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Transient turbulence in plane Couette flow

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Plane Couette flow, the flow between two parallel plates moving in opposite directions, belongs to the group of shear flows where turbulence occurs while the laminar profile is stable. Experimental and numerical studies show that at intermediate Reynolds numbers turbulence is transient and that the lifetimes are distributed exponentially. However, these studies have remained inconclusive about a divergence in lifetimes above a critical Reynolds number. The extensive numerical results for flow in a box of width 2π and length 8π presented here cover observation times up to 12 000 units and show that while the lifetimes increase rapidly with Reynolds number, they do not indicate a divergence and therefore no transition to persistent turbulence.

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Plane Couette flow (pCf), the flow driven by two parallel plates moving in opposite direction, belongs to a class of shear flows where transition to turbulence may be observed at flow speeds where the laminar profile is linearly stable against infinitesimal perturbations [1]. In these systems, which also include pressure driven plane Poiseuille and pipe flow, laminar and turbulent dynamics coexist for the same flow speed [2]. Linear stability of the laminar flow implies that a finite amplitude perturbation is needed to drive the system out of the laminar fixed point's basin of attraction and to trigger turbulence.

In the transition region the reverse process, whereby the turbulent flow relaminarizes without external stimulus or any noticeable precursor, has been seen both experimentally and numerically [3,4]. Therefore, the turbulent state must be associated not with a turbulent attractor but rather with a turbulent saddle. An important question is whether there is a critical Reynolds number above which the lifetimes diverge and the system undergoes a transition to an attractor [5]. Previous evidence in pCf indicated a divergence [3,6], but the reanalysis of data in [7] failed to reproduce the divergence. Pipe flow is another flow where the situation is undecided [7–12]. Beyond the immediate interest of the transition to turbulence in linearly stable shear flows, these studies may also contribute to our understanding of transiently chaotic systems in general [13].

The critical Reynolds number in pCf has long been studied experimentally. Ever present fluctuations and perturbations in the flow can trigger a "natural" transition without explicit external stimulus. Experiments such as [14] indicate that this happens near Re=370, with the usual definition of the Reynolds number as $\mathrm{Re}=U_0 h/\nu$ with U_0 is the half the velocity difference between the plates, h half the gap width and ν the kinematic viscosity of the fluid. However, since the "natural" transition depends on uncontrolled background fluctuations, experiments with reproducible finite amplitude perturbations were developed. Early experiments [15] using a jet injection perturbation give $\mathrm{Re}_c=370\pm 10$ above which turbulent spots are reported to be self-sustained for arbitrarily long times while for $\mathrm{Re}<\mathrm{Re}_c$ they decay in a short time. In order to capture the dependence on the amplitude Dauchot and Daviaud [16] measured the critical amplitude A_c of an injected jet as a function of Re. They found a divergent behavior $A_c(\mathrm{Re})\propto(\mathrm{Re}_c-\mathrm{Re})^{-\alpha}$ with $\alpha\in[0.3,0.8]$ and a critical value $\mathrm{Re}_c=325\pm 5$ below which no perturbation of any strength could trigger sustained turbulence.

Bottin and co-workers [3,6] then introduced a statistical approach that also removes dependencies on the spatial structure of the applied perturbation. They considered an ensemble of many experimental runs where the flow was perturbed by jet injection, or the Reynolds number of developed turbulence was suddenly reduced ("quenching experiments"). From several runs at the same Re, lifetime distributions were measured, whose characteristic decay time τ strongly grows with Re. They concluded that the lifetime varied like $\tau\propto(\mathrm{Re}_c-\mathrm{Re})^{-1}$ with a critical Reynolds number $\mathrm{Re}_c=323\pm 2$. Another approach to determine the critical Reynolds number was followed by Bottin and co-workers [17,18]. Adding a localized fixed perturbation to the flow (a wire or a small bead) allowed introduction of an additional control parameter. Studying the system's phase diagram in the limit of the embedded unperturbed pCf sustained turbulence was found above $\mathrm{Re}\approx 325$.

In summary, previous experimental results seem to indicate the existence of a critical Reynolds number, but they vary in the value between 320 and 370, and differ in the way this divergence is approached. Moreover, a reanalysis of the data, which takes finite time effects in the distribution of lifetimes into account, does not point to a divergence [7].

On the numerical side available lifetime data are very limited. Based on extrapolating median lifetimes measured in a small periodic box Schmiegel reports a critical Re between 290 and 325 [19]. An extended domain was later studied by Barkley and Tuckerman [20]. They observed periodically arranged turbulent stripes for Re>350 which begin to localize at Re<350 and whose detailed features depend on the size, aspect ratio and orientation of the computational domain [21], but did not explicitly study the long-term statistical behavior of these states.

