



Quark fragmentation as a probe of dynamical mass generation

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ABSTRACT

We address the propagation and hadronization of a struck quark by studying the gauge invariance of the color-averaged cut quark propagator, and by relating this to the single inclusive quark fragmentation correlator by means of new sum rules. Using suitable Wilson lines, we provide a gauge-invariant definition for the mass of the color-averaged dressed quark and decompose this into the sum of a current and an interaction-dependent component. The latter, which we argue is an order parameter for dynamical chiral symmetry breaking, also appears in the sum rule for the twist-3 \tilde{E} fragmentation function, providing a specific experimental way to probe the dynamical generation of mass in Quantum Chromo Dynamics.

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1. Introduction

One of the crucial properties of the strong force is confinement, namely the fact that partons cannot exist as free particles outside of hadrons. As a consequence, any individual parton struck in a high-energy scattering process and freed from its parent hadron must transform into at least one hadron. During this hadronization process, the colored and much lighter parton interacts with the surrounding matter and the vacuum to produce massive and colorless hadrons. Hadronization is thus tightly connected to the dynamical generation of the mass, the spin, and the size of hadrons, but the details of the quark-to-hadron transition are still unknown. Unraveling hadronization dynamics is not only of fundamental importance to understand the nature of visible matter, but also to tackle hadron tomography studies at current and future facilities, such as the 12 GeV upgrade at Jefferson Lab [1] and a future US-based Electron-Ion Collider [2], where measuring the transverse momentum of one final state hadron is crucial to provide a handle into the transverse motion of quarks and gluons in the hadron target [3–10].

In this letter, we address the propagation and hadronization of a struck quark by studying the gauge invariance properties of the “inclusive jet correlator” defined in Ref. [11–14], *i.e.*, the color-averaged cut quark propagator supplemented by suitable Wilson lines, and by relating this by means of sum rules to the fragmentation correlator [15] utilized in Quantum Chromo Dynamics (QCD) to describe the semi-inclusive transition of a quark into a single hadron.

In particular, through the Dirac decomposition of the jet correlator, we provide a gauge-invariant definition of the jet mass M_j that was previously introduced in Ref. [14]. In this letter, we elucidate for the first time its nature and properties, recognizing that M_j can be decomposed into the sum of the current quark mass and of an interaction-dependent “correlation mass”. Thus the jet correlator can also be interpreted as a color-averaged propagator for a *dressed quark*.

We find that the jet mass and the correlation mass are experimentally accessible through sum rules for the unpolarized collinear twist-3 E and \tilde{E} fragmentation functions (FFs). A similar sum rule for the \tilde{D}^\perp FF also supplies information on the average transverse momentum of the produced hadrons. This letter focuses on those FFs whose sum rules encode in a quantitative way the intimate connection between hadronization and the dynamical generation of mass and momentum, providing one with a novel way to quan-

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tify the quark dressing process, the dynamical breaking of the chiral symmetry, and the nature of the QCD vacuum.

2. The inclusive jet correlator

Let us consider the unintegrated color-averaged gauge-invariant cut propagator for a quark of momentum k :

$$\Xi_{ij}(k; w) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{ik \cdot \xi} \times \frac{\text{Tr}_c}{N_c} \langle \Omega | \mathcal{T} W_1 \psi_i(\xi) \bar{\psi}_j(0) W_2 | \Omega \rangle, \quad (1)$$

where $|\Omega\rangle$ is the interacting QCD vacuum, and, for simplicity, we omitted the flavor index of the quark field ψ .

Gauge invariance is implemented via the Wilson lines $W_1 \equiv W(\infty, \xi; w)$ and $W_2 \equiv W(0, \infty; w)$, and the time-ordering operator \mathcal{T} exchanges, when needed, the ψ and $\bar{\psi}$ fields as well as the ending point ξ of the Wilson line W_1 and the starting point 0 of W_2 . The vector w will be used to specify the shape of the path in spacetime.

Thanks to the color trace, we can perform a cyclic permutation of the fields. The operator becomes $\psi_i(\xi) \bar{\psi}_j(0) W(0, \xi; w)$, where the two Wilson lines are now combined into a single one: $W(0, \xi; w) = W_2 W_1$. We consider $W = W_2 W_1$ to be a staple-like Wilson line, and the vector w defines the longitudinal direction. Let us now introduce the light-like vectors n_+ and n_- such that $n_+ \cdot n_- = 1$. For sake of simplicity, in this treatment we restrict our attention to the case $w = n_+$ [15], but the techniques discussed in this paper can in principle be applied also to an off-the-light-cone vector w [12,16]. Accordingly, W is a staple-like Wilson line running along the n_+ direction to infinity, then jolting in the n_- direction and the plane transverse to n_{\pm} , and running back from infinity again along the n_+ direction:

$$W(0, \xi; n_+) = \mathcal{U}[0^+, 0^-, \mathbf{0}_T; \infty^+, 0^-, \mathbf{0}_T] \times \mathcal{U}[\infty^+, 0^-, \mathbf{0}_T; \infty^+, \xi^-, \xi_T] \times \mathcal{U}[\infty^+, \xi^-, \xi_T; \xi^+, \xi^-, \xi_T]. \quad (2)$$

