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Mathematical Platonism Meets Ontological Pluralism?

Mathematical platonism is the view that abstract mathematical objects exist. Ontological pluralism is the view that there are many modes of existence. This paper examines the prospects for plural platonism, the view that results from combining mathematical platonism and ontological pluralism. I will argue that some forms of platonism are in harmony with ontological pluralism, while other forms of platonism are in tension with it. This shows that there are some interesting connections between the platonism-antiplatonism dispute and recent debates over ontological pluralism.

1. Mathematical Platonism and Ontological Pluralism

Mathematical platonism is the doctrine that abstract mathematical objects exist (see Linnebo 2013). Ontological pluralism is the doctrine that there are different modes of existence (McDaniel 2009,Turner 2010) and (usually) that abstract objects have a different mode of existence than concrete ones (McDaniel 2009, Tuner 2010 both attribute to abstracta and concreta different modes of existence). Ontological monism, by contrast, is the view that there is just one mode of existence.

The purpose of this paper is to evaluate whether pluralist platonism, the combination of mathematical platonism and ontological pluralism, might reveal itself to be superior to monist platonism, the combination of mathematical platonism and ontological monism. I will argue that some versions of platonism are in harmony with ontological pluralism, while other forms of platonism are in tension with it. Kris McDaniel has argued that "accepting that there are different ways of being might impact certain ontological disputes, such as the dispute between nominalists and realists over mathematical entities" (McDaniel 2009, 291). I am going to argue that also the reverse holds: the dispute between platonists and nominalists might impact recent debates over ontological pluralism, in the sense that whether platonists find ontological pluralism attractive depends on the kind of platonist account they favor.

I am going to proceed as follows. In section 2, I sketch one way of combining mathematical platonism and ontological pluralism. The view is inspired by Hilbert's conception of truth and

existence in mathematics and contains also elements of alethic pluralism. As Shapiro (2014b) points out, a natural extension of the view also includes a form of logical pluralism. I label such a combination of mathematical platonism and ontological, logical, and alethic pluralism *plural platonism*. In section 3 I discuss how plural platonism solves some puzzles associated with platonism, in particular the epistemological challenge against platonism. In section 4, I introduce a version of mathematical platonism called *heavy duty platonism* (Knowles 2015) and argue that heavy duty platonism and plural platonism might be in tension with each other. Some philosophers hold that the indispensability argument should convince us to embrace platonism. In section 5, I argue that supporters of the indispensability argument have some reasons not to adopt plural platonism. Section 6 discusses the connection between the platonism-antiplatonism dispute and the debate over ontological pluralism.

2. From Hilbert to plural platonism

Frege thought that the axioms of geometry were consistent because they were true (of space). In his correspondence with Frege, Hilbert points out that he sees things differently:

> ...As long as I have been thinking, writing and lecturing on these things, I have been saying the exact reverse: if the arbitrarily given axioms do not contradict each other with all their consequences, then they are true and the things defined by them exist. This is for me the criterion of truth and existence. (Frege 1980, 42).

Hilbert's position is usually summarized in the slogan "Consistency is the criterion for truth and existence". The slogan, which I will call the consistency criterion, calls for a qualification. Hilbert's point is not that, in every domain of discourse, consistency is the criterion of truth and existence. Rather, Hilbert is saying that *in mathematics*, consistency is the criterion for truth and existence. In other areas of discourse, consistency is not the criterion for truth and existence (Shapiro 2000, p. 156). Everyday claims about ordinary objects might be consistent but false.

Hilbert adopted the consistency criterion to vindicate the legitimacy of incompatible, but internally consistent, mathematical theories. He did not use the consistency criterion to defend a

version of mathematical platonism. And yet it is possible to use the consistency criterion to support a form of platonism.

A version of platonism based on the idea that every consistent mathematical theory is true of some abstract mathematical objects has been presented by Mark Balaguer under the label of *plenitudinous platonism* (1995, 1998a, 1998b see also Restall 2003 and Rabin 2007).

If Hilbert's idea that consistency is the criterion for truth and existence is meant to apply to the mathematical domain of discourse but not to the concrete domain, then plenitudinous platonism might be presented, in contemporary jargon, as a form of alethic and ontological pluralism.[1]

Alethic pluralism is the view that there are different ways of being true and ontological pluralism is the view that there are different ways of being. The central thought of alethic pluralism is "that there may be different things to say about the nature of truth in different cases, or different 'domains of discourse'" (Cotonoir and Edwards 2014, 118). The central thought of ontological pluralism is "that different things constitute existence in different cases" (Cotonoir and Edwards 2014, 23).[2]

Plenitudinous platonism is at home with both alethic pluralism and ontological pluralism. Plenitudinous platonists hold that there is something to be said about the notions of existence and truth in the mathematical discourse that is different from what we say about truth and existence in other domains of discourse: in mathematics consistency is the criterion for truth and existence.

