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Jaime MONTANA DONCEL

Three essays on skills mismatch in labor markets

Directeurs de thèse :

- M. David N. MARGOLIS, Research Director CNRS and Authorized Thesis Director Université Paris 1
- M. Francesco DEVICIENTI, Professor University of Turin and Collegio Carlo Alberto

Composition du jury :

- M. Thibault DARCILLON, Professor, Université Paris 8
- M. François FONTAINE, Professor Université Paris 1
- M. Ignacio MONZON, Associate Professor with Tenure Collegio Carlo Alberto
- Mme. Eva MORENO-GALBIS, Professor Université Aix-Marseille
- Mme. Véronique SIMONNET, Professor Université Grenoble-Alpes
- Mme. Joanne TAN, Assistant Professor, School of Economics, Singapore Management University

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Three essays on skills mismatch in labor markets

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Jaime MONTANA DONCEL [1]

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[1]Paris School of Economics, University Paris-1 Panthéon-Sorbonne and Turin University.

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Introduction

General Introduction

Work is fundamental to most of us, where we spend a considerable part of our lives. Work is essential not only because we invest a lot of time and energy in our jobs, but also because work shapes us. Consider the sort of career you have picked or the job you expect to work. The path you had to take to arrive at such a job, the company where you landed, and the compensation path in that career are entirely different from job to job. The exchange of our time and energy for a wage determines, in a broad sense, the scope of labor economics. Labor economics seeks to understand the phenomena attached to the functioning of the market for labor and its compensation. Labor economics studies this market and its evolution and considers its effects in other aspects of our lives, both as individuals and as a society. As in other economic disciplines, labor economics utilises mathematical models to describe the mechanisms behind the market. Generally, labor services are simplified to a unique and homogeneous factor of production, which is used by the firm to produce goods or services. One individual equals one homogeneous labor factor.

This dissertation studies the role of skills in labor markets and how multiple and heterogeneous skills complement our understanding of the labor market. A first, natural question is to which extent complicating the existing models by including the concept of skills would complement our knowledge. In economic terms, does the cost of considering skills and thus more complex mathematical methods surpass the the benefits in terms of insights?

To answer, one can look to one of the most influential essays of economic thought, written more than fifty years ago by Friedman (1953). In it, Friedman lays the foundations of economic analysis. In particular, he championed the use of simplified mathematical models to describe economic phenomena. Consider reality as the limit model, which includes all complexities that we can build or consider. A simple, representative, and consistent model is thus informative on the reality it approximates. Economic modelling allows one to isolate relevant forces and mechanisms, which leads to a deeper understanding of the phenomenon at hand, be it individual behaviors or outcomes of social interaction. The principles presented in Friedman's essay are still ubiquitous across all fields of economics.

These principles apply to labor economics as well. For instance, labor is still conceived of as a homogeneous and undifferentiated production factor that enters the firms' production function. In the real world, though, labor is far from homogeneous: skills are multidimensional, individuals differ in each skill, and jobs require different combinations or amounts of skills (Lazear, 2009). In Friedman's words, a *more* realistic model is useful insofar it provides us with a sounder understanding of the modelled phenomenon.[3]

There are several reasons to consider skills nowadays. The first is rather pragmatic: we currently have at our disposal detailed data that allow us to consider skills and their relevance for policy at a granular level. Skills measures have been incorporated into surveys (PIAAC, STEP) to assess cross-country discrepancies. Classification and harmonisation of job-specific skills and occupational information has greatly improved as well (ESCO, O*NET). We have also improved our ability to collect and organize public and administrative information. For example, the collection, categorization, and content of posted vacancies and employment services records now provides crucial information to help us understand workers' and firms' behavior. Therefore, we are now empirically equipped to test the role of skills in different labor market processes. More importantly, we can inform labor market programs for employability and workforce planning on solid empirical grounds. Secondly, skills are important[4], and their existence causes and alters some of the phenomena studied in labor economics.

But, what is a skill? How is it defined? Depending on the context, there might be different definitions. When we consider the educational system, skills are frequently associated with qualifications (technical, vocational, university), or with fields of study (law, medicine, economics), or even through the measurement of cognitive and non-cognitive skills (mathematics

[3]In Friedman words: "Complete "realism" is clearly unattainable, and the question whether a theory is realistic "enough" can be settled only by seeing whether it yields predictions that are *good enough for the purpose in hand*" (Friedman, 1953, emphasis added)

[4]We use the word important to indicate that skills play a major causal role. It can be seen then as an ontological reason.

or soft skills). In labor markets, skills are an individual ability to perform well in a particular job and are associated with occupational characteristics.

The Organization for Economic Cooperation and Development (OECD, 2016) surveys the different uses and meanings to which the term skill refers and how they have been used in different contexts. First, skills can be understood from a generic perspective as a developed capacity to perform a job. Skills are categorized in different groups, concerning the scope of such capacity. Cognitive skills refer to abilities that require intellectual development (arithmetic, literary or problem-solving). In contrast, non-cognitive skills refer to developed abilities that require organization and social perception (teamwork, perseverance, and soft skills). Finally, manual skills refer to capacities that require physical abilities, either in strength or precision. It is also common to refer to specific capabilities within the firm, which correspond uniquely to its organization and its way of production. These are specialized skills, distinguished by the fact that they are not transferable between employers, professions, or industries and are exemplified by firm-specific knowledge of the organization's functioning, its community, technological knowledge, or sector-specific competencies.

The scope of this dissertation is to complement our understanding of three simple situations in the labor market with respect to skills. The three cases analyzed are the following: how people sort into jobs, whom matches with whom, and who is fired in a mass layoff. We analyze such questions through the lens of skill mismatch. In a world full of heterogeneity, grasping the intuition on mismatch is straightforward, since it subsumes the distaste of dissimilarities. When we consider the firms' side, the heterogeneity rests in requirements. Conversely, when we consider the workers' side, the heterogeneity is with respect to their endowments. Considering mismatch then boils down to a cost, in productivity for the employer or in well-being for the worker. The three situations analyzed in the present dissertation highlight the importance skills and their mismatch plays in labor market outcomes.

Skills mismatch has drawn a good deal of policy interest recently. Several reports from international organizations stress the policy relevance of mismatch in labor markets, since it directly affects unemployment and productivity (Mcgowan and Andrews, 2015b; McGowan and Andrews, 2015a; McGuinness et al., 2017; Co-operation et al., 2011; Stoevska, 2017; Hatos, 2014). Broadly speaking, skills mismatch occurs when workers' skills exceed or do not meet current market labor requirements. Mismatches refer mainly to workers having higher or lower qualifications than the requirements of their current jobs. A natural question is how to correctly measure mismatch. The answer depends on the level of aggregation we use, or on the characterization of the skills or qualifications we take into account. For example, Brunello and Wruuck (2019) compares mismatch at the macro level and the micro-level. Mismatch at the macro level originates from the combination of demand and supply for skills in a region or a country: comparing the demand characteristics (average occupational composition) against specific group characteristics (e.g., employability, educational attainment) provides an intuitive approximation for mismatch. From the micro standpoint, skills mismatch happens when individual workers do not possess the characteristics that their job requires. This conceptual difference affects how we measure mismatch. We can understand mismatch to indicate different skill gaps and imbalances, such as overeducation, under-education, overqualification, low qualification, excess skills, shortage of skills and surpluses, and obsolescence of skills or technologies.

This dissertation is composed of three chapters. In all chapters, we use the notion of skills. These are individuals' multiple and heterogeneous characteristics and are incorporated by firms in their production activity. However, the methods and the type of data used in each chapter vary. The data also differ in their sources: throughout the dissertation we employ surveys, administrative data, or publicly available information (scraped from job boards on the internet). Even if the dissertation's approach is quantitative in all chapters, the methods used differ, too. In the first chapter, we combine surveys and web-scraped vacancy data. Using numerical methods, we investigate the efficiency of the allocation of workers to firms. In the second chapter, we develop a random search model with two-sided multidimensional heterogeneity. Firms choose and post a wage with commitment, independent of the worker type who eventually accepts the job. Posted wages determine the set of acceptable jobs for each worker and a unique *applicant pool* for each firm. Using data from France, we take the model to the data with structural estimation to find that non-cognitive skills' disutility is higher in a mismatch. At the same time, employers value more good matches on cognitive skills. We also find that multidimensionality plays an important role, being another source for frictions. The last chapter employs social security records (administrative data) and a reduced form econometric approach to investigate how French firms reorganize during a mass layoff. Crucially, we also shed light on the types of workers most exposed to displacement during such deep restructuring.

This dissertation contributes to the literature in labor economics. It develops a theoretical background and collects empirical evidence on the importance of skills and the role that skills mismatch plays in the process of job allocation and separations.

Summary of chapters

Chapter 1 is the product of joint work with David Margolis. In this chapter we consider how skills and skills mismatch affect workers sorting into jobs. Our effort is to answer '*what behavior underlies the matching process?*' Frictions on the workers' side originate from searching for jobs that provide the highest utility, for given skill set. Conversely, search on the employer side refers to firms seeking workers whose skill set most closely matches the requirements of the post being offered. Workers accurately predict the skills demanded by the market, and invest accordingly: thus supply and demand of skills match at the aggregate level. But workers and firms fail to instantly and optimally match, because workers' knowledge of their competitors' skill sets is imperfect, or equivalently their limited rationality does not allow them to solve the full multidimensional sorting problem, preventing the market from reaching and equilibrium in which workers only apply to the jobs that will hire them.

This chapter theoretically models the worker-side search process when workers have full information about offered jobs and the skills available in the population, but whose their of sophistication is limited.[5] It then numerically solves the equilibrium allocation of skills to jobs and the time to job finding using data on cognitive, non-cognitive, and technical skills supplied and demanded (as announced in on-line job postings) in Colombia. It first establishes that this allocation of workers to jobs is inefficient, in the sense that there are over-qualified workers in medium-skilled jobs and under-qualified workers who require significant skill upgrading in high-skill jobs. Then, the chapter introduces a counterfactual simulation in which firms are subsidised for training (thereby reducing the cost of hiring workers with skills below the minimum threshold for a job), a simulation in which the long-term unemployed receive training and a simulation in which all unemployed receive training, so that their skills increase to a level that makes them

[5]Solving for the multidimensional matching equilibrium is only starting to attract attention in the economics literature (see Dupuy and Galichon (2014)), and the large literature demonstrating the important significant role of job search assistance and placement for the unemployed (Card et al., 2010) suggests that the complexity of the decision problem is non-trivial.

eligible for jobs that otherwise they would be unable to occupy. The policy of training the long-term unemployed is the most effective in moving the equilibrium towards a more efficient labor allocation.

Chapter 2 proposes a new answer to an old question in labor economics, *"Who matches with whom?"*. It presents a setting where firms and workers are heterogeneous in many dimensions, and workers can be over- and under- qualified for the jobs they end up occupying. In the chapter we present a random search model with two-sided multidimensional heterogeneity in which firms choose and post a wage with commitment, i.e. maintaining the posted wage, independent of the productivity of the new worker. Posted wages determine the set of acceptable jobs for each worker and a unique *applicants pool* for each firm. The composition of these sets varies in size and composition across workers and firms. The optimal posted wage level takes into consideration the requirements of each firm and the characteristics of the applicants pool. In equilibrium, sorting is assortative but mismatches can occur across all skills dimensions. Using French data on workers' observed skills and matches, we structurally estimate the parameters associated with the model for the French economy. We find that the disutility of non cognitive skills is higher when mismatched, while employers value more good matches on cognitive skills. We also find that the number of dimensions along which mismatch can occur plays an important role, since it is another source of frictions.

This chapter makes a number of contributions. First, the chapter contributes to the (limited) literature on multidimensional search (Lindenlaub and Postel-Vinay, 2016; Lise and Postel-Vinay, 2015; Tan, 2017; Lazear, 2009), proposing a microfoundation for matching and sorting with multiple and heterogeneous skills. Moreover, this chapter contributes to the literature of multidimensional skills, providing theoretical evidence for how wages depend on multidimensional skills and requirements (Deming, 2017; Deming and Kahn, 2018; Speer, 2017). Lastly, the chapter introduces vectorial calculus notation in a random search model, allowing the use of a multidimensional Leibniz rule. This permits a clear interpretation of the wage determination optimality conditions, given that wage determination is the main strategic decision of the firm. Changes in the posted wage will induce changes in the composition of *the applicants pool*, thereby changing the size and composition of this set. In equilibrium we know who matches with whom, and can characterize the set of acceptable jobs for each worker and a unique *applicants pool* for each firm. One novelty of our study is that sorting is not homogeneous in the economy. Matching and sorting are distribution dependent, and are fully characterized for each point in the support of both the endowment and requirement distributions.

The last chapter of the dissertation is the product of joint work with David Margolis. It makes use of a combination of employer-employee administrative data and survey data on skills. In this chapter, we show that companies that experienced a mass layoff used this event to restructure their workforce. We observe a small but significant increase in the use of social skills, a decrease in manual skills and a non-significant increase in cognitive skills within the firm. Restructuring occurs over a relatively short period (two years) compared to the long-term analysis of previous macro literature. The results are consistent with those findings from the macroeconomic strand of literature. The restructuring of the workforce highlights that firms use layoffs strategically, and that selection into displacement plays an important role.

When we investigate selection into displacement directly, we find that skill mismatch and relative wage cost play an important role in determining who is displaced. The coefficients for cognitive and social skill mismatch are both significant and positive, implying that higher mismatch increases the likelihood of being displaced. The result is robust across samples and specifications, even if we control by other demographic characteristics, firm characteristics, and firm and year fixed effects. The findings on firm characteristics also demonstrates how firms' performance indicators impact differently displacement choices.

With the ongoing economic downturn, the findings discussed here highlight the value of re-employment initiatives for recently unemployed people. This demographic has the greatest levels of mismatch, thus programs based on skills upgrades will speed up re-employment. Specifically, it would be necessary for policy makers to identify the occupations that are most in demand, identify their skill requirements and up-skill the unemployed workforce *in the correct dimensions* in order to prevent lengthy periods of unemployment, as in past recessions.

Introduction générale

Le travail est l'un des éléments les plus essentiels, car nous passons une partie considérable de notre vie au travail. Le travail est essentiel non seulement parce que nous y investissons beaucoup de temps et d'énergie, mais aussi parce que le travail, d'une manière, nous façonne. Considérez le type de carrière que vous avez choisi ou l'emploi que vous comptez occuper. Le chemin que vous avez dû emprunter pour arriver à un tel emploi, l'entreprise qui vous accueille et la rémunération dans cette carrière sont entièrement différents d'un emploi à l'autre. L'échange de notre temps et de notre énergie contre un salaire détermine, au sens large, la portée de l'économie du travail. L'économie du travail est une branche de l'économie qui cherche à comprendre les phénomènes attachés au fonctionnement du marché du travail et de sa rémunération. L'économie du travail étudie ce marché et son évolution et considère ses effets dans d'autres aspects de notre vie, à la fois en tant qu'individus et en tant que société. Comme dans d'autres disciplines économiques, l'économie du travail fait appel à des modèles mathématiques pour décrire le mécanisme de son fonctionnement. En général, les services du travail sont représentés à l'aide d'une simplification de base : un facteur de production unique et homogène qui est utilisé par l'entreprise pour produire des biens ou des services. Un individu est égal à un facteur de travail homogène.

Cette thèse étudie le rôle des compétences sur les marchés du travail et comment l'introduction de compétences multiples et hétérogènes pourrait améliorer notre compréhension du marché du travail. Une première question qui se pose naturellement en considérant ce sujet est de savoir dans quelle mesure le fait de compliquer les modèles existants en incluant le concept de compétences améliorerait nos connaissances. En termes économiques, si les coûts de la prise en compte des compétences, qui nous contraindront, en général, à utiliser des outils et des méthodes mathématiques plus complexes, surpassent les gains aux contributions de notre compréhension, ou ne feront que compliquer sans un gain suffisant de connaissance.

Pour répondre à ces questions, il faut considérer l'un des essais les plus influents de la pensée économique, écrit il y a plus de cinquante ans par Friedman (1953). Dans cet essai, Friedman jette les bases de la méthodologie économique. En particulier, il présente comme l'un des principaux outils de l'économie l'utilisation de modèles mathématiques simplifiés pour décrire les phénomènes économiques. La cohérence et la représentativité du modèle permettent de le comparer à la réalité, ce qui le rend crédible et explicatif en soi (Hausman, 1994). La différence entre le modèle et la réalité s'affiche sous les yeux de tous. Considérons la réalité comme le modèle limite, qui comprend toutes les complexités que nous pouvons construire ou envisager. La modélisation économique permet de comprendre le phénomène, le comportement individuel et les résultats sociaux par déduction à partir d'un modèle plus simple. Les principes présentés dans l'essai de Friedman sont toujours omniprésents dans tous les domaines de l'économie.

Par exemple, en économie du travail, la notion de fonction de production dans laquelle un facteur de production unique et homogène est utilisé par l'entreprise est encore l'outil théorique fondamental. Dans le monde réel, les qualifications sont multidimensionnelles, les individus possèdent différents niveaux de chaque qualification, et chaque type d'emploi peut exiger une combinaison différente de qualifications et dans des quantités différentes (Lazear, 2009). Suivre Friedman en étant *plus* réaliste n'apporte aucun gain si nous n'obtenons pas une compréhension plus solide du phénomène que nous étudions [6], et dans ce cas, une bonne compréhension du marché du travail.

Il y a plusieurs raisons pour lesquelles il peut être intéressant de considérer le rôle des compétences. La première est pragmatique, dans le sens où nous disposons aujourd'hui de données plus détaillées qui nous permettent de considérer le rôle des compétences à un niveau granulaire et sa pertinence pour la politique économique. Nous avons amélioré notre capacité à générer, collecter et organiser les données en mettant l'accent sur leur granularité. Des mesures des compétences ont été intégrées dans des enquêtes (PIAAC, STEP) afin de déterminer les différentes capacités des pays. Nous avons amélioré la manière dont nous classons et organisons les informations sur les professions et les compétences associées aux emplois (ESCO, O*NET). Nous avons également amélioré notre capacité à collecter et à organiser les informations publiques et les informations administratives. Par exemple, la collecte, la catégorisation et l'analyse du contenu des offres d'emploi publiées et des dossiers des services de l'emploi contiennent des informations précieuses pour comprendre les comportements des travailleurs et des entreprises. Le fait de disposer de toutes ces nouvelles informations permet donc de tester le rôle des compétences dans les différents processus du marché du travail, mais surtout de savoir comment adapter les programmes du marché du travail pour l'employabilité et la planification de la main-

[6] Dans les mots de Friedman : "Complete "realism" is clearly unattainable, and the question whether a theory is realistic "enough" can be settled only by seeing whether it yields predictions that are *good enough for the purpose in hand*" (Friedman (1953), emphases ajoutées)

d'œuvre. La deuxième raison est que les compétences sont importantes[7], et leur hétérogénéité cause et modifie certains des phénomènes étudiés en économie du travail.

Mais, qu'est-ce qu'une compétence ? Comment la définir ? Selon le contexte, il peut y avoir différentes définitions. Lorsque nous considérons le système éducatif, les compétences sont fréquemment associées à des qualifications (techniques, professionnelles, universitaires), ou à des domaines d'études (droit, médecine, économie), ou encore par la mesure de compétences cognitives et non cognitives (mathématiques ou soft skills) qu'elles intègrent. Sur le marché du travail, les compétences sont une faculté individuelle de bien accomplir un travail particulier associée à des caractéristiques professionnelles.

L'Organisation pour la Coopération et le Développement Économiques (OECD, 2016) étudie les différentes utilisations et significations auxquelles renvoie le terme "compétence" et la manière dont il a été utilisé dans différents contextes. Tout d'abord, les compétences peuvent être comprises d'un point de vue générique comme une capacité développée à accomplir une tache. Les compétences sont classées en différents groupes, dépendant de l'extension de cette capacité. Les compétences cognitives font référence aux capacités qui nécessitent un développement intellectuel (arithmétique, littéraire ou résolution de problèmes). En revanche, les compétences non cognitives font référence à des capacités qui nécessitent une organisation et une perception sociale (travail d'équipe, persévérance et compétences non techniques). Enfin, les aptitudes manuelles font référence à des capacités qui requièrent des aptitudes physiques, que ce soit en force ou en précision. Il est également courant de faire référence à des capacités spécifiques à l'intérieur de l'entreprise, qui correspondent uniquement à son organisation et à son mode de production. Il existe des compétences spécialisées qui se distinguent par le fait qu'elles ne sont pas transférables d'un employeur, d'une profession ou d'un secteur d'activité à l'autre et qui sont liées à la connaissance du fonctionnement de l'entreprise et de la communauté, les connaissances technologiques ou les compétences spécifiques au secteur.

Le but de cette thèse est de compléter notre compréhension de trois situations simples sur le marché du travail lorsque nous considérons les qualifications. Les trois situations analysées sont les suivantes : comment les agents sont alloués aux emplois, quel est le mécanisme d'appariement, et qui est licencié dans un licenciement collectif. Nous analysons ces questions

[7] Nous utilisons le mot important pour indiquer que les compétences jouent un rôle causal majeur. On peut donc le considérer comme une raison ontologique.

à travers la perspective de l'appariement imparfait. Dans un monde où l'hétérogénéité est abondante, il est très facile de comprendre l'appariement imparfait puisqu'il decoule de l'aversion pour la variété. Du côté des entreprise, il s'agit de l'lhétérogénéité dans les exigences qu'elles cherchent. Du côté des travailleurs, l'hétérogénéité par rapport à leur dotation de capacités. Introduire un appariement imparfait implique alors l'introduction d'un coût, en productivité pour l'employeur ou en bien-être pour le travailleur. Les trois situations analysées dans le cadre de la présente thèse montrent l'importance des qualifications et mettent en évidence le rôle que l'appariement imparfait des qualifications joue dans ces résultats.

L'appariement imparfait des qualifications a suscité un intérêt politique ces derniers temps. Plusieurs rapports d'organisations internationales ont souligné l'importance politique de l'appariement imparfait sur les marchés du travail, puisqu'il a une incidence directe sur le chômage et la productivité (Mcgowan and Andrews, 2015b; McGowan and Andrews, 2015a; McGuinness et al., 2017; Co-operation et al., 2011; Stoevska, 2017; Hatos, 2014). De manière générale, l'appariement imparfait des qualifications se produit dans des situations où les qualifications des travailleurs dépassent ou ne répondent pas aux exigences actuelles du marché du travail. Principalement, les appariement imparfaits se réfèrent aux travailleurs ayant des qualifications supérieures ou inférieures aux exigences de leurs emplois actuels. La façon de mesurer l'appariement imparfait a fait l'objet de nombreux débats dans la littérature. La réponse dépend du niveau d'agrégation que nous utilisons ou de la caractérisation des compétences ou qualifications que nous prenons en compte. Par exemple, Brunello and Wruuck (2019) considèrent la différence entre l'appariement imparfait au niveau macro- et micro-économique. L'appariement imparfait au niveau macro-économique compare la demande et l'offre de qualifications dans une région ou un pays, en comparant les caractéristiques de la demande (composition professionnelle moyenne) aux caractéristiques de groupes spécifiques (par exemple, l'employabilité, le niveau d'éducation). Du point de vue micro-économique, l'appariement imparfait des qualifications se produit lorsque les travailleurs individuels ne possèdent pas les caractéristiques que leur emploi exige. Cette différence conceptuelle et de perspective affecte la mesure des effets de l'appariement imparfait. Nous pouvons comprendre l'appariement imparfait comme indiquant différents écarts et déséquilibres de compétences, tels que la sur-éducation, la sous-éducation, la sur-qualification, la faible qualification, l'excès de compétences, la pénurie de compétences et les excédents, et l'obsolescence des compétences ou des technologies.

Cette thèse est divisé en trois chapitres. Dans tous les chapitres, nous utilisons la notion de qualifications. Nous considérons que celles-ci sont des caractéristiques multiples et hétérogènes des individus et utilisées par les entreprises dans leur activité de production. Cependant, les méthodes et le type de données utilisées dans chaque chapitre ne sont pas les mêmes. Les données sont différent par leur sources, puisque les données des différents chapitres proviennent d'enquêtes, de données administratives ou d'informations accessibles au public (sites d'emploi en ligne). Même si l'approche de la thèse est quantitative dans tous les chapitres, les méthodes utilisées diffèrent. Dans le premier chapitre, j'utilise des données d'enquête et des données sur les offres d'emploi récupérées sur le Web. À l'aide de méthodes numériques, nous étudions l'efficacité de l'allocation des travailleurs aux entreprises. Dans le deuxième chapitre, je développe un modèle de recherche aléatoire avec une hétérogénéité multidimensionnelle bilatérale. Les entreprises choisissent et affichent un salaire avec engagement, indépendamment du type de travailleur qui accepte l'emploi. Les salaires affichés déterminent l'ensemble des emplois acceptables pour chaque travailleur et un *bassin de candidats* unique pour chaque entreprise. À l'aide de données françaises, j'estime le modèle empiriquement en utilisant l'économétrie structurelle pour constater que la désutilité des qualifications non cognitives est plus élevée lorsqu'elles sont mal assorties. Au même temps, les employeurs valorisent davantage les bonnes correspondances sur les qualifications cognitives. Je constate également que la multidimensionnalité joue un rôle important, étant une autre source de frictions. Le dernier chapitre utilise des enregistrements auprès de la sécurité sociale (données administratives) et une approche économétrique de forme réduite pour étudier si les entreprises en France se recomposent lors d'un licenciement massif, et pour savoir comment les entreprises choisissent les travailleurs à licencier.

Cette thèse contribue à la littérature sur l'économie du travail, en développant un contexte théorique et en recueillant des preuves empiriques sur l'importance des qualifications et le rôle que joue l'appariement imparfait des qualifications dans le processus d'attribution des emplois et des séparations.

Synthèse des chapitres

Le chapitre 1 est le fruit d'un travail en collaboration avec David Margolis. Dans cet article, nous examinons comment les qualifications et l'appariement imparfait des qualifications peuvent affecter la façon dont les travailleurs se rangent dans les emplois. Nous essayons de répondre à la question "*qu'est-ce qui se passe derrière le processus d'appariement?*" Dans ce contexte, les frictions de recherche du côté des travailleurs font référence aux personnes qui essaient de trouver les emplois qui leur procurent la plus grande utilité, compte tenu de leurs qualifications. La recherche du côté des employeurs fait référence aux entreprises qui essaient de trouver les travailleurs dont les qualifications correspondent le mieux aux exigences technologiques du poste proposé. Même si les travailleurs prédisent avec précision les qualifications dont le marché aura besoin et investissent de conséquence (et donc que l'offre et la demande de qualifications dans la population correspondent), les travailleurs et les entreprises peuvent ne pas parvenir à un appariement optimal instantané, lorsque la connaissance que les travailleurs ont des caractéristiques de leurs concurrents est imparfaite, ou que le degré de sophistication de leur raisonnement ne leur permet pas de résoudre l'équilibre d'appariement multidimensionnel complet de sorte que les travailleurs ne postulent qu'aux emplois qui les embaucheront en équilibre.

Cet article modélise théoriquement le processus de recherche côté travailleur lorsque les travailleurs disposent d'informations complètes sur les emplois proposés et les qualifications disponibles dans la population, mais dont le niveau de sophistication de raisonnement est limité[8]. Il résout ensuite numériquement l'allocation d'équilibre des travailleurs aux emplois et le délai de recherche d'emploi en utilisant des données sur les qualifications cognitives, non cognitives et techniques offertes et demandées (telles qu'annoncées dans les offres d'emploi en ligne) en Colombie. Après avoir établi que cette allocation des travailleurs aux emplois est inefficace, car il y a des travailleurs surqualifiés dans les emplois moyennement qualifiés et des travailleurs sous-qualifiés qui ont besoin d'une mise à niveau importante de leurs compétences dans les emplois hautement qualifiés, l'article présente une simulation contrefactuelle dans laquelle la formation au sein des entreprises est subventionnée (réduisant ainsi le coût de l'embauche de travailleurs

[8] La résolution de l'équilibre d'appariement multidimensionnel commence seulement à attirer l'attention dans la littérature économique (voir Dupuy and Galichon (2014)), et l'importante littérature démontrant le rôle significatif de l'aide à la recherche d'emploi et du placement des chômeurs (Card et al., 2010) suggère que la complexité du problème de décision est non triviale.

ayant des compétences inférieures au seuil minimum pour un emploi), une simulation dans laquelle les chômeurs de longue durée reçoivent une formation et une simulation dans laquelle tous les chômeurs reçoivent une formation afin que leurs compétences puissent augmenter jusqu'à un niveau qui les rende admissibles à des emplois qu'ils ne pourraient pas occuper autrement. Il s'avère que la politique de formation des chômeurs de longue durée s'approche davantage d'une allocation efficace de la main-d'œuvre qu'une politique d'amélioration généralisée des qualifications ou une subvention à la formation pour les entreprises.

Le chapitre 2 propose une nouvelle réponse à une vieille question en économie du travail, *"Qui s'apparie avec qui ?"*. Nous introduisons un cadre où les entreprises et les travailleurs sont différents dans de nombreuses dimensions et nous permettons aux travailleurs d'être sur- et sous-qualifiés pour les emplois qu'ils finissent par occuper. Je présente un modèle de recherche aléatoire avec une hétérogénéité multidimensionnelle bilatérale dans lequel les entreprises choisissent et affichent un salaire avec engagement, c'est-à-dire en maintenant le salaire affiché, indépendamment de la productivité du nouveau travailleur. Les salaires affichés déterminent l'ensemble des emplois acceptables pour chaque travailleur et un *bassin de candidats* unique pour chaque entreprise. La taille et la composition de ces ensembles varient selon les travailleurs et les entreprises. Le niveau optimal du salaire affiché tient compte des exigences de chaque entreprise et des caractéristiques du pool de candidats. À l'équilibre, l'appariement est assortatif mais des appariement imparfaits peuvent se produire dans toutes les dimensions des qualifications. En utilisant des données françaises sur les qualifications et les appariements observés des travailleurs, j'estime les paramètres structurels associés au modèle pour la France. Je constate que la désutilité des qualifications non cognitives est plus élevée en cas d'appariement imparfait, tandis que les employeurs accordent une plus grande valeur aux bonnes qualifications cognitives. Je trouve également que le nombre de dimensions joue un rôle important, puisqu'il est une autre source de frictions.

Tout d'abord, l'article contribue à la littérature (limitée) sur la recherche multidimensionnelle (Lindenlaub and Postel-Vinay, 2016; Lise and Postel-Vinay, 2015; Tan, 2017; Lazear, 2009), en proposant une microfondation pour comprendre comment l'appariement se produise dans le cas de qualifications multiples et hétérogènes. De plus, cet article contribue à la littérature sur les qualifications multidimensionnelles, en proposant une théorie de la manière dont les salaires dépendent des qualifications et des exigences multidimensionnelles (Deming, 2017; Deming and Kahn, 2018; Speer, 2017). Enfin, l'article introduit la notation du calcul vectoriel dans un modèle de recherche aléatoire, permettant l'utilisation d'une règle de Leibniz multidimensionnelle. Cela permet une interprétation claire des conditions d'optimalité de détermination des salaires, ce qui est la principale décision stratégique de l'entreprise. Les changements de salaire induisent des changements dans la composition du *bassin de candidats*, modifiant ainsi la taille et la composition de cet ensemble. À l'équilibre, nous connaissons les pairs appairés, ce qui caractérise un ensemble d'emplois acceptables pour chaque travailleur et un *bassin de candidats* unique pour chaque entreprise. Une nouveauté de cette approche est que le tri n'est pas homogène dans l'économie. L'appariement dépend de la distribution des besoin des entreprises et des capacités des travailleurs, et sont entièrement caractérisés pour chaque point du support des distributions des dotations et des exigeances.

Le dernier chapitre de cette thèse est le produit d'un travail conjoint avec David Margolis. Il utilise une combinaison de données administratives des employeurs et des employés et de données d'enquête sur les qualifications. Dans cette contribution, nous montrons que les entreprises qui ont connu un licenciement collectif ont utilisé cet événement pour recomposer leur main-d'œuvre. Nous observons une augmentation faible mais significative de l'utilisation des qualifications sociales, une diminution des qualifications manuelles et une augmentation non significative des qualifications cognitives au sein de l'entreprise. La recomposition se produit dans un laps de temps très court (deux ans) par rapport à l'analyse à long terme de la littérature macroéconomique précédente. La recomposition de la main-d'œuvre prouve que les entreprises utilisent le licenciement de manière stratégique pour se réorganiser. De plus, la sélection dans le déplacement joue un rôle important.

Lorsque nous examinons la sélection dans le licenciement, nous constatons que l'appariement imparfait des qualifications et le coût salarial relatif jouent un rôle important pour déterminer qui est licencié. Les coefficients des appariements imparfaits cognitives et sociales sont à la fois significatifs et positifs, ce qui implique que l'appariement imparfait des qualifications augmente la probabilité d'être licencié. Le résultat est robuste aux échantillons et aux spécifications, même si nous contrôlons par d'autres caractéristiques démographiques, les caractéristiques de l'entreprise et les effets fixes de l'entreprise et de l'année. Les résultats sur les caractéristiques de l'entreprise montrent également comment des performances différentes de l'entreprise pourraient jouer sur les licenciements pratiqués.

Avec le ralentissement économique actuel, les résultats discutés ici peuvent servir à souligner la valeur des initiatives de réemploi pour les chômeurs récents. Ce groupe démographique présente les plus hauts niveaux d'appariement imparfait et les programmes basés sur l'amélioration des qualifications accéléreront le réemploi. Plus précisément, il serait nécessaire que les décideurs politiques caractérisent les professions qui sont plus employables, identifient les compétences les plus importantes et améliorent les qualifications de la main-d'œuvre au chômage *dans ces dimensions* afin d'éviter de longues périodes de chômage, comme lors de la dernière récession.

Chapter 1

Matching heterogeneous skills demand with heterogeneous skills supply under limited rationality[1]

1.1 Introduction

How workers match to jobs has implications for productivity and affects both workers and firms. The allocation mechanism has been treated conceptually via the aggregate matching function in macroeconomics and search and matching models (Mortensen and Pissarides, 1994; Postel–Vinay and Robin, 2002; Lindenlaub and Postel-Vinay, 2016; Lindenlaub, 2017). In this literature, the sorting and match stability results are backed by mathematical properties of the matching and the production functions. Still, few papers model the allocation process from a microeconomic perspective. This paper provides a microeconomic model of the allocation mechanism of workers to jobs when skills and requirements are multidimensional. The model aims to reproduce congestion, coordination failures, and other aspects that characterize the (macroeconomic) matching function.

Close to the concept of the allocation mechanism is skills mismatch. Skills are a fundamental component when analyzing the labor market (Deming and Kahn, 2018; Deming, 2017; Lindenlaub and Postel-Vinay, 2016; Guvenen et al., 2020; Lise and Robin, 2017; Lise and Postel-Vinay, 2015;

[1] This chapter is the product of joint work with David Margolis.

Tan, 2017). It is common now to think of the labor market in terms of skills, in which individuals possess different levels of each skill, and each job can require a different combination of skills and in different amounts (Lazear, 2009). In this context, search frictions on the worker side refer to people trying to find the jobs that provide them with the highest utility given their skill set, while search on the employer side refers to firms trying to find the workers whose skill sets most closely matches the technological requirements of the post being offered. Even if workers accurately predict the skills that will be needed by the market and invest accordingly (and thus the supply and demand of skills in the population match), workers and firms can fail to instantly and optimally match when worker knowledge of the characteristics of their competition is imperfect, or the degree of sophistication of their reasoning does not allow them to solve the full multidimensional matching equilibrium so that workers only apply to the jobs that will hire them in equilibrium.

This paper theoretically models the worker-side search process when workers have complete information about the jobs on offer and the skills available in the population, but whose level of sophistication in their reasoning is limited. Limited rationality in this paper is taken to mean that workers find it impossible to solve for the optimal behavior of the other participants in the market, so they cannot directly account for it when deciding on their own behavior. This is operationalized in our model by assuming that although workers know their own skill endowments and the population distribution of skills, they apply to jobs as if they believe their competitors are drawn from the full population and not just those who would find it optimal to apply to a specific job. This implies that workers cannot accurately calculate the actual chances of obtaining a job to which they apply, creating congestion and the potential for inefficiency in the allocation.

Using survey data on worker skills and combining this with labor demand data on skills requirements from Colombia, we use simulated method of moments to recover the allocation mechanism of workers to jobs. In the estimation, we numerically solve the equilibrium allocation of skills to jobs and the time to job finding using data on cognitive, non-cognitive, and technical skills supplied and demanded (as announced in online job postings). After establishing that this allocation of workers to jobs is inefficient, in the sense that there are over-qualified workers in medium-skilled jobs and under-qualified workers who require significant skill upgrading in high-skill jobs, the paper introduces a counterfactual simulation in which firm training is subsidized (thereby reducing the cost of hiring workers with skills below the minimum threshold for a job) and a simulation in which long term unemployed receive training so that their skills can increase to a level that makes them eligible for jobs that otherwise they would be unable to occupy. All of the policy simulations improve the allocation of workers to jobs, but the policy of training the long-term unemployed is found to do the best job of approaching an efficient labor allocation.

The rest of this paper is structured as follows. Section 1.2 describes the state of the literature concerning the allocation of workers to jobs and skills mismatch. Section 1.3 lays out the theoretical model that describes the matching process which skills are multidimensional and workers have limited rationality. Section 1.4 describes the data used in the estimation while section 1.5 details the estimation procedure, results and policy simulations. Section 1.6 concludes.

1.2 Workers allocation and skills mismatch

Policymakers are concerned with the efficiency of the allocation of workers to jobs, in that one goal they have is to ensure that firms with specific skill needs hire workers with the appropriate set of skills. There is evidence that some workers do not possess the technical, cognitive, and socio-emotional skills to fill current vacancies, and those that do possess these skills may not be able to match the jobs for which they are best suited. The World Bank report on skills notes that 45% of the current employers worldwide claim that they can not fill entry-level jobs, while a similar share of working youth state that their jobs do not use their acquired skills (Almeida et al., 2012).

An inappropriate allocation of workers to jobs might have negative aggregate impacts. A review of the literature in Europe finds that it affects directly not only firms and overall productivity, but also the levels of unemployment and individual income, and could hamper innovation and growth. (Brunello and Wruuck, 2019).

Skills Mismatch Generally, skills mismatch happens when a worker's skills surpass or fall short of the employer's requirements. Skills mismatch affects both sides involved in the match formation. The adverse effects affect both the worker and the firm differently.

Existing evidence suggests that skills mismatch negatively affects workers' well-being. This includes dimensions related directly to workers' jobs, diminishing workers' wages (Badillo-Amador and Vila, 2013), increasing labor turnover, and decreasing job satisfaction (Quintini, n.d.). Vieira (2005), using panel data from Portugal between 1994 and 1999, provide evidence that over-qualification harms job satisfaction when measured. Recent studies highlight how skills mismatch can also negatively affect dimensions that are not related directly to the job, as when Shevchuk, Strebkov and Davis (2019) show that skills mismatch creates stress and lower personal confidence in a sample of British workers. Such findings pin down the effect that being over- or under-skilled has on work-life balance.

The other side of the match is also affected. Understanding the adverse effects of mismatch is straightforward since firm production is lower when they do not employ workers the proper skill levels. From the economic point of view, since the firm does not find the right inputs for the production function, the firm does not use the overall capacity. When skills are not assigned efficiently in the market, it implies reduced productivity in the firm. Using PIAAC data, Mcgowan and Andrews (2015b) provide evidence that the effect of skill mismatch in productivity is negative. The source of such an effect can be found in a less efficient resource allocation across firms. When the allocation of skills to the firm is inappropriate, workers adjust their skills to the job. As this can take time, firms can invest in on-the-job training to update specific and general skills, although such training is costly and can impact the firm's profit.

The cost to the firm of worker mismatch can be considerable regardless of whether or not the firm invests in the skills of its workforce. If it does not invest in the worker, the opportunity cost of using its technology suboptimally is a cost for the firm. If it invests in adapting the workers' skills to the firms' technology, it has to be paid by the firm. Framing our analysis in an environment of imperfect competition (Acemoglu and Pischke, 1999, 2003), the training leads to wage compression, and the firm can profit from training for the duration of the employment relationship. This is in line with the findings of Bassanini and Brunello (2008), who find that training is correlated with a smaller wage premium using the European Community Household Panel.

Finally, the relation between inefficient allocation and the role of multiple and heterogeneous skills is not straightforward. Skill multi-dimensionality, specifically how wages are set by the firm when dealing with multiple and heterogeneous skills, can play a role in how workers are allocated to jobs. When multiple skills exist, the cost associated with mismatch can be different for each side of the match and can vary by skill. The weights assigned to mismatch on each skill in the worker utility function can be completely different from their importance in the firm's production function. In the case in which only one side of the market directs its search, such differences in valuations could result in an inefficient allocation. To complicate the situation in case there are several skills, how the weights are distributed and if they are substitutes or complements will also change the allocation. A general solution to this problem, which is akin to the optimal transport problem (Villani, 2008), has been adapted to the context of labor economics by Lindenlaub (2017) using an aggregate matching function. However, it is still unclear how the allocation mechanism works or is modeled [2].

