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# Bargaining over a Divisible Good in the Market for Lemons[†]

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*We study bargaining with divisibility and interdependent values. A buyer and a seller trade a divisible good. The seller is privately informed about its quality, which can be high or low. Gains from trade are positive and decreasing in quantity. The buyer makes offers over time. Divisibility introduces a new channel of competition between the buyer's present and future selves. The buyer's temptation to split the purchases of the high-quality good is detrimental to him. As bargaining frictions vanish and the good becomes arbitrarily divisible, the high-quality good is traded smoothly over time and the buyer's payoff shrinks to zero.* (*JEL* C78, D82, L15)

In many economic environments, agents bargain over goods that are divisible. Negotiations in financial markets typically involve both the amount of an asset and its price. Banks and institutional investors (e.g., pension funds) routinely bargain over how much of a securitized asset (pool of mortgages, credit-card debts, automotive loans) to trade and at what price. Similarly, after restructuring a company, an equity firm negotiates what fraction to sell and at what price. These negotiations are generally dynamic and decentralized. One party, typically the seller, is better informed about the quality of the asset.[1]

We study bargaining over a divisible good with asymmetric information, interdependent values and positive gains from trade. Gains from trade of a divisible good may depend not only on the quality of the asset, but also on how much of it has already been traded. We focus on the case of decreasing gains from trade, which leads to new insights into bargaining. Consider a bank negotiating the sale of a pool of mortgages to a pension fund. The quality of the asset can be either low or

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[1] Consider the classic example of the synthetic CDO Hudson Mezzanine. As explained in McLean and Nocera (2011), Goldman Sachs selected all the securities in that CDO, aimed to sell it as fast as possible and simultaneously bet against that security by taking a short position. See also Ashcraft and Schuermann (2008), Downing, Jaffee, and Wallace (2009), and Gorton and Metrick (2013).

high, depending on its future cash flows from homeowners. As the pension fund is more interested in owning these promises of future cash flows, there are gains from trade. These gains are decreasing in the amount of the asset already traded between the parties, as they reflect the pension fund's desire to diversify its portfolio. The bank is directly involved in the process of securitization and hence has better information about the quality of these assets.

The main message of this paper is that divisibility introduces a new channel of competition between the buyer's present and future selves, and that this new channel has stark implications for the pattern of trade and for parties' payoffs. When assets are arbitrarily divisible and bargaining frictions vanish, high-quality assets are traded gradually. Divisibility is detrimental to the buyer; the competition between his present and future selves drives his payoffs to zero. This is in contrast to the outcome when the asset is indivisible. In that case, only the low-quality asset is traded in the beginning of the relationship. A market freeze then follows, and only afterwards the high-quality asset is traded. The buyer of an indivisible asset obtains a positive payoff.

We extend the canonical bargaining model with incomplete information (Fudenberg, Levine, and Tirole 1985; and Gul, Sonnenschein, and Wilson 1986) to account for interdependencies in values (Deneckere and Liang 2006—henceforth, DL) and divisibility. A buyer purchases a durable good from a seller who is privately informed about its quality. A high-type seller provides a high-quality good, while a low-type seller provides a low-quality good. The good is divided into finitely many units. There are positive gains from trade, which are decreasing in the number of units already traded by the parties. In every period, the buyer makes a take-it-or-leave-it offer that specifies a price and a number of units to be traded. The bargaining process continues until the parties have traded all available units. We assume that all learning is strategic. The buyer learns about the good's quality only through the seller's behavior; owning a fraction of the good does not provide the buyer with additional information about its quality.[2]

In equilibrium, the buyer employs only two types of offers: *screening* and *universal*. Screening offers are for all remaining units at a price lower than the high-type seller's cost. Universal offers are for some (or all) of the remaining units, at a price equal to the high-type seller's cost. The buyer alternates between screening the seller and purchasing some units through universal offers. The low-type seller randomizes between accepting and rejecting screening offers, while the high-type seller always rejects them. The rejection of screening offers makes the buyer more optimistic that the good is of high quality. Eventually, he is optimistic enough to purchase some (or all) of the remaining units through a universal offer. Both seller types accept this offer. After the purchase, the units that remain (if any) are less valuable, so the buyer returns to screening the seller.