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[1] The claim that according to plenitudinous platonism, consistency is the criterion for truth calls for some qualifications. According to plenitudinous platonism, every consistent mathematical theory is true of some mathematical objects. Given well known independence results in set theory, this means that ZFC+ CH is true of some objects and ZFC+ ¬CH is true of some objects. This does not mean that both the Continuum Hypothesis and its negation are true simpliciter. They are not true of the same objects, according to the version of plenitudinous platonism endorsed by Balaguer (see Balaguer 1995, Rabin 2007). Moreover, according to Balaguer, there might be consistent theories whose models are unintended models. For instance, according to Balaguer, there are consistent arithmetical theories whose models do not respect what he calls "our full conception of the natural numbers" (Balaguer 1998b, section 2): these theories are not true *simpliciter*; they are true of some objects (different from the natural numbers). In what follows, I will continue to speak of consistency as the criterion for truth, taking such important qualifications as understood. Thanks to an anonymous referee for pointing this out.

[2] See Cotonoir and Edwards 2014 and Pedersen 2014 on the connections between alethic and ontological pluralism.

I said that plenitudinous platonism *might* be presented as a form of alethic and ontological pluralism. But it need not be presented in such a way. As a matter of fact, Balaguer did not mention ontological or alethic pluralism in his presentation of plenitudinous platonism (1995, 1998a).

Yet, there is a reason why presenting plenitudinous platonism as a form of onto/alethic pluralism might be convenient for the plenitudinous platonist. Plenitudinous platonism draws a distinction between the epistemology of mathematics and the epistemology of other disciplines: in mathematics, consistency is a sufficient condition for the acceptance of a theory, whereas this does not hold in other disciplines. Ontological pluralism draws a distinction between modes of existence of abstract and concrete objects which parallels the epistemological distinction drawn by plenitudinous platonism. The two views are in harmony with each other. According to plenitudinous platonism, all possible abstract objects exist, whereas not all possible concrete objects exist: this explains why, according to the plenitudinous platonist, consistency is a criterion for truth in mathematics but not in physics. This is coherent with the idea, typical of ontological pluralism, that abstract and concrete objects have different modes of existence. McDaniel has claimed that "[t]he hypothesis that […] epistemological differences are grounded in different ways of existing is both viable and intellectually satisfying" (McDaniel 2009, 316). Perhaps the claim that the epistemological theses proper of plenitudinous platonism need to be *grounded* in the meta-ontological theses proper of ontological pluralism is too strong. All I want to defend is the idea that plenitudinous platonism and ontological pluralism can be combined in a way that is "both viable and intellectually satisfying" as McDaniel says.[3][4]

[3] McDaniel was not talking about plenitudinous platonism in the quoted passage, but he was talking about a similar principle of plenitude "for every way that something actual could be, there is something possible that is that way"(McDaniel 2009, 316).

[4] Note that according to alethic pluralism (a doctrine distinct form, but connected to, ontological pluralism), mathematics is not the only area of discourse in which the relevant truth property is a form of coherence rather than correspondence (ethics and aesthetics are often cited as other areas of discourse where truth behaves differently than it does in our discourse about ordinary objects). Alethic pluralism might help the plenitudinous platonist to place the consistency criterion within a broader account of how the notions of truth and existence behave in different domains of discourse.

In sum: there is a form of platonism, plenitudinous platonism, that makes a neat distinction between the epistemology of mathematics and the epistemology of other disciplines. Such a version of platonism can be fruitfully combined with ontological/alethic pluralism.[5]

