UNIVERSITÀ DEGLI STUDI DI TORINO

Dipartimento di Matematica
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Scuola di Dottorato in
Scienze della Natura e Tecnologie Innovative
Dottorato di Ricerca in
Matematica Pura e Applicata
Ciclo XXXIII

MATHEMATICAL MEMES: FROM INTERNET PHENOMENON TO DIGITAL EDUCATIONAL RESOURCE

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Settore scientifico-disciplinare di afferenza: MAT/04 - Matematiche complementari

AKNOWLGEDEMENTS

This thesis is the end product of a collective adventure: it would not have been possible without the support of inspired professors, colleagues, teachers, friends and family. I can't thank everyone here, but certain people require a special mention.

Firstly, I must thank Professor Ornella Robutti for being my enlightened supervisor, embracing this daring project and supporting me throughout it with useful suggestions, openness and attention, and for guiding me through the PhD not just from a technical perspective, but also as a mentor. You trusted me to manage this work whilst giving me the freedom to pursue my own research interests, and this has been really invaluable.

A second but not less important thank you goes to my co-supervisor Professor Angelika Bikner-Ahsbahs, who has been listening with sincere interest, patience and positivity to my embryonal (and unavoidably confused) ideas since the very beginning of this research journey, and has taught me so much welcoming me into her home and meeting me in endless Zoom calls. I will never forget our *déjeuner sur l'herbe* in Montpellier that made me realise that my daily quest for mathematical memes on the Web could become a substantial part of this research.

The third thank you goes to my husband Gabriele, who never failed to believe in me and supported me throughout these years, with harsh (albeit sometimes due) critiques and endless dinners.

Being this a project that tackles a subject very far from the usual research perspectives of the Mathematics Education community, I am grateful to all scholars who have accepted it with an open and unbiased mind. A special thank you must be reserved for professor Zsolt Lavicza and his group of PhD students at JKU Linz School of Education, who have welcomed me many times and made me feel like I belonged there. I am grateful for your enthusiasm, which nurtured fruitful exchanges of thoughts and ideas with Bea and Diego and other fellow students and professors and contributed to making Altenberger Strasse 54 my home away from home in these years.

Next, in dispassionate chronological order, my thanks go to Professor Paolo Boero, who (surprisingly for me) endorsed this project and provided valuable ideas at the Young Researchers' Seminar in Rimini in 2019 and to all the young researchers who encouraged me there and after, among which I cannot fail to mention my best supporter Domenico Brunetto. I am also deeply thankful to Professors Nathalie Sinclair, Ron Kenett and Ricardo Nemirovsky who listened to my research ideas in their initial and shapeless form, offering useful suggestions and support, and to

Professor Pietro Di Martino, who trusted this project enough to include it in the official recommendations for distance learning education of the MaddMaths website, resulting in a precious dissemination that allowed me to reach teachers throughout the Italian territory.

I am also very grateful to my Master student and co-author Martina: I enjoyed working with you and seeing my project through Generation Z eyes.

Finally, not enough can be said to express my deep sense of thankfulness towards Silvia and Cristiano, and all the other teachers who so generously offered their time (and students) to participate in the school experiments, and towards my own student Mattia and all the 00F class group: I can only say that without you none of this would even exist.

Then, since a PhD is a life experience and not just a study experience, a big thank you goes to my fellow PhD students Carola and Riccardo, who made me feel accepted despite the generation gap, guiding me through the meanders of Palazzo Campana and sharing many *caffè gianduiato* between chats and laughter. Special thanks are also due to my dearest friends Paolo and Elení who have supported me as friends and as scholars, to Maura who convinced me to embrace this challenge, and to all my friends who provided nourishing non-doctoral interruptions: to Marina for being always there when I needed a chatty pause and to all my friend colleagues Fred, Renata, Daniela, Donata, Paola, Nicoletta, Laura and Laura who rejoiced with me in these years.

As a last note, my thought goes to Ughetto e Orsina, the feline company that anchored me to reality reminding me of their (and my) physiological needs and providing some healthy distraction from my daily quest for the right word during the endless pandemic dissertation-writing months.

Giulia Bini

Milano-Torino, 13 March 2021

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DECLARATIONS

Declaration of previous publication

This thesis includes the following six original papers that have been previously published/accepted for publication in peer reviewed journals or conference proceedings. I certify that I have obtained a written permission from the copyright owners to include the published materials in my thesis. I certify that the material describes work completed during my registration as a doctoral student at the University of Turin.

CH.	REFERENCE	STATUS
1	Bini, G., & Robutti, O., (2019). Meanings in Mathematics: using Internet Memes and Augmented Reality to promote mathematical discourse. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*, Freudenthal Group & Freudenthal Institute, Utrecht University and ERME https://hal.archives-ouvertes.fr/hal-02422152	Published 9/2019
2	Bini, G., & Robutti, O. (2019). Thinking inside the post: Investigating the didactic use of mathematical Internet memes. In A. Shvarts (Ed.), *Proceedings of the PME and Yandex Russian conference "Technology and Psychology for Mathematics Education"* (pp. 106–113). HSE Publishing House http://www.igpme.org/wp-content/uploads/2020/01/PMEYandex2019Final.pdf	Published 9/2019
3	Bini, G., & Robutti, O. (2020). Is this the real life? Connecting mathematics across cultures. In *Proceedings of the CIEAEM 71 conference, Quaderni di Ricerca in Didattica, 7.* G.R.I.M. (Dipartimento di Matematica e Informatica, University of Palermo, Italy) http://math.unipa.it/~grim/quaderno_2020_numspec_7.htm	Published 7/2020
5	Bini, G. (2021). How Spiderman Can Teach You Math: The Journey of Memes from Social Media to Mathematics Classrooms. In J. H., Kalir, & D. Filipiak (Eds.), *Proceedings of the 2020 Connected Learning Summit* (Version 2). (pp. 20–27) Carnegie Mellon University, ETC Press, USA. https://doi.org/10.1184/R1/13530038.v2	Published 1/2021
6	Bini, G., Robutti, O., & Bikner-Ahsbahs, A. (2020). Maths in the time of social media: conceptualizing the Internet phenomenon of mathematical memes. *International Journal of Mathematical Education in Science and Technology*. https://doi.org/10.1080/0020739X.2020.1807069	Published 9/2020
7	Bini, G., Robutti, O., Montagnani, M. (forthcoming). When They Tell You That $i^{56} = 1$: Affordances of Memes and Geogebra in Mathematics. *The International Journal for Technology in Mathematics Education*	Accepted for publication

Declaration of Co-authorship

I hereby declare that this thesis incorporates material that is result of joint research with the following authors:

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Author 2 Angelika Bikner-Ahsbahs, University of Bremen (Germany)
Author 3 Martina Montagnani, MSc graduate, University of Torino (Italy)

The extent of my contribution to the manuscripts as declared hereafter is based on the following scale:

A. The candidate did the majority of the work independently (67-100 %)
B. The candidate has made a substantial contribution (34-66 %)
C. The candidate has contributed to the work (0-33 %)

Chapter 1: Meanings in mathematics: Using Internet memes and augmented reality to promote mathematical discourse

Authors: Candidate, Author 1

Individual elements	Candidate's contribution
Formulation/identification of the scientific problem	B
Development of the key methods	B
Planning of the experiments and methodology design	A
Conducting the experimental work	A
Conducting the analysis of data	B
Interpretation of the results	B
Writing of the first draft of the manuscript	A
Finalisation of the manuscript and submission	B

Chapter 2: Thinking inside the post: Investigating the didactical use of mathematical Internet memes

Authors: Candidate, Author 1

Individual elements	Candidate's contribution
Formulation/identification of the scientific problem	B
Development of the key methods	B
Planning of the experiments and methodology design	A
Conducting the experimental work	A
Conducting the analysis of data	B
Interpretation of the results	B
Writing of the first draft of the manuscript	A
Finalisation of the manuscript and submission	B

Chapter 3: Is this the real life? Connecting mathematics across cultures

Authors: Candidate, Author 1

Individual elements	Candidate's contribution
Formulation/identification of the scientific problem	A
Development of the key methods	B
Planning of the experiments and methodology design	A
Conducting the experimental work	A
Conducting the analysis of data	A
Interpretation of the results	B
Writing of the first draft of the manuscript	A
Finalisation of the manuscript and submission	B

Chapter 4: Some like it social: Looking into the interplay between math and Internet memes

Authors: Candidate, Author 1

Individual elements	Candidate's contribution
Formulation/identification of the scientific problem	A
Development of the key methods	B
Planning of the experiments and methodology design	A
Conducting the experimental work	A
Conducting the analysis of data	A
Interpretation of the results	B
Writing of the first draft of the manuscript	A
Finalisation of the manuscript and submission	B

Chapter 6: Maths in the time of social media: Conceptualizing the Internet phenomenon of mathematical memes

Authors: Candidate, Author 1, Author 2

Individual elements	Candidate's contribution
Formulation/identification of the scientific problem	B
Establishing the theoretical framework	A
Developing the methodology and key methods	B
Collecting, aggregating and preparing data	A
Conducting the ethnographic work	A
Conducting the analysis of data	B
Interpretation of the results	B
Writing of the first draft of the manuscript	A
Finalisation of the manuscript and submission	B

Chapter 7: When they tell you that i^56=1: Affordances of memes and GeoGebra in mathematics

Authors: Candidate, Author 1, Author 3

Individual elements	Candidate's contribution
Formulation/identification of the scientific problem	A
Development of the key methods	B
Planning of the experiments and methodology design	A
Conducting the experimental work	A
Conducting the analysis of data	B
Interpretation of the results	B
Writing of the first draft of the manuscript	A
Finalisation of the manuscript and submission	B

Chapter 8: How to meme it: Mathematical memes as creative acts

Authors: Candidate, Author 2, Author 1 Manuscript in preparation

Individual elements	Candidate's contribution
Formulation/identification of the scientific problem	B
Development of the key methods	A
Planning of the ethnographic work and methodology design	A
Conducting the ethnographic work	A
Conducting the analysis of data	A
Interpretation of the results	B
Writing of the first draft of the manuscript	A

I certify that I have properly acknowledged the contribution of other researchers to my thesis, and have obtained written permission from each of the co-author(s) to include the above materials in my thesis.

We, the undersigned, endorse the above stated contribution of work undertaken for each of the co-authored manuscripts contributing to this thesis:

Co-authors signatures			
Date	Name	Title	Signature
13/3/21	Ornella Robutti	Full Professor of Mathematics Education, supervisor	
13/3/21	Angelika Bikner-Ahsbahs	Professor (em.) of Mathematics Education, co-supervisor	
13/3/21	Martina Montagnani	MSc graduate	

General statement

I certify that, with the above qualification, this thesis, and the research to which it refers, is the product of my own work. I declare that, to the best of my knowledge, my thesis does not infringe upon anyone's copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my thesis, published or otherwise, are fully acknowledged in accordance with the standard referencing practices.

Giulia Giovanna Maria Bini

ABSTRACT

This thesis aims at providing insights into the process of transforming the Internet phenomenon of mathematical memes as it appears naturally on the Web into a digital resource for teaching mathematics. This aim is pursued in a hybrid perspective where resources for school mathematics stretch beyond traditional educational tools, with the purpose of contributing to bridging the cultural and generational divide that separates informal out-of-school learning environments and traditional school-based learning environments.

Mathematical Internet memes are a nearly unexplored research topic in the field of Mathematics Education. Therefore, it was not possible to expand the results of already existing research, nor to rely on previously used theoretical frameworks or established methodologies. This particular condition requested to sample adequate theoretical foundations and develop appropriate methodologies for investigating this phenomenon as a manifestation of the Web 2.0 culture, guided by the main research question asking how can we conceptualize mathematical Internet memes and what could their educational potentialities be.

The investigation process tackling this question has progressed according to two research approaches during the three-year of my doctoral studies from 2018 to 2020. One is an ***exploratory*** approach, grounded on experimental data collected observing mathematical memes created by students in several designed school experiments conducted with secondary school students, university students and teachers, and subsequently analysed through some of the most significant theoretical lenses in Mathematics Education, with a stance aimed at maximizing the understanding of an unknown phenomenon by observing it from different standpoints. The other research approach is ***systematic***, grounded on a three-year-long ethnographic fieldwork conducted observing the spontaneous social phenomenon in its natural habitat, and collecting a data set of nearly 2000 memes that are analysed through the theoretical framework used to investigate the Web 2.0 culture. The ethnographic research follows an innovative methodological approach developed for this purpose, consisting of the adaptation of the research in mathematics education to the context of the Web, and yielding to a new research methodology that had never been used before to study digital phenomena from the point of view of mathematics education.

In the process of the research, the two approaches overlap and intertwine, producing the results presented in this thesis that contribute from the theoretical, methodological and empirical point of view in building a body of knowledge about exploring Internet phenomena with an educational purpose.

From the theoretical point of view, the research succeeds in conceptualising mathematical memes as representations of mathematical statements with an epistemic power that nurtures mathematical discussions, and in devising a semiotic tool to interpret the levels of meanings embedded in a meme, distinguishing the meanings pertaining to the Web 2.0 digital culture from those pertaining to the mathematics culture. Methodically, the contribution of the research consists in the development of an innovative use of ethnography framing multi-case focus studies, that now can be used to explore Internet phenomena in general. Empirically, the research accomplishes the building of a heuristic model for the creation of mathematical memes, that can be used by educators in different contexts, and the sampling of the educational potentialities of mathematical memes to mobilise emotions, channel visual skills and digital culture, and foster changes in students' praxeologies.

Drawing all the results together, the research shows that the contamination between mathematics and memes empowers both cultural realms: it upgrades the use of memes beyond their original subculture, and it expands the range of disciplinary signs traditionally pertaining to the domain of mathematics. Mathematical memes sketch a portrait of mathematics and mathematicians different from the boring and emotionless stereotype, resulting in a new kind of crossover educational resources to be used in a new way, different from the traditional teaching mode. These resources may contribute to foster the cultural change aimed at bridging the discontinuity between out-of-school and school learning environments, as embraced by the recent literature about the future themes of mathematics education research.

INTRODUCTION

Today, after more than a century of electric technology,
we have extended our central nervous system itself in a global embrace,
abolishing both space and time as far as our planet is concerned.
Marshall McLuhan, *Understanding Media*, 1964, p. 5

In 1964, Marshall McLuhan opened his book "Understanding Media" with the quote above, delineating a visionary prediction of what he later called a *global village* (p. 43): an interconnected world brought together by the unifying effect of the developing electric communication technology.

At the time his insight seemed revolutionary, today the diffusion of the Internet communication technology has transformed McLuhan's global village into a reality. At present, the Internet connects more than half of the world inhabitants (5 billion users in December 2020, 63.2 % of the world population)[1], a number that has progressed in just 25 years from the initial 16 million users (0,4% of the world population) of December 1995, the date when the military-purposed ARPAnet was opened to home users. This interconnected global village has not only changed our way of working, learning and socialising (as we all have experienced during the Covid-19 pandemic), but it has also "placed a new and interesting semiotic pressure on language" (Zappavigna, 2012, p. 2), fostering the emergence and propagation of new languages as emojis, hashtags or Internet memes (Crystal, 2004; Jablonka, 2012; Zappavigna, 2012; McCulloch, 2019).

The focus of this thesis lies in providing insights into the spontaneous mathematical mutation of one of these new languages, represented by mathematical Internet memes, with the purpose of envisaging how these digital objects that appeared naturally on the Web can be transformed into resources for teaching mathematics in a traditional educational environment.

Internet memes and their mathematical mutations are a spontaneous digital phenomenon almost uncharted by academic research in Mathematics Education. In fact, the inspiration for this

[1] https://www.Internetworldstats.com/

research comes from my 25-year experience as a secondary school Mathematics and Physics teacher, during which I have been introduced to this new language by my own students. Therefore, conducting this research required a preparatory study of cultural backgrounds and technologies which are outside the usual research perspectives of the Mathematics Education community, but are essential to understand the environment that originated the observed phenomenon.

The opening Chapter 0 aims at giving the reader some introductory information about this innovative cultural and technological background and the about the overarching progress of the research, describing (1) the context of the research, sketching the *zeitgeist* that fostered the appearance, diffusion and establishment of the new language of Internet memes which is the object of the research, (2) the rationale of the research, outlining why this new language could be profitably exploited for educational purposes, and (3) the development of the research, with the theoretical and methodological challenges implied by tackling an almost unprecedented study for the field of Mathematics Education. The research will then be described in Chapters 1-8 through the most significant peer-reviewed manuscripts that I authored or co-authored in the period of my doctoral studies, from 2018 to 2020. The closing Chapter 9 will summarise the results achieved in this thesis, discuss them and outline possible further research paths.

CHAPTER 0: OVERVIEW OF THE RESEARCH

1 The context of the research: McLuhan's global village

1.1 The pillars of the global village: the Internet and the Web

The terms Internet and Web (short for World Wide Web, also known as WWW) are commonly used as synonyms, but they are not: the Internet supplies McLuhan's *electric technology* providing the setting for the communication, while it is the Web, created in 1989 by researcher T. Berners-Lee and his group at CERN in Geneva, that provides the contents of the communication in terms of webpages, files, videos and so on. If we imagine the Internet as a rail network, the Web can be figured as a high-speed train that circulates thanks to the infrastructures supplied by the Internet railway. Nevertheless, embracing McLuhan's famous saying, "the medium is the message" (1964, p. 9), meaning that the content transferred (the message) changes and adapts to the technology used (the medium), we understand that the Internet and the Web are deeply interrelated. For the matter of this research, it is particularly relevant to gain deeper insight into how the development of the Internet technology has affected the development of the Web, yielding to what is commonly known as Web 2.0.

1.2 The evolution of the Web: Web 2.0 and the birth of the digital culture

In the initial version of the Web (subsequently retro-named as Web 1.0), creating and sharing virtual content online required specific programming skills; therefore, end-users' activities were generally limited to processing virtual content passively "in essentially static screenfuls" (DiNucci, 1999, p. 32). After 2001, the development of simplified interfaces lowering the technological barrier to communicate online expedited the evolution of this initial static version of Web services into a new generation of dynamic Web services. This new generation of software, popularised as Web 2.0 in 2004 by O'Really[2]., allowed users to actively interact with the virtual content and environment, becoming digital content creators, aggregating in virtual communities and collaborating online with each other.

Following McLuhan motto, the differences between Web 2.0 and Web 1.0 are cultural as well as technological: in fact, this easier technical accessibility inspired a new view of the Web as a community (a platform) and not simply as a collection of websites. In 2005 O'Really summarised this new culture in the map in Figure 0.1, which illustrates the set of principles and practices governing the new "architecture of participation", thus placing the basis for what is

[2] https://www.oreilly.com/pub/a/web2/archive/what-is-Web-20.html

commonly known as *digital culture.* Among these principles and practices, we find "The Right to Remix", the invitation to "Play" and the core competence of "Harnessing collective intelligence", which will be foundational in shaping the culture of Internet memes I will be delving into in the course of this thesis.

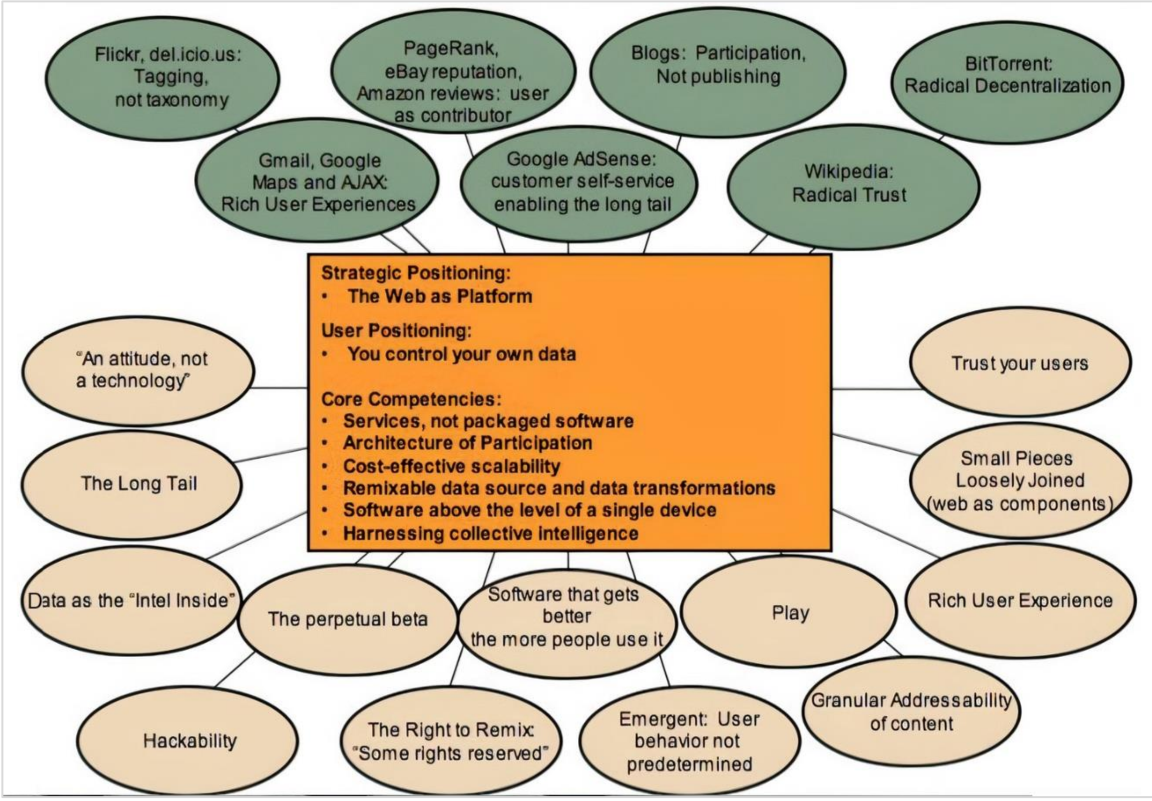

Figure 0. 1 O'Really's Web 2.0 map

Nowadays, the Web 2.0 *architecture of participation* has become a common feature, spreading from amateur blogs to commercial and institutional websites (Hardey, 2007; Harrison & Barthel, 2009; Baricco, 2018). The worldwide diffusion of Web 2.0 websites and of the related digital culture is commonly linked to two innovations that have changed the way people use the Web: the development of social network sites as Facebook, established in 2004, and the diffusion of the mobile technology, introduced by Steve Jobs in January 2007, that allows ubiquitous access to Internet services.

1.3 New meeting places in the global village: Social Network Sites

"The Web 2.0 is *inherently* social so that users are central to both the content and form of all material and resources" (Hardey, 2007, p. 870, emphasis in the original). Thus, in the Web 2.0

global village where the Internet technology is used to "enact relationships rather than simply share information" (Zappavigna, 2012, p. 2), social network sites, i.e. websites that allow individuals to connect with other users and share their own content (boyd & Ellison, 2007), play the role of virtual meeting places.

In this research, I will focus on three social network sites: Reddit, Facebook and Instagram. All three websites allow the practice of commenting and reacting to user-generated content, but they differ substantially in terms of internal architecture and accessibility, and consequently in their role in the diffusion of the digital culture. Their characteristics can be briefly outlined as follows:

- *Facebook*[3] (2,7 billion users in October 2020[4]) is a closed social network site: it is accessible only to registered users, who generally login with their real names. Users can view and post content in the form of text, pictures, videos and links, comment, and react to other people's contributions. It hosts thematic pages and public groups, whose content is open to all registered users, and closed groups that are accessible only to admitted members; pages and groups can allow filtered or unfiltered content upload by members.
- *Instagram*[5] (1,2 billion users in October 2020[4]) is a closed picture and video sharing social network site, where only registered users, who generally use nicknames, can post content. Enrolled users can upload images and videos to their profiles only, and view, comment and like other people's contributions.
- *Reddit*[6] (430 million users in October 2020[4]) an open American news aggregator, i.e. a collection of communities (called subreddits) where users share content and news, comment on other people's posts and express their appreciation for uploaded content through a system of upvoting and downvoting. It is an open platform, meaning that everyone can see the content, but only registered users, who generally use nicknames, can post, comment or vote.

1.4 Old languages in the global village: Memes as units of culture

The concept of meme originated in 1976 when evolutionary biologist Richard Dawkins

[3] www.facebook.com
[4] https://www.statista.com/statistics/272014/global-social-networks-ranked-by-number-of-users/
[5] www.instagram.com
[6] www.reddit.com

invented this term to identify the cultural correspondent of the gene. In his book *The Selfish Gene*, he describes memes as “units of culture” (Dawkins, 1976, p.249) that multiply and spread prevailing upon others, in a cultural parallel of Darwinian selective evolution. Later, Csikszentmihalyi summarised the idea of meme as "any permanent pattern of matter or information produced by an act of human intentionality" (1993, p. 120), clarifying that “memes come into being when the human nervous system reacts to an experience, and codes it in a form that can be communicated to others” (ibid.), and putting the focus on intentionality and communicability as key characteristics for memes.

In the pre-digital era, memes needed time to evolve and spread *leaping from brain to brain*, therefore successful pre-Internet memes (as dominant genes) stretched across diverse generations and traditions. Examples of these successful memes can be found in heraldry, with symbols like the Fleur-de-Lys (Rowley, 2015) evolved in time from the medieval French monarchy to the contemporary flag of Quebec to represent the ties of the Canadian province to this ancestry, and in art, with motifs like the *Three Hares* in Figure 0.2.

Figure 0. 2 Examples of a pre-Internet meme: the *Three Hares* motif

In its evolution, the *Three Hares* motif became a meme multiplying virally and travelling across time and space from 6th century China (Figure 0.2 left, from the Dunhuang Museum in north-western China[7]) to 13th century Europe (Figure 0.2 right, from Paderborn Cathedral in

[7] www.en.dha.ac.cn/

Germany, source The Three Hares Project © Chris Chapman[8]), thus becoming part of the shared heritage of the medieval civilized world.

1.5 New languages in the global village: Internet Memes as units of digital culture

In 1994, American attorney and Internet guru Mike Godwin was the first to use the term "meme" to describe a viral idea spreading through the Internet, in his article "Meme, Counter-meme"[9] published in the online magazine Wired. Nowadays, the term Internet meme has become part of the common Web 2.0 language to indicate a "digital image, video or piece of text, typically humorous in nature, that is copied, personalised and shared online by Internet users"[10]. Internet memes are memes in Csikszentmihalyi's sense, since they are produced by human *intentionality* and can be *communicated* (1993), epitomising the Web 2.0 digital culture (Shifman, 2014).

The common thread linking genes, Dawkins' memes, and Internet memes is that all three disseminate leaping from one host to another and that they prevail and survive in the meme/gene pool if they are strong enough and can mutate and adapt to new contexts. The distinct feature of Internet memes is their humoristic stance, which embodies the "Play" principle of the Web 2.0 map in Figure 0.1. Thus, they prevail and survive if they are sufficiently funny, relatable and "spreadable" (Jenkins et al., 2013, p. 20), i.e. fitting within the Web 2.0 digital culture, having both the technical and cultural potential to be mutated and customised to share personal content.

Internet memes are built through a process of copying, representing the "Right to Remix" leading principle in the Web 2.0 culture, a culture where copies "are the raison d'être of digital communication" (Shifman, 2014, p.12). Nevertheless, Internet memes are intrinsically different from viral digital objects that are simply shared without any personal customisation: they are more akin to Queneau's or Warhol's postmodern conception of copy, which is a sort of rewriting that brings the authorial skills of its interpreter into play (Bollini, 2017). They are also different from genes, whose mutations happen by chance. The characterising features of Internet memes are *designed mutation*, as Richard Dawkins himself acknowledged in his 2013 talk: "an Internet meme is a hijacking of the original idea, instead of mutating by random chance before spreading by form of Darwinian selection, Internet memes are altered deliberately by human creativity; in the hijacked version, mutations are designed not random, with the full knowledge of the person

[8] http://www.chrischapmanphotography.co.uk/hares/page7.htm
[9] https://www.wired.com/1994/10/godwin-if-2/
[10] https://www.oxfordlearnersdictionaries.com/definition/english/meme

doing the mutating" (Dawkins, 2013, 00:04:16).

Shifman, in the first book-length study on the topic (2014), classifies nine meme genres, distinguishing among video-based memes and image-based memes.

Figure 0. 3 Examples of Internet meme genres

Examples of video-based Internet memes are flash mobs: videos depicting strangers who gather in a public place to perform a particular act. Figure 0.3 (left, source YouTube[11]) shows an example of a flash mob meme: a screenshot from the White House version of the *Mannequin Challenge* video, where a moving camera films people frozen in action like mannequins. Examples of image-based Internet memes are photo fads, staged photos in which the subject imitates a specific action, and image macros, digital images wittily reinterpreted with superimposed captions. Figure 0.3 (centre, source Bored Panda[12]) shows an example of photo fad, the *Leaning Tower of Pisa* forced perspective. Figure 0.3 (right, source Reddit[13]) shows an example of an image macro, created remixing a popular memetic image known as *Two Buttons*, showing a guy struggling with a problematic choice denoted by the inscriptions on two red buttons. Here the author's "designed mutation" implied adding Shakespeare's portrait onto the guy's face and the "To Be" and "Not to Be" labels on the two buttons to represent Hamlet's monologue famous incipit.

Internet memes are everywhere on the Web: we get the idea of their diffusion from the growth of the number of occurrences of the hashtag #memes in Instagram shown in Figure 0.4. I have been collecting these data since August 2018, and I have observed a steadily growing

[11] https://youtu.be/5ZzklOEGW0w
[12] https://www.boredpanda.com/funny-tourist-photos-leaning-tower-of-pisa/
[13] https://www.reddit.com/r/HistoryMemes/comments/ctlyrx/the_allusive_shakespeare_meme/

progression from nearly 50 million in August 2018 to 184 million in March 2021.

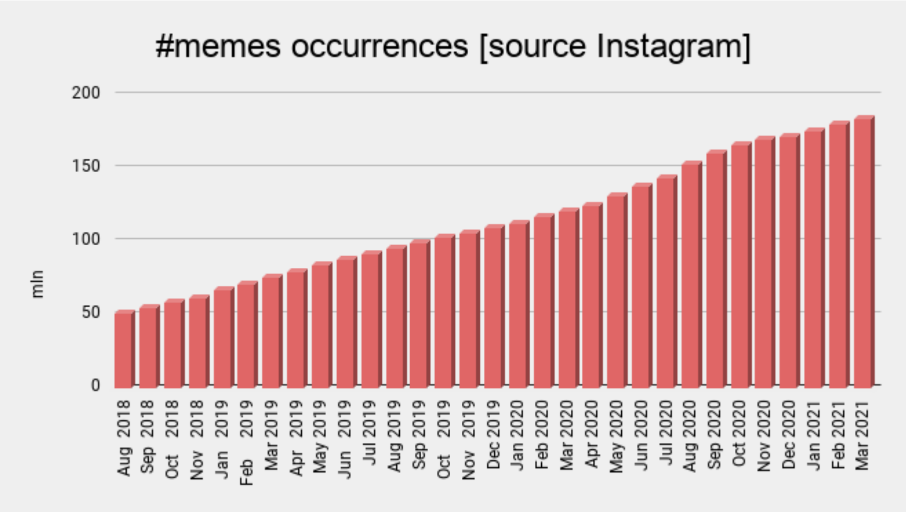

Figure 0. 4 The hashtag #*memes* on Instagram (data collected by the author)

1.6 The champions of Internet memes: Image Macros

Among all genres of Internet memes, image macros (Figure 0.3, right) stand out because they are easy and fast to produce, and highly *spreadable*. Image macros are built on existing images with codified meanings, that are humorously reinterpreted by Internet users to convey a wide variety of messages, from political opinions (Huntington, 2017; Denisova, 2019) to personal feelings (Milner, 2013; Miltner, 2014).

The term *Image Macro* itself reflects this practice: it was coined in the forum of the comedy website Something Awful[14], originating from the term *Macro*, used in Computer Science. In computer programming, a macro is a single command that expands into a complex pre-written sequence of computing instructions; macros are language-targeted and function only if processed by a compiling program that recognises them and expands them into the codified sequence. Similarly, an image macro is a meme whose pictorial part expands into a complex pre-written meaning, provided that the reader has sufficient background knowledge to "compile" it.

In a nutshell, image macros are interpretative puzzles (Dzani & Berberovic, 2017) that the reader solves grasping the pictorial and verbal cues and connecting them, as happens in the Shakespeare example in Figure 0.3 (right).

[14] https://forums.somethingawful.com/dictionary.php?act=3&topicid=83

Due to their spreadability, image macros are by far the most popular genre of Internet memes, and the terms Internet meme and meme are usually intended as synonyms for image macro. To capture the idea of the diffusion of the practice of creating, sharing and commenting image macros, Table 0.1 reports some of the most popular online communities in Reddit, dedicated to memes of this genre on general topics or typical educational subjects. We notice that all communities' names incorporate the term "meme", but the contents shared are almost uniquely image macros.

Table 0. 1 Popular meme communities in Reddit (March 2021)

Topic	Name	Web address	Numbers of followers
General	r/memes	https://www.reddit.com/r/memes/	14,9 million
History	r/HistoryMemes	https://www.reddit.com/r/HistoryMemes/	2,3 million
Mathematics	r/mathmemes	https://www.reddit.com/r/mathmemes/	110k
Physics	r/Physicsmemes	https://www.reddit.com/r/physicsmemes/	102k
Philosophy	r/PhilosophyMemes	https://www.reddit.com/r/PhilosophyMemes/	59,2k
Chemistry	r/chemistrymemes	https://www.reddit.com/r/chemistrymemes/	52,5k
Geography	r/geographymemes	https://www.reddit.com/r/geographymemes/	7,2k

In reason of their diffusion and popularity, the majority of research studies on Internet memes, focus on this genre (Huntington, 2013; Yus, 2018; Denisova, 2019), as will this thesis, and the terms Internet meme and meme used hereafter are to be intended as synonyms for image macro unless otherwise indicated.

1.7 The stuff that image macros are made on: Templates

The digital image encapsulating the complex codified meanings used to build an image macro is commonly referred to as template, another term derived from Computer Science where it is used to designate a file that serves as a starting point for a new document. Similarly, image macro templates serve as starting points to create new image macro memes.

Digital images circulating on the Web become templates conveying codified meanings as a consequence of their remixing and sharing on the Web. Once established, templates acquire social names and are listed in official repositories as the r/MemeTemplateOfficial[15] community in Reddit, and in meme aggregator websites (see 1.8).

Figure 0. 5 Examples of Image Macro templates

This "memeification[16]" of digital images is a process of "*resemiotisation*, whereby content is lifted from a text in the source domain and recast in a modified form during the production of a subsequent derivative text in the target domain." (Laineste, & Voolaid, 2016, p. 28, emphasis in the original). It can apply to a wide range of existing images, some of which are exemplified in Figure 0.5:

- pictures of objects, as the *Left Exit 12 Off Ramp* (Figure 0.5 left, source KnowYourMeme[17]), a screenshot from a YouTube video used to create memes about situations where an impromptu unfortunate choice is made
- comics or cartoons, as the *Two Buttons* (Figure 0.5 centre, source KnowYourMeme[18], elaborated from a comic posted by the animator Jake Clark in his blog in 2014 and used to create memes about contradicting choices or moral conundrums as in the Hamlet image macro in Figure 0.3 (right)
- pictures of real-life people, whose facial expressions become templates for their archetypical value (as Commedia dell'Arte masks, Mina, 2020), and "wake up one day as a meme hero" as happened to Andras Arato, a 73-year-old Hungarian electrical engineer. Arato narrates his

[15] https://www.reddit.com/r/MemeTemplatesOfficial/
[16] https://www.urbandictionary.com/define.php?term=Memeification
[17] https://knowyourmeme.com/memes/left-exit-12-off-ramp
[18] https://knowyourmeme.com/memes/daily-struggle

memeification in a TEDx video[19], describing how he became the world-known meme template *Hide The Pain Harold* (Figure 0.5 right, source KnowYourMeme[20]), used to create memes about situations of hidden internal discomfort.

Successful templates "go viral" (Jenkins et al., 2013, p. 21) very quickly, i.e. they are shared, mutated and remixed at a great speed fuelled by their spreadability and eased by the Web 2.0 technological environment. They also fade away just as fast, as confirmed by my own ethnographic observation and by academic literature modelling memes' lifespan (Notarmuzi & Castellano, 2018; Lonnberg et al, 2020), which shows that popular templates peak and decline within a few months. This means that keeping up with up-to-date templates requires being regularly connected to the Internet, a condition which is natural for young people (nearly 90% of US teen are habitually online at least "several times a day", according to the Pew Research Centre "Teen, Social Media & Technology 2018 Report"[21]).

1.8 Internet memes' cultural environment: The Memesphere

On the Web 2.0, the virtual cultural environment where Internet memes are created and shared is commonly indicated with the terms *Memesphere* (Stryker, 2011) or *memescape* (Wiggins & Bowers, 2015), *portmanteau* terms derived from the linguistic blend of the words meme and landscape or meme and sphere. The memesphere is constituted by social networking sites and websites expressly dedicated to Internet memes, known as meme aggregators (Ţăran, 2014). Meme aggregators span from user-generated meme encyclopaedias as KnowYourMeme[22] and Meming Wiki[23], that give information about Internet memes' use, development and meanings, to meme generator websites as Imgflip[24] or Kapwing[25] that provide user-friendly interfaces to generate image macros and host repositories of users' productions, or else websites as Meme Market[26] which combines the two things, supplying the interface to create and share a meme, and giving some abridged information about the template "history" and established social use.

[19] https://youtu.be/FScfGU7rQaM
[20] https://knowyourmeme.com/memes/hide-the-pain-harold
[21] https://www.pewresearch.org/internet/2018/05/31/teens-social-media-technology-2018/
[22] https://knowyourmeme.com/
[23] https://en.meming.world/wiki/Main_Page
[24] https://imgflip.com/memegenerator
[25] https://www.kapwing.com/meme-maker
[26] https://meme.market/meme-add

Meme generator websites have desktop and mobile-app versions and are well known to young people who usually have one installed onto their mobile devices. In the school-based experiments of meme creation featured in this thesis, I suggested the use of the Imgflip website, because it allows adding mathematical formulas during the creation of the meme, but students were free to choose otherwise if they felt so.

Figure 0.6 shows the page in the Imgflip website[27] that can be used to reproduce the Shakespearian image macro in Figure 0.3 (right), starting from its template already archived in the meme generator website. To generate the meme, it is sufficient to (1) type in the desired text in the two blank spaces indicated by the horizontal arrows (the website interface provides automatically the correct text font and positioning), (2) insert Shakespeare's portrait with the "add image" button indicated by the vertical arrow and (3) hit the blue "Generate meme" button[28].

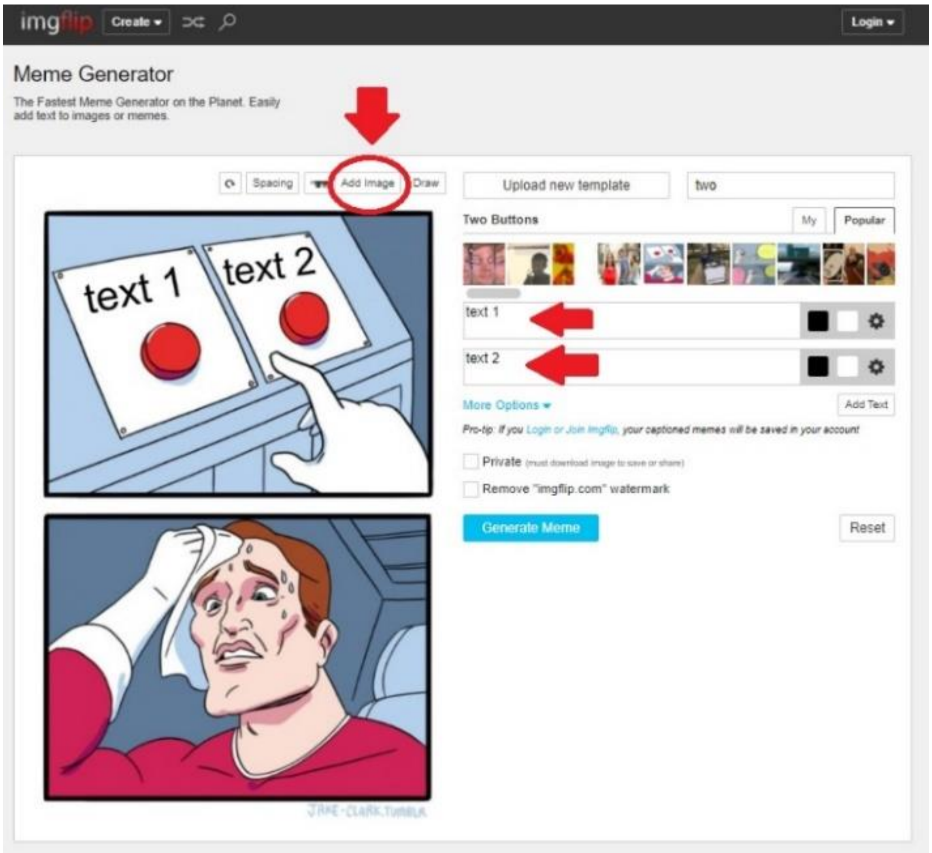

Figure 0. 6 Imgflip website interface to generate an image macro (desktop version)

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[27] https://imgflip.com/memegenerator/Two-Buttons

[28] For a thorough explanation of how to use the Imgflip website see the YouTube video tutorial https://youtu.be/4t3ikPOCmDE I recorded for teacher training purposes.

2 The rationale of the research: embracing a cultural change

2.1 A tale of two cultures: How the digital culture challenges traditional education

The evolution from Web 1.0 to Web 2.0 outlined in the previous paragraphs explains why people of different ages engage with Internet technology in different ways. Older people are considered *digital immigrants* (Prensky, 2001): they were born before the advent of the Web and got accustomed to it in its initial Web 1.0 version, and they usually retain the passive approach typical to this original architecture, surfing the Web in an "information-focused" way (Jenkins, 2009, p. 5). Users born after 1985 are *digital natives* (Prensky, 2001): they naturally embrace the Web 2.0 digital culture, diving into the Web as both authors and consumers of digital content (boyd, 2007; Lenhart & Madden 2007; Lenhart, 2013) and engaging in activities that, although entertainment-focused, contribute to their "personal development, identity and expression" (Jenkins, 2006, p. 22).

Indeed, the differences between digital immigrants and digital natives are not limited to how they *use* technology but go up to how they *perceive* it (Howe & Strauss, 2003; Herring, 2008). Technology is a relative concept, as summarised by a famous quote attributed to computer scientist Alan Kay *"technology is anything that was invented after you were born, everything else is just stuff*". This means that digital immigrants "see" the Internet technology as an addition to their lives that they can choose to embrace or reject, while digital natives do not "see" it, as it is *stuff* that has always been there, whose acceptance is not subject to discussion (no more than we can imagine discussing whether or not to accept the use of the wheel). Digital natives naturally embrace the Internet technology and the Web 2.0 digital culture that goes along with it and include it in their lives so that it becomes seamlessly integrated into their way of thinking, communicating and interacting. This is even more factual with mobile technology, with which digital natives interact as an extension of their bodies and cognitive systems (Ward, 2013; Barr et al., 2015; Nijssen, 2018) in a way that echoes' Clark and Chalmers' extended mind theory, where "a subject's cognitive system extends beyond his/her bodily boundaries into the environment [whenever] the human organism is linked with an external entity in a two-way interaction, creating a coupled system that can be seen as a cognitive system in its own right" (1998, p. 8).

21st-century learners born after 1995 are even more than *digital natives,* they are *connected natives* (Buckleitner, 2009), as they belong to what is known as Generation Z (see

Figure 0.7, source Pew Research Center[29]). This generation is the first to have no memory of life before the Internet and the first to use mobile technology since a very young age, thus getting accustomed to having ubiquitous and immediate access to information and communication, and to being completely immersed into the Web 2.0 digital culture (95% of US teens have access to a smartphone, according to the Pew Research Centre "Teen, Social Media & Technology 2018 Report"[30]).

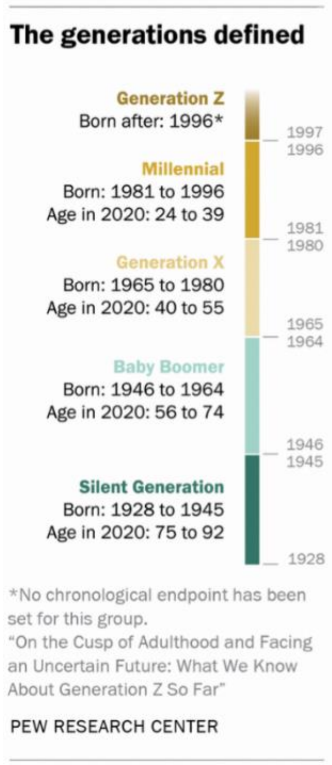

Figure 0. 7 Overview of the established demographic cohorts in the past 100 years

This technical accessibility to information, paired with the digital culture tendency to "Harnessing the Collective intelligence", has changed the way youngster access information and knowledge inside spontaneous Internet-harboured informal learning contexts. This is not simply a technical matter, it is a cultural change that poses a challenge for traditional education and requires a paradigm shift, in Kuhn's "revolutionary" sense (1962). The technological discontinuity among generations has produced a cultural discontinuity between teachers and

[29] https://www.pewsocialtrends.org/essay/on-the-cusp-of-adulthood-and-facing-an-uncertain-future-what-we-know-about-gen-z-so-far/
[30] https://www.pewresearch.org/internet/2018/05/31/teens-social-media-technology-2018/

learners, and between school and out-of-school learning contexts (Bronkhorst & Akkerman, 2016). This cultural discontinuity is particularly evident when we look at the difference between how 21st-century *connected-native*-learners access and share information and knowledge outside the school environment and how they are exposed to them inside schools (Clark et al., 2009). The focal point in this difference is not simply that notions are easily accessible through a simple Google search, but stands in terms of being involved in a Web 2.0 participatory way in the construction of knowledge and not simply being exposed to it as Web 1.0 end-users (Jenkins, 2006, 2009; Ito, 2010; Ito et al., 2013, 2018; Thomas & Seely Brown, 2011).

The proportions of this cultural discontinuity have been recently dramatically exposed by the global switch to distance learning imposed by the Covid-19 pandemic in 2020. Feedbacks collected from teachers posting in social media groups and from personal acquaintances evidenced that the majority of Italian teachers simply moved their usual teacher-centred lesson from in-person to distance mode, resulting in one-way lessons with teachers lecturing passive students (often with no camera on), which dramatically failed in engaging and motivating students. Recent studies acknowledge this challenging situation worldwide (Bakker & Wagner, 2020) and studies about distance education (Moore & Kearsley, 1996; Gorsky & Caspi, 2005; Saba, & Shearer, 2017) blame the failure not on the physical distance imposed by the online learning environment, but on the *transactional distance*, i.e. the "the gap in what a student understands about a reality, and the understanding of that same reality by the person charged with helping that student in the development of his or her knowledge" (Moore, 2018, p. 34). Teacher-centred one-way lessons are inherently transactionally distant and struggle to deliver their educational content regardless of the format of the learning environment. In a distance learning setting, they simply fail more evidently, and teachers are forced to face the evidence. In this situation the pandemic acted as a PCR, the Polymerase Chain Reaction used in molecular biology to amplify DNA samples to make them observable: it amplified malfunctions that were already there; now they are evident, and we can observe and act on them.

Moreover, this discontinuity is not stable, but it floats and is re-shaped at a fast rate by the ever-changing technological and cultural landscape young learners are immersed in. This casts doubts on the very essence of school-centred education characterised by a rigid curriculum and a fixed succession of learning (Bauman, 2005), and on its adequacy to teach non-routine skills suitable to preparing learners "for jobs that have not yet been created, for technologies that have not yet been invented, to solve problems that have not yet been anticipated" (Schleicher, in the preface of Howells, 2018).

To bridge the discontinuity and meet the needs of an evolving society, we should challenge the dichotomy between the two cultures represented by in-school formal learning and out-of-school informal learning (Kruskopf et al., 2020; Poshka, 2020). This could be done, among others, embracing Jenkins' suggestion to overcome the stereotype that places "big C" Culture dispensed inside the school setting above "small c" cultures (or *sub*cultures) dwelling in the out-of-school environment, and "use popular culture as tools and resources to think *with* and to think *about* the changing environment around us" (Jenkins, 2010, 00:02:30[31]).

2.2 What Internet memes represent to the digital culture

I will elucidate what Internet memes represent to the digital culture through some examples I collected during my exploration of the memesphere. The first comes from a Facebook group dedicated to *Meme Research and Development* where, in October 2020, a user posted the image in Figure 0.8 with no further explanation.

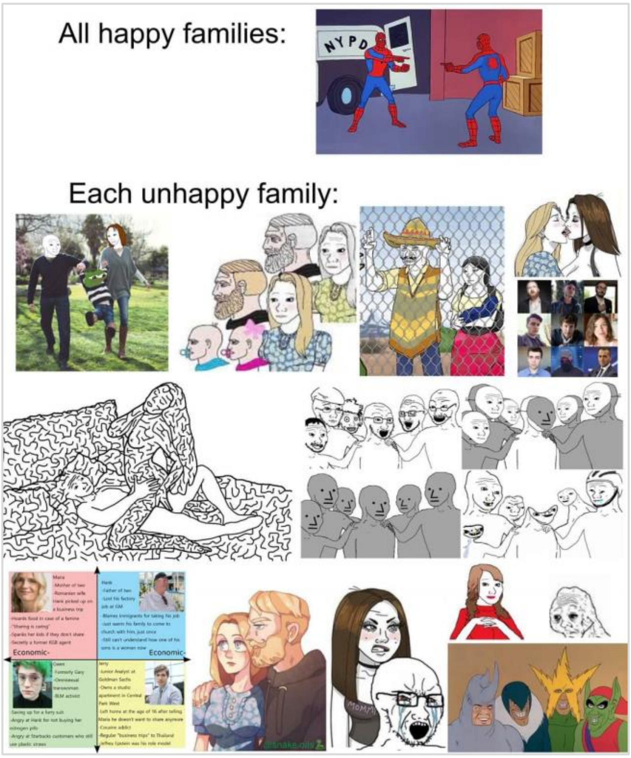

Figure 0. 8 The Karenina meme

[31] https://youtu.be/AFCLKa0XRlw

This image is a meme made up of various meme templates; it encodes Tolstoy's famous Anna Karenina incipit "All happy families resemble one another, but each unhappy family is unhappy in its own way". The famous quote is encrypted in a combination of textual and memetic signs to which the community promptly reacted. The post's thread quickly filled up with comments of users asking for explanations and others showing off that they had correctly cracked the hidden message, hinting at the answer with insider slang as "Based and Tolstoypilled" (an adaptation of the popular *based and redpilled*[32] appraisal phrase used in the memesphere), or explicitly citing Tolstoy's novel or else posting pictures of the famous incipit as in Figure 0.9.

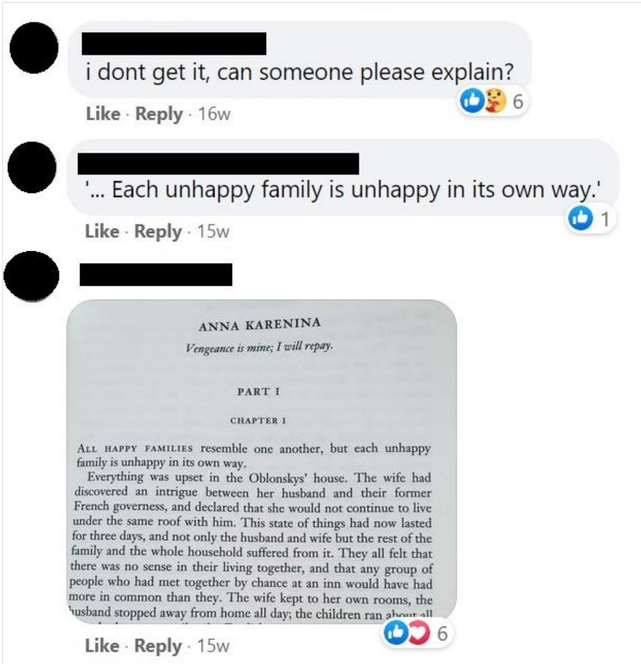

Figure 0. 9 The decoding of the Karenina meme in the comments

The episode gives us the gist of what these objects represent to the digital culture: the image itself show us that memes function as a new language that is multimodal in the sense that it involves "common semiotic principles [which] operate in and across different modes" (Kress & van Leeuwen, 2001, p. 2), as already acknowledged by the literature on Internet memes (Davison, 2012; Zappavigna, 2012; Dancygier & Vandelanotte, 2017; Osterroth, 2018; Yus, 2018). The thread of comments shows us that memes as interpretative puzzles "subconsciously build community"[33], brought together by the act of understanding these signs, that qualifies as a

[32] https://knowyourmeme.com/memes/based-and-redpilled
[33] https://www.theverge.com/2018/8/27/17760170/memes-good-behavioral-science-nazi-pepe

marker of "semiotic belonging" (Zappavigna, 2012, p. 103) and has to be exhibited as a passcode to the memesphere (Nissenbaum & Shifman, 2017).

In the case of the Karenina meme in Figure 0.8, the belonging is proved by recognising the image complementing the "All happy families" part of the text. This image is a well-known meme template, known as *Spiderman pointing as Spiderman* (Figure 0.10), broadly popular in the memesphere where it is used to create memes representing the concept of similarity. Its use is so ingrained in the digital culture (as further investigated in Chapter 5), that it is easily recognised by anyone with adequate inside knowledge of the memes' language.

Figure 0. 10 The *Spiderman pointing as Spiderman* template

In the Karenina meme, this image provides the code to decipher the "All happy families resemble one another" part of the famous opening. The decoding process continues if this first part unlocks the reminiscence of the remaining part of the well-known beginning, "but each unhappy family is unhappy in its own way", which is hinted at by the text and by the juxtaposition of other templates in the lower part of the meme. There is no need to scrutinise the other templates, as they are not there for their metaphorical meaning as the Spiderman template in the upper part, but just to represent the diversity of unhappiness. Nevertheless, recognising them as other popular templates gives a sense of accomplishment to the decoding, reinforcing the feeling that memes "create a private language that makes us feel like we belong" (Marsh, 2019, para. 1).

This first example also shows how memes' private language embodies some of the focal Web 2.0 digital culture principles illustrated in the map in Figure 0.1: the "Right to Remix" other images to convey your own message, the invitation to "Play", even with a masterpiece of the world literature, and the power of "Harnessing collective intelligence" to decode the hidden message, as does the first commenter asking for (and obtaining) help in Figure 0.9.

At the core of successful memetic communication are the *spreadability* (Jenkins, 2013) and *relatability* of the message conveyed, which grounds on the recognition of signs pertaining to different knowledge domains: in the case of the Karenina meme, these domains are the digital culture knowledge enabling the recognition of the meme templates (or at least, of the Spiderman one), and the literary knowledge related to the recognition of the Tolstoian novel. In other cases, the relatability grounds on a shared feeling, as illustrated by the second example in Figure 0.11, the "how everyday feels" meme that went viral on Twitter during the worldwide pandemic lockdown in Spring 2020. The decoding of this meme requires also some digital culture knowledge to recognise the image as a "designed mutation" (Dawkins, 2013, 00:04:16) of the *Spiderman pointing at Spiderman* template that maintains and amplifies the similarity message of the original template.

Figure 0. 11 The pandemic meme

In the multimodal language of Internet memes, templates constitute an apparatus of signs that authors are allowed to use and remix, stretching and bending them in order to convey their own messages, provided that the reinterpretation is done coherently with the original established meaning of the template, as in Figure 0.11. It is the proper use of (for authors) and the ability to interpret (for readers) this apparatus of signs that builds the collective identity of the inhabitants of the memesphere (Nissenbaum & Shifman, 2017), reinforced through the social validation provided by shares and likes, which are described in social studies as *tribal rewards*, (Eyal, 2014) and confirmed by neuroscience as stimuli acting on nucleus accumbens, the brain reward centre (Meshi et al., 2013).

The sense of belonging activated by the sharing of a common private language is not only cultural-situated but also age-situated, meaning that older people's skills are usually unfit to capture the innuendos required to decode a meme. This generational divide is well described by the third examples, the meme in Figure 0.12 (source Facebook), that depicts older people's inadequateness in dealing with memes from the point of view of their physical aptitude ("Well, let me put my glasses on…"), cultural dexterity ("In what order am I supposed to look at the images"; "I don't understand honey… who's that person?"; "who took that picture?") and technical handiness ("Oops, the image is gone, what happened?").

Figure 0. 12 Memes as generationally characterised objects

To sum up, Internet memes represent a new language that allows users to represent their feelings, opinions or knowledge about a specific topic, while at the same time positioning themselves as belonging to a culturally and generationally characterised group.

2.3 What Internet memes can represent to education

The previous paragraphs give an overarching view of the main characteristics of the new language of Internet memes: their visual components, the humoristic stance, the appeal as interpretative puzzles, the social value as bonding signs, the representativeness in the digital culture. All these features have acknowledged educational potentialities that have been already investigated by educational literature and, in some cases, by mathematics education, as briefly summarised hereafter:

Visual components: the educational affordances of visual imagery are widely recognized by educators (Kress, 2003; Riddle, 2009) and mathematics educators (Presmeg, 1986, 2006; Duval, 1999; Arcavi, 2003; Nardi, 2014), not only because the ability to give a visual representation to events in a symbolic or graphic form is a fundamental stage of human cultural development since cave paintings (Cecchinato, 2009), but also in light of the 21st-century culture "pictorial turn" (Mitchell, 1995, p. 15), that knocked over centuries-long domination of texts and words in Western culture and has promoted visual resources to the centre of contemporary "communication and meaning-making" (Felten, 2008, p. 60).

Humour: the potentialities of humour to enhance learning in the classroom lie in the power to stimulate imagination and positive emotions, as generally acknowledged in education (Cornett, 1986; Herbert, 1991) and in mathematics education (Shmakov & Hannula, 2010; Menezes, 2017), where humour is also accredited as an effective means to reduce math anxiety (Bakar & Amran, 2020).

Puzzle effect: educational literature agrees that attuned "desirable difficulties" (Bjork, 1994) can support learning and long-term retention by forcing students into more elaborate decoding and retrieval processes. Among these desirable difficulties are those hindering the straightforward readability of information as hard-to-read fonts (Diemand-Yauman et al., 2010) or puzzles (McDaniel et al., 1994). Also in the learning of mathematics, since the Rhind Papyrus and the tower of Hanoi, the act of cracking puzzles is renowned as a playful way to engage in serious mathematical reasoning, and riddles and puzzles are acknowledged didactic tools (Parker, 1955; Michalewicz & Michalewicz, 2008; Farnell, 2017; Danesi, 2018).

Social value and representativeness in the digital culture: these features, characterising objects from the Web 2.0 digital culture, have been explored from an educational standpoint as bridges across the aforesaid cultural discontinuity between students' out-of-school and in-school experiences (Kruskopf et al., 2020; Poshka, 2020). The research in this perspective, as Ito et al.'s Connected Learning framework (2013), "focuses attention on the spaces of integration and translation between divergent domains of knowledge, culture, and social practice" (p. 63), in a bridging effort where discontinuities and boundaries are no longer perceived as dividing chasms but become "catalysts for innovation" (Thomas & Seely Brown, 2011, p. 23). In Mathematics Education these qualities are not yet investigated, despite the shared ideas that they play an important part in 21st-century students' learning processes.

As per Internet memes, some of the said educational potentialities have been probed and confirmed by studies in different educational fields, as listed in Table 0.2.

Table 0. 2 Educational literature on Internet memes

Internet memes characterising features	**Explored educational fields**			
	General education	**New literacies**	**Language Learning**	**Pharmacy education**
Visual component	Yoon (2016)	Lankshear & Knobel (2003) Silva (2016) Harvey & Palese (2018)	Romero & Bobkina (2017; 2021) Han (2019)	
Humour	Reddy et al. (2020)		Harshavardhan et al. (2019)	
Puzzle effect				
Social value and representativeness	Wells (2018) Purnama, (2017) Taddeo & Tirocchi (2019)	Knobel & Lankshear, (2007, 2018)		Brown (2020)

Much is still to be explored: Table 0.2 includes all the literature currently (March 2021) available on the topic and shows that many educational fields are still uncharted, including that of Mathematics Education. Indeed, at least at the beginning of this research, no studies on Internet memes with mathematical content could be found, despite the popularity of these digital objects within dedicated online communities, and despite it seems consistent to infer that Internet memes with mathematical content have the same educational potentialities accredited to Internet memes since they all share the same typifying characteristics.

To clarify this point and focus on the object of this research, I present in Figure 0.13 two examples of Internet memes with mathematical content (i.e. mathematical memes), taken from online communities inside Facebook.

Figure 0. 13 Examples of mathematical memes

Both examples are built on the *Spiderman pointing at Spiderman* template used in the Karenina meme (Figure 0.8) and in the pandemic meme (Figure 0.11). Here the template is used accordingly to its conventional meaning addressing the concept of similarity to represent (left) the relation of equality standing between the values of the trigonometric functions cos(0°) and sin(90°), while it undergoes a "designed mutation" (right) to convey the mathematical idea that the equality between sin(theta) and theta holds only if the angle theta is sufficiently small.

These examples show (1) that mathematical memes can convey proper mathematical content whose understanding requires specialised knowledge, and (2) that they vehiculate their content leveraging on memes' characterising features: the *visual component* (represented by the template) contributes to providing the socially established meaning of similarity, the apparently incongruous combination between this meaning coming from the digital culture and the meaning coming from the mathematical culture endows these objects with their *humorous* tang, and the fact that neither meaning is openly stated, but requires some inside knowledge to be interpreted, contributes to the *puzzle effect* whose decoding has a *social value* and makes the whole process *representative* inside the digital culture.

The result is a new form of representation of mathematical ideas, a representation that requires the ability to see a mathematical structure in a non-mathematical object, one of the Key Understandings of Mathematical Reasoning according to the PISA 2021 Mathematics Framework[34]. This representation is not merely catchy and humorous: as reported by a commenter in the *Meme Research and Development* Facebook group, "a meme can transcend comedy and be used to teach things too. It adds relatability to the topic being taught, I myself am in meme groups related to science and engineering related to my degree, which has helped a lot in memorization of terminology and context".

[34] https://pisa2021-maths.oecd.org/#Mathematical-Reasoning

This suggests that mathematical memes have an epistemic potential that deserves to be explored and that can be exploited if we understand it and figure out how to funnel it to transform mathematical memes into digital educational *resources*. Here I intend the term *resource* in Adler's *hybrid practice* sense, where "resources for school mathematics extend beyond basic material and human resources to include a range of other human and material resources, as well as mathematical, cultural, and social resources" (2000, p. 210). In Adler's view, following Lave and Wenger (1991), to be such a resource must be simultaneously *visible* and *invisible*. A resource is *visible* if it is recognised as such and used in school mathematics practice, and it is *invisible* if it can be seen through to illuminate the mathematical content. These two characteristics add up to what Adler, also following Lave and Wenger, calls the *transparency* of the resource. Transparency is necessary to enable learning in a hybrid practice, but it is not "a property of the resource, but a function of how the resource is used and understood within the practice in context" (p. 217). Therefore, the process of transformation of mathematical memes from Internet phenomenon to digital educational resource requires research effort to investigate how to make these objects used and understood, so that they can become simultaneously *visible* and *invisible* and achieve *transparency*.