The matching function Within labor economics, the aggregate matching function plays a fundamental role. Conceptually, the matching function encompasses all the activities performed by agents to find each other and form a match that, after hiring, allows them to produce. The activities that compose its narrative are varied. For example, it includes agents actions such as worker searching, employer job posting, how they meet with each other, how they negotiate, how they screen each other, and how they form their matches[3].

The function also captures the possible frictions in the market: heterogeneity, congestion, or information asymmetries between the agents. Petrongolo and Pissarides (2001) discuss the micro-foundations on the matching function. In particular, *mismatch* and *coordination failures* are the relevant features that help explain the role of multiple and heterogeneous skills in the allocation process. In the presence of heterogeneity, the difference between the skills possessed and required would delay the time that it takes to match efficiently. *Mismatch* and *coordination failures* are interrelated since coordination failures could aggravate mismatch. Moreover, with each additional dimension along which mismatch could occur, the risk of coordination failure increases and one would expect that the overall match quality to get qualitatively worse. The

[2] Villani's transport map function represents all optimal couplings, while the optimal allocations in Lindenlaub (2017) are represented through the concept of a matching function which "*denotes the firm's productivity bundle that is matched to the worker* [...] *skill bundle*" (Lindenlaub, 2017).

[3] The survey by Petrongolo and Pissarides (2001), describes it as the function that summarizes the behavior of agents that "*place advertisements, read newspapers and magazines, go to employment agencies, and mobilize local networks that eventually bring them together into productive matches*".

matching function can also incorporate *ranking* behavior of firms. In an environment where the firm ranks applicants, some applicants might see no congestion when they are at the top of the rank.

The matching function approach can be applied to differentiated labor markets. From a spatial perspective, this would correspond to different matching functions for different local labor markets. When we consider that vacancies require different qualifications, tasks, skills, and technological requirements, one could transpose this approach in terms of occupational labor markets. According to recent evidence, Stops (2014) finds that markets are interdependent when using the notion of an *occupational topology*, for which the proximity between the occupational requirements are taken into consideration, using German data. Similar markets are affected by what happens in any specific market. Such a notion of similarity implies that workers search in markets where their skills are similar, and the cost of mismatch is not high enough.

1.3 Model

This section describes the allocation mechanism proposed in the form of a dynamic game. The environment of the model is composed of two types of agents, firms and workers, and skills are multidimensional with continuous support. The section first details the objective functions of the workers and firms, then discusses the structure of the game, how firms decide on their wage posting strategy, how workers deciding on their optimal job application strategy and, finally, how firms choose workers from the applicant pool and the remaining (unmatched) firms and workers restart the process.

1.3.1 Workers

Denote the population of workers I. A worker is an agent endowed with a set of skills that characterize its type, represented by a vector of size k. The skill bundle of worker i is described $\boldsymbol{s}^i \in \mathbb{R}^k$ with the elements denoted as $\boldsymbol{s}^i = (s_1^i, ..., s_k^i)$. A probability density function $g(\boldsymbol{s}) : \mathbb{R}^k \to \mathbb{R}$ characterizes the distribution of workers skills in the population, and skills are assumed to be untradeable among workers.

Workers are assumed to be risk neutral and they optimize their expected utility, which depends exclusively on their income. They are aware the employers have different skill requirements and that they are potentially in competition with other job seeks for any job to which they apply. Each job seeker must therefore form beliefs about the probability of employment for each type of job j to which they apply, which depend on the their own skills, the number of vacancies available for that type of job v^j and the amount of time since the start of the game z according to the function $p_z(v^j, \boldsymbol{s}^i)$.

1.3.2 Firms/Occupations

A firm is an agent, denoted j, characterized by two vectors: a vector of requirements $\boldsymbol{r}^j = (r_1^j, ..., r_l^j)$ and a vector that weights the importance of each requirement in the production technology $\boldsymbol{\omega}^j = (\omega_1^j, ..., \omega_l^j)$. In this paper, we will refer to firms and occupations interchangeably. Each firm is characterized by a technology that combines the requirements and worker skills in production $f(\mathbf{r}^j, \boldsymbol{\omega}^j, \boldsymbol{s}^i)$ *s.t.* $f : \boldsymbol{s} \times \boldsymbol{r} \times \boldsymbol{\omega} \to \mathbb{R}_+$, that produces a unique homogeneous good. A probability density function $\gamma(\boldsymbol{r}, \boldsymbol{\omega}) : \mathbb{R}^{2k} \to \mathbb{R}$ characterizes the joint distribution of firm skills requirements and importance weights. Firms requirements and importance weights are assumed to be inherent in the installed technology and thus unchangeable (in the short run).

In this setting, assume that the firm's production function links skill requirements, weights and worker endowments through a parametric specification which is a generalization of the firm specific human capital model of Lazear (2009). Production is reduced when the workers skill endowments are below requirements (with a weight of ω_k^j for skill k in firm j), but there is no extra production if worker skills exceed requirements. Specifically, we assume

$$q_j = f(\boldsymbol{s}^i, \boldsymbol{r}^j, \boldsymbol{\omega}^j) = \sum_{k=1}^{K} \omega_k^j \min\left\{\left(\frac{s_k^i}{r_k^j}\right), 1\right\} \tag{1.1}$$

with $\sum_{k=1}^{K} \omega_k^j = 1$.

Firms post wages and are assumed to ignore the externalities of their wage posting behavior on worker application decisions (i.e. they do not compete across occupations through their wage posting strategies) and only choose wages to maximize profits. Each occupation is exogenously endowed with a fixed number of vacancies v^j at the beginning of the game. A density function $h(v) : \mathbb{R} \to \mathbb{R}$ characterizes the distribution of vacancies in the economy.

1.3.3 The structure of the game

Before the game starts, each firm posts[4] a contract, specifying the wage that it commits to pay to each worker it hires (which represents the complete utilization of the skill endowments by the worker that fills the position[5]). Labor markets are assumed to be competitive, in that there is a market price p_k for each unit of skill k provided, so the posted wage reflects an aggregate of the amount of skills demanded by each firm.

Vacancies are filled by rounds[6]. Round z of hiring consists of workers applying simultaneously to firms followed by firms selecting workers from among their applicants. If there are available vacancies after a round is over, a new round starts until either all vacancies in the economy are filled or all unemployed workers find jobs. If a firm receives more applicants than vacancies, it will fill all of its vacancies in that round[7]. If a firm has more vacancies than applicants, it will only fill a portion of their vacancies, and the remaining vacancies become available for the next round.

Within a round, unmatched workers observe the posted wage w^j, the number of vacancies available v^j, and the technology used in the firm (skill requirements $\boldsymbol{r}^j$ with weights $\boldsymbol{\omega}^j$). Given this information, workers construct their beliefs about the probability of being hired, i.e. a subjective probability assessment. The construction of this probability has an information constraint: the worker knows his/her skills, but does not know the distribution of skills of the other job seekers who will be applying to each job[8]. Using the subjective probability, the worker calculates the hypothetical expected value of applying to firm j. Such information allows him to rank all the possibilities and choose the one with the largest expected utility.

After all workers have applied to jobs, firms rank the applicants and choose the "best" among the candidates. Our definition of the best is the applicant whose set of skills $\boldsymbol{s}^i$ best matches its production technology (skill requirements and importance vectors, $\boldsymbol{r}^j$ and $\boldsymbol{\omega}^j$). As noted above, skills in excess of the minimum level of requirements do not generate additional profits for firms, so they use other criteria to break ties. Note that this technological constraint and selection

[4]Unlike other papers with wage posting mechanisms (Burdett and Mortensen, 1998), the uncertainty in our model that underlies the non-degenerate posted wage distribution is derived from which workers apply for jobs and not from poaching risk.

[5]Individuals are assumed to be unable to split the use of their skill endowment across jobs.

[6]When we simulate the model we also use the word iteration to define a round.

[7]This implies that there is no optimal search behavior on the part of firms, i.e. firms cannot choose to leave some vacancies open in hopes of finding better matches in future rounds.

[8]The construction of this subjective probability will be detailed in section 1.3.5.

process implies that some skills could go unused, which could be a source of inefficiency for the economy.

To understand how the allocation occurs is necessary to explain in detail the wage setting mechanism, worker job applications, and firm selection. The next sections explain in detail each of them.

1.3.4 Wage setting

The wage posting problem for the firm is non trivial, since the firm does not know ex ante which workers will apply and it has specific technology (represented by the importance vector $\boldsymbol{\omega}$ and the requirement vector $\boldsymbol{r}$) that can make the value of a hire change with the characteristics of the person hired. Firms will target individuals whose skill sets at least meet the requirements of the posted job, so the problem is to find the wage that allows the firm to hire the selected individuals at the lowest cost, while still meeting production requirements for a given workforce size (the number of vacancies is exogenous). To find the unique wage posted, the firm proceeds as if it could choose the level of each specific skill individually, solving the following cost minimization problem:

$$\min_{\boldsymbol{s}} \Gamma = \min_{\boldsymbol{s}} \sum_{k=1}^{K} p_k s_k \qquad s.t. \;\; f(\boldsymbol{s}, \boldsymbol{r}^j, \boldsymbol{\omega}^j) \geq \bar{q}^j \tag{1.2}$$

The $k+1$ first order conditions for the problem in equation 1.2 are:

$$\begin{aligned} \frac{\partial L(\cdot)}{\partial s_1} &= 0 \implies p_1 = \lambda f'_{s_1}\left(\boldsymbol{s}, \boldsymbol{r}^j, \boldsymbol{\omega}^j\right)) \\ &(..) \\ \frac{\partial L(\cdot)}{\partial s_k} &= 0 \implies p_k = \lambda f'_{s_k}\left(\boldsymbol{s}, \boldsymbol{r}^j, \boldsymbol{\omega}^j\right)) \\ \frac{\partial L(\cdot)}{\partial \lambda} &= 0 \implies \bar{q} = f(\hat{\boldsymbol{s}}, \boldsymbol{r}^j, \boldsymbol{\omega}^j) \end{aligned} \tag{1.3}$$

Given that the firm can not compensate each skill by its individual price, the optimal wage that ensures the profit maximization under the output constraint (solves the dual problem) is therefore given by the sum of the individual skills compensation, and equal to the sum of the marginal product of each skill, weighted by the optimal amount of the skill and rescaled by the shadow price of satisfying the output constraint.

$$w^{j} = \sum_{1}^{k} p_{k} \hat{s}_{k}^{j} = \hat{\lambda}^{j} \sum_{1}^{k} \hat{s}_{k}^{j} f'_{s_{k}} \left(\hat{\boldsymbol{s}}, \boldsymbol{r}^{j}, \boldsymbol{\omega}^{j} \right) \tag{1.4}$$

The firm may, however, not receive any applicants whose available skill set meets the required skill level. In this case, training will be required to bring the individual's skill level up to the minimum requirements. The production function given in equation 1.1 reflects this cost as a lower net output for individuals whose skill level $\boldsymbol{s}$ is less than the required amount $\boldsymbol{r}^{j}$. The firm anticipates this training cost and reduces the offered wage so that, in expectation at the start of the game, the worker pays the full cost of the training. This implies that the final posted wage is reduced from the optimal wage by an amount reflecting the risk of having to make a suboptimal selection. The final wage posted ex-ante by firm j is thus defined as:

$$w^{j,p} = w^{j} - \delta^{j} \tag{1.5}$$

Where delta is the difference between the skill perfect match and the average skill in the economy.

$$\delta^{j} = \hat{\lambda}^{j} \sum_{1}^{k} \hat{s}_{k}^{j} \min \left[f'_{s_{k}} \left(\hat{\boldsymbol{s}}, \boldsymbol{r}^{j}, \boldsymbol{\omega}^{j} \right)) - \int f'_{s_{k}} (\boldsymbol{s}, \boldsymbol{r}, \boldsymbol{\omega}) dF(s_{k}), 0 \right] \tag{1.6}$$

This process has two relevant implications: first, each firm can value each skill differently. This can be seen in the fact that the optimal skill level $\hat{s}_{k}^{j}$ and the marginal productivity for a given skill $f'_{s_{k}}$ in two firms will be different when the technological parameters $\boldsymbol{r}$ and $\boldsymbol{\omega}$ differ. This implication is interesting since two persons with the same endowments can have different wages in different jobs, and represents a firm effect in the sense of Abowd et al. (1999). Second, using this setup, even skill supply in the unemployed population was homogeneous, there would be differences in income across jobs. Again, the differences come from the heterogeneity in production technologies across jobs. This last fact has an implication for policy making and planning in that it suggests that training alone cannot eliminate wage inequality (although it could eliminate the δ^{j} component of equation 1.5), as technological differences would drive wage dispersion even if the skill level of the entire workforce could be increased to the maximum possible skills endowment (through education and training). This is a direct implication of the wage posting assumption, in that firms are allowed to minimize cost through unilateral wage variation, as opposed to being pure price takers on the labor market.

1.3.5 Worker job application process

In this subsection we detail how workers select the firm to which they apply for a job from among the different firms with vacancies still open. In each round, unmatched workers observe the posted wage w^j, the number of vacancies available v^j, and the technology used (skill requirements $\boldsymbol{r}^j$ and weights of each skill $\boldsymbol{\omega}^j$) for each firm. With such information, the worker constructs a subjective probability, to calculate the expected value of applying to each job. One of the constraints that the worker faces is that he/she does not know the distribution of skills of applicants to each job (because it is too hard of a problem to solve), but only has information on the joint distribution of skills in the population.

$$p_z^{i,j}(v^j, \boldsymbol{s}^i) = \frac{v_z^j}{V_z} \times \frac{f(\boldsymbol{s}^i, \boldsymbol{r}^j, \boldsymbol{\omega}^j)}{\int\limits_{\boldsymbol{s}} f(\boldsymbol{s}, \boldsymbol{r}^j, \boldsymbol{\omega}^j) g(\boldsymbol{s}) d\boldsymbol{s}} \tag{1.7}$$

where V_z is the total number of vacancies in round z. The subjective probability of worker i being hired by firm j in round z is composed by two parts. The first reflects the individual's belief that the chances of getting higher increases if he/she applies to firms with higher share of vacancies with respect to other firms. The second part reflects the fact that the individual assumes his chances of receiving a particular job are related to his relative performance with respect to the market average. It is important to note that this probability reflects the worker's naiveté, in that he/she does not account for strategic behavior of other job seekers when considering the set of potential competitors for a job. Here, the individual assumes he potentially faces all unemployed workers for each job to which he applies, i.e; he/she uses $g(\boldsymbol{s})$ instead of the actual distribution of competitors in equation 1.7.

With these beliefs, the expected value for individual i of occupation j when the posted wage is $w^{j,p}$ is equal to:

$$E_z^{i,j}\left[w^{j,p}\right] = p_z^{i,j}(v^j, \boldsymbol{s}^i) w^{j,p} = \left(\frac{v_z^j}{V_z} \times \frac{f(\boldsymbol{s}^i, \boldsymbol{r}^j, \boldsymbol{\omega}^j)}{\int\limits_{\boldsymbol{s}} f(\boldsymbol{s}, \boldsymbol{r}^j, \boldsymbol{\omega}^j) g(\boldsymbol{s}) d\boldsymbol{s}} \right) \times w^{j,p} \tag{1.8}$$

The worker then compares the expected value of working for each firm and identifies the firm with the highest value, to which he/she applies.

1.3.6 Hiring from the applicant pool

The objective of the firm is to maximize its profit level. Given the collection of applicants $A_z^j \subset I$ for whom the job j maximizes their subjective expected utility in round z, the firm will rank candidates by their productivity (which depends on its skill requirements and importance vector), breaking ties among equally productive individuals using other criteria besides skills, and hire the v_z^j most productive candidates when the number of applicants $a_z^j = \text{card}\left(A_z^j\right) \geq v_z^j$. When there are fewer applicants than vacancies, all applicants are hired and $v_{z+1}^j = v_z^j - a_z^j$, i.e. firms would rather hire the entire available applicant pool than forego production and wait until the next round. When all of the vacancies are filled, job seekers in later rounds will have to search for a job in a different occupation.

Stable match Under this process, a hire thus represents a statically stable coalition[9] for which the job seeker i maximizes his utility by choosing firm j and firm j maximizes its profit by choosing the job seeker i among the candidates in round z. This match is a stable coalition since no occupation other than j can provide higher subjective expected utility (and thus induce a deviation from the worker) and no refused applicant can generate higher profits (and thus induce a deviation from the firm).

1.4 Data

For the empirical analysis, we take advantage of three different data sources. We use the information on both the supply and demand sides of the labor market in Colombia. Beyond any intrinsic interest its labor might possess, the fact that it has data sets with the information on multidimensional skills supply and demand needed for this analysis gives Colombia a distinct advantage. Three different datasets are used to characterize the Colombian labor market in terms of occupational structure, skill requirements by occupation, and skill endowments by the job seeker. This section describes the sources of information used and how they are combined for our estimation purposes.

[9] Recall that the model is a repeated static game with no dynamic considerations, i.e. individuals cannot decide to forego applying for jobs in a round in anticipation of better outcomes in subsequent rounds while firms cannot decide to intentionally leave vacancies open when there are enough candidates available in a round in the hopes of having a better applicant pool in later rounds. An extension of this model would explicitly explore this dynamically optimizing behavior.

STEP Survey 2012 - Colombia The World Bank's *Skills Toward Employment and Productivity* (STEP) survey is used to capture the supply structure of the Colombian labor market. The STEP program's activity in Colombia consists of a household survey with a complete module to assess the skill endowments of the working population. The survey aims to provide internationally comparable, quantitative data on employment-related skills in three domains: cognitive, non-cognitive, and technical skills. In order to select the skills to include in the measurement, the World Bank ran a survey to identify the skills by their relevance for employment and worker employability. The relevance of cognitive and non-cognitive skills coincides with findings from previous studies (Felstead et al., 2007; John and Srivastava, 1999). Those also have been relevant to explain differences in wage determination (Heckman et al., 2006).

The survey includes direct, test-based measures of cognitive skills (reading, writing, and numeracy) and self-declared measures of used to build indicators of socio-emotional skills (personality, behavior, and preferences). The survey samples the working-age population (between the ages of 15 and 64), active and inactive. Data collection for the survey began in March 2012, the results were processed and cleaned, and the final database was published officially in February 2013.

The survey has several modules relevant for this study. The first part collects household-level information, including basic roster information for all household members such as relationship to the household head, characteristics (academic and self-declared level of literacy), and labor market status (employed, unemployed or inactive). The second part contains information about household assets such as household size, building materials, facilities, appliances, number of books, and income sources. The later modules gather information on a randomly selected individual from the household. It covers education and training (quantity and type of education), health status, employment status, job skill requirements, personality and behavior measures, family background, and tests to measure cognitive skills (reading and numeracy) directly.

The methodology for collecting the data of the survey is based on a random representative sample of households in urban areas of the country. The information of the first module is collected by asking questions to the main household respondent concerning the income, size, and other characteristics of the household. The scores for reading and numeracy result from a test and are calculated based on the number of correct answers. We concentrate on the skills of the economically active population, according to the Colombian definition. This decision also implies inclusion of underage workers.

The main descriptive statistics for the underlying data are shown in table 1.1.

Table 1.1: STEP survey Colombia (2012) - summary statistics

Variable	**Mean**	**Std. Dev.**	**Min**	**Max**
Read	1.889	1.005	0	3
Write	1.223	0.838	0	3
Numeric	1.779	0.830	0	3
Interpersonal	2.053	1.174	0	3
Presentation	0.233	0.422	0	1
Supervise	0.338	0.473	0	1
Computer	1.340	1.354	0	3
Computer type	0.559	0.850	0	2
Drive	0.106	0.308	0	1
Repair	0.053	0.224	0	1
Operate	0.100	0.300	0	1
Think	1.289	1.176	0	3
Learn	1.820	1.207	0	3
Cognitive Challenge	1.557	0.940	0	3
Autonomy	2.015	0.861	0	3
Physical	1.901	1.013	0	3
Extroversion	3.047	0.640	1	4
Conscientiousness	3.326	0.498	1.67	4
Openness	3.238	0.513	1	4
Emotional Stability	2.543	0.726	1	4
Agreeableness	3.176	0.554	1.33	4
Grit	2.990	0.613	1	4
Decision making	3.118	0.599	1.25	4
Hostile bias	1.710	0.603	1	4
Risk	1.640	1.080	1	4
Gender	0.543	0.498	0	1
Age	34.96	13.16	15	64

Source: STEP survey Colombia (2012).

Vacancy database The Colombian vacancy dataset combines information from different sources. It contains information on posted vacancies from the major online job boards and public employment databases from Colombia. The Colombian Ministry of Labor collected the information during 2014 to monitor jobs and job requirements. The database used contains information on 1,892,219 vacancies.

The database was homogenized between the different sources in order to define variables according to common categories. Using the information from the title of the posting and its description, it also standardizes the sectoral and the occupational classification and retrieves other job-relevant information. The process for the homogenization of the information and how the database was constructed can be found in Guataquí et al. (2014). The final version of the database contains information on the geographical location, the wage, the sector of the firm that posts the job, the required occupation, the educational level required, and other information. Table 1.2 shows the variables and the description of data. One of the main concerns when using such data is that it could have selection issues. Rubio et al. (2015) presents the comparison between the database and the Colombian household survey, showing that the moments of the data coincide by groups of education, geographical location, and occupation. When we compare the vacancy database with the Colombian household survey, it has broader coverage.

Table 1.2: Variables - Colombian posted vacancy database 2014

Variable	**Description**
ID	Number of the job vacancy (Requisition ID in the data warehouse. This number is unique and the role is to identify the vacancy within the warehouse
Title	”Title” of the vacancy, i.e., the name given to the occupation. This provides information for categorization, clustering and the basis for splitting the identification of skills and competencies of occupations
Company Name	Company name
Sector	Sector of the company
Position	Area where the person performs
Total years of experience	Total experience required
Experience in the position offered	Total required experience in the position
City	Location of the vacancy
Professional title	Title of the person requesting the vacancy i.e. economist
Wage	Wage proposed for work
Level of education	Degree (i.e. Technical, University, Bachelor)
Type of contract	Type of contract
Language	Language requirements for the position
Number of vacancies per offer	Number of vacancies that the job posting offers.
Publication date	Date of publication of the vacancy
Expiring Date	Expiration date of the vacancy
Description	Description of the occupation
Occupation ISCO08	ISCO 08 classification of occupation
Occupation O*NET	O*NET classification of occupation

Variables from the vacancy database.

O*NET The O*NET taxonomy of occupations is used to quantify the demand for specific skills in each occupational job posting. O*NET is a database that contains detailed information for 965 occupations in the United States and was developed to replace the Dictionary of Occupational Titles (DOT). The project started in 1991, and the idea was to collect detailed information on the different aspects of occupations in order to be able to describe and analyze them with a quantitative approach. The methodology for collecting the information is based on continuous surveys of employers, research studies by sector and occupation, continuous revision of the estimates and updating of the information, and occupational analysis. The database has information on many occupational dimensions, including tasks, generalized work activities, knowledge, education and training, work styles, work context, skills, and abilities.

O*NET is a publicly accessible online database, so all the available dimensions of occupations can be accessed through the web. Table 1.3 presents the descriptive statistics for the occupation skills. The O*NET database skills are grouped into two broad categories: basic skills and cross-functional skills. The basic skills are the ones that facilitate the acquisition of knowledge, while the cross-functional skills are the ones that facilitate the performance of activities, and thereby the performance of specific tasks inherent to each occupation.

The O*NET skill content of the broad categories is divided into 35 skills. The basic skills are subdivided into content skills (reading comprehension, active listening, writing, speaking, mathematics, and science) and process skills (critical thinking, active learning, learning strategies, monitoring). The cross-functional skills are subdivided into social skills (social perceptiveness, coordination, persuasion, instructing, service orientation), complex problem solving, technical skills (operation analysis, technology design, equipment selection, installation, programming, operations monitoring, operations and control, equipment maintenance, troubleshooting, repairing, quality control), system skills (judgment and decision making, system analysis, system evaluation) and resource management skills (time management, management of financial resources, management of material resources, management of personnel resources).

The skill taxonomy of O*NET presupposes that skills are the characteristics that an individual has to have to perform a task well. Thus the presence of a certain skill level in an individual can make him perform the different activities associated with a particular occupation. An important implication of this assumption is that employers value skills in the hiring decision: they do not decide solely whether a worker can already perform a particular set of tasks, but rather whether the person possesses the skills needed to perform those tasks. The O*NET database characterizes skill requirements along two dimensions: the level, referring to the minimum amount of the skill level required by the employer to perform the tasks associated with a specific occupation, and the importance, referring to the relative mix of skills needed in order to perform an occupation well. The analysis undertaken here only uses 29 of the 35 listed skills since these were the only skills present in both the STEP survey and the O*NET taxonomy[10].

Data limitations

Using multiple sources of data has some limitations that we would like to consider up front. We highlight four main questions from among the multiple concerns of taking different data sets and using them together: How compatible is the use of O*NET in the case of Colombia? Is the data from the vacancies representative of the Colombian labor market? Can the different timings between demand and supply information affect the results? Is it possible to compare survey data with web scraped data?

Concerning the relevance of O*NET, it is important to note that there exists no comparable taxonomy relating skill requirements to occupations for Colombia. Using O*NET data for the analysis undertaken here requires the additional assumption that the relative skill content of occupations is comparable between Colombia and the United States. This does not imply that the same technologies are necessarily used in each country, which would be particularly unrealistic given the different levels of development. It does, however, require that technological differences across countries result in a homogeneous shift in skill requirements between countries and that the relative importance of each skill type for each occupation is preserved. Some advances in filling this data gap include Chaparro (2012), who has replicated data collection for some occupations of the IT sector (business process outsourcing and knowledge process outsourcing) in Colombia. The results show a shift in the levels of requirements, but the variation in requirements across occupations is comparable to what is found in the US data.

Regarding the vacancy data, which is scraped from the internet and combined with administrative records, one may be concerned about how representative the data is with respect to the universe it represents. In this case, we should examine the extent to which the collected vacancy data

[10]The unused skills were related to resource management, a skill type not quantified by the STEP survey.

Table 1.3: Skill Requirements: Descriptive Statistics

Variable		Mean	Std. Dev.	Min	Max
Active Learning	Importance	50.83	12.54	19	78
	Level	44.09	11.09	16	80
Active Listening	Importance	64.34	11.05	35	97
	Level	49.09	9.42	29	84
Critical Thinking	Importance	61.95	10.81	31	94
	Level	49.80	9.00	29	80
Learning Strategies	Importance	42.46	14.46	3	85
	Level	39.17	12.08	0	77
Mathematics	Importance	36.97	14.26	0	100
	Level	34.68	13.40	0	87
Monitoring	Importance	57.17	8.99	31	85
	Level	47.40	8.21	27	70
Reading Comprehension	Importance	59.54	13.80	25	97
	Level	50.38	12.09	20	86
Science	Importance	23.13	21.57	0	91
	Level	19.71	19.99	0	84
Speaking	Importance	62.90	12.27	31	94
	Level	47.85	10.49	25	77
Writing	Importance	52.37	15.28	10	97
	Level	45.54	12.31	7	75
Coordination	Importance	53.01	9.22	25	81
	Level	44.71	7.16	27	68
Instructing Others	Importance	44.86	15.04	0	91
	Level	40.71	11.30	0	70
Negotiation	Importance	40.20	11.79	13	91
	Level	35.91	9.61	12	71
Persuasion	Importance	43.46	11.75	16	81
	Level	39.02	9.77	14	68
Service Orientation	Importance	47.74	13.22	0	91
	Level	40.08	9.05	2	73

Source: O*NET.

Table 1.3 (continued)

Variable		**Mean**	**Std. Dev.**	**Min**	**Max**
Social Perception	Importance	54.30	10.96	0	94
	Level	43.31	9.59	5	84
Complex problem Solving	Importance	53.91	11.38	22	81
	Level	43.72	8.94	21	73
Equipment Maintenance	Importance	18.32	21.64	0	81
	Level	15.28	18.32	0	68
Equipment Selection	Importance	18.45	17.77	0	75
	Level	14.67	15.38	0	57
Installation	Importance	6.14	12.17	0	78
	Level	4.65	10.86	0	60
Operations and control	Importance	30.94	22.46	0	97
	Level	25.40	18.22	0	80
Operations and monitoring	Importance	39.58	19.16	0	94
	Level	32.42	14.51	0	70
Operation Analysis	Importance	27.12	15.98	0	75
	Level	24.67	16.02	0	73
Programming	Importance	12.34	11.81	0	88
	Level	9.484	11.41	0	68
Quality control	Importance	35.28	17.34	0	78
	Level	30.56	14.99	0	57
Repairing	Importance	17.49	21.80	0	85
	Level	14.75	18.47	0	61
Tech Design	Importance	15.93	9.94	0	60
	Level	12.72	10.51	0	60
Troubleshooting	Importance	26.16	19.67	0	81
	Level	22.34	16.78	0	75
Judgment Decision Making	Importance	55.50	10.28	25	85
	Level	44.51	9.250	23	71

Source: O*NET.

is representative of the Colombian labor market, and in particular the posted wages for each occupation. To calculate the wages for each occupation, we use only the vacancies that refer to full-time employment and the reported monthly wage, thereby eliminating concerns about part-time work. Rubio (2020) recently examined the internal and external validity of these data. He finds that the vacancy database does not represent agricultural and public sector employment well, as it covers mainly the urban labor market (as does the STEP data). The data is not representative of self-employment either, which is closely related to informality in Colombia[11]. For wage employment, it is unlikely that informal employers use the search channels collected in this data. Nevertheless, informal employers tend to offer lower paid jobs than formal employers due to the latter's need to cover payroll taxes (Perry et al., 2007). The lack of coverage of informal wage employers would be problematic if informality were to be concentrated in some particular occupations, but this is not the case in Colombia, where informality is present over all levels of labor income and occupations. Moreover, when Rubio (2020) compares the vacancy data with the Colombian household survey at the occupational level, he finds that it correctly represents the wages and the occupational distribution.

Table A.1 presents an additional check of the representativity of the data. This table shows the channels used for a job search in Colombia, calculating them from the Colombian household survey (GEIH). Since the vacancy database comprises public and private job boards, employment services, and headhunters (that use in general online job boards), our data covers more than 50% of the channels used for job searching. The use of informal networks for job finding is common, even in developed countries (Montgomery, 1991), but there is no consensus as to whether the jobs found through networks are qualitatively different from those found through formal channels (see also Margolis and Simonnet (2002)).

Another concern arises from the fact that the data collected do not correspond to the same year. As figure A.2 shows, however, the distribution of occupations in the Colombian economy was remarkably stable across the set of years from which the data are drawn, suggesting that there are unlikely to have been major shifts in the types of jobs offered from year to year. Finally, we consider the challenge to use two datasets, one a survey with sample weights that provides information on labor supply, and the other a database of job offers scraped from

[11] Informality in Colombia represents a large share of overall employment, although much of it is self-employment (Perry et al., 2007).

the web. In order to make the scraped data comparable to the effective demand (hires), we designed a reweighting scheme so that the collected job offer distribution across occupations, once weighted, was representative of recent hires (the structure of the vacancy data can be seen in table 1.4).

Table 1.4: Structure of the vacancy database

O*NET occupation	**Occupation Title**	**Average wage**	**Number of vacancies**	**Weight**
41-2031	Retail Salespersons	843525	324494	4.552409
43-4051	Customer Service Representatives	856134	130709	4.77578
41-9011	Demonstrators and Product Promoters	734387	92029	5.392699
43-5081	Stock Clerks	749143	63231	5.573521
51-9198	Helpers - Production Workers	745251	47480	4.96724
15-1152	Computer Network Support Specialists	1246871	33200	4.882156
41-2011	Cashiers	821517	32066	3.648313
15-1131	Computer Programmers	1121887	30627	4.594952
43-3031	Bookkeeping, Accounting, and Auditing Clerks	922857	24903	4.935306
43-5021	Couriers and Messengers	743303	18867	5.924564
51-6052	Tailors, Dressmakers, and Custom Sewers	720115	17817	4.733252

Source: Vacancy database.
Note: Wages are expressed monthly in Colombian pesos. Top 10 occupations sorted by the number of vacancies.

1.5 Estimation

Since solving for the equilibrium allocation of workers to occupations is analytically intractable, numerical techniques are used. What follows is a brief description of the algorithm used to simulate the allocation process.

- *Construction a similarity matrix.* For the demand side of the market, we merge information of the vacancy and the skills requirements of O*NET. For the supply side we use information on the STEP survey. Using the functional specification of equation 1.1 along with the information from the demand side and supply side, we define a measure of similarity for each observation and each occupation, that is used to compute the subjective probability. The index synthesizes the value of each worker type in each occupation, combining the different skill dimensions as specified by the $\boldsymbol{r}^j$ and $\boldsymbol{\omega}^j$ vectors for the occupation and the $\boldsymbol{s}^i$ vector for the worker type.

- *Estimation of a tiebreaker index.* In the case where multiple individuals have equal values of the similarity index, other worker characteristics that can affect the job search process and the firm's selection decision are used to break the tie. This tiebreaking index is calculated by estimating a probit model of the probability of employment as a function of the average skill-based similarity index for the individual, the individual's demographic characteristics and other job-related characteristics. Table A.3 presents the descriptive statistics of the variables used in the construction of the tiebreaker index, and table A.4 presents the estimated coefficients for the probit model and the associated marginal effects.

- *Iteration.* For each round of the model presented, we simulate the model from the worker side, and the firm side.

- *Stopping.* Stop the algorithm when the implied unemployment rate based on unmatched individuals is the same as that in the overall economy.

One of the main advantages of this model is that counterfactual policies can easily be simulated, in particular active labor market policies that affect the level of skills[12]. Three main counterfactual policies are of interest:

- The first policy is training in the firm, where job seekers are hired by the firm even if their skill endowment is under the desirable level, and the firm pays the cost of the training without passing it on to workers in the form of a lower posted wage.

- The second policy is a direct subsidy to firms for technological investment. This is formalized in the context of the model as a reduction in the minimal skill requirements to perform a job well. The simulated policy lowers skill requirements for all firms, extrapolating from any firm decision to take up the policy.

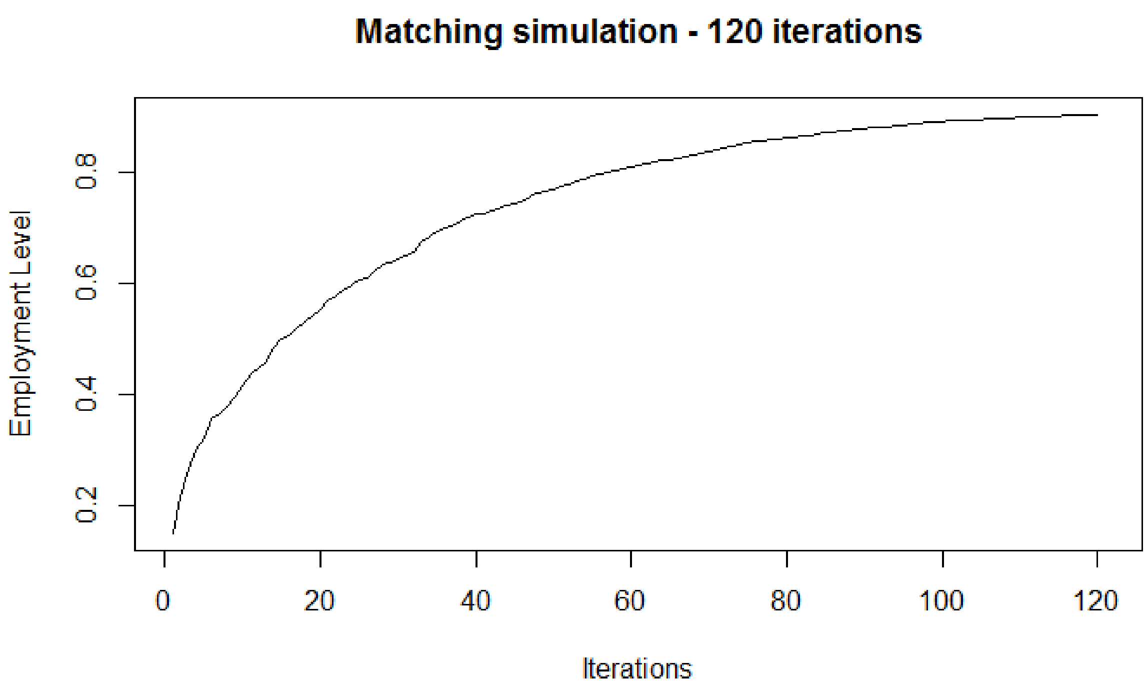
- The third policy is an increase in spending on training for the unemployed. Given that we identify the individuals that remain in unemployment in the simulation, we increase the skills for those and rerun the simulation, comparing the outcome.

[12] For example, a program to encourage training in the firm, that in Colombia was approved after 2014, has been implemented in 2015 with the name UVAES. Under this plan, firms provide spaces for learning the tasks required by the company, and the national vocational education training institute - SENA - certifies the competencies of the set of skills learned in the firm for future recognition.

Results of the simulation of the model

Numerical resolution of the equilibrium allocation for the model was attained after 120 iterations (see figure 1.1). By design, the simulation matches perfectly the unemployment rate. Table 1.5 presents the allocation results of the simulation. It also does a good job reproducing the observed allocation, since it reproduces under-qualification and over-qualification for low and medium skill occupations. Nevertheless, the model over estimates the under-qualification for high skill occupations. In order to have alternative measures to evaluate the fit of the simulated allocation mechanism, we also examine misallocation based on education (Allen and De Weert, 2007). The results are presented in table A.5.

Figure 1.1: Convergence of the simulation



Clearly, in both cases, the model generates an equilibrium allocation of workers to jobs that, although individually optimal at the point in time the match occurs, is socially inefficient. Workers whose average skill level is high are regularly allocated to jobs that only need a low level skills, while some high-skilled occupations undertake costly investment to improve the skills of their low-skilled recruits.

The main reason for this result comes from the means by which workers calculate the

Table 1.5: Fit of the allocation (simulation vs. observed)

Job Skill type	Worker Skill Type	Observed Match	Under- qualification	Over qualification	Output Simulation - Worker Skill level based on assessment
High Skill Occupation	**High skill Worker**	0.559			0.2902
	Medium Skill Worker	0.338	0.441		0.3426
	Low skill Worker	0.103			0.3674
Medium Skill Occupation	**High skill Worker**	0.288		0.288	0.3116
	Medium Skill Worker	0.361			0.3790
	Low skill Worker	0.351	0.351		0.3093
Low skill Occupation	**High skill Worker**	0.275		0.677	0.3746
	Medium Skill Worker	0.402			0.3255
	Low skill Worker	0.324			0.2999

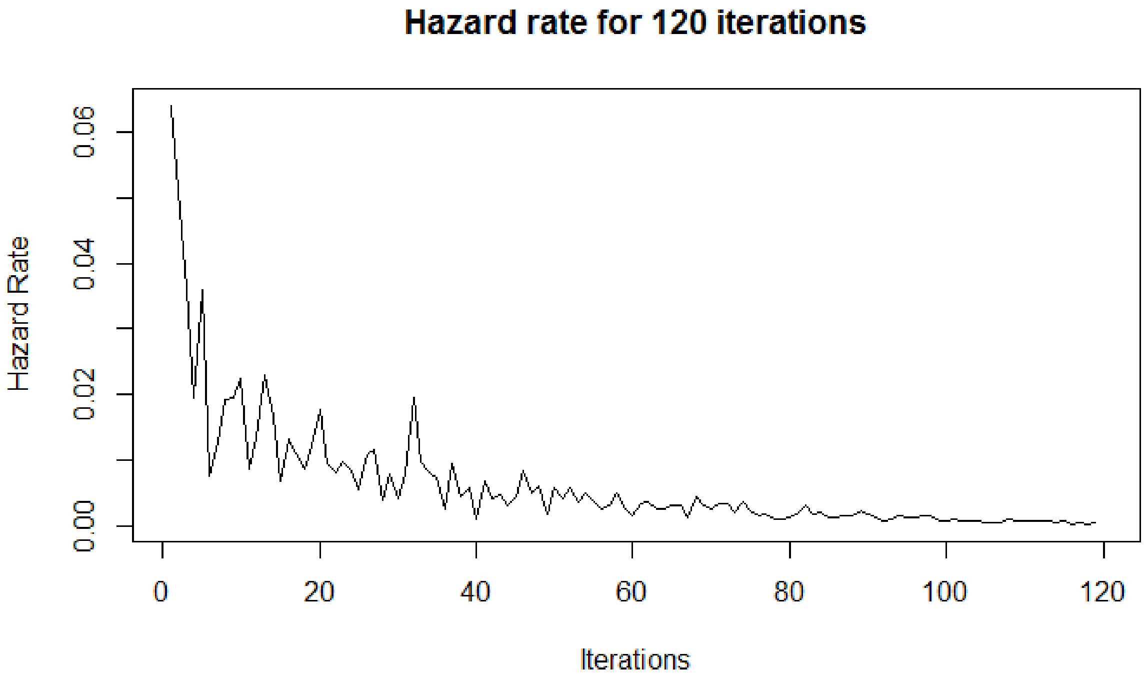
Note: The table compares the fit of the simulation against the observed allocation in the STEP data using the skills as reference. Individuals and jobs were categorize in high three skill intensity categories using the 33th and 66th deciles of the distribution of requirements and skills.

subjective probability of obtaining a job. Although being more skilled than average increases the likelihood of hiring in any job and occupations with higher skill requirements pay more, it appears that the availability of vacancies is a more dominant factor in determining an individual's occupational choice. If the amount of vacancies for a particular occupation is really large in comparison to other occupations, the results suggest that this increased likelihood of finding a job outweighs the variation in wages or any job-specific productivity advantage an individual might have. In this data analyzed here, nearly 10% of all vacancies are in retail or related occupations. The easy availability of such jobs make these occupations particularly appealing.