It is worth noting that plenitudinous platonism might be fruitfully combined not only with ontological and alethic pluralism but also with logical pluralism, the idea that there is more than one legitimate notion of logical consequence, i.e., more than one acceptable logic (see Beall and Restall 2006). The idea, typical of plenitudinous platonism, that consistency is the criterion for truth and existence in mathematics might also be naturally combined with logical pluralism.[6] Such a combination has been explored by Shapiro (2014a, 2014b). As Shapiro notes, consistency is a logical notion. A theory is consistent if no contradiction follows from it.[7] But what follows from what depends on the notion of logical consequence (that is, on the logic) we adopt. A theory might be inconsistent if a certain notion of logical consequence is adopted, but consistent if another notion of logical consequence is adopted. Shapiro (2014b) discusses some mathematical theories that are inconsistent with classical logic, but consistent with intuitionistic logic. The theories discussed by Shapiro are interesting and mathematically fruitful. Their interesting character and fruitfulness constitutes a reason to count them as acceptable mathematical theories. Adopting logical pluralism along with the idea that consistency is the only criterion for the acceptability of a mathematical theory provides a way to vindicate the feeling that the theories discussed by Shapiro are indeed acceptable. Such theories are intuitionistically consistent (consistent with intuitionistic logic) and, according to logical pluralism, being intuitionistically consistent is one way of being consistent; according to the hilbertian criterion, being consistent is enough for being an acceptable mathematical theory;

[5] There are really three theses here. A metaphysical distinction: all possible abstract objects exist, not all possible concrete objects do. An epistemological distinction: consistency is the criterion for truth in mathematics, not in physics. A (meta)ontological distinction: abstract and concrete objects have different modes of existence. I do not want to take stance on which distinction is more fundamental. I just want to point out that the epistemological distinction might be more plausible when accompanied by the other two distinctions. Thanks to an anonymous referee for pressing me on this issue.

[6] On how to combine alethic, ontological and logical pluralism, see Pedersen 2014.

[7] In the case of paraconsistent logics, consistency might be replaced with non-triviality: a theory is non-trivial iff not every sentence expressible in its language follows from the theory.

therefore combining ontological pluralism and the hilbertian criterion we get that the theories discussed by Shapiro are acceptable.

3. Epistemological objections to platonism and plural platonism

In the previous section I have argued that plenitudinous platonists have reasons to endorse plural platonism. In this section I am going to review some reasons for mathematical platonists to adopt plenitudinous platonism. The resulting picture will be that mathematical platonists have reasons to find plural platonism attractive.

The reason why plenitudinous platonism might be an attractive version of platonism is that it offers a neat reply to one of the most famous objections against platonism, the so-called epistemological challenge against platonism (see Liggins 2010).

The core of the epistemological objection to platonism is this: according to platonism, we can form accurate beliefs about a domain of abstract mathematical objects; but abstract objects are mind-independent and causally isolated (we cannot see them, nor touch them, nor smell them, etc.), so how is it possible for us to form accurate beliefs about them? In sum, the objection is that mathematical platonism fails to explain how mathematical knowledge is possible (Field 1989, Introduction).

Plenitudinous platonists reply to this objection appealing to the principle that consistency is the criterion for truth and existence. The outline of the reply that plenitudinous platonists offer to the epistemological objection against platonism is the following explanation of how we form reliable beliefs about abstract mathematical objects (see Balaguer 1995):

(E1) we can form reliable beliefs about the consistency of some mathematical theories

and

(E2) If plenitudinous platonism is true, then if a mathematical theory is consistent, the objects it quantifies over or refers to exist.

Combining (E1) and (E2) we reach the conclusion that when we believe a mathematical theory because we believe it to be consistent, then usually the theory really is consistent and hence true (of some objects). This is how plenitudinous platonists explain the reliability of our mathematical beliefs.

The explanation of the reliability of our mathematical beliefs offered by the plenitudinous platonist appeals to (E2), the idea that in mathematics consistency suffices for truth. We saw that alethic/ontological pluralism might be used to support (E2). This means that alethic and ontological pluralism support one promising reply to one of the most pressing objections against platonism.[8] This gives a reason to platonists to adopt the version of plural platonism that combines plenitudinous platonism and onto/alethic pluralism.

It might be worth noting that one way to support (E1) might be to adopt logical pluralism. If there are various admissible notions of consistency, it might be reasonable to hold that a successful mathematical theory must be consistent in at least one of the admissible notions of consistency. I say this to highlight how natural the combination of plenitudinous platonism with ontological, alethic, and logical pluralism is.[9]

4. Heavy duty platonism and ontological pluralism

In the previous sections I have argued that there is a version of platonism, i.e. plenitudinous platonism, that is in harmony with ontological and alethic pluralism. Moreover, plenitudinous platonism has certain nice features which are enhanced by the combination with ontological, alethic, and logical pluralism. Given that plenitudinous platonism is an attractive position and that it combines well with onto/alethic pluralism, platonists have reason to adopt plural platonism, the combination of onto/alethic pluralism and plenitudinous platonism.