These are propellants that ignited this research, together with the idea that bringing a culturally representative Web 2.0 object into the school environment can be an important step to foster the cultural change aimed at bridging the discontinuity between out-of-school and school learning environments. I embrace the position that educators should "remain open to innovation, make an attempt to understand it" (Levy, 2001, p. X), and "search out the ways in which literacy and communicative discourses mediate and are mediated by social contextual forces" (Appelbaum,1995, p. 24).

I'm aware that navigating the unstable sea of the multiform modes that young people develop in their digital interactions is challenging, but I believe that we cannot ignore the fact that our students do not remember a world without the Internet and memes, and that "learners walking through our doors will continue to be shaped by an ever-changing technological milieu" (Lenhart, 2013, 00:52:17). The globalization and technological progress foreseen by McLuhan (1964) drive unrelenting social, economic and environmental changes, but they also present new opportunities that educators can take on (Schleicher, in the preface of Howells, 2018). The purpose of this thesis is to show that mathematical Internet memes are one of these opportunities, as embraced by the recent literature about the new themes for mathematics education (Bakker et al., 2021).

3 The development of the research: theoretical and methodological challenges

3.1 Object of the research, general aim and main research question

As highlighted in the previous paragraphs, the object of this research is the Internet phenomenon of mathematical memes, which is investigated with the aim of envisaging how these objects can be transformed into digital resources in traditional educational environments. Thus, the main research question driving this research can be summarised as follows:

How can we conceptualize mathematical Internet memes and what are their educational potentialities, if any*?*

This question has a double nature: the first part "*How can we conceptualize mathematical Internet memes*" is necessary to frame a topic which is new for the field of Mathematics Education. It will be fully answered in this thesis with an answer that allows seeing through the meme illuminating its mathematical content, thus building to the *invisibility* of the resource. The second part "*What are their educational potentialities, if any*?" is spiralling throughout the whole work at it is never fully answered, leaving space for further research well beyond this thesis. It sparks a variety of answers in the different chapters, contributing to the usability of mathematical memes in the mathematics classroom, thus building the *visibility* of the resource. The investigation process tackling the main question above intermingled two research approaches whose focuses and methodologies can be briefly outlined as follows:

<u>Ethnographic research</u>: aimed at understanding the phenomenon of mathematical memes and its educational potentialities within the culture that originates it, grounding on the data I collected since February 2018 within online communities dedicated to mathematical memes. This research draws from the established ethnographic methodologies, adapting the objectives of the research in Mathematics Education to the new context of investigation and developing a new research methodology to study digital phenomena from the point of view of Mathematics Education.

<u>Experimental research</u>: aimed at investigating the educational potentialities of mathematical memes by analysing data collected during school experiments that were designed and refined through cycles of implementations in the period from May 2018 to May 2020. In these experiments, I observed communities of students and teachers in experiments built to collect data on the product and on the process of creating mathematical memes, data on the elaboration and understanding of mathematical ideas represented by memes, and short-term and long-term feedbacks.

3.2 The state of the art: Literature review and language barriers

When this research project started in February 2018, I was able to find the literature reported in the previous paragraphs about the cultural value of Internet memes and their possible didactical use in various educational fields, but I could not find anything published on the subject of mathematical memes, neither from the point of view of Mathematics Education nor from other perspectives. On the one hand, the lack of existing literature suggested that this research could fill a relevant gap and be useful in the academic world by opening new research paths, and on the other hand this presented a challenge because I did not have anything to rely on for guidance in terms of suitable theoretical frameworks and research methods.

In August 2018, Benoit's Doctor of Education dissertation was published. Although the focus of this work is more on the vision of mathematics in popular culture carried by mathematical memes, than on their educational potentialities, it confirmed some of the observations about mathematical memes' engaging potentialities I had made by that time. Later on (November 2019), when a substantial part of the experimental research was already completed, I was able to find four other studies, one written in Spanish (Beltrán-Pellicer, 2016), and three in Portuguese (Gonçalves & Gonçalves, 2015; Gonçalves, 2016; Friske, 2018), and even later (December 2020 and March 2021), I found four more Portuguese studies (Felcher & Folmer, 2018; Brito et al., 2020; Friske, 2020; Friske & Rosa, 2021). Table 0.3 reports the existing literature at the present date (March 2021) in chronological order.

Table 0. 3 A survey of the existing literature on mathematical memes

Information about the Study	Research Focus	Theoretical Framework
2015 Ethnographic study Gonçalves & Gonçalves, Um Retrato da Matemática Segundo Os Memes: Potencialidade Para O Ensino-Aprendizagem Short Journal Article, Portuguese	Analyse a dataset of 30 mathematical memes taken from the Web, investigate how mathematics is represented, and infer some possible educational potentialities	Symbols in virtual communication Jablonka (2012) Linguistic analysis Possenti (1998)
2016 Ethnographic study Gonçalves, Memes e educação matemática: um olhar para as redes sociais digitais Conference paper, Portuguese	Analyse a dataset of 30 mathematical memes taken from the Web, categorise them according to their content, and infer some possible educational potentialities	Ethnomathematics D'Ambrosio (2005) Image analysis Panofsky, E. (1955)
2016 Classroom note Beltrán-Pellicer, Utilizando memes con tus alumnos	Use 4 examples of mathematical memes created by the author to depict possible didactical uses of	Onto-semiotic approach Godino et al. (2007)

Short report, Spanish	mathematical memes	
2018 Ethnographic & experimental study Benoit, Mathematics in Popular Culture: An Analysis of Mathematical Internet Memes Dissertation, English	Investigate what messages students are receiving about mathematics from mathematical memes found on the Web and how they affect students' mathematics identities.	Mathematics in popular culture Appelbaum (1995) Mathematics identity Martin (2003, 2012) Martin et al. (2010)
2018 Experimental study Felcher & Folmer A criação de memes pelos estudantes: uma possibilidade para aprender matemática Short Journal Article, Portuguese	Use data collected in a school experiment with 6 y.o. students discussing and creating mathematical memes to investigate the use of mathematical memes as didactical resources	Research on mathematical memes Gonçalves (2016)
2018 Experimental study Friske, Memes e matemática: processos de ensino e de Aprendizagem Conference paper, Portuguese	Use data collected in an experiment with in-service teachers creating mathematical memes to investigate the potentialities of mathematical memes as didactical resources	Cyberformation Vanini et al. (2013)
2020 Experimental study Friske, Memes e matemática: a formação com professores/as na perspectiva da cyberformação Master Dissertation, Portuguese	Use data collected in a meme creation teacher training course involving in-service teachers creating mathematical memes to investigate the potentialities of mathematical memes as didactical resources	Cyberformation Vanini et al. (2013)
2020 Experimental study Brito, Sant'Ana, & Sant'Ana, Memes com viés matemático e suas potencialidades para o ensino de Matemática Short Journal Article, Portuguese	Use data collected in a school experiment with 8th-grade students discussing and creating mathematical memes to investigate the use of memes with mathematical bias/content (?) as didactical resources a	Ethnomathematics D'Ambrosio (2005) Research on mathematical memes Gonçalves (2016)
2021, Experimental study Friske & Rosa, Memes, Matemática e formação com professores/professoras: uma perspectiva sociopolítica Journal Article, Portuguese	Use data collected in a meme creation teacher training course involving in-service teachers creating mathematical memes to investigate the relationship between mathematics education and democracy.	Cyberformation Vanini et al. (2013)

Finding and using non-English literature has been hindered by a substantial language barrier: these studies are not retrieved by search engines when the search is done using English keywords, which was my initial approach. Moreover, automated translation provided by Google translate struggles with some terms, as happened with the Brito et al. study, where the expression "memes com viés matemático" is rendered as "memes with a mathematical bias" but reading the whole Google-translated paper I think it should be better interpreted as "memes with a mathematical content" which substantially changes the research focus.

For these reasons non-English literature was unfortunately retrieved too late to substantially inform this research, nevertheless, it offered ex-post confirmations of the validity of approaching the object of the research balancing an ethnographic and an experimental perspective. From the ethnographic perspective, it also evidenced that the existing ethnographic research on mathematical memes (Gonçalves & Gonçalves, 2015; Gonçalves, 2016) focuses on the objects and not on the interactions initiated by the objects, thus leaving a significant research gap that has been addressed by the study in Chapter 6. From the experimental perspective, the two studies reporting school experiments with students (Felcher & Folmer, 2018; Brito et al., 2020) confirmed the appropriateness of the structure and components of the activity designed for our school experiments.

Going through the Portuguese studies is evident that they build on each other in terms of methodology and theoretical basis, a process that is foundational to set out the background against which a study takes place. Therefore, although the majority of these studies is constituted by short essays, I think it is important to include them here, with the idea that this thesis could offer a comprehensive starting point for future research on mathematical memes.

3.3 Theoretical backgrounds: theoretical challenges and research foci

Due to the aforementioned difficulties in the retrieval of previous studies, the theoretical and methodological choices for this research were made without any guidance from prior works in Mathematics Education. Consequently, in the development of the research, the main research question exposed before "opened up" in more focused subquestions that were investigated through various theoretical lenses with diverse research goals. Different theoretical frameworks have been sampled not with the aim to systematically *combine* them in the sense of *networking* theories (Bikner-Ahsbahs & Prediger, 2009; Prediger et al., 2008), but with a "bricolage" stance (Cobb, 2007) aimed at maximizing the understanding of an unknown phenomenon by observing it from different standpoints. In this perspective, the two research approaches described in 3.1 intermingled throughout the research, although with different relevance.

In fact, due to my experience as a teacher and to the fact that research through studies based on school experiments is way more usual in Mathematics Education as opposed to ethnographic research, the first *exploratory* phase in the study of mathematical memes was based on the data collected in the experimental research, and the data coming from the ethnographic research was used only to briefly introduce the topic. Only at a later time I realised that, since memes are *digital artefacts*, in the sense that they are objects created by humans (hence *artefacts*) expressly for the Web (hence *digital*), we have to understand how young people do mathematics with memes on the Web, to cross the boundary and figure out how we can do mathematics with memes in the classroom. This brought to a more *systematic* phase of the research where the analysis focused on the data collected in the *ethnographic research* to shed light on the phenomenon in itself.

Therefore, in the chronological development of the research, the "*What are mathematical memes educational potentialities, if any*" part of the main research question had been addressed before the "*How can we conceptualize mathematical Internet memes*" part.

The research foci have evolved with the progress of the research phases: in the exploratory phase the main research question was addressed observing experimental data from various points of view, beginning with the investigation of the meanings carried by a mathematical meme, aimed at identifying patterns that could be used to ease their understanding. This first investigation yielded to the formulation of a semiotic tool to interpret memes, which we called the *triple-s construct of the three partial meanings* of a meme, which is presented in Chapter 1. This semiotic tool has become part of our theoretical tools in the subsequent stages of the research, evolving with the development of the research to reach the more worked out stage described in Chapter 8.

Further steps of the exploratory investigation included observing mathematical memes through some of the most significant theoretical lenses in the literature in Mathematics Education, to investigate if and how memes' characterising features listed in 2.3 could reveal educational potentialities for the teaching of mathematics to:

- initiate mathematical discourse (Sfard, 2008)
- ignite emotions (Zan et al., 2006; Radford, 2015)
- activate learning mechanisms at the boundary (Star & Griesemer, 1989; Akkerman & Bakker, 2011)
- channel students' visual skills (Presmeg, 2006, 2019)
- channel students' digital culture knowledge (Ito et al., 2013, 2018)
- foster changes in students' praxeologies in synergy with known educational resources

(Chevallard, 1992, 1999).

In the systematic phase, the main research question has been addressed by observing ethnographical data to:

- reveal what mathematical memes represent inside the digital culture for members of the online communities who act on them (Jenkins, 2006, 2009)
- reconstruct a heuristic model of a mathematical meme creative mechanism (Koestler, 1964) that could support teachers in introducing memes into their teaching practices.

These research phases will be presented chronologically in this thesis, to enable the reader to follow the process of the research as it developed through time.

3.4 Data sources

3.4.1 Ethnographic research: data sources

Data for the ethnographic research has been collected since February 2018 through a covert nonparticipant observation (Liu & Maitlis 2010) from a selection of online communities exchanging mathematical meme inside the memesphere. The observed communities are hosted in the three social networking websites presented in 1.3. Full information about the process of field mapping that brought to the selection criteria is given in Chapter 6.

3.4.2 Experimental research: data sources

Data for the experimental research originates from different sources in the period May 2018 – May 2020. Part of the data has been collected during six meme-creation experiments that I designed and that took place under my guidance between May 2018 and March 2019: three experiments with learners at the secondary school level, two experiments at the university level, and one experiment with learners from a teacher training course. These experiments followed (entirely or at least in part) the general structure that will be exposed in the next paragraph.

Other data came later (December 2019 – May 2020), from school activities organised and conducted independently by teachers who spontaneously joined the project after the teacher training course or from distance learning activities spontaneously implemented by teachers during the Spring 2020 pandemic lockdown. These distance learning experiment followed the guidelines included in the project dissemination website (see #lifeonmath Mathematical Memes Project[35])

[35] https://lifeonmathmeme.wordpress.com /

that gained particular popularity in April 2020 when it was shared by MaddMaths![36], an Internet site about mathematical culture endorsed by the Italian Mathematical Society UMI.

The school experiments were set in STEM-oriented public secondary schools (grade 6-13) in Piedmont and Lombardy. All schools, apart from the first, are part of the Scuole Secondarie Potenziate in Matematica (SSPM)[37], a public engagement project of the University of Turin, designed to enhance the mathematical culture of students and the didactical approaches in mathematics of teachers, recently selected as a case study for the national assessment of the quality of research performed by ANVUR (Agenzia Nazionale di Valutazione del Sistema Universitario e della Ricerca). These enhanced approaches - new teaching methods, cutting-edge technologies, and in-depth mathematical contents - are introduced by Mathematics Education academics and are justified by research results. The academics act as teachers' educators, in a free professional development programme conducted under the responsibility of Professor Ornella Robutti. In turn, teachers develop new professional roles as teacher-researcher and teacher-experimenter and offer quality education to students interested in expanding their knowledge of mathematics during additional curricular hours (at least 33 hours/year) dedicated to the learning of mathematics from a laboratory and interdisciplinary perspective. This cultural background, which makes these students and teachers particularly receptive to experimenting with new educational resources, is the reason for the choice of these schools as settings for the experiments. Students' creations have been taken into account as formative assessment by the teachers, and no grade was attributed to the productions.

I will sketch here the settings of the six research-designed (RD) experiments that took place under my guidance in chronological order:

May 2018 – RD Secondary School experiment: the first school experiment was conducted in the Liceo Scientifico" Leonardo da Vinci", located in Milan, capital of Lombardy in northern Italy. This is one of the oldest secondary school in Milan (established in 1946), located in the city centre and has been recently ranked first in Milan by the secondary school ranking system Eduscopio 2020[38]. About 1000 students attend school, coming from a high socio-economic background, with a low incidence of students from disadvantaged

[36] http://maddmaths.simai.eu/didattica/meme-matematici-didattico/
[37] https://www.liceomatematico.it/torino/
[38] https://eduscopio.it/

families. The school has a specific focus on training students to develop critical thinking skills with the use of technology and laboratories, and a long tradition of excellence, with students attending regularly national competitions in STEM subjects. It also offers many extracurricular educational and cultural activities. The experiment involved those who were my own students at the time: a class group of 28 students attending the 12th year, to whom I had been teaching mathematics and physics since the 9th grade. The mathematical topic for the meme creation activity was left to the choice of the students.

October 2018 - RD University experiment: the first university experiment was conducted in the Departments of Mathematics of the University of Turin, involving Bachelor students enrolled in "Introduzione al pensiero matematico", a first-year introductory course about the philosophy and epistemology of mathematics held by Professor Ornella Robutti. The experiment was conducted in a distance learning setting, with the researcher interacting with the students through an online platform (Moodle) or dedicated digital spaces (Padlet). The subject suggested was false proofs in mathematics; participation in the experiment was on a voluntary basis, and only 11 students (of the total 200 attending the course) submitted a meme uploading it in the provided digital spaces.

October 2018 - RD Secondary School experiment: the second school experiment took place in the Liceo Scientifico "Marie Curie", located in the small town of Pinerolo in the province of Turin and open to students from the town and from the neighbouring territory. About 1000 students attend school, coming from a mixed socio-economic background, with an incidence of students from disadvantaged families higher than the national average. The school was established in 1963 and has a long tradition of very well-trained teachers who collaborate with the research group in Mathematics Education of the University of Turin. The experiment involved 22 10th grade students and took place in the presence of the researcher and the class teacher, who had been teaching mathematics and physics to this class group since the 9th grade. The mathematical topic for the meme creation activity (linear systems) was chosen by the teacher.

March 2019 - RD Secondary School experiment: the third school experiment was conducted in the Liceo Scientifico "Galileo Ferrari" in Turin. The school is the eldest in the city of Turin, established in 1923 by Gentile, the minister of education responsible for the first organic school reform in Italy. It is located in the city centre, near the Politecnico and has been ranked first in Turin by the recent Eduscopio 2020 report. About 1600 students attend school, coming from a high socio-economic background, with a low incidence of students from disadvantaged families. The school has a long tradition of excellence, with students participating regularly in national STEM competitions, and it offers various extracurricular activities. The experiment involved 29 12th grade students and took place in the presence of the researcher and the class teacher, who had been teaching mathematics and physics to this class group since the 9th grade, and of a master student from the Department of Mathematics of the University of Turin as an observer. The mathematical topic for the meme creation activity (complex numbers) was chosen by the teacher.

March 2019 - <u>RD Teacher training experiment</u>: this experiment was set within the SSPM teacher training course, it involved approximately 150 teachers (50 teachers teaching grades 6-8, and 100 teachers teaching grades 9-13), and took place in the presence of the research and other collaborators of the SSPM program. The mathematical topic for the meme creation activity was left to the choice of the teachers, among the curricular topics.

April 2019 - <u>RD University experiments</u>: the second university experiment was conducted in the Departments of Mathematics of the University of Turin, involving Master students enrolled in "Didattica della Matematica 1", a course in Mathematics Education, held by Professor Ornella Robutti. The experiment involved around 40 students and took place in the presence of the researcher and professor Robutti. The mathematical topic for the meme creation activity was left to the choice of the students, but it was restricted to 9th-13th grade topics.

The teacher-designed (TD) experiments took place without my guidance in the following settings:

May/September 2019 - <u>TD Middle School experiment</u>: it involved a pair of teachers of the Istituto Comprensivo D'Azeglio Nievo in Turin who had attended the April 2019 SSPM teacher training course. This school is an important complex hosting student from grade 1 to grade 8. Students come from a high socio-economic background, with a low incidence of students from disadvantaged families and the school offers a teaching environment responding to the families' high-quality requests. The experiment involved two teachers and two class groups attending the last period of the 6th grade and included a follow-up with the same class groups at the beginning of the 7th grade.

December 2019 - <u>TD Secondary School experiment</u>: involved a group of three teachers who had attended the April 2019 SSPM teacher training course. All teachers work at the_well-reputable Liceo "Galileo Galilei", established in 1942 in Legnano, a small town in the province of Milan. About 1300 students attend school, coming from a high socio-economic background, with a low incidence of students from disadvantaged families. The school has a high standard teaching tradition with particular care to critical thinking skills. The experiment involved three class groups of 10th-grade students.

March/May 2020 - TD Distance Learning experiments: these experiments involved seven teachers who spontaneously adhered to the project through the dissemination website #lifeonmath Mathematical Memes Project[39]). The experiments took place as distance-learning activities during the Spring 2020 pandemic lockdown in seven different secondary schools in the Italian territory: two lower secondary schools (8th grade) and five higher secondary schools (grades from 9th to 13th).

3.5 Data collection: methodological challenges

3.5.1 Ethnographic research: filling a methodological gap

The methodological challenge in the ethnographic research consisted in finding the way to adapt the existing ethnographic methodologies to the aim of observing a digital phenomenon spontaneously appearing within an online environment from the point of view of Mathematics Education. The method had to be built from scratch, as it was at the empty intersection of different methodological domains:

- ethnographic research (Sunstein & Chiseri-Strater, 2012; Harwati, 2019, Liu & Maitlis 2010)
- ethnographic research in mathematics education (Eisenhart, 1988)
- online research in social networked websites (Sloan & Quan-Haase, 2016)
- ethnographic research about online learning with digital media (Jenkins, 2006; Ito et al., 2010; Thomas & Seely Brown, 2011).

The new method was built intersecting adequate practices from the existing literature in the different fields: from the established ethnographic methodologies I took the use of field notes to capture key-case and rituals of the observed communities. From the ethnographic methodologies employed in mathematics education, I captured the search for artefacts and the use of participant observation for data collection. Since I am a digital immigrant in the observed culture, I “choose to be primarily an observer and less of a participant” (Eisenhart, 1988, p. 105),

[39] https://lifeonmathmeme.wordpress.com /

and I choose not to disclose my presence within the observed communities to avoid the *Hawthorn effect* (Landsberger, 1958), the possible changes in behaviour occurring when subjects are aware of being under study. In a nutshell, my role was that of a "covert nonparticipant observer" (Liu & Maitlis, 2010). The literature about conducting online research in social networked websites, and specifically on researching images (Hand, 2016), informed me about the ethics of doing research in these environments and about the practices related to the selection and classification of visual digital objects (described in detail in Chapters 6 and 8). Finally, the literature reporting ethnographic research about online learning with digital media enlightened me about the fact that I had to collect not only the products (i.e. the mathematical memes posted inside online communities) but also the threads of comments initiated by the memes, that could support the understanding how the community perceives and acts on these objects, as we have seen in the previous paragraphs.

3.5.2 Ethnographic research: collecting the data

In February 2018, I started my "search for artifacts" (Eisenhart, 1988, p. 106) by observing a selection of online communities as a lurker, the term identifying a nonparticipant observer in the digital culture. There I began to populate the *meme pool* downloading mathematical memes related to the Italian secondary school mathematics curriculum and memes about creating and interacting with mathematical memes (later called meta-memes, see Chapter 6). In parallel, I saved in my social accounts the active links corresponding to the downloaded memes, to build a *comment pool* corresponding to the meme pool that could enable following the developing of the threads of comments.

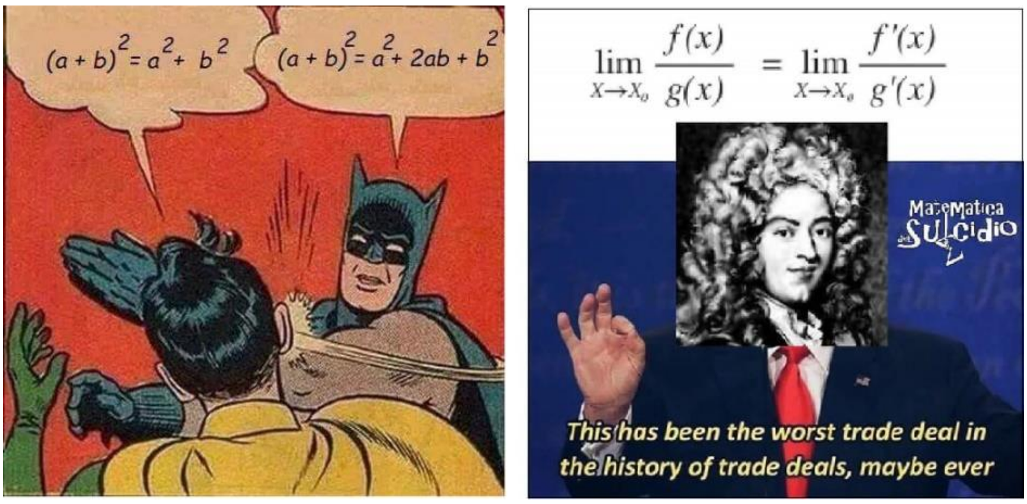

Figure 0. 14 First mathematical memes collected in the ethnographic research

Figure 0.14 presents the first two mathematical memes I collected: on the left, a mathematical meme focused on the typical mistake of forgetting the middle term in the binomial quadratic expansion, a popular subject in the mathematical memesphere, and on the right a mathematical meme hinting to the historical circumstance that the so-called Hopital rule on limits' indeterminate forms was in fact discovered by Bernoulli who made "the worst trade deal in the history of trade deals" selling it to l'Hopital for a small sum.

I knew these were memes because they were *designed mutations* of known templates, which I had encountered before in other customizations: the quadratic expansion meme is a mutation of the *Batman Slapping Robin*[40] template (Figure 0.15 left, source KnowYourMeme) and the Hopital-Bernoulli meme is a mutation of *The Worst Trade Deal*[41] template (Figure 0.15 right, source KnowYourMeme).

Figure 0. 15 Templates of the memes in the previous Figure

The next Figure 0. 16 shows an example of meta-meme, that uses the *Galaxy Brain* template[42] to represent a progression of strategies to tackle the learning of mathematics. The strategies provokingly escalate from the small-brained "Focus in class to learn mathematics" to the star-studded "Join Mathematical Mathematics Memes to learn mathematics", alluding to one of the liveliest mathematical memes communities inside Facebook. The meme and the excerpt of the comment thread reported aside, with the "Learning math from meme is a thing now" remark, show how important are meta-memes and comments for a full understanding of the Internet phenomenon which the object of the study.

[40] https://knowyourmeme.com/memes/my-parents-are-dead-batman-slapping-robin
[41] https://knowyourmeme.com/memes/the-worst-trade-deal
[4242] https://knowyourmeme.com/memes/galaxy-brain

Figure 0. 16 An example of meta-meme

The process of ethnographic data collection has been going on since then, and the population of the meme pool (and of the corresponding comment pool) has been growing steadily reaching the total number of 1989 items in March 2021, as represented by Figure 0.17.

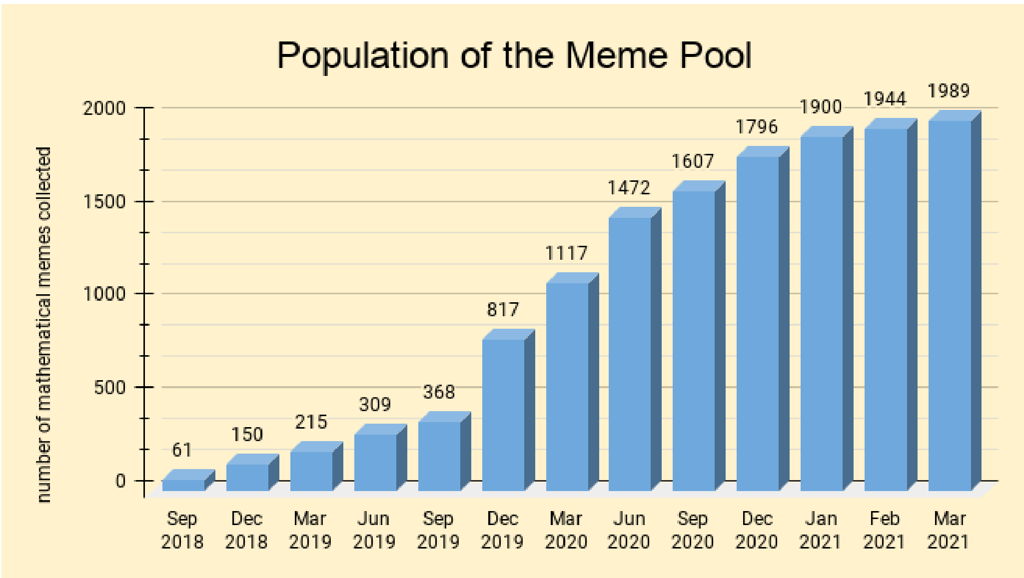

Figure 0. 17 Ethnographic research: development of the meme pool

3.5.3 Experimental research: the task design

The design of the task for the experimental research grounded on the awareness that the process of importing memes into the school environment had to be pursued respecting their natural essence to preserve their educational potentialities. Since memes representativeness is related to the Web 2.0 participatory culture (Shifman, 2014) and creativity (Willmore & Hocking, 2017), it seemed appropriate to insert them into school activities that engage students as meme creators. At the same time, memes' social value is profoundly connected to the process of *tribal reward* activated by shares and likes in social networking sites (Eyal, 2014), thus it was meaningful for the research to be able to observe the reactions of the class group as a community to this creation activity. Therefore, once created, memes could not be dispersed in the Web but had to be kept in a contained and easily observable environment.

Starting with this awareness in designing the school experiments, the biggest methodological challenge was to find a way to preserve the social value embodied by Internet memes in students' out-of-school digital life, while at the same time making the memes' mathematical content sufficiently evident to enable teachers to express some kind of assessment of the students' knowledge. In other terms, I had to find some balance between the out-of-school informal nature of memes and their school use in the mathematics classroom. This balance was reached providing an adequate contained social environment to collect students' productions and asking them to create something more than the mathematical meme. In particular, the out-of-school social vibe was maintained supplying a digital sharing space for students' productions that mimicked memes' natural habitat in the memesphere. For this purpose, I used the webapp Padlet[43], which allows to create and share digital boards where students can post their productions, upvote, downvote and possibly comment on classmates' productions.

In the experiments, memes' educational potentialities were investigated in three different ways (1) giving students a *creation task* asking to create a mathematical meme plus something else that made the mathematical content evident in a more traditional way, thus engaging students in handling different semiotic representations of mathematical objects, (2) giving students a *decoding task* asking to interpret the mathematical content of memes created by others (the researcher in school experiments or other students in university experiments) and (3) involving students in a whole-class discussion.

[43] www.padlet.org

1. The *creation task*, in its final form, was:

- for students: *Create a mathematical meme on an aspect of your choice in the topic XXX, a video and a GeoGebra applet decoding the mathematical content of the meme. Post your products in the Padlet and like your classmates' productions.* (assigned verbally or through a post on the Padlet).
- For teachers *Create a mathematical meme using one or more of these templates and describe its possible didactical use* (assigned through a worksheet, see Appendix Q).

2. The *decoding task* was prompted through printed worksheets (Appendix J, Appendix K, Appendix O, Appendix P) or Padlet board (Appendix G) and requested learners to write down the mathematical meanings of memes that were not their own creations.
3. The *whole-class discussion* focused on the mathematical meaning of the memes previously created and decoded, and it was conducted by the teacher with the assistance of the researcher and possibly other collaborators.

As anticipated, the design of the experiments has been refined through cycles of implementations. In particular, the first secondary school and university experiments (that took place at the very beginning of this research) were limited to the creation task assigned to single students. Subsequently, to enable the observation of the creation processes, students have been grouped in pairs for the creation task and focus pairs have been video-recorded (VR), and a written decoding task and a whole-class discussion (also VR) have been added. In addition to that, to enable the observation of the synergy of mathematical memes with an established didactical tool, the request to create the GeoGebra applet was added for the 3rd secondary school experiment. Several questionnaires and personal interviews have complemented the data collection.

To sum up, in their final form experiments have been organised as follows:

- Before the activity: administration of entry forms and worksheets (single)
- The activity:
  - Part 1) creation task (in pairs, 2/3h)
  - Part 2) feedback forms (single, 5 minutes approx.), decoding task (single, 15 minutes approx.) and whole-class discussion (1,5h approx.)
- After the activity: administration of students' evaluation forms (single) and feedback forms for teachers

Table 0.4 summarises the structure and components of the school experiments, together with the research aim and arrangement of the different components.

Table 0. 4 Experimental research: structure & components of the school experiments

When	What	Ref	Research Aim	Arrangement
Before the Activity	Entry Form	Appendix A	Assess students' familiarity with memes and social media	Self-administered Online
	Entry worksheet	Appendix C	Assess students' familiarity with mathematical memes and check the a-priori hypotheses about memes' partial meanings	Teacher
Activity Part 1: Creation Task	Memes	Appendix E Appendix F Appendix G Appendix H Appendix Q	Investigate how students express known mathematical concepts in a memetic form	Teacher + researcher
	Videos		Investigate how students express the mathematical concept encoded in the meme in a verbal/algebraic/graphical register	Teacher + researcher
	GeoGebra Applets		Investigate how students express the mathematical concept encoded in the meme in a dynamic geometry environment	Teacher + researcher
Activity Part 2: Decoding Task & Discussion	Worksheets	Appendix J Appendix K Appendix O Appendix P	Investigate if students decode the mathematical content of mathematical memes created by others	Teacher + researcher
	Students' Feedback Form	Appendix L	Collect students' short-term impressions about the first part of the activity (creation)	Self-administered Online
	Whole class discussion	/	Share and discuss the students' productions Discuss the mathematical content of the created and decoded mathematical memes	Teacher + researcher
After the Activity	Students' Evaluation Form	Appendix M	Collect students' long-term overall recollection and impressions about the activity	Self-administered Online
	Students' personal Interviews	/	Collect students' impressions about mathematical memes and their use at school	Researcher
	Teachers' Feedback Form	Appendix N	Collect teachers' impressions about mathematical memes and their use at school	Self-administered Online

3.5.4 Experimental research: collecting the data

The timeline of the experiment is summarised in Table 0.5, with indications about the role of the researcher and the type of data collected. Teacher-designed experiments did not allow to gather data about the processes of creation and discussion, nevertheless both students' productions and teachers' feedbacks provided information to investigate the implications of the research for educational practice.

Table 0. 5 Experimental research: timeline of school experiments & collected data (grey background=with researcher, white background=without researcher)

Date		May 2018	Oct 2018		Mar 2019		Apr 2019	May-Sep 2019	Dec 2019	Mar-May 2020
Grade		12th	Bachelor Degree	10th	12th	Teachers training	Master Degree	6^{th}-7^{th}	9^{th}	Various grades
Reacher / Teacher Designed		RD	RD	RD	RD	RD	RD	TD	TD	TD
Products	Memes	☑	☑	☑	☑	☑	☑	☑	☑	☑
	Videos	☑		☑	☑					☑
	GeoGebra Applets				☑					
	Reflective worksheets			☑	☑	☑	☑			
Processes	Creation VR (Focus Pairs)			☑	☑	☑	☑			
	Discussion VR (Whole class)			☑	☑		☑			
Responses	Students' Entry Form			☑	☑					
	Students' Entry worksheet			☑	☑					
	Students' Feedback Form			☑	☑	☑				☑
	Students' Evaluation From			☑	☑			☑		
	Students' personal Interviews	☑		☑	☑		☑			
	Teachers' Feedback Form			☑	☑			☑	☑	☑

During the development of the research, I sampled further different ways to observe students' engagement beyond the activity described here:

- In October 2018 (2nd secondary school experiment), memes created by students were shared not only in the virtual digital space provided by Padlet but also in a printed billboard (see Figure 0.18 and Appendix I: Creation task billboard (2st secondary school exp.)), where they were linked to the explaining videos through the mobile-activated Augmented Reality technology explained in Chapter 1 (now unfortunately discontinued). The research aim here was to investigate the effect of sharing on long-term memories.

Figure 0. 18 The printed sharing space for the 2nd secondary school experiment

- In March 2019 (3rd secondary school experiment), a promotional leaflet (see Appendix B) was shared with the students before the activity, to present the project and foster students' engagement.

- In March-May 2020 (distance learning experiments), memes created by the students were shared also outside the school environment through the Instagram page of the project @lifeonmath[44], to experiment with students-created memes leaving the school environment and going back to their natural habitat.

[44] https://www.instagram.com/lifeonmath/

3.6 Data analysis: methods and timeline

Methods for the data analyses are detailed in the single chapters: Table 0.6 gives an overview of the chapters, with the research foci and the correspondent analysed data. Data from the two university experiments and from the Distance learning experiment are not analysed in this thesis, nevertheless they contributed to the research: the university experiments confirmed that the task should include the decoding part and the class discussion to enable the observation of processes and corroborated the adequateness of our semiotic tool to interpret memes, and the distance learning experiment suggested possible further research paths.

Table 0. 6 Outline of the thesis: correspondence between the chapters, research foci and analysed data

Chapter and Research Focus		ETHNOGRAPHIC DATA	EXPERIMENTAL DATA								
			May 2018	Oct 2018		Mar 2019		Apr 2019	May-Sep 2019	Dec 2019	Mar-May 2020
			12th	Bachelor Degree	10th	12th	Teachers training	Master Degree	6^{th}-7^{th}	10^{th}	Various grades
1	Meanings and mathematical discourse	✓	✓								
2	Emotions	✓			✓						
3	Learning mechanisms at the boundary	✓			✓						
4	Visual skills				✓	✓	✓		✓		
5	Educational value of the digital culture	✓			✓	✓			✓		
6	Conceptualization of the Internet phenomenon	✓									
7	Synergy and changes in students' praxeologies	✓				✓					
8	Memes' creative mechanism	✓									

3.7 Outline of the thesis

The thesis will report on the research process addressing the overarching question stated in 3.1 by the means of the research approaches, theories, methodologies, and data exposed insofar. Figure 0.19 incorporates all the steps of this research process: the thicker blue and pink lines represent the backbones of the research: the exploratory phase (blue line) with the experimental observations, and the systematic phase (pink line) grounded on the ethnographic observations. Different theories and foci that have been sampled are represented here by coloured thinner lines that cross the main exploratory line, leading to several studies observing the object of the research from different stances.

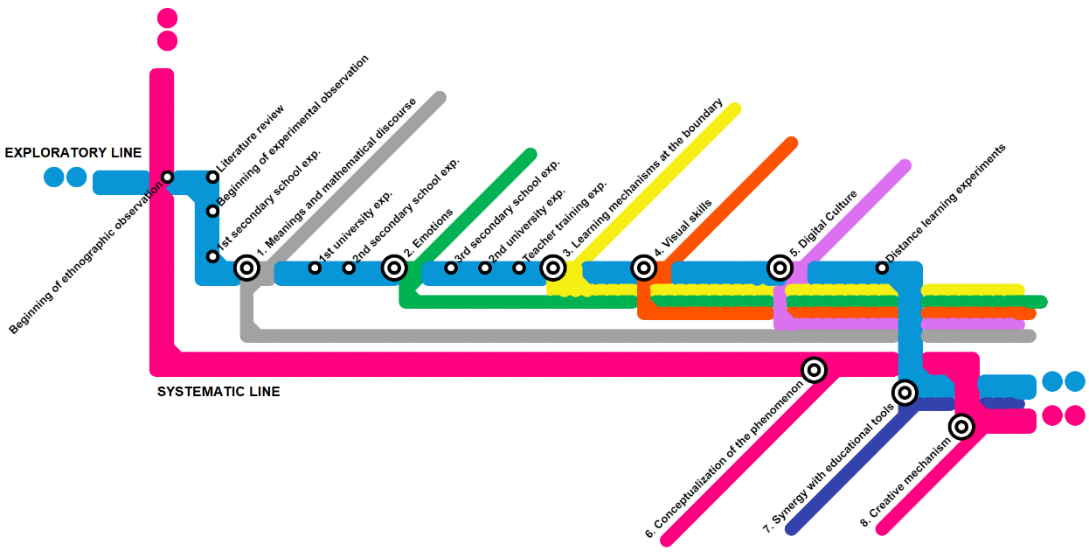

Figure 0. 19 The timeline of the research

The image shows that, towards the end of the research, the exploratory and systematic phases started to intertwine. This reflects how much this work requested to go back and forth from the school to the out-of-school environment to gather knowledge to understand mathematical Internet memes and their possible role in education. To preserve this complexity, the steps of the research will be exposed chronologically in chapters 1 to 8. A final chapter will hold a reflection on the results achieved, on their limitations and on the implications of the research for mathematics education.

Connections among the different steps of the research are summarised in Figure 0.20 and outlined in the following Table 0.7. The image highlights how the different research paths evolved and intermingled converging in the final Chapter 8 that draws from all the previous findings with the aim of providing educators with the knowledge to transform mathematical Internet memes into digital resources for teaching mathematics. The following Table 0.7 gives an overview of the content of each chapter, summarising the research approach, research focus and the subquestions addressed, and presenting some junctions notes explaining how the research proceeded from one study to the following ones.

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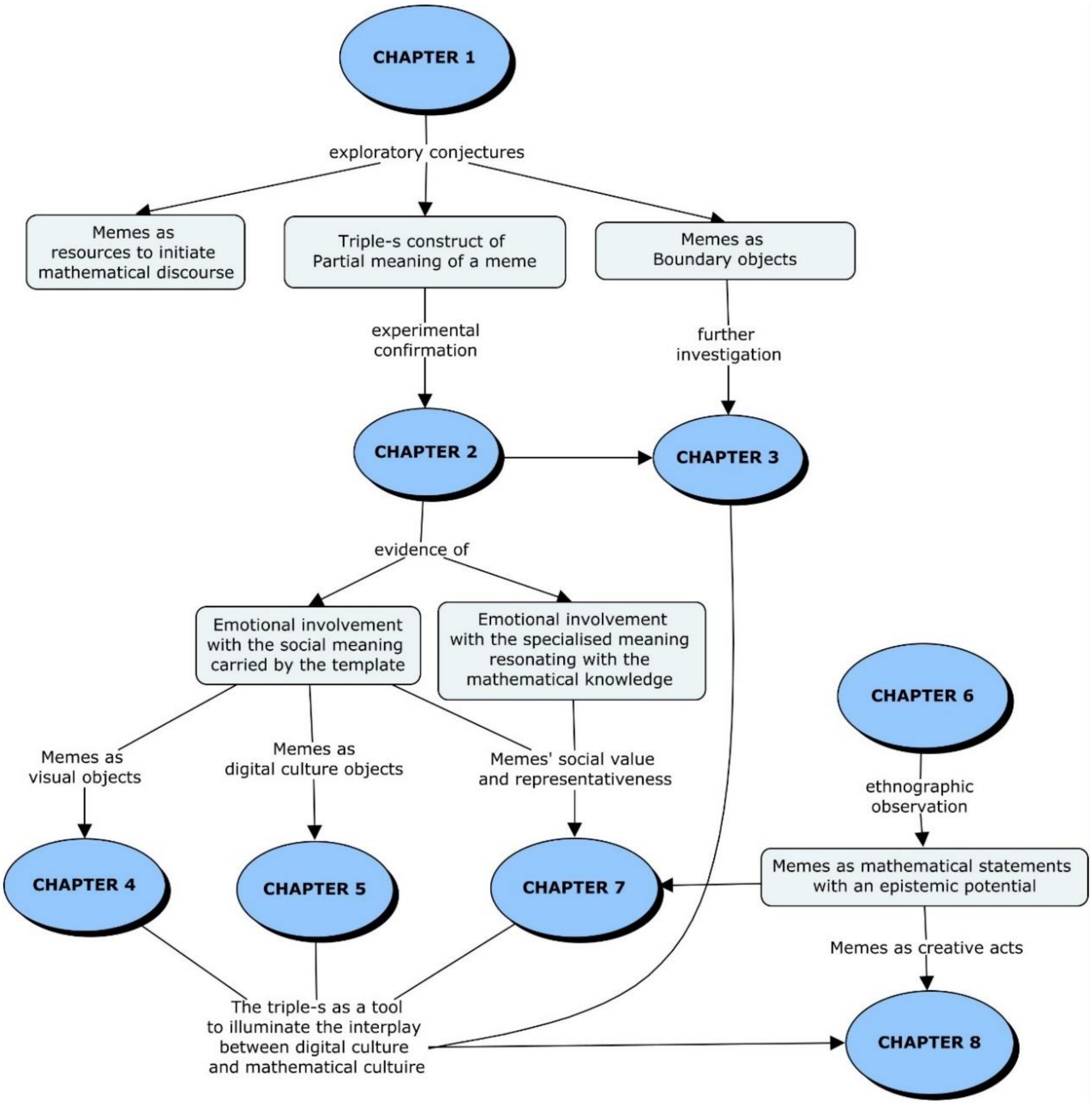


Figure 0. 20 Connections among the steps of the research

Table 0. 7 The development of the research project

<table>
<tr><th colspan="2">CHAPTER 1
MEANINGS IN MATHEMATICS: USING INTERNET MEMES AND AUGMENTED REALITY TO PROMOTE MATHEMATICAL DISCOURSE</th></tr>
<tr><td>Research Approach</td><td>Ethnographic + Experimental</td></tr>
<tr><td>Research Focus</td><td>Meanings of mathematical memes and educational potentialities as boundary objects initiating a mathematical discourse</td></tr>
<tr><td>Theoretical Framework</td><td>Boundary Objects, Star & Griesemer (1989)
Boundary crossing, Akkerman & Bakker (2011)
Commognition, Sfard (2008)</td></tr>
<tr><td>Subquestions</td><td>1. What are the epistemic affordances, if any, of mathematical Internet memes?
2. Does creating and/or interacting with these memes implies/determines learning?
3. Which characterizations identify a boundary object in this context? Which interactions among the communities of students, teachers and researchers are triggered by these boundary objects?
4. Can memes, students' explanations vehiculated through the narrative virtual content and classroom discussion be identified as parts of a mathematical discourse?</td></tr>
<tr><td colspan="2">This first chapter approaches the object of the research from a wide-ranging perspective, with diverse and broad questions reflecting the exploratory stance we assumed at the beginning of the research, and the fact that available data are the products and not the processes. In the study we find partial answers to our questions on the possible educational potentialities of mathematical memes and their value as boundary objects, and we also find an answer without having a question, represented by the a-priory hypothesis about the triple-s construct of the partial meaning. These answers spark new questions, which are addressed in the following chapters: the confirmation of the a-priori hypothesis about the partial meaning is addressed in Chapter 2 and the study of mathematical memes as boundary objects in Chapter 3.</td></tr>
<tr><th colspan="2">CHAPTER 2
THINKING INSIDE THE POST: INVESTIGATING THE DIDACTIC USE OF MATHEMATICAL INTERNET MEMES</th></tr>
<tr><td>Research Approach</td><td>Ethnographic + Experimental</td></tr>
<tr><td>Research Focus</td><td>Mathematical memes as links between emotions and mathematics</td></tr>
<tr><td>Theoretical Framework</td><td>Emotions in learning, Zan et al., (2006); Radford (2015)
Triple-s, Bini & Robutti (2019a)</td></tr>
</table>

<table>
<tr><td>Subquestions</td><td>1. What is the students’ familiarity with social platforms and memes?
2. Are the meanings of didactical memes recognised by students the same as we described in the a-priori analysis?
3. What is the role of emotions in learning processes involving didactical memes?</td></tr>
<tr><td colspan="2">The second step of the research answers the question sparked by the a priori hypothesis about the triple-s construct formulated in Chapter 1. Moreover, available data about the creation and interaction processes allow to understand that the hypothesised sequence of the three partial meanings (1. structural, 2. social and 3. specialised) is not to be considered rigidly, but that students access these three meanings in different orders depending on their knowledge and emotional involvement with the mathematical subject targeted by the specialised meaning and with the social meaning carried by the template. These results open the way to the next steps of the research, using the triple-s as an investigation tool to study mathematical memes as boundary objects (Chapter 3), to look into the educational potentialities of memes as visual objects (Chapters 4) as digital culture objects (Chapter 5), and to explore how memes’ social value and representativeness pair synergically with known educational resources (Chapter 7).</td></tr>
<tr><td colspan="2">CHAPTER 3
IS THIS THE REAL LIFE? CONNECTING MATHEMATICS ACROSS CULTURES</td></tr>
<tr><td>Research Approach</td><td>Ethnographic + Experimental</td></tr>
<tr><td>Research Focus</td><td>Mathematical memes as boundary objects between the communities of students and teachers activating learning mechanisms at the boundary</td></tr>
<tr><td>Theoretical Framework</td><td>Boundary Objects, Star & Griesemer (1989)
Boundary crossing, Akkerman & Bakker (2011)
Triple-s, Bini & Robutti (2019a)</td></tr>
<tr><td>Subquestions</td><td>1. Which characterizations identify a boundary object in this context?
2. Which learning mechanisms emerge from our observations?
3. How can school mathematics take into account the culture developed by young people in their everyday lives?</td></tr>
<tr><td colspan="2">The third chapter investigates and confirms the conjecture formulated in Chapter 1 about mathematical memes as boundary objects between the communities of students and teachers. The study focuses on a meme I created that was administered to students in the decoding activity. The following discussion shows that the transmission of the specialised meaning is hindered by the fact that the template has been chosen for its visual appearance and not following its established social meaning. This is seen as evidence that memes can be considered as boundary objects with the social meaning as the robust component that crosses the boundary</td></tr>
</table>

<table>
<tr><td colspan="2">maintaining its social value as recognised by the digital culture and the specialised meaning as the element that needs to be adapted to enable the boundary crossing. The study also sheds light on the learning mechanisms activated at the boundary crossing, evidencing the importance of taking into account students' cultural background to establish effective communication.</td></tr>
<tr><td colspan="2">CHAPTER 4
SOME LIKE IT SOCIAL: LOOKING INTO THE INTERPLAY BETWEEN MATH AND INTERNET MEMES</td></tr>
<tr><td>Research Approach</td><td>Experimental</td></tr>
<tr><td>Research Focus</td><td>Mathematical memes as educational resources channelling students' visual skills</td></tr>
<tr><td>Theoretical Framework</td><td>Visualisation, Presmeg (2006, 2019)
Triple-s, Bini & Robutti (2019a)</td></tr>
<tr><td>Subquestions</td><td>1. Can young learners' social visual expertise be channelled by researchers and teachers to foster and support the understanding of mathematical meanings?
2. How may the affect generated by personal imagery be harnessed by teachers to increase the enjoyment of learning and doing mathematics?</td></tr>
<tr><td colspan="2">Chapter 4 grounds on the previous findings about the triple-s construct of partial meanings (Chapters 1 & 2) and about students' emotional involvement with the social meaning of the meme (Chapter 2). The social meaning is carried by the template of the meme, which has two very intertwined characteristics: it provides memes' visual component, and it resonates with students' digital culture. The aim of this chapter is to investigate more thoroughly the first characteristic, looking into the educational potentialities of mathematical memes as visual objects. The study reports the use of the triple-s construct as a tool to keep track of students' cognitive processes and as a task design-tool, evidencing that students' visual expertise can become an asset in creating school activities that foster the understanding of mathematics through non-standard skills.</td></tr>
<tr><td colspan="2">CHAPTER 5
HOW SPIDERMAN CAN TEACH YOU MATH: THE JOURNEY OF MEMES FROM SOCIAL MEDIA TO MATHEMATICS CLASSROOMS</td></tr>
<tr><td>Research Approach</td><td>Ethnographic + Experimental</td></tr>
<tr><td>Research Focus</td><td>Mathematical memes as educational resources channelling students' digital culture</td></tr>
<tr><td>Theoretical Framework</td><td>Connected Learning Framework, Ito et al. (2013, 2018)
Triple-s, Bini & Robutti (2019°)</td></tr>
<tr><td>Subquestions</td><td>1. How do mathematical memes provide opportunities for connected learning experiences?
2. How can the connected learning framework and triple-s</td></tr>
</table>

	construct support educators in designing effective learning scenarios involving mathematical memes?
Chapter 5 picks up the loose end left by Chapter 4 and looks into the second characteristic connected to the memes' template which carries the social meaning, investigating the educational potentialities of mathematical memes' as digital culture objects. The study positions its result within the connected learning framework and uses the triple-s construct as an investigation tool to follow learners' cognitive processes activated by the interaction with a meme in formal and informal learning contexts and as a task design tool. The results of the study reinforce the educational potentialities of memes' connection with students' digital culture, but at the same time they show that these features cannot be clearly distinguished from the educational potentialities connected to the visual component examined in Chapter 4. This result, combined with the role of the social meaning as the robust element crossing the boundary (Chapter 3), suggests that memes can become digital educational resources only if created identifying and respecting the core idea carried by the template.	
CHAPTER 6 MATHS IN THE TIME OF SOCIAL MEDIA: CONCEPTUALIZING THE INTERNET PHENOMENON OF MATHEMATICAL MEMES	
Research Approach	Ethnographic
Research Focus	Conceptualize the Internet phenomenon of mathematical memes
Theoretical Framework	Convergence culture, Jenkins (2006) Participatory culture, Jenkins (2009)
Subquestions	1. How can the phenomenon of mathematical memes be characterised as a representation of mathematical ideas within the Internet culture? 2. How do mathematical memes activate and guide interactions among members of online communities?
This Chapter opens the systematic phase of the research, in which the observation moved from explorations conducted in the classrooms to the scenario on the Web, with the aim of conceptualizing the Internet phenomenon. For this new onset, we adopted an ethnographic methodological approach, adapting the research in mathematics education to the new context. In the study we look at the cultural context in which mathematical memes are created and shared, and at how communities act on them through the practice of commenting on social media. The analysis of these threads of comments shows that, within these communities, mathematical memes are perceived as representations of mathematical statements and that they activate epistemic needs aimed at determining their truth value. These results sparked the following systematic question about the research of a possible model of creating a mathematical meme, which is addressed in Chapter 8.	

<table>
<tr><td colspan="2">CHAPTER 7
WHEN THEY TELL YOU THAT $i^{56}=1$: AFFORDANCES OF MEMES AND GEOGEBRA IN MATHEMATICS</td></tr>
<tr><td>Research Approach</td><td>Ethnographic + Experimental</td></tr>
<tr><td>Research Focus</td><td>The synergy between mathematical memes and other educational resources to activate students’ ZPDs and foster changes in students’ praxeologies</td></tr>
<tr><td>Theoretical Framework</td><td>Social constructivism & ZPD, Vygotsky (1978)
Anthropological Theory of Didactics, Chevallard (1992, 1999)
Agents, Prodromou et al. (2018)
Triple-s, Bini & Robutti (2019a)</td></tr>
<tr><td>Subquestions</td><td>1. At a macro level, what elements of mathematical memes and GeoGebra applets activate changes in students' praxeologies or some of their components?
2. At a micro-level, how are these changes activated in terms of mentors and agents?</td></tr>
<tr><td colspan="2">This Chapter builds on the findings from Chapter 3 about students’ emotional involvement with the social and specialised meaning of the meme to investigate how these elements, paired with memes’ social value and representativeness, combine synergically with GeoGebra in activating students’ ZPD and promoting changes in their praxeologies. The study follows three students with different learning styles combining observations from a school experiment and interviews conducted one year later, and uses the triple-s as an investigation tool keep track of the emergence of mentors and agents fostering changes in students’ praxeologies. The results show that mathematical memes can blend with other educational resources, adding a social component to the activity that reaches students usually not engaged in mathematics.</td></tr>
<tr><td colspan="2">CHAPTER 8
HOW TO MEME IT: MATHEMATICAL MEMES AS CREATIVE ACTS</td></tr>
<tr><td>Research Approach</td><td>Ethnographic</td></tr>
<tr><td>Research Focus</td><td>Heuristic modelling of the mechanism of mathematical memes creation</td></tr>
<tr><td>Theoretical Framework</td><td>Bisociation theory of the creative act, Koestler (1964)
Triple-s, Bini & Robutti (2019a)</td></tr>
<tr><td>Subquestions</td><td>1. How can we model the mechanism of creating a mathematical meme?
2. How does the memetic component of a mathematical meme contribute to the represented mathematical statement?</td></tr>
</table>

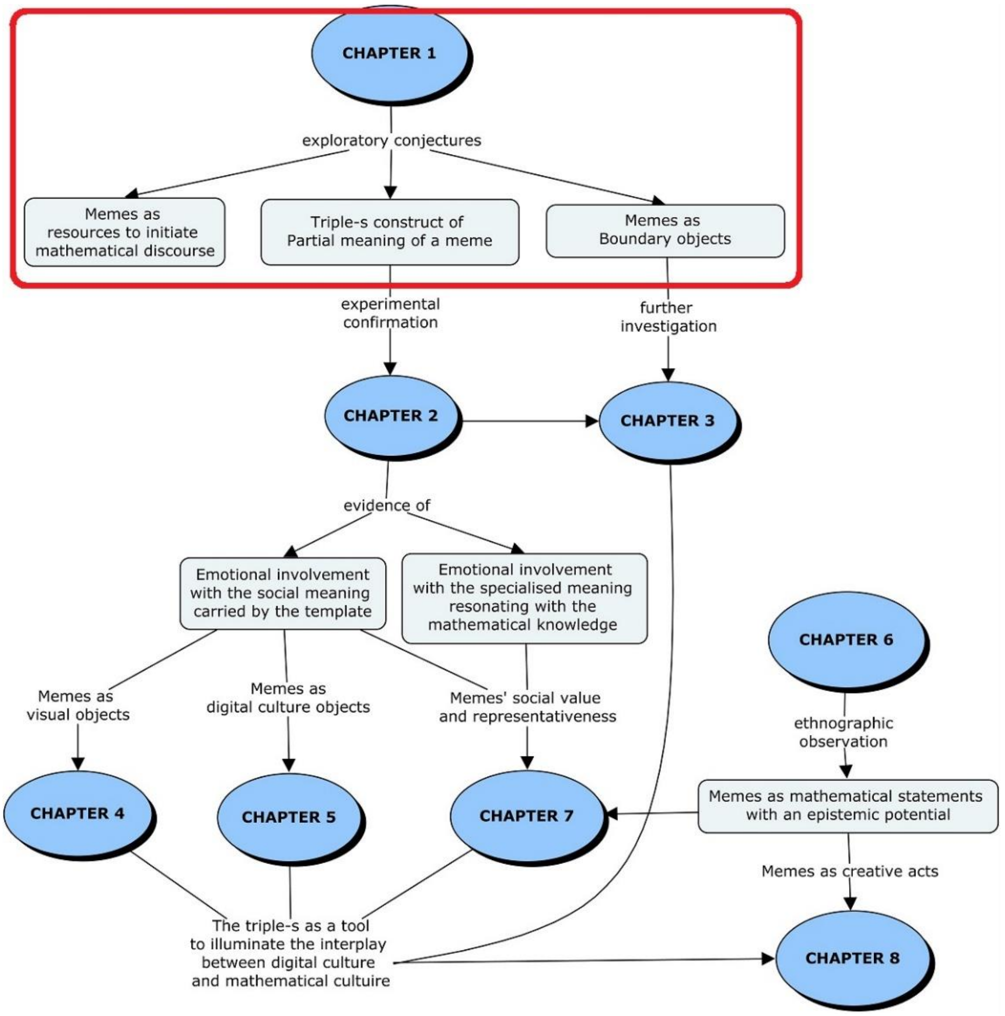
The final Chapter draws from all the previous research steps: it builds on the interpretation of mathematical memes as representations of mathematical statements conceptualized in Chapter 6, and on the triple-s as a tool to illuminate the interplay between digital and mathematical culture emerged from the experimental Chapters. The study aims at providing educators with the knowledge to transform mathematical Internet memes into digital resources for teaching mathematics. This is done firstly assuming a widened concept of mathematical creativity that allows considering mathematical memes as products of creativity in mathematics, then using a suitable theory of the creative act to build a heuristic model of the mechanism of meme creation and identify how the digital and mathematical cultures intermingle to develop the hybrid language that characterises memetic mathematical statements. These results are subsequently interpreted to show how mathematical memes can be transformed into educational resources

CHAPTER 1: MEANINGS AND MATHEMATICAL DISCOURSE

MEANINGS IN MATHEMATICS: USING INTERNET MEMES AND AUGMENTED REALITY TO PROMOTE MATHEMATICAL DISCOURSE

This is the Accepted Manuscript of an article published in U. T. Jankvist, M. van den Heuvel-Panhuizen & M. Veldhuis (Eds.), *Proceedings of the eleventh congress of the European society for research in mathematics education*. Freudenthal Group & Freudenthal Institute, Utrecht University and ERME in September 2019 available at https://hal.archives-ouvertes.fr/hal-02422152

Positioning the Chapter in the research



This first chapter approaches the object of the research from a wide-ranging perspective, with diverse and broad questions reflecting the exploratory stance we assumed at the beginning of the research, and the fact that available data are the products and not the processes. In the study we find partial answers to our questions on the possible educational potentialities of mathematical memes and their value as boundary objects, and we also find an answer without having a question, represented by the a-priory hypothesis about the triple-s construct of the partial meaning. These answers spark new questions, which are addressed in the following chapters: the confirmation of the a-priori hypothesis about the partial meaning is addressed in Chapter 2 and the study of mathematical memes as boundary objects in Chapter 3.

Abstract: Levering on the influence and use of social media in students' perception of learning mathematics, this project has a twofold purpose: the research aim is to contribute in breaking away from the idea that digital culture can be considered as opposed to school culture, investigating the epistemic and didactical affordances of mathematical Internet memes, combined with Augmented Reality technology. The coupled educational aim is to cultivate classroom discourse and maximize linguistic and cognitive meta-awareness in high school students. The project is a work in progress of which this paper presents the theoretical background used to frame the study, the results of a first exploratory teaching experiment - conducted with a group of 24 students attending the 12th grade of Liceo Scientifico in Milan - and the methodology and the next steps of the study.

Keywords: digital culture, meme, augmented reality, boundary object, discourse

1 Introduction

Internet memes have become a viral form of Web 2.0 communication, with an ever-increasing number of occurrences on social platforms as Instagram (from 37 million in March 2018 to 76 million in March 2019). Nevertheless, Knobel & Lankshear claim that "understanding successful online memes can contribute much to identifying the limitations of narrow conceptions of literacies and new technologies in classrooms" (2007, p.221) still remains almost unanswered. In fact, up to today, the possible didactic relevance of Internet memes is nearly unexplored by academic research and their meaning and use in mathematics education is an unmapped territory. Even if popular culture is playing an increasingly important role in the lives and learning opportunities of young people, Peter Applebaum's regrets that "little work, if any, has been done by math educators to probe the efficacy of mass culture criticism for education in math" (1995, p. 24) strikes for its up-to-dateness.

Levering on the influence and use of social media in students' perception of learning mathematics, this project has a twofold purpose: the research aim is to contribute in breaking away from the idea that digital culture can be considered as opposed to school culture, investigating the epistemic and didactical affordances of mathematical Internet memes, combined with Augmented Reality technology. The coupled educational aim is to cultivate classroom discourse and maximize linguistic and cognitive meta-awareness in high school students. The project is a work in progress of which this paper presents the theoretical background used to frame the study and the results of a first exploratory teaching experiment - conducted with a group of 24 students attending the 12th grade of Liceo Scientifico in Milan - together with the methodology and the next steps of the study.

2 From Memes to Internet Memes

In 1976, well before the digital era, evolutionary biologist Richard Dawkins coined the term *meme* as "a unit of cultural transmission", examples of which are "tunes, ideas, catch-phrases, clothes fashions, ways of making pots or of building arches" that "propagate themselves in the meme pool by leaping from brain to brain via a process which, in the broad sense, can be called imitation." (Dawkins, 1976, p.249). Web culture[45] gives credit to Mike Godwin for revamping the concept of memes in 1993, identifying Internet memes as a subset of memes and describing them as an activity, concept, or piece of media that spreads through social channels, evolving in the hands of the digital users and reaching a large audience.

Internet memes can be in the form of viral images, videos o files: in this work, we will focus on those made of "verbal and pictorial parts, which unfold their meaning through collective semiosis" (Osterroth, 2018, p.6); as they integrate different modes of communication, we think they can fall into the category of multimodal artefacts. They usually have a humorous or satirical intent and are widely shared by young people through social platforms (Facebook or Instagram), since they connect the participatory potential of the Internet with the quest for user-generated content that - through shares and likes - provides a form of social validation.

