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ORIGINAL RESEARCH

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When philosophy (of science) meets formal methods: a citation analysis of early approaches between research fields

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Abstract
The article investigates what happens when philosophy (of science) meets and begins to establish connections with two formal research methods such as game theory and network science. We use citation analysis to identify, among the articles published in *Synthese* and *Philosophy of Science* between 1985 and 2021, those that cite the specialistic literature in game theory and network science. Then, we investigate the structure of the two corpora thus identified by bibliographic coupling and divide them into clusters of related papers by automatic community detection. Lastly, we construct by the same bibliometric techniques a reference map of philosophy, on which we overlay our corpora to map the diffusion of game theory and network science in the various sub-areas of recent philosophy. Three main results derive from this study. (i) Philosophers are interested not only in using and investigating game theory as a formal method belonging to applied mathematics and sharing many relevant features with social choice theory, but also in considering its applications in more empirically oriented disciplines such as social psychology, cognitive science, or biology. (ii) Philosophers focus on networks in two research contexts and in two different ways: in

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the debate on causality and scientific explanation, they consider the results of network science; in social epistemology, they employ network science as a formal tool. (iii) In the reference map, logic—whose use in philosophy dates back to a much earlier period—is distributed in a more uniform way than recently encountered disciplines such as game theory and network science. We conclude by discussing some methodological limitations of our bibliometric approach, especially with reference to the problem of field delineation.

Keywords Formal methods in philosophy · Citation analysis · Game theory · Network science · Bibliometric mapping

1 Preliminary remarks

In the last decades, the philosophical toolbox has increasingly expanded beyond logic to other formal methods. Among them, for example, probability theory and statistics, but also graph theory and computer simulation. Arguably, the introduction of formal methods different from logic partly originates from the need for more flexible tools in philosophy. It is conceivable that philosophers sought in such formal disciplines, especially probability theory, the resources to deal with complex philosophical problems that could not be easily addressed using solely traditional, strictly deductive logic. These non-deductive methods thus played mainly an instrumental role in philosophy. Other formal disciplines, including game theory and network analysis, made their way into philosophy not as an instrument but as an independent body of knowledge by stimulating, reshaping and setting the terms of the philosophical debate in a certain area. At any rate, nowadays philosophers commonly employ a wide array of formal methods and, although logic is perhaps still dominant, other formal disciplines have become deeply influential in philosophy (Hansonn & Hendricks, 2018).

According to Horsten and Pettigrew (2011), three main phases can be distinguished in the use of formal methods in philosophy. In the first phase philosophers striving to attain rigor and clarity employed logic, often to find out that the philosophical problem under consideration was in fact ill-conceived. This phase was almost exclusively dominated by the logic which had been developed in the works of the fathers of analytic philosophy, especially Frege and Russell. The second phase was characterized by the development, from the 1960s onwards, of possible-world semantics for the analysis of intensional notions and it coincides with the intense development of modal logics. During this phase, the formal semantics for classical first-order logic brought about in the 1930s by Tarski (and successfully employed in the analysis of truth and semantic paradoxes) was extended in the 1960s to modal and non-classical logic, with many applications to philosophical problems concerning issues such as necessity, knowledge, and time. The ensuing phase is more difficult to assess. One of its main traits is surely the emergence, besides logic, of mathematics as a formal tool. While possible-world semantics can still be thought of as part of the logical toolkit, formal methods such as probability theory or game theory does not traditionally belong to logic and their adoption constitutes a significant expansion of the realm of methods employed in philosophical analysis. In the philosophy of science, for example, formalizing the confirmation relation between hypotheses and evidence in purely deductive logic leads to paradoxes. It is not surprising, then, that over the decades probabilistic models of confirmation have made their way into the philosophical debate around science (Crupi & Tentori, 2016).

The idea that in the last decades formal methods different from logic has made their entrance into philosophy and that their role is increasingly relevant is straightforwardly assumed by the present article, and we are going neither to argue for it, nor to "measure" it. The aim of the article is rather that of investigating what happens when two disciplines—previously foreign to each other—meet and begin to establish mutual connections. On one side of this connection there is, of course, philosophy (especially philosophy of science). On the other side we chose two instances of formal methods different from logic: game theory and network theory. The motivation for our choice is fourfold: their role in philosophy is relatively little studied; their disciplinary boundaries are sufficiently easy to draw; they have different dates of entrance into philosophy; their presence in philosophy is not equally large.

A recent article investigated a related issue: the presence and role of logic in analytic philosophy from 1941 to 2010 (Bonino et al., 2020). The issue was investigated by using quantitative methods such as distant reading, inspired by the work of Franco Moretti, and the building and testing of interpretative models, based on the work of Arianna Betti and Hein van den Berg (Moretti, 2013; Betti & van den Berg, 2014, 2016). In particular, a corpus of five journals of analytic philosophy—*The Journal of Philosophy*, *Mind*, *The Philosophical Review*, *Philosophy and Phenomenological Research*, and *The Proceedings of the Aristotelian Society*—was annotated and assessed by domain experts against three main criteria: the presence of logic, its role, and the level of technical sophistication. The analysis revealed that (1) logic was not present at all, even in an informal manner, in nearly three-quarters of the corpus; (2) the instrumental role of logic—i.e., logic as a methodological tool for achieving results in metaphysics, moral philosophy, philosophy of language, and so forth—increasingly prevailed over the consideration of logic as a subject matter of philosophical analysis; and (3) the level of technical sophistication increased over time, although it has always remained rather low.

The present paper shares some trivial features with the aforementioned article: it deals with formal methods in philosophy; it focuses on the analytic philosophical tradition; it is based on the application of quantitative methods. However, the differences between the two articles are more significant than their similarities. Besides the obvious fact that the older article focuses on logic, whereas the present one focuses on game theory and network theory, there are more significant differences. First, the corpus that is selected is dissimilar: different journals (*Synthese* and *Philosophy of Science*), and a different timespan (1985–2021). Second, the methodology that is applied is different: citation analysis, rather than distant reading and model building and testing. But the most important point is that the two articles have different aims: as already hinted at, here we are not aiming to quantify the presence of formal tools in philosophy over time, but we are trying to identify and recognize the different modes of encounter between philosophy (especially of science) and such tools. One could almost say that, by means of quantitative methods, we are trying to achieve a qualitative description of the field in which this encounter takes place. In the light of these differences, in the following part of this introduction we will provide a motivation for the selection of *Synthese* and *Philosophy of Science* as representatives of contemporary analytic philosophy (of science) and we will briefly introduce, both conceptually and historically, game theory and network analysis, making explicit the reasons for which we consider them two interesting case-studies. Then, in the next section, we will present the methodology that we used to answer our research questions.

The choice of the philosophical corpus—*Synthese* and *Philosophy of Science* from 1985 to 2021—was determined by several considerations. First of all, these journals jointly constitute a reasonably rich corpus: 7976 articles, of which 5159 belong to *Synthese*, 2817 to *Philosophy of Science*; the articles contain 231,142 references, of which 118,083 are unique. This corpus provides a considerable amount of data, yet still manageable within the time and computing power limitations that always constrain this kind of quantitative inquiries. Second, they cover the whole period under consideration. More specific reasons for our choice are the following. We were not interested in investigating what happens in quite specialized philosophical journals: it is clear, to make a completely trivial example, that if one examines a logic journal, she is going to find a lot of logic; only slightly less obviously, if one examines a journal devoted to ontology, she is probably going to find an appreciable amount of mereology. Unless a very wide and well-balanced selection of such journals were considered, the results would be overly sensitive to the initial choice. Moreover, we wanted to investigate the circulation of certain formal methods in *philosophy*, conceived in a rather general way, not in some of its specialized niches. Therefore, it would have been natural to look at generalist journals. Yet, journals that are *fully* generalist, such as *The Journal of Philosophy*, *The Philosophical Review* or *Mind* raised another difficulty. The presence of references to specific formal methods ran the risk of being too sparse, so that the gathered data would not have been enough to detect some fine-grained phenomena, unless we took in consideration a very large number of journals, which, however, would have been too onerous. *Synthese* and *Philosophy of Science* seemed to be a Goldilockean midway between generalism and specialization: on the one hand they are devoted to the philosophy of science, an area in which references to formal methods are likely to be found with reasonable frequency,[1] on the other hand their approach to the philosophy of science is rather broadly oriented and their range of interests is quite large, so that they could nearly be regarded as generalist journals (and as both authoritative and representative journals). We are aware that a different selection could have produced different results, but it seems to us that our choice is the most fruitful for a first exploratory survey of this theme, which can certainly be refined by future studies based on different corpora.[2]

[1] In *Synthese*'s self-description it is stated that one of the journal's areas of interest is "formal methods in philosophy, including methods connecting philosophy to other academic fields", which seems a perfect illustration of our subject matter.

[2] In the Supplementary Materials, we provide the descriptive statistics of five generalist journals commonly considered the top journals in analytic philosophy (*Journal of Philosophy*, *Philosophical Review*, *Mind, Philosophy and Phenomenological Research, Noûs*) and a journal specialized in philosophy of science (*British Journal for the Philosophy of Science*). Following our method of field delineation (see the Methodology below), *Synthese* and *Philosophy of Science* result to be the journals in which the contact between philosophy and the formal methods was more appreciable (see Table S6).

Let us now come to the presentation of game theory and network science. Game theory is the mathematical investigation of strategic interactions, i.e., situations where the outcome of individuals' actions depends not only on their own actions but also on what (they expect) others will do. Defined in this way, game theory covers nearly all sorts of interactions and it is hardly surprising to find game-theoretic reasoning applied across different areas such as economics, political science, sociology, evolutionary biology, and philosophy. Throughout this paper game theory will be understood in a broader and slightly unconventional way, so as to include subjects that are not normally considered purely game-theoretic. Topics from social choice theory such as Arrow's impossibility theorem, preference aggregation, etc. will count here as game theory. Although game theory and social choice theory share many relevant features, our choice, admittedly unorthodox, needs to be motivated. And the reason is that for several applications of game theory in philosophy, especially those in the field of ethics and political philosophy, game theory is seldom employed in isolation, but it is often supplemented with methods from social choice theory. To be sure, social choice theory methods are no less formal than the purely game-theoretic ones, so our taking them into account does not fall short of the scope of the present work. Of course, one can argue that the disciplinary boundaries of game theory and social choice theory are sufficiently clear and demand a more fine-grained analysis that considers the distinction between them. However, for the purpose of the present paper it is enough that there exists a common core of notions that are relevant to both fields and that they can be considered fully-fledged formal methods when employed in philosophy.

Game theory was officially established as an independent discipline in 1944, when John von Neumann and Oskar Morgenstern published their *Theory of Games and Economic Behavior* (von Neumann & Morgenstern, 1944). By providing an extensive analysis of finite two-player zero-sum games, as well as by settling the axioms of utility theory in rigorous mathematical terms, von Neumann and Morgenstern shaped game theory as a fully-fledged discipline and set the terms of the debate for several decades. In the 1950s game theory grew rapidly with the work of John F. Nash on non-cooperative games and the development of the fundamental notion of equilibrium point—now called Nash equilibrium (Nash, 1951). While game theory keeps growing at a steady pace under the impetus of an increasing number of applications, the connections with philosophy are not always easy to identify and a systematic account of the relationships between game theory and philosophy is still to be developed (de Bruin, 2005). One of the main underlying assumptions, often made unwittingly in the early days of game theory and explicitly theorized by Aumann (1976), is the so-called principle of common knowledge, according to which all players know a piece of information in a game and they also know that they know, and so on ad infinitum. As a matter of fact, in many games the existence of an equilibrium heavily depends on the common knowledge principle. Interestingly, the game-theoretic notion of common knowledge was applied in philosophy by David K. Lewis to provide a rigorous account of the concept of convention (Lewis, 1969). The systematic investigation of the epistemic context behind game-theoretic reasoning gave rise to the field of epistemic game theory which is still flourishing today (Perea, 2012). Other applications of game theory to philosophy include ethics and political philosophy. In the 1950s cooperative game theory and bargaining theory inspired the project of a new foundation for the theory of social contract (Barry, 1965; Braithwaite, 1955; Gauthier, 1986; Harsanyi, 1955; Rawls, 1971). Distributive justice and equity have also been analysed in game-theoretic terms (Skyrms, 1996). In the history of philosophy, game theory has been often invoked to account for Hobbes's state of nature or Hume's theory of justice as based on social conventions (Skyrms, 2004). Moreover, game-theoretic notions such as the famous prisoner's dilemma have been extensively applied in philosophy to better understand the notion of social dilemma, and its generalization known as iterated prisoner's dilemma can be used to explain the possibility of altruistic behaviour on the background of the self-interest character of standard game theory (Axelrod, 1984).

Network science or science of networks is the scientific study of the natural and social systems that can be modelled as networks. In its simplest form, a network is a collection of nodes, or vertices, joined together in pairs by edges, or links. The topology of the network defines how nodes are connected to each other through links, i.e., the wiring of the network, whereas the dynamic of the network specifies the rules that govern the behaviour of nodes and edges. For instance, the rule of preferential attachment dictates that links are attracted to nodes that already have many links. Together, the topology and the dynamic determine the global properties of the network (Lewis, 2009, pp. 6–7). This basic formal structure can be used to model a great variety of physical and non-physical systems, from the World Wide Web to metabolic processes in cells. Network science has developed in the last twenty years from the convergence of research programmes in diverse scientific fields, including social network analysis in sociology, complexity theory in physics and biology, and graph theory in mathematics (Lewis, 2009; Newman, 2018). Modern network science has roots as far back as the 18th century, when Swiss mathematician Leonhard Euler solved the problem of the seven bridges of Königsberg by applying graph theory.[3] After more than two hundred years, in which networks remained confined in the esoteric realm of discrete mathematics, in the late 1960s and 1970s social scientists started to use them to model social relationships and groups behaviour, developing the field of social network analysis (Wasserman & Faust, 1994). Popular concepts such as "six degrees of separation" or the "strength of weak ties" were forged in that period, in the context of the first experiments on the structure and function of social networks. Network science became a scientific discipline on its own only in the late 1990s, when scientists in other fields began to use networks as models of various physical and biological phenomena. The pioneering works of Duncan Watts, Steven Strogatz, and Albert-László Barabási inaugurated the "new science of networks", developing important concepts such as small-world, scale-free network, preferential attachment, and modularity (Lewis, 2009). The contemporary development and increasing relevance of technological networks, such as the Internet, gave further momentum to these studies (Newman, 2018). Today the tools of network science are applied in several disciplines, from marketing to neuroscience, to model a wide array of phenomena, from the stability of electric power grids to the synchronization patterns in mammals to the spreading of epidemics (Barabási, 2014). The new science of networks sets forth the fundamentals that all these disparate systems have in common, combining ideas from

[3] A short history of network science from the 18th century to the "new science of networks" can be found in the first chapter of Lewis (2009).

mathematics, physics, biology, computer science, statistics, social sciences, and many other fields. In the last ten years, networks have found their way also in philosophy, especially in social epistemology. Starting with the pioneering work of Zollman (2007, 2010), who brought to philosophy of science the network epistemology framework developed by Bala and Goyal (1998) in economics, network models have been used to investigate the effects of communication structures on inquiry in epistemic communities. Networks have since then been used to address, among others, topics such as conformity in scientific communities (Weatherall & O'Connor, 2020), the influence of industrial propaganda in science (Weatherall et al., 2020), how social patterns affect the epistemic performance of groups (Hahn et al., 2020), and how scientific innovation spreads within and across scientific communities (Herfeld & Doehne, 2019). Network analysis has also been advocated as a promising tool for doing digital philosophy of science (Pence & Ramsey, 2018).

