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Federigo Enriques (1871–1946) and the Training of Mathematics Teachers in Italy

Livia Giacardi

What has the greatest influence on the effectiveness of teaching, more than differences in methods or the guidelines of programs, is the skilfulness of the teachers: their mentality, their ability to communicate, the passion they bring to the subjects they teach, the breadth of interests that make it possible for them to put themselves in the students' place and feel with them.

Federigo Enriques[1]

Abstract This essay will illustrate Federico Enriques' vast, multifaceted efforts to improve the preparation of mathematics teachers, situating them in their historic context and within the framework of the cultural project that formed the basis of his whole scientific output. The first part of the essay is dedicated to a brief presentation of the principal steps in the history of Italy's *Scuole di Magistero* (teacher training schools), with reference to the most significant legislative measures, to the contribution of teachers' associations, and to debates among mathematicians. The second part will show how Enriques' cultural project for the creation of a scientific *humanitas*, which was rooted in the philosophy and history of science, developed gradually during his years in Bologna, and how this was reflected in

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[1] See [53, p. 188]: "Più che le differenze dei metodi o le indicazioni dei programmi influisce sull'efficacia dell'insegnamento il valore degli insegnanti: la loro mentalità, la comunicativa, la passione che portano nelle cose insegnate, la larghezza degli interessi che li fa capaci di mettersi al posto degli allievi e di sentire con essi."

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his vision of mathematics teaching. The influence of Felix Klein will also be highlighted. The third part examines Enriques' involvement in teacher training and the various strategies he adopted, and frames his initiatives and methodological assumptions within his cultural program. Finally, three appendices containing previously unpublished letters and documents conclude the essay.

1 Introduction

Immediately following the constitution of the Kingdom of Italy and the establishment of an educational system at a national level, the Italian political class, which included many high-level mathematicians, understood the importance of having corps of adequately trained teachers in order to guarantee the formation of the future ruling class of the new nation. The problem was urgent, given the fact that at the time people without a degree were permitted to teach. It was only in 1906 that the legislation was approved regarding the legal status of teachers, making it official that only those who had won a competition could teach, and that a degree was required for admission to the competition [GU 1906, 106, p. 2085]; it was not until 1914 [GU 1914, 174, pp. 4086–4101] that mathematics teachers were placed on an equal footing with teachers of Italian, rectifying the inequality that had existed in the system since the Casati legislation (1859). Among those who were personally involved in the effort to improve Italian schools and in training teachers for various levels of schools were many members of the well-known Italian school of algebraic geometry. Many factors led them to embrace this commitment, although for each of these mathematicians the various factors had different overtones. First of all, it was not incidental that geometry is the discipline that best makes it possible to bring into focus the problems of methods that are inherent in mathematics teaching, and to clarify the delicate relationship between *formation* and *information*, which has always played a particular role in education. Secondly, of indubitable importance was the influence of Felix Klein, a mathematician who was not only active in advanced research, but was also involved in the reform of secondary and university mathematics teaching in Germany, and who had been president of the International Commission on Mathematical Instruction since 1908. Klein's influence (see [71, 75, 117]), which can be seen in the trends and methods of research – it is sufficient to think of the number of young mathematicians who gathered around him to perfect their scientific training – also affected the way that mathematics teaching and curriculum reform was conceived. One sign of this is the translation into Italian of Klein's *Erlanger Programm*,[2] as well as some of his other writings which were more specifically concerned with didactics, in particular his 1895

[2]See [86]. The translation was by Gino Fano at the suggestion of Corrado Segre.

Vorträge über ausgewählte Fragen der Elementargeometrie translated by Francesco Giudice with the title *Conferenze sopra alcune questioni di geometria elementare*,[3] and the 1895 lecture entitled *Über Arithmetisirung der Mathematik*, translated by Salvatore Pincherle with the title *Sullo spirito aritmetico nella matematica* [89]. Another important reason why many Italian algebraic geometers were committed to teacher training is that they belonged to a school that shared a well-defined program for research and a common vision of mathematics and its teaching, a strong school that was at the forefront of the international scene at the turn of the century. Finally, political and social motivations cannot be overlooked: the mathematicians working in the period immediately following Italian unity were animated by the strong spirit of the Risorgimento, and believed that the creation of national identity depended on an efficient school system and adequate preparation of an educated ruling class.

1.1 The Scuole di Magistero

The *Scuole di Magistero*, or teacher training institutes, were established by Minister of Public Instruction Ruggero Bonghi in 1875 to respond to the need to train future teachers and thus guarantee a higher level of secondary schools, and they survived with successive modifications until 1920, when they were abolished by the minister Benedetto Croce. Their history was especially troubled, as shown by the great number of decrees that concerned them.[4]

The initial purpose of the Scuole di Magistero was fundamentally ambiguous, emphasising both research and professional teacher training as can be seen from articles 32 and 33 of the Royal Decree of 2 November 1875 (R. Bonghi):

> The program of the Scuole di Magistero consists, in addition to the studies required for the corresponding degree, in special exercises aimed at instilling in the students an aptitude for research and the original exposition of that discipline that they wish to profess . . . [and students] will take a course on the limits and methods of teaching of the sciences instituted by the Minister . . .[5]

We need only consider the fact, for example, that Francesco Faà di Bruno, who was responsible for teaching mathematics in the Scuole di Magistero of the

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[3] See [88]. It was Gino Loria who was behind the translation; see the letter from Loria to Klein dated Genova, 22 July 1895, SUB Göttingen, *F. Klein* 10.

[4] All of the legislative measures cited here can be consulted in the section *Teacher Training* of [76].

[5] See [GU 1875, 255, p. 6835]: "Il corso delle scuole di Magistero consiste, oltre che negli studi richiesti per la corrispondente laurea, in esercitazioni speciali dirette a produrre negli studenti l'attitudine alla ricerca e alla esposizione originale e propria di quella disciplina che vogliono professare . . . Di più egli seguirà un corso sui limiti e sui metodi dell'insegnamento delle scienze instituito dal Ministro."

University of Torino, in the year 1882–1883 dealt only with the theory of elliptic functions.

In order to clarify the aims of the Scuole di Magistero and to improve their effectiveness, a commission was created in 1885 by the *Consiglio Superiore della Pubblica Istruzione* (High Council for Public Instruction). Its members were Luigi Cremona, Eugenio Beltrami and Sebastiano Richiardi. In its report, the commission placed particular emphasis on the “practical preparation for secondary teaching,” which had up to that time been neglected in favour of a specifically scientific preparation, and insisted on the need for practical training dedicated to the study of the foundations of mathematics and critical analysis of the methods.[6] These suggestions were reflected in article 2 of the Royal Decree of 30 December 1888 (P. Boselli), which states that “The Scuola di Magistero is aimed at the practical training for secondary teaching” and underlines the importance of a preparation for teaching by means of practical training that “consists in the examination of the postulates of science, in written works, and in lessons by students on subjects chosen by them at the suggestion of their professor and with his approval. The discussion of the didactic rules to be applied to the aforementioned subject in secondary teaching will be included.”[7] Further, in article 4, a certain emphasis was given to mathematics by assigning to it four years of courses, while only two years were assigned to other scientific disciplines.

The nature of the lessons were further defined by the historian and statesman Pasquale Villari in the Royal Decree of 29 November 1891, which underlined from the beginning that the primary aim of the courses (of a minimum of two years) was to “render the students expert in the art of teaching the different disciplines” in the various kinds of secondary schools. In particular, article 6 regarded lectures of a didactic nature:

> In these, the professor should: 1. set forth the method to be used in Secondary Schools for teaching the subject assigned to him, assessing its extents and limits; 2. make the students perform appropriate practical exercises that serve to accustom them to applying the method being taught. Among these practical exercises there are also actual lessons given in the Scuole di Magistero, and, when possible, in a Secondary School as well; 3. present and examine the best textbooks for Secondary Schools.[8]

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[6]See *Sull’istruzione secondaria classica. Notizie e documenti presentati al Parlamento Nazionale dal Ministro della Pubblica Istruzione Paolo Boselli* (Roma, Sinimberghi, 1889), pp. 266–269 and 273–274.

[7]See [GU 1889, p. 219]: “La Scuola di Magistero ha per fine la preparazione pratica all’insegnamento secondario” . . . “consistono nell’esame di postulati della scienza, in lavori scritti e in lezioni degli studenti sopra soggetti scelti da loro con approvazione del professore, indicati da questo. Vi sarà compresa la discussione delle regole didattiche da applicarsi alle suddette materie nell’insegnamento secondario.”

[8]See [GU 1892, p. 80]: “In esse il professore dovrà quindi: 1. esporre il metodo da seguirsi nelle Scuole secondarie per l’insegnamento della materia a lui affidata, determinandone l’estensione ed i limiti, 2. fare eseguire agli alunni opportune esercitazioni che valgano ad abituarli alla applicazione del metodo insegnato. Fra queste esercitazioni vi sono anche saggi di lezioni date nelle Scuole di

It also recommended holding lectures on education in general, to be given by only those who had acquired long years of practical experience in secondary teaching.

In many cases, the Scuole di Magistero were completely inadequate for reliably addressing the problem of teacher training. There were many reasons for this: above all, the professors who taught there were the same ones who taught institutional courses at university, and because these had, with rare exceptions, no experience in secondary teaching, they were unprepared to address questions about pedagogy and method. Furthermore, supporting structures (libraries, laboratories, etc.) and teaching materials were practically nonexistent, the number of assigned course hours was inadequate, and there was scant funding.

There are various testimonies to these shortcomings by both pedagogists and mathematicians. For example, Saverio De Dominicis, professor of pedagogy at the University of Pavia, wrote in 1882:

> The Scuola di Magistero does not exist in many faculties, although these create professors; where it exists it simply provides an illusion, because it has no distinct purpose; it is always, even the blind can see it, incomplete. . . . The Scuola di Magistero should come after the specialised studies in this or that faculty; . . . it should be, not the Scuola di Magistero of one faculty or another, but the Scuola di Magistero for secondary teaching. . . . This would be a serious school: a school that would oblige various professors to ponder the problems of pedagogy . . . It is the Scuola di Magistero, not the faculty, that can create good teachers: the faculty has always created and will continue to create erudite young people, but erudite young people are not professors.[9]

In his report to the Senate on the reorganisation of university teaching, Luigi Cremona, at that time a professor at the University of Rome, wrote:

> The great interest that the State has in the formation of qualified teachers demands that these be trained in only a few centres . . . under the guidance of men who are not only scientists but also masters of the art of teaching. No country has an abundance of such men, and we fewer than others, because for a long time no one seems to have cared about this. It is precisely in this that the regulations of 1875 were mistaken, relying too much on the pedagogical-didactic training of our professors.[10]

magistero, e, quando si possa, anche in una Scuola secondaria; 3. far conoscere ed esaminare i migliori libri di testo per le Scuole secondarie."

[9]See [18, pp. 184–185]: "La scuola di magistero dunque in molte facoltà, che pure creano professori, manca; dove trovasi è una semplice illusione, perché non ha scopo a sé e distinto; sempre, anche ad essere ciechi, è incompleta. . . . La scuola di magistero . . . dovrebbe essere non la scuola di magistero di questa o di quella facoltà, ma la scuola di magistero per l'insegnamento secondario . . . scuola seria sarebbe questa; scuola che obbligherebbe professori vari a ponderare i problemi pedagogici . . . È la scuola di magistero, non la facoltà, che può fare de' bravi insegnanti: la facoltà ha fatto e farà sempre de' giovani dotti, ma i giovani dotti non sono i professori."

[10]See [16, p. 85]: "L'alto interesse che lo Stato ha per la formazione di valenti maestri, esige che questi siano educati soltanto in pochi centri . . . sotto la direzione d'uomini che non siano solo scienziati, ma anche maestri nelle arti educative. D'uomini siffatti non v'à abbondanza in alcun paese, e in casa nostra meno che altrove, perché da gran tempo pare che nessuno se n'occupi In ciò appunto errarono i regolamenti del 1875, facendo soverchia fidanza con la scienza pedagogico-didattica de' nostri professori."

The lack of interaction between the university world and secondary teachers was a further reason why the Scuole di Magistero were inadequate. This chasm was highlighted by Gino Fano in 1894, in an article written upon his return from a year of professional development spent in Göttingen with Klein. Fano pointed out the initiatives promoted by Klein to address this problem:

> ...each year during the Easter holidays the secondary school teachers are invited to convene, those of the eastern provinces in Berlin, those in the western provinces in Göttingen; and there they stay for about fifteen days, in contact with university teachers. Lectures and lessons make it possible on one hand for the numerous participants to stay up-to-date with the many, many advancements that are continually being made by science, while on the other, the university professors as well have a way to understand fully the needs and desires of the secondary school teachers.[11]

From the very beginning, the problem of the professional training of teachers was one of the most hotly debated topics in the *Associazione Mathesis*, an association of teachers of mathematics founded in Torino in 1895–1896 by Rodolfo Bettazzi, Aurelio Lugli and Francesco Giudice with the aim of "improving the school and the training of teachers, from the points of view of science and didactics."[12] From the association's very first congress, held in Torino in 1898, the Mathesis Association sponsored an enquiry among the members regarding the theme "Modifications to be introduced in the regulations of university mathematical studies, intended to produce good secondary teachers." The theme was taken up again in subsequent meetings and congresses, and different proposals were formulated.[13]

Within the multiplicity of presentations, it is possible to identify two main lines of thought. Some, such as Salvatore Pincherle, proposed the separation of curricula, and more precisely, the institution, after the first two years of university, of a special course leading to a degree in education (*laurea didattica*) to be attended by all those who intended to pursue a career in secondary teaching; this was to be distinct from the degree in pure mathematics, which was instead to be sought by those who intended to pursue a career in research. According to Pincherle, the future teachers should, in a biennial course dedicated to mathematical methodology, inspect and analyse in depth all of the chapters of elementary mathematics [108, p. 86]. This proposal was shared by Guido Castelnuovo, as well as by Enriques, as we shall see in greater depth in a moment. What Pincherle intended can be clearly seen in the classes he held at the Scuole di Magistero of the University of Bologna from 1899–1900 to 1920–1921. All of his annual reports show the importance he attached to

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[11] See [67, pp. 181–182]: "ogni anno nelle vacanze Pasquali gli insegnanti delle scuole secondarie sono invitati a riunirsi, quelli delle province orientali a Berlino, quelli delle province occidentali a Gottinga; e lì rimangono circa quindici giorni, a contatto degli insegnanti universitari. Conferenze e lezioni permettono da un lato ai numerosi convenuti di tenersi al corrente dei tanti e tanti progressi che la scienza va continuamente facendo, mentre d'altra parte anche gli insegnanti di Università hanno modo di rendersi conto esatto dei bisogni e dei desideri dei primi."

[12] See "Statuto dell'Associazione," *Bollettino dell'Associazione Mathesis*, 1, 1896–1897, in *Periodico di Matematica*, 1896, p. 161.

[13] See [72, 101], and the section *Mathesis' Congresses* in [76].

elementary mathematics from an advanced standpoint, and his growing interest in questions regarding the principles and foundations of mathematics, in all probability a result of time spent with Enriques, who at that time was teaching in Bologna.[14]

Others, such as Alessandro Padoa, supported by Gino Loria and Giuseppe Peano, disapproved of the separation into two different curricula, and instead believed that it was urgent to strengthen the Scuole di Magistero. In particular, they proposed instituting, in addition to an obligatory period of practice teaching in a secondary school, a two-year course of Mathematical Methodology in place of the didactic lectures in the Scuole di Magistero, which would make it possible to address not only topics of arithmetic, algebra and geometry useful for the future teacher, but also include an examination of teaching methods, an analysis of school textbooks, as well as show the educational usefulness of the history of mathematics and mathematical games [96]. Loria and Padoa wrote that the history of mathematics should permeate the entire program, aiming above all to reconstruct the various phases of development of each theory, as well as to render the subject "less arid and more attractive" [96, pp. 4, 6]:

> The new university course we are suggesting would serve, in our opinion, to fill the deplorable abyss that separates university teaching from secondary teaching today, . . . which F. Klein has recently referred to as "a system of double forgetting" [n.b. *doppelte Diskontinuität*]: the university student's forgetting what he studied in secondary school, and the secondary school teacher's forgetting all that he studied while he was at university.[15]

References to Klein emerge in all the Mathesis congresses, and are an index of the influence he exerted in the Italian debates, an influence that can be also perceived at the base of the project for the *Enciclopedia delle matematiche elementari* presented by Luigi Berzolari and Roberto Bonola during the congress in Padua in 1909. Intended for mathematics teachers as well as the students of the Scuole di Magistero, the encyclopaedia was aimed at addressing elementary mathematics from an advanced standpoint as well as contain suitable remarks regarding the history of mathematics and questions of education.[16]

When the Minister Croce abolished the Scuole di Magistero with the Royal Decree of 8 October 1920 [BUMPI 1920, p. 2064], some of the most vigorous opposition came from the Mathesis Association and the two members of the Italian school of geometry, Loria and Fano. In a lecture to the Liguria section of the Mathesis Association, Loria expressed indignation for this "sudden and violent measure," saying that the Scuole di Magistero represented "a bridge, the only

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[14] ASUB, *Scuole di Magistero* (pos. 53/b), busta 3 (1880–1921).

[15] See [96, pp. 3–4]: "Il nuovo corso universitario da noi suggerito servirebbe, a parer nostro, a colmare il deplorevole abisso che oggi separa l'insegnamento universitario dall'insegnamento secondario, la cui esistenza venne segnalata da uno di noi sin dal 1898 e che F. Klein ha recentemente designato come 'sistema del duplice oblio': oblio da parte dello studente universitario di quanto studiò nelle scuole secondarie, oblio dell'insegnante secondario di tutto quello che lo occupò mentre trovavasi all'università."

[16] For more about this, see the article by Erika Luciano in this present volume.

one that exists between upper-level and middle-level teaching." He criticised the identification of scientific training with educational training, the lack of interest in questions of methodology, and the fact that "future teachers were not put in front of school students in the way the future health worker is put in contact with human suffering" [95, p. 163]. During the 1921 Mathesis congress in Naples, Fano formulated an item for the agenda in which he asked for "the reinstatement of the Scuole di Magistero for mathematics, in a broader and more comprehensive form than the previous one." Convinced that "*knowing more* than what you teach is worthless, if this *more* does not make you know better what should be taught," he energetically proposed the establishment of courses of Elementary Mathematics from an Advanced Standpoint, with an emphasis on the historical, critical, methodological, and didactical aspects, citing the lessons of Corrado Segre and Enriques as examples. He also invited the faculties to accept as dissertations for degree theses in complementary mathematics (*matematiche complementari*), that is concerning those sectors of mathematics more strictly connected to elementary mathematics, and urged his colleagues to establish, without awaiting ministerial decrees, practice teaching programs in secondary schools for the future teachers [68, pp. 103, 109].

The proposals were accepted at least in part by the Minister for Public Instruction Orso Mario Corbino, who in 1921 established "combined" degrees (*lauree miste*) in physical and mathematical sciences [BUMPI 1922, p. 22] aimed at qualifying young people to teach scientific subjects in secondary schools, and in 1922 instituted a course in complementary mathematics, accompanied by didactic and methodological exercises [BUMPI 1922, p. 349].

2 The Emergence of Enriques' Cultural Project and the Project's Effects on Mathematics Education

2.1 *The Teaching of Projective Geometry in Bologna*

After earning his degree at the Scuola Normale di Pisa in 1891, in 1892 Enriques obtained a Lavagna scholarship and although he had hoped to study in Torino with Corrado Segre,[17] he was sent to the University of Rome. This was where the extraordinary friendship with Guido Castelnuovo was born, a friendship that would last for the rest of his life and lead to the publication of the well-known works on algebraic surfaces.[18] However, the young Enriques went to Torino anyway in

[17] See Enriques to Castelnuovo, s.l. 6 November 1892, in [8, p. 3].

[18] For Enriques' contribution to algebraic geometry, and related bibliography see the paper by P. Gario and C. Ciliberto in this present volume.

November 1892, staying there for some weeks and then returning there a year later, in November 1893, at the end of a year spent in Rome perfecting his studies, in hopes of becoming an assistant to Luigi Berzolari and thus being able to work with Segre.[19] The months in Torino between November 1893 and January 1894 were very intense for his scientific research, and stimulating for his reflections on the foundations of geometry. It should be recalled that in 1889 Segre had been behind Mario Pieri's translation of K. G. Staudt's *Geometrie der Lage*, as well as Fano's 1890 translation of Klein's *Erlanger Programm*, and had urged Fano and Federico Amodeo to study the foundations of projective geometry. In contrast to Segre, who left physical or philosophical aspects of the problem aside,[20] Enriques was attracted by these very aspects, and explicitly said as much in a 1894 paper on the foundations of projective geometry:

> The route followed by them [Fano and Amodeo] is quite different from that we intend to take, especially in that, while the two esteemed authors propose to establish an arbitrary system of hypotheses that is capable of defining a linear space to which the results of ordinary geometry be applicable, here we will seek to establish the postulates deduced from experimental intuition of the space that appear to be the simplest for defining the object of projective geometry.[21]

In a note he added, "It only seems to us that geometry's experimental origins must not be forgotten in the search for the hypothesis on which it is founded."[22]

Enriques' interest in problems connected to mathematics teaching was born in close connection to his philosophical, historical, and interdisciplinary interests, and in particular, to research into the foundations of geometry, stimulated by the course in projective geometry at the University of Bologna, which, thanks to a series of lucky events, he was assigned to teach on 16 January 1894.[23] A month after the course began, Enriques wrote to Castelnuovo about the difficulty of reconciling the need for rigour with that of intuition in his lessons:

> . . . whether I already have or will yet sin in aiming too high in the course, depends on the fact that I don't yet have an adequate idea of the difficulty that young people run up against. I can only realise it during the lesson, as I explain, when by then the order of the topics to cover is fixed and the notes written; but I believe I compensate for the difficulties of some

[19] See the letters from Enriques to Castelnuovo in [8, pp. 39 e 44].