Our main result characterizes the limit equilibrium outcome when bargaining frictions vanish and the good becomes arbitrarily divisible. We first let the length of each period converge to zero and we then let the number of units grow to infinity. In the limit, the buyer continuously makes both screening offers and universal offers

[2] We elaborate on the assumption of strategic learning in Section IB.

for infinitesimal fractions of the good. At each point in time, he breaks even with either type of offer, so he obtains a payoff equal to zero. The high-type seller only accepts universal offers and thus sells the good smoothly over time. The low-type seller is indifferent between the two offers (screening and universal). He sells the good smoothly (pooling with the high-type seller) until a certain random time, and then concedes by selling the remaining fraction of the good at once.

Our work provides novel and testable predictions for markets for lemons. Our model highlights a rationale for markets of divisible goods (like markets for securities) to be more efficient than markets for indivisible goods (like real estate markets). We also show that these markets differ sharply on how parties split the gains from trade. Divisibility is detrimental to the buyer (uninformed party with bargaining power) and beneficial to the seller of lemons.

In order to understand the driving forces behind our main result, we first describe the pattern of trade when parties bargain over an indivisible good, as in DL. When bargaining frictions vanish, if the buyer can obtain a positive payoff, the usual Coasean forces imply that trade occurs without delay. In one of their main contributions, DL show that if the buyer must screen the seller, he does it through an impasse. During an impasse the market freezes: trade occurs with probability zero. After the impasse, the buyer is optimistic enough to pay the cost of the high-quality good. The impasse introduces delay, which is necessary to lower the price of screening offers before the impasse. In their path-breaking *double delay* result, DL show that the delay is twice the time necessary to make the low-type seller indifferent between the price after the impasse (which is the low-type seller's continuation payoff then) and the buyer's valuation of the low-quality good. This result has two important implications. First, before the impasse, the price of screening offers is strictly lower than the buyer's valuation of the low-quality good, so the buyer obtains a strictly positive payoff. Second, the larger the price after the impasse, the lower the price of screening offers before the impasse.

The driving force behind the gradual sale of the high-quality good when the good is divisible is that the buyer benefits from splitting his purchases. To see this, consider a simple example with ten remaining units. Suppose that the buyer is optimistic enough so that by making a universal offer, he obtains a positive payoff from the first five units (which are more valuable), a negative payoff from the last five units (which are less valuable), and overall, obtains a positive payoff from purchasing all ten units. If the buyer could only make offers for ten units, then he would purchase all of them through a universal offer. When the good is divisible, the buyer can instead purchase the more valuable units through a universal offer and by doing so essentially commit to pay a low price for the less valuable ones. Intuitively, when only the less valuable units remain, the buyer obtains a negative payoff from a universal offer, and so he must screen the seller. As in DL, screening occurs through impasses when the good is divisible. We extend their double delay result and show that the buyer obtains a strictly positive payoff before impasses. The buyer thus prefers to split the purchases of the high-quality good, instead of purchasing all remaining units through one transaction.

The temptation to split the purchases of the high-quality good generates a new channel of competition between the buyer's present and future selves. This new channel of competition is the driving force behind the buyer's zero payoff from trading an arbitrarily divisible good. To see this, consider again the simple example from the previous paragraph. Suppose now that the buyer is so pessimistic that he suffers a loss from a universal offer even for the most valuable of the ten remaining units. He must then screen the seller through an impasse. After this impasse, the buyer splits the purchases of the high-quality good, and so the low-type seller's payoff is lower than the one he would obtain if the buyer could only make offers for ten units. As the low-type seller's payoff after the impasse is lower, then the delay is shorter, which means that the price of the screening offers for ten units before the impasse must be larger. To sum up, since the buyer splits the purchases of the high-quality good *after* the impasse is resolved, then he must pay a higher price for screening offers *before* the impasse.