2 Methodology

The methodology we used to investigate the interaction between philosophy and the two formal methods combined several techniques drawn from the field of citation analysis (reviews can be found in Mingers & Leydesdorff, 2015; Nicolaisen, 2007; Petrovich, 2020). In particular, we focused on the analysis of the *cited references* of publications. The cited references listed in the bibliographies of scientific and scholarly articles offer precious insights on how knowledge cumulates over time, as they are the links that connect new contributions to the accepted body of knowledge (Hyland, 1999; Leydesdorff & Amsterdamska, 1990). Following the links in the bibliographies it is possible to travel back in time and reach the foundations of scientific fields (Marx et al., 2014), whereas by tracing the documents that recur most frequently in bibliographies, we can reconstruct the "paradigms" of a research area (Bornmann et al., 2020; Chen, 2013; Small, 2003). Since references play this function of "conduits" through which knowledge circulates, they are particularly suited to shed light on the diffusion of formal methods in philosophy.

2.1 Delineation of knowledge cores

Our first application of cited references analysis was to identify a set of scientific documents that reasonably represent the *scientific nucleus* of game theory and network science. These documents are the common base shared in the scientific communities that actively develop and use these formal methods. The task of identifying these "paradigms" is known in bibliometrics as "field delineation" (Zitt et al., 2019).

In the bibliometric literature, three main strategies for field delineation are distinguished. The first is based on ready-made classification systems, such as those provided by libraries and other information management systems. In the case of philosophy, they include the taxonomies provided by archives such as PhilPapers or the Philosopher's Index. Even if these databases may be useful to retrieve philosophical publications related to the formal methods we target in this study, their main limitation is that they do not provide citation data for the publications they index.[4] The types of quantitative analysis that can be performed on them are thus significantly reduced. Moreover, since they focus only on philosophical literature, these archives cannot be used for retrieving the specialist literature on the formal methods developed in fields other than philosophy. Therefore, we did not follow this strategy. The second strategy for field delineation is based on producing a list of keywords associated to the field in question. This approach, however, is limited by the fact that keywords may be used with different meanings by different scientific communities. The term "modularity", for instance, recurs both in the philosophy of mind and in network science but with completely different meanings. In the light of this impasse, we discarded this option. The third and last strategy for field delineation relies on citation analysis and it is the one adopted in this study. In particular, we implemented it in a two-step procedure.

First, for each of our case studies, we selected a list of journals which are specialized in the method in question. We identified these journals on the basis of the description of scope found in their official websites. They represent the specialized outlets where new research results on the formal methods are communicated to the specialized scientific communities. For game theory, the list comprises: *International Journal of Game Theory*, *Social Choice and Welfare*, and *Games and Economic Behavior*. We also considered *Theory and Decisions* and *Economics and Philosophy*, but these journals turned out to be scarcely cited by the other three. *Economics and Philosophy*, for instance, receives only 10 citations in IJGT, 19 citations in GEB, and 59 citations in SCW. By contrast, GEB receives 697 citations in IJGT and 450 citations in SCW; IJGT is cited 484 times in GEB and 236 times in SCW; and SCW is cited 169 times in GEB and 697 in IJGT. Thus, the three journals that we considered are characterized by an intense exchange of citations.[5] For network science, we considered two recent journals that are explicitly devoted to this discipline—Cambridge University Press *Network Science,* started in 2013, and Oxford Academic *Journal of Complex Networks,* started in the same year—and the more established but broader journal *Social Networks*, started in 1979. Also in this case, the citation profiles of these three journals confirm that they intensively cite each other (Maltseva & Batagelj, 2021).

The cited references mentioned in these journals constitute the *knowledge base* that is shared by the specialists (Börner et al., 2005). To access and further analyse this knowledge base, we downloaded from Clarivate Analytics Web of Science database all the documents published in these journals from 1985 until present or, for more recent journals, from their date of foundation. All the cited references mentioned in the bibliographies of these documents were retrieved as well, along with the number of times each of them was cited in the considered journals. In Web of Science, each reference is associated with several metadata including the name of the first author, the publication year (known as RPY, reference publication year), the journal of

[4] PhilPapers has recently started to collect citation data of publications, but they are still too sparse to be useful.

[5] In the Supplementary Materials, we present a bibliometric mapping of an expanded set of game theory journals, which incudes, in addition to the three we selected, also *Theory and Decision*, *Decision Sciences*, and *Group Decision and Negotiation* (see Figure S5). Journal co-citation analysis shows that our three journals receive more citations compared to the other three also in this expanded set and that they are more frequently co-cited together than the other three.

publication (or the abbreviated title in the case of books), volume, page, and sometimes the DOI. These metadata allowed us to further analyse the properties of the knowledge base. However, as reference data in Web of Science frequently contain variants of the same reference and other errors, the downloaded data were cleaned with the software CRExplorer (Thor et al., 2016). Variants were identified and reduced to standard form using CRExplorer algorithm, manually supervising the process in order to correct for wrong merging. In this way, reliable and cleaned datasets representing the knowledge base were produced for further processing.

In particular, since not all the references in the journals' knowledge bases are directly related to the method in question, in the second step of field delineation we retained only those references that occurred in *all* the journals considered, i.e., we retained the references included in the *intersection* among the knowledge bases of the journals. References that do not belong to this intersection were excluded, so as to obtain a *knowledge core* consisting of the scientific documents that are most representative of each of our case-studies. For game theory journals, the knowledge core comprised 1971 distinct references; for network theory journals, it amounted to 455 distinct references.

Even if this reference-based approach to field delineation does not completely eliminate false positives (i.e., references that are common to all journals but do not really belong to the knowledge core), it has the great advantage of constructing the knowledge cores on the basis of a reproducible and completely bottom-up process. In fact, determining manually which of the references belong to the knowledge core was not only practically infeasible given the size of the reference lists, but also potentially prone to subjective biases. Moreover, the knowledge cores thus identified often contain many references that are highly cited in all the journals considered, confirming that this method captures accurately enough not merely what is shared among the journals' knowledge bases, but what is scientifically central for the community investigating or employing the method in question. From this point of view, the focus on the cited references mitigates the potential selection bias in the choice of the starting list of journals, as we can expect the knowledge core to be reasonably stable over different choices of journals in the same area, provided that the area in question is sufficiently homogeneous (for a further discussion of this issue see the methodological considerations in the last section of the paper).

2.2 The knowledge junction: intersection with the philosophical corpus

The assessment of the impact of the formal methods on philosophy included several steps. First, the knowledge core associated with each case-study was intersected with the set of references extracted from *Synthese* and *Philosophy of Science*, i.e., the knowledge base of the two journals. We call this intersection "knowledge junction". Thus, for each case-study we determined a first raw indicator of its diffusion in the philosophical literature, which is equal to the size of the knowledge junction.

Within the knowledge junction, three types of references can be distinguished: "method-specific references", "philosophical references", and "other references". Since in this paper we are focusing on the role of the two formal methods in philosophy, rather than vice versa, we will call "narrow" knowledge junction the set

Table 1 Summary statistics for the two datasets. All statistics are expressed in number of unique cited references, except for the corpus size, which is expressed in number of citing papers and percentage on total philosophy papers = 7976. The Jaccard coefficient is computed using the size of the philosophy knowledge base = 118 083 unique cited references

	Game theory	Network science
Knowledge core (KC)	1971	455
Knowledge Junction (KJ)	314	77
Jaccard coefficient (× 1000)	2.6	0.7
KJ over KC	16%	17%
Narrow KJ (Method-specific CR) (%)	233 (74%)	62 (81%)
Philosophy CR (%)	8 (3%)	0 (0%)
Other CR (%)	73 (23%)	15 (19%)
Dataset size (citing papers)	401 (5%)	61 (0.8%)

of method-specific references. Accordingly, knowledge junction *simpliciter* will also be referred to in terms of "broad" knowledge junction. Yet, the group of philosophical references can also be interesting for our research questions, as they may be the sign of a two-way communication between philosophy and the specialist communities. Even the group of "other references", which are neither method-specific nor philosophical and may belong to deeply diverse disciplines, can reveal something interesting about the relationship between philosophy and formal methods.

Moreover, we annotated for each reference the type of document to which it pointed to, distinguishing between books, research articles, companions and text-books, collections, and so on. References in the narrow knowledge junction (that is, the method-specific references) were further classified as "empirical" or "formal", as we explain in more detail in the next sections. This classification was based on our expert knowledge (in particular, two authors were competent on game theory and one author on network science). The manual annotation was performed by browsing the full text of the cited works and in some cases by taking into account the publication venue.[6]

Uncertain cases were discussed among all the four authors until consensus was reached. To statistically assess the inter-coder agreement, we ran a test on a sample of 62 references belonging to the game theory narrow knowledge junction, which were coded by the two authors competent in game theory. The resulting Cohen's Kappa = 0.82 showed a very good level of agreement between annotators (Landis & Koch, 1977).

After the classification step, only the narrow knowledge junction was retained and all the papers that cited at least one of these references were extracted from the two philosophical journals. In this way, we obtained a philosophical dataset for each of our case-studies. These datasets included all the philosophical publications that have at least one method-specific reference link with the specialistic knowledge core associated with each formal method. The size of these datasets in terms of publications is another raw indicator of the diffusion of the method in the philosophical literature.

Table 1 displays the summary statistics of the two datasets.

[6] For each reference in the dataset, the annotators read the title and the abstract, in the case of articles, or the back cover, in the case of books; when necessary, they also read the first and last paragraph of the article, as well as a page randomly selected, or the index of the book.

2.3 Internal structure of the philosophical datasets

To capture the internal structure of each philosophical dataset, we measured the similarity between the pairs of papers within each dataset by applying a technique that depends again on the analysis of cited references, namely *bibliographic coupling* (Kessler, 1963). Bibliographic coupling is based on the idea that the more references two publications share, the more likely it is that they are "similar" in some respect, for instance they may be about the same topic. Bibliographic coupling has been proved to be a powerful technique for generating science maps that can be used to visualize the internal structure of scientific fields (Boyack & Klavans, 2010; Petrovich, 2020). Recently, it was used to produce an overall map of philosophy (Noichl, 2019) and to map the fields of history and philosophy of science (Weingart, 2015).

When bibliographic coupling is applied, however, it is recommended not to use directly the raw number of shared references to compute the similarity among pairs of publications because papers with long reference lists tend to have more references in common with other publications compared to papers with shorter bibliographies. To avoid these distortions due to difference in bibliography lengths, the raw number of shared references should be replaced by a normalized coefficient of similarity (van Eck & Waltman, 2009).

In particular, in this study we used the classical Jaccard similarity coefficient between sets (Jaccard, 1912; see Leydesdorff, 2008 for a discussion of the advantages of the Jaccard Index in bibliometrics). If A and B represent two non-empty sets, the Jaccard coefficient is defined as:

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

where $|\cdot|$ denotes the cardinality of a set. Clearly, it holds $0 \leq J(A, B) \leq 1$. In the present case, the similarity between two articles is proportional to the number of references they share. If two articles have exactly the same set of references, i.e., when $A = B$, the maximum similarity $J(A, B) = 1$ is achieved. By contrast, if two papers do not have any reference in common, i.e., when $A \cap B = \emptyset$, the coefficient will equal 0. Normalizing by the union of the reference sets allows to avoid the distortions mentioned above.

Similarities were computed for each pair of papers in each philosophical dataset and arranged in a square similarity matrix with the software *Pajek* (Batagelj & Mrvar, 2004). Then, each matrix was visualized as a similarity network using the software *Gephi* (Bastian et al., 2009) and, specifically, the nodes were laid out in the two-dimensional space using the layout algorithm ForceAtlas2 (Jacomy et al., 2014). The algorithm turns the structural proximities contained in the similarity matrix into visual proximities, so that papers frequently citing the same literature appear as nodes that are close to one another on the map.

Lastly, each similarity network was partitioned into communities by using the classic Louvain algorithm as implemented in *Gephi* (Blondel et al., 2008). The algorithm identifies communities of similar papers by optimizing the modularity of the network, allowing to detect sub-disciplinary or topical differences within the philosophical datasets.[7]

2.4 Diffusion in the philosophical literature

The same procedure was applied also to the entire corpus of publications that appeared in *Synthese* and *Philosophy of Science* from 1985 to 2021 to obtain a *reference map* of the philosophical literature. The complete bibliographic records, including the cited references, were downloaded from Web of Science and manually cleaned using CRExplorer to merge variants and correct errors in data.[8] Then, the similarity between each pair of papers was computed on the basis of bibliographic coupling normalized with the Jaccard coefficient, obtaining a network made of 7976 nodes and 928,485 edges. Edges with a weight < 0.02 (61.89% of the total) and nodes with less than 5 links (9.74%) with other nodes in the network were removed to reduce noise in the visualization. Lastly, the network's layout was produced with ForceAtlas2 and communities extracted with the Louvain algorithm.

In this way, a *reference map* was produced, on which the papers in the two corpora could be highlighted in order to analyze the diffusion of game theory and network science in the main subareas of the philosophical literature.

The flowchart in Fig. 1 overviews the whole process of data-gathering, cleaning, filtering and processing that was used to produce the datasets.

3 Philosophy meets game theory

3.1 Broad and narrow knowledge junction

The knowledge junction between the game theory knowledge core and the 118,083 unique references of the philosophical journals resulted into a set of 314 references, the "broad" knowledge junction. By means of manual annotation, the broad knowledge junction was subdivided in the following way: narrow knowledge junction (method-specific references), 233; philosophical references, 8; other references, 73. The number of citing articles (that is, articles published in *Synthese* or *Philosophy of Science* in which at least one of the references belonging to the narrow knowledge junction is cited) is 401. The number of citations (that is, the overall number of references made in the 401 articles to items belonging to the narrow knowledge junction) is 907.

Among the references to research areas different from game theory and philosophy, the most common are those to mathematics (especially statistics and probability theory, but also algebra, topology and mathematical logic) and economics (microeconomics, econometrics, etc.). Both cases can be reasonably accounted for: economics has obvious and close connections with game theory, while mathematics has a clear

7 We tested the VOS algorithm too and the clustering solutions obtained were not significantly different from those generated with the Louvain algorithm. The resolution parameter of the algorithm was set always to the default value of 1. Slight changes in the parameter did not cause relevant changes in the overall structure of the maps.