[20] See for example [118, p. 61].

[21] See [21, p. 551]: "L'indirizzo da essi *[Fano e Amodeo]* seguito è alquanto diverso da quello a cui noi intendiamo attenerci, specialmente per ciò che, mentre i due egregi autori si propongono di stabilire un qualunque sistema di ipotesi capace di definire uno spazio lineare al quale siano applicabili i resultati dell'ordinaria geometria, noi cerchiamo qui di stabilire i postulati desunti dall'intuizione sperimentale dello spazio che si presentano più semplici per definire l'oggetto della geometria proiettiva."

[22] See [21, p. 551]: "Ci sembra soltanto che l'origine sperimentale della geometria non debba essere dimenticata nella ricerca delle ipotesi su cui essa è fondata." See also [1, pp. 391–401].

[23] For the reconstruction of the events based also on archival documents, see [105, pp. 72–84].

> points by insisting on them vigorously, since it is precisely on those that it occurs to me to be more energetic in my explanation.[24]

The first steps in Enriques' path towards the research on the psychological and physiological origin of the postulates of geometry are found in the lessons in higher geometry that he gave at the invitation of several students in the year 1894–1895,[25] and which were collected in the lithograph printed that same year, entitled *Conferenze di geometria tenute nella R. Università di Bologna. Fondamenti di una geometria iperspaziale.* The discussion is preceded by an introduction, four aspects of which we want to underline here, because they will be examined in depth and clarified by Enriques in his later works. First of all, Enriques affirms the importance of the history of mathematics. Second, he reflects on the importance of the study of the foundations, whose value derives from the fact that "in mathematics every step forward has drawn attention back to an analysis of the foundations and, vice versa, such an analysis has often resulted in new and important concepts that made it possible to extend known results to a more general area" [22, p. 1].[26] The third aspect is that he underlines the importance of comparing mathematics with other sciences, because that is the only way to ascertain the true significance of the scientific importance of mathematical research, in keeping with the conviction that science is "an organic whole" [22, p. 2]. The fourth and final aspect is that he addresses the problem of what geometry is in order to arrive, at the end of the introduction, at an explanation of what is meant by abstract geometry, thus tying in with the guiding thread that Segre had provided for algebraic geometry:

> Bearing in mind from the very beginning the extension that we want to give to the results obtained in a given field by applying them to other fields, it avails us to consider the fundamental elements of Geometry as objects of an abstract nature connected by purely

[24] See Enriques to Castelnuovo, Bologna (?) 22 February 1894 [8, p. 77]: "... se io ho peccato o peccherò di troppa elevatezza del corso, ciò dipende da che non ho ancora un'idea adeguata delle difficoltà che incontrano i giovani. Io me ne accorgo soltanto nella lezione, spiegando, quando ormai è fissato l'ordine delle cose da svolgere e sono scritti gli appunti; ma credo di compensare le difficoltà di alcuni punti con una vibrata insistenza, poiché su quelli appunto mi avviene di animarmi di più nell'esposizione."

[25] See Enriques to Castelnuovo, 23 November 1894 [8, p. 151]: "Some young people have asked me to give a class in Higher Geometry: I am not against the idea of satisfying them in part with a series of weekly lectures that I will, however, only begin later (after January). Just in case, I'll tell you my plan for these: they would be concerned with a general principle that completes that of Klein (Programm) in order to encompass ideas about various other kinds of research ... which for now escapes it, at least directly" ("Alcuni giovani mi chiedono che faccia un corso di G*[eometria]* Sup*[eriore]*: non sono alieno dall'idea di contentarli in parte con un seguito di conferenze settimanali che però comincerei solo più tardi (dopo Gennaio). Ti dirò in caso il piano di queste: s'informerebbero ad un principio generale che completa quello di Klein (Programm) per far rientrare in quell'ordine d'idee vari altri tipi di ricerche ... che ad esso sfuggono, almeno direttamente").

[26] See [22, p. 1]: "... nella Matematica ogni passo avanti ha richiamato l'attenzione all'analisi dei fondamenti, e viceversa da una tale analisi sono scaturiti spesso concetti nuovi ed importanti che hanno permesso di estendere i resultati noti ad un campo più generale."

> logical relations, and in this sense conceive the science founded as an *Abstract Geometry*. This way of thinking, to which we are naturally led by the previous observations, on the other hand makes no difference in the mathematical development of Geometry.
>
> The importance that we attribute to Abstract Geometry is not (as may be believed) in opposition to the importance attributed to intuition: rather, it lies in the fact that *Abstract Geometry can be interpreted in infinite ways as a concrete (intuitive) Geometry by fixing the nature of its elements: so in that way Geometry can draw assistance in its development from infinite divers forms of intuition.*[27]

However, Enriques – as Castelnuovo observed[28] – also affirmed that in analysing the genesis of the postulates of geometry it is useful to take into account the psychological criteria, and to conduct an investigation of the sensations and experiences that led to the formulation of those postulates. This kind of investigation would carry Enriques to study German physiological psychology in 1896, and would have a systematic presentation in his 1906 publication *Problemi della scienza*.

The correspondence between Enriques and Castelnuovo makes it possible to retrace his steps. In January 1896, he began to study biology;[29] in February, he undertook the study of the physiology of cells;[30] in May that same year he gathered information on the studies in psychology and physiology of Hermann von Helmholtz, Ewald Hering, Ernst Mach, and above all the German psychologist and physiologist Wilhelm Wundt. He tried unsuccessfully to involve his friend in the discussion:

> While the mathematical questions are sleeping until a better day, I have been occupied for several days with a high question that only takes its pretext from mathematics . . . It is the 'philosophical problem of space'. Books of psychology and logic, of physiology and of comparative psychology, of critique of knowledge, etc., all cross my desk, where I savour them with sensuous delight in the attempt to extract the essences that concern my problem. . . . Since included in my program is the question of the genesis of the concept of space on the basis of physiological psychology (especially from the eye and the sense of touch) of Helm[h]oltz, Wundt, etc.[31]

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[27] See [22, pp. 9–10]: "Tenendo di mira fin da principio la estensione che vogliam dare ai resultati ottenuti in un dato campo applicandoli ad altri campi, ci converrà considerare gli elementi fondamentali della Geometria come enti di natura astratta legati da relazioni puramente logiche e concepire in questo senso la scienza fondata come una Geometria astratta. Tale modo di considerare a cui si è naturalmente condotti dalle precedenti osservazioni, è d'altronde indifferente nello sviluppo matematico della Geometria.
L'importanza che attribuiamo alla Geometria astratta non è (come si potrebbe credere) da contrapporsi all'importanza attribuita all'intuizione: essa sta invece nel fatto che la Geometria astratta si può interpretare in infiniti modi come una Geometria concreta (intuitiva) fissando la natura dei suoi elementi: sicché in tal modo la Geometria può trarre aiuto nel suo sviluppo da infinite forme diverse d'intuizione."

[28] See [14, p. 7] and the letter of Enriques to Castelnuovo dated 4 May 1896 [8, p. 261].

[29] See Enriques to Castelnuovo, Firenze 19 January 1896 [8, p. 237].

[30] See Enriques a Castelnuovo, Firenze 9 February 1896 [8, p. 246].

[31] See Enriques to Castelnuovo, s. l., 4 May 1896 [8, pp. 260–261]: "Mentre le questioni matematiche sonnecchiano fino al miglior tempo, io mi sto occupando da più giorni di un'alta

Four days later he observed:

> I draw the elements of physiology of the sensations from Wundt, who reproduces and corrects the experiments of his predecessors, and especially of Helm[h]oltz. In many points his ideas correspond to mine, but, for example, his observation that "the idea of the straight line comes from the sense of touch and from the sensation of muscular motion because the mechanical conditions of the organism favour rectilinear motion of the muscles" does not seem to me to be correct. . . . Instead, the notion of the straight line comes directly from the eye, like all other graphic notions of shape. Likewise, it is strange that W[undt] admits that the notion of "distance" comes (also) from the eye, while the experiments he cites prove the opposite, that is, that "the eye is never capable of perceiving the equality of two distances that are not equally situated."[32]

The theme of the psychogenesis of geometrical properties is also mentioned in the introduction to Enriques' *Lezioni di Geometria proiettiva*, which came out in 1898 and were the fruit of four years' experience in teaching at the university.[33] Here, along with the problem of the scientific presentation of the subject,[34] he also addresses that of the educational presentation, as evidenced in the dense correspondence with Castelnuovo, and as Enriques himself writes in the preface:

> Having resolved the problem as far as the scientific aspect was concerned, it was still necessary to articulate the form of the exposition and carry it out more completely in its details, in order to make it acceptable from an educational point of view. It seems to me that, during the past three years, the lessons that I am now publishing in print have come ever closer to this educational end. In them I have sought to reconcile the needs of the logical mind with the advantages and the attractions that intuition confers on studies of geometry, . . . observations of an intuitive nature . . . appear in any case to illuminate some of the more abstruse concepts or explanations, and in some places can even take the place of the rigorous procedure of proof to the advantage of didactics.[35]

questione che dalla matematica prende solo il pretesto ... Si tratta del "problema filosofico dello spazio". Libri di psicologia e di logica, di fisiologia e di psicologia comparata, di critica della conoscenza ecc. passano sul mio tavolino dove li assaporo con voluttà tentando di estrarne il succo per ciò che concerne il mio problema. ... Giacché vi è nel mio programma la questione della genesi dei concetti di spazio sopra i dati della psicologia fisiologica (specie dell'occhio e del tatto) di Helm[h]oltz, di Wundt ecc."

[32]See Enriques to Castelnuovo, s. l., 8 May 1896, [8, pp. 264–265]: "Io traggo gli elementi di fisiologia delle sensazioni dal Wundt che riproduce e corregge le esperienze dei predecessori e specialmente di Helm*[h]*oltz. In molti punti le sue idee collimano con le mie, ma non mi sembra p*[er]* e*[sempio]* accettabile la sua osservazione che 'l'idea di retta viene dal tatto e dalla sensazione di movimento muscolare perché le condizioni meccaniche dell'organismo favoriscono il moto rettilineo dei muscoli'. . . . Invece la nozione di retta proviene direttamente dall'occhio, come ogni altra nozione grafica di forma. Similmente è strano che il W*[undt]* ammetta che la nozione di 'distanza' proviene (anche) dall'occhio mentre le esperienze che cita provano il contrario, e cioè che 'l'occhio non sa mai apprezzare l'uguaglianza di due distanze se non sono ugualmente poste."

[33]See [24, pp. 3–4]; see also [23, pp. 4–5].

[34]For more on this, see [1, §§ 6 and 7].

[35]See [24, pp. V–VI]: ". . . risoluto il problema sotto l'aspetto scientifico, occorreva ancora elaborare la forma della trattazione e svolgerla più compiutamente nei suoi dettagli, in guisa da renderla accettabile nella scuola. A questo scopo didattico mi sembra si sieno venute avvicinando, durante i tre anni scorsi, le lezioni che ora pubblico per le stampe. Nelle quali ho cercato di contemperare le

Among the educational instruments used by Enriques the history of mathematics had already a place: he inserted an appendix[36] about history at the end of the book, in order to show his students the genesis of the fundamental concepts of projective geometry and to make clear how

> the various branches of pure and applied Mathematics interweave and connect to each other in unexpected ways; and [how] the ideas, which arise from elementary practical problems, seem to require long process of thought in order to mature, in the highest regions of theory, before they can descend and bear fruit in the field of daily activity.[37]

Enriques' letters to Castelnuovo, the lecture notes, and the class registers for these years show what a tight mix Enriques' activities were of the study of foundations, the history of mathematics, and the needs of education. These can be summarised in the following points:

- The refusal to resort to artifices in the proofs. He wrote to Castelnuovo: "I am disposed for educational reasons even to the greatest compromises in order not to oblige one to introduce artifices. For me any proof that is not remembered once understood is artificial. Such proofs do not illuminate, and the students prefer them precisely because there is nothing substantial in them to understand: thus I hold them to be educationally futile: we might as just as well give the student only the statement".[38]

esigenze dello spirito logico coi vantaggi e colle attrattive che l'intuizione conferisce agli studi geometrici ... osservazioni di carattere intuitivo ... compariscono tuttavia a lumeggiare alcuni concetti o ragionamenti più astrusi, ed in taluni punti possono anzi sostituire con vantaggio didattico il procedimento rigoroso della dimostrazione." See also the comment by Segre [119, p. 11]: "In his book he presents his research, taking due account of the needs of education. Some intuitive observations lead to stating certain postulates with precision, chosen so that from them it is possible to deduce, not only with rigour, but also with simplicity, all of the fundamental propositions of the geometry of position. This condition of simplicity is essential for teaching purposes. It would not be satisfied by an author who wanted to break each postulate into its most minute parts, discarding those that could be deduced logically from others, and demonstrating the logical independence of the remaining propositions; the result would be a bad *educational* job even if it were excellent from a scientific point of view" ("Nel suo libro egli si serve di quelle sue ricerche, tenendo il debito conto delle esigenze didattiche. Alcune osservazioni intuitive conducono ad enunciare con precisione certi postulati, scelti per modo che da essi si possan poi trarre, non solo con rigore, ma anche con semplicità, tutte le proposizioni fondamentali della geometria di posizione. Questa condizione della semplicità è essenziale per la scuola. Ad essa non soddisfarebbe, e quindi riuscirebbe ad una cattiva opera didattica, se anche ottima dal punto di vista scientifico, chi volesse scindere ogni postulato nelle sue più minute parti, scartando quelle che si possono dedurre logicamente dalle altre, e dimostrando l'indipendenza logica delle proposizioni rimaste").

[36] See [24, pp. 358–371].

[37] See [24, p. 371]: "... i vari rami della Matematica pura ed applicata si annodano e si collegano fra loro per vie inaspettate; e le idee, che traggono origine da elementari problemi della pratica, sembra debbano maturarsi per lunga elaborazione di pensiero, nelle regioni più alte della teoria, prima che possano discendere feconde nel campo di attività della vita."

[38] See Enriques to Castelnuovo s. l, 24 November 1895 [8, p. 224]: "Io sono disposto per ragioni didattiche a quelle maggiori transazioni che non obbligano a introdurre artifici. È per me artificiosa

- The importance of using intuition.
- The digressions into higher mathematics; Oscar Chisini, for examples, recalled that Enriques had a habit of amplifying his lessons in elementary projective geometry with frequent digressions into advanced geometry, topology, logic and economy [15, p. 119].
- The use of the history of mathematics as a tool for understanding the genesis of the concepts presented.[39]
- A unified vision of science and culture.

The open course in the philosophy of science that Enriques taught in 1902–1903 is emblematic, because the program interweaves scientific, philosophical and educational aspects of the subject.[40] The correspondence with Giovanni Vailati shows that although the program for the course proposed by Enriques was not approved at first, he did not give up, and asked the High Council for Public Instruction to decide whether or not philosophy of science could be part of the open courses of the faculty of science at the University of Bologna. The answer was positive, under the condition that not too much space was given to philosophy. The letters also show that the topics addressed in the course were the objects of six lectures given by Enriques in March 1902 at the *Univérsité Nouvelle* in Brussels.[41] Significant is the fact that Enriques expressly asked the rector for and was granted permission to open his course entitled *Filosofia scientifica*, "Scientific Philosophy," to students in the Faculty of Philosophy and Letters and that of Law as well as to students of mathematics. Moreover, all of the 366 lire that he was paid for that course was spent in buying books for the library of the Scuola di Magistero where Pincherle was teaching at the time.[42]

Enriques' cultural project, of which his vision of mathematics teaching was part, was beginning to take shape, and a few years later, it would lead Enriques to formulate his proposal for a reform of the university, as an expression of a unified vision of knowledge.

ogni dimostrazione che capita una volta non si ricorda senz'altro. . . . Siffatte dimostrazioni non illuminano, e gli studenti le preferiscono appunto perché non vi è in esse nulla di sostanziale da capire: quindi io le ritengo inutili didatticamente: tanto varrebbe dare gli studenti il solo enunciato."

[39] The history of mathematics, among other things, is also found in the courses in higher analysis that Enriques was assigned to teach at the University of Bologna; for example, in the course of 1917–1918 no fewer than fifteen lessons were dedicated to it; see ASUB, *Enriques prof. Federigo. Fascicolo personale.*

[40] See Appendix 1.

[41] See the letters of Enriques to Vailati dated 11 November 1901, 1 January 1902, 24 January 1902 [92, pp. 570–571, 575–576].

[42] See Enriques to Castelnuovo s. l, 31 October 1902 [8, p. 503].

2.2 Klein's Influence

In the evolution of Enriques' cultural project and his vision of mathematics teaching, along with his experience in teaching which we have described above, an important role was played by the influence of Klein,[43] to whom Enriques refers often, and to whom Enriques reserved a special place in the section devoted to teaching in the 1934 entry entitled *Matematica* that he wrote in the *Enciclopedia Italiana*. Here, we will briefly mention some of the characteristics of Klein's vision that were taken up and reinterpreted by Enriques.

For Klein, theoretical research had to be very strictly connected to experimental research:

> From the point of view of pure mathematical science I should lay particular stress on the *heuristic value* of the applied sciences as an aid to discovering new truths in mathematics. . . . Such separation [between abstract mathematical science and its scientific and technical applications] could only be deplored; for it would necessarily be followed by shallowness on the side of the applied sciences, and by isolation on the part of pure mathematics [87, pp. 46, 50].

He classifies geometry as one of the applied sciences, and he affirms that the mathematical treatment of any applied science "substitutes exact axioms for the approximate results of experience, and deduces from these axioms the rigid mathematical conclusions" [87, p. 47].[44] Klein also shows a refusal of the axiomatic point of view and a conviction that progress in science originates from the combined use of intuition and logic:

> The science of mathematics may be compared to a tree thrusting its roots deeper and deeper into the earth and freely spreading out its shady branches to the air. Are we to consider the roots or the branches as its essential part? Botanists tell us that the question is badly framed, and that the life of the organism depends on the mutual action of its different parts [90, pp. 248–249].

As far as intuition is concerned, Klein distinguishes between *naïve intuition* and *refined intuition* and highlights the fact that naïve intuition is important in the discovery phase of a theory (as an example Klein cites the genesis of differential and integral calculus) and at the time when its foundations are being established, refined intuition (shown, for example, in Euclid's *Elements*) intervenes in the elaboration of data furnished by naïve intuition, and in the rigorous logical development of the theory itself: "The naïve intuition is not exact, while the refined intuition is not properly intuition at all, but arises through the logical development from axioms considered as perfectly exact" [87, pp. 42]. In the article "The Arithmetizing of Mathematics," Klein further hypothesises that the clarification of the relationship

[43] See, for example, [106] and [83]. See also the four letters from Enriques to Klein (SUB Göttingen, *F. Klein* 4A, 8, 34 and 51) and one letter from Klein to Enriques (SUB Göttingen, *F. Klein* 51).

[44] See also [91, II vol., pp. 201–202].

between the intuitive process and the logical process may be achieved through physiology and experimental psychology [90, p. 247], a theme he discussed with Enriques during his second visit to Italy in 1899. On that occasion Enriques wrote to Castelnuovo:

> . . . I passed two splendid days with Klein; the first in Florence where (except for a two-hours visit to the Institute of Geography) I had him all to myself, and the second in Bologna where I was again able to talk with him at length. . . . Saturday during the visit to the galleries, I told him in detail about the outline for my article on the Foundations of Geometry, and I was very pleased to see that he was satisfied. He took very detailed notes about what I told him. . . . But the problem we discussed at greatest length was that regarding the psychological issues relating to mathematics. Yesterday morning, as he took leave of me, he said, "We must take up our conversation on these subjects again, which I will not forget."[45]

In fact, Klein had invited him to write a chapter on the foundations of geometry for the *Encyklopädie der mathematischen Wissenschaften*. This was the principal theme discussed during Enriques' stay in Göttingen in 1903:

> As far as my conversation with Klein goes, you already know how interesting it was. In addition to talking about the foundations of geometry, we discussed educational issues at length, and in just a few hours I learned a great deal from him about a lot of things I knew nothing about – specifically about the way in which mathematics teaching is developing in England and Germany.[46]

Enriques would make many of Klein's pedagogical assumptions his own. These can essentially be summarised as follows. First, he desired to bridge the gap between secondary and higher education. In particular, he proposed transferring the teaching of analytic geometry and, above all, of differential and integral calculus, to the middle school level, even in those schools which did not specialise in the sciences. The concept of function would pervade the whole mathematics curriculum: the famous expression "functional thinking" (*funktionales Denken*) was adopted as a slogan for his reform program. Furthermore, he favoured a genetic teaching method, that is, one that takes account of the origins and evolution of the subject, and

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[45]See Enriques to Castelnuovo, s. l, 28 March 1899 [8, p. 404]: ". . . ho passato col Klein due giornate bellissime: la prima a Firenze ove (tranne due ore di visite all'Istituto geografico) me lo sono goduto interamente, e la 2^a a Bologna dove pure ho conferito lungamente con lui . . . Sabato durante la visita alle gallerie, gli ho esposto dettagliatamente il programma del mio Art*[icolo]* sui Fondamenti della Geometria, e sono stato lieto di vederlo soddisfatto. Egli ha preso note assai minute su ciò che gli ho esposto. . . . Ma il soggetto di cui abbiamo discorso più lungamente è quello che si riferisce ai problemi psicologici matematici. Ieri mattina congedandosi da me, mi ha detto: riprenderemo la nostra conversazione su questi argomenti, che non dimenticherò." Klein had already been in Italy the first time in 1878, and on the occasion of this second visit he stopped in Florence, Bologna, Rome and Padova, meeting amongst others, Enriques, Castelnuovo, Cremona, Veronese and Fano.

