Rethinking reflectional symmetry through Bee-Bots

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In this paper we propose an approach to reflectional symmetry that uses symmetric synchronic movements of two Bee-Bots as a way of experiencing dynamic and functional aspects of symmetric relationships. Drawing on Barad's agential realism, we investigate how the use of the two robots opens up new ways of thinking about the concept of symmetry. We present and analyse some episodes coming from a teaching experiment that involved a class of grade-4 students to highlight their entanglement with the mathematical concept and the different characterizations of reflectional symmetry that emerge through their gestures and speech.

Keywords: Reflectional symmetry, coding, agential realism, primary school, spatial reasoning.

Introduction

In primary school, reflectional symmetry is usually approached through activities focused on drawing symmetric shapes with the support of a squared grid or folding papers to produce mirror images. Most of the time, school activities involve just horizontal/vertical lines of symmetry internal to the figures, focusing students' attention mainly on intrafigural qualities of symmetry. According to Bruce et al. (2017), symmetry in school is often presented "as a static property of two-dimensional images—that is, a quality that is already manifest in a stable form" (p. 152), while a dynamic approach is rarely pursued. A static approach pushes in the background central features of the concept, in particular the fact that symmetry is a plane transformation, hence a map from a Euclidean space to itself.

Research in mathematics education provides evidence that technological instruments can promote a functional approach to the learning of symmetry. Ng and Sinclair (2015) proposed a pre-constructed sketch (the "symmetry machine") within a dynamic geometric environment and have observed a shift from a static conception of symmetry focused mainly on intrafigural qualities of shapes to a more dynamic one, which takes into account the functional relationship between a figure and its symmetric image. Faggiano et al. (2018) investigated the positive effects of the combined use of concrete-manipulative and virtual artefacts in the transition from a mainly perceptual to a more functional approach to symmetry. More generally, technological-based activities on symmetry contribute to the development of a fundamental ability for STEM studies and careers: spatial reasoning, namely "the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects" (Bruce at al., 2017, p. 146). Concerning the teaching and learning of geometric transformations, different authors (e.g., Jagoda and Swoboda, 2011; Clements et al., 2001) stress the importance of reflecting on the movements that change the initial figure into the final one. In particular, Clements et al. (2001) consider a figure and its mirrored image within the Logo microworld as paths along which turtles can move: symmetry, then, can be thought of as the correspondence between the components of each path and as the correspondence between the logo commands. Drawing on these studies, we explore the use of a pair of Bee-Bots, educational robots that can be programmed to move along specific paths, to study how relationships between movements, codes and paths of the robots can open up new ways of thinking about reflectional symmetry.

Theoretical framework

As briefly discussed in the introduction, the concept of symmetry is usually approached in a static, deterministic way in the classroom. In this paper, our interest is in discussing mathematical activities that might disrupt this vision of symmetry.

To do so, we draw on *agential realism* by Karen Barad, as it embraces mutual relationships between epistemological and ontological commitments in science. Influenced by feminist theories, Barad claims: "Realism is not about representations of an independent reality, but about the real consequences, interventions, creative possibilities, and responsibilities of intra-acting within the world" (Barad 2007, p. 37). The term *intra-action* is used to replace "interaction" and to query the assumption that matter and meaning need a priori distinction, and to challenge classic perspectives that postulate objects primary to relations. This is well illustrated by examples from the physics field, like the double-slit experiment, which made apparent to scientists the wave-particle duality of light, i.e., that light manifests particle behaviour under certain circumstances (and wave behaviour, by means of interference patterns, under different circumstances). This example sheds light not just on how the measure depends on the instrument we are using to perform such a measurement, but on the ontology of the process of measurement itself. If we put it in Barad's words, "it is not so much the case that things behave differently when measured differently; rather, the point is that there is only the phenomena – the intra-action of "apparatus" and "object" in their inseparability" (Barad, 2011, p. 14). In analogy with the physical phenomena, she proposes diffraction as a method opposed to reflection for the fact that the latter is founded on representationalism, while the former attends to specific material entanglements. This approach also implies a different ethic of knowing, which takes into account also the researcher's entanglement with methodology and data–while we acknowledge this aspect, we do not have the space to delve into it in this paper.

In mathematics education, de Freitas and Sinclair (2014) rely on Barad's realist approach to reconceive the materiality of mathematical concepts in their *inclusive materialism*. Chorney (2017) aligned with their post-human perspective and looked at students intra-acting with a digital tool to challenge the perspective that objects precede interactions. Drawing on Barad, de Freitas (2016) argues that design experiments in mathematics education field operate as diffractive apparatuses, that is, as an instrumental device that makes interference behaviour (diffraction) emerge out of specific practices (like in the case of the double-slit experiment).