In this section I am going to argue that there is another form of platonism, heavy duty platonism, which might be in tension with ontological pluralism and that this tension gives to

[8] Field himself (2001, 325-6) acknowledges that Balaguer's reply to the epistemological challenge against platonism is a good one.
[9] Perhaps some issues touched by Beall (1999) might be relevant for this point.

supporters of heavy duty platonism a reason not to endorse ontological pluralism. I will first introduce heavy duty platonism and then explain why it is in tension with ontological pluralism.

Heavy duty platonism (HDP) is a view about the metaphysics of physical magnitudes, like temperature and mass. The most recent discussion of HDP is Knowles (2015), which characterizes HDP as "the view that physical magnitudes, such as mass and temperature, are cases of physical objects being related to numbers" (Knowles 2015, 1255).

It is important to distinguish HDP from weaker versions of platonism. All platonists believe in platonic relations, i.e. "relations of physical magnitude that relate physical things and numbers" (Field 1989: 186). The difference between HDP and weaker versions of platonism has to do with:

> what they tell us about these relations: weaker forms tell us they are derivative of more fundamental properties or relations that hold of physical objects alone, while HDP says these relations are fundamental (Knowles 2015, 1256)

According to weaker versions of platonism this chair bears the has-mass-in-kg relation to the number 8 because this chair has-mass-8Kg (mind the hyphens!) and *has-mass-8-kg* is a nominalistic property: a property that holds 'of the physical object alone' as Knowles says. The obtaining of platonic relations between concreta and abstracta, on standard versions of platonism, is grounded in the nominalistic properties possessed by concreta (see Liggins 2016).[10]

As Knowles (2015) notes, in the literature it is usually assumed that HDP is incorrect, but the usual arguments against HDP can be criticized: Knowles 2015 is precisely a powerful critique of usual arguments against HDP.[11] I think that Knowles' paper succeeds in showing that "HDP has been unfairly ignored in the literature, and the arguments […]that take the falsity of HDP as

[10] Weaker forms of platonism might also "explain the obtaining of quantitative relations [like the *has-mass-in kilograms* relation] by appeal to the obtaining of nominalistic relations. In the case of mass, these nominalistic relations are likely to be comparative relations, such as *is-more-massive-than* and *is just- as-massive-as*." (Liggins 2016, 536). See Liggins 2016 for discussion.

[11] See in particular the first section of Knowles (2015), where it is argued that "HDP is not threatened by the intuition that some physical magnitudes are intrinsic properties" (Knowles 2015, 1269).

a key premise should be re-assessed" (Knowles 2015, 1256). Moreover, even though I do not know of any philosopher who explicitly defended HDP, there are some philosophers who might be attracted to such a view (see Knowles 2015, fn. 1 for a list). In sum: HDP deserves to be taken seriously.

I submit that heavy duty platonism is in tension with ontological pluralism. This is because both ontological pluralism and heavy duty platonism are theses about the metaphysical structure of reality, but they characterize such a structure differently. Here is how ontological pluralists explain to others the way they think about the structure of reality:

> I am sure you will distinguish between different "ontological categories": abstracta and concreta, or objects and events, or possibilia and actualia, perhaps. Since you are not an ontological pluralist, you will think that the only way to make these distinctions is to say that things are some way or another […] But as an ontological pluralist, I hold that thinking of reality as having a single ontological structure—a single pegboard—is a mistake, as is thinking of ontological categories as divisions within this single structure. Rather, reality has multiple ontological structures—multiple, *independent* pegboards—with, say, the abstract things on this one and the concrete things on that one (Turner 2010, 3, my italics).

What bothers me here is not the idea that reality might be composed by different pegboards: this is perfectly compatible with HDP. What bothers me is the idea, endorsed by Turner's pluralist, that those pegboards are "independent". According to the heavy duty platonist "there is a robust metaphysical connection between physical objects and numbers that renders the latter explanatorily relevant to physical phenomena" (Knowles 2015, 1269). But it is hard to see how there could be a robust metaphysical connection between abstracta and concreta, as the heavy duty platonist holds, if abstract and concrete entities belonged to "independent pegboards", as the ontological pluralist holds. At least it is fair to ask in which sense a platonist attracted by HDP might accept the thesis that the concrete and the abstract realm are independent.[12]

[12] Thanks to an anonymous referee for pressing me on this point.

One might reply that Turner's talk of "multiple, independent pegboards" is just metaphorical talk and should not be taken too seriously. Turner's talk of pegboards is indeed metaphorical, but Turner explains that the pegboard-metaphor "lies at the heart of ontological pluralism" (2010, 9). So, even though claiming that ontological pluralism and HDP are incompatible might be too much, it is fair to say that the metaphor that "lies at the heart of ontological pluralism" is in tension with the central tenet of HDP.