8 All documents were retrieved, including editorials and reviews.

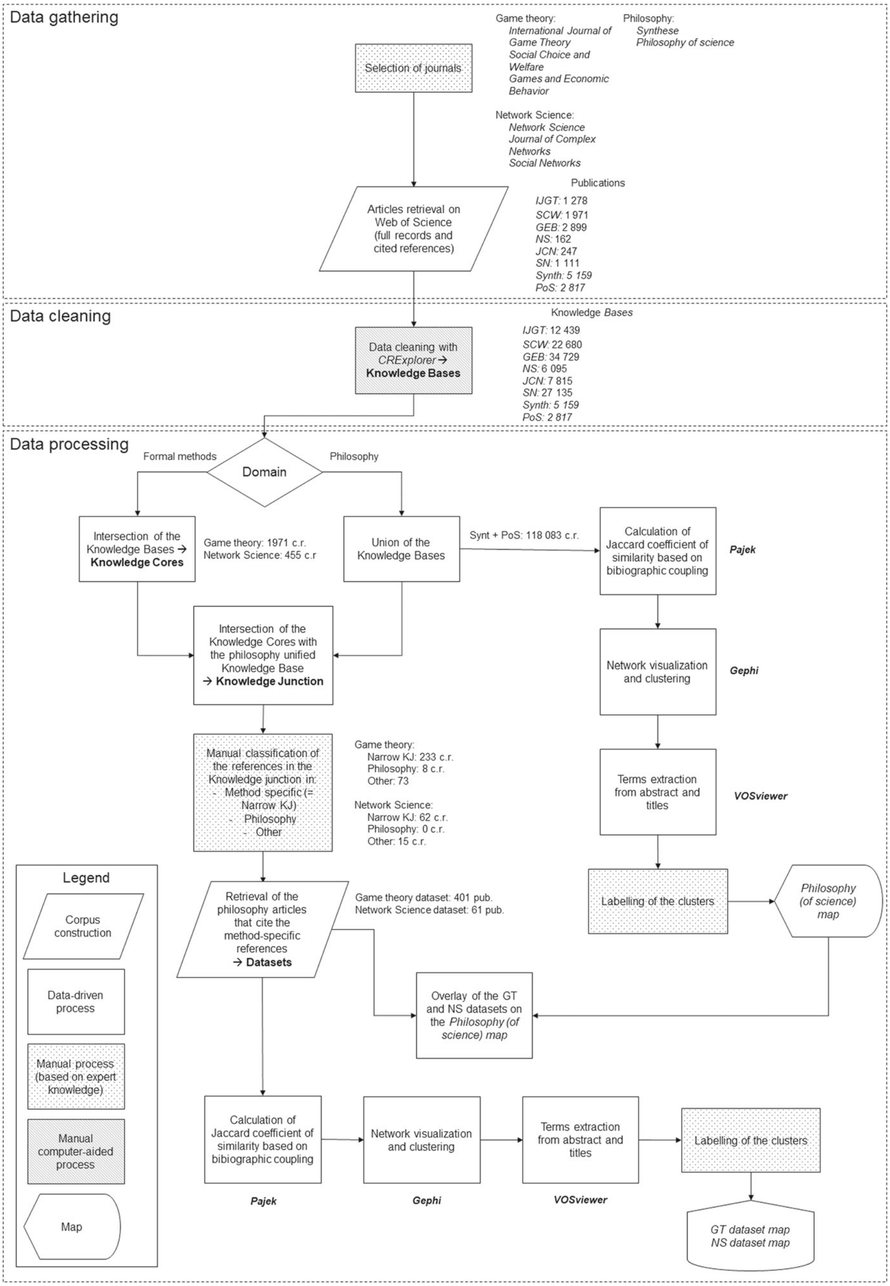

Fig. 1 Overview of the methodology

instrumental role. It is interesting that a significant part of such references are handbooks or "classical" texts. Among the cited economics works, for instance, one can find Paul Samuelson's famous *Foundations of Economic Analysis* (1947), or Joseph Schumpeter's *Capitalism, Socialism and Democracy* (1942). One of the few references to mathematical logic is that of Herbert Enderton's classic textbook *A Mathematical Introduction to Logic* (1972). It seems that, when philosophers or game theorists cite works belonging to other disciplines, they preferably cite "reference" works, maybe because of their limited knowledge of the most up to date research, or just because the reasons for citing them are rather "generic", so that a reference work is the most appropriate citation in that context. At a slightly different level, among the few references that strictly belong to biology one can find Richard Dawkins's semi-popular *The Selfish Gene* (1976).

The references to philosophy works are rather few. This probably reflects the relative lack of interest for philosophy among game theorists, or at least among game theorists intent on their technical work and trying to publish on their specialist journals. There is a sense in which this is not surprising: one is not expecting to find many references to philosophy in—say—electrical engineering journals. Yet game theory seems to be different from electrical engineering with respect to its relationship with philosophy, which is certainly closer, as is witnessed by the reverse interest *of* philosophers *in* game theory, which is certainly more intense than their interest in electrical engineering. All this raises the rather general issue of the somewhat asymmetrical character of the relationships between philosophy and special disciplines, which—however—should be investigated from a broader point of view, by taking into account a large number of different scientific areas. We leave this for future research. It is worth noting that two among the most cited philosophic references in the knowledge junction are John Rawls's *A Theory of Justice* (1971), which is cited in 9 of the 401 citing articles, and David K. Lewis's *Counterfactuals* (1973), with 4 citing articles. Both of them are very important and influential books, in the fields of ethics and philosophy of language (or metaphysics) respectively, and their connections with game theory are rather evident.[9]

We have also annotated both the broad and the narrow knowledge junction according to the kinds of works referred to. We distinguished references to articles, books, and handbooks (which include companions, textbooks, etc.). These are the results for the broad knowledge junction: 217 articles, 61 books, 36 handbooks. These are the results for the narrow knowledge junction: 181 articles, 44 books, 8 handbooks. What immediately stands out is the larger share of handbooks in the broad knowledge junction with respect to the narrow. This is accounted for by a phenomenon that has already been pointed out: the relatively common practice of citing "reference" works of disciplines different from both game theory and philosophy.

3.2 Empirical and formal references

The narrow knowledge junction was internally analyzed by classifying each article as "empirical" or "formal".[10] The two categories of empirical and formal employed

[9] For David Lewis's crucial role in the history of late analytic philosophy see Buonomo and Petrovich (2018, p. 166).

[10] For a more detailed description of the process of annotation, see footnote 6 above.

in the classification can be explicated as follows. Preliminarily, the two notions were "modelled" in the sense of Betti and van den Berg (2014 and 2016). A cited work is said to be formal just in case game theory is investigated or used in it as a formal discipline, i.e., as part of (applied) mathematics. In other words, the purpose of game theory in the formal sense is that of introducing, discussing or reforming some mathematical models that can be applied to rational behaviour, abstractly conceived. A work is regarded as empirical if it aims at testing a game-theoretic model empirically or if it uses some game-theoretic notions in the context of an empirical research. In other words, empirical works focus on the measurable interactions among empirical subjects such as for example psychological subjects, rather than on the mathematically modelled cooperation or competition among abstract agents. Operatively, empirical and formal references were detected in the dataset by applying some guiding instructions or annotation rules. Following this procedure, we first identified and recognized the works in which the purpose was above all empirical, i.e., the cases in which empirical data were collected and discussed in order to test a game-theoretic mathematical model or a theoretical view, whose formulation required the use of some game-theoretic notions. We labelled "formal" all the remaining game-theoretic works. Notice that for the sake of the present investigation the two notions were treated as mutually exclusive, though in principle it could be argued that they are not so.

Here is the main result of the classification. Among the 907 citations to the 233 unique cited references in our dataset, 212 (23.4%) turned out to be empirical, whereas 695 of them (76.6%) were classified as formal. The different percentages of empirical and formal references are a quite expected result: after all, game theory was born as a formal discipline, so it is not surprising that philosophers use and investigate it, above all, as a formal discipline. However, the very presence of empirical references is a relevant fact in itself, which deserves consideration.

Most cited works belonging to the formal subset are cases of game theory (in the broad sense we use in this paper, which includes social choice theory as well), conceived of as part of mathematics and applied to traditional economic questions such as the problem of social choice in competitive conditions, the calculation of preferences and the issue of equilibrium. Examples of this kind of references are the classical works by John Nash: "The Bargaining Problem" (1950), "Non-Cooperative Games" (1951), and "Equilibrium Points in N-Person Games" (1950). Different kinds of examples are "Independence, Rationality, and Social Choice" by Charles R. Plott (1973); Gerard Debreu, "Continuity Properties of Paretian Utility" (1964); or Amartya K. Sen, "Choice Functions and Revealed Preference" (1971). A relevant and well-represented sub-topic concerns Arrow theorems. Kenneth Arrow occurs in this dataset both as an author (see for example his *Social Choice and Individual Values*, 1951; *Essays in Risk Bearing*, 1971; *Social Choice and Multicriterion Decision-Making*, 1986; "Rational Choice Functions and Orderings", 1959, and "Existence of an Equilibrium for a Competitive Economy", coauthored with Debreu in 1954) and as a subject of investigation (see, for instance, Christian List and Philip Pettit, "Aggregating Sets of Judgments: An Impossibility Result", 2002, or Mark Allen Satterthwaite, "Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions", 1975). Yet another sub-topic is decision theory, represented for example by Duncan Black, "On the Rationale of

Group Decision-Making" (1948), or Faruk Gul and Wolfgang Pesendorfer, "Temptation and Self-Control" (2003). Within this subgroup**,** a frequently investigated topic is Condorcet jury theorem: see for example "Information, Aggregation, Rationality, and the Condorcet Jury Theorem" by David Austen-Smith and Jeffrey S. Banks (1996), or "Condorcet Social Choice Functions" by Peter C. Fishburn (1977).

The empirical works that are cited by philosophers in *Synthese* and *Philosophy of Science* in the considered period try to complement mathematical game theory with frameworks and especially results taken from more empirically oriented disciplines such as social psychology, cognitive sciences, or biology. Usually, such works present some empirical, above all behavioural or psychological, data, in order to test experimentally a theorem, a model or just a view belonging to or stemming from more mathematically oriented game theory. As some examples can easily show, social psychology and cognitive sciences are the most representative cases. "The Framing of Decisions and the Psychology of Choice" (1981) by Amos Tversky and Daniel Kahneman ultimately belongs to the cognitive sciences, arguing that "the psychological principles that govern the perception of decision problems and the evaluation of probabilities and outcomes produce predictable shifts of preference when the same problem is framed in different ways" (Tversky & Kahneman, 1981, p. 453); similarly, "Thinking Through Uncertainty: Nonconsequential Reasoning and Choice" (1992) by Tversky and Eldar Shafir focuses on cognitive reactions under uncertainty conditions, i.e., on cases in which, as it often happens, people do not consider appropriately each of the relevant branches of a decision tree. Other slightly different though related cases are for example "An Experimental Analysis of Ultimatum Bargaining" (1982) by Werner Güth et al., an empirical study of behaviour in games; "Behavioral Game Theory: Experiments in Strategic Interaction" (2003) by Colin F. Camerer and "Learning in Extensive-Form Games: Experimental Data and Simple Dynamic Models in the Intermediate Term" by Alvin Roth and Ido Erev, two different attempts to complement game theory with psychology. A relevant subgroup is paradigmatically represented by John Maynard Smith's *Evolution and the Theory of Games* (1982), in which the interaction is between theory of games and evolutionary theory; similarly, Jack Hishleifer studied "Economics from a Biological Viewpoint" (1977). Yet other kinds of empirical application and testing of game theory concern economics (e.g., "Social Preferences Reveal About the Real World?", 2017, by Steven D. Levitt and John A. List, which tests a game-theoretic model using laboratory experiments in economics) and the problem of voting (as it happens, for example, in "Standard Voting Power Indexes Do Not Work: An Empirical Analysis" by Andrew Gelman et al., 2004). As to politics, *Game Theory and Politics* (1975) by Steven J. Brams uses plenty of real-life examples to show how game theory can explain and elucidate complex political situations.

Another interesting aspect concerns the distribution over time of formal and empirical references. Two kinds of data, both represented in Figs. 2 and 3 below, deserve interpretation.

It clearly turns out that philosophers initially dealt with formal game theory, whereas reference to empirical works became relevant only later, arguably from the mid-2000s. That is accounted for partly by the fact that empirical applications or testing of game theory occurred with some delay with respect to the development of game theory

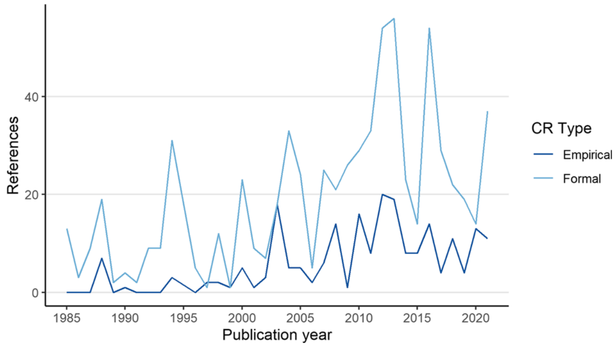

Fig. 2 Citations to formal and empirical references by publication year of the citing paper

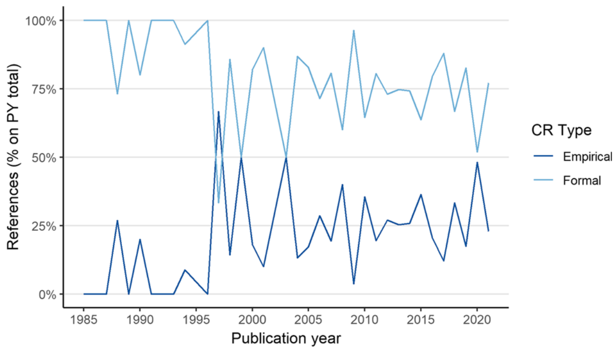

Fig. 3 Quota of citations to formal and empirical references on the publication year's total, by publication year

as a formal discipline, and partly by the fact that the reception of these studies by philosophers may have added to the delay.[11] The same phenomena may also account for that the fact that the cited references belonging to the formal subset and those belonging to the empirical subset have a slightly different average age (the number of years between the publication of a work and its citation): 31.5 years for formal references, and 26.9 for the empirical ones.

3.3 Cluster analysis

To analyse the internal structure of the citing papers corpus, we employed the techniques of bibliographic coupling and cluster detection via the Louvain algorithm, as explained in the Methodology section.

[11] At a first glance, Fig. 3 might be taken to suggest that the year 1997 represented a significant exception, since it is the only year in which philosophers cited more empirical works than formal ones. However, this statistical data simply depends on the fact that in 1997 the number of overall references was extremely small, hence statistically irrelevant. Something similar can be said for 1999, when there were as many empirical references as formal ones. The first significant year is perhaps 2003, with 18 articles published in *Synthese* and *Philosophy of Science* containing empirical references, and 18 containing formal references.