They are deeply rooted in visual media culture and catch users' attention with their puzzling vibe that calls for the active contribution of the viewer to unlock the meaning and thus adds a gratifying flavour. Internet memes are created by users according to collectively established and shared rules that govern the so-called *memesphere*: these rules dictate the conventional meaning of the pictorial parts and the position, font, syntax and narrative structure of the text, they "cannot be enforced by a specific person, but the community sanctions wrong uses by downvoting, not liking or simply not spreading the misused meme" (Osterroth, 2018, p.7). These same rules shape meme generators websites as https://imgflip.com/memetemplates or https://makeameme.org/: there are several accepted structures, two of the most common being those in Figure 1.1.

[45] https://en.wikipedia.org/wiki/Internet_meme

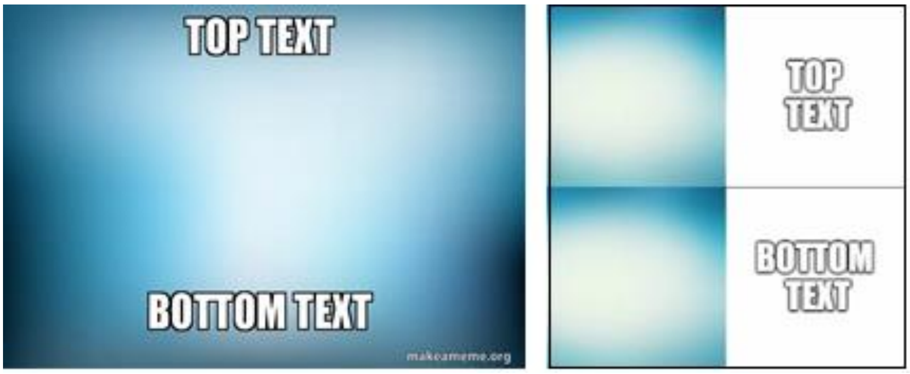


Figure 1. 1 Two prototypical meme structures

Pictorial parts, usually identified with names, might have local or global recognisability: in Figure 1.2, a blank template of the *Success Kid* meme - globally used to boast of something good - is shown (top), followed by a general example of the meme propagation by imitation and an example of its psycho-pedagogical adaptation, to end with its mathematical variation (bottom), source Google search, Sept 18.

Figure 1. 2 Template (top) and propagation (bottom) of the *Success Kid* meme

The last example is representative of a rare situation in which mathematics *spontaneously* leaves the school context and is used to identify the belonging to a group and to set the author's position in the group. This kind of memes, in fact, are usually shared within specialized Internet groups, where products are scrutinized by skilled peers and acceptance grants the author the cited social validation.

3 Mathematical Internet Memes as Knowledge Carriers

To focus more on the concept of mathematical Internet memes and illustrate their epistemic potentiality, we shall analyse two examples of the *Drowning kid in the pool* meme (Figure 1.3), used to describe a situation where something is typically forgotten. On the left a correct version posted in May 2018 within a Reddit thread[46], where was identified as "the quintessential math joke", on the right its wrong variation, posted in another Reddit thread[47] in June 2018.

Figure 1. 3 *Drowning kid in the pool* correct (left) and wrong (right) memes

Taking a look at the exchange triggered by the wrong one (some excerpts are listed below), we see how memes can act as knowledge carriers and starters of meaningful mathematical discussions:

bike0121 This is not really correct. The function sqrt(x) is always greater or equal to zero. The inverse of x^2 is indeed ±sqrt(x), but sqrt(4) is always +2, not -2.

Dat_J3w Yea it's positive by convention; the square root of four is still plus/minus two

[46] https://www.reddit.com/r/EngineeringStudents/comments/8i4ojo/every_goddamn_time/
[47] https://www.reddit.com/r/EngineeringStudents/comments/8pjvvm/i_tend_to_be_forgetful/

functor7 Absolutely not. Sqrt(4) is 100% only the value 2. The solutions to the equation $x^2=4$ are ±2. Sqrt(x) is a function, which means it only has one value. Sqrt(x) is a number, not a collection of numbers.

Dozens of Reddit and Facebook mathematically-themed groups (some with evocative names like "Complex Analysis Memes For Holomorphic Teens" and "The Name Of This Group Is Left As An Exercise For Its Members") with thousands of users showing off their mathematical knowledge, discussing, posting and sharing mathematical Internet memes on a daily basis, suggest that the meaning of a meme can be looked for in a "sphere of practice (SP), […] defined as the 'community' adhering to a common set of rules, within which mathematical meanings are constructed" and could be a suitable setting to address the issue of "communicating, transforming and negotiating the social meaning of school mathematics" (Kilpatrick et al., 2005, p.10).

4 Partial and Full Meanings of a Meme

As a first step of the study on mathematical memes, we tried to figure out how memes act as carriers of meanings and we identified a *triple-s construct* of the three partial meanings that contribute in building up the full meaning of a mathematical Internet meme.

- The first meaning of a meme lies in its being a meme, namely, to have a specific and shared structure and graphics (font, colour and text position). It can be considered at a *structural* level (Figure 1.1).
- The second meaning of a meme is conveyed by the shared conventions connected to viral images, compositional setups and accepted syntaxes. It is at a *social* level (Figure 1.2, top image).
- The third meaning of a meme is borne by images, symbols or text referring to a specific topic (mathematical, but also political, physical or other). This is at a *specialised* level (Figure 1.2, last bottom image, textual part).

The first two meanings ground in the popular culture rules that govern the *memesphere*, acting as Kilpatrick's sphere of practice, while the third calls some mathematical skills into action. The interplay of all three *partial meanings* unlocks what we call the *full meaning* of the meme, that triggers the surprise/membership effect. There are people who can only access the first meaning, merely recognising the artefact as a meme (and not a cartoon, for example). Others will stop at the second level, identifying the viral elements only. On the other hand, those who understand the specialized mathematical signs in the meme, but are not aware of the other meanings, will equally miss the full meaning. But those who succeed in appreciating all three meanings will crack the full meaning of the meme: they will laugh and feel part of a community. In fact, “meaning mobilizes feeling, and emotions only translate if we process images and captions in the same way” (Benoit, 2018, p.41).

To sum up, we think that it would be fruitful to investigate whether mathematical Internet memes can contribute to the construction of mathematical meanings and explore the possible epistemic affordances implied by their multiple referents (one of many being their visual component that can cater to different intelligences and learning styles). To pursue this research path we imagined that, having mathematics spontaneously already crossed the boundary between formal and informal learning, it could be worth exploring how to facilitate the crossing in the opposite direction and test mathematical Internet memes in a standard school context.

5 Theoretical Framework

It is beyond the scope of this study to evaluate why Internet memes in general, and mathematical Internet memes in particular, have this success in the social arena, although that “aha” moment we all experience when we finally grasp the joke resonates with what Mason (2014, p.1) describes as a “disturbance, experienced as surprise, as puzzlement or perplexity” that somehow “provoke[s] learners into taking initiative” and “call[s] upon learners to make use of their undoubted powers of making sense.” Zooming in, we will focus our attention on the learning dynamics connected to the use of mathematical memes within the classroom: in this optics we will try to read our data through the lenses of the Boundary Objects framework as introduced by Star & Griesemer (1989) and further developed by Akkerman & Bakker (2011) and of Anna Sfard’s (2001, 2008) theory about discourse and communicational approach to cognition in a Vygotskian sociocultural perspective.

According to Star & Griesemer, (1989, p.393) “boundary objects are objects which are both plastic enough to adapt to local needs and the constraints of the several parties employing them, yet robust enough to maintain a common identity across sites. […] They have different meanings in different social worlds, but their structure is common enough to more than one world to make them recognizable, a means of translation.” Our point is that mathematical Internet memes can be looked at as boundary objects between social media and mathematics, two separate worlds with separate rules and languages, whose “intersections of cultural practices open up third spaces that allow negotiation of meaning and hybridity—that is, the production of new cultural forms of dialogue” (Akkerman & Bakker, 2011, p.135), and therefore they can be fruitfully trialled in boundary crossing activities. In a complementary way we argue that memes, together with the inner discourse brought into being to make sense of them and the collective discussion they trigger in the class group, are aligned with Sfard's statement that "the term discourse will be used to denote any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system […] learning mathematics may now be defined as an initiation to mathematical discourse, that is, initiation to a special form of communication known as mathematical." (Sfard, 2001, p.28).

The research questions we are searching answers to are the ones below, they are big and complex ones, and we are aware that this study just put a small dent into them, but we think they deserve to be stated because they outline the bigger picture, we strongly believe is worth looking for:

- RQ1: What are the epistemic affordances, if any, of mathematical Internet memes?
- RQ2: Does creating and/or interacting with these memes implies/determines learning?
- RQ3: Which characterizations identify a boundary object in this context? Which interactions among the communities of students, teachers and researchers are triggered by these boundary objects?
- RQ4: Can memes, students' explanations vehiculated through the narrative virtual content and classroom discussion be identified as parts of a mathematical discourse?

6 Methodology: The Building of the Teaching Experiment

The beginning of this project dates back to a few years ago when one high school student (of the first author) provokingly said that he had finally grasped a math concept only when coming across a mathematical Internet meme on the same subject in a social platform. Moving forward from this first naïve appearance of memes in the school environment and trying to figure out how to build a significant task to test our assumptions in an almost unexplored territory is, of course, a challenge that can be tackled only by small steps. We started observing students' reaction to mathematical Internet memes found in the Web (examples in Figure 1.4) and used by the teacher to catch and hold attention, to stress specific aspects of already taught topics and to trigger classroom discussion.

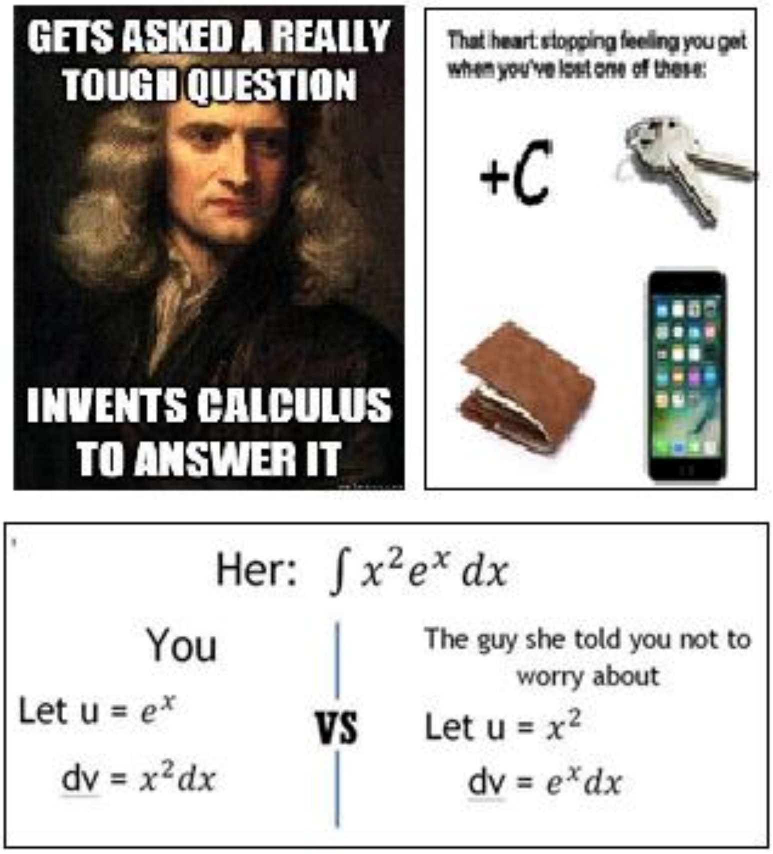


Figure 1. 4 Examples of Web-found mathematical Internet memes

The quality of the mathematical discourse enthused by these mathematical Internet memes convinced us that interacting with them calls mathematical competencies into play but, to dig deeper into the epistemic of memes, we realised that students had to create the memes themselves and shoot videos to keep track of the explanations of the mathematical meanings.

This is when Augmented Reality (AR) comes into action: vision-based AR technology refers to the triggering of a superimposed computer-generated layer (usually a video) when pointing a smartphone or other GPS-enabled device to a precise spot; it allows the connection of the meme to the corresponding explanatory video, that can be then viewed scanning the meme into the phone. Hopefully, the bonded couple meme+video will thus give origin to a multimodal and multimedia learning object, providing opportunities for the single student to maximize his linguistic and cognitive meta-awareness and systematize knowledge, for the class group to connect and revise knowledge and for the teacher to assess students' comprehension of previously studied concepts, to single out possible misunderstandings and misconceptions and give a formative assessment. Last but not least, it could cross the threshold of the school environment, be shared on the Web and become part of the global network heritage as a viral meme.

Having students create memes and videos themselves matches different goals: it allows taking advantage of the aforesaid participatory thrust of the digital world, it leverages on the notion of identity as "a perfect candidate for the role of "the missing link" in the researchers' story of the complex dialectic between learning and its sociocultural context" (Sfard & Prusak, 2005, p. I-43) and finally it will hopefully facilitate the emergence of the students' linguistic and cognitive meta-awareness, which are the declared educational aims. This last expectation grounds in the concepts of "active response" and "meaningfulness" advocated by Burbules in his description of the virtual, that "should not be understood as a simulated reality exposed to us, which we passively observe, but a context where our own active response and involvement are part of what gives the experience its veracity and meaningfulness" (2006, p. 38).

7 A First Teaching Experiment

At the end of May 2018, a first exploratory teaching experiment was conducted with a group of 12th-grade students, aiming at testing the potential educational effectiveness of memes. The assigned task, to be done individually at home, was to create a meme on one of the year's math course topics (optionally using one of the said meme generator websites) and shoot a video with the smartphone explaining the mathematical concept recalled by the meme. Students then were asked to bond meme and video through HP reveal (a free augmented reality Web app[48]) and post the meme in the collective space, using the free Web app Padlet[49], set up for the occasion to mimic the social media environment, allowing the sought-after tribal reward reactions (Appendix D).

Connecting students' memes to our *triple-s* analysis, we can see that all productions comply with the structural meaning and that they all call for the viewer's engagement mobilizing an emotional reaction due to a misalignment between different parts of text or between text and image. Looking at the social meaning, that seems the element that crosses the boundary "maintaining the common identity" (Star & Griesemer, 1989, p.393), we have identified three main categories (examples in Figure 1.5).

[48] Now discontinued (Dec 2020)
[49] www.padlet.org

Figure 1. 5 Example 1 (left), Example 2 (centre), Example 3 (right)

Finally, examining the memes' specialized meaning, further clarified in the coupled videos, we completed the possible analysis in Table 1.1 (videos – in Italian - can be seen downloading the HP Reveal app to a smartphone, following *lifeonmath* and then scanning the images in Figure 1.5 – due to monitor light reflections, it works better with printed images[50]).

Table 1. 1 Analysis of the produced memes and videos

CATEGORIES	SOCIAL LEVEL	SPECIALIZED LEVEL
Figure 1.5 Example 1	meme caption starting with a "*when you/they*" statement	mathematical terms in the meme caption.
Figure 1.5 Example 2	meme describing the effect of an action or operation	image connected to the mathematical action; mathematical symbols used to label elements in the image
Figure 1.5 Example 3	meme creating a pun based on a bivalency of meanings	meme caption with terms that have a mathematical and a common use meaning

Considering these data, a first observation is that technology seems to intervene with different roles: it acts both as an instrument for students to create the learning objects and as an environment with rules that informs designers and users. From the educational point of view, their creations seem to confirm that students knowingly selected a topic in which they felt comfortable (cognitive meta-awareness) and made an effort to explain it clearly using appropriate lexis (linguistic awareness). Looking this preliminary analysis through the lenses of our framework theories, we ventured that the boundary object could be looked for in the social level that "maintain[s] a common identity across sites" (Star & Griesemer, cited), while the couple

[50] Due to the discontinuance of the AR app, videos are no longer available in augmented reality mode

meme+video appears effective in triggering a special form of mathematical communication in which meta-cognition and language accuracy play a relevant role. This is, of course, a first rough attempt that has to be refined, breaking it down and trying to identify the steps of the boundary crossing (identification, coordination, reflection and transformation) and the commognitive constructs (word use, visual mediators, narratives, and routines), once the observation of the cognitive processes will be implemented.

8 Next Moves

Although the results of this first exploratory teaching experiment seem encouraging, there are still many shadows hanging over this project, one among others being its replicability. In order to dispel at least this first doubt, in the upcoming months, we are planning to set up a focus group observing 2/3 students, coming from a different school environment, during the process of creating memes and videos on a fixed topic. In the light of the data collected, we will hopefully structure a larger study in a class group, giving space also to class discussions, teacher's feedbacks and possible students' revisions of the products, together with the assessment of the learning after a few months.

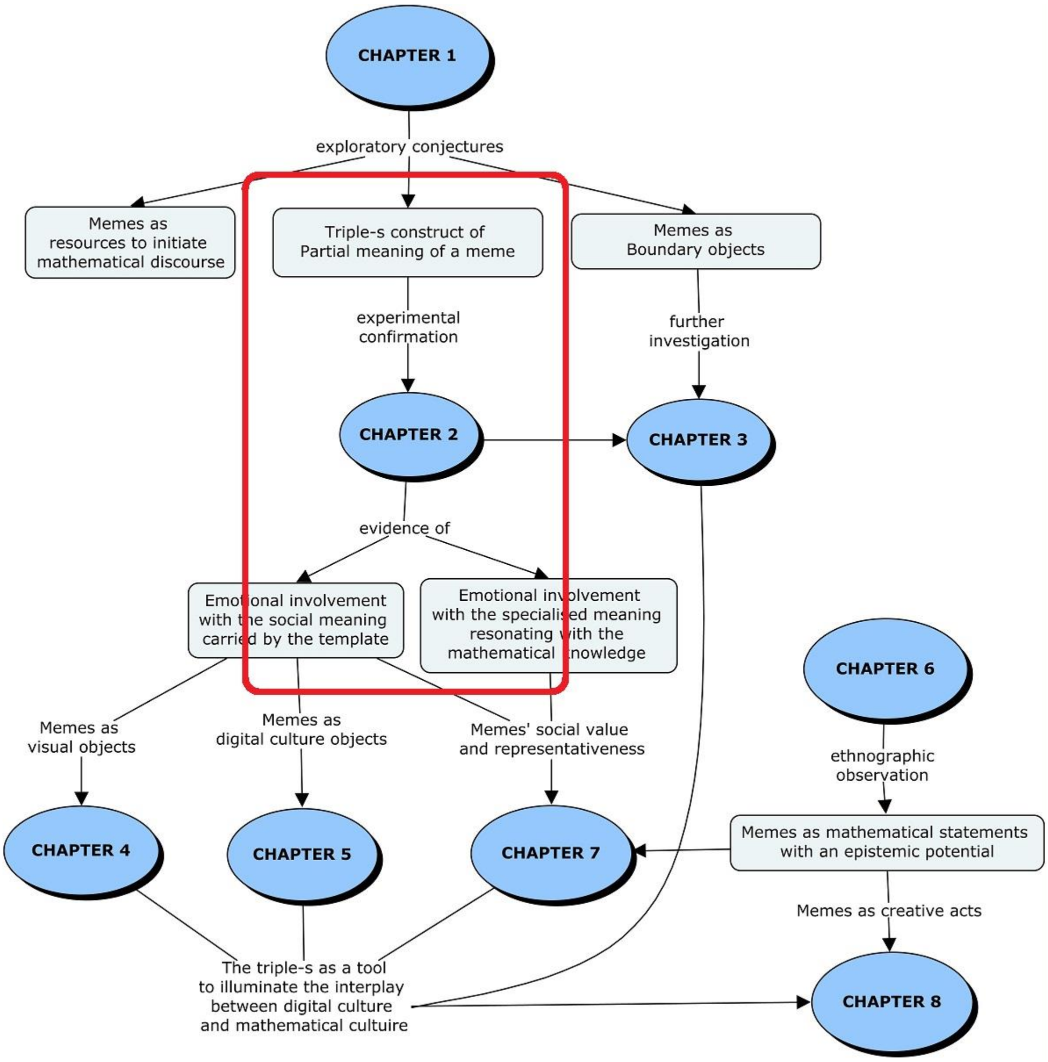
As we recalled in the opening paragraph, this work is still in progress, with no assessed conclusions and many unanswered questions, but we strongly believe it is worth to keep venturing in this almost unmapped territory because this is something more than a traditional technological learning experience, as memes produced by students can leave the school environment and spread virally on the Web. It is a unique situation in which the virtual world comes into contact with the school world, expectantly enabling us to learn something new about the impact of the digital revolution and new communication technologies on epistemological pathways and learning practices.

CHAPTER 2: THE ROLE OF EMOTIONS

THINKING INSIDE THE POST: INVESTIGATING THE DIDACTICAL USE OF MATHEMATICAL INTERNET MEMES

This is the Accepted Manuscript of an article published in A. Shvarts (Ed.), *Proceedings of the PME and Yandex Russian conference "Technology and psychology for mathematics education"* (pp. 106–113). HSE Publishing House in September 2019 available at http://www.igpme.org/wp-content/uploads/2020/01/PMEYandex2019Final.pdf

Positioning the Chapter in the research



The second step of the research answers the question sparked by the a priori hypothesis about the triple-s construct formulated in Chapter 1. Moreover, available data about the creation and interaction processes allow to understand that the hypothesised sequence of the three partial meanings (1. structural, 2. social and 3. specialised) is not to be considered rigidly, but that students access these three meanings in different orders depending on their knowledge and emotional involvement with the mathematical subject targeted by the specialised meaning and with the social meaning carried by the template. These results open the way to the next steps of the research, using the triple-s as an investigation tool to study mathematical memes as boundary objects (Chapter 3), to look into the educational potentialities of memes as visual objects (Chapters 4) as digital culture objects (Chapter 5), and to explore how memes' social value and representativeness pair synergically with known educational resources (Chapter 7).

Abstract: We venture in the almost unexplored field of mathematical Internet memes, with the aims of investigating their didactical features in a teaching and learning setting. The work is framed within the research field studying the links between emotions and mathematical thinking and takes off with a schematization of the meanings carried by a meme, formulated through an a-priori analysis of spontaneous Web productions and results of an exploratory experiment. The analysis is then compared to the data collected in a teaching experiment conducted at high school level. Results sustain the conjectured meanings' structure and elicit evidence of students' emotions and of their role in the learning process initiated by the interaction with memes.

1 Introduction: Memes as New Learning Objects

We argue that mathematical Internet memes can morph into effective learning objects if paired with adequate teaching practices, pointed at harnessing memes' social, emotional and communicative potentials and funnel them into teaching and learning assets. The present study aims at verifying the robustness of an a-priori schematization of the meanings carried by memes and test their didactical use in a learning setting.

2 What is a Mathematical Internet Meme

Mathematical Internet memes are a special kind of Internet memes, which in turn are a subset of memes, "unit[s] of cultural transmission" that propagate themselves by imitation (Dawkins, 1976, p.249). The common feature that characterises Internet memes is that they are pieces of digital media that spread virally through social channels, reaching a large audience in a very short time. They are built of "verbal and pictorial parts, which unfold their meaning through collective semiosis" (Osterroth, 2018, p.6); they can be in the form of viral images, videos or files created by users following collectively established rules that govern the so-called *memesphere* and they are widely shared through social platforms with a satirical or humorous intent. According to Shifman (2014, p.15) "while seemingly trivial and mundane artefacts", memes "reflect deep social and cultural structures" and "epitomize the very essence of the so-called Web 2.0 era". Although they score an ever-increasing number of appearances on social platforms (the hashtag #memes hit 67 million of occurrences on Instagram in January 2019), they can be called *famous strangers:* well known to net citizens worldwide, but totally foreign for those who are not familiar with social media culture. As a matter of fact, up to now, they are understudied by academic research.

We start with a Web found example: in Figure 2.1 (left) we see the original viral image of the *Who Killed Hannibal?* meme, Figures 2.1 centre and right show two mathematical variations.

Figure 2. 1 Original template (left) and mathematical variations (centre and right)

Which information is necessary to understand them? First, we need to recognize them as memes. Second, we have to connect some evidence to the background image: we should know the original image to identify the remixing in the mathematical variations. In Figure 2.1 (centre) Hannibal is unaffected by the shooter and in 2.1 (right) he doubles himself as a consequence of being shot at. Third, we have to understand the mathematical meanings represented symbolically: in Figure 2.1 (centre) the notion that the exponential function $y=e^x$ remains untouched by the differential operator and in Figure 2.1 (right) the fact that $y=e^{2x}$ doubles when the first derivative is taken. In our preliminary Web survey on social media, we have encountered dozens of mathematically themed groups, with hundreds of users reacting, commenting and questioning about the image or the mathematical part of memes like the one analysed here. This suggests that only those who succeed in grasping all levels fully understand the meme, laugh and feel part of this mathematically skilled community that emerged spontaneously in the digital world.

3 The Meanings of a Meme

In a previous study (Bini & Robutti, 2019a), through the a-priori analysis of Web productions and the results of an exploratory experiment, we identified three partial meanings that build up the full meaning of an Internet meme:

- The first partial meaning is *structural* and lies in its being a meme, namely to have a specific and shared structure and graphics (font, colour, text position).
- The second partial meaning is *social* and lies in the shared conventions of viral images,

compositional setups and syntaxes. (Figure 2.1 left).

- The third partial meaning is *specialised* and lies in images, symbols or text referring to a specific topic (mathematical, or other). (Figure 2.1 centre and right).

The first two meanings ground in the popular culture rules that govern the *memesphere*, while the third calls some mathematical knowledge and skills into action. The interplay of all three *partial meanings* is needed to unlock what we call the *full meaning* of the meme, which triggers a sense of surprise and fun. Here we intend meaning within a "sphere of practice", adhering to a common set of rules, where "mathematical meanings are constructed" (Kilpatrick et al., 2005, p.10).

For students, who are fully fledged net citizens and access the first two meanings easily, the obstacle in grasping the full meaning usually lays in understanding its specialised meaning (the mathematical content). In an educational setting, we hypothesise that this final hurdle, that makes the act of cracking the meme even more rewarding, could turn out as one of the meme's significant didactical feature. In fact, the introduction of some attuned *desirable difficulties* in the learning process can improve long-term retention, since "in responding to the difficulties and challenges, the learner is forced into more elaborate encoding processes and more substantial and elaborate retrieval processes" (Bjork, 1994, p.192). On the other hand, teachers - who are usually not familiar with social media trends – may be shut out of the first two meanings, and therefore of the full meaning of the meme. This can be called an *undesirable difficulty* that creates a barrier between teachers and students, grounding on the digital culture vs. school culture cliché. This paper aims at opening a breach in this barrier, introducing the idea of *didactical meme*: a mathematical Internet meme used in the classroom for teaching and learning purposes.

4 Theoretical Framework

Memes are a totally new phenomenon in mathematical education research and there is no history in literature of suitable theories to frame them. Our first exploratory approach (Bini & Robutti, 2019a) was based on the results of an a-priori analysis of memes and aimed at describing their role in education from a cognitive point of view. The Boundary Object perspective, as introduced by Star & Griesemer (1989), and Sfard's (2008) theory about discourse and communicational approach to cognition seemed appropriate to ground our analysis on.

Further investigations, involving new data on the memes design process (described in the *Data and analysis* paragraph), steered our focus from the cognitive to the emotional aspects of students' interplay with memes. We were faced with evidence that "emotions also affect cognitive processing in several ways: they bias attention and memory and activate action tendencies" (Zan et al., 2006, p.118). The cultural-historical approach introduced by Radford (2015), a "cultural conception of emotions and their role in thinking in general and mathematical thinking in particular" (p.26), seems fit for our case. In fact, according to Radford, emotion and thinking are strictly connected: "from a cultural-historical perspective, emotions are both subjective and cultural phenomena simultaneously; they are entrenched in physiological processes and conceptual and ethical categories through which individuals perceive, understand, reflect, and act in the world" (p.35). In particular, his idea that "contemporary cultural ideas of learning and learners are conveyed by schools and other social institutions, family, and mass culture." (p.46) valorize the social value of memes, which, through shares and likes, act as social currency in the memesphere.

To sum up, using the words of an anonymous Reddit user, memes are "like inside jokes between millions of people": they find their reason for being in reactions and root deeply into emotions. We argue that a didactical meme can be a conveyor of cognitive and emotional elements, taking advantage of a fact neural scientists agree on, i.e. that "emotional arousal often leads to stronger memories" (LeDoux, 2007), as memories about emotional situations are normally stored both in explicit and implicit memory systems. The research questions of the study are: RQ0) What is the students' familiarity with social platforms and memes? RQ1) Are the meanings of didactical memes recognised by students the same as we described in the a-priori analysis? RQ2) What is the role of emotions in a learning process involving didactical memes?

5 Methodology

This work presents the pilot study of a PhD thesis (one of the authors'), involving a class group of 22 10th grade students, who created their own didactical memes on the topic of linear systems and recorded videos with the explanations of the specialised meaning. Due to page restriction, didactical memes involved in the study will be now on referred to simply as memes. Data collected are: individual entry forms and worksheets (Appendices A and C), memes and videos created at school by students working in pairs (3h), screencast and video recordings of the memes and videos production processes by two selected pairs, individual feedback forms (Appendix L) and reflective worksheets (Appendix J), video recordings of the collective discussion guided by the teacher (2h). Observed pairs were picked out coupling students with mixed mathematical and linguistic abilities, to facilitate the emergence of the expected meanings and their interaction. Students' creations have been gathered in collective spaces (Padlet walls shared via Google Classroom) that mimicked the social media environment, allowing the coveted reactions (Appendix E).

6 Data and Analysis

The entry online form (Appendix A) answered RQ0, assessing that 100% of the students were familiar with social platforms (83% declared to visit them "several times a day"), with memes ("an image with funny text") and had some interaction with general purpose (i.e. non-didactical) memes (100% declared to like and/or share them on social platforms, one student identified himself as a meme creator). The entry worksheet (Appendix C), administered by the teacher, used the meme in Figure 2.2 to check if our a-priori identified partial meanings matched those recognised by students.

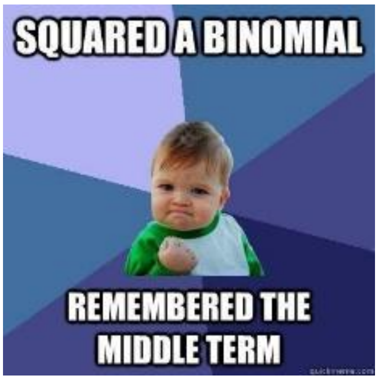


Figure 2. 2 The entry worksheet mathematical meme

All answers support our a-priori analysis of the partial meanings of a meme (RQ1). In Table 2.1 we show expected partial meanings (not revealed in the questionnaire), questions, and samples of students' answers (selected because particularly effective).

Table 2. 1 Assessment of the meanings of a meme

Meaning	Questions	Answers
Structural	In your opinion, what leads you to say that this is a meme and not a cartoon?	S1: From the classic meme font, that is in capital letters, and the fact that it is divided into two parts, a sentence above and one below that do not cover the image
Social	What do you think is the purpose of the image chosen as the background of the meme?	S2: The child's gesture means "hurray I've done it" and implies that, as finding the middle term is difficult, when you do you are happy

Specialised	Which mathematical topic is referred to in this meme?	95% identified and explained the rule applied to square a binomial and 81% agreed on the fact that the middle term is usually forgotten

Subsequently, we analysed the products (Figure 2.3) and the production processes of the two focus pairs, to identify the interaction between partial meanings.

Figure 2. 3 Pilot study focus couples' memes (captions translated by authors)

All memes were created through a meme generator website[51] that automatically imposes the compositional rules, so we shall take the structural meaning for granted and focus on the interplay of the social and specialised ones. Hereafter we summarize the key moments of the selected pairs' memes production processes.

Figure 2.3 left: pair 1 (two girls) started with the idea of creating a meme on what they identified as the most difficult aspect of the assigned topic (specialised meaning: Cramer's rule and fractional equations). In the Meme Generator website, they looked for a template whose social meaning matched the emotion that these mathematical difficulties stirred. In the explanatory video, they connected the two meanings, clarifying that "the expression of the old woman in the meme represents our faces when we see [simultaneous] fractional equation to be solved with Cramer's rule".

Figure 2.3 right: pair 2 (two boys) browsed through the various templates in the website, laughing and quoting a variety of possible captions related to their feelings and experienced difficulties in mathematics (fractions, binomial expansions, systems), to create something funny and original ("in my opinion everybody will use the first [images], we could differentiate

[51] https://imgflip.com/memegenerator

ourselves…"), because if the template and/or subject were already used by someone else, there would be less chance of gaining likes.

In both cases, grounding on the shared structural level, we witness something that we did not consider in our a-priori analysis: a dynamic interplay between social and specialised meanings in the design activity, deeply rooted on emotions (Figure 2.4).

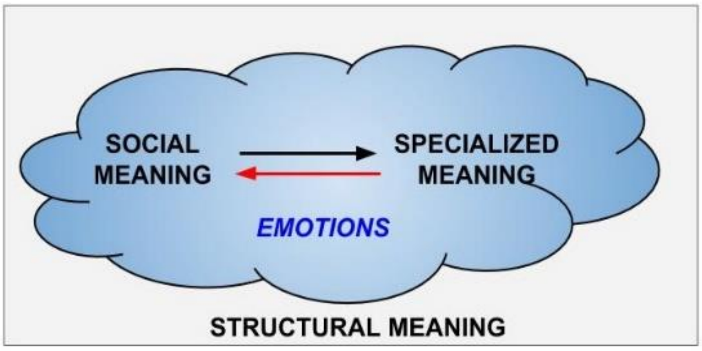


Figure 2. 4 The meanings interplay and the role of emotions

Pair 1 follows the left-pointing red arrow and pair 2 the right-pointing black arrow, but in both cases students' facial expressions, choice of words and physical reactions showed that emotions - aroused by a dynamic coaction of mathematical content and cultural constructs (Radford, 2015, p.29) - enabled the connection between meanings and acted as origin and ultimate goal of the interaction with the meme (RQ2).

A similar double path illustrates the class group dealing with memes in Figure 2.5, created by the researcher and presented in a worksheet the following lesson (Appendix J) – asking students to describe their specialised meanings - as a start off for the discussion.

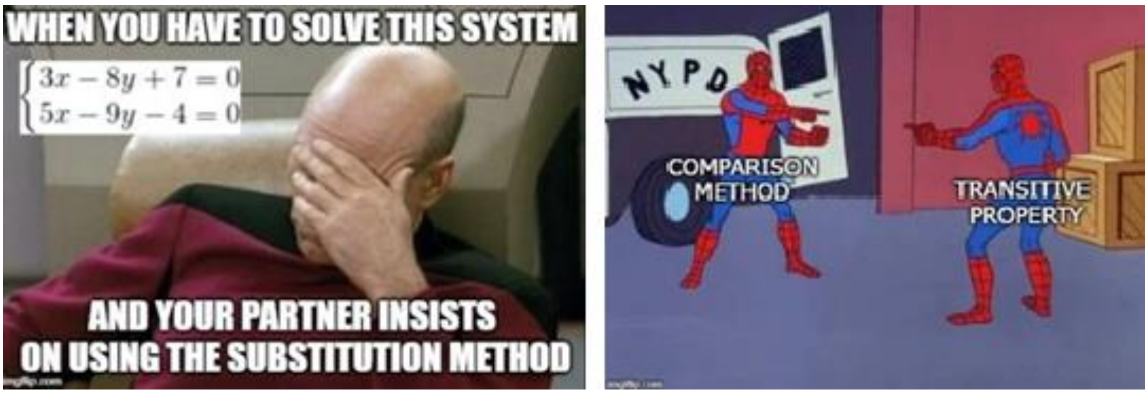


Figure 2. 5 The reflective worksheet memes

The meme in Figure 2.5 left was greatly appreciated (“this is beautiful”): its specialised meaning was immediately understood and connected to the social meaning through an emotional interlacing (“it looks like Viola – the smartest student in the class group – when we solve problems together”). Here the specialised meaning, i.e., the ability to recognize that the given problem is best solved using the elimination method, is processed first and then connected to the social meaning resulting in a successful didactical meme that prompted a deep collective discussion: we are moving along the left-pointing red arrow in Figure 2.4 scheme.

On the other hand, the meme in Figure 2.5 right puzzled students: in the worksheet a significant share (64%) correctly described the mathematical meaning ("for the comparison method we need to apply the transitive property", "the comparison method is justified by the transitive property" [in the Italian curriculum, the comparison method refers to a 2x2 linear system solving technique where the same unknown is obtained from both equations and then the right-end sides expressions are joined to get a single variable equation]). Discussing the meme later, they showed mixed feelings (“I did not understand it so well because of this transitive property”). The following excerpt clarifies the unfolding of the interplay between the different meanings:

Student: No, I did not remember very well what the transitive property was

Teacher: But looking at the meme image, what would you say?

Student: That they are two similar things, two equal things [the social meaning of the image is to describe situations in which two very similar elements meet]

Upon further inspection, the majority of the students admitted they did not remember what the transitive property was and a recap of the property was given by the teacher.

Teacher [after the recap, addressing the whole class group]: The things you wrote [in the worksheet], those of you who wrote them correctly, did you write them because you remembered the transitive property or just to make sense of this meme and say that they are almost the same thing because the image tells us this?

Students yelling in chorus: Because of the meme!

Teacher: I do not know whether to be happy or not, though ...

Researcher: Now that we have used this meme to recall that they are the same thing, do you think you will remember when you use the comparison method, do you think you will associate it with the transitive property more consciously?

Students in chorus: Yes, definitely.

In this case, knowledge is built moving along the black arrow of Figure 2.4, from the social to the specialised meaning and emotions are deeply tangled with the whole path. Finally, in the feedback questionnaire (Appendix L), 81% of the students answered positively to the question whether they had learnt or understood something better ("yes, also checking at the other memes created"), 86% scored more than 7 in a 1-10 rating scale question about "having created the meme will help you remember this topic better?". In the following days, the teacher reported that "we are working on the transitive property of equality: I was amazed by my pupils' attention to names... when I explain I do not give much importance to names. But I think it's for the idea of *making a good impression*".

7 Discussion: A New Cognitive Opportunity

To sum up, we started with an a priori analysis that led to the three meanings structure of a meme and was confirmed by students' entry worksheets (RQ1). When we observed the processes of memes' creation and interaction with them, we saw that the three partial meanings were not handled in a fixed order, but that the development is more complex and dynamic, with different access points. Students' physical reactions and utterances showed that emotions drive the relation with memes (both as creators and users) and allow shifting from the specialised to the social meaning and vice versa (RQ2). This emotional involvement turns out as the meme's most powerful affordance: from the learning point of view, it guides and motivates students to understand the meme's specialised meaning. From the teaching point of view, it can be exploited by teachers who can use memes to reply to students' memes, resulting in a *memetic* adaptation of the *semiotic game* (Arzarello et al., 2009), focused on the memes' mathematical specialised meanings. This use of memes can foster language awareness and further mathematical reflections, as shown by the teacher's testimony.

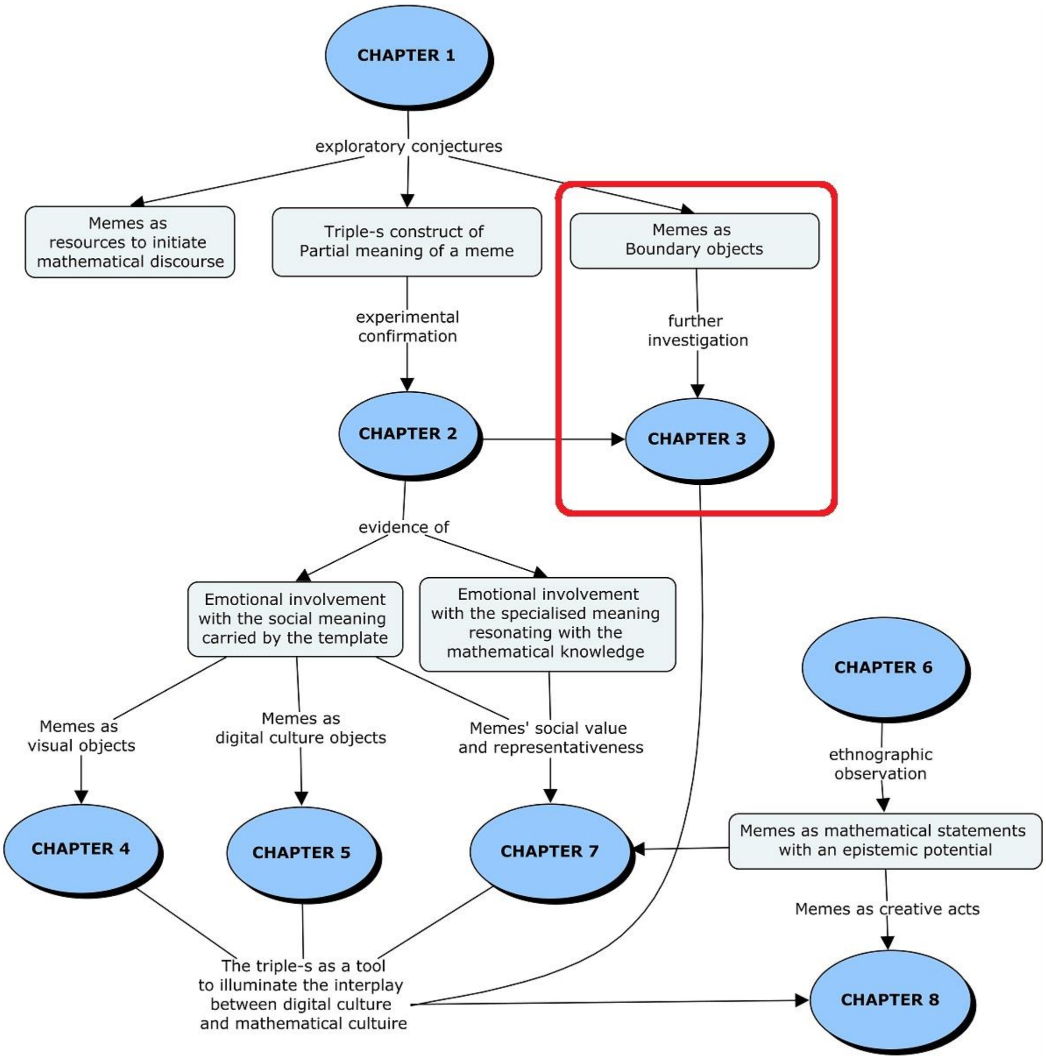
Although these results seem encouraging, our work is far from complete, more investigation is needed to dig deeper into the affordances of memes: for example, a mid and long-term assessment to evaluate the connection between the emotional situations aroused by memes and students' retention of the associated knowledge. Anyway, we think that this almost uncharted territory is worth exploring, because, even if digital culture can be labelled by someone as a non-culture, facing the evidence that our students are emotionally embedded in it, we think it would be educationally valuable to embrace it and turn it into a cognitive opportunity.

CHAPTER 3: LEARNING MECHANISMS AT THE BOUNDARY

IS THIS THE REAL LIFE? CONNECTING MATHEMATICS ACROSS CULTURES

This is the Accepted Manuscript of an article published in Proceedings of the CIEAEM 71 conference, “Quaderni di Ricerca in Didattica (Mathematics)”, Numero speciale n. 7, 2020. G.R.I.M. (Dipartimento di Matematica e Informatica, University of Palermo, Italy) pp. 455-461. in July 2020 available at http://math.unipa.it/~grim/quaderno_2020_numspec_7.htm

Positioning the Chapter in the research



The third chapter investigates and confirms the conjecture formulated in Chapter 1 about mathematical memes as boundary objects between the communities of students and teachers. The study focuses on a meme I created that was administered to students in the decoding activity. The following discussion shows that transmission of the specialised meaning is hindered by the fact that the template has been chosen for its visual appearance and not following its established social meaning. This is seen as evidence that memes can be considered as boundary objects with the social meaning as the robust component that crosses the boundary maintaining its social value as recognised by the digital culture and the specialised meaning as the element that needs to be adapted to enable the boundary crossing. The study also sheds light on the learning mechanisms activated at the boundary crossing, evidencing the importance of taking into account students' cultural background to establish effective communication.

Abstract. Internet memes are hilarious virtual objects widely created and shared by young people in social media, with the purpose of gaining social endorsement by showing wittiness. Mathematical Internet memes are a mathematically themed variation of memes that stemmed spontaneously on the Internet. In this study we test these as means to engage students, connecting school mathematics to young people everyday culture. We present here a teaching experiment carried out in a 10th-grade class group, who created mathematical memes on a given subject and reacted to similarly themed memes produced by the authors. We describe this exchange as an example of boundary crossing, involving two communities – students and teachers - that fruitfully traded knowledge across the increasingly permeable boundary between young people popular culture and institutional scholastic culture.

Résumé. Les memes d’Internet sont des objets virtuels hilarants largement créés et partagés par les jeunes dans les médias sociaux, dans le but d’obtenir un soutien social en montrant de l’esprit. Les memes mathématiques d’Internet sont une variante mathématique de memes issus spontanément dans l'Internet. Dans cette étude, nous les testons comme moyen d'impliquer les étudiants, en reliant les mathématiques scolaires à la culture quotidienne des jeunes. Nous présentons ici une expérience pédagogique réalisée dans un groupe d'élèves de 10e année, qui a créé des memes mathématiques sur un sujet donné et réagi à des memes sur un sujet pareil produits par les auteurs. Nous décrivons cet échange comme un exemple de boundary crossing, impliquant deux communautés - des étudiants et des enseignants - qui ont échangé des connaissances de manière fructueuse à travers la frontière de plus en plus perméable entre la culture populaire des jeunes et la culture scolaire institutionnelle.

Keywords: Internet memes; popular culture; virtual artefact; boundary object; boundary crossing

1 Introduction

Successful connections are established when the exchange takes place in both directions. Thus, to build effective links between school and real life, moving mathematics out of school and plunging it into reality (as in real-world problems), is only half of the answer. To search for the bond in the opposite direction, we think that a first step is taken by looking into what young people acknowledge as their *real* world. We may end up surprised realising that a significant part of teens “real life” is, in fact, virtual.

According to two recent surveys on American teens’ familiarity and experiences with technology and social media, conducted in the Spring and Autumn of 2018 by the Pew Research Center, “fully 95% of teens have access to a smartphone, and 45% say they are online 'almost constantly” (Figure 3.1 left) and “majorities of teens say social media helps peers talk to a diverse group of people, support causes” (Figure 3.1 right). Interviewed teens (ages 13-17) declared that social media had a positive impact on their lives “because a lot of things created or made can spread joy.” (boy, age 17), “[social media] allow us to communicate freely and see what everyone else is doing. [It] gives us a voice that can reach many people.” (boy, age 15) and “it has given many kids my age an outlet to express their opinions and emotions and connect with people who feel the same way.” (girl, age 15). (Pew Research Centre, Spring 2018 Survey).

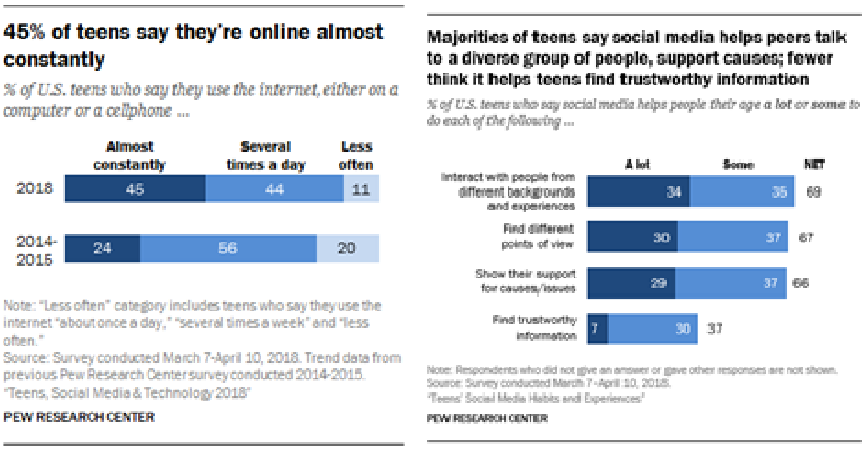


Figure 3. 1 Pew Research Center findings about teens social media use

Our aim is to show that the sought-after *everyday life to school life* connection can be initiated by taking something representative of young people’s digital habitat and plunging it into the school environment. With their 100 million Web occurrences (source: Instagram, Sep 2019), Internet memes could be the right digital artefact to establish this connection.

Memes are humorous virtual objects, widely created and shared by young people in social media, with the purpose of showing wittiness to gain social endorsement. According to Shifman (2014, p.15), memes “reflect deep social and cultural structures” and “epitomize the very essence of the so-called Web 2.0 era”. What makes us think they might be our missing link is that, besides being funny and relatable, they have already been spontaneously used to share mathematics knowledge. In fact, social media abounds with mathematically themed groups acting as communities of practice, where knowledge is shared in a process of collective learning (Wenger, 1998).

In Figure 3.2 left we see an example of a mathematical meme shared within the Italian Facebook Group "L'Agorà del Superuovo[52]", paired with the explanation of its mathematical content made by the author himself (Figure 3.2 centre) and a mathematically improved version suggested by another user (Figure 3.2 right).

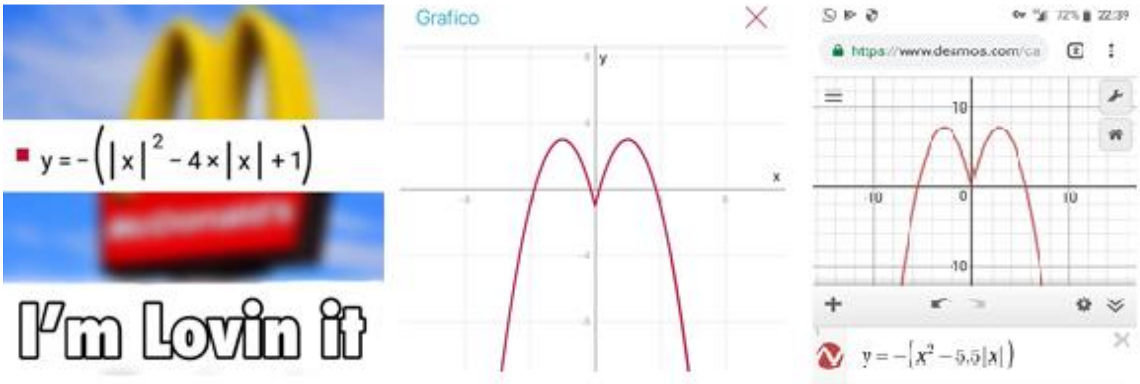


Figure 3. 2 Mathematics knowledge sharing on social media

2 Meanings of Internet Memes

Like jazz variations of classic standards, memes change and evolve in the hands of network users, who personalize and reinterpret them to create humorous snippets, while preserving their recognisability.

They are densely layered objects: to unpack their message, in a previous study we have identified a **triple-s construct** of the partial meanings necessary to grasp the full meaning of a meme (Bini & Robutti, 2019a).

Figure 3. 3 Partial meanings of Internet memes

[52] https://www.facebook.com/groups/1049296555091373/?ref=group_header

- The first partial meaning is **structural**, and lies in its being a meme, namely, to have a specific and shared structure and graphics (font, colour, text position, Figure 3.3 left).
- The second partial meaning is **social**, and lies in the shared conventions of viral images, compositional setups and syntaxes. (Figure 3.3 centre, in How to use memes from the website 9gag.com]).
- The third partial meaning is **specialised**, and lies in images, symbols or text referring to a specific topic, in our case mathematical, to be framed within a "sphere of practice", adhering to a common set of rules, where "mathematical meanings are constructed" (Kilpatrick et al., 2005, p.10). [Figure 3.3 right, from the website www.quickmeme.com].

Although in an informal, pop-culture based way, creating a meme on a specific topic implies finding the right match between humour and subject knowledge. For this reason, we believe that memes have some interesting educational potential, that remains almost unexplored to the present day. In fact, besides our previous works (Bini & Robutti, 2019a), only a limited number of studies about memes can be found in general education research (Knobel & Lankshear, 2007, Romero & Bobkina, 2017, Wells, 2018) and in mathematics education research (Benoit, 2018).

3 Theoretical Framework

Making connections it's all about crossing boundaries, therefore we framed our study within the theoretical frameworks of boundary objects (Star & Griesemer, 1989, Star, 2010) and boundary crossing (Akkerman and Bakker, 2011). We interpret mathematical memes as boundary objects at the same time "both plastic enough to adapt to local needs [...], yet robust enough to maintain a common identity across sites" (Star & Griesemer, 1989, p. 393). They are "artefacts doing the crossing by fulfilling a bridging function" between social media and school mathematics, whose "intersections of cultural practices open up third spaces that allow negotiation of meaning and hybridity" that "carry potential for learning" (Akkerman & Bakker, 2011, pp.133-135). Akkerman and Bakker identified four learning mechanisms activated by interacting with boundary objects: identification, coordination, reflection, and transformation.

Following a recent study applying the boundary crossing framework in mathematics education, we focused on the learning mechanism of transformation, the one that "more than the other learning mechanisms, [...] leads to profound changes in practices" (Robutti et al, expected publication date November 2019, p.3). In particular, we looked for some of the steps Akkerman and Bakker unfolded transformation into confrontation (taking place when different communities, encountering at the boundary, compare their practices on the boundary object), hybridization, (when a new hybrid object emerges from the involved actors' collaboration), and crystallization, (when the new object stabilizes as part of the counterparts' acknowledged practices).

The research questions we address in this work can be summarized as follows:

- RQ1 Which characterizations identify a boundary object in this context?
- RQ2 Which learning mechanisms emerge from our observations?
- RQ3 How can school mathematics take into account the culture developed by young people in their everyday lives?

4 Methodology

The example we present was collected during a teaching experiment conducted with a 10th-grade class group, the first author was present and collaborating with the teacher throughout the activity. After completing the topic of linear systems, students in pairs created memes and recorded connected explanatory videos on that theme in a 3h school-based activity aimed at reviewing and systematizing knowledge on the subject. In the following 2h class, they discussed their memes and reacted to similarly themed ones produced by the first author, in a memetic variation of Arzarello's semiotic game (Arzarello et al., 2009).

Collected data include memes and videos made by the students (Appendix E), videotapes of the creative processes of two selected couples, memes created by the authors (Appendix J), videotaping of the following discussion, and entry and feedback questionnaires (Appendix A, Appendix C, Appendix L). The chosen example is taken from the second part of the experiment (the class discussion), where we think that the interaction between the students' and teachers' communities is more evident.

5 Data and discussion

In the passage analysed in Table 3.1, the whole group is involved in a collective discussion, orchestrated by the teacher (indicated with T in the excerpt). The discussion focused on the meme in Figure 3.4 (top), created by the first author (indicated with R in the excerpt).

Using the previously described triple-s construct, we identify the meme partial meanings as follows:

- Structural meaning: *image macro* consisting of an image with superimposed top and bottom text in the typical white Impact font

- Social meaning: the *Kermit drinking tea* image is conventionally used to make fun at awkward situations [source: Know Your Meme[53]]
- Specialized meaning: modelling phone price plans as linear equations, using linear systems as tools for comparing plans

The topic for this meme was chosen among those that were not touched by students in their meme creation activity but was considered worthy of recalling by both researcher and teacher.

Figure 3. 4 The meme discussed in Table 3.1 (top) and a close-up of the inserted cartesian diagram (bottom)

In the following table we illustrate the connection between excerpts from the group

[53] https://knowyourmeme.com

discussion, partial meanings of the meme and Akkerman and Bakker boundary learning mechanisms. All quotations refer to Akkerman and Bakker cited 2011 study on Boundary crossing and boundary objects.

Table 3. 1 Class group discussion.

Utterings	Remarks
S1 It's one of those that did not make me laugh T Why didn't it make you laugh? S1 I do not understand the connection between the image and the meme T What does the image tell us? The image is the one you have chosen too S1 Oh yes but my meme was more beautiful [laughter] T What does the image tell us? S1 It conveys serenity, but I do not understand why: what does it have to do with the telephone operator? T Maybe it is not the correct image S2 Maybe it's not really the correct image related to what it wants to express R Maybe	The structural meaning is naturally acknowledged, the discussion starts on the social meaning, the *robust* element that is crossing the boundary. *Confrontation* as "a disruption in the current flow of work" (p. 147).
S3 Yes, it is the correct image […] because phone rates are equations, if you put them on GeoGebra, you can find them immediately when one is cheaper than the other and it's easy and you do not have to lose time thinking S4 And so it's easy T And when does one become cheaper than the other? S3 Before the intersection the blue is cheaper and after the other is cheaper S5 We also did some exercises R Do you remember having solved similar problems? [all students nod] […] T Perhaps you do not see my satisfied expression… how did we solve this? S5 We draw the lines and then we found that when one line was below the other is cheaper than when it was above	The focus shifts to the specialized meaning, the *adapted* element: students look for it in connection with the social meaning *Hybridization*: "ingredients from different contexts are combined into something new" (p. 148)

We interpret this meme as an example of a boundary object between the communities of students and teachers, initiating the dialogic learning process analysed in Table 3.1.

We think that is also worth pointing out that in the opening confrontation episode about the social meaning, only two students are in dialogue with the teacher and the researcher. Once the hybridization takes over and the social and specialized meanings act synergically, students who had remained silent up to that moment intervene with appropriate mathematical arguments, and finally the whole class group acknowledges the mathematical fact that was the primary goal.

Looking at the whole process, we see how, through Akkerman and Bakker's dialogic steps, the different partial meanings complement each other, generating the full meaning that finalizes the boundary crossing and enthuses learning. Sometime after this teaching experiment, we received the following message from the teacher: 'Today we solved a physics problem about uniformly accelerated motion and we ended up with a system... when I asked if we were on the right path, the answer was: sure "two equations, two unknowns", mimicking the meme...'. This suggests that a sort of crystallization as "means of developing new routines or procedures that embody what has been created or learned" (p. 148) has taken place within the two communities.

6 Conclusion

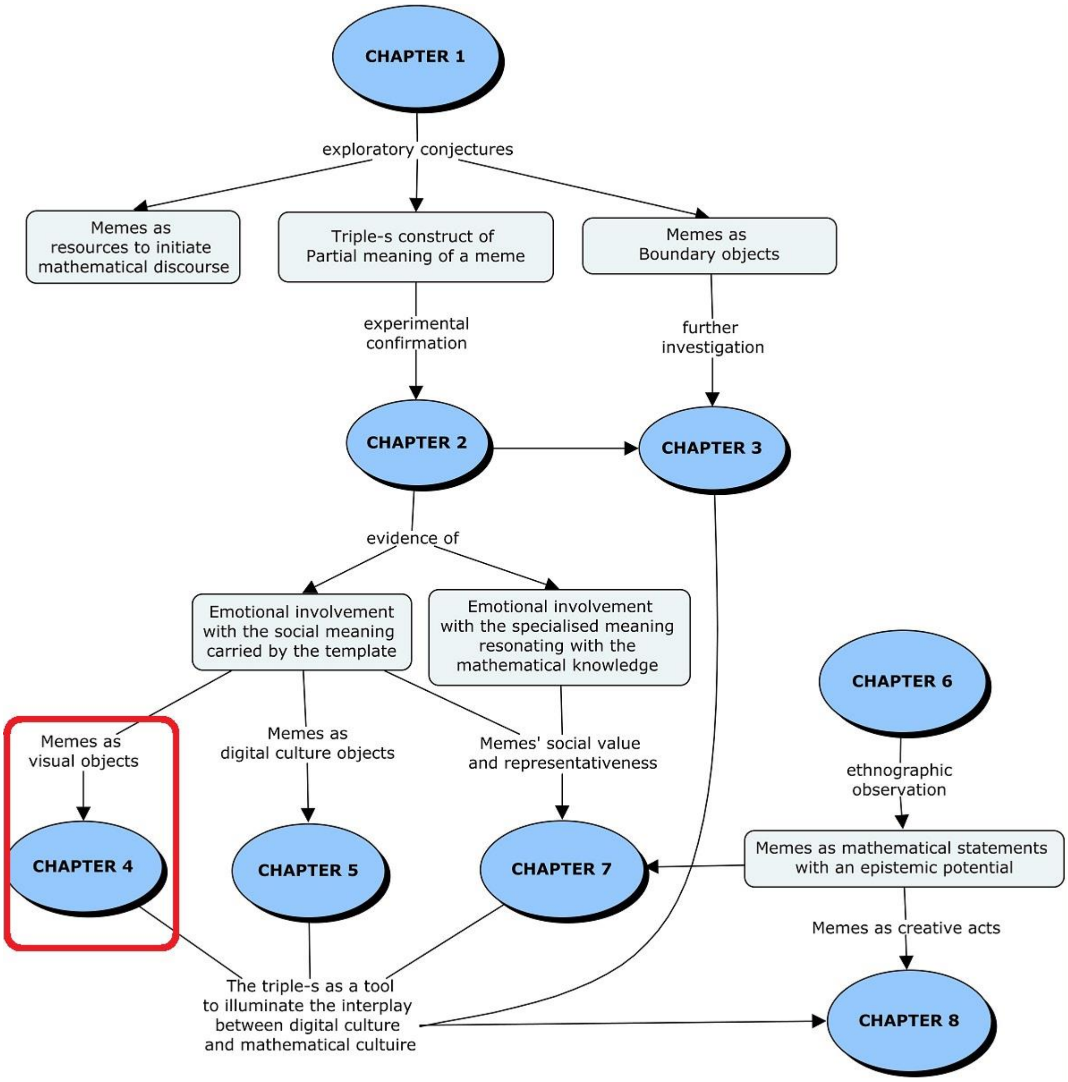
We think that this example provides important insights into the potentialities of memes as new learning objects, aimed at enriching the teaching and learning of mathematics taking cultural aspects into account. Incorporating memes into didactic practice requires "a shift in our thinking about education" to embrace a "new culture of learning [that] allows us to recognize, harness, and institutionalize" these unconventional resources (Thomas & Seely Brown, 2011, p. 7). We believe this is an important issue for future research, and that memes might be powerful means to create hybrid learning spaces at the boundary between school mathematics and young people's everyday lives.

CHAPTER 4: THE ROLE OF VISUAL SKILLS

SOME LIKE IT SOCIAL: LOOKING INTO THE INTERPLAY BETWEEN MATH AND INTERNET MEMES

This is the authors' Manuscript of an article accepted for presentation as long paper at the 14th International Congress on Mathematical Education, Shanghai, 12th –19th July, 2020 postponed to July 2021

Positioning the Chapter in the research



Chapter 4 grounds on the previous findings about the triple-s construct of partial meanings (Chapters 1 & 2) and about students' emotional involvement with the social meaning of the meme (Chapter 2). The social meaning is carried by the template of the meme, which has two very intertwined characteristics: it provides memes' visual component, and it resonates with students' digital culture. The aim of this chapter is to investigate more thoroughly the first characteristic, looking into the educational potentialities of mathematical memes as visual objects. The study reports the use of the triple-s construct as a tool to keep track of students' cognitive processes and as a task design-tool, evidencing that students' visual expertise can become an asset in creating school activities that foster the understanding of mathematics through non-standard skills.

Abstract: With emoji, GIFs and Internet memes crowding chats and social media newsfeeds every day, XXI century learners are highly capable visual communicators. Mathematical Internet memes are one example of this collective ability to distil theoretical knowledge into images: they are viral visual artefacts, created by users who remix and caption popular pictures giving them an encoded mathematical meaning, and share them through social media challenging others to unlock the puzzle. Looking at mathematical memes through the theoretical lens that acknowledges the role of imagery and visualization in mathematics education, we argue that this communication model can be channelled by researchers and teachers to foster and sustain the understanding of mathematical meanings. To support our claim, we present a theoretical tool devised to unfold and keep track of the interplay between memes and mathematical ideas, and we apply this tool to evidence from a teacher training course and three school experiments conducted with 10th and 12th-grade students.