> Insofar as an experiment involves a diffractive device, the experiment becomes a means of mutating concepts and reassembling the world. Such an experiment has consequential meaning and cannot be described as simply a means to test hypotheses. These quantum experiments are wonderful examples of how creative and generative experiments can be. (de Freitas, 2016, p. 157)

Therefore, mathematical concepts are not entirely determined or preformed prior to a teaching experiment: in this paper, we draw on the concepts of diffractive apparatus and intra-action to study how the concept of symmetry, together with the tool and the students' bodies is a vibrant apparatus that is put into motion inside the mathematics classroom.

Method and participants

The teaching experiment (Steffe & Thompson, 2000) we designed and analyse in the present study consisted of three two-hour meetings and involved a class of 22 grade-3 students in Italy. Each meeting alternated moments of collective discussion (when a square grid was placed in the middle of the class to host robots' and students' movements) with group activities on written worksheets. Due to space limitations, in this paper we will only focus on the first two meetings in which vertical and horizontal line reflectional symmetry was explored using two robots, which were to be programmed in order to move symmetrically to each other on the square grid. The robots involved were called Maya (a Bee-Bot) and Blue (a Blue-Bot). The students had already used the tool the previous year and knew how to make the robot move along specific paths; they also have had some experience concerning reflectional symmetry including the drawing of symmetric figures or folding symmetric configurations using a paper sheet. In order to work on reflectional symmetry with the robots, we designed an introductory "symmetric dance" that Maya and Blue would perform in front of the students. Figure 1 shows the paths traced by the two robots (Maya on the left; Blue on the right) and the corresponding codes.

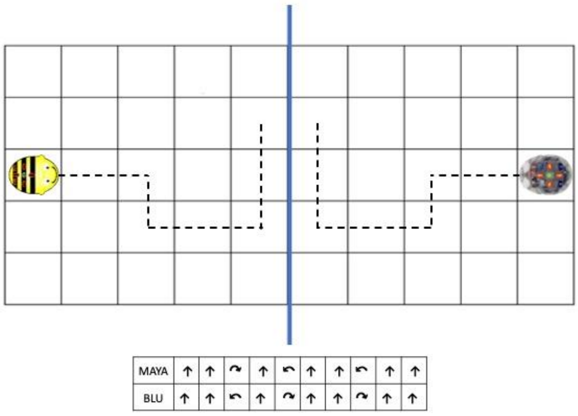

Figure 1: Diagram and codes representing the "symmetric dance" of Maya and Blue

As it can be observed in the drawing, the paths traced by the robots are symmetric and this can also be read by looking at the relationships between the sequence of instructions for each bee. In fact, forward instructions are preserved in the corresponding positions of the sequences, while each turning arrow corresponds to a turning arrow in the opposite direction. Differently from the usual way in which symmetry is generally approached in primary school, static symmetric figures are not present, but symmetric paths can be observed dynamically, explored through embodied experiences and imagined. Once the researchers secretly implemented the codes in the robots and put them in symmetric positions (with respect to the blue line) onto the grid, the two robots were simultaneously put into motion. The robot started "dancing symmetrically" in front of the students, who were seated close to the grid. We envisioned the "symmetric dance" to involve simultaneously: the paths described by the robots' movements; the coordination of movements; the sequence of instructions. Symmetry emerges out of the relationships among these elements: the shape of the paths is symmetrical; forward movements of one robot correspond to simultaneous forward movements of the other, turning left movements of one robot corresponds to simultaneous turning right movements of the other (and vice-versa); the two codes are almost identical, except for the direction of the turning arrows. Each relationship can be explored at least in two ways: observing the properties of symmetric paths from the "outside", as if we were flying above of the robots'; or, from the "inside", as if we were inhabiting the robots' bodies. We believe that this is a rich environment for developing activities which open up a perspective on symmetry more in terms of relationships (or functional correspondence) rather than the usual shape recognition and embodied experiences rather than passive observation. Along this line of thought, the didactical aim of the intervention was to create occasions for the children to experience these aspects of symmetry through other means and/or materials (like movement, embodied and/or imagined paths and codes).

The researchers (authors of the paper) co-conducted the meetings of the teaching experiment with the class teacher acting as participant-observer. Collective discussions and group work were video recorded and constitute our data for the analysis. In the next section, we present four brief episodes which illustrate different material entanglement with reflectional symmetry that have emerged during the experiment. Images that capture still frames of the video are used to support the transcript and to illustrate how students' bodies and multimodal utterances are partaking in the diffractive apparatus. Focus is on the ways in which gestures, movements, speech, and the dynamic coordination of all human and non-human components make symmetry emerge out of classroom discourse.

