? Suppose $m = 1$. Suppose further that

$$(1-x)V + t \geq V \Rightarrow t \geq Vx \,. \tag{C.1}$$

Condition (C.1) implies that a single bribe of value $t$ is sufficient to make $a = 0$ IC for P. In the baseline model, even if (C.1) holds, P requires the payment of two bribes for a total amount of $2t$ to set $a = 0$ because they have the power to commit to that, and of course, $2t > t$. In this model, if (C.1) holds, the optimal mechanism for the merged firms is:

$$\begin{cases} l_i^*(0) = 1 \\ l_j^*(0) = 0 \,. \end{cases} \tag{C.2}$$

[41] We can see this by looking at Appendix A.4, where we derive the properties of the optimal merger policy relying only on the impact of $a$ and $m$ on prices.

Since the merged firms know that P is willing to set $a = 0$ in exchange for $t$, there is no incentive to offer more and pay $2t$. In this case, only one firm lobbies, and P does not implement the policy. This equilibrium exists if and only if $t \in [Vx, \Delta\pi(1)]$. If $t \in \left[\frac{Vx}{2}, Vx\right)$, the merged firms know that they need to pay a total of $2t$ in order to make $a = 0$ IC for P, as in the baseline model.

Now, suppose that $m = 0$. As in the previous case, if $t \in \left[\frac{Vx}{2}, Vx\right)$, both firms lobby in equilibrium. When $t \in [Vx, \Delta\pi(0)]$, there exist two equilibria in pure strategies. In the first equilibrium, firm $i$ commits to the offer $l_i^*(0) = 1$ and firm $j$ commits to $l_i^*(0) = 0$, and P chooses $a^* = 0$. In the second equilibrium, firm $j$ lobbies, and firm $i$ does not.[42]

The multiplicity of equilibria generates a coordination problem between the two firms. To illustrate this, let us consider the following profile of mixed strategies. Suppose that firm 1 makes the offer $l_1(0) = 1$ with probability $k \in (0,1)$ and commits not to lobby with probability $1-k$. Let $\tilde{l}_1$ denote this mixed strategy. If firm 2 makes the offer $l_2(0) = 1$, its expected payoff is:

$$\mathbb{E}\left[l_2(0) = 1, \tilde{l}_1\right] = \pi_2\left(p^{m=0}(\eta)\right) - t\ . \tag{C.3}$$

If firm 2 commits to $l_2 = 0$, its expected payoff is:

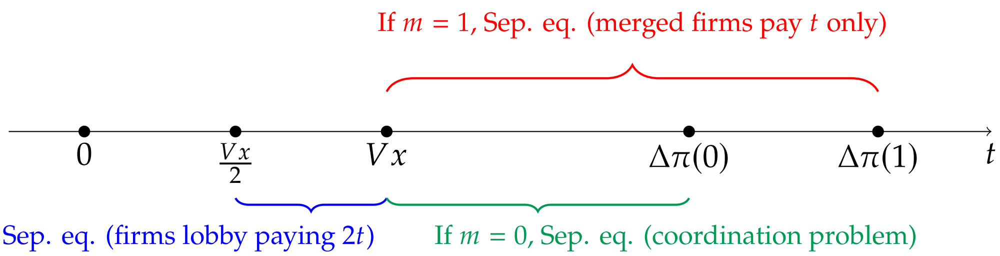
$$\mathbb{E}\left[l_2 = 0, \tilde{l}_1\right] = k\left(\pi_2\left(p^{m=0}(\eta)\right)\right) + (1-k)\,\pi_2\left(p^{m=0}(0)\right)\ . \tag{C.4}$$

It follows that if $k = \frac{\Delta\pi(0)-t}{\Delta\pi(0)} := k^*$, there exists an equilibrium in mixed strategies where both firms lobby with probability $k^*$. In this equilibrium, with probability $(1-k^*)^2$, neither firm lobbies and P does not implement the policy. This coordination failure arises from the incentive to free-ride on the competitor's lobbying effort.

There are two main takeaways from this robustness check. First, for all $m$, when firms have the power of commitment on the offer to P, there exist equilibria where only one bribe worth $t$ is paid. Firms can extract more surplus from P, as they can get protection ($a = 0$) at a lower price ($t$ rather than $2t$). However, if the two firms do not merge ($m = 0$), the potential multiplicity of equilibria generates a coordination problem. The merger solves the coordination problem. Second, we have shown that our main results are robust to this alternative bargaining structure. Figure C.1 summarizes the results of this Section. By comparing

[42] There is no equilibrium where both firms lobby because, given that firm $i$ is lobbying, firm $j$ does not want to lobby.

Figure C.1: Existence of Separating Equilibrium when firms have commitment power




Figures C.1 and 3.1, we can see that this alternative bargaining structure does not affect the likelihood that lobbying emerges in equilibrium, but only the extent of lobbying. The welfare analysis of the merger is also analogous to the baseline model.

## C.2 Endogenous $t, \phi, V$ in a Two-Period Model

In this Section, we introduce two key novelties. First, the political game is repeated twice, as in most political agency models. This allows us to endogenize the cost for consumers to re-elect a Bad P and the value of the office for a Bad P. Second, we allow P to specify $t$ as part of the contract. We show that the main results of the baseline model extend to this more general setup. For the sake of simplicity, we assume perfect monitoring ($x = 1$). Then, we can think of the consumer and the voter as the same player.

The timing is as follows. With a slight abuse of notation, we refer to the time between Stage 1′ and Stage 5′ as the First Period and to the time between Stage 6′ and Stage 8′ as the Second Period. As in the baseline model, firms 1, 2 merge if $m = 1$, and do not merge if $m = 0$.

Stage 1′ Nature draws $\theta_P, \theta_C$.

Stage 2′ P commits to a mechanism $a(l_1, l_2, t)$.

Stage 3′ Firms observe the mechanism and choose whether to lobby ($l_i = 1$) or not ($l_i = 0$). The reform $a$ is implemented.

Stage 4′ Firms observe $p_3(a)$ and simultaneously set prices $p_1, p_2$. The consumer observes prices and chooses a consumption plan $d_1, d_2, d_3$. First-period payoffs are realized.[43]

Stage 5′ The consumer chooses whether to re-elect P ($r = 1$) or not ($r = 0$).

Stage 6′ The elected politician (with type $\theta^*$) commits to a mechanism $a'\left(l_1', l_2', t'\right)$.

Stage 7′ Firms observe the mechanism and choose whether to lobby ($l_i' = 1$) or not ($l_i' = 0$). The second-period reform $a'$ is implemented.

Stage 8′ Firms observe $p_3(a')$ and simultaneously set prices $p_1', p_2'$. The consumer observes prices and chooses a consumption plan $d_1', d_2', d_3'$. Second-period payoffs are realized.

During the first period, players discount the future at a common discount factor $\delta \in (0, 1)$. The parameter $\delta$ can be interpreted as the exogenous probability that the game ends before the second period.

The consumer's lifetime utility is then:

$$u(d_1, d_2, d_3) + \delta\left[u\left(d_1', d_2', d_3'\right) - s\right] , \tag{C.5}$$

where $u(d_1, d_2, d_3)$ is defined as in (2.1) and

$$s = \begin{cases} \phi' & \text{if } \theta^* = \text{Bad} \\ 0 & \text{otherwise} , \end{cases} \tag{C.6}$$

for some $\phi' \geq 0$. The parameter $\phi'$ captures the consumer's anticipated cost from appointing a Bad P, in addition to the market's partial equilibrium effects. If $\phi' = 0$, the consumer only cares about their utility within the market. If $\phi' > 0$, re-electing a Bad PM has a negative effect beyond the market. The lifetime payoff for a Bad P is:

$$V' + t(l_1 + l_2) + r\delta\left(V' + t'\left(l_1' + l_2'\right)\right) . \tag{C.7}$$

In this case, $V' \geq 0$ may be seen as the value of re-election, on top of bribes. For instance, $V'$ may be the politician's salary.[44] As in the baseline model, the Good P has a behavioral type and always implements the policy. Firms' lifetime profits are:

$$\pi_i(p_i, d_i, l_i) + \delta\left(\pi_i\left(p_i', d_i', l_i'\right)\right) , \tag{C.8}$$

[43] This assumption is not consequential. We would obtain analogous results by assuming that payoffs are realized at the end of the game.

[44] In the baseline model, we assume that $V$ is already expressed in present-value terms.

where $\pi_i(p_i, d_i, l_i)$ is defined as in (2.4). To characterize the equilibrium, we proceed by Backward Induction.

**Second Period** Let us start from Stage 8′. This stage is analogous to the baseline model. The consumer's consumption plan is as in (3.2). Equilibrium prices are as in Appendix A.1. Let us now consider the lobbying equilibrium (Stage 6′, Stage 7′). Suppose that a Good P is in office. Then, they always implement the policy. Suppose that a Bad P is in office. Their optimal contract is:

$$a'^{*}\left(l_1' = l_2' = 1, t'^{*} = \Delta\pi(m)\right) = 0\,. \tag{C.9}$$

A Bad P commits not to implement the policy $a' = 0$ if and only if both firms lobby. It is optimal for them to set the price of lobbying so as to extract all the surplus from the two firms: $t'^{*} = \Delta\pi(m)$. Since the game ends at the end of the period, a Bad P has no re-election incentives, and they never implement the pro-competitive reform. Firms are indifferent between lobbying or not, and, as in the baseline model, we assume that the tie is broken in favor of lobbying.[45]

**First Period** Let us start from the voting stage (Stage 5′). If the consumer appoints a Bad P, the reform is never implemented, and they also bear a penalty $-\phi'$. If the consumer appoints a Good P, the reform is always implemented. Hence, the re-election rule described in Section 3.1 is optimal. The consumer re-elects the incumbent P if and only if $\hat{q} \geq q$, where $\hat{q}$ is the posterior belief that $\theta_P$ = Good.