[46]See Enriques to Castelnuovo, Brussels 24 October 1903 [8, p. 536]: "Quanto alla conversazione di Klein sai già quanto era interessante; oltre che delle questioni sui principii abbiamo discorso molto di questioni didattiche e da lui solo in poche ore ho imparato tante cose interessanti, di cui non avevo mai avuto notizia, sullo sviluppo dell'istruzione matematica in Inghilterra e in Germania."

he believed that teachers should capture the interest and attention of their pupils by presenting the subject in an intuitive manner. He stressed the importance of showing the applications of algebra to geometry and vice versa. He suggested highlighting the applications of mathematics to all the natural sciences. He believed in looking at the subject from a historical perspective. In addition, he argued that more space should be dedicated to the "mathematics of approximation" (*Approximationsmathematik*), that is, "the exact mathematics of approximate relations." Lastly, he firmly believed that it was crucial that elementary mathematics viewed from an advanced standpoint play a key role in teacher training.

It was thanks to Klein's intervention that a German translation of Enriques' *Lezioni di geometria proiettiva* was published in 1903. In his introduction to this book, Klein expresses particular appreciation for Enriques' treatment of the subject, which "is always intuitive, but thoroughly rigorous," and underlines the impact of this kind of research on didactics, writing:

> Italian researchers are also well ahead of us from a practical point of view. They have by no means disdained exploring the educational consequences of their investigations. The high quality textbooks for secondary schools which came out from this exploration could be made available to a broader audience through good translations. And it would seem particularly desirable in Germany when we consider that our own textbooks are completely out of touch with active research.[47]

As Enriques would write twenty years later in his review of Klein's *Gesammelte mathematische Abhandlungen*, it was precisely the "tendency to consider the objects to be studied in the light of visual intuition" [*Periodico di Matematiche,* (4), 3, p. 55] that brought Klein and the Italian geometers so close together intellectually.

Klein's example, and in particular that of the 1895 *Vorträge über Ausgewählte Fragen der Elementargeometrie*, inspired Enriques to begin to collaborate with his friends and followers on a series of monographs on elementary geometry from an advanced standpoint for the students of the Scuole di Magistero. In the spring of 1899, he wrote to Castelnuovo:

> Now I shall tell you about a project that I hope to turn into a reality without a great deal of effort. It would be a book dedicated to all the questions that concern elementary geometry (included in which are also the problems not of the second-degree which are dealt with by Klein, but there are very many questions). I do not propose to do it myself, but to have it done by students newly graduated and by secondary school teachers, reserving for myself, or for any other mathematician who wants to take it on, the treatment of some of the more delicate arguments.[48]

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[47] F. Klein, "Zur Einführung," in [27, p. *III*]: "Aber die italienischen Forscher sind längst nach praktischer Seite weitergegangen: sie haben es nicht verschmäht aus ihren Forschungen pädagogische Folgerungen zu ziehen. Die sehr bemerkenswerten Lehrbücher für Hoch- und Mittelschulen, welche solcherweise entstanden sind, können den weiten Kreisen, für die sie Interesse haben, nur durch geeignete Übersetzungen zugänglich gemacht werden. Und daß dies geschieht, erscheint gerade in Deutschland um so erwünschter, als unsere Lehrbüchliteratur den Kontakt mit der vorwärts drängenden Forschung gar zu sehr verloren hat."

[48] See Enriques to Castelnuovo, undated [8, p. 419]: "Ora vengo a parlarti di un progetto, che spero di attuare con poca fatica. Si tratta di un libro dedicato a tutte le questioni che interessano

The idea of a collective work aimed at teacher training in Italy was actually not completely new. Cremona had already thought of it when in 1865–1868 he edited the translation of Richard Baltzer's *Elemente der Mathematik*. In fact, Cremona, as he wrote to Genocchi, considered that book too difficult to be used as a manual for secondary schools, while he believed it could be quite useful for teachers.[49]

Among the friends and colleagues that Enriques involved in the realisation of his project were Ugo Amaldi, Ettore Baroni, Roberto Bonola, Benedetto Calò, Castelnuovo, Alberto Conti, Ermenegildo Daniele, Amedeo Giacomini, Alfredo Guarducci and Giuseppe Vitali.[50] In 1900 the volume entitled *Questioni riguardanti la geometria elementare* was published.[51] It was a work specifically aimed at teacher training: although for Enriques Euclidean geometry remained "the most effective tool for educating the mind, the most consistent with geometric reality," he, like Klein, nevertheless believed that the teaching of geometry could "take advantage of the progress made, in the field of the elements as well, by a more mature criticism and recent developments in higher mathematics," and that "the teacher entrusted with secondary school education must possess a much broader knowledge of such progress so that his work is inspired by much larger perspective" [25, p. *II*]. [52] The topics treated were congruence, equivalence, the parallel theory, problems that could or could not be solved with straightedge and compass, the constructibility of regular polygons. Enriques' own contribution to the volume regarded algebraic equations

la G*[eometria]* elementare (fra queste vi sono anche quei problemi non di 2^o gr*[ad]*o trattati dal Klein, ma le questioni sono moltissime). Mi propongo non di farlo, ma di farlo fare a giovani laureati e ad insegnanti delle scuole secondarie, serbando a me, o a qualche altro matematico che volesse occuparsene, la trattazione di qualche argomento più delicato." See also the introduction to *Questioni riguardanti la geometria elementare* [25, p. VII] where Enriques writes: "Such questions were recently addressed in a series of lectures by Mr Klein, to whom we are indebted for the idea behind this collection;' ("Tali questioni sono state svolte recentemente in una serie di conferenze del signor Klein, alla quale dobbiamo in parte l'idea di questa raccolta").

[49] See the letter of L. Cremona to A. Genocchi, Milano, 6 November 1867 [10, p. 110].

[50] Many of these were teachers in secondary schools: Ettore Baroni (1866–1918) taught from 1901 at the Liceo E. Q. Visconti in Rome; Benedetto Calò (1869–1917) taught from 1900 at the Istituto tecnico in Naples; Alberto Conti (1873–1940) taught in secondary schools in Florence, and in 1900 founded the *Bollettino di Matematica*, a journal mainly for teachers; Amedeo Giacomini (1873–1948) was from Pisa; Alfredo Guarducci was professor of mathematics at the Liceo classico in Prato.

[51] See [25]. The German edition, entitled *Fragen der Elementargeometrie* (vol. II, Leipzig: Teubner, 1907; vol. I, Leipzig: Teubner 1910) contained an additional article by Giovanni Vailati on the theory of proportions. The reviews by M. Grossmann and H. Fehr appeared in *L'Enseignement mathématique* (vol. 11 (1909): p. 322; vol. 13 (1911): pp. 427–428) underlined the high level of research and teaching in geometry in Italy.

[52] "... possa avvantaggiarsi dei progressi portati, anche nel campo degli elementi, da una critica più matura e dagli sviluppi recenti delle alte Matematiche"; '... di tali progressi debbano possedere una cognizione assai larga gli insegnanti cui la scuola secondaria è affidata, affinché l'opera loro possa ispirarsi a più larghe vedute' [25, p. II].

and the constructibility of regular polygons,[53] but he also prefaced the anthology with an essay on the scientific and educational importance of the questions that refer to the principles of geometry,[54] which merits discussion because provides us with a clear picture of how Enriques conceived mathematics teaching and the training of mathematics teachers.

The essay is divided into two parts. In the first he outlines his vision of geometry as an "experimental science" just like physics (p. 5), and then he once again addresses the concept of abstract geometry, already presented in his 1894–1895 *Conferenze di geometria*, making evident its merits, but also cautioning:

> Abstract Geometry can be variously interpreted and thus draw new aid from various forms of intuition. But where, in contrast, it is desired leave aside any consideration of possible ways of interpreting it, and construct an edifice that is purely logical, on the basis of criteria that are exclusively logical, there is a danger of falling into a void. . . . It should not be forgotten that this science is a science of facts, physical or intuitive, however we want to consider them. The logical formalism must be conceived, not as an end to achieve, but as a means aimed to use and increase the faculty of intuition. The results themselves, logically established, no matter how far-reaching, must still not be considered as mature achievements until they can be in some way comprehended intuitively. But *in the principles the intuitive evidence must shine brightly*.[55]

Enriques situates geometry in a central position in mathematics because he considers it the most fertile terrain for reconciling abstract formal procedures with experimental procedures, as he will say more clearly in his 1906 *Problemi della Scienza*, which is an organic formulation of the ideas born when he was teaching in Bologna, as we have seen; here he shows his refusal of dogmatic Kantism and his divergence from Poincaré's conventionalism.[56]

He then addresses the problem of the psychological acquisition of fundamental concepts of geometry and, on the basis of his study of physiological psychology that he had been pursuing for a number of years, he states that the three branches into which Geometry is divided, that is, topology, metric geometry, and projective geometry, appear to be connected to three orders of sensations: respectively, to

[53]See F. Enriques, "Sulle equazioni algebriche risolubili per radicali quadratici e sulla costruibilità dei poligoni regolari" in [25, pp. 353–396].

[54]See F. Enriques, "Sull'importanza scientifica e didattica delle questioni che si riferiscono ai principii della Geometria" in [25, pp. 1–31].

[55]See [25, p. 12]: "La Geometria astratta può ricevere varie interpretazioni e trarre così nuovi aiuti da varie forme di intuizione. Ma ove, all'opposto, si voglia prescindere affatto da ogni maniera d'interpretarla, costruendo un edifizio puramente logico, in base a criterii esclusivamente logici, si corre il pericolo di cadere nel vuoto. . . . non bisogna dimenticare che tale scienza è scienza di fatti, fisici o intuitivi, che vogliano considerarsi. Il formalismo logico deve essere concepito, non come un fine da raggiungere, ma come un mezzo atto a svolgere e ad avanzare le facoltà intuitive. Gli stessi resultati più lontani, logicamente stabiliti, non debbono ancora considerarsi come un acquisto maturo, fino a che non possano essere in qualche modo intuitivamente compresi. Ma nei principii l'evidenza intuitiva deve risplendere luminosa."

[56]See [28, Chapter IV]. See also the article by G. Lolli in this present volume, and [105, pp. 87–127].

general tactile-muscular sensations, special tactile sensations (like the hand that allows man to measure objects) and those of vision (p. 19); The detailed explanation will be provided by Enriques in his 1901 article entitled "Sulla spiegazione psicologica dei postulati della geometria," and in the 1906 *Problemi della scienza*.[57]

In the second part of the essay Enriques examines educational questions in light of the reflections set out in the first part. He addresses himself directly to mathematics teachers, exhorting them to enter more deeply into the "philosophical spirit of [their] science: that spirit of relation that coordinates everything into a synthesis, and makes the great light of the general idea shine on the humble details!" (p. 23).[58] The teachers he is addressing are above all those at the gymnasiums-lyceums and those teaching physics and mathematics at the technical schools, but he says in a note (p. 24) that the teachers at the Normal Schools, which specialised in primary schoolteacher training, could also benefit from his reflections.

Here, we will underline only a few salient points to which Enriques returns more than once. First of all, the object of secondary teaching is not merely to provide useful notions, but to train the mind to reason, and to foster the spirit of initiative in young people; the teacher should be familiar with critical analyses and philosophical investigations, and although these should not enter into the practice of teaching, because the students are not capable of appreciating them, they should nevertheless enlighten their lessons; artificial technical developments and abstruse problems are to be avoided. Regarding geometry, Enriques observes:

> . . . it seems to us that the essential goal of teaching is achieved *when we are able to make it understood how the logical development of Geometry rests on an empirical basis, destroying the strange illusion that the postulates founded on immediate experience appear to have a degree of certainty that is inferior to theorems, even though* [the theorems] *depend on the* [postulates].[59]

He then specifies what the method of teaching should be, a method which will later be called rational-inductive: the teachers should begin with a series of observations, and then on the basis of these present the fundamental concepts as "ideal representations of objects of reality" and state the postulates "as expressions of elementary facts." From these they will then deduce the theorems, beginning with the simplest and going on to consider the most complex. The rigorous proof of the theorems can be followed by experimental verification. Enriques then invites the teachers to keep empirical facts and logical facts well separated, and remarks that "a new datum of intuition, which has been neglected in the premise, should never insinuate itself in a hidden way in the reasoning of the proof" (p. 29). This, he says,

[57] See [28, pp. 177–187]; see also [1, pp. 401–406].

[58] See [25, p. 23]: ". . . lo spirito filosofico della vostra scienza; quello spirito di relazione che tutto coordina in una sintesi, e fa brillare sugli umili particolari la grande luce dell'idea generale!"

[59] See [25, p. 28]: ". . . ci pare che il fine essenziale dell'insegnamento sia raggiunto, se si riesce a far comprendere come lo sviluppo logico della Geometria riposi sopra una base empirica, distruggendo la strana illusione per cui i postulati fondati sopra un esperimento immediato, sembrano quasi avere un grado di certezza inferiore ai teoremi, che pur da quelli dipendono."

"is the only, important, even necessary, condition for rigour," while "for rigour it is not at all important to seek the independence of the postulates, and indeed, in terms of education it is preferable to draw a greater number of evident principles from the observation" (pp. 29, 30).[60]

On the basis of Enriques' reflections over time, gathered in the *Problemi della scienza*, this essay was later fleshed out and split into two chapters ("Sull'importanza filosofica delle questioni che si riferiscono ai principii della Geometria" and "Sull'insegnamento della Geometria razionale") in the second, enlarged edition of the *Questioni*, which appeared in 1912 with the title *Questioni riguardanti le matematiche elementari*. This second edition extended to questions of arithmetic and algebra, and also featured new collaborators: his disciple Oscar Chisini, and three capable secondary school teachers (at that time), Duilio Gigli, Alessandro Padoa and Umberto Scarpis.

2.3 Epistemological Assumptions at the Basis of Enriques' Vision of Mathematics Teaching

From the research projects carried out from 1896 to 1906 emerge a rather clear picture of Enriques' particular vision of mathematics and its influence on teaching. There also emerges a very precise cultural program in which active research in the field of algebraic geometry and philosophical, psychological and historical reflections are all closely intertwined. Enriques' aim was to communicate to his intended audience – scientists, philosophers, and educators – his vision of a scientific *humanitas* in which the boundaries between disciplines were overcome and the abyss between science and philosophy was bridged. The history of science constituted the path of first choice for achieving this end, or at least it was the tool used by Enriques, as we have seen, in his university teaching from the very first years, and had over time gradually become the most important one in the various initiatives aimed at teacher training.

Broad, rich, and sometimes contradictory, it is impossible within the limits of this paper to outline the epistemological vision on which all of Enriques' scientific work was founded, so I will confine myself to indicating the most important factors which inspired his idea of mathematics education.

[60]See [25, p. 30]: "...per il rigore non importa affatto cercare l'indipendenza dei postulati, ed anzi didatticamente è preferibile trarre dall'osservazione un maggior numero di principi evidenti."

2.3.1 A Genetic and Dynamic Vision of the Scientific Process and the Significance of Error

First of all, Enriques held a dynamic and genetic view of the scientific process, which he described as one

> . . . at once inductive and deductive, which ascends from specific observations to abstract concepts, only to descend again to practical experience. It is a process of continuous development, which establishes a generative relationship between theories and perceives in their succession only an approximation of the truth.[61]

Science was therefore not conceived by Enriques as a closed system of definitive propositions. He writes:

> . . . if the truth is only one step towards truth, the value of science would consist in moving forward rather than in stopping at a terminus reached provisionally. The facts, laws, theories will become meaningful not so much as a finished and static system, as in their reciprocal concatenation and their development.[62]

In such a vision of science, errors become valuable as well, because in the dynamic process of science truth and error are constantly mixed: "every error always contains a partial truth that must be kept, just as every truth contains a partial error to be corrected."[63]

According to Enriques, the error/gap (which is found when there is a missing link in the deduction that leads to a true statement) and the error proper (when a false proposition is stated as true) are errors that are almost necessarily encountered in the psychological acquisition of a theory, and are often reflected in the historical development of science. They do not appertain to either the faculty of logic or to the faculty of intuition, but are introduced "at the delicate moment of their juncture," that is, when the abstract concepts are developed from the objects effectively perceived [55, pp. 64–65]. The correction of errors leads to scientific progress, and from this derives their heuristic value.[64]

This vision is necessarily reflected in mathematical education. Enriques in fact criticised the tendency to present a mathematical theory in a strictly deductive

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[61] See [34, p. 132]: ". . . processo induttivo e deduttivo, che dalle osservazioni particolari sale ai concetti generali ed astratti per ridiscendere all'esperienze di fatto, processo di sviluppo continuo, che pone fra le teorie un rapporto generativo e scorge nel loro succedersi un'approssimazione alla verità."

[62] See [52, p. 3]: ". . . se la verità è solo un passo verso la verità il valore della scienza consisterà piuttosto nel camminare che nel fermarsi ad un termine provvisoriamente raggiunto. I fatti, le leggi, le teorie riceveranno il loro senso non tanto come sistema compiuto e statico, quanto nella loro reciproca concatenazione e nel loro sviluppo."

[63] See [33, p. 417]: ". . . ogni errore contiene sempre una verità parziale da mantenere, come ogni verità un errore parziale da correggere."

[64] See also Enriques' criticism of the theory of error proposed by Croce, according to which "error is the product of practical motives that deter the spirit from contemplation of the truth. Thus error is to be corrected with thrashing" ("l'errore è il prodotto di motivi pratici che distolgono lo spirito dalla contemplazione della verità. Dunque l'errore si corregge con le bastonate") [33, p. 417].

manner at school, as in this way it appears something closed and already perfect, leaving no room for further discovery. Instead, teachers should approach problems with a number of different methods, paying attention to the errors which have allowed science to move forward, and indicating open questions and new fields of discovery.

On the other hand, the good teacher must also take into account the errors of his students and quickly learn "to distinguish the significant errors from those that are not actually errors – rather gratuitous statements by insolent [learners] who try to guess – where no effort is made to think."[65] Since, in Enriques' opinion, "errors proper" represent "natural steps along the way of thought in search of truth," the teacher must attribute an educational value to them:

> . . . they are educational experiences that he pursues, encouraging the student to discover for himself the difficulties that impede right judgment, and thus also to err in order to learn to correct himself. Every kind of possible errors is also a kind of opportunity for learning.[66]

2.3.2 Inductive Aspects of Scientific Research and the Dialectic Between Intuition and Rigour

These views on science are connected to Enriques' conception of the nature of mathematical research – typical of the Italian school of algebraic geometry – as something aiming above all at discovery and particularly emphasising the inductive aspects and intuition:

> The main thing is *to discover*. . . . A posteriori it will always be possible to give a proof, [which,] translating the intuition of the discoverer into logical terms, will provide *everyone* with the means to recognise and verify the truth.[67]

Much has been written on the working method of the Italian geometers, and about Enriques in particular, so here I will limit myself to underlining by means of a quotation the importance that he attached to intuition in scientific research:

> The faculty which comes into play in the construction of science and which thus expresses the actual power of the mathematical spirit is intuition. . . . There are in any case different forms of intuition. The first is the intuition or imagination of what can be seen. . . . But there is another form of intuition that is more abstract, that – for example – which makes it possible for the geometer to see into higher dimensional space with the eyes of the mind.

[65] See [52, p. 14]: ". . . a distinguere gli errori significativi da quelli, che non sono propriamente errori – affermazioni gratuite di sfacciati che cercano d'indovinare – dove manca lo sforzo del pensiero."

[66] See [52, p. 14]: ". . . sono esperienze didattiche che egli persegue, incoraggiando l'allievo a scoprire da sé la difficoltà che si oppone al retto giudizio, e perciò anche ad errare per imparare a correggersi. Tante specie di errori possibili sono altrettante occasioni di apprendere."

[67] See [63, *II*, p. 307, 318]: "La cosa essenziale è di regola scoprire . . . a posteriori si riesce sempre a darne una dimostrazione . . . *[che]* traducendo l'intuizione dello scopritore in termini logici, vuol dare a tutti il mezzo di riconoscere ed appurare la verità."