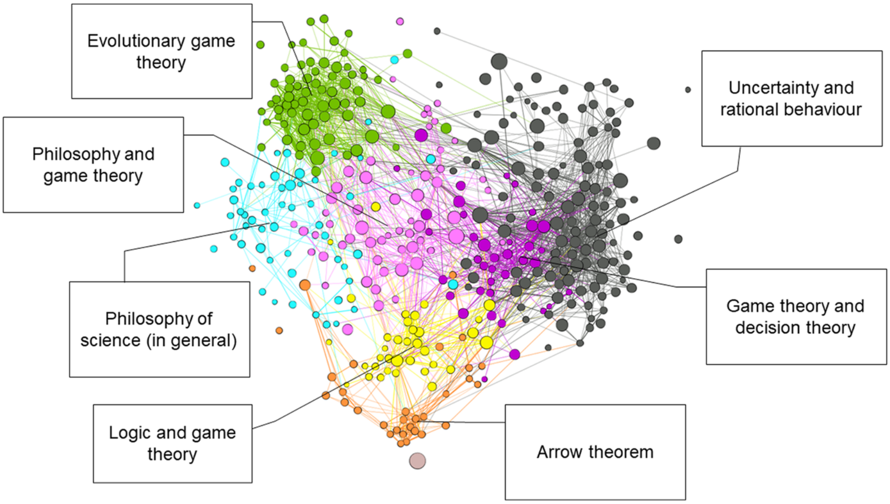

Fig. 4 Map of game theory corpus: bibliographic coupling network of philosophy papers citing game theory references ($n = 401$)

The resulting network is visualized in Fig. 4. Nodes represent the 401 papers citing the narrow knowledge junction associated with game theory, whereas links represent the bibliographic coupling links between them. The thickness of the links is proportional to the strength of the bibliographic coupling, i.e., the number of shared references normalized via the Jaccard coefficient. Nodes are positioned on the map based on their bibliographic coupling similarity, so that papers with many common references are placed closer on the map. Their size is proportional to the number of connections they have with other nodes in the network (i.e., the node degree) and their colour indicates the community to which they are attributed by the community detection algorithm. Labels were attributed to clusters based on the data presented below (Table 2).

The Louvain algorithm partitions the network into 7 main communities (plus 1 minor community containing only 1 article). Table 2 shows, for each community and in general, the top five most cited references in each cluster (note that we consider all the references in the bibliographies of the articles, not only those present in the knowledge junction). Table 3 shows the most frequent terms extracted from the abstracts of the papers in each cluster. Terms are defined as n-grams that consist exclusively of nouns and adjectives and that end with a noun (e.g., "game theory", "text mining", "network analysis"). Occurrences were calculated by counting the number of abstracts that mentioned each term, independently of whether the term occurred multiple times in the same abstract.[12] A threshold of 5 occurrences was set for inclusion. The set of terms was further refined by including only the 60% most relevant terms according to VOSviewer relevance score (van Eck and Waltman 2018).[13] To complete the clusters' profile, Table 4 presents several statistics associated with them.

[12] This method is called "binary counting" in VOSviewer terminology.

[13] Roughly, the algorithm attributes a relevancy score to each term considering how it is diffused among different documents (in our case, different abstracts). The idea is that common terms that appear in many

Table 2 Top five most cited references in the overall game theory corpus and by cluster (number of citations in the corpus in squared brackets)

Rank	Overall	Uncertainty and rational behaviour	Evolutionary game theory	Philosophy and game theory	Philosophy of science (in general)	Game theory and decision theory	Logic and game theory	Arrow theorem
1	Savage L., 1954, FDN STAT [42] **N**	Savage L., 1954, FDN STAT [36] **N**	Skyrms B, 1996, EVOLUTION SOCIAL CON [24] **N**	Lewis D., 1969, CONVENTION PHILOS ST [24] **N**	Woodward J., 2003, MAKING THINGS HAPPEN [11] **N**	Luce R. D., 1957, GAMES DECISIONS [25] **F**	Osborne M. J., 1994, COURSE GAME THEORY [18] **F**	List C, 2002, ECON PHILOS, V18, P89, https://doi.org/10.1017/S0266267102001098 [19] **F**
2	Lewis D., 1969, CONVENTION PHILOS ST [37] **N**	KAHNEMAN D, 1979, ECONOMETRICA, V47, P263, https://doi.org/10.2307/1914185 [33] **E**	Smith J.M., 1982, pi [23] **E**	Schelling T.C., 1960, STRATEGY CONFLICT [19] **E**	Schelling T., 1978, MICROMO-TIVES MACROBE [9] **F**	Jeffrey Richard C., 1983, LOGIC DECISION [6] **N**	Fagin R, 1995, REASONING KNOWL-EDGE [13] **N**	Arrow K. J., 1951, SOCIAL CHOICE INDIVI [7] **F**
3	KAHNEMAN D, 1979, ECONO-METRICA, V47, P263, https://doi.org/10.2307/1914185 [34] **E**	ELLSBERG D, 1961, Q J ECON, V75, P643, https://doi.org/10.2307/1884324 [28] **F**	Skyrms B, 2004, STAG HUNT EVOLUTION [18] **N**	AUMANN RJ, 1976, ANN STAT, V4, P1236, https://doi.org/10.1214/aos/1176343654 [13] **F**	Hausman Daniel M., 1992, INEXACT SEPARATE SCI [6] **N**	ARROW KJ, 1959, ECONOMICA, V26, P121, https://doi.org/10.2307/2550390 [4] **F**	ALCHOURRON CE, 1985, J SYMBOLIC LOGIC, V50, P510, https://doi.org/10.2307/2274239 [12] **N**	Pauly M, 2006, J PHILOS LOGIC, V35, P569, https://doi.org/10.1007/s10992-005-9011-x [7] **N**
4	Skyrms B, 1996, EVOLUTION SOCIAL CON [33] **N**	Allais M, 1953, ECONOMETRICA, V21, P503, https://doi.org/10.2307/1907921 [20] **F**	TAYLOR PD, 1978, MATH BIOSCI, V40, P145, https://doi.org/10.1016/0025-5564(78)90077-9 [18] **N**	Gauthier D., 1986, MORALS AGREEMENT [10] **F**	KITCHER P, 1990, J PHILOS, V87, P5, https://doi.org/10.2307/2026796 [6] **N**	KRANTZ DH, 1971, F MEA-SUREMENT, V1 [4] **N**	Blackburn P., 2001, MODAL LOGIC [9] **N**	Pigozzi G, 2006, SYNTHESE, V152, P285, https://doi.org/10.1007/s11229-006-9063-7 [7] **N**
5	Luce R. D., 1957, GAMES DECISIONS [31] **F**	Levi I, 1974, J PHILOS, V71, P391, DOI [10.2307/2025161, https://doi.org/10.2307/2025161] [20] **N**	Lewis D, 1969, CONVENTION [17] **N**	AUMANN RJ, 1987, ECONO-METRICA, V55, P1, https://doi.org/10.2307/1911154 [7] **F**	Weisberg M, 2006, PHILOS SCI, V73, P730, https://doi.org/10.1086/518628 [6] **N**	BERNHEIM BD, 1984, ECONO-METRICA, V52, P1007, https://doi.org/10.2307/1911196 [3] **F**	Plaza J., 1989, P 4 INT S METH INT S, P201 [9] **N**	Arrow K.J, 1963, SOCIAL CHOICE INDIVI [6] **F**

Table 3 Top ten most occurring terms by cluster (number of occurrences in the corpus in squared brackets)

Rank	Uncertainty and rational behaviour	Evolutionary game theory	Philosophy and game theory	Philosophy of science (in general)	Game theory and decision theory	Logic and Game theory	Arrow theorem
1	Behavior [16]	Case [12]	Paper [19]	Account [14]	Paper [11]	Logic [23]	Aggregation [12]
2	Person [16]	Signal [12]	Condition [11]	Agent [10]	Choice [10]	Game [13]	Account [8]
3	Study [16]	Agent [11]	Account [9]	Case [10]	Problem [9]	Game theory [12]	Belief [8]
4	Analysis [15]	Cooperation [8]	Model [9]	Question [10]	Case [8]	Agent [10]	Arrow [7]
5	Set [14]	Kind [8]	Rationality [9]	Argument [9]	Order [8]	Situation [8]	Case [7]
6	Article [13]	Question [8]	Case [7]	Article [9]	Belief [6]	Player [7]	Collective judgment [7]
7	Assumption [13]	Account [7]	Sense [7]	Claim [8]	View [5]	Complete axiomatization [6]	Set [7]
8	Question [13]	Emergence [7]	Player [6]	Information [8]			Theorem [7]
9	Rule [13]	Equilibrium [6]	Principle [6]	Way [8]			Group [6]
10	Condition [12]	Information [6]	Common knowledge [5]	Effect [7]			Member [5]

Table 4 Summary statistics of the clusters in the game theory corpus

Cluster	Label	Articles	Avg. PY	SD PY	N formal CR	N empirical CR
Dark grey	Uncertainty and rational behaviour	116	2011,6	7,7	62	22
Green	Evolutionary game theory	67	2009,4	8,9	45	19
Pink	Philosophy and game theory	58	2003,6	11,4	69	18
Light blue	Philosophy of science (in general)	55	2013,8	4,7	28	21
Violet	Game theory and decision theory	38	2003,2	10,6	36	7
Yellow	Logic and game theory	35	2008,8	6,5	39	2
Orange	Arrow theorem	31	2011,3	9,0	18	2
Overall	*GT corpus*	*401*	*2009,3*	*9,2*	*179*	*54*

Here is a brief illustration of the labels that we attached to the clusters, based on the analysis of the most cited references of each cluster (Table 2) and terms occurring in abstracts (Table 3).

Papers in the dark grey cluster—the largest in the dataset—refers to the issue of uncertainty and rational behaviour: in it we find a mix of empirical and formal references, and the works by Kahneman and Tversky are the most cited ones. Some probability is present—not only Savage's *Foundations of Statistics*, but even Keynes's *Treatise of Probability*. This set of papers uses some formal methods—logic, probability theory, game theory—together with empirical results obtained in social psychology and cognitive sciences. Apart from the most commonly used terms, which occur in every cluster and are to a large extent irrelevant, the most frequently used terms in this cluster are 'behaviour', 'person', and 'uncertainty'.

The green cluster concerns evolutionary game theory. The most cited author is Brian Skyrms, with a special focus on his works on evolutionary game theory and in particular on *Signals* (Skyrms, 2010). The most frequently used terms are 'signal' (the single most frequent term), 'agent', 'cooperation', 'emergence', 'equilibrium', 'information' and 'convention' (a word that clearly refers to the work of David Lewis, as confirmed by the references). Here again there is a mix of formal and empirical references. The story can be approximately reconstructed as follows (Ross, 2019). Initially, biologists and statisticians who worked on the theory of evolution started to use game-theoretic instruments. Later, after game theory had acquired a recognized status, game theorists themselves used in turn tools borrowed from evolutionary theory,

Footnote 13 continued

documents carry less information about the specific content of a document, compared to terms that are concentrated in few documents. The relevance of the former is then lower than that of the latter.

extending their application from biology to the social sciences (this process structurally resembles the so-called “reverse imperialism” in the history of economics and social sciences; for the notion of “reverse imperialism” see for instance Davis, 2010).

The pink cluster can be labeled “philosophy and game theory”. The most cited author in this cluster is David Lewis (other frequently cited authors are the economist Thomas Schelling and the mathematician Robert Aumann, but also the moral philosopher David Gauthier); most cited references are philosophical or game-theoretic in the formal sense; the most frequently used terms are ‘model’, ‘rationality’, ‘player’, ‘principle’, ‘convention’.

The light blue cluster refers to philosophy of science (in general). Its key terms are ‘information’, ‘effect’, ‘individual’, and ‘probability’, and many terms associated with conceptual tools typically employed by analytic philosophers are also present. Its most cited authors are James Woodward, Michael Weisberg, Thomas Schelling, Daniel Hausmann, Philip Kitcher. Narrow game-theoretic references seem not to be central.

The violet cluster can be interpreted as “game theory and decision theory”. The most frequently used terms are ‘choice’, ‘order’, ‘belief’. Most references are game-theoretic in the formal sense, or formal though not game-theoretical (probability, mathematics, etc.). The most cited articles are R. Duncan Luce, *Games and Decisions* (1957) and Richard Jeffrey, *The Logic of Decision* (1965); Arrow occupies a central position too.

The yellow cluster is “logic and game theory”, or “game theory for logicians”. Key terms are ‘logic’, ‘game’, ‘game-theory’, ‘agent’, ‘player’. Most references are logical or game-theoretic in the formal sense. The most cited work is Martin J. Osborne’s handbook of game theory; other frequently cited authors are logicians such as Jaakko Hintikka or Johan van Benthem.

The orange cluster concerns Arrow theorem, which is often compared to other theorems. Christian List is the most cited author. There is much Arrow. Significant terms are ‘aggregation’, ‘belief’, ‘Arrow’, ‘theorem’. Most cited articles are formal, either game-theoretic or broadly mathematical.

3.4 Diffusion of game theory in the reference map of philosophy

As said in the Methodology, bibliographic coupling was used to produce a *reference map* of philosophy including all the 7976 articles published in *Synthese* and *Philosophy of Science* from 1985 to 2021. By overlaying our corpus of interest on this reference map, we can better understand where philosophical papers citing game theory are localized in the general structure of philosophy.

The map is visualized in Fig. 5. Again, each node represents a paper and links the bibliographic coupling similarity between them.[14] To simplify the visualization, the size of the nodes is fixed, i.e., it does not reflect any property of the nodes. The colour, by contrast, is based on the community to which the nodes are attributed by the Louvain algorithm. For this reference map, the algorithm finds 11 different clusters.

[14] As said in the Methodology, links with strength < 0.02 and nodes with < 5 links were removed from visualization.

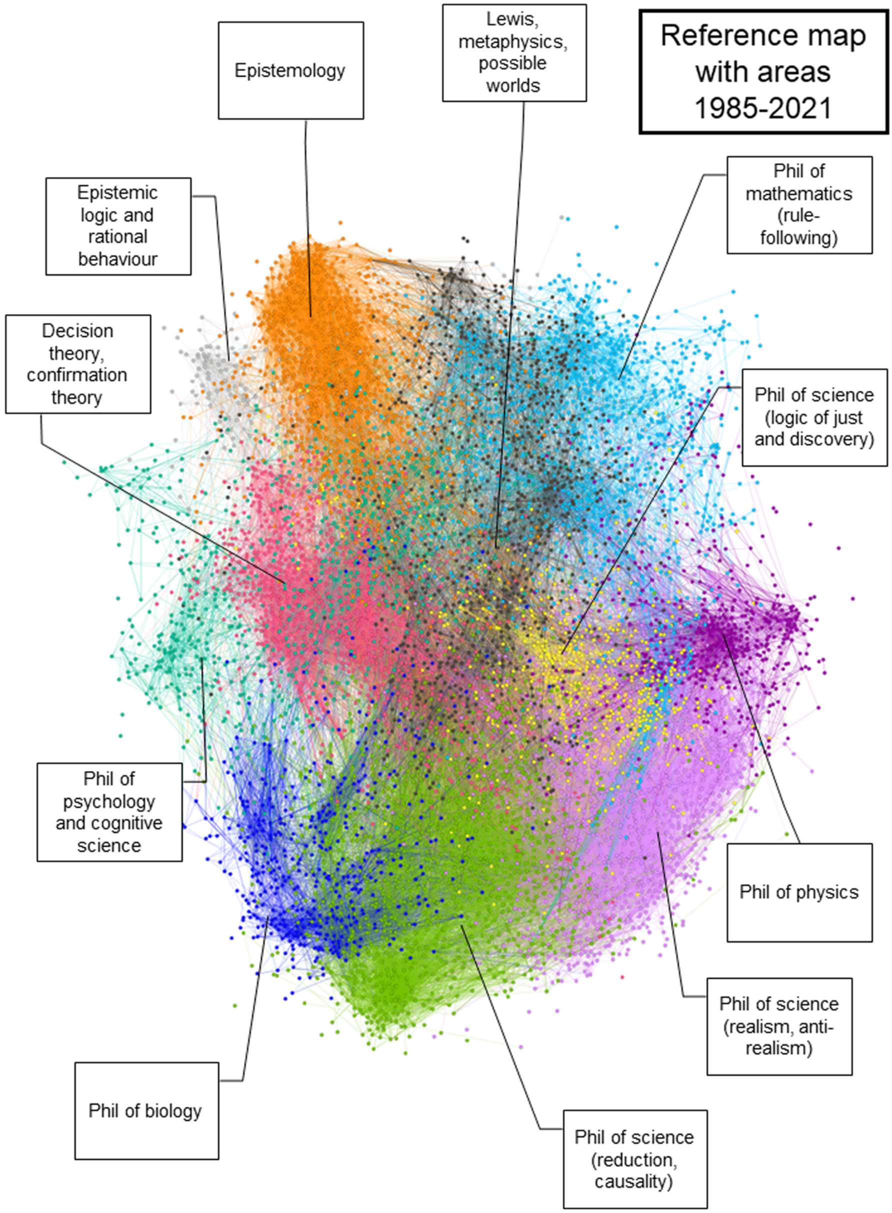

Fig. 5 Reference map of philosophy: bibliographic coupling network of philosophy papers ($n = 7976$). Links with weight < 0.02 and nodes with degree < 5 are not shown. The size of the nodes is fixed. Clusters are represented by colors

To better characterize and label them, we used again VOSviewer text mining utility to extract the most frequent and relevant terms from the abstracts of each cluster (Table 5).