We show that the competition between the buyer's present and future selves is fierce when the good becomes arbitrarily divisible. Formally, as the good becomes arbitrarily divisible, the number of impasses goes to infinity but each of them becomes short: the price of screening offers before and after each impasse are close to each other, and thus screening does not take long. Between two consecutive impasses, the buyer purchases a vanishing fraction of the good through a universal offer. The driving forces described in the previous paragraphs lead to stark results: the high-quality good is traded smoothly over time and the buyer's payoff is zero.

Our analysis highlights the importance of the shape of gains from trade. If gains from trade are constant in the number of units already traded, the buyer cannot benefit from splitting the purchases of the high-quality good. Intuitively, the buyer cannot commit to pay a lower price for the last units by purchasing the first ones through a universal offer. All units are equally valuable, so if the buyer is willing to pay the cost of the high-quality good for the first ones, he must also be willing to pay that price for the last ones. Thus, with constant gains from trade all units are traded at the same time.[3]

*Related Literature*.—There is a large literature that studies bilateral bargaining with interdependent values (Samuelson 1984; Evans 1989; Vincent 1989; Deneckere and Liang 2006; Fuchs and Skrzypacz 2013; Gerardi, Hörner, and Maestri 2014; Hwang 2018b; Hwang 2018a; and Daley and Green 2020). Deneckere and Liang (2006) solves the one-unit version of the model in our paper. We take DL's construction as a stepping stone and extend the analysis to multiple units when there are two types of sellers.[4] Our paper uncovers a new role for divisibility in bargaining. The buyer gradually learns the seller's type as he makes two kinds of offers. On the one hand, he gradually makes universally accepted offers for small pieces of the good at large per-unit prices. On the other, he makes offers for all remaining units at large discounts.

Most of the literature focuses on bargaining between long-run players. Gerardi, Hörner, and Maestri (2014) study the role of commitment in negotiation environments under adverse selection.[5] Hwang (2018a) shows that bargaining deadlock

[3] The pattern of trade with increasing gains from trade is equal to that with constant gains from trade.

[4] We restrict attention to the case of interdependent values because if instead values are private, divisibility plays no role; the Coase conjecture holds.

[5] In our paper, as in most of the literature, the uninformed party has all the bargaining power. This modeling choice reduces the informed party's ability to signal his type and yields strong predictions. See Gerardi et al. (2014) for the case of an informed party with all the bargaining power.

arises when the seller receives random outside options over time. Hwang (2018b) focuses on a buyer who randomly becomes informed about the seller's type, while Daley and Green (2020) present a model with correlated but imperfect news that arise over time. Our findings of a continuous pattern of trade and of a zero payoff for the uninformed party are reminiscent of Fuchs and Skrzypacz (2013) and Ortner (2017). Fuchs and Skrzypacz (2013) bridge the gap from DL by letting the gains from trade from the weakest type shrink to zero. Ortner (2017) studies a durable goods monopolist whose cost of production evolves stochastically over time.

Our paper is also related to the burgeoning body of literature that studies the effects of adverse selection in dynamic markets. Following the pioneering work of Inderst (2005), an important stream of this literature focuses on markets where a long-run player faces a sequence of short-run players. Hörner and Vieille (2009) analyze the role of private of offers in the market for lemons. Philippon and Skreta (2012) focus on optimal government interventions in such markets. Daley and Green (2012) study how noisy information about the value of a good is revealed to the market. Kim (2017) analyzes the role of time-on-the-market information in the market for lemons. Fuchs and Skrzypacz (2019) characterize optimal market design policies. Beyond the issue of divisibility, our paper differs from the above studies by analyzing the strategic effects that arise when two long-run players bargain under adverse selection.