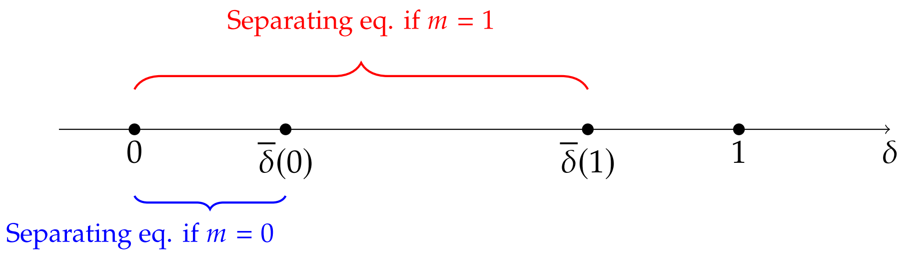
In Stage 4′, the equilibrium is analogous to the second period, and to the baseline model.[46] We now turn our attention to the first-period lobbying equilibrium (Stage 2′-Stage 3′). A Good P always implements the pro-competitive reform. A Bad P faces a trade-off between accepting bribes today or ensuring re-election and bribes tomorrow. The solution of this trade-off depends on the discount factor $\delta$.

The optimal contract is analogous to (C.9). This contract is IC for the two firms. In particular, as $\delta < 1$, firms have no incentives to give up protection

[45]If that was not the case, the optimal contract for P would be such that $t'^{*} = \Delta\pi(m) - \epsilon$, with $\epsilon \to 0_+$.

[46]Firms and the consumer have no incentives to change their market behavior to induce changes in the election stage. The two firms have no price strategy that may change the consumer/voter's beliefs. The consumer's payoff is additive-separable across the two periods, so their maximization problem can be seen as two separate ones.

Figure C.2: Existence of Separating Equilibrium with endogenous $t, \phi, V$.




today ($a = 0$) to induce the re-election of the Bad P and gain protection tomorrow ($a' = 0$). However, for this to be IR for P, it must be that:

$$2\Delta\pi(m) \geq \delta \left(2\Delta\pi(m) + V'\right) . \tag{C.10}$$

Accepting bribes today must be better than winning the office and getting bribes tomorrow. Unsurprisingly, this is optimal for P if and only if they are impatient enough. The IR constraint (C.10) implies:

$$\delta \leq \frac{2\Delta\pi(m)}{2\Delta\pi(m) + V'} := \overline{\delta}(m). \tag{C.11}$$

As in the baseline model, if (C.10) is not satisfied, the equilibrium is Pooling: both Ps types implement the policy and firms do not lobby. If instead (C.10) holds, the equilibrium is Separating. This more general model produces a result equivalent to Proposition 1. The merger increases the likelihood that lobbying emerges in equilibrium as $\overline{\delta}(1) > \overline{\delta}(0)$. If $m = 1$, firms have more incentives to lobby, and a Bad P can ask for higher bribes. Since bribes today are more valuable than bribes tomorrow, the merger increases lobbying in the first period. Figure C.2 illustrates this finding.

**Merger Assessment** We now briefly discuss the welfare analysis of the merger. In particular, we show that if there are no benefits from separation other than those coming from the market ($\phi' = 0$), the optimal merger policy from the consumer's perspective is always stricter than the consumer welfare standard ($\mu^* \leq \overline{\mu}(0)$).

To see this, suppose by contradiction that $\mu^* > \overline{\mu}(0)$. Given the intuition presented in the main text, this is possible if and only if the benefits from separation are high enough. In this two-period model, we can micro-found these benefits. Benefits from separation come from the fact that, in the second period, the anti-competitive policy ($a' = 0$) is approved with a low probability: $(1-q)^2$ in a Separating equilibrium and $1-q$ in a Pooling one. However, this comes at the cost of having the same policy with probability $1-q$ in the first period (versus a zero probability in the Pooling equilibrium). Since the value of today is always higher than the value of tomorrow ($\delta < 1$), then this cannot be optimal. On the contrary, if $\phi' > 0$, it can be the case that $\mu^* \geq \overline{\mu}(0)$ when $\phi'$ is high enough. The intuition is analogous to the baseline model.

There are three main takeaways from this extension. First, a two-period model with endogenous $\phi, V$, and $t$ displays the same intuitions of the simple baseline model presented in Section 2: the merger increases lobbying. Second, when P is allowed to specify $t$ as part of the mechanism, the optimal contract is such that all the surplus from trade is extracted from the two firms. P chooses the highest possible $t$ so that firms are indifferent between lobbying or not. Third, the optimal merger policy may be less strict than the consumer welfare standard if and only if there are benefits from screening other than those coming from the narrowly defined market, for example, if corruptible politicians can implement regulations detrimental to consumers/voters in other parts of the economy (out-of-market externalities).

# Chapter 3

Table of Contents

# Betting on the Right Horse: Corporate Campaign Contributions and Closeness of Elections*

Matteo Broso† and Tommaso Valletti‡

February 24, 2025
– Preliminary–

**Abstract**

We study how closeness of elections affects campaign contributions from corporations. We propose a model in which a corporation can offer contributions to office-seeking political candidates in exchange for favors. The model predicts that candidates in close elections may receive more funding because of both demand and supply effects. We use U.S. House election data from 1974 to 2014, and leverage the quasi-random variation in districts' competitiveness induced by incumbents' deaths, to provide empirical evidence supporting this claim. We find that corporate campaign contributions increase in districts following an incumbent's death relative to other districts in the same electoral cycle.

**Keywords** Lobbying, Campaign Contributions, Political Competition.
**JEL Classification** D72, D73, L13, L51, P16.

---

*We thank Andrea Gallice, Gabriele Gratton, Brett McCully, Alessandra Moresi, Ignacio Monzón, Jeanne Pinay, Lorien Sabatino, Margherita Tambussi, Richard Van Weelden, and Silvia Vannutelli for useful comments and discussions. The figures of this paper are best appreciated on screen or when printed in color.

†University of Turin and Collegio Carlo Alberto.
‡Imperial College London and CEPR.

# 1 Introduction

Corporations' influence on politics in the U.S. is on the rise. In the 1980 electoral cycle, corporate Political Action Committees (PACs) contributed approximately USD 55 million to congressional campaigns. During the 2020 electoral cycle, PACs raised approximately USD 13 billion, with corporate PACs accounting for a significant portion of this amount.[1]

As corporations gain power, it might not be the same for citizens. In a recent and influential paper, Gilens and Page [2014] analyze data from 1981 to 2002 on 1779 policy issues discussed in Congress. They find that the preferences of U.S. citizens have little to no impact on whether a policy becomes law. In contrast, political support from economic elites, such as corporations and interest groups, strongly correlates with a policy's probability of approval. Policies favored by economic elites are likely to pass and become law, while those popular among "ordinary" citizens are not.

Corporations influence politics through lobbying and campaign contributions. Lobbying operates as a form of *quid pro quo* exchange, where firms offer bribes to politicians in exchange for favors. Bribes can be either illegal or legal, such as through revolving doors (Blanes i Vidal et al. [2012]). Campaign contributions work similarly. In this case, however, corporations pay for politicians' electoral campaigns rather than private consumption goods. Campaign contributions are generally legal across all democratic countries, albeit they are subject to different regulations.[2]

While bribes are more common in non-democratic countries due to the lack of political accountability, campaign contributions are widespread. In other words, when there is no free competition for power, and political incumbents are sure to be re-elected, they face no cost of corruption. This is not necessarily true for campaign contributions.

In this paper, we study the impact of competitiveness of elections on firms' campaign contributions. We answer this question both theoretically and empirically.

[1] Sources: Brookings.edu, FEC.gov.

[2] For example, on September 17, 2015, Brazil's Supreme Federal Court ruled that corporate contributions to political campaigns are unconstitutional. Moreover, some countries, such as Germany and France, allocate public funds to political parties to reduce reliance on private donations. In the United States, federal law prohibits corporations from making direct contributions to candidates' campaigns or political parties. However, corporations can influence elections through Political Action Committees (PACs). Following the 2010 Supreme Court decision in Citizens United v. FEC, corporations can make unlimited contributions to Super PACs.

We propose a model where a corporation can offer campaign contributions to office-seeking political parties in exchange for favors. From the voters' point of view, the two parties are both horizontally and vertically differentiated. The extent of vertical differentiation captures the closeness of the election.[3] When voters perceive one party as of much higher quality than the other (vertical differentiation is high), the election is not competitive, as the high-quality party has a very high probability of winning. When vertical differentiation is low, both parties have similar probability of winning, and the election is close.

Before voters make their decision, political parties interact with the corporation. The corporation can offer a campaign contribution to one of the two parties, to both parties, or to none. Then, parties who receive an offer decide whether to accept it or not. Parties face a trade-off. On the one hand, accepting contributions allows them to buy targeted ads and increase turnout among their supporters (Spenkuch and Toniatti [2018]). On the other hand, campaign contributions come with a cost. Parties prefer winning the office without committing to pay favors to the corporation.

We solve the demand side and the supply side of the model separately. On the demand side, the model shows that parties' demand for contributions is positive if and only if they are not too strong. When parties are sufficiently likely to win, they do not need to rely on contributions to win the office. On the supply side, when the election is close, the corporation has incentives to hedge its bets by funding both parties. On the contrary, when the election is not close, the corporation wants to *bet on the right horse* and fund the strong candidate only.

Our model delivers an additional result. When a party accepts a contribution, its competitor faces a lower cost from accepting a contribution too. We name this the "normalization effect". When the election is not close, the corporation wants to fund the strong party only. However, it may need to offer a contribution to both parties in order to trigger the normalization effect, reduce the cost of acceptance, and thereby induce the strong party to accept. In this case, increased closeness of the election can make the strong party willing to accept contributions regardless of its opponent's strategy, ultimately reducing corporate campaign contributions in equilibrium.