Considering both the recurring terms and the titles of the articles in each cluster, we associated the eleven clusters with eleven philosophical sub-areas or topics. Starting from the northern part of the map and in clockwise order, they are:

- Epistemology
- Lewis, metaphysics, possible worlds
- Philosophy of mathematics (rule-following considerations)
- Philosophy of science (logic of justification and logic of discovery)
- Philosophy of physics
- Philosophy of science (realism, anti-realism)
- Philosophy of science (reduction, causality)
- Philosophy of biology
- Philosophy of psychology and cognitive science
- Decision theory and confirmation theory
- Epistemic logic and rational behaviour

The game-theoretic clusters that were identified before can be then highlighted on the reference map (Fig. 6). In this way, we can better understand how the corpus of philosophical articles citing the game theory references are distributed among different philosophical sub-areas.

Two main features of Fig. 6 are worth mentioning. The first is that game theory is especially present in the following philosophical clusters: the cluster 'philosophy of psychology and the cognitive sciences' contains in particular references of evolutionary game theory and some empirical/cognitive investigations on uncertainty and rational behaviour; the cluster 'decision theory and confirmation theory' contains the game theory clusters 'game theory and decision theory', 'philosophy and game theory', 'uncertainty and rational behaviour', 'Arrow theorem'; not surprisingly, the cluster 'epistemic logic and rational behaviour' especially includes references concerning game theory and logic. The second result is that game theory is nearly absent in many philosophical clusters, such as for example general philosophy of science, philosophy of physics, philosophy of biology. A more thorough discussion of this point will be provided in the final section.

4 Philosophy meets network theory

4.1 The knowledge core of network science

As said in the first section of this article, network science is an interdisciplinary research field, to which both natural and social scientists contribute (Hidalgo, 2016). To identify the knowledge core associated with network science, hence, we used a set of journals that reflects both the communities of specialists. For the social science community, the established journal *Social Networks*, started in 1979, was selected. *Social Networks* is the most prestigious journal for research in social networks and their application

Table 5 Top ten most occurring terms in the reference map by cluster. The number of abstracts in which the terms are mentioned is shown in square brackets. The size of the cluster is expressed in number of papers.

Rank	Phil of psychology and cognitive science	Phil of science (reduction, causality)	Phil of mathematics (rule-following)	Phil of science (realism, antirealism)	Phil of science (logic of justification and discovery)	Epistemology	Decision theory and confirmation theory	Phil of physics	Phil of biology	Lewis, metaphysics, and possible worlds	Epistemic logic and rational behaviour
Size	118	161	139	177	65	159	132	72	65	137	21
Colour	Dark green	Green	Light blue	Violet	Yellow	Orange	Red	Dark violet	Blue	Dark grey	Grey
1	Model [80]	Mechanism [116]	Philosophy [96]	Argument [141]	Value [60]	Philosophy [54]	Measure [78]	Quantum mechan-ics [74]	Evolution [74]	Null [103]	Agent [46]
2	Content [79]	Causation [100]	Sentence [64]	System [93]	Role [58]	Analysis [51]	Hypothesis [70]	Result [48]	Selection [58]	Semantic [80]	Knowledge [36]
3	Property [69]	Property [75]	Science [63]	Process [82]	Model [55]	Person [48]	sSate [57]	Space [39]	Natural selection [57]	Proposition [72]	Way [30]
4	Article [60]	Physic [67]	Predicate [45]	Data [74]	Result [49]	Practice [48]	Decision [53]	Work [36]	Process [47]	Metaphysic [63]	Problem [28]
5	Information [60]	Cause [61]	Paradox [43]	Debate [73]	Scientist [48]	Context [47]	Choice [52]	State [35]	Question [47]	Context [62]	Result [28]
6	Perception [57]	Research [57]	Wittgenstein [43]	Realism [73]	Article [44]	Concept [44]	Method [49]	Spacetime [33]	Term [42]	Language [59]	Case [26]
7	Experience [56]	Biology [56]	Aim [41]	Scientific realism [71]	Philosopher [37]	Solution [44]	Confirmation [48]	Case [32]	Function [41]	Sentence [57]	Analysis [22]

Table 5 (continued)

Rank	Phil of psychology and cognitive science	Phil of science (reduction, causality)	Phil of mathematics (rule-following)	Phil of science (realism, antirealism)	Phil of science (logic of justification and discovery)	Epistemology	Decision theory and confirmation theory	Phil of physics	Phil of biology	Lewis, metaphysics, and possible worlds	Epistemic logic and rational behaviour
8	Behavior [47]	Method [56]	Existence [41]	Relation [70]	Term [33]	Virtue [42]	Context [48]	Formulation [31]	Population [41]	Law [50]	Approach [22]
9	Strategy [45]	Effect [54]	Objection [41]	Version [69]	Practice [31]	Attitude [38]	Credence [47]	Possibility [30]	Species [41]	System [47]	Question [22]
10	Cognitive science [44]	Function [54]	Article [40]	Truth [66]	Sense [30]	Experience [37]	Study [45]	law [29]	Science [39]	Event [45]	Reasoning [22]

LThe number of abstracts in which the terms are mentioned is shown in squared brackets. The size of the cluster is expressed in number of papers

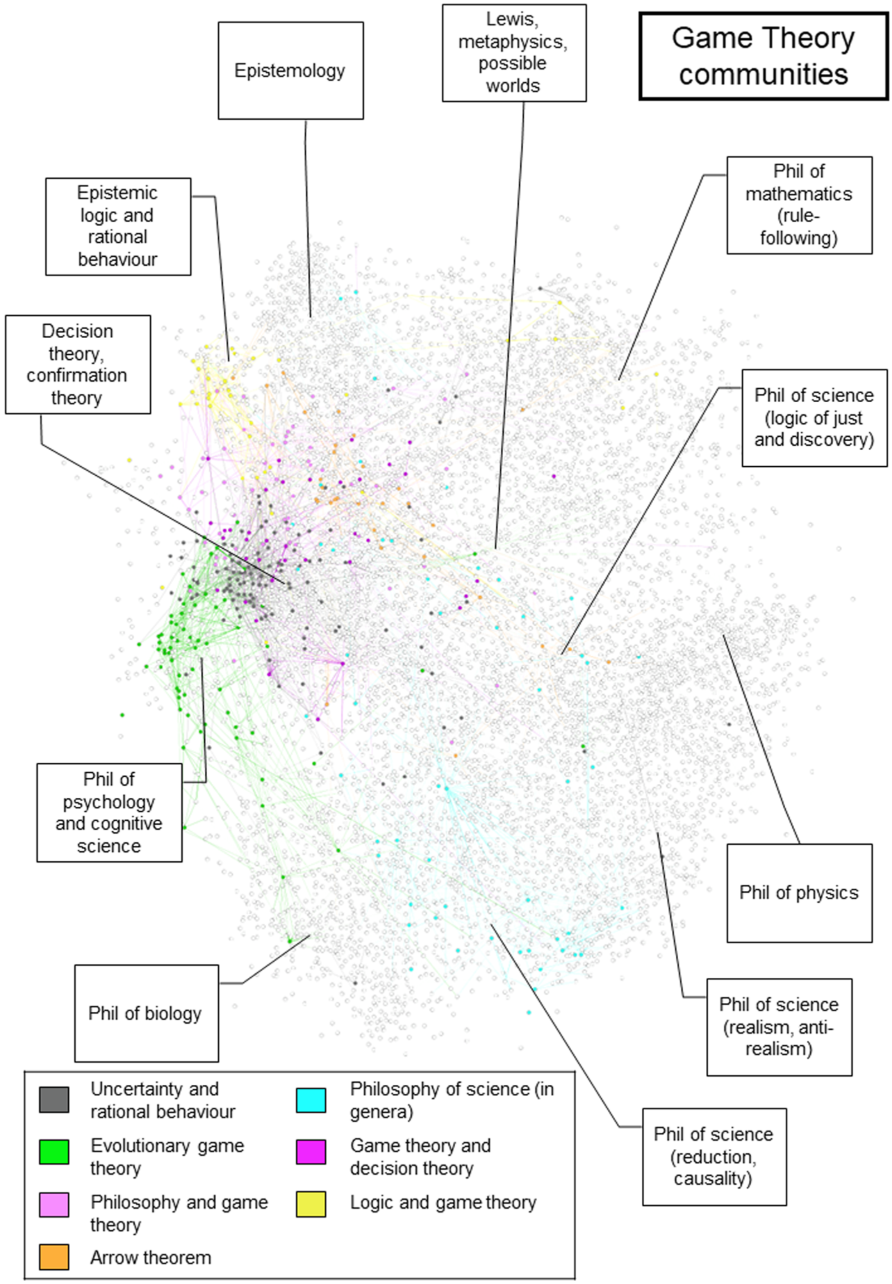

Fig. 6 Game-theoretic clusters overlaid on the reference map of philosophy

to social sciences, mainly sociology (Maltseva & Batagelj, 2021). For the natural science community, which has begun to work with networks in the last two decades, we selected more recent journals: *Network Science*, started in 2013, and *Journal of Complex Networks*, started in the same year. The presence of two journals that approach network science from a point of view that is closer to natural sciences compensates for the longer publication life of *Social Networks*.

The intersection of the knowledge bases of these three journals resulted in a knowledge core (KC) of 455 cited references. The oldest reference in the KC dates 1938, the most recent 2018. Most of the references are concentrated in the period 1999–2014, when 72% of the references where published. The average RPY (reference publication year) of the KC equals 2000. These data are consistent with standard historical reconstruction that dates the establishment of the "new science of networks" after 2000 (Lewis, 2009, Chapter 1).

4.2 Weight of the network science knowledge core in philosophy

Of the 455 references in the KC of network science, 77 (17%) appear also in the philosophy knowledge base, constituting the knowledge junction between the two. Of these, 62 (81%) are method-specific, i.e., are directly related to network science, 15 (19%) do not regard network science, but other topics, mainly game theory. No reference can be ascribed to the field of philosophy. The 62 method-specific references point mainly to research articles (42), reviews (10), and handbooks (6). The temporal distribution of these 62 references is not significantly different from the temporal distribution of the entire KC (Fig. 7). Hence, even if philosophers cite only a portion of the network science KC, their overall citing behaviour is similar to that of network science specialists in terms of temporal distribution. However, the most recent segment of the KC is scarcely absorbed by philosophers: of the 15 KC references with RPY = [2015–2018], only 2 appears in the philosophical subset. In fact, the ratio between the number of total KC reference per RPY, on the one hand, and the number of KC references cited by philosophers per RPY, on the other hand, is slightly decreasing (from 0.2 in 1999 to 0 in 2018, see Fig. 8).

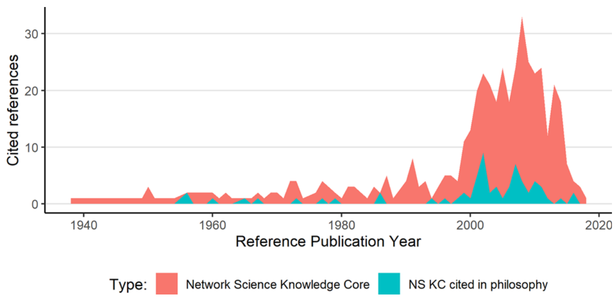

Fig. 7 RPY spectroscopy (Thor et al., 2016) of the network science knowledge core, compared with the RPY spectroscopy of the network science references cited in philosophy

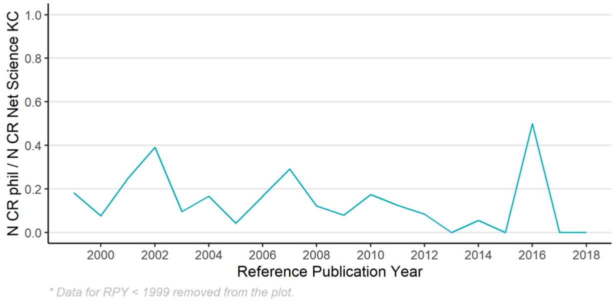

Fig. 8 Ratio between the number of network science references cited in philosophy and the number of network science references in the network science knowledge core, over reference publication year

Within the set of method-specific references, we further distinguished between "formal" and "empirical" references, as said in the Methodology. Following Hidalgo (2016), a reference is classified as formal when the work it points to focuses on the universal properties of networks, i.e., those properties that depend purely on the topology of the network. By contrast, a reference is classified as empirical when the work it points to focuses on context-specific features of networks. Usually, natural scientists, mainly physicists, work on the formal side of network science, whereas social scientists are more interested in the empirical examination of real (social) networks and their mechanisms (Hidalgo, 2016). For instance, Newman (2006) was classified as formal, as it describes an algorithm for community detection that can be used on any network, independently of the entities the nodes and edges stand for. Barabási et al. (2002), by contrast, was classified as empirical, as it studies the properties of a specific network, that of scientific collaborations expressed by paper co-authorship. This use of the distinction between formal and empirical references largely overlaps with the one made for game theory, even if it was clearly slightly adapted to the case of network science.

4.3 Properties of the method-specific network science references in philosophy

The 62 method-specific references were cited in a total of 61 articles in the philosophy corpus (40 published in published in *Synthese*, 21 in *Philosophy of Science*). The oldest article dates 1994, the most recent 2021. Most of corpus, however, is concentrated in the period [2012–2021] (Fig. 9).