The rest of the paper is organized as follows. We describe the model and the equilibrium concept in Section I. In Section II we present our main result and discuss the economic implications of divisibility. In Section III we discuss equilibrium existence and present the intermediate results leading to our main result. In Section IV we present extensions to our framework. Section V concludes. Proofs are relegated to the online Appendix.

## I. The Model

A buyer and a seller bargain over a good of size one. The seller is of one of two types $i \in \{L,H\}$. A seller of high type $(i = H)$ provides a high-quality good, while a low-type seller $(i = L)$ provides a low-quality good. The seller knows his own type, but the buyer does not. The seller is of high type with prior probability $\hat{\beta}$ that satisfies $0 < \hat{\beta} < 1$.

The buyer and the seller can trade fractions of the good. Let $z \in [0,1]$ denote an infinitesimal unit of the good. We index units in *reverse order*. The buyer's first purchase consists of units $z \in [\bar{z},1]$, for some $0 \leq \bar{z} \leq 1$. A buyer who has already acquired units $z \in [\bar{z},1]$ can then buy subsequent units $z \in [\underline{z},\bar{z}]$ from the seller, with $0 \leq \underline{z} \leq \bar{z}$.[6]

[6] We solve the game (with the good divided into finitely many units) using backward induction on the number of units *left* for trade, which is one of the state variables (see Section III). The reverse order thus allows us to use the same index both for the state variable and for the unit number.

### A. *Parties' Valuations*

The buyer's valuation for the units $z \in [\underline{z}, \bar{z}]$ when the seller is of type $i$ is equal to $\int_{\underline{z}}^{\bar{z}} \lambda(z) v_i dz$, where $\lambda(z)$ is a smooth function and $\lambda(z) > 0$ for all $z \in [0,1]$. This valuation is higher if the seller is of high type: $0 < v_L < v_H$. The cost of the units $z \in [\underline{z}, \bar{z}]$ to the seller of type $i$ is equal to $(\bar{z} - \underline{z})\, c_i$. The constant marginal cost of providing the good is higher for the high-type seller: $0 = c_L < c_H = c$.

We focus on the case with decreasing gains from trade. Since we index units in reverse order, this corresponds to a strictly increasing function $\lambda(z)$.[7] Without loss of generality we assume that $\min_{z \in [0,1]} \lambda(z) = \lambda(0) = 1$. We also assume that $0 < v_L < c < v_H$, so there are always gains from trade. Furthermore, we assume that

$$\left[\hat{\beta} v_H + \left(1 - \hat{\beta}\right) v_L\right] \lambda(1) < c. \tag{1}$$

The buyer's expected valuation from the first infinitesimal unit is lower than the high-type seller's cost. This assumption allows us to focus on the most interesting case: the buyer must screen the seller even to purchase the most valuable unit.[8]

We study the equilibrium behavior of the buyer and the seller as the good becomes arbitrarily divisible. We divide the good into $m$ equally sized units and study the equilibrium behavior as $m$ grows large. As with $z \in [0,1]$, we also index units in reverse order, by $s \in \{1, \ldots, m\}$: $s = 1$ indicates the last unit, while $s = m$ indicates the first unit. The cost of each unit to the seller of type $i$ is simply $c_i/m$. The buyer's valuation for the $s$'th unit when the seller is of type $i$ is $\Lambda_s^m v_i$ with

$$\Lambda_s^m \equiv \int_{(s-1)/m}^{s/m} \lambda(z) dz.$$

Figure 1 illustrates the buyer's valuation coefficients $\Lambda_s^m$ of successive units of the good. In panel A of Figure 1, the good is divided into 3 units. Assume that the seller is of type $i$. The buyer's valuation for the first unit is $\Lambda_3^3 v_i$. The second unit gives the buyer intermediate valuation $\Lambda_2^3 v_i$. The last unit is the one with the lowest valuation to the buyer: $\Lambda_1^3 v_i$. Panel B of Figure 1 illustrates the valuation coefficients of successive units of the good when it is divided into 6 units.