Therefore, our model predicts that closeness of elections can increase (via demand or supply effects) or decrease (via the normalization effect) campaign contributions from corporations. We bring this question to the data.

[3]Throughout the paper, we refer to *closeness* of elections as the difference in the (two main) competing candidates' probability of winning.

We collect data from the DIME Database. This dataset contains matched contributor-recipient information on the universe of campaign contributions to U.S. House elections from 1980 to 2014. We focus on corporate contributions made by CEOs and directors of Fortune 500 companies.[4]

We start by presenting descriptive evidence of the relationship between closeness of elections and corporate campaign contributions. First, we observe that, across all electoral cycles, most contributions are concentrated in competitive districts. Second, we show that strong candidates and candidates in highly competitive races attract more contributions than weak candidates. We interpret this result through the lenses of our model. The first effect is driven by supply side incentives, with strong candidates attracting many contributions despite their possibly low demand. The second effect is driven by both demand and supply effects. When elections are close, candidates have strong demand for contributions, and contributors have incentives to fund all candidates to secure their political influence. To this end, we also provide evidence of corporations' hedging. When the district is won by a small margin, corporations are likely to fund both parties. These "Bi-Partisan" contributions are less likely in districts that are won with large margins.

To clean our analysis from reverse causality, we leverage the quasi-random variation in closeness of elections induced by the sudden deaths of House members. Our identification strategy builds on a widely acknowledged fact about U.S. Congress elections: it is extremely difficult to defeat an incumbent.[5] We implement a Difference-in-Differences (DiD) design. We define as treated those districts where an incumbent died while serving as the district's representative. Our estimates show that corporate campaign contributions increase significantly in a district following the death of the district's representative as compared to other districts in the same electoral cycle.

To rule out alternative explanations and anticipation effects, we repeat the analysis considering as treated only those districts wherein an incumbent died because of an unpredictable event (e.g., an heart attack or an accident). We obtain qualitatively analogous results. We also show that the effect is driven by competitive districts, while strongly Republican or Democratic districts show no statistically significant change in corporate contributions following an incumbent's death.

[4]We plan to extend our analysis to PACs contributions. See Section 4.

[5]For instance, more than 95% of incumbents were re-elected in the recent 2024 general election.

We contribute to the theoretical and empirical literature about money in politics. To the best of our knowledge, ours is the first attempt to assess the causal effect of closeness of elections on corporate campaign contributions.

The theory of corporate influence in elections starts with Snyder Jr [1990]. He models the market for campaign contributions as an investment market where candidates sell political favors, and firms allocate funds based on candidates' probabilities of winning.

The literature on campaign contributions tries to explain *why is there so little money in U.S. politics* (Ansolabehere et al. [2003]). This question is sometimes referred to as the Tullock's Paradox (Tullock [1972]). The paradox is that the amount of money in U.S. politics is small relative to the stakes. Two main alternative explanations have emerged. First, political contributions might simply be assets with relatively small returns, possibly reflecting the strong competitiveness of the market for contributions (see Battaglini et al. [2024] and the papers cited therein). Second, campaign contributions might be purely consumption goods, similar to charitable giving (Ansolabehere et al. [2003]). If this is the case, donors contribute to politics without expecting nothing in return.

Very few papers - however - have contributed to this debate by analyzing the impact of plausibly exogenous shocks to the demand or supply of political contributions. Roberts [1990] shows that the sudden death of a senior senator was associated with a decline in the stock market value of firms connected to the senator. Battaglini et al. [2024] show that the unexpected death of large donors significantly reduces candidates' election chances and changes their legislative behavior. Jayachandran [2006] studies the sudden decision of Senator Jeffords' to change party in 2001, which shifted Senate control from the Republicans to the Democrats. The paper finds that firms that had contributed to the Republicans saw significant declines in stock market value, while those that had supported the Democrats experienced the opposite effect. Kalla and Broockman [2015] provide experimental evidence that political donors are more likely to obtain meetings with senior policymakers.

We show that corporate campaign contributions respond to shocks to closeness of elections. Therefore, our results support a *quid pro quo* interpretation of campaign contributions.

The remainder of the paper is organized as follows. In Section 2, we present our theory. The empirical analysis is presented in Section 3. Section 4 concludes and discusses our next steps.

# 2 Theory

We start by introducing the main ingredients of the voting model. We introduce a Hotelling-type model with sincere voting. Parties are both horizontally and vertically differentiated. The extent of vertical differentiation captures closeness of the election. Then, we introduce *quid pro quo* campaign contributions from a corporation, and we study how the closeness of the election shapes the demand and the supply of contributions.

## 2.1 Baseline Model

Two political parties compete for office. Let $P = \{A, B\}$ be the set of parties. There is a continuum of voters with mass normalized to one. Each voter has a type $v$, with $v \sim U[0,1]$, which captures the voter's political preferences. In particular, the voter obtains utility

$$u(p \mid v) = \begin{cases} 1 - v + a & \text{if } p = A \\ \epsilon + v + b & \text{if } p = B \end{cases} \tag{2.1}$$

from voting for party $p$. The parameters $a, b$ capture the "quality" dimensions of the two parties. For further reference, we define $\Delta := a - b$. To allow uncertain election outcomes, we introduce a shock to preferences. In particular, $\epsilon \sim U[-1,1]$ captures an unexpected (from parties' point of view) redistribution of votes across the two parties.[6]

## 2.2 Baseline Model Solution

Voters' preferences are monotone in the type. Therefore, the voter who is indifferent between the two parties $\hat{v}(\Delta)$ is:

$$u(A, \hat{v}(\Delta)) = u(B, \hat{v}(\Delta)) \Rightarrow \hat{v}(\Delta) = \frac{1 - \epsilon + \Delta}{2}. \tag{2.2}$$

Let $p_p(\Delta)$ denote the probability that party $p$ wins the office. Then,

$$p_A(\Delta) = \frac{1}{2} + \frac{\Delta}{2}; \quad p_B(\Delta) = \frac{1}{2} - \frac{\Delta}{2}. \tag{2.3}$$

[6]The shock $\epsilon$ will be important in the model with campaign contributions, where parties will have to decide whether to accept contributions or not before the realization of the shock.

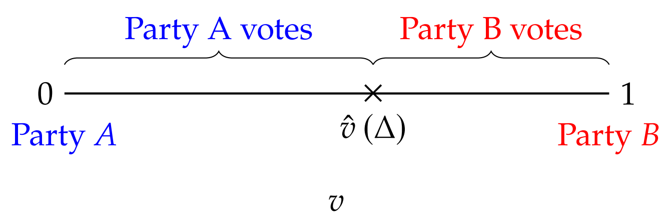



Figure 2.1: **Illustration of the Baseline Model**

Figure 2.1 illustrates the solution of the baseline model.

## 2.3 Model with Campaign Contributions

**Changes to the Baseline Model** There are two main changes with respect to the baseline model. First, there is a corporation. The corporation chooses which parties to offer campaign contributions to. Formally, the corporation chooses a subset $P_C \in \mathcal{P}(P)$, where $\mathcal{P}(P)$ is the power set of $P$.

Second, there are two types of voters. There are $\delta_p$ *partisan uninformed* voters who turn out and vote for party $p$ if and only if party $p$ uses targeted ads. Parties can use ads when they receive a contribution from the corporation. We assume $\delta_A = \delta_B = \delta$, with $\delta \in \left(0, \frac{1}{2}\right)$. The remaining $1 - 2\delta$ *swing* voters behave as in the baseline model.

**Timing**

Stage (1) The corporation chooses $P_C \in \mathcal{P}(P)$.

Stage (2) If $p \in P_C$, party $p$ chooses whether to accept the contribution ($m_p = 1$) or not ($m_p = 0$). If $m_p = 1$, party $p$ uses targeted advertising.

Stage (3) The shock $\epsilon$ is realized. Voters choose which party to vote for. The party that obtains the majority of votes is elected.

**Payoffs** Parties have commitment power. Whenever a party accepts a contribution, it commits to granting a favor to the corporation. The favor is potentially bad for voters. Swing voters' payoff is:

$$u(p \mid v) = \begin{cases} 1 - v + a - m_A \alpha & \text{if } p = A \\ \epsilon + v + b - m_B \alpha & \text{if } p = B \end{cases}, \tag{2.4}$$

where $\alpha \geq 0$ captures the anticipated cost for the voter of the party's ties with the corporation.[7]

The corporation obtains the favor if and only if they pay a contribution to the winning party. Let $W$ denote the winning party. The corporation's payoff is:

$$\pi_C(P_C) = \mathbb{1}\{W \in P_C\} - c \mid P_C \mid, \tag{2.5}$$

where $c \geq 0$ is the cost of a single contribution.

Parties' payoff is:

$$U_p(m_p) = \mathbb{1}\{p = W\} - \beta \mathbb{1}\{m_p = 1 \quad \text{and} \quad p = W\}, \tag{2.6}$$

where $\beta \in (0, 1)$ captures the cost for parties of winning the office while committing to protect the corporation's interests. This is the crucial assumption of our model. Parties are office-seeking, but they prefer winning the election while keeping their hands free.[8]

## 2.4 Results

Our solution concept is Subgame Perfect Nash Equilibrium in pure strategies (equilibrium henceforth). Throughout this section, starred variables denote equilibrium strategies.

In the voting stage, if one party accepts a contribution while its competitor does not, it gains $\delta$ partisan uninformed voters but it loses the support of swing voters with "intermediate" types. This is shown in Figure 2.2. If the competitor party receives money from the corporation, there is no direct cost from accepting a contribution, too. Swing voters split across parties as in the baseline model. There is, however, an opportunity cost. By declining the contribution, the party would gain swing voters with "intermediate" types. If neither party receives a contribution, the equilibrium is as in the baseline model.

In the Appendix, we obtain parties' probability of winning as a function of $(m_A, m_B)$. This allows us to state our demand and supply side results.