> And there is also a sense of formal analogies which, in the work of many analysts, takes the place of the visual representation of things. . . . [I]ntuition protracts and surpasses itself in the unifying power of reason, which is not something exclusive to the mathematician, but – in every field of science and application – marks the greatest reaches of the spirit.[68]

This belief is naturally reflected in the style of teaching, which should, according to Enriques, take into account the inductive as well as the rational aspect of theories. Logic and intuition are not two distinct faculties of the human intellect; rather, they represent two inextricable aspects of the same process. Teachers should therefore find the right balance between the two. The important thing is to distinguish clearly between empirical observation and intuition on the one hand, and logic on the other. On this subject, Enriques distinguishes between what he calls “small scale logic,” the refined and almost microscopically accurate analysis of thought, and “large scale logic,” which considers the organic connections in science. He maintained that teaching should above all take “large scale logic” into account, gradually preparing young people to develop a more refined and rigorous approach. He writes:

> It is of no use to develop with impeccable deduction the series of theorems of Euclidean geometry, if the teacher does not go back to contemplate the edifice constructed, inviting the students to distinguish the truly significant geometric properties from those which are valuable only as links in the chain.[69]

At the first level of teaching it is convenient to keep to a method which appeals to intuition and calls for active work on the part of the students:

> a logical education (indeed the most appropriate one for minds little disposed to abstraction) is also comprised in the exercise of intuition, when this is put to the test by making the students *work*. Thus, for example, the construction of a geometric figure requires not only the attitude of passively seeing a model . . . but also the capacity to shape a possible model, on which are imposed, a priori, certain conditions: and this kind of constructive activity which orders the data of observations and past experience, is not pure imagination . . . but rather true logical activity.[70]

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[68]See [53, pp. 173–174]: “La facoltà che viene in opera nella costruzione della scienza e che esprime perciò il reale potere dello spirito matematico è l’intuizione. . . . Vi sono del resto più forme d’intuizione. La prima è l’intuizione o immaginazione del visivo. . . . Ma c’è poi un’altra forma d’intuizione più astratta, quella – per esempio – che consente al geometra di vedere con gli occhi dello spirito negli spazi a più dimensioni. E c’è ancora un senso delle analogie formali che, presso molti analisti rimpiazza la rappresentazione visiva delle cose. . . . *[L’]* intuizione stessa si prolunga e si supera nel potere unificatore della ragione che non è qualcosa di esclusivo del matematico, ma – in ogni campo della scienza e della pratica – contrassegna la maggiore altezza dello spirito.”

[69]See [38, p. 10]: “Non giova sviluppare con impeccabile deduzione la serie dei teoremi della geometria euclidea, se non si ritorni a contemplare l’edifizio costruito, invitando i discepoli a distinguere le proprietà geometriche veramente significative da quelle che hanno valore soltanto come anelli della catena.”

[70]See [38, p. 8]: “Un’educazione logica (anzi la più appropriata alle menti poco disposte ad astrarre) è pur contenuta nell’esercizio dell’intuizione, quando questa venga messa alla prova facendo lavorare il discepolo. Così, per esempio la costruzione di una figura geometrica, importa – non solo – l’attitudine a vedere passivamente un modello . . . ma anzi la capacità di foggiare . . . un

With regard to the fact that many Italian teachers resisted the introduction of methods that were more intuitive and empirical, lamenting that a certain incompleteness and a non-rigorous way of reasoning is inherent in these, Enriques observed with a touch of humour:

> Resisting the ideas that… relate to the eye, the ear, the sense of touch, and seeing in sensations, not the doors to knowledge, but only occasions for sinful errors, this strange chastity of mathematical logicians brings to mind Plotinus and those Christian ascetics of the Middle Ages who were ashamed of having a body.[71]

Teaching how to reason abstractly without recourse to intuition must be done gradually, so that the student is able to grasp its importance. Enriques suggested, for example, beginning by presenting some proofs *ad absurdum* from which, he wrote, logic "draws its historical origins," adding:

> Only at the end of a course in geometry, looking at the system of science, is it useful to explain the logical structure, pointing out the significance of the primitive concepts and the postulates which must come at the beginning of a written treatise … but not in a lively lesson, in which those principles should be left aside, informing the student that [the principles] contain only a precise recapitulation of things already known, and they will be introduced along the way as need arises.[72]

Enriques also believed it was counterproductive educationally to persist in proving everything that is intuitively evident because of the danger of depriving intuition of its value and leading the student to doubt the importance of reasoning. Further, a good teacher should not overindulge in the search for generality:

> … a too abstract form of the statement can obscure the true meaning of the theorem, concealing its origins, and – in the second place – awakens in the young scholars the allurement of easy, purely formal generalisations.[73]

modello possibile, cui s'impongono, a priori, talune condizioni: ed una tale attività costruttiva che ordina i dati di osservazioni ed esperienze passate, non è pura fantasia … bensì vera attività logica."

[71]See [53, p. 145]: "Respingere le idee che hanno … rapporto con l'occhio, o con l'orecchio, o col tatto, vedendo nelle sensazioni non le porte della conoscenza, ma soltanto l'occasione di errori peccaminosi, questo strano pudore dei logici matematici ci richiama alla memoria Plotino e quegli asceti cristiani del Medio Evo che si vergognavano di avere un corpo."

[72]See [38, p. 11]: "Solamente al termine d'un corso di geometria, riguardando al sistema della scienza, gioverà spiegarne l'organismo logico, rilevando il significato dei concetti primitivi e dei postulati, coi quali si deve cominciare un trattato scritto … ma non la lezione viva, che lascia dietro di sé quei principii, avvertendo il discepolo che contengono soltanto una ricapitolazione precisa di cose note, da richiamare di mano in mano che se ne presenti il bisogno"; see also [36] and [49].

[73]See [63, *I*, p. *XI*]: "la forma troppo astratta dell'enunciato riesce ad oscurare il vero significato del teorema nascondendone le origini, ed – in secondo luogo – crea nei giovani studiosi la lusinga delle facili generalizzazioni, puramente formali"; see also [53, p. 153].

2.3.3 Science as a "Conquest and Activity of the Spirit" and Unified Vision of Culture

For Enriques, science is the "conquest and activity of the spirit, which ... merges in the unity of the spirit with the ideas, feelings and aspirations which find expression across all the different aspects of culture" [54, p. 130].[74] In this, Enriques thus ran counter to Croce and Gentile, the leading proponents of Italian neo-idealism, who tended to devalue science, recognising in it only a practical function and a role that was completely instrumental, and separating it from the world of philosophy and culture.[75] He was aware of the grave danger that cultural isolation poses to science, so he continually emphasised the importance of "cultivating one's own field of study as a segment of the greater body of science!."[76] He held that:

> The end that should be sought today is a scientific education that allows a person working in any given field to understand how the object of his own research is subordinated to more general problems. ... Nothing is as dangerous as enclosing oneself in a circle from which everything that does not agree with the results of limited experience is banished according to rigorous logic.[77]

Furthermore, for Enriques the fact that science does not have goals that are purely utilitarian does not imply a separation between pure and applied science, but means that scientific research is valuable in itself, and does not necessarily have to aim at applications. Like Klein, he believed it was useful and necessary to maintain close ties between abstract science and applied sciences because pure sciences offer instruments that are needed for the purposes of applied science, and in their turn, applied sciences perform functions that are essential for stimulating the development of theoretical sciences, as history makes amply clear.[78]

Such a unitary vision of culture found expression in Enriques' constant efforts to bridge the gap between mathematics and other scientific and scholarly fields, such as physics, biology, psychology, physiology, philosophy, and history. Only by overcoming narrow specialisation could the sciences, and especially mathematics, realise their true humanistic and educational value.

From this viewpoint derive some of the fundamental tenets of Enriques' idea of education: the importance of establishing links between the various parts of mathematics and between mathematics and the other intellectual activities, because

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[74] See [54, p. 130]: "conquista e attività dello spirito ... *[che]* si fonde nell'unità dello spirito colle idee, coi sentimenti, colle aspirazioni che si esprimono nei vari aspetti della cultura."

[75] See for example, [83, 110, 112].

[76] See [35, p. 35]: "...coltivare il proprio ramo di studii come un frammento della scienza generale!."

[77] See [28, pp. 3–4]: "...il fine a cui oggi si deve tendere è un'educazione scientifica, la quale faccia meglio comprendere a colui che lavora in un campo qualsiasi come l'oggetto della propria ricerca venga subordinato a problemi più generali. ... Nulla è così pericoloso come il rinchiudersi in un cerchio, donde si bandisca con una logica rigorosa ciò che non si accorda coi resultati di un'esperienza ristretta!."

[78] See [43, p. 4].

these are simply different moments within a single cognitive process; the need for active teaching; the importance of training teachers who are capable of transmitting a vivacious knowledge to students' minds, "like sparks from one fire ignite other fires" [38, p. 15];[79] and finally, his firm belief in the educational and cultural value of mathematics.

According to Enriques, the duty of the teacher consists in communicating to the student "his need for knowledge before allowing him to possess it," and attaining anew that knowledge along with the student, with the joy of discovery; this kind of teaching is certain more difficult, but is much more effective for the student as well as more gratifying for the teacher [46, p. 68]. He, therefore, proposes that teachers adopt the method that Socrates used with his students, which consists in conversing with them, acting "a little ignorant" and, through dialogue and a guided search, leading them to a personal discovery of mathematical truth:

> The greatest advantage of this method is, in my opinion, its sincerity, because the postulate of ignorance is infinitely closer to the truth than the presupposition of knowledge already certain in the mind of the student, which the pedantic lesson starts off with [38, p. 14].[80]

It is only through personal conquest that the student can arrive at the true comprehension of mathematics. Enriques writes:

> Teaching should not be a gift from a teacher to a person who comes to hear his perfectly prepared lessons ... but rather it should be an aid given to the person who wants to learn by himself or is, at any rate, disposed not merely to absorb passively, but to attain to knowledge, as if it were a discovery or a product of his own spirit.[81]

To stimulate the students to active participation, the teacher must not limit himself to repeating mechanically the old lessons he himself learned when was a student, but must show himself capable of offering a clearer and broader point of view born from a mastery of higher mathematics:

> ... there is no gap or schism between elementary and higher mathematics, because the latter is a development of the former, as a tree develops from a seedling. And as by studying the tree we discover new aspects of the seedling, and understand characteristics whose meaning had escaped our understanding, so the development of mathematical problems will throw light on the elementary theories in which they have their roots.[82]

[79] See [38, p. 15]: "... come scintilla di fuoco ad accendere altro fuoco."

[80] See [38, p. 14]: "Il più grande vantaggio di questo metodo è, a mio avviso, la sincerità, perché il postulato dell'ignoranza è infinitamente più vicino al vero che la presupposizione di conoscenze già sicure nella mente dell'allievo, da cui muove la lezione cattedratica."

[81] See [38, p. 6]: "... l'insegnamento non può essere un regalo che il maestro faccia a qualcuno che viene ad ascoltare le sue ben tornite lezioni (che, se sta disattento, merita di essere rimproverato per la sua ingratitudine!); ma è piuttosto un aiuto a chi voglia imparare da sé e però sia disposto, anziché a ricevere passivamente, a conquistare il sapere, come una scoperta o un prodotto del proprio spirito."

[82] See [38, pp. 15–16]: "... non vi è iato o scissura fra matematiche elementari e matematiche superiori, perché queste si sviluppano da quelle, al pari dell'albero dalla tenera pianticina. E come, riguardando l'albero, potremo scoprire nella pianticina nuovi aspetti o comprendere caratteri di

Further, in order for mathematics teaching to improve the faculty of logic, the teacher must be able to coordinate the various aspects of mathematics and relate them to each other:

> ... we are pleased to see recognised today the rights of education, on the condition that this fact leads the teachers ... to account for the psychology of the students and the usefulness of reconciling mathematical doctrines, too separated by purist concerns, which the history of science shows to be related.[83]

2.3.4 The History of Science

There are three methods that belong to Enriques' "positive gnoseology": historical, for retracing the genesis and development of scientific theories; psychological, for studying the formation of concepts; and scientific, which "consists in the direct critical examination of Science, regarding science itself as a fact to be explained."[84] Of the three, the historical method, which is also closely connected to the dynamic vision of science, was to assume an increasingly important role for Enriques.[85] In his own words:

> A dynamic vision of science leads us naturally into the territory of history. The rigid distinction that is usually made between science and history of science is founded on the concept of this [history] as pure literary erudition; ... But a very different meaning is obtained by the historical comprehension of scientific knowledge that aims at ... clarifying the progress of an idea. ... Such a history becomes an integral part of science.[86]

History is in fact intended as a science in itself:

> The history of science ... must be constructed thanks to the scientific reasoning which is useful for coordinating and evaluating the traditions, the testimonies, the sources, investigating first the *possibility* in order to infer the *reality*. In this manner the antithesis of

cui ci era sfuggito il significato, così anche lo sviluppo dei problemi matematici recherà luce sulle dottrine elementari in cui essi profondano le loro radici."

[83]See [39, p. 123]: "... non ci dispiace di vedere riconosciuti oggi i diritti della didattica, a condizione che l'indirizzo così affermato conduca gli insegnanti ... a rendersi conto della psicologia degli alunni e dell'utilità di ravvicinare dottrine matematiche, troppo separate da preoccupazioni puristiche, di cui la storia della scienza è atta a metter in luce la parentela."

[84]See [28, p. 78]: "... consiste nell'esame critico diretto della Scienza, riguardata essa stessa come il fatto da spiegare."

[85]See for example [113], especially the essays by G. Israel, M. Galuzzi and P. Freguglia; see also [105, pp. 150–173 and pp. 186–226].

[86]See [63, *I*, p. *XI*]: "Una visione dinamica della scienza porta naturalmente sul terreno della storia. La rigida distinzione che si fa di consueto fra scienza e storia della scienza, è fondata sul concetto di questa come pura erudizione letteraria; ... Ma assai diverso significato ha la comprensione storica del sapere che mira a ... chiarire il cammino dell'idea ... Una tale storia diviene parte integrante della scienza."

> science and history is reconciled into a collaboration regarding the concrete progress of our knowledge.[87]

It should be emphasised that the kind of historiography that Enriques proposed required an in-depth knowledge of scientific theories, including their technical aspects, and this couldn't help but render it unpalatable for pure historians and philosophers.

Furthermore, history also offers the cultural legitimisation of the function of mathematics, and thus for Enriques has a central educational role in both teacher training as well as in teaching proper. He rues the fact that too often,

> ... mathematics has been studied as an organism in itself, looking at the abstract formulation achieved after centuries of development, rather than at the profound historical reasons. Therefore the concrete problems that confer interest on the theories are forgotten, and the facts by then long since acquired are no longer visible behind the formula or the development of the reasoning, but only the concatenation into which we have artificially restrained them.[88]

For this reason, according to Enriques, future teachers should study the origins of each theory, together with its relationships and developments, not some static formulation;[89] they should be familiar with the work of ancient mathematicians, analysing the ways they addressed problems and the methods used to solve them, in order to better understand the more general and complex developments in modern science. Young people too should be "educated in the masterpieces of the masters" by means of readings of significant passages from their works during class:

> For developing culture that is serious and effective, it is necessary that [the students] be put in touch with the great thinkers, and thus set on the path to knowing the historic genesis of scientific ideas. The poets develop their knowledge in the company of poets, merchants in the company of merchants, philosophers in the company of philosophers. For that particular philosophy which is science, it is also time to turn from the textbooks and anthologies to the sources.[90]

[87]See [53, p. 166]: "La storia della scienza ... deve essere costruita mercé il ragionamento scientifico che vale a coordinare e valutare le tradizioni, le testimonianze, le fonti, indagando prima la possibilità per inferire la realtà. In tal guisa l'antitesi scienza-storia si risolve in una collaborazione per riguardo al progresso concreto del nostro sapere."

[88]See [29, p. 71]: "... le matematiche sieno state studiate come un organismo a sé, riguardandone piuttosto la sistemazione astratta conseguita dopo uno sviluppo secolare, che non l'intima ragione storica. Si dimenticano per tal modo i problemi concreti che conferiscono interesse alle teorie, e sotto la formula o lo sviluppo del ragionamento non si vedono più i fatti ormai da lungo tempo acquisiti, ma soltanto la concatenazione in cui noi artificialmente li abbiamo stretti."

[89]See [38, p. 16].

[90]See [64, p. 11]: "Per una cultura seria e veramente fattiva è necessario che questi vengano messi a contatto coi grandi pensatori, e avviati così a conoscere la genesi storica delle idee scientifiche. I poeti sviluppano la loro coscienza in compagnia dei poeti, i mercanti in compagnia dei mercanti, i filosofi dei filosofi. Anche per quella filosofia che è la scienza è tempo di volgersi dai manuali e dalle compilazioni alle fonti."

He also writes:

> The school is not a place in which individual imagination can do what it likes in attempting arbitrary experiments, indeed, the more it aims at grasping the spirits and voices of the society around it, the more it is nourished by the tradition in which it is rooted: not by preserving outdated forms and repeating dead words, but reconnecting . . . past culture to the present, in striving towards the future. And as in school, so too in science. Also for science there is no real progress if new generations do not frame their vision of problems within the continuity of scientific thought, honing their skills in the study of the great models.[91]

The history of science, furthermore, can also constitute an important auxiliary tool for education in making it possible to better understand certain concepts or properties. Here, I will only cite by way of example the use that Enriques himself ~~made~~ of Pythagorean figurate numbers to facilitate comprehension of some arithmetic properties in one of his texts for middle schools:

> If the student is to participate in an active way in this study, he cannot be given definitions and rules without explanations, like gifts rained down from above, which he would not be able to use. . . . The history of science comes to our aid here, showing us how arithmetical truths were recognised by the Pythagoreans by means of the geometric models of numbers, which are the figurate numbers: square and rectangular numbers, triangular numbers, etc.[92]

In discussing Enriques' cultural project, mention must be made of another brilliant exponent of the scientific movement in Italy at the beginning of the twentieth century, Giovanni Vailati, who shared the idea of promoting a scientific *humanitas* and who even proposed creating a unified front of all Italian scientists, especially including Enriques, Volterra and Peano, to fight against the separation of science and philosophy (see [81]). His premature death, and the fact of having underestimated the evident differences in the various methodological and epistemological approaches to mathematics, led to the failure of this project. It is emblematic, for example, that Enriques and Vailati were never able to reach an effective ~~understanding on~~ the nature of logical and philosophical research, even though their ideas regarding the role of philosophy and history within science

[91]See [45, p. 8]: "La scuola non è un campo in cui la fantasia individuale abbia a sbizzarrirsi tentando esperimenti arbitrarii, anzi tanto più è atta ad accogliere gli spiriti e le voci della società circostante, quanto più si alimenti della tradizione in cui anche questa prolunga le sue radici: non già serbando viete forme e ripetendone la morta parola, ma riattaccando . . . il passato al presente della cultura, in uno sforzo verso l'avvenire. E come la scuola la scienza. Anche per questa non vi ha un vero progresso, dove le nuove generazioni non attingano alla continuità del pensiero scientifico la visione dei problemi, facendosi valenti nello studio dei grandi modelli." Enriques had certainly absorbed and interiorised into his own vision of science the teaching of his own mentors Segre and Beltrami. From Beltrami he had received the conviction that study of the history of mathematics can assume "the interest and value of scientific research" and he quotes verbatim [64, p. 11] the invitation to young people to study "the masterpieces of the great masters." See [77].

[92]See [50, pp. IX–XI]: "Se l'allievo deve partecipare in modo attivo a questo studio, non si può dargli definizioni e regole senza spiegazione, come doni piovuti dal cielo, di cui poi quegli che riceve il dono non saprebbe servirsi. . . . La storia della scienza viene qui in soccorso, mostrandoci come le verità aritmetiche siano state riconosciute dai Pitagorici mediante modelli geometrici dei numeri, quali sono i numeri figurati: numeri quadrati e rettangolari, numeri triangolari, ecc."

were quite similar, and they also shared many pedagogical assumptions regarding mathematics teaching. Their correspondence sheds a great deal of light on this, as historical studies have shown.[93]

3 The Battle for a Scientific *Humanitas*: Strategies and Teacher Training

Enriques used several strategies to make his vision of a scientific *humanitas* clear to and accepted by mathematicians, philosophers and teachers. They were aimed in many directions – cultural, institutional and editorial – in addition to the channel of university courses, where research and the history of science were intertwined in a significant way. This is made evident by the registers of lessons given in both Bologna[94] and Rome.[95]

In particular, his efforts and commitment to the training of teachers, and thus more generally to the improvement of mathematics education in secondary schools, are truly remarkable.

3.1 The Textbooks for Secondary Schools

In 1903, Enriques inaugurated a long and successful series of textbooks for secondary schools in collaboration with Ugo Amaldi. This was the year which saw the publication of the very well known textbook *Elementi di geometria* [57], successive editions of which were published up to 1992,[96] and various adaptations released for schools of different levels and specialties: middle and high schools, classical and technical schools, normal (*normali*) schools for primary school-teachers training, and *scuole complementari*. The historical catalogue of Zanichelli, the famous Bologna publisher who brought out all the textbooks Enriques wrote for schools, show that before the Gentile reform (1923) eight different kinds of textbooks were published, while seventeen were published after the reform with

[93] See [92, pp. 559–602; 1, pp. 406–411].

[94] See ASUB, Enriques prof. Federigo. *Fascicolo personale* and Appendix 1.

[95] See ASUR, Facoltà di Scienze Matematiche Fisiche e Naturali, *Libretti delle lezioni*. See also [105, Appendice 4, Le lezioni di Storia delle scienze a Roma].

[96] See also [62], with a preface by G. Israel [84].

various later editions.[97] Most of these are textbooks for geometry, but there are also textbooks for algebra, trigonometry and calculus.