On the RHS we have specified the light-cone coordinates of the starting and ending points and we have left the dependence on n_+ implicit. In Eq. (2) each \mathcal{U} represents one of the straight segments of the staple:

$$\mathcal{U}[a; b] = P \exp \left(-ig \int_a^b dz^\mu A_\mu(z) \right), \quad (3)$$

where z runs in a straight line from a to b , P denotes the path-ordering operator, and the square brackets are used to emphasize the straightness of the path.

In this letter, we study the cut propagator Ξ as an object of intrinsic theoretical interest in a formal twist expansion in powers of $1/k^-$. This can be justified in scattering processes with a hard scale, such as the 4-momentum transfer Q in deep inelastic scattering. In such processes, the n_{\pm} vectors are determined by the initial or final state momenta, and $k^- \propto Q \gg k_T \gg k^+$. Our considerations will be, however, process independent.

By taking the discontinuity (Disc) of the propagator – or its cut, in diagrammatic terms – Eq. (1) can be interpreted as a gauge-invariant inclusive quark-to-jet amplitude squared, or “inclusive jet correlator” in short, in which all the quark hadronization products cross the cut and are on-shell [11–14,17]. The adjective “inclusive”

is to stress that none of the jet's constituents is actually measured, hence the absence of an axis and radius in Eq. (1), contrary to semi-inclusive definitions of jets [18–23].

The color averaging of the initial-state quark, implemented as $\text{Tr}_c[\dots]/N_c$ in Eq. (1), arises naturally in QCD factorization theorems and in the definition of the fragmentation correlator, to which the cut quark propagator Ξ is connected through the sum rules we will prove later. In our analysis, color-averaging plays a crucial role, technically, as it enables a spectral decomposition of the cut propagator Ξ . At the conceptual level, it implements the color neutralization that has to take place in order for all the states propagating through the cut to be on-shell hadrons, as confinement dictates for a physical process. Therefore, Eq. (1) can also be interpreted as a *color-averaged* (or *color-screened*) version of the gauge-invariant Feynman propagator for a dressed quark with four-momentum k .

3. Spectral decomposition

The cut propagator Ξ can be given a spectral representation by rewriting Eq. (1) as a convolution of a quark bilinear \tilde{S} and the Fourier transform \tilde{W} of the Wilson line W :

$$\Xi_{ij}(k) = \text{Disc} \int d^4p \frac{\text{Tr}_c}{N_c} \langle \Omega | \tilde{S}_{ij}(p) \tilde{W}(k-p) | \Omega \rangle, \quad (4)$$

where

$$\tilde{S}_{ij}(p) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot p} \mathcal{T} \psi_i(\xi) \bar{\psi}_j(0), \quad (5)$$

$$\tilde{W}(k-p) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot (k-p)} \mathcal{T} W(0, \xi). \quad (6)$$

In Eq. (5), the operator \mathcal{T} acts as an ordinary time-ordering operator, while in Eq. (6) it acts only on the endpoints of the path for the Wilson line, orienting this from 0 to ξ or vice versa.

The quark operator \tilde{S} can furthermore be decomposed in Dirac space assuming invariance under Lorentz and parity transformations:

$$\tilde{S}_{ij}(p) = \hat{s}_3(p^2) \not{p}_{ij} + \sqrt{p^2} \hat{s}_1(p^2) \mathbb{1}_{ij}, \quad (7)$$

where we refer to $\hat{s}_{1,3}$ as “spectral operators”. In principle, when working in an axial $v \cdot A = 0$ gauge, we should also add a structure proportional to \not{v} to the right hand side of Eq. (7) [24], where v is the vector used to specify the gauge choice, which, in principle, can be different from the vector w used to construct the Wilson line. However, in our explicit calculations we adopt the light-cone gauge, $v = n_+$ ($= w$), and, in this specific case, this additional gauge-fixing term would only contribute at twist-4 level, which is not relevant to the matter discussed in this letter.