[7] The parameter $\alpha$ captures a combination of different factors: the time discount factor (higher $\alpha$ when future policies matter more), the transparency of the political system ($\alpha = 0$ if contributions are unobservable), and the utility cost of favors that parties must grant to corporations in exchange for contributions (e.g., antitrust or privacy regulations). We assume $\frac{\alpha}{2\alpha+1} \leq \delta < \frac{1}{2}$ to guarantee that parties can have positive demand for contributions in equilibrium.

[8] See Section 2.5 for a detailed discussion on the model's assumptions.

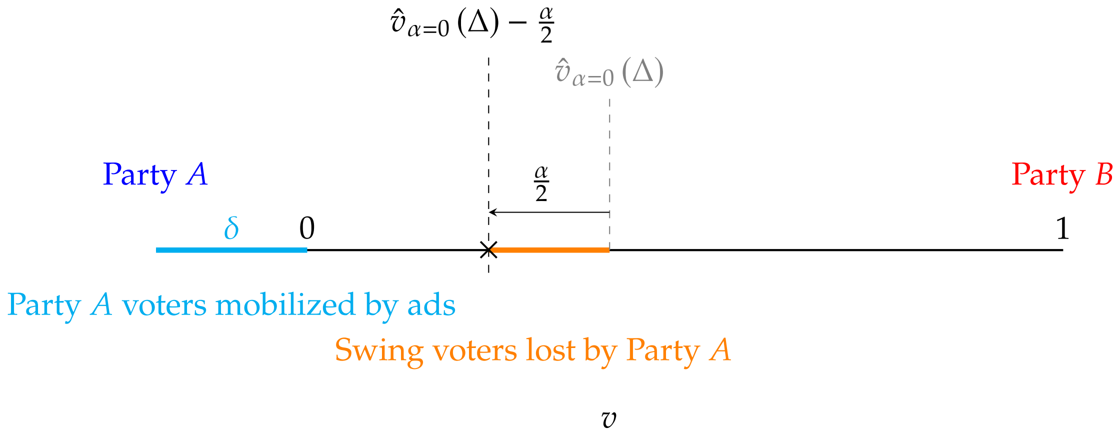



Figure 2.2: **Example of Election Stage Equilibrium (if $m_A = 1; m_B = 0$)**

**Proposition 1.** *(Demand for Campaign Contributions)*
*Suppose $p \in P_C$. There exist two thresholds $\hat{\Delta}_A(m_B), \hat{\Delta}_B(m_A)$ such that, for $p = A$ ($p = B$), the party accepts the contribution if and only if $\Delta \leq \hat{\Delta}_A(m_B)$ ($\Delta \geq \hat{\Delta}_B(m_A)$). Moreover, $\hat{\Delta}_A(1) \geq \hat{\Delta}_A(0)$ and $\hat{\Delta}_B(1) \leq \hat{\Delta}_B(0)$.*

*Proof.* See Appendix A.1 □

**Proposition 2.** *(Supply of Campaign Contributions)*
*Suppose parties always accept contributions. As $\Delta$ increases (decreases), the corporation's expected gain from funding both parties - rather than Party A (Party B) only - decreases.*

*Proof.* See Appendix A.2 □

The above propositions summarize our main arguments. The closeness of the election can increase campaign contributions through demand and supply effects.

In Proposition 1, we derive the demand side of the model. Parties reject contributions when they are sufficiently strong. When elections are close ($\Delta$ is close to zero), parties accept contributions, as this may be their only path to victory.

Moreover, a party's use of ads increases the competitor's incentive to use ads, too. This happens through two effects. First, when party $p$ accepts money from the corporation, party $p' \neq p$ faces a lower cost from doing the same, as swing voters would not punish it. Second, when party $p$ uses ads to mobilize

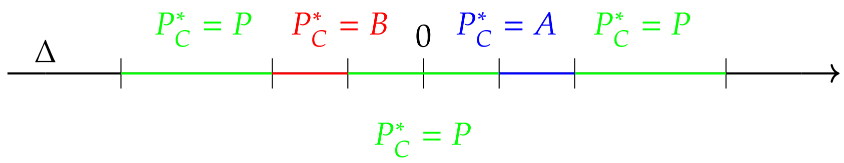



Figure 2.3: **Example of Equilibrium (Strong Complementarity Effects)**

its $\delta$ supporters, party $p'$ loses relative consensus. Then, it has a stronger incentive to use ads as well and mobilize its own supporters. We name this the "normalization" effect.

In Proposition 2, we show that when elections are close, the corporation has an incentive to hedge and offer a contribution to both parties. Supporting only one party is risky, as the likelihood of losing influence if that party fails to win is high. By financing both parties, the corporation ensures its political influence.

We now prove an existence and uniqueness result.

**Proposition 3.** *(Equilibrium)*
*For all parameter values, there exists a unique equilibrium.*

*Proof.* See Appendix A.3. □

Figure 2.3 shows an example of the equilibrium. In equilibrium, the corporation funds both parties in two different cases.[9] First, when $\Delta$ is close to zero, hedging is optimal for the corporation as the election is close. There is no other possible profile of players' strategies that would make the corporation better off.

The second case is when $\Delta$ is either sufficiently low or sufficiently high. In this case, the corporation would be better off by funding the strong party only (Party $A$ when $\Delta > 0$ and Party $B$ when $\Delta < 0$). However, the strong party would accept a contribution if and only if the weak competitor receives one too. We have shown in Proposition 1 the existence of a normalization effect. Parties are more willing to accept contributions when their competitors do the same. Therefore, in this second case, the corporation has to fund both parties just to make their preferred one willing to accept the money.

As $\Delta$ gets closer to zero, strong parties' demand for contributions increases. Therefore, they accept contributions even if their competitors do not. This happens in equilibrium in the red and blue regions of Figure 2.3.

The interplay of the demand and supply effects generates an interesting trade-off. As the election is closer, campaign contributions in equilibrium can

[9] In both cases, it must be that $c$ is low enough.

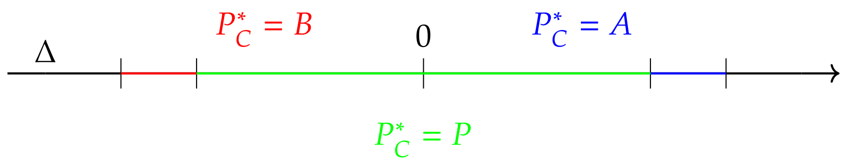



Figure 2.4: **Example of Equilibrium (Weak Complementarity Effects)**

increase (via hedging-supply effects or via competitive-demand effects) or decrease (via the normalization effect). The latter outcome depends on $\delta$. When $\delta$ is high, complementarity effects in the contributions' acceptance stage are strong. When $\delta$ is low, the green regions at the extremes of the $\Delta$ axis disappear. We show this in Figure 2.4.

Before focusing on the the empirical analysis, in the next section, we offer a brief discussion on the assumptions of the model.

## 2.5 Discussion on Assumptions

Our model, albeit simple, delivers a strong prediction. Closeness of elections can increase corporate camapaign contributions via demand and supply effects. However, the model hinges upon a series of assumptions which are worth discussing.

First, in our static setting, we assume parties have commitment power. In practice, where candidates could "take the money and run," repeated elections and reputational concerns can sustain similar outcomes without formal commitment.

Second, we assume corporations and voters' interests are not aligned. This is the crucial assumption of the demand side of the model. Political candidates face a cost when accepting corporate contributions. Otherwise, raising money would always be optimal, and a party's electoral strength would have no impact on its decisions.

Third, we assume the model is symmetric. Both parties can use ads to mobilize the same number of voters, and swing voters punish them equally if a commitment with the corporation is observed. In reality, parties may differ in their ideologies or reputational costs from accepting corporate money. Nonetheless, the main demand and supply insights from the model are qualitatively robust to an introduction of such asymmetries.

Fourth, we assume that advertising affects election outcomes by altering the the partisan composition of the electorate, as in Baron [1994] and as motivated by the empirical evidence in Spenkuch and Toniatti [2018]. However, our results qualitatively extend to a setting where advertising is helpful to persuade swing and undecided voters, as in Sides et al. [2022] and Fujiwara et al. [2024].

Fifth, we assume that contribution costs are exogenous. We make this assumption to isolate the strategic interaction between parties and corporations. Endogenous costs of contributions would add complexity to the model without altering the its core qualitative insights.

Finally, it is worth mentioning that players' risk preferences and the electoral rule (whether winner-takes-all or proportional voting) do not play a significant role for the results.

# 3 Empirical Analysis

## 3.1 Data and Descriptive Evidence

We use campaign contributions data from the DIME Database (Bonica [2024]). This database contains contributor-recipient matched data on all campaign contributions in the U.S. from 1980 to 2024. We assemble a dataset on the universe of corporate contributions to candidates in U.S. electoral campaigns for the U.S. House of Representatives for 18 election cycles from 1980 to 2014.[10]

Our definition of corporate contributions follows Bonica [2016]. We consider as corporate the contributions from Fortune500 corporations' CEOs and directors.[11] The Fortune500 is an annual list compiled by Fortune magazine that ranks the 500 largest U.S. corporations based on their total revenue for the respective fiscal year.[12]

Our unit of observation is the contribution. For each contribution, we have detailed information on both the contributor and the recipient. This includes the amount donated (in USD), the date of the contribution, and key electoral characteristics of the recipient. In particular, we record the recipient's primary and general election vote share, as well as the district-level vote share of the

[10]We do not observe corporate campaign contributions after 2014 as the specific section of the DIME website has not been not updated after that electoral cycle.

[11]At the current stage of the project, we do not observe corporate PACs contributions. We plan to do this as a next step. See Section 4 for a discussion.

[12]Table B.1 in the Appendix shows an example selection of Fortune500 corporations that saw at least one board member contribute to U.S. House races in 2012, along with the occupations of their contributors.