1 Introduction

Norma Presmeg, in her PME & Yandex talk in Moscow in March 2019, reminded us that, if we want to take individual differences into account when we investigate mathematical abilities, visual and logical thinking must not be considered as contrasting skills, but that they have to be looked at as two separate dimensions, to be positioned on orthogonal axes (Nardi, 2014, Presmeg, 2019). Stemming from this assumption and acknowledging that XXI century learners are "bombarded by visual cues, [and that] they seem to translate images and information effortlessly" (Riddle, 2009, p.1), in this paper we claim that mathematical memes can be effective means to connect visual and mathematical meanings, leveraging on students' shared social culture. Mathematical memes are viral visual artefacts, created by users who remix and caption popular pictures giving them an encoded mathematical meaning, and sharing them through social media challenging others to unlock the puzzle. They can be considered iconic examples of Web 2.0 users' ability to distil theoretical knowledge into images.

With 121 million occurrences of the hashtag #memes on Instagram in March 2020, these artefacts are a viral phenomenon acknowledged as a significant part of the digital culture that shapes young people media literacy (Shifman, 2014; Wiggins & Bowers, 2015; Cannizzaro, 2016; Danesi, 2019). Their relevance in the online discourse is so widely established that, recently, research efforts have been aimed at developing a new technology to make memes accessible for people with visual impairments (Gleason et al., 2019). Despite this massive diffusion and accredited potentialities, memes remain understudied in educational research (Knobel &

Lankshear, 2005, 2007, 2019; Romero & Bobkina, 2017) and even more in research focusing on mathematics education, where at the present date, only a few exploratory studies have been conducted (Bini & Robutti, 2019a; Bini & Robutti, 2019b; Benoit, 2019). None of these works looks at mathematical memes from the point of view of the role of imagery and visualization in mathematics education, a perspective which is interesting for two different reasons: on one hand XXI century learners are highly capable visual communicators, dealing with emoji, GIFs and Internet memes in chats and social media newsfeeds every day, and on the other hand Presmeg (2019) reminded us also that "imagery, especially if it's vivid, has mnemonic advantages: you remember well what you have a good image of", but that we have to be aware of the tendency of visual people to memorize without understanding. We think that memes can fill this gap, because they are popular and engaging, but to be unlocked they require processing not only of the visual part but also of the verbal or symbolic part. In this work we investigate this issue, trying to give answers to the following questions:

- RQ1 Can young learners' social visual expertise be channelled by researchers and teachers to foster and support the understanding of mathematical meanings?
- RQ2 "How may the affect generated by personal imagery be harnessed by teachers to increase the enjoyment of learning and doing mathematics?" (Presmeg, 2006, p.234, Big Research Question no.11).

2 Theoretical Background

To unfold and keep track of the interplay between memes and mathematical ideas, we use the ***triple-s meme construct*** (Bini & Robutti, 2019a), a theoretical tool to guide the understanding of the memes and of teachers' and learners' interaction with them. According to this construct, the comprehension of the *full meaning* of a meme is achieved through the understanding and intersecting of three *partial meanings*: the *first partial meaning* is **structural**, and lies in having a consistent aesthetic (text font, colour and position, and overall visual impact); the *second partial meaning* is **social**, and is conveyed by shared rules about the message carried by images and templates; the *third partial meaning* is **specialised** and is carried by elements referring to a specific topic, in our case mathematical.

The examples we shall address in this work are designed as *objects labelling memes* (Figure 4.1), in which subjects in the image are labelled to convey a metaphorical meaning, in Lakoff & Johnson's sense of "understanding and experiencing one kind of thing in terms of another" (1980, p. 5). This metaphorical meaning is strictly codified at a social level (see the website Know Your Meme[54], the online meme encyclopaedia). The *Distracted Boyfriend* meme (Figure 4.1 left) represents situations where an assumption (the boyfriend, text1), instead of being followed by the correct conclusion (the girl on the right, text 2), is diverted to a misleading wrong one (the girl on the left, text 3). The *Spiderman pointing* meme (Figure 4.1 centre) describes situations in which two similar things (text 1 and 2) meet. The *Drowning kid in the pool* meme (Figure 4.1 right.1) portrays circumstances where something (text 3) is typically forgotten. Memes in Figure 4.2 achieve the third specialised level with the superposition of mathematical elements.

Figure 4. 1 Structural and social meanings

Figure 4. 2 Specialised meanings [source: Facebook]

We think that the use of images in mathematical memes described above can be viewed as an example of Presmeg's pattern imagery, defined as "pure relationships stripped of concrete details" (2006, p. 211). Therefore, memes might have significant cognitive affordances, relying on the fact that "pattern imagery, and use of metaphor via an image, are two significant ways by means of which a static image may become the bearer of generalized mathematical information for a visualizer" (2006, p.212).

3 Methodology

To substantiate our claim that memes can turn popular culture into an asset to convey mathematical ideas, we chose four different examples: the first two examples focus on students,

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[54] https://knowyourmeme.com

showing how strong social meanings can easy the delivery of the specialised meanings, the third and the fourth example focus on teachers, one proving how difficulties in grasping the social meaning hinders the transfer of the specialised meaning, and the other showing how familiarity with the triple-s differentiation of meanings and knowledge of the social meanings can support in designing and assessing meme-based tasks. The examples were collected during four meme-creation experiments: two school experiments conducted with 10^{th} and 12^{th}-grade students (first author and teacher present), a teacher training course (authors and young assistant present), and one school experiments conducted with 10^{th} students (teacher present). The teacher training course was set within the project Scuole Secondarie Potenziate in Matematica (SSPM)[55], a public engagement project of the University of Turin, designed to enhance the mathematical culture of students and the didactical approaches in mathematics of teachers, recently selected as a case study for the national assessment of the quality of research performed by ANVUR (Agenzia Nazionale di Valutazione del Sistema Universitario e della Ricerca).

During the first two school experiments, designed by the first author, in the class after the meme creation activity, students received handouts with memes on a given topic produced by the first author (Appendix J and Appendix K), whose mathematical meanings they were asked to elicit; answers were debated in the following discussion. In the third experiment, teachers where given worksheets with selected blank templates to be filled in with specialised meanings (Appendix Q), while the fourth experiment, designed and conducted by the teacher (no researcher present), included the meme creation and discussion activities only. Collected data include created memes, complemented, when possible, by the videotaping of the creation processes for selected couples and of the following group discussion, questionnaires and individual interviews. The first and second excerpts are taken from the discussion of students' interpretation of the researcher's memes, while the third is an excerpt from the teachers' meme creation activity, supported by the young assistant. Memes are originally in Italian and have been translated into English by the first author. Data analysis, conducted through the triple-s construct, will elicit the interplay between memetic images (social meanings) and mathematical ideas (specialized meanings).

4 Data and Analysis

All memes in the experiments were created either using a meme generator website or printed blank templates, thus compositional rules that pertain to the structural meaning were

[55] https://www.liceomatematico.it/torino/

automatically imposed. Our analysis will therefore focus on the interplay of the social and specialised meanings. In Tables 4.1 and 4.2 we present the analyses of the excerpts recorded during the two school experiments, and in Table 4.3 the analysis of the excerpts recorded during the teacher training course. The analysis of the fourth example is limited to the memes created during the activity, since no recording was taken.

4.1 Examples 1 and 2: Strong social meaning smooths the delivery of specialised meaning

The meme in the first example refers to what is known in the Italian curriculum as the comparison method, a 2x2 linear system solving technique where the same unknown is obtained from both equations and then the right-end sides expressions are joined to get a single variable equation.

Table 4. 1. Example 1

	Example 1: 10th grade, October 2018 Topic: linear systems Meme created by the first author Excerpt from the class discussion
Data	Analysis
Student: No, I did not remember very well what the **transitive property** was	Student declares a lack of knowledge about the **specialised meaning**
Teacher: But looking at the **meme image**, what would you say?	Teacher asks about the **social meaning**
Student: That **they are two similar things**, two equal things	Student shows knowledge of the **social meaning**
Teacher [gives a recap of the property, then addresses the whole class group]: The things you wrote [in the handouts], those of you who wrote them correctly, **did you write them because you remembered the transitive property or just to make sense of this meme** and say that they are almost the same thing because the image tells us this?	Teacher investigates **which meaning** was accessed first by students
Students: [in chorus] **Because of the meme**!	Students declare they retrieved the specialised meaning from the social meaning

Table 4. 2 Example 2

	Example 2: 12th grade, March 2019 Topic: complex numbers Meme created by the first author Excerpt from the class discussion
Data	Analysis
TeacherWhat does it **mean**?	Teacher asks about the **specialised meaning**
Students [in chorus]: that **they are the same thing**: on a graphic level multiplying by *i* is equivalent to a pi/2 rotation	Student shows knowledge of the **specialised meaning**
Teacher: Ok	Teacher agrees on the **specialised meaning**
Researcher: And did you already know this or **did you gather it from the meme**?	Researcher investigates **which meaning** was accessed first by students
Students answer indistinctly, some voices can be heard saying "**from the meme**" [...]	Some students declare they retrieved the specialised meaning from the social meaning
Researcher: Who understood it thanks to the **meme**? [Five students raise their hands]	Researchers focuses on the **social meaning** as access point
Student: [pointing to the meme and addressing his classmate] **you rely on the meme** ... it means that **they must be equal!**	Students describe which specialised knowledge was derived from the social knowledge

These two examples show how students' visual expertise enables them to easily envisage the mathematical message. In addition to that, the emotional involvement triggered by the meme *puzzle effect* (the social meaning implies only that the two things are connected, but it's up to the viewer to unravel why) kept their interest hooked during the following mathematical explanation, which was conducted in a traditional way by the teacher (example 1) and one of the students (example 2). Quite predictably, it turned out that students from the first example did in fact know what the transitive property was, although not associating it with this name: this confirms that when we refer to the mathematical meanings conveyed at the specialized level, these have to be framed within a "sphere of practice" where "mathematical meanings are constructed" (Kilpatrick et al., 2005, p.10).

4.2 Example 3: Weak social meaning hampers specialised meaning

Table 4. 3 Example 3

	Example 3: teacher training, March 2019 Topic: simplifying algebraic fractions Meme created by the teachers Excerpt from the creation activity
Data	Analysis
First teacher Wait, **I do not understand the meme**... because she (the girl on the right,) is not glaring at the girl (on the left), she is looking at the boy	First teacher declares a lack of understanding of the **social meaning**
Second teacher She looks at what he said and **she corrects him**	Second teacher gives a personal (wrong) interpretation of the **social meaning**
First teacher That's right, so if she corrects him, **that's him who is wrong**	Fist teacher agrees on the misinterpretation of the **social meaning**
Second teacher Yes, we must put the error (i.e. the wrongly reduced fraction 1+b) on him	Second teacher connects the misinterpreted **social meaning to the specialised meaning**
First teacher I would put the error on him too, **but she** [referring to the young assistant] **is saying no**	First teacher observes that the young assistant does not agree with the shared interpretation of the **social meaning**
Young assistant **no, because I saw it on volleyball memes** […] so he is the one tempted by her [pointing to the girl on the left], the classic error, while she [the girl on the right], is the correct answer.	Young assistant explains the established correct **social meaning** of the meme referring to the distracted boyfriend volleyball meme here below

	
Second teacher Ok, then **we must put the error here** [points to the girl on the left]	Second teacher connects the correct social meaning to the specialised meaning

This contrasting example shows how teachers failing to grasp the established social metaphorical meaning of the image (boyfriend=assumption / girl on the right = correct conclusion / girl on the left = wrong conclusion) and giving it a more teacher-familiar mistake/stick-eye interpretation, hinders the visual representation of the mathematical meanings, until the young (and more socially proficient) assistant comes at rescue, using an example from a different subject domain.

4.3 Example 4: The triple-s construct as a task-design tool

Memes in the fourth examples have been produced by 10th-grade students in a meme-creation activity designed by the teacher in December 2019. The activity involved two different meme-creation assignments: in the first tasks the teacher chose the images to be used for the memes, thus freezing the social meaning, and the students filled them in with their specialized meanings, and in the second task the teacher chose the specialised meanings leaving students free to express them through their preferred social conveyor.

Figure 4.3 shows three memes from the first task: here the teacher chose the *Drowning kid in the pool* meme. The choice was made knowingly to prompt students to reflect metacognitively on their recurrent errors and misconceptions.

Figure 4. 3 Assigned social meaning, free specialised meaning

Memes in the next figure are from the second task, designed by the teacher choosing algebra 1 as specialised meaning: it is therefore interesting to analyse students' image (i.e. social meaning) choice.

In these examples, the selected social meaning can give the teacher insight into the student's emotional relationship with the common misconception of mixing up the zero solution and no solution cases. Students' feelings escalate from the *Philosoraptor* in Figure 4.4 left, conveying a sense of uncertainty in dealing with the equation, to the *Pun Dog* in Figure 4.4 centre with its anticlimactic punchline revealing that the author is deliberately teasing the reader with the wrong answer, to the *Me and the boys* in Figure 4.4 right where the knowledge of the right solution is proudly shown off as a bonding sign between mates.

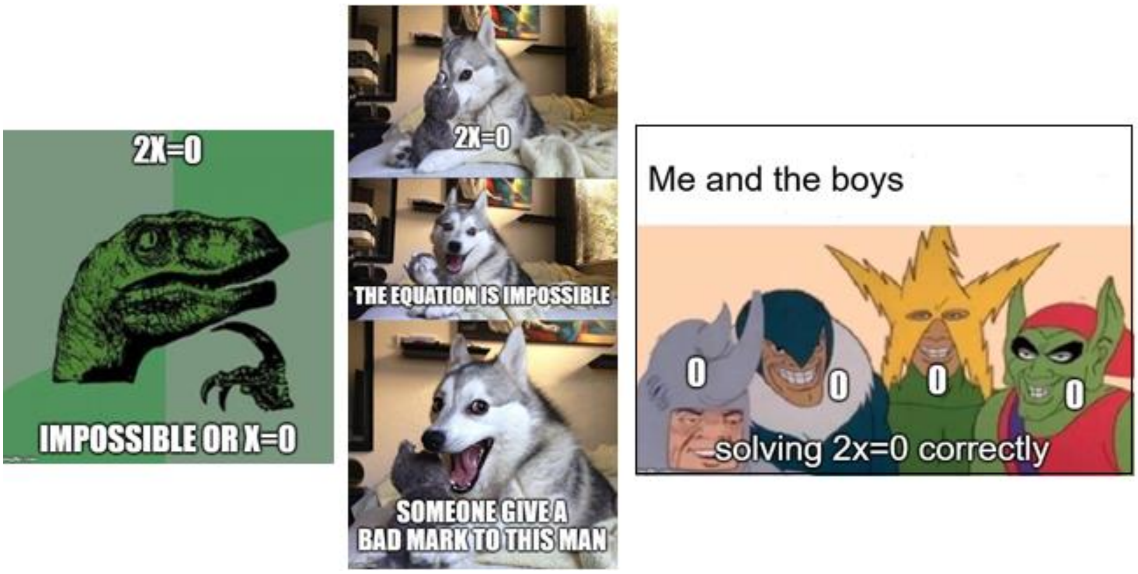


Figure 4. 4 Free social meaning, assigned specialised meaning

This last example shows us how the triple-s construct can be used as a valuable task-design tool, if complemented with the teacher's willingness to widen the traditional concept of literacy to accommodate visual talents and popular culture knowledge.

We conclude emphasising that, in all school experiences, the meme-creation activity has always been followed by a class discussion guided by the teacher, aimed at digging deeper into the specialised meanings touched by the memes and eliciting possible misconceptions. It is, in fact, our experience that mathematical memes are effective in engaging less involved students in these group discussions.

5 Discussion and Conclusion

To sum up, in this work we showed evidence of how the multiplicity of meanings carried by mathematical memes can harness the affect generated by personal imagery to increase the enjoyment of learning and doing mathematics (RQ2), and how the triple-s construct can help researchers and teachers to channel young learners' social visual expertise to foster and support the understanding of mathematical meanings (RQ1). Further research must be done to substantiate our claims, both in the Web, which is mathematical memes' natural habitat, and in more traditional learning environments at different school levels, to look deeper into the cognitive and emotional affordances of these artefacts.

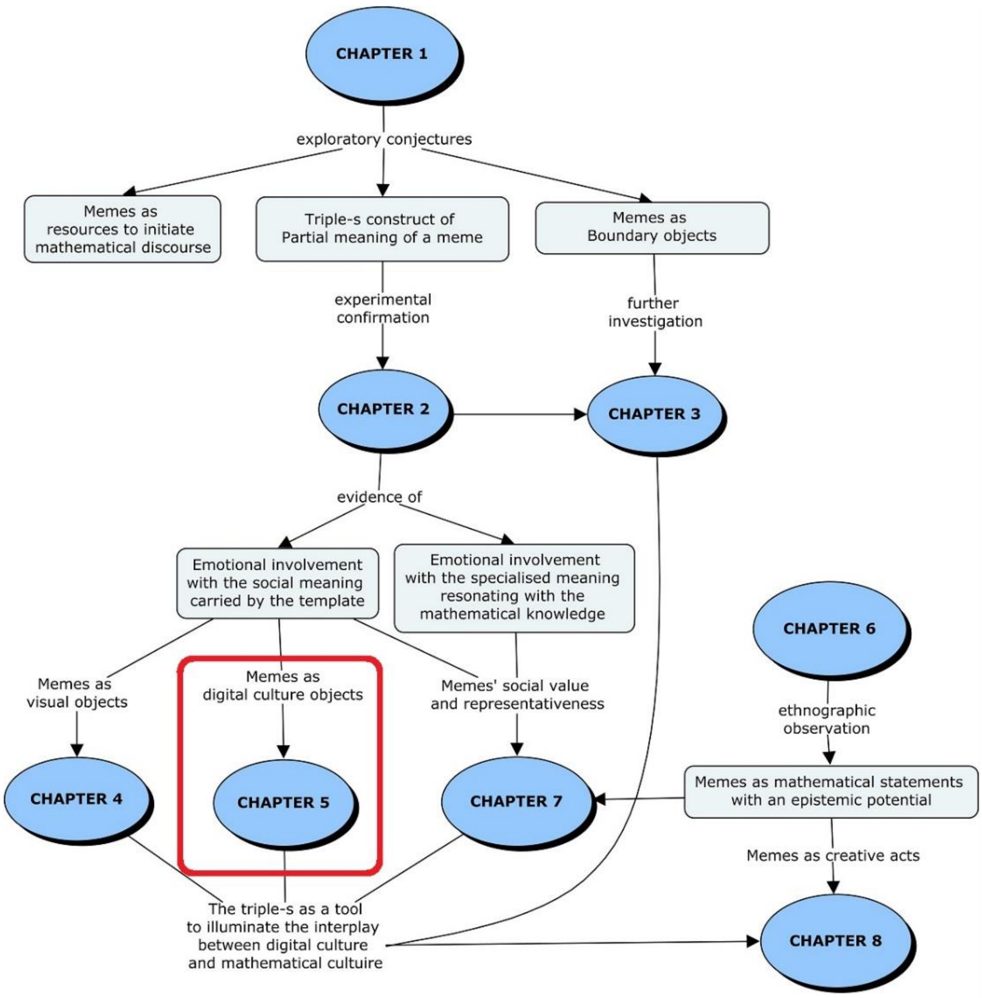
Since cave paintings, the ability to create meanings through images has always been part of humans' way of communicating, but now visual literacy has become a competence in its own, involving the skill to translate abstract thoughts into concrete graphic forms and vice-versa. Memes are visual artefacts that come from a different world: our goal is far from suggesting they are a better method in comparison with other teaching aids or that can substitute textbooks and traditional classroom practices, but we think that, if properly used, they can be fruitfully exploited in education, building on what students know (and teachers can learn) enriching the modes of communications and increasing the pleasure of doing mathematics.

CHAPTER 5: THE ROLE OF THE DIGITAL CULTURE

HOW SPIDERMAN CAN TEACH YOU MATH: THE JOURNEY OF MEMES FROM SOCIAL MEDIA TO MATHEMATICS CLASSROOMS

This is the Accepted Manuscript of an article published in Kalir, J. H., & Filipiak, D. (2021). *Proceedings of the 2020 Connected Learning Summit* (Version 2). Carnegie Mellon University. https://doi.org/10.1184/R1/13530038.v2

Positioning the Chapter in the research



Chapter 5 picks up the loose end left by Chapter 4 and looks into the second characteristic connected to the memes' template which carries the social meaning, investigating the potentialities of mathematical memes' as educational resources built on objects coming from students' digital culture. The study positions its result within the connected learning framework and uses the triple-s construct as an investigation tool to follow learners' cognitive processes activated by the interaction with a meme in formal and informal learning contexts and as a task design tool. The results of the study reinforce the educational potentialities of memes' connection with students' digital culture, but at the same time it shows that these features cannot be clearly distinguished from the educational potentialities connected to the visual component examined in Chapter 4. This result, combined with the role of the social meaning as the robust element crossing the boundary (Chapter 3), suggests that memes can become digital educational resources only if created identifying and respecting the core idea carried by the template.

Abstract: Memes are humorous digital artefacts created by Web users copying an image and overlaying a personal funny caption. They are virally shared on the Web and represent an important part of the online discourse young learner are exposed to on a daily basis. The aim of this paper is to show how memes on mathematical subjects can inspire learning activities that harness the participatory and playful nature of these digital artefacts, connecting positive emotions, focal for learning achievement, to serious mathematical reasoning. The paper presents a collection of three examples taken from different learning scenarios: one from a spontaneous out-of-school learning environment, and two from intentionally designed school experiences conducted with 6th and 12th-grade students, all pivoting on the use of a popular meme based on a Spiderman cartoon. The analysis elicits the core properties of these experiences, in order to show how they fit within the connected learning framework. The outcome of this research can hopefully shed some light on how educators can leverage on students' popular culture embodied in memes to foster interest-powered learning outcomes in mathematics.

1 Introduction

Students' emotions and interest are acknowledged as key internal factors for learning achievement in the mathematics classroom. They can affect cognitive processing in several different ways, from the bias on attention and memory to the activation of reactive behaviors, like that of math anxiety, that can hinder mathematical thinking (McLeod, 1992; Zan et al., 2006; Hannula, 2015; Schukajlow et al., 2017). For these reasons, fostering positive emotions in math class can be a winning practice that extends its effects well beyond the timespan of the actual lecture. According to the connected learning framework (Ito et al., 2013-2018), an effective way to associate positive feelings to learning experiences is building connections between academic goals and other areas of expertise and interest of the learners.

The aim of this paper is to show how memes on mathematical subjects can support in building these connections, providing the basis for creative learning activities that harness the participatory and playful nature of these digital artefacts and bond positive emotions to serious mathematical reasoning. The paper presents a collection of three examples, taken from different learning scenarios: one from a spontaneous out-of-school learning environment, and two from intentionally designed school experiences, all pivoting on the use of a popular meme based on a Spiderman cartoon, chosen for its relevance which will be outlined in the next paragraph. The analysis elicits the core properties of these experiences, in order to show how they fit within the connected learning framework. The paper concludes discussing how educators can leverage on students' popular culture embodied in memes to foster interest-powered learning outcomes in mathematics.

2 Background: from cartoons to meme icon

The image in Figure 5.1, currently known as *Spiderman pointing at Spiderman*, is a snapshot from a 1967 cartoon that in the last decade has been promoted to a new status. It became the template (i.e., base image) of an extremely popular meme, a humorous digital artefact created by Web users overlaying an image with a personal funny caption, widely shared in networking websites like Reddit or Instagram.

Figure 5. 1 The *Spiderman pointing at Spiderman* template [source Reddit]

In the process of meme genesis, whose mechanisms lie far beyond the scope of this paper, the *Spiderman pointing at Spiderman* template acquired the metaphorical meaning of two similar things meeting (see Know You Meme[56], the Internet meme database). In this sense it triggered a wide range of modifications and personalisations, all aimed at capturing the humorous side of the acknowledged metaphor, with the personalized caption either in a white strip above the image (Figure 5.2, left) or superposed on the two Spiderman figures (Figure 5.2, right).

Figure 5. 2 *Spiderman pointing at Spiderman* meme examples [source Know Your Meme]

[56] https://knowyourmeme.com/,

The popularity of this template is so widely recognized that the above mentioned Know Your Meme website recently (2019) enlisted it among 'The Top 50 Memes of the Decade'. Its fame propagates even outside memes' natural habitat (i.e., social media websites), inspiring a handful of recreations, like the real-life Halloween *Spiderman pointing at Spiderman* kids posted on Reddit (Figure 5.3, left) and the digital 3D Spidermen in the advertisement of a new superhero-inspired video game (Figure 5.3, right).

Figure 5. 3 *Spiderman pointing at Spiderman* meme-inspired recreations [sources Reddit and Twitter]

All these evidences attest how deeply this meme has been interiorized by Web users, who easily recognize and decode it even when it is taken out of context. From the point of view of mathematical reasoning, the metaphorical meeting of two similar things described by this meme resonates with the idea of the commonality of meanings across different semiotic representations of mathematical objects, whose recognition and understanding are acknowledged as cornerstones of an effective mathematical activity aiming beyond the mere memorization of facts and procedures (Duval, 2006; Etkind, Kenett & Shafrir, 2015). In fact, this template is widely exploited within online communities exchanging memes on mathematical subjects, where its *puzzle effect* (the template implies that the two things are connected, but it's up to the viewer to unravel why) challenges users providing openings for spontaneous learning.

3 Literature and theoretical framework

With 144 million occurrences of the hashtag #memes on Instagram in July 2020, memes are a viral phenomenon acknowledged as a significant part of the digital culture that shapes young people media literacy (Shifman, 2014; Wiggins & Bowers, 2015; Danesi, 2019). Their relevance in the online discourse is so widely established that researchers at the Carnegie Mellon University recently disclosed the results of a research aimed at developing a new technology to make memes accessible for people with visual impairments (Gleason et al., 2019). Despite this massive diffusion and accredited potentialities, memes remain understudied in educational research (Knobel & Lankshear 2005, 2007, 2018; Romero & Bobkina, 2017) and even less in research focusing on mathematics education, where at the present date, only a few exploratory studies have been conducted (Benoit, 2018; Bini & Robutti, 2019a; Bini & Robutti, 2019b). Since memes on mathematical topics dwell at the intersection of the spheres of learning involving personal interests, peer culture and mathematical academic content, I believe that the connected learning framework (Ito et al., 2013, 2018), could be a suitable lens to observe the learning experiences involving these artefacts. According to the framework, spontaneous connected learning experiences share the core properties of being production-centered, organized around a common goal and openly networked. These characteristics inform the design principles for the intentional creation of connected learning environments, that pivots around participation, experiential learning, interest cultivation and challenge, and interconnection among learners and between learners and teachers. The common thread linking these experiences, whether spontaneous or intentional, is that they are emotionally satisfying, thus connecting the sought-after positive emotions to the learning process.

Zooming in from the learning setting to the artifacts, I will use the *triple-s construct* of the *partial meanings* of a meme (Bini & Robutti, 2019a) to guide the understanding of the memes and of teachers' and learners' interaction with them. According to this construct, the comprehension of the *full meaning* of a meme is achieved through the understanding and intersecting of three *partial meanings:*

- the *first partial meaning* is **structural**, and lies in having a consistent aesthetic: text font, color and position, and overall visual impact (see Figure 5.4 left for the examples analyzed in this work);
- the *second partial meaning* is **social**, and is conveyed by shared rules about the message carried by images and templates (see Figure 5.4 right for the examples analyzed in this work and the Know Your Meme website for other memes);
- the *third partial meaning* is **specialized** and is carried by elements referring to a specific topic, in our case mathematical (see Figures 5.5 5.6 and 5.7 in the Data and analysis paragraph)

Figure 5. 4 Structural meaning (left) and social meaning (right) of the Spiderman meme

This study is therefore designed to address the following questions:

- How do mathematical memes provide opportunities for connected learning experiences?
- How can the connected learning framework and triple-s construct support educators in designing effective learning scenarios involving mathematical memes?

4 Method

This paper draws on my doctoral research on memes on mathematical subjects, developed throughout the last two years under the supervision of Ornella Robutti, exploring online communities and conducting school-based experiments with students creating and discussing memes. Data collected are memes and related comments for the online research, memes and questionnaires for the school-based research, complemented, when possible, with the videotaping of the creation processes and of the following discussion. This study presents three examples, chosen for their representative use of the *Spiderman pointing at Spiderman* template. The first is an example of spontaneous interest-driven learning, taken from r/mathmemes, the largest mathematical memes public community in the Reddit website, commonly acknowledged as the birthplace of memes (83.3k members in July 2020). The second example was collected during the discussion that followed a meme-creation experiment on complex numbers conducted with a class group of 29 12th-grade students (17 y.o). The third example comes from a meme-creation experiment on exponents and powers conducted with two class groups of 19 6th-grade students (11 y.o.) each. In both school examples, the meme-creation activity was proposed after completing the indicated topic, with the didactical aim of fostering the reorganization of cornerstone ideas, possibly eliciting doubts and misconceptions.

5 Data and analysis: from meme icon to learning object

5.1 Example 1: the Internet Spiderman

The meme in this example (see Figure 5.5 left) was uploaded to the popular r/mathmemes Reddit public community with the title “unit circle baby”, and in the two following days it was commented by three different commenters (none of which was the author of the meme). To protect the identity of commenters in the quoted excerpt, publishing dates are not shared and nicknames are replaced with the initials C1 C2 C3; all comments are unredacted and originally in English.

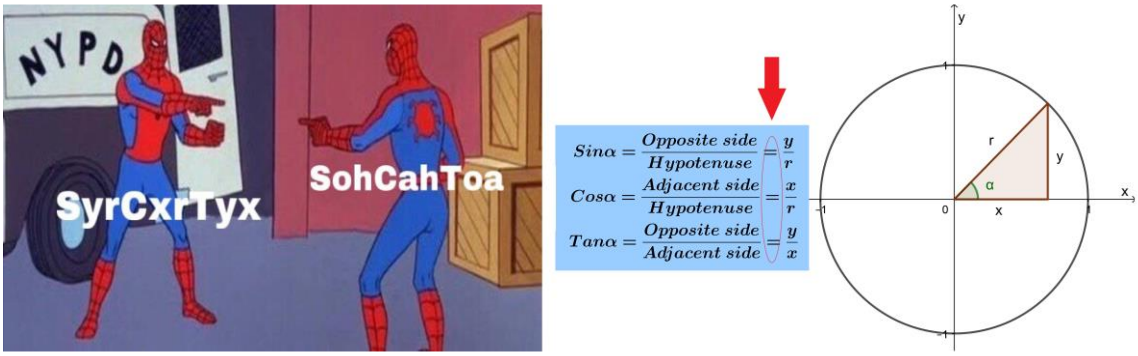


Figure 5. 5 The Internet Spiderman meme (left) and my representation of the explanation (right)

C1: [day 1, 9:40] Explain pls

C2: [day 1, 13:44] SohCahToa is a mnemonic for trigonometry students to remember the right triangle definitions of the trig functions: soh is “sine is opposite over hypotenuse” coh is “cosine is adjacent over hypotenuse” and toa is “tangent is opposite over adjacent” SyrCxrTyx is using the circle definitions: sin is y/r, cos is x/r, tan is y/x

C1: [day 1, 13:46] Oh thx didn't know that

C3: [day 2, 00:29] It defeats the purpose of the mnemonic. If you can remember where the y,x and r are supposed to go you already know the formulas.

C2: [day 2, 01:52] I absolutely agree. I was just explaining the joke to OP [Original Poster in the Reddit jargon, i.e. C1]

This example shows how the sharing of a meme in an openly networked environment, where people communicate across national boundaries, provides a connected learning opportunity for commenter C1, who explicitly seeks for help, going back to the website after a while to check for answers. Research in the Reddit website revealed that C1 is a non-native English speaker, thus not acquainted with the SohCahToa mnemonic device for trigonometric functions, which is language-sensitive and therefore diffused in English-speaking learning environments only. C2, a native English speaker, gives a thorough explanation of the SohCahToa acronym and its connection to SyrCxrTyx (as summarized in Figure 5.5 right, image elaborated by the author of the present study), confirming that mathematical meanings have to be framed within the "sphere of practice" where they are constructed (Kilpatrick et al., 2005, p.10). C2 subsequently agrees with C3 in contesting the advantages of the second representation, whose clarification was nevertheless due in order to testify that he successfully cracked the meme. The challenge represented by the puzzle effect of the meme offered C2 an opening to show off his mathematical knowledge, attesting his position as member of the community, and at the same time resulted in a connected learning opportunity for C1. As a final observation, we note how the *Spiderman pointing at Spiderman* popularity previously described led C2 to assume without hesitation that C1's explanation request was referring to the specialized partial meaning and not to the social one.

5.2 Example 2: the school Spiderman (teacher version)

This example focuses on a meme I created, that was proposed in the class following the students' meme creation activity (Appendix K), in a peer-based exchange of memetic productions between students and educators. Students received printed worksheets with the meme in Figure 5.6 (among others), and were asked to explain the specialized partial meanings, teacher and author were present. All students turned in correct answers for this meme, with the exception of one student who wrote 'I have no idea'. This student, while filling in the handout, had asked me for support in decoding the meme, and some effort was needed to persuade him that he could simply leave it blank or write that he did not know the answer. He was convinced only after being reassured that he would eventually receive the requested explanation during the discussion. This behavior is diametrically opposite to what commonly happens in a math class, where students, unfortunately, rarely care for explanations of unsolved questions: I believe that this can be interpreted as evidence of the connected learning features of interest cultivation and challenge associated to the meme.

Figure 5. 6 The teacher's Spiderman

In the discussion that followed, the equivalence implied by the meme between the multiplication by *i* of a complex number and the rotation of the associated vector in the complex plane was furtherly examined:

Teacher: What does it mean?

Students [in chorus]: that they are the same thing: on a graphic level multiplying by *i* is equivalent to a pi/2 rotation

Teacher: Ok

Author: And did you already know this or did you gather it from the meme?

Students answer indistinctly, some voices can be heard saying "from the meme" [...]

Author: Who understood it thanks to the meme? [Five students raise their hands]

Student [pointing to the meme and addressing his classmate] you rely on the meme ... it means that they must be equal!

This excerpt shows another connected learning feature of memes, in this case how they succeed in linking peer culture and academic content. In fact, in this example and in other experiments involving this same meme, students' proficiency in recognizing the *Spiderman pointing at Spiderman* meme and identifying its social partial meaning, ascribable to the exposure to this type of artifact outlined in the background section, allowed mathematical specialized meanings to be easily vehicolated. As a complement, the mentioned puzzle effect connected to the meme created a challenge that kept students hooked to the following review of the motivations supporting the equivalence, eventually leading to some shared learning.

5.3 Example 3: the school Spiderman (student version)

This last example incorporates two different memes, created by students in a meme-creation activity where the mathematical topic and templates were assigned. Due to their younger age, and consequent lower exposure to social media, students where not all acquainted with the *Spiderman pointing at Spiderman* template and its partial meanings, which were illustrated by the teachers at the beginning of the activity using the triple-s construct. Students' creations were then shared and discussed in the class groups. The experience was followed after several months by a test and a feedback questionnaire.

Figure 5. 7 The students' Spidermen

In this activity the use of memes, instead of traditional exercises, positively challenged learners, allowing each student to participate and contribute according to his/her own personal mathematical expertise. This constructive climate fostered metacognitive processes that enabled assessing the basic knowledge about powers (see Figure 5.7 left), and in some cases elicited misconception (see Figure 5.7 right). The sharing and discussion of the memes orchestrated by the two teachers, provided immediate feedback from peers in a non-judgmental way and supported the learning process initiated by the memes. In the feedback questionnaire, 80% of the students declared themselves pleased with the experience "because we invented the jokes", and described it as "a fast funny, different, easy way to learn math".

After the summer break, the entry test administered to all parallel classes in the school showed that students exposed to the meme activity scored significantly better results in questions about exponents and powers than other students in the school. Surely this cannot be considered as the result of a proper quantitative research with control groups, nevertheless it argues in favor of the educational effectiveness of the engagement and motivation component given by the meme.

6 Results and discussion

This work sampled the learning potential of a particular meme, but the overall investigation revealed that the affordances of the *Spiderman pointing at Spiderman* meme occur in many other templates: in fact, a moderator of one of the most popular mathematical memes groups in Facebook, acknowledged in a public post in the group page that "without them [the mathematical memes] I wouldn't have discovered all the fun I want to learn and know!".

This is consistent with the idea that mathematical memes can provide opportunities for learning experiences characterised by the creative, participatory, challenging and interconnected features that typify the connected learning model. I think that the common thread bringing together all examples is that they provide evidence of learning practices "linking deep 'vertical' expertise with horizontal expertise and connection to other cultural domains and practices" (Ito et al., 2013, p. 56). In this case the deep vertical expertise is the mathematical academic knowledge (the specialized meaning of the meme) and the broad horizontal one is learners' acquaintance with memes and popular culture (the structural and social meanings). Reversing the order of the factors, the connected learning framework can support educators in designing new learning experiences that incorporate mathematical memes. For instance, the interconnection feature, which was restricted to the class group in the reported examples, can be amplified sharing students' memes at school level or ever further within online communities, and the challenge feature can be enhanced allowing students to team and compete in a different way, not only encoding mathematical meanings into their memetic creations, but also decoding the mathematical meanings of memes created by classmates. In agreement with the connected learning stance, in these activities teachers should fall in the background, giving space to peer-based learning experiences where students and students' creations are central. The triple-s construct can therefore be an effective tool for educators to survey and facilitate the process of didactic transposition (in the sense of Chevallard, 1988) that allows the transformation and adaptation of knowledge to the new means of communication.

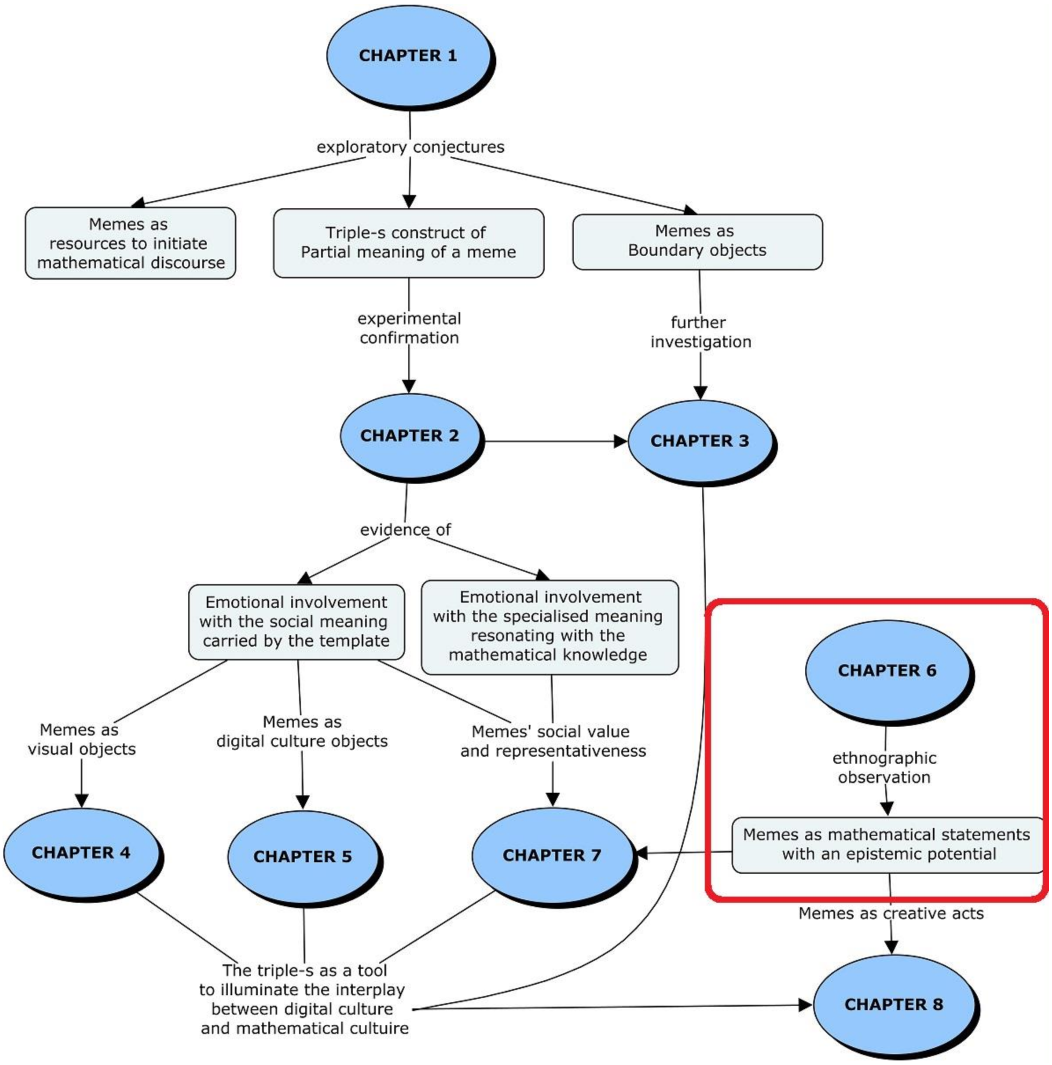
I believe that the results of this work point to the potential of mathematical memes in associating positive feelings to learning experiences and in fostering interest-powered learning outcomes in mathematics and show that the connected learning design principles together with the triple-s construct can support educators in creating activities that successfully link school culture and popular culture.

CHAPTER 6: THE CONCEPTUALIZATION OF THE INTERNET PHENOMENON

MATHS IN THE TIME OF SOCIAL MEDIA: CONCEPTUALIZING THE INTERNET PHENOMENON OF MATHEMATICAL MEMES

This is the authors original manuscript of an article published as the version of record in International Journal of Mathematical Education in Science and Technology© Taylor & Francis 2020 Informa UK Limited, trading as Taylor & Francis Group, https://doi.org/10.1080/0020739X.2020.1807069.

Positioning the Chapter in the research



This Chapter opens the systematic phase of the research, in which the observation moved from explorations conducted in the classrooms to the scenario on the Web, with the aim of conceptualizing the Internet phenomenon. For this new onset, we adopted an ethnographic methodological approach, adapting the research in mathematics education to the new context. In the study we look at the cultural context in which mathematical memes are created and shared, and at how communities act on them through the practice of commenting on social media. The analysis of these threads of comments shows that, within these communities, mathematical memes are perceived as representations of mathematical statements and that they activate epistemic needs aimed at determining their truth value. These results sparked the following systematic question about the research of a possible model of creating a mathematical meme, which is addressed in Chapter 8.

Abstract: Mathematical memes are an Internet phenomenon with an epistemic potential noteworthy for the teaching and learning of mathematics. The aim of this paper is to conceptualize this phenomenon on an empirical base, elucidating its educational potential. To pursue this goal, a two-year ethnographic research has been conducted in a selection of social media communities that exchange mathematical memes. In the paper, a two-fold theoretical lens is used to investigate the phenomenon from a memetic perspective focused on the general characteristics of Internet memes, and from a cultural perspective considering the Web 2.0 environment. Analysis tools derived from the memetic and cultural theoretical background are applied to an instrumental case study, to reconstruct how mathematical memes activate the interaction among members of an online community. The investigation elicits key characteristics of the phenomenon, provides grounds to conceptualize mathematical memes as hybrid representations of mathematical statements, and sheds light on the traits of the epistemic culture they establish in the community.

Keywords: Internet meme, mathematics, social media, learning communities, epistemic need, mathematical statement, Web 2.0 culture

Though this be madness, yet there is method in it.

Hamlet Act 2, scene 2, 203–204

1 Introduction: purpose and object of the study

Memes are humorous digital objects created by Web users copying an existing image and overlaying a personal caption. Since their appearance in the late 90's, memes have been colonizing the Web and their use spontaneously extended from the initial entertaining goals (Miltner, 2014) to encompass different subjects, including serious themes as politics (Huntington, 2017) and educational topics as mathematics (Benoit, 2018).

This research focuses on mathematical memes, an Internet phenomenon attracting communities of thousands of followers, still significantly under researched. The purpose of the study is to reach a conceptualization of the phenomenon that sheds light on its educational potentialities. To clarify the object of the study, we start with an example. The meme known as *Me and the boys* appeared in May 2019 on the popular social media website Reddit, when users started sharing posts with the same figurative element, a screenshot showing the villains from a 60's Spider-Man animated movie. This image, complemented by different captions all with the same *Me and the boys* incipit, was used to metaphorically represent a group of students doing things together, with a view to evoking nostalgic memories of schoolboys' lives (Figure 6.1).

Figure 6. 1 *Me and the boys* mutations with different captions [source Reddit]

These posts were immediately widely liked and shared, and rapidly followed by others partaking the same evocative plot, differing in the textual part and in some alterations of the basic picture (Figure 6.2).

Figure 6. 2 *Me and the boys* mutations with edited images [source Reddit]

At the beginning of June 2019, two other users posted a couple of mathematical mutations of the *Me and the boys* post (Figure 5.3), shifting the narrative from commonly shared schoolboys' memories to mathematical graduates' knowledge.

Figure 6. 3 *Me and the boys* first mathematical mutations [source Reddit and Facebook]

At the end of June 2019, another mathematical mutation (Figure 6.4) appeared. This mutation, although implying the acquaintance with the iconographic part (the original *Me and the boys'* horizontal image), distances itself from the initial storyline, bending it (or we could say *rotating it*) to convey a hidden meaning, hinted by the text and accessible only to mathematically savvy viewers.

Figure 6. 4 ***Me and the boys*** **second mathematical mutation [source Facebook]**

In the example described, a popular image becomes a meme changing in the hands of Web users who reinterpret it, adding a personal humorous touch. The process suddenly turns towards specialized mathematical contents when the memetic narrative device is edited to encode a mathematical idea. This mutation is further followed by users, who consequently become involved in the construction of a new form of representation of mathematical ideas, different from all other known representations, such as diagrams, graphs, number lines, manipulatives or formulas (Goldin, 2014). Reconsidering this process, we can grasp the gist of what memes are: a type of visual entertainment in the form of an indefinitely editable narrative device, composed of an image and a text. In the networked Internet environment, a meme spreads from user to user with changes in the text and possibly in the image, maintaining an overall recognizability. Mathematical memes are a type of memes in which the textual part refers to a mathematical idea, that can be identified combining the information carried by the text with those carried by the image: in other words, a mathematical meme is a kind of riddle that can be decoded with the help of mathematics. The encoding/decoding mechanism mentioned above enlightens the educational potential of these objects for the learning of mathematics.

2 Rationale and research questions

Internet has produced a digital environment in which 21st-century students are immersed and where knowledge is easily accessible. In this culture, memes are recognized as building blocks (Milner, 2013; Marwick, 2013; Miltner, 2014; Shifman, 2014). Until now, they have been studied across different perspectives: digital culture (Stryker, 2011; Davison, 2012; Börzsei, 2013; Shifman, 2014; Danesi, 2019), genre studies (Wiggins & Bowers, 2015), semiotics (Cannizzaro, 2016; Osterroth, 2018), rhetoric and affect (Huntington, 2013, 2017). Their significance in the online culture is so widely established that, recently, research efforts have been aimed at developing a new technology to make memes accessible for people with visual impairment (Gleason et al., 2019). However, memes are greatly understudied in educational research written in English: we have found examples in new literacy (Lankshear & Knobel, 2003; Knobel & Lankshear, 2005, 2007, 2018; Silva, 2016; Harvey & Palese, 2018; Wells, 2018), racial discourse (Yoon, 2016) and visual literacy in language education (Romero & Bobkina, 2017). Portuguese-speaking academia seems more open to integrating memes into educational settings: an updated survey of Portuguese educational literature on memes, referencing 17 studies on the subject, can be found in Santos and Carvalho's work (2019). In the field of mathematics education, apart from Bini and Robutti's (2019a, 2019b, 2019c) conference papers and Ward-Penny's work (2011) that focuses on pre-Internet mathematical memes, we have found only five studies on mathematical memes. One in Spanish (Beltrán-Pellicer, 2016), three in Portuguese (Gonçalves & Gonçalves, 2015; Gonçalves, 2016; Friske, 2018) and one in English (Benoit, 2018). Benoit looks into mathematical memes to study the perception of mathematics in popular culture. Beltrán-Pellicer (2016) and Friske (2018) describe their class-use with students and prospective teachers. In Gonçalves & Gonçalves (2015) and Gonçalves (2016), beyond a categorization according to teaching aspects (such as theme or purpose) of 30 mathematical memes, no further clarification of the phenomenon of mathematical memes themselves is revealed.

All cited studies agree on the fact that memes can activate educational situations where some knowledge, e.g. mathematical, is learned by users. However, they achieve their findings by focusing on memes only, separated from their natural habitat. This approach leaves two significant aspects aside: the study of the connections between mathematical memes and their ecological cultural environment, and the study of the interactions they initiate within online communities. Mathematical memes are *media*: they represent, and therefore communicate, mathematical ideas. According to Jenkins and Gitelman, media are defined through a two-level model "on the first, a medium is a technology that enables communication; on the second, a medium is a set of associated 'protocols' or social and cultural practices that have grown up around that technology" (Gitelman, 2006 as cited in Jenkins, 2006, pp. 13–14). Referring to these two levels, we want to deepen our understanding of the phenomenon of mathematical memes as representations of mathematical ideas and investigate how it is shaped and how it is used within online communities. To be more specific we ask:

- How can the phenomenon of mathematical memes be characterised as a representation of mathematical ideas within the Internet culture?
- How do mathematical memes activate and guide interactions among members of online communities?

These research questions are answered by investigating mathematical memes with a two-fold theoretical lens: a *memetic* background, focused on characterizing mathematical memes as Internet artefacts; and a *cultural* background, dedicated to characterizing mathematical memes as cultural objects of the Web 2.0. Based on the innovative character of our research, we have entered the field with an ethnographical approach, deciding to collect mathematical memes and their threads of comments in social media communities. In the paper, we start our empirical investigation by mapping the field in order to identify appropriate data sources to study the phenomenon in relation to its online communities. Then we identify paradigmatic cases for an instrumental case study to reconstruct in detail how a community deals with the mathematical idea encoded in a meme. We finally discuss the results with respect to the original questions, providing a conceptualization of the phenomenon and reflect on the educational potential of mathematical memes for the teaching and learning of mathematics.

3 Theoretical background

3.1 The memetic perspective: mathematical memes as Internet artefacts

As a first step to understand mathematical memes, we address the fact that they are part of the general Internet phenomenon of memes, following a line of investigation aimed at eliciting *what* constituent elements of Internet memes occur in mathematical memes and *how* these elements are edited to encode a mathematical idea.

The first definition of memes as a contemporary Internet phenomenon is given by Davison (2012, p. 122): "an Internet meme is a piece of culture, typically a joke, which gains influence through online transmission". A more recent and detailed definition, frequently referred to in literature, is the online Oxford Dictionary entry[57] (n.d. para. 2) where a meme is described as "an image, a video, a piece of text, etc. that is passed very quickly from one Internet user to another, often with slight changes that make it humorous". Memes are generally shared in social media platforms and dedicated websites (meme aggregators): these environments, identified in literature with the terms *memesphere* (Stryker, 2011) or *memescape* (Wiggins & Bowers, 2015), are governed by collectively established and shared rules (Osterroth, 2018), and constitute the virtual and cultural habitat where Internet memes are created and consumed.

Although memes have become common ground in the Internet era, the term meme dates back to the mid 70's, when evolutionary biologist Dawkins derived it from the Greek word μίμημα, meaning *imitation*, to identify "a unit of culture that is passed from one generation to another and can be understood as the cultural equivalent of a gene […] examples of memes are tunes, ideas, catch-phrases, clothes fashions, ways of making pots or of building arches" (Dawkins, 1976, p. 249). Merging the various definitions, we can say that Internet memes are comprised of the following elements: a digital image and/or video or text, a typically humorous nature, a very high changeability and spreadability (Jenkins, 2013), and a capacity of being a unit of culture. Due to their digital nature, Internet memes spread across space and cultures at an extremely faster pace than their offline ancestors and have reached a massive worldwide diffusion (Figure 6.5, data collected by the first author).

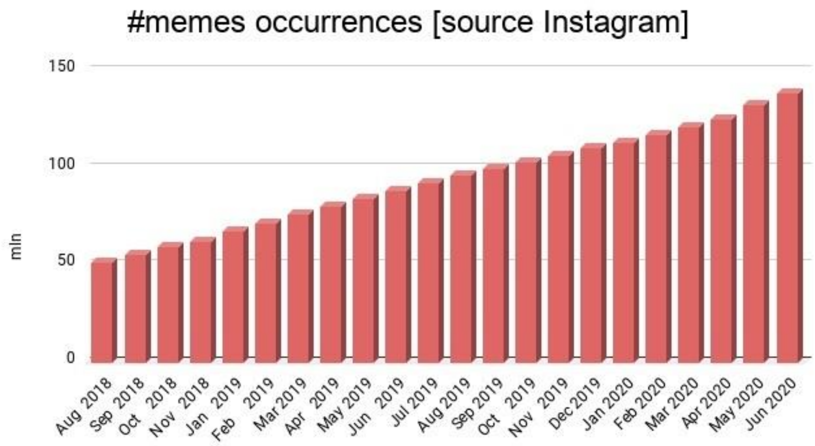


Figure 6. 5 Diffusion of the hashtag #memes in millions [source Instagram]

[57] .https://www.oxfordlearnersdictionaries.com/definition/english/meme

Artefacts corresponding to the hashtag #memes belong to all categories listed in the dictionary definition: they can be images, videos or texts. In fact, Shifman (2014, pp. 99-117) classifies nine meme genres between video and image-based memes. Video-based memes in Shifman's classification are (1) "flash mobs", videos depicting strangers who gather in a public place to perform a particular act, (2) "lip-synch", videos in which a popular song or movie clip is dubbed with a user-created soundtrack, (3) "misheard lyrics", captioned videos with deliberately erroneous translations of spoken sounds, and (4) "recut trailers", reinterpreted movie trailers. Image-based memes are (5) "photo fads", staged photos in which the subject imitates a specific action, (6) "reaction photoshops", user-created images incorporating a specific recurring element, (7) "LOLCats", images of cats with misspelled captions, (8) "rage comics", amateur-looking comics with particularly expressive characters, and (9) "stock character macros" (now known as *image macros*), existing digital images wittily reinterpreted with superimposed captions. Image macros are by far the most diffused kind of meme, to the extent that, in a process of semantic narrowing (Wilkins, 1996), the term *meme* is now commonly used as a synonym for image macro, and the original wider meaning is almost forgotten.

Mathematical memes come also mostly in the form of image macros, so we focus on this genre of memes, exemplified by the *Me and the boys* opening example. In Figure 6.6 the relation between mathematical memes and other memetic artefacts is represented, showing a nearly nested structure.

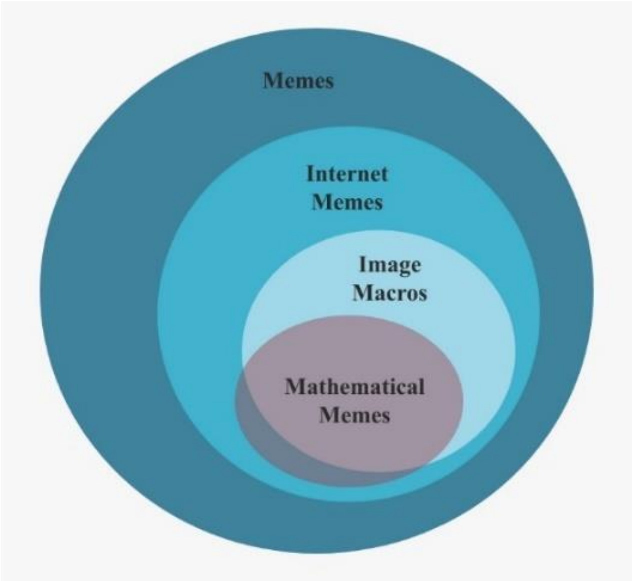


Figure 6. 6 Embedding mathematical memes into the general phenomenon of memes

As we now take image macros as our core focus, we give a description of their constituent elements and of the process that originates them, based on literature (Davison, 2012; Shifman, 2014; Wiggins & Bowers, 2015) and on the first author's two years' experience observing online communities dedicated to mathematical memes and collecting around 1000 mathematical memes. As already noted, memes are basically jokes made up of a pictorial and a textual part. The pictorial element is usually an eye-catching picture, which appeared on the Web in a different context: it can be the portrait of a celebrity or a common person, a famous logo, a snapshot from a video or a comic strip, as in the *Me and the boys* example. The creator of the meme extracts it from its original context and encodes a new meaning in it, adding a coherent humorous caption and possibly editing the image, and uploads it to a sharing website. The meme, thus, begins its virtual roaming in the memesphere, where it can inspire new mutations. Some images prompt only a few personalisations, while others become established *meme bases*, acquiring names and codified meanings, with the initial figure clearly recognizable as a characteristic trait. Here is where the meme acquires its value as a unit of culture, where the image becomes able to transmit meanings for those who have the knowledge to read it. The cultural level achieved is confirmed by the circumstance that established memes and their meanings are categorized in the Know Your Meme website[58], an online meme encyclopaedia created and edited by volunteer contributors around the world. In a nutshell, the core characteristics of these memes are *recognizability* of the meme base and *sharpness* of the joke, attained with a right match of humour and subject knowledge. Capturing the meaning of the meme base and connecting it to the text allows to decode the meme, grasp the joke and laugh, as happens in the *Me and the boys* example.

Memes' constituent elements are maintained in mathematical memes, with the textual and pictorial part edited to convey a particular mathematical idea. This allows us to summarize the constituent elements in the following compositional structure. Mathematical memes are artefacts where a meme base and a mathematical idea are merged through a humorous/emotional link (Figure 6.7) that embodies the mathematical content in a rigorous sense. This merger creates a new kind of encoded representation of a mathematical idea that can be decoded only if the mathematics and the meme base are understood, and the mathematical meme is consequently interpreted through the lens of this understanding.

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[58] www.knowyourmeme.com

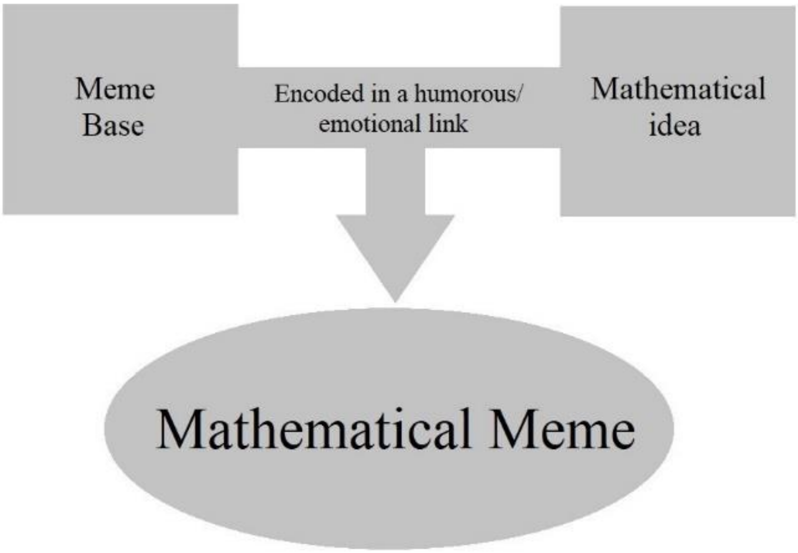


Figure 6. 7 The compositional structure of a mathematical meme

To understand how these constituent elements are edited to encode a mathematical idea, we apply the compositional structure in Figure 6.7 to some cases of popular mathematical memes, starting with the initial *Me and the Boys* examples.

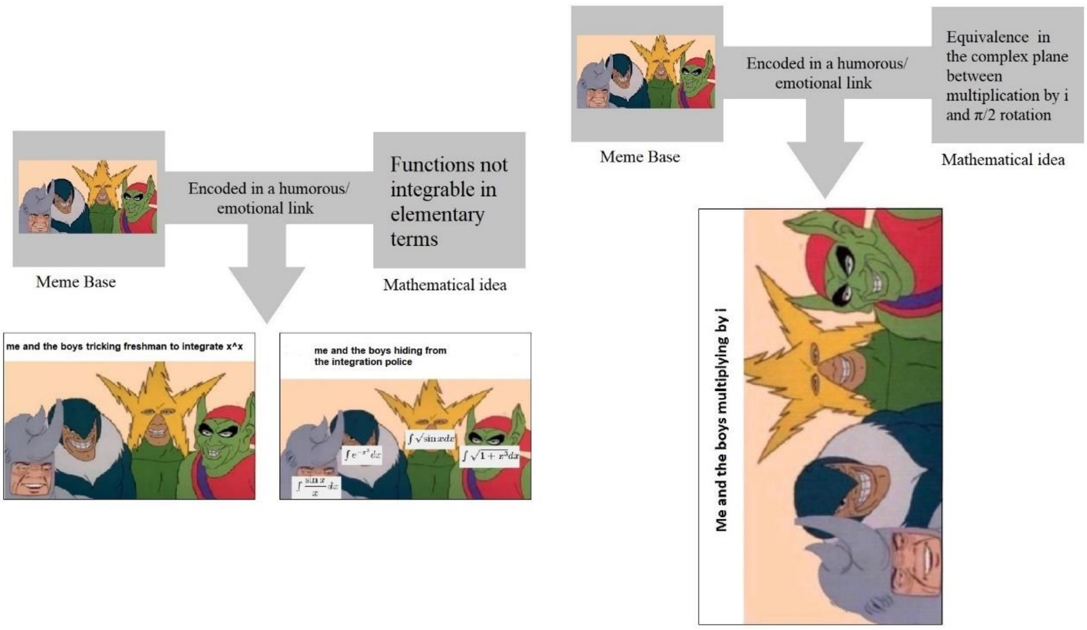


Figure 6. 8 *Me and the boys* mathematical memes [see Figures 6.3 and 6.4 for enlarged images]

Both memes in Figure 6.8 (left) use the metaphorical meaning of the *Me and the boys* meme base as shared school experiences to connect a goliardic feeling to the mathematical idea of functions that are not integrable in elementary terms. In the meme in Figure 6.8 (right) the conventional meaning is overruled, for the image is iconically used to encode the mathematical idea of the rotation in the complex plane, hinted by the caption. However, the link between the meme base and the mathematical idea draws its humorous strength from the popularity of the original horizontal image.