It is worthwhile to describe briefly at least two of these in order to show how Enriques' vision of mathematics teaching was translated into practice. The edition of the 1903 geometry textbook had been carefully prepared from the scientific and methodological points of view with the preliminary publication of *Questioni riguardanti la geometria elementare* (1900), which is often referred to in the notes. Amaldi, who that same year had been appointed professor of algebra and analytic geometry at the University of Cagliari, made good use of the studies undertaken for the *Questioni* on basic concepts of geometry and the equivalence theory, but the methodological vision which underpins the book is, without a doubt, that of Enriques. The preface opens with a clear indication of the method its two authors will follow:

> An elementary geometry textbook must satisfy two sets of needs: the scientific and the didactic. A mistaken idea of scientific rigour leads some mathematicians to believe that the ideal of the science of geometry consists in a systematic exclusion of intuition. According to this premise one would arrive at an abstruse treatment of the elements which would be inaccessible to a beginner and irreconcilable with the educational purpose of geometry. Geometry is a science of observation and reasoning. It should educate young people in both of these faculties. Scientific rigour, as we understand it, has a formative value because it accustoms students to distinguishing between the activity of one faculty and that of the other.[98]

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[97] Before the Gentile reform: *Elementi di geometria* [*scuole normali*], *Elementi di geometria* [*scuole secondarie superiori*], *Elementi di geometria elementare* [*ginnasi superiori*], *Elementi di geometria* [*scuole tecniche*], *Nozioni di geometria* [*ginnasi inferiori*], *Nozioni di geometria* [*scuole complementari*], *Geometria elementare* [*scuole secondarie superiori*], *Nozioni di matematica* [*licei moderni*]. After the Gentile reform: *Elementi di geometria* [*edizione ridotta*], *Elementi di geometria* [*scuole complementari*], *Elementi di geometria* [2 vols. *scuole secondarie superiori*], *Elementi di geometria con esercizi* [*istituti tecnici*], *Elementi di geometria con esercizi* [*edizione ridotta*], *Geometria elementare* [*scuole secondarie superiori*], *Geometria elementare con esercizi* [*edizione ridotta*], *Nozioni di geometria* [*ginnasi inferiori*], *Nozioni intuitive di geometria* [*istituti magistrali inferiori*], *Nozioni di geometria* [*scuola media*], *Algebra elementare* [*ginnasi superiori e corso inferiore degli istituti tecnici*], *Algebra elementare* [*licei classici*], *Algebra elementare* [*corso ordinario degli istituti tecnici*], *Algebra elementare* [*primo biennio dei licei scientifici*], *Complementi di algebra e nozioni di analisi* [*secondo biennio del liceo scientifico*], *Elementi di Algebra* [*scuole medie superiori*], *Elementi di trigonometria piana* [*licei*]. See *Le Edizioni Zanichelli 1859–1939* (Bologna: Zanichelli, 1984) and the *Catalogo storico* on the Zanichelli website (http://www.catalogo.zanichelli.it/Page/t01?siteLang=IT&idp=24).

[98] See [57, p. 1]: "Un trattato elementare di geometria deve soddisfare a due ordini di esigenze: scientifiche e didattiche. Un falso concetto del rigore scientifico, fa ritenere a taluno che l'ideale della scienza geometrica consiste nel bandire sistematicamente l'osservazione intuitiva, onde si sarebbe condotto ad una trattazione astrusa degli elementi, inaccessibili ai principianti, ed inconciliabili collo scopo educativo della geometria. La geometria è scienza d'osservazione e di ragionamento; essa deve educare nei giovani queste due facoltà. Il rigore scientifico come lo intendiamo noi, serve allo scopo educativo, insegnando a discernere l'esercizio dell'una dall'esercizio dell'altra."

The subject is presented using a "rational-inductive" method, with the aim of avoiding the shortcomings typical of Euclidean-style exposition, which by "presenting propositions which are analysed at length in their logical connections and coordinated in a deductive system, hides the process of discovery under a rigidly dogmatic framework" [35, p. 24].[99] The procedure is as follows: beginning with a series of observations, the authors enunciate certain postulates from which the theorems that depend on them are developed by logical reasoning; from these theorems, they then continually return to observations or intuitive explanations. In this case as well Enriques acknowledged Klein's influence; in fact, he wrote to him:

> I am sending you a copy of the 2nd edition of my *Elementi di geometria*. In the explanation of a method which, while remaining rational, lays emphasis on the inductive aspects, you will recognise the influence of your own ideas and our conversations in Göttingen.[100]

Among the textbooks that Klein would mention in his essay on geometry teaching in Italy, *Der Unterricht in Italien*, he refers to the *Enriques–Amaldi*, which he praises for having taken didactic requirements into consideration, thus reconciling logical rigor and intuition [91, *II*, pp. 245–250]. Similar praise is found in the long, in-depth review of the textbook written by Vailati, who goes so far as to observe that some of the theorems whose proofs lead to conclusions which for the student are no less evident than the postulates they use, could have been stated in the form of a postulate, because the student has to learn *"as soon as possible* to see in the process of demonstrating a means to go from the known to the unknown."[101] In contrast, Beppo Levi was not in agreement with the "philosophical" part; he believed that too much emphasis had been placed on observation and experience in the explanation of geometric concepts, and he was equally unenthusiastic about the approach to the theory of congruence, which was developed in part by following Hilbert's formulation.[102] In fact, Enriques and Amaldi had assumed the notion of congruence as a primitive for segments and angles, and used movement, intended as a "physical operation," to explain its meaning and check its first properties. They then define it case by case for the more complex figures as they arise. Particular attention is given to the constructions and use of the instruments for making them in order to achieve the aim of "stirring up in young people the spirit of geometrical research" [57, p. 5]. The textbook is supplemented with some 600

[99] See [35, p. 24]: "…presentando coordinati in un sistema deduttivo dei resultati lungamente analizzati nei loro rapporti, nasconde sotto la forma dommatica il cammino della scoperta."

[100] See Enriques to Klein, 10 January 1905, SUB Göttingen, *F. Klein* 34: "Le invio una copia della 2a ediz.e dei miei Elementi di geometria. Nell'avviamento ad un metodo che, pur essendo razionale, accentua il carattere induttivo, Ella potrà riconoscere una influenza delle sue idee e delle conversazioni di Gottinga." See also Appendix 2.

[101] See [124, p. 24]: "…il più presto possibile a vedere nel processo di dimostrazione un mezzo per passare dal noto all'ignoto."

[102] See the letter of B. Levi to U. Amaldi, Piacenza, 19 October 1902 [103, pp. 28–31]. For details regarding technical aspects of the textbook, and for a comparison with textbooks of the time, see [73, pp. CXV-CXIX]. See also [84, 125] and [122].

exercises, between problems to solve and propositions to prove. In later editions, the manual was gradually refined and simplified, especially in the parts regarding equivalence and proportions; above all, the texts were enriched with numerous notes about history of mathematics.[103] In some cases, as mentioned earlier, the history of mathematics is also used in order to facilitate understanding of certain concepts. Moreover, in the textbooks for middle schools and normal schools, frequent use is made of experiments with folded or cut paper, sand, or small models.[104]

Another textbook which became a classic was the two-volume *Nozioni di matematica ad uso dei licei moderni* (1914–1915), written with Amaldi for use in the modern secondary school instituted by the Minister of Education Luigi Credaro in 1911. The mathematics programs, formulated by Guido Castelnuovo, introduced the concepts of function, derivative and integral, and gave greater emphasis to numeric approximation.[105] The *Nozioni di matematica* opens with a chapter on approximate measures and irrational numbers, discusses the calculation of areas and volumes from an elementary point of view, establishing connections between geometry and algebra, introduces the concept of function with ample use of grid paper, presents the elementary functions, and trigonometry with particular attention to practical problems, and introduces the concepts of limit, derivative and integral. With respect to the manuals of geometry, this one reveals other characteristic aspects of Enriques' vision of mathematics education. First of all, the various theories are seen as parts of a single organism, and thus the authors try to re-establish the unity of mathematics, making evident the connections between the various branches, especially algebra and geometry in keeping with their historic development; they "abolish the boundary" (vol. 1, p. III) between elementary and higher mathematics and between mathematics and the other sciences, from which are drawn problems, exercises and examples, especially to illustrate the concept of function. The history of mathematics makes its appearance in some digressions intended to show how science had moved forward, as in the note on the history of *pi* from the Egyptians to Lindemann (vol. 1, pp. 35–36). It is also used as a means of approach to certain concepts; for example, to calculate the volumes of the pyramids, of the cone, and the sphere, the authors "set forth in an elementary fashion the classical procedure of integration used by the precursors of infinitesimal calculus, which goes back to Archimedes" (vol 1, p. V).

[103] See, for example, [59] and [60].

[104] See, for example, [61, pp. 64, 78, 88–89, 96–97]. The importance of developing the students' faculty of intuition "with *drawing*, with *cutting*, and with *folding paper*, with the *construction of solid models*" starting in the very first grades of teaching is explicitly stated in [65].

[105] See [75, pp. 6–8], and the website [76].

3.2 *The Initiatives of the First Decades of the Twentieth Century*

The first two decades of the twentieth century were extremely busy for Enriques. He took an active part in the congresses of the Italian National Federation of Middle School Teachers (the *Federazione Nazionale Insegnanti Scuola Media*, or FNISM) beginning with the first one in Florence in 1902; at the fifth one in Bologna in 1906 he spoke on the topic of teacher training. From 1912 to 1915, he was president of the Italian National Association of University Professors, and presented a project for a university reform. From 1908 to 1920, he was one of the Italian delegates, together with Guido Castelnuovo and Giovanni Vailati, to the International Commission on Mathematical Instruction (ICMI) under Klein's presidency.[106] In 1906, Enriques was one of the founders of the reorganised Zanichelli publishing house, with which he collaborated not only by publishing his own works but also by soliciting publications by esteemed scientists (see [66]). In that same year, he founded the Italian Philosophical Society, and was its president until 1913; in that capacity he organised and presided over the fourth international congress of philosophy, which took place in Bologna in 1911, and provoked the well-known, harsh criticism of Croce and Gentile.[107]

The idea of bringing together philosophy and mathematics was not the product of a extemporaneous improvisation on the part of Enriques; rather, it was the primary concept underlying an entire intellectual movement in Europe, one that spread above all in France, and which found an expression in the first four international congresses of philosophy.[108] As we shall see, Enriques declared more than once that this movement should influence the ordering of schools and universities. For example, it is emblematic that Enriques wanted to organise an international meeting of philosophy in Paris in 1914 to coincide precisely with the congress of the International Commission on Mathematical Instruction (1–4 April 1914).[109]

In 1907, together with Eugenio Rignano, he founded the *Rivista di Scienza* (*Scientia* from 1910 on), "international organ of scientific synthesis," aimed at fighting the excessive specialisation in the field of science and putting an end to the hegemony of literary and historic studies (see [93]).

Here, we will mainly focus on the initiatives directly aimed at improving mathematics teaching and teacher training.

Enriques' position on institutional ways of providing an adequate scientific and educational training for teachers emerges from the report prepared on the occasion of the fifth congress of FNISM in 1906 [29]. In a rather long introduction, he presented his vision of scientific teaching, and his idea of a philosophical university based on the German model, which makes possible "the free and full development

[106] See [74], and L. Giacardi, *Timeline* 1908–1910, in the website [70].

[107] See for example [80, 94, 110, 112, 120, and 105, pp. 139–150].

[108] See for example, [114].

[109] See the letters of Enriques to Xavier Léon in [115, pp. 311–315].

of all the elective affinities among the various branches of knowledge" (p. 73). He then suggested the establishment of a *pedagogical degree* in addition to the *scientific degree*: the first two years of study would be dedicated to acquiring basic knowledge of the discipline, and by the end of that time, a distinction would be made between those who intended to dedicate themselves to research and those who wanted to teach. For the future teachers, the next two years would take place in the Scuole di Magistero and would be aimed at providing professional training by means of "(1) courses on those parts of science that aim at a more profound understanding of the elements, (2) lectures on concrete questions of pedagogy that interest the various areas of teaching, particular in relation to the analysis of the textbooks, (3) exercises comprising practice teaching, partly in the university and partly in secondary schools, drawing, and experimental technique" (p. 78).[110] He further expressed his hope that those called to teach in the Scuole di Magistero would include all the professors of the scientific faculties and the best of the secondary school teachers; he also proposed that the selection of teachers be based on the results of a competition comprising both written and oral exams in order to make evident the candidates' attitudes towards science and education.

Enriques' proposals, as he himself emphasised at the beginning of his presentation, were directly related to his project for university reform,[111] which had grown out of the ascertainment of the defects of the Italian university system. Above all he criticised the lack of interaction between the various faculties, the excessive fragmentation, and the separation of disciplines with programs that were obligatory and too heavy:

> Heaven help you if you pass from one laboratory to another, interrupt the process to meditate or study, or worse still, to attempt research that goes beyond the limits set in the definition of the chair!
> The rash one who dares set foot in new territory, investigating the relationship between two *different* disciplines, knows well the fate that awaits him.[112]

In addition, the tendency of each professor to defend his own discipline favoured the pre-eminence of already consolidated areas of research over those which were interdisciplinary or unexplored, with serious repercussions for research, teaching and the work world:

[110]See [29, p. 78]: "(1) corsi su quelle parti della scienza che si riattaccano ad una più profonda visione degli elementi, (2) conferenze sulle questioni di pedagogia concreta che interessano i varii rami d'insegnamento, particolarmente in rapporto colla critica dei testi (3) esercitazioni comprendenti il tirocinio parte nell'università e parte in una scuola secondaria, il disegno e la tecnica sperimentale."

[111]See the two articles [31, 32] and Enriques' paper in *Atti dell'Assemblea della Associazione nazionale fra i professori universitari*, Torino, 1911, pp. 122–141; rpt. in [121, pp. 91–132].

[112]See [121, p. 99]: "Guai a passare da un laboratorio ad un altro, a interrompere la produzione per meditare o studiare, o peggio ancora per tentare ricerche che oltrapassino i limiti stabiliti nella definizione delle cattedre! Il temerario che si sarà avventurato sopra un terreno nuovo, indagando i rapporti fra due discipline diverse, sa bene quale sorte l'attenda."

> Now all of these deficiencies and difficulties are directly reflected in middle school teaching . . . The exaggerations of rigour – in the form of minutia and senseless pedantry – in schools of mathematics, the empiricism of physics teaching . . . , the morphological erudition that suffocates the natural sciences . . . , all these defects – so often lamented – are in direct correlation with the conditions of the university training of middle school teachers.[113]

He also predicted that future workers would be "devoid of initiative, . . . ready at any moment to take refuge in the excuse of procedure and the observance of form."[114]

Enriques, in accordance with his strategy, explained his point of view to the philosophers of the first philosophical congress in Milan in 1906, and later that same year to the middle school teachers in Bologna, to the mathematicians and scientists in his 1908 article in the *Rivista di Scienza*, and finally to university professors in 1911. The solution he proposed was that of conjoining in a single faculty of philosophy all of the theoretical disciplines: mathematics, physics, physiology, history, law, economy, etc. He also proposed the institution of "special schools of Application" which were to group together professional teaching aimed at a specific career, the polytechnical schools for engineers and the polyclinical schools for physicians, and the Scuole di Magistero for the training of teachers. With more specific regard to the programs, the courses and the examinations, Enriques believed that it was necessary to "reduce the science to be learned to a minimum" (in [121, p. 97]); to give the students the freedom to choose the courses to attend within a given number established by the faculty, which, however, would be responsible for guaranteeing the reliability and coherence of the courses; and to introduce a different way of testing what knowledge had been acquired and the capacity for putting it to use.

To sum up, Enriques wrote:

> The reform of the Italian university
>
> (1) Must correspond to the synthesis required by renewed philosophical consciousness and practical life, as opposed to the scientific-educational particularism of the previous era
> (2) Must give new life to the spirit of initiative of our universities, promoting their free differentiation
> (3) Must sanction the principle of the freedom of study and, emancipating young people from the weight of formal erudition, prepare them for professions and for life through a more active exertion of their faculties.[115]

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[113] See [121, p. 94]: "Ora tutte queste deficienze ed angustie si rispecchiano direttamente nell'insegnamento medio, . . . Le esagerazioni del rigore – sotto forma di minuzie e di pedanterie senza scopo – nelle scuole di Matematica, l'empirismo dell'insegnamento fisico . . . , l'erudizione morfologica che soffoca i corsi di scienze naturali . . . , tutti questi difetti – spesso lamentati – sono in correlazione diretta colle condizioni della preparazione universitaria dei docenti delle scuole medie."

[114] See [121, p. 96]: ". . . fiacchi, . . . pronti a rifugiarsi ogni momento nelle scuse della procedura e nell'osservanza della forma."

[115] See [121, p. 114]: "La riforma dell'Università italiana 1) deve corrispondere alle esigenze sintetiche della rinnovata coscienza filosofica e della vita pratica, avverso il particolarismo scientifico-didattico dell'epoca precedente; 2) deve ravvivare lo spirito d'iniziativa dei nostri

From the same need to combat excessive specialisation was born the *Rivista di Scienza*, which Enriques co-directed with Rignano until 1915, and then again from 1930 to 1938, when he had to quit because of the racial laws. The vision underlying the *Rivista* was that of a scientific philosophy which, "free of direct ties to traditional systems, arises to promote the coordination of work of science, the criticism of its methods and theories, and to assert a broader appreciation of its problems."[116] It was precisely for this reason that from the very beginning the *Rivista* had an international dimension: it came out in two editions, one Italian, the other foreign, which was distributed by prestigious publishers, William & Norgate in England, F. Alcan in France, and W. Englemann in Germany. Moreover, thanks to his personal prestige, Enriques was able to count on the collaboration of well known scholars – mathematicians, physicists, chemists, geologists, historians of science, sociologists, linguists, economists – including, just to name a few, Einstein, Mach, Michelson, Ostwald, Picard, Russell, and Volterra. Enriques himself, from 1907 to 1938, wrote twenty-three articles and critical notes, sixty-three reviews and twenty-five surveys of journals. His imprint is particularly noticeable in the early years, and it is no coincidence that in addition to aspects of history, philosophy and methodology, attention was also given to aspects of education: in 1907, articles appeared by G. Castelnuovo, J. Tannery, T. Bonnesen; in 1908 were published Enriques' own articles on university reforms [31, 32] and his review of the book by A. Galletti and G. Salvemini entitled *La riforma della scuola media* (1908); between 1913 and 1915 there were three reviews by G. Scorza of works aimed at mathematics teaching in secondary schools; and in 1915 there was another article by Enriques on the art of writing a mathematics treatise.

3.3 Enriques' *Mathesis* Presidency and Direction of the *Periodico di Matematiche*

In 1919, Enriques was nominated president of the *Mathesis* association, a position he held until 1932. Then, since in 1921 the *Periodico di Matematiche* had gone back to being the association's publishing venue, he assumed its direction together with Giulio Lazzeri. The imprint of the fourth series, which began with the 1921 volume, is exquisitely Enriques', starting with the title – *Periodico di Matematiche. Storia-Didattica-Filosofia* – and from the introductory sentence that appears on the inside of the front cover of each issue of the journal:

Atenei, promuovendone la libera differenziazione; 3) deve sancire il principio della libertà degli studi ed, emancipando i giovani dal peso di un'erudizione formale, prepararli alle professioni ed alla vita con un esercizio più attivo delle loro facoltà."

[116] See ["Preface"], *Rivista di Scienza*, 1, 1907, pp. 1–3, at p. 2: "... libera da legami diretti con i sistemi tradizionali, sorge appunto a promuovere la coordinazione del lavoro, la critica dei metodi e delle teorie, e ad affermare un apprezzamento più largo dei problemi della Scienza."

> The *Periodico* publishes above all articles regarding elementary mathematics in a broad sense, and others that tend towards a wider comprehension of the spirit of mathematics. It also contains reports on movements in mathematics abroad, notes on bibliographies and treatises, miscellany (problems, games, paradoxes, etc.) as well as news of a professional nature, and finally, the Proceedings of the Italian Matematical Society "Mathesis."[117]

According to Enriques' project, the *Periodico* was intended to disseminate the idea of mathematics as an integral part of the philosophical culture, an idea he had always supported, as well as to fill the gap that existed in scientific education at that time in Italy. In the letter to the readers that opened the 1921 issue, he presented an actual working program for the journal, which was at the same time a working program for teachers. The cardinal points are: teachers should study the science that they are teaching in depth from various points of view, so as to master it from new and higher points of view, and thus make evident the connections between elementary mathematics and higher mathematics; use the history of the science seeking to attain, not so much erudite knowledge as a dynamic consideration of concepts and theories, through which students can recognise the unity of thought; bring out the relationships between mathematics and the other sciences, and physics in particular, in order to offer a broader vision of science and of the aims and meanings of the many different kinds of research [40, pp. 3–4].