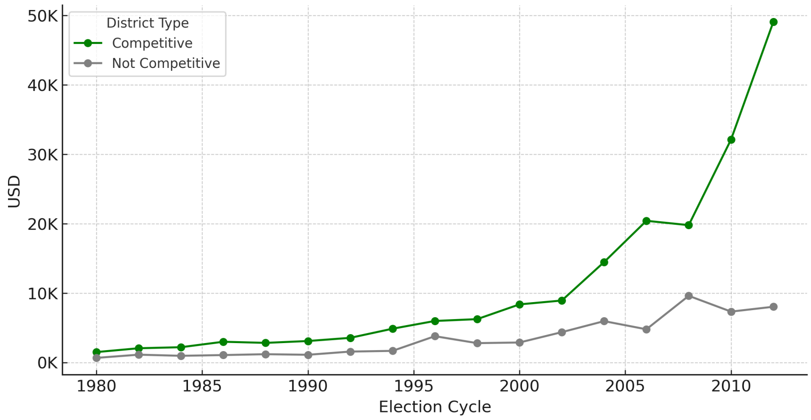



Figure 3.1: **Average Corporate Contributions to U.S. House Districts, 1980-2012.** Note: Districts are classified as Competitive if the average vote share of Democratic Presidential nominees across election cycles in the district is between 40% and 60%, and as Not-Competitive otherwise. The dataset includes 272 Competitive districts and 209 Not Competitive districts. Data from 2014 are not included as they are incomplete.

Democratic presidential nominee, among other election-related variables.[13]

Our empirical strategy relies on the quasi-random variation in races' competitiveness induced by the death of incumbents. We gather information about all U.S. House members who died in office between 1980 and 2014 from Wikipedia.[14] For each congress member who died in office, we collect the date of the death, the age at death, the party, the district, and the cause of the death. We exploit the latter information to create a dummy variable to distinguish between plausibly expected and unexpected deaths.[15] We use information on the district to match this data with the DIME database.

Before detailing our empirical strategy, we present some evidence, albeit descriptive, to support our central hypothesis: closeness of elections can drive corporate campaign contributions.

First, Figure 3.1 shows that competitive districts collect the majority of corporate contributions. This might happen because of demand and supply effects.

We can comment on both sides of the market with the help of Figure 3.2. In this figure, we plot the average of the total contributions per candidate over election cycles, distinguishing by candidates' electoral strength. We interpret these patterns through the lenses of our model. Contributions to strong candi-

[13]See Table B.2 in the Appendix for descriptive statistics of the key variables used in the analysis.

[14]See Wikipedia.org.

[15]See Table B.3 in the Appendix for a comprehensive list of Congress members' causes of death.

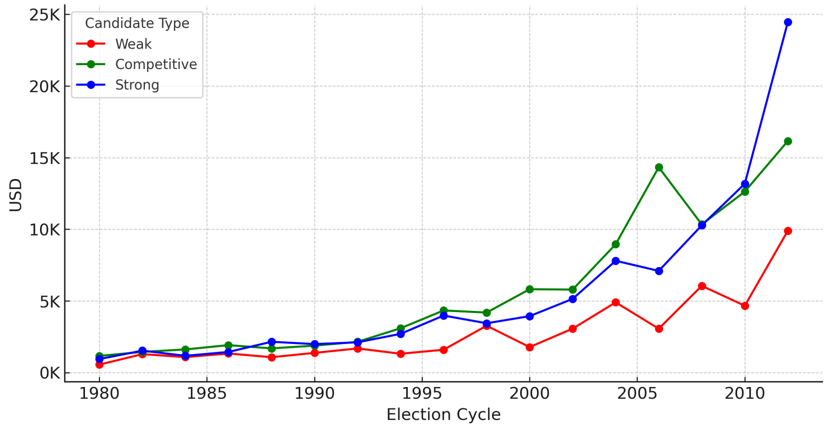



Figure 3.2: **Average Corporate Contributions to U.S. House Candidates, 1980-2012.**
Note: Candidates are classified based on the general election vote share: Weak (<40%), Competitive (40-60%), and Strong (>60%). Data for the 2014 cycle are not included as they are incomplete.

dates are high (more than USD 15.000 on average in 2012) because of supply side incentives. Corporations prefer to fund politicians who are likely to win and influence policy. Competitive candidates also receive substantial contributions (also more than USD 15.000 on average in 2012), because of both demand and supply effects. On the demand side, competitive candidates may actively seek contributions to avoid losing tight races. On the supply side, when elections are close, corporations may fund multiple candidates to hedge their bets. In contrast, weak candidates receive little funding (less than USD 10.000 on average in 2012) because donors are unwilling to invest in unlikely winners, not because of their lack of demand.

To discuss further the supply side of the market, Figure 3.3 plots the distribution of contributions across the vote share obtained by the funded candidate. We distinguish by two types of contributions. A contribution is classified as Bi-Partisan if the same corporation funded candidates from different parties in the same district and cycle. We compare the distribution of Bi-Partisan contributions with that of all other contributions. The vast majority of Bi-Partisan contributions are given to candidates that obtain between 40% and 60% of the district's votes. On the contrary, the remaining contributions are more evenly spread across the entire vote share range. This might be evidence of hedging. When the election is not close, corporations have little incentives to fund more than one candidate, as predicted by our model.

Descriptively, we have shown that competitive candidates and districts at-

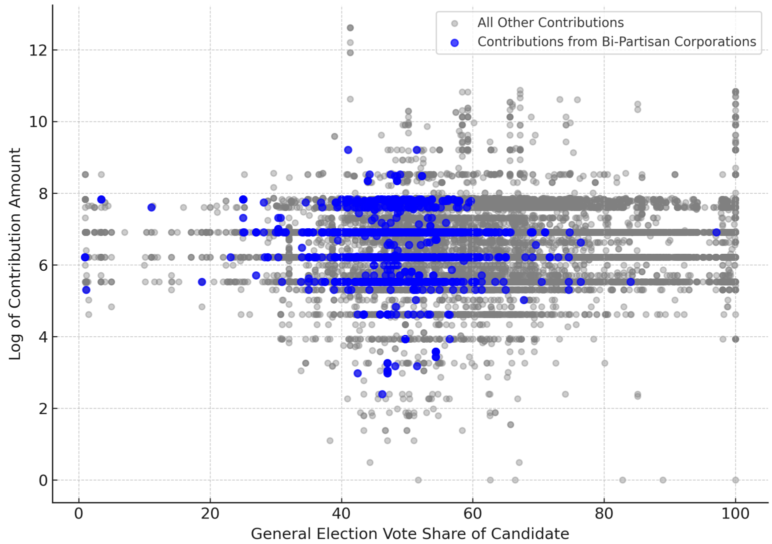



Figure 3.3: **Bi-Partisan and Partisan Contributions.**
Note: Bi-partisan contributions are contributions made by a corporation in a district where the same corporation funded more than one party in the same cycle. To improve clarity, we exclude candidates who received exactly 0% or 100% of the votes.

tract most corporate contributions. However, the direction of the causal relationship is not obvious. It could be that contributions make districts and candidates competitive, rather than vice-versa, with other factors driving the supply and demand of this market. To clean our analysis from reverse causality and other possible sources of endogeneity, we leverage the quasi-random variation in races' competitiveness induced by the death of incumbents.

## 3.2 Empirical Strategy

Our identification approach builds on a widely acknowledged fact about U.S. politics: it is extremely hard to defeat an incumbent. For instance, in the recent 2024 general election, 96.6% of incumbents were re-elected. Since 1980, when our data begins, the incumbency re-election rate was always higher than 85%.[16]

The incumbency advantage in U.S. House elections arises from several factors. Incumbents cultivate personal connections with constituents through services and engagement, securing a "personal vote" (Alford and Brady [1989]). Due to this advantage and their potentially higher quality (Kawai and Sunada [2024]), incumbents may deter strong challengers from entering the race (Gelman and King [1990]). Additionally, incumbents usually raise more money than

[16] Source OpenSecrets.com.

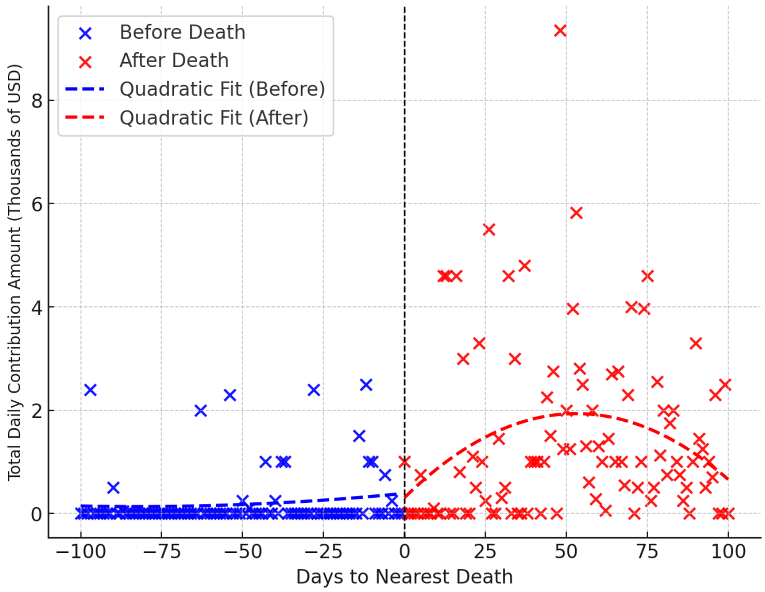



Figure 3.4: **Total Daily Contributions Before and After the Death of an Incumbent (-100;+100 days).**
Note: For treated districts only, days are indexed relatively to the day of the incumbent's death. For each day, we sum all contributions received across all treated districts on that day.

challengers (Cox and Katz [1996]).[17]

The sudden death of an incumbent offers a plausibly exogenous shock to a district's competitiveness. Following this intuition, we propose a Difference-in-Differences (DiD) approach, defining "treated" as those districts where an incumbent died in office and "control" as those districts where no incumbent died.