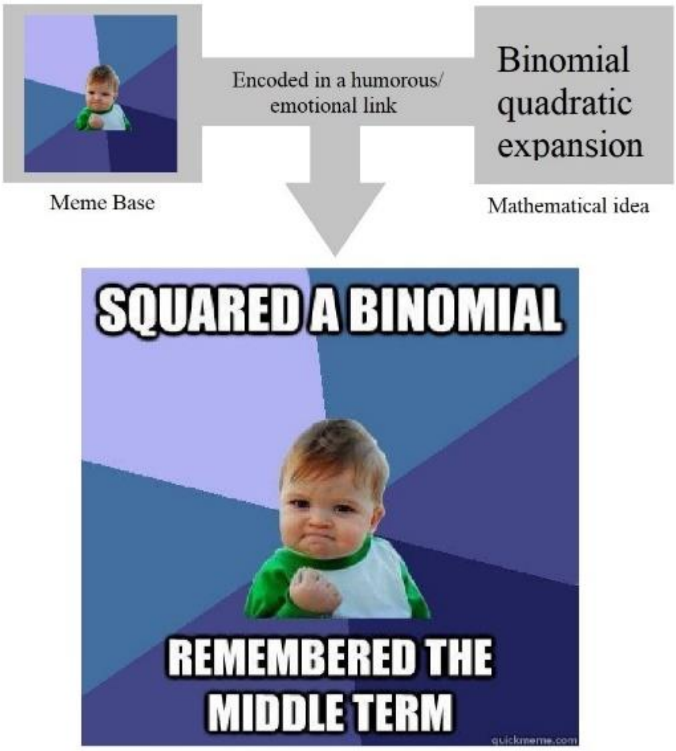


Figure 6. 9 *Success kid* mathematical meme [source Facebook]

In the example in Figure 6.9, the metaphorical meaning of the *Success kid* meme base to boast about something good is used to connect a positive feeling to the correct computation of the binomial quadratic expansion. This meme encodes the mathematical idea in a didactical way, focusing on the recurring mistake of forgetting the middle term. The result is an artefact that can be decoded only if the viewer knows the binomial expansion, is aware that the middle term is often forgotten and thus can relate the mathematical message to the meme base metaphorical meaning.

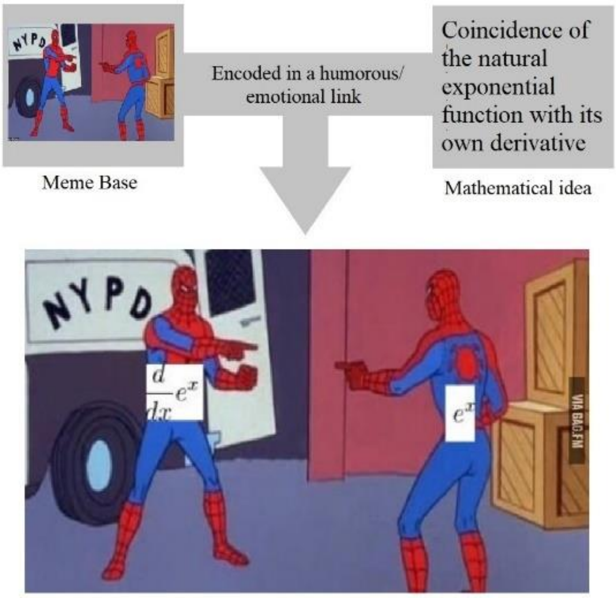


Figure 6. 10 *Spiderman pointing at Spiderman* mathematical meme [source Reddit]

In the example in Figure 6.10 we see how the metaphorical meaning of the *Spiderman pointing at Spiderman* meme base, i.e., the meeting of two similar elements, is used to convey the mathematical idea of the coincidence between the exponential function and its derivative. In this case the meme base is used to encode part of the mathematical idea, namely the equal sign that links the two mathematical expressions.

Going back to the line of investigation at the beginning of this paragraph, the memetic theoretical background allows to understand that humour and emotions, spreadability and diffusion, and image macros structural and compositional rules are the constituent elements of Internet memes that occur in mathematical memes, and that, in mathematical memes, images are edited to encode mathematically rigorous effects (tangible or emotional) or elements of the mathematical idea hinted by the text.

3.2 The cultural perspective: mathematical memes as Web 2.0 cultural objects

As a second step to understand the phenomenon of mathematical memes, we address the fact that they are part of the culture that typify the Web 2.0, the development of the World Wide Web that emphasizes collaboration between users, social networking and user-generated content. This is done following a line of investigation aimed at eliciting *what* features of the Web 2.0 culture relate to the production and use of mathematical memes and *how* the perception of mathematical memes is shaped by these features.

The culture nurtured in the Web 2.0 environment is characterised by Jenkins as being *participatory* (2009) and *convergent* (2006, a term which has to be interpreted in a non-mathematical way). Participatory culture is typified by a "strong support for *creating* and *sharing creations* with others, [and] some type of *informal mentorship* whereby what is known by the most experienced is passed along to novices" (Jenkins, 2009, pp. 5-6, emphasis added). Building on Jenkins' idea, Bruns (2008) introduces the hybrid term *produsage*, a portmanteau of the words *production* and *usage,* to summarize the participatory environment nurtured by social software, whose protagonists not only *use* digital content, but also *produce* it, as happens for example with online encyclopaedias like Know Your Meme or Wikipedia.

In this environment, "Internet users seemed to have grasped […] that *the meme is the best concept to encapsulate some of the most fundamental aspects of the Internet* in general, and of the so-called participatory or Web 2.0 culture" (Shifman, 2014, p. 18, emphasis in original) because they "harness the participatory potential of the Internet and typify modern popular culture" (Marwick, 2013, p. 13). So, the participatory culture inspires Internet users to create memes in general (Wiggins & Bowers, 2015) and mathematical memes in particular. It fuels the creative activity, which reveals itself both in the making of the mathematical meme and in the interaction around it, through which people with good math skills offer their "informal mentorship" (Jenkins, 2009, p. 6) to less skilled individuals. The qualifying value of participation in the Web 2.0 ecosystem explains also the role that memes have in this culture. In an environment where users "care what other people think about what they have created" (p. 6), memes are exchanged as "passcodes" (Schmidt, 2014, 00:08:55) to regulate the entrance to the memesphere or the "status in the community" (Stryker, 2011, p. 76).

The first author's observation of online communities dedicated to mathematical memes, revealed that these communities form a sub-region of the memesphere, whose citizens are proficient in memes and mathematics. Symbolic admission is granted only by sharing OC (original content), i.e. mathematical memes not copied from elsewhere, that have to be appropriate at both memetic and mathematical level. We take as an example the meme in Figure 6.11 and an excerpt of its comments[59]:

[59] To protect the privacy of commenters, personal identifiers have been substituted with codes throughout the study.

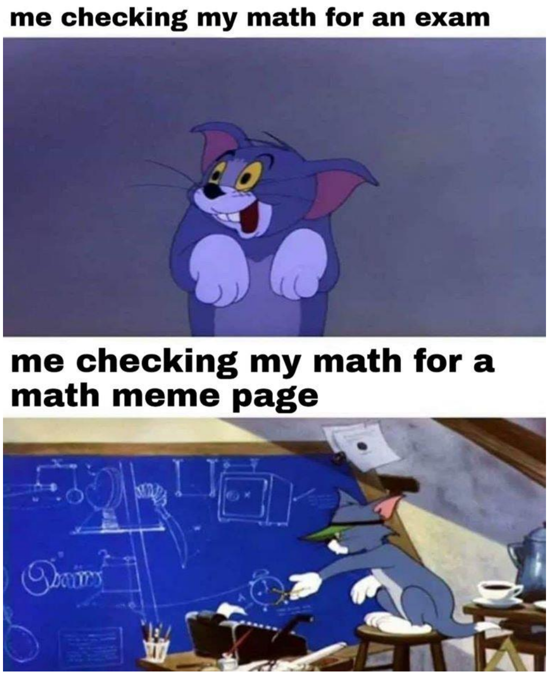


Figure 6. 11 Checking the mathematics behind a meme [source Facebook]

1-C1: if your math is wrong in a meme, the repercussions are a lot more severe lol

[…]

2-C2: Well that's because the teacher is a single person who will ridicule you in private if you're wrong. Lol. I haven't shared anything to this group because the small pool of things I could come up with have already been done

3-C3: It's only right. You're risking your good name throwing math memes out there

This is a meme *about* mathematical memes (i.e. a *meta*-meme), that is taken as a source to gain information about the cultural practices (in the sense of Gitelman, 2006) grown up around mathematical memes. It describes the users' contrasting attitudes with respect to school mathematics and meme mathematics. Produsers of mathematical memes show a responsible attitude in checking their mathematical content, as opposed to a more naïve attitude shown towards school mathematics. Comments indicate the reasons why this checking has to be thorough (#1), that the content has to be original (#2), and that mathematical memes are perceived as personal identifiers in the community, acting as passcodes or as status qualifiers (#3). This example is one of many from which we can infer that mathematical memes are perceived as qualifying proofs of produsers' mathematical knowledge, because memes in these communities are publicly scrutinized by a cultured, merciless and potentially worldwide audience.

Besides participation, the second typifying characteristic of the Web 2.0 culture is *convergence*. Convergence is not simply the wide availability of information funnelled in a single device but is "a cultural shift [in which] consumers are encouraged to seek out new information and make connections among dispersed media" (Jenkins, 2006, p. 3). In a convergent perspective, academic and popular culture are perceived as a continuum. For example, a recent discovery in number theory has been inspired by an episode of the popular TV series The Big Bang Theory (Pomerance & Spicer, 2019), and an anonymous post on the social platform 4chan has given an essential contribution in solving a 25-year-old problem in combinatorics (Anonymous 4chan Poster et al., 2018). In the memesphere, convergence is "a central pattern" (Danesi, 2019, p. 57): this implies, among other things, that the traditional boundaries of education are challenged. Thus, Web 2.0 users are ready to learn even if they are not at school (Thomas & Seely Brown, 2011; Ito et al., 2013), in a process of "*collective meaning-making* […] within the brains of individual consumers and through their *social interactions with others"* (Jenkins, 2006, pp. 3–4, emphasis added). Convergence describes how mathematical memes act as "cultural activators, setting into motion their decipherment, speculation, and elaboration" (p. 95). They spark a process of collective meaning-making, providing opportunities for non-curricular learning experiences, a practice which is widely confirmed by evidence shared on the Web. In Figure 12 we see two meta-memes expressing appreciation for the use of mathematical memes to support the understanding of mathematical concepts (left) and to foster engagement (right).

Figure 6. 12 Mathematical memes as learning objects

However, the meta-meme in Figure 6.13 shows that mathematical memes produsers are also aware that, to build proper mathematical knowledge non-curricular, meme-induced learning is not enough, but it has to be put together with curricular learning, in a truly convergent perspective where “old and new media collide” (p. 2).

Figure 6. 13 Don’t forget to read some actual maths [source Facebook]

To sum up, the cultural theoretical background provides evidence to characterise mathematical memes as cultural objects, showing that participatory culture inspires the production of mathematical memes and shape their perception as membership identifiers, and that convergence culture inspires ways of using them as learning opportunities.

4 Methodology and methods

Based on the theoretical background, we were able to characterise mathematical memes as memetic artefacts encoding mathematical ideas, and as cultural objects in dedicated participatory and convergent online communities. These characterizations give some important information to start building up our answers to the research questions. What is still missing is a characterization of mathematical memes as *representations of mathematical ideas* and a deeper investigation of how they *activate and guide interactions* within the community. Since such a representation not only differs from all usual mathematical representations but is also in constant change in the mathematical memesphere, it can only be characterised from inside the meme community, for example analysing the meme’s decoding process in the situated interactions among its members.

To address the missing information, we have approached the field ethnographically (Eisenhart, 1988; Harwati, 2019), considering practices as cultural habits shown and re-established in the interaction among the members of the community (Gitelman, 2006). This ethnographic approach is chosen because it is aligned with our aim to investigate “what people’s opinions are about a particular phenomenon occurred in their social context and what their action is". (Harwati, 2019, p. 152). The study began in February 2018 and is still in progress. It is conducted by participant observation of the first author being a read-only (a *lurker* in the Internet jargon) member of the communities without interacting with them. This role allowed the first author to become "part of the situation being studied in order to be able to feel the way people do in that situation" (p. 152), taking care not to perturb the environment while gathering data.

Data collected in the ethnographic research are chronological field notes, memes (constituting a *meme pool*) and their threads of comments (constituting a *comment pool*). Focusing on the comment pool, we reconstruct the interaction activated by the memes from inside the observed communities, considering the meaning-making processes. Because of the novelty of our topic, an instrumental case study that aims at theorizing through comparing cases is an appropriate approach (see Grandy, 2010), coherently combined with the ethnographic approach (Harwati, 2019). This case study follows the principle of comparison between two contrasting empirical cases, which represent polar types of mathematical memes, one being mathematically right and the other mathematically wrong. Data are analysed based on the tools provided by the theoretical background. The final step is theorizing the results with respect to the original aim of reaching a conceptualization of the phenomenon of mathematical memes. Since our study addresses an under-researched field of a fluid nature with a huge amount of available data sources on the Web, our first step is to systematically map the field to identify appropriate data sources that could provide paradigmatic cases to investigate the community from the inside.

4.1 Mapping the field and identifying appropriate data sources

Memes are everywhere on the Web, from blogs to meme aggregator websites and social media, but mathematical memes are shared mainly in social media websites. Therefore, to organize the research, we referred to the shared methodologies of qualitative research in social media as exposed by the Social Media Research Group (2016), the SAGE Handbook of Social Media Research Methods (Sloan & Quan-Haase, 2016) and, in particular, to Hand's work on researching images in social media, included in the latter. According to Hand (2016), the primary methodological issue in investigating images in social media is due to the large amount of available data, which necessarily implies a selection that will affect the research and therefore must be made knowingly. This selection is conducted beginning with websites, funnelling towards particular communities as data sources: Choices were made applying combined criteria of popularity and relevance for the research as described below.

4.2 Selecting social media websites

Within social media websites, the choice fell on Reddit[60], Facebook[61] and Instagram[62]. Reddit (355 million users) was chosen for its relevance for the meme culture, being the commonly acknowledged birthplace of most memes (as happened to the *Me and the boys* example in the opening section). Facebook (2.26 billion users) and Instagram (1 billion users) were chosen for their popularity, being respectively first and second in the global social networks ranking (Statista[63], 2019).

4.3 Selecting communities

In social media websites, we looked for active (i.e., sharing new content at least weekly) communities interested in mathematics *and* memes, using both the intern search engines with keywords *meme+mathematics* or *meme+math/maths* and the *similar pages* suggestion option. Among them, the second level of selection was again made according to popularity, based on the number of followers, expected to imply deeper, wider and more meaningful interactions. Thus, communities with more than 10,000 (10k) followers were chosen (Table 6.1).

Table 6. 1 Observed communities ordered according to the number of followers (June 2020)

Reddit		Instagram		Facebook	
r/dankmemes	3.7million	@bad science jokes	388k	Educational Memes	597k
r/math	1.2million	@juicy mathematical memes	128k	Mathematical Mathematics Memes [group]	226k
r/theydidthemath	601k	@meme for mathematicians	113k	I love Mathematics 2.0	110k
r/mathmemes	82.6k	@mathsforyou	62.6k	L’Agorà Del Superuovo	97.6k
r/physicsmemes	77k	@maths.meme	58.2k	Math Memes	72k
r/mathpics	21.4k	@memes educational	46.6k	Matematica del Suicidio	30k
		@mathematical jokes	27.6k	Matemáticas y Café	30k

60 www.reddit.com
61 www.facebook.com
62 www.instagram.com).
63 .https://www.statista.com/statistics/272014/global-social-networks-ranked-by-number-of-users/

		@puremaths memes	22.8k	>implying we can discuss mathematics	26k
		@calculuscious	21k	The Name Of This Group Is Left As An Exercise For Its Members	12k

4.4 Selecting data

Communities in Table 6.1 were followed daily by the first author to collect data for the research. Considering the prospective idea of using the results of this research to inform teaching in school, we selected mathematical memes encoding mathematical ideas on 8th to 13th grade mathematical topics according to the Italian national curriculum[64], and meta-memes, i.e. memes *about* mathematical memes. Memes adhering to these criteria were downloaded and archived using a double tagging criterium (meme base name + brief math reference). In the period of observation (February 2018–current) this process produced a meme pool of around 1400 items, filtering reposts. This meme pool was complemented by a comment pool of saved active links connected to the first author's personal accounts on the three websites to follow real-time comments engagement. Occasionally these communities share other image-based humorous digital objects addressing mathematical themes, like memes dealing with emotions connected to doing mathematics and mathematical cartoons. These other objects were also inspected to gather information for comparison that could contribute to characterizing mathematical memes.

4.5 Selecting cases

As noted in the participant observation and described by Jenkins (2006), mathematical memes communities are coherent in the way they interact with memes. Therefore, our selected cases are present not for themselves but as representative characteristics of a class, in a sense echoing that of Balacheff's "generic examples" (1988, p. 57). Further, there is no theoretical knowledge about the nature of this kind of interaction in the context of mathematical memes communities. In order to disclose such theoretical knowledge we, therefore, have chosen the instrumental case study approach built on the "particularly important theoretical sampling

[64] http://www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/_Liceo%20scientifico.pdf

strategy” (Eisenhardt & Graebner, 2007, p. 27) of polar types, “in which a researcher samples *extreme* […] cases in order to more easily observe contrasting patterns in the data", [relying of the fact that] this sampling leads to very clear pattern recognition of the central constructs, relationships, and logic of the focal phenomenon” (p. 27, emphasis added).

Communities usually share correct mathematical memes, while wrong memes are extremely rare. For that reason, we choose a mathematically correct and a mathematically wrong meme as the two extreme polar cases in our sampling strategy.

The choice of the environment of the polar cases was based on a quantitative criterion of popularity, i.e., the number of followers. Selected cases were chosen from Facebook, confirmed as the most popular social media among teens (13-17 y.o.) with 113 million users (Hootsuite – We are Social Global Digital Report[65], 2019; Edison Research, The Infinite Dial[66], 2019). Within Facebook, we chose the most popular among the communities in Table 6.1: Educational Memes (597k users) and Mathematical Mathematics Memes (226k users). Among mathematical memes shared within these communities, we applied a qualitative criterium of choice, selecting the two memes among those where mathematics is deeply used, a characteristic expected to imply richer threads of comments and where highly relevant aspects are discussed, embracing Pettigrew’s suggestion recalled by Eisenhardt (1989) that “if the phenomena to be observed is to be contained within a single or relatively small number of cases, then choose cases where the progress is transparently observable” (Pettigrew, 1988/1990, p. 275). In particular, our memes stood out because they dealt with two key issues for the teaching and learning of mathematics: the ability to identify circular arguments (Freudenthal, 1971; Hanna & de Villiers, 2008) in the correct meme, and the misconceptions that could arise from the polysemy of mathematical symbols (Durkin & Shire, 1991; Gray & Tall, 1994; Priss, 2018) in the wrong one. We excluded a quantitative selection centred on the number of likes or comments because not fitting with the qualitative criterium of finding memes with deep mathematical content. The selection was carried out using a sorting criterion, to identify the two polar types, so that the emerging characterization is expected to be well grounded in their comparison (Eisenhardt, 1989; Eisenhardt & Graebner, 2007).

[65] .https://wearesocial.com/global-digital-report-2019
[66] https://www.edisonresearch.com/wp-content/uploads/2019/03/Infinite-Dial-2019-PDF-1.pdf

4.6 Analysing data

Data in the meme and comment pools have been explored with an ethnographical attitude, taking as a unit of analysis the memes referring to their threads of comments. Mathematical memes were taken as a source to gain information about the interaction with the mathematical idea represented in the meme, and meta-memes were taken as a source to gain information about the cultural value of mathematical memes and of the related interaction inside the community.

In the empirical case study, theoretical concepts previously provided in the paper are merged in the analyses. In the first step, the encoding of the mathematical idea in each case is shown according to the compositional structure presented in Figure 6.7. In the second step, a sequential analysis of the thread of comments is conducted, following Jenkins's (2009) idea that the interaction in a community is a collective process where each contribution is related to the others. In the analysis, the process of meaning-making is reconstructed following the interactive threads of comments, where each comment is taken as an interpretation of previous comments and of the meme itself. Comments convey two levels of information: an explicit information, carried by the text openly expressing something, and an implicit information, carried by the way the comments are articulated, conveying information about the commenters themselves, their relationship with others and with the meme. These two levels of information are traced through the excerpts by re-interpreting comments in a turn-by-turn-analysis (Jungwirth, 2003).

We begin with a case whose mathematical content is correct (and recognized as such by the commenters), and then we analyse the mathematically incorrect case. As the culture in the community is re-shaped in the interaction around the mathematical meme, the cultural codes *participatory* and *convergence* are added in the analyses when evidenced. The analyses with the coded excerpts are then used to identify and synthesize the emergent mathematical themes. Separate analyses of each of the two polar types are subsequently compared and contrasted to answer the research questions. This will show how the meme drives the interaction and how the interaction keeps going while decoding the mathematical idea hereby encoded.

Complying with the ethical principles of social media research (Social Media Research Group, 2016), we do not include nor analyse produsers' personal information, replacing identifiers in all quoted excerpts (A = author of the mathematical meme, C1 C2 C3… = commenters) and adopting an inclusive and bias-free language in our analysis. All comments are unredacted and originally in English. To follow the development of the interaction across time, the excerpts are reported in chronological order with the timestamp of the meme and of the first and last comment, and the indication, whenever present, of who is replying to who. If needed, slang terms are interpreted using the Urban Dictionary website[67].

5 Analyses and results

To understand the phenomenon of mathematical memes with respect to their use as representations of mathematical ideas inside the community and investigate how they activate and guide interactions among members, we first summarize the results of our participant observation while collecting the meme and comment pools.

5.1 Participant observation: memes and mathematical ideas

The ethnographic research, documented by the ethnographic representative in the author group with chronological notes, has provided a data pool organised for the specific purpose of this part. From this data pool, we have extracted information about the practices around mathematical memes in the inspected communities.

Our observation revealed that almost all mathematical memes are followed, in the same social media page, by threads of comments where the meme's mathematical content is discussed by the meme's author and other users. It revealed also that the practice of exchanging mathematical comments is specific of mathematical memes: in fact, in the case of memes about emotions connected to doing mathematics (Figure 6.14 left) there are usually no mathematical comments, but just common emojis or friends tagging. The same applies to cartoons about mathematics (Figure 6.14 right) which are different digital objects having, sometimes, a proper mathematical content.

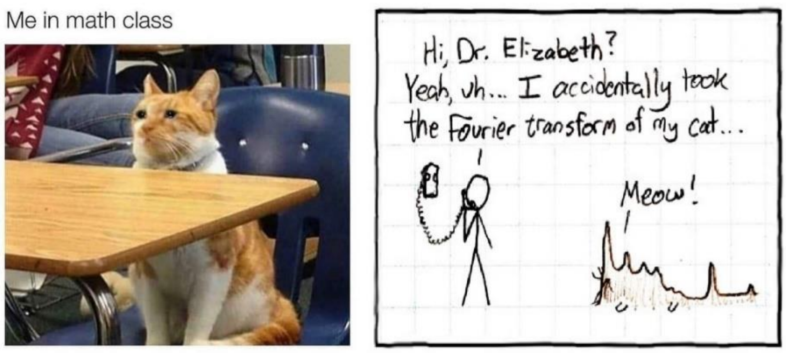


Figure 6.14 Emotional memes and cartoons [sources Facebook and xkcd.com]

67 www.urbandictionary.com

Thus, we can say that, among image-based humorous digital objects exchanged within the observed communities, threads of *explicated mathematics* stand out as a characterizing practice of the interaction with mathematical memes as representations of mathematical ideas.

The importance of mathematical comments in the community is confirmed by the high value given to this habit, as shown for example in the meta-meme in Figure 6.15.

Figure 6.15 The importance of mathematical comments [source Facebook]

The observation of these interactions proves that threads of explicated mathematics are not simply openings for more educated members of the community to show off some knowledge. In fact, they allow the collective meaning-making of the mathematical idea encoded in the meme to take place. This is clearly acknowledged in the comments to the meta-meme in Figure 6.16.

Figure 6.16 The importance of understanding mathematical memes [source Facebook]

This meme, posted in the Mathematical Mathematics Memes (MMM) page, addresses the issue of high school students' understanding of university level mathematical memes. The image of a hand moving fast towards a blue button is a popular meme base used to symbolise situations that trigger impulsive reactions. In this case the triggering situation is described by the top text "*When someone posts a university level mathematics meme in the university level mathematics meme facebook page*" and the impulsive reaction is represented by the caption superimposed on the button "*I really really wish I understood this*". The author's note accompanying the meme "*if (o)nly people would (c)onsider the high school students in MMM*" and the number of likes (1,9k) tell us how much this feeling is experienced within the community, while the following excerpt from the 149 comments gives us insight into how the community relies on the threads of explicated mathematics to overcome these difficulties.

1-C1: Every time I've seen someone actually explain the mathematics behind a meme, it's been a great experience. … [I] feel like going and learning more. Thank you to those wonderful mathmemeaticians who help others appreciate the joy of glimpsing new realms of mathematics.

[…]

2-C2: I think this group is the very reason why i am even more interested in mathematics than i was before, even though i dont understand more than half of the stuff here...i m still learning and its really fun!

Here we see that explaining *the mathematics behind a meme* is accredited to activate the learning mechanisms (#1) and that mathematical memes are recognized as resources to foster learning and interest in mathematics (#2).

This characterizing practice is connected to both features of the Web 2.0 culture: it is participatory in the sense that it gives commenters the opportunity to contribute offering the characterizing informal mentorship, and it is convergent because the interplay between mathematical memes and comments allows the collective meaning-making to happen. Indeed, these communities operate as spontaneous communities of practice (Wenger, 1998), and build group meanings of mathematical concepts, acting as a "sphere[s] of practice" where "mathematical meanings are constructed" (Kilpatrick et al., 2005, p. 10).

The educational value of mathematical memes and their threads of mathematical comments is not only recognized by members of the community, but it may also be the reason for the very existence of the community. This is the case of MEME4000W, two twin mathematical memes pages created by graduate students on Facebook and Instagram, to teach mathematical concepts using memes as a medium. The following interview excerpt illustrates the origin of the project:

At the start of last year, I joined a mathematics meme group and realized that *I learn so much from the interactions on the group, through the memes and the comments* [emphasis added]. I wanted to share this experience with others and show them that maths isn't as threatening as people make it out to be: It can be experienced in a very fun and conducive environment. (founder of the pages, interviewed by the first author via chat, April 2019).

This evidence indicate that threads of explicit mathematical comments connected to mathematical memes are exactly the loci where the characteristics of mathematical memes as representations of mathematical ideas become evident, and where the educational potentialities of these objects can be observed. In the next paragraph we deepen our understanding of the phenomenon by analysing the interaction in two contrasting cases selected according to the criteria explained in the methodology section.

5.2 Instrumental case study: Threads of comments on mathematical memes

5.2.1 Case 1: the correct meme

- Meme base name: *Mega Wrong!*
- Meme base use: stigmatize situations where something significantly wrong occurs
- Source: Facebook Total comments: 96
- Topic: Calculus
- Subtopic: Circular reasoning, Limits, Derivatives proofs, L'Hôpital's rule
- Publication year: 2019

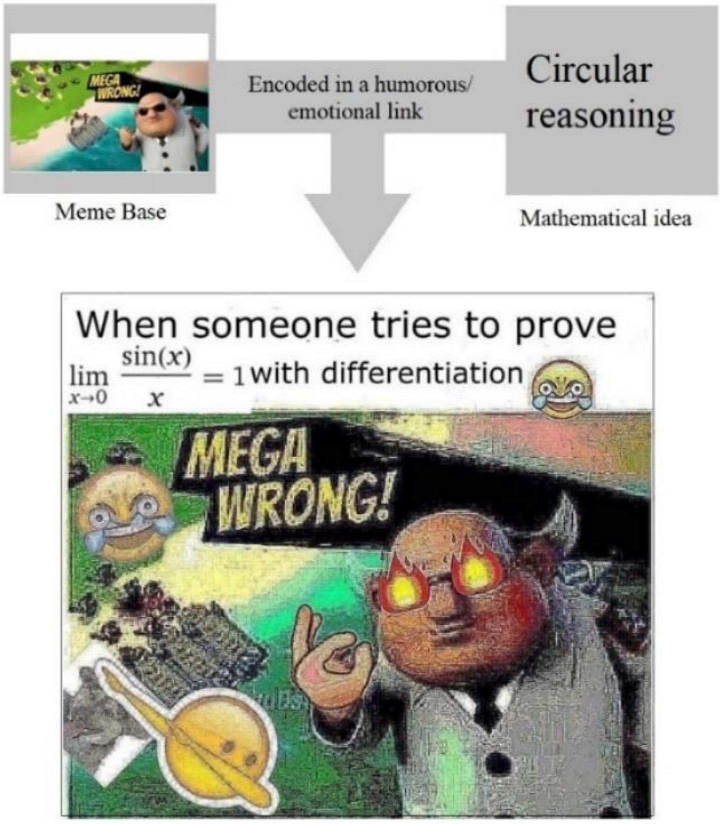


Figure 6.17 Case 1: the memetic analysis

Contextual description: The meme in this example (Figure 6.17) is used to label as *Mega Wrong* the proof of the indicated limit using differentiation (i.e., L'Hôpital's rule). The proof is considered *Mega Wrong* because it is circular, involving a logical fallacy that occurs if we assume that the derivative of the sine function is obtained using the result of the $\lim_{x\to 0}\frac{sinx}{x}$, which is the usual development of the topic in high school. This same procedure would be logically sound if, for example, we accept power-series definitions for trigonometric functions, and obtain the derivative of the sine function from these premises. This knowledge is mastered and disseminated by the author of the meme and by several commenters, as will be shown in the analysis. We chose key parts of the exchange taking a large section from the beginning where the engagement is established. We are taking further parts following the same mathematical topic. The selection is also made according to indicators related to the analysis perspective. The development of the comments goes in various directions, several proofs are offered, only one is included in the analysed excerpt (Table 6.2).

Table 6.2 Case 1 – the cultural analysis

Comments	Explicit information	Implicit information	Environmental culture	Emerging mathematical themes
1-C1: [Day 1; 16:29] How is lhopitals not correct	Asks for reasoning (*need*) about what is labelled as “mega wrong” in the meme	*How* means “how come”, an *informal* way to ask about the reason, (*need for reason*). C1 shows knowledge (recognizing the limit and that differentiation refers to L'Hôpital's rule); C1 is open for broadening knowledge (why this procedure is incorrect)	C1 addresses the community as a knowledge holder for the meaning-making (convergence culture)	Why using differentiation to solve the limit is incorrect
[…]				
2-C3: That would be a circular argument	Gives a key word that it is “mega wrong” because of a logical fallacy	C2 leaves deeper reasons unexplained, acknowledges the expertise shown by C1	Informal mentorship (participatory culture)	
3-C3: um why?	Asks for reasons why (*need for reasons*)	The expression “*um”* indicates uneasiness, probably due to the *lack of reason* in C2’s answer, thus C3 explicitly asks “why”	C3 needs help from the community for the meaning making (convergence culture)	
4-C4: You usually use sin(x)/x -> 1 to prove sin'=cos	Provides a missing reason, a brief *explanation* (reacts to the need) about the circularity	C4 shows experience and assumes C3 knows that applying L'Hôpital's rule to	C4 explains, hence, practices informal mentorship (participatory	

		the limit means using the derivative of y=sinx, recognizes sin' as the derivative	culture)	
5-C3: Oh, right. Yeah that	Acknowledgment	The acknowledgement without further questions indicates that C3 was already familiar with all information shared, but C4's comment helped to connect them	Meaning-making is complete (convergence culture)	
[…]				
6-C5: but i thought we used taylor series to prove that sin'=cos tho	Asks about the use of Taylor series instead of the limit for the proof of the derivative of sinx	C5 indicates a lack of information (*need*) C4's answer does not satisfy C5, C5 does not (seem to) recognize the role of the limit in proving sin'=cos	C5 expresses the *need* for more information (convergence culture)	Digging deeper into the usual steps of proofs and what makes the reasoning in the meme circular
7-C4: [replying to C5] you begin with their Taylor series definitions?	Asks about the first step of the topic (*need for clarity*)	C4 is not evaluating C5's comment, but changes the focus of the discussion to the steps of the topic	Informal (didactic) mentorship (participatory culture)	
8-A: As long as you explicitly show that d/dx sinx = cosx then you can use l'hopitals rule	A intervenes to *clarify* (to an implicit *need*) which are the mutual dependencies of the steps	A brings the community back to the original problem Here the adverb "*explicitly*" referred to the proof of the derivative is to be	Informal mentorship giving a rule (participatory culture)	

		read as independently of the limit in the meme		
9-C5: ok nice ! 😊	Acknowledgement and acceptance of the explanations	C5 reacts with appreciation	C5 completes the meaning-making (convergence culture)	
10-C4: [replying to A] yeah, but in rigor, depending on how you begin, you *might need* first to prove sin(x)/x. In the usual development of the topics I suppose it's the case	Replies to A referring to the established steps of proofs: first the limit and then the derivative (as is implied by the meme)	C4 focuses again on the *usual* steps of proofs with the limit as the first step of the *usual development*. C4 connects this interpretation to the meme *("I suppose it's the case")*, meaning this is the case where the "*mega wrong"* circularity occurs (clarifies the usual context of learning the maths of the meme, reacts to an implicit *need to clarify*)	C4 (coming back to the meme) acknowledges the fact that the meme is constructed according to what is mathematically consistent in the usual way of proving (convergence culture)	
[…]				
11-A: [replying to C4] that's what makes it circular reasoning because you'll use sinx/x to show sin'x = cosx and then go use that to prove sinx/x	A explains in details the logical steps of the circular reasoning occurring if the steps of proofs starts with the limit (reacts to need for reasoning)	A comes back to the original meme and acknowledges C4's explanation of the fallacy ("*that's what makes it circular reasoning"*) and gives a detailed explanation ending with a didactical	A is self-positioning in the community as an expert, owning the meme acting toward a *need* to decode (participatory culture)	

notice how that reasoning goes in circles		observation "*notice how that reasoning goes in circles*"		
12-C4: Yeah, I said that in my very first comment, lol [laugh out loud]	Acknowledgement	C4 points out that the same clarification had been already provided and endorses A's	C4 is self-positioning at the same level of expertise (participatory culture)	
[…]				
13-C6: How is it proved??	Clarification request about the proof of the limit sinx/x	Based on the previous comments and on the meme, C6 expresses the *need for a proof* of the limit in the meme. It may be part of the sentence: if it is "mega wrong" to prove it with differentiation, then *how is it proved*?	C6 addresses the community as a knowledge holder (convergence culture)	Proof of the limit using the sandwich theorem
14-A: [replying to C6] Geometric argument is the easiest way	A replies hinting to the geometrical proof of the limit	A reacts to the *need for proof*, stressing: there are other not mentioned proving techniques and A is able to select the *easiest* one	A is self-positioning in the community as an expert, offering informal mentoring (participatory culture)	
15-C4: You can geometrically prove sin(x) <= x <= tan(x), divide by sin(x) and use the reciprocal inequalities:	Description of the steps of the geometric proof using the sandwich (squeeze) theorem to prove the limit (reacts to *the need to prove* by offering one)	The explanation is consistent although many elements are left to the reader to fill in: how to "*geometrically prove*" the first inequality and how	Informal mentorship is worked out (participatory culture)	

cos(x) <= sin(x)/x <= 1 and then the sandwich theorem (plus geometrical demonstration that cos(x) -> 1)		the sandwich theorem actually works		
[…]				
16-C7: So the reason why this doesn't work is dx/dy sinx=cosx is proven by lim x->0 sinx/x =1?	Questions about the wrap up of the reasoning (reacts to a *need for certainty*)	C7 summarizes the reasoning towards the others to confirm, showing that C7 followed the discussion and correctly grasped "*why this doesn't work"*	C7 addresses the community as a knowledge holder (convergence culture)	Final consideration about the correctness of the mathematical idea represented by the meme
17-C4: [Day 1, 17:11] Generally	Acknowledgement according to the usual way of proving	C4 does not notice C7 final typo on the derivative (dx/dy instead of dy/dx) possibly because the focus is on the wrap up of the reasoning	Final informal mentorship (participatory culture)	

The convergence culture is driven by different *epistemic needs* (Kidron et al., 2011) towards the understanding of the mathematics encoded in the meme: need for reasons (#1, #3), need for more information (#6), need for clarity (#7), need for argumentation premises (#10), need for proof (#13), need for certainty (#16), where the latter seems to be the main one that underlies all the others. Kidron et al. (2011) focus on the epistemic orientation of needs, Drodge and Reid (2000) point to their emotional orientation in a mathematical activity, both indicate the relevance of the social when (epistemic) needs appear in a mathematical activity as in the participatory culture shown in the thread of comments.

The participatory culture is driven by the informal mentorship activated by the epistemic needs, even if sometimes implicitly: reacting to a need for explanation (#2, #4), clarity (#8, #10, #16), and reasoning (#11). Doing this, the commenters also show, keep, and address expert levels of mathematics according to the meme, offering as much information as needed, but no more, unless someone expresses an epistemic need (#2). Sometimes commenters explain without someone asking for an explanation, in this case they play the role of mentors towards an implicit need for explanation in the group (#8, #11), similarly they clarify the argumentation spontaneously and refer to the original meme (#10). We are aware that some of the users might be people with good math skills or even mathematics teachers, willingly offering their knowledge. What is new here is that less-skilled individuals do not hold back from interacting with more competent people, spontaneously showing epistemic needs and taking advantage of the informal opportunity to learn something.

Throughout the exchanges, the mathematical idea encoded in the meme is reconstructed producing and using information and knowledge by commenters, naturally enacting the different roles of *teacher* (A C2 C4) and *learner* (C1 C3 C5 C6 C7). A meaning-making process emerges, activated by the meme and kept by the epistemic culture, revealing an argumentation progression with different mathematical themes. The process is summarized in C7's final comment: applying L'Hôpital's rule to solve the limit means taking the derivative of sinx, *AND* the proof of the derivative of sinx is based on the knowledge of the same limit (in the high school developing of the topic), *HENCE* the reasoning is circular. *THEREFORE*, applying L'Hôpital's rule to solve the limit is *mega wrong*. It is interesting to note that C7 was not among the commenters who asked the initial questions, nonetheless the final remark shows us that C7 read the preceding comments, and elaborated the information shared to produce some mathematical knowledge. In the progressing of the reasoning, some side knowledge is also produced, stemming from the acquired correctness of the meme in a mathematically consistent way: *IF* applying L'Hôpital's rule is *mega wrong*, *THEN* what is the correct solving procedure?

Drawing together, the meaning-making process takes the form of an argumentation nurtured by an epistemic culture. This epistemic culture is established and re-established by the interplay between the convergence culture driven by epistemic needs and the participatory culture where the community members react towards explicit or implicit epistemic needs by explaining, clarifying, and reasoning. In this epistemic culture, the negotiation of knowledge is expert-like, recognizing the other as experts in the community, mutually offering and asking for mentorship.

5.2.2 Case 2: the wrong meme

- Meme base name: Spiderman pointing at Spiderman
- Meme base use: describe situations in which similar elements meet
- Source: Facebook Total comments: 131
- Topic: Functions
- Subtopic: Direct and inverse trig functions
- Critical point: misinterpreted equivalence between $f^{-1}(x)$ and $\frac{1}{f(x)}$
- Publication year: 2018

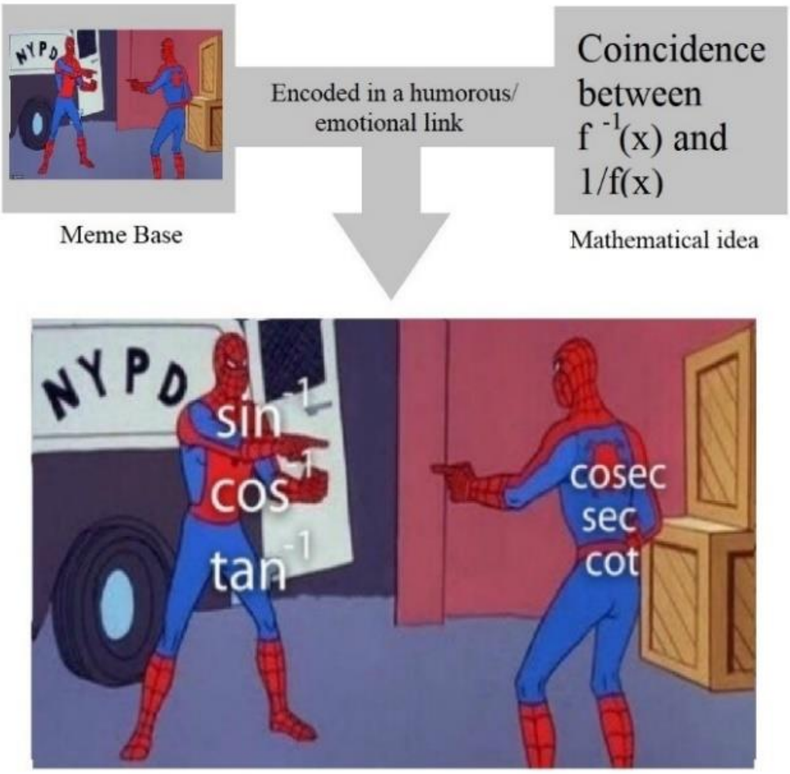


Figure 6.18 Case 2: the memetic analysis

Contextual description: The image of this meme in Figure 6.18 is commonly used to describe situations in which similar elements meet: in this case, the intent of the author is to represent the equivalence between the triplets of functions sin^{-1} cos^{-1} tan^{-1} and *cosec sec cot.* This equivalence is, of course, incorrect: the excerpt (Table 6.3) shows how swift and enduring the online community is in singling out and discussing the mistake.

Table 6.3 Case 2 - the cultural analysis

Comments	Explicit information	Implicit information	Environmental culture	Emerging mathematical themes
1-C1 [Day 1, 9:22]: This is mathematically wrong	C1 declares the meme as incorrect	By using the adverb *mathematically* C1 makes clear that the problem is a mathematical one and not a memetic one	C1 addresses A as a teacher, pointing to the mistake (participatory culture)	Difference between $\frac{1}{f(x)}$ and $f^{-1}(x)$
[…]				
2-A: Some people are trying to be buzzkills by saying sin^-1 is angle and 1/sin is the actual ratio(cosec). However, those smart alecks don't realise the first mistake of the meme is that I didn't even mention the angle unit or anything lol. Try scrutinizing a little harder.	A defends the position, sharing information about what is considered as *the first mistake in the meme* which was not captured by the community	A attacks C1 making fun of C1's mathematical smartness (*smart aleck* = geek) The use of the slang word *buzzkill* (someone who ruins something enjoyable) tells us that A perceives memes as jokes rather than as mathematical objects	A refuses to be treated as a student and enters a competition about mathematical knowledge (participatory culture)	
3-C1: nice comeback after the mistake 😃😃	Ironically confirms the mistake	C1 gives a patronizing comment that positions A at a lower level of expertise The emoji says "don't take it too seriously"	C1 is self-positioning as an expert (participatory culture)	
4-C2: Arcsin can be 0 while csec can not. same for	Observes that the value 0 is in the range of the	C2 offers an explanation of why the meme is	Informal mentorship (participatory	

the arccos and sec	function arcsin (or arcos) while it is not in the range of the function csec (or sec) (reacts to an implicit *need for a disprove)*	*mathematically wrong*: if the codomains of the two functions are different *therefore* the functions cannot be in the relation implied by the meme (one example is enough)	culture)	
5-C3: The ^-1 doesnt necessarily means that they are a ratio it a sign that its inverse of that trig fn	Explains the misunderstanding connected to the different meanings of the notation ^-1	The use of the term *necessarily* gives the idea that C3 acknowledges the common meaning of the ^-1 sign for ratios (building on A's comment), but wants to make clear that there is another meaning	Informal mentorship expressly directed towards A (participatory culture)	
[…]				
6-C4: [replying to A] Whatever u say it's wrong even if there's no angle. An inverse trig function is not the same as raising it to -1 making it a reciprocal of itself. This meme is wrong, accept it!!! 😆 [in a separate comment] Also you can't raise a function to -1 and say that it's a reciprocal. You only do that to	C4 confirms the mistake and explains that the notation ^-1 means reciprocal only when applied to variables	C4 replies to A's defense confirming the mistake *even if there's no angle* C4'sconclusion about these comments being for *dumb alecks* (nice but not very clever guys) implies that C4's observations are well known mathematical facts that A does not know.	Mathematical memes as membership identifiers, thus they must follow the norm to be mathemati-cally correct (participatory culture)	

variables especially the independent ones like (x). Hahahha just commenting in behalf of the dumb alecks 😆 […]				
7-C5: [replying to A] Imagine trying to justify the conflation of different mathematical identities by saying you weren’t using units. Nice try, but have you ever heard of a functions domain? Trig functions have domains from -1 to 1 (except tangent) unless you’re using complex analysis to define them in some other way, which most people aren’t. Just accept that you’re wrong	Reply to A’s defense about the missing angle units, confirming the mistake	C5 supports A’s comment bringing in a not well related argument about functions range (erroneously referred to as domain)	C5 is self-positioning as an expert, allowed to mentor (participatory culture)	Domain and codomain of a function
8-A: [Day 3, 17:10] I did admit I was wrong lol. If you think I don't know anything about trigonometric ratios and angle,	A acknowledges the mistake, shows what has been learned and identifies C5’s mistake about the domain	Given the time span between the initial comment and this, A likely uses external resource to straighten the issue. A comes back to recognize the	A accepts the norm and achieves the meaning-making (convergence culture)	Final consideration about the incorrectness of the mathematical idea represented

then hear this: sin=1/cosec cos=1/sec tan=1/cot Whenever there's an inverse (^-1), it becomes the value of an angle. Also, (sin or cos or tan)^-1 =arc(sin or cos or tan), which I learnt after making the mistake. Moreover, another admin made this meme. The idea was just mine. And idk [I don't know] what you are saying about domain. You probably wanted to say codomain. I hope I don't have to justify myself anymore. I am just in school NIBBA. [a slang word for "guys"]		mistake and show off the newly acquired knowledge, *learnt after making the mistake*, scoring a point towards C5's domain/ codomain jumble. The new knowledge is exposed with emphasis, both verbal (*hear this*) and structural, in the layout of the writing (as opposed to the usual in line comments) to make concepts stand out. Dilemma: A wants to be acknowledged vs. uses the school background as an excuse for the mistake	A generalizes the norm to the behaviour in the community addressing C5's mistake (participatory culture)	by the meme
9-C5 [Day 4, 04:22] Understandable	C5's closing comment replies only to A's excuse for the mistake and accepts it	When A shows some willingness to learn and reveals as a high school student, C5 recognizes A the right to make mistakes	Final informal mentorship (participatory culture)	

C1 shows the need to check the correctness of the meme mathematically. This initiates a negotiation on how a meme and the encoded mathematics should be perceived, either as a mathematical object or as a joke (#2, #3). A's defence is initially not accepted (#3, #6, #7) as A's comment does not show any epistemic need to reason or ask for reasons necessary when something is incorrect. In the following interactions the epistemic culture is re-established by the participatory culture which is usually implemented in the community. The comments express actions towards needs aimed at disproving the mathematical idea encoded in the meme (#4), giving reasons (#4, #5, #6, #7), explaining (#5, #6), and mentoring others (#6, #7). The community is strongly committed to the fact that the mathematics encoded in the meme must be mathematically correct (#6, 7#). The commenters' participation can be taken as expressing what experts do with incorrect mathematical ideas: they take them in and disprove them, showing a need for proof and its underlying need for certainty.

The incorrectness of the meme is finally recognized by A (#8). By finding mistakes in the experts' comments, too, A appears not fully acquainted with the epistemic culture established in the community, which tolerates mistakes in the epistemic interaction but not in the meme. By stressing that the meme was another admin's creation, A refuses to take responsibility for the mistake (apologizing), showing a need to explain the background situation rather than an epistemic need. A uses the declared status of high school students as an excuse for the incorrectness of the meme. However, as soon as A shows some willingness to learn, the mistake is accepted (#9).

The convergent meaning-making process about the incorrectness of the mathematical idea represented by the meme comes about only in the end, hindered by A's initial reluctance (#2). This can be interpreted as an evidence of the perceived value of these digital objects as *identifiers of the belonging to the community*. Three commenters (C2 C3 C4) provide diversified and mathematically consistent information, from counterexamples to notation conventions, aimed at sustaining the first negative comment (C1). Finally, some mathematical knowledge is constructed: A acknowledges the mistake made but at the same time tries to reaffirm the position in the group by making in turn note C5's domain/codomain error.

Drawing all together, we have reconstructed the practices of convergence and participation in this community. Community members share the norm that the mathematics in the meme must be correct. Epistemic needs related to the memes drive the meaning-making (convergence), from there members act fulfilling the needs based on the norm of keeping a certain expert level but being also open to include those who express epistemic needs. We call this the *meme epistemic culture*, fostering a process of argumentation, driven by epistemic needs, resulting in proofs and clarifications around the mathematical idea in the meme.

These two polar types epitomize a practice that our participant observation revealed as characteristic for the interaction with mathematical memes in dedicated online communities. Indeed, our comment pool investigation showed that, apart from emojis and friends tagging, comments to mathematical memes are always at the mathematical level, aimed at proving or disproving the mathematical idea encoded in the meme, and never at the memetic level (e.g. asking explanations about the meaning of a particular image). We take this as evidence of the fact that users are familiar with the rules of the memesphere and therefore do not need support in this field. However, this indicates also that these communities perceive mathematical memes as mathematical objects rather than as entertaining objects, as suggested by name choices as Mathematical *Mathematics* Memes. Another relevant point is that, even if the understanding of a mathematical meme requires the decoding of the encoded link, comments do not make this decoding process explicit. E.g., in case 2, C1 does not write “the equalities sin^{-1}=cosec, cos^{-1}=sec and tan^{-1}=cot are mathematically wrong”, but simply “the meme is mathematically wrong” (#1), assuming that everybody gets the equalities encoded in the meme. Comments are all directed towards giving arguments to prove or disprove the mathematical idea expressed by the meme, explaining *why* the functions are different or explaining *why* using L'Hôpital's rule to solve the limit is incorrect.

To sum up, interacting with a mathematical meme implies a two-step process: the first step is the decoding of the mathematical idea, and the second step is the argumentation needed to determine whether this mathematical idea is correct or incorrect. The first step requires memetic and mathematical knowledges: this step is not externalized in the comments; thus, we assume that users achieve it on their own. The second step requires mathematical knowledge only and is what the community actively engages in: a collective meaning-making process of the correctness of the mathematical idea, activated by an epistemic culture. This meme epistemic culture even enriches the epistemic process with further clarifications about the mathematical theme, for example offering more than just one proof although one would be enough or explaining concepts at the periphery of the meme.

6 Results and theorizing towards a conceptualization of the phenomenon

Our research questions aimed at characterizing the Internet phenomenon of mathematical memes as a new form of representation of mathematical ideas and at exploring how they activate interactions among members of online communities. We have investigated these issues applying a twofold theoretical background to a participant observation of the field and an instrumental case study. Results are summarized in Table 6.4.

Table 6.4 A summary of the findings

Theoretical background	Memetic	The memetic constituent elements of mathematical memes are: humour and emotions, spreadability and diffusion, and image macros structural and compositional rules. In mathematical memes images are used to represent effects (tangible or emotional, always mathematically rigorous) or elements of the mathematical idea hinted by the text
	Cultural	The cultural features of the Web 2.0 that inspire the production and use of mathematical memes are participatory culture and convergence culture. These features shape the perceptions of mathematical memes as membership identifiers and as learning opportunities inside the communities
Empirical study	Participant observation	Threads of comments with explicated mathematics are a characterising practice of the interaction with mathematical memes as representations of mathematical ideas. This practice is highly respected inside the community, where mathematical memes and the accompanying comments are recognised for their educational value.
	Instrumental case study	The interaction with mathematical memes as representations of mathematical ideas revealed in the threads of comments focuses on discussing the correctness of the mathematical idea encoded in the meme. In these interactions, epistemic needs are activated. They drive the discussion, handled with rigor, resulting in a process of transfer of mathematical knowledge, where engagement is highly valued

The memetic and cultural perspectives provide grounds to characterise mathematical memes as humorous and spreadable artefacts where images and texts are combined to represent mathematical ideas. They are cultural objects, created respecting both the laws of the memesphere and those of mathematics, and shared within dedicated communities to acquire a status. Participant observation and a case study illustrate mathematical memes' educational potential, provided by the epistemic culture activated by the meme itself and leading to a collective meaning-making (cf. Kruglanski, 2010). The meme epistemic culture in the communities is (re-)established by an embedding participatory culture of acting epistemically, and by a convergence culture driven by epistemic needs. Members of the communities spontaneously tend to organize themselves according to a social order depending on mathematical skills and interact following shared sociomathematical as well as social norms (Yackel & Cobb, 1996). The key sociomathematical norm requires mathematical memes to be correct, a key social norm implies, among others, an assumption of expertise, which manifests itself in the fact that, unless epistemic needs are expressed, commenters never overexplain.

These results allow us to move further in conceptualizing the phenomenon. We have seen that comments focus mainly on the rigorous assessment of the correctness or incorrectness of the mathematical idea encoded in the meme. Our explanation is that mathematical memes represent mathematical ideas by their properties that can be logically proved or disproved. Therefore, we hypothesize through abduction (Peirce, 1931, CP 5.189; cf. Arzarello et al., 1998; Douven, 2011) that mathematical memes represent *mathematical statements* in the sense of propositional logic (Durand-Guerrier, 2008; Cook, 2009), that is declarative propositions whose truth-value can be determined as either true or false through logically valid arguments (i.e. arguments that originate from premises assumed as true and that develop in a truth-preserving way).

From this, we propose a *conceptualization* of the observed phenomenon:

> *Mathematical memes represent mathematical statements. Elements that compose the statement can come from the textual part, from the image or from a combination of the two, in a new hybrid language which merges mathematical elements with memetic elements, rigorously abiding by the rules of both fields. The need to determine the truth-value of the statement evokes a negotiation shaped by an epistemic culture on the mathematics encoded.*

To validate this hypothesis, we tested it on our meme pool. For each and all of the collected memes we were able to express the matching statement. Due to space constraints, we will only show here the correspondence between mathematical memes previously presented in this paper and the related statements, along with their truth-value (Table 6.5).

Table 6.5 Mathematical memes as statements

Figure	Meme	Statement	Truth-value
6.3 left	Me and the boys	$y = x^x$ cannot be integrated in elementary terms	True
6.3	Me and the boys	$y = \frac{sinx}{x}, y = e^{-x^2}, y = \sqrt{sinx},\ y =$	True

right		$\sqrt{1+x^3}$ cannot be integrated in elementary terms	
6.4	Me and the boys	In the complex plane, the effect of multiplying by i is a $\frac{\pi}{2}$ rotation	True
6.10	Success kid	$(a+b)^2 = a^2 + 2ab + b^2$	True
6.11	Spiderman pointing at Spiderman	$\frac{d}{dx}(e^x) = e^x$	True
6.17	Mega wrong	Evaluating $\lim_{x \to 0} \frac{sinx}{x}$ with L'Hôpital's rule is a logical fallacy	True (assuming certain premises)
6.18	Spiderman pointing at Spiderman	$sin^{-1}(\alpha) = cosec\alpha;\ cos^{-1}(\alpha) = sec\alpha;\ tan^{-1}(\alpha) = cot\alpha$	False

This conceptualization fits with all the characteristics that emerged from our empirical analysis: it explains that

- mathematical memes are rigorously formulated from the memetic and mathematical standpoint because they are mathematical statements,
- they are taken as membership identifiers as they display mathematical expertise,
- the discussion focuses on their truth-value and, thus, mobilizes mathematical information and knowledge as prerequisites for (dis-)proving and understanding.

It also explains why memes are not explicitly decoded in the comments, as the community members are aware that paraphrasing a statement does not count in the process of argumentation.

Given that mathematical memes represent mathematical statements, we can differentiate what is and what is not a manifestation of the phenomenon, distinguishing these objects from other digital objects shared within these communities, as emotional memes and cartoons. In fact, the emotional meme seen in Figure 6.14 (left) conveys the message *math is puzzling*, which is not a mathematical statement, because its truth-value is subjectively evaluated. While the cartoon in Figure 6.14 (right), offers a *cartoonised* version of the effect of the Fourier transform that cannot be translated into a provable or disprovable mathematical statement, as opposed to the *Me and the boys multiplying by i* meme in Figure 6.4.

Going back to Gitelman's two-level model for media cited at the beginning (2006), we can say that a mathematical meme is firstly a technology that enables the communication of mathematical statement. Secondly, a mathematical meme is embedded in a set of social practices and cultural norms, implying that the mathematical statement must be true, and shaping an epistemic culture where the meme activates and guides epistemic processes about the mathematics addressed in the statement.

7 Discussion, reflection, concluding remarks

Ordinary mathematical representations can take the form of numerical, algebraic, graphical, verbal mathematical expressions, sometimes including logic operators (Goldin, 2014). They are formal, relieved from any possible emotion that mathematics can spark. Not so in the representation of mathematical memes, where mathematical statements are taken as rigorously as in mathematics, but their decoding requires additional knowledge from the memesphere. Only a deep understanding of the encoded mathematics unfolds the power of the joke in the meme. Such a hybrid representation of a mathematical statement is no longer dry, it addresses emotions although being rigorous with respect to mathematics. This may provide widened access to mathematics for an enlarged range of learners.

Our conceptualization resulted in the insight that mathematical memes represent statements and activate an epistemic culture in the online community. This is an important finding pointing to their educational potential for constructing mathematical knowledge. It expands and updates our semiotic capital to represent mathematics. Following O'Halloran’s idea that “a culture may be understood as a typical configuration of choices from a variety of semiotic resources” (2005, p. 6), this can open new research paths of investigating hybrid representational systems in mathematics. Another research path may address learning in everyday classrooms, investigating there how mathematical memes activate and drive mathematical discussions and how this fosters interest in mathematics.

Our findings can also be a first step to figure out how mathematical memes can be used for teaching in traditional educational settings. In fact, statements are the building blocks of mathematics and proving statements is the building block of any mathematical activity. Thus, mathematical memes could be used in argumentation-based learning scenarios, where their manifestation as objects closer to students’ experience than to classical mathematical registers can leverage on learners’ popular culture to engage students in the essential mathematical practice of justification.

Naturally, this work has some limitations. As a case study, although taken from and supported by a big meme pool, our findings do not offer information about the scope of the study, e.g. how other online communities treat these memes or if there are online communities creating mathematical memes that cannot be taken as statements. Further, we do not know the members of these communities and the time they invest in making the communities work as shown in the investigation. But our study has also a certain strength: our conceptualizing is ecologically valid, because it is rooted in the ethnographic approach which made us gain data of high ecological validity. As a lurker, the observer knew what was relevant for community members but also was controlling this by comprehensive data collections and analyses. Ecological validity was further substantiated by mapping the field with a systematic funnelling procedure to gain paradigmatic cases for our theorizing, which added ecologically significant data to be analysed on a micro level from inside the interaction in the communities.

This empirically founded conceptualizing could provide a well-established basis for further investigations of the memesphere and testing of our findings. Since we have gained a new kind of representation, semiosis related to discussing the meme' mathematical ideas could reveal deeper insight into the mechanisms of knowledge construction activated by the meme, and semiosis related to meme mutations could reveal scientific as well as educational knowledge of the encoding mechanisms, which may inform teaching.

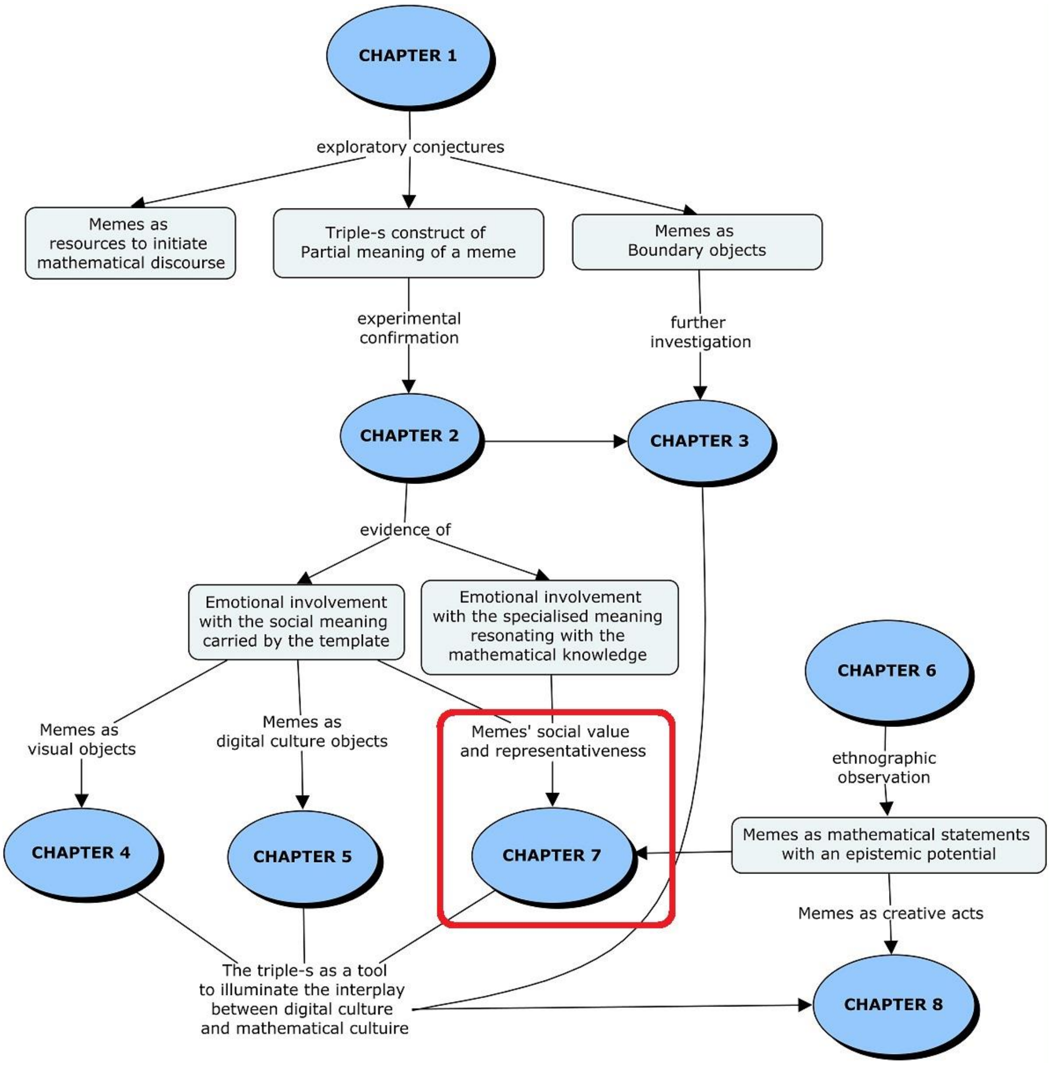
Observing that Internet culture is in constant change, we conclude that we cannot predict if, a few years from now, mathematical memes will still have the same characteristics as those described in this study or will exist altogether. Nonetheless, even if memes, as we know them, will go out of fashion or morph into something different, the present work aspires to be a first step in the investigation of the evolving epistemic mechanisms of the new generation of convergent and participant learners, with a value that may go beyond memes and their possible transience.

CHAPTER 7: THE SYNERGY OF MEMES' SOCIAL VALUE AND REPRESENTATIVENESS WITH KNOWN EDUCATIONAL TOOLS

WHEN THEY TELL YOU THAT $i^{56}=1$: AFFORDANCES OF MEMES AND GEOGEBRA IN MATHEMATICS

This is the Accepted Manuscript of an article to be published in the International Journal for Technology in Mathematics Education, Volume 16, No 3 (forthcoming).