### B. *Timing and Payoffs*

The buyer and the seller trade sequentially over time. Time is discrete and periods are indexed by $t = 0, 1, \ldots$. In each period the buyer makes an offer $\varphi_t = (k, p)$, where $k \in \mathbb{Z}_+$ is the number of units requested and $p \in \mathbb{R}_+$ is the total payment offered. Without loss of generality, we assume that the number of units requested cannot exceed the number of remaining units. The seller can either accept $(a_t = A)$ or reject $(a_t = R)$ the offer. If the seller accepts, $k$ units are traded and the buyer pays $p$ to the seller. The game ends when all $m$ units are traded.

[7] There are two natural alternative environments: $\lambda(z)$ constant and $\lambda(z)$ strictly decreasing. In Section IV we describe how divisibility plays no role in those cases.

[8] In Section IV we extend our analysis to cases where equation (1) does not hold.

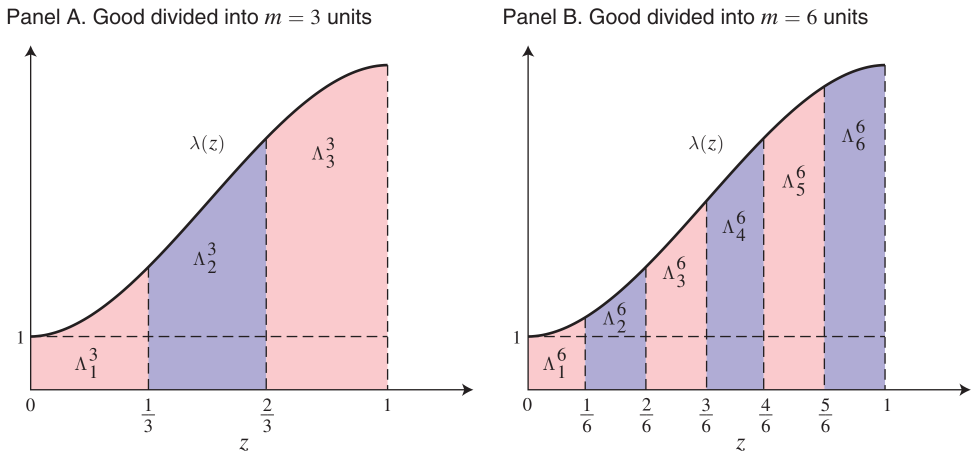


FIGURE 1. VALUATION COEFFICIENTS OF SUCCESSIVE UNITS OF A DIVIDED GOOD

The buyer and the seller share a discount factor $\delta = e^{-r\Delta}$ where $\Delta > 0$ represents the length of each period and $r > 0$ represents the discount rate. Suppose that the buyer and the seller of type $i$ agree on trading a total of $D$ times, indexed by $d \in \{1, \ldots, D\}$. In the first trade $(d = 1)$, which takes place at time $t_1$, the buyer pays the seller $p_1$, in exchange for $k_1$ units, so the set of traded units is $S_1 = \{m, \ldots, m - k_1 + 1\}$. A generic trade $d > 1$ takes place at time $t_d$ and involves a total payment $p_d$ in exchange for $k_d$ units. The set of traded units is $S_d = \{m - k_1 - \ldots - k_{d-1}, \ldots, m - k_1 - \ldots - k_d + 1\}$. Then, the total payoff to the buyer is

$$\sum_{d=1}^{D} \delta^{t_d} \left[ \sum_{s \in S_d} \Lambda_s^m v_i - p_d \right].$$

The seller, in turn, obtains

$$\sum_{d=1}^{D} \delta^{t_d} \left[ p_d - \frac{c_i}{m} k_d \right].$$

The buyer does not learn about the quality of the good upon purchasing units of it. Therefore, all learning is strategic; the buyer only updates his belief based on the seller's behavior.