Table 6 shows the most cited method-specific references in the corpus. The first two are classics of the new science of networks, published in *Nature* and *Science* by four of the key figures of the field, Watts and Strogatz, and Barabási and Albert. They are the classic references for major concepts such as small-world and scale-free networks and are highly cited also within the three network science journals considered in this study. The third work, by Vincent D. Blondel and colleagues, introduced the famous Louvain method for the detection of communities in large networks. Interestingly, the fourth in the ranking (David Easley, *Networks, Crowds, and Markets*) is a handbook that is frequently adopted in economics undergraduate courses to teach network analysis

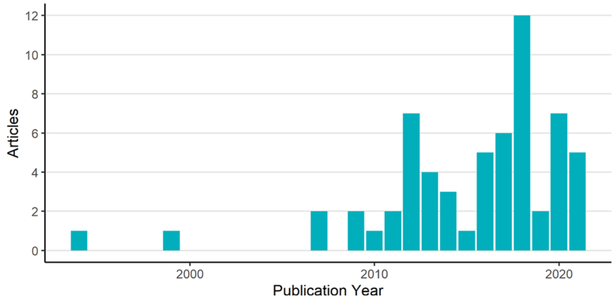

Fig. 9 Number of philosophy articles citing network science over time

applied to economics. It also includes chapters on game theory. Note that it is however scarcely cited in the network science specialist journals. A similar pattern occurs also for the book *Linked. The New Science of Networks* by László Barabási, that is an introductory book intended for the general public. The presence of 3 reviews is also notable, as it may show that philosophers connect to network science not only through research articles but also through overviews of specific areas, such as biological networks or community detection algorithms.

As to the balance between formal and empirical references, the former are two times and half the latter (43 against 16), showing that philosophers are more interested by the formal properties of networks rather than by specific networks.[15]

If we define the *age* of a reference as the difference, measured in years, between the year of publication of the citing article (PY) and the year of publication of the reference (RPY), we can investigate how philosophers relate to the network science literature from a temporal point of view. Do philosophers concentrate their references to network science on recent or older literature? The average age of a reference to network science in our corpus is 15.7 years, i.e., the average network science reference is cited in the philosophical corpus after about 15 years from its original publication. Empirical references are on average older than formal references (18.9 vs. 15.2 years). The reference age however is not constant in time and has increased from 5 for papers published in 2007 to 20.6 for papers in 2020 (Fig. 10). No significant patterns can be distinguished in the trends of empirical versus formal references over time.

4.4 Clusters of philosophy articles citing network science

To better understand the internal structure of the corpus of 61 papers citing the network science references, we generated their bibliographic coupling network and used the Louvain algorithm for partitioning it into distinct communities, as explained in the Methodology section. The result is shown in Fig. 11. In this visualization, nodes correspond to the 61 papers and links to the presence of a bibliographic coupling link between them. The size of the nodes is proportional to the number of connections they have with other nodes in the network (i.e., the node degree), whereas their relative

[15] Even if, as we will see below, the formal properties of specific networks, viz. neural and biological networks, have attracted significant philosophical attention.

Table 6 Network science cited references with at least 3 citations in the philosophy corpus

Cited reference	Title and document type	RPY	Citing articles	%	Cits in NS journals*
Watts DJ, 1998, NATURE, V393, P440, https://doi.org/10.1038/30918	Collective dynamics of "small-world" networks [Research article]	1998	14	23%	151
Barabasi AL, 1999, SCIENCE, V286, P509, https://doi.org/10.1126/science.286.5439.509	Emergence of scaling in random networks [Research article]	1999	12	20%	149
Blondel VD, 2008, J STAT MECH-THEORY E, https://doi.org/10.1088/1742-5468/2008/10/P10008	Fast unfolding of communities in large networks [Research article]	2008	4	7%	59
Easley D., 2010, NETWORKS CROWDS MARK	Networks, Crowds, and Markets: Reasoning about a highly connected world [Handbook]	2010	4	7%	19
laszlo Barabasi A., 2002, LINKED NEW SCI NETWO	Linked. The new science of networks [Book]	2002	3	5%	11
Newman MEJ, 2006, P NATL ACAD SCI USA, V103, P8577, https://doi.org/10.1073/pnas.0601602103	Modularity and community structure in networks [Research article]	2006	3	5%	46
Newman MEJ, 2001, P NATL ACAD SCI USA, V98, P404, https://doi.org/10.1073/pnas.021544898	The structure of scientific collaboration networks [Research article]	2001	3	5%	28
Ravasz E, 2002, SCIENCE, V297, P1551, https://doi.org/10.1126/science.1073374	Hierarchical Organization of Modularity in Metabolic Networks [Research article]	2002	3	5%	9
Nowak MA, 2006, SCIENCE, V314, P1560, https://doi.org/10.1126/science.1133755	Five Rules for the Evolution of Cooperation [Research article]	2006	3	5%	5

Table 6 (continued)

Cited reference	Title and document type	RPY	Citing articles	%	Cits in NS journals*
Barabasi AL, 2004, NAT REV GENET, V5, P101, https://doi.org/10.1038/nrg1272	Network biology: understanding the cell's functional organization [Review]	2004	3	5%	11
Strogatz SH, 2001, NATURE, V410, P268, https://doi.org/10.1038/35065725	Exploring complex networks [Review]	2001	3	5%	14
Fortunato S, 2010, PHYS REP, V486, P75, https://doi.org/10.1016/j.physrep.2009.11.002	Community detection in graphs [Review]	2010	3	5%	63
Jackson MO, 2008, SOCIAL AND ECONOMIC NETWORKS, P1	Social and Economic Networks [Handbook]	2008	3	5%	20

*Sum of the citations received in Social Networks, Network Science, and Journal of Complex Networks

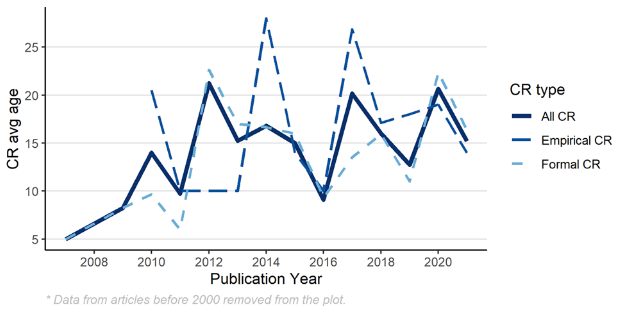

Fig. 10 Average age of network science cited reference by citing paper publication year by cited reference type

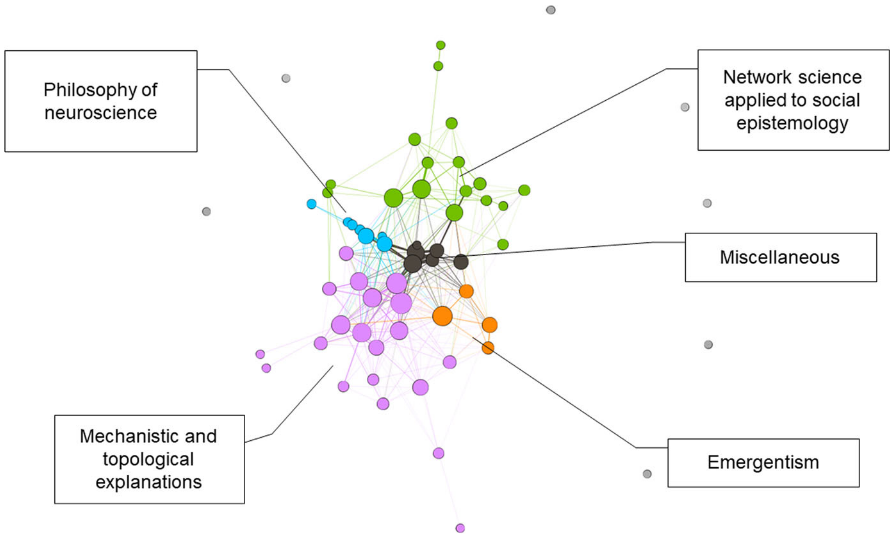

Fig. 11 Map of network science corpus: bibliographic coupling network of philosophy papers citing network science references ($n = 61$)

position reflects their bibliographic similarity, so that papers that share many references are placed closer on the map. The thickness of the link is proportional to the strength of the bibliographic coupling similarity between nodes (i.e., the number of shared references normalized with the Jaccard coefficient). Lastly, the colour of the nodes indicates the community to which they are attributed by the community detection algorithm.

The algorithm identifies five main communities, which correspond to five philosophical sub-areas in which network science is cited.[16] Table 7 shows, for each cluster and in general, the top five most cited references in each cluster (note that we consider

[16] 7 are attributed by the algorithm to 7 distinct 1-node communities. They are the grey nodes in the network visualization.

Table 7 Top five most cited references in the overall network science corpus and by cluster (number of citations in the corpus in squared brackets)

Rank	Overall	Cluster				
		Violet	Green	Light Blue	Orange	Black
		Mechanistic and topological explanation	Network theory applied to social epistemology	Philosophy of neuroscience	Emergentism	Miscellaneous
1	Watts DJ, 1998, NATURE, V393, P440, https://doi.org/10.1038/30918 [14]	Machamer P, 2000, PHILOS SCI, V67, P1, https://doi.org/10.1086/392759 [8]	Zollman KJS, 2010, ERKENNTNIS, V72, P17, https://doi.org/10.1007/s10670-009-9194-6 [6]	Blondel VD, 2008, J STAT MECH-THEORY E, https://doi.org/10.1088/1742-5468/2008/10/P10008 [4]	Barabasi AL, 1999, SCIENCE, V286, P509, https://doi.org/10.1126/science.286.5439.509 [3]	Barabasi AL, 1999, SCIENCE, V286, P509, https://doi.org/10.1126/science.286.5439.509 [5]
2	Barabasi AL, 1999, SCIENCE, V286, P509, https://doi.org/10.1126/science.286.5439.509 [12]	Huneman P, 2010, SYNTHESE, V177, P213, https://doi.org/10.1007/s11229-010-9842-z [7]	Strevens M, 2003, J PHILOS, V100, P55, https://doi.org/10.5840/jphil2003100224 [4]	Newman MEJ, 2006, P NATL ACAD SCI USA, V103, P8577, https://doi.org/10.1073/pnas.0601602103 [3]	Corning P. A., 2002, Complexity, V7, P18, https://doi.org/10.1002/cplx.10043 [3]	Erdos P., 1960, PUBL MATH I HUNG, V5, P17, DOI https://doi.org/10.2307/1999405 [2]
3	Machamer P, 2000, PHILOS SCI, V67, P1, https://doi.org/10.1086/392759 [8]	Watts DJ, 1998, NATURE, V393, P440, https://doi.org/10.1038/30918 [7]	Bala V, 1998, REV ECON STUD, V65, P595, https://doi.org/10.1111/1467-937X.00059 [3]	Albert R, 2000, NATURE, V406, P378, https://doi.org/10.1038/35019019 [2]	Kauffman S., 1995, HOME UNIVERSE SEARCH [3]	Newman MEJ, 2003, SIAM REV, V45, P167, https://doi.org/10.1137/S003614450342480 [2]

Table 7 (continued)

Rank	Overall	Cluster				
		Violet	Green	Light Blue	Orange	Black
		Mechanistic and topological explanation	Network theory applied to social epistemology	Philosophy of neuroscience	Emergentism	Miscellaneous
4	Huneman P, 2010, SYNTHESE, V177, P213, https://doi.org/10.1007/s11229-010-9842-z [7]	Craver C, 2007, EXPLAINING BRAIN [6]	Bonjour L., 1985, STRUCTURE EMPIRICAL [3]	Fortunato S, 2010, PHYS REP, V486, P75, https://doi.org/10.1016/j.physrep.2009.11.002 [2]	Kauffman S.A., 1993, ORIGINS ORDER SELF O [3]	Watts DJ, 1998, NATURE, V393, P440, https://doi.org/10.1038/30918 [2]
5	Zollman KJS, 2010, ERKENNTNIS, V72, P17, https://doi.org/10.1007/s10670-009-9194-6 [6]	Kaplan DM, 2011, PHILOS SCI, V78, P601, https://doi.org/10.1086/661755 [5]	Easley D., 2010, NETWORKS CROWDS MARK [3]	Watts DJ, 1998, NATURE, V393, P440, https://doi.org/10.1038/30918 [2]	ANDERSON PW, 1972, SCIENCE, V177, P393, https://doi.org/10.1126/science.177.4047.393 [2]	Alexander JM, 2003, PHILOS SCI, V70, P1289, https://doi.org/10.1086/377408 [1]

Table 8 Top five most occurring terms in the network science corpus and by cluster (number of occurrences in the corpus in squared brackets)

Rank	Overall	Cluster				
		Violet	Green	Light Blue	Orange	Black
		Mechanistic and topological explanation	Network theory applied to social epistemology	Philosophy of neuroscience	Emergentism	Miscellaneous
M1	Theory [27]	System [18]	Knowledge [10]	debate [4]	Phenomenon [4]	Information [5]
2	Property [25]	Property [15]	Enemy [8]	Approach [3]	Emergent phenomena [3]	Model [4]
3	Agent [22]	Science [13]	Friend [7]	Cognitive architecture [3]	Question [3]	Complex phenotype [3]
4	Community [20]	Theory [13]	Superstar [7]	Functional stability [3]	Collective phenomenon [2]	Fitness [3]
5	Topological explanation [20]	Icm [9]	Group [6]	Higher level causation [3]	Determinism [2]	Mutation [3]

all the references in the bibliographies of the articles, not only the network science references). Table 8, analogously, shows the most cited terms extracted from the abstracts and titles of the papers belonging to each cluster and in general. Terms were extracted with the same method detailed above for game theory.[17] Lastly, Table 9 shows several descriptive statistics for each cluster, namely its size in terms of articles, the average publication year of its articles along with the standard deviation, and the number of formal versus empirical network science unique references occurring in each corpus.

The biggest cluster in the network, the violet cluster, comprises 20 articles and it is the most recent one, with an average PY = 2017.1 (st. dev. = 3.8). These papers belong to general philosophy of science and discuss the epistemological status of *topological explanations*. The debate was spurred by Huneman (2010), the second most cited reference in the cluster, that argued for the existence of explanations that relies upon topological properties of systems and not upon mechanisms or causal processes. Huneman focuses in particular on how ecologists explain ecological stability and notes that their explanations are based on the topological features of the ecological

[17] Abstracts were present in 35 (57%) of the papers, titles in all papers. We considered only those terms with 2 or 3 occurrences in the corpus associated with each cluster. Given the small size of the corpora, each occurrence of a term was counted, even if the term appeared multiple times in the same abstract. From the overall set of terms, then, we retained the 60% most relevant terms according to VOSviewer relevancy algorithm.