Let me try to get to a similar (if not the same) conclusion from another angle. HDP is in line with the idea that there is no sharp demarcation between concrete and abstract entities. The central tenet of ontological pluralism, the idea that abstract and concrete entities have different modes of existence, seems to presuppose a neat distinction between abstract and concrete entities. HDP might reasonably be included in a group of platonist positions that reject such a neat distinction: Maddy (1990) is often cited as an example. Burgess and Rosen mention other positions of this kind in Burgess and Rosen (1997, III.B.2.c). The important point is that one route to platonism might consist in the rejection of a sharp distinction between the abstract and the concrete: "perhaps the most important consequence of the difficulty of characterizing abstractness […] is that it leaves room for an anti-nominalist position of the kind being contemplated [i.e. positions like that of Maddy]" (Burgess and Rosen 1997, 31). The moral of this section is that to the extent that ontological pluralism relies on the idea that there is a sharp abstract/concrete distinction, ontological pluralism undercuts one possible route to platonism. [13]

5. Applied mathematics, mixed mathematical sentences and alethic pluralism

In the previous section I argued that ontological pluralism might be in tension with some versions of platonism. In this section I am going to argue that the versions of platonism that are in tension with ontological pluralism are also in tension with alethic pluralism.

[13] One way to challenge the abstract/concrete distinction would be to point out to examples like the property of spin: according to Morrison (2007, 552, quoted by Knowles and Liggins 2015, 3406), spin is "perhaps best viewed as a curious hybrid of the mathematical and the physical". See also Putnam (2012a, 195).

Alethic pluralism is the view that sentences belonging to the mathematical domain of discourse are true in virtue of one property, say coherence or consistency, whereas sentences belonging to the empirical domain of discourse are true in virtue of another property, say correspondence. A problem for this view is how to classify applied mathematical sentences: to which domain do applied mathematical sentences belong?[14] Call this the problem of mixed mathematical discourse.

The problem of mixed mathematical discourse is particularly pressing if one agrees with Frege that the applicability of mathematics is the key reason to consider mathematics a body of *truths* and not merely a formal game.[15] And it seems that many modern platonists, including those inspired by the so-called Quine-Putnam indispensability argument, agree that the role of mathematics in our best scientific theories is the main reason to accept mathematical theories and their (apparent) ontological commitments.[16]

The problem I want to draw attention to might be presented in the form of a dilemma that confronts a platonist who (i) takes applicability of mathematics as the key reason to believe in the truth of standard mathematics and (ii) at the same time wants to distinguish neatly sentences belonging to the mathematical domain of discourse from sentences belonging to the empirical domain of discourse and assign different truth properties to sentences belonging to different domains of discourse.

On the one hand, mere consistency does not seem a sufficient condition for the truth of applied mathematical sentences, so applied mathematical sentences do not seem to be true in virtue of the property usually associated with the mathematical domain of discourse.

[14] I am thinking of mathematical physics as the key example of applied mathematical theory. See Field (2016, P-5, i.e. page 5 in the introduction to the new edition) for discussion.
[15] "it is applicability alone which elevates arithmetic from a game to the rank of a science" (Frege, 1893–1903, Vol. II, §91]).

On the other hand, if applied mathematical statements are the paradigm case of mathematical sentences that we consider true, it would be strange to hold that applied mathematical sentences do not belong to the mathematical domain of discourse. Assigning pure and applied mathematics to completely different domains of discourse would render their interaction mysterious. Moreover, it seems natural to assume that the empirical domain of discourse is the area of discourse that is about empirical objects. If this is the case, then holding that applied mathematics belongs to the empirical domain of discourse would entail that applied mathematical sentences are not about abstract mathematical objects and so do not entail the existence of mathematical objects.

In principle one could hold that applied mathematics does not entail the existence of any abstract mathematical object, but that pure mathematics does and that we have reason to believe that pure mathematical sentences are true and mathematical objects exist. This is of course an option, but it is worth pointing out that if applied mathematical sentences do not entail the existence of abstract mathematical objects, then one standard argument for platonism (applied mathematical sentences are true, hence mathematical objects exist) is blocked. As remarked above, many platonists take this applicability-based argument for platonism as the master argument for platonism. If adopting the distinction between domains of discourse proposed by the alethic pluralist meant renouncing to one prominent argument for platonism, this would be a serious cost. So the plural platonist would better look for another solution to the problem of classifying applied mathematical sentences.