Episodes from the classroom

First episode

As outlined in the methodology section, during the first meeting the two researchers programmed the two robots so that they would describe symmetrical paths and put them onto the grid. The children were asked to observe "what the two robots do together" while moving. Some students noticed that the grid was divided in the middle by a blue tape and guessed that it might be a line of symmetry.

As the robots moved, the discussion started. We highlight in the following some interventions by the children in the discussion led by the researcher (R) that allow for a first characterization of reflectional symmetry emerging from the observation of the robots' movements in this first experiment:

Alberto:	They were synchronised
Adele:	I noticed that, first, when … they went in the opposite direction and then that Maya was a little ahead of Blue
Aurora:	They move … symmetrically
Adele	No! They clash, they move in opposite directions (*flatted hands pointing one another*). Because if they both moved like this (*hands moving farther from her torso, parallel to each other*) it would be the same direction [...] it's like a mirror!
Leo:	I wanted to say that they don't go in the same direction but they go in the same way because they are put like this (*hands points one another, Figure 2a*) so for Maya the right is the one (*look at the robot than places himself in Maya's direction and points to his right, Figure 2b*) and for Blue the right is the other (*turns his torso and points to his right, Figure 2c*), so Maya will turn right (*turns his torso again and point to his right, Figure 2d*) and Blue will turn left (*turns 180° and points to the left with left arm, Figure 2e*)
Gaia:	In my opinion, they drew something symmetrical because they followed the same path only in the opposite way... so, I don't know, maybe they drew a drawing and drew one side there and the other there (*points to each region of the grid separated by the axis, Figure 2f*)

Figure 2: a-b-c-d-e-f Students' gestures characterising reflectional symmetry movements

Until that moment, the children had only used one robot at a time, while now they are asked to focus on the relationship between the two robots' movements. The first utterance in the discussion brings forth the synchronism among the two robots. From this first experiment, symmetry emerges as synchronic movements which can happen along the same direction (expressed by students through hands moving parallel to each other) or in opposite directions (expressed by students through hands pointing to and approaching each other as shown in Figure 2a and through the expression "they clash"). Hands are used to reproduce the robots' movements and seem to allow students to catch a top view of the "symmetric dance". Focusing on the movement in opposite directions, Leo "steps inside" the robots' bodies, taking their points of view, to show by turning his torso that the right and left of the two bees are swapped and concludes that the turning movements should be swapped (Figures 2b-e). Focusing on the imagined paths traced by the two robots, Gaia seems to condense the top view and the bee-bots eye's views in the observation that the path is "the same [...] but in the opposite way". Looking at students' speech and gesturing, the emerging concept of symmetry is characterised by a type of correspondence between the synchronic discrete movements performed by the two robots and a specific relationship between the imagined paths traced by them. This is probably emphasised by the tool's rhythmic way of moving, which separates with a pause two consecutive movements. In the first part of the collective discussion, symmetry then emerges as coordinated movements along parallel or opposite directions.

Second episode

A second replay of the "symmetric dance" opens room for a new round of discussion:

Gaia: They drew a symmetrical path because it was not that one went forward *(points to Maya's area, Figure 3a)* and the other turned *(points at Blue's area, Figure 3b)* and they went *(parallel hands, Figure 3c)* always in time with each other *(gestures synchronous opposite paths, Figure 3d-e)*, both of them

R: So, when one went forward, what did the other do?

Gaia: It went forward, if one turned, the other did it, too

Gaia: They have the same instructions

Adele: But the body position is not the same

R: So, are the movements actually the same?

Anna: Only when they go forward they have the same instructions because when they turn, they turn in opposite ways. Because they always go in the same direction, they always make the same shape, but when they turn one goes right, the other goes left

Figure 3: a-b-c-d-e Gaia's gestures describing Maya and Blue's movements

In this second excerpt, adopting a top view, Gaia expresses the forward movements parallel and perpendicular to the axes with parallel hands and hands approaching each other's. The body position, the direction of movement, the shape, along with the instructions are all integral parts of the ways in which movements are described in the discussion. Gaia and Anna's utterances highlight how symmetry can also be a type of correspondence between synchronic instructions, because "go(ing) forward" instructions are coupled with "the same instructions" while "turning" instructions are coupled with a "turn in opposite way(s)". It is intriguing here that the word "same" is used with each of the previous elements and offers new ways to interpret the relationships at play in the experiment.

Third episode

Slightly after, Maya's code is written by a child on the blackboard. In order to touch on the idea of symmetry as correspondence between instructions, the researcher asks: "If this is Maya's code, what will Blue's code look like?". Both in words ("invert right with left, left with right") and with gestures (by crossing hands or moving forearms from left to right) students answer that a left turn instruction should be switched with a right turn instruction and vice-versa, while forward instructions do not change. Then, Gaia goes to the blackboard and writes down Blue's code, by translating one by one Maya's commands. Blue's code is not written anew but is written in function of Maya's code.