This open letter was followed by his famous article, "Insegnamento dinamico" [38], which is almost a manifesto of Enriques' working program, and of his particular vision of mathematics education: active teaching, Socratic method, learning as discovery, the right balance between intuition and logic, the importance of error, the historic view of problems, the connections between mathematics and physics, elementary mathematics from an advanced standpoint, and the educational value of mathematics.[118] A look through the issues shows above all an increase in the number of articles about physics and history of physics (mostly written by Enrico Persico, Umberto Forti and Enrico Fermi), and those dealing with history of mathematics and science in general: the principal collaborators are the mathematics historians Ettore Bortolotti, Gino Loria and Amedeo Agostini, but there are also contributions by Ugo Cassina, Giulio Vivanti, Alpinolo Natucci (a secondary school teacher in Pisa), Emilio Artom (a secondary school teacher in Torino), and Maria Teresa Zapelloni, among others. Noteworthy are the articles written by Oscar Chisini, which clearly show Enriques' influence. Enriques had made Chisini editorial secretary of the *Periodico* in 1921, and it was Chisini who, after Enriques' death, succeeded him as director. Chisini's articles mostly concern the elementary aspects of mathematics which show the connections with the recent progress in mathematics, making

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117 "Il Periodico pubblica soprattutto articoli riguardanti le matematiche elementari intese in senso lato, ed altri tendenti ad una più vasta comprensione dello spirito matematico. Esso contiene inoltre relazioni del movimento matematico straniero, note di bibliografia e di trattatistica, varietà (problemi, giuochi, paradossi, etc.) nonché notizie di carattere professionale, ed infine gli Atti della Società Italiana di matematiche 'Mathesis'."

118 See the new edition [56] accompanied by essays by F. Ghione and M. Moretti published by the Centro Studi Enriques.

reference to history and to educational aspects. Enriques himself wrote no fewer than twenty-seven articles and brief notes, and thirty-four reviews, most of which regarded history of science or mathematics teaching.[119]

The desire to open up to other sciences is also evident in the new charter for the Mathesis association, which, on 7 May 1922, welcomed teacher of physics into its ranks, and led the society to assume a new name: *Società italiana di scienze fisiche e matematiche "Mathesis"*. Under the leadership of Enriques, the number of members in 1920 grew from 775 to 895; by 1924 there were more than 1, 200. During his presidency, the society organised six national congresses (Trieste, 1919; Naples, 1921; Leghorn, 1923; Milan, 1925; Florence, 1929; ~~Milano~~, 1931). The congresses of 1929 and 1931 were organised in collaboration with the Italian Society for the Progress of the Sciences (SIPS),[120] directed at that time by phycisist and geologist Gian Alberto Blanc. The SIPS had also the aim of contrasting excessive specialisation and stimulating interdisciplinary dialogue, but its project was complementary to that of Enriques because it was primarily addressed to the world of technology and industry.

The inaugural lectures that Enriques gave at the congresses were all aimed at upholding the educational and cultural value of mathematics and the sciences.[121] The problems that he had to grapple with were not simple, not least because they were contingent on historical and political situations, but a strong point of Enriques was his constant attention to the opinions of the teachers and the various local sections of the association, as emerges for example from the unpublished correspondence with Giacomo Furlani, president of the Trieste section.[122] In particular, after the first World War, it was necessary to solve the delicate problem of how to harmonise the mathematics programs of the provinces of Trento and Trieste, recently annexed from Austria, with those of the Kingdom of Italy.[123] After the advent of Fascism, it was necessary to address the problems related to the Gentile reform: the devaluation of mathematics and of sciences in general, the reduction of the number of teaching hours, the combination of mathematics and physics, and teacher training.

[119]On the historiography of mathematics in Italy see [20].

[120]See "Congresso della Società Italiana *Mathesis*," *Periodico di matematiche*, (4) 11, 1931, pp. 322–325. See also *Atti della Società Italiana per il Progresso delle Scienze, Firenze 18–25 Settembre 1929*. Roma 1930, and *Atti della Società Italiana per il Progresso delle Scienze, Milano 12–18 Settembre 1931*. Roma 1932. Enriques had already interacted with the SIPS at the beginning of the twentieth century; see [105, pp. 134–139].

[121]Enriques gave the following inaugural lectures: Trieste, 1919: *Il valore delle Matematiche nella Filosofia italica* [37]; Naples, 1921: *Evoluzione del concetto della Scienza nei pensatori matematici* [42]; Livorno, 1923: *Il significato umanistico della scienza nella cultura nazionale* [43]; Milan, 1925: *L'essenza della matematica* (see [*Periodico di Matematiche*, (4) 1, 1925, p. 378]; Florence, 1929: *La geometria non-euclidea e i presupposti filosofici della Teoria della Relatività* [47].

[122]See Appendix 3.

[123]See Giacardi, L., ed. 2006. Da Casati a Gentile. *Momenti di storia dell'insegnamento secondario della matematica in Italia*. Lugano: Lumières Internationales, pp. 47–53 and [127].

In 1923, in the space of a single year, Giovanni Gentile, minister for education, put into effect a complete and systematic reform of the Italian scholastic system in keeping with neo-idealist philosophy. Secondary education was divided into two branches: classical-humanistic and technical-scientific. The classical-humanistic branch was intended to train the ruling class and was considered overwhelmingly superior to the scientific-technical one, which, moreover, made access possible to only a limited number of university degrees. The principles of Fascism and neo-idealist ideology were opposed to the widespread diffusion of scientific culture and, above all, to its interaction with other cultural sectors. Humanistic disciplines were to form the main cultural axis of national life and, in particular, of education; it was symptomatic that even the courses of history of science introduced into the scientific high schools were taught by philosophers.[124] In addition, Gentile, who identified knowledge with knowing how to teach, paid no attention at all to professional training of teachers. This point of view was, of course, opposed to the scientific *humanitas* to which Enriques aspired. As president of the Mathesis Association, he engaged in intense negotiation with Gentile, both before and after the law on secondary education was enacted, in the hope of avoiding the devaluation of science teaching. However, the pleas of the Mathesis fell on deaf ears.[125] Unlike Vito Volterra and Guido Castelnuovo, who were in absolute opposition to the Gentile Reform, Enriques assumed and maintained a conciliatory position. In fact, he agreed with Gentile on many points: he was convinced that among the various kinds of secondary schools, those which best performed the function of education were the *ginnasi-licei* (classical schools); he conceived of knowledge as a personal conquest; he was in agreement with the need to fight encyclopaedism and he considered education to be the free and unfettered development of inner energy. Moreover, he did not want to renounce his idea of the fusion of scientific knowledge and humanistic idealism which was the basis of the cultural program he had dedicated his whole life to: the creation of a scientific *humanitas* which would express and make manifest the universality of human reason.[126]

Enriques' position emerges clearly from his correspondence with Gentile [80], as well as from the report on the reform which he prepared for the ICMI in 1929 [48]. His account appears less critical than might be expected: he limited himself to pointing out the reduction in the number of the hours devoted to mathematics, and the unsolved problem of teacher training. Instead, he gave ample space to the flourishing of new textbooks, citing the manuals which he himself had written with Amaldi and two series, one directed by Roberto Marcolongo and Onorato Nicoletti, the other by Francesco Severi. He also presented his many initiatives aimed at teacher training: in addition to the *Questioni riguardanti le matematiche elementari*, of which the third edition had just been published, he cited the school

[124]See "Atti della Società Italiana di Scienze fisiche e matematiche "Mathesis". Relazione del Congresso di Milano," *Periodico di matematiche*, (4) 5, 1925, pp. 374–383, at p. 383.

[125]This is discussed in greater detail in Giacardi, L., ed. 2006. Da Casati a Gentile. cit., pp. 54-63.

[126]See [43, p. 4]. See also [80, 83, 94, 112].

for specialisation in history of the sciences annexed to the *Istituto Nazionale per la Storia delle Scienze*, which he had created after the Gentile reform and the book series on the history of mathematics, *Per la storia e la filosofia delle matematiche*, which he had begun in 1925 and was expressly intended for teacher training. These are precisely the ~~initiative;~~ we will examine in the next section.

3.4 *The Roman Initiatives and Teacher Training*

In December 1920 Alberto Tonelli died; Tonelli had held the chair in algebraic analysis at the University of Rome. Many were interested in succeeding him, including Enriques and Severi. In the end, it was Severi who prevailed; Enriques' being called to Rome was only made possible thanks to Castelnuovo's having given up the chair in higher geometry, as has been recently shown.[127] In fact, Enriques, who had been called for a temporary position at the University of Rome in 1921–1922 to "teach lessons in mathematics for the [Scuola di] Magistero," and in 1922–1923 to teach the then newly established course in complementary mathematics, had not even been successful in obtaining a transfer to the chair of complementary mathematics.[128]

His lessons and related practical exercises in complementary mathematics of that year are the translation into practice of Enriques' way of conceiving teacher training: the history of mathematics is interwoven with the mathematical theory, elementary mathematics are linked to higher mathematics; mention is made of the theory of relativity, and a comparative examination is proposed of textbooks.[129] With regard to this course he wrote to Gentile:

> I should add that the difference between this course and the other two in advanced mathematics given during our second biennium (higher analysis and advanced geometry) is this: that here come into play precise arguments – such as the problems of the trisection of the angle and the squaring of the circle, etc. – which we believe the teacher needs to know about, and which cannot be dealt with in courses in higher analysis and advanced geometry, the only ones in our university which are aimed at pure mathematics! . . . Further, by means of those problems that are closer to elementary mathematics and which have a history that is twenty centuries old, we aim to reach young people with a vocation for teaching, who . . . must be protected from the risk of becoming mechanical propagators of a culture that they have received from outside and is truly foreign to their spirit: this is a conclusion to

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[127] See [105, Appendice 2, *Il trasferimento di Enriques a Roma*].

[128] See the documents of 17 February 1922, 4 September 1922 in ASUB, *Enriques prof. Federigo. Fascicolo personale*, and the document of 30 December 1922 in ASUR, *Enriques, Federigo. Fascicolo personale*.

[129] See the *Libretto delle lezioni di Matematiche complementari* and the *Libretto delle esercitazioni di Matematiche complementari* of Enriques, 1922–1923, ASUR *Facoltà di Scienze Matematiche Fisiche e Naturali. Libretti delle lezioni*. See also Appendix 1.

> which you have arrived by means of metaphysical premises, but to which I have also – as far as my powers allow – contributed with the actions of my life.[130]

History of mathematics was also used, sometimes quite extensively, in the lessons in higher geometry,[131] and its centrality in Enriques' program is clearly shown by the many initiatives that went hand in hand with his leadership of the Mathesis association during this period. Effectively the campaign Enriques was conducting amounted to a genuine battle aimed at projecting an image of science both as a unified whole and as an integral part of culture.

In 1923, he founded the *Istituto nazionale per la Storia delle Scienze fisiche e matematiche*, with the aim of giving an impetus to studies in the history of the physical and mathematical sciences, and in particular, to promote:

> the collection, in some of the most suitable centres, of books and documents that are necessary for the pursuit of serious and wide-reaching research projects; the diffusion of research ..., the arrangement and publication of unpublished manuscripts ..., the publication ... of works either classic or representative of some special interest.[132]

In connection with the Rome Institute, the following year Enriques founded the *Scuola universitaria per la Storia delle scienze*, annexed to the University of Rome, whose threefold aim was to provide incentives for historical research, train future teachers, and promote the consolidation of the idea of scientific *humanitas*. In 1924–1925, Enriques taught a course on the history of scientific concepts, while Giovanni Vacca taught one on history of mathematics; the next year Enriques and Vacca taught the same courses again, broadening and enriching them with new material, while Aldo Mieli taught history of chemistry, Federico Raffaele gave lectures on the evolution of cellular theory, Silvestro Baglioni taught history of medicine and Roberto Almagià taught history of geography. In the following years,

[130] See the letter of Enriques to Gentile, Rome, 23 December 1922 [80, pp. 149–150]: "Aggiungo che la differenza fra questo corso e gli altri due di matematiche superiori del nostro secondo biennio (analisi superiore e geometria superiore) è questa: che qui entrano argomenti precisi – come i problemi della trisezione dell'angolo o della quadratura del cerchio, ecc. – intorno a cui si ritiene che l'insegnante debba essere informato, ed a cui non si può costringere i corsi di analisi superiore e di geometria superiore, i soli che mirino presso di noi alla pura scienza matematica! ... Inoltre attraverso quei problemi che toccano più da vicino le matematiche elementari e che hanno una storia venti volte secolare, si mira soprattutto ai giovani chiamati all'insegnamento, i quali ... debbono essere preservati dal pericolo di diventare ripetitori meccanici di una cultura ricevuta dal di fuori e però estranea veramente al loro spirito: che è una tesi a cui Ella giunge da premesse metafisiche, ma a cui io ho pur dato da parte mia – nella misura delle mie forze – il contributo dell'azione della mia vita."

[131] See [105, Appendice 4].

[132] See "Istituto Nazionale per la Storia delle Scienze fisiche e matematiche," *Periodico di matematiche*, (4), 3, 1923, pp. 149–153, at p. 151: "... la raccolta, in alcuni centri più adatti, dei libri o dei documenti che occorrono per proseguire serie e larghe ricerche; la divulgazione delle ricerche ...; l'ordinamento e la pubblicazione di manoscritti inediti ...; la pubblicazione ... di opere classiche o rappresentanti qualche speciale interesse." For the later fusion of the *Istituto* with Aldo Mieli's *Federazione nazionale fra le Società, gli Enti, gli Insegnanti ed i Cultori di Storia della Scienze*, see [105, pp. 160–163].

the courses were almost all continued, and in 1934–1935 were added courses in history of astronomy taught by Pio Emanuelli and history of biology taught by Giuseppe Montalenti; in that same year, Ettore Carruccio and Attilio Frajese joined the school as volunteer assistants.[133] In order to bolster and consolidate the school, in a lecture at the Accademia dei Lincei in 1938 Enriques asked for the institution of a chair in history of mathematics:

> A minister who is a philosopher …had the merit of understanding the educational and didactical value of the history of science, and to introduce its teaching in several orders of Italian middle schools [but there being no] adequate preparation of teachers, his reform could not be carried out seriously. But the idea remains, and more than the idea, the incumbent duty to translate it into action [54, p. 134].[134]

Enriques' proposal was thwarted by Bortolotti and by Severi.[135] In any case, in that same year the the racial laws excluded Enriques from teaching. In 1938–1939 the course in history of mathematics was taught by Fabio Conforto, who had helped Enriques with this course the previous year and was then collaborating with him on the treatise *Le superfici razionali* (1939). Baglioni taught a course in the history of discoveries in biology and physiology, and Adalberto Pazzini taught history of medicine.[136] In February 1939,[137] Severi was named director of the School, and this marked the end of an important period for the history of science in Italy.

Among the initiatives collateral to the School, two deserve special mention. The first was the book series created in 1925 entitled *Per la storia e la filosofia delle matematiche*, the second was the *Settimana della Scuola di Storia delle scienze* organised by Enriques and his collaborators in 1935 in Rome.

The idea for the book series had been suggested to him "from practical experience in the Scuola di Magistero" [45, p. 7]; primarily intended for a readership of educators, it also aimed at students and educated people in general. Twelve volumes were published from 1925 to 1938:[138] a look at the titles shows that

[133] For more about Enriques' courses, see [105, "Appendice 4"]; for the others see ASUR, *Facoltà di Scienze Matematiche Fisiche e Naturali. Libretti delle lezioni*, 1924–1941.

[134] See [54, p. 134]: "Un ministro filosofo …ha avuto il merito di comprendere il valore educativo e didattico della storia della scienza e d'introdurne l'insegnamento in alcuni ordini della scuola media italiana [ma, mancando] un'adeguata preparazione degli insegnanti, la sua riforma non ha potuto essere ancora seriamente attuata. Ma l'idea rimane; più che l'idea il dovere incombente di tradurla in atto." See also the letter of Enriques to Gentile dated 20 December 1924 in [80, pp. 151–153].

[135] On Enriques' relation with Severi and with Fascism, see for example [102] and the essays of E. Vesentini, C. Ciliberto, A. Brigaglia and S. Linguerri in [111].

[136] See ASUR *Facoltà di Scienze Matematiche Fisiche e Naturali. Libretti delle lezioni*, 1938–1939.

[137] See the letters of Pietro De Francisci to Francesco Severi, Rome 16 February 1939 and 13 January 1943, ASUR *Personale docente. Severi, Francesco.*

[138] 1) F. Enriques (ed.), *Gli Elementi d'Euclide e la critica antica e moderna*, (Libri I–IV) (Rome: Alberto Stock, 1925); 2) J. L. Heiberg, *Matematiche, scienze naturali e medicina nell'antichità classica*, Gino Castelnuovo, trans. (Rome: Alberto Stock, 1924); 3) F. Enriques and U. Forti (eds), *I. Newton: Principii di Filosofia naturale, teoria della gravitazione* (Rome: Alberto Stock,

Enriques particularly favoured translations with commentaries, often accompanied with historical notes, of works by important authors of the past (Euclid, Archimedes, Bombelli, Galileo, Newton, Dedekind, etc.) which might be of relevance to mathematics teaching. Collaborators on the book series included colleagues, followers, students and friends working in various areas: Ettore Bortolotti, Guido Castelnuovo, Umberto Forti (professor of mathematics in secondary schools and historian of science), Amedeo Agostini, Oscar Zariski, Enrico Rufini (teacher at *Liceo Tasso* in Rome), Ettore Carruccio, Attilio Frajese, Maria Teresa Zapelloni, Gino Castelnuovo (son of Guido Castelnuovo, at that time a student in the school for engineering in Rome), Maria Lombardini (from the geophysics observatory at Rocca di Papa), Guido Rietti, Ruth Struik (wife of Dirk Struik). The first volume was dedicated to the first four books of Euclid's *Elements*, a text which, in Enriques' opinion, all teachers should know. The second was Gino Castelnuovo's Italian translation of the Danish Johan Ludvig Heiberg's treatise on mathematics, natural science and medicine in antiquity. Heiberg embodied Enriques' ideal of the historian of science: a philologist with profound knowledge of the sources, but capable of "hiding all burdensome erudition," thus writing inspired, panoramic works capable of shedding light on the relationships between science of the past with contemporary and later culture, and ready to collaborate with scholars in other fields. In Enriques' words:

> ...it is well known that Heiberg worked, especially in the history of mathematics, alongside the great geometer Zeuthen, and Zeuthen alongside Heiberg, with a communion of spirit that constitutes a splendid example of collaboration between scholars differently trained, and thus to the benefit of both, and above all fortunately for our knowledge.[139]

The importance that Enriques attributed to the history of science in teacher training is twofold: it not only helps to understand the genesis of the ideas and problems, but also makes it possible to participate in scientific research:

> The training of mathematics teachers who are capable of carrying out their educational responsibilities requires, generally speaking, that they understand science not only in its

1925); 4) E. Rufini, *Il Metodo di Archimede e le origini dell'analisi infinitesimale nell'antichità* (Rome: Alberto Stock, 1926); 5) O. Zariski (ed.), *Riccardo Dedekind: Essenza e significato dei numeri. Continuità e numeri irrazionali* (Rome: Alberto Stock, 1926); 6) M. Lombardini (ed.), *A. C. Clairaut: La teoria della forma della terra dedotta dai principi dell'idrostatica* (Bologna: Zanichelli, 1928); 7) E. Bortolotti (ed.), *L' Algebra, opera di Rafael Bombelli da Bologna, Libri IV e V comprendenti "La Parte geometrica" inedita tratta dal manoscritto B. 1569, Biblioteca dell'Archiginnasio di Bologna* (Bologna: Zanichelli, 1929); 8) F. Enriques (ed.), *Gli Elementi d'Euclide e la critica antica e moderna*, (Libri $V - IX$) (Bologna: Zanichelli, 1930); 9) U. Forti, *Introduzione storica alla lettura del "Dialogo sui massimi sistemi di Galileo Galilei"* (Bologna: Zanichelli, 1931); 10) F. Enriques (ed.), *Gli Elementi d'Euclide e la critica antica e moderna*, (Libro X) (Bologna: Zanichelli, 1932); 11) F. Enriques (ed.), *Gli Elementi d'Euclide e la critica antica e moderna*, (Libri $XI - XIII$) (Bologna: Zanichelli, 1936); 12) G. Castelnuovo, *Le origini del calcolo infinitesimale nell'era moderna* (Bologna: Zanichelli, 1938).

[139] See [82, pp. 6–7]: "...è ben noto come Heiberg abbia lavorato, particolarmente nella storia delle matematiche, accanto al grande geometra Zeuthen, e Zeuthen accanto ad Heiberg, con una comunione di spiriti che costituisce un esempio splendido di collaborazione fra studiosi diversamente educati, e così con profitto di entrambi e soprattutto per fortuna del nostro sapere."

> static aspect, but also in its developing state; and thus that the scholar learn from history to reflect on the genesis of the ideas, and on the other hand, take an active interest in research.[140]

The second initiative, the *Settimana della Scuola di Storia delle Scienze* (Rome, 15–22 April 1935) organised by Enriques and the teachers of the School (Almagià, Baglioni, Montalenti and Vacca), deserves mention because it documents Enriques' aperture to other countries in oppostion to all forms of nationalistic isolation and distortion in the field of history of science. The participants included Castelnuovo, Bompiani and Giuseppe Armellini and twenty-six members from London's Unity History School as well as scholars from other European countries, including the Belgian Paul Libois, who would draw various aspects of his own vision of mathematics teaching from Enriques,[141] and the French historian Hélène Metzger,[142] who shared Enriques' unitary concept of science. The topics addressed went from philosophy to the history of physics, astronomy, biology and technology, and the debate was lively, as can be seen from the detailed summary of the week's activity written by Metzger and published in Aldo Mieli's journal *Archeion* [99].