Here we present data from direct numerical simulations of

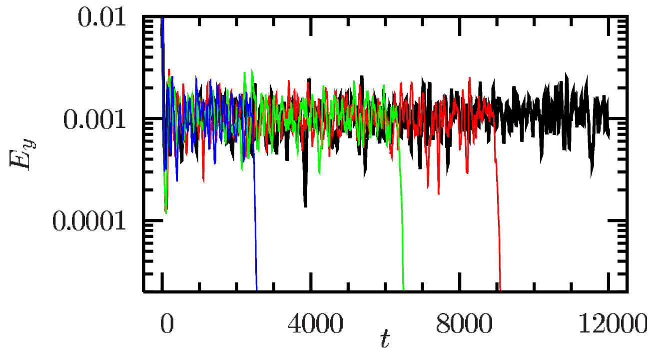

FIG. 1. (Color online) Evolution of four different but similar initial conditions. Plotted is the kinetic energy stored in the wall-normal velocity component E_y as a function of time. Note the sudden decay without prior indication. The chosen criterion for decay is $E_y < 10^{-7}$

pCf. Units are chosen such that half the gap width is $h=1$, half the velocity difference of the plates is $U_0=1$ and time is measured in units of h/U_0. Adopting periodic boundary conditions in downstream and spanwise direction, the flow domain is set to be 2 units high, L_x units long and L_z units wide. The Navier-Stokes equations $\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mathrm{Re}^{-1} \Delta \mathbf{u}$ were solved using a spectral Fourier-Chebyshev-τ scheme [22–24]. In particular, the vertical (wall-normal) component of velocity and vorticity are solved for, and the remaining components are reconstructed by incompressibility, according to the scheme extensively described and discussed in [23]. For time advancement, a semi-implicit integrator was used. Viscous dissipation was solved with an implicit second-order Crank-Nicholson scheme, while a second-order Runge-Kutta scheme was used for the nonlinear terms (for a similar integration method, see also [22,25,26]). Results discussed here are for a computational domain of size $(L_x, 2, L_z) = (8\pi, 2, 2\pi)$ and a grid resolution of $(N_x, N_y, N_z) = (128, 65, 64)$ points. For a few runs, we increased the resolution further to confirm the accuracy of our numerics, but even with the mainly used resolution the computational costs are enormous. The presented statistical data is based on following more than 5000 trajectories for a maximal integration time of $T_{\max} \geq 3000$ which required more than 15 CPU years on a 2.4 GHz AMD Opteron based cluster. In addition for part of this calculation we used a grid technology based distributed computing architecture to harvest results from many stand alone CPUs [27].

Following [28], characteristic lifetimes are extracted from the asymptotic behavior of lifetime distributions for long times: The distributions are obtained from integrating ensembles of initial conditions until they decay or until the maximal integration time is reached. Figure 1 shows the evolution of four different but very similar initial conditions consisting of a pair of counterrotating downstream oriented vortices and differing only by the realization of a low-energy three-dimensional noise applied to break translational invariance. As an indicator for the turbulent intensity and persistence we take the kinetic energy stored in the wall-normal velocity component v because, like in other cases, the decay first shows a reduction in transverse velocity fluctuations that are captured by v.