We can obtain a connection with the usual Källén-Lehman spectral representation of the (gauge-variant) quark propagator [13,25,26] by noticing that the quark's Feynman propagator in momentum space is given by the expectation value of \tilde{S} on the interacting vacuum. It is then possible to write

$$\frac{\text{Tr}_c}{N_c} \langle \Omega | \tilde{S}(p) | \Omega \rangle = \int_{-\infty}^{+\infty} \frac{d\mu^2}{(2\pi)^4} \frac{\mathbf{i}}{p^2 - \mu^2 + i\epsilon} \times \left\{ \not{p} \rho_3(\mu^2) + \sqrt{p^2} \rho_1(\mu^2) \right\} \theta(\mu^2). \quad (8)$$

Using the operator decomposition for \tilde{S} given in Eq. (7) and the Cutkosky rule [27,28], we can then connect the spectral operators

$\hat{s}_{1,3}$ to the chiral-odd and -even Källén-Lehman spectral functions $\rho_{1,3}$:

$$(2\pi)^3 \text{Disc} \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{s}_{1,3}(p^2) | \Omega \rangle = \rho_{1,3}(p^2) \theta(p^2) \theta(p^-). \quad (9)$$

4. The TMD inclusive jet correlator

It is useful to consider the cut quark propagator integrated over the plus component of the partonic momentum [14],

$$J_{ij}(k^-, \mathbf{k}_T) \equiv \frac{1}{2} \int dk^+ \Xi_{ij}(k), \quad (10)$$

where the time ordering in Eq. (1) is now trivial since $\xi^- = 0$ [29, 30]. Likewise, the Wilson line (2) will be evaluated at $\xi^- = 0$. As explained later, this integrated correlator is also of interest for the derivation of sum rules for fragmentation functions.

The Transverse Momentum Dependent (TMD) correlator J can be decomposed in Dirac structures, with coefficients determined by Dirac projections of Ξ defined as

$$J^{[\Gamma]} \equiv \frac{1}{2} \int dk^+ \text{Tr} \left[\Xi \frac{\Gamma}{2} \right], \quad (11)$$

where Γ is a generic Dirac matrix and Tr represent the trace on the Dirac indexes. The structures of interest for the present discussion are:

$$\alpha(k^-) \equiv J^{[\gamma^-]} = \frac{k^-}{\mathbf{k}_T^2} J^{[\gamma'^1]}, \quad (12)$$

$$\zeta(k^-) \equiv \frac{k^-}{\Lambda} J^{[\mathbb{I}]}, \quad (13)$$

where Λ is a scale with the dimension of a mass introduced for power counting purposes. The kinematic dependence of α and ζ on k^- can be justified as follows. The $\Xi(k)$ correlator can be expanded on a basis of Dirac matrices, where the coefficients are, in principle, functions of all the independent scalars that one can build with the available Lorentz vectors, k , n_\pm [9,31], for example $k \cdot n_+$ and k^2 . Calculating $J^{[\gamma^-]}$ corresponds to integrating over k^2 the coefficient of Ξ associated to the \not{k} Dirac structure. Since this coefficient is only a function of k^- and k^2 , the result is a simple k^- dependence for α . A similar argument applies to the definition of ζ .

The integrated cut quark propagator can then be decomposed up to terms of $\mathcal{O}(\Lambda^2/(k^-)^2)$ as:

$$J(k^-, \mathbf{k}_T) = \frac{1}{2} \alpha(k^-) \left[\gamma^+ + \frac{\not{\mathbf{k}}_T}{k^-} \right] + \frac{\Lambda}{2k^-} \zeta(k^-) \mathbb{I}. \quad (14)$$

The coefficient α can be calculated directly from the definition (12), using the convolution (4) and the spectral decomposition (9). The details are collected in the Appendix. The transverse Wilson line drops out after integration over $d^2 \mathbf{p}_T$ in Eq. (4). Moreover, choosing the light-cone gauge $A \cdot n_+ = 0$ further simplifies the calculation because the longitudinal Wilson lines reduce to the unity matrix in color space. The result is:

$$\alpha(k^-) = \frac{\theta(k^-)}{2(2\pi)^3} \left\{ \int_0^{+\infty} dp^2 \rho_3^{lcg}(p^2) \right\} = \frac{\theta(k^-)}{2(2\pi)^3}, \quad (15)$$

where we have used the gauge-independent normalization property $\int_0^{+\infty} ds \rho_3(s) = 1$ [26] and the superscript *lcg* stresses the use of the light-cone gauge. This confirms that α is a function of k^- only. The $\theta(k^-)$ function factored out in Eq. (15) corresponds to

four-momentum conservation, namely to assuming that the particles passing the cut all have physical four-momenta.

Analogously, we can rewrite the coefficient of the chiral-odd Dirac structure as

$$\zeta(k^-) = \frac{\theta(k^-)}{2(2\pi)^3} \frac{M_j(k^-)}{\Lambda}, \quad (16)$$

where M_j is a gauge-invariant “jet” mass term, potentially function of k^- . As we are going to discuss, this term characterizes the non-perturbative generation of mass in the fragmentation of the quark to the inclusive jet.¹