Figure 3.4 shows contributions before and after the death of an incumbent.

Contributions in treated districts increase sharply after the incumbent's death. There are, however, potentially "mechanical" explanations for this result. First, the death could represent a saliency shock rather than a competition shock, temporarily boosting contributions due to media attention and without affecting the full election cycle. Second, if the death was expected, particularly for aging or ill incumbents, donors might have been reluctant to contribute to their campaigns.

To address these concerns, we propose a DiD specification, comparing total contribution changes in treated districts with respect to other districts within the same cycle. We also distinguish between expected and unexpected deaths.

[17] In Figure B.1 in the Appendix, we show that incumbents collect more money than challengers from corporations in our sample across all election cycles.

Since incumbents' deaths occur at different times across districts, our treatment follows a staggered adoption pattern. We then use the Callaway and Sant'Anna [2021] estimator Figure B.2 in the Appendix shows the spatial distribution of treated and control districts.

We estimate the following Event-Study model:

$$TotalContributions'_{d,c} = \sum_{r=-T, r\neq -1}^{T'} \beta_r Death_{r,d,t(d)} + \alpha_d + \gamma_c + \epsilon_{d,c} \ , \qquad (3.1)$$

where:

- $d = 1, \cdots, 481$ indexes districts;[18]
- $c = 1980, 1982, \cdots, 2014$ indexes electoral cycles;
- $TotalContributions'_{d,c}$ is a log-like transformation of the total amount of contributions in district $d$ in cycle $c$;[19]
- $Death_{d,t(d)}$ is a dummy equal to 1 if district $d$ is $r$ electoral cycles away from the cycle of the death $t(d)$, and equal to 0 otherwise;
- $\beta_r$ are the coefficients of interest. They represent the dynamic Average Treatment effects on the Treated (ATT). They have a causal interpretation under the assumption that total contributions in treated and control districts would have evolved similarly absent the death of the incumbent;
- $\alpha_d, \gamma_c$ are district and cycle fixed effects, respectively;
- $\epsilon_{d,c}$ is the error term.

## 3.3 Results

Figure 3.5 shows the estimation results. First, on average, the dynamics of campaign contributions in treated and control districts follow a similar trend before treatment, suggesting that the parallel trends assumption is likely to

[18] We observe more districts than the 2025 number due to redistricting.

[19] We construct our dependent variable as follows. For each district-cycle pair, we compute $TotalContributions'_{d,c} = \ln(TotalContributions_{d,c} + 1)$; When an incumbent dies, special elections are often held in the district, potentially leading to multiple elections within the same cycle. To account for this, in treated districts, and only for the first cycle of treatment, we divide $TotalContributions_{d,c}$ by two before taking the log. Under the assumption that the first, special election is the really "treated" one, we will then only recover a lower bound of the true effect. See Section 4 for a discussion on how we plan to refine this measurement.

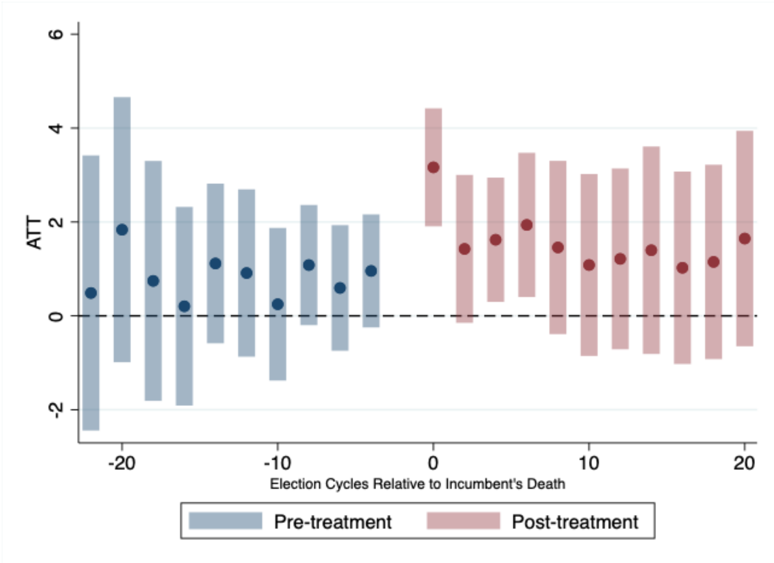



Figure 3.5: **The Effect of an Incumbent's Death on** $TotalContributions'_{d,c}$ **(Callaway and Sant'Anna [2021]- Estimator).**
Note: The figure reports the estimated coefficients for $\beta_r$ from equation (3.1) (for $T = -20, T' = 20$). Confidence Intervals are at the 99%. Number of observations: 8658.

hold. Second, there is a sharp increase in contributions during the first electoral cycle after the incumbent's death. This result is consistent with our theory. When an incumbent dies, the subsequent open-seat election is more competitive, and it attracts more contributions. Third, this effect appears to persist, albeit to a smaller extent, across electoral cycles. If incumbency advantage stems from relationships and social connections with constituents, then building this advantage may take time. As a result, districts could remain more competitive for some cycles after an incumbent's death, as the new incumbent may be weaker than their predecessor.

However, we cannot claim that the effect of an incumbent's death on campaign contributions is entirely attributable to changes in races' competitiveness. For instance, corporate donors might be reluctant to donate to ill politicians, or ill politicians may have lower time or ability to raise funds. If this is the case for the period before an incumbent's death, the DiD estimator could capture a positive effect in later electoral cycles driven by changes in candidates' strength or fundraising efforts rather than district competitiveness.

To clean our results from these possible confounding factors, we perform the following tests. First, we focus on unexpected deaths only. That is, we take districts where an incumbent dies after a presumably long illness out of the sample.[20] We consider as treated only those districts wherein an incumbent died because of an unpredictable event, such as an heart attack or an accident. As we can see from Figure B.3 in the Appendix, our result is qualitatively robust to this refinement of the treatment group.

Second, we distinguish the results by districts' level of competitiveness. Figures 3.6 and 3.7 show that the effect is entirely driven by competitive districts. We define competitive districts as districts where the average vote share for the Democratic Party across electoral cycles is neither too high nor too low. When an incumbent dies unexpectedly in a competitive district, corporate campaign contributions increase. On the contrary, in strongly Republican or Democratic districts, there is no statistically significant effect. This heterogeneity supports our findings.

# 4 Conclusions

We study how closeness of elections affects corporate campaign contributions. We propose a model where corporations can offer financial support to political candidates in exchange for favors. The model predicts that close elections increase corporate contributions through demand and supply effects.

Empirically, we test this hypothesis using U.S. House election data from 1974 to 2014, leveraging the quasi-random variation in races' competitiveness caused by incumbents' deaths. Our results show that corporate contributions increase significantly in districts where an incumbent dies, relative to others in the same electoral cycle. We further refine the analysis by distinguishing between expected and unexpected deaths to rule out alternative explanations. Additionally, we find that the effect is driven by competitive districts.

In the next steps of this paper, we will strengthen our findings as follows. We will repeat the analysis using corporate PACs contributions' data and refine our outcome measure by focusing only on contributions to special elections. We will also test whether districts effectively become more competitive after an incumbent's death by analyzing vote shares changes. We will perform a placebo test, checking whether small donors react similarly to corporate donors to incumbents' deaths. Finally, using the DIME+ database, we plan to investigate whether politicians elected in close races vote and behave differently than those elected with large margins.

[20]See Table B.3.

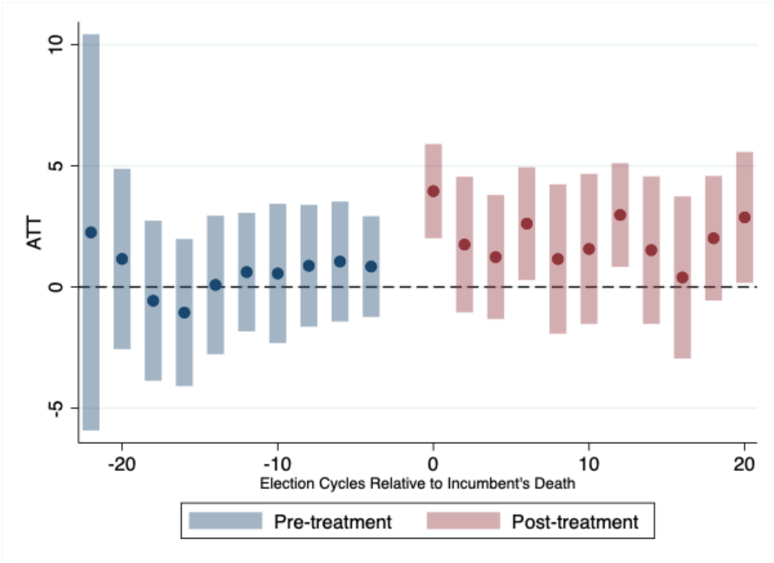



Figure 3.6: **The Effect of an Incumbent's Death on** $TotalContributions'_{d,c}$ **in Competitive Districts (Callaway and Sant'Anna [2021]- Estimator, Unexpected Deaths only).** Note: The figure reports the estimated coefficients for $\beta_r$ from equation (3.1) (for $T = -20, T' = 20$). Confidence Intervals are at the 99%. Observations from districts with expected deaths are out from the sample. Number of observations: 8028. Districts are classified as Competitive if the average vote share of Democratic Presidential nominees across election cycles in the district is between 35% and 65%, and as Not-Competitive otherwise.