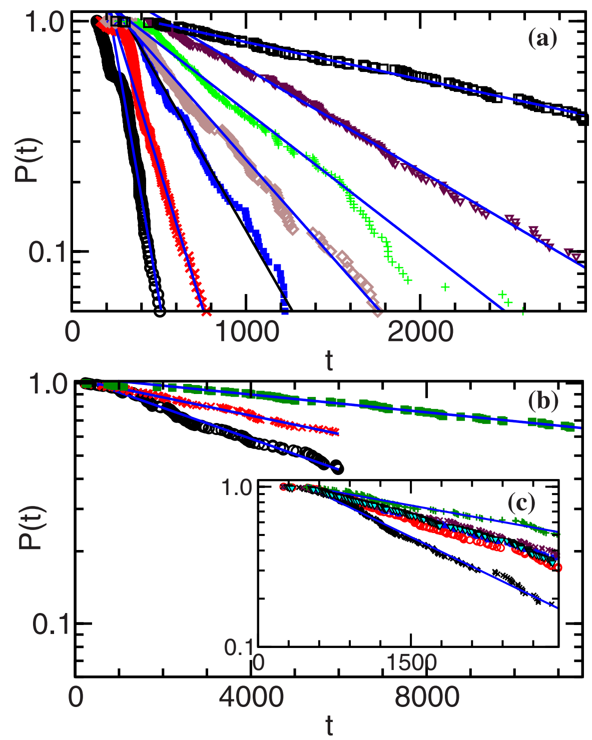

FIG. 2. (Color online) Probability $P(t)$ to still be turbulent after some time t as a function of t for different Re. Ordered by decreasing slope (from left to right): (a) Re=250, 280, 320, 340, 360, 380, 400; (b) Re=420, 430, 440. Each data set is compatible with an exponential variation indicated by the solid lines. (c) Resolution test: Four distributions for Re=400 together with Re=390 (lowest curve) and Re=410 (upper curve) data. With respect to the standard resolution the number of gridpoints is doubled in streamwise, spanwise or wall-normal direction.

Since a flow field only asymptotically reaches the laminar profile exactly, we define "decay" as reaching a situation where the perturbations are so small that the dynamics is described by the linearized equations and the further evolution follows an exponential drop off due to the linear stability of the base flow. Technically we introduces a cutoff threshold on the kinetic energy of the perturbation field's y-component ($E_y < 10^{-7}$). Once the chosen decay criterion is met turbulence never revived in long control runs. Thus, any perturbation may be characterized by a lifetime—the time it takes to reach a small target region around the laminar profile.

Now consider a set of several different but similar initial conditions. From their individual lifetimes one can estimate the probability $P(t)$ to still be turbulent after some time t. As for radioactive decay this probability should be exponential if the escape probability is constant and independent of when turbulence started. Thus, a chaotic saddle should give rise to exponential asymptotic tails $P(t) \propto \exp(-t/\tau)$ with a characteristic lifetime τ which is inversely proportional to the slope of $P(t)$ in a semilogarithmic representation. This is indeed confirmed by Fig. 2 where we show the survival probability $P(t)$ as a function of t for various different Reynolds numbers. The probability is estimated from ensembles consisting of 200 trajectories for each Re. For Re<420 these trajectories where followed up to a maximum time $T_{\max}=3000$

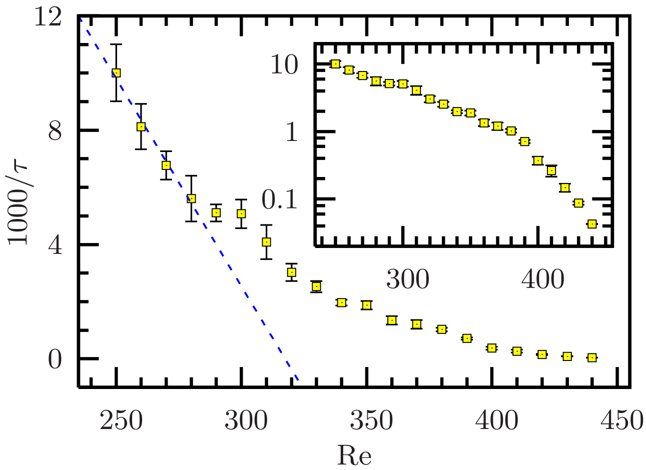

FIG. 3. (Color online) Inverse characteristic lifetime τ^{-1} as a function of Re on a linear (main) and a logarithmic scale (inset). The blue dashed line indicates a linear fit to the low Re part of the data.

[panel (a)], for larger Re, the time $T_{\max}$ was increased to 6000 and 12 000, respectively [panel (b)]. Each data set it fully compatible with an exponential variation as indicated by the exponential fits included as solid lines. To estimate statistical uncertainties, we chose three additional independent sets of initial conditions to compute $P(t)$ for Re=400. The results are shown in inset (c) together with the data for Reynolds numbers of 390 and 410. For each of the additional sets of runs the number of gridpoints was doubled in one direction. The comparison shows that statistical fluctuations due to the limited number of events are considerably smaller than effects from changing Re by 2.5%. Moreover, within the limits of statistical accuracy the slope of the curves does not depend on numerical resolution.

The slopes of the exponential tails in the survival probability give the inverse characteristic lifetimes $1/\tau$ shown in Fig. 3 as a function of Re both on a linear (main) and a logarithmic scale (inset). As discussed in the introduction, the strong increase in τ with Re led earlier studies [3,6] to suggest a divergent scaling like $\tau(\mathrm{Re}) \propto (\mathrm{Re}_c - \mathrm{Re})^{-1}$ with a critical value $\mathrm{Re}_c \approx 323$. If the lifetime diverges, its inverse $1/\tau$ should cross zero at the critical value. Our datapoints for the lowest Re are indeed compatible with the suggested scaling and a least-squares fit to the first four datapoints (blue dashed line in Fig. 3) gives a critical Re of about 325 which is surprisingly close to the value given in [3,6]. While the datapoints for low Re are thus quantitatively compatible with the suggested scaling, the data for large Re clearly deviate from it. There is no evidence for a divergence in τ. Spontaneous decay of turbulence can still be observed for Reynolds numbers much larger than the suggested “critical Re” estimated from fitting the low Re part of the data to a linear scaling law (cf. Fig. 1 which presents spontaneously decaying turbulence at Re=440).