In our model, the buyer enjoys the benefits from a unit from the period in which he purchases it. However, realized payoffs do not provide additional information about the quality of the good to the buyer. By shutting down the possibility of learning from experiencing the good, this modeling choice allows us to focus on the effects of strategic learning.

Strategic learning is key in many environments, including markets for securities. To see this, consider a bank negotiating the sale of a pool of mortgages to a pension fund in a context of macroeconomic uncertainty. The bank has private information about the quality of its mortgages. In every period, an i.i.d. macroeconomic shock may occur. While no macroeconomic shock materializes, the housing market booms and borrowers associated to both high and low-quality mortgages are able to honor their debts. Thus, both types of mortgages provide the same cash flow to the pension fund. As soon as a shock occurs, both types of borrowers may become delinquent, with low type borrowers being more likely to default. Furthermore, after the shock, rating agencies downgrade mortgage securities, which prevents the pension fund from further negotiating them. Our model describes this environment with a slight reinterpretation of the parameters, where in particular $\delta$ incorporates the exogenous probability that a macroeconomic shock materializes in each period.

## C. *Strategies*

The public history $h^t$, with $t \geq 1$, lists all offers made, together with all responses by the seller, from period 0 through period $t-1$: $h^t = \left((\varphi_0, a_0), \ldots, (\varphi_{t-1}, a_{t-1})\right)$. We let $h^0 = \varnothing$ denote the initial public history and we let $H^t$ denote the set of all possible histories $h^t$ at the beginning of period $t$. Intermediate histories $(h^t, \varphi_t)$ include the offer made after history $h^t$, but not the subsequent action chosen by the seller.

A buyer's (behavior) strategy $\sigma_B = \left(\sigma_B^t\right)_{t=0}^{\infty}$ assigns a random offer to every public history $h^t$, with $\sigma_B^t(h^t) \in \Delta\Phi(h^t)$, where $\Phi(h^t)$ is the set of available offers at $h^t$. A seller's (behavior) strategy $(\sigma_L, \sigma_H) = \left(\sigma_L^t, \sigma_H^t\right)_{t=0}^{\infty}$ assigns a random decision ($A$ or $R$) to each intermediate history $(h^t, \varphi_t)$, so $\sigma_i^t(h^t, \varphi_t) \in \Delta\{A, R\}$ for every $i \in \{L, H\}$. The system of beliefs $\beta(\cdot)$ is as follows. We let $\beta(h^t)$ and $\beta(h^t, \varphi_t)$ denote the buyer's belief that the seller is of high type after an arbitrary public history $h^t$, and an arbitrary intermediate history $(h^t, \varphi_t)$, respectively.

## D. *Equilibrium Concept and Preliminary Results*

We work with *stationary perfect Bayesian equilibria*. In this model, at any public history $h^t$ there are two state variables: the number of remaining units $K(h^t)$ and the buyer's belief $\beta(h^t)$. A strict notion of stationarity would require strategies and value functions to depend only on the two state variables $K(h^t)$ and $\beta(h^t)$. As is standard in bargaining, there is no equilibrium that satisfies this strict notion. We then use a notion that places restrictions only on the seller's strategy. We require the seller's strategy to be a supply function and to depend only on state variables. In what follows, we describe our definition in detail.

We first present some preliminary results which facilitate the exposition of our notion of stationarity. In any perfect Bayesian equilibrium (PBE), the buyer's system of beliefs $\beta(\cdot)$ must satisfy the following properties. Beliefs $\beta(h^t, \varphi_t, a_t)$ are derived from $\beta(h^t)$ according to Bayes' rule whenever action $a_t$ occurs with positive probability after intermediate history $(h^t, \varphi_t)$. Moreover, beliefs after intermediate histories are not affected by the buyer's offer: $\beta(h^t, \varphi_t) = \beta(h^t)$.