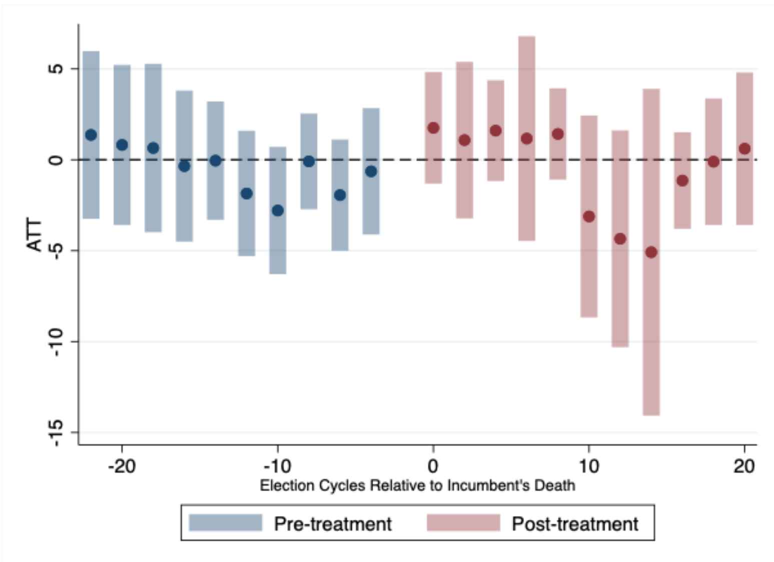



Figure 3.7: **The Effect of an Incumbent's Death on** $TotalContributions'_{d,c}$ **in Not Competitive Districts (Callaway and Sant'Anna [2021]- Estimator, Unexpected Deaths only).** Note: The figure reports the estimated coefficients for $\beta_r$ from equation (3.1) (for $T = -20, T' = 20$). Confidence Intervals are at the 99%. Observations from districts with expected deaths are out from the sample. Number of observations: 8028. Districts are classified as Competitive if the average vote share of Democratic Presidential nominees across election cycles in the district is between 35% and 65%, and as Not-Competitive otherwise.

# References

J. R. Alford and D. W. Brady. The incumbency advantage in US elections: An analysis of state and federal offices, 1942–2000. *Journal of Politics*, 51(2):337–349, 1989.

S. Ansolabehere, J. M. De Figueiredo, and J. M. Snyder Jr. Why is there so little money in US politics? *Journal of Economic Perspectives*, 17(1):105–130, 2003.

D. P. Baron. Electoral competition with informed and uninformed voters. *American Political Science Review*, 88(1):33–47, 1994.

M. Battaglini, V. L. Sciabolazza, M. Lin, and E. Patacchini. Unobserved contributions and political influence: Evidence from the death of top donors. *NBER Working Paper*, 2024.

J. Blanes i Vidal, M. Draca, and C. Fons-Rosen. Revolving door lobbyists. *American Economic Review*, 102(7):3731–3748, 2012.

A. Bonica. Avenues of influence: On the political expenditures of corporations and their directors and executives. *Business and Politics*, 18(4):367–394, 2016.

A. Bonica. Database on ideology, money in politics, and elections: Public version 4.0. *[Computer file]. Stanford, CA: Stanford University Libraries.*, 2024.

B. Callaway and P. H. Sant'Anna. Difference-in-differences with multiple time periods. *Journal of Econometrics*, 225(2):200–230, 2021.

G. W. Cox and J. N. Katz. Why did the incumbency advantage in US house elections grow? *American Journal of Political Science*, 40(2):478–497, 1996.

T. Fujiwara, K. Müller, and C. Schwarz. The effect of social media on elections: Evidence from the United States. *Journal of the European Economic Association*, 22(3):1495–1539, 2024.

A. Gelman and G. King. Incumbency advantage: Evidence from a regression discontinuity design. *Journal of Politics*, 52(4):1148–1161, 1990.

M. Gilens and B. I. Page. Testing theories of American politics: Elites, interest groups, and average citizens. *Perspectives on Politics*, 12(3):564–581, 2014.

S. Jayachandran. The Jeffords effect. *Journal of Law and Economics*, 49(2):397–425, 2006.

J. L. Kalla and D. E. Broockman. Campaign contributions facilitate access to congressional officials: A randomized field experiment. *American Journal of Political Science*, 60(3):545–558, 2015.

K. Kawai and T. Sunada. Estimating candidate valence. *Econometrica*, 2024. Forthcoming.

B. E. Roberts. A dead senator tells no lies: Seniority and the distribution of federal benefits. *American Journal of Political Science*, 34(1):31–58, 1990.

J. Sides, L. Vavreck, and C. Warshaw. The effect of television advertising in united states elections. *American Political Science Review*, 116(2):702–718, 2022.

J. M. Snyder Jr. Campaign contributions as investments: The US house of representatives, 1980-1986. *Journal of Political Economy*, 98(6):1195–1227, 1990.

J. L. Spenkuch and D. Toniatti. Political advertising and election results. *The Quarterly Journal of Economics*, 133(4):1981–2036, 2018.

G. Tullock. The purchase of politicians. *Western Economic Journal*, 10(3):354–55, 1972.

# Appendix

# A Theory

## A.1 Proof of Proposition 1

We proceed in three steps.

**Step** 1. **Parties' Probability of Winning** We start by obtaining parties' probability of winning as a function of $(m_A, m_B)$. To this end, let $V_p(m_A, m_B)$ be the number of votes for party $p$. There are four possible cases.

- Both parties receive a contribution ($m_A = m_B = 1$).

  In this case, the position of the indifferent swing voter does not change with respect to the baseline model. The indifferent voter is $\hat{v}(\Delta) = \frac{1-\epsilon+\Delta}{2}$. As both parties mobilize an equal number of partisan uninformed voters, parties' probability of winning are as in (2.3).

- Both parties do not receive a contribution ($m_A = m_B = 0$).

  This case is analogous to the previous one. As in the baseline model, parties probability of winning are as in (2.3).

- Party $A$ only receives a contribution ($m_A = 1; m_B = 0$).

  The position of the swing indifferent voter is:

$$\hat{v}(\Delta) = \frac{1+\Delta-\epsilon-\alpha}{2}\ . \tag{A.1}$$

  The two parties number of votes are:

$$\begin{aligned} V_A(1,0) &= (1-2\delta)\frac{1+\Delta-\epsilon-\alpha}{2} + \delta \\ V_B(1,0) &= (1-2\delta)\left(1 - \frac{1+\Delta-\epsilon-\alpha}{2}\right)\ . \end{aligned} \tag{A.2}$$

  Figure 2.2 shows the equilibrium of the voting stage in this case.

  Then,

$$V_A(1,0) \geq V_B(1,0) \Leftrightarrow \epsilon \leq \frac{\Delta+\delta-\alpha+2\delta(\alpha-\Delta)}{1-2\delta} := \tilde{\epsilon}_{1,0}\ . \tag{A.3}$$

Since $\epsilon$ is uniformly distributed, the two parties' probability of winning are:

$$\begin{aligned} p_A(1,0) &= \frac{1+\tilde{\epsilon}_{1,0}}{2} = \frac{\alpha + a(2\delta-1) - 2\delta(\alpha+b) + b + \delta - 1}{4\delta - 2} \\ p_B(1,0) &= 1 - p_A(1,0) = \frac{1}{4}\left(2\alpha - 2\Delta + \frac{1}{2\delta-1} + 3\right) . \end{aligned} \tag{A.4}$$

- Party $B$ only receives a contribution ($m_A = 1; m_B = 0$).

  This case is analogous to the previous one. In particular,

$$\begin{aligned} p_A(0,1) &= \frac{1}{4}\left(2\alpha + 2\Delta - \frac{1}{1-2\delta} + 3\right) \\ p_B(0,1) &= \frac{1}{4}\left(-2\alpha - \Delta + \frac{1}{1-2\delta} + 1\right) . \end{aligned} \tag{A.5}$$

**Step 2. Acceptance Decisions.** We assume that parties break indifference in favor of acceptance. We now study the two parties' demand for contributions separately.

- Party $A$.
  - $m_B = 1$. Party $A$ accepts the contribution if and only if

$$\begin{aligned} &p_A(1,1)\left(1-\beta\right) \geq p_A(0,1) \Leftrightarrow \\ &\Delta \leq \frac{-2\alpha\delta + \alpha - 2\beta\delta + \beta - \delta}{2\delta\left(1-\beta\right)} \end{aligned} \tag{A.6}$$

  - $m_B = 0$. Party $A$ accepts the contribution if and only if

$$\begin{aligned} &p_A(1,0)\left(1-\beta\right) \geq p_A(0,0) \Leftrightarrow \\ &\Delta \leq \frac{2\alpha\beta\delta - \alpha\beta - 2\alpha\delta + \alpha - \beta\delta + \beta - \delta}{2\delta\left(1-\beta\right)} \end{aligned} \tag{A.7}$$

- Party $B$.
  - $m_A = 1$. Party $B$ accepts the contribution if and only if

$$\begin{aligned} &p_B(1,1)\left(1-\beta\right) \geq p_B(1,0) \Leftrightarrow \\ &\Delta \geq \frac{2\alpha\delta - \alpha + 2\beta\delta - \beta + \delta}{2\delta(1-\beta)} \end{aligned} \tag{A.8}$$

– $m_A = 0$. Party $B$ accepts the contribution if and only if

$$\begin{aligned} & p_B(0,1)\left(1-\beta\right) \geq p_B(0,0) \Leftrightarrow \\ & \Delta \geq \frac{-2\alpha\beta\delta + \alpha\beta + 2\alpha\delta - \alpha + \beta\delta - \beta + \delta}{2\delta(1-\beta)} \end{aligned} \tag{A.9}$$

**Step 3. Thresholds Comparative Statics.** We conclude this proof by showing that the condition

$$\delta \geq \frac{\alpha}{1+2\alpha} \tag{A.10}$$

guarantees that (A.6) is greater or equal than (A.7) and that (A.8) is lower or equal than (A.9).