To illustrate the physical meaning of M_j , we calculate ζ in the light-cone gauge $A \cdot n_+ = 0$. The calculation follows closely the strategy discussed for α in the Appendix. We obtain:

$$\zeta(k^-) = \frac{\theta(k^-)}{2(2\pi)^3 \Lambda} \left\{ \int_0^{+\infty} dp^2 \sqrt{p^2} \rho_1^{lcg}(p^2) \right\}, \quad (17)$$

which justifies the normalization chosen in Eq. (16). The gauge-invariant M_j mass has a particularly simple and k^- -independent form in this gauge, and is completely determined by the first moment of the spectral function ρ_1^{lcg} :

$$M_j = \int_0^{+\infty} d\mu^2 \sqrt{\mu^2} \rho_1^{lcg}(\mu^2). \quad (18)$$

The integral at the right hand side is summing over all the discontinuities of the quark propagator. In the light-cone gauge, therefore, M_j can be interpreted as the average mass generated by the chirality flipping component of the quark-to-jet amplitude squared (however, one has to be careful with a probabilistic interpretation because, in a confined theory, the spectral functions are not guaranteed to be positive definite). Choosing another gauge, the right hand side of this equation would also receive contributions from the Wilson line, and its physical interpretation would be less immediate. Interpretation aside, we stress that M_j is gauge-invariant, thanks to the gauge invariance of the operator Ξ in Eq. (1), and distinct from the average invariant mass of the fragmented hadrons. We also note that under renormalization M_j would acquire an additional scale dependence.

Putting all elements together, up to twist 3 we find

$$4(2\pi)^3 J(k^-, \mathbf{k}_T) = \left\{ \gamma^+ + \frac{M_j}{k^-} \mathbb{I} + \frac{\not{\mathbf{k}}_T}{k^-} \right\} \theta(k^-). \quad (19)$$

The jet correlator can thus be directly compared to the cut quark propagator with current quark mass m_q , but now depends on the non-perturbative dressed quark mass M_j .

Being related to the trace of the cut propagator, the mass M_j is intrinsically different from the mass function which appears in non-perturbative treatments of the quark propagator [32,33]. M_j is gauge invariant and scale dependent, whereas the mass function is gauge dependent, but renormalization group invariant. Nonetheless, being a scale that characterizes the physics of a color-screened dressed quark, it also provides a window on color confinement.

Remarkably, the gauge-invariant mass M_j is also experimentally accessible because it contributes explicitly to the mass sum rules for the twist-3 fragmentation functions E and \tilde{E} that we shall prove in the following.

¹ The “jet mass” M_j was labeled M_q in Ref. [14] to stress its quark flavor dependence.

5. Sum rules for single-hadron fragmentation functions

The unintegrated correlator describing the fragmentation of a quark into a single unpolarized hadron is [15,29,31,34–36]:

$$\Delta_{ij}^h(k, P_h) = \sum_{S_h} \int \frac{d^4\xi}{(2\pi)^4} e^{ik \cdot \xi} \quad (20)$$

$$\times \frac{\text{Tr}_c}{N_c} \langle \Omega | \mathcal{T} W_1 \psi_i(\xi) a_h^\dagger(P_h S_h) a_h(P_h S_h) \bar{\psi}_j(0) W_2 | \Omega \rangle,$$

where a_h^\dagger and a_h create and destroy a hadron h with momentum P_h and spin S_h .

Integrating over the suppressed quark momentum component k^+ [29,37,38], one defines the TMD quark-to-single-hadron fragmentation correlator:

$$\Delta_{ij}^h(z, \mathbf{P}_{h\perp}) = \int \frac{dk^+}{2z} \text{Disc}[\Delta_{ij}^h(k, P_h)]_{k^- = P_h^-/z}, \quad (21)$$

where, in the parton frame, $\mathbf{k}_T = 0$ and $\mathbf{P}_{h\perp}$ is the hadronic transverse momentum relative to the quark [16,36], and $0 < z < 1$ if the considered discontinuity is in the s -channel of the associated scattering amplitude [29,38]. The parametrization of this correlator in terms of TMD FFs is known up to twist 3 [15,31]. Here we are only interested in

$$\Delta_{ij}^h(z, \mathbf{P}_{h\perp}) = \frac{\gamma^+}{2} D_1^h + \frac{M_h}{2P_h^-} E^h + \frac{\mathbf{P}_{h\perp}}{2zP_h^-} D^{\perp h}, \quad (22)$$

where D_1^h is the unpolarized twist-2 TMD FF and E^h and $D^{\perp h}$ are the unpolarized twist-3 TMD FFs.

We can now derive a master four-momentum sum rule for unintegrated correlators, from which we can later obtain sum rules for specific fragmentation functions. Let's consider the quantity

$$\sum_h \int \frac{d^4 P_h}{(2\pi)^4} (2\pi) \delta(P_h^2 - M_h^2) P_h^\mu \Delta^h(k, P_h), \quad (23)$$

which, naively, can be interpreted as the average hadronic four-momentum produced during the hadronization of the parton. Introducing the operator associated with the vector P_h^μ [39],

$$\hat{P}^\mu = \sum_{h, S_h} \int \frac{dP_h^- d^2 \mathbf{P}_{h\perp}}{2P_h^- (2\pi)^3} P_h^\mu \hat{a}_h^\dagger(P_h, S_h) \hat{a}_h(P_h, S_h), \quad (24)$$