Fourth episode

During the following meeting, after a group activity, the students are seated around the square grid where the two robots are placed in symmetric cells and facing each other. The teacher prompts the students with a new question:

Teacher: I ask you one thing. If I put Blue turned like this *(rotates Blue 180 degrees counterclockwise, Figure 4a)* is it symmetrical with respect to Maya?
Students: Noooo [in chorus]
Teacher: Could it [Blue] do the same path but not with symmetric movements?
Iago: No, [Blue] goes backward *(traces with his arm Blue's direction of movement).*
Teacher: So, would it be symmetrical with respect to Maya?
Leo: The path is the same because if you go forward or backward it's the same… It's the same path. Instead, for symmetrical movements [the robots] must really make the same movements, that is, if one goes two forward, the other must go two forward, if one turns right, the other turns left… and so it goes on… instead for the same path Blue can go two backward *(points at Blue and moves the finger towards the*

axis, Figure 4b) and Maya two forward *(points at Maya and moves the finger toward the axis, Figure 4c)* and this is the same path but movements aren't symmetrical

Teacher: Filippo, what do you think?

Filippo: It would be right because if I do the same path backward, it should be the same thing, but not... the path is symmetrical except for the position [of the robot] badly put *(touches Blue and rotates it, Figure 4d)*

Leo: To make symmetric movements we should also put Maya backward

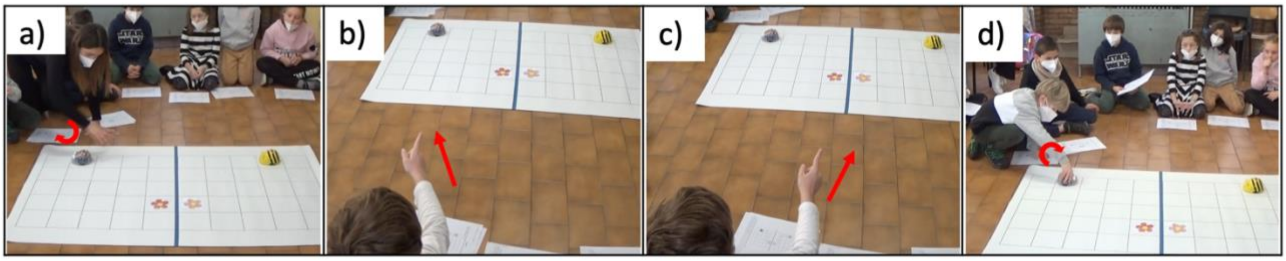

Figure 4: a-b-c-d. Characterization of almost-symmetrical movements

The teacher's intervention is meant to focus students' attention on the fact that Maya and Blue are in symmetric cells, but their orientation is not symmetric. Unexpectedly, Iago proposes to move Blue backward, and Leo observes that the path would be the same even though the robots' movements are not symmetric. In this case symmetry is observed in the imagined paths but not entirely preserved in the robots' movements. We notice a variation of the students' first way of characterising symmetry, since there is no longer a rigid correspondence between types of movements: the forward movement of Maya can correspond to a backward movement of Blue. We refer to this type of movements' correspondence for describing symmetry as *almost-symmetrical* movements.

Discussion

Following a dynamic and technology-based approach, in our study we proposed the use of a pair of Bee-Bots "dancing symmetrically" to create new possibilities for encountering reflectional symmetry in the mathematics classroom. This diffractive apparatus engages a rethinking of symmetry in terms of movements, which involves robots, children's bodies and corresponding codes. In the four brief episodes, we tried to highlight how, in different moments of the teaching experiment, the concept of symmetry is diffracted through students' speech and gestures in light of synchronic movements, directions and opposite turns. The intra-action among tools, concepts and students operates both in the rethinking of symmetry for the students but also for the teacher as–in the fourth episode–we saw how her prompt actually opens room for a new type of symmetric relationship, which takes into account simultaneous movements, symmetric paths but different orientations for the two robots. As researchers, we envisioned that correspondence between movements, codes and paths might account for approaching symmetry more in terms of a functional correspondence rather than the recognition of a static shape that possesses two "equal halves". Such correspondence is both between commands and between paths traced by the two bees. In line with de Freitas (2016), the paper illustrates the generativity of the teaching experiment, and the new nuances concerning symmetry that have been evoked through it. For example, we observed how weakening the conditions for the starting positions (the robots' point of view) created a new type of symmetry, described in terms of almost-symmetrical movements. Further research is needed to enlarge our understanding of this phenomenon, which raises interesting issues concerning a dynamic approach to symmetry, and to extend the presented approach both to the case of oblique lines of symmetry and of other congruent transformations.

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