Another interesting aspect of the model is its ability to produce a declining hazard in job finding rates (figure 1.2). This effect is due in part to the fact that the most common vacancies are the ones to which there are more applications, leading many workers to match to jobs quite rapidly. Once those vacancies are filled, the opportunities for employment in the remaining occupations are less unbalanced across occupations in which case wage variation and relative productivity advantages both have the potential to become more important. The fact that workers still find jobs at a relatively high rate from iterations 10-30 may suggest that, once the easy-to-find jobs are gone, workers do sort to the types of jobs where they have a productivity advantage. As the number of simulations increases, however, wage variation becomes more important. The relatively high paying remaining jobs have few vacancies but will be oversubscribed, leading to even lower hazard rates into employment.

Analyzing the actual matching process, the most skilled individuals have the shortest unemployment

Figure 1.2: Variation of the hazard into employment



spells, while low skilled workers have the longest spells. High skilled workers apply to, and are matched with, the very common medium and low skilled vacancies first. Medium and low skilled individuals match in the middle and latter iterations of the game, once the competition from high-skilled workers clears out. This is a direct implication of of the limited rationality of the job seekers. If high skilled workers considered that they would beat all other workers whose skill sets are less well adapted for the high productivity jobs, then they would not apply for the medium skilled jobs. Likewise, medium skilled workers would see the competition from high skilled workers evaporate for medium productivity jobs, and would thus apply there, leaving the low productivity jobs to the low skilled workers.

As seen in table 1.6, all three of the policy simulations substantially reduce misallocation of workers to jobs. In all three cases, the share of high-skilled workers in high-skilled jobs, medium-skilled workers in medium-skilled jobs and low-skilled workers in low-skilled jobs increases relative to the reference scenario. Nevertheless, in all cases there remains a large share of jobs that remain held by workers with inappropriate qualifications. Even though all of these policies operate on other dimensions of the application and selection process than the simple vacancy rate (which is the main source of misallocation in the reference scenario), the assumption that

Table 1.6: Results of policy simulations

		Reference Scenario	Training in the Firm Without Overqualification	Technological change	Training for unemployed
High Skill Job	**High Skill Worker**	29.02	52.05	52.04	54.77
	Medium Skill Worker	34.23	31.09	31.09	30.92
	Low Skill Worker	36.74	16.86	16.87	14.32
Medium Skill Job	**High Skill Worker**	31.16	34.67	34.67	31.90
	Medium Skill Worker	37.90	39.35	39.35	41.01
	Low Skill Worker	30.93	25.98	25.98	27.09
Low Skill Job	**High Skill Worker**	37.46	20.89	20.89	22.54
	Medium Skill Worker	32.55	31.29	31.29	30.29
	Low Skill Worker	29.99	47.82	47.82	47.16

Note: The table compares the base scenario, and the policy simulations.

over-qualified workers do not generate extra output implies that the tie-breaking mechanism is invoked increasingly frequently, especially in the technological change and training simulations. Despite the similarity index entering the tie-breaker function, the presence of demographic characteristics unrelated to similarity that improve one's chances of being hired can lead to inappropriately-skilled being hired for a position.

Among the three policies evaluated, training residual unemployed workers (who can be thought of as long-term unemployed) does the best job at reducing the allocative inefficiency of the labor market. This occurs because these workers, initially low skilled, become medium-skilled and increasingly apply to medium skilled jobs. As there will be more workers who enter the tie-breaker setting (the high-skilled workers and previous medium-skilled workers), the likelihood of a medium-skilled worker getting a medium skilled job increases. The high-skilled workers who do not get these jobs then apply to the remaining vacancies for high-skilled jobs, leading to a higher share of these workers being hired in high-skilled jobs.

The policy of eliminating the wage penalty of hiring an under-qualified worker is the second-most effective. This is mainly because the wage gain is biggest for the high-skill jobs (where the training requirements would be higher), and the higher wage has a bigger impact in attracting high-skilled individuals to high-skilled jobs than it does for low-skilled individuals (for whom their subjective probability of being hired will be lower due to their position in the skills distribution). As higher skilled workers apply for high-skilled jobs, they leave vacancies at lower skilled jobs open for lower skilled individuals, thereby further reducing the misallocation.

The last policy, introducing technological change that results in a decrease in skill requirements, has similar effects for lower skilled jobs but is less effective at getting high-skilled workers into high-skilled occupations. This is likely because the requirements shift, as simulated, affects the whole requirements distribution, making all jobs more accessible to everyone. Since high-skill workers believe that they will beat a randomly drawn competitor with high or low requirements, the vacancy effect continues to dominate their decision making. However, more workers will be tied at high-skilled jobs than without the policy, increasing the likelihood that the firm resorts to alternative methods for ranking candidates and hiring lower-skilled workers when high-skilled workers are available.

1.6 Conclusion

This paper has presented a model in which skills are multidimensional and skills mismatch occurs as a result of optimizing behavior of workers and firms. Workers apply for jobs in a way that maximizes their subjective expected utility, although they behave in a naive manner by not taking into account the strategic job application decisions of their competitors. Firms post wages for a given number of vacancies and select among applicants based (initially) on the appropriateness of their skill sets for the job on offer in a manner that maximizes profits. Numerically solving for the equilibrium allocation shows that although each agent behaves optimally, the socially optimal allocation of workers to jobs is not reached, primarily due to the naive behavior of workers. This result helps explain why job search assistance is among the most effective types of active labor market program, as it allows workers to better assess their chances of finding a job and to better target vacancies for job applications.

Although the model studied here relies on limited rationality and relatively straightforward behavior on behalf of all labor market participants, it does a reasonably good job of reproducing certain stylized facts (higher wages for more skill-intensive occupations, decreasing hazard rates into employment, longer unemployment durations for less skilled workers). Moreover, the model specification makes implementation of skills-based policy simulations straightforward. The main drawback of the model, however, is the lack of an analytical solution to the equilibrium allocation of workers to firms. Introducing fully rational agents could also extend the model, although it is less clear whether such an extension would render the model more realistic. On the data side, estimating the model for Colombia has some major advantages (availability of data), but also so disadvantages (coverage of job offers). Nevertheless, this paper has shown that one can gain interesting insights into labor market behavior and outcomes even when introducing the complexity of multidimensional skills, and many extensions of the model can be envisioned to both make it more realistic and more easy to solve analytically.

Appendix:

A.1 Additional tables and figures

Figure A.1: **Vacancy distribution in Colombia**

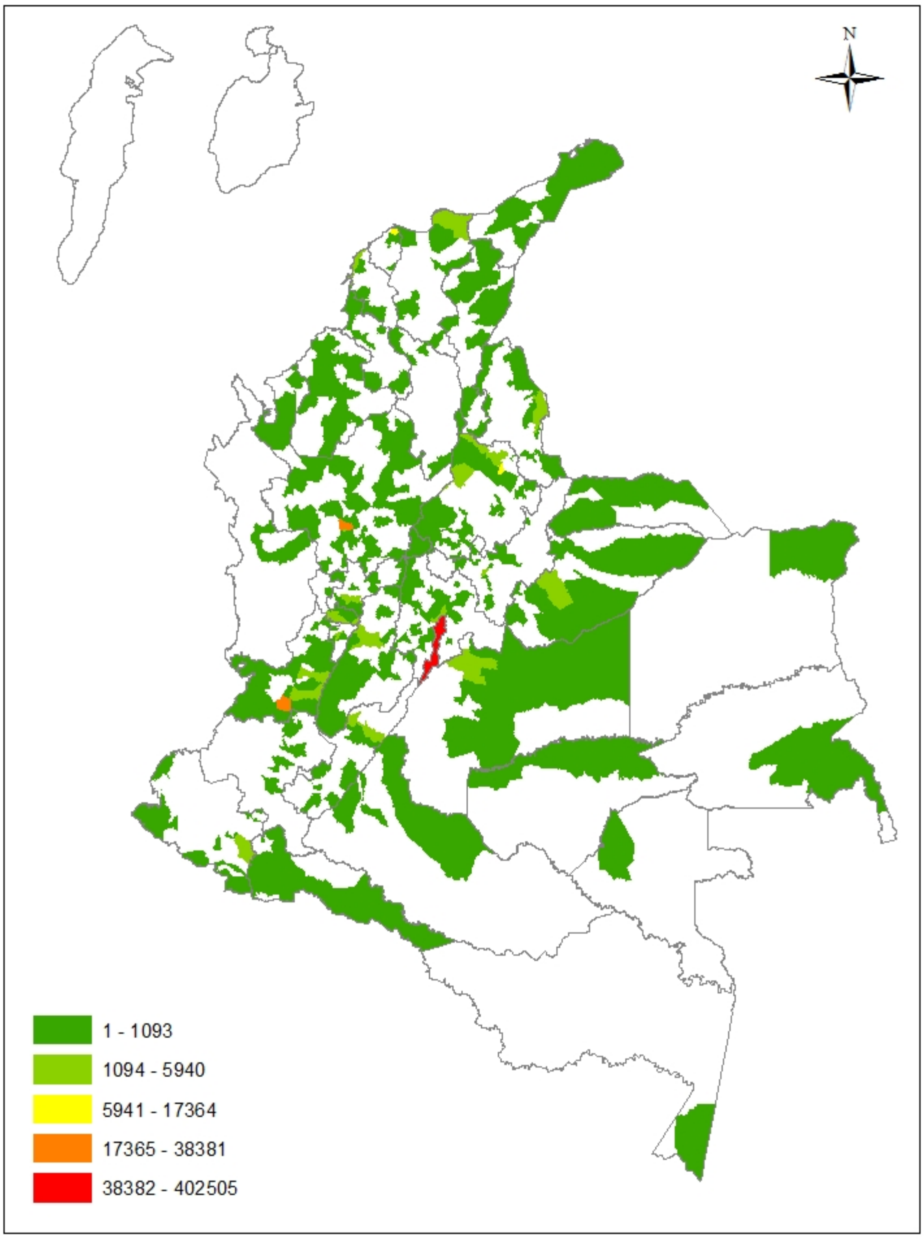
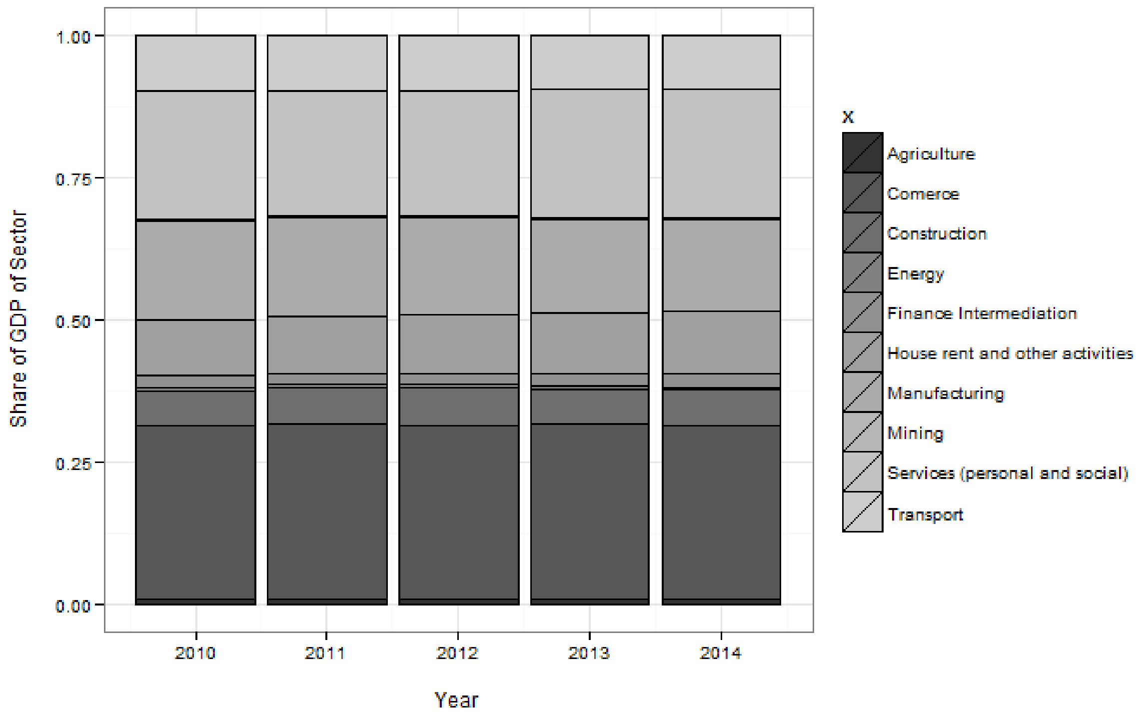


Table A.1: Job search channels

Means of searching	Industry	Trade	Services
Informal networks	23.80%	26.90%	18.00%
Databases / own records	17.40%	18.30%	18.70%
Web job boards	*16.70%*	*13.70%*	*20.20%*
National Apprenticeship Service (SENA) - Public Employment Service	*12.30%*	*13.70%*	*10.40%*
Advertising on media	*12.20%*	*10.80%*	*10.40%*
Job Boards of Universities and other organizations	*8.40%*	*6.90%*	*10.80%*
Headhunters / Job boards	*6.70%*	*6.50%*	*6.70%*
Contact with other educational institutions	2.10%	2.70%	4.00%
Job Fairs	0.50%	0.50%	0.80%

Source: Colombian household survey (GEIH) - 2014.
Note: The means of searching in bold text are covered by the vacancies database.

Figure A.2: **Occupation by sector 2010 - 2014**



Source: DANE - Household Survey (GEIH)

Table A.2: Employment structure by activity

	2012	**2013**	**2014**
Agriculture	0.92%	0.88%	0.80%
Mining	0.32%	0.32%	0.33%
Manufacturing	16.98%	16.28%	16.10%
Energy	0.51%	0.53%	0.55%
Construction	6.59%	6.19%	6.29%
Comerce	30.55%	30.69%	30.58%
Transport	9.69%	9.48%	9.29%
Finance Intermediation	2.02%	2.21%	2.18%
House rent and other activities	10.29%	10.79%	11.17%
Services (personal and social)	22.15%	22.61%	22.71%

Source: Colombian household survey (GEIH) - 2014.

Table A.3: Summary statistics for the variables input of the tiebreaker index

	Average		St. Deviation		t-test
	Unemployed	Employed	Unemployed	Employed	p-value
Average skills ind.	573.54	671.77	100.55	67.92	0.000
Age	31.02	36.27	12.80	12.13	0.000
Years education	10.37	10.58	3.31	3.63	0.347
Female	0.68	0.50	0.47	0.50	0.000

Note: The table presents the summary statistics for the variables used in the calculation of tie breaking index. We present the mean and standard deviation both for employed and unemployed workers in the sample. The last column present the *p-value* for the t-test on means between the employed and unemployed sample. The only variable for which the means are not statistically different is the years of educations.

Table A.4: Probit coefficients and marginal effects for the tie breaking model

	Probit	$\frac{dy}{dx}$
Intercept	-6.02^{***}	
	(0.56)	
Average skills ind.	0.01^{***}	0.0015^{***}
	(0.00)	(0.0001)
Age	0.09^{***}	0.0162^{***}
	(0.02)	(0.0034)
Age^2	-0.00^{***}	-0.0002^{***}
	(0.00)	(0.0000)
Years of education	0.06	0.0097
	(0.05)	(0.0095)
(Years education)^2	-0.01	-0.0009
	(0.00)	(0.0005)
Female	-0.18^{*}	-0.0314^{*}
	(0.08)	(0.0147)
AIC	1167.75	1167.7464
BIC	1206.52	1206.5196
Log Likelihood	-576.87	-576.8732
Deviance	1153.75	1153.7464
Num. obs.	1880	1880

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Note: The first column in the table presents the estimated coefficients of the model for the tie breaking rule. We also present the estimated marginal effects on the means.

Table A.5: Fit of the allocation (simulation vs. observed)

Job Skill type	**Worker Skill Type**	**Observed Match**	**Under-qualification**	**Over qualification**	**Output Simulation - Worker Skill level based on assessment**
High Skill Occupation	**High skill Worker**	0.559			0.2902
	Medium Skill Worker	0.338	0.441		0.3426
	Low skill Worker	0.103			0.3674
Medium Skill Occupation	**High skill Worker**	0.288		0.288	0.3116
	Medium Skill Worker	0.361			0.3790
	Low skill Worker	0.351	0.351		0.3093
Low skill Occupation	**High skill Worker**	0.275		0.677	0.3746
	Medium Skill Worker	0.402			0.3255
	Low skill Worker	0.324			0.2999

Note: The table compares the fit of the simulation against the observed allocation in the STEP data. Individuals and jobs were categorize in high three skill intensity categories using the 33th and 66th deciles of the distribution of requirements and years of education.

Chapter 2

Wage posting and multidimensional skills mismatch

2.1 Introduction

Policy makers are often concerned about the possible losses of productivity for firms that do not find a suitable workforce, since this slows firms' growth and competitiveness. Contemporaneously, there have been increasing concerns about workers' well-being and dissatisfaction due to over-qualification and under-qualification. These two phenomena are just two sides of the same coin and are the consequences of mismatch in the labor market. An old question then re-arises: "*Who matches with whom?*". This paper provides a new answer to this old question by providing a microeconomic explanation based on worker and firm behaviour when skills are multidimensional, which allow us to understand how workers and firms sort in an environment where mismatches are the norm.

This paper makes the following contributions. First, the paper contributes to the (limited) literature on multidimensional search literature (Lindenlaub and Postel-Vinay, 2016; Lise and Postel-Vinay, 2015; Tan, 2017; Lazear, 2009), proposing a microfoundation for understanding how matching and sorting occurs in the case of multiple and heterogeneous skills. Moreover, this paper contributes to the literature of multidimensional skills, providing a theoretical evidence for how wages depend on multidimensional skills and requirements (Deming, 2017; Deming and Kahn, 2018; Speer, 2017). When we take the model to the data, we also find that workers suffer greater disutility for mismatches in the non cognitive skill dimension. Firms, on the other hand, value better matches on the cognitive skill dimension more highly. Lastly, the paper provides a technical contribution by introducing vectorial calculus notation into a random search model, allowing the use of a multidimensional Leibniz rule. This permits a clear interpretation of the wage determination conditions. Wage determination is the main strategic decision of the firm. Changes in the posted wage will induce changes in the composition of *the applicants pool*, thereby changing the size and composition of this set. In equilibrium we know who matches whom, characterizing a set of acceptable jobs for each worker and a unique *applicants pool* for each firm. One novelty of our takeaway, is that sorting is not homogeneous in the economy. Matching and sorting are distribution dependent, and fully characterized for each point in the supports of the skills endowments and skills requirements distributions.

Matching and sorting have long been of interest to economists. Two sided matching markets (Gale and Shapley, 1962) and their implications for on sorting and stability have been well studied under various settings. In labor economics, multidimensional matching markets and sorting has been tackled only very recently (Galichon, 2018; Chiappori et al., 2016). Using the notion of multidimensional skill endowments and requirements, based on the results of the optimal transport problem (Villani, 2008), Lindenlaub (2017) presents how the production technology determines sorting in the presence of multiple dimensions, and under which specifications the equilibrium allocation and wages exist. These compelling results were rapidly incorporated into the random search model (Mortensen and Pissarides, 1994) to characterize a labor market equilibrium in multiple heterogeneous dimensions by Lindenlaub and Postel-Vinay (2016). The paper is comprehensive on the possible wage mechanisms. It characterizes the equilibrium using a whole range of wage settings: sequential auction, sequential auction with bargaining, Nash bargaining (surplus splitting), and wage posting as in Burdett and Mortensen (1998). Both papers have two things in common: sorting in both papers results from some technical characteristics of the production function and the distributions of the multidimensional vectors. In such an environment, wages and mobility depend on the surplus function (match productivity adjusted by the individual's outside option). Sorting and its sign are then an endogenous result independent of workers' and firms' preferences, a practical and powerful result.

One of the challenges that policy makers face is how to use economic models for real life policy analysis. For example, generalizations like Nash wage bargaining, in which the wage is a function of both firm requirements and worker endowments, imply that potentially all workers can perform any job. In many cases this is not sufficient if we want to know the horizontal employability of workers (how they change work across occupations or sectors). This paper aims to construct a model in which for any point in the distribution of skills endowments, one can determine a specific set of jobs that are compatible with that vector of skills. This feature is very informative in policy analysis for the case of skills retraining, workforce adaptation, and other policies aimed at reallocating the workforce. Equivalently for firms, the model allows us to characterize the segment of workers who would be willing to work for the firm at any given wage. The model then presents a new perspective for analyzing the equilibrium in labor market beyond the individual relationship, considering the *segment* of each joint distribution specified along all the dimensions that are important to workers.

Although the resulting equilibrium can be expressed in a surprisingly parsimonious fashion , it takes a high-level view of the essential heterogeneity of agents on both sides of the market and the implications that it has for decision making. The conditions imposed on the production function duly determine sorting behavior and mismatch. In contrast to previous work, this paper deals with a setting in which sorting is not homogeneous in the economy, and in which the distribution of attributes and workers' and firms' preferences define the mismatch outcome at the individual level. This paper models the microeconomic mechanism underlying mismatch in a setting that allows us to understand matching and sorting and provide an economic narrative to understand what is behind the technical properties of the equilibrium solution.

The value of such an approach is twofold: first, it complements our understanding of matching and sorting. It unveils the mechanisms that underlie technical assumptions, creating a narrative that allows one to gain a better understanding of such phenomena. Second, it provides a framework to consider sorting and matching at a more granular level, in which mismatch is a microeconomic feature.

The paper relies on a theoretical model to develop such a narrative. The starting point is the random search model of Mortensen and Pissarides (1994). In this setting, we assume that workers are willing to accept an offer only if the posted wage compensates for the utility cost of mismatch. Firms know this, and along with full information on the technology and worker type distributions, they determine the set of workers willing to accept a job offer for a given wage level. This determines a set we call *the applicants pool*. The worker's choice behaviour introduces heterogeneity in reservation wages through mismatch, in which the existence of a cost that affects utility makes it undesirable to work in a job where skills are distant from requirements. The previous literature on monopsony has used a similar mechanism, as when Bhaskar et al. (2002) and Manning (2003) introduced commuting cost into preferences, creating heterogeneity depending on workers' distance to the firm. In their models, firms compensate the average worker, so a worker who travels very long distances is not adequately compensated and decides to turn down the work. This paper's approach is conceptually similar, since the firm posts a single wage that maximizes the expected profit over workers (endogenizing who would be willing to accept the wage offer), so workers who have the highest mismatch cost in this set are worst off. As in Manning's article, this gives market power to firms and allows them to segment the market.

Firms internalize workers' behavior and use this information for optimal wage determination. Changes in the posted wage will induce changes in the composition of *the applicants pool*, thereby changing the quantity and quality[1] of this set, impacting expected earnings and productivity.

This market segmentation property of the firm's optimal strategy has links to a longstanding theme in the economic literature. In a seminal industrial organization paper, Spence (1975) discusses how a monopoly must choose not only price and quantity, but also has other instruments to select, such as quality. When it includes another instrument such as quality, the model leads to multiple equilibria. This paper relates to Spence's idea in its setting: the firm tries to maximize profits by choosing a price which affects two other features, the quantity (demand) and quality of workers it can hire. We can draw a parallel between Spence's paper and the proposed approach of this paper. In this paper, the price is the posted wage, the quantity is the size of the applicants pool, and the quality is the composition of such a pool. Like in Spence's paper, such features will determine the expected profit of each locally monopsonistic firm. There exists, however, a key difference when considering both models: in the Spence model, price does not determine quality, while in this paper, both the size and quality (composition) of the applicant pool will depend on the selected wage. This idea has recently been explored in a

[1] It should be noted that the a given *applicant pool* will be judged to be of higher or lower quality depending on the skill requirements of the firm, so there is no absolute measure of "quality" in this sense.

multidimensional setting in a series of papers on optimal platform and product design (Veiga and Weyl, 2012, 2016; Veiga, Weyl and White, 2017). Their approach influences substantially the modelling strategy presented here in two ways: first, we use their notation, based on vectorial calculus, which we adapt for the search and matching environment. Like them, this allows us to calculate the derivative under the integral sign in a compact way. The second and more valuable contribution is that we consider market segmentation and analyze the marginal acceptance set. As a result, we can study how these market segmentation ideas lead to a behavioral mechanism that implies sorting without imposing restrictions on the production technology.

A key characteristic of the proposed setting is that wages are posted with commitment, even if firms are uncertain about the worker type with which they will match. This type of framework has been used before under direct search (Galenianos and Kircher, 2009) and seems natural in many settings. First, consider a situation in which the firm needs to comply with laws or conventions concerning equality and non-discrimination. In this case, the firm may not wish, or be able to, vary its wage ex-post as a function on its potential hire. A second case could arise when the dimensions along which applicants vary are (at least partially) unobserved by the firm prior to hiring. This case is widespread and particularly relevant in online labor markets (HIT[2] and freelance markets). In such markets, the employer does not know workers' quality and makes an offer to execute a specific task. This kind of behavior has been linked to monopsony, mostly due to concentration (Dube, Jacobs, Naidu and Suri, 2018).

The idea that posted wages could affect the scope of search has been experimentally tested recently in a directed search framework (Belot, Kircher and Muller, 2019). The role of posted wages has also been described in various theoretical papers: Moen (1997) demonstrates that when firms advertise a vacancy along with the offered wage, this leads to the competitive search equilibrium. Burdett et al. (2001) also model a framework in which a firm posts a unique price to attract buyers, and consider the strategic interaction of buyers and firms. In Galenianos and Kircher (2009) the role of posting wages is the same as in direct search models, in that prices guide search behaviour. In this model, we introduce posted wages into a random search model, enriching the random search framework with a strategic mechanism for the firms. This assumption seems appropriate for modern labor markets, in which there is

[2]HIT is an acronym for Human Intelligence Task. Evidence is beginning to emerge that this kind of behavior is also present in crowdsourcing job markets (Kingsley, Gray and Suri, 2014).

evidence that employers have the power to set wages without bargaining, especially when hiring from unemployment (Hall and Krueger, 2012; Brenzel et al., 2014).

The paper is structured as follows: Section 1 details the theoretical model, describing behaviours on both sides of the market, deriving the optimal wage posting strategy and characterizing multidimensional mismatch in equilibrium. Section 2 presents the data and section 3 presents the estimation method and the results. Section 4 concludes and presents some final considerations.

2.2 The Model

2.2.1 Basic setup

The environment of the model is composed of two types of forward-looking agents, firms and workers. All agents discount the future at a common rate of r. There is a continuum of workers and firms. Workers are endowed with skills that they can offer to the market, and firms have requirements specific to their production technology. A worker is an agent endowed with a set of skills[3] that characterize its type, represented by a vector of size k. The skill bundle of worker i is described $\boldsymbol{\theta}_i \in \mathbb{R}^K$ with the elements of $\boldsymbol{\theta}_i$ denoted as $\boldsymbol{\theta}_i = (\theta_1^i, ..., \theta_K^i)$. Each skill has a known support, $\theta_k \in (\underline{\theta_k}, \bar{\theta_k})$. Each firm j is characterized by a vector of requirements $\mathbf{r}_j = (r_1^j, ..., r_l^j)$. In this paper, we will refer to firms and occupations interchangeably. Each firm is characterized by a technology that combines the requirements and worker skills in production $m(\mathbf{r}_j, \boldsymbol{\theta}_i) = m_j(\boldsymbol{\theta}_i)$ $s.t.$ $m : \boldsymbol{\theta} \times \mathbf{r} \rightarrow \mathbb{R}_+$, that produces a unique homogeneous good. In this setting, neither firms or workers can exchange or trade the vector of endowments or requirements. A probability density function $f(\boldsymbol{\theta}) : \mathbb{R}^K \rightarrow \mathbb{R}$ characterizes the distribution of workers skills in the population; a probability density function $\gamma(\mathbf{r})$ characterizes the distribution of firm skills requirements. Workers and firms know the densities. These functions are C^2, there are no mass points, and have finite moments.

The model is framed as a random search model. Agents maximize the income they receive. In unemployment, workers face a search cost $b(x) = -\bar{b}$, which for simplicity is constant[4]. This

[3]The vector of worker endowments could contain multiple dimensions of workers' characteristics, such as demographics (age, gender, race, schooling), qualifications (skills, abilities, technologies, work values), or preferences (flexibility on schedule, or importance of remote work) as long as the characteristics affect firm productivity and are embodied in the worker. The presentation of the model purely in terms of skills should thus be thought of as a simplification ease of exposition.

[4]We extrapolate from the participation decision and only consider workers who are active on the labor market.

cost represents, for example, the monthly fee of usage of a job board and embeds the cost or stigma of being unemployed. Once the cost is paid, the agent will start to receive offers. The arrival rate denoted by λ is the probability by unit of time of receiving an offer. An employment relation ends at a constant exogenous rate, and thus separations are modeled with a constant risk of η.

Firms make offers to unemployed workers[5], sampled at random from a sampling distribution $s(\mathbf{r}_j)$. We will refer to such offers as *posted vacancies*. Each posted vacancy contains two pieces of information: the requirements of the firm j that makes the offer, and a proposed wage. A posted vacancy for occupation j is then a pair $(\mathbf{r}_j, w_j)$.

One of the main assumptions of the model is that wages are set ex-ante, and each firm can post only one wage, committing to it *independently of the characteristics of the worker sampled.* This assumption differs from the partial equilibrium search literature with wage bargaining, in which there is a distribution of wages that are a direct function of workers' ability. Instead, in this setting, each firm j posts a unique wage w_j. Firms select the posted wage strategically considering the distribution of $\boldsymbol{\theta}$, since an increase in the posted wage makes the job more more attractive for a larger pool of workers, increasing the probability of a match but also potentially changing the level of expected production given the heterogeneous applicants pool. Such a mechanism suggests that firms can segment the labor market via the wage posting strategy (given their requirements), and thus have market power. Workers do not have any bargaining power, so they accept the posted wage if they decide to be employed or receive their outside option in the case they prefer unemployment.

We characterize workers' and firms' dynamic behavior in the next sections, followed by the definition of equilibrium. The next section succinctly presents some definitions, which are useful since we use vectorial calculus notation.

2.2.2 Definitions

We begin defining the instantaneous utility of worker i at time t when employed at occupation j. It is given by:

[5]In this setting, we do not consider job to job mobility since the scope of the paper is to present a new wage-setting mechanism. The proposed setting can be enriched, including job to job mobility in the traditional posted wage form (Burdett and Mortensen, 1998) or through renegotiation (sequential auction) (Postel–Vinay and Robin, 2002), but such an extension is beyond the scope of this paper.

$$U(\mathbf{r}_j, w_j; \boldsymbol{\theta}_i) = w_j - c(\mathbf{r}_j, \boldsymbol{\theta}_i)$$

Where $c(\mathbf{r}_j; \boldsymbol{\theta}_i)$ is a distance function that represents the cost of being mismatched. The cost arises, for example, by a distaste of performing the task for which one's skill does not coincide. Since we are in a model of posted wages and do not allow for a menu of wages, the cost function is the only element that incorporates mismatch into utility[6].

One of the mechanisms that is important in our setting is the worker's willingness to participate in certain occupations. We denote the set of all workers willing to participate in a job j as the *applicants pool.* When workers consider a posted vacancy $(\mathbf{r}_j, w_j)$, they can calculate their instantaneous utility if they were to be employed in occupation j, and compare it to the sure outside option (unemployment). A worker is willing to join the *applicants pool* whenever the instantaneous utility is larger or equal to the outside option.

$$\begin{aligned} U(\mathbf{r}_j, w_j; \boldsymbol{\theta}_i) &\geq -\bar{b} \\ U(\mathbf{r}_j, w_j; \boldsymbol{\theta}_i) + \bar{b} &\geq 0 \\ \tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}_i) &\geq 0 \end{aligned}$$

This determination takes into consideration only the static information of each period and assumes that workers can distinguish correctly the requirements and the wages proposed by the firm j.

Definition 1 : *The applicants pool (AP) is the set of workers that choose to participate in the market for occupation j. This set is represented as:*

$$\boldsymbol{\Theta}_j \equiv \{\boldsymbol{\theta} : \tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0\}$$

Definition 2 : *The marginal applicants pool (MAP) is the set of workers who are indifferent between working or not in a job in occupation j. For them, the instantaneous utility is equal to the outside option. They are in the boundary of the set and are represented by:*

[6] In other wage-setting mechanisms, wages can compensate for such distaste because they are a function of worker type.

$$\partial\boldsymbol{\Theta}_j \equiv \{\boldsymbol{\theta} : \tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) = 0\}$$

The number of workers in the set $\boldsymbol{\Theta}_j$ increases when the posted wage w_j increases, diminishes when $c(\cdot)$ increases and increases with unemployment stigma. Intuitively, $\boldsymbol{\Theta}_j$ is the k-dimensional space of $\boldsymbol{\theta}$ for which the acceptance is assured for a given wage value, and $\partial\boldsymbol{\Theta}_j$ is the boundary surface of such space ($k-1$ dimensions).

Theorem 1 in Appendix A.2 presents the Divergence theorem, a commonly known result in vector calculus. We introduce some of the notation used, and provide intuition on the elements of the applicants pool: one as a volume and the other as the surface of that volume. We are going to use also the following notation: the sign $\nabla_{\boldsymbol{\theta}} H$ is the gradient of the function H with respect to the variables $\boldsymbol{\theta}$. The $\|a\|$ is the Euclidean norm of a.

Definition 3 : *Acceptance probability. The firm calculates the mass of participating individuals as:*

$$N_j \equiv \int_{\boldsymbol{\Theta}_j} f(\boldsymbol{\theta}) d\boldsymbol{\theta}$$

Definition 4 : *Marginal acceptance probability. The mass of marginal acceptance individuals, is defined as:*

$$M_j \equiv \int_{\partial\boldsymbol{\Theta}_j} \frac{f(\boldsymbol{\theta})}{\|\nabla_{\boldsymbol{\theta}} \tilde{U}\|} d\tau$$

M_j then captures the responsiveness of the marginal participants, all of whom have a common reservation value in the boundary. As mentioned in the introduction, these definitions are similar to the definitions presented in recent IO literature (Veiga and Weyl, 2012, 2016; Veiga et al., 2017).

Using the definitions $1-4$, we can construct the conditional operator for the AP and MAP. For any given function $Q(\boldsymbol{\theta})$, the conditional operator is defined by:

$$\mathbb{E}[Q(\boldsymbol{\theta})|\boldsymbol{\Theta}_j] = \frac{\int_{\boldsymbol{\Theta}_j} Q(\boldsymbol{\theta}_j) f(\boldsymbol{\theta}_j) d\boldsymbol{\theta}_j}{N_j}$$

$$\mathbb{E}[Q(\boldsymbol{\theta})|\partial\boldsymbol{\Theta}_j] = \frac{\int\limits_{\partial\boldsymbol{\Theta}_j} \left(Q(\boldsymbol{\theta}) \frac{f(\boldsymbol{\theta})}{\|\nabla_{\boldsymbol{\theta}}\tilde{U}\|} \right) d\tau}{M_j}$$

Using these definitions we present worker and firm behaviours.

2.2.3 Workers

The derivation of the continuous value of unemployment V_u is standard[7].

$$V_u = \frac{1}{1+r\Delta t}\left[-b\Delta t + (1-\lambda\Delta t)V_u + \right.$$
$$\left. +\lambda\Delta t\mathbf{E}\max\{V_e, V_u\}\right] + o(\Delta t)$$

The first term inside the square brackets is the discounted value of search. The second term corresponds to not receiving a proposal and continuing in unemployment, which occurs with probability $(1-\lambda\Delta t)$, and the third part represents what occurs when an offer arrives and the individual chooses among two possibilities, taking the job or leaving it. The last term is a negligible component of order $o(\Delta t)$ that comes from the Bellman optimality principle. After some rearranging and passing the limit of Δt to 0, the latter equation is then:

$$rV_u = -b + \lambda\mathbf{E}_{\{\tilde{U}(\cdot)\geq 0|w_j;\boldsymbol{\theta}\}}\{V_e - V_u\} \tag{2.1}$$

We use $\tilde{U}(\cdot)$ instead $\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta})$ to improve readability.

The value of being employed is given by the discounted value of flow utility while working, the value of employment continuation and the value of separation and going in to unemployment.

$$V_e(\mathbf{r}_j, w_j, \boldsymbol{\theta}) = \frac{1}{1+r\Delta t}\left[(w_j\Delta t - c(\mathbf{r}_j, \boldsymbol{\theta})\Delta t) + \right.$$
$$\left. +(1-\eta\Delta t)V_e(\mathbf{r}_j, w_j, \boldsymbol{\theta}) + \eta\Delta t V_u\right] + o(\Delta t)$$

[7] The reader can find the detailed derivation of all equations in appendix A.2

After some algebra the value of being employed yields:

$$V_e(\mathbf{r}_j, w_j, \boldsymbol{\theta}) = \frac{(w_j - c(\mathbf{r}_j, \boldsymbol{\theta})) + \eta V_u}{(r + \eta)} \tag{2.2}$$

Workers will decide to take an offer if the value of employment is larger than or equal to the value of unemployment, so when $V_e(\mathbf{r}_j, w_j, \boldsymbol{\theta}) \geq V_u$.

Workers face two decisions that rely on different information sets. Unemployed workers choose to belong to the *applicants pool* (AP) considering only static information; i.e. they only consider the value of the instantaneous utility relative to the outside option. However, unemployed workers that receive the offer from firm j make their decision considering the dynamics, so the expected value of taking the job to search continuation. Given that each of the decisions is taken using a different information set, there are individuals that decide to participate in the the pool of applicants but not will accept a contract proposal, even though the information sets are related.

2.2.4 Firms

Let the constant returns to scale matching function $M(u, v)$ describe the technology that matches unemployed workers to vacancies. We define $M(u, v)/v = q(\omega)$ as the rate at which a vacant job is matched with an unemployed worker. The rate is dependent on the tightness in the market. We make the standard assumptions with respect to the matching function: $M(u, v)$ has constant returns to scale, $q(\omega)$ is decreasing in ω, and the $\lim_{\omega \longrightarrow 0} q(\omega) = \infty$, and that the $\lim_{\omega \longrightarrow \infty} q(\omega) = 0$. Firms are rational and maximize current and future profits. The present value of a vacant job Υ_j is formed by the discounted cost of posting the vacancy k, the continued value of maintaining the vacancy open, and the probability that a match is made times the larger value of the value of the filled job and the value of the unfilled vacancy, conditional on the person contacted being in the acceptance set.

$$\begin{aligned} \Upsilon_j = \frac{1}{1 + r\Delta t} \Bigg[& -k\Delta t + (1 - q(\omega)\Delta t)\Upsilon_j + \\ & + q(\omega)\Delta t \int\limits_{\boldsymbol{\Theta}_j} \max\{J(\mathbf{r}_j, \boldsymbol{\theta}), \Upsilon_j\} d\boldsymbol{\theta} \Bigg] + o(\Delta t) \end{aligned}$$

After some manipulation the flow value of a vacant job is written as:

$$r\mathrm{Y}_j = -k + q(\omega)N_j\mathbb{E}\left[\max\{J(\mathbf{r}_j,\boldsymbol{\theta}) - \mathrm{Y}_j\}|\boldsymbol{\Theta}_j\right] \tag{2.3}$$

where the conditional expected value with respect to firm j's AP is multiplied by the acceptance probability and the matching rate.