and relying on its commutation relation $[\mathcal{O}(\xi), \hat{P}^\mu] = i\partial^\mu \mathcal{O}(\xi)$, one can relate Eq. (23) to the Fourier transform of the derivative of the matrix element in Eq. (1). Furthermore, integrating by parts with vanishing boundary conditions for the fields, one obtains:

$$\sum_h \int \frac{d^4 P_h}{(2\pi)^3} \delta(P_h^2 - M_h^2) P_h^\mu \Delta^h(k, P_h) = k^\mu \Xi_{wc}(k), \quad (25)$$

where the subscript wc implies considering Ξ in Eq. (1) without the discontinuity. The relation holds true for all the components μ , but only for $\mu = -, 1, 2$ there is a connection to the fragmentation functions.

We can now obtain sum rules for the fragmentation functions by integrating both sides of Eq. (25) in the parton frame over the suppressed momentum component k^+ , calculating their discontinuity [29,37,38], and using suitable Dirac projections of the result. Since the sum over the hadron spin S_h is fundamental in the manipulations involving the momentum operator (24) [39], we can only obtain sum rules for FFs involving unpolarized hadrons. In

particular, for $\Gamma = \{\gamma^-, \mathbb{I}, \gamma^i\}$, and suitably choosing $\mu = -$ or $\mu = i$, we obtain:

$$[\Gamma = \gamma^-] \sum_{h, S_h} \int dz z D_1^h(z) = 1, \quad (26)$$

$$[\Gamma = \mathbb{I}] \sum_{h, S_h} \int dz M_h E^h(z) = M_j, \quad (27)$$

$$[\Gamma = \gamma^i] \sum_{h, S_h} \int dz M_h^2 D^{\perp(1)h}(z) = 0. \quad (28)$$

Note that we identified the integrated TMD FFs $f = D_1^h, E^h$ with their collinear counterpart, i.e., $f(z) \equiv \int d^2 \mathbf{P}_{h\perp} f(z, P_{h\perp}^2)$, which is formally correct in the context of models of QCD at low energies (given specific regularization prescriptions for the integral over the transverse momentum) and for bare, i.e. non-renormalized, distributions in perturbative QCD. We furthermore defined the first moment of any FF f as $f^{(1)}(z) = \int d^2 \mathbf{P}_{h\perp} [\mathbf{P}_{h\perp}^2 / (2z^2 M_h^2)] f(z, P_{h\perp}^2)$ [36].

Since these arguments are related to the conservation of the partonic four-momentum encoded in Eq. (25) and on the symmetry properties of the correlators Ξ and Δ , we expect these momentum sum rules to be valid *in form* also at the renormalized level in perturbative QCD. In fact, renormalization is known to preserve Eq. (26) [16,40], and we will address quantitatively the renormalization of the other sum rules in future works.

Eq. (26) is the well-known momentum sum rule for the unpolarized twist-2 FF, and encodes the conservation of the longitudinal momentum in the fragmentation process [40]. The sum rule (27) was first introduced in Ref. [14], but it is proven here for the first time in field theory. It generalizes the sum rule proposed in Ref. [41], which however neglected the non-perturbative component of the dressed quark mass. We refer to Eq. (27) as the “mass sum rule” because of its physical interpretation: the non-perturbative color-screened dressed quark mass M_j corresponds to the average of all the possible masses of the particles produced in the hadronization of the quark, weighted by the chiral-odd collinear twist-3 fragmentation function $E^h(z)$. The sum rule (28) is new, and encodes the conservation of the hadronic transverse momentum relative to the momentum of the fragmenting quark.

6. Twist-three fragmentation and the dynamical component of the jet mass

Let us now consider the equations of motion relations discussed in Ref. [15], that relate the twist-2 and the twist-3 fragmentation functions. In particular, we need

$$E^h = \tilde{E}^h + z \frac{m_q}{M_h} D_1^h, \quad D^{\perp h} = \tilde{D}^{\perp h} + z D_1^h, \quad (29)$$

where the functions with a tilde are related to the parametrization of the dynamical twist-3 quark-gluon-quark correlator of type $\tilde{\Delta}_A^\alpha$ [15] and m_q is the current quark mass.