Positioning the Chapter in the research



This Chapter builds on the findings from Chapter 3 about students' emotional involvement with the social and specialised meaning of the meme to investigate how these elements, paired with memes' social value and representativeness, combine synergically with GeoGebra in activating students' ZPD and promoting changes in their praxeologies. The study follows three students with different learning styles combining observations from a school experiment and interviews conducted one year later and uses the triple-s as an investigation tool keep track of the emergence of mentors and agents fostering changes in students' praxeologies. The results show that mathematical memes can blend with other educational resources, adding a social component to the activity that reaches students usually not engaged in mathematics.

Abstract: This work aims at investigating the educational potentialities of mathematical memes, digital objects belonging to students' out-of-school culture, in combination with classic educational objects as GeoGebra applets. To pursue our goal, we observed a group of 29 12th-grade students who design, create and interact with mathematical memes and GeoGebra applets on complex numbers, during an experiment carried out in Turin in March 2019. Our analysis follows three students, from the design and production phases to the group discussion and the interviews conducted the following year. Specifically, we looked for changes in students' praxeologies (following Chevallard's ATD, 1992, 1999) and we traced them back to ZPD mentors (Vygotsky, 1978; Wood et al., 1976) and micro-level agents (Prodromou et al., 2018). Results show that the social value of mathematical memes promotes the appearance of new micro-level facilitators that enhance the educational potentialities of GeoGebra and contribute to foster changes in students' praxeologies.

1 Introduction

On a daily basis, mathematics educators face the challenge to design class activities that cater for the maximum scope of learning styles (Chinn, 2004; Devlin, 2000), involving all students, from the less motivated who typically linger at the borders of what happens during class, to the more attentive and focused ones. One acknowledged way to reach a wider scope of learning styles is accomplished by seeking a continuity between school and out-of-school learning contexts (Bronkhorst & Akkerman, 2016). Building on this assumption, we investigate if mathematical memes, digital objects belonging to students' out-of-school digital culture, combined in a specifically designed task with school objects as GeoGebra dynamic geometry applets, succeed in building this continuity, engaging students with different learning styles and promoting changes in their practices of learning mathematics.

The research is focused on three students in a 12th-grade class group of twenty-nine: two usually low-engaging learners and a high-achiever. We looked for macro evidence indicating a change in students' praxeologies (Chevallard, 1992, 1999) and we connected it to the different components of the activity: the mathematical meme, the GeoGebra applet, and the class discussion. Building of the fact that "as in thermodynamics macro quantities such as internal energy or temperature are due to micro quantities as particles' velocity" (Prodromou et al., 2018, p. 447), this macro-level correlation is investigated through a micro-level analysis. This micro-level analysis is conducted via the lens of the triple-s construct of the partial meanings of a meme (Bini & Robutti, 2019a), that allowed to distinguish the semiotic components of the meme and connect them to the observed changes in praxeologies, and to the appearance of facilitators as mentors (Vygotsky, 1978; Wood et al., 1976) and micro-level agents (Prodromou et al., 2018).

2 Literature

The importance of mathematical reasoning, critical thinking and creativity for future career plans of current students are at the core of PISA 2021 mathematics framework[68]. Therefore, educational efforts should be aimed at making mathematics education as inclusive as possible, in order to maximise the scope and effectiveness of the learning of mathematics for all students.

The affordances of a dynamic geometry software such as GeoGebra, and in particular of its characteristic feature of continuous dragging, are widely recognised in mathematics education (Arzarello et al., 2002; Sinclair & Robutti, 2012), and their benefits for different kinds of students are well investigated (Gawlick, 2002), while Internet memes, although considered emblematic products of the XXI century digital culture (Shifman, 2014; Wiggins, 2019), are significantly understudied in educational literature (Knobel & Lankshear, 2007, Harvey & Palese, 2018).

Internet memes, or simply memes, are humorous pieces of digital visual content created by Internet users adapting and mutating popular images (referred to as templates), according to a practice well established in the digital culture (Milner, 2016; Shifman, 2014). Some memes can be just representations of anecdotes or jokes, but many of them go way beyond their frivolous appearance, conveying meanings that cover a wide range of topics, from personal emotions and political protest to mathematical ideas (Benoit, 2018; Bini et al., 2020). Memes are created captioning templates that comply with a commonly recognised structure, according to well-established insider rules (see KYM www.knowyourmeme.com, the online encyclopaedia of Internet Memes). Thus, they are endowed with a social value as bonding signs among users. Once created, they spread in their digital cultural environment, known as memesphere (Stryker, 2011), typically constituted by social networking websites like Facebook or Instagram.

In particular, memes conveying mathematical ideas (Figure 7.1), are very popular on the Web, where they are shared inside dedicated online communities that read them as hybrid representations of mathematical statements, connecting the spheres of digital culture and mathematics (Bini et al., 2020). In the example in Figure 7.1, from a mathematical memes' community inside Facebook, the author of the meme added the text "constants when they see

[68] https://pisa2021-maths.oecd.org

d/dx” to the popular template known as Black Guy disappearing (source KYM). This template is widely used in the memesphere (without any racist intent) to describe circumstances where someone leaves an undesirable situation (same source). Here the author used it to convey the mathematical idea that constants disappear when the derivative is taken. The mathematical meme can be therefore be seen as the hybrid representation of the mathematical statement d/dx (c)=0. This is evidence of the author’s ability to see a non-mathematical sign (the act of disappearing performed by the guy in the meme) through the eyes of mathematics, linking it to a proper mathematical idea (the nullification of a constant).

Figure 7. 1 An example of a mathematical meme from the Web

Since mathematical memes subsume elements from diverse spheres of culture inside and outside school, we think that they have unexplored educational potentialities to reach students otherwise less involved in the learning of mathematics, initiating interests linked to students’ social life (create an original and funny meme, be liked, gain visibility inside the class group), that can work synergically with the acknowledged educational potentialities of GeoGebra for the learning of mathematics.

3 Theoretical Background

Given that our interest is on students, their learning styles, their connections with the Web 2.0 culture, and their possible changes in conducts and responses, the choice of framing our research within Vygotsky’s sociocultural approach and Chevallard’s (1992, 1999) Anthropological Theory of Didactics (ATD) seemed appropriate.

In particular, we focus on the tenet of Vygotsky’s theory regarding how learning happens as a result of the interactions within a social environment. According to this theory, learning takes place if the sociocultural environment surrounding the learner conducts him/her from the level of actual development, where students can successfully perform tasks independently, to a different level named the Zone of Proximal Development (ZPD), where students can successfully perform tasks “under adult guidance or in collaboration with more capable peers." (Vygotsky, 1978, p. 86). This guidance is usually referred to as scaffolding (Wood et al., 1976), and the more capable peers that surround the learners and provide it as mentors, who, like the character from Homer’s Odyssey where the term originates from, nurture and support the learning process. In more recent times, the advent of digital technology in education added new dimensions of scaffolding, widening the range of possible mentors from the original Vygotskian physical mentors (peers, teachers or other adults) to technological mentors (related to the use of technological artefacts) (Pea, 2004; Faiella, 2005).

Focusing more specifically on the learning of mathematics, Chevallard’s ATD theory (1992, 1999) provides a model of mathematical knowledge in the school, conceived “within the set of human activities and of social institutions” (Chevallard, 1999, p. 223). The main theoretical principle of ATD is the notion of praxeology, which is structured into two levels:

- The practical level or praxis, i.e. know-how, composed by tasks (assignments) and techniques (procedures needed to carry out the assignments).
- The theoretical level or logos, i.e. know-why, composed by technologies (explanations and justifications supporting the techniques) and theories (general structures supporting the technologies).

This theory offers us a framework to distinguish the practical and theoretical aspects of the observed activity, in order to be able to spot at macro-level how the students interact with them, and to detect if any changes occur at each level. Nevertheless, if students adopt new praxeologies or change their acquired ones, ATD does not shed light at a micro-level about the possible starting causes. Therefore, we integrated this theory with the notion of emergence and agents as described by Prodromou et al. (2018) in their work, built on the meta-didactical transposition, the extension of ATD to mathematics teachers' professional development (Arzarello et al., 2014, Robutti, 2020). The term emergence is used by Prodromou et al. for the interaction of independent micro-level agents, i.e. causes that shape, support, and influence the macro-level components of the praxeologies. Prodromou et al. in their work identify four kinds of agents: methodological (related to teaching methods), institutional (related to curriculum, national assessment, and syllabuses), material & technological (related to physical and virtual tools), and motivational (related to the influence of the communities), whose interaction contributes to shaping the observed praxeologies and their possible changes. Being this a theory born to investigate teachers' praxeologies, we do not expect to find all these same agents in the observation of the students; nevertheless, we think that the concepts of emergence and agents fit with the goal to identify the causes of change or adoption of new praxeologies in the sense of ATD.

To sum up, mentors provide scaffolding in areas that students are not yet familiar with, while agents prompt individuals to autonomously initiate changes in praxeologies. We consider both mentors and agents as micro-level elements because they were not part of the macro variables, we had direct access to during the experiment (i.e. the components of the activity and the students' praxeologies). Nevertheless, we expect to infer their emergence looking deeper into the macro-level connection between activity and praxeologies through the lens given by the triple-s construct of the partial meanings of a meme (Bini & Robutti, 2019a). According to this construct, the full meaning carried by a meme can be analysed by taking apart three partial meanings:

- A social meaning, conveyed by the message carried by the template, established by shared rules in the memesphere (in Figure 7.1, it is the conventional use of the template to represent situations where the subject vanishes as a consequence of a particular condition);
- A structural meaning, that lies in the meme's characteristic aesthetic: text font, colour and position, and overall visual impact (in the example in Figure 7.1, it is linked to the reading path of the sequence of images in the template and to the role of the caption);
- A specialised meaning, carried by textual or pictorial elements in the meme referring to a specific topic, in our case mathematical (in Figure 7.1, it is the rule for differentiating constant functions, represented in the text by the word "constant" and by the mathematical symbol d/dx, and in the template by the act of disappearing that becomes a sign standing for the =0 part of the statement).

Separating the partial meanings of the meme allows us to differentiate the elements of the meme that relate to the sphere of digital culture (structural and social meanings) from those afferent to mathematics (specialised meaning), in order to get insight into how these elements relate to the emergence of mentors and agents, and to the components of the praxeologies, as represented in Figure 7.2.

To sum up, our research questions can be articulated as follows:

- RQ1: At a macro level, what elements of mathematical memes and GeoGebra applets activate changes in students' praxeologies or some of their components?
- RQ2: At a micro-level, how are these changes activated in terms of mentors and agents?

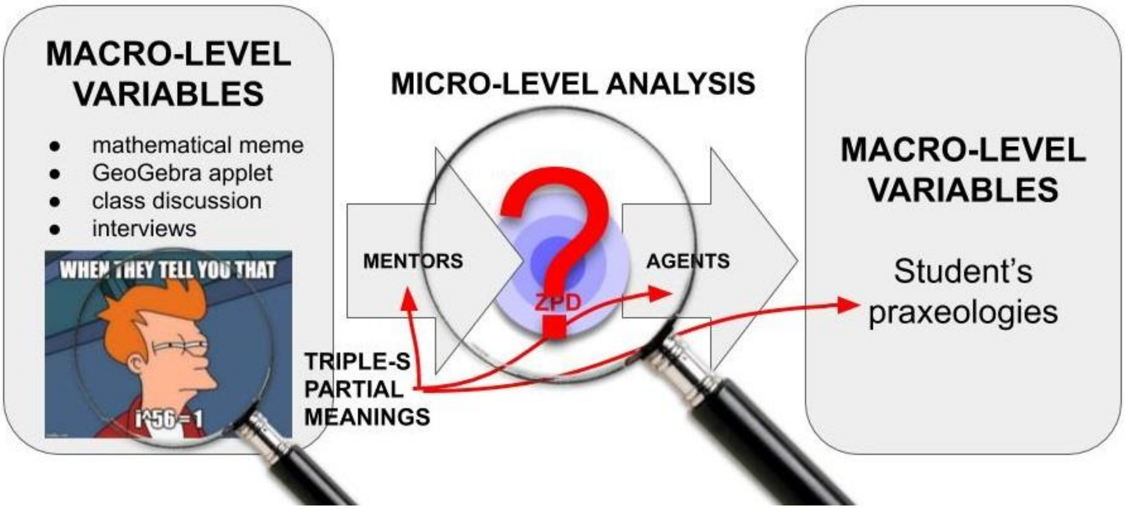


Figure 7. 2 Connecting the macro and micro levels of the research

4 Methodology

This work draws on the first author's doctoral research on mathematical memes, developed throughout the last three years exploring online communities and conducting school-based experiments, and on the third author's Master thesis, both developed under the supervision of the second author, Ornella Robutti. This study presents the cases of three students, observed during an experiment conducted in March 2019 in Turin with a class group of 29 12th-grade students (17 years old). The first author was present and collaborating with the teacher, while the third was present as a nonparticipant observer. (Liu & Maitlis 2010). Following our purpose to observe the engagement of students with different learning styles, we selected students at the opposite ends of the spectrum of mathematical involvement. We decided to observe the extremes because these students are the most challenging to reach for educators, the less interested being at risk of disengaging from mathematics, and the more acute at risk of getting bored when not sufficiently stimulated. The selection was made according to the teacher's evaluation based on criteria of participation and assessment: the first two being low-engaging learners (a faithful translation of the term used by the teacher himself) and the third being a high achiever. The experiment took place after the topic of complex numbers had been completed and aimed at encouraging students to reorganize knowledge on the subject. It was divided into two parts: one 3h class dedicated to the creation activity and a 2h class the following week dedicated to a whole-class discussion of the students' productions and to the reaction to mathematical memes on the same topic created by the first author, in a memetic variation of Arzarello's semiotic game (Arzarello et al., 2009).

The task (in the sense of ATD) assigned to students was designed with the purpose of capturing the interest of a wider scope of students by seeking a continuity between out-of-school context (the meme belonging to the digital culture) and school context (the GeoGebra package). Thus, we asked students to "Create a mathematical meme on a particular aspect of the topic of complex numbers and a GeoGebra applet decoding the mathematical content of the meme". The task included also the request to create a video explaining the mathematical content of the meme; for reasons of space constraint, the video will not be analysed in this work and will not be mentioned further. Technically, we relied on the fact that students already owned the techniques to create a meme, thanks to their exposure to these objects in their out-of-school life, as well as the techniques to create a dynamic geometry applet, thanks to the school use of GeoGebra encouraged by the teacher since grade 9. The mathematical meme was created with a popular meme maker Web app[69] and the dynamic geometry applet with the GeoGebra software already downloaded onto the devices in the school computer lab. Students worked in pairs producing one meme and one GeoGebra applet for each pair, and creations were uploaded on the Web in a collective digital space (www.padlet.com), shared inside the class group, that allowed comments and likes, mimicking the memesphere.

Data collected during the activity include mathematical memes, GeoGebra applets made by the students (Appendix E), screencast of the creative processes of two pairs selected on the recommendation of the teacher, memes created by the first author (Appendix K), and videotaping of the following discussion. In addition to these data, we collected feedback questionnaires immediately after the activity (Appendix L), and videotaping and transcripts of the interviews conducted one year after by the third author with the students featured in this study. It is interesting to note that the teacher had so little faith that the low-engaging students could produce something interesting that they were not in the screencast pairs; nevertheless, we managed to reconstruct

[69] https://imgflip.com/memegenerator

their creative path through our field notes and the students' own recollections in the following interview.

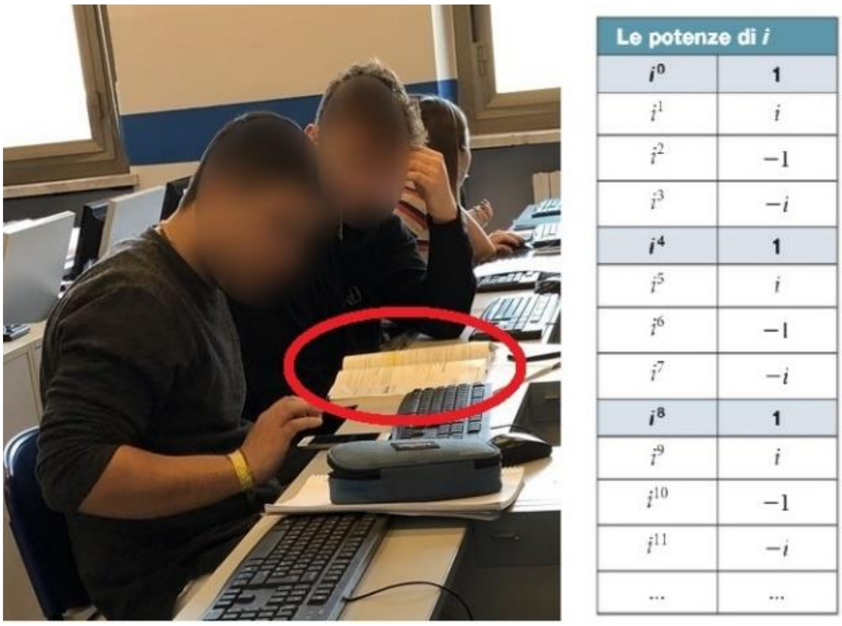
Using the theoretical tools provided by the framework, we will focus our analysis on the creative process of the two low-engaging students, and on the outcome of their work on their mathematical reasoning and on that of the high achiever during the following discussion, in terms of techniques, technologies and theories. We will start with the analysis of the partial meanings of the meme they created, and through the analysis of some excerpts of the interviews and of the following discussion, we will connect these partial meanings to the different mentors and agents that emerged during the activity showing how these facilitators are involved in the scaffolding of the students in their ZPDs and in the change of students' praxeologies.

5 Data and Analysis

5.1 The case of the low-engaging students

Our low-engaging students (who shall be referred to as L1 and L2 from now on) are friends, as well as desk mates at the time of the implementation of the activity: it is, therefore, no coincidence that they paired together for it. In the class group they are considered funny, but not too interested in school activities, especially mathematical ones, and, according to the teacher, they do not usually take notes during mathematics lessons.

In fact, when assigned with the task described in the methodological section, they did not have a personal notebook to refer to as the majority of their classmates. They used the textbook (Bergamini et al., 2016, pp. 941-942) highlighted by the red oval in Figure 7.3 (left, (faces are blurred for privacy), which looked pristine, and rummaged through it extensively before selecting the topic of the powers of the imaginary unit, which in the textbook is presented via the table shown in Figure 7.3 (right), accompanied by a verbal explanation. We think this is the first macro evidence that the combination of a mathematical meme and a GeoGebra applet prompted in the students the adoption of a new technique to perform the task.



Le potenze di i	
i^0	1
i^1	i
i^2	-1
i^3	$-i$
i^4	1
i^5	i
i^6	-1
i^7	$-i$
i^8	1
i^9	i
i^{10}	-1
i^{11}	$-i$
...	...

Figure 7. 3 L1 and L2 consulting the textbook during the activity (left) and the selected topic as represented in the textbook (fight)

Once settled on the mathematical topic, L1 and L2 begun their production with the meme shown in Figure 7.4 (left, translated by the authors), whose partial meaning can be analysed as follows:

Figure 7. 4 The mathematical meme created by L1 and L2 (left) and its social (centre) and structural meaning (right)

- Social meaning: the template, known as Futurama Fry (Figure 7.4 centre, source KYM), is used to represent an internal monologue sparked by a puzzling situation;
- Structural meaning: Interior Monologue Captioning (source KYM), meme with top/bottom text following a phrasal template "when they tell you that…" in the typical Impact white font (Figure 7.4 right);
- Specialised meaning: the cyclic pattern of the powers of the imaginary unit in the complex field.

It is significant to highlight that the case of i^56 was not among the examples presented in the textbook, that simply stated: "from the table [see Figure 7.3 right] we note that the powers cycle with period 4". Therefore, we can say that L1 and L2 grasped the technique from the textbook and applied it to a power sufficiently high to justify the emotion conveyed by the social meaning of the chosen template. In other words, they adopted a new technique in order to align the social and specialised meanings of the meme.

We see this as the emergence of the interaction of three agents: the institutional, connected to the mathematical topic addressed by the specialised meaning, the material & technological, connected to the use of the meme-creating platform, and a new kind of agent, directly connected to the digital culture represented by the meme, that we identified as a social agent.

The GeoGebra applet created by L1 and L2 shows that the students, driven by the task, independently moved from the praxis to the logos components of the praxeology. L1 and L2 used their techniques regarding the use of GeoGebra to build a convincing representation of the technology, i.e., the explanation supporting the techniques. Again, it is relevant to note that the textbook presents the topic from the algebraic perspective only, therefore L1 and L2, probably accessing other online resources through their smartphones as can be seen in Figure 7.3 (left), spontaneously adopted new technologies to build the geometric representation of the cycle of powers in the complex plane.

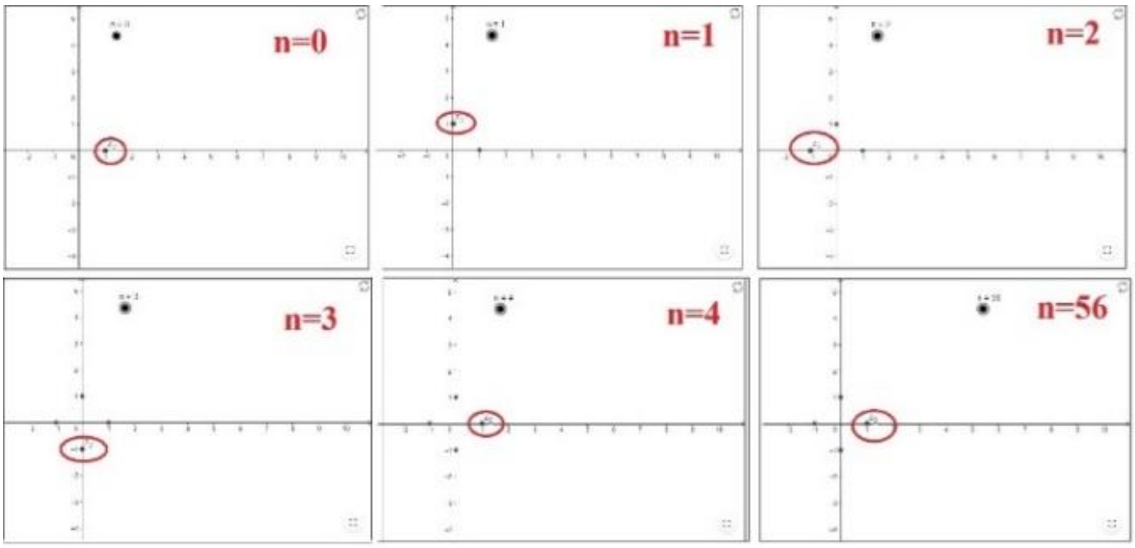


Figure 7. 5 Screenshots of the dynamic applet by L1 and L2

The creation of the applet underwent an interesting process: L1 and L2 initially built it as a static representation of the four powers of the imaginary unit on the Gauss plane, and subsequently made it dynamic with the addition of the slider n, following the suggestion of the first author, who was summoned by the students to check the ongoing work. The slider n itself is the result of a development: L1 and L2 initially created it with GeoGebra's default increment (0.1), therefore it moved the point z, defined as z=i^n, along a circular path constituted by the intermediate positions in addition to the four intended cardinal points. Later the teacher intervened suggesting adjustments of the increment of the slider n in order to have integer powers only, and L1 and L2 arrived at the final version, visible at https://www.GeoGebra.org/m/cbvr89sn and in Figure 7.5, where screenshots of the positions of z=i^n for n= 0, 1, 2, 3, 4 and 56 are shown.

Here we can observe that the specialised meaning of the mathematical meme facilitated the emergence of the institutional agent, activating the adoption of new technologies (the theoretical justification). It also enhanced the affordances of the GeoGebra dynamic geometry software, enabling the emergence of the material & technological agent, that prompted the adoption of new techniques (the correct use of the slider), scaffolded by physical mentors (the researcher and the teacher).

Altogether, the task resulted in an increase of the time dedicated to mathematics by L1 and L2, if compared with their standard class participation as reported by the teacher. This is a first evidence of the effectiveness of the activity in engaging them. To sample the long-term efficacy of the activity and gain further insight into the possible mentors and agents acting at the micro-level, L1 and L2 have been interviewed by the third author one year after the experiment. We present here a few excerpts of the interview (Table 7.1), which was conducted with the two students together, to maximize the exchange of recollections between the two.

Table 7. 1 L1 and L2's one year after interview (excerpts)

Questions	Answers
What do you remember about that activity? If you remember it of course	1- L1: I remember we went to the lab and worked in groups to create a meme… 2- L2: …and our topic: we had chosen the powers of complex numbers, if I remember correctly ... [...] 3- L1: And ... so we explained how these complex numbers work, these powers and ... their repetition with ... precisely ... the numbers ... so every time ... for every number divisible by 4, the same power returns and … you get the same result

How did you choose them [the powers of i] among all the possible topics? was there a reason or ... " instinctively"?	4- L1: instinctively 5- L2: but... in my view we started from the template and then we added something that seemed appropriate...
Do you think that this activity helped you to understand something better? Or do you think that your knowledge is still the same?	6- L1: no, I think we understood certain things better, because to create the meme we have looked into the topics and so we have... let's say, studied them thoroughly to understand better how to use them, so it was useful […] 7- L2: yes, maybe [the activity helped] just to be able to review it [the topic], or review it without giving weight to the fact that you are reviewing it […] anyway, you have to know things to be able to explain them and... this probably helps because... you don't bother too much and you do it more lightly and... you don't notice it, in my opinion
Are there other things that struck you?	8- L1 I don't know, as I said before I found the whole "meme thing" very nice and, in my view, another thing that pushed us to do better was the fact that all the class could see it [the meme] and could react with like or dislike ... and that, in my view, was the added value for... to prompt us to do something better 9- L2: yes, sharing with friends ... maybe it could be a stimulus, to see who is able to me more [mathematically] precise, who could make it funnier... yes, let's say that competition helps...

In this excerpt, we see that L1 and L2, despite their reputation as low-engaging students, remember the activity quite clearly one year after (#1): they remember both the memetic element (i.e. the template used #5) and the mathematical content (#2 #3). This is a first evidence that the activity engaged them effectively enough to produce long-term memories. Further on, L1 and L2 convene first about the techniques adopted to perform the task (#4 #5): they have chosen the social meaning first, based on their emotional connection with the image, and then the specialised mathematical meaning. We interpret this as a confirmation of the emergence of the social agent noted before. We see the same social agent emerging when L1 and L2 acknowledge the importance of sharing their productions (#9): this social agent works synergically with the motivational and institutional ones, prompting them to adopt new praxeologies in order "to do something better", from the point of view of the memetic and mathematical culture (#6 #7). Lastly, both students reported that the "meme thing" enhanced their understanding of the topic

(#8 #9): since they worked mainly on their own, we can deduct that they were scaffolded in their ZPDs by a new kind of mentor that we call a virtual mentor. The virtual mentor is an abstract mentor who is located the memesphere that young people virtually inhabit and is strictly connected to the memetic element of the activity. His scaffolding comes from the social and sociomathematical norms (Yackel & Cobb, 1996) shared in the memesphere, that require the structural and social meanings of the meme to adhere to the established codes and the specialised meaning to be correct (Bini et al., GeoGebra2020).

5.2 The case of the high-achiever

We complete this section briefly describing how the third student, the high-achiever whom we shall now on refer to as H, reacted to this activity. H is one of the brightest students in the class group, very participatory and always full of questions to ask (sometimes even too much, according to the teacher). He is very interested in mathematics and aims at reaching complete knowledge and deep understanding of each topic addressed in class. H did not work with L1 and L2 in the creation activity but interacted with L1 and L2's productions during the whole class discussion held one week after. When the teacher was showing to the class group the GeoGebra applet created by L1 and L2 representing z=i^n (Figure 7.5), the following exchange took place:

H: But between one [integer] number and the other, does it [the point z] describe a circumference or...?

Teacher: Are you asking if it describes a circumference?

H: Between 55 and 56 there are decimal numbers... and for those decimal numbers which curve does it describe? Where does it go?

Teacher [addressing the authors]: They already know it because I told them...

L1: A circumference

Teacher: Why?

L1: [laughing]: Because it is so!

In his question, H spontaneously applied the variation theory (Marton et al., 2004, Arzarello, 2016), an approach well practised by the teacher. The adoption of this praxeology is activated by the interaction of the institutional agent (the curricular knowledge) with a new agent that we shall call epistemic, connected to H's drive toward a conceptual understanding of the topic. His question sparked a lively exchange among the classmates: L1 and L2 remembered the answer, thanks to their first version of the applet with the decimal slider, but as we can see they were not able to give appropriate technologies, i.e. theoretical justifications ("Because it is so"). The justifications were collectively built with the contribution of H and other students in the group, guided by the teacher. In this situation, we can say that H was scaffolded in his ZPD by a technological mentor (the GeoGebra applet) and a physical mentor (the teacher). It is relevant to note that H reacted only to the more traditionally scholastic part of the task, i.e. the GeoGebra applet. In the one year after interview (Table 7.2), this aspect was further investigated.

Table 7. 2 H's one year after interview (excerpts)

Questions	Answers
What do you think [about the benefits of this kind of activity]?	1- H: well… I would distinguish the observation and the creation of the memes, because, when you look at it, you see the meme, you find it nice and, if you have all the mathematical background to understand I, you laugh. However, I found interesting to create it, because creating it makes you think a lot and you have to find something nice but also something about mathematics and it is not so immediate, at least for me ... well, I like math anyway, but precisely I did not find it so immediate, we thought about it a lot because in mathematics there are many paradoxes and things that still make you laugh… However, once it is done, it is also interesting to do all the explanations ... that is, I found the part with all the explanations behind it more interesting, that is, the explanation of the meme
How did you come up with it? [referring to the question about the slider in the discussion]	2- H: well, I always ask myself in general about mathematics and everything, I ask myself for the graphic explanation because so I can understand it much better. I have always done this, mostly because our teacher also does so…

Unlike L1 and L2, H does not need social agents to prompt the adoption of new praxeologies or virtual mentors to scaffold him in his ZPD he is already interested in learning mathematics (#1) and the epistemic agent has already facilitated the internalisation of valid praxeologies to explore and unpack mathematical knowledge (#2). He relies on the teacher as a physical mentor (#2) and on the graphical representation as a technological one. He confirmed to be intrigued by the stimuli introduced by the memes; however, his attention remains tied to the mathematical components of the activity (which he refers to as "the explanations"), more than the memetic component.

Figure 7. 6 Our reconstruction of H's false memory

Nevertheless, the memetic component of this activity offered us openings for further investigations. In fact, during the interview, H shared a revealing false memory about L1 and L2's meme. He clearly remembered the specialised meaning (i^56=1), but he combined it with another template, the Spiderman pointing at Spiderman, with different structural and social meanings (Figure 7.6). The social meaning of this template, in fact, affirms the equivalence of the objects superimposed on the two Spidermen, while the original Futurama Fry template expresses a sense of doubt and perplexity about the equivalence. H's memory slip tells us two things: first, that mathematical memes convey a mathematical content that can reach learners independently of the memetic component, i.e., that mathematical memes do not only activate a social agent but also an institutional agent connected to the curriculum, and second it gives us insight into H's emotional relationship with the specialised meaning of the meme showing that it is different from that of L1 and L2.

6 Discussion

At the macro-level, our investigation showed that the activity with mathematical meme and GeoGebra applets, blending elements from the different cultural realms, succeed in engaging and promoting changes in students' praxeologies as described by Chevallard (1992, 1999). Evidence shows that this happened for students with very different learning styles, from the low-engaging L1 and L2 to the high-achiever H. In particular, using the triple-s construct as a magnifying lens to differentiate the elements of the meme, we observed that the less engaged students were reached by the components more directly related to the digital culture (social and structural meaning) that activated changes in their praxeologies, while the specialized meaning of the meme, in connection with the GeoGebra applet, activated the high achiever's already internalised praxeologies.

At the micro-level, we observed the intervention of some of the already known facilitators as mentors (physical Vygotsky, 1978, and technological, Pea, 2004; Faiella, 2005) scaffolding students in their ZPDs, and agents (institutional, material & technological, and motivational, Prodromou et al., 2018) fostering changes in students' praxeologies. We observed also that the social value of memes, which was preserved in the activity through the sharing of students' creation in the collective digital space that allowed reactions, promoted the appearance of two new micro-level facilitators, a social agent and a virtual mentor, that enhanced the educational potentialities of GeoGebra for the low-engaging learners. The activity offered stimuli and opportunity for growth also to the high-achiever, through the sharing of students' production and through the collective discussion that favoured the emergence of the epistemic agent.

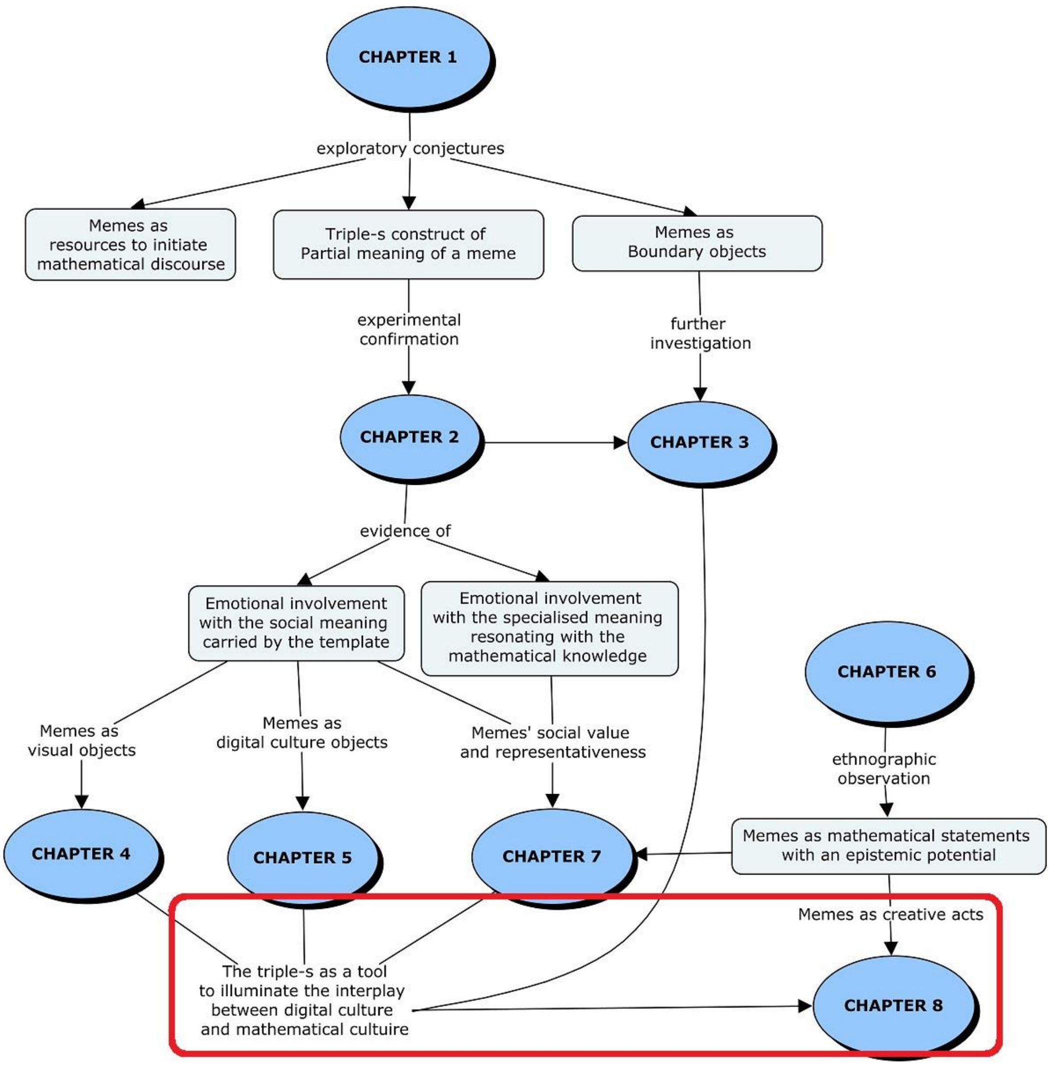
Although the current study is based on a small sample of participants, these results suggest that mathematical memes can successfully play the role of virtuous trojan horses, carrying their mathematical content in a form that makes it appealing for a broader scope of students and learning styles. We are aware that memes have affordances and constraints, and we are not suggesting that they should replace traditional teaching practices or the use of occasional anecdotes or jokes, but we think that they could enrich the range of teaching resources available to teachers, establishing a fruitful continuity between school and out-of-school learning contexts. Further research in this field would be of great help in assessing the solidity of our findings and explore in more depth the affordances and constraints of these objects.

CHAPTER 8: THE CREATIVE MECHANISM

HOW TO MEME IT: MATHEMATICAL MEMES AS CREATIVE ACTS[70]

[70] Manuscript in preparation

Positioning the Chapter in the research



The final Chapter draws from all the previous research steps: it builds on the interpretation of mathematical memes as representations of mathematical statements conceptualized in Chapter 6, and on the triple-s as a tool to illuminate the interplay between digital and mathematical culture emerged from the experimental Chapters. The study aims at providing educators with the knowledge to transform mathematical Internet memes into digital resources for teaching mathematics. This is done firstly assuming a widened concept of mathematical creativity that allows considering mathematical memes as products of creativity in mathematics, then using a suitable theory of the creative act to build a heuristic model of the mechanism of meme creation and identify how the digital and mathematical cultures intermingle to develop the hybrid language that characterises memetic mathematical statements. These results are subsequently interpreted to show how mathematical memes can be transformed into educational resources

Abstract: The Web 2.0 offers openings for displaying user-generated creativity on a variety of subjects, from the most futile to those linked to serious disciplinary knowledge. Mathematical Internet memes are an example of how this creativity reaches the field of mathematics, translating mathematical statements into a new digital form endowed with an epistemic potential capable to initiate a process of mathematical argumentation. The research presented in this paper aims at providing educators with the knowledge to transform mathematical Internet memes into digital resources for teaching mathematics. Theoretically, this is done assuming a widened concept of mathematical creativity and focussing on the creative act that links the digital culture to mathematics by distinguishing three different perspectives to interpret mathematical memes. Methodologically, mathematical memes' creative mechanism is explored in a dataset of circa 1500 memes, collected during a nearly three-year-long ethnographic research (February 2018–current) conducted within online communities in social media networking websites. An ethnographic analysis of the whole dataset is combined with an embedded study on exemplary cases, revealing a heuristic model for creating a mathematical meme. The results show how the Web 2.0 digital culture merges with and enriches the mathematics represented by the meme and shed light on the possible educational use of these digital objects.

"Mathematicians are like a certain type of Frenchman: when you talk to them, they translate it into their own language and then it soon turns into something entirely different."

Johann Wolfgang von Goethe, Maxims and Reflections 1279

1 Introduction

In September 2019, the image in Figure 8.1 was posted within an online community dedicated to exam tips for the UK General Certificate of Secondary Education, hosted in the popular social media website Reddit.

Figure 8. 1 ***I think I forgot something*** **mathematical meme**

The post was tagged as "revision resources", and sparked a lively thread of comments of which we report here an excerpt (comments are anonymised and unredacted):

1-Commenter 1: Is it bad that I'm doing a level maths and dont get it?

2-Commenter 2: Not gonna say yes or no but it's basically when expanding $(x+y)^2$ people always forget to multiply x by y and y by x so miss out the 2xy

3-Commenter 1: Ohhh yeah I get what it means now thanks

4-Author: I'm doing A-Level and it's been two weeks. Yet I barely understand anything. Basically $(x+2)^2$ means (X+2)(x+2) which then expands to x^2 +4x+4. But because it's $(x+2)^2$ instead of (X+2)(x+2) people forget

This is an example of a *mathematical Internet meme:* it is an *Internet meme* (or simply a *meme*), a humorous user-generated digital object created through the repetition and mutation of an image taken from the Web (Davison, 2012; Shifman, 2014), and it is *mathematical* because it addresses a mathematical topic, identified by a coalescing of key disciplinary signs [the symbols $(x+y)^2$, x^2+y^2, $2xy$] that pertain to the domain of mathematics (Love & Pimm, 1996; O'Halloran, 2005). Memes are so popular on the Web that there are websites expressly conceived to generate them (Imgflip[71], Kapwing[72]), and specialised online encyclopaedias [KnowYourMeme[73] (KYM), Meming Wiki[74] (MW)]. In particular, mathematical memes are so widespread that communities in social networking websites are dedicated to them, constituting the cultural environment where mathematical memes are created, mutated, shared and commented, that we identify as the mathematical *memesphere* (Stryker, 2011). A recent study (Bini et al., 2020) showed that, to members of these communities, a mathematical meme is not simply a meme (i.e. a funny joke), but it is perceived as the representation of a *mathematical statement* (p. 33), which in the case of Figure 8.1 is the formula of the square of the binomial $(x+y)^2 = x^2 + y^2 + 2xy$. The perception of memes as mathematical statements activates an epistemic culture inside the community, resulting in threads of comments dedicated to argumentations, proofs and clarifications of the mathematical content, as in the excerpt above.

Hence, a mathematical meme as in Figure 8.1 is a product of the creative participatory

[71] https://imgflip.com/memegenerator
[72] https://www.kapwing.com/meme-maker
[73] www.knowyourmeme.com
[74] https://en.meming.world/wiki/Main_Page

thrust characterising the Web 2.0 digital culture (Jenkins, 2009; Shifman, 2014; Bini et al., 2020), endowed with an epistemic potential capable to initiate a process of mathematical argumentation. This epistemic potential could be fruitfully exploited for teaching, as research has shown with respect to new literary practices (Lankshear & Knobel, 2003; Silva, 2016; Knobel & Lankshear, 2018; Harvey & Palese, 2018; Wells, 2018; Reddy et al., 2020; Brown, 2020). However, memes' educational potentialities are still to be unfolded, especially in mathematics education where only four studies reporting experiments of class-use of mathematical memes can be found (Beltrán-Pellicer, 2016; Friske, 2018; Bini & Robutti, 2019a, 2019b).

The main hurdle that hinders the school use of mathematical memes is that they express statements "in a new *hybrid language* which merges mathematical elements with memetic elements" (Bini et al., 2020, p. 33, emphasis added). Members of online communities master this hybrid language and its rules, as we can tell noting that, in the threads of comments, they do not discuss the meme but focus on the truth-value of the represented mathematical statement.

We catch a glimpse of memes' hybrid language following the interpretation of the statement represented by the meme in Figure 8.1. The process requires the reader first to identify the pictorial part as the memetic image known as *I Think I Forgot Something* (Figure 8.2 left). This image is a four-panel composition of screenshots from a popular TV sitcom, hence the subtitles "I Think I Forgot Something – If you forgot, then it wasn't important – Yes, you're right". It is used in the memesphere to create memes metaphorically addressing the idea of forgetting something vitally important (KYM).

Figure 8. 2 *I think I forgot something* original image (left) text positioning (centre) and mathematical content (right)

Secondly, the metaphorical meaning of the image is connected to the position of the text: according to the encyclopaedias, this is an *object labelling* meme, with texts superimposed onto the characters in the image, who thus embody the object in the text (Figure 8.2 centre). The punchline is delivered by the text in the lower right panel (text 3 in Figure 8.2 centre), representing the *something* which is being forgotten, personified by the child left at soccer practice. The third step of the interpretative path is achieved recognising the meaning of the mathematical symbols in the texts (Figure 8.2 right). The interpretation is completed connecting all this information to produce the statement $(x+y)^2 = x^2 + y^2 + 2xy$, which is the object of the question posed by commenter 1 (#1) and of the given argumentations (#2, #4).

The memetic component (i.e., the image and the text position) provides the linking signs that connect the mathematical elements in Figure 8.2 (right) to compose the final statement, namely the equal sign between the $(x+y)^2$ and $x^2 + y^2$ components, and the + sign before the $2xy$. It also adds to the prosody of the represented statement, emphasising the $+2xy$ element and endowing it with an emotional value: this is why Bini et al. (2020) say that memetic mathematical statements are written in a *hybrid language* that merges the two semiotic realms of digital and mathematical culture. Indeed, Zappavigna refers to the practice of creating memes as "inherently heteroglossic" (2012, p. 101), recalling a neologism coined by Bakhtin (1981) to describe the coexistence of different languages into a single narrative style. In Bakhtin view, this coexistence happens because the language is not a static corpus governed by abstract rules, but a living thing responding to the "semantic shifts and lexical choices" (p. 326) characterising the sociohistorical moment. The hybrid language of memes can, therefore, be seen as the result of new *semantic shifts and lexical choices* expedited by the social and technological environment of the Web 2.0, and memes creators as authors who "perform a particular syncretic expression of social heteroglossia [where] the originality is in the combination, not [in] the elements" (Robinson, 2011, para. 4). Paraphrasing Goethe's quotation opening this article, we can say that creators of mathematical memes translate a statement "into their own [hybrid] language and it soon turns into something entirely different" (Goethe, Maxims and Reflections 1279). It becomes the encrypted representation of a mathematical statement, conceived through an innovative connection between *online* digital culture (the image and its metaphorical meaning) and *offline* mathematical knowledge.

2 Rationale: rethinking creativity in mathematics

The act of making innovative connections at the genesis of mathematical memes corresponds to the definition of creativity widely accepted by scholars, but not by mathematicians and mathematics educators. Studies in the field of general education define creativity as the ability to "recombin[e] ideas or seeing new relationships among ideas" (Torrance, 1964, p. 4), psychological studies as the ability of "making unfamiliar combinations of familiar ideas" (Boden, 2004, p. 3), neurological studies affirm that "creative thinking involves searching memory to connect concepts that are 'farther away' from each other in memory" (Beaty & Kenett, 2020, p. 219) and machine intelligence studies reckon that "moments of insight are merely the result of the brain making connections between weakly and strongly activated bits of information, and then bringing them to consciousness". (Carpenter, 2019, p. 1).

Whereas, in mathematics and mathematics education, the definition of creativity is heavily influenced by the Gestalt four-stage model of creativity (Preparation, Incubation, Illumination, Verification) derived by Wallas (1926) from Poincaré's work (1910), and subsequently embraced by Hadamard (1945). This model induced to consider fluency as a parameter to measure creativity, thus interrelating mathematical creativity and mathematical ability (Krutetskii, 1976). Presently mathematical creativity is defined as the act of discovering something new (or subjectively new) in the mathematical field: new concepts, new solutions to open questions, or new angles of observation of problematic situations (Hiele, 1986; Sriraman, 2009; Sriraman et al., 2011; Nadjafikhah et al., 2012). This definition is more selective than the commonly shared one and implies that the performance of a creative act in mathematics involves skills which typically are the prerogative of gifted students. Indeed, studies about mathematical creativity are usually paired with studies on giftedness, as in Sriraman (2005) and a recent ICME-13 Monograph (Singer, 2018). This feature puts the narrower definition of creativity in mathematics in contrast with the recommendations of OECD's "Future of Education and Skills 2030"[75] project and of the PISA 2021[76] mathematics framework, both enlisting creativity among the 21st Century core capacities to be powered *in all students*, following Vygotsky's idea that "creativity creates a lifelong zone of proximal development" (Moran & John-Steiner, 2003, p. 3) and Piaget's convincement that "the principal goal of education is to create men who are capable of doing new things, not simply of repeating what other generations have done" (Duckworth, 1964, p. 499).

To pursue the educational goal of fostering creativity *in all students*, we assume here a widened concept of mathematical creativity, aligned with the general definition recalled above. We consider as mathematically creative not only acts that produce new mathematical objects but

[75] https://www.oecd.org/education/2030/E2030%20Position%20Paper%20(05.04.2018).pdf
[76] https://pisa2021-maths.oecd.org/

also acts that *create new representations of known objects obtained by making new connections*. In this way we could count mathematical memes as manifestations of mathematical creativity since they are representations of known mathematical objects (statements) created making new connections between digital culture and mathematics, where, as said, “the originality is in the combination, not [in] the elements” (Robinson, 2011, para. 4). As youngsters are generally at ease with the practice of meme creation, this could be a step towards making mathematical creativity more accessible and therefore more democratic (Prabhu & Czarnocha, 2014), allowing *all students* to experience the sense of accomplishment accompanying a creative act in mathematics, that has recognised empowering effects on students’ motivation (Barnes, 2000; Liljedahl, 2004, 2005; Riling, 2020).

If widening the concept of mathematical creativity to accommodate mathematical memes can help in including a larger portion of students in creative mathematical activities, it could be at the same time discriminating for teachers, who normally are not familiar with the digital culture infusing the memesphere. Therefore, teachers might not be able to create mathematical memes themselves nor anticipate the students’ way of acting with them; hence, teachers are not prepared to use memes in their classrooms. If we want to enable them to take advantage of this educational potential, we need scientific knowledge about the creative mechanism at the basis of mathematical memes, and about the possible use of these objects in a teaching and learning environment. The latter has been done in Bini et.al. (2020), that showed that mathematical memes are used as hybrid representations of mathematical statements, around which an epistemic culture is established. In this paper we expand this research investigating mathematical memes’ creative mechanism, guided by the following research questions:

- RQ1: How can we model the mechanism of creating a mathematical meme?
- RQ2: How does the memetic component of a mathematical meme contribute to the represented mathematical statement?

To answer these questions, we are challenged by crucial practical limitations: we cannot directly investigate the creation process, as creators do not disclose it, nor can we approach memes’ creators (who usually hide behind nicknames in social media) to ask how they create them. Indeed, as Moran and John-Steiner write recalling Vygotsky, “a researcher cannot work back from the end product to the artistic process because the product has crystallized the process in such a way that obscures the process” (2003, p. 7). Hence, it is not our aim to find out how authors *really create* memes, but to find out how memes *could be created*, accepting that authors have their own way of creating them. We pursue this aim by investigating the results of these creations based on two approaches, a methodical and a theoretical one. Methodically we use a large dataset comprehending around 1500 mathematical memes gathered during a nearly three-year-long ethnographic research (February 2018–current) within online communities dedicated to mathematical memes. Theoretically, we use Koestler's *bisociation* theory of the creative acts (1964), which coherently frames our widened concept of mathematical creativity by defining as creative the act that makes new connections among separate cultural realms. The study will adopt an *ethnographic* approach (Eisenhart, 1988) using the *triple-s construct of partial meanings of a meme* (Bini & Robutti, 2019a) to interpret memes' hybrid language: this will allow reconstructing the mechanism of creating a mathematical meme and will show how the memetic component contributes to the mathematical statement.

3 Theoretical background

We summarise here Koester's *bisociation* theory and the *triple-s construct of partial meanings* (Bini & Robutti, 2019a). Koestler theory, firstly exposed in *The Act of Creation* (1964), has been already adopted in mathematics education to expand the accessibility of mathematical creativity (Prabhu & Czarnocha. 2014; Czarnocha et al., 2016; Baker, 2016),

3.1 Koestler's bisociation theory

According to Koestler, all creative acts in humour, science and art follow the same pattern of "perceiving a situation or idea […] in two self-consistent but habitually incompatible frames of reference" (1964, p. 35) as illustrated in Figure 8.3. He also reckons that the act of understanding a creative product corresponds to re-creating it in our mind and is therefore a creative act in itself (p. 263). Koestler names his theory *bisociation*, a neologism that reflects his view that the situation or idea perceived in the two frames of reference is "not merely linked to one associative context, but bisociated with two" (p. 35). The term *bisociation* is a counterpoint to the known concept of *association*, which Koestler interprets as the standard connection among elements from a single frame of reference. In Koestler's view, a creative act always brings into contact two frames of reference, while the result of "routine skills of thinking" (p. 35) is an associative thought connecting ideas in a single frame. In his work, Koestler uses the terms planes and matrices as synonyms for frames of reference; to prevent misunderstanding we shall stick to frames of reference and amend further quotations accordingly.

In bisociation, each frame of reference retains its identifying rules and logic. When two frames are linked in a creative act, they "can be adapted to environmental conditions; but the rules […] must be observed and set a limit to flexibility" (p. 38). In fact, Koestler underlines that bisociation is accomplished through the "amalgamation of two realms as wholes, and the integration of the laws of both realms" (p. 658), producing a greater unified language.

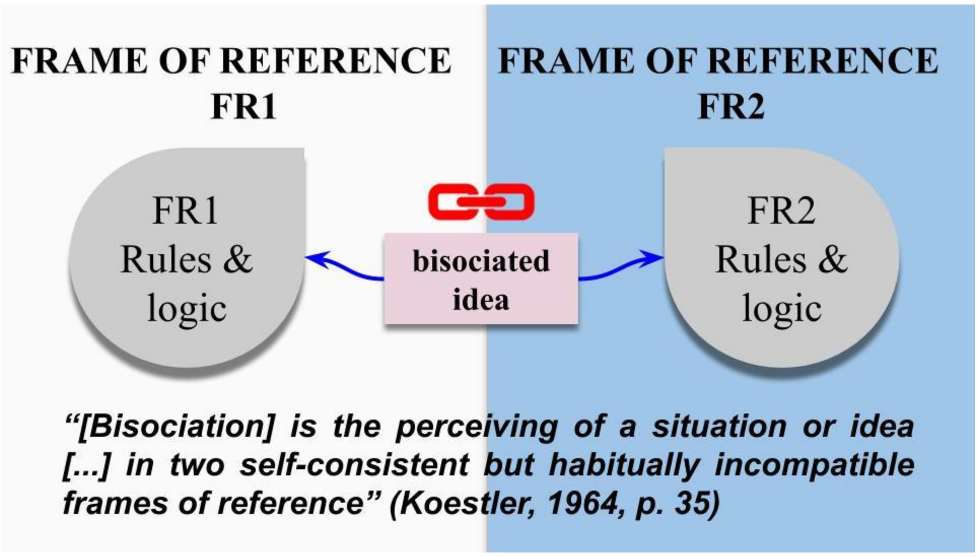


Figure 8. 3 Koestler's bisociation theory

To exemplify his theory in science, Koestler mentions the frames of reference of magnetism and electricity, physics and chemistry, and corpuscles and waves, which were all initially "developed separately and independently, both in the individual and the collective mind, until the frontiers broke down" (p. 658), shattered by a creative act that generates a new bisociated scientific idea. The two separated frames of reference pre-existed, and the new bisociated idea came into being only when "the time was ripe for that particular synthesis" (p. 658). In our case, bisociation may explain the memes' cited *heteroglossia* (Bakhtin, 1981) fostered by the Web 2.0 sociocultural environment, and the superposed perceptions of mathematical memes as memes and as statements revealed by investigations inside dedicated communities (Bini et al., 2020): as memes, they make the reader laugh and provide bonding experiences, and as statements, they engage the reader in the process of argumentation to determine their truth-value.

Another interesting tenet of Koestler's theory is that bisociation applies equally for creative acts in the domains of humour, scientific discovery and art. Koestler's idea is that these three domains are not to be intended as separated, but "shad[ing] into each other without sharp boundaries" (1964, p. 27). In particular, he describes how humorous riddles can blur into scientific problems and, crossing the fluid boundary between them, "the task of 'seeing the joke' becomes the task of 'solving the problem'. And when we succeed, […] laughter gradually shades into [the] quiet glow of intellectual satisfaction” (p. 91). According to Koestler, this happens because riddles and problems follow a principle of economy, leaving gaps that force the reader to co-operate in *“an imaginative, re-creative effort”* (p. 263) that emotionally charges the interaction. In our experience, the same spectrum of feelings, from laughter to intellectual satisfaction, characterises the interaction with mathematical memes. In fact, memes follow the same principle of economy of riddles and problems: they all are *lazy mechanisms* (Eco, 1979, p. 52) that leave the interpretative initiative to the reader, activating precise narrative strategies to be deciphered with a sufficient margin of uniqueness.

3.2 The triple-s construct of partial meanings of a meme

Memes’ narrative strategies are evident to readers accustomed to their hybrid language but can be obscure to those not familiar with the logic of the memesphere. Yet, to apply Koestler’s theoretical framework to mathematical memes, we need to unveil these narrative strategies to understand what is bisociated and how. To make them visible, we adopt the *triple-s construct of partial meanings* (Bini & Robutti, 2019a), a semiotic tool that provides criteria to extract and interpret meaningful data to understand image-based memes (not only mathematical) and be able to reprocess these data to create new mutations. According to this construct, the interpretation of a meme is achieved through the understanding of *three partial meanings*, described in Table 8.1, and their subsequent interconnection to build the *full meaning* of the meme which, for mathematical memes, corresponds to the represented mathematical statement.

Table 8. 1 The triple-s construct of the partial meaning of an image-based meme (Bini & Robutti, 2019a)

Partial meaning	Description	Connection to the opening example
Social	Information carried by the value conventionally attributed in the memesphere to the pictorial component in the meme	Figure8. 2 left
Structural	Information carried by the value conventionally attributed in the memesphere to the graphical composition of the meme: font style, position, phrasal pattern of the captions, overall arrangement of the composition	Figure 8.2 centre
Specialised	Information carried by texts, pictorial elements, addition or alterations of the original image referring to a specific topic	Figure 8.2 right

In the memesphere, the pictorial component carrying the social meaning is known as *template*, a terminology we shall adopt from now on. A template is *something that is used as a pattern for producing other similar things*[77]: this term reveals that the image in the meme is not simply an image, it is a picture already containing the seed of its reproducibility. Templates are images taken from the Web, but not all images become templates: they achieve the status of templates when they become signs, loaded with a recognisable meaning, assigned through a process of collective semiosis (Osterroth, 2018). Rules that codify the social meaning develop spontaneously in the memesphere; once established, templates acquire names, categorised in encyclopaedias and meme generator websites: for example, the meme in Figure 8.1 can be found in all websites as *I Think I Forgot Something*. Thus, templates become a "cultural capital" (Bourdieu, 1986, p. 243) inside Web-based communities, who harshly condemns their misuse with a vocabulary that "marks the 'breaches' as outsiders" (Nissenbaum & Shifman, 2017, p. 491). In the memesphere, templates are symbols standing "for something other than themselves" (Nemirovsky & Monk, 2000, p. 180): to interpret them, the reader has to recognise them as symbols and know what these symbols stand for.

Conventions established within the memesphere, and legitimated by the encyclopaedias, dictate also the rules for the graphical composition specific to each template, corresponding to the *structural* meaning. These rules are so ingrained in the memesphere, that meme-generator websites provide automatically the correct structural set up according to the chosen template. In Table 8.2 we summarise the currently (November 2020) known values of the structural meaning, assuming the terminology from KYM and giving examples for each category.

[77] https://dictionary.cambridge.org/dictionary/english/template

Table 8. 2 Structural meaning values [sources KYM and MW]

Reaction images	Object labelling	Exploitable	Multi-pane	Interior Monologue Captioning
Top or top/bottom text arrangement, usually in Impact or Arial font. Brief texts possibly following a *phrasal template* (Zappavigna, 2012) with fixed parts to be completed with suited variable elements	Brief texts or symbols are superimposed onto the image, with elements in the picture embodying the object in the superimposed label, according to the social meaning of the template	Brief texts or symbols are inserted in specific empty spots of the frame, according to the social meaning of the template	Different lines of brief texts or symbols are inserted in each panel, usually in Impact or Arial font uppercase and lowercase, in a sequence modulated according to the social meaning of the template	Randomly dispersed brief texts or symbols representing what the subject is thinking or feeling, usually in Comic Sans font uppercase and lowercase
Example 1: Annoyed Picard 	Example 2: I think I forgot something 	Example 3 Two buttons 	Example 4: Drakeposting 	Example 5: Obama Cell Phone Photo 

As evident from Table 2, the structural meaning follows some overarching rules: added textual elements must be clearly recognisable (hence the font), readable (hence the font size) and captivating (hence the brevity of the text). Moreover, the narrative of the meme always develops in a top/bottom direction, leaving to the bottom part the role of the punchline. We interpret these aesthetic features as imposed by practical needs of readability via the quick top/down scrolling gesture on mobile devices which are the natural habitat of memes. In other words, it is the technology providing the support for the meme that imposes these structural rules, to which the meme adapts. This causation between technology and patterns of human interactions has been theorised by McLuhan in his famous saying "the medium is the message" (1964, p. 9): here the medium is the touch-screen technology, and the message is the structural meaning of the meme (and, indeed, in McLuhan's intentions message and meaning are considered as synonyms).

The last partial meaning is the *specialised*, framing the meme in a disciplinary domain, in our case mathematical. This meaning can leverage on elements of the template, alterations of the template that do not prevent its recognition, or additions as symbols, diagrams, other images or textual elements (as in the example in Figure 8.1).

To sum up, when Internet users create a meme, they accept the social and structural meaning established in the memesphere for the chosen template and match them to a suitable specialised meaning. This process produces the *mutation* of the original template that characterises memes. In Figure 8.4 we see three mutations of the *I Think I Forgot Something* template used in Figure 8.1, where the social meaning of forgetting something vitally important matches with specialised meanings in the topics of complex numbers (left), definite integrals (centre) and quadratic equations (right).

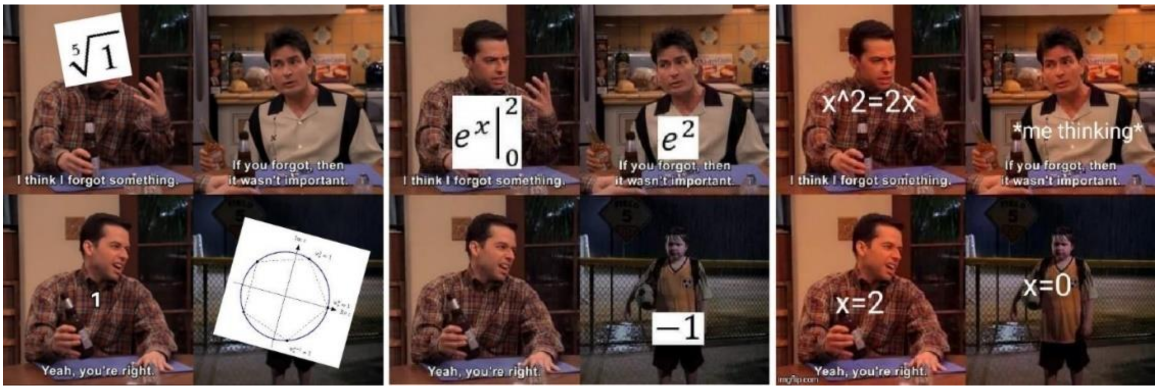


Figure 8. 4 Mutations of *I Think I Forgot Something* [sources Reddit and Facebook]

The understanding of the three partial meanings is a necessary but not sufficient step to grasp the *full meaning* of a meme, corresponding to the translation of the represented mathematical statement from the hybrid language of memes to the formal language of mathematics. To *see the statement*, the reader has to bridge by his/her own effort and expertise the gaps left by the meme creator, taking the different meanings and combining them into a single unit loaded with cognitive content. It is a process of synthesis in the sense of Kant, where a collection of elements is "gone through, taken up and combined in a certain way in order for a cognition to be made out of it" (Kant, A77/B103), and it can only happen in the mind of the reader. It is "the sudden emergence of a new insight, or [the] short-circuits of reasoning"

(Koestler, 1964, p. 211) that characterises the act of creating a meme as well as the act of understanding it, which means re-creating the meme as a reader, and endows mathematical memes with their engaging appeal.