In pipe flow the variation with Reynolds number was found to be compatible with an exponential variation in $\tau(\mathrm{Re})$ [7], while for the plane Couette system the lifetime increases clearly faster than exponentially. Note, however, that while a chaotic saddle scenario predicts exponential tails of individual lifetime distributions for fixed Re there is no theoretical prediction on how the characteristic lifetime scales with Re [29]. Thus, pipe flow and pCf show a very similar behavior: as Re increases the characteristic lifetime rapidly grows but is does not follow a divergent $\tau \propto (\mathrm{Re}_c - \mathrm{Re})^{-1}$ scaling. Instead in a linear representation of the inverse characteristic lifetime, the data bends toward larger Re and appears to only asymptotically approach zero. Consequently there is no evidence for a divergence that would mark the transition from a chaotic saddle to a permanent chaotic attractor, at least not close to the Reynolds numbers where critical values have been reported previously. This conclusion is further supported by noting that there is a finite probability to observe spontaneous decay at Re much larger than the previously reported critical value of Re≈325 The observation of exponentially varying survival probabilities for Re=440 evidently rule out a transition to a permanent attractor for any Re≤440.

Our results clearly indicate the absence of a transition to permanent turbulence close to Re≈325. However, the conclusions are based on considering a periodically continued domain that does not capture the full spatiotemporal structure of a turbulent spot observed in large experimental setups. There is no coexistence of turbulent and laminar domains and no boundary between these regions. Such an approach is encouraged by the observation that turbulent spots decay from within, and not by collapsing boundary fronts [30]. While the example of pipe flow however shows that lifetimes may scale with the turbulent domain size [28] that example also demonstrates that lifetimes increase with the size of the domain but do not diverge at some finite size. For pCf domain size effects have been studied in [21]. In domains of large spanwise extension both localized turbulent spots and periodic stripes can be found in addition to uniform turbulence. On reducing Re turbulent bands suddenly disappear without first shrinking in size. This suggests that the decay of turbulent spots might be due to their internal dynamics instead of the interaction with the contiguous laminar bands. However, turbulence in one part of the domain may trigger turbulence in a close-by region which previously laminarized. Thus, full relaminarization of an extended domain requires a simultaneous decay in all regions suggesting lifetimes that increase with the size of the domain and the number of turbulent bands. While the minimal Re where turbulent transients are first observed is reported to decrease with increasing spanwise extension of the domain [21] a statistical analysis of lifetimes within full direct numerical simulations of extended domains is currently out of reach. However, a low dimensional approximation is available that has been related in its phenomenology to spatially extended pCf. Lagha and Manneville considered a 2.5 dimensional model using full resolution in the *x*-*z*-plane but a low-order truncation in the wall-normal direction to study spatiotemporal features of pCf [31]. Lifetime studies were carried out in a domain slightly longer and much wider than the one studied here. The authors found a deviation from an exponential variation in $\tau(\mathrm{Re})$ used in [7] as a heuristic fit to pipe data and concluded based on the deviation that there is a divergence at a critical Re_c. However, in view of the absence of a theoretical prediction for the variation in τ with Re we note that a deviation from an exponential scaling law alone does not allow to conclude that there is a divergence. Instead the functional form of their τ-Re data (cf. Fig. 6 in [31]) is very similar to the data presented in this study: The characteristic lifetime grows slightly faster than exponentially with Re but appears not to diverge. Clearly the data is not compatible with the divergent $\tau(\mathrm{Re}) \propto (\mathrm{Re}_c - \mathrm{Re})^{-1}$ scaling proposed in previous studies. Since also a reanalysis of available experimental pCf data in a large box confirms their compatibility with a nondivergent scaling [7], we conclude that the features of the periodically continued box discussed in this Rapid Communication carry over to extended systems and that the absence of a critical Re is a general feature of pCf.

Together with results from pipe flow, both from numerical studies in small periodic pipe segments and laboratory experiments in long pipes and pCf studies in large systems our results suggest that transience is a common property for turbulence in linearly stable systems.

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