During the same period Enriques also participated in the meetings (Paris, Vienna, Berlin) and congresses (Heidelberg, 1927; Barcelona, 1929; Paris, 1933; Budapest, 1934; Zurich, 1938) of the *Institut International de Coopération Intellectuelle* (IICI) inaugurated on January 1925 in Paris,[143] in addition to various other international congresses of philosophy, history of philosophy and philosophy of science: it was no coincidence that Enriques remained in contact with the IICI, whose aim was to promote international cultural exchange between scientists, researchers, teachers, artists and other intellectuals. He also directed two sections of the book series *Actualités scientifiques et industrielles* published by Hermann in Paris: "Philosophie et histoire de la pensée scientifique"[144] and "Histoire de la pensée scientifique."[145]

[140] See [53, p. 190]: "La formazione di docenti di matematiche, che siano all'altezza dei loro compiti didattici, richiede, in genere, che la scienza sia da loro appresa non soltanto nell'aspetto statico, ma anche nel suo divenire. E quindi che lo studioso apprenda dalla storia a riflettere sulla genesi delle idee, e d'altro lato partecipi all'interesse per la ricerca."

[141] See [98].

[142] See [104].

[143] See ASUR, *Enriques, Federigo. Fascicolo personale* and [97].

[144] F. Enriques, *Signification de l'histoire de la pensée scientifique* (1934); G. Castelnuovo, *La probabilité dans les différentes branches de la science* (1937); F. Gonseth, *Qu'est-ce que la logique?* (1937); H. Metzger, *Attraction universelle et religion naturelle chez quelques commentateurs anglais de Newton. Première partie, Introduction philosophique* (1938); H. Metzger, *Attraction universelle et religion naturelle chez quelques commentateurs anglais de Newton. Deuxième partie, Newton, Bentley, Whiston, Toland* (1938); H. Metzger, *Attraction universelle et religion naturelle chez quelques commentateurs anglais de Newton. Troisième partie, Clarke, Cheyne, Derham, Baxter, Priestley* (1938); F. Enriques, *La théorie de la connaissance scientifique de Kant à nos jours* (1938); F. Enriques, *Causalité et déterminisme dans la philosophie et l'histoire des sciences* (1941).

Between 1934 and 1939 eight volumes were published in the first series, with the collaboration of Hélène Metzger, Ferdinand Gonseth and Guido Castelnuovo, and six in the second series, written in collaboration with de Santillana; these were developed on the bases of the book they had published together in 1932 entitled *Storia del pensiero scientifico*. *Il mondo antico* (Milan, Treves). The first volume to appear was *Signification de l'histoire de la pensée scientifique* in 1934, and in the opening chapter, titled "La science et son histoire" Enriques once again presented his dynamic vision of science and the conviction that it is precisely from history that science draws its meaning.

In 1937, together with de Santillana, he published the *Compendio di storia del pensiero scientifico dall'antichità ai tempi moderni* (Bologna, Zanichelli), which aimed at filling the gap in the teaching of philosophy and history in secondary schools. Although some parts now appear dated, Paolo Casini has written that "the two authors" political commitment and their efforts to overcome the impasse of then current trends in textbook writing' is above all evident in the "brief sections concerning nineteenth century, positivism, pragmatism and neo-idealism" [11, pp. XIV, XV].[146] The following year was published the volume *Le matematiche nella storia e nella cultura* [53], aimed primarily at students in secondary school and the first two years of university. As the title indicates, the objective was to show the significance and place of mathematics in the context of other sciences and in its relations with technology, art, history and philosophy in order to reconstruct the unity of thought in the face of increasing specialisation. A few dense pages dedicated to mathematics teaching (pp. 184–191) gave Enriques the chance to reaffirm the educational and cultural value of mathematics and the importance of having adequately prepared teachers.

It is abundantly clear that history played an increasingly central role in the struggle for a scientific *humanitas*, and that the teachers were a very important channel for Enriques. This can also be seen in the third edition of the *Questioni riguardanti le matematiche elementari* (1924–1927), which was republished in a reorganised form and enriched with new material[147] drawn principally from the courses Enriques had taught at the University of Rome in the previous two years.

[145] *Les Ioniens et la nature des choses* (1936); *Le problèmes de la matière: Pythagoriciens et Eléates* (1936); *Les derniers "Physiologues" de la Grèce* (1936); *Le problème de la connaissance. Empirisme et rationalisme grecs* (1937); *Platon et Aristote* (1937); *Mathématiques et astronomie de la période hellénique* (1939).

[146] See [11, pp. XIV, XV]: "…l'impegno politico dei due autori e il loro sforzo di superare le impasses della manualistica corrente"' …"nei rapidi scorci concernenti il diciannovesimo secolo, il positivismo, il pragmatismo e il neoidealismo."

[147] Enriques himself wrote six articles: "L'evoluzione delle idee geometriche nel pensiero greco: punto, linea e superficie" [III ed., I.1 pp. 1–40]; "I numeri reali" [III ed., I.1 pp. 231–389]; "Spazio e tempo davanti alla critica moderna" [III ed. I.2 pp. 429–459]; "Sulle equazioni algebriche risolubili per radicali quadratici e sulla costruibilità dei poligoni regolari" [III ed., II, pp. 263–305]; "Alcune osservazioni generali sui problemi geometrici" [III ed., II pp. 575–596]; "Massimi e minimi nell'Analisi moderna" [III ed., III pp. 311–471].

New collaborators flanked the original ones, including Enrico Bompiani, Alfredo Sabbatini e Vittoria Notari Cuzzer, Enriques' assistant first in Bologna and then in Rome, and his collaborator on questions of didactics in the *Periodico di matematiche* as well. In the preface Enriques affirmed that the aim was that of "giving scientific theory a basis in history," at the very moment when various circumstances threatened "to diminish science and mathematical culture ... precisely among those whose highest duty it is to diffuse them in the schools." The work is thus addressed to teachers, to the students of the course of complementary mathematics, and to those who were preparing for the state examinations, but Enriques underlined that another aim was that of "opening the fruitful field of historical investigation to a greater number of scholars."[148]

In his 1931 preface to the index of the first ten years of the second series of the *Periodico di matematiche* that he himself had inaugurated, Enriques underlined with pride the role played by the journal in teacher training:

> No other journal of this sort, in no other country in the world, has been able to realise a program that is as lofty and attuned to the exigencies of education and culture of teachers of middle schools.[149]

Once again, Enriques highlighted the effectiveness of using advanced mathematics to improve comprehension of elementary questions, the importance of criticism of the principles which avoids logical subtleties and makes evident the philosophical meanings of the problems, and the use of history of mathematics to cultivate in teacher the idea of "becoming" in science.

Enriques' aspiration of diffusing his unitary vision of science and avoiding the cultural isolation of mathematics also lay at the basis of his collaboration with the *Enciclopedia Italiana*, and was joined to the need for a wider dissemination. As he expressed it, addressing members of the Mathesis Association:

> Nor should this work of dissemination and propaganda appear superfluous ... And, even if the need for propaganda distracts us for a time from other useful work, we must not regret it, because, by strengthening our scientific faith and recreating the need [of science] in the society around us, we prepare younger and more daring energies for scientific progress.[150]

In 1925 Gentile, with the financial support of Giovanni Treccani, relaunched an earlier project of the *Società Italiana per il Progresso delle Scienze* for a national

[148]See [44, vol. I, Prefazione]: "dare alla teoria scientifica una base storica," "diverse circostanze minacciano oggi di menomare la scienza e la cultura matematica ... nella schiera di coloro che hanno l'alto compito di diffonderla nella scuola," "illuminare la ricerca più elevata e aprire anche il campo fruttifero dell'investigazione storica ad un più vasto numero di studiosi."

[149]See "Indice generale Serie IV – Volumi I a X – Anni MCMXX-MCMXXX", *Periodico di matematiche*, (4) 11, 1931, pp. 3–21: "Nessuna rivista dello stesso genere, in nessun paese del mondo, ha saputo realizzare un programma così alto e intonato alle esigenze formative e culturali dei docenti delle scuole medie."

[150]See [43, p. 3]: "Né quest'opera di divulgazione e di propaganda deve apparirvi superflua ... E, se anche la necessità della propaganda ci distragga per alcun tempo da altro utile lavoro, non dobbiamo rammaricarcene, perché, rinfrancando in noi la fede scientifica e ricreandone il bisogno nella società circostante, prepariamo pure al progresso della scienza più giovani e balde energie."

encyclopaedia. After Volterra refused to collaborate, Enriques joined the enterprise, and enthusiastically accepted the direction of the scientific part, with the help of Fermi and Amaldi. The significance of his contribution and his relationship with Gentile have been the subjects of recent studies (see [5–7]). Here we will only underline the fact that it is above all the mathematical entries (see [3]) that reflect Enriques' cultural project and his vision of mathematics, where the theoretical results are seen in connection to the problems from which they originated, even outside of the specific disciplinary field, and are closely related to aspects of history, epistemology and, of more specific interest to us here, education. This is already evident in the guidelines sent by Enriques to the collaborators, the salient points of which are: address both mathematicians and non-mathematicians; present fundamental problems, shedding light on their scientific and philosophical significance and the mutual connections; use history to illustrate the development of ideas and to connect mathematics with other aspects of culture; avoid too minute technical details; develop the elementary questions in greater depth with respect to the more advanced questions because they are of interest to a wider public; reduce symbolism to a minimum.[151] Emblematic of this point of view is the entry "Matematica" [51], where, among others, there is a paragraph specifically dedicated to teaching: the approach to the topic is historical, and Enriques manages to reaffirm the educational value of mathematics, which

> reveals itself not only in the elevation and strengthening of those minds which, by means of classical instruction, want to prepare themselves for more advanced studies, but also in the early grades of education of children and the working classes.[152]

Here, Enriques also makes explicit reference to the renowned pedagogists Pestalozzi and Fröbel,[153] attributing to them above all the merit of having introduced mathematics into the education of children as an important element for their intellectual development. Instead, in spite of evident points of contact, no reference is made to Adolphe Ferrière, the father of the "école active," nor to Ovide Decroly, both of whom were well known in Italy.

In all, from 1925 to 1935 Enriques wrote a total of thirty-eight entries for the *Enciclopedia*.[154]

All of these activities were brought to a sudden halt after the enactment of the racial laws in 1938. Nevertheless, Enriques continued his involvement as far as possible, writing in the *Periodico* under a pseudonym, and giving courses in

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[151] See "Norme per la collaborazione dei matematici all'Enciclopedia Italiana," *Periodico di matematiche*, (4) 6, 1926, pp. 46–47.

[152] See [51, p. 553]: "...si palesa non soltanto nell'elevamento e nel potenziamento delle intelligenze che, attraverso l'istruzione classica, vogliono abilitarsi ai più alti studi, bensì anche nei primi gradi di educazione dell'infanzia e delle classi popolari."

[153] Enriques had already cited the two pedagogists in [25, p. 26] and would cite them again in [53, p. 185].

[154] See the list in [6, pp. 129–130].

geometry and history of mathematics in the so-called clandestine university in Rome. This university had been organised by Castelnuovo beginning in 1941 and continued under his direction until 1943 to offer courses to enable Jewish students who had been banned from the Italian university to take the examinations at the *Institut Technique Supérieur* in Fribourg (see [12]). One of his students recalled:

> The course that [Enriques] gave in the history of mathematics was a memorable event, which drew not only students of engineering. This handsome old man, this fascinating gentleman . . . spoke with the soft and direct voice of the great persuaders. He guided the listeners to the limpid comprehension of complex relations, to the identification of connections never dreamed of.[155]

After the Liberation in 1944, as Castelnuovo wrote, Enriques "resumed teaching, but his body was by then worn out, and He no longer had the strength to take up fighting positions" [14, p. 12],[156] but he never abandoned his interest in educational questions. He was one of the supporters of the *Instituto Romano di Cultura Matematica* founded at the beginning of 1945 by Tullio Viola and Emma Castelnuovo to foster discussion of educational problems and teacher training, and he gave two lectures on topics that were dear to him: the significance of mathematics in general culture, and the significance of mathematics for physics (see [107]). As Emma Castelnuovo recalled, he also organised meetings in his own home for students and teachers, with the aim of improving teaching of geometry in secondary schools:

> . . . in addition to these meetings [of the Roman Institute for Mathematical Culture] of about a hundred people, there were also small meetings at the home of the mathematician Enriques, we were about eight or ten at most. Enriques had proposed to study the books . . . of $1700-1800$ of elementary geometry in order to have an idea of how the school courses could be modified by moving away from Euclid.[157]

It was in one of these meetings that Emma learned about the 1741 *Eléments de Géométrie* by Alexis Clairaut, which led her to change her way of teaching by introducing the active method of teaching intuitive geometry: "In a single stroke I change," she writes, "the class changes in my hands."[158]

In those years, Enriques also influenced Carleton Washburne, who had been sent to Italy by the United States in the summer of 1943 to eliminate all traces

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[155] See [19, p. 96]: "Il corso che tenne di storia delle matematiche fu un memorabile avvenimento, che richiamò non soltanto gli studenti d'ingegneria. Il bel vecchio, l'affascinante signore . . . parlava con la voce piana e diritta dei grandi persuasori. Conduceva gli ascoltatori alla comprensione limpida di relazioni complesse, all'individuazione di nessi mai sospettati."

[156] See [14, p. 124]: ". . . riprese l'insegnamento, ma l'organismo era ormai stanco ed Egli non sentiva più la forza di assumere posti di combattimento."

[157] See [13, p. 25]: ". . . oltre a queste riunioni di circa cento persone [dell'Istituto Romano di Cultura matematica] c'erano riunioni in piccolo, a casa del matematico Enriques, eravamo 8,10 al massimo. Enriques aveva proposto di studiare dei libri . . . del 1700–1800, di geometria elementare per avere un'idea di come si poteva forse modificare il corso allontanandosi da Euclide."

[158] See [13, p. 26]: "Di colpo cambio. . . . la classe mi cambia fra le mani."

of Fascist propaganda from the schools and to begin the process of democratising the country. A well known pedagogist who had created the "Winnetka School" and a supporter of the active method of teaching, Washburne was the director of the Allied Forces Education Review Board of the Allied Control Commission in Italy which, with the help of a subcommittee of Italian experts, produced new programs for elementary and secondary schools and for the *Istituti magistrali* (primary schoolteacher training schools).[159] The methodologies that inspired the mathematics programs of the Allied Commission reflect Enriques' influence: the new programs stressed the importance of a teaching that is intuitive-dynamic in close connection with the historic process, and invited teachers to pay greater attention to the psychological needs of the students.

Enriques died suddenly in Rome on 14 June 1946. Up to the end he was involved in teacher training, which he believed to be the crucial element for the formation of good schools and one of the channels for achieving his cultural project. In his own words:

> These ideas were defended by us, even with battles, in the social science area of scientific institutions and in the ordering of studies; and we have not given up hope that they are about to leave some fertile seeds.[160]

Acknowledgements I am very grateful to Aldo Brigaglia, Philippe Nabonnand, Pietro Nastasi, Ornella Pompeo Faracovi and Gabriele Lolli for reading the text and making suggestions. I also want to express my gratitude to Cristina Chersoni (*Archivio storico dell'Università di Bologna*), Gian Paolo Brizzi and Carla Onesti (*Archivio storico dell'Università di Roma*), Helmut Rohlfing (*Niedersächsische Staats-und Universitätsbibliothek Göttingen*), Sandra Linguerri, Emma Sallent and Gert Schubring for their help. My most sincere thanks go to Kim Williams, who with intelligence and professionalism translated the text. Thank you also to the personnel of the *Biblioteca "Giuseppe Peano"* of the Department of Mathematics of the University of Torino for invaluable help in bibliographic research.

[159] See [126]. See also the Decree of 9 February 1945 [BUMPI I 1945.1, pp. 253–313]; the *Piano di Studi per gli Istituti magistrali superiori 1944–1945*. Roma: Signorelli, 1945; and Commissione Alleata in Italia (Sotto-Commissione dell'Educazione), *La politica e la legislazione scolastica in Italia dal 1922 al 1943 con cenni introduttivi sui periodi precedenti e una parte conclusiva sul periodo postfascista* (Milan: Garzanti, 1947), in particular pp. 382–386.

[160] See [41, p. 287]: "Queste idee sono state sostenute da noi, anche con battaglie, nel campo sociale delle istituzioni scientifiche e dell'ordinamento degli studi; e non abbiamo perduto la speranza che esse sieno per lasciare qualche seme fruttifero."

Appendix 1: *Class Registers* [161]

1. ASUB, *Enriques prof. Federigo. Registri delle lezioni: Corso libero di Filosofia delle Scienze. Programma per l'anno 1902–1903*
Introduzione – Rapporti della Filosofia colle scienze fisico-matematiche da una parte e colle scienze biologiche dall'altra
I problemi filosofici attinenti ai principii della geometria
Questioni pedagogiche che ne dipendono
I problemi filosofici attinenti ai principii della meccanica
Questioni pedagogiche
Maggio 1902
Federigo Enriques

2. ASUR, Facoltà di Scienze Matematiche Fisiche e Naturali, *Libretti delle lezioni*: *Libretto delle lezioni di Matematica Complementare dettate dal Sig. Prof. Enriques Federigo nell'anno scolastico 1922–1923*
16.11.1922 Gli Elementi di Euclide
18.11.1922 Sulle origini della geometria greca: i pitagorici
21.11.1922 Critica eleatica
23.11.1922 Segue: origini dell'analisi infinitesimale
25.11.1922 Def.^{i} assiomi e postulati in Euclide
28.11.1922 Concetti primitivi e post.^{i} nella geom. moderna
30.11.1922 Analisi di Pasch dei primi post.^{i} della geom. piana
02.12.1922 Segue
05.12.1922 I numeri naturali
07.12.1922 I numeri fratti e negativi
09.12.1922 Non fatta per chiusura dell'Università
12.12.1922 Numeri irrazionali: potenza del continuo
14.12.1922 Numeri non archimedei
16.12.1922 Varie forme del post. della continuità
19.12.1922 Applicazioni elem.^{i} del post. di continuità
21.12.1922 Segue: intersez.^{i} di rette e circoli
11.01.1923 Sviluppo storico della geom. proiettiva
13.01.1923 Teoria fondamentale della pr.
16.01.1923 Eq.^{ne} funzionale di Darboux
18.01.1923 Omografie piane punti uniti
20.01.1923 Om.^{e} particolari metriche del piano
23.01.1923 Omogr.^{e} nello spazio
25.01.1923 Rappresentazione delle quadriche
27.01.1923 Class.^{e} p.^{ti} uniti omogr.^{e} spaziali
30.01.1923 Cubica gobba e om.^{e} con p.^{ti} un.^{i} multipli

[161] All transcriptions are by L. Giacardi unless otherwise noted.