We define the flow value of a filled job with $J(\cdot)$, composed of the present value of flow profits of the match for the firm, its continuation and its termination. The flow value of a filled job will depend on firm requirements, worker endowments, and the posted wage.

$$J(\mathbf{r}_j,\boldsymbol{\theta}) = \frac{[m(\mathbf{r}_j,\boldsymbol{\theta}) - w_j]\Delta t}{1 + r\Delta t} + \frac{(1-\eta\Delta t)}{1+r\Delta t}J(\mathbf{r}_j,\boldsymbol{\theta}) + \frac{\eta\Delta t}{1+r\Delta t}\mathrm{Y}_j + o(\Delta t)$$

which after some algebra yields:

$$J(\mathbf{r}_j,\boldsymbol{\theta}) = \frac{m_j(\boldsymbol{\theta}) - w_j + \eta(\mathrm{Y}_j)}{(r+\eta)} \tag{2.4}$$

In equilibrium in the Mortensen and Pissarides (1994) framework, a firm accepts any match for which the value of the filled vacancy is larger than or equal to the discounted value of the vacancy $J(\mathbf{r}_j,\boldsymbol{\theta}) \geq r\mathrm{Y}_j$ (hence the max in equation 2.3). The firm also faces a strategic decision on wage-setting since the posted wage determines the flow value of a vacant job and the flow value of a filled job, as well as the applicant pool. Inserting (**??**) into (**??**) we obtain the flow value of an unfilled job conditional on the posted wage. Equation (**??**) describes how the posted wage w_j might change the value of an unfilled job.

$$r\mathrm{Y}_j = \frac{-k(\eta+r) + q(\omega)N_j\mathbb{E}\left[\left[m(\mathbf{r}_j,\boldsymbol{\theta}_i) - w_j\right]|\,\boldsymbol{\Theta}_j\right]}{(\eta+r) + N_j q(\omega)} \tag{2.5}$$

Equation 2.5 shows that the posted wage is the only decision variable available to the firm that can affect the flow value of an unfilled vacancy. An increase in the posted wage will reconfigure the applicant pool, increasing the applicants pool size and thus increasing the acceptance probability N_j. The increase is also induces a change in the expected flow net value of the match. The sign and magnitude of such change will depend on the technology and its sensitivity to the changes in the composition of the applicant pool, and it will also depend on the characteristics of the population and its distribution (on the shape of the distribution of types $f(\boldsymbol{\theta})$).

2.2.5 Equilibrium & Wage-setting

Balanced flows and worker densities

In equilibrium, the flows are balanced between states, the number of posted vacancies is optimal for each firm, and the posted wage is optimal. In this section we present the conditions under which such an equilibrium exists, emphasizing how the wage-setting mechanism responds to a behavioral optimal response in equilibrium.

For the aggregate flows to be balanced in the steady state, the flow into employment is equal to the flow out of unemployment. We assumed that an employment relation has an exogenous constant rate of termination η, and that the probability per unit of time of being matched is λ, such that unemployed meetings are equal to employed terminations, $\lambda u = \eta(1-u)$. Solving for u we have:

$$u = \frac{\eta}{\lambda + \eta} \tag{2.6}$$

For the distributions of skill endowments of unemployed workers to be stationary we know that outflow of of workers of each type must be equal to the inflow. The outflow of workers from the firm is defined by the share of employed workers in the firm by skill type, multiplied by the rate of termination of the contracts $\eta(1-u)\ell(\boldsymbol{\theta}, \mathbf{r}_j)$, where $\ell(\boldsymbol{\theta}, \mathbf{r})$ is the density of workers with skill bundle $\boldsymbol{\theta}$ employed at firm $\mathbf{r}_j$. The inflow of workers into employment must be equal to the arrival of unemployed workers per type, adjusted by the probability of sampling, $u\lambda f(\boldsymbol{\theta})s(\boldsymbol{r}_j)$. By equalizing the inflows and outflows and using equation 2.6, we have that the density of workers with skill bundle $\boldsymbol{\theta}$ employed at firm $\mathbf{r}_j$, unconditional and conditional on the AP, is given by:

$$\ell(\boldsymbol{\theta}, \mathbf{r}_j) = f(\boldsymbol{\theta})s(\boldsymbol{r}_j) \tag{2.7}$$

$$\ell(\mathbf{r}_j) = \ell(\boldsymbol{\theta}, \mathbf{r}_j | \boldsymbol{\Theta}_j) = \int_{\boldsymbol{\Theta}_j} f(\boldsymbol{\theta}) s(\boldsymbol{r}_j) d\boldsymbol{\theta} = N_j s(\boldsymbol{r}_j) \tag{2.8}$$

With these equations we can calculate the search intensity $\frac{s(\boldsymbol{r}_j)}{\gamma(\mathbf{r}_j)}$, and firm size $\frac{\ell(\mathbf{r}_j)}{\gamma(\mathbf{r}_j)}$ for each of the firms in the economy. Those values depend on the AP, whose composition is determined completely by the posted wage w_j. We can use equation 2.5 to determine the missing components of the equilibrium, the optimal vacancies per firm, and the optimal wage posting. To retrieve the optimal vacancies per firm we use the free entry condition $r\Upsilon_j = 0$, while for the definition of the optimal wage we use the profit maximization condition $\frac{\partial r\Pi_j}{\partial w_j} = 0$.

The calculation of the last derivative presents some complications since the posted wage is in the integration limit. In order to calculate such values, we use the Leibniz rule to obtain the equilibrium of the model.

Profit maximization, wage determination and equilibrium

The value of an unfilled job, conditional on the posted wage, is presented in equation 2.5. From this equation we derive two of the equilibrium conditions for which the employer assures that the posted wage is optimal. The first condition is free entry. Imposing free entry implies that the number of vacancies in the market is endogenous to the model, so that firms can make no additional profit by posting an additional vacancy. As can be seen, the number of vacancies can affect the profits of the firm, as it affect $q(\omega) = q(\frac{v}{u}, 1)$ the matching function. The arrival rate of offers also is affected by the increase in the number of vacancies, and in equilibrium $\lambda(v)$ is an concave increasing function on vacancies.

$$\lambda = \frac{q(v,u)}{u} = q\left(\frac{v}{u}, 1\right) = q(\omega) \equiv \lambda(v) \tag{2.9}$$

Following Mortensen (1998), we impose free entry, meaning that the value of of an unfilled vacancy is zero, and insert equation 2.9 into equation 2.5. After some algebra we get:

$$kv_j = \lambda(v_j) \frac{N_j}{\eta + r} \mathbb{E}\left[[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j] | \boldsymbol{\Theta}_j \right] \tag{2.10}$$

Given an equilibrium posted wage $(w)^*$ equations 2.1-2.8 are well defined. Assuming Inada conditions hold and that $\lambda(v)$ is increasing and concave, equation 2.10 has a stable equilibrium for a positive $v_j^* > 0$.

Using the preceding condition, the optimal value of the posted wage will satisfy the boundary condition (equation 2.10). After taking the derivative with respect to the posted we get:

$$\frac{\partial k v_j}{\partial w_j} = \frac{\partial}{\partial w_j}\left(\lambda(v_j)\frac{N_j}{\eta + r}\mathbb{E}\left[\left[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j\right] | \boldsymbol{\Theta}_j\right]\right) \tag{2.11}$$

Applying the Leibniz rule, and reorganizing terms, we get the expression for the optimal posted wage. After some manipulation, the optimal posted wage rule solves the following equation.

$$\begin{aligned} M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w} \mid \partial \boldsymbol{\Theta}_j\right](w)^* = -N_j + & \\ + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w} \mid \partial \boldsymbol{\Theta}_j\right]\mathbb{E}\left[m(\cdot) \mid \partial \boldsymbol{\Theta}_j\right] + & \\ + M_j Cov\left[\frac{\partial \tilde{U}}{\partial w}, m(\cdot) \mid \partial \boldsymbol{\Theta}_j\right] & \end{aligned} \tag{2.12}$$

The optimal posted wage equation gives the posted wage that the firm commits to offer, taking into consideration the multidimensional skill distribution in the economy, the mismatch cost associated with work in a job, and the pool of candidates that will accept the offer. The decision depends on the size of the pool N_j, the sensitivity of the infra marginal workers to changes in wage and its relationship to their productivity.

It is worth noting, that we can calculate the optimal posted wage both from condition 2.10 and the steady state profit flow. Maximizing either equation leads to the same definition. To show this fact we derive the optimal wage from the steady state profit flow, as is done in the basic Burdett and Mortensen (1998) wage posting model.

We write profit as the expected flow value of production net of cost, calculated over the density of the employer's applicants pool.

$$\Pi_j(\mathbf{r}_j, \boldsymbol{\theta}_i; w_j) = \max_w \mathbb{E}\left[m(\cdot) - w | \boldsymbol{\Theta}_j\right] \ell(\boldsymbol{\theta}, \mathbf{r}_j | \boldsymbol{\Theta}_j)$$

Replacing the value of the joint density (eq. 2.8) in the above equation, we can rewrite profits in terms of the *quantity* and *quality* of the AP. In this framework, we define quality as the expected match productivity net of cost evaluated over the pool of applicants. Profits are related to quality since they depends on the distribution of skills within the applicants pool. They are related to quantity since the quality of the applicants pool is multiplied by the mass of workers that belong to the pool. The firm will select the wage that defines the best pool of candidates that will accept the offer, and by doing so it will maximize it's profits.

$$\Pi_j(\mathbf{r}_j, \boldsymbol{\theta}_i; w_j) = \max_w \underbrace{\mathbb{E}\left[m(\cdot) - w|\boldsymbol{\Theta}_j\right]}_{\text{Quality}} \underbrace{N_j}_{\text{Quantity}} s(\mathbf{r}_j) \tag{2.13}$$

Using the differential under the integral sign (eq. A.1), the firm chooses the posted wage that maximizes the steady-state profit flow. The first order condition of this problem is given by[8]:

$$\frac{\partial \Pi_j(\mathbf{r}_j, \boldsymbol{\theta}_i; w_j)}{\partial w} = s(\mathbf{r}_j)\left[-N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w}(m(\cdot) - w)\,|\boldsymbol{\Theta}_j\right]\right] = 0$$

Which after some manipulation, and solving for the posted wage, yields the optimal posted wage rule.

$$\begin{aligned} M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w}\,|\,\partial\boldsymbol{\Theta}_j\right](w)^* = -N_j + & \\ + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w}\,|\,\partial\boldsymbol{\Theta}_j\right]\mathbb{E}\left[m(\cdot)\,|\,\partial\boldsymbol{\Theta}_j\right] + & \\ + M_j Cov\left[\frac{\partial \tilde{U}}{\partial w}, m(\cdot)\,|\,\partial\boldsymbol{\Theta}_j\right] & \end{aligned} \tag{2.14}$$

As mentioned before the optimal wage can be retrieved both from profit maximization or by equalizing flows, reconciling the older (Burdett and Mortensen, 1998) and newer models (Ellingsen and Rosén, 2003). With the equilibrium values v^* and w^*, the model is closed and all the equations of the model are well defined.

It is essential to note some results from the theoretical model. First, consider the size of the marginal applicants pool. We calculate expected productivity with respect to this set. Its size and composition define the participation patterns, including the firm's mismatch. An increase in the wage will increase the number of people willing to accept the job and increase

[8] Derivation in the appendix

the mismatch for that firm type. In equilibrium, the optimal wage posting strategy integrates such information and offers the wage that corresponds to the marginal group composition that maximizes expected marginal match productivity, adjusted for the co-variance between preferences and the production function.

Another important result from the theory comes from the role of multidimensionality. Recent literature shows that omitting multidimensionality could lead to wrong results (Lise and Postel-Vinay, 2015; Lindenlaub and Postel-Vinay, 2016). In the presented model, multidimensionality plays different roles: through mismatch, skills substitution plays an important role because firms can still substitute one skill for another. More importantly, the marginal set effects becomes relatively more important when the number of dimensions increases, which will have consequences for the proposed wage schedules.

2.2.6 Narrative of the model: order of events

Here we would like to emphasize the narrative behind the matching and sorting process in the proposed model. Such a narrative helps us to understand how both types of agents use the available information. In order to construct this narrative, we propose to analyze the model at two specific points in time. These points in time occur simultaneously, but we present them separately since the agents use different sets of information.

In the first moment, the firm understands the participation rule that workers have. Workers are passive at this point, in the sense that they only accept or reject based on an already defined set of acceptable postings. Recall that we have defined each posting as the combination of information about an offered wage with commitment and specific requirements. The firm internalizes how the worker processes this information and sees how different types accept and reject the offer for different posted wage levels. Using the wage rule, firms optimize the size and composition of the applicant pool. The wage corresponds to the point at which a marginal increase in the posted wages changes the composition in the applicants pool in a way that decreases the firm's steady-state profit.

The second moment is when the offer arrives and the worker decides whether or not accept the match. This part is similar to the classic interpretation. To create a narrative, we imagine that a firm randomly sends a posting (wage and requirements) to a worker sampled from the skill distribution. Workers compare the flow value of employment and unemployment to accept or reject the match, but this value depends not only in the wage but also on the requirements of the firm relative to the skill endowment of the worker. If the employment value is larger than the value of unemployment, the worker accepts the match until the match dissolves with an exogenous probability.

Comparing the information sets at these two points in time, we can observe that the choice is dynamic in the second one, while in the first, the worker chooses based on information that determines flow utility. Given that there is no job to job mobility, skills depreciation, or learning, the two sets of information are compatible and lead to the same outcome.

Following this narrative, what are the possible shocks that can modify the worker's decision? The most evident is the value of the outside option. Changes in the distribution of skills will also change the expected value of the match, the posted wage, and thus worker decisions. Changes in the production technology will also affect the posted wage, and thereby the acceptance set.

In the next sections, we present the data and estimate the model for France.

2.3 Data

We use the data for France from the Programme for the International Assessment of Adult Competencies (PIAAC). The survey was developed by the OECD and data collection for France was undertaken between September and November 2012. The PIAAC provides internationally comparable data about skills of the adult population in 24 countries. The sample consists of adults between 16 and 65 years of age. Even if sampling schemes are different between countries, the data provides post-sampling weightings which allows one to fit the principal moments of labor market indicators, earnings, demographics and the skills distribution. In order to match the measured skills, a multiple imputation method is proposed, and 10 plausible values are provided for both literacy and numeracy. For each plausible value a weight is also provided.

The survey includes a direct assessment of cognitive skills in two main domains: literacy and numeracy. For literacy, the survey measures the ability to understand written texts; For numeracy, it quantifies the ability to access, use, interpret, and communicate mathematical information and ideas. An optional dimension is also measured, Problem solving in technology-rich environments, which is understood as the ability to use digital technology. The latter was not measured for France, so we use only the literacy and numeracy measures. It is important to note that these measures are not self declared, but rather directly assessed through a test administrated by the interviewer.

The non cognitive skills measures are derived from the answers to the background questionnaire of the survey. In this part, six questions about attitudes and interest toward learning are asked. These measures are related to personality and intelligence and can be linked to one of the big 5 personality traits: openness to experience (Goff and Ackerman, 1992).

To assess job requirements, we use O*NET data. Specifically, the O*NET is a U.S.-based system which provides up-to-date and detailed descriptors of the requirements for each occupation in terms of the knowledge, skills, and abilities required by workers, as well as how the work is performed in relation to tasks, work activities, work context and other descriptors (Onetcenter, 2016). In this paper I used the skill information on requirements and I construct two vectors of skill requirements for cognitive and non cognitive skills using factor analysis.

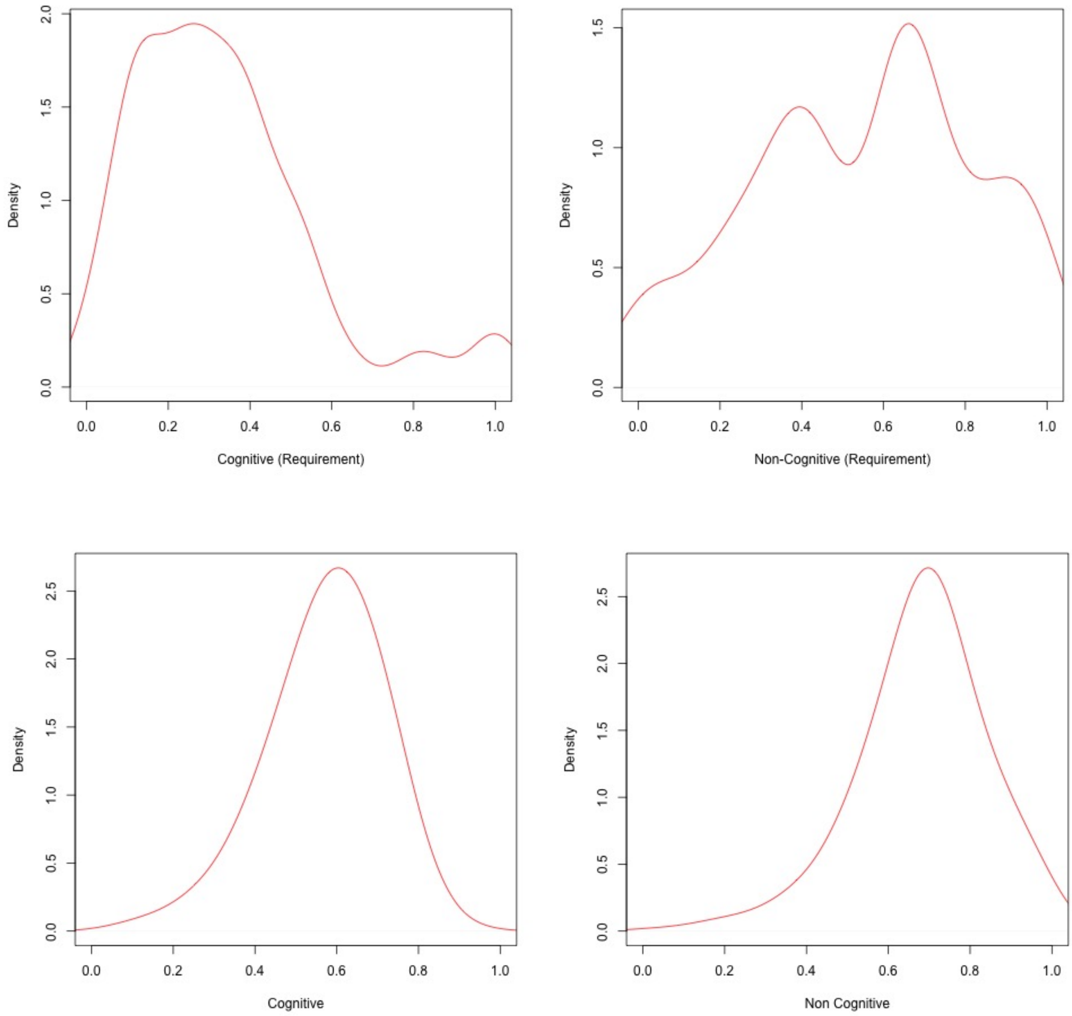
Figure 2.1 presents the cognitive and non-cognitive skills supply (top figures) and demand (bottom figures) after having applied a linear transformation to each set of measures in order to make the supports of the distributions fit in the $[0, 1]$ interval. These figures highlight the main challenge the model is designed to address: how to allocate such heterogeneous demand to the heterogeneous supply of skills, especially when the distributions have different shapes, and how do wages and preferences affect the allocation.

Table A.1 presents the moments used in the paper for the estimation. The values for the wages are calculated directly from the data.

2.4 Estimation

I estimate the model by indirect inference using the simulated method of moments (SMM). I calculate the moments of the distribution of skill endowments, requirements, the value of the unemployment and wage deciles using the survey data. These are the observed (sample) moments z^o. Using a set of proposed parameters π, we then solve the model starting from equation 2.14 and equation 2.10. We then calculate the moments $z^s(\pi)$, generated by the set of parameters π, for the simulated population. The estimation procedure minimizes the distance between the empirical observed moments and the simulated moments, so that the wage

Figure 2.1: Requirements and skills



distribution, skills endowment distribution, skills requirements distribution and unemployment rate are replicated as close as possible. The minimization problem an be written as:

$$P(\hat{\pi}) = \min_{\hat{\pi}} \sum_{I} \omega_i (z_i^o - z_i^s(\pi))^2 \tag{2.15}$$

where ω_i is a weight that values the importance of the moment in the sample. For the actual exercise, the unemployment level and the percentiles of the wage distribution are assigned double the importance of the skill percentiles.

2.4.1 Parametric specification

One of this paper's main contributions is to model, in the most granular way possible, the decision of each agent. From the theoretical results, we saw that the model's main equations involve the distributions of skill endowments and requirements. The way we model these multivariate distributions will have an impact on our results. Papers that used similar estimation procedures (Lise and Postel-Vinay, 2015) combine two beta marginals for the skills and requirements distribution with a Gaussian copula.

The dependence between the dimensions is constant when using Gaussian copulas, while is likely that such dependence changes in the different parts of the distribution. We therefore model the skills and endowment joint distributions using a Frank copula, which allows the skill distribution to be non-symmetric and allows the dimensions to be locally dependent. Our Frank copula has beta marginals, allowing different shapes on the margin. This copula can also provide the LDF (Local Dependence Function), which describes the point correlation of two random variables, (x_1, x_2), at each point of the common support. In this way, we can model and understand the joint variation of the strength of association at each point of the support. The Local Dependence Function is generally interpreted as a "local Pearson correlation" and is defined as:

$$\gamma(x_1, x_2) = \frac{\partial^2 \log f(x_1, x_2)}{\partial x_1 \partial x_2}$$

If the x_1, x_2 are independent random variables, the LDF is 0 over the support of x_1 and x_2. In the case of a bivariate Gaussian, the value is constant and equal to $\frac{r}{1-r^2}$. The shape of the Frank copula is defined by:

$$F(x_1,x_2) = -\frac{1}{\bar{\alpha}} \log \left(1 + \frac{\left(e^{-\bar{\alpha}F_1(x_1)} - 1\right)\left(e^{-\bar{\alpha}F_2(x_2)} - 1\right)}{e^{-\bar{\alpha}} - 1}\right)$$

For the Frank copula the LDF is:

$$\gamma(x_1,x_2) = 2\bar{\alpha} f(x_1,x_2)$$

The estimation of the multidimensional densities is the first step of the simulation. In the results the estimation points with more mass will have a larger correlation. The sign and size of the $\bar{\alpha}$ parameter will determine the correlation in each point of the simulated grid and determines the weight assigned by the joint density to each point.

For the purposes of estimation, we can define a grid and calculate the value of all functions of the model for each point to determine the applicants pool that each firm faces, and simulate the complete model. In the simulation we use only two skills. We also need to specify functional forms for the cost utility and the production function in order to generate the remaining simulated moments.

We model the cost function as the weighted euclidean distance between the skills endowment of the worker and the skills requirements of the job. In this formulation, the weights ξ_k, which are estimated, value the mismatch cost for the individual in each dimension.

$$c(\boldsymbol{r}_j, \boldsymbol{\theta}_i) = \left(\sum_{k=1}^{2} \xi_k (\boldsymbol{\theta}_{ik} - \boldsymbol{r}_{jk})^2\right)^{0.5}$$

Using this specification, we can calculate the flow utility function. Recall that flow utility is the difference between the posted wage and the worker's disutility from the mismatch. The function used is then given by $u(r_j, \theta_i, w_j) = w_j - c(r_j, \theta_i)$. Given a wage, we can calculate the density of the applicants pool for each firm and its marginal pool of applicants. All the operations on expected productivity and expected costs are then calculated to determine the posted wage for each requirements vector in the grid.

We also specified the production function using a constant elasticity of substitution functional form. We use this functional form since we would like to test skills complementarity in production.

$$m(\boldsymbol{r}_j, \boldsymbol{\theta}_i, \xi) = \left(\phi_c \left(\frac{\theta_i^c}{r_j^c}\right)^{\mu} + \phi_{nc} \left(\frac{\theta_i^{nc}}{r_j^{nc}}\right)^{\mu}\right)^{\frac{1}{\mu}}$$

The simulation generates work histories of individuals and firms sampled from the skill distributions and matches the observed unemployment, earnings, and skills distribution. For comparing simulated unemployment in the optimization to observed unemployment, we consider the average unemployment of the last 50 simulated periods after burning the first 100 simulated periods.

The final functional form we need to define is the matching function. One caveat of our estimation is that we do not have the duration of employment or unemployment in our data, so estimating the separation rate does not have an observed counterpart and can only be fit through its influence on other moments that are matched in the simulation. The matching function is specified by the following functional form $\lambda(v) = m(v, u) = \psi\sqrt{uv}$.

The set of parameters to estimate is then:

$$z = [\alpha_c, \alpha_{nc}, \beta_c, \beta_{nc}, \bar{\alpha}, \alpha_c^s, \alpha_{nc}^s, \beta_c^s, \beta_{nc}^s, \bar{\alpha}^s, \xi_c, \xi_{nc}, \phi_c, \phi_{nc}, \mu, \psi, \lambda, \eta, b, r]$$

In the next section we present the main results of the estimation and discuss their implications.

2.4.2 Results

Table 2.1 presents the estimation results. The first four parameters in the table determine the shape of the estimated copula's beta marginals. The fifth parameter is the Frank copula's strength correlation parameter and determines the degree of association between cognitive and non-cognitive skills endowments. This parameter is positive (1.233), implying a positive correlation, especially in the distribution's more dense parts. The next five parameters have the same interpretation, but for the distribution of the requirements. The correlation between cognitive and non-cognitive requirements is stronger (1.816). Considering both estimates together gives us a hint about how these distributions are different and points to the difficulties that the allocation mechanisms face: on the workers side, the distribution is flatter and even if positively correlated, less correlated than the distribution of the requirements. A visual representation of such copulas is presented in figure 2.2.

Considering the cost function parameters, workers assign a higher disutility to being mismatched

Table 2.1: Estimated parameters

	Estimate	C.I. (95%)	Type	Description
α_c	1.360	$[1.357, 1.364]$		Shape parameter from the beta distribution. Cognitive skill endowment.
α_{nc}	1.971	$[1.969, 1.973]$		Shape parameter from the beta distribution. Non cognitive skill endowment.
β_c	2.618	$[2.616, 2.620]$		Shape parameter from the beta distribution. Cognitive skill endowment.
β_{nc}	1.416	$[1.414, 1.418]$		Shape parameter from the beta distribution. Non cognitive skill endowment.
$\bar{\alpha}$	1.233	$[1.226, 1.238]$		Strengh of correlation parameter.
α_c^s	6.790	$[6.784, 6.797]$		Shape parameter from the beta distribution. Cognitive skill requirement.
α_{nc}^s	6.196	$[6.194, 6.198]$		Shape parameter from the beta distribution. Non cognitive skill requirement.
β_c^s	2.978	$[2.975, 2.981]$		Shape parameter from the beta distribution. Cognitive skill requirement.
β_{nc}^s	3.464	$[3.146, 3.648]$		Shape parameter from the beta distribution. Non cognitive skill requirement.
$\bar{\alpha}^s$	1.816	$[1.812, 1.821]$		Strengh of correlation parameter.
ξ_{nc}	90.71	$[86.40, 94.12]$		Cost function Non Cognitive
ξ_c	105.31	$[97.22, 112.91]$		Cost function Cognitive
ϕ_{nc}	335.12	$[333.09, 338.14]$		Weight production Non Cognitive
ϕ_c	311.35	$[308.04, 315.58]$		Weight production Cognitive
μ	-0.429	$[-0.430, -0.428]$		Elasticity production function
ψ	0.542	$[0.541, 0.544]$		Matching function parameter
λ	0.055	$[0.040, 0.063]$	*	Finding rate - Offer arrival
η	0.006	$[0.006, 0.006]$	*	Separation rate
b	250		Not calc.	
r	0.004		Not calc.	Discount rate
k	$0.6E[m(r.\theta)]$		Not calc.	Cost of a vacancy

in cognitive skills than non cognitive skills. When we consider the firm side, we can see the production technology assigns higher weights to non-cognitive skills. We also find that the elasticity of both skills in the production function is negative, even if small. This result suggests that there is a degree of complementarity in production between cognitive and non-cognitive skills.

The next part of the table shows the estimated labor market parameters: the matching function scale parameter, arrival rate and the exogenous separation rate. We can see that the values of the finding rate are higher than the usually in the reported literature. This is because, in our model, the arrival of an offer does not imply acceptance. An offer from a firm can arrive at a worker outside its applicants pool. In this way, the mismatch costs creates additional frictions which do not depend on the traditional parameters.

Using our estimates, we can recover the mismatch across the support of worker's skill distributions from the simulation results. Figure 2.3 presents the difference between the requirements and the skills endowments over the support of the distribution of cognitive and non-cognitive skill endowments. We can observe a clear decreasing pattern in the figures: low skill workers are underqualified and high skill workers are overqualified. In the first part (until the 25$^{\text{th}}$ percentile), workers are underskilled in cognitive skills. In the last part of the distribution (after the 75$^{\text{th}}$ percentile), they are over-skilled. In the middle of the distribution, we can not reject the null hypotheses of the absence of mismatch, in some cases positive, while negative in others. This slope is by construction negative, since there can be no values in the requirements distribution below the smallest value of compact support of cognitive skills endowment distribution (hence a positive difference) and no values in the requirements distribution above the largest value in the support of the endowments distribution (hence a negative difference). Nevertheless, it is worth noting the share of the population correctly matched in each of the dimensions, and how this result can be inferred from the estimated distribution of skills and requirements. The lower panel of the figure reveals the same decreasing pattern of mismatch across the whole distribution. Even if the degree of mismatch is quantitatively lower than in the previous case, we can see that the share of workers for whom we cannot reject the null of no mismatch is much smaller, with only about 10 percent of workers being correctly matched on the non cognitive skills dimension.

Figure 2.2: Copula requirements and skills

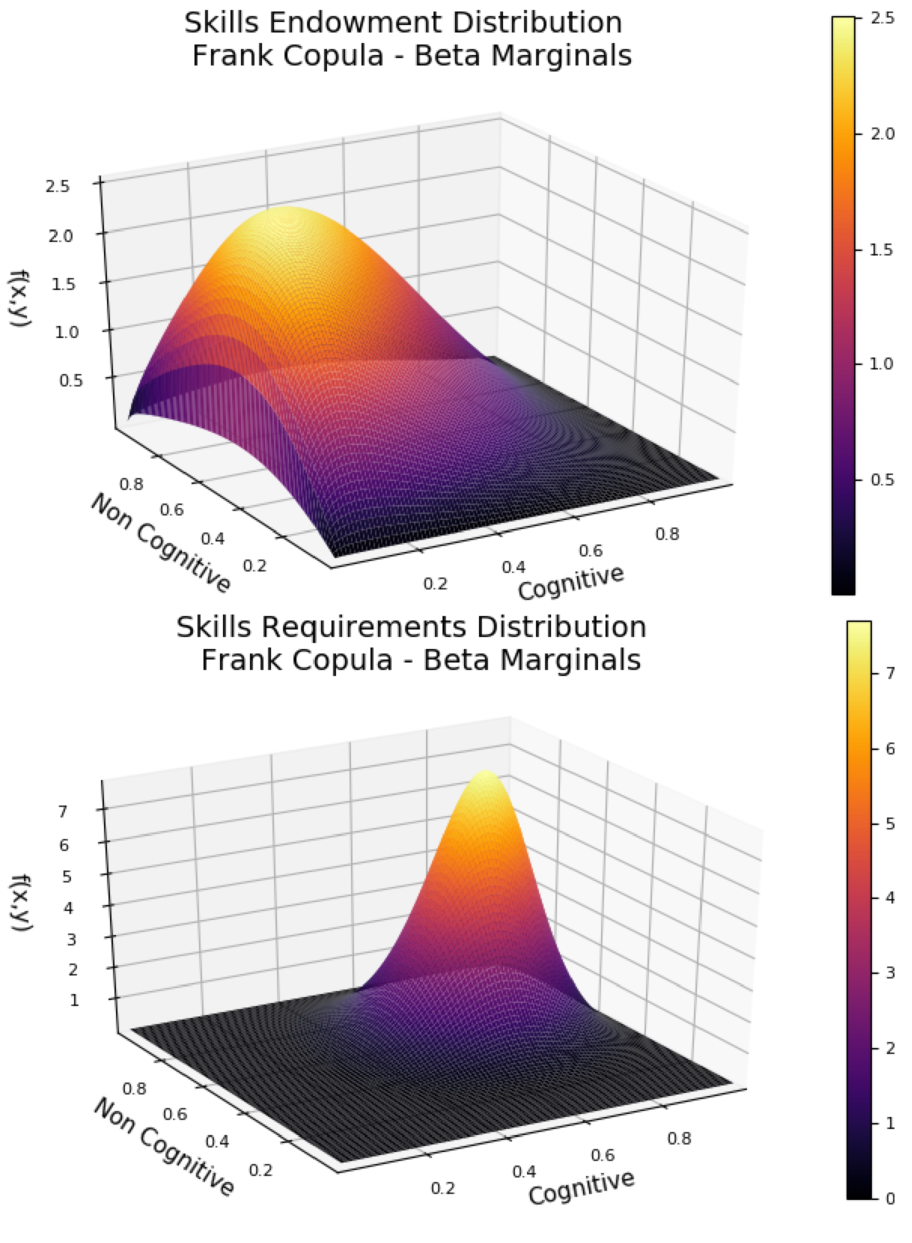
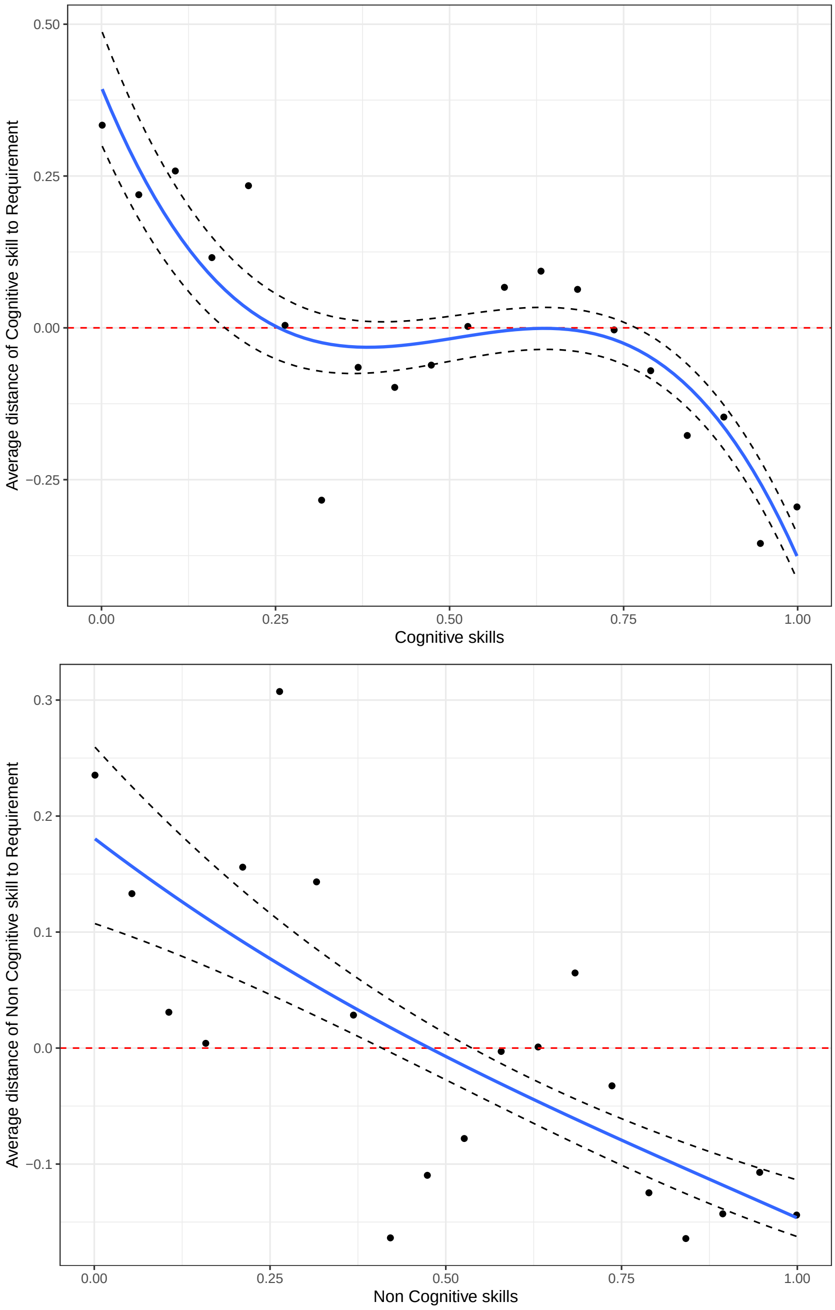


Figure 2.3: Average difference between skills by type



Panels (a) and (b) display the 3^{rd} order polynomial interpolation of the average difference between the requirements and endowments along the support of each type of skill. Panel (a) presents the information for cognitive skills, while panel (b) for non cognitive skills.

2.4.3 Skills importance in wage determination:

The wage posting strategy derived here can be used to characterize the wage at each point of the support of the multivariate distribution. To do this we need to know the primitive parameters of preferences and the production function, which can only be recovered via structural estimation. What would happen if, in the absence of such parameters, we estimated directly a reduced form regression the effect of skills on wages?

To answer this question we estimate a simple OLS regression on the simulated data using the estimated data generating process of the form:

$$Y_i = \beta X_i + \epsilon_i \tag{2.16}$$

where Y_i represents the wage of individual i in our simulated sample. The explanatory variables are the different skills measures available in the career simulation: first the worker skills, second the skill requirements, and last the mismatch. The sample in consideration is last simulated period. We use this cross-section to perform our regressions, since is similar to the databases from which researchers typically calculate their estimates. We estimate the model for each of the skills available.

Table 2.2 presents the results of the different estimations. In the first column, worker skills are regressed on the wage. The weights of the cognitive skills are 50% higher than the non-cognitive skills. Nevertheless, the explanatory power of the model using worker skills is lower than the one that uses skill requirements. When considering skill requirements, we can not distinguish between the effect of cognitive and non cognitive, but the explanatory power is the highest. These two findings could be explained by the shapes of the distributions (see figure 2.2), as the joint density of worker skills is flatter and covers the whole support, while the the requirement distribution is concentrated in higher values. The last column in Table 2.2 presents the contribution of mismatch to wages. The effect of cognitive mismatch is not significant, while the non cognitive mismatch is significant. This could be reflecting the fact that we estimate that a smaller share of workers are actually mismatched on the cognitive skill dimension than the non cognitive dimension, and that skills mismatch on the non cognitive dimension is more penalizing for firms than on the cognitive dimension. Both values are small, in comparison to the regressions when we use worker skills and skills requirements.

Table 2.2: Contributions of skills endowments and requirements to wages

	$\hat{w}$ - Simulated Wage		
	(1)	(2)	(3)
Cognitive Skill	6.347^{***} (0.4574)		
Non Cognitive Skill	4.267^{***} (0.4523)		
Cognitive Requirement		10.259^{**} (4.1615)	
Non Cognitive Requirement		13.909^{***} (0.9974)	
Cognitive mismatch			-0.714 (0.6128)
Non Cognitive mismatch			-0.922^{***} (0.1477)
R^2	0.140	0.187	0.061

Note: $*\ \ P < 0.1; **\ \ P < 0.05; ***\ \ P < 0.01$

These results highlight the importance of accounting for mismatch in understanding the role of skills in wage determination. They suggest that a linear model that does not take into consideration the complete dynamic of the wage rule, would be unable to reproduce important sources of wage variation. In particular, neglecting the skills of either side of the employment relation ends up estimating only an average effect over the whole distribution, omitting the importance of firms' ability to segment the market. Even if the values are significant, they are unable to capture the rich story behind the way workers and firms match when skills are multidimensional.

2.5 Conclusion

In this paper, we provide a micro founded model of matching, sorting, and mismatch in a random search environment in which we introduce multidimensional heterogeneity of worker endowments and firm requirements. We provide a microeconomic narrative that extends our understanding of the matching and sorting process, adapted to a setting in which heterogeneous and multidimensional skill endowments and requirements characterize agents. In this setting, firms post a wage with commitment, independent of the type of workers that accept it. Wage setting becomes a strategic decision and the optimal wage rule considers the distribution of the types, the complementarity of skills in the production function, and the size and composition of the set of workers willing to accept the job. An increase in the posted wage increases the applicants pool size but might increase mismatch depending on the endowment distribution and preferences. We derive the wage rule both from the steady-state profits and the flow value of an unfilled job conditional on posted wage. From both equations, we get an equivalent result.

We then estimate the model for France. We find that the correlation between skills endowments is lower than the correlation for requirements. We also find that cognitive and non-cognitive skills are weak complements in production, which makes mismatch more costly. The estimation allows us to calculate mismatch at a granular level. When we analyze the degree of mismatch along the distribution of cognitive and non-cognitive skills endowments, we observe that the intensity of mismatch is larger for non cognitive skills than cognitive skills for the case of France. Good matches occur for the middle two quartiles of the cognitive skills distribution, while they are closer to the 10 percent of workers closer to the median for non cognitive skills distribution.

In sum, multidimensionality of skills plays an important role in matching and wage determination. Multidimensionality can aggravate problems of mismatch since cognitive and non cognitive skills are found to be complements in production and this affect wages. Finally because mismatch is costly to workers, it represent an additional friction in the labor market. The estimated offer arrival rate must therefore increase to compensate for such inefficiency.

A.1 Appendix I

In order to consider the margin and the interior solution, we will use a multidimensional version of differentiation under the integral, or the Leibniz Rule. This approach is well known in mathematics and physics (Flanders, 1973; Dieudonné, 1959), and recently has been used also in other fields of economics (Veiga and Weyl, 2012, 2016; Veiga et al., 2017)[9].