Within the “Wandzura-Wilczek (WW) approximation” [42], namely assuming that the $\tilde{E}, \tilde{D}^{\perp}$ FFs are negligible compared to the quark-quark FFs without the tilde, M_j reduces to the current quark mass, as can be seen by setting $\tilde{E} = 0$ in Eq. (29) and using the sum rules (26) and (27). This suggests a decomposition of M_j into the sum of the current quark mass and of an interaction-dependent (or dynamical) mass m_q^{corr} generated by quark-gluon-quark correlations, which we name “correlation mass”:

$$M_j = m_q + m_q^{corr}. \quad (30)$$

In analogy with the generation of the quark mass induced by dynamical chiral symmetry breaking [32,33], we expect $m_q^{corr} = O(\Lambda_{CQD})$. For light quarks, therefore, M_j might be substantially larger than the current quark mass m_q . Combining the sum rules (26) and (27) with Eq. (29) and (30) we obtain:

$$\sum_{h S_h} \int dz M_h \tilde{E}^h(z) = m_q^{corr}, \quad (31)$$

and one clearly sees that m_q^{corr} is generated by the interaction-dependent quark-gluon-quark correlations encoded in the \tilde{E} function. Thus, Eq. (31) generalizes the sum rule $\int dz \tilde{E} = 0$ discussed in Ref. [15] to the non-perturbative case.

The correlation mass vanishes when neglecting quark-gluon-quark correlations, as in the already discussed WW approximation. However, when interactions are considered, the spectral representation (18) shows that m_q^{corr} and M_j receive contributions from all possible propagating hadronic states, including baryons. As a consequence, contrary to m_q , we expect that m_q^{corr} and M_j do not vanish even in the chiral limit. Therefore, they can also be considered order parameters for dynamical chiral symmetry breaking and are moreover experimentally accessible, at least in principle, via the mass sum rule for the E and \tilde{E} FFs.

Finally, combining the sum rules (26) and (28) with the D -type equation of motion in Eq. (29), we obtain:

$$\begin{aligned} \sum_{h S_h} \int dz M_h^2 \tilde{D}^{\perp(1)h}(z) &= - \sum_{h S_h} \int dz z M_h^2 D_1^{(1)h}(z) \\ &\equiv -\frac{1}{2} \langle \mathbf{P}_{h\perp}^2 / z \rangle. \end{aligned} \quad (32)$$

This new sum rule provides an experimental way of accessing the average squared transverse momentum acquired by unpolarized hadrons fragmenting off an unpolarized quark, and probes the dynamical nature of the hadronization process in analogy with the way the sum rule for \tilde{E} probes dynamical chiral symmetry breaking. Indeed, the transverse momentum acquired during the hadronization is generated by the quark-gluon-quark correlations and would vanish in the absence of these, as it can be directly checked assuming the WW approximation. Similar sum rules involving transversely polarized quarks or hadrons have been discussed in Refs. [36,43].

7. Summary and outlook

With the jet mass M_j defined in Eqs. (13) and (16), we have for the first time proposed a *gauge-invariant* definition of a color-screened dressed quark mass, which is furthermore experimentally accessible by using the proposed sum rules (27) and (31) for twist-3 collinear FFs. In perturbation theory, M_j is proportional to the current quark mass, but in the full theory it remains non-zero also in the chiral limit. Therefore we also recognize M_j as an order parameter for dynamical chiral symmetry breaking. This provides a novel connection between hadronization and the dynamical generation of mass in QCD.

The dynamical component of the jet mass, quantified by the correlation mass m_q^{corr} in Eq. (30), is generated by the quark-gluon-quark correlations encoded in the \tilde{E} fragmentation function. The \tilde{E} FF, and thus the sum rule (31) connecting this to m_q^{corr} , can be experimentally accessed by looking at chiral odd observables at twist-3 level and higher in semi-inclusive hadron production in polarized electron-hadron scattering, electron-positron annihilation, and hadronic collisions. As proposed in Ref. [14,44], the correlation mass itself may also directly contribute to the inclusive DIS g_2

structure function at large Bjorken x , and to an analogous asymmetry in electron-positron induced dihadron production. More generally, in scattering processes, the jet mass M_j (rigorously defined as the trace of the gauge-invariant cut quark propagator) appears to play a similar phenomenological role to that of the constituent mass in quark models.

With the sum rule (32) we are also suggesting that the transverse momentum acquired during the hadronization process is a fully dynamical quantity, namely it is generated by the quark-gluon-quark correlations and it vanishes in the absence of these. However, like for Eq. (31), this interpretation needs to be corroborated by a study of the renormalization properties of the involved operators. Such a sum rule could be, in principle, experimentally accessed at twist-2 level through the TMD FF D_1^h , about which limited information is so far available [8,45–48].

Finally, we note that the master sum rule (25) and the calculational techniques we have introduced have a general applicability. In particular, we expect the convolutional spectral representation of the quark correlator to be also applicable to the study of the gauge invariance of other correlators, for example the recently introduced virtuality-dependent parton distributions [49] that play an important role in the direct lattice QCD calculation of PDFs in momentum space [50,51]. Moreover, our methods are not limited to Wilson lines on the light-cone, making their domain of applicability potentially wide.

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Appendix. Calculation of the α coefficient

We outline here the steps involved in the calculation of the γ^- projection of the correlator J in Eq. (10).