One natural reply is that applied mathematical sentences are the result of combining sentences that belong to two domains of discourse: the empirical and the mathematical. One way to defend this view might be called the splitting strategy. According to the splitting strategy, we can divide the content of an applied mathematical sentence into two components. One component belongs to the empirical domain of discourse and could be expressed by a purely empirical sentence. The other component belongs to the mathematical domain of discourse and

could be expressed by a purely mathematical sentence. Applied mathematical sentences might be considered equivalent to the conjunction of the two sentences expressing their empirical and mathematical components. When dealing with very simple applied mathematical sentences, the splitting strategy looks promising. Consider a sentence like:

(A) The number of dragons = 0.

The content of (A) might be split into two components. A purely empirical component:

(E) There are no dragons.

And a purely mathematical component:

(M) The number of dragons = 0 if and only if there are no dragons.

According to the splitting strategy A is equivalent to E&M.

The problem with the splitting strategy is that, if successful, it would achieve a key anti-platonist goal. It would amount to the realization of a key step in Hartry Field's nominalization program (1989, 2016). Field makes the point this way:

> the task of splitting up mixed statement into purely mathematical and purely nonmathematical components [...] i s precisely the same as the task of showing that mathematics is dispensable in the empirical sciences [...] Certainly, then, no one doubtful of the possibility of carrying out the nominalist program of showing the dispensability of all reference to mathematical entities in science could consistently advocate the strategy (Field 1989, 235)

Field's nominalization program (Field 1989, 2016) consists in showing that (i) a scientific theory T can be divided in two sub-theories, a nominalistic theory N (i.e. a theory which does

not quantify over nor refer to abstracta) and a mathematical theory M, and in showing that (ii) M is conservative over N. The most difficult part of the program is step (i): formulating a nominalistic theory N with the same expressive power of the original scientific theory T (which usually quantifies over and refers to abstracta). The main obstacle in the realization of Field's program is that "there may be observations that we want to formulate that we don't see how to formulate without reference to numbers, or there may be explanations that we want to state that we can't see how to state without reference to numbers" (Field 1989, 161-2). Friends of the so-called indispensability argument (Colyvan 2015) hold precisely that there are observations and explanations of such a kind: they think step (i) of Field's program cannot be realized. But step (i) of Field's program is equivalent to the splitting strategy, which means that friends of the indispensability argument cannot support the splitting strategy.

So the problem of the splitting strategy is that it requires the realization of a technical program that is very difficult, that platonists persuaded by indispensability considerations consider unrealizable, and that if successful would bring support to a nominalistic position, i.e. Field-style nominalism.[17] This suggests that adopting the splitting strategy to solve the problem of classifying mixed mathematical statements is not a promising option for platonists persuaded by the indispensability argument.

Of course a simple solution to the problem of mixed mathematical discourse is to deny the idea of a sharp separation between the mathematical and the empirical discourse. That seems precisely to be the spirit of the indispensability argument. But the idea that there are different areas of discourse, which are associated with different truth properties and are about objects that enjoy different modes of existence, plays an important role in motivating alethic and ontological

[17] Here is a sentence that can create problems for the splitting strategy, according to Field: "there is a differentiable function ψ from points of space to real numbers such that the gradient of ψ gives the gravitational force on any object per unit mass of that object" (Field 1989, 234) Let me stress that what Field is saying here is that if the splitting strategy could be successfully applied also to cases like the one just mentioned, this would be good news for the nominalist, not for the platonist.

pluralism (see Cotonoir and Edwards 2014). Alethic and ontological pluralists should not take lightly the idea of abandoning the distinction between domains of discourse.

The other simple solution to the problem of mixed mathematical discourse, i.e. assigning applied mathematics to the empirical domain, was discussed above and I argued that platonists who take very seriously the applicability of mathematics to the empirical sciences are likely to reject such a view: their view is precisely that in order to say some of the things we want to say about the empirical world we need to talk about mathematical objects too, so that the mathematical and the empirical discourse cannot be sharply separated.

Admittedly, I did not show that it is logically inconsistent to combine forms of platonism inspired by the indispensability argument with alethic and ontological pluralism. I have just given some reason to think that platonists persuaded by the indispensability argument are likely to reject a thesis that alethic and ontological platonists are likely to accept, i.e. the idea that there is a sharp distinction between the empirical and the mathematical areas of discourse. That is why I speak of a tension between indispensability-driven versions of platonism and alethic/ontological pluralism.