Definition : ***(Leibniz Rule)*** *Consider the function:*

$$G(w_j) = \int_{\boldsymbol{\Theta}:\tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} g(\boldsymbol{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta}$$

Differentiating this function with respect to w_j yields:

$$\begin{aligned} \frac{dG(w_j)}{dw_j} = & \int_{\boldsymbol{\Theta}:\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} \left(\frac{\partial}{\partial w_j} g(\mathbf{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) \right) d\boldsymbol{\theta} + \\ & + \int_{\boldsymbol{\Theta}:\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) = 0} \left(\frac{\partial \tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta})}{\partial w_j} \frac{1}{\left\| \frac{\partial \tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right\|} g(\mathbf{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) \right) d\tau \end{aligned} \tag{A.1}$$

The effect of a change in wages, following equation A.1, can be divided in two parts:

- The first part is the average effect in the acceptance set with respect to wages and implies a transfer of the match productivity. It measures how the function is sensitive to changes in wages for a given set of multidimensional requirements.
- The second part is the effect on the boundary, and measures the effect on marginal individuals to accept or not the proposed offer, proportional to the marginal reservation of participation.

Is important to remark that under this setting, the maximization is in terms of the marginal profitable accepted worker, and the wage is the instrument that the firm has to segment the pool of unemployed to maximize profits. We follow the equilibrium description, presenting the results from the free entry condition and number of vacancies, and optimal wage.

[9] See derivation 7 in the appendix.

A.2 Appendix II

Theorem 1 (Divergence theorem) *We assume that* $\boldsymbol{\Theta}_j$ *is a compact domain of integration with a piecewise smooth boundary* $\partial\boldsymbol{\Theta}$*. If there is a function* $\boldsymbol{H}$ *that is continuous and differentiable vector field defined on the boundary of* $\boldsymbol{\Theta}_j$*, we then have:*

$$\int_{\underline{\theta}_K}^{\bar{\theta}_K} \cdots \int_{\underline{\theta}_1}^{\bar{\theta}_1} (\nabla \cdot \boldsymbol{H}) d\theta_1 ... d\theta_K = \int_{\boldsymbol{\Theta}_j} (\nabla \cdot \boldsymbol{H}) d\boldsymbol{\Theta}_j =$$

$$= \int_{\partial\boldsymbol{\Theta}_j} \boldsymbol{H} \cdot d\boldsymbol{\tau} =$$

$$= \int_{\partial\boldsymbol{\Theta}_j} \boldsymbol{H} \cdot \boldsymbol{n} d\tau$$

where $\boldsymbol{n}$ is the outward unit vector normal to the acceptance boundary surface $\boldsymbol{\Theta}$, and $d\tau$ is the element of the set. The first equality highlights the notation used in the paper. We use just use one integral even if dealing with a multidimensional space since we indicate that we are integrating over a n-dimensional set. The second line contains the definition of the theorem. The third line takes into consideration that outward-pointing normals orient the surface (closed manifold $\partial\boldsymbol{\Theta}$). The change of variable is made to consider that the space of the n-surface is $1-n$ dimensions, and the rotation makes it possible to write it in terms of the element scalar of the set. This definition is adapted from Weisstein (2002).

Moreover, to define $d\tau$ we take the definition of a general formula in Flanders (1973). In this definition, $d\boldsymbol{\tau}$ is the vectorial element on the boundary surface of $\partial\boldsymbol{\Theta}$, such that the resulting surface is oriented by outward pointing normals, so $d\boldsymbol{\tau} = \boldsymbol{n} d\tau$.

Derivation 2 (Unemployment value) *We start from the definition of the value of being unemployed:*

$$
\begin{aligned}
V_u &= \frac{1}{1+r\Delta t}\left[-\bar{b}\Delta t + (1-\lambda\Delta t)V_u + \right. \\
&\quad \left. + \lambda\Delta t \mathbf{E} \max\left\{_e, V_u\right\}\right] + o(\Delta t)(1+r\Delta t) \\
V_u + r\Delta t V_u &= -\bar{b}\Delta t + V_u - \lambda\Delta t V_u + \\
&\quad + \lambda\Delta t \mathbf{E} \max\left\{V_e, V_u\right\} + o(\Delta t)(1+r\Delta t) \\
r\Delta t V_u &= -\bar{b}\Delta t - \lambda\Delta t V_u + \\
&\quad + \lambda\Delta t \mathbf{E} \max\{V_e, V_u\} + o(\Delta t)(1+r\Delta t)
\end{aligned}
$$

We divide each side by Δt and we make $\Delta t \to 0$. Given the indeterminacy we apply L'Hôpital's rule so we can operate and introduce the value of being unemployed into the expectation.

$$
\begin{aligned}
rV_u &= \frac{-\bar{b}\Delta t - \lambda\Delta t V_u + \lambda\Delta t \mathbf{E} \max\{V_e, V_u\} + o(\Delta t)(1+r\Delta t)}{\Delta t} \\
rV_u &= -\bar{b} + \lambda \mathbf{E} \max\{V_e - V_u, V_u - V_u\} \\
rV_u &= -\bar{b} + \lambda \mathbf{E} \max\{V_e - V_u, 0\} \\
rV_u &= -\bar{b} + \lambda \mathbf{E}_{\{\tilde{U}(\cdot)\geq 0 | w_j; \boldsymbol{\theta}_i\}}\{V_e - V_u\}
\end{aligned}
$$

Which is equal to equation 2.1 in the paper. Note that here the value of the expectation is with respect to the cases when $\{\tilde{U}(\cdot) \geq 0 | w_j; \boldsymbol{\theta}\}$, that is when the value of the instantaneous utility is larger than or equal to the value of the outside option, conditional to the full information of firm types and their posted wages. The participation set then is defined by the set of all occupations in which the job seeker will accept to work given a proposed wage.

Derivation 3 (Employment value) *Taking the definition of the employment value in the* *text*

$$
\begin{aligned}
V_e(\boldsymbol{r}_j, \boldsymbol{\theta}, w_j) &= \frac{1}{1 + r\Delta t}\left[(w_j \Delta t - c(\boldsymbol{r}_j, \boldsymbol{\theta})\Delta t) + \right. \\
&\left. +(1 - \eta\Delta t)V_e(\boldsymbol{r}_j, \boldsymbol{\theta}, w_j) + \eta\Delta t V_u\right] + o(\Delta t) \\
(1 + r\Delta t)V_e(\boldsymbol{r}_j, \boldsymbol{\theta}, w_j) &= w_j \Delta t - c(\boldsymbol{r}_j, \boldsymbol{\theta})\Delta t + \\
+(1 - \eta\Delta t)V_e(\boldsymbol{r}_j, \boldsymbol{\theta}, w_j) &+ \eta\Delta t V_u + o(\Delta t)(1 + r\Delta t) \\
V_e(\boldsymbol{r}_j, \boldsymbol{\theta}, w_j) + r\Delta t V_e(\boldsymbol{r}_j, \boldsymbol{\theta}, w_j) &= w_j \Delta t - c(\boldsymbol{r}_j, \boldsymbol{\theta})\Delta t + V_e(\boldsymbol{r}_j, \boldsymbol{\theta}, w_j) - \\
&- \eta\Delta t V_e(\boldsymbol{r}_j, \boldsymbol{\theta}, w_j) + \eta\Delta t V_u + o(\Delta t)(1 + r\Delta t)
\end{aligned}
$$

We then group similar terms and divide by $(r + \eta)\Delta t$.

$$
\begin{aligned}
(r + \eta)\Delta t V_e(\mathbf{r}_j, \boldsymbol{\theta}, w_j) &= (w_j - c(\mathbf{r}_j, \boldsymbol{\theta}_i) + \eta V_u)\Delta t + \frac{o(\Delta t)(1 + r\Delta t)}{(r + \eta)\Delta t} \\
V_e(\mathbf{r}_j, \boldsymbol{\theta}_i, w_j) &= \frac{(w_j - c(\mathbf{r}_j, \boldsymbol{\theta}) + \eta V_u)\Delta t}{(r + \eta)\Delta t} + \frac{o(\Delta t)(1 + r\Delta t)}{(r + \eta)\Delta t}
\end{aligned}
$$

Which, after simplification, and sending Δt to 0 is equal to equation 2.2 in the text.

$$
V_e(\mathbf{r}_j, \boldsymbol{\theta}, w_j) = \frac{w_j - c(\mathbf{r}_j, \boldsymbol{\theta}) + \eta V_u}{(r + \eta)}
$$

Derivation 4 (Unfilled job value) *Starting from the definition in the text:*

$$
\begin{aligned}
\Upsilon_j = \frac{1}{1 + r\Delta t}&\Bigg[-k\Delta t + (1 - q(\omega)\Delta t)\Upsilon_j + \\
&+ q(\omega)\Delta t \int \max\{J(\boldsymbol{r}_j, \boldsymbol{\theta}), \Upsilon_j\} d\boldsymbol{F}(\boldsymbol{\theta})\Bigg] + o(\Delta t)
\end{aligned}
$$

$$
\begin{aligned}
(1+r\Delta t)\Upsilon_j &= -k\Delta t + (1-q(\omega)\Delta t)\Upsilon_j + \\
&+ q(\omega)\Delta t \int \max\{J(\mathbf{r}_j,\boldsymbol{\theta}),\Upsilon_j\} d\boldsymbol{F}(\boldsymbol{\theta}) + o(\Delta t) \\
\Upsilon_j + r\Delta t\Upsilon_j &= -k\Delta t + \Upsilon_j - q(\omega)\Delta t\Upsilon_j + \\
&+ q(\omega)\Delta t \int \max\{J(\mathbf{r}_j,\boldsymbol{\theta}),\Upsilon_j\} d\boldsymbol{F}(\boldsymbol{\theta}) + o(\Delta t)(1+r\Delta t) \\
r\Delta t\Upsilon_j &= -k\Delta t - q(\omega)\Delta t\Upsilon_j + \\
&+ q(\omega)\Delta t \int \max\{J(\mathbf{r}_j,\boldsymbol{\theta}),\Upsilon_j\} d\boldsymbol{F}(\boldsymbol{\theta}) + o(\Delta t)(1+r\Delta t)
\end{aligned}
$$

Dividing by Δt and passing it to the limit we get that:

$$
r\Upsilon_j = -k - q(\omega)\Upsilon_j + q(\omega)\int \max\{J(\mathbf{r}_j,\boldsymbol{\theta}),\Upsilon_j\} d\boldsymbol{F}(\boldsymbol{\theta})
$$

$$
\begin{aligned}
r\Upsilon_j &= -k + q(\omega)\int \max\{J(\mathbf{r}_j,\boldsymbol{\theta})-\Upsilon_j,\Upsilon_j-\Upsilon_j\} d\boldsymbol{F}(\boldsymbol{\theta}) \\
r\Upsilon_j &= -k + q(\omega)\int\limits_{\boldsymbol{\Theta}_j} \max\{J(\mathbf{r}_j,\boldsymbol{\theta})-\Upsilon_j,0\} d\boldsymbol{F}(\boldsymbol{\theta}) \\
r\Upsilon_j &= -k + q(\omega)N_j\mathbb{E}\left[\max\{J(\mathbf{r}_j,\boldsymbol{\theta})-\Upsilon_j\}|\boldsymbol{\Theta}_J\right]
\end{aligned}
$$

$$
r\Upsilon_j = -k + q(\omega)N_j\mathbb{E}\left[\max\{J(\mathbf{r}_j,\boldsymbol{\theta})-\Upsilon_j\}|\boldsymbol{\Theta}_j\right]
$$

This last expression provides the value function of an unfilled vacancy, presented in equation 2.3 in the text. An equivalent definition without the conditional expectation is written below.

$$
r\Upsilon_j = -k + q(\omega)\int\limits_{\boldsymbol{\Theta}_j} [J(\mathbf{r}_j,\boldsymbol{\theta})-\Upsilon_j] f(\boldsymbol{\theta}) d\boldsymbol{\theta}
$$

Derivation 5 (Filled job value) *We start by the definition of the filled value of the vacancy* *presented in the text.*

$$J(\mathbf{r}_j,\boldsymbol{\theta}) = \frac{[m(\mathbf{r}_j,\boldsymbol{\theta}) - w_j]\Delta t}{1+r\Delta t} + \frac{(1-\eta\Delta t)}{1+r\Delta t}J(\mathbf{r}_j,\boldsymbol{\theta}) + \frac{\eta\Delta t}{1+r\Delta t}\Upsilon_j + o(\Delta t)$$

We multiply both sides by $(1+r\Delta t)$ and regroup all terms that have $J(\mathbf{r}_j,\boldsymbol{\theta})$ on the left hand side.

$$\begin{aligned}(1+r\Delta t)J(\mathbf{r}_j,\boldsymbol{\theta}) &= [m(\mathbf{r}_j,\boldsymbol{\theta}) - w_j]\Delta t + (1-\eta\Delta t)J(\mathbf{r}_j,\boldsymbol{\theta}) + \\ &+ \eta\Delta t\Upsilon_j + o(\Delta t)(1+r\Delta t)\\ J(\mathbf{r}_j,\boldsymbol{\theta}_i) + r\Delta t J(\mathbf{r}_j,\boldsymbol{\theta}) &= [m(\mathbf{r}_j,\boldsymbol{\theta}) - w_j]\Delta t + J(\mathbf{r}_j,\boldsymbol{\theta}) - \\ &- \eta\Delta t J(\mathbf{r}_j,\boldsymbol{\theta}) + \eta\Delta t\Upsilon_j + o(\Delta t)(1+r\Delta t)\end{aligned}$$

Grouping common terms and then solving for $J(\mathbf{r}_j,\boldsymbol{\theta})$, we have:

$$\begin{aligned}(\eta+r)\Delta t J(\mathbf{r}_j,\boldsymbol{\theta}) &= \{[m(\mathbf{r}_j,\boldsymbol{\theta}) - w_j] + \eta\Upsilon_j\}(\Delta t) + o(\Delta t)(1+r\Delta t)\\ J(\mathbf{r}_j,\boldsymbol{\theta}) &= \frac{[m(\mathbf{r}_j,\boldsymbol{\theta}) - w_j] + \eta\Upsilon_j}{(\eta+r)} + \frac{o(\Delta t)(1+r\Delta t)}{(\eta+r)\Delta t}\end{aligned}$$

After passing the limit Δt to 0, we get equation 2.4 in the text.

$$J(\mathbf{r}_j,\boldsymbol{\theta}) = \frac{m_j(\boldsymbol{\theta}) - w_j + \eta(\Upsilon_j)}{(r+\eta)}$$

Derivation 6 (Conditional Flow Vacancy) *Take equation 2.4 and replace the value of a filled job in equation 2.3. We get:*

$$\begin{aligned}r\Upsilon_j &= -k + q(\omega)N_j\mathbb{E}\left[\frac{[m(\boldsymbol{r}_j,\boldsymbol{\theta}) - w_j] + \eta\Upsilon_j}{(\eta+r)} - \Upsilon_j|\boldsymbol{\Theta}_j\right]\\ r\Upsilon_j &= -k + \frac{q(\omega)N_j}{(\eta+r)}\mathbb{E}\left[[m(\boldsymbol{r}_j,\boldsymbol{\theta}) - w_j] - r\Upsilon_j|\,\boldsymbol{\Theta}_j\right]\end{aligned}$$

Equation 2.5 is the flow value of a vacancy given the posted wage w_j and it can be derived easily with a simple manipulation. Since the expected value can be operated linearly, and since the value of the vacant job does not depend on the participation set, we can write:

$$
\begin{aligned}
r\mathrm{Y}_j &= -k + \frac{q(\omega)N_j}{(\eta+r)}\mathbb{E}\left[\left[m(\mathbf{r}_j,\boldsymbol{\theta}_i) - w_j\right]|\boldsymbol{\Theta}_j\right] - \\
&\quad - \frac{q(\omega)N_j r}{(\eta+r)}\mathrm{Y}_j \\
r\mathrm{Y}_j\left(\frac{(\eta+r)+N_j q(\omega)}{\eta+r}\right) &= -k + \frac{q(\omega)N_j}{(\eta+r)}\mathbb{E}\left[\left[m(\mathbf{r}_j,\boldsymbol{\theta}_i) - w_j\right]|\boldsymbol{\Theta}_j\right] \\
r\mathrm{Y}_j\left((\eta+r)+N_j q(\omega)\right) &= -k(\eta+r)+ \\
&\quad + q(\omega)N_j\mathbb{E}\left[\left[m(\mathbf{r}_j,\boldsymbol{\theta}_i) - w_j\right]|\boldsymbol{\Theta}_j\right]
\end{aligned}
$$

Finally, solving for $r\mathrm{Y}_j$ we arrive at equation 2.5 presented in the text.

$$
r\mathrm{Y}_j = \frac{-k(\eta+r) + q(\omega)N_j\mathbb{E}\left[\left[m(\mathbf{r}_j,\boldsymbol{\theta}_i) - w_j\right]|\boldsymbol{\Theta}_j\right]}{(\eta+r)+N_j q(\omega)}
$$

Derivation 7 (Leibniz Rule) *Consider the function:*

$$
G(w_j) = \int_{\boldsymbol{\Theta}:\tilde{U}(\boldsymbol{r}_j,w_j,\bar{b};\boldsymbol{\theta})\geq 0} g(\boldsymbol{r}_j, w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})d\boldsymbol{\theta}
$$

For the defined function $G(w_j)$, using the Leibniz multidimensional rule of differentiation under the integral leads to[10]:

$$
\begin{aligned}
\frac{dG(w_j)}{dw_j} &= \int_{\boldsymbol{\Theta}:\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\geq 0}\left(\frac{\partial}{\partial w_j}g(\mathbf{r}_j, w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})\right)d\boldsymbol{\theta} + \\
&+ \int_{\boldsymbol{\Theta}:\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})=0}\left(g(\mathbf{r}_j, w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})\right)\left(\nabla_{w_j}\boldsymbol{\theta}\cdot d\boldsymbol{\tau}\right)
\end{aligned}
$$

Here the gradient $\nabla_{w_j}\boldsymbol{\theta}$ is the velocity at which the boundary changes when changing w_j. In this definition, $d\boldsymbol{\tau}$ is the vectorial element on the boundary surface of $\partial\Theta$ such that $d\boldsymbol{\tau} = \boldsymbol{n}d\tau$, $\boldsymbol{n}$ is the outward unit vector normal to the acceptance boundary surface $\boldsymbol{\Theta}$, and $d\tau$ is the element of the set. Replacing the above equivalence, we get:

[10] In this definition we follow the appendix of (Veiga and Weyl, 2012, 2016), but we take the definition of a general space formula in Flanders (1973)

$$
\begin{aligned}
\frac{dG(w_j)}{dw_j} &= \int_{\boldsymbol{\theta}:\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\geq 0} \left(\frac{\partial}{\partial w_j} g(\mathbf{r}_j,w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})\right) d\boldsymbol{\theta}+ \\
&+ \int_{\boldsymbol{\theta}:\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})=0} \Big(g(\mathbf{r}_j,w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})(\nabla_{w_j}\boldsymbol{\theta}\cdot\boldsymbol{n})\Big)\, d\tau
\end{aligned}
$$

Considering the outward velocity (divergence times the outward unit normal) of the boundary at each point, and with the definitions above we can write:

$$
\begin{aligned}
\nabla_{w_j}\boldsymbol{\theta}\cdot\boldsymbol{n} = \nabla_{w_j}\boldsymbol{\theta}\cdot \frac{\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\|\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\|} &= \\
= \frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial w_j}\frac{\nabla_{\tilde{U}}\boldsymbol{\theta}\cdot\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\|\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\|} &= \\
= \frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial w_j}\frac{1}{\|\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\|} &= \\
= \frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial w_j}\frac{1}{\left\|\frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right\|}&
\end{aligned}
$$

Replacing the result in the main equation leads us to equation A.1 in the paper.

$$
\begin{aligned}
\frac{dG(w_j)}{dw_j} &= \int_{\boldsymbol{\theta}:\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\geq 0} \left(\frac{\partial}{\partial w_j} g(\mathbf{r}_j,w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})\right) d\boldsymbol{\theta}+ \\
&+ \int_{\boldsymbol{\theta}:\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})=0} \left(g(\mathbf{r}_j,w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})\frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial w_j}\frac{1}{\left\|\frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right\|}\right) d\tau
\end{aligned}
$$

Derivation 8 (Optimal posted wage) *Using the definition of the conditional operator for the applicants pool in equation 2.3, we can rewrite it as:*

$$
kv = \frac{\int_{\boldsymbol{\theta}:\tilde{U}(\boldsymbol{r}_j,w_j,\bar{b};\boldsymbol{\theta})\geq 0} g(\boldsymbol{r}_j,w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})d\boldsymbol{\theta}}{N_j}\lambda(v_j)\frac{N_j}{\eta+r}
$$

Using the result in eq. A.1

$$
\frac{\partial kv}{\partial w_j} = \int_{\boldsymbol{\Theta}_j} -f(\boldsymbol{\theta})d\boldsymbol{\theta} + \int_{\partial\boldsymbol{\Theta}} \left((m(\mathbf{r}_j,\boldsymbol{\theta})-w_j)\frac{\partial\tilde{U}}{\partial w_j}\frac{1}{\left\|\frac{\partial\tilde{U}}{\partial\boldsymbol{\theta}}\right\|}\right) d\tau = 0
$$

Using the definition of the conditional expected value we have that:

$$
\begin{aligned}
0 &= -N_j + M_j \frac{\int_{\partial\boldsymbol{\Theta}} \left((m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \frac{\partial \tilde{U}}{\partial w_j} \frac{1}{\left\| \frac{\partial \tilde{U}}{\partial \boldsymbol{\theta}} \right\|} \right) d\tau}{\int_{\partial\boldsymbol{\Theta}_j} \frac{f(\boldsymbol{\theta}_j)}{\left\| \nabla_{\boldsymbol{\theta}_j} \tilde{U} \right\|} d\tau} \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} (m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \middle| \partial\boldsymbol{\Theta}_j \right] \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial\boldsymbol{\Theta}_j \right] - M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} w_j \middle| \partial\boldsymbol{\Theta}_j \right] \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial\boldsymbol{\Theta}_j \right] - w_j M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial\boldsymbol{\Theta}_j \right]
\end{aligned}
$$

Using the definition of the covariance we can rewrite $E[XY] = Cov[X,Y] + E[X]E[Y]$ for the second term and get:

$$
\begin{aligned}
0 = -N_j + M_j Cov\left[\frac{\partial \tilde{U}}{\partial w_j}, m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial\boldsymbol{\Theta}_j \right] + \\
+ M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial\boldsymbol{\Theta}_j \right] \mathbb{E}\left[m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial\boldsymbol{\Theta}_j \right] - w_j M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial\boldsymbol{\Theta}_j \right]
\end{aligned}
$$

Derivation 9 (Optimal posted wage - Alternative) *Using the definition of the conditional operator for the applicants pool in equation 2.13, we can rewrite it as:*

$$
\Pi_j(\boldsymbol{r}_j, \boldsymbol{\theta}_i; w_j) = \max_{w} \frac{\int_{\boldsymbol{\theta}: \tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} g(\boldsymbol{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta}}{N_j} N_j s(\boldsymbol{r}_j)
$$

Using the result in eq. A.1

$$
\frac{\partial \Pi_j}{\partial w_j} = \int_{\boldsymbol{\Theta}_j} -f(\boldsymbol{\theta}) d\boldsymbol{\theta} + \int_{\partial\boldsymbol{\Theta}} \left((m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \frac{\partial \tilde{U}}{\partial w_j} \frac{1}{\left\| \frac{\partial \tilde{U}}{\partial \boldsymbol{\theta}} \right\|} \right) d\tau = 0
$$

Using the definition of the conditional expected value we have that:

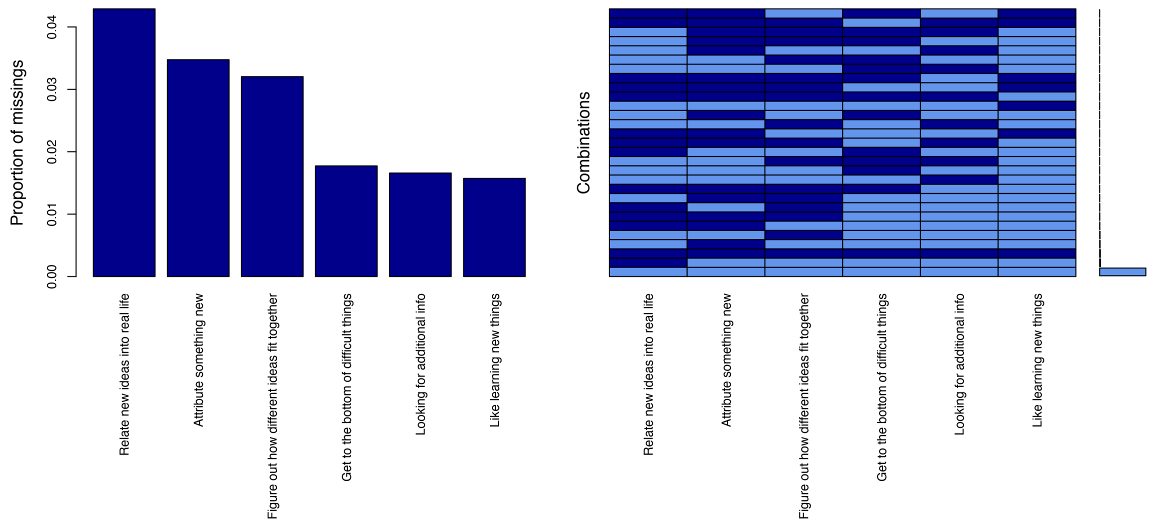
$$
\begin{aligned}
\frac{\partial \Pi_j}{\partial w_j} &= -N_j + M_j \frac{\int_{\partial \boldsymbol{\Theta}} \left((m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \frac{\partial \tilde{U}}{\partial w_j} \frac{1}{\left\| \frac{\partial \tilde{U}}{\partial \boldsymbol{\theta}} \right\|} \right) d\tau}{\int_{\partial \boldsymbol{\Theta}_j} \frac{f(\boldsymbol{\theta}_j)}{\left\| \nabla_{\boldsymbol{\theta}_j} \tilde{U} \right\|} d\tau} \\
&= -N_j + M_j \mathbb{E} \left[\frac{\partial \tilde{U}}{\partial w_j} (m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \middle| \partial \boldsymbol{\Theta}_j \right] \\
&= -N_j + M_j \mathbb{E} \left[\frac{\partial \tilde{U}}{\partial w_j} m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial \boldsymbol{\Theta}_j \right] - M_j \mathbb{E} \left[\frac{\partial \tilde{U}}{\partial w_j} w_j \middle| \partial \boldsymbol{\Theta}_j \right] \\
&= -N_j + M_j \mathbb{E} \left[\frac{\partial \tilde{U}}{\partial w_j} m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial \boldsymbol{\Theta}_j \right] - w_j M_j \mathbb{E} \left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial \boldsymbol{\Theta}_j \right]
\end{aligned}
$$

Using the definition of the covariance we can rewrite $E[XY] = Cov[X, Y] + E[X]E[Y]$ for the second term and get:

$$
\begin{aligned}
\frac{\partial \Pi_j}{\partial w_j} = -N_j + M_j Cov \left[\frac{\partial \tilde{U}}{\partial w_j}, m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial \boldsymbol{\Theta}_j \right] + \\
+ M_j \mathbb{E} \left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial \boldsymbol{\Theta}_j \right] \mathbb{E} \left[m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial \boldsymbol{\Theta}_j \right] - w_j M_j \mathbb{E} \left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial \boldsymbol{\Theta}_j \right] = 0
\end{aligned}
$$

A.3 Appendix III

Figure A.1: Patterns of missingness for Non Cognitive questions



Source: PIAAC France 2012

Dimension	Variable	Weight
Plausible value - Numeric	PVNUM1	0.763
Plausible Value - Literacy	PVLIT1	0.646

	Variable	Factor1
Relate new ideas into real life	I_Q04b	0.581
Like learning new things	I_Q04d	0.681
Attribute something new	I_Q04h	0.485
Get to the bottom of difficult things	I_Q04j	0.723
Figure out how different ideas fit together	I_Q04l	0.728
Looking for additional info	I_Q04m	0.612

Table A.1: Weighted and Unweighted moments - France

	$Moment_w$	$Moment_u$
Cognitive skill - Q_{10}	0.382	0.380
Cognitive skill - Q_{25}	0.483	0.488
Cognitive skill - Q_{50}	0.583	0.586
Cognitive skill - Q_{75}	0.669	0.673
Cognitive skill - Q_{90}	0.738	0.743
Non-cognitive skill - Q_{10}	0.477	0.485
Non-cognitive skill - Q_{25}	0.592	0.596
Non-cognitive skill - Q_{50}	0.685	0.685
Non-cognitive skill - Q_{75}	0.775	0.779
Non-cognitive skill - Q_{90}	0.881	0.881
Cognitive Requirement - Q_{10}	0.096	0.096
Cognitive Requirement - Q_{25}	0.138	0.138
Cognitive Requirement - Q_{50}	0.280	0.280
Cognitive Requirement - Q_{75}	0.415	0.415
Cognitive Requirement - Q_{90}	0.558	0.558
Non - Cognitive Requirement - Q_{10}	0.156	0.205
Non - Cognitive Requirement - Q_{25}	0.328	0.328
Non - Cognitive Requirement - Q_{50}	0.612	0.634
Non - Cognitive Requirement - Q_{75}	0.700	0.732
Non - Cognitive Requirement - Q_{90}	0.947	0.947
Hourly wages - Q_{10}	7.910	7.972
Hourly wages - Q_{20}	9.891	9.971
Hourly wages - Q_{30}	10.889	10.944
Hourly wages - Q_{40}	11.310	11.455
Hourly wages - Q_{50}	11.857	12.171
Hourly wages - Q_{60}	12.689	13.117
Hourly wages - Q_{70}	15.807	16.315
Hourly wages - Q_{80}	17.544	17.712
Hourly wages - Q_{90}	21.481	22.478
Mean Employment	0.861	0.881

Chapter 3

Selective displacement and workforce restructuring during a mass layoff[1]

3.1 Introduction

In the aftermath of the great recession, the low-skilled have been affected considerably more than those with more skills at the macro level, both in terms of job loss and longer unemployment spells (OECD, 2013). The current economic downturn due to COVID-19 has led to a similar situation, with a potentially higher incidence for low skilled workers Mongey and Weinberg (2020). Policymakers need to know which factors determine job separations in order to create tailored programs for the displaced workers. This paper presents evidence of workforce restructuring at the firm level when downsizing occurs and characterizes the selective displacement of workers during a mass layoff, focusing on the role of skill mismatch.

A long stream of literature had studied the effects of job displacement on workers' labor market outcomes. The results of the seminal work of Jacobson, LaLonde and Sullivan (1993), where displacement had huge long term effects in labour earnings, have been confirmed across time (Davis and Von Wachter, 2011) and countries (Bertheau et al., 2021; Seim, 2019). The long-term effects on displaced workers' earnings have also been studied previously in France

[1]This chapter is the product of joint work with David Margolis.

(Bender et al., 2002; Bertheau et al., 2021), and recent research tries to unveil the sources of such losses (Schmieder et al., 2018; Brandily et al., 2020). Previous literature has focused mainly on worker outcomes (except some research in which value-added per worker that is also an outcome that is taken into account). However, less is known on the firm's role and decisions around displacement when the firm does not disappear entirely (Gibbons and Katz, 1991). If displacement affects the composition of a firm's workforce, it will also affect the firm's productivity and its ability to absorb different types of labor.

This paper discusses mass layoffs from the firm's perspective and addresses two different questions. First, do firms use mass layoffs to restructure their workforces? Second, how do firms choose which workers to layoff? We focus on worker skills and, for this second question, we outline the importance of the factors that directly affect the value of the employment relationship and specifically the role of skills mismatch, defined as the difference between the skills the worker provides and the requirements of the job that he performs in the firm.

A comparison of workforce composition between 30 years ago and today shows that the skill composition of the workforce has changed. For example, there is evidence at the macro level that medium-skill routine jobs have disappeared (Autor and Dorn, 2009). Such a restructuring of the labor force is often explained by a change in the economic activity at the sector level (Goos et al., 2011). However, given that the firm's occupational structure plays an essential role in its productivity (Simon, 1962; Michaels et al., 2014), one could imagine that within variation should also be important. How the firm organizes the human capital it employs has an impact on how productive and competitive it is, and reorganization of the firm might occur due to a multitude of factors: the firm's life-cycle, its use of technology, offshorability, or managerial styles, for example. There is also evidence of workforce restructuring across Europe. Harrigan et al. (2020) shows that ICT occupations have increasing weight in the structure of occupations, and France is likely not an exception.

Often, long periods of time are required to evaluate changes in organization and the structural composition of employment. However, if firm uses mass layoff periods to adjust and restructure its workforce, we could see reorganization occur more rapidly. The strategic use of mass layoff to adjust workforce composition has been less studied, but given the legal constraints and the high cost of firing, once a firm has concluded that it is optimal to incur adjustment costs (especially fixed adjustment costs), it can use such moments to undertake adjustments that would have been too costly to make on continuous basis. In France, where the firing cost function is concave in the number of layoffs (Abowd and Kramarz, 2003), such behavior seems natural.

In order to examine the firm's strategic behavior during a mass layoff, we first test if the firm restructures its workforce in a shorter period when undergoing a mass layoff, or iff it lays off all workers with equal probability.To do this, we identify a set of mass layoff firms using french administrative data on the universe of private sector jobs and firms (DADS postes). In selecting this sample, we do not differentiate between separations for economic or other reasons, but we identify the mass layoff based on changes in the firm's workforce size. We then study how the occupational composition and average skill use within a firm changes during a mass layoff. We find evidence of firm reorganization in the firm's skill structure, finding a small and significant increase in the use of social skills, a small and significant reduction of manual skills, and a positive and non-significant increase of cognitive skills. To explore how this mechanism operates, we then explore selective displacement.

How do firms decide which workers are fired when they decide they need to downsize? If firms use mass layoffs to re-organize their workforce, selecting the workers that must leave the firm becomes a strategic decision. What are the factors that enter into this decision? Bender et al. (2002) investigate the importance of age, tenure, and education for selective displacement for France. Seim (2019) studies the role of skills in determining layoff risk, finding that an increase in one standard deviation in cognitive and non-cognitive skills reduces the likelihood of being laid off by around 1%. Our results complement the literature on selective displacement, finding that skill *mismatch* and compensation cost plays an important role in determining who is fired. The result is robust to different specifications, even when we control for demographic characteristics, firm characteristics, and firm and year fixed effects. The results are also robust when calculated in a sub sample (one third of full sample size), where we observe family characteristics.

The paper proceeds as follows. Section 3.2 presents an economic analysis of why firms want to displace workers. Section 3.3 describes the data sources used, and describes the samples under consideration. Sections 3.4 and 3.5 presents the empirical results on firm re-organization and the role of skills mismatch in determining the selective displacement. Section 3.6 concludes the paper.

3.2 Why do firms want to downsize?

All firms face ups and downs during their life cycle. What are the main factors that induce firms to reduce their workforce? The economic and management literature has provided various explanations for the factors that determine mass layoffs, ranging from productivity shocks, a need for firm re-organization, to cost structure changes. Depending on the reason for the mass layoff, the firm will make different actual decisions and these will determine the resulting productivity and workforce structure after a layoff process.

How displacement works depends on the stability of the match. A static model of separations would consider firms and workers, calculating in each period the value of match continuation. Each party would compare its surplus share against its outside option of terminating the employment relationship. While employed, workers compare their share of the match surplus to the outside option value, which is the value of unemployment in a model without on-the-job search. Firms compare the value of production from the match to its net cost (wage plus other employment costs).

The stability of this relation can change with a productivity shock. When wages are negotiated in each period and the total value of production from the match is still positive, the firm will be willing to continue the employment relationship (after renegotiating wages and seeing the post-shock value of production) as long as its share of the surplus is positive. However, it may be the case that an acceptable renegotiated wage for the firm, although positive, would be lower than the outside option and would result in a voluntary separation. When dealing with multiple and heterogeneous skills, further consideration must be taken into account. How skills enter into a production function and how they affect productivity also influences the likelihood of separation, especially when workers have heterogeneous skills (Lise and Robin, 2017). Wage renegotiation might happen using several mechanisms that depend on the expected productivity, worker inputs considered in the match, and firm inputs that enter the match value function. For example, Postel-Vinay and Turon (2010) consider that the renegotiation will happen if one of the parties has a credible outside option and the new surplus generated is higher than the sum of outside options.

In practice, however, such types of wage adjustments may not be feasible due to regulation, long-term contracts, and the existence of internal labor markets. First, consider the case of regulation. A binding minimum wage will prevent too large of a downward wage adjustment, resulting in a layoff. As such, competitive labor market models in the literature (Mortensen and Pissarides, 1994) imply that an increase in minimum wages will increase separations since there will be fewer profitable matches. Such a view contradicts the findings of more general models such as Dube et al. (2016), where an increase in the minimum wage decreases the number of layoffs.

A wage cut could be also unfeasible in the presence of an agreement between the two parties. In case of a formal agreement (collective or individual), the contract establishes a level compensation that can not be unilaterally modified. Such agreements can be informal, i.e. implicit contracts in which the worker expects a wage increase conditional on good effort or performance and/or investment of specific human capital (Jovanovic, 1979). Internal labor markets are an example of informal contracts, where incentive mechanisms result in vertical mobility within the firm and increasing wage profiles (Dohmen et al., 2004). In face of a negative productivity shock and the absence of wage cuts or wage renegotiation, worker displacement may be a rational option for the firm. This behavior implies that firm employment over time fluctuates with the overall conditions of the economy (Davis et al., 2012).

Nevertheless, firms do not change size only because of productivity shocks. The life-cycle of the firm may also play a role in the composition and size of its workforce. The type of knowledge that the firm requires in each phase of its development would determine the optimal occupational structure, the organization of work, and its labor productivity. For example, consider a firm that was recently established. It would be reasonable to think that it would invest a lot of resources in research and development, hiring high-skill workers with that objective in the first phase. A later phase of production would require different types of tasks and skills for the production of goods and services, thus having a different occupational composition. How the firm is composed of self-organized elements and how these elements interact have consequences for firm performance. This is not a new idea in economics, and is pervasive to the management literature. It can be traced back to Simon (1962). The management decision of corporate structure and strategy would thus have an impact on workforce composition and firm size.

Another factor that could explain a modification in the structure of the firm is technological change. Implementing new technology requires the adaptation of workers' skills and knowledge and can potentially impact how the firm is organized. For example, Michaels, Natraj and Van Reenen (2014) document the occupational structure change due to the adoption of ICT in 11 countries (including France) during 25 years. Blinder and Krueger (2013) also analyze the effect of technology and offshorability on the structure of occupations, finding significant effects for both, with the effects being larger for technology. In France's case, Harrigan, Reshef and Toubal (2020) show an occupational shift in the composition of workers in the period 1994 - 2007, where firms that employed "techies" in 1994 realized an overall skill upgrade at the end of the analyzed period.

Given that structural reorganization is a slow process, it has always been analyzed over a long time span. A mass layoff, in which a large share of the firm is displaced in a limited period of time, could serve as an opportunity to change the composition and structure of the firm's workforce more rapidly. Thus, the selective displacement can play an important role in re-organizing the firm, especially in changing the skill composition.

Of course, mass layoffs entail adjustment costs. It is a known result that an increase in termination costs, in the form of employment protection legislation, tends to reduce layoffs, but at the same time can reduce job creation. Such costs provide an incentive for labor hoarding, in which non-profitable employment relationships are maintained because the separation costs exceed the present discounted value of the profit gains from ending the employment relation. If the cost of displacement is a function that exhibits decreasing returns to scale, a mass layoff is an opportunity for the firm to get rid of expensive matches. Abowd and Kramarz (2003) investigate the incidence of firing and hiring cost in France and find that the separation function cost is indeed concave, and therefore it makes more sense for the firm dismiss workers by in large groups as opposed to individually.

These last three factors that might influence displacement have a common characteristic: all of them highlight situations where a worker's productivity is low compared to his/her cost. These considerations provide firms with an incentive to monitor the quality of the match between workers and jobs. One measure to calculate *match quality* is in terms of the opportunity cost of a filled job. For instance, firms can identify if the worker is *"too expensive"*, comparing his/her wage to that of the best alternative worker, or comparing the requirements of a job with the capacity of its occupant to perform these tasks. This idea is at the heart of our calculation of *skills mismatch*, used throughout this paper. Using the notions of cognitive and social skills required for each job, we measure the extent to which workers' skills coincide with skill requirements and the degree to which such differences influence the probability of displacement during a mass layoff.

3.3 Data

This section describes the sources of information used and how they are combined for our estimation purposes.

3.3.1 French administrative data

The analysis presented here relies on French social security records (DADS - Déclaration Annuelle des Données Sociales) collected by social security and tax authorities and covering the universe of non public sector workers and firms. We use a sample covering the 2003-2015 period, comprised of the administrative declarations that all employers complete for each employment spell for each worker in each establishment in each year. The data set contains detailed information at the level of the firm, establishment, and worker, and includes the stand and end dates of employment spells, measured to the day. We use two different administrative data sources in this paper: the DADS postes, and DADS-EDP panel.