Using the definition of α given in Eq. (12) and the convolution representation for Ξ in Eq. (4) and (7), we find:

$$\begin{aligned} \alpha(k^-) &= \int dk^+ \text{Disc} \int_M d^4 p \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{s}_3(p^2) p^- \tilde{W}(k-p) | \Omega \rangle \quad (33) \\ &= \text{Disc} \int_{\mathbf{R}} dp^2 \int_{\mathbf{R}} \frac{dp^-}{2} \\ &\quad \times \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{s}_3(p^2) \int \frac{d\xi^+}{2\pi} e^{i\xi^+(k^- - p^-)} W_{\text{coll}}(\xi^+) | \Omega \rangle, \end{aligned}$$

where the integration domain for p is the whole Minkowski space (M) and we decompose the integral as $d^4 p = dp^2 d^2 \mathbf{p}_T dp^- / 2p^-$. From the first to the second line we used Eq. (6) and performed the integrations over k^+ and \mathbf{p}_T , fixing $\xi^- = 0$ and $\xi_T = 0$ so that the Wilson line reduces to the collinear Wilson line $W_{\text{coll}}(\xi^+) \equiv W(0^+, 0^-, \mathbf{0}_T; \xi^+, 0^-, \mathbf{0}_T)$. Accordingly, we lose any k_T dependence for the α coefficient, and the same argument applies to the calculation of the ζ coefficient. Next, we choose the light-cone gauge $A^- = 0$ so that the collinear Wilson line reduces to the unity

matrix in color space. Finally, performing the integration over ξ^+ we obtain:

$$\begin{aligned} \alpha(k^-) &= \text{Disc} \int_{\mathbf{R}} dp^2 \int_{\mathbf{R}} \frac{dp^-}{2} \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{s}_3(p^2) | \Omega \rangle \delta(k^- - p^-) \\ &= \int_{\mathbf{R}} dp^2 \int_{\mathbf{R}} \frac{dp^-}{2} \delta(k^- - p^-) \{ (2\pi)^{-3} \rho_3(p^2) \theta(p^2) \theta(p^-) \} \\ &= \frac{1}{2(2\pi)^3} \left\{ \int_0^{+\infty} dp^2 \rho_3(p^2) \right\} \theta(k^-) = \frac{\theta(k^-)}{2(2\pi)^3}, \end{aligned} \quad (34)$$

where we have used the representation for the spectral operator \hat{s}_3 given in Eq. (9), and in the last step we used the normalization property for $\int_0^{+\infty} d\mu^2 \rho_3(\mu^2) = 1$. We remark that the only dependence on k^- resides in the theta function.