## A.2 Proof of Proposition 2

If the corporation funds both parties ($P_C = P$), it obtains an expected payoff of[21]

$$\pi_C(P) = 1 - 2c\ . \tag{A.11}$$

If the corporation funds party $A$ only ($P_C = A$), it obtains a payoff of

$$\pi_C(A) = p_A(1,0) - c\ . \tag{A.12}$$

If the corporation funds party $B$ only ($P_C = B$), it obtains a payoff of

$$\pi_C(B) = p_B(0,1) - c\ . \tag{A.13}$$

It is easy to see that (A.12) is increasing in $\Delta$, while (A.11) is constant. In the same way, (A.13) is decreasing in $\Delta$.

## A.3 Proof of Proposition 3

Existence follows directly from the game's structure and can be established by backward induction.

[21] We assume the corporation minimizes contributions when indifferent. If indifferent between two and one offer (or between one and zero), it always chooses the lower number. This assumption comes with no loss of generality.

In the voting stage, there is no strategic interaction: each voter has a finite and well-defined payoff.

In the acceptance stage, if only one party receives an offer, there is again no strategic interaction. If both parties are offered a contribution, they play a $2 \times 2$ simultaneous game with finite payoffs, which admits a unique equilibrium in pure strategies (the equilibrium is characterized by Proposition 1).

In the first stage, there is no strategic interaction and the corporation simply chooses the strategy that maximizes its expected payoff (which is finite and well-defined).

Uniqueness follows directly from our assumption on players' tie-breaking rules.

# B Empirics

## B.1 Tables, Descriptive Statistics and Additional Results

Table B.1: **Example Selection of Corporate donors in 2012 and the occupations of the contributors.**

| **Corporation Name** | **Donor Occupations** |
|---|---|
| Allegheny Technologies Inc | President Ceo, Retired, VP Admin and Finance, Chairman, Investment banker |
| Alliant Techsystems Inc | Self-employed, Dentist, Retired, Consultant, Engineer |
| Amazon.com Inc | Attorney, Venture investor, Banker, Partner at law firm, Executive |
| Apple Inc | Chairman of the board, Chairman, Executive, Retired, Investor |
| Bank of New York Mellon Corporation | Financial executive, Executive, CEO, Chairman, Director |

Table B.2: **Descriptive Statistics of Main Variables Used in the Analysis.**

| Statistic | Min | Max | Mean | Median | Std |
|---|---|---|---|---|---|
| Recipient's General Vote Share | 0.00 | 100.00 | 57.61 | 58.21 | 20.01 |
| Contribution Amount (USD) | 0 | 300000.00 | 1112.56 | 1000.00 | 3604.53 |
| Days to Nearest Death | -15165.00 | 11769.00 | 145.99 | 0.00 | 1973.47 |
| District's Democratic Vote Share | 17.00 | 95.00 | 52.46 | 51.50 | 12.93 |

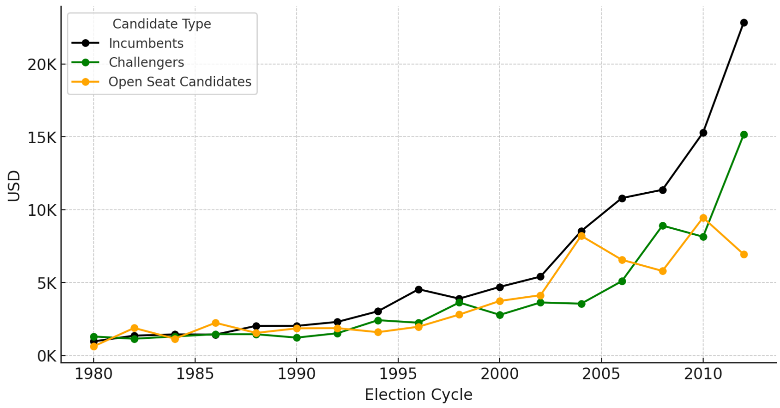



Figure B.1: **Average Corporate Contributions to U.S. Candidate by Election Cycle and Incumbency Status, 1980-2012.**
Note: Across cycles, the dataset includes varying numbers of candidates in each category. For example, in 2012, there were 191 incumbents, 368 challengers, and 79 open-seat candidates. Across all electoral cycles, the 55.27% of contributions' recipients are incumbents, 34.16% are challengers, and 10.57% are candidates running for an open seat.

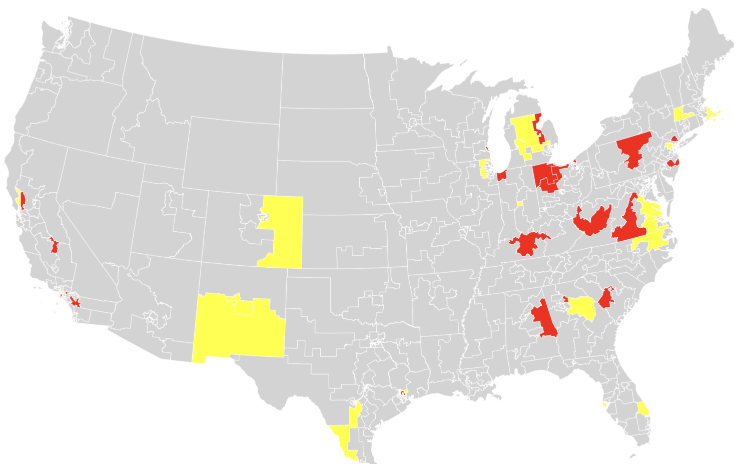


Figure B.2: **Treated and Control Districts.**
Note: This map highlights continental U.S. congressional districts where an incumbent died while in office. Districts with expected deaths are shown in yellow, while those with unexpected deaths are in red. If a district experienced both types of deaths, it is split into a left-yellow and right-red section. The base map corresponds to the 2018 congressional district boundaries. Expected deaths occurred in: NY07, IL21, FL18, NJ13, IL04, NM02, TX02, IL14, NY06, CA05, CO04, CO01, MI08, NJ11, TX28, MI03, MA01, NM01, NC01, MI04, GA10, VA01, MA09, CA37, IN07, VA04, CA12, FL13, NJ10. Unexpected deaths occurred in: IN01, KY07, WI04, VA05, LA08, NJ03, OH17, WV03, MI05, GA07, TX18, CA05, OH04, AL03, CA42, CA44, NY17, CA22, KY02, SC02, CA32, HI02, OH05, OH11, PA12. The 2018 map does not include the following districts due to redistricting: IL21 (expected), KY07 (unexpected), LA08 (unexpected), NJ13 (expected), OH17 (unexpected). HI02 (unexpected) is also not shown to improve readability, as Alaska and Hawaii are excluded from the map.

Table B.3: **Causes of Death of U.S. House Members (1974-2014)**

•

| Cause of Death | Expected Deaths | Unexpected Deaths |
|---|---|---|
| Cancer-related | 24 | 0 |
| Myelodysplastic syndrome | 2 | 0 |
| Unspecified natural causes | 2 | 0 |
| Natural causes | 1 | 0 |
| Complications from nutritional deficiencies | 1 | 0 |
| AIDS-complicated pneumonia | 1 | 0 |
| Heart attack | 0 | 13 |
| Plane crash | 0 | 3 |
| Post-surgery complications | 0 | 2 |
| Fall down stairs | 0 | 1 |
| Peptic ulcer | 0 | 1 |
| Cerebral thrombosis | 0 | 1 |
| Cerebral hemorrhage | 0 | 1 |
| Skiing accident | 0 | 1 |
| Thrombosis | 0 | 1 |
| Viral pneumonia (chickenpox) | 0 | 1 |
| **Total** | **31** | **24** |

Note: We classify deaths as expected vs. unexpected using ChatGPT 4o. We instruct the LLM to identify expected deaths as those happening after a sufficiently long ill.

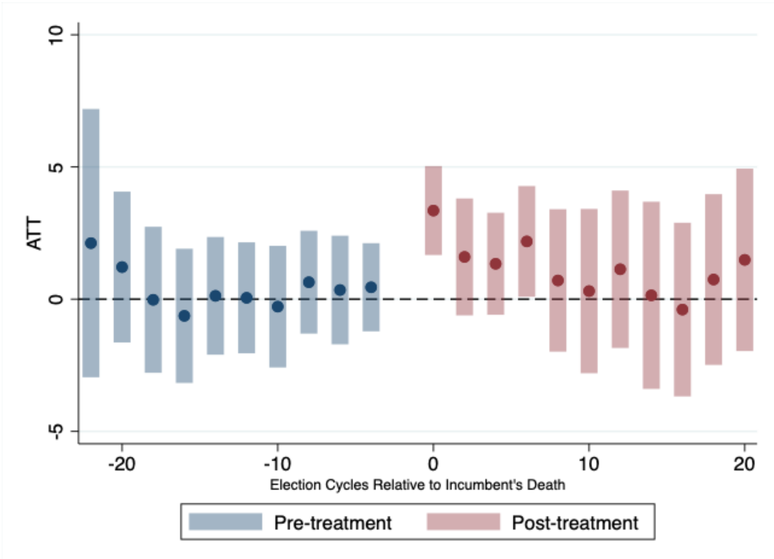



Figure B.3: **The Effect of an Incumbent's Death on** $TotalContributions'_{d,c}$ **(Callaway and Sant'Anna [2021]- Estimator, unexpected deaths only).**
Note: The figure reports the estimated coefficients for $\beta_r$ from equation (3.1) (for $T = -20, T' = 20$). Confidence Intervals are at the 99%. Observations from districts with expected deaths are out from the sample. Number of observations: 8028.

# Concluding Remarks

In this Thesis, I study the interaction between markets and politics in three different contexts.

The results of the three chapters highlight the importance of considering markets and politics as interconnected systems, where changes in one domain can influence the outcomes in the other.

Chapter 1 shows that public firms' optimal privatization policy strictly depends on the level of political connections.
Chapter 2 shows that changes in market concentration have political economy effects. When firms acquire market power, they also acquire political power, which they can use to influence elections and, in turn, the market structure.
Chapter 3 shows that the extent of closeness of elections determines firms' ability to sway elections through campaign contributions.

Table of Contents