DADS postes This database contains the universe of employed individuals in the non-public sector and uniquely identifies each worker, firm and establishment. Each observation describes the employment relationship in the current and previous year[2], allowing us to follow employment relationships through time[3]. An observation in the data to which we had access presents one employment relation per year, in which each registry provides information on up to two employment spells during the year with the same worker-establishment combination. It also contains information on the overall duration of employment, sex, occupational information, and

[2]The current or previous year information is missing if the person left the firm in the previous year or was hired by the firm in the current year, respectively.

[3]We are considering the firm true employment ("postes non annexes"). According to the information in the DADS guide, a job is considered in the DADS as non annex if the net remuneration is higher than 3 minimum wages (SMIC) per month, and the employment relationship is longer than 30 days, with an intensity of more than 120 hours.

wage [4]. The establishment-level data is aggregated to the level of the firm in all of our analyses.

We use DADS postes for two purposes: first, it allows us to determine the sample of firms undertaking mass layoffs. We follow them before and after an event to evaluate a change in their skill composition using changes in occupational structure. Second, we use the unique identifiers to identify displaced and not displaced workers involved in a mass layoff.

The variables used in our analysis are:

- The firm's unique identifier (SIREN).
- The individual's unique identifier.
- The start and end dates of each job spell.
- The total duration of job spells in the year.
- The number of job spells per year.
- The occupation[5].

Using the above information, we construct a measure of skill requirements for each occupation using the skill contents of the Occupational Information Network (O*Net), which contains information of job characteristics at the level of occupation. We merge a vector of skill requirements for each occupation into the DADS data, based on three types of skills: cognitive skills, social skills, and manual skills[6]. Aggregated to the firm level, these measures can give a sense of the firm's skill structure.

[4]We correct the start and end dates of the observed spells in case the spells are not consistent with the reported duration. Since each registry reports up to two employment spells per worker and firm, total employment duration does not coincide for some observations (around 5% of the total). As we know the number of distinct spells for each match, we correct such registries by adding the correct number of (approximately) equal length spells such that the total length of spells coincides with the reported length without making them overlap. Such correction allows us to calculate more precisely firm size and its variations.

[5]We construct a correspondence table that relates the french national occupation classification and the international occupation classification. We then re-categorize the occupation from PCS-82 and PCS-2003 to ISCO-08.

In cases when the occupation was missing or had errors, we use the information from the previous year. In case it is not available or it has errors, we rely on the socio-professional category (cs) (either in the year or the previous year). The CS is a more aggregated categorization that can be related to the occupation at an aggregate level. This makes missings in the occupation variable rare and sparse.

[6]We build the skill measures using all the skills information from O*NET, following Lise and Postel-Vinay (2020). Using principal component analysis (PCA) to reduce dimensionality, we construct a skill vector that describes every occupation's cognitive, social, and manual skill requirements. More details on how the occupation skills requirements are built can be found in appendix C.2.2.

DADS-EDP panel This data set merges the panel version of DADS and the permanent demographic sample (EDP - Echantillon Démographique Permanent). The DADS panel contains around 1/12 of the workers, formed by retaining all workers born in October, following them through all of their jobs and organizing the observations into a panel. Apart from the worker demographic variables (age, sex, seniority) and job characteristic variables (firm characteristics, wage, and occupation) that come from the panel, the data provides additional information on the educational attainment, civil status, and birth age of children collected from the census or other administrative records, such as birth and marriage certificates.

BIC -RN The BIC-RN (Bénéfice Industriels et Commerciaux - Régime Normal) data includes fiscal year information from the tax declarations and balance sheets of firms. Using this information, we calculate some fundamental financial indicators: value-added, return on investment, return on equity, and EBITDA. The BIC-RN data shares the firm identifier with the DADS data, allowing us to merge these sources.

3.3.2 PIAAC

The French workers' skill endowment information comes from the Programme for the International Assessment of Adult Competencies (PIAAC). The OECD developed the survey, and the data was collected for France between September and November 2012. The PIAAC provides internationally comparable data about the skills of the adult populations in 24 countries. The sample consists of adults between 16 and 65 years of age. The survey assigns 10 plausible values to each individual in the survey for both literacy and numeracy. A weight accompanies each plausible value.

The survey includes an assessment of cognitive skills in two main domains: literacy and numeracy. For literacy, the survey assesses how well people comprehend, evaluate, use, and engage with written texts. For numeracy, it assesses a person's ability to solve a problem in a real-world setting by relating it to mathematical data and ideas. It is worth noting that these are not self-declared measures but are derived from directly assessed raw test responses and other personal characteristics. The test was designed to accurately assess cognitive abilities by adjusting the questions' complexity and specifying the thresholds based on the individual's educational level and whether or not they are a native speaker. To evaluate each cognitive component, the test is divided into two stages, the first with nine tasks and the second with eleven tasks. PIAAC is based on an incomplete balanced block design, so not all individuals are evaluated on the same components.

Furthermore, since the test is adaptive and the respondent's results determine the questions' complexity, raw responses have missing values by design. The OECD suggests that the plausible values be used. Social skills measures are derived from the answers to the background questionnaire (BQ) of the survey. In this part, six questions about attitudes and interest toward learning are asked. These measures are related to personality and interpersonal skill areas.

We build a person's vector of cognitive skills by combining knowledge on literacy and numeracy. The questions in the BQ are combined to form a social skills assessment. A Factor Analysis was used to determine the composition's weights[7]. By combining the information on the identified questions from the BQ, we construct a unique vector that expresses each individual's social skill ability in the survey. Using a principal component analysis, we find the optimal weights that capture the largest part of the variance (see appendix C.2.1 for details).

3.3.3 Adding skills endowments into the DADS-EDP panel data

Due to the lack of skills measures in the French administrative data, direct calculation of the size of mismatch is practically impossible. To overcome such shortcomings, we therefore use the observable individual and firm characteristics common to the DADS-EDP and PIAAC data to combine the skill endowments of the individuals in the DADS - panel EDP. In order to combine the information of both data sets we follow Ridder and Moffitt (2007).

Such a proposal is made under the assumption that the joint distribution of skills and observable variables in the DADS-EDP and PIAAC samples is the same. Several reasons support this assumption. First, the PIAAC survey (the donor database) represents the French working population (as does the DADS-EDP data), so the relation between skills and observable characteristics should be maintained across the samples. Moreover, the PIAAC survey incorporates additional sources of uncertainty and variability, given that it provides plausible values and weights for the variables of interest. This allow us to avoid the risk that the imputed data variance is too small, as would be the case if the imputation were on conditional means and did not incorporate uncertainty (Little and Rubin, 2019)). Intuitively, the uncertainty derives from the error of the estimated combination model on the donor data set. In the case of multiply

[7]See appendix C.2.1 for details.

imputed surveys, we obtain the same number of estimated vectors as imputations. This makes PIAAC design more suitable for such combination, given that the plausible values account for uncertainty in the measurement. A final important consideration is that the two bases have common variables or variables that can be easily harmonized [8] across samples.

Using a stochastic regression imputation, we impute a conditional draw form the individual specific joint skills distribution into the DADS panel. The procedure is divided into the following steps:

(i) For each of the m multiple imputation samples in the PIAAC data, estimate a model that relates each one of the skills to the observable characteristics for each record. We select the observable characteristics that are available in both data sets and can be harmonized to the same categories.

$$S_i^m = \beta X_i + \epsilon_i$$

For this regression, we take into account worker demographics, job and firm characteristics. The model includes as demographic characteristics sex, a sixth degree polynomial on age, a third degree polynomial on seniority, and educational level. As job characteristics we include the logarithm of monthly earnings and the occupation (2-digit ISCO-08 level); as firm characteristics we include the size of the firm. Note that this model is intended to be descriptive and not causal, so the endogeneity of earnings and occupation are less problematic in this setting.

(ii) As result of this imputation we obtain a vector of estimated residuals $\hat{e}_i^m$ for each one of the 10 plausible values m. We also obtain m vectors of estimated coefficients β. For the imputation we used the average of the 10 models calculated $\tilde{\beta}$. Tables C3 - C4 (in the appendix) report the estimated coefficients and calculate the adjusted standard errors[9].

(iii) For each individual in the DADS-EDP sample we draw a value from $\hat{e}_i^m$. We indicate such draw with $\bar{e}_i^p$. We then combine the samples as in the two sample instrumental variable

[8] When referring to harmonization, we are taking into consideration the fact that both sources have the same categorizations and groupings and can be compared across samples. We also use the same level of detail of classification information and other adjustments.

[9] We adjust the standard errors to incorporate within and between variance.

approach (Ridder and Moffitt, 2007). The value of skills introduced into the DADS-EDP data then is:

$$\hat{S}_{it} = \tilde{\beta} X_{it}^{p} + \bar{e}_{i}^{p}$$

The combination in our case is divided then in two components. The first part correspond to the observable individual, job and firm characteristics in the DADS panel X^p, multiplied by $\tilde{\beta}$, the average coefficient across plausible values (this is the same approach that is advised by Avvisati and Keslair (2020), following the design of the PIAAC data). Even if this part seems deterministic, note that it already incorporates the uncertainty of the plausible values and their weights. The second part is stochastic and allows us to avoid the risk that the imputed data variance is too small (Little and Rubin, 2019).

(iv) Considering the missingness patterns of the data in the DADS-EDP panel, we run five different models. One model includes all the explanatory variables common to both data sets, and the four others capture the most common patterns of missingness in the data: missing hourly wages[10], missing occupation, missing firm size, and missing education. We repeat steps (i) to (iii) for each of the five models.

Skills cannot be imputed for some observations in the DADS-EDP panel due to a pattern of missingness that is not considered. These observations account for the 3% of the values in the worker sample and are excluded from the subsequent analysis.

3.3.4 Sample Description and Estimation

In order to investigate the two hypotheses of the paper, we construct two different samples. To test the composition change within the firm, we use a panel of mass layoff firms. To study the selective displacement, we use a panel of workers that worked in firms prior to a mass layoff event. Identification of mass layoffs is very important for the construction of such samples.

[10] In the PIAAC, the monthly wage is calculated from the hourly wage. To have an equivalent measure in the DADS panel, we use reported hours and wages. When reported hours are missing this can not be calculated and the value is missing.

What is a mass layoff?

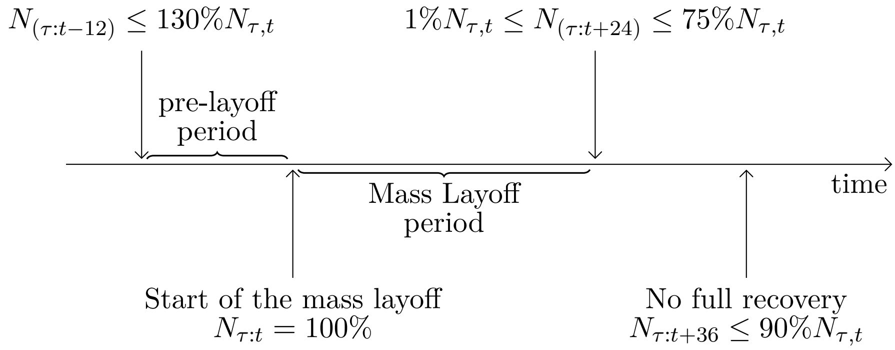
Both samples hinge entirely on the definition of mass layoff that is used. In this paper, we consider a mass layoff to have occurred when the following conditions are met: i) a firm at the start of the layoff period must have 50 employees or more[11], and ii) the firm's workforce contracts by between 25% and 99% in a two year period. The last condition avoids the possibility that we consider firms that disappear from the administrative records because they are merged or acquired by other firms, or for other problems in the processing and compilation process of the data (for example, a change in the firm identification number in the sample). iii) Among these firms, we only consider those for which the maximum employment the year before the start of the layoff period is less than 130% of the employment level at the start of the layoff. Using this condition, we take out firms in a steady decline, which helps us avoid classifying them in the mass layoff event. iv) To avoid capturing temporary fluctuations in firm employment level, we consider only firms which do not recover recent employment levels a year after the end of the layoff period. In particular, we consider only firms for which the employment a year after the mass layoff is less than 90% of the employment level one year before the start of the mass layoff period. In case a firm presents multiple layoff events, we consider only the first four. These conditions are very similar to those considered in the displacement literature (Lachowska et al., 2018; Davis and Von Wachter, 2011). It is important to note that this definition relies exclusively on employment stocks and flows, and not on whether the firm designates a separation as a layoff or not, as firms may choose to spread layoffs over time to avoid needing to apply the layoff legislation and incur extra costs[12]. The description of the selected firms is summarized in figure 3.1.

Such definition is also comparable with the recent literature on separations in France, which defined a mass layoff as occurring when the workforce reduces year to year by 10% or more (Royer, 2011; Brandily et al., 2020). It is also comparable with the management literature in which the 10% threshold is a reference point. This threshold usually describes severe workforce reduction (Datta et al., 2010). We chose a 25% threshold, however, to remain

[11] According to Davis and Von Wachter (2011) it is more challenging to identify mass layoffs in smaller firms as they are subject to higher percentage fluctuations. Since this paper is concerned with the firm's structure and composition, dropping small firms is less problematic.

[12] Not focusing on declared layoffs means that some employment variation can be due to voluntary departures, but the size threshold (at least a 25% reduction) should eliminate the risk of misclassification of voluntary departures as mass layoffs.

Figure 3.1: Mass layoff definition



close to the definition in Davis and Von Wachter (2011) (30%) and close to the cited literature when considered as a yearly change. Figures C.1-C.2 in the appendix show how variations in the threshold change the size of the sample with respect to the universe of firms in DADS postes. These figures also make clear that mass layoffs events are not distributed uniformly across months, especially when such thresholds are low, suggesting that low thresholds might disproportionately capture the seasonality of workforce variation.

Legal definition of a mass layoff in France

When we consider a mass layoff as a function of the size of the firm, there is not an equivalent definition in the French legislation. This makes that finding strictly comparable official statistics on firms that downsize impossible. The most similar legal indicator associated with a mass layoff, is the Employment Saving Plan ("Plan de Sauvegarde de l'emploi", or PSE). A PSE is an employment protection legislative requirement that is a function of the number of economic displacements in the firm that occur during a fixed period of time and the size of the firm. An economic displacement ("licencement economique") is a separation initiated by the firm, without the worker's consent, in which the firm must justify that the separation occurs for economic reasons (see Appendix C.3.1 for a detailed description of economic displacement). In practice, economic displacement is very costly.

To be required to propose a PSE, the firm must displace 10 or more employees for economic reasons during a period of 30 days. In order to reduce the risk that firms split their layoffs over a longer time span so as to remain under the threshold, the mechanism also requires a PSE if the firm lays off 10 workers in a 90 day period for economic reasons, or 18 during a calendar year. When the firm meets such conditions must put in place a PSE[13].

A PSE is composed by all the actions that the firm must put in place to limit the number of layoffs, in particular through re-qualification, re-skilling, and the creation of favorable conditions in local labor markets. It includes the internal reallocation of employees to jobs in the same or equivalent categories (within the firm or other firms with the same company group), measures to create better conditions of employment in local labor markets, the redistribution of overtime hours across the shifts of all the workers of the firm, and programs for skill upgrading for the affected workers. The implementation of a PSE is costly in time and resources for the firm. It is even more expensive when the costs associated to the economic displacement and the potential legal costs are taken in consideration.

When we compare the number of mass layoffs using the size of the firm (see table C.5), and the number of PSEs in firms with more than 50 employees (table C.15) it seems that firms might use other mechanisms to reduce their workforce, perhaps due to the high cost of the mechanism. But what could be the other alternatives? In particular, firms might adjust their workforces using other channels due to the high cost that economic displacements imply for the firm. It has been previously suggested that the firm might adjust its size by reducing its hiring rate and not by increasing its separations rate (Abowd and Kramarz, 2003; Fraisse et al., 2015). Given this option is available to many firms, downsizing might take place through a combination of economic displacements and the adjustment of in- and outflows from the firm.

The economic displacement definition involves taking into consideration only involuntary separations. By using adjustments in firm size, we are considering all types of separations, including voluntary (worker initiated quits), accidental (deaths), or legal (termination of a fix term contract, by worker leaving the firm because he arrived to the pension age, or separation with cause). In all cases, we observe the destruction of a job in a specific occupation that is not filled again by other worker.

Sample description

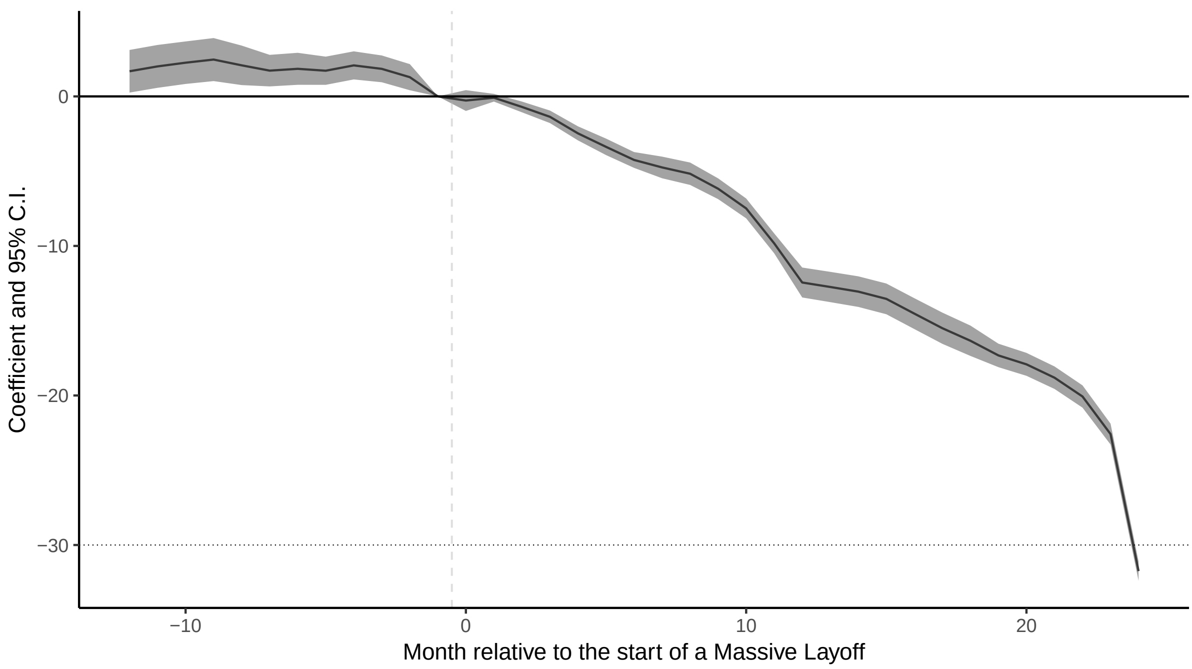
To calculate the firm size, we use the information on the start and end of each employment spell reported in the DADS postes data. We aggregate this information to the firm level to

[13] Section C.3 in the Appendix, presents a detailed description of the institutional framework of economic displacements and its relation to mass layoffs in France.

obtain the daily number of employees per firm[14]. With this information, we can calculate the day-to-day variation of the workforce. It is again worth mentioning that we observe only the size, and not the type, of separations that result in downsizing. Recent literature focuses on the identification of mass layoffs using involuntary separations only Brandily et al. (2020); Seim (2019). Such a choice is associated to the level of analysis and the research question, which in both cases is the displaced worker. This degree of specificity is less relevant when the unit of analysis is the firm, and when we want to understand skill restructuring during mass layoffs.

We use this data on the firm's daily size over the period 2004 - 2014 and conditions i) to iv) to identify the firms that undertake a mass layoff and assign a date to the mass layoff. We then construct a firm and a worker sample. The firm sample allows us to evaluate if there are changes in the firms' composition and structure. To examine selective displacement, we construct a worker sample containing worker demographic characteristics and firm characteristics.

Figure 3.2: Employment evolution in the mass layoff sample



Control group We construct a control group for the firms that experienced a layoff by selecting comparable units based on employment structure, firm sector, firm financial indicators

[14]Note that our algorithm for introducing spells to the DADS postes data can lead to measurement error in this variable when observations refer to more than 2 spells within a year. This situation concerns less than 3% of the DADS postes observations.

two years prior to the start of displacement. The observable characteristics used to assess the employment structure are the size of the firm, the occupational composition and the number female of workers in the firm. We also use a set of financial indicators calculated using the balance sheet data that characterize the firm productivity (value added and labor productivity), profitability (fiscal year results), the wage profile of the firm (compensation costs), and the degree of indebtedness (debt ratio).

For each year, we match units on all firms that never experienced a mass layoff. We perform match with replacement, so the order of the matching does not change the result of the algorithm Imbens (2015). The matching method used is nearest neighbor on the propensity score, which is calculated using a logistic regression. Table C.1 and figure C.3 present as an example the balance for the year 2009, where the quality of the matching can be assessed. The figures show that the selection method reduces the difference in covariates between the two constructed samples. Under conditional independence, an appropriate matching makes the robust estimation of the average treatment effects feasible, since the methods will not be exposed to specification choices or outliers. In the tables we present both the t-statistic and the standardized difference, since the latter is more adequate to assess the difference in the covariates (Imbens, 2015). Tables C.2-C.3 present the mean differences and the p-value of the t-statistic for the matching in all years in the sample. Table C.7 presents the difference for the treated and control samples for each covariate The normalized difference is under the 0.10 threshold, implying overlap of the covariates.

Firms characteristics The mass layoff sample contains information on 16.185 firms. Table 3.1 reports some financial indicators in the different years considered in the sample. Mass layoffs are known to impact such financial indicators (Reynaud, 2010). Following the criteria summarized in figure 3.1, firm size in our mass layoff sample evolves as shown in Figure 3.2. Two years after the start of the layoff event, the firms in our mass layoff sample shrink their workforce by 35% on average. As can be seen in the figure, on average, this change is gradual. The layoff happens slowly in the first part and accentuates in the second half of the layoff period. This contrasts with the idea of a mass layoff as an event in which all the workers are displaced at the same time, and is visible in our data due to the precise dating of the start and end dates of employment at the match level. When we consider our sample's sector composition, the 55,1%

of the observations belong to the service sector, 5.8 construction, 13.2% Retail, and 26.9% to Manufacturing.

Workers characteristics We filter the observations identified in our mass layoff firms sample from DADS-EDP panel to construct the worker sample. The worker sample includes all workers employed at the firm at some point during the layoff process and contains information on both displaced and not displaced workers. The sample contains information on 161.293 workers. Table 3.2 presents the sample's main characteristics, calculated both for displaced and non displaced workers[15].

3.4 Firm restructuring

This section provides evidence that firms that experience a mass layoff use this opportunity to restructure their skill requirements.

To identify their change, we perform an event study type analysis. The outcomes of interest are the average firm requirements for cognitive, social, and manual skills. Using such an approach allow us to identify changes in the firm's skill structure.

To understand how we capture skills change, imagine two identical firms: same sector, size, and occupational distribution. Imagine that ten managers compose the firms. Each of them supervises a team of ten workers (110 workers in each firm). The only difference between the firms is their behavior during a mass layoff. During a mass layoff, one firm had to downsize and laid off five of its managers and the teams under their supervision. At the end of the mass layoff, the final number of employees decreased by half, but its organization and structure did not change. For the second firm, instead, the mass layoff impacted mainly the team workers and not the managers, since it decides to keep the ten managers but only five of the workers' teams (60 workers). Even if workforce downsizing remains similar, its occupational structure and how the firm is organized has changed.

The model used to evaluate the hypothesis is standard to the displaced workers literature.

[15]The construction of the cognitive mismatch index, social mismatch index, and wage cost is presented in section 3.5.

Table 3.1: Firm financial indicators for mass layoff sample

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Commercial margin	−0.074	0.056	0.466	−0.019	0.015	0.144	0.175	0.041	0.178	−0.077	0.017	−0.196
Productivity	−0.235	−0.245	0.042	0.026	0.081	−0.267	−0.074	−0.276	−0.278	1.937	−0.283	−0.315
Value added	−0.336	−0.284	0.398	0.160	0.222	−0.341	−0.165	−0.321	−0.342	1.533	−0.348	−0.397
Gross operating surplus	−0.172	−0.109	0.014	0.132	0.083	−0.192	−0.043	−0.240	−0.251	1.994	−0.255	−0.283
Operating Results	−0.204	−0.192	0.205	0.346	0.105	−0.258	−0.043	−0.253	−0.251	1.692	−0.282	−0.307
Earnings before taxes	−0.193	−0.161	0.189	0.515	0.300	−0.189	−0.012	−0.231	−0.252	1.460	−0.255	−0.281
Exceptional Income	0.104	0.070	0.102	0.168	0.029	0.090	0.115	0.099	0.106	−3.004	0.102	0.104
Profits	0.027	0.021	0.188	0.443	0.240	−0.011	0.099	−0.023	0.008	−2.643	−0.013	−0.019
ROA	−0.002	0.014	0.015	0.144	0.069	−0.010	0.061	0.007	−0.060	−0.072	0.155	−0.173
ROE	−0.028	−0.044	−0.022	0.134	0.057	−0.036	0.024	−0.006	0.027	0.109	0.320	0.004
Sales	−0.215	−0.204	0.303	0.013	0.062	−0.251	−0.061	−0.270	−0.271	1.790	−0.269	−0.342
Purchase/Sales	0.298	0.251	−0.023	−0.109	−0.071	0.233	0.152	0.089	0.149	−0.011	0.342	0.424
Export/Sales	0.284	0.141	−0.113	−0.095	0.011	0.246	0.029	0.002	−0.038	−0.122	−0.102	−0.413
Debt Ratio	0.113	0.069	−0.016	0.007	−0.030	0.079	0.018	0.021	0.065	−0.075	0.140	0.476

Source: DADS-EDP panel merged with BIC-RN. The statistics are calculated relative to the start of the layoff event. The variables are winsorized and standardized for ease of interpretation in the regression.

Table 3.2: Descriptive statistics for the worker sample

	Non Displaced		**Displaced**		**Differences**	
	Mean	**St. Dev.**	**Mean**	**St. Dev.**	**t-stat**	**p-value**
Worker and job characteristics						
Age	33,614	11,447	37,621	10,890	-20.82	0.00
Tenure	2,206	2,886	3,387	3,976	-39.50	0.00
$\log(w_{ijt}/\tilde{w}_{to})$	-0,012	0,153	-0,005	0,142	-4.83	0.00
Sex (Female)	0,391	0,488	0,364	0,481	7.98	0.00
Family characteristics						
Has an under age children	0,993	0,084	0,993	0,081	-0.09	0.93
Occupation						
Managers	0,046	0,210	0,077	0,267	-13.04	0.00
Professionals	0,107	0,309	0,136	0,343	-11.73	0.00
Technicians	0,450	0,497	0,429	0,495	3.08	0.00
Clerical support	0,012	0,111	0,012	0,103	-1.10	0.27
Service and sales workers	0,088	0,284	0,073	0,258	2.57	0.01
Skilled agri. workers	0,003	0,059	0,002	0,046	3.00	0.00
Craft and related workers	0,092	0,290	0,098	0,297	-0.53	0.59
Plant and machine operators	0,065	0,246	0,076	0,266	-1.97	0.05
Elementary occupations	0,136	0,343	0,099	0,299	13.57	0.00
Armed Forces	0	0	0	0	1	0
Education						
Lower secondary or less	0,184		0,187		8.64	0.00
Upper and Post Secondary	0,347	0,476	0,355	0,479	-7.14	0.00
Bachelor	0,340	0,474	0,332	0,471	-10.27	0.00
Higher Tertiary	0,129	0,335	0,126	0,332		
Mismatch						
Cognitive mismatch index	0,025	0,049	0,020	0,054	-7.43	0.00
Social mismatch index	0,087	0,124	0,100	0,136	-14.48	0.00
Number of workers per layoff episode						
1st mass layoff	20209		130311			
2nd mass layoff	13543		59826			
3rd mass layoff	13070		34181			
4th mass layoff	4975		14248			

Source: DADS-EDP panel. The descriptive statistics are calculated for demographic and firm characteristics relative to the start of the layoff event. The bottom part of the table presents the the number of workers when a firm has multiple layoff events.

We use an event study design of the form:

$$Y_{jt} = \alpha_j + \omega_t + \sum_{k=-12}^{24} \gamma_k 1_{\{K_{jt}=k\}} \times G_j + \epsilon_{jt} \tag{3.1}$$

where the outcome of interest Y_{jt} is the firm's average skills, the coefficient γ_k captures the change in the outcome variable with respect to the beginning of the mass layoff event[16]. We also include firm fixed effects α_j and year fixed effects ω_t. In the model we indicate the start of the layoff event with K_{jt}. Treatment (having a mass layoff) is indicated with the letter G_j, which is a dummy that takes the value of 1 for the mass layoff group ($G_j = 1$), and ($G_j = 0$) for the the control. We investigate the skill requirements (cognitive, social, and manual) associated with the occupations using the mass layoff sample from the DADS postes.

Figures 3.3, 3.4 and 3.5 illustrate the results of our analysis. They present the changes in the outcome variable and its 95% confidence interval for each month after the start of the mass layoff period. We can see that the restructuring effect is small but significant when observing the average effect of the difference in difference estimation (horizontal red line in the plots). We observe that, on average, the firm uses more social skills (+1.2% standard deviations) and less manual skills (−0.5% standard deviations). The effect on cognitive skills is also positive and small (ranges from 0.25% − 0.8% standard deviations). The difference in difference estimates are all significant, and all the p-values are under the 0.05 threshold. The magnitude of such results is expected to be small since we are analyzing the composition of large firms in a short time frame (24 months).

Matching has been under scrutiny recently in the statistical literature, since the method bases unit selection in observable characteristics. Non observable characteristics, when present and not homogeneous between samples, have the potential to make unfeasible the estimation of robust effects. The design used here has two components that help to deal with such unobserved characteristics: first, we match control units each single year, and assign to each unit the event date of the corresponding treated unit, so in the regression we use the same calendar with respect to the event. This allow us to include year and firm fixed effects, controlling for unobserved characteristics in the regression. Second, in order to test the robustness of our estimates, in the

[16] Following Borusyak and Jaravel (2017) we drop the period $k = -1$ and $k = -12$ (the period most negative and distant to $k = -1$) are taken as reference for the estimation and are not included into the regression, so $\gamma_{k=-1}$ and $\gamma_{k=-12}$ are not identified.

Figure 3.3: Firm social skills per capita (full dynamic specification)

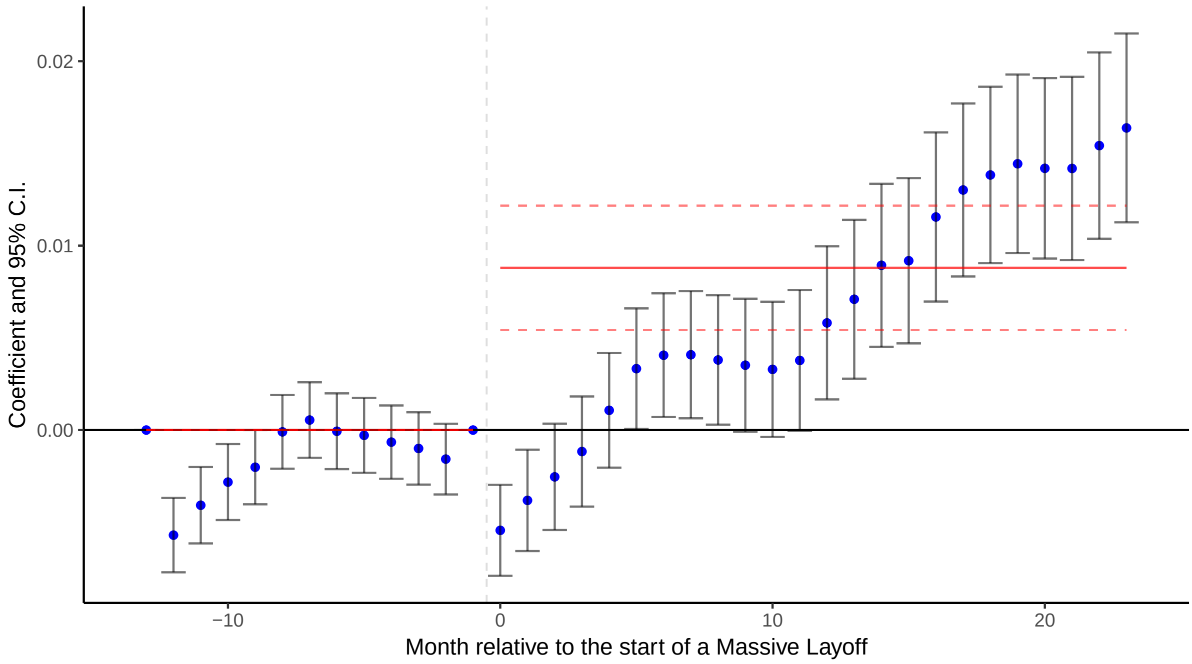


Figure 3.4: Firm manual skills per capita (full dynamic specification)

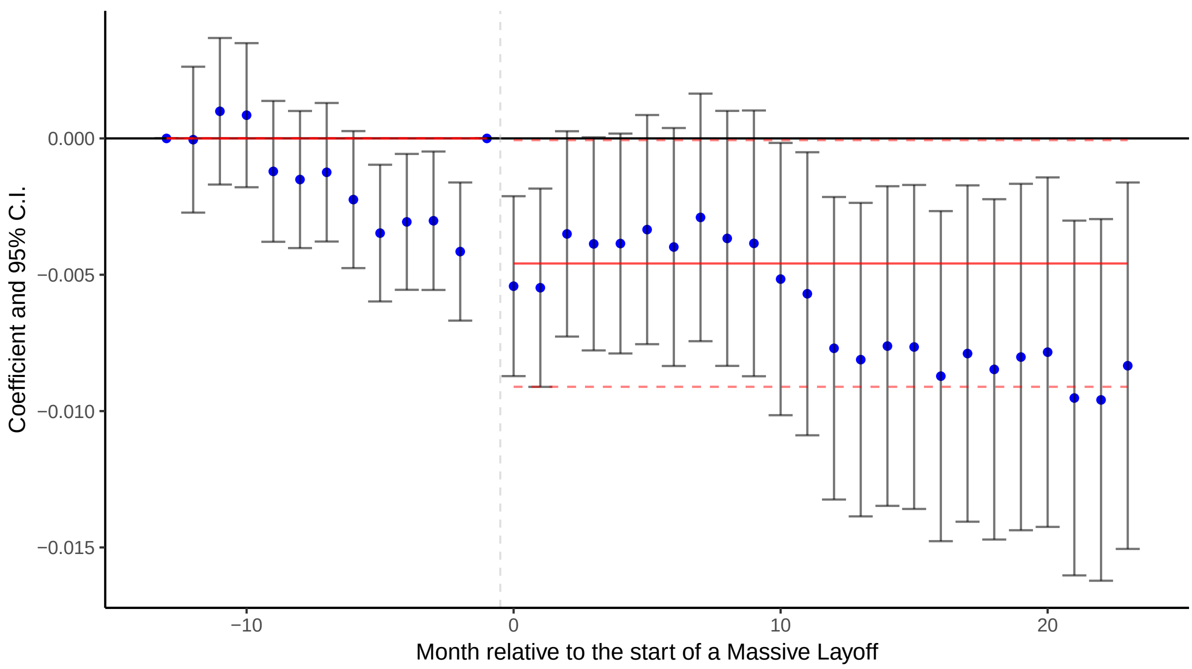
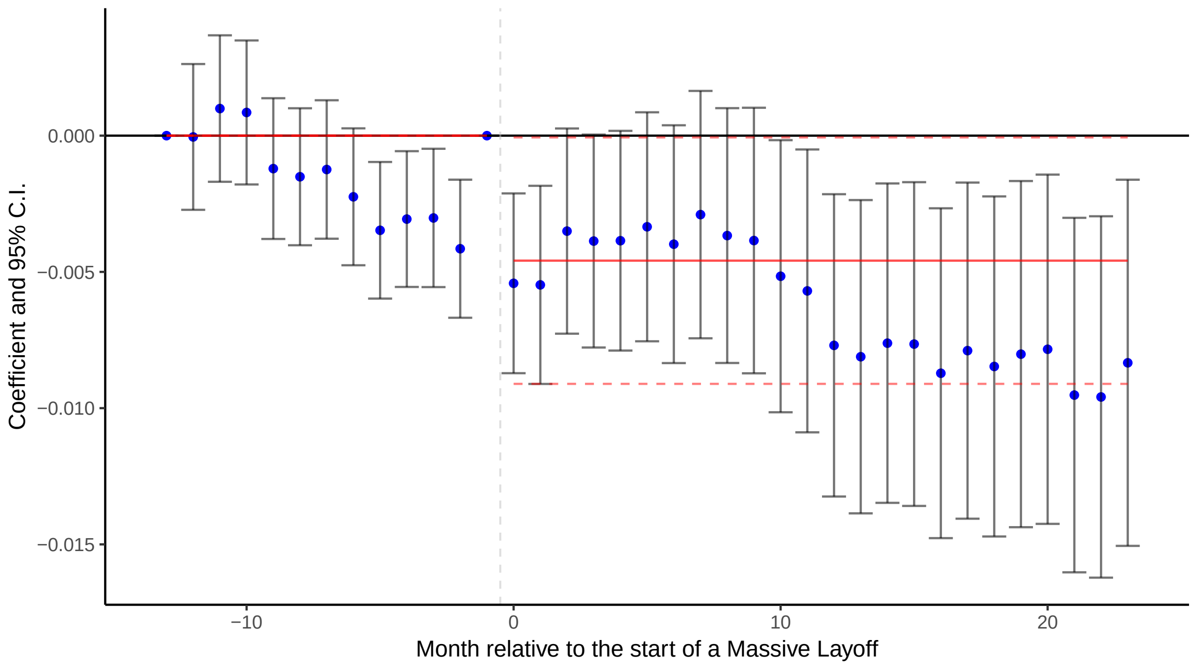


Figure 3.5: Firm cognitive skills per capita (full dynamic specification)



design we also calculate different weights[17] to make comparable the layoff firms and the matched control firms[18]. The coefficients of the difference in difference estimates, both unweighted and weighted, are significant and stable both in magnitude and sign across all the weighting schemes (see table C.7).

The positive coefficients for social skills are in line with several sets of results in the literature, including the macro results on the growth of services in the overall economy. They are also consistent with the literature on changes in skill composition within sectors, such as the results for France, where Harrigan et al. (2020) find evidence of a change in the occupational composition at the macro and sector level and Crozet and Milet (2017) find changes within-firm for the manufacturing sector.

[17] We calculate weights on different target populations. For the calculation of the weights we follow Li, Morgan and Zaslavsky (2018), for which we calculate ATE, ATT, ATC, ATO weights. We use the formulas in Table 1 of Li's paper.

[18] Combining weighting and matching is known as the *Tudor solution* in the statistical literature (Li et al., 2018).

3.5 Skills mismatch and selective displacement

Understanding selective displacement is relevant for multiple domains. For policy, it is essential to understand who is displaced in order to formulate targeted programs for reemployment. From a theoretical point of view, understanding selective displacement complements our understanding of separations.

The selection of which workers to keep and which workers to lay off is a strategic decision. Considering the mechanism behind an employment separation, the layoff choice is associated with the elimination of matches whose cost must exceed their benefits. This section proposes two channels to define a "*too expensive match*" in terms of the match surplus-value and the worker's potential to perform his/her job. The first channel uses the notion of *skills mismatch*, considering the worker's skill endowments relative to the job's skill requirements. The second considers the extra cost that a firm pays in terms of compensation for a worker.

To investigate the role of expensive match characteristics on the layoff decision, we estimate the following linear probability model:

$$P_{ijt} = \alpha_j + \eta_t + \rho_r + \omega_a + \mathbf{x}_{itj}\beta + \epsilon_{ijt} \tag{3.2}$$

where P_{ijt} is an indicator function that describe if the worker is has been displaced or not for each period observed. α_j is a firm fixed effect, which takes into account the time-invariant firm characteristics[19]. We also include year fixed effects (η_t) that capture macroeconomic events that could affect our estimates. This is very important for our sample since it covers the great recession. Another concern is that we identify layoffs using the firm level ("*entreprise*") measures, and not measures at the establishment level ("*établissement*"). To account for different labor market conditions that vary with a jobs' geographical location, we also included a worker region of residence fixed effect (ρ_r). Finally, the $\mathbf{x}_{itj}$ term includes all the variables of interest and additional time-varying controls. Recognizing that there could be also differences in the procedures for separations across collective agreements, we also include a set of collective agreement fixed effects (ω_a) to capture such differences.

[19]We include this since different sectors and sizes will imply a different productive organizations, and thus a different skill composition. Different management styles and human resources practices can also affect our estimates, and insofar as they are invariant over time, the firm fixed effects absorb such practices.

We are interested in examining the impact of skills mismatch on the layoff decision. We construct an index of cognitive and social mismatch for each individual, taking into account the worker's skill level and his/her job requirements. When the worker's skill level is below the occupation's skill requirement, we calculate its euclidean distance. When the worker skills endowments are above the required level, the mismatch assigned is 0, since it does not represent a cost for the firm. Our index is not therefore symmetric around zero in the difference between skill requirements and endowments.

$$M\left(s_{it}, r_{ot}\right) = M_{it} = \begin{cases} \sqrt{\left(s_{it} - r_{ot}\right)^2} & \text{if} \quad s_{it} \leq r_{ot} \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

We then scale the M_{it} to lie between 0 and 1, where 0 is no mismatch, and 1 is the maximum mismatch level observed in the data. The resulting index I_{it} is calculated for cognitive and social skills. In order to assess the effect of labor cost, we also include a variable that measures the percent difference between the wage and the average wage in the same occupation that year.