References

- [1] J. Dudek, et al., *Eur. Phys. J. A* 48 (2012) 187, arXiv:1208.1244.
- [2] A. Accardi, et al., *Eur. Phys. J. A* 52 (2016) 268, arXiv:1212.1701.
- [3] A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori, *J. High Energy Phys.* 06 (2017) 081, arXiv:1703.10157.
- [4] I. Scimemi, A. Vladimirov, *Eur. Phys. J. C* 78 (2018) 89, arXiv:1706.01473.
- [5] R. Angeles-Martinez, et al., *Acta Phys. Pol. B* 46 (2015) 2501, arXiv:1507.05267.
- [6] T.C. Rogers, *Eur. Phys. J. A* 52 (2016) 153, arXiv:1509.04766.
- [7] A. Bacchetta, *Eur. Phys. J. A* 52 (2016) 163.
- [8] A. Bacchetta, M.G. Echevarria, P.J.G. Mulders, M. Radici, A. Signori, *J. High Energy Phys.* 11 (2015) 076, arXiv:1508.00402.
- [9] D. Boer, S. Cotogno, T. van Daal, P.J. Mulders, A. Signori, Y.-J. Zhou, *J. High Energy Phys.* 10 (2016) 013, arXiv:1607.01654.
- [10] C. Shi, I.C. Cloët, *Phys. Rev. Lett.* 122 (2019) 082301, arXiv:1806.04799.
- [11] G.F. Sterman, *Nucl. Phys. B* 281 (1987) 310.
- [12] J.C. Collins, T.C. Rogers, A.M. Stasto, *Phys. Rev. D* 77 (2008) 085009, arXiv:0708.2833.
- [13] A. Accardi, J.-W. Qiu, *J. High Energy Phys.* 07 (2008) 090, arXiv:0805.1496.
- [14] A. Accardi, A. Bacchetta, *Phys. Lett. B* 773 (2017) 632, arXiv:1706.02000.
- [15] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P.J. Mulders, M. Schlegel, *J. High Energy Phys.* 02 (2007) 093, arXiv:hep-ph/0611265.
- [16] J. Collins, *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.* 32 (2011) 1.
- [17] P.-y. Chen, A. Idilbi, X.-d. Ji, *Nucl. Phys. B* 763 (2007) 183, arXiv:hep-ph/0607003.
- [18] D. Neill, I. Scimemi, W.J. Waalewijn, *J. High Energy Phys.* 04 (2017) 020, arXiv:1612.04817.
- [19] D. Neill, A. Papaefstathiou, W.J. Waalewijn, L. Zoppi, *J. High Energy Phys.* 01 (2019) 067, arXiv:1810.12915.
- [20] D. Gutierrez-Reyes, I. Scimemi, W.J. Waalewijn, L. Zoppi, *Phys. Rev. Lett.* 121 (2018) 162001, arXiv:1807.07573.
- [21] Z.-B. Kang, X. Liu, F. Ringer, H. Xing, *J. High Energy Phys.* 11 (2017) 068, arXiv:1705.08443.
- [22] Z.-B. Kang, J.-W. Qiu, F. Ringer, H. Xing, H. Zhang, *Phys. Rev. Lett.* 119 (2017) 032001, arXiv:1702.03287.
- [23] Z.-B. Kang, A. Prokudin, F. Ringer, F. Yuan, *Phys. Lett. B* 774 (2017) 635, arXiv:1707.00913.
- [24] H. Yamagishi, *Phys. Lett. B* 173 (1986) 465.
- [25] J.D. Bjorken, S.D. Drell, *Relativistic Quantum Fields*, 1965.
- [26] S. Weinberg, *The Quantum Theory of Fields. Vol. 1: Foundations*, Cambridge University Press, ISBN 9780521670531, 2005, 9780511252044.
- [27] R.E. Cutkosky, *J. Math. Phys.* 1 (1960) 429.
- [28] S. Bloch, D. Kreimer, arXiv:1512.01705, 2015.
- [29] M. Diehl, *Phys. Rep.* 388 (2003) 41, arXiv:hep-ph/0307382.
- [30] J. Levelt, P.J. Mulders, *Phys. Rev. D* 49 (1994) 96, arXiv:hep-ph/9304232.
- [31] P.J. Mulders, R.D. Tangerman, *Nucl. Phys. B* 461 (1996) 197, Erratum: *Nucl. Phys. B* 484 (1997) 538, arXiv:hep-ph/9510301.
- [32] C.D. Roberts, *J. Phys. Conf. Ser.* 706 (2016) 022003, arXiv:1509.02925.
- [33] C.D. Roberts, M.S. Bhagwat, A. Holl, S.V. Wright, *Eur. Phys. J. Spec. Top.* 140 (2007) 53, arXiv:0802.0217.
- [34] K. Goeke, A. Metz, P.V. Pobylitsa, M.V. Polyakov, *Phys. Lett. B* 567 (2003) 27, arXiv:hep-ph/0302028.
- [35] S. Meissner, A. Metz, K. Goeke, *Phys. Rev. D* 76 (2007) 034002, arXiv:hep-ph/0703176.
- [36] A. Metz, A. Vossen, *Prog. Part. Nucl. Phys.* 91 (2016) 136, arXiv:1607.02521.
- [37] R.L. Jaffe, *Nucl. Phys. B* 229 (1983) 205.
- [38] L.P. Gamberg, A. Mukherjee, P.J. Mulders, *Phys. Rev. D* 83 (2011) 071503, arXiv:1010.4556.
- [39] S. Meissner, A. Metz, D. Pitonyak, *Phys. Lett. B* 690 (2010) 296, arXiv:1002.4393.
- [40] J.C. Collins, D.E. Soper, *Nucl. Phys. B* 194 (1982) 445.
- [41] R.L. Jaffe, X.-D. Ji, *Phys. Rev. Lett.* 71 (1993) 2547, arXiv:hep-ph/9307329.
- [42] S. Bastami, et al., arXiv:1807.10606, 2018.
- [43] J.-w. Qiu, G.F. Sterman, *Phys. Rev. Lett.* 67 (1991) 2264.
- [44] A. Accardi, A. Signori, *PoS DIS2018* (2018) 158, arXiv:1808.00565.
- [45] H.H. Matevosyan, W. Bentz, I.C. Cloët, A.W. Thomas, *Phys. Rev. D* 85 (2012) 014021, arXiv:1111.1740.
- [46] W. Bentz, A. Kotzinian, H.H. Matevosyan, Y. Ninomiya, A.W. Thomas, K. Yazaki, *Phys. Rev. D* 94 (2016) 034004, arXiv:1603.08333.
- [47] M. Boglione, J.O. Gonzalez-Hernandez, R. Taghavi, *Phys. Lett. B* 772 (2017) 78, arXiv:1704.08882.
- [48] E. Moffat, T.C. Rogers, N. Sato, A. Signori, arXiv:1909.02951, 2019.
- [49] A. Radyushkin, *Phys. Lett. B* 767 (2017) 314, arXiv:1612.05170.
- [50] K. Orginos, A. Radyushkin, J. Karpie, S. Zafeiropoulos, *Phys. Rev. D* 96 (2017) 094503, arXiv:1706.05373.
- [51] J. Karpie, K. Orginos, A. Radyushkin, S. Zafeiropoulos, *EPJ Web Conf.* 175 (2018) 06032, arXiv:1710.08288.