Table 9 Summary statistics of the clusters in the network science corpus (only main clusters are shown)

Cluster	Label	Articles	Avg PY	St. Dev. PY	N Formal CR	N Empirical CR
Violet	Mechanistic and topological explanation	20	2017,1	3,8	16	7
Green	Network theory applied to social epistemology	17	2015,2	6,3	14	7
Light Blue	Philosophy of neuroscience	7	2016,0	4,0	12	0
Black	Miscellaneous	6	2013,3	2,7	5	3
Orange	Emergentism	4	2012,0	0,0	4	0
Overall	*NS corpus*	*61*	*2015,3*	*5,1*	*43*	*16*

networks (in particular, the size and density of the species' network). In this context. Huneman presents several formal properties of networks, discussing network science literature on random and scale-free networks. Most of the other papers in the cluster take part in this debate, discussing topological explanations and comparing them with the traditional mechanistic explanations based on causal interaction in mechanisms. Accordingly, the most cited reference in the cluster is Machamer et al. (2000), a paper on the concept of mechanism. Interestingly, the debate focuses mostly on case studies from biology, neuroscience, and medicine, the disciplines in which topological explanations based on networks seem to be more frequent (at least, to the eyes of philosophers). In this sense, it is important to note that philosophers working on topological explanations do not directly use network science as a formal method. Rather, they reflect upon some results of network science, assessing in particular what is the epistemological status of explanations based upon the formal properties of networks. Hence, it is not surprising that formal network science references, such as Watts and Strogatz (1998), are more frequent than empirical references. At the same time, network science is rarely discussed as such[18] but contextualized within disciplinary case studies.

This approach to network science recurs in the light blue and orange clusters as well. They share with the violet cluster the disciplinary focus on biological sciences but are more specialized in neuroscience (light blue) and emergent properties (orange). In the light blue cluster, discussed topics include, among others, cognitive architecture and how it can be explained by means of the modularity of brain networks, neural reuse and brain networks, multilayer networks applied to neuroimaging. Again, the large number of formal references seems to depend on the fact that formal properties of brain networks, such as their scale-free property, are considered. The orange cluster, on the other hand, discusses the concepts of emergence, downward causation, and

[18] The only case is a paper by Rathkopf (2018), that explicitly focuses on a methodological discussion of network science per se.

self-organization in the light of network science, considering in particular Barabási and Albert (1999), a paper in which the two network scientists used the notion of emergence to describe the scale-free property of complex networks. Note that the orange cluster is, on average, older than the violet and light blue cluster.

To sum up, the violet, light blue, and orange clusters are all characterized by the epistemological reflection upon network science and its results, rather than by the direct use of networks to address philosophical problems. Networks appear as an object of philosophical scrutiny, relevant in the discussion of classic topics of philosophy of science, such as scientific explanation and emergent properties, but not as a first-hand tool for developing philosophical results. This direct use of networks, however, is what we find in the green cluster, the second biggest cluster of the network with 17 articles. The papers in this cluster share a common focus on social epistemology, showing that this is the area of philosophy where networks are mostly used as a formal method. Accordingly, the most cited references in the cluster include Zollman (2010) and Bala and Goyal (1998). Interestingly, the kind of social epistemology that is done in these papers is strictly connected to the philosophy of science, and the epistemic communities that are modelled by networks are frequently made of interacting scientists, especially in the more recent contributions. However, more general papers devoted to dynamics of beliefs in social networks are also present. Another notable feature of this cluster is the contextual presence of game theory. Indeed, many papers frame their problem of interest as a game played by a set of actors, mutually connected in a social network, that revise their beliefs considering both the information they gather from their neighbours and the evidence they collect by themselves. By altering various parameters of these models, among which usually there is also the connectivity of the social network, different epistemic scenarios are investigated and sometimes norms for the collective organization of the cognitive labour are derived.

In sum, the map of the philosophical papers citing network science can be divided into two main areas along the North–South axis. In the North, network science is directly applied to social epistemological problems, as a formal method in the proper sense. In the South, network science is rather discussed in the context of philosophy of science topics. In between, the black cluster is a sort of miscellaneous cluster that hosts papers belonging to both groups.

4.5 Network science in the reference map of philosophy

In the reference map of philosophy, the 61 papers citing network science references are concentrated in three general philosophy of science areas (reduction and causality: 26 papers; logic of justification and discovery: 11 papers; realism and anti-realism: 6 papers), in philosophy of psychology and cognitive sciences (7 papers), and in philosophy of biology (5 papers). These three macro-areas account for 90% of the papers in the corpus. The distribution over research areas shown in Table 10 is consistent with the topics described in the previous section. In particular, the social epistemology cluster, in which networks are used as a formal method, is spread over general philosophy of science (10 papers), epistemology (3 papers), and philosophy of psychology

Table 10 Distribution of network science clusters over the philosophy reference map's research areas

Research Area	Cluster						
	Green	Violet	Light Blue	Orange	Black	[Grey]	*Overall*
Phil of science (reduction, causality)	0	14	3	4	2	3	*26*
Phil of science (logic of just and discovery)	8	0	0	0	1	2	*11*
Phil of psych & cog sci	2	2	1	0	1	1	*7*
Phil of science (realism, anti-realism)	2	1	2	0	1		*6*
Phil of biology	0	3	0	0	1	1	*5*
Epistemology	3	0	0	0	0	0	*3*
Decision th, conf th	1	0	0	0	0	0	*1*
Epistemic logic and rational behaviour	1	0	0	0	0	0	*1*
Lewis, metaphysics, and possible worlds	0	0	1	0	0	0	*1*
Tot	*17*	*20*	*7*	*4*	*6*	*7*	*61*

The grey cluster includes the isolated nodes of network science bibliographic coupling network

and cognitive sciences (2 papers). The assessment of the epistemological status of networks (violet, orange, and light blue clusters), on the other hand, is concentrated in the philosophy of science (reduction and causality) area (21 papers) and in the philosophy of biology (3 papers).

These results suggest that the career of networks in philosophy, both as formal methods and as research objects, has begun in the philosophy of science and, for the moment, it is still confined in rather specialized sub-areas such as social epistemology. We will see whether, in the future, they will diffuse over new philosophical areas and, in particular, more traditional areas such as metaphysics.

5 Concluding remarks

5.1 Comparative results

In the previous sections we presented and interpreted some data concerning the diffusion of game theory and network theory, respectively, in the philosophical literature. It may be interesting, here, to consider these data in a comparative way, with the addition of an external foil. This role is played by logic, that is the formal method *par excellence* in philosophy. In this case the set of references representing logic was obtained by intersecting the references cited from 1985 to 2021 in two "core" logic journals such as *The Journal of Symbolic Logic* and *The Journal of Philosophical Logic*. The resulting set was in its turn intersected with the references cited in the philosophical articles published in *Synthese* and *Philosophy of Science* in the same period. As can be easily seen, we followed the same procedure we used in the determination of the knowledge cores and knowledge junctions for game theory and network theory. In this case, however, we deliberately avoid using the expressions "knowledge core" and "knowledge junction" for methodological reasons concerning the issue of field delineation, which will be discussed in the next subsection.

Let us consider the following Table 11 and the association plot in Fig. 12.

Table 11 shows how the papers in the three corpora are distributed over the research areas of the philosophy reference map. Both the absolute number of papers and their

Table 11 Distribution of corpora over research areas

Philosophy research areas	Logic		Game Theory		Network science		Total
	Abs	Perc	Abs	Perc	Abs	Perc	
Decision theory and confirmation	224	29%	152	20%	1	0%	761
Epistemology	67	9%	24	3%	3	0%	788
Lewis (metaphysics, possible worlds)	248	31%	6	1%	0	0%	811
Philosophy of biology	5	1%	10	2%	3	1%	419
Philosophy of physics	72	16%	1	0%	0	0%	456
Phil of psychology & cogn scien	102	15%	71	10%	8	1%	690
Phil of science (logic of just and disc)	43	9%	37	8%	9	2%	470
Phil of science (realism, anti-realism)	109	7%	32	2%	6	0%	1597
Phil of science (reduction, causality)	77	8%	20	2%	27	3%	946
Epistemic logic and ration behave	103	66%	31	20%	1	1%	157
Philosophy of mathematics (rule foll)	472	54%	17	2%	0	0%	881
Total	1522	19%	401	5%	58	1%	7976

Note that papers belonging to multiple corpora are counted in all the corpora in which they appear

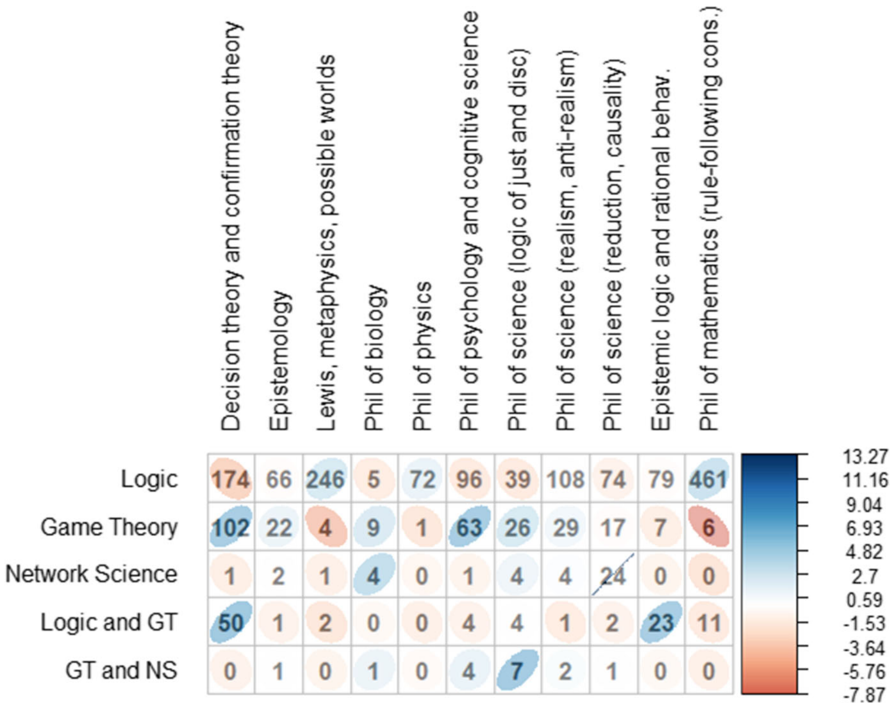

Fig. 12 Association plot between corpora and research areas. The strength and direction of the association is shown by the size, orientation, and color of the ellipses

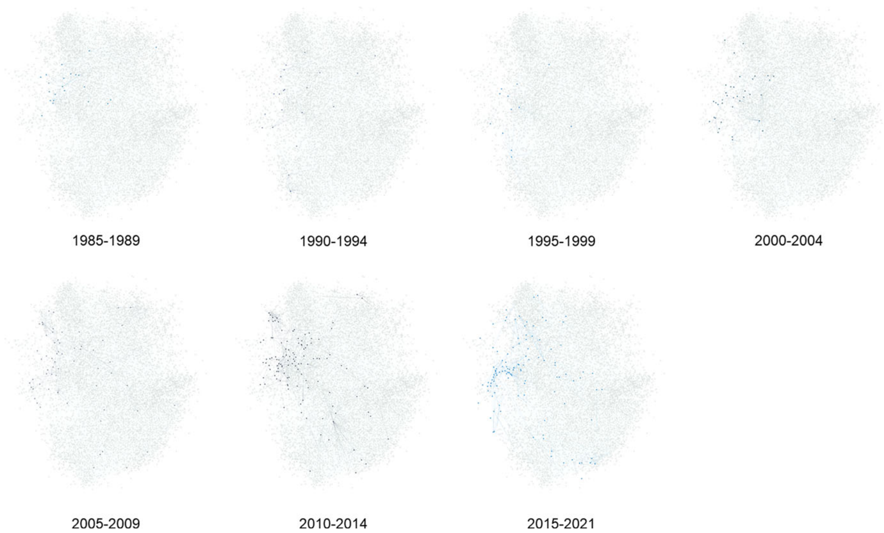

Fig. 13 Diffusion over time of papers citing game theory in the reference map of philosophy

percentage over the total number of papers in the research area are shown. Figure 12 provides the same information, with the addition of a measure of statistical association between the variables, namely the Pearson residuals obtained from a Chi-squared test. Pearson residuals are calculated from the difference between the observed frequency of a cell (reported in black on the plot) and the expected frequency we would obtain under the null hypothesis that the rows and column variables on the table were statistically independent (Agresti, 2007).[19] A Chi-squared test of independence (χ^2 (40, $N = 1861$) = 807.17, $p < 0.001$) rejects the null hypothesis that the corpora and philosophical research areas are independent, i.e., that no association exists between rows and columns in Table 11.[20] Therefore, we can use Pearson residuals to measure how far observed frequencies deviate from expected frequencies under the null hypothesis. Pearson residuals are shown in the cells of the table with a colour code in order to assess the strength and direction of the association between corpora and research areas. Specifically, when the observed frequency is higher than expected (positive association), the ellipse in the cell is blue and right-oriented; when the observed frequency is lower than expected (negative association), the ellipse is red and left-oriented (Fig. 13). If it is equal to the expected frequency (no association), it is white. Lastly, the strength of the association is inversely proportional to the size of the ellipse: stronger the association, thinner the ellipse.[21]

From these Table and Figure, four facts are worth pointing out. (i) In the reference map logic is distributed in a more uniform way than both game theory and network theory: it is substantially present in almost every cluster, with the exception of the cluster Philosophy of biology. (ii) Game theory is strongly associated with two clusters: Decision theory and confirmation theory, and Philosophy of psychology and cognitive science. (iii) Network science is strongly correlated with the cluster Philosophy of science (reduction, causality). (iv) Game theory is negatively correlated with the two clusters—Lewis, metaphysics, possible worlds, and Philosophy of mathematics (rule-following)—with which logic has a slight positive correlation.

The last point seems to suggest a sort of division of labour among logicians, on the one hand, and game-theorists on the other hand. Results (i), (ii) and (iii), jointly considered, can perhaps be accounted for by the fact that the use of logic in philosophy dates far more back than the use of other formal methods, and therefore logic itself has had plenty of time to spread over different areas of philosophy. This conjecture can be corroborated by considering the diffusion of game theory in the reference map over time.

[19] More precisely, the Pearson residuals are the difference between the two, normalized over the square root of the expected frequency:$r = \frac{f_o - f_e}{\sqrt{f_e}}$ Note that the Chi-squared requires non-overlapping categories, so that the total of both rows and columns are equal to the total number of observations. This is the reason why, differently from Table 11, in Fig. 12 the intersections between corpora (e.g., logic and game theory) are considered separately.

[20] We considered the test significant at the 0.001 level. Cramer's V yields an effect size of 0.33, showing a weak to medium association strength between corpora and research areas.

[21] Post-hoc testing with Bonferroni correction on the Chi-squared results shows that most of the associations in the cell are statistically significant at last at the level of 0.05. In the Supplementary Materials, Figure S8, the association plot with the significance code of each cell is reported. We thank an anonymous Reviewer for suggesting us to conduct post-hoc testing.

The maps show a gradual propagation of game theory in philosophy as time passes. It is reasonable to suppose that with more time at its disposal game theory would spread further on the map, thus acquiring a distribution pattern closer to that of logic.

5.2 Methodological remarks

As we hope to have shown in this study, reference analysis is a powerful technique for shedding light on the diffusion of game theory and network science in philosophy. However, it is not immune from some limitations, like any empirical method. In these final remarks, we discuss three of them in order to contextualize the validity of our results and suggest directions for future research as well.

The first limitation has to do with the phenomenon of "obliteration by incorporation" (Merton, 1988). Obliteration by incorporation happens when a concept or method becomes so well-known or integrated into a research area that the original work in which it was proposed in the first place is not cited anymore. Eponymous phrases are used instead, such as "Planck constant", "Arrow theorem", or similar. This phenomenon clearly hinders the complete retrieval of the knowledge core, especially if only recent literature is considered.