Our models also include individual-specific demographic characteristics that have been shown to be related to worker displacement. Specifically, we are interested in seeing the role of sex, age and tenure on selective displacement. We are also interested in understanding if the firm considers other variables that do not directly affect the match-specific surplus. We thus include a variable related to the worker's family composition, namely whether the household includes members under age 18. We only see this variable for a subset of the observations, mainly one-third of the sample. We also include a set of firm financial indicators: value-added, return on assets, return on equity, and EBITDA.

Table 3.3 presents the results for the estimation of Equation 3.2. The results indicate that the likelihood of being displaced increases with the skill mismatch. This relationship is particularly strong, positive and significant for cognitive skills mismatch in all the models compared. Social skills mismatch is also a good predictor when controlling for demographic characteristics and the relative wage. The coefficients for both skill mismatch indices remain significant and positive in all models once controlling for the relative wage. The magnitude of cognitive skills coefficient is comparable to that of social skills in the more complete specifications, however when we take in consideration the average mismatch for cognitive (0.021) and social skills (0.084), the expected

Table 3.3: Selective displacement - Linear probability model

	(1)	(2)	(3)	(4)	(5)
Mismatch variables					
Mismatch Cognitive Skill	0.072**	0.042	0.174***	0.172***	0.137*
	(0.035)	(0.044)	(0.046)	(0.046)	(0.074)
Mismatch Social Skill	−0.026	−0.051**	0.161***	0.155***	0.187***
	(0.020)	(0.025)	(0.027)	(0.027)	(0.048)
Personal Characteristics					
Sex (Female)		−0.152	−0.152	−0.145	−0.084
		(0.094)	(0.097)	(0.098)	(0.169)
Age		0.014***	0.010**	0.009*	0.024**
		(0.005)	(0.005)	(0.005)	(0.011)
Age^2		−0.000***	−0.000	−0.000	−0.000***
		(0.000)	(0.000)	(0.000)	(0.000)
Seniority		0.058***	0.055***	0.055***	0.067***
		(0.002)	(0.002)	(0.002)	(0.003)
Seniority^2		−0.003***	−0.003***	−0.003***	−0.003***
		(0.000)	(0.000)	(0.000)	(0.000)
Upper and Post Secondary		0.186**	0.216**	0.220**	−0.327*
		(0.095)	(0.096)	(0.096)	(0.189)
Bachelor		0.061	0.058	0.060	−0.168
		(0.089)	(0.090)	(0.091)	(0.174)
Higher Tertiary		0.032	−0.040	−0.041	−0.079
		(0.118)	(0.124)	(0.125)	(0.233)
Perceived Cost					
$\log\left(w_{ijt}/\tilde{w_{to}}\right)$			0.442***	0.440***	0.429***
			(0.020)	(0.020)	(0.035)
Firm Characteristics					
Added value				−0.102***	−0.065***
				(0.010)	(0.018)
ROA				−0.003***	−0.000
				(0.001)	(0.002)
ROE				−0.007***	−0.009***
				(0.001)	(0.002)
Purchases/Sales				−0.097***	−0.087***
				(0.005)	(0.009)
Family Characteristics					
Children under 18					−0.108
					(0.075)
Num. obs.	803543	546835	542657	537490	172418

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Source: DADS-EDP panel. The dependent variable is a dummy variable that indicates if the worker is displaced in period t. All the columns control for firm, year, collective agreement, and worker region of residence fixed effects. Robust standard errors are clustered at the worker level.

effect on the probability of displacement in the sample is larger for social skills ($0.187 \times 0.084 = 1.58\%$) than for cognitive skills ($0.137 \times 0.021 = 0.29\%$). This result is consistent with the findings of Montana (2021), where the production function's structural coefficients are calculated and social skills are found to have a higher weight. One of the reasons social skills are more valuable for the firm is because they depend heavily on the worker endowment, which cannot easily be adjusted since social skills are difficult to learn and transfer (Deming and Kahn, 2018; Deming, 2017).

The second channel that we study is the perceived cost of the worker by the firm. This is expressed as the percent deviation of the observed wage for individual i, working at firm j at time t, versus its market reference, i.e. the average wage of the occupation o the same year t[20], controlling for demographic characteristics. When a worker's wage is 10% over the market wage, his/her likelihood to be displaced increases 4.29%.

This paper is not the first to consider the impact of skills on job displacement. Seim (2019) investigates how cognitive and not cognitive skills affect the displacement decision. His paper finds that cognitive and non cognitive skills are good predictors of displacement. An increase in one standard deviation of cognitive or non cognitive skills decreases the probability of being laid off by 1%. Even if Seim's result highlights the importance of skills in selective displacement, it does not account for the firm's skill structure and the worker's occupation. Seim's result further differs from ours since we consider the mismatch with respect to the occupation requirements and wage costs, thus controlling for the extra cost incurred in maintaining expensive employment relationships.

The effect of age, seniority, and education on the likelihood of selective displacement is in line with previous literature for France and Germany from almost 20 years ago (Bender et al., 2002). Even though we are not considering only economic separations in our sample (and thus some separations may actually be retirements), the effect of age on separations is negative, in contrast to Bender et al. (2002) but similar to the estimates of Seim (2019) for Sweden. When considering education levels, the likelihood of being displaced decreases with high education levels conditional on the degree of skills mismatch. The effect of seniority is also non-linear, initially increasing to reach a maximum at around 10 years (in the most complete specification) before falling for workers with higher tenure.

[20] Formally, we define the variable as: $\log\left(\frac{w_{ijt}}{\tilde{w}_{ot}}\right)$, where $\tilde{w}_{ot}$ is the average wage in the occupation o in year t.

The coefficient for sex is not significant in the proposed specification that includes time, firm, region, and collective agreement fixed effects. The gender dummy, whose coefficient implies that women have a lower risk of being displaced in a mass layoff event, is not significant in the specification when errors are clustered at the individual level, but it is when using standard robust standard errors (see Table C.8). When we control for the number of children under 18, the effect on sex disappears (see column 5 in Table C.8). This combination of results might be due to the effect of regulations since women cannot be fired while on maternity leave, but the gender effect disappears once we control for the presence of children. Even if the presence of children under 18 reduces layoff risk, it is not significant across specifications when we include clustered robust errors at the worker level. In the case we use robust heteroscedastic errors, the coefficient is significant at the 1% level. We investigate further the effect that gender plays in displacement by interacting it with the mismatch indices and the relative wage cost variables. Results in table C.9 suggest that women have a higher likelihood than men of being displaced when they are mismatch in social skills or if the wage is high with respect to the occupation average. When we interact the dummy with mismatch in cognitive skills the estimate is negative, implying a lower probability of being displaced, but it is not significant at the 5% threshold when standard errors are clustered at the individual level.

When we look at the influence of financial indicators[21] on the likelihood of displacement, all of them have negative and significant effects. When working in a firm with 1 additional standard deviation of value-added, the likelihood of being displaced is reduced by 10%. For the return on assets and equity the results are smaller in magnitude, and also negative, decreasing the likelihood in 0.3% and 0.7% respectively. We include an additional financial indicator that measures the ratio of purchases to sales in the firm, which is positively associated with the outsourcing of production[22]. A 1 additional standard deviation in the purchase over sales indicator decreases the likelihood of displacement by 9%.

Given that we can only observe the number of children under 18 for one-third of our sample[23], this variable's inclusion also serves as a robustness check for our results. The coefficients remain

[21] The financial indicators are winsorized. Moreover they are standardized for ease of interpretability of the results.

[22] The balance sheet item of purchases considers also the imports in the firm, so it controls for both domestic and foreign outsourcing activities.

[23] For the remaining two thirds of the sample the value is missing. Missing information on children does not necessarily imply that the individual does not have children.

stable and significant in this sub-sample for skills mismatch, relative wage costs, firm and personal characteristics except for sex. Both of the mismatch coefficients and the relative wage cost are not significantly affected by restricting the sample and the inclusion of the additional household composition variable.

To further investigate the robustness of our results, we run the regression by sector to alloww all coefficients to vary across sectors. Table C.10 presents the results by sector, highlighting the heterogeneity, and the difference that the existing occupational structure might have in the results[24]. The result on mismatch on cognitive skills seems to be driven by the services sector, while social skills affect the displacement probability in all sectors.

Another source of heterogeneity is the collective agreement. Each collective agreement might have particularities that affect the process and selection into displacement. As such, we also estimated the model on subsamples divided by an aggregate grouping of collective agreements Table C.11 shows the result by aggregated collective agreement. These results align with those by sector, except that workers in firms covered by the agriculture, commerce and (to a lesser extent) construction collective agreements are not more protected from displacement when their social skills are more aligned with the needs of their jobs. However, those workers in firms not covered by a collective agreement or whose collective agreements are not more constraining than standard labor law are the most subject to selective displacement when their cognitive skill mismatch is high, while the impact of social skills mismatch on their risk of selective displacement is similar to that of workers employed by firms covered by manufacturing or construction collective agreements.

3.6 Conclusion

Using a combination of linked employer-employee administrative data and survey data on skills,we have found that restructuring occurs in a time span that is very short (two years) compared to the long-term analysis of previous macro literature, although our results are consistent with those findings. The restructuring of the workforce provides evidence that firms use layoffs strategically, and selective displacement plays an important role.

When we investigate selective displacement directly, we find that skills mismatch and relative

[24]The results for the agricultural sector are not reported due to a relatively small sample and collinearity of covariates with the fixed effects.

wages play an important role in determining who leaves the firm. The coefficients for both cognitive and social skills mismatch are significant and positive, implying that being mismatched increases the likelihood of being displaced. The result is robust across samples and specifications, even if we control for other demographic characteristics, firm characteristics, and firm and year fixed effects. The findings for firm characteristics also demonstrate how difference in firm performance can affect the likelihood of displacement.

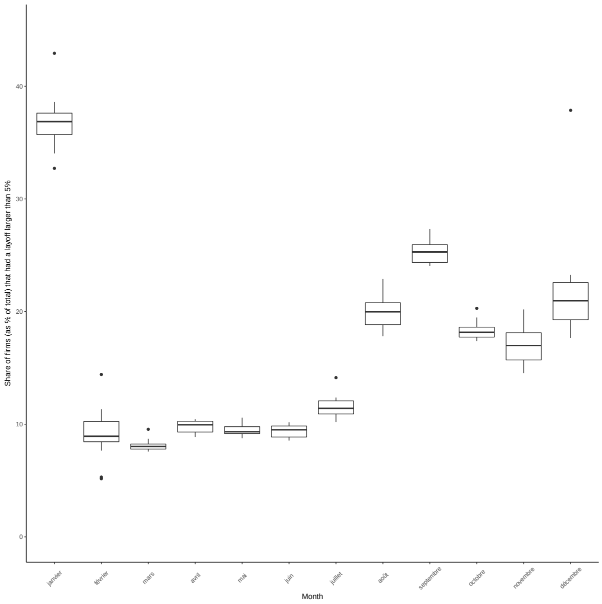
Our findings may serve to highlight the value of re-employment initiatives for recently unemployed people. This group has the greatest levels of mismatch and programs based on skill upgrades could speed up re-employment into jobs similar to the ones that were lost. Moreover, policy makers could attempt to identify the occupations that are more employable and up-skill the unemployed workforce in order to reduce susceptibility to future mass layoffs.

This paper confirms the relevance of skills in selective displacement. It also opens the door to study how other dimensions could affect displacement risk beyond the characteristics of the specific match. These channels should be explored still further.

Appendix: Selective displacement and workforce restructuring during a layoff

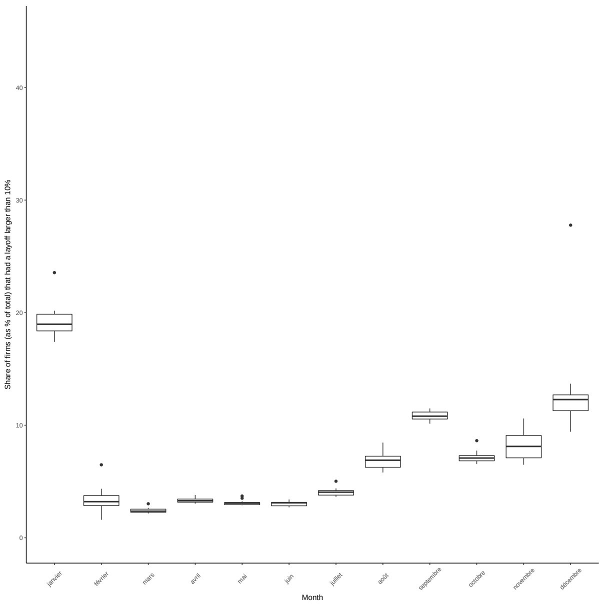
C.1 Additional tables and figures

Figure C.1: Firms that downsize - 5% threshold



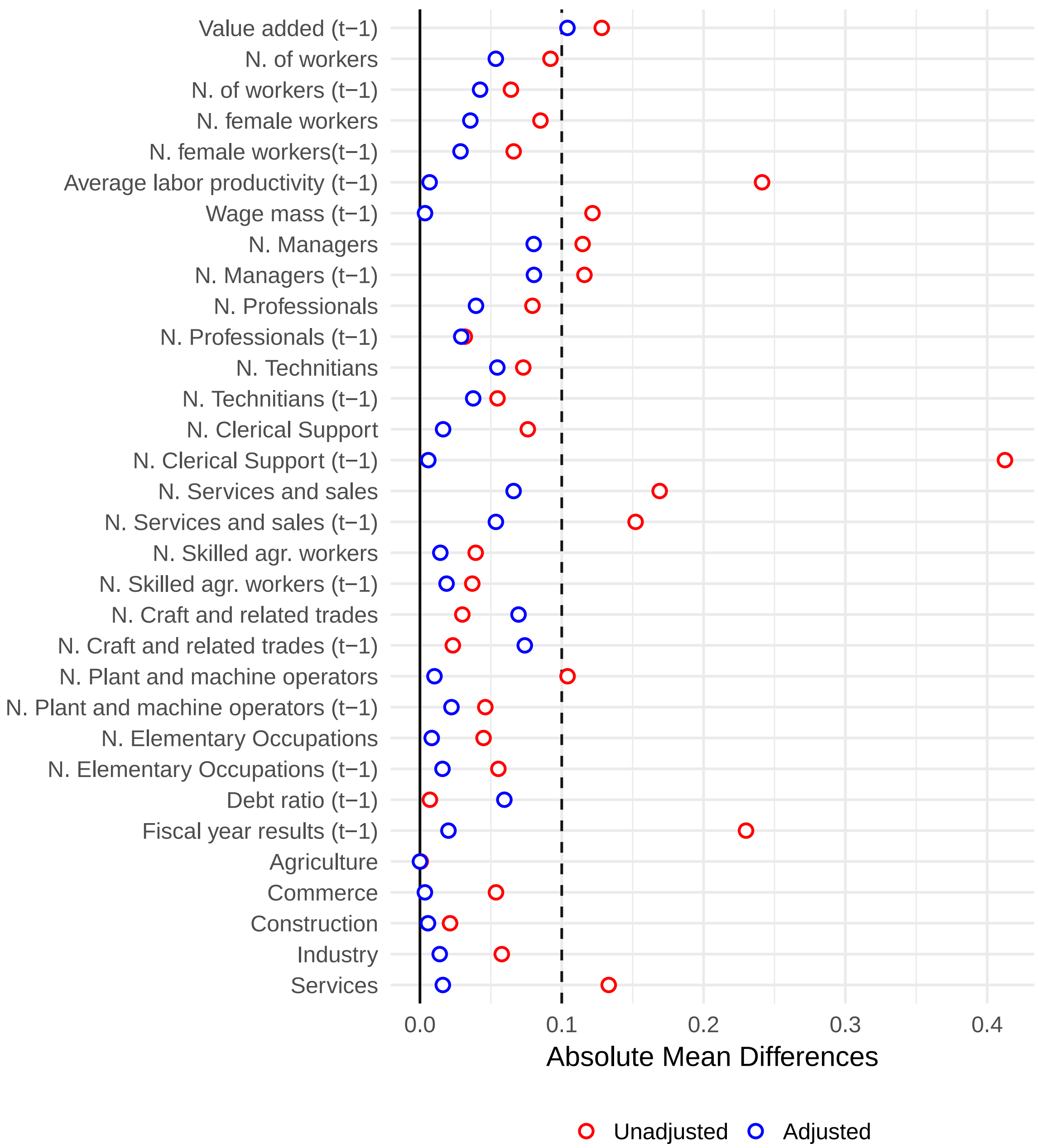
Source: DADS Postes

Figure C.2: Firms that downsize - 10% threshold



Source: DADS Postes

Figure C.3: Matching balance for selected covariates 2009 - Absolute Standarized mean



The figure presents the absolute mean differences for the all the firms in DADS (red) and the matched units in year 2009 (blue). The vertical dashed line propose a 0.1 threshold to evaluate the distance. This threshold is very conservative, since in general the 0.25 threshold is used (Imbens, 2015).

Table C.1: Matching balance for selected covariates 2009 - Standarized mean

Variable Name	Mean Control Unweighted	Mean Treated Unweighted	Difference Unweighted	t-test Unweighted	p-value Unweighted	Mean Control Adjusted	Mean Treated Adjusted	Difference Adjusted	t-test Adjusted	p-value Adjusted
Distance	0.07	0.09	0.36			0.09	0.09	-0.00		
N. of workers	186.28	151.68	-0.09	3.47	0.00	171.80	151.68	-0.05	-17.57	0.00
N. female workers	73.99	58.12	-0.08	3.30	0.00	64.75	58.12	-0.04	-13.63	0.00
N. Managers	19.88	13.86	-0.11	3.81	0.00	18.07	13.86	-0.08	-11.31	0.00
N. Professionals	28.49	21.74	-0.08	3.10	0.00	25.10	21.74	-0.04	-11.11	0.00
N. Technicians	74.50	61.68	-0.07	2.90	0.00	71.28	61.68	-0.05	-15.20	0.00
N. Clerical Support	7.31	2.59	-0.08	2.85	0.00	1.57	2.59	0.02	-1.90	0.06
N. Services and sales	17.16	10.20	-0.17	3.85	0.00	12.92	10.20	-0.07	-10.58	0.00
N. Skilled agr. workers	0.12	0.18	0.04	-1.78	0.08	0.20	0.18	-0.01	-5.49	0.00
N. Craft and related trades	14.19	12.96	-0.03	1.13	0.26	15.83	12.96	-0.07	-13.46	0.00
N. Plant and machine operators	12.66	9.39	-0.10	2.97	0.00	9.07	9.39	0.01	-13.27	0.00
N. Elementary Occupations	11.62	18.96	0.04	-2.05	0.04	17.60	18.96	0.01	-5.16	0.00
Agriculture	0.00	0.00	-0.00	3.87	0.00	0.00	0.00	0.00		
Commerce	0.21	0.16	-0.05	6.65	0.00	0.16	0.16	-0.00	-19.33	0.00
Construction	0.09	0.07	-0.02	3.71	0.00	0.07	0.07	0.01	-12.54	0.00
Industry	0.23	0.17	-0.06	6.99	0.00	0.16	0.17	0.01	-20.22	0.00
Services	0.46	0.59	0.13	-12.47	0.00	0.61	0.59	-0.02	-53.02	0.00
N. of workers (t-1)	183.01	158.57	-0.06	2.42	0.02	174.73	158.57	-0.04	-18.17	0.00
N. female workers(t-1)	72.08	59.99	-0.07	2.55	0.01	65.22	59.99	-0.03	-14.38	0.00
N. Managers (t-1)	19.39	14.22	-0.12	3.72	0.00	17.81	14.22	-0.08	-13.66	0.00
N. Professionals (t-1)	27.43	23.84	-0.03	1.35	0.18	27.15	23.84	-0.03	-9.17	0.00
N. Technicians (t-1)	76.19	65.97	-0.05	2.15	0.03	72.98	65.97	-0.04	-15.37	0.00
N. Clerical Support (t-1)	5.50	1.16	-0.41	5.16	0.00	1.23	1.16	-0.01	-4.89	0.00
N. Services and sales (t-1)	17.99	11.17	-0.15	3.53	0.00	13.57	11.17	-0.05	-10.69	0.00
N. Skilled agr. workers (t-1)	0.09	0.15	0.04	-1.65	0.10	0.19	0.15	-0.02	-3.93	0.00
N. Craft and related trades (t-1)	14.73	13.74	-0.02	0.86	0.39	16.90	13.74	-0.07	-13.79	0.00
N. Plant and machine operators (t-1)	12.19	10.28	-0.05	1.51	0.13	9.36	10.28	0.02	-11.06	0.00
N. Elementary Occupations (t-1)	9.34	17.90	0.06	-2.53	0.01	15.43	17.90	0.02	-5.17	0.00
Value added (t-1)	20183815.73	15076454.75	-0.13	5.80	0.00	19219910.01	15076454.75	-0.10	-16.29	0.00
Fiscal year results (t-1)	1221970.75	371842.29	-0.23	10.39	0.00	446163.08	371842.29	-0.02	-4.37	0.00
Average labor productivity (t-1)	153568.57	109908.05	-0.24	10.87	0.00	111142.95	109908.05	-0.01	-26.79	0.00
Wage mass (t-1)	43796.07	40928.15	-0.12	5.60	0.00	40844.05	40928.15	0.00	-76.05	0.00
Debt ratio (t-1)	1.15	1.18	0.01	-0.33	0.74	1.45	1.18	-0.06	-11.45	0.00

Source: DADS-EDP panel. The table show the difference in means for all the units in the DADS sample, and for the selected matching units. The treated sample are the firms who have a layoff in the year 2009, and the control the set of firm who do not. In the unadjusted sample the control are all firms in the DADS that do not have a mass layoff under the definition proposed. The adjusted control group consist of all the matched firms based on nearest neighbor matching. Column 3 and 8, compute the standardize mean difference for each of the selected observable covariates. Columns 4 and 9 present the t-statistics (the null hypothesis that there is no difference between the mean of both samples), and the corresponding p-value (columns 5 and 10).

C.2 Cognitive and social skills

C.2.1 PIAAC

Cognitive skills In order to construct the cognitive skills measure we use the information on two dimensions evaluated in the Programme for the International Assessment of Adult Competencies (PIAAC) survey: literacy and numeracy. We use the PIAAC's constructs instead of the raw responses due to the test administration methodology.

The definition of literacy is broad. It includes the evaluation of the comprehension of texts at different levels, from the most basic (understanding) to the most complex (how to use information from a text for self development). The design of the questions that evaluate literacy take into account the ability to interpret texts in different contexts (personal, health, or occupation related), trying to capture literacy level in job related activities.

The definition of numeracy evaluates not only the comprehension of mathematical concepts, but also the ability to locate, interpret and communicate mathematical ideas in real contexts (among them work contexts).

Table C.2: p-values for the corresponding t-statistic - difference in means for matched and layoff units

Variable Name	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Distance										
N. of workers	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. female workers	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. Managers	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04
N. Professionals	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
N. Technicians	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. Clerical Support	0.02	0.00	0.05	0.00	0.00	0.06	0.00	0.00	0.00	0.00
N. Services and sales	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. Skilled agr. workers	0.02	0.00	0.05	0.08	0.09	0.00	0.00	0.00	0.00	0.00
N. Craft and related trades	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
N. Plant and machine operators	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.05
N. Elementary Occupations	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
Agriculture	0.08		0.41	0.37	0.19		0.17	0.01	0.01	0.00
Commerce	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Construction	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industry	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Services	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. of workers (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. female workers(t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. Managers (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
N. Professionals (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
N. Technicians (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. Clerical Support (t-1)	0.05	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. Services and sales (t-1)	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N. Skilled agr. workers (t-1)	0.01	0.00	0.04	0.04	0.12	0.00	0.00	0.00	0.00	0.00
N. Craft and related trades (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
N. Plant and machine operators (t-1)	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.06
N. Elementary Occupations (t-1)	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
Value added (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fiscal year results (t-1)	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15
Average labor productivity (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Wage mass (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Debt ratio (t-1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Source: DADS-EDP panel. The table shows the p-values for the corresponding t-statistic, that calculates the difference in means for matched and mass layoff units for all periods between 2004 - 2015. The adjusted control group consists of all the matched firms based on nearest neighbor matching.

Table C.3: Standardized difference in means for matched and layoff units

Variable Name	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Distance	0.00	0.00	0.00	0.01	0.00	-0.00	0.00	0.00	0.00	-0.00
N. of workers	0.01	0.05	0.02	0.03	0.01	-0.05	0.02	0.02	-0.01	0.02
N. female workers	0.01	0.04	0.02	0.02	0.01	-0.04	0.01	0.01	-0.00	0.02
N. Managers	-0.03	0.02	-0.02	-0.00	-0.04	-0.08	-0.04	-0.06	-0.03	0.02
N. Professionals	0.01	0.03	0.03	0.03	-0.02	-0.04	-0.06	-0.00	-0.06	0.02
N. Technicians	0.01	0.05	0.03	0.03	0.01	-0.05	0.01	0.02	-0.01	0.02
N. Clerical Support	-0.00	-0.01	-0.02	0.01	0.00	0.02	-0.02	0.01	-0.00	-0.01
N. Services and sales	-0.03	0.02	-0.03	0.02	-0.02	-0.07	0.02	-0.07	0.01	0.02
N. Skilled agr. workers	0.03	-0.03	0.03	0.02	0.03	-0.01	0.03	0.02	-0.00	-0.03
N. Craft and related trades	0.01	0.07	0.03	0.03	0.02	-0.07	0.03	0.03	-0.04	0.03
N. Plant and machine operators	0.00	0.00	0.03	0.03	0.02	0.01	0.04	0.02	-0.01	0.01
N. Elementary Occupations	0.01	0.02	0.04	0.02	0.02	0.01	0.05	0.02	0.01	0.03
Agriculture	0.00	0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	0.00	-0.01
Commerce	0.02	0.03	0.01	0.00	0.01	-0.00	-0.00	-0.00	0.00	0.02
Construction	-0.00	-0.00	-0.00	0.00	0.01	0.01	-0.00	0.01	0.01	-0.00
Industry	-0.01	0.03	0.03	0.02	0.02	0.01	0.02	-0.00	-0.01	0.01
Services	-0.01	-0.05	-0.04	-0.03	-0.03	-0.02	-0.01	-0.00	-0.01	-0.02
N. of workers (t-1)	0.01	0.01	0.02	0.03	0.01	-0.04	0.02	0.02	-0.01	0.02
N. female workers(t-1)	0.01	0.01	0.02	0.03	0.01	-0.03	0.01	0.02	0.00	0.02
N. Managers (t-1)	-0.02	-0.01	-0.02	-0.00	-0.04	-0.08	-0.05	-0.06	-0.03	0.02
N. Professionals (t-1)	0.01	-0.03	0.03	0.04	-0.01	-0.03	-0.07	0.03	-0.04	0.02
N. Technicians (t-1)	0.01	0.02	0.03	0.03	0.01	-0.04	0.01	0.02	-0.01	0.02
N. Clerical Support (t-1)	0.01	-0.02	-0.02	0.01	0.01	-0.01	-0.01	0.01	0.00	-0.01
N. Services and sales (t-1)	-0.02	0.03	-0.03	0.02	-0.02	-0.05	0.02	-0.07	0.01	0.02
N. Skilled agr. workers (t-1)	0.03	0.01	0.03	0.02	0.02	-0.02	0.03	0.03	0.00	-0.03
N. Craft and related trades (t-1)	0.02	0.05	0.03	0.04	0.02	-0.07	0.03	0.04	-0.03	0.03
N. Plant and machine operators (t-1)	0.01	0.00	0.03	0.03	0.02	0.02	0.04	0.03	0.00	0.01
N. Elementary Occupations (t-1)	0.02	0.01	0.04	0.02	0.02	0.02	0.05	0.02	0.01	0.03
Value added (t-1)	-0.07	0.06	-0.03	-0.01	-0.02	-0.10	-0.03	-0.03	-0.06	-0.02
Fiscal year results (t-1)	-0.02	-0.03	-0.03	-0.02	-0.01	-0.02	-0.04	-0.06	-0.03	-0.04
Average labor productivity (t-1)	-0.01	0.03	-0.01	-0.03	-0.02	-0.01	-0.01	-0.01	-0.01	-0.00
Wage mass (t-1)	0.02	0.04	0.02	0.04	0.02	0.00	0.02	0.07	0.06	0.03
Debt ratio (t-1)	-0.03	-0.02	-0.00	-0.05	-0.06	-0.06	0.00	0.01	-0.04	-0.01

Source: DADS-EDP panel. The table shows the standardized difference in means for matched and mass layoff samples for all periods between 2004 - 2015. The adjusted control group consists of all the matched firms based on nearest neighbor matching.

Table C.4: Standardized difference in means for matched and layoff units

Variable Name	Mean Control	Mean Treated	Normalized Difference
N. of workers	173.19	215.55	0.02
N. female workers	65.91	77.22	0.02
N. Managers	16.31	15.23	-0.01
N. Professionals	22.87	23.91	0.01
N. Technicians	72.08	92.14	0.02
N. Clerical Support	2.79	2.54	-0.00
N. Services and sales	15.50	14.70	-0.00
N. Skilled agr. workers	0.24	0.49	0.02
N. Craft and related trades	15.00	21.54	0.03
N. Plant and machine operators	11.60	16.74	0.02
N. Elementary Occupations	16.57	27.14	0.03
Agriculture	0.00	0.00	-0.00
Commerce	0.14	0.14	0.00
Construction	0.05	0.05	-0.00
Industry	0.20	0.20	0.01
Services	0.61	0.60	-0.01
N. of workers (t-1)	173.34	219.48	0.02
N. female workers(t-1)	65.82	78.68	0.02
N. Managers (t-1)	16.19	15.37	-0.00
N. Professionals (t-1)	23.71	25.20	0.01
N. Technicians (t-1)	73.54	96.09	0.02
N. Clerical Support (t-1)	2.62	2.34	-0.00
N. Services and sales (t-1)	15.10	14.68	-0.00
N. Skilled agr. workers (t-1)	0.22	0.48	0.03
N. Craft and related trades (t-1)	14.64	21.26	0.03
N. Plant and machine operators (t-1)	11.57	16.97	0.02
N. Elementary Occupations (t-1)	15.47	25.99	0.03
Value added (t-1)	15606849.62	14432028.29	-0.03
Fiscal year results (t-1)	531332.06	365594.83	-0.05
Average labor productivity (t-1)	104622.31	99609.86	-0.03
Wage mass (t-1)	38142.44	38382.08	0.01
Debt ratio (t-1)	1.25	1.16	-0.02

Source: DADS-EDP panel. The table shows the standardized difference in means for matched and mass layoff sample. The control group consists of all the matched firms based on nearest neighbor matching.

Table C.5: Number of firms that start a mass layoff periods

Finalization year of layoff	Total number of firms
2006	1,999
2007	1,982
2008	2,272
2009	2,870
2010	2,697
2011	1,997
2012	1,932
2013	2,227
2014	2,132
2015	1,690

Table C.6: Difference in difference estimates for all weighted and unweighted specifications

	Dependent variable:				
	Average Cognitve requirement				
	Unweighted	ATE	ATT	ATC	ATO
after × treatment	−0.0061***	−0.0058**	−0.0057**	−0.0058**	−0.0059**
	(0.0023)	(0.0025)	(0.0025)	(0.0024)	(0.0025)
	Average Manual skills				
	Unweighted	ATE	ATT	ATC	ATO
after × treatment	−0.0053**	−0.0053**	−0.0054**	−0.0050**	−0.0052**
	(0.0022)	(0.0023)	(0.0023)	(0.0023)	(0.0023)
	Average social skills				
	Unweighted	ATE	ATT	ATC	ATO
after × treatment	0.0116***	0.0115***	0.0115***	0.0115***	0.0116***
	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)

Note: *p<0.1; **p<0.05; ***p<0.01

Source: DADS-EDP panel. Each value presents the estimate of the difference in difference models. The top of the table presents the estimate for the model in which the dependent variable is the average cognitive skills requirement in the firm, in the center the dependent variable is the average manual skills in the firm, and in the bottom the average social skills requirements in the firm. The formulas to calculate the different weightings follow table 1 in Li, Morgan and Zaslavsky (2018).

Table C.7: Standardize difference in means for matched and layoff units

Variable Name	Mean Control	Mean Treated	Normalized Difference
N. of workers	173.19	215.55	0.02
N. female workers	65.91	77.22	0.02
N. Managers	16.31	15.23	-0.01
N. Professionals	22.87	23.91	0.01
N. Technicians	72.08	92.14	0.02
N. Clerical Support	2.79	2.54	-0.00
N. Services and sales	15.50	14.70	-0.00
N. Skilled agr. workers	0.24	0.49	0.02
N. Craft and related trades	15.00	21.54	0.03
N. Plant and machine operators	11.60	16.74	0.02
N. Elementary Occupations	16.57	27.14	0.03
Agriculture	0.00	0.00	-0.00
Commerce	0.14	0.14	0.00
Construction	0.05	0.05	-0.00
Industry	0.20	0.20	0.01
Services	0.61	0.60	-0.01
N. of workers (t-1)	173.34	219.48	0.02
N. female workers(t-1)	65.82	78.68	0.02
N. Managers (t-1)	16.19	15.37	-0.00
N. Professionals (t-1)	23.71	25.20	0.01
N. Technicians (t-1)	73.54	96.09	0.02
N. Clerical Support (t-1)	2.62	2.34	-0.00
N. Services and sales (t-1)	15.10	14.68	-0.00
N. Skilled agr. workers (t-1)	0.22	0.48	0.03
N. Craft and related trades (t-1)	14.64	21.26	0.03
N. Plant and machine operators (t-1)	11.57	16.97	0.02
N. Elementary Occupations (t-1)	15.47	25.99	0.03
Value added (t-1)	15606849.62	14432028.29	-0.03
Fiscal year results (t-1)	531332.06	365594.83	-0.05
Average labor productivity (t-1)	104622.31	99609.86	-0.03
Wage mass (t-1)	38142.44	38382.08	0.01
Debt ratio (t-1)	1.25	1.16	-0.02

Source: DADS-EDP panel. The table shows the standardized difference in means for the matched and mass layoff samples. The control group consists of all the matched firms based on nearest neighbor matching.

Table C.8: selective displacement - Linear probability model with robust standard errors

	(1)	(2)	(3)	(4)	(5)
Mismatch variables					
Mismatch Cognitive Skill	0.072^{***}	0.042	0.174^{***}	0.172^{***}	0.137^{***}
	(0.022)	(0.028)	(0.029)	(0.029)	(0.046)
Mismatch Social Skill	-0.026^{**}	-0.051^{***}	0.161^{***}	0.155^{***}	0.187^{***}
	(0.013)	(0.015)	(0.017)	(0.017)	(0.029)
Personal Characteristics					
Sex (Female)		-0.152^{***}	-0.152^{**}	-0.145^{**}	-0.084
		(0.059)	(0.060)	(0.061)	(0.134)
Age		0.014^{***}	0.010^{***}	0.009^{***}	0.024^{***}
		(0.003)	(0.003)	(0.003)	(0.009)
Age^2		-0.000^{***}	-0.000^{**}	-0.000^{*}	-0.000^{***}
		(0.000)	(0.000)	(0.000)	(0.000)
Seniority		0.058^{***}	0.055^{***}	0.055^{***}	0.067^{***}
		(0.001)	(0.001)	(0.001)	(0.002)
Seniority^2		-0.003^{***}	-0.003^{***}	-0.003^{***}	-0.003^{***}
		(0.000)	(0.000)	(0.000)	(0.000)
Upper and Post Secondary		0.186^{***}	0.216^{***}	0.220^{***}	-0.327^{**}
		(0.062)	(0.062)	(0.063)	(0.139)
Bachelor		0.061	0.058	0.060	-0.168
		(0.062)	(0.063)	(0.063)	(0.141)
Higher Tertiary		0.032	-0.040	-0.041	-0.079
		(0.075)	(0.076)	(0.076)	(0.205)
Perceived Cost					
$\log\left(w_{ijt}/\tilde{w_{to}}\right)$			0.442^{***}	0.440^{***}	0.429^{***}
			(0.012)	(0.012)	(0.021)
Firm Characteristics					
Added value				-0.102^{***}	-0.065^{***}
				(0.005)	(0.008)
ROA				-0.003^{***}	-0.000
				(0.001)	(0.001)
ROE				-0.007^{***}	-0.009^{***}
				(0.001)	(0.001)
Purchases/Sales				-0.097^{***}	-0.087^{***}
				(0.003)	(0.006)
Family Characteristics					
Children under 18					-0.108^{***}
					(0.042)
N. Obs.	803543	546835	542657	537490	172418

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Source: DADS-EDP panel. The dependent variable is a dummy variable that indicates if the worker is displaced in period t. All the columns control for firm, year, collective agreement, and worker region of residence fixed effects. Robust standard errors are reported.

Table C.9: Gender heterogeneity in selective displacement - Linear probability model

	Clustered error		*Robust errors*	
	(1)	(2)	(3)	(4)
Mismatch variables				
Mismatch Cognitive Skill	0.172^{***}	0.227^{***}	0.172^{***}	0.227^{***}
	(0.046)	(0.056)	(0.029)	(0.036)
Mismatch Social Skill	0.155^{***}	0.103^{***}	0.155^{***}	0.103^{***}
	(0.027)	(0.034)	(0.017)	(0.021)
Personal Characteristics				
Sex (Female)	-0.145	-0.154	-0.145^{**}	-0.154^{**}
	(0.098)	(0.098)	(0.061)	(0.061)
Perceived Cost				
$\log\left(w_{ijt}/\tilde{w_{to}}\right)$	0.440^{***}	0.404^{***}	0.440^{***}	0.404^{***}
	(0.020)	(0.025)	(0.012)	(0.014)
Interactions with gender				
Female × Mismatch Cognitive		-0.165^{*}		-0.165^{***}
		(0.098)		(0.062)
Female × Mismatch Social		0.149^{***}		0.149^{***}
		(0.056)		(0.035)
Female × $\log\left(w_{ijt}/\tilde{w_{to}}\right)$		0.100^{**}		0.100^{***}
		(0.042)		(0.025)
Num. obs.	537490	537490	537490	537490

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Source: DADS-EDP panel. The dependent variable is a dummy variable that indicates if the worker is displaced in period t. All the columns control for firm, year, collective agreement, and worker region of residence fixed effects. Columns (1) and (2) report the errors clustered at the individual level. Columns (3) and (4) reports robust standard errors. All the model includes all the covariates for individual, firm characteristics (Column (4) of table 3.3. The estimated models include firm, year, collective agreement, and worker region of residence fixed effects.)

Table C.10: selective displacement by sector - Linear probability model

	Dependent variable: Worker is displaced			
	Industry	Services	Construction	Commerce
	(1)	(2)	(3)	(4)
Mismatch Cognitive Skill	-0.058	0.201^{***}	0.305	0.101
	(0.089)	(0.061)	(0.191)	(0.112)
Mismatch Social Skill	0.232^{***}	0.149^{***}	0.273^{**}	0.145^{*}
	(0.060)	(0.034)	(0.135)	(0.076)
$\log\left(w_{ijt}/\tilde{w_{to}}\right)$	0.413^{***}	0.473^{***}	0.582^{***}	0.329^{***}
	(0.047)	(0.024)	(0.083)	(0.065)
R^2	0.563	0.517	0.595	0.565
Adjusted R^2	0.451	0.404	0.485	0.442

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Source: DADS-EDP panel. The dependent variable is a dummy variable that indicates if the worker is displaced in period t. All the columns control for firm, year, collective agreement, and worker region of residence fixed effects. Robust standard errors are reported.

Table C.11: selective displacement by collective agreement - Linear probability model

	Dependent variable: Worker is displaced						
	Missing	No Binding Agreement	Agriculture and wood	Manufacturing	Services	Construction	Commerce
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Mismatch Cognitive Skill	-0.169	0.411^{***}	0.089	-0.029	0.285^{***}	0.138	-0.081
	(0.313)	(0.105)	(0.142)	(0.079)	(0.084)	(0.160)	(0.241)
Mismatch Social Skill	0.243	0.156^{***}	0.122	0.155^{***}	0.339^{***}	0.187^{*}	0.094
	(0.178)	(0.052)	(0.090)	(0.057)	(0.055)	(0.104)	(0.136)
$\log\left(w_{ijt}/\tilde{w_{to}}\right)$	0.283^{***}	0.676^{***}	0.339^{***}	0.362^{***}	0.476^{***}	0.574^{***}	0.302^{***}
	(0.102)	(0.045)	(0.069)	(0.043)	(0.038)	(0.067)	(0.098)
R^2	0.819	0.514	0.576	0.545	0.561	0.585	0.617
Adjusted R^2	0.629	0.393	0.480	0.423	0.452	0.473	0.480

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Source: DADS-EDP panel. The dependent variable is a dummy variable that indicates if the worker is displaced in period t. All the columns control for firm, year, and worker region of residence fixed effects. Robust standard errors reported.