4 Methodology and methods

This research is imprinted to an approach that merges ethnography (Eisenhart, 1988; Harwati, 2019) with grounded theory (Glaser & Strauss, 1967; Strauss & Corbin, 1998; Teppo, 2015). This is an established methodological approach in studies on memes (Katz & Shifman, 2017, Nissenbaum & Shifman, 2017; Denisova, 2019; Stöckl et al., 2019), that takes into account the facts that data are openly available in the field and not in structured experiments, and the novelty of the investigation, implying the lack of previous theories that could inform the research, direct the data collection and support the subsequent analysis. Following a basic principle of anthropology (Powdermaker, 1967), we have made an effort to balance our *inside* and *outside* stance on the observed culture: The first author is a nonparticipant observer (Liu & Maitlis 2010) *inside* online communities dedicated to mathematical memes, while other authors remain *outside* acting as controllers questioning the first author's reporting. The fieldwork started in February 2018 and is still going; throughout it, chronological field notes and memos are taken in real-time by the first author, following the guidelines of ethnographic research (Sunstein & Chiseri-Strater, 2012), noting key cases, impressions and behaviours, insider phrases and rituals of the observed communities. These observations are shared between authors, following a reflexive approach to grounded theory (Mruck & Mey, 2007), enabling to harness new ideas as the research proceeds.

We address our research questions observing data from two different standpoints: an ethnographic study embracing the whole dataset, and an embedded modelling study on sliced data. The ethnographic study aims at gaining an overview of the variety of the mathematics addressed by memes, organising them into a categorical landscape, observing the types of templates used in the mathematical memesphere, and identifying the various kinds of bisociation with the help of the triple-s construct. Results from the ethnographic study inform the embedded modelling study aimed at eliciting memes' creative mechanism and the contribution of the memetic component (i.e. the template and its social and structural meaning) to the represented statement. In particular, to understand the scope of the mathematics that can be used for a specific template and identify the strategies used by memes' creators to make the content readable, we slice our data keeping the social and structural meanings constant and letting the specialised meaning free to vary. Then, to investigate how the memetic component contributes to the mathematical statement, we reverse the fixed and variable dimensions slicing our data by fixing the specialised meaning and observing the contribution of the different social and structural meanings. These two cases will finally inform the building of a theoretical model of meme creation showing two perspectives of meme creation and revealing what either perspective adds to the other when expressing a mathematical statement in a hybrid way.

Methodically, Koestler provides us with instructions to sort and analyse our data in light of his theory. His observation that "we learn by assimilating experiences and grouping them into ordered schemata, into stable patterns of unity in variety" (1964, p. 44) sustains our choice of slicing data to perform a systematic comparison that supports the identification of the *unity in variation*. His directions about how to "dissect any specimen of humour" (p. 63) suggest us how to proceed to *bisociate* our mathematical memes which are, in their original essence, humoristic creations. "[F]irst, determine the nature of [the two frames of reference] […] by discovering the type of logic, the rules of the game, which govern each [frame of reference]. Often these rules are implied, as hidden axioms, and taken for granted […]. The rest is easy: find the 'link' - the focal concept, word, or situation which is bisociated with both [frames of reference]; lastly, define the character of the emotive charge and make a guess regarding the unconscious elements that it may contain" (p. 63).

4.1 Data collection and coding

The fieldwork was preceded by a plan that consisted in: (1) mapping of the territory for social sites and community identification and selection and (2) identifying the dataset. The first step was guided by criteria of popularity, expected to imply richer habitats to explore: we chose to observe the 25 most popular communities (in terms of followers) sharing mathematical memes within 3 of the most popular social media websites[78] (Facebook, Instagram and Reddit). The second step was guided by the long-term purpose of using the results of this research to inform the teaching of mathematics, consequently, we focused on memes representing topics from the 8^{th}-13^{th} grade mathematics syllabus (according to the Italian national curriculum[79]). We collected also meta-memes, i.e. memes about memes, which reveal how the practice of meme creation is

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[78] https://www.statista.com/statistics/272014/global-social-networks-ranked-by-number-of-users/

[79] http://www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/_Liceo%20scientifico.pdf

perceived inside the memesphere. Data collection entailed downloading memes to create a *meme pool* and saving the associated active links to create a *comment pool* that gives access to the threads of comments, where discussions about the mathematics in the meme take place. This gave us insight into how the specialised meaning of the meme is approached inside the community, as in the opening example. In the period from February 2018 to September 2020, we collected a meme pool of around 1500 memes (filtering reposts) and a corresponding comment pool. All memes and threads of comments in the dataset were collected from publicly available groups and pages, and no personal information was the object of research, thus abiding by the ethical use of social media data in research (Sloan & Quan-Haase, 2016; Townsend & Wallace, 2016).

Following the grounded theory approach, memes have been coded as they were collected. Each meme has been saved and renamed with a two-part code: the first part identifies the template via its social name as catalogued in the encyclopaedias KYM or MW (or by another clear identifier), and the second part identifies the mathematical topic. Thus, the first part is connected to the social and structural meanings, and the second part to the specialised meaning: for example, the code for the meme in Figure 8.1 is *I think I forgot something / binomial square.* Constant comparison of new and old data allowed this coding activity to proceed coherently.

4.2 Methods for the ethnographic study: data categorisation and analysis

For the ethnographic study, codes of the mathematical memes collected from February 2018 to September 2020 are combined and related to one another taking into account the connection between the template, the mathematical topic addressed by the specialised meaning, and the represented mathematical statement, until 14 *categories* (e.g. Properties, Objects or Typical Mistakes, see Figure8.7) emerge and are saturated. These categories are subsequently clustered into four *macro-themes,* as Mathematical Concepts or Mathematical Knowledge (see Figure 8.7), allowing to gather a holistic view of the mathematics addressed in the meme pool.

Data analysis in the ethnographic study applies the *triple-s* to identify the *frames of reference* bisociated in a mathematical meme. Then we proceed giving for each category a description of the corresponding statement, the number of entries in our meme pool (signalling the relevance within the mathematical memesphere), and a representative sample meme. For each sample meme we provide the *triple-s* analysis, drawing from KYM and MW (for the social and structural meanings) and from the authors' mathematical knowledge (for the specialised meaning and represented statement), and the bisociation analysis following Koestler's own guidelines. Finally, we make some overarching observations about the connection between the two *frames of* *reference* which is at the core of mathematical memes' creative mechanism.

4.3 Methods for the modelling study: data slicing, selection and analysis

The modelling study requires a method to investigate systematically the *stable patterns of unity in variety* advocated by Koestler. This is done slicing our data by fixing alternatively the partial meanings: the first assortment is built slicing our data fixing the social and structural meanings (i.e. the template), and leaving the specialised mathematical meaning free to vary (Case A); the second case is built keeping the specialised meaning fixed and letting the social and structural meaning vary (Case B). Interpreting the three meanings as categorical (non-metric) dimensions, we can represent the mathematical memesphere as a 3D space and a mathematical meme as a point whose coordinates are the three meanings (Figure8.5 left). Thus, Case A corresponds to points in a line, resulting from the intersection of the two planes identified by the fixed social and structural meanings, with different point heights identifying different mathematical topics addressed by the specialised meaning. (Figure 8.5 centre), and Case B corresponds to points in a plane at the fixed specialised meaning (Figure 8.5 right).

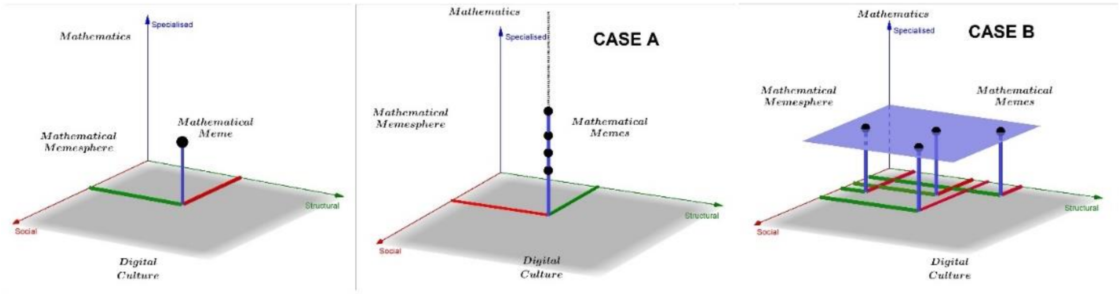

Figure 8. 5 Partial meanings as dimensions (left) and analysed cases (centre and right)

To illustrate our modelling study, we need to choose a template corresponding to a specific line (Case A) and a mathematical topic corresponding to a specific plane (Case B). For Case A the selection is conducted adopting quantitative criteria of longevity and popularity, which in the memesphere are discerning characteristics for significant templates (see KYM[80]). Longevity is estimated in relation to the average meme lifespan, appraised around 4 months (Veix, 2018), and popularity is measured in relation to our meme pool, where the most popular template appears in 65 mutations. Applying these criteria, we select the template known as *They Are the Same Picture*, appeared in February 2018 and still in use, and highly popular in our meme pool

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[80] https://knowyourmeme.com/editorials/meme-review/the-top-10-memes-of-the-decade

with 62 mutations.

Data selection for Case B entails combining quantitative and qualitative criteria: the popularity of the mathematical topic in our meme pool and the relevance of the subject from a didactical point of view. The chosen topic is the typical mistake of forgetting the middle term in the quadratic binomial expansion (as in Figure 8.1), which is very popular in our meme pool with 40 mutations. This mistake, commonly known as “the freshman’s dream”, is didactically relevant because it is not merely a problem of forgetting a piece of an expression but is related to the misinterpretation of the role of the parentheses at a conceptual level. This is connected to an assumption of applicability of the distributive law to algebraic operations other than sum and multiplication (Vergnaud, 1983), known as the “illusion of linearity” (De Bock et al., 2007) or the “over-reliance on linearity” (Verschaffel et al., 2015). This is a cornerstone in the developing of students' algebra sense (Mok, 2010); therefore, showing the nuances in the way this topic can be expressed through this new kind of representation is significant for our long-term purpose to inform teaching.

For each case, we present four examples, selected to represent different uses of the template (Case A) and interpretations of the mathematical topic (Case B). For these examples we perform the *triple-s* and *bisociation* analyses with the same methods described in the ethnographic part, then we give a narrative and descriptive analysis of the connection between the three partial meanings, the represented mathematical statement and the bisociated idea.

5 Findings

5.1 The ethnographic study

The ethnographic part of the study investigates the meme pool as a whole with a triple aim: (1) draw on the partial meanings to identify the frames of reference connected through the bisociative act by a mathematical meme; (2) use the coding of our data to categorise the mathematical topics addressed by memes, gathering information on the composition of our meme pool in terms of mathematics and bisociated ideas; (3) observe the templates of our memes (first part of the code) to make some observations about the use of templates in the mathematical memesphere.

The first analytical step is pursued, following Koestler suggestion “by discovering the type of logic, the rules of the game” (1964, p. 63) governing each frame of reference. Since the social and structural meanings of a mathematical meme follow the logic of the digital culture, while the specialised meaning respects the rules of mathematics, we can say that these two are the frames of reference brought together creating a mathematical meme and summarise this with the image in Figure8. 6. The specific bisociated idea linking the two frames of reference depends on the partial meanings of the mathematical meme taken into consideration: We will detail this level of analysis when we focus on particular examples.

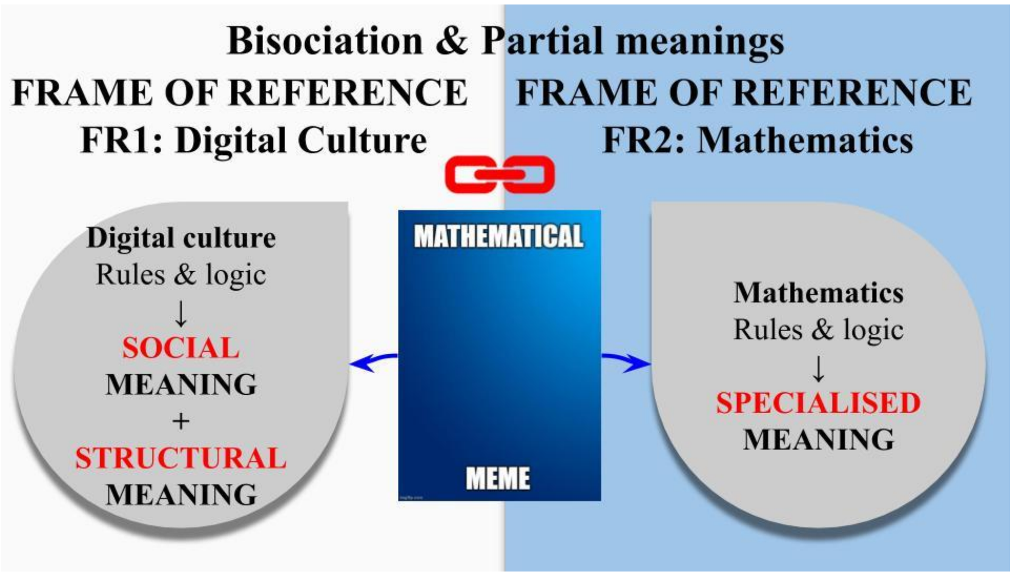


Figure 8. 6 Bisociation and partial meanings

As a second step, we analyse the categories that emerged through the process of comparison of the codes. Following the inductive nature of grounded theory methodology, categories names have been chosen from the clustering codes, and subsequently organised in four macro-themes (Mathematical Relationships, Mathematical Content, Mathematical Signs and Mathematical Knowledge, see Figure 8.7).

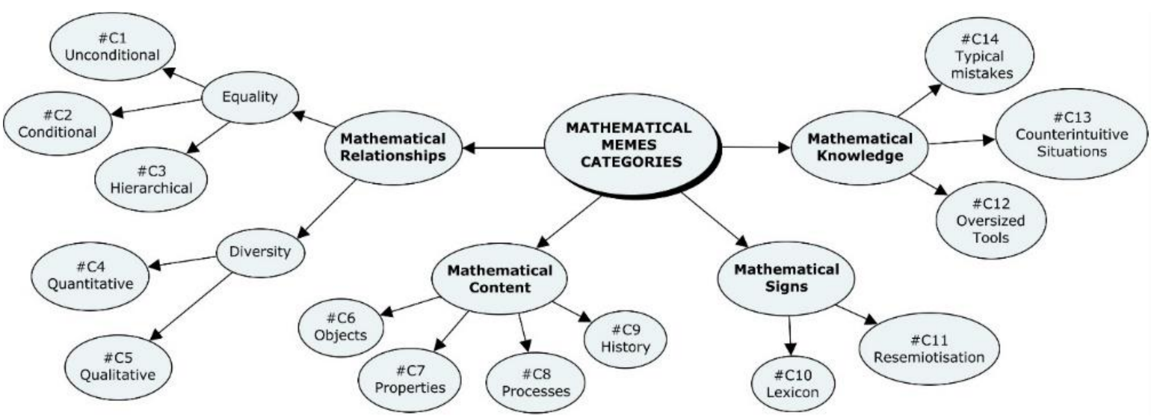


Figure 8. 7 Categories of mathematical memes in the meme pool

For each category we chose a clearly representative sample meme that is analysed through the triple-s, interpreting the represented statement, and through the lens of Koestler's bisociation theory to identify the bisociated idea, assuming digital culture and mathematics as the frames of reference linked in bisociation by mathematical memes (see Figure 8.6).

In Figure 8.8 we illustrate the procedure applied to find the bisociated idea, i.e., "the 'link' - the focal concept, word, or situation which is bisociated [and] made to vibrate simultaneously on two different wavelengths" (Koestler, 1964, p. 63), in the case of #C1 sample meme, *Spidermen pointing*. In the image, *a* and *b* stand for the elements that authors add to the template when creating a meme mutation. We search for the *bisociated idea* by comparing the interpretations of these elements in the two frames of reference: on the left side is the interpretation of *a* and *b* in the frame of reference of the digital culture, and on the right side is the interpretation of *a* and *b* in the frame of reference of mathematics. The linking concept between the two interpretations corresponds to the *bisociated idea*, which in this case we call *sameness*. This example shows that the bisociated idea does not depend upon the specific elements inserted in the place of *a* and *b*, indicating that the bisociated idea is not connected to a specific mutation, but to the core idea of the social and structural meanings of the template and to the core idea of the linked specialised mathematical meaning.

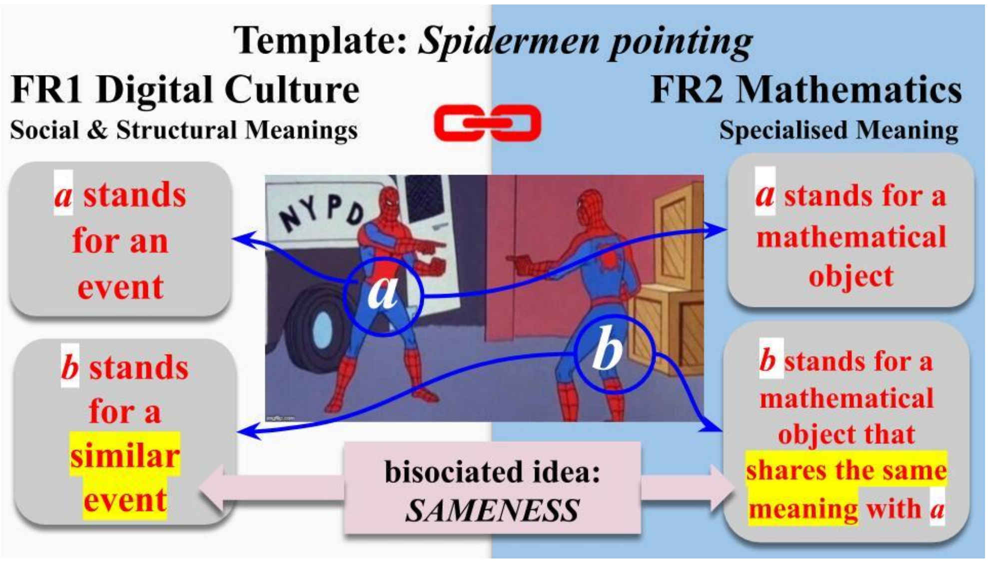
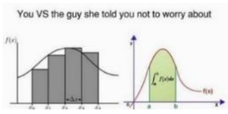
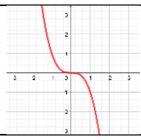


Figure 8. 8 #C1 sample meme: bisociation analysis

In Table 8.3 we analyse each category, giving a narrative description of the statement represented by memes in the category, the number of entries and the sample meme with its partial meanings, represented statement and bisociated idea.

Table 8. 3 Categorization of mathematical memes

Macro-themes and Categories		Description of the represented statement	Entries	Sample meme: FR1: Digital culture	Sample meme: Mathematical Meme and coding	Sample meme: FR2: Mathematics
Mathematical Relationships	#C1: Unconditional Equality	The elements in the meme are equal	87	Social: represent situations where similar things meet Structural: object labelling	 Spidermen pointing / rationalisation [source Facebook]	Specialised: arithmetic, equivalence of rationalised fractions
				Represented statement: $\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$		
				Bisociated idea: Sameness		
	#C2: Conditional Equality	Under specific circumstances, the elements in the meme are equal	54	Social: represent situations where an opinion is expressed Structural: two-panel exploitable	 They're the same picture / topology [source Reddit]	Specialised: invariants in topology
				Represented statement: In topology, a mug and a torus are equivalent		
				Bisociated idea: Reasoning		
	#C3: Hierarchical Equality	The elements in the meme are equal, with nuances of mathematical finesse or correctness	262	Social: represent equal ideas expressed in progressing finesse Structural: multi-pane	 Tuxedo Pooh / interval notation [source Reddit]	Specialised: real numbers, equivalent notations for subsets
				Represented statement: The two expressions represent equal subsets in different notations (inequality and interval)		
				Bisociated idea: Elegance		

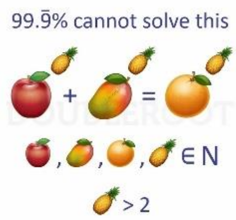
	#C4: Quantitative Diversity	The output of one of the operations in the meme is different from that of the others	30	Social: single out one element based on a specific diversity Structural: object labelling	 Bullied girl / 1+1 [source Instagram]	Specialised: arithmetic, different outputs of operations
				Represented statement: $1^1 = 1 \times 1 = \frac{1}{1} \neq 1 + 1$		
				Bisociated idea: Separation		
	#C5: Qualitative Diversity	The nature of one of the operations in the meme is different from that of the others	57	Social: compare two different but related elements, with the one on the right being the better one Structural: exploitable	 VS / Riemann sums [source Facebook]	Specialised: calculus, connection between Riemann sums and Riemann integral
				Represented statement: The Riemann integral of a function gives the actual area underneath the graph of f, while the Riemann sums give an approximation		
				Bisociated idea: Comparison		
Mathematical Content	#C6: Objects	A mathematical object is directly represented or hinted by some element of the meme	138	Social: part of a series where the shapes of the cow's horns refer to functions' graphs Structural: object labelling	 Cow / Cubic function [source Instagram]	Specialised: graphs of polynomial
				Represented statement: The graph of the function $y = -x^3$ is 		
				Bisociated idea: Shape		

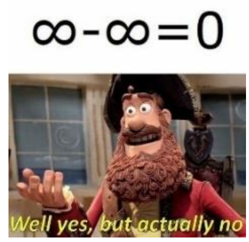
<table>
<tr><td rowspan="12"></td><td rowspan="4">#C7: Properties</td><td rowspan="4">A property of one or more mathematical objects is represented in the meme</td><td rowspan="4">223</td><td>Social: represent situations where two subjects share something</td><td rowspan="2">

Business handshake / trig functions [source Facebook]</td><td rowspan="2">Specialised: common values of goniometric functions’</td></tr>
<tr><td>Structural: object labelling</td></tr>
<tr><td colspan="3">Represented statement: The functions $y = sinx$ and $y = cosx$ intersect for y=$\frac{\sqrt{2}}{2}$</td></tr>
<tr><td colspan="3">Bisociated idea: Commonality</td></tr>
<tr><td rowspan="4">#C8: Processes</td><td rowspan="4">A mathematical procedure or operation (possibly accompanied by its output) is represented</td><td rowspan="4">220</td><td>Social: represent situations where an action has to be repeated</td><td rowspan="2">

Do it again / Hopital [source Facebook]</td><td rowspan="2">Specialised: calculus, solving limits applying l’Hopital theorem</td></tr>
<tr><td>Structural: reaction image</td></tr>
<tr><td colspan="3">Represented statement: Solving the limit $\lim_{x \to \infty} \frac{e^x}{x^3}$ requires applying l’Hopital theorem three times</td></tr>
<tr><td colspan="3">Bisociated idea: Repetition</td></tr>
<tr><td rowspan="4">#C9: History</td><td rowspan="4">The meme represents a famous anecdote in the history of mathematics</td><td rowspan="4">24</td><td>Social: represent situations where someone gets rid of something annoying</td><td rowspan="2">

Griffin / Pythagoras & Hippasus [source Reddit]</td><td rowspan="2">Specialised: Algebra, irrational numbers</td></tr>
<tr><td>Structural: object labelling</td></tr>
<tr><td colspan="3">Represented statement: According to the legend, Pythagoras throws Hippasus off a ship to keep irrationality secret.</td></tr>
<tr><td colspan="3">Bisociated idea: Rejection</td></tr>
</table>

Mathematical Signs	#C10: Lexicon	The statement revolves around the polysemy of a mathematical term	111	Social: represent situations where two subjects share something Structural: object labelling	 Epic handshake / imaginary [source Instagram]	Specialised: imaginary unit in complex numbers
				Represented statement: $\sqrt{-1} = i$, the imaginary unit in the complex field		
				Bisociated idea: Commonality		
	#C11: Resemiotisation	The statement of a famous mathematical theorem is represented, with mathematical elements replaced by other objects	26	Social: part of a series that parodies the “fruit salad” approach to algebra teaching (MacGregor, 1986) replacing elements of higher mathematical statements with fruits and flowers Structural: exploitable	 Fruits and flowers / Fermat [source Reddit]	Specialised: number theory, Fermat’s last theorem
				Represented statement: Fermat’s last theorem		
				Bisociated idea: Replacement		
Mathematical Knowledge	#C12: Oversized Tools	A situation /problem can be dealt with a simpler tool/ procedure than the one represented in the meme	49	Social: represent situations where a more powerful tool than needed is used Structural: reaction image	 Giant ping pong paddle / quadratic formula [source Reddit]	Specialised: algebra, quadratic equations solving techniques
				Represented statement: The equation $x^2 - 1 = 0$ can		

				be solved by factorization		
				Bisociated idea: Exaggeration		
	#C13: Counterintuitive Situations	The result of the procedure/ operation in the meme is different than expected	81	Social: represent unexpected situations	 Actually no / infinity - infinity [source Facebook]	Specialised: calculus, indeterminate forms
				Structural: reaction image		
				Represented statement: When computing limits, $[\infty - \infty]$ is an indeterminate form		
				Bisociated idea: Puzzlement		
	#C14: Typical Mistakes	The result of the procedure/ operation in the meme is wrong (possibly accompanied by its rectification)	163	Social: represent situations where something vitally important is forgotten	 I think I forgot something / binomial square [source Facebook]	Specialised: algebra, the middle term in the square of the binomial
				Structural: object labelling		
				Represented statement: $(x+y)^2 = x^2 + y^2 + 2xy$		
				Bisociated idea: Carelessness		

Firstly, we can say that our meme pool covers a broad variety of mathematical ideas, of which a vast majority (98.5%) are correct and correctly expressed. Some preferred categories stand out, as #C3 (Hierarchical Equality, 262 entries), #C7 (Properties, 223), #C8 (Processes, 220) and some are *niche* categories such as #C9 (History, 24) and #C11 (Resemiotisation, 26). A higher number of entries can depend on the popularity of a template or of a mathematical topic; for instance, #C3 is a very popular category thanks to the diffusion of the *Tuxedo Pooh* template that counts 65 mutations in our meme pool, while #C8 owe its popularity to that of l'Hopital theorem, which appears 41 times as specialised meaning in our meme pool. These preliminary observations show that mathematical memes in our meme pool not only link but also balance the two frames of reference they are bisociated into, merging and respecting "the laws of both realms"

(Koestler, 1964, p. 658). Addressing our first research question about the creative mechanism of mathematical memes, this observation suggests that the creative act from which a mathematical meme emerges can initiate equally from the frame of digital culture (a popular template) or from the frame of mathematics (a popular mathematics subject), eventually reaching for the other frame and linking the two in the final object.

Keeping in mind our second research question on the contribution of the memetic component to the represented mathematical statement, we notice that each template has specific parts that combine with explicit mathematical elements added by the creator to produce the mathematical statement. For example, in *Spiderman pointing* (#C1), the template represents the sign $=$ connecting the two expressions $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$, as corroborated by the comments in Figure 8.9, from another mathematical meme using the same template. In *Bullied girl* (#C4), the girls in the template background represent the sign $=$ connecting the expressions $1^1, 1 \times 1$, $\frac{1}{1}$ while the girl in the foreground represents the sign $\neq$ linked to the 1+1.

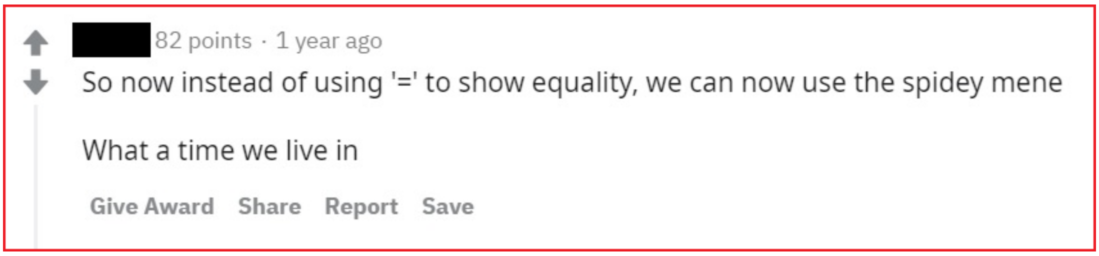

[redacted] 82 points · 1 year ago
So now instead of using '=' to show equality, we can now use the spidey mene
What a time we live in
Give Award Share Report Save

Figure 8. 9 Comments evidencing the contribution of the memetic component [source Reddit]

These shared interpretations of templates' contributions show how the culture of the mathematical memesphere forges the hybrid language of mathematical memes, implementing "semantic shifts and lexical choices" (Bakhtin, 1981, p. 326) that connect the language of memes to the language of mathematics. In addition to that, the memetic component emphasises parts of the statement with an emotional charge carried by the social meaning of the template, in the entertaining way that characterises this means of communication (Shifman, 2014).

Moving to the third purpose of this ethnographic part of the study, we examine our meme pool taking into account the general knowledge of the memesphere acquired by the first author during the fieldwork and the results of the analysis in Figure 8.8 that evidenced that the bisociated idea is connected to the template and not to the specific mutation. This will allow expressing some overarching observations about the use of templates in the mathematical memesphere, which will inform the building of our model.

- Except for special cases which are evergreen, as *Spidermen pointing* (#C1) or *They're the same picture* (#C2), templates are used in waves: once the fashion has run out, the template becomes "dead" and using it is a sign of not belonging to the culture (Nissenbaum & Shifman, 2017).
- Templates carrying the same bisociated idea can coexist in the memesphere.

The latter happens, for instance, for *Epic Handshake* (#C10) and *Business Handshake* (#C7), both carrying the same bisociated idea of *commonality* as classified in Table 8.3 and acknowledged by the meta-meme in Figure 8.10 left (source KYM). The use of one template rather than the other is just a matter of taste of the creator. In fact, the same specialised meaning addressing the imaginary unit appears in our meme pool matched with *Epic Handshake* (#C10) and with *Business Handshake* (Figure 8.10 right, source Reddit).

Figure 8. 10 A meta-meme about sharing the bisociated idea (left) and a mutation of *Business Handshake* (right)

- Generally, templates are used to create mathematical memes with specialised meanings belonging to one category, but some templates appear in more than one category:

As a rule, most templates stick to one category, as happens for example to *Spidermen pointing* used exclusively to create memes in #C1, where it is highly popular (58 mutations). However, some templates appear in more than one category; this is the case of *Business Handshake*: this template appears in #C7 (Table 8.3) with a specialised meaning describing the intersection of two goniometric functions, and in #C10 with a specialised meaning based on the polysemy of the term 'imaginary' (Figure 8.10 right).

We can explain this discrepancy using the bisociated ideas: in Table 8.3 we identified *sameness* as the bisociated idea in *Spidermen pointing*, and *commonality* in *Business Handshake*.

Comparing these bisociated ideas, we find that *sameness* relates distinctively to the relationship of unconditional equality, therefore the template carrying this bisociated idea is used exclusively to create memes in #C1. Whereas *commonality* can accommodate a wider range of specialised mathematical meanings, thus the template carrying this bisociated idea can be used to create memes belonging to different categories: #C10 (lexical commonality, see Figure 8.10 right), or #C7 (analytical commonality, see Table 8.3).

- Not all popular templates are used for mathematical memes and not all mathematical memes use popular templates.

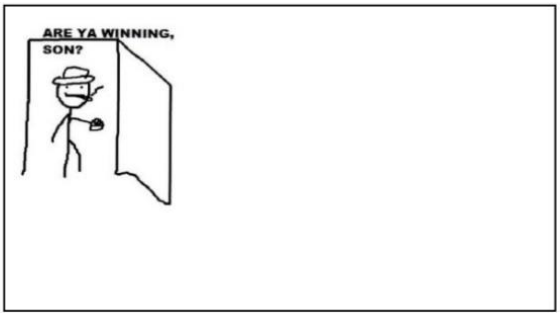


Figure 8. 11 A template not used for mathematical memes (left) and one used only for mathematical memes (right)

For example, the template in Figure 8.11 left, known as *Are ya winning, son*, is highly popular (source KYM, October 2020), but has not been used so far in the mathematical memesphere. It is an *exploitable*: the creator adds something in the blank space of the template, following the social meaning to represent situations where older people unsuccessfully try to socialise with youngsters, which points to *embarrassment* as the possible bisociated idea. Conversely, in Figure 8.11 right we see the template for *Bullied girl*, (#C4 in Table 8.3), whose bisociated idea is *separation*: This image, originally coming from websites discussing school bullying problems, is used exclusively inside the mathematical memesphere where it is mildly popular (12 mutations).

Considering that the mathematical memesphere is “a sub-region of the memesphere” (Bini et al., 2020, p. 11) meaning that templates inside it are used coherently to their general social and structural meanings. This evidence allows us to make some conjectures about how the two frames of reference, digital culture and mathematics, get linked in the creation of a mathematical meme. From the fact that the popular *Are ya winning, son* template does not spread inside the mathematical memesphere, we can gather that the mathematical memesphere has selective boundaries and does not automatically inherit templates from outside, but picks those templates conveying ideas that can be effectively bisociated to link the social and structural meaning to some mathematical specialised meaning. From the fact that the *Bullied girl* template is diffused only inside the mathematical memesphere, we can gather that creators can start new templates, provided that these templates are images already on the Web and that they respect the general aesthetic of memes. Nevertheless, when a template is born and diffused inside the mathematical memesphere, it remains inside it, i.e., this sub-region acts as an enclave, including creators who "are proficient in memes *and* mathematics" (p. 11, emphasis added) and excluding others.

5.2 The modelling study on sliced data

Embedded in our ethnographic study we focus on the sliced data to follow mutations to gather information on the creative mechanism. This will be done by analysing sequences of memes from two different categories as illustrated in Figure 8.12.

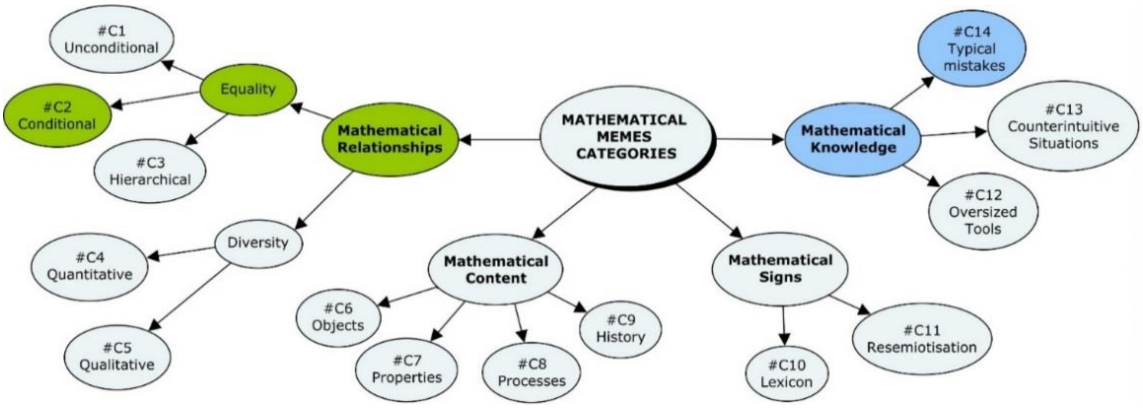


Figure 8. 12 Analysed cases in the meme pool categorisation

5.2.1 Case A. Fixed social and structural meanings, variable specialised meaning: *They're the Same Picture*

This template, used in the sample meme of #C2, is a composition of screenshots from a popular TV sitcom, hence the subtitles "Corporate needs you to find the differences between this picture and this picture - They're the same picture*"*: in Table 8.4 the template and its fixed meanings are described in detail.

Table 8. 4 ***They're the Same Picture*****: social and structural meanings**

	First use: June 22th, 2018
	Mutations in the meme pool: 62
	Social: represent situations in which two things (displayed in the upper panel) are considered as equal from the point of view of the user in the lower panel. The subtitles in yellow reinforce this meaning [source KYM].
	Structural: *two-panel exploitable*, requiring two things (texts, symbols, images) to be inserted into the blank spaces in the upper panel, and possibly an identifier (text, symbol, image) to be superimposed on the person in the lower panel.

The template is commonly used to represent the opinion that someone (the creator or another subject specified in the lower panel) has about something. This is done by inserting the object of the opinion in the left blank space of the upper panel, and a second object symbolising the opinion in the right blank space. If the meme is meant to convey the creator's opinion no further information is added, otherwise, some identifier is inserted on the lower panel. From the syntactic standpoint, the upper panel in the template provides the linking verb "is" and the lower panel the "according to" part of the sentence. Entering the frame of reference of mathematics, the idea of opinion conveyed by the template morphs into the mathematical topic of conditional equality "if c, then a=b". The equality a=b is illustrated in the upper panel, and the lower panel possibly includes information about the premises c that have to be assumed as true to validate it (Figure 8.13). The morphing from opinion to conditional equality supports our choice of *reasoning* as the bisociated idea indicated in Table 8.3, perceived as "opinion" in the frame of reference of the digital culture and as "conditional equality" in mathematics.

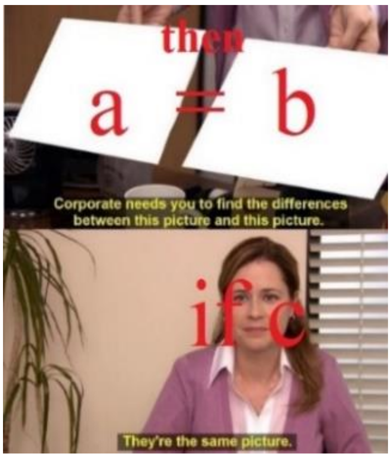


Figure 8. 13 Case A: represented mathematical statement

Our previous observation about the McLuhan-inspired causal connection between the top/bottom visualisation movement in mobile devices and the structure of a meme is confirmed by this template that reverses the natural flow of a conditional mathematical statement, placing the consequence (then a = b) *before* the premises (if c). In the following examples, we will see that all memes revolve around equalities which are sufficiently counterintuitive or startling, thus giving to the *if* part the role of the closing punchline.

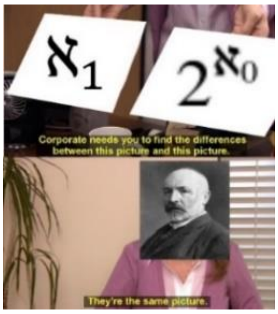
In Table 8.5 we present four examples, giving for each one the coding, the specialised meaning and the represented mathematical statement.

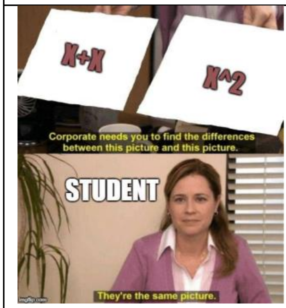
Table 8. 5 Case A: examples

<table>
<tr><td rowspan="6">

They're the Same Picture /
repeating decimals</td><td>Example #A1: no further information about premises</td></tr>
<tr><td>Publication date: 15/02/2020</td></tr>
<tr><td>Source: Reddit</td></tr>
<tr><td>Mathematical topic: Arithmetic</td></tr>
<tr><td>Specialised: The symbol $0,\overline{9}$ represents the repeating decimal 0,999 ..., to which the following chain of equalities applies, with no need of further premises:
$0,\overline{9} = 0,\overline{3} \cdot 3 = \frac{1}{3} \cdot 3 = 1$</td></tr>
<tr><td>Represented statement: $0,\overline{9} = 1$</td></tr>
<tr><td rowspan="6">

They're the Same Picture /
projective geometry</td><td>Example #A2: textual information about premises</td></tr>
<tr><td>Publication date: 27/11/2019</td></tr>
<tr><td>Source: Facebook</td></tr>
<tr><td>Mathematical topic: Geometry</td></tr>
<tr><td>Specialised: The two drawings represent families of parallel and intersecting lines. The identifier Projective Students in the lower panel suggest that these objects have to considered from the point of view of projective geometry, where any two distinct lines are incident. Therefore, parallel lines and intersecting lines cannot be distinguished</td></tr>
<tr><td>Represented statement: If considered in projective geometry, then families of parallel straight lines and of intersecting straight lines cannot be distinguished</td></tr>
</table>

<table>
<tr><td rowspan="6">

They're the Same Picture /
continuum hp</td><td>Example #A3: pictorial information about premises</td></tr>
<tr><td>Publication date: 30/10/2019</td></tr>
<tr><td>Source: Reddit</td></tr>
<tr><td>Mathematical topic: Set theory</td></tr>
<tr><td>Specialised: The added image represents Georg Cantor and the symbols correspond to Cantor's transfinite cardinals. On the left is $\aleph_1$, the transfinite cardinal immediately following $\aleph_0$. On the right is $2^{\aleph_0}$, the cardinality of the continuum. Cantor's continuum hypothesis implies their coincidence</td></tr>
<tr><td>Represented statement: If Cantor's continuum hypothesis is assumed as true, then $\aleph_1 = 2^{\aleph_0}$</td></tr>
<tr><td rowspan="6">

They're the Same Picture /
Monomials</td><td>Example #A4: the premises overturn the full meaning of the meme</td></tr>
<tr><td>Publication date: 05/03/2019</td></tr>
<tr><td>Source: Facebook</td></tr>
<tr><td>Mathematical topic: Algebra</td></tr>
<tr><td>Specialised: The expression $x + x$ is the sum of two like terms and the expression x^2 is the power (i.e. the product) of the same terms. These expressions are different, but they are commonly mistaken as equal by students.</td></tr>
<tr><td>Represented statement: $x + x \neq x^2$</td></tr>
</table>

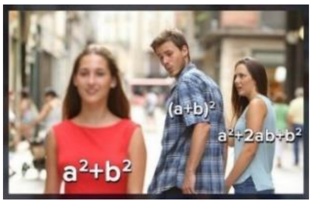
The slicing allows us to follow how memes (and represented mathematical statements) vary following the different nuances of equality, adapting the structural element in the lower panel to its scope of validity. If the equality holds universally (thus becoming unconditional), no further information is added (#A1), if the equality holds in a particular mathematical field, textual information referring to the field is added (#A2), if the equality is a consequence of a particular mathematical statement, an identifier of the adequate premises is added, here in the form of a portrait of the mathematician, which implies some non-trivial specialised knowledge (#A3). In the last example (#A4) the structural and social meanings in the upper panel imply that the two monomials are equal, but the mathematical specialised meaning is in contrast with this conclusion. To grasp the creator's message and interpret the statement, this information has to be combined with the bisociated idea and with some pedagogical content knowledge (Shulman, 1986), evoked by the structural addition "STUDENT" in the lower panel, and with the digital culture interpretation of the bisociated idea of *reasoning* as *opinion*. Taking all these data into account we obtain that the equality $x + x = x^2$ has to be read as the students' *opinion* corresponding to *wrong reasoning*, and the represented statement is, therefore, $x + x \neq x^2$.

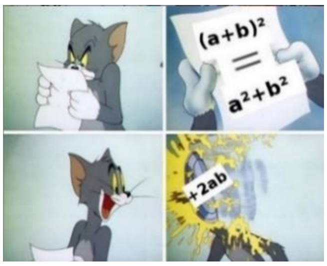
These examples show that the bisociated idea of *reasoning* is used by memes' creators in both bisociated senses (opinion *and* conditional equality) to represent mathematical statements addressing different levels of generality of the mathematical relationship of conditional equality and possible students' misconceptions. Indeed, the emotional charge that gives these memes their humoristic appeal is the fact that the premises endorsing the equality are embodied by the character in the lower panel and are perceived at the same time as mathematical premises and as opinions expressed by the character.

5.2.2 Case B. Fixed specialised meaning, variable social and structural meanings: Square of the binomial

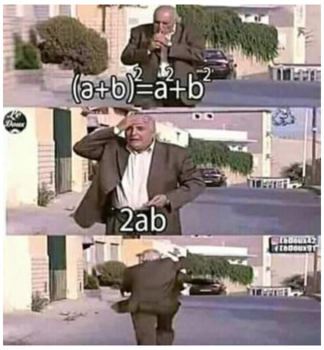
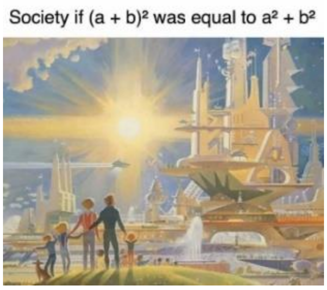
Table 8.6 shows four examples of this kind of mutation: for each meme we give the code, the social and structural meanings and the represented statement.

Table 8. 6 Case B: examples

<table>
<tr><td rowspan="8">

Distracted / binomial square</td><td colspan="2">Example B1</td></tr>
<tr><td colspan="2">Publication date: 01/12/2017</td></tr>
<tr><td colspan="2">Source: Facebook</td></tr>
<tr><td colspan="2">Template name: Distracted boyfriend</td></tr>
<tr><td colspan="2">Mutations in the meme pool: 25</td></tr>
<tr><td>Social</td><td>Structural</td></tr>
<tr><td>Represents situations where an assumption, instead of being followed by the correct conclusion, is diverted to a misleading wrong one.</td><td>Object labelling: the assumption is placed on the boyfriend, the correct conclusion on the girl on the right, and the misleading wrong conclusion on the girl on the left.</td></tr>
<tr><td colspan="2">Represented statement: $(a + b)^2 = a^2 + b^2$ is incorrect, the correct expansion is $(a + b)^2 = a^2 + 2ab + b^2$</td></tr>
<tr><td rowspan="7">

Tomcat pie / binomial square</td><td colspan="2">Example B2</td></tr>
<tr><td colspan="2">Publication date: 14/08/2018</td></tr>
<tr><td colspan="2">Source: Instagram</td></tr>
<tr><td colspan="2">Template name: Tom Cat Pie to face</td></tr>
<tr><td colspan="2">Mutations in the meme pool: 1</td></tr>
<tr><td>Social</td><td>Structural</td></tr>
<tr><td>Represents situations where some unrealistic expectation, described in</td><td>Four-panel exploitable, the unrealistic expectation is inserted in the sheet of paper in the top right</td></tr>
</table>

	the sheet of paper in the upper panel, is overturned by reality, symbolized by the pie thrown into Tom's face in the lower panel.	image, and the contrasting evidence is superposed onto the thrown pie in the bottom right image.
	Represented statement: $(a+b)^2 = a^2 + b^2$ is incorrect, the correct expansion includes the term $+2ab$	
 Forgetful / binomial square	Example B3	
	Publication date: 8/03/2019	
	Source: Instagram	
	Template name: Forgetful old man	
	Mutations in the meme pool: 1	
	Social	Structural
	Represents situations were something, initially forgotten, is suddenly remembered	*Three-panel interior monologue captioning*: the incomplete expression is placed on the character in the upper panel, and the forgotten element in the middle panel.
	Represented statement: $(a+b)^2 = a^2 + b^2$ is incorrect, the correct expansion includes the term $+2ab$	
 Society / binomial square	Example B4	
	Publication date: 16/02/2020	
	Source: Reddit	
	Template name: *Society if*	
	Mutations in the meme pool: 14	
	Social	Structural
	Represents ironically a utopian future that would have resulted if certain things (usually erroneous or futile things) were different.	*Reaction image*, an erroneous expectation is added in the white strip above the image, following the phrasal templates *Society if* ______
	Represented statement: $(a+b)^2 = a^2 + b^2$ is incorrect	

In this case, the positioning of the examples at the same specialised level implies that all the represented statements revolve around the same mathematical fact, namely that $(a+b)^2 = a^2 + b^2$ is incorrect, and that the correct expansion is $(a+b)^2 = a^2 + 2ab + b^2$. Notwithstanding this uniformity in terms of specialised meaning, the represented statements vary slightly from one example to the other, in connection with the variable social and structural meanings. The analysis of these variable meanings allows us to appreciate the contributions of the memetic component to the represented statement and to shed light on the bisociated idea at the core of the meme creation. In fact, templates carry different bisociated ideas, emphasising different parts of the mathematical statement as summarised in Figure 8.14.

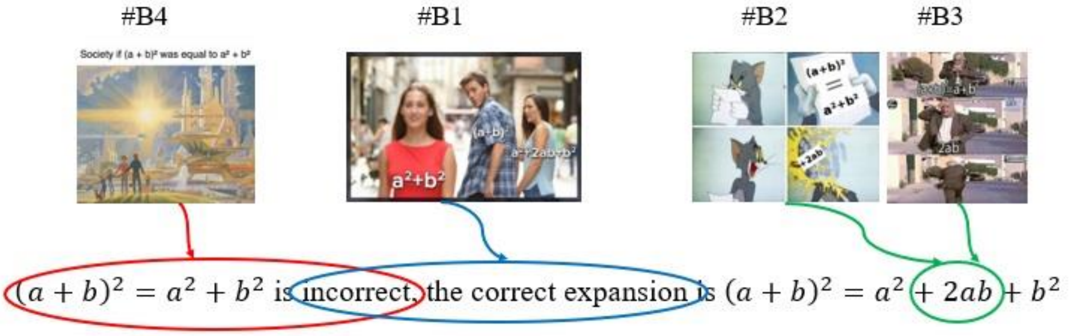


Figure 8. 14 Case B: the contribution of the memetic component to the mathematical statement

We identify the bisociated idea in #B1 as *betrayal*: the meme presents this idea literally by juxtaposing the two girls, and figuratively as the betrayal of the rules of mathematics embodied by the wrong equality $(a+b)^2 = a^2 + b^2$. The competition between the girls in the template emphasises the incorrect/correct dialectics in the statement, and the distracted boyfriend's gaze conveys the impression that the incorrect one is somehow more alluring than the correct one, probably because it embodies the fallacy of universal linearity. #B4 shares the same message about the attractiveness of the wrong equality, but its bisociated idea, that we identify as *delusion*, tells us that the creator is aware of the deceitfulness of the linearised expansion: therefore, in this case, the emphasis is on the "$(a+b)^2 = a^2 + b^2$ is incorrect" part of the statement. Examples #B2 and #B3 emphasized the +2ab part in the expansion, but in two opposite ways. #B2, whose bisociate idea we identify as *disillusion*, represents situations in which the realisation of the missing piece comes abruptly from the outside (the pie), and #B3, through its bisociated idea we recognise as *remembrance*, represents situations in which the realisation of the missing piece comes from the inside (the interior monologue) and allows correcting the mistake autonomously.

In #B1 and #B4, the represented statements are accessible only to viewers who are deeply embedded into the memesphere, where these templates are widely popular, while the templates used in #B2and #B3 make the statements immediately accessible. The template in #B2, although not much used in the mathematical memesphere, is quite common elsewhere, and its social meaning is easily adapted to the specialised meaning. #B3 is an example of a stand-alone meme, built from scratch from an existing image (a composition of screenshots from a video posted in a satirical Facebook page) which is not a standard template, but is relatable because connected to the feeling permeating the mathematical mistake addressed in the specialised meaning.

All examples subsume two non-written hypotheses: an implicit universal quantifier, meaning that the equality $(a+b)^2 = a^2 + b^2$ is incorrect if intended $\forall\, a, b\ \in \mathbb{R}$, whereas it is correct if restricted to the case $a = b = 0$, and an implicit immersion field whose characteristic is different from 2, which would make the linearised expression correct. These are limit cases, that often provide critical moments for learning mathematics (Janßen et al., 2020). Memes' creators are well aware of these situations, that are often discussed in the comments accompanying mathematical memes as in Case B and are used as specialised meanings for memes that cast a new light on the equality $(a+b)^2 = a^2 + b^2$, moving it from Typical Mistakes to Counterintuitive Situations (Figure 8.15 left and right). Therefore, at a deeper level, these memes also address the nuances of wrongness in a mathematical equation, and the awareness that being correct is a relative concept.

Figure 8. 15 Case B: addressing the nuances of wrongness of the linearised expansion [source Facebook]

5.3 Results: towards a heuristic for meme-creating

Summarising our findings with respect to the research questions, we can say that, with respect to RQ1, the mechanism of creating a mathematical meme stems from a bisociated idea that links a template with a mathematical statement. The bisociated idea connects the core idea of the social meaning of the template to the core idea of the mathematical statement, that assumes the role of the specialised meaning of the meme. The connection is implemented through textual or pictorial additions complying with the structural meaning of the template. Templates' social meanings can convey broader bisociated ideas subsuming a variety of specialised meaning or narrower bisociated ideas linking only to a specific category of specialised meaning. With respect to the second research question RQ2, the memetic component contributes to the represented mathematical statement in two complementary ways: it provides specific disciplinary signs that appear in the final statement, and the social and structural meanings create a deeper level of significance, adding nuances to the represented statement, and endowing it with a humorous tang reflecting the feelings that the creator of the meme connects to doing mathematics in an entertaining way.

Putting all this knowledge together, we reinterpret our findings as a *heuristic for meme creating*, with an approach echoing Polya's heuristic for problem-solving (1945). This provides grounds to formulate the heuristic model for the creation of mathematical memes represented in Figure 8.16,

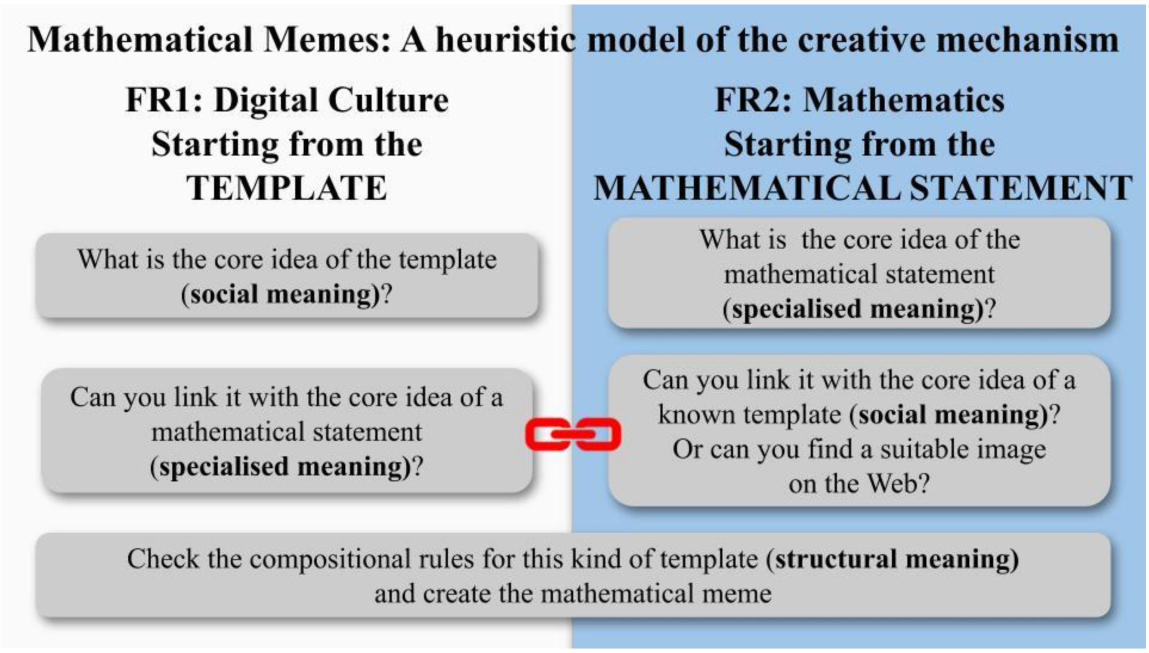


Figure 8. 16 A heuristic model for mathematical memes' creation mechanism

This heuristic model comprises all explored cases: the question *What is the core idea of the template?* on the left side (starting from the template) models Case A, the question *What is the core idea of the mathematical statement?* on the right side (starting from the mathematical statement) models Case B, and the question *Can you find a suitable image on the Web?* models the case of *Forgetful old man* (example #B3 in Table 6) created from a new image expressly inside the mathematical memesphere.

6 Discussion and conclusion

Our investigation started with the long-term purpose to shed light on the mechanism of creating mathematical memes to provide educators with knowledge to use these objects for the teaching of mathematics. The final heuristic action model in Figure 8.16 achieves this purpose, as it may inform educators about *how* to create a mathematical meme and anticipate *what* can be expected when students create one. It also offers strategies to plan educational tasks with students creating mathematical memes, that can be designed selecting a specific mathematical idea or a particular template. This latter has great potential for mathematics education, as students can be encouraged to "see with mathematician eyes" objects that dwell in their out-of-school culture, recognising mathematical structures in the templates, as the *repetition* of a procedure in *Do it again* (#C8) or the idea of *sameness* that bisociates into equality in *Spidermen pointing* (#C1). The model shows also that, notwithstanding memes' *hybrid language*, the creative mechanism can be initiated within whatever frame of reference the creator is more familiar with, giving educators a key to enter the mathematical memesphere through the mathematical statement they might feel more comfortable with.

The study showed also that mathematical memes, as bisociated creative acts, are created "rigorously abiding by the rules of both fields" (Koestler, 1964, p. 33), i.e., the rules shared in the memesphere that regulate the social and structural meaning (Bini et al., 2020) and the mathematical rules ensuring that a meme's specialised meaning is correct. This fact implies that these rules must be understood in their essence and not only reproduced automatically, as summarised by Koestler's description of the creative act as "the defeat of habit by originality" (p. 96). In mathematics education, this means that interacting with (i.e., creating or understanding) a mathematical meme requires conceptual knowledge of the topic and not only procedural knowledge, a fact that is widely recognized as important for success in mathematics (Canobi, 2009; Rittle-Johnson & Schneider, 2015). Moreover, the fluidity of boundaries between humour and science advocated by Koestler (1964, p. 91) suggests that, in an educational environment, mathematical memes can smoothly shift from being humoristic riddles (*seeing the joke*) to being mathematical problems (*solving the problem,* i.e. proving the validity of the represented statement) maintaining their nature as creative objects and engaging students in active participation to interpret them.

Mathematical memes are mathematical statements, yet at the same time, they are more than that. The memetic component adds to the original statement – pertaining to the domain of pure mathematical reasoning – translating it into “something entirely different” (Goethe, Maxims and Reflections 1279) enriched with humour and emotions – pertaining to other aspects of human life. All mathematical statements represented by memes in this work are valid assuming a proper system of axioms, but memes’ hybrid language adds something that, coupled with mathematical memes’ already known epistemic potential, shows us the way these digital objects can become interesting educational resources.

Methodologically, our work opens a new path in ethnographic research, drawing the attention of academic research upon how Web-users spontaneously employ communication tools of the digital culture to convey serious ideas such as mathematics, suggesting that “the time [is] ripe for that particular synthesis” (Koestler, 1964, p. 658). This research aims to pave the way for other research of the same kind and challenges scholars and educators to pay attention to what is going on out there where the students are. At the same time, the study suffers from all the limitations connected to ethnography and case studies. First, it investigates in an in-depth manner only a small number of exemplary cases within a more comprehensive ethnography. However, these cases are carefully and systematically chosen with respect to relevant criteria and to the aim of gaining theoretical knowledge in a completely new research area in mathematics education. Second, ethnography is a time-consuming research approach where the interpretation and analysis of data rely mainly on the knowledge gained by the researcher during the fieldwork. This means that a complete neophyte might encounter difficulties in using the model in Figure 8.16 to create a mathematical meme autonomously, especially in the identification of an up-to-date template conveying a suitable bisociated idea, which requires good confidence with the memesphere. Nevertheless, we think that the model could be effective in guiding him/her to *pose the right questions* to follow and understand someone else’s creative process. This point is paramount if we accept the idea that the Web 2.0 participatory culture that memes epitomise (Shifman, 2014) belongs to the generation of students and not to that of teachers. Therefore, memes would be more fruitfully used at school as *opportunities for students to speak their language* within the school environment, rather than as openings for teachers to show that they speak the students’ language, a behaviour that could be perceived as *cultural appropriation*, leading to negative outcomes (Coutts-Smith, 1976/2006). To avoid *cultural appropriation* in favour of *cultural appreciation*, school activities with mathematical memes would profit from putting students at the centre of the creative act, creating their own memes and re-creating them as readers of memes created by classmates, with the teacher possibly joining in at a later time with his/her memes, respecting the students' expertise on the matter. This approach would contribute to *fostering creativity in all students* as indicated by OECD 2030 and PISA 2021, by nurturing an environment promoting and acknowledging creativity that, as Csikszentmihalyi (1996) has shown, is a necessary condition for creative productions.

Further investigation should be conducted to corroborate these points, testing the efficacy of the model with educators and experimenting with the creative and emotional added values of mathematical memes in school activities. Indeed, although mathematics is commonly considered dry and relieved from any possible feelings, mathematics educators know that emotions are very much involved in mathematical engagement and learning (Di Martino & Zan, 2011; Goldin, 2014) and "rhetorical form plays an essential part in the expression and acceptance of all mathematical knowledge" (Ernest, 1999, p. 9). In fact, Aristotle in his *Rhetoric* (I.3, 1358a37ff as cited in Rapp, 2010, para. 2) ascribes the effectiveness of communication to three elements: the reliability of the speaker (ethos), the soundness of the argument (logos) and the emotions evoked (pathos). Our work shows that mathematical memes reach all the vertices of the rhetorical triangle: they are reliable as they are expected to be correct with respect to the digital culture and mathematical rules (ethos) (as shown in this study and in Bini et al., 2020), they establish an epistemic culture requiring proofs and argumentations to determine the truth-value of the represented statement (logos) (shown in Bini et al., 2020) and they convey positive emotions added to the statement by the memetic component (pathos) (as shown in this study). Therefore, they could be effective means of communication in the mathematic classroom, making mathematics less austere with their bisociated *ethos*, accurate *logos* and playful *pathos*

CHAPTER 9: SUMMARY OF RESULTS, DISCUSSION AND FURTHER RESEARCH

1 Summary of results

This research took off with the purpose of addressing the main research question presented in Chapter 0:

How can we conceptualize mathematical Internet memes and what are their educational potentialities, if any?

Table 9.1 summarises the subquestions addressed in Chapters 1-8 and the corresponding answers and key findings that will contribute to answering this question.

Table 9. 1 Research subquestions and corresponding answers and key findings

Ch	Subquestions	Answers	Key findings
1	1. What are the epistemic affordances, if any, of mathematical Internet memes? 2. Does creating and/or interacting with these memes implies/determines learning? 3. Which characterizations identify a boundary object in this context? Which interactions among the communities of students, teachers and researchers are triggered by these boundary objects? 4. Can memes, students' explanations vehiculated through the narrative virtual content and classroom discussion be identified as parts of a mathematical discourse?	The observation of the students' products seems to confirm memes' educational potentialities in fostering cognitive meta-awareness (students selected a topic based on their self-assessed knowledge) and linguistic awareness (students made an effort to explain it clearly using appropriate lexis). We formulate an a priori hypothesis about the levels of partial meaning composing the full meaning of a meme (triple-s) and we conjecture that the boundary object could be looked for in the social meaning that maintains its common identity across the communities of students and teachers. The couple mathematical meme+video appears effective in triggering a special form of mathematical communication, to be further investigated.	A priori hypothesis on the triple-s construct Conjecture about mathematical memes as boundary objects between the communities of students and teachers with the social meaning as the robust component

Ch	Subquestions	Answers	Key findings
2	1. What is the students' familiarity with social platforms and memes? 2. Are the meanings of didactical memes recognised by students the same as we described in the a-priori analysis? 3. What is the role of emotions in learning processes involving didactical memes?	The study confirmed students' expected acquaintance with social networking sites and memes and corroborated the a priori hypothesis about the partial meanings of a meme. The observation of the students' creation and interaction processes showed that the three partial meanings can be accessed in different orders. Students' physical reactions and utterances showed an emotional involvement that can be an educational potentiality: it guides and motivates students to understand the meme's specialised meaning, and it can be exploited by teachers to foster language awareness and further mathematical reflections.	Experimental confirmation of the a priori hypothesis about the triple-s construct Finding about the interplay among the social and specialised meanings. Sampling of the memes educational potentialities related to students' emotional involvement
3	1. Which characterizations identify a boundary object in this context? 2. Which learning mechanisms emerge from our observations? 3. How can school mathematics take into account the culture developed by young people in their everyday lives?	Mathematical meme act as boundary objects between the communities of students and teachers, initiating dialogic processes that carry potential for learning. The study confirmed our initial hypothesis that social meaning is the *robust* element crossing the boundary, and the specialised meaning is the *adapted* element. The boundary crossing learning mechanisms of confrontation, hybridization, and crystallization can be observed in the students' and teacher's interaction on the meme. Mathematical memes can enrich the teaching and learning of mathematics by taking into account cultural elements from young people's everyday lives.	Identification of mathematical memes as boundary objects between the communities of students and teachers. Identification of the social meaning as the robust component that represents the social values of the digital culture inside the school environment.