The second limitation, closely related to the previous one, stems from the citation behaviour of philosophers. Reference analysis assumes that philosophers cite most, if not all, the documents that had an intellectual influence on their articles. However, this assumption may not always hold. Especially when a formal method is used at a very basic level, the authors may not cite a reference from the specialist literature, not even a textbook. In these cases, reference analysis fails to capture these works. To roughly esteem the number of the papers in which this "not referenced use" of formal methods occurs, we searched on Web of Science all the publications in *Synthese* that mentioned the term "game theory" in their title, abstract or keywords. The query resulted into 51 publications. They amount to less than one fifth of the 308 *Synthese* publications, which we retrieved with the reference-based method, showing the limited retrieval efficiency of keywords. However, 5 papers retrieved via keywords remained outside the corpus we selected via reference analysis. These papers make *some* use of game theory but did not have any explicit contact with the game theory knowledge core.

The third limitation concerns the delineation of the knowledge cores associated with formal methods. This limitation is perhaps the most radical and difficult to address properly, as it is related with deep issues concerning the structure of scientific fields that, in the context of the present paper, we can only touch upon. In the Methodology section, we said that the general structure of the knowledge core should remain constant over different journals, provided that the journals have a sufficiently similar scope. In this sense, the knowledge core should be relatively robust to changes in the choice of journals. However, this may be false for scientific fields that are very fragmented or, in some sense, not homogeneous. Fields that are divided between qualitative and quantitative approaches, such as sociology, may represent a case of this type (Traag & Franssen, 2016). It is likely that these two traditions in sociology have in fact two distinct knowledge cores. Therefore, it would be a mistake to search for a unique knowledge core when two (or many) are present. Logic seems to be another case. Originally, we planned to delineate the knowledge core for logic as well and include it as a further case study in this work. However, preliminary analyses of the knowledge bases of different logic journals revealed that logic is in fact fragmented into different communities gravitating around different journals. Hence, it was not possible to delineate a stable knowledge core for logic, or, otherwise said, the knowledge core of logic was heavily dependent on the choice of logic journals considered. In fact, the only stable feature of the logic case study—and the only one that we included in this paper—was the spread of philosophy articles citing logic over the various areas of the map of philosophy. Besides this qualitative finding, any quantitative measure would have given a false sense of precision and certainty. Research areas associated with other formal methods, such as probability, may present similar issues. Relatively homogeneous research fields such as game theory and network science, by contrast, are relatively immune to them, as they are more "self-contained". Still, we think that an important topic for future research is the refinement of the methodology for knowledge core delineation. A comparison with other methods, such as distant reading, may provide important insights in this sense.

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References

Agresti, A. (2007). *An introduction to categorical data analysis*. John Wiley & Sons.

Aumann, R. (1976). Agreeing to disagree. *The Annals of Statistics, 4*(6), 1236–1239.

Axelrod, R. (1984). *The evolution of cooperation*. Basic Books.

Bala, V., & Goyal, S. (1998). Learning from neighbours. *Review of Economic Studies, 65*(3), 595–621. https://doi.org/10.1111/1467-937X.00059

Barabási, A.-L. (2014). *Linked: How everything is connected to everything else and what it means for business, science, and everyday life*. Basic Books.

Barabási, A.-L., Jeong, H., Néda, Z., Ravasz, E., Schubert, A., & Vicsek, T. (2002). Evolution of the social network of scientific collaborations. *Physica A: Statistical Mechanics and Its Applications*, *311*(3–4), 590–614. Doi: https://doi.org/10.1016/S0378-4371(02)00736-7.

Barabási, A.-L., & Albert, R. (1999). Emergence of scaling in random networks. *Science, 286*(5439), 509–512. https://doi.org/10.1126/science.286.5439.509

Barry, B. (1965). *Political argument*. Routledge.

Bastian, M., Heyman, S., & Jacomy, M. (2009). *Gephi: an open source software for exploring and manipulating networks*. International AAAI Conference on Weblogs and Social Media. http://www.aaai.org/ocs/index.php/ICWSM/09/paper/view/154.

Batagelj, V., & Mrvar, A. (2004). Pajek. In M. Jünger & P. Mutzel (Eds.), *Graph drawing software* (pp. 77–103). Springer.

Betti, A., & van den Berg, H. (2014). Modelling the history of ideas. *British Journal for the History of Philosophy, 22*(4), 812–835.

Betti, A. & van der Berg, H. (2016). Towards a computational history of ideas. In Wieneke, L., Jones, C., Düring, M., Armaselu, F. & Leboutte, R. (Eds.), *CEUR Workshop Proceedings*.

Blondel, V. D., Guillaume, J.-L., Lambiotte, R., & Lefebvre, E. (2008). Fast unfolding of communities in large networks. *Journal of Statistical Mechanics: Theory and Experiment, 2008*(10), P10008. https://doi.org/10.1088/1742-5468/2008/10/P10008

Bonino, G., Maffezioli, P., & Tripodi, P. (2020). Logic in analytic philosophy: A quantitative analysis. *Synthese*. https://doi.org/10.1007/s11229-020-02770-5

Börner, K., Chen, C., & Boyack, K. W. (2005). Visualizing knowledge domains. *Annual Review of Information Science and Technology, 37*(1), 179–255. https://doi.org/10.1002/aris.1440370106

Bornmann, L., Wray, K. B., & Haunschild, R. (2020). Citation concept analysis (CCA): A new form of citation analysis revealing the usefulness of concepts for other researchers illustrated by exemplary case studies including classic books by Thomas S. Kuhn and Karl R. Popper. *Scientometrics*, *122*(2), 1051–1074. https://doi.org/10.1007/s11192-019-03326-2

Boyack, K. W., & Klavans, R. (2010). Co-citation analysis, bibliographic coupling, and direct citation: Which citation approach represents the research front most accurately? *Journal of the American Society for Information Science and Technology, 61*(12), 2389–2404. https://doi.org/10.1002/asi.21419

Braithwaite, R. (1955). *Theory of games as a tool for the moral philosopher*. Cambridge University Press.

Buonomo, V., & Petrovich, E. (2018). Reconstructing late analytic philosophy. A quantitative approach. *Philosophical Inquiries*, *6*(1), 152–182. Doi: https://doi.org/10.4454/philinq.v6i1.184.

de Bruin, B. (2005). Game theory in philosophy. *Topoi, 24*(2), 197–208.

Chen, C. (2013). *Mapping scientific frontiers: The quest for knowledge visualization* (2nd ed.). Springer.

Crupi, V., & Tentori, K. (2016). Confirmation theory. In A. Hájek & C. Hitchcock (Eds.), *Oxford handbook of philosophy and probability* (pp. 650–665). Oxford University Press.

Davis, J. B. (2010). *Individuals and identity in economics*. Cambridge University Press.

Gauthier, D. (1986). *Morals by agreement*. Clarendon Press.

Hahn, U., Hansen, J. U., & Olsson, E. J. (2020). Truth tracking performance of social networks: How connectivity and clustering can make groups less competent. *Synthese, 197*(4), 1511–1541. https://doi.org/10.1007/s11229-018-01936-6

Hansonn, S. O. (2018). Formalization. In S. O. Hansonn & V. Hendricks (Eds.), *Introduction to formal philosophy* (pp. 3–61). Springer.

Harsanyi, J. (1955). Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. *Journal of Political Economy, 63*(4), 309–321.

Herfeld, C., & Doehne, M. (2019). The diffusion of scientific innovations: A role typology. *Studies in History and Philosophy of Science Part A, 77*, 64–80. https://doi.org/10.1016/j.shpsa.2017.12.001

Hidalgo, C. A. (2016). Disconnected, fragmented, or united? A trans-disciplinary review of network science. *Applied Network Science*, *1*(6).

Horsten, L., & Pettigrew, R. (2011). Mathematical methods in philosophy. In L. Horsten & R. Pettigrew (Eds.), *The continuum compendium in philosophical logic* (pp. 14–26). Bloomsbury.

Huneman, P. (2010). Topological explanations and robustness in biological sciences. *Synthese, 177*(2), 213–245. https://doi.org/10.1007/s11229-010-9842-z

Hyland, K. (1999). Academic attribution: Citation and the construction of disciplinary knowledge. *Applied Linguistics, 20*(3), 341–367. https://doi.org/10.1093/applin/20.3.341

Jaccard, P. (1912). The distribution of the flora in the alpine zone. *New Phytologist, 11*, 37–50.

Jacomy, M., Venturini, T., Heymann, S., & Bastian, M. (2014). ForceAtlas2, a continuous graph layout algorithm for handy network visualization designed for the Gephi software. *PLoS ONE, 9*(6), e98679. https://doi.org/10.1371/journal.pone.0098679

Kessler, M. M. (1963). Bibliographic coupling extended in time: Ten case histories. *Information Storage and Retrieval, 1*(4), 169–187. https://doi.org/10.1016/0020-0271(63)90016-0

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*, 159–174.

Lewis, D. (1969). *Convention: A philosophical study*. Harvard University Press.

Lewis, T. G. (2009). *Network science: Theory and practice*. John Wiley & Sons.

Leydesdorff, L. (2008). On the normalization and visualization of author co-citation data: Salton's Cosine versus the Jaccard index. *Journal of the American Society for Information Science and Technology, 59*(1), 77–85. https://doi.org/10.1002/asi.20732

Leydesdorff, L., & Amsterdamska, O. (1990). Dimensions of citation analysis. *Science, Technology, and Human Values, 15*(3), 305–335. https://doi.org/10.1177/016224399001500303

Machamer, P., Darden, L., & Craver, C. F. (2000). Thinking about mechanisms. *Philosophy of Science, 67*(1), 1–25. https://doi.org/10.1086/392759

Maltseva, D., & Batagelj, V. (2021). Journals publishing social network analysis. *Scientometrics, 126*(4), 3593–3620. https://doi.org/10.1007/s11192-021-03889-z

Marx, W., Bornmann, L., Barth, A., & Leydesdorff, L. (2014). Detecting the historical roots of research fields by reference publication year spectroscopy (RPYS). *Journal of the Association for Information Science and Technology, 65*(4), 751–764. https://doi.org/10.1002/asi.23089

Merton, R. K. (1988). The Matthew effect in science, II: Cumulative advantage and the symbolism of intellectual property. *Isis, 79*(299), 606–623.

Mingers, J., & Leydesdorff, L. (2015). A review of theory and practice in scientometrics. *European Journal of Operational Research, 246*(1), 1–19. https://doi.org/10.1016/j.ejor.2015.04.002

Moretti, F. (2013). *Distant reading*. Verso.

Nash, J. (1951). Non-Cooperative Games. *Annals of Mathematics Journal, 54*, 286–295.

von Neumann, J., & Morgenstern, O. (1944). *The theory of games and economic behavior*. Princeton University Press.

Newman, M. E. J. (2006). Modularity and community structure in networks. *Proceedings of the National Academy of Sciences, 103*(23), 8577–8582. https://doi.org/10.1073/pnas.0601602103

Newman, M. E. J. (2018). *Networks* (2nd ed.). Oxford University Press.

Nicolaisen, J. (2007). Citation analysis. *Annual Review of Information Science and Technology, 41*(1), 609–641. https://doi.org/10.1002/aris.2007.1440410120

Noichl, M. (2019). Modeling the structure of recent philosophy. *Synthese*. https://doi.org/10.1007/s11229-019-02390-8

Perea, A. (2012). *Epistemic game theory: Reasoning and choice*. Cambridge University Press.

Pence, C., & Ramsey, G. (2018). How to do digital philosophy of science. *Philosophy of Science, 85*(5), 930–941. https://doi.org/10.1086/699697

Petrovich, E. (2020). Science mapping. *ISKO Encyclopedia of Knowledge Organization*. https://www.isko.org/cyclo/science_mapping.

Rathkopf, C. (2018). Network representation and complex systems. *Synthese, 195*(1), 55–78. https://doi.org/10.1007/s11229-015-0726-0

Rawls, J. (1971). *A theory of justice*. Harvard University Press.

Ross, D. (2019). Game Theory, *The Stanford Encyclopedia of Philosophy*.

Shafir, E., & Tversky, A. (1992). Thinking through uncertainty: Nonconsequential reasoning and choice. *Cognitive Psychology, 24*(4), 449–474. https://doi.org/10.1016/0010-0285(92)90015-T

Skyrms, B. (1996). *Evolution of the social contract*. Cambridge University Press.

Skyrms, B. (2004). *The stag hunt and the evolution of social structure*. Cambridge University Press.

Skyrms, B. (2010). *Signals: Evolution, Learning, and Information*. Oxford University Press.

Small, H. (2003). Paradigms, citations, and maps of science: A personal history. *Journal of the American Society for Information Science and Technology, 54*(5), 394–399. https://doi.org/10.1002/asi.10225

Thor, A., Marx, W., Leydesdorff, L., & Bornmann, L. (2016). Introducing CitedReferencesExplorer (CRExplorer): A program for reference publication year spectroscopy with cited references standardization. *Journal of Informetrics, 10*(2), 503–515. https://doi.org/10.1016/j.joi.2016.02.005

Traag, V., & Franssen, T. (2016). Revealing the quantitative-qualitative divide in sociology using bibliometric visualization. *CWTS Blog*. https://www.cwts.nl/blog?article=n-q2v294.

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science, 211*(4481), 453–458. https://doi.org/10.1126/science.7455683
van Eck, N. J., & Waltman, L. (2009). How to normalize cooccurrence data? An analysis of some well-known similarity measures. *Journal of the American Society for Information Science and Technology, 60*(8), 1635–1651. https://doi.org/10.1002/asi.21075
van Eck, N. J., & Waltman, L. (2011). Text mining and visualization using VOSviewer. *ISSI Newsletter, 7*(3), 50–54.
Wasserman, S., & Faust, K. (1994). *Social network analysis: Methods and applications*. Cambridge University Press.
Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *Nature, 393*(6684), 440–442. https://doi.org/10.1038/30918
Weatherall, J. O., & O'Connor, C. (2020). Conformity in scientific networks. *Synthese*. https://doi.org/10.1007/s11229-019-02520-2
Weatherall, J. O., O'Connor, C., & Bruner, J. P. (2020). How to beat science and influence people: Policymakers and propaganda in epistemic networks. *The British Journal for the Philosophy of Science, 71*(4), 1157–1186. https://doi.org/10.1093/bjps/axy062
Weingart, S. B. (2015). Finding the history and philosophy of wcience. *Erkenntnis, 80*(1), 201–213. https://doi.org/10.1007/s10670-014-9621-1
Zitt, M., Lelu, A., Cadot, M., & Cabanac, G. (2019). Bibliometric Delineation of Scientific Fields. In W. Glänzel, H. F. Moed, U. Schmoch, & M. Thelwall (Eds.), *Springer Handbook of Science and Technology Indicators* (pp. 25–68). New York: Springer. Doi: https://doi.org/10.1007/978-3-030-02511-3_2.
Zollman, K. J. S. (2010). The epistemic benefit of transient diversity. *Erkenntnis, 72*(1), 17–35.
Zollman, K. J. S. (2007). The communication structure of epistemic communities. *Philosophy of Science, 74*(5), 574–587. https://doi.org/10.1086/525605