Table C.12 shows the result of the factor analysis for the two PIAAC-constructed interest variables. The factor analysis methodology allows us to reduce the dimensions and express the information in a unique vector of weights that captures the largest amount of variance. In this calculation, the resulting vector is rotated such that the weights can be interpreted easily[25]. The results suggest that the numeracy value explains a larger proportion of the total variability, and thus is attributed a higher weight in the composite cognitive skill measure.

In the publicly available PIAAC data, literacy and numeracy are provided as plausible values and a set of 10 values is proposed for each dimension. Following the multiple imputation methods (Little and Rubin, 2019), from the set of ten plausible values of each sub-measure proposed, we can calculate a set of 10 cognitive skills measures for each observation in the sample.

Table C.12: Factor loadings for the construction of cognitive skills

Dimension	PIAAC variable name	Weight
Numeric	numer	0.763
Literacy	liter	0.646

Source: PIAAC France 2012.

Social skills As stated previously, the social skills measures are derived from the answers to the background questionnaire (BQ) of the survey. In this part, six questions about attitudes and interest toward learning are asked. These measures are related to personality and interpersonal skill areas. Following the same methodology as before, we combine the results of the six questions into a unique vector using principal component analysis (PCA). The only difference between the PCA and the factor analysis (FA) is that FA implies a rotation of the components that might help one to interpret the role of each component. Since in this case the interpretation is not straightforward, we use the PCA weights directly. Table C.13 presents the estimated loadings for the first factor on each of the questions.

One of the worries in the construction of the social measure is the rate of the missingness for some questions in the background questionnaire. Unlike the numeracy and literacy measures,

[25] I used the standard rotation option, 'varimax'.

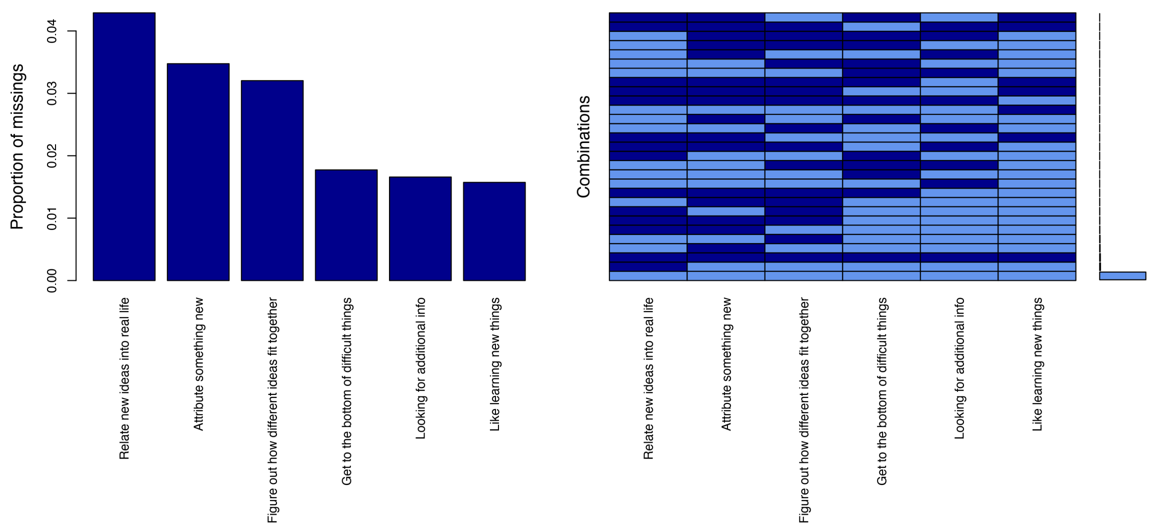
Table C.13: Factor loadings for the construction of social skills

	Variable	Factor1
Relate new ideas into real life	I_Q04b	0.581
Like learning new things	I_Q04d	0.681
Attribute something new	I_Q04h	0.485
Get to the bottom of difficult things	I_Q04j	0.723
Figure out how different ideas fit together	I_Q04l	0.728
Looking for additional info	I_Q04m	0.612

Source: PIAAC France 2012.

these are self-reported responses, and a systematic pattern of missing values could be problematic when building a unique measure of social skills. Figure C.4 presents a visualization that helps analyze the distribution of missing values across questions. The rate of missing values is very low. If we analyze separately each of the questions, the maximum rate of missing values is around 4%. When considering patterns for missingness (right part of the figure), we can see there are no visible patterns.

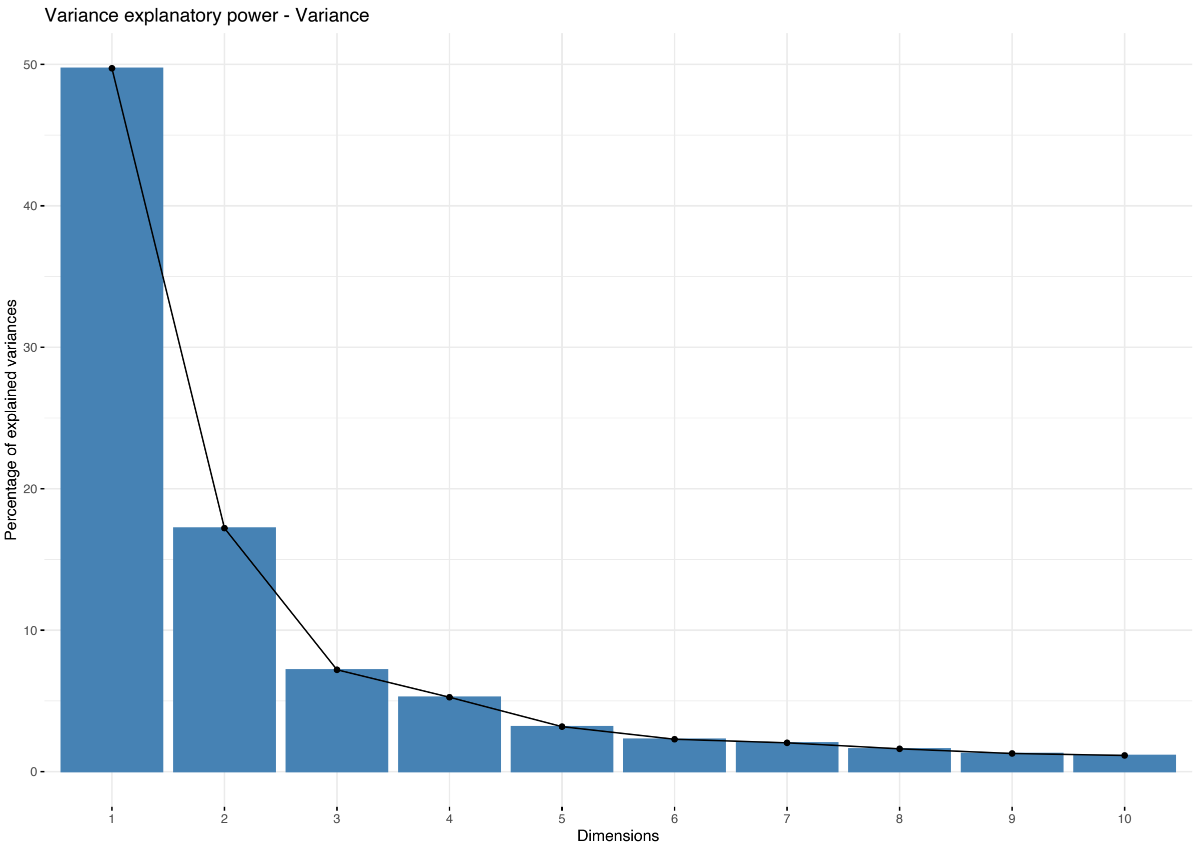
Figure C.4: Patterns of missingness for Non Cognitive questions



Source: PIAAC France 2012

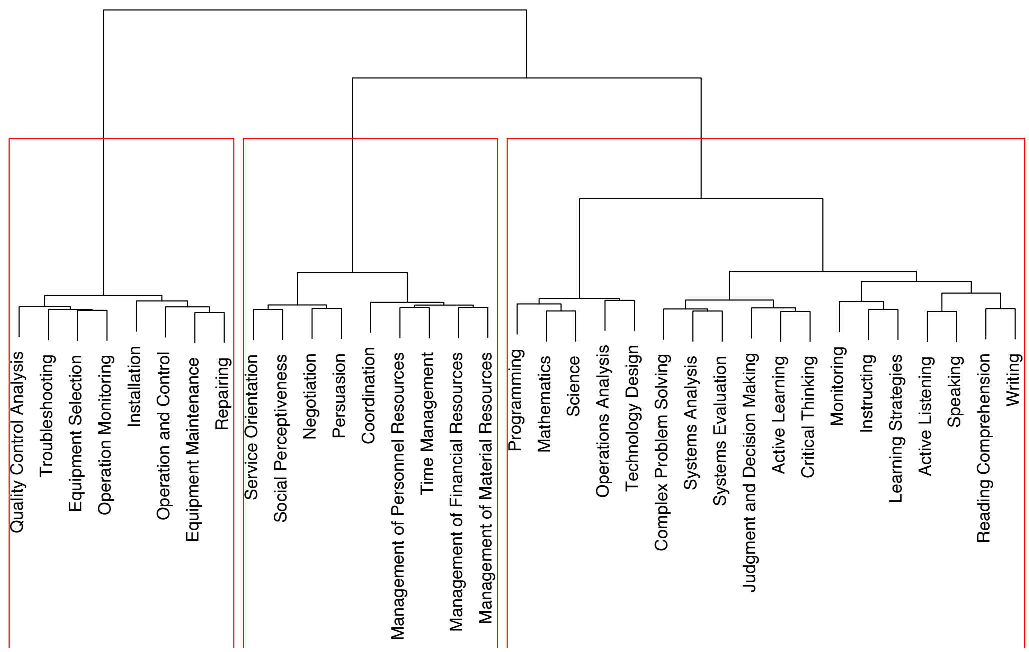
C.2.2 O*NET - Skills requirements

Figure C.5: Explanatory power of variance - PCA



Source: O*NET Skills

Figure C.6: Cluster selection based on PCA and hierarchical clusters based on Ward distance



Source: O*NET Skills

Table C.14: Factor loadings for three principal components (PCA) on skills measures - O*NET

	Comp.1	Comp.2	Comp.3
Active Learning	0.219	0.066	0.092
Active Listening	0.217	−0.045	0.027
Complex Problem Solving	0.208	0.143	0.101
Coordination	0.180	0.026	−0.291
Critical Thinking	0.217	0.084	0.085
Equipment Maintenance	−0.122	0.302	−0.130
Equipment Selection	−0.105	0.319	−0.056
Installation	−0.055	0.229	−0.082
Instructing	0.192	0.033	−0.049
Judgment and Decision Making	0.217	0.095	0.045
Learning Strategies	0.197	0.034	−0.010
Management of Financial Resources	0.135	0.065	−0.215
Management of Material Resources	0.130	0.123	−0.235
Management of Personnel Resources	0.186	0.086	−0.245
Mathematics	0.132	0.135	0.277
Monitoring	0.195	0.097	−0.113
Negotiation	0.188	−0.043	−0.241
Operation and Control	−0.130	0.249	−0.127
Operation Monitoring	−0.105	0.304	−0.083
Operations Analysis	0.157	0.123	0.198
Persuasion	0.199	−0.041	−0.191
Programming	0.068	0.140	0.338
Quality Control Analysis	−0.073	0.343	−0.035
Reading Comprehension	0.214	0.020	0.152
Repairing	−0.116	0.300	−0.131
Science	0.128	0.128	0.299
Service Orientation	0.159	−0.111	−0.209
Social Perceptiveness	0.189	−0.086	−0.199
Speaking	0.219	−0.055	0.011
Systems Analysis	0.204	0.151	0.073
Systems Evaluation	0.207	0.147	0.051
Technology Design	0.066	0.224	0.239
Time Management	0.196	0.064	−0.190
Troubleshooting	−0.107	0.341	−0.087
Writing	0.213	−0.005	0.112

Source: O*NET.

C.3 Institutional framework of mass layoff in France

This section describes and compiles the institutional information concerning how the layoff process works in France in the case of displacement for economic reasons - (ED). It presents the legal environment of economic displacement, the process timing, and the implications for the identification of mass layoffs in the project from the data point of view.

The process for layoffs for economic reasons is heterogeneous and includes numerous thresholds. First, a firm has more or fewer obligations depending on its size. Second, the size of the layoff can affect the timing of various obligations. As noted by Cahuc and Carcillo in 2007:

"The individual redundancy procedure is not very different from other individual redundancy procedures, and lasts on average 15 days. However, it involves informing the labor administration, in order to avoid "saucissonnage". The procedure for collective layoffs of less than ten employees over a period of 30 days lasts at least 3 days longer, as it entails, in addition to the individual procedures and the information of the administration, a consultation for opinion and the information of the staff representatives, who must be provided with a summary document explaining the reasons for the layoffs and specifying the details (persons and positions concerned, timetable, etc.) On the other hand, the procedure for large-scale economic layoffs is particularly complex (see Cahuc and Kramarz, 2005, for a detailed description), and lasts much longer: a minimum of three months, in practice around six months, and can reach nine or twelve months for a large company when negotiations are difficult or when there is a failure to fulfill the requirements."[26]

We begin by presenting the definition of economic displacement, followed by the definition of a mass layoff.

C.3.1 Definition of economic displacement

This type of separation involves some particularities in the definition:

- It is an *involuntary* separation (the decision follows the employer's will and not the employee).
- The displacement happens because the job is destroyed or *transformed in its nature* (by changing

[26] "La procédure individuelle de licenciement économique se distingue peu des autres procédures de licenciement individuel, et dure en moyenne 15 jours. Elle implique néanmoins d'informer l'administration du travail, afin d'éviter le "saucissonnage". La procédure de licenciement collectif de moins de dix salariés sur 30 jours dure au minimum 3 jours de plus, car elle entraîne, outre les procédures individuelles et l'information de l'administration, une consultation pour avis et l'information des représentants du personnel auxquels il faut fournir un document de synthèse motivant et précisant les licenciements (personnes et postes concernés, calendrier, etc.) En revanche, la procédure en cas de grand licenciement économique est particulièrement complexe (voir Cahuc et Kramarz, 2005, pour une description détaillée), et dure beaucoup plus longtemps : au minimum trois mois, en pratique autour de six mois, et pouvant atteindre neuf ou douze mois pour une grande entreprise lorsque les négociations sont difficiles ou qu'il y a eu constat de carence." (Cahuc and Carcillo (2007) - page.8-9, own translation)

previous mutual agreements reflected in the job contract). The worker does not accept such changes.[27]

Both points share a characteristic of economic displacement. It is *non-consensual*. From the economic point of view, the surplus of the employment relation changes, and the employer no longer benefits from continuing the match. In the paper, we examined how changes in productivity could explain value of production from a match could change. Given that the legal arrangement happens between the firm and the employer, such a process occurs at the firm level. From the legal standpoint, such change could arise from:

i Economic performance was poor in comparison to the previous years;

ii The firm's technology changed;

iii The firm made a strategic decision to reorganize to improve its competitiveness[28]. According to the jurisprudence, it may not be used to improve it but only to maintain it;

iv The firm will shut down operations and will disappear.

Another level of complexity in the application of the law has to be considered since, conditions (i) to (iii) could happen and be calculated at a level different from the firm's, including that of the conglomerate to which it belongs. Judges could consider the level of the group that controls the firm or the performance of the sector as a whole, and examine its performance to justify the ability of the firm to use the mechanism. There have been cases in which a firm that is having economic difficulties but belongs to a group that is performing well has found it difficult to motivate an economic displacement. Consider for example some recent jurisprudence of the Court de Cassation: *"But whereas the economic cause of a dismissal is assessed at the level of the company or, if it is part of a group, at the level of the sector of activity of the group in which it operates; whereas the perimeter of the group to be taken into consideration for this purpose is all of the companies united by the control or influence of a dominant company under the conditions defined in article L. 2331-1 of the Labor Code, without there being any reason to restrict the group to the companies located on national territory."* (Court de Cassation, 6

[27]" A dismissal for economic reasons is a dismissal carried out by an employer for one or more reasons not inherent to the person of the employee resulting from the elimination or transformation of a job or from a modification, refused by the employee, of an essential element of the employment contract" (Article L1233-3 - Code du travail) https://www.legifrance.gouv.fr/codes/article_lc/LEGIARTI000036762081/ .["Constitue un licenciement pour motif économique le licenciement effectué par un employeur pour un ou plusieurs motifs non inhérents à la personne du salarié résultant d'une suppression ou transformation d'emploi ou d'une modification, refusée par le salarié, d'un élément essentiel du contrat de travail"]

[28]This aspect is crucial in the conception of the law, but is very difficult to interpret. Following Cahuc (2012), the French case is extreme when compared to other European countries, since an interpretation of the law that does not allow firms to fire to improve productivity, but just to maintain it, is jurisprudence in labor courts. Still, the *maintenance* of productivity is very difficult to proof and is conditional on the judge's interpretation.

novembre 2016, 14-30.063)[29]. The definition of the reach (perimeter) of the group in this sense is far from the context of the firm, which could make the mechanism difficult to access. A firm, to be able to use the economic separation mechanism, has to comply with any of the conditions listed above.

The accessibility of the economic displacement mechanism in France has three barriers. First, the motivation of the reasons to layoff can be easily disputed since they have to be interpreted by an authority using a concept which can be subject to subjective interpretation. Second, the perimeter of the group can be disputed, and this can limit the ability to access the mechanism. Finally, the mechanism can not be used to improve productivity, but only to maintain it, which could make it unsuitable for firm reorganization.

The next section details the process of economic displacement. It differs by the size of the firm, the number of workers involved in the layoff, and the concentration of layoffs in time.

C.3.2 The process of economic displacement

There is a well established timeline for firms that intend to use economic displacement. The procedure differs slightly if the firm is large or by the number of employees firing. Below a summary of the process, which depends on the number of layoffs by the firm.

In the case of an individual layoff

Ind.1 A firm recognizes itself in a situation where an economic displacement could be justified (conditions (i) to (iv). It is crucial that it can demonstrate such a condition in front of a judge since the employee could contest it, increasing the time and cost of the layoff. Fraisse et al. (2015) provide evidence that the legal procedure affects the job flow of firms. An increase in the amount of litigation decreases firings. Such evidence suggests that firms might adopt this mechanism essentially in cases where the underlying economic motivation can not be contested at all.

Ind.2 The firm must organize an interview in which it informs the employee that she will be fired. The law defines the minimum contents of the interview. The firm notifies the employee of the interview at least five days in advance[30].

Ind.3 In this meeting, the employee is told the decision and the causes. The firm offers him the possibility of getting a “contrat de sécurisation professionnelle (CSP)”. When the separation is for economic

[29] https://www.legifrance.gouv.fr/juri/id/JURITEXT000033429110/[*Mais attendu que la cause économique d'un licenciement s'apprécie au niveau de l'entreprise ou, si celle-ci fait partie d'un groupe, au niveau du secteur d'activité du groupe dans lequel elle intervient ; que le périmètre du groupe à prendre en considération à cet effet est l'ensemble des entreprises unies par le contrôle ou l'influence d'une entreprise dominante dans les conditions définies à l'article L. 2331-1 du code du travail, sans qu'il y ait lieu de réduire le groupe aux entreprises situées sur le territoire national*].

[30] Article L1233-11 - Code du travail. https://www.legifrance.gouv.fr/codes/article_lc/LEGIARTI000006901023/

reasons, some rules must also be considered, specifically which employees to lay off in which order, accounting for family responsibilities, seniority, age and disabilities, and others[31]. If there exists a collective agreement, it also needs to be taken into consideration.

Ind.4 Seven days after the meeting, the employer sends a letter of dismissal. The employee has 12 months to dispute this decision with the authorities. The letter offers him the "contrat de sécurisation" professionnelle (CSP) if the firm has less than 1000 employees or a retraining period if the firm (or economic group) has more than 1000 employees[32]. If the employee accepts the option of retraining, it can last from 4 to 12 months .

Ind.5 The firm communicates the decision to the french administration (Dirrecte).

Ind.6 The interruption of the contract occurs when the notification arrives, after a specified advanced notice period ('preavis') that changes as a function of the seniority of the employee[33].

Layoff of two or more employees (below nine)

A similar procedure as the one stated before should be implemented. Still, before the interview with the employer, the firm must also meet with the employee's representatives and communicate to them all the details of the workforce restructuring. In case the firm has more than 50 employees, it must notify the Ministry of Labor.

The communication involves the design and presentation of a restructuring plan. It requires the economic reasons that motivate the plan to be well described (financial, economic, or technical reasons). There is a precise number of separations proposed, the occupations considered, and the expected calendar.

Mass layoff (over ten economic displacements)

If the firm has less than 50 employees (strictly) and wants to perform a mass layoff, it must comply with the above conditions. Additionally, the consultation procedure with the employee representative changes and must be done twice in 14 days before proceeding to the interview. This has to be communicated to the administrative authorities (DIRECCTE), and 30 days after that, the firm can send the letters to the employees.

If the firm has 50 or more employees, the firm has to put in place an Employment Saving Plan, PSE (plan de sauvegarde de l'emploi). The content of a PSE has to be in agreed upon with the employee

[31] Article L1233-5 - Code du travail https://www.legifrance.gouv.fr/codes/article_lc/LEGIARTI000036261856/.

[32] These requirements cost around 65% of the wage in addition to the cost of the training. More details can be found in https://travail-emploi.gouv.fr/emploi/accompagnement-des-mutations-economiques/article/conge-de-reclassement.

[33] The length of the *preavis* is one (1) month for a worker with less than two years of seniority and two (2) months for a seniority equal or superior to two years.

representatives. It has to be presented to them in (at least) 2 meetings, and the employee representatives have some time to reply to its points and evaluate its contents (they have a window of 2 to 4 months to respond to the proposed content). The proposal and response are communicated to the administration before the layoffs can continue. The administration validates the plan (it has around 21 days to do it), during which the firm can organize the interviews and proceed with the process. The firm can send the letters around 30 days after it communicates the PSE to the Direccte (French Ministry of Labor).

We can thus use the number of PSEs to have a sense of what could be the order of magnitude of mass layoffs in France. According to information of the French ministry of labor, table C.15 presents the number of PSE for the period 2005 to 2013. As we can see, the number of events is pretty low compared to the reported number of events per year using our definition based on the size of the firm, suggesting that the the economic displacement is not the principal channel by which a firm reduces its workforce. A revision of the legislation suggests that the cause for this is related to the barriers to use the mechanism, and the high cost that it has (which includes the cost in time).

Table C.15: Number of PSE notifications to the French Ministry of labor 2005 - 2013

Year	Number of PSE notifications (more 50)	All PSE notifications
2005	396	1270
2006	412	1305
2007	351	957
2008	393	1061
2009	764	2245
2010	372	1195
2011	270	952
2012	307	914
2013	237	583
2014		772
2015		768
2016		721
2017		562
2018		561
2019		491
2020		871

Source: French Ministry of Labor. The second column indicates the number of PSEs notified to the French Ministry of labor for firms with more than 50 employees at the moment of the notification. Column 3 presents the total number of notifications including small firms. There is a series break in 2013 since the source of the data changes.

C.4 Coefficients for multiple imputation samples in PIAAC

Table C.16: Average coefficient for multiple imputation samples, PIAAC - FR, Cognitive skills

	No seniority	No firm size	No occupation	No education	No wage	Complete
Intercept	−3.017	−0.063	−2.423	1.641	1.286	−2.072
	(9.668)	(9.692)	(10.146)	(9.526)	(9.516)	(9.750)
Sex (female)	0.084	0.089	0.057	0.034	0.109	0.085
	(0.027)	(0.027)	(0.024)	(0.028)	(0.028)	(0.027)
Monthly earnings	−0.132	−0.134	−0.228	−0.237		−0.124
	(0.032)	(0.030)	(0.029)	(0.040)		(0.031)
Isco Group 02	−0.343	−0.425		0.275	−0.396	−0.361
	(0.253)	(0.250)		(0.288)	(0.256)	(0.256)
Isco Group 03	−0.540	−0.584		0.139	−0.564	−0.554
	(0.213)	(0.211)		(0.213)	(0.228)	(0.212)
Isco Group 11	−0.335	−0.353		−0.121	−0.360	−0.355
	(0.194)	(0.197)		(0.213)	(0.223)	(0.198)
Isco Group 12	−0.335	−0.373		−0.016	−0.422	−0.364
	(0.178)	(0.179)		(0.201)	(0.195)	(0.182)
Isco Group 13	−0.386	−0.436		−0.137	−0.453	−0.416
	(0.166)	(0.166)		(0.189)	(0.187)	(0.169)
Isco Group 14	−0.333	−0.348		0.193	−0.416	−0.362
	(0.183)	(0.180)		(0.197)	(0.203)	(0.184)
Isco Group 21	−0.326	−0.395		−0.031	−0.385	−0.357
	(0.175)	(0.177)		(0.199)	(0.196)	(0.179)
Isco Group 22	−0.175	−0.248		0.056	−0.213	−0.210
	(0.175)	(0.180)		(0.195)	(0.193)	(0.184)
Isco Group 23	−0.358	−0.373		−0.135	−0.365	−0.375
	(0.167)	(0.170)		(0.183)	(0.186)	(0.171)
Isco Group 24	−0.262	−0.314		0.055	−0.333	−0.294
	(0.163)	(0.164)		(0.183)	(0.187)	(0.169)
Isco Group 25	−0.412	−0.478		−0.170	−0.459	−0.444
	(0.173)	(0.181)		(0.201)	(0.192)	(0.181)
Isco Group 26	−0.063	−0.100		0.134	−0.125	−0.090
	(0.174)	(0.181)		(0.194)	(0.197)	(0.182)
Isco Group 31	−0.102	−0.151		0.508	−0.134	−0.131
	(0.164)	(0.168)		(0.180)	(0.187)	(0.169)
Isco Group 32	−0.163	−0.208		0.213	−0.197	−0.196
	(0.177)	(0.184)		(0.193)	(0.196)	(0.183)

Isco Group 33	−0.226	−0.280	0.224	−0.253	−0.257
	(0.166)	(0.170)	(0.183)	(0.190)	(0.172)
Isco Group 34	−0.174	−0.210	0.237	−0.210	−0.205
	(0.168)	(0.170)	(0.190)	(0.187)	(0.173)
Isco Group 35	−0.165	−0.217	0.243	−0.208	−0.191
	(0.211)	(0.214)	(0.229)	(0.225)	(0.216)
Isco Group 41	−0.144	−0.192	0.369	−0.133	−0.175
	(0.168)	(0.172)	(0.189)	(0.188)	(0.175)
Isco Group 42	−0.266	−0.271	0.157	−0.266	−0.290
	(0.194)	(0.199)	(0.214)	(0.217)	(0.201)
Isco Group 43	−0.169	−0.221	0.303	−0.204	−0.199
	(0.166)	(0.168)	(0.179)	(0.188)	(0.169)
Isco Group 44	−0.140	−0.170	0.386	−0.083	−0.157
	(0.224)	(0.224)	(0.242)	(0.244)	(0.226)
Isco Group 51	0.239	0.170	0.892	0.238	0.206
	(0.175)	(0.181)	(0.183)	(0.191)	(0.180)
Isco Group 52	0.046	−0.008	0.625	0.032	0.013
	(0.171)	(0.177)	(0.189)	(0.191)	(0.178)
Isco Group 53	0.078	−0.006	0.731	0.076	0.040
	(0.180)	(0.184)	(0.195)	(0.199)	(0.186)
Isco Group 54	−0.118	−0.191	0.552	−0.136	−0.153
	(0.192)	(0.200)	(0.215)	(0.212)	(0.202)
Isco Group 61	0.211	0.140	0.879	0.214	0.175
	(0.194)	(0.200)	(0.220)	(0.215)	(0.202)
Isco Group 62	0.142	0.079	0.805	−0.142	0.104
	(0.444)	(0.450)	(0.456)	(0.362)	(0.442)
Isco Group 71	0.566	0.497	1.403	0.500	0.528
	(0.199)	(0.202)	(0.211)	(0.217)	(0.203)
Isco Group 72	0.087	0.035	0.749	0.065	0.057
	(0.193)	(0.198)	(0.208)	(0.215)	(0.199)
Isco Group 73	−0.032	−0.092	0.607	−0.067	−0.061
	(0.231)	(0.241)	(0.246)	(0.248)	(0.241)
Isco Group 74	−0.272	−0.341	0.359	−0.311	−0.307
	(0.221)	(0.229)	(0.221)	(0.236)	(0.226)
Isco Group 75	0.262	0.194	0.979	0.283	0.227
	(0.185)	(0.198)	(0.201)	(0.207)	(0.196)
Isco Group 81	0.306	0.255	1.062	0.256	0.272

	(0.178)	(0.180)		(0.196)	(0.201)	(0.184)
Isco Group 82	0.067	0.054		0.817	0.023	0.038
	(0.213)	(0.211)		(0.222)	(0.230)	(0.217)
Isco Group 83	0.190	0.136		0.895	0.162	0.155
	(0.183)	(0.186)		(0.204)	(0.204)	(0.189)
Isco Group 91	0.401	0.382		1.166	0.419	0.365
	(0.172)	(0.179)		(0.190)	(0.194)	(0.180)
Isco Group 93	0.132	0.108		0.846	0.127	0.100
	(0.180)	(0.178)		(0.201)	(0.196)	(0.183)
Isco Group 94	0.479	0.371		1.266	0.508	0.437
	(0.239)	(0.236)		(0.276)	(0.249)	(0.244)
Isco Group 95	−0.124	0.046		0.484	0.378	−0.179
	(0.276)	(0.374)		(0.292)	(0.420)	(0.287)
Isco Group 96	−0.043	−0.091		0.634	−0.029	−0.072
	(0.179)	(0.178)		(0.192)	(0.209)	(0.180)
age	0.958	0.451	0.959	0.239	0.111	0.794
	(1.669)	(1.674)	(1.754)	(1.662)	(1.637)	(1.686)
age^2	−0.072	−0.036	−0.069	−0.036	−0.016	−0.061
	(0.116)	(0.116)	(0.122)	(0.116)	(0.113)	(0.117)
age^3	0.003	0.001	0.002	0.002	0.001	0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
age^4	−0.000	−0.000	−0.000	−0.000	−0.000	−0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
age^5	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
age^6	−0.000	−0.000	−0.000	−0.000	−0.000	−0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Lower secondary	−0.545	−0.616	−0.597		−0.588	−0.551
	(0.078)	(0.081)	(0.081)		(0.074)	(0.078)
Upper and Post Secondary	−0.911	−0.996	−1.048		−0.955	−0.918
	(0.069)	(0.072)	(0.072)		(0.063)	(0.069)
Bachelor	−1.343	−1.441	−1.668		−1.413	−1.355
	(0.074)	(0.078)	(0.073)		(0.068)	(0.074)
Higher Tertiary	−1.468	−1.584	−1.869		−1.568	−1.490
	(0.079)	(0.088)	(0.081)		(0.077)	(0.083)
11 to 50 workers	0.026		0.004	0.001	0.025	0.028
	(0.032)		(0.034)	(0.033)	(0.032)	(0.033)

51 to 250 workers	−0.029		−0.037	−0.061	−0.030	−0.023
	(0.036)		(0.037)	(0.038)	(0.035)	(0.037)
250 to 1000 workers	−0.039		−0.052	−0.085	−0.035	−0.029
	(0.039)		(0.040)	(0.042)	(0.039)	(0.040)
More than 1000 people	−0.129		−0.162	−0.142	−0.136	−0.115
	(0.046)		(0.050)	(0.050)	(0.046)	(0.048)
Tenure		−0.007	−0.006	0.003	−0.005	−0.004
		(0.010)	(0.010)	(0.010)	(0.009)	(0.009)
Tenure^2		0.000	−0.000	−0.000	0.000	0.000
		(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Tenure^3		−0.000	0.000	0.000	−0.000	−0.000
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R2	0.456	0.461	0.413	0.376	0.444	0.456
BIC (null)	-1791	-1880	-1816	-1293	-1797	-1765
N	3702	3772	3700	3698	3875	3697

Table C.17: Average coefficient for multiple imputation samples, PIAAC - FR, Social skills

	No seniority	No firm size	No occupation	No education	No wage	Complete
Intercept	21.392	21.812	22.503	16.921	21.667	23.323
	(13.035)	(12.968)	(13.693)	(13.003)	(12.896)	(13.240)
Sex (female)	0.018	0.025	0.048	0.043	0.001	0.024
	(0.035)	(0.035)	(0.030)	(0.036)	(0.036)	(0.036)
Montly earnings	0.069	0.113	0.151	0.135		0.099
	(0.033)	(0.031)	(0.029)	(0.032)		(0.032)
Isco Group 02	−0.654	−0.764		−0.983	−0.756	−0.777
	(0.406)	(0.407)		(0.413)	(0.412)	(0.409)
Isco Group 03	−0.051	−0.160		−0.413	−0.157	−0.165
	(0.423)	(0.425)		(0.428)	(0.423)	(0.427)
Isco Group 11	−0.033	−0.175		−0.234	0.000	−0.166
	(0.386)	(0.386)		(0.405)	(0.385)	(0.392)
Isco Group 12	−0.015	−0.186		−0.284	−0.113	−0.176
	(0.357)	(0.362)		(0.382)	(0.356)	(0.366)
Isco Group 13	−0.050	−0.219		−0.292	−0.196	−0.206
	(0.357)	(0.360)		(0.382)	(0.349)	(0.364)
Isco Group 14	0.006	−0.144		−0.315	−0.116	−0.144
	(0.382)	(0.374)		(0.389)	(0.374)	(0.382)
Isco Group 21	0.044	−0.123		−0.214	−0.106	−0.116
	(0.368)	(0.372)		(0.392)	(0.362)	(0.375)
Isco Group 22	−0.031	−0.189		−0.283	−0.197	−0.211
	(0.364)	(0.369)		(0.387)	(0.363)	(0.372)
Isco Group 23	0.059	−0.077		−0.128	−0.091	−0.063
	(0.354)	(0.357)		(0.379)	(0.352)	(0.362)
Isco Group 24	0.021	−0.149		−0.251	−0.134	−0.145
	(0.359)	(0.362)		(0.381)	(0.356)	(0.366)
Isco Group 25	−0.402	−0.546		−0.621	−0.530	−0.541
	(0.350)	(0.359)		(0.378)	(0.353)	(0.363)
Isco Group 26	0.092	−0.096		−0.141	−0.008	−0.065
	(0.374)	(0.375)		(0.396)	(0.363)	(0.379)
Isco Group 31	−0.139	−0.304		−0.492	−0.298	−0.292
	(0.358)	(0.363)		(0.375)	(0.354)	(0.366)
Isco Group 32	−0.230	−0.401		−0.520	−0.411	−0.401
	(0.353)	(0.356)		(0.373)	(0.350)	(0.360)

Isco Group 33	−0.192	−0.353	−0.499	−0.351	−0.348
	(0.346)	(0.351)	(0.369)	(0.346)	(0.355)
Isco Group 34	−0.078	−0.229	−0.378	−0.289	−0.240
	(0.356)	(0.357)	(0.376)	(0.354)	(0.363)
Isco Group 35	0.021	−0.115	−0.249	−0.065	−0.118
	(0.394)	(0.393)	(0.411)	(0.377)	(0.396)
Isco Group 41	−0.304	−0.462	−0.634	−0.511	−0.470
	(0.368)	(0.375)	(0.390)	(0.371)	(0.379)
Isco Group 42	−0.081	−0.211	−0.360	−0.207	−0.222
	(0.361)	(0.365)	(0.384)	(0.358)	(0.370)
Isco Group 43	−0.346	−0.493	−0.644	−0.515	−0.491
	(0.356)	(0.362)	(0.378)	(0.356)	(0.365)
Isco Group 44	−0.261	−0.365	−0.562	−0.386	−0.375
	(0.365)	(0.370)	(0.388)	(0.366)	(0.375)
Isco Group 51	−0.323	−0.499	−0.715	−0.564	−0.493
	(0.359)	(0.362)	(0.382)	(0.363)	(0.369)
Isco Group 52	−0.246	−0.425	−0.610	−0.445	−0.415
	(0.351)	(0.355)	(0.369)	(0.357)	(0.360)
Isco Group 53	−0.217	−0.421	−0.619	−0.443	−0.401
	(0.369)	(0.368)	(0.386)	(0.369)	(0.376)
Isco Group 54	−0.280	−0.455	−0.680	−0.454	−0.458
	(0.351)	(0.356)	(0.369)	(0.352)	(0.360)
Isco Group 61	−0.239	−0.422	−0.629	−0.429	−0.415
	(0.374)	(0.380)	(0.393)	(0.376)	(0.384)
Isco Group 62	0.793	0.580	0.415	0.306	0.645
	(0.554)	(0.576)	(0.590)	(0.570)	(0.585)
Isco Group 71	−0.591	−0.784	−1.052	−0.755	−0.773
	(0.371)	(0.374)	(0.387)	(0.369)	(0.379)
Isco Group 72	−0.351	−0.511	−0.734	−0.512	−0.508
	(0.358)	(0.360)	(0.375)	(0.357)	(0.364)
Isco Group 73	−0.207	−0.364	−0.579	−0.403	−0.362
	(0.389)	(0.387)	(0.406)	(0.386)	(0.395)
Isco Group 74	−0.230	−0.396	−0.614	−0.392	−0.400
	(0.395)	(0.399)	(0.410)	(0.388)	(0.401)
Isco Group 75	−0.394	−0.577	−0.813	−0.592	−0.573
	(0.368)	(0.369)	(0.382)	(0.362)	(0.372)
Isco Group 81	−0.522	−0.689	−0.954	−0.699	−0.693

	(0.352)	(0.354)		(0.365)	(0.354)	(0.358)
Isco Group 82	−0.459	−0.567		−0.860	−0.601	−0.610
	(0.387)	(0.385)		(0.404)	(0.380)	(0.393)
Isco Group 83	−0.389	−0.556		−0.808	−0.591	−0.561
	(0.370)	(0.374)		(0.390)	(0.372)	(0.379)
Isco Group 91	−0.298	−0.491		−0.745	−0.525	−0.480
	(0.370)	(0.373)		(0.387)	(0.369)	(0.377)
Isco Group 93	−0.371	−0.498		−0.792	−0.571	−0.539
	(0.373)	(0.379)		(0.394)	(0.375)	(0.381)
Isco Group 94	−0.467	−0.660		−0.930	−0.664	−0.663
	(0.420)	(0.415)		(0.431)	(0.410)	(0.425)
Isco Group 95	−0.751	−0.959		−1.198	−1.102	−1.006
	(0.376)	(0.383)		(0.407)	(0.372)	(0.388)
Isco Group 96	−0.319	−0.480		−0.710	−0.486	−0.478
	(0.421)	(0.434)		(0.444)	(0.430)	(0.436)
age	−4.142	−4.173	−4.438	−3.400	−4.091	−4.443
	(2.197)	(2.194)	(2.322)	(2.195)	(2.187)	(2.237)
age^2	0.311	0.309	0.327	0.262	0.307	0.329
	(0.150)	(0.150)	(0.159)	(0.150)	(0.149)	(0.153)
age^3	−0.012	−0.012	−0.012	−0.010	−0.012	−0.012
	(0.005)	(0.005)	(0.006)	(0.005)	(0.005)	(0.005)
age^4	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
age^5	−0.000	−0.000	−0.000	−0.000	−0.000	−0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
age^6	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Lower secondary	0.000	−0.024	−0.000		0.004	−0.014
	(0.084)	(0.082)	(0.082)		(0.084)	(0.082)
Upper and Post Secondary	0.214	0.192	0.247		0.241	0.195
	(0.084)	(0.081)	(0.081)		(0.083)	(0.082)
Bachelor	0.388	0.354	0.536		0.421	0.350
	(0.090)	(0.087)	(0.083)		(0.085)	(0.086)
Higher Tertiary	0.442	0.386	0.620		0.472	0.369
	(0.096)	(0.091)	(0.082)		(0.091)	(0.092)
11 to 50 workers	0.082		0.089	0.107	0.088	0.093
	(0.038)		(0.037)	(0.037)	(0.034)	(0.037)

51 to 250 workers	0.074		0.072	0.107	0.089	0.094
	(0.049)		(0.050)	(0.049)	(0.047)	(0.050)
250 to 1000 workers	0.058		0.063	0.108	0.090	0.089
	(0.048)		(0.047)	(0.047)	(0.044)	(0.048)
More than 1000 people	0.053		0.081	0.108	0.099	0.096
	(0.064)		(0.064)	(0.066)	(0.064)	(0.066)
Tenure		−0.018	−0.018	−0.024	−0.023	−0.022
		(0.012)	(0.012)	(0.011)	(0.011)	(0.011)
Tenure^2		0.001	0.001	0.001	0.001	0.001
		(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Tenure^3		−0.000	−0.000	−0.000	−0.000	−0.000
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R2	0.114	0.117	0.095	0.109	0.122	0.119
BIC (null)	29	2	-199	43	-3	34
N	3542	3594	3540	3538	3699	3537

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