01.02.1923 Movimenti dello spazio: traiettorie dei [. . .] eliche.
03.02.1923 Sup. con ∞^2 movimenti. L'immaginario: introduzione
06.02.1923 Eq. di gr. n e rad. complesse
10.02.1923 L'imm.^{o} e la teoria delle funz., condiz.^{e} di monogeneità
13.02.1923 Segue: integrali e teor. di Cauchy
15.02.1923 Sviluppi in serie di potenze
17.02.1923 Principio di cont. e immaginario in geometria
20.02.1923 Segue: l'imm.^{o} nella teoria delle coniche
22.02.1923 Discussione sul sistema delle coniche omofocali
24.02.1923 Linee di lunghezza nulla
29.02.1923 Applic.^{i}: teor. Beltrami trasf. conformi nello spazio
01.03.1923 Principio di dualità e di trasporto logico
03.03.1923 Introd.^{e} coor. proiettive. Trasf.^{i} che mutano sfere in sfere
06.03.1923 Trasf.^{e} quadratica
08.03.1923 Post. d'Euclide sulle parallele
10.03.1923 Teor. di Saccheri-Legendre ecc.
13.03.1923 Principii di Geom. non euclidea
15.03.1923 Interpretaz. su sup. a curv. cost. neg. e in p.^{no} rispetto a conica
17.03.1923 Cerchi, ipercicli, oricicli in geom. non-euclidea
20.03.1923 Non fatta per funerale prof. Semeraro
22.03.1923 Sul valore fisico della geom. non euclidea
24.03.1923 Segue: cenni sulla teoria della relatività
12.04.1923 Geometria ellittica
14.04.1923 Principio di corr. sulle rette
17.04.1923 Involuzioni [. . .] teor. di Lüroth.
19.04.1923 Il birapporto e l'inv. ass. F
24.04.1923 Gruppi finiti di proiett.: analisi di Klein
26.04.1923 Segue
28.04.1923 Non fatta perché aula occupata per libera docenza
01.05.1923 Poliedri regolari
03.05.1923 Le coniche e le prop. focali elementarmente
05.05.1923 Non fatta per esami scritti di cultura per lauree miste
08.05.1923 Sezioni circolari del cono quadrico
12.05.1923 Critica comparativa dei testi di geometria
15.05.1923 Segue: criteri di ug.^{a} dei Δ
17.05.1923 Segue: equivalenza
19.05.1923 Non fatta per motivi personali
22.05.1923 Segue: teor. di Pitagora
24.05.1923 Schiarimenti riassuntivi
26.05.1923 Segue
29.05.1923 Segue
09.06.1923 Schiarimenti
14.05.1923 Schiarimenti
Visto: Il Preside

3. ASUR, Facoltà di Scienze Matematiche Fisiche e Naturali, *Libretti delle lezioni*: *Libretto delle lezioni di Esercitazioni di Matematiche Complementari dettate dal Sig. Prof. Enriques Federigo nell'anno scolastico 1922–1923*

21.11.1922 Indicazioni bibliografiche

23.11.1922 Proporzioni

25.11.1922 Uguaglianza dei triangoli

28.11.1922 Eq.^{i} di 2^{o} gr.^{o} in Euclide

30.11.1922 Volume della sfera

2.12.1922 Segue

5.12.1922 Costruzione del triang.^{o} date le mediane e le altezze

7.12.1922 Impossibilità di costruire in generale il triang. date le bisettrici

9.12.1922 Non fatta per chiusura dell'Un^{à}.

11.12.1922 Segue: discussione dei problemi di 2^{o} grado

14.12.1922 Sviluppi in serie delle funzioni

16.12.1922 Segue: campo di convergenza

19.12.1922 Caduta dei gravi

21.12.1922 Frazioni continue

11.01.1923 Irraz.^{i} quadratici e fraz.^{i} cont.^{e} periodiche

13.01.1923 Duplicazione del cubo

16.01.1923 Volume del tetraedro

18.01.1923 Geometria del compasso

20.011923 Costruzioni del tr. eq. e del pentagono regolare

23.01.1923 Numeri primi

25.01.1923 Poligoni equivalenti

27.01.1923 Costr.^{i} 1^{o} gr. con riga

30.01.1923 Probl. 1^{o} grado con riga e cerchio fisso

01.02.1923 Analisi indeterminata di 1^{o} gr.

03.02.1923 Geom. del compasso

06.02.1923 Trasf.^{i} per raggi vettori reciproci

10.02.1923 Geom. sferica

13.02.1923 Segue: dualità

15.02.1923 Segue: confronto colla geom. euclidea

17.02.1923 Trigonometria sferica

20.02.1923 Sist.^{i} di eq.^{i} di 1^{o} grado

22.02.1923 Angoli nel cerchio

24.02.1923 Logaritmi

27.02.1923 Prima lez.^{ne} sulle equazioni

01.03.1923 Segue

03.03.1923 Interpolazioni

06.03.1923 Sist.^{a} d'eq.^{i} di 1^{o} e 2^{o} gr.

08.03.1923 Interpretazione di geom. analitica

10.03.1923 Frazioni decimali periodiche

13.03.1922 Eq.^{ne} di 4^{o} grado come resultante di due eq.^{i} di 2^{o} gr. con 2 incognite

15.03.1923 Eq.^{ne} di 3^{o} grado

17.03.1923 Segue: soluz.^{ne} geometrica. Progressioni

20.03.1923 Non fatta per funerale Prof. Semeraro

27.03.1923 Sistemi di eq.^{i} di 2^{o} grado risolubili con l'eq. di 2^{o} gr.
24.03.1923 Ciclometria
12.04.1923 Massimi e minimi in algebra
14.04.1923 Segue
17.04.1923 Teoria elementare degli isoperimetri
19.04.1923 Sul resultante di due eq.^{i}
24.04.1923 Massimi e minimi in analisi
26.041923 Segue
28.04.1923 Segue: massimi e minimi di funz.^{i} di 2 e più var.^{i}
01.05.1923 Decimali illimitati
03.05.1923 Eq.^{i} alg.^{e} e funz.^{i} simm.^{e} delle radici
05.05.1923 Non fatta per lauree miste
08.05.1923 Funzioni simmetriche delle rad. d'un'eq.^{ne} nel piano complesso
12.05.1923 Numeri negativi
15.051923 Segue
17.05.1923 Numeri frazionari
19.05.1923 Non fatta per motivi personali
22.05.1923 Segue: num.^{i} frazionari, teorie sintetiche
24.05.1923 Esercitazioni riassuntive
26.05.1923 Segue
Visto: Il Preside

Appendix 2: *Letters to Felix Klein*

1. *SUB Göttingen. F. Klein 34, F. Enriques to F. Klein, Bologna 10 January 1905*

Illustre Sig. Professore

In risposta alla sua lettera Le ho spedito stamani i programmi ufficiali dei nostri Licei, Ginnasii e Istituti tecnici. Quello per i Licei e Ginnasii subisce in questi giorni un rinnovamento, ma il nuovo testo non è stato ancora pubblicato a parte. Quando vedrà la luce glie ne manderò una copia. La modificazione introdotta è assai profonda perché si tratta di rendere possibile la scelta agli studenti fra il Greco e la Matematica dal primo anno di Liceo in su.

Insieme ai programmi anzidetti Le invio una copia della 2^{a} ediz.^{e} // dei miei Elementi di Geometria. Nell'avviamento ad un metodo che, pur essendo razionale, accentua il carattere induttivo, Ella potrà riconoscere una influenza delle sue idee e delle conversazioni di Gottinga.

Aspetto dal Fleischer comunicazione delle osservazioni intorno al mio art. per l' Enciclopedia, osservazioni che terrò nel massimo conto.

Coi migliori e più distinti saluti, mi abbia per

Suo dev.mo

Federigo Enriques

Bologna 10/1/1905

2. *SUB Göttingen. F. Klein 51, F. Enriques to F. Klein, Bologna 19 July 1920*

Bologna viale Gozzadini 9: 19 Luglio 1920

Caro ed illustre professore,

col prossimo anno mi propongo di riprendere la pubblicazione di un Periodico di Matematiche diretto agli insegnanti secondari, a cui vorrei dare nuova vita, valendomene per promuovere la cultura dei detti insegnanti, specie col richiamare la connessione fra i campi più elevati delle matematiche e gli elementi, nonché dando sviluppo alle questioni storiche. Non ho bisogno di spiegare a Lei l'interesse ed anche la difficoltà di una tale impresa, che risponde proprio ad una delle vedute che Lei stesso ha fatto brillantemente valere con tanti modi diversi di operosità. Ma non Le dispiaccia che, ricordando appunto il Suo interesse per tali questioni, io venga a chiederle il dono della Sua collaborazione, ed il Suo prezioso consiglio.

Se Ella può collaborare alla nuova Rivista con un suo proprio scritto (che procureremo di volgere, nel miglior modo in italiano, e che – per tale motivo – vorrei pregare fosse scritto, possibilmente a macchina, o almeno con caratteri latini molto leggibili) questo sarà effettivamente un grosso regalo per i nostri lettori.

Oltre a ciò io Le sarei pur grato di additarmi questioni che, a suo avviso meriterebbero di attrarre l'attenzione del Periodico, ed anche il nome di qualche collaboratore che ritenga specialmente adatto, per un tale lavoro.

Ringraziandola in ogni caso per qualsiasi contributo che Ella voglia recare al disegnato Periodico mi abbia, coi migliori devoti saluti suo Federigo Enriques

2. *SUB Göttingen. F. Klein 4A, F. Enriques to F. Klein, Bologna 18 January 1921*

Bologna viale Gozzadini 9: 18 Gennaio 1921

Caro ed illustre professore,

ebbi a suo tempo la Sua lettera e La ringrazio dell'appoggio indiretto che Ella promette al Periodico, di cui ho ora il piacere di inviarle in omaggio la prima copia.

Per quello che concerne le condizioni dello spirito pubblico italiano verso la Germania, e segnatamente nel mondo della cultura, credo di poterle affermare che la grande maggioranza è favorevole alla migliore ripresa dei rapporti; il tempo vincerà le riluttanze di coloro che credono di essere ancora in stato di guerra ammesso come fatto, e non concesso secondo il mio sentimento, che la guerra delle nazioni debba estendersi al campo intellettuale!).

Inviandole ora, come ho detto, la prima copia – che ha appena veduto la luce – del periodico di matematiche, vorrei chiederle in pari tempo, se il nostro programma non le sembri tale che il Periodico stesso meriti di essere diffuso anche in Germania, presso le biblioteche delle scuole di magistero o quelle che sono alla portata degli insegnanti. Ma se così è, io non mi nascondo tuttavia la difficoltà che a questa diffusione crea l'altezza dei cambi. L'editore Zanichelli, tenuto conto del prezzo di trasporto ecc., cede la rivista all'estero per fr. (francesi) 20 all'anno, e questa somma, che probabilmente non varrà a compensare le spese della pubblicazione, riesce ora un po' alta se deve esser pagata // in marchi. In considerazione di ciò, ho ottenuto dal detto editore Zanichelli il consenso ad una proposta, che – se potrà attuarsi – avrà un carattere simpatico, come quella che tende a facilitare la ripresa degli scambi intellettuali dei nostri paesi. Il pagamento degli abbonamenti al Periodico di matematiche potrà essere fatto in libri (da scegliere dall'editore Zanichelli). Siccome ora i libri tedeschi, venduti all'estero a prezzi assai più alti che all'interno, vengono – per noi – a costare molto cari, sicché non trovano più quel largo mercato che ebbero in passato, e che – nell'interesse della cultura – dovrebbero nuovamente acquistare, sarebbe questo un mezzo assai atto a promuovere ciò che si ha in vista. Non è escluso poi che la cosa possa estendersi dal Periodico, anche ai libri italiani in cambio dei quali si accetterebbe sempre, come pagamento, libri anziché denaro.

Io sottopongo questa proposta all'editore Teubner. Ma se essa potesse avere il Lei un patrocinatore, e se Lei stesso credesse di raccomandare il Periodico di Matematiche a biblioteche ecc., la cosa avrebbe grande probabilità di riuscire.

Ringraziandola intanto La prego gradire l'espressione dei miei sentimenti devoti, Suo

Federigo Enriques[162]

3. *SUB Göttingen. F. Klein 8, F. Enriques to F. Klein, Bologna 1March 1921*

Bologna 1 marzo 1921

Caro e illustre Collega,

ebbi la gentilissima Sua e – soltanto ieri, dati i soliti ritardi postali – il libro inviato dall'editore Springer, che mi è giunto molto gradito. Mi è caro avere così sott'occhio la raccolta dei suoi lavori, iniziata con questo primo volume. Da parte mia molto volentieri ne parlerò in qualche rivista, ma sono in dubbio se io sia il recensore più adatto per scriverne nel Circolo di Palermo con quella diffusione che ivi è desiderabile, e specialmente per ciò che riguarda gli ultimi lavori sulla fisicomatematica che a Lei giustamente preme di vedere messi in rapporto colle antiche ricerche di geometria non euclidea. Nel caso dunque che veda la cosa riuscire meno facile per me (e dati i molteplici impegni che mi tolgono di dedicarvi eventualmente tutto il tempo necessario) resta inteso che io stesso cercherò chi si occupi della cosa, e frattanto – come ho detto – mi procurerò il piacere di fare un cenno più breve del libro sopra qualche altra rivista. In ogni modo temo che la pubblicazione nel Circolo non potrà essere tanto sollecita, perché le condizioni della stampa da noi sono ora difficili, e le riviste matematiche sovraccariche d'impegni; ma in proposito scriverò alla redazione.

[162]The following manuscript note by Klein appears in the margin of the letter: "Hrn. [Herrn] Koll.[egen] Krazer zur fr.[eundlichen] Kenntnisnahme, mit der Bitte um spätere Rücksendung. Die Sache fügt sich sehr gut in unsere allgemeinen Austauschpläne ein!", that is: "To my colleague Krazer for his kind consideration, with the request of a subsequent return. The matter fits perfectly into our general plans for an exchange!". The note means that Klein forwarded Enriques' letter, dated Bologna 18 January 1921, to Adolf Krazer (1858–1926) on 25 January 1921. This is made clear by Klein's note in the third line: "G.(öttingen) 25 I 21," which is the date he sent off Enriques' letter to Krazer. I thank Helmut Rohlfing for help in interpretating the note.

L'editore Zanichelli è in massima disposto all'accordo per organizzare uno scambio di libri tedeschi ed italiani, specie di matematica: attendiamo perciò le proposte del Geh. Krager (sic).

Mi abbia intanto cordialmente e devotamente Suo.

F. Enriques

4. *SUB Göttingen. F. Klein 8, F. Enriques to F. Klein, Bologna 25 June 1923*

Roma 25 giugno 1923

Caro e illustre Professore,

Le esprimo i più vivi ringraziamenti per l'invio del 3º Volume delle Sue Opere, che porta tanti concetti e resultati interessanti nel campo delle funzioni algebriche intese nel più vasto senso, e a cui conferisce mirabile unità di pensiero e singolare pregio l'insieme delle Sue note e spiegazioni.

Naturalmente anche di questo Volume, come dei due primi, sarà dato un cenno nel Periodico di Matematiche.

Mi è grata l'occasione per ricordarmi a Lei col sentimento di devota e reverente amicizia.

F. Enriques

Appendix 3: *Correspondence with Giacomo Furlani President of the Trieste Section of Mathesis Association*[163]

BDMIUT. Fondo Mathesis, Serie II "Carteggio"

1. *F. Enriques to G. Furlani, Bologna 8 July 1920*

Bologna 8 luglio 1920

Caro prof. Furlani,
ebbi la gentile cartolina che Ella mi ha inviato con Amaldi, e Le son grato del buon ricordo.

Ricevo ora i verbali e le relazioni della Sezione della Mathesis; e prima di tutto rivolgo un saluto e un ringraziamento ai colleghi del cessato consiglio direttivo della detta Sezione per l'opera da loro prestata, ed un cordiale saluto ai nuovi eletti: mentre mi compiaccio, in special modo, che Lei – caro amico e solerte presidente – abbia a continuare nell'ufficio tenuto per il bene della nostra società e della scuola.

Passo ora a rispondere ad alcune sue domande.

[163] My most sincere thanks to Luciana Zuccheri who provided me with copies of the letters. See the catalogue of the *Fondo Mathesis* of Trieste in *Animi divisi. Vicende dell'insegnamento della matematica nella Venezia Giulia dal 1918 al 1923*, L. Zuccheri, V. Zudini, eds. Trieste: EUT, pp. 47–71. The correspondence is entirely transcribed in [122, pp. 354–379].

1) Ho comunicato al Franchi (ditta Zanichelli) la sua lettera circa il testo del Battelli; ma egli dice di non poter sobbarcarsi al rifacimento da loro desiderato. Egli ritien dubbio che la cosa possa convenirgli in massima (poiché ha avuto l'avviso in contrario da fisici); ma – in ogni caso – si trova nella impossibilità pratica di realizzare la cosa, per quest'anno.
2) Quanto alle Nozioni di matematiche, compilate dall'Amaldi e da me, la ditta Zanichelli ha già risposto a scuole che gliene han domandato, di avere a disposizione le copie richieste. Ma, poiché par di comprendere che vi sarà una richiesta più larga, provvede ad una immediata ristampa.

Però a proposito di libri, mi consenta una preghiera. Se loro, nella Sezione vogliono discutere dei libri di testo, questa discussione non potrà che illuminare le questioni didattiche; ma non mi par compito della nostra società, o delle sue Sezioni, di raccomandare o meno l'adozione di dati libri di testo. So bene che voi avete fatto e fareste questo nel modo più elevato, per il puro interesse della scuola; pure non si può dimenticare che a tali questioni si legano interessi personali, sicché – almeno negli interessati – potrebbe ingenerarsi qualche malumore. Se, domani, una discussione di questo genere si porta in una Sezione dove si trova qualche autore di libro di testo, la cosa viene ad assumere un aspetto imbarazzante.

Aggiunga che, nel concetto italiano – secondo la tradizione – i proff. considerano che la scelta dei libri deve esser fatta con scelta individuale libera, e però non amano di vedere raccomandazioni aventi l'aspetto di inviti, più o meno officiosi.

Per tali motivi, La prego consentirmi di non pubblicare nel Bollettino l'elenco dei libri raccomandati: tanto più che, fra questi ce n'è anche uno mio.

Mi abbia, intanto, coi più cordiali saluti, Suo aff.mo

F. Enriques

2. *F. Enriques to G. Furlani, Torino 9 January 1920*

Torino, 9 Gennaio 1921

Carissimo Furlani,

ricevo qui la Sua lettera e mi compiaccio per l'acquisto dei nuovi soci. Ora Le raccomando il *Periodico*: è proprio necessario che la gran maggioranza dei soci si abbonino! Anche in questo caso l'editore ha preventivato una perdita.

Ha ricevuto il primo numero, che impressione ne ha riportato?

Può fare un pò di propaganda?

Ho scritto all'ufficio per le nuove province del Ministero, nel senso convenuto per la visita a Trieste e Trento (studio de visu delle scuole e anche conferenze): ho atteso per far ciò la rielezione del Cons. dir. in nome del quale è fatta la domanda. Lei può ora appoggiare la cosa presso gli amici dell'ufficio?

Saluti cordiali suo

F. Enriques

3. *G. Furlani to F. Enriques, Trieste 23 January 1921*

Trieste, 23 gennaio 1921

Carissimo e stimatissimo professore,

in risposta alla Sua gradita del 9 corr. confermo il ricevimento del periodico. Incaricandosi dell'incasso dei canoni pro 1921 sarà facile al C. D. della sezione di fare abbonati per il periodico.

Questo fu accolto qui con simpatia. Con piacere Le comunico impressioni e idee raccolte. Io lessi con vivo compiacimento il Suo articolo, dove ravviso delle direttive nell'insegnamento che sole possono contribuire alla diffusione estensiva della cultura matematica. Mentre per la maggior parte dei lettori delle nuove province sono interessanti gli articoli riguardanti le matematiche antiche o la critica dei fondamenti, mi pare che per quelli delle vecchie province sarebbero interessanti la trattazione di argomenti che si riferiscono all'insegnamento nelle scuole medie di capitoli della matematica moderna. Io eccitai più di uno dei miei colleghi a dare la propria collaborazione. Un desiderio espresso da qualcuno, che non mi pare di facile attuazione è questo: che fossero pubblicati degli articoli sintetici sugli ultimi progressi fatti dalla scienza matematica nei vari rami compilati da maggiori cultori specialisti in quei rami. Forse delle recensioni di pubblicazioni recenti fatte con queste intenzioni gioverebbero a questo bisogno.

O' raccomandato tosto in via amichevole a Roma quella domanda avanzata dalla Direz. della Mathesis

Vivissimi saluti, anche dai miei colleghi e famigliari

[Giacomo Furlani]

4. *F. Enriques to G. Furlani, Roma 10 March*

Roma 10 Marzo
Park Hotel

Caro Furlani,

ringrazio cordialmente Lei e il prof. Cermeli!

Per quel che riguarda la traduzione dei lavori del Boscovich, mi consulterò con Chisini nello spazio che avremo disponibile nel Periodico, ma temo che – al solito – non ne abbiamo molto; certo sarebbe interessante pubblicare lavori di questo genere, ma essi andrebbero a scapito di ciò che già dobbiamo, e non possiamo fornire nella misura in cui vorremmo ai lettori.

Però io sto pensando al modo di dar vita ad un *istituto per la storia delle scienze matematiche*; se la cosa (che è stata interrotta dalla crisi bancaria) mi riesce (e al momento solleciterò anche il vostro appoggio!) potremo pubblicare appunto, // e in primo luogo memorie e trattati di classici, specie italiani.

Saluti cordiali
dal suo
F. Enriques

5. *G. Furlani to F. Enriques, Trieste 4 June 1923*

Trieste, 4.VI.23

Chiarissimo prof Enriques,

Le notizie contenute nell'appello e implicite nel memoriale per il Ministero che sono stati inviati dalla Presidenza della "Mathesis" alle sezioni sono tali da suscitare un vivo allarme e richiedere la più pronta e vivace azione. Io avevo deciso quindi di convocare i soci della mia sezione e provocare tale azione. Senonché quasi contemporaneamente mi occorse di // leggere qua e là che il decreto sulla riforma era già bello e fatto e vi era fissato anche la distribuzione della materia. In tal caso piuttosto che fare una azione affrettata varrebbe meglio prepararla in modo più opportuno e fondarla su dati più precisi.

Considerate le difficoltà di radunare i soci, per non sciupar tempo in un'azione poco utile, mentre ò già provvisto a suscitar l'interessamento dei colleghi col diffondere le informazioni avute, vorrei ulteriori chiarimenti // per convocare la sezione. In particolare vorrei sapere se e quando sia stato pubblicato quel decreto oppure che cosa si conosca in particolare di preciso da cui partire per un'azione.

La ringrazio per la gent. cartolina.

[Giacomo Furlani]

6. *F. Enriques to G. Furlani, Viareggio 10 August [1923]*

Viareggio, Pens. Margherita al [...]
20 Agosto

Caro Furlani,

sarebbe bene che tutte le Sezioni fossero rappresentate al Congresso di Livorno (25 – 27 Sett) ove si discuteranno i problemi sollevati dalla riforma e si prenderanno accordi d' importanza pratica, anche per le disposizioni transitorie ecc. Verrà qualcuno di voi? Faccia propaganda e procuri // che i probabili congressisti inviino fin d'ora l'adesione al prof. Lazzeri (via Indipendenza 15)

Cordiali saluti
suo
F. Enriques

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Abbreviations

ASUB: Archivio Storico dell'Università di Bologna
ASUR: Archivio Storico dell'Università di Roma
BUMPI: Bollettino Ufficiale del Ministero della Pubblica Istruzione
BDMIUT: Biblioteca del Dipartimento di Matematica e Informatica dell'Università di Trieste

GU: Gazzetta Ufficiale

SUB Göttingen: Niedersächsische Staats-und Universitätsbibliothek, Göttingen.

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