Ch	Subquestions	Answers	Key findings
4	1. Can young learners' social visual expertise be channelled by researchers and teachers to foster and support the understanding of mathematical meanings? 2. How may the affect generated by personal imagery be harnessed by teachers to increase the enjoyment of learning and doing mathematics?	Mathematical memes' visual component can harness the affect generated by imagery to engage students in doing mathematics. The triple-s construct of partial meanings of a meme can help researchers and teachers in creating educational activities that channel young learners' social visual expertise to foster and support the understanding of mathematical meanings	Sampling the educational potentialities of memes' visual components. The triple-s as an investigation tool to look into students' cognitive processes and as a task design tool
5	1. How do mathematical memes provide opportunities for connected learning experiences? 2. How can the connected learning framework and triple-s construct support educators in designing effective learning scenarios involving mathematical memes?	Mathematical memes can provide opportunities for connected learning experiences as they link expertise represented by the mathematical academic knowledge (the specialised meaning) with expertise related to learners' acquaintance with memes and popular culture (the structural and social meanings) The connected learning framework and the triple-s can support educators in designing new learning experiences that incorporate mathematical memes	Sampling the educational potentialities of memes' role in the digital culture The triple-s as an investigation tool to look into students' cognitive processes and as a task design tool
6	1. How can the phenomenon of mathematical memes be characterised as a representation of mathematical ideas within the Internet culture? **2.** How do mathematical memes activate and guide interactions among members of online communities?	Mathematical memes are representations of mathematical statements, expressed in a hybrid language that combines memetic and mathematical signs. They activate an epistemic culture in the online community directed toward the fulfilment of the need to determine the truth-value of the statement	Conceptualization of mathematical memes as representations of mathematical statements activating an epistemic culture Development of new methods

Ch	Subquestions	Answers	Key findings
7	1. At a macro level, what elements of mathematical memes and GeoGebra applets activate changes in students' praxeologies or some of their components? 2. At a micro-level, how are these changes activated in terms of mentors and agents?	The observation evidenced that the activity with mathematical meme and GeoGebra applets, blending elements from the different cultural realms, succeed in engaging and promoting changes in the praxeologies of students with very different learning styles. At the macro level, the meme components related to the digital culture (social and structural meaning) activated changes in the praxeologies of low-engaging students, and the meme specialised meaning, in connection with the GeoGebra applet, activated the high achiever's already internalised praxeologies. At the micro-level, the synergy between GeoGebra and memes promoted the appearance of new facilitators: a virtual mentor, scaffolding students in their ZPDs, a social and an epistemic agent fostering changes in students' praxeologies.	Sampling the educational potentialities of memes' social value and representativeness The triple-s as an investigation tool to look into students' cognitive processes at a micro level to identify the causes of changes in praxeologies
8	1. How can we model the mechanism of creating a mathematical meme? 2. How does the memetic component of a mathematical meme contribute to the represented mathematical statement?	The mechanism of creating a mathematical meme stems from a bisociated idea linking the template (social meaning) with the mathematical statement (specialised meaning). The connection is implemented through textual or pictorial additions complying with the structural meaning. The memetic component provides disciplinary signs that appear in the final statement, and creates a deeper level of significance, adding humorous nuances to the statement.	Investigation of the creative process and building of the heuristic model of memes' creative process. Development of new ethnographic methods

The key findings reported in Table 9.1 shed light on the object of the research from different standpoints:

Theoretical:

- Identification of the triple-s construct of the three partial meanings of a meme, a semiotic tool to interpret Internet memes (Chapters 1 & 2);
- Conceptualization of mathematical memes as representations of mathematical statements activating an epistemic culture, built on the theoretical framework used to investigate the Web 2.0 digital culture (Chapter 6);
- Investigation of the creative process, conducted through the lens of the bisociation theory of the creative act (Chapter 8);
- Identification of mathematical memes as boundary objects between the communities of students and teachers, with the social meaning as the robust component (Chapters 1 & 3).

Methodological:

- Development of an innovative use of ethnography framing multi-case focus studies, that now can be used to explore Internet phenomena in general (Chapters 6 & 8).

Empirical:

- Building of a heuristic model of the memes' creative process that can be used by educators in different contexts (Chapter 8);
- Sampling of how the triple-s can be used by researchers to look into learners' cognitive processes (Chapters 4 5 & 7) and by educators to decode mathematical memes and to design meme-based tasks that activate learning mechanisms (Chapters 4 & 5);
- Sampling of the educational potentialities of mathematical memes to mobilise emotions, channel visual skills and digital culture, and foster changes in students' praxeologies (Chapters 2 4 5 & 7).

Finally, the synthesis of these findings allows to address the main research question recalled at the beginning of this paragraph as follows:

> *Mathematical memes are representations of mathematical statements expressed in the new hybrid and multimodal language of Internet memes. They embed three levels of meanings: a social and structural meaning carried by the template and pertaining to the Web 2.0 digital culture, and a specialised meaning carried by disciplinary signs and pertaining to the mathematics culture. They can be created and interpreted by unlocking*

the encoded link between the social and structural meanings of the template and the specialised meaning of the mathematical statement.

Their hybrid nature positions them as boundary objects between the communities of students and teachers, with the educational potentialities to initiate learning mechanisms at the boundary crossing. In particular, their mathematical nature as statements has the educational potentiality to initiate epistemic needs aimed at determining the truth-value of the statement, and their characterising features as Internet memes (visual component, humour, puzzle effect, social value and representativeness) have the educational potentialities to engage students, mobilising positive emotions, fostering creativity and non-standard abilities, and activating learning facilitators.

Recalling Adler's notion of *transparency* as a necessary feature for effective educational resources (2000, p. 214), which depends on how much the resource is seen (*visible)* and seen through (*invisible*), we see that the first part of the answer sheds light on how mathematical memes can be seen through to illuminate their mathematical content (attaining *invisibility*), and the second part of the answer suggests possible ways to see mathematical memes in school mathematics practice (contributing to their *visibility*). Therefore, the findings of this research offer guidance to transform mathematical memes from Internet phenomenon into digital educational resources that can be used in a hybrid pedagogy, where "learners are provided the means to enact the task themselves, bringing to it their own meanings and interpretations from which to construct their mathematical knowledge" (p. 209).

Besides the findings addressing the main question, the research reached a theoretical finding represented by the triple-s construct, the semiotic tool to unpack the layers of meanings carried by a meme. Initially found as an answer without a proper corresponding question (and in fact in Chapter 1 there is no subquestion focusing on the meanings of a meme), the experimental confirmation of the initial a-priori conjecture about the triple-s opened the way to new questions in the following steps of the research. Thus, the triple-s progressed from being an interpretative semiotic tool providing criteria to extract and read data to decode the meanings carried by a meme, as initially imagined, to being a tool enabling the investigation of learners' cognitive processes as well as a task-design tool for educators, as shown in Figure 9.1. Furthermore, filling in the specialised meaning with other disciplinary subjects, the triple-s is applicable to more than mathematical memes, and could become a tool to ease the introduction of memes in other educational fields as well.

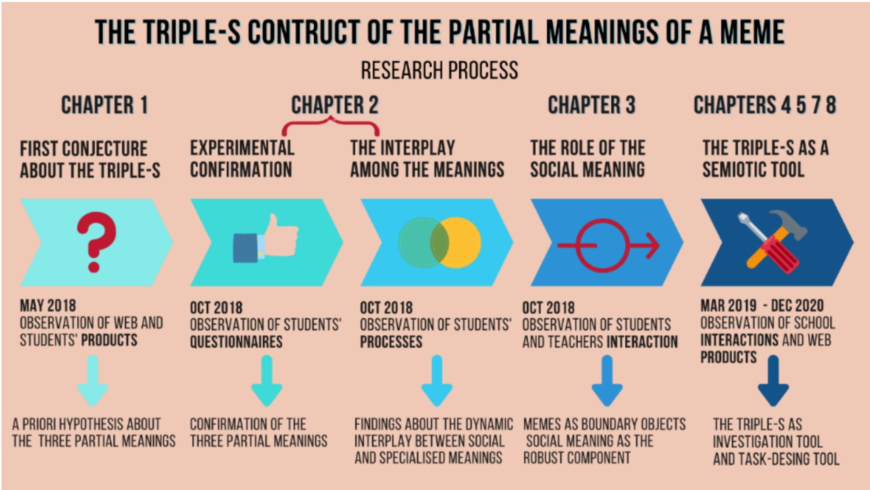


Figure 9. 1 The Triple-s research process and findings

2 Discussion

2.1 Wider implications: a new kind of resource

This research has provided knowledge to build the *visibility* and *invisibility* of mathematical memes as educational resources, nevertheless mathematical memes exceed Adler's categorisation of resources in school mathematics. For obvious reasons they cannot be considered "Basic Resources – For the Maintenance of Schooling" nor "Human resources" (p. 212). They also do not fit exactly within Adler's categorisation of Material and Social & Cultural resources represented in Figure 9.2, for the reason indicated in the following page.

Resources	Exemplars	Issues
Material		
Technologies	chalkboard, calculator, computer, copier	need for invisibility to see through techno-logy to practice
School mathematics materials	textbooks, other texts, cuisenaire rods, geoboards, computer software	mathematical meaning not obvious; mathe-matical meaning and pedagogical possib-ility is built-in; in use "learner centred" pedagogy – can become too visible
Mathematical objects	proofs, number lines, magic squares	specifically mathematical, but with social history, also need to be visible and invisible
Everyday objects	money, newspapers, stories, calculators, rulers	uses outside of maths, so need to be visible and invisible
Social & cultural		
Language	L1, L2, code-switching (CS), verbalisation, communication	assumptions that CS and talk are enabling; need to be visible and invisible
Time	time-table; length of periods; homework	structuring of time needs to be visible and invisible; with new pedagogies or when schooling breaks down, can become too visible

Figure 9. 2 Categorisation of mathematics resources [source Adler, 2000, p. 213]

- The specialised meaning of a mathematical meme can be considered a *Mathematical object*, which for Adler "arise[s] in the context of the discipline and the academy" (p. 211);
- The social meaning of a mathematical meme can be considered an *Everyday object*, whose "determining context has no direct relation to the mathematics classroom, but is constituted by everyday cultural practices" (ibid);
- The structural meaning of a mathematical meme can be considered a feature of the *Language* of Internet memes, contributing at a morphosyntactic level to the transmission of the social and the specialised meanings.

If we accept that the findings reached by this thesis point to the possibility of a successful transformation of mathematical Internet memes into digital educational resources, then we must devise a new category to accommodate them. As mathematical memes attain within a single object the "mixture of everyday and academic mathematics" (p. 207) advocated by Adler for a hybrid practice of school mathematics, I suggest that they can be categorised as "*Crossover*" resources, borrowing a term born outside the educational context to indicate "an instance of breaking into another category"[81], and already used by educational literature to identify learning settings connecting formal and informal education (Sharples et al., 2015).

Therefore, an educational activity with mathematical memes can be more than an educational experience with a digital tool, it is a *crossover* educational experience where the digital culture meets the school culture and shifts the paradigm (Kuhn, 1962), thus contributing to bridging the cultural discontinuity between teachers and learners, and between curricular mathematics and learners' background culture, language and heritage.

I am aware that, even with the theoretical and practical support provided by this thesis, using mathematical memes as crossover resources requires an effort from the teachers. It entails that is up to the teachers to step out of the traditional educational context and "move to meet them [the students]" (Lenhart, 2013). It also entails that teachers could find themselves in the position of knowing something less than the students, as the signs and semiotic rules of the memesphere naturally belong to the culture of the students and not to that of the teachers. These two features (stepping out of the known context and being at a disadvantage compared to students) might sound scary, but I think they are both necessary to shift the paradigm. To substantiate this claim, I recall here the words of McLuhan's disciple and media art theorist Gene Youngblood (1989, para. 5):

[81] https://www.merriam-webster.com/dictionary/crossover#h1

> "Communication (from the Latin "a shared space") is interaction in a common context ("to weave together"), which makes communication possible and determines the meaning of all that's said: the control of the context is the control of language is the control of reality. To create new realities, we must create new contexts, new domains of consensus. This cant' be done through communicating. You can't step out of the context that define communication by communicating: it will lead only to trivial permutations, repeatedly validating the same reality. Rather, we need a creative conversation (from the Latin, "to turn around together") that might lead to new consensus and hence new realities".

In his writing, Youngblood was debating about the future of computer graphics, but I think that his etymological distinction between *communication* and *conversation* enlightens the process of "stepping out of the context" in a way that applies to different fields. In my view, in education this means that the new reality that could shift the paradigm and bridge the cultural gap can only be created if teachers "step out of the context", withdrawing from the total control of the language and accepting the idea that they might need to learn something from the students. Thus, teachers and students can pair up to *turn around together* the established system of mutually understood signs and semiotic rules, enabling traditional school-centred communication to make space for new common contexts and crossover resources as mathematical memes.

2.2 Methodological contributions

While addressing the main content-based research question, this thesis also resulted in the development of a new research method that uses ethnography to investigate spontaneous digital objects in their ecological habitat from the point of view of Mathematics Education.

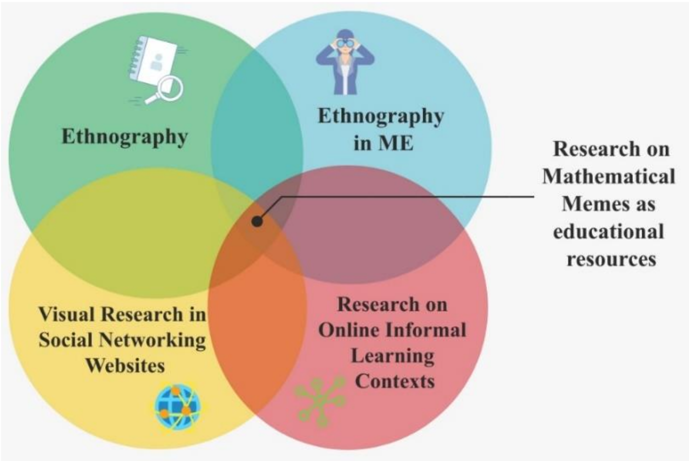


Figure 9. 3 Ethnographic research: The building of a new methodology

The new methodology (described and used in Chapters 6 and 8) is built drawing from different research domains as represented in Figure 9.3. It uses covert nonparticipant observation as a data collection method, restraining the collected data to publicly shared digital objects and comments to take into account the issues related to the ethics of conducting an ethnographic observation in a virtual environment. The ethnographic observation is then combined with a multi-case study approach to produce broad and deep knowledge of the observed phenomenon. The result is a completely new research methodology, stemming from the adaptation of the research in mathematics education to the context of ethnographic research, that had never been used before to study digital phenomena from the point of view of mathematics education and that can now be used to investigate Internet phenomena in general as *crossover* educational resources,

2.3 Scope and limitations

The findings of this study have to be seen in light of some methodological and study design limitations, which I will discuss in this paragraph.

2.3.1 Experimental research limitations

The experimental part of the study provided relevant data to explore the educational potentialities of mathematical memes. Nevertheless, data collected and analysed in this thesis have to be read in light of some limitations, which are due to practical research constraints. Methodologically, data are drawn from a small range of experiments all conducted in the same country (Italy). In particular, the secondary school experiments that provided data for the analyses of the processes (Chapters 2 3 4 5 7) involved a limited number of participants (a total of around 60 students) and took place in similar schools (STEM-oriented secondary schools) all located in the same part of the country (North). Moreover, the first experiment (Chapter 1) has been conducted on my own students, which could imply that results could have been biased by my personal expectations producing self-fulfilling prophecies. Another methodological limitation is that, besides a couple of personal interviews (Chapter 7), I did not systematically check for long-term effects of meme-based educational experiences on students' learning. Also, for reasons of time and length constrain, I did not analyse all the available data as evidenced in Table 0.6.

From the point of view of the study design, this thesis has not fully sampled all the educational potentialities of mathematical memes in terms of visual component, humour, puzzle effect, social value and representativeness. In particular, the connection between humour and learning is an aspect that has not been investigated, mainly because the humorous component is something that memes share with other educational objects that have been already widely explored in education (Cornett, 1986; Herbert, 1991) and mathematics education (Shmakov & Hannula, 2010; Menezes, 2017; Bakar & Amran, 2020).

From the theoretical point of view, the exploratory nature of this research which suggested investigating the object of the research through various theoretical lenses with diverse research goals unavoidably limited the depth of analysis attained from each perspective. Nevertheless, the choice of a single theoretical lens to observe an unexplored subject like this would have prevented from taking advantage of the different views offered by the multiplicity of perspectives.

Lastly, the theoretical finding about mathematical memes as representations of mathematical statements occurred after the conduction of the school experiments. Therefore, the tasks therein could not be designed according to these findings. But the *meme-statement correspondence* is exactly the new knowledge that enlightens the educational potential for future task designs on mathematical Internet memes.

2.3.2 Ethnographic research limitations

The ethnographic research included in this thesis sheds light on an unexplored phenomenon, notwithstanding all the acknowledged limitations typical of this research method, which is time-consuming (nearly three years in this case), and it is deeply centred on knowledge of the observed environment and phenomenon gained by the ethnographer in the course of the research. Even if all possible efforts have been made to ensure systematic and rigorous approaches to sampling, field notes, and data collection, the issues related to the length of the research and to the centrality of the ethnographer make this research difficult to replicate and therefore make the corresponding results hard to check for an outsider, that could challenge the objectivity of the observer or consider the illustrating examples chosen as case studies as cherry-picked. However, main results can be validated by other researchers using other threads of comments within public online communities on mathematical memes.

A second limitation is connected to the methodological choice of being a covert nonparticipant observer within online communities, excluding the possibility to conduct personal interviews with some of the memes' authors and commenters. Even if knowing more about the cultural, geographical and social background of memes' author and commenters surely is an interesting research focus, the choice to remain undisclosed was made believing that preliminary information to understand the phenomenon of mathematical memes as it appears spontaneously on the Web could be only captured from unprompted interactions in the phenomenon natural context, keeping the ecological validity of the research high.

3 Recommendations for further research

This thesis has highlighted many topics requiring further research, some of which are given below:

Building upon findings of the research. From the theoretical point of view, further steps can be aimed at using the conceptualization of mathematical memes as statements and the triple-s construct to look into mathematical memes from a semiotic or aesthetic perspective. From a practical point of view, new research could be aimed at providing educators with hands-on tools to adopt mathematical memes as educational resources. The latter could be done, among others, designing and testing a mathematical meme database open to teachers' contributions that could foster the gathering of a community of practice around the new crossover resource, or creating a mathematical memes generator website supporting the addition of mathematical formulas.

Addressing limitations of the research. The limitations of this research summarised in the previous paragraph leave space for further research in many directions. Gathered data related to feedback questionnaires which are not analysed in this thesis can be an interesting source for new studies that take into account the students' and teachers' voices about this new educational resource. Furthermore, new school experiments can be conducted, aimed at expanding the study of mathematical memes' educational potentialities in connection with their humorous nature or deepening the aspects which have been just sampled in this thesis. In particular, the connection between memes and mathematical discourse (in Sfard's sense) which has just been touched in Chapter 1 deserves a thorough investigation. The ethnographic research could be expanded by gathering data about the socio-cultural backgrounds of the users acting and interacting on mathematical memes, possibly conducting interviews with the purpose of collecting deeper knowledge about the creation acts.

Constructing the same research in a new context, location and/or culture. More research is needed to validate the experimental results of this thesis: new studies can be conducted with students of other grades, or with students attending non-STEM secondary school in different geographic, social and cultural contexts.

Re-assessing and expanding theories, frameworks or models addressed in the research. Future studies can assess the usability of the heuristic model for the creation of mathematical memes and probe the ethnographic methodology devised in this thesis to investigate other Internet phenomena from an educational point of view in the field of Mathematics Education or can make use of the triple-s construct on Internet memes with different specialised meanings as an investigation tool in other educational fields.

4 Conclusion

The initially stated overarching aim of this research to providing insight into the process of transforming the Internet phenomenon of mathematical memes as it appears naturally in the Web into a digital resource for teaching mathematics.

The results show that, in a way echoing Max Ernst famous description of surrealism as "a linking of two realities that by all appearances have nothing to link them", the linking of two apparently clashing realities as mathematics and memes has produced a coherent unified system for meaning making, which is accessible only to users who are proficient in both domains and which can be a useful educational resource. The findings also prove that this contamination between mathematics and memes empowers both cultural realms: it upgrades the use of memes beyond their original subculture, enriching them with an epistemic power that nurtures mathematical discussions, and it expands the range of disciplinary signs traditionally pertaining to the domain of mathematics. Pairing with mathematical content, memes move from their original purpose of disseminating feelings and opinions to that of stating mathematical truths, while maintaining their ironic *reductio ad absurdum* humour (Suler, 2015) that endows them with their comic twist mobilising positive emotions. Thus, they weave a *social tapestry* (Milner, 2017) that sketches a portrait of mathematics and mathematicians different from the boring and emotionless stereotype and possess a *discursive power* to aggregate users in a community sharing an "alignment of meanings" (Wiggins, 2019, p. 23), which this research has shown is preserved in a traditional educational context, transcending the physical boundaries of the classroom and transforming mathematical memes in effective crossover educational resources that "emphasize the movements and connections between mathematics education and other practices" (Bakker et al., 2021, p. 8).

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APPENDICES

Appendix A: Entry online form

Administered before the 2nd and 3rd secondary school experiments and before Distance Learning experiments

Research aim: Assess students' familiarity with memes and social media

#lifeonmath MEME PROJECT - ENTRY FORM

a project by Giulia Bini and Ornella Robutti

*Campo obbligatorio

#lifeonmath MEME PROJECT is a research project on mathematical memes conducted by the Department of Mathematics of the University of Turin. Do you like memes and want to help us? Answer the questions below!

1. Your school *

2. Your town and country *

3. Your age *

Tick one choice only

- 14
- 15
- 16
- 17
- 18
- 19
- 20
- > 20

4. What is a meme for you? *

5. How would you describe a meme to someone who does not know what it is? *

6. What is the main purpose of a meme in your opinion? *

7. What do you do with memes? (you can choose more than one option) *

Tick all that apply

- [ ] I create them and share them through social media
- [ ] I share those that I like through social media
- [ ] I share those that I like with my friends on Whatsapp or other message apps
- [ ] I like or react to those that I find on the Internet
- [ ] I comment those that I find on the Internet
- [ ] I do not interact with memes

Altro: [ ] ____________

8. Have you ever seen a mathematical meme? *

Tick one choice only

- ( ) Yes
- ( ) No
- ( ) I don't know

9. If so, what did you think? *

10. Which social websites do you use? (you can choose more than one option) *
Tick all that apply

- [ ] I do not use any social website
- [ ] Facebook
- [ ] Instagram
- [ ] Reddit
- [ ] Twitter
- [ ] Pinterest
- [ ] Tumblr
- [ ] 9GAG
- [ ] 4chan

Altro: [ ]

11. How often do you connect to social websites? *
Tick one choice only

- ( ) Every day, several times a day
- ( ) Every day, just once
- ( ) About 3 times a week
- ( ) Once a week
- ( ) Once a month
- ( ) Never
- ( ) Altro:

Appendix B: Promotional leaflet

Shared with the class before the 3rd secondary school experiment)

Research aim: engage students and administer the entry questionnaire (in the QR code)

Appendix C: Entry worksheet

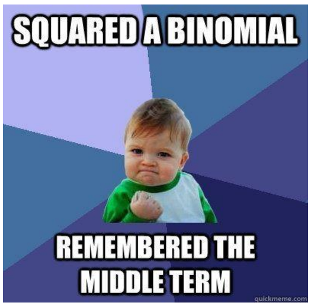
Administered before the 2nd and 3rd secondary school experiments

Research aim: Asses students' familiarity with mathematical memes and check the a-priori hypotheses about memes' partial meanings

DIPARTIMENTO DI MATEMATICA GIUSEPPE PEANO UNIVERSITÀ DI TORINO — UNIVERSITÀ DEGLI STUDI DI TORINO

#lifeonmath MEME PROJECT ENTRY WORKSHEET

a project by Giulia Bini and Ornella Robutti



Do you know this meme? YES NO

Does it make you laugh? YES NO

In your opinion, what leads you to say that this is a meme and not a comic or a cartoon?

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What is the meaning of the image in the meme in your opinion?

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Which mathematical topic is recalled by this meme?

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Do you think the meme can help you remember the mathematical rule better?

YES NO

How would you explain, using words and examples, the mathematical content necessary to understand the meme?

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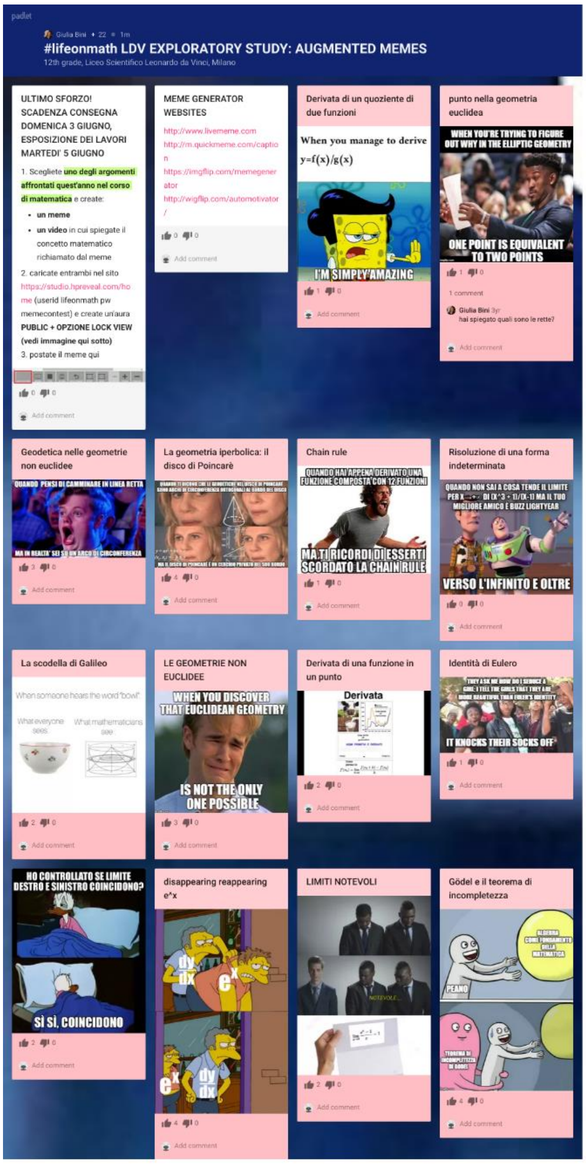
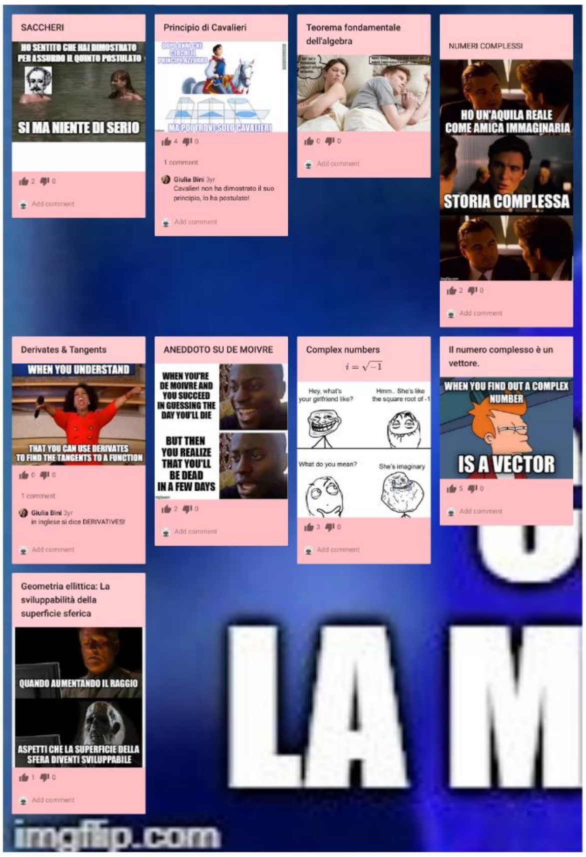
Appendix D: Creation task (1st secondary school exp.)

Digital sharing space for students' productions collected during the activity in Padlet walls

Research aim: investigate the role of sharing and likes

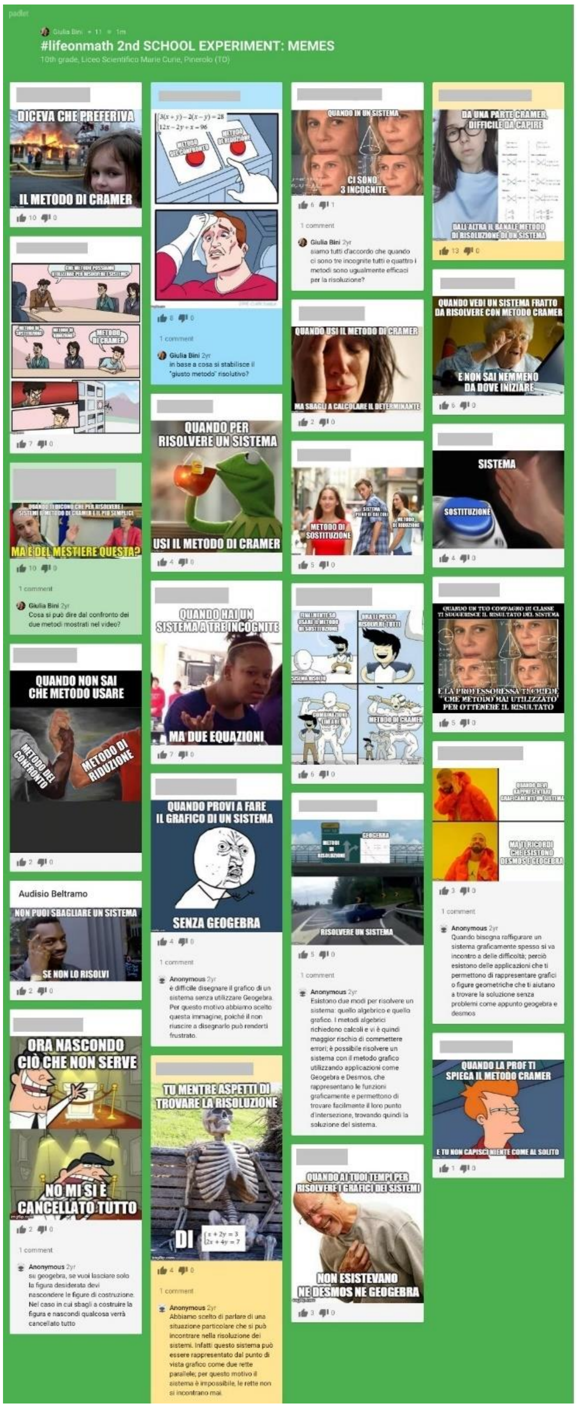


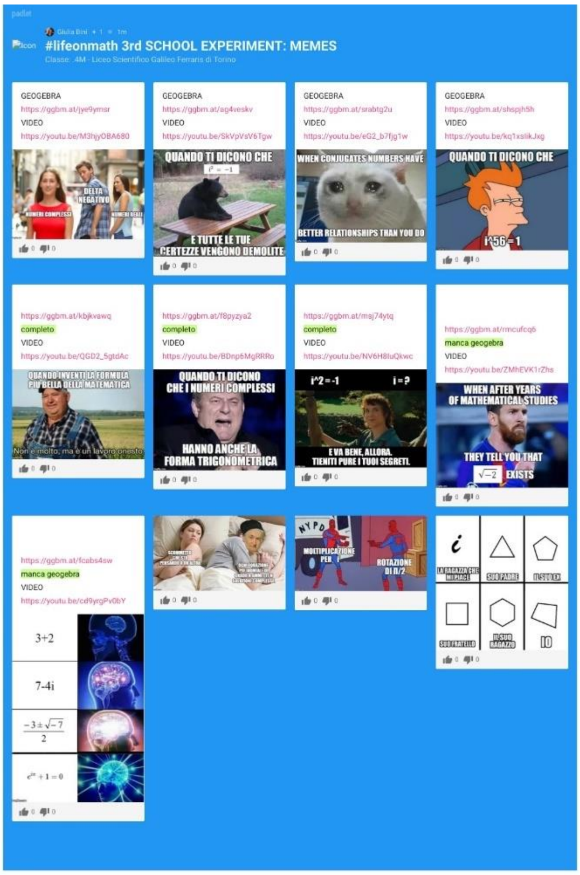


Appendix E: Creation task (2nd & 3rd secondary school exp.)

Digital sharing spaces for students' productions collected during the activity in Padlet walls 1st experiment (left) 2nd experiment (right)

Research aim: investigate the role of sharing and likes

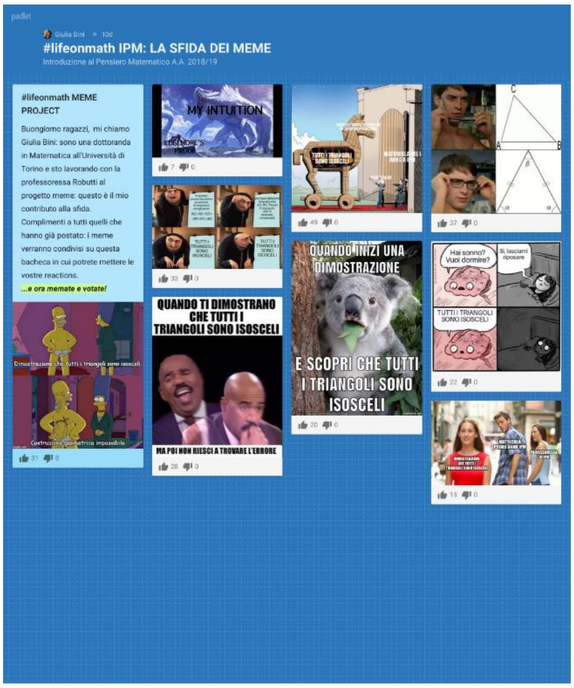




Appendix F: Creation task (1st university exp.)

Digital sharing spaces for students' productions collected during the activity in Padlet walls

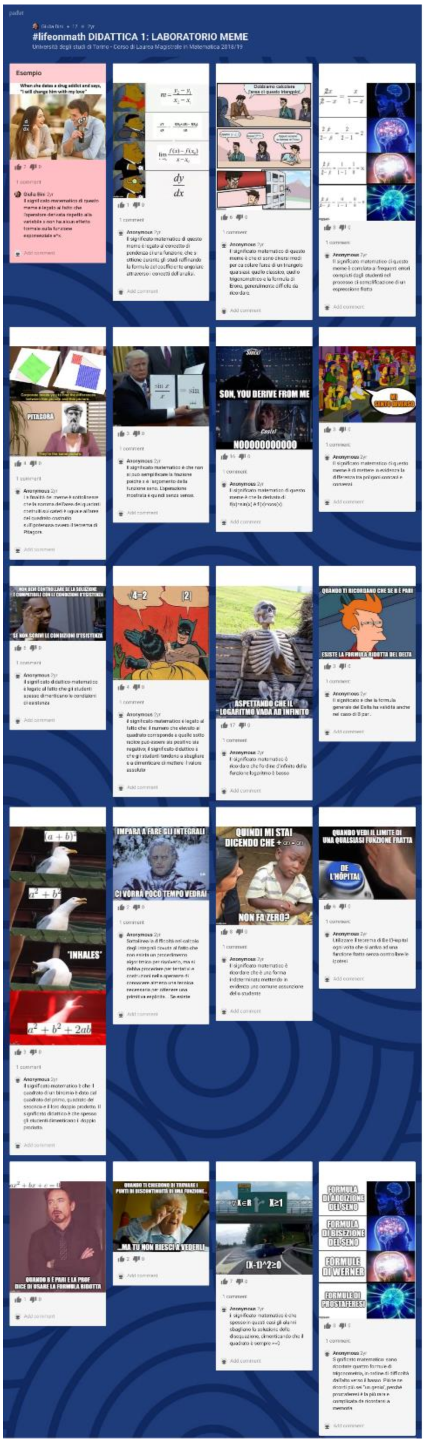
Research aim: investigate the role of sharing and likes

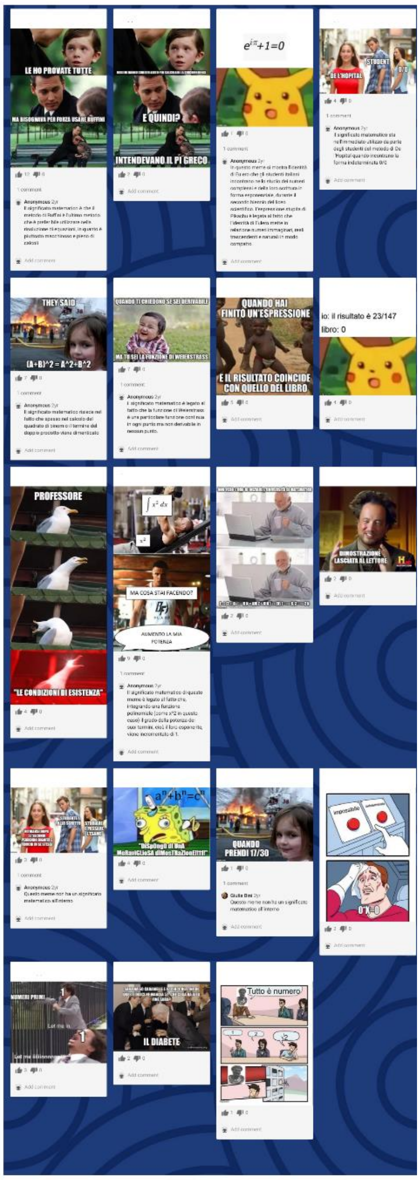


Appendix G: Creation task (2nd university exp.)

Digital sharing spaces for students' productions collected during the activity in Padlet walls

Research aim: investigate the role of sharing and likes

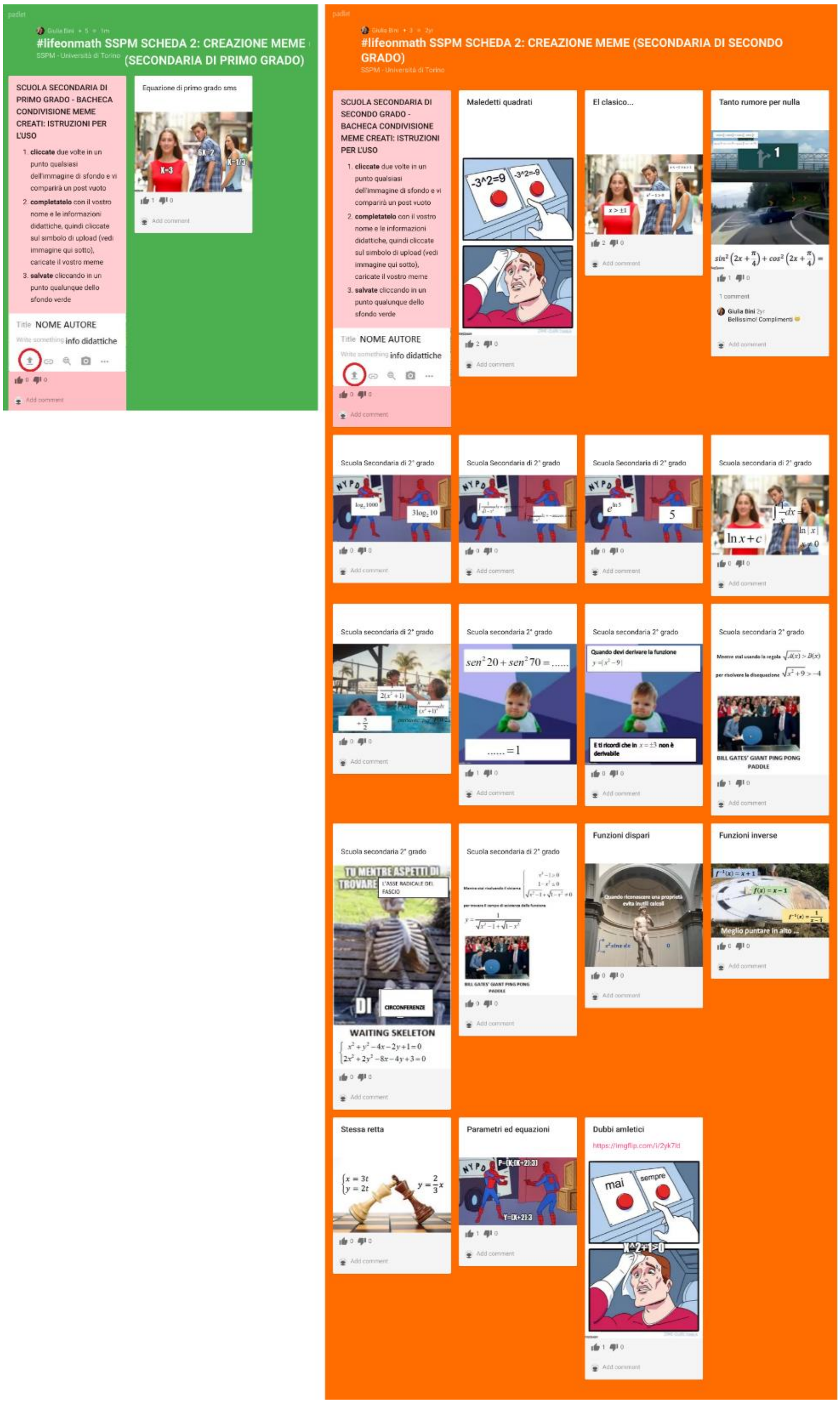




Appendix H: Creation task (teacher training course)

Digital sharing spaces for students' productions collected during the activity in a Padlet walls

Research aim: investigate the role of sharing and likes



Appendix I: Creation task billboard (2st secondary school exp.)

Printed sharing space for students' production shared with the class before the whole class discussion

Research aim: investigate the effect of sharing on long-term memories

Appendix J: Decoding task worksheet (2nd secondary school exp.)

Administered before the whole class discussion

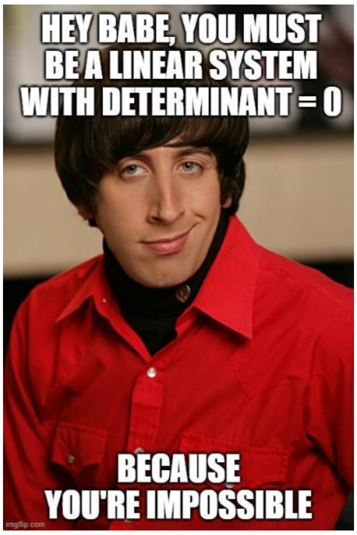
Research aim: Investigate if students decode the mathematical content of mathematical memes created by others

DIPARTIMENTO DI MATEMATICA GIUSEPPE PEANO UNIVERSITÀ DI TORINO
UNIVERSITÀ DEGLI STUDI DI TORINO

#lifeonmath MEME PROJECT REFLECTIVE WORKSHEET

un progetto di Giulia Bini e Ornella Robutti

Guys, congratulations on your work! Here below you find my memes: do you like them? In your opinion, what are the mathematical meanings of these memes?



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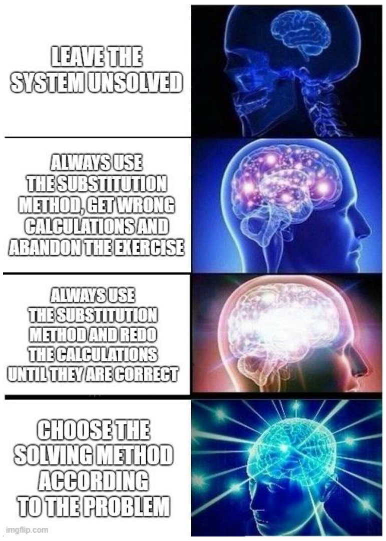

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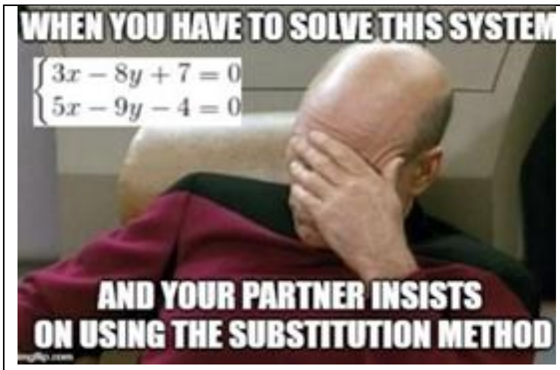
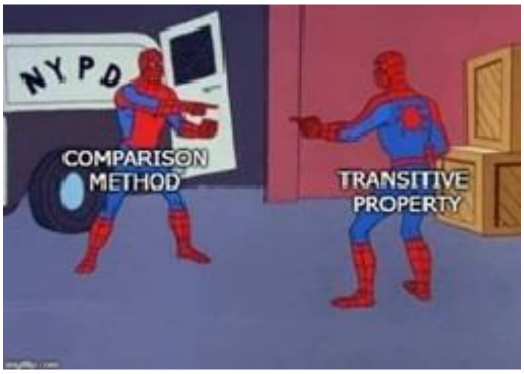
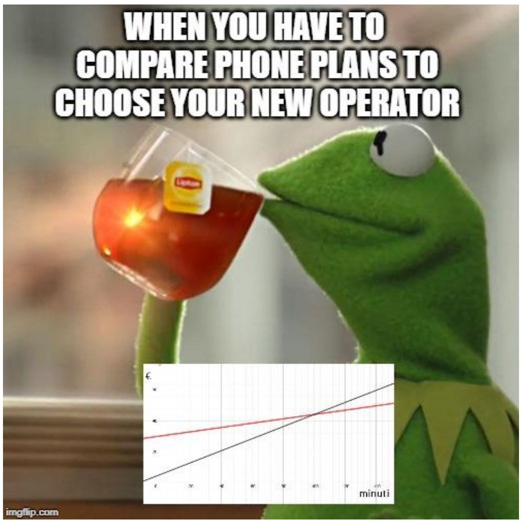

WHEN YOU HAVE TO SOLVE THIS SYSTEM
$\begin{cases} 3x - 8y + 7 = 0 \\ 5x - 9y - 4 = 0 \end{cases}$
AND YOUR PARTNER INSISTS
ON USING THE SUBSTITUTION METHOD


NYPD
COMPARISON METHOD
TRANSITIVE PROPERTY


WHEN YOU HAVE TO
COMPARE PHONE PLANS TO
CHOOSE YOUR NEW OPERATOR
€
minuti
imgflip.com

Appendix K: Decoding task worksheet (3rd secondary school exp.)

Administered before the whole class discussion

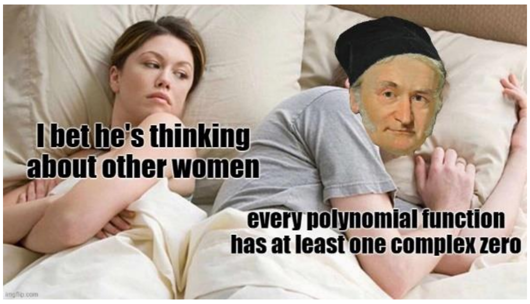
Research aim: Investigate if students decode the mathematical content of mathematical memes created by others

DIPARTIMENTO DI MATEMATICA GIUSEPPE PEANO UNIVERSITÀ DI TORINO   UNIVERSITÀ DEGLI STUDI DI TORINO

#lifeonmath MEME PROJECT REFLECTIVE WORKSHEET

un progetto di Giulia Bini e Ornella Robutti

Guys, congratulations on your work! Here below you find my memes: do you like them? In your opinion, what are the mathematical meanings of these memes?



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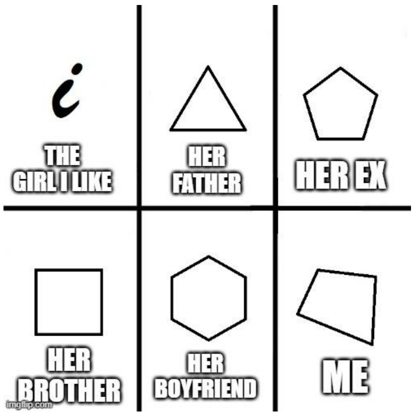

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Appendix L: Online feedback form for students

Administered before the whole-class discussion in the 2nd and 3rd secondary school experiments

Research aim: Collect students' short-term impressions about the first part of the activity (creation)

#lifeonmath MEME PROJECT – FEEDBACK FORM

a project by Giulia Bini and Ornella Robutti

*Campo obbligatorio

UNIVERSITA DEGLI STUDI DI TORINO

DIPARTIMENTO DI MATEMATICA GIUSEPPE PEANO UNIVERSITÀ DI TORINO

1. Your school *

2. Your town and country *

3. Did you like this activity? *

Tick one choice only

	1	2	3	4	5	6	7	8	9	10	
not at all	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	very much

4. Are you satisfied with your work? * *

Tick one choice only

	1	2	3	4	5	6	7	8	9	10	
not at all	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	very much

5. What guided your choice of the image for your meme? *

6. What specific meaning did you want to bring out? *

7. Did you want to make people laugh? *

Tick one choice only

◯ Yes
◯ No

8. If you answered yes, what did you use to create humor? *

9. Did you learn or understood something better? *

10. Will the meme creation help you remember this topic better? *

Tick one choice only

	1	2	3	4	5	6	7	8	9	10	
not at all	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	very much

11. Did you tell someone about this activity? *

Tick one choice only

◯ Yes
◯ No

12. Did you show someone your meme? *

Tick one choice only

◯ Yes
◯ No

Appendix M: Online evaluation form for students

Administered three months after the activity in the 2nd and 3rd secondary school experiments

Research aim: Collect students' long-term overall recollection and impressions about the activity

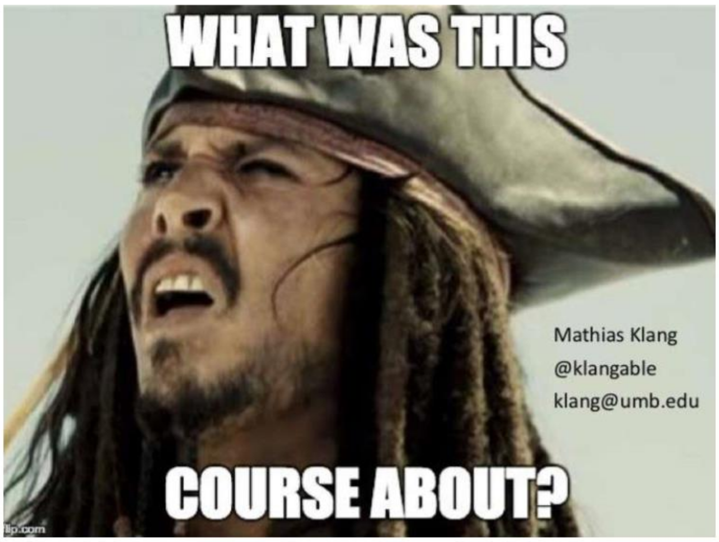
#lifeonmath MEME PROJECT – STUDENTS' EVALUATION FORM

a project by Giulia Bini and Ornella Robutti

*Campo obbligatorio

UNIVERSITA DEGLI STUDI DI TORINO

DIPARTIMENTO DI MATEMATICA GIUSEPPE PEANO UNIVERSITÀ DI TORINO



1. Your school *

2. Your town and country *

3. What do you remember about the experience we did on memes? You can choose more than one answer *

Tick all that apply

- [ ] The meme
- [ ] The video
- [ ] The GeoGebra applet
- [ ] The class discussion
- [ ] The mathematical topic
- [ ] I do not remember anything

What did you feel? Describe below your impressions on the various aspects of the experience: you do not need to write long reflections, just a brief remark or a few keywords

4. the meme *

5. the video *

6. the GeoGebra applet *

7. the class discussion *

8. The mathematical topic *

9. Do you think this experience has allowed you to express your creativity more than a typical maths class? *

Tick one choice only

- Yes
- No
- I don't know

10. Why? How? *

11. Do you think this experience has changed your attitude about math a little? * *

Tick one choice only

- Yes
- No
- I don't know

12. Why? How? *

13. Have you imagined, created or noticed any other mathematical memes in this period? You can choose more than one answer *

Tick all that apply

- [ ] Immagined
- [ ] Created
- [ ] Notices
- [ ] None of the above

Thanks a bunch!

Questi contenuti non sono creati né avallati da Google.

Google Moduli

Appendix N: Online feedback form for teachers

Administered shortly after the activity in the 2nd and 3rd secondary school experiments and spontaneously filled in for distance learning experiments

Research aim: Collect teachers' impressions about mathematical memes and their use at school

#lifeonmath MEME PROJECT - TEACHER'S EVALUATION FORM

a project by Giulia Bini and Ornella Robutti

*Campo obbligatorio

UNIVERSITA DEGLI STUDI DI TORINO

DIPARTIMENTO DI MATEMATICA GIUSEPPE PEANO UNIVERSITÀ DI TORINO

1. Name and Surname *

2. Email *

3. Town and country *

4. Name and type of school *

5. Class with which the activity was carried out *

6. Has the teacher created any meme? *

   Tick one choice only

   - Yes
   - No

Sharing of products: please send the works of the students (and any memes created by the teacher) to lifeonmathmeme@gmail.com.
If they were collected in a Padlet, you can enter the link below directly.

7. Link to the Padlet with the students' works *

____

Overall report and comments

8. What is your overall assessment of the experience? *

Tick one choice only

	1	2	3	4	5	
very bad	◯	◯	◯	◯	◯	excellent

9. Are you satisfied with the students' work? *

Tick one choice only

	1	2	3	4	5	
not at all	◯	◯	◯	◯	◯	very much

10. After this experience, do you think that memes can contribute in opening a new communication channel between students and teachers? *

____

11. Can the interaction with memes in the classroom change something in the attitude of the students towards mathematics? *

____

12. Do you think you will use memes again in your teaching practice? You can choose more than one answer *

Tick all that apply

- [ ] Yes, by replicating this experience with another class
- [ ] Yes, replicating this experience with the same class on another topic
- [ ] Yes, I will use ready made memes as an incentive for class discussion
- [ ] Yes, I intend to use it with impaired students
- [ ] Yes, but I still do not know how
- [ ] No

Altro: [ ] ____________________

13. Other comments

____
____
____
____
____

Questi contenuti non sono creati né avallati da Google.

Google Moduli

Appendix O: Decoding task worksheet (teacher training exp. grade 6 to 8)

Administered during the activity

Research aim: investigate how teachers interact with the mathematics encoded in memes

Piano Lauree Scientifiche — SSPM — DI. FI. MA. in Rete

#lifeonmath UTILIZZARE I MEME MATEMATICI NELLA DIDATTICA

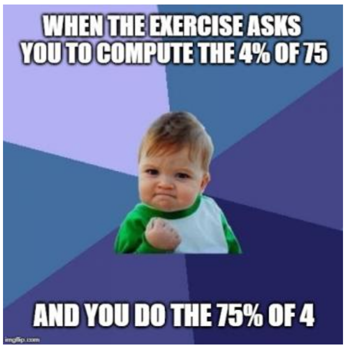
Giulia Bini & Ornella Robutti - 28 Marzo 2019

Meme decoding worksheet

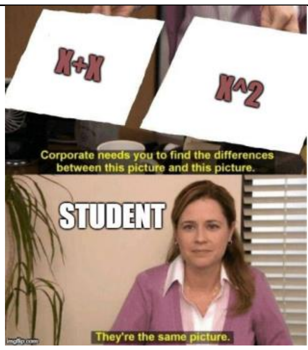
SPIDERMAN POINTING AT SPIDERMAN

DISTRACTED BOYFRIED

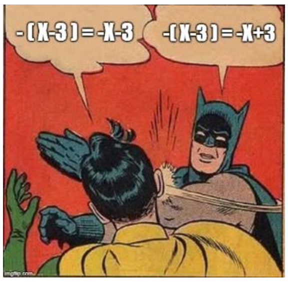
DROWING KID IN THE POOL



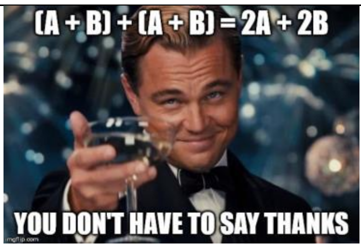
SUCCESS KID



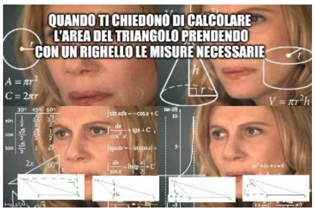
THEY'RE THE SAME PICTURE



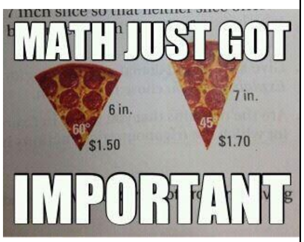
BATMAN SLAPPING ROBIN



LEONARDO DICAPRIO CHEERS



CONFUSED MATH LADY



PIZZA MATH

Decoded meme: ……………………………………………………………………………………
Structural meaning ……………………………………………………………………………
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Social meaning …………………………………………………………………………………
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Specialised meaning ……………………………………………………………………………
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POSSIBLE DIDACTICAL USE
Grade ……………………………………………………………………………………………
Topic ……………………………………………………………………………………………
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Prerequisites ……………………………………………………………………………………..
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Methodology ……………………………………………………………………………………
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References to the National Curriculum ………………………………….…………………..
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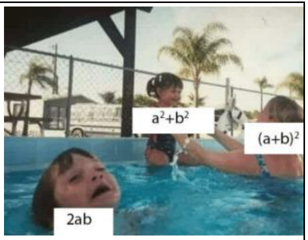
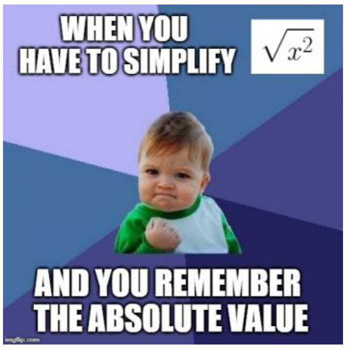
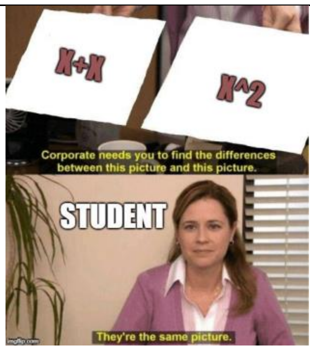
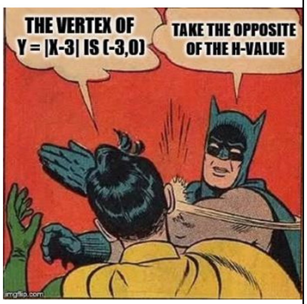
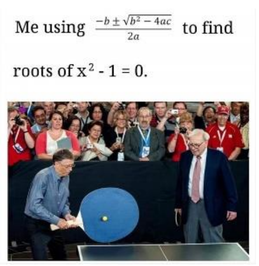
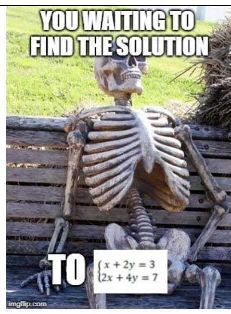
Appendix P: Decoding task worksheet (teacher training exp. grade 9 to 13)

Administered during the activity

Research aim: investigate how teachers interact with the mathematics encoded in memes

Piano Lauree Scientifiche — SSPM — DI. FI. MA. in Rete

#lifeonmath UTILIZZARE I MEME MATEMATICI NELLA DIDATTICA
Giulia Bini & Ornella Robutti - 28 Marzo 2019
Meme decoding worksheet

 **SPIDERMAN POINTING AT SPIDERMAN**	 **DISTRACTED BOYFRIED**	 **DROWING KID IN THE POOL**
 **SUCCESS KID**	 **THEY'RE THE SAME PICTURE**	 **BATMAN SLAPPING ROBIN**
 **BILL GATES' GIANT PING PONG PADDLE**	 **IPHONE X MEME**	 **WAITING SKELETON**

Decoded meme: ……………………………………………………………………………………
Structural meaning ……………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………
Social meaning …………………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………
Specialised meaning ……………………………………………………………………………
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POSSIBLE DIDACTICAL USE
Grade ……………………………………………………………………………………………
Topic ……………………………………………………………………………………………
…………………………………………………………………………………………………
Prerequisites …………………………………………………………………………………
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Methodology …………………………………………………………………………………
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Appendix Q: Creation task worksheet (teacher training exp.)

Administered during the creation activity

Research aim: investigate how teachers interact with memes

#lifeonmath UTILIZZARE I MEME MATEMATICI NELLA DIDATTICA

Giulia Bini & Ornella Robutti - 28 Marzo 2019

Meme creation worksheet

Create a mathematical meme using one or more of these templates, and describe its possible didactical use.

EXAMPLE 1: SPIDERMAN POINTING AT SPIDERMAN

Grade ……………………………………………………………………………………

Topic ……………………………………………………………………………………

Prerequisites ……………………………………………………………………………

Methodology ……………………………………………………………………………

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References to the National Curriculum ……………………………………………………

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EXAMPLE 2: DISTRACTED BOYFRIEND MEME

Grade ……………………………………………………………………………………

Topic ……………………………………………………………………………………

Prerequisites ……………………………………………………………………………

Methodology ……………………………………………………………………………

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References to the National Curriculum ……………………………….………………….

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EXAMPLE 3: DROWNING KID IN THE POOL MEME

Grade ………………………………………………………………………………………
Topic ………………………………………………………………………………………
Prerequisites ……………………………………………………………………………….
Methodology ………………………………………………………………………………

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References to the National Curriculum ……………………………….………………….

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EXAMPLE 4: SUCCESS KID MEME

Grade ………………………………………………………………………………………
Topic ………………………………………………………………………………………
Prerequisites ……………………………………………………………………………….
Methodology ………………………………………………………………………………

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References to the National Curriculum ……………………………….………………….

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PUBLICATIONS

Articles in Scientific Journals

Bini, G., Robutti, O., & Bikner-Ahsbahs, A. (2020). Maths in the time of social media: conceptualizing the Internet phenomenon of mathematical memes. *International Journal of Mathematical Education in Science and Technology.* https://doi.org/10.1080/0020739X.2020.1807069

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Bini, G. (2021). How Spiderman Can Teach You Math: The Journey of Memes from Social Media to Mathematics Classrooms. In J. H., Kalir, & D. Filipiak, *Proceedings of the 2020 Connected Learning Summit* (Version 2). (pp. 20-27) Carnegie Mellon University, ETC Press, USA. https://doi.org/10.1184/R1/13530038.v2

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