I also think that in principle one could maintain a distinction between areas of discourse without assigning mixed mathematical sentences to just one of those areas.[18] In principle, that is possible.[19] But this strikes me as contrary to the spirit, if not the letter, of the indispensability

[18] One referee suggested that acknowledging that sentences like "this number is heavy" strike us as category mistakes might motivate a version of alethic pluralism. Perhaps, but this would require a substantial argument. For instance, I do not think that the discussion of category mistakes in Magidor (2014) supports alethic pluralism. "My toothbrush is pregnant" is recognized as a category mistake, but this does not support the idea that our discourses about toothbrushes and about women form two separate areas of discourse, each associated with a different truth property.

[19] See for instance Wyatt 2012, where it is argued that the alethic pluralist might accept that the same proposition belongs to more than one domain of discourse. It should be noted that Wyatt does not discuss the problem of applied mathematical statements and it is not clear how to apply his account to that case. In any case, I do not think that friends of the indispensability argument could be attracted by Wyatt's proposal, for reasons mentioned below. What friends of the indispensability argument find unattractive is not just the idea that propositions belong to a single domain of discourse, but also the idea that there is a neat distinction between mathematical and empirical concepts and between the mathematical and the empirical domains of discourse. Wyatt says that propositions can be "sorted into *strict* (that is, *well-defined*) domains" (2012, S225): friends of the indispensability argument hold that the mathematical and the empirical domains of discourse cannot be separated into two well-defined domains.

argument. The spirit of the indispensability argument is that the mathematical and the empirical discourses are tightly interwoven and cannot be neatly divided. They are naturally considered as parts of a unified discourse: scientific discourse. This has to do with the epistemology endorsed by friends of the indispensability argument:

> According to a tradition stemming from Quine and Putnam, we have the same broadly inductive reason for believing in numbers as we have for believing in electrons: certain theories that entail that there are numbers are better, qua explanations of our evidence, than any theories that do not. (Dorr 2010, 133)

Plural platonists are likely to disagree with this: the reason we have for believing that there are numbers, according to them, is that we believe that our mathematical theories are consistent and we believe that consistency is the criterion for truth and existence in mathematics. We don't believe in electrons because we believe our physical theories to be consistent and believe that consistency is the criterion for truth in physics.

Again, I am not saying that the two kinds of epistemological viewpoint are in contradiction. In principle, we might have more than one reason to believe that there are numbers.[20] I am saying that platonists following the Quine-Putnam tradition as portrayed by Dorr in the quote above defend an account of our mathematical knowledge that does not require the endorsement of the view that consistency is the criterion for truth and existence in mathematics. And they might find more economical not to adopt an epistemological principle if they do not need to. Moreover, note that Quine's rejection of what he called "mathematical recreation" (Quine 1981, 400), i.e. the parts of set theory that are not necessary for the applications of mathematics, is hard to reconcile with the idea that consistency is the only criterion for the acceptability of a mathematical theory.

[20] Sereni (2015) has some interesting proposals about how to combine ideas from the versions of platonism inspired by Quine with ideas from the version inspired by Frege. Of course this at most shows that sometimes different brands of platonism are combinable, not that they always are.

Incidentally, I also see a tension between an endorsement of the indispensability argument and logical pluralism. According to Putnam (2012a) the indispensability argument was an argument for the objectivity of mathematics and against those who identify mathematical truth with provability, like the intuitionists (see Putnam 2012a,182; see also Liggins 2007). Of course, one could reject the philosophy of intuitionism, i.e. the idea that provability coincides with truth, while accepting intuitionistic logic. But clearly the claim that Putnam is defending, i.e. that we should accept the principle of bivalence in order to account for the applicability of mathematics, might be used against logical pluralism: there is a privileged logic and it is the logic used in applied mathematics. Again, an ontological pluralist might reply that there is a plurality of logics used in applied mathematics. However, even if we grant this, the point is that, in its argument against intuitionism, Putnam is rejecting the Hilbertian idea that the only criterion for the acceptability of a mathematical theory is its consistency and the Hilbertian conception is central to the case for logical pluralism as articulated by Shapiro (2014a, 2014b).

6. Ontological pluralism and ontological disputes

Platonism and nominalism are theses on what there is (on whether there are or not abstract mathematical objects): they are ontological theses. Ontological pluralism and ontological monism are theses about whether the modes of existence are one or many: they are meta-ontological theses.

The central task for every platonist account of mathematics is to motivate the admission of abstract mathematical objects into our ontology. One way to make abstract objects acceptable is to adopt ontological pluralism and argue that abstract objects exist in a way that is different from the way concrete objects exist. But that need not to be the only way to make abstracta acceptable. Another way is to argue that we have the same broadly empirical reasons to accept sets in our ontology as we have to accept electrons. Platonists arguing in this way might also

add that there is no sharp distinction between abstract and concrete objects.[21] Which is the best strategy to make a case for platonism? I think this is a substantial issue and I do not want to take a stance here, but I have tried to defend the idea that the two strategies pull in different directions.

Let me say something more about how the present discussion connects with recent debates on ontological pluralism.

There is a temptation to think that the meta-ontological dispute between ontological monism and ontological pluralism has priority over the dispute between platonists and nominalists. Call such an approach the *meta-ontology first approach*. Some supporters of ontological pluralism come very close to endorsing meta-ontology first. In his sympathetic reconstruction of Heidegger's thoughts on ontology, McDaniel says that "According to Heidegger, one will not successfully engage in first order ontological inquiry unless one engages in meta ontological inquiry, and determines the meaning of 'being'" (McDaniel 2009, 2912).

Here is an argument for the meta-ontology first approach. By drawing a distinction between the mode of existence proper to abstract objects and the mode of existence proper to concrete ones, the ontological pluralist is able to show that nominalism is actually compatible with platonism: platonists are right in claiming that numbers exist abstractly, whereas nominalists are right in maintaining that numbers do not exist concretely. Once ontological monism is abandoned, the thought goes on, the platonism/nominalism debate evaporates: this means that the meta-ontological debate about pluralism vs. monism is more important than the ontological one (see McDaniel 2009, section 6.1).

[21] A platonist who maintains that there is no sharp distinction between abstract and concrete objects might still maintain that numbers and sets are abstract objects.

This line of thought is seductive, but can be resisted. The choice between ontological monism and ontological pluralism partly depends on the assessment of classical arguments for nominalism and platonism.

Why? One reason is that the choice between ontological monism and ontological pluralism will be of little significance for anyone persuaded by classical arguments for nominalism. Even if a nominalist were converted to ontological pluralism, the same considerations that led her to believe that abstract objects do not exist tout court in her monist days might lead her to believe that there is nothing that exists abstractly in her pluralist days.

To see why, consider again one of the classic arguments for nominalism: the epistemological challenge to platonism (Field 1989, Liggins 2010). The core idea behind the challenge is that the causal isolation of abstract mathematical objects seems to preclude any account of our ability to form accurate beliefs about them.

Merely saying that mathematical objects have their own form of existence does not help with this problem. Rather, the opposite is the case: if numbers exist in a radically different way from that in which concrete objects do, this seems to make it harder to understand how flesh and blood mathematicians manage to know about them. If the nominalist is correct in considering abstract objects mysterious entities, replying that abstract objects exist abstractly would be like replying that mysterious entities exist mysteriously.

Even if the mere adoption of ontological pluralism is not enough to respond to the usual anti-platonist argument, certain versions of platonism might be fruitfully combined with ontological and alethic pluralism. The combination of plenitudinous platonism with ontological pluralism is natural and provides a neat reply to the epistemological challenge, for reasons discussed in sections 2 and 3.

That said, plenitudinous platonism is just one version of platonism. Other versions of platonism, like heavy duty platonism, or versions of platonism based on the contribution of mathematics to the empirical sciences, are harder to combine with ontological pluralism or alethic pluralism.

The tentative moral I draw from this is that whether platonists might be attracted to ontological pluralism depends on which version of platonism they subscribe to. If they consider plenitudinous platonism the stronger version of platonism, then they might find ontological platonism attractive. On the other hand, platonists attracted to HDP or other versions of platonism mentioned in section 4 have no reason to adopt ontological pluralism and a few reasons to reject it.

This might suggest adopting an *ontology first* view: in order to successfully engage in meta-ontological debates and determine the sense of being, we should first engage in ontological inquiry and determine which version of platonism is the strongest.[22] But this is probably too much. There might be arguments in favor of (or against) ontological pluralism which are independent from the platonist-nominalist debate and those arguments can support one kind of platonist position rather than others.

I am content to argue that there are fruitful interactions between ontological and meta-ontological debates: the platonist-antiplatonist debate might influence the debate between ontological monists and ontological pluralists *and viceversa.*

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[22] McDaniel in his (2010) seems to take the ontology first approach. Thanks here to an anonymous referee.

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