Lemma 1 provides a partial characterization of equilibria whenever the seller's strategy depends only on state variables. Let $V_H(h^t)$, $V_L(h^t)$ and $V_B(h^t)$ denote the continuation payoffs for, respectively, a seller of high type, a seller of low type and the buyer.

LEMMA 1 (Partial Characterization): *Let* $(\sigma_B, (\sigma_L, \sigma_H), \beta)$ *be an arbitrary PBE. Assume that whenever histories* $h^t$ *and* $\tilde{h}^{t'}$ *have the same state variables:* $\beta(h^t) = \beta(\tilde{h}^{t'})$ *and* $K(h^t) = K(\tilde{h}^{t'})$, *then* $\sigma_i(h^t, \varphi) = \sigma_i(\tilde{h}^{t'}, \varphi)$ *for all* $\varphi \in \Phi(h^t) = \Phi(\tilde{h}^{t'})$ *and for both* $i \in \{L, H\}$. *Then,*

(i) *Whenever* $\beta(h^t) = 0$, *the low-type seller gets zero payoffs:* $V_L(h^t) = 0$.

(ii) *The buyer's continuation payoff* $V_B(h^t)$ *depends only on* $\beta(h^t)$ *and on* $K(h^t)$.

(iii) *The high-type seller gets zero payoffs:* $V_H(h^t) = 0$ *for all* $h^t$.

(iv) *The low-type seller's payoffs are bounded:* $V_L(h^t) \leq (c/m)K(h^t)$ *for all* $h^t$.

See online Appendix A.1 for the proof.

Lemma 1(i) states that the low-type seller cannot obtain positive payoffs after his type has been revealed. This result holds true in any PBE, so does not rely on stationarity.[9] Lemmas 1(ii) and (iii) are direct results of the seller's strategy depending only on state variables. Lemma 1(ii) states that the buyer's continuation payoff must depend only on beliefs and on the number of remaining units. Lemma 1(iii) states that the high type seller always obtains zero profits. Hence, any offer of a payment larger than $(c/m)K(h^t)$ would be accepted with probability one by the high-type seller, and so also by the low type. This implies Lemma 1(iv): the low-type seller continuation payoff is bounded above by $(c/m)K(h^t)$.

Our definition of stationary PBE incorporates the results from Lemma 1. The behavior of both types of sellers must be consistent with the payoffs that they obtain in a stationary environment. Following Lemma 1(iii), a high-type seller accepts any offer that leads to nonnegative payoffs. Similarly, following Lemma 1(iv), a low-type seller accepts any offer that the high-type seller also accepts.[10] But, does the low-type seller ever accept offers that the high-type seller rejects? If he does so, he immediately reveals his own type to the buyer. Moreover, if the low-type seller mixes, then a rejection increases the belief that the seller is of high type. Then, the behavior of the low-type seller is more subtle than that of the high-type seller. We impose that the acceptance decision of the low-type seller be governed by a function $\mathcal{V}_L(K, \beta)$ that depends on the number of remaining units $K$ and on the beliefs $\beta$ induced by a rejection.

DEFINITION (Stationary Perfect Bayesian Equilibrium): *A PBE is stationary if there exists a (left-continuous) function* $\mathcal{V}_L(K, \beta) : \{1, \ldots, m\} \times [\hat{\beta}, 1] \rightarrow \mathbb{R}$ *such that*

1. *The high-type seller accepts with probability one any payment greater or equal than* $(c/m)k$ *in exchange for any number of remaining units* $k \leq K(h^t)$. *The high-type seller rejects any other offer with probability one.*

[9] This standard result is analogous to that in a model with common knowledge of types and a buyer who always makes the offer.

[10] This in turn implies that beliefs never decrease over time, and so they are bounded below by $\hat{\beta}$.

2. *The behavior of the low-type seller is as follows. Take any history $h^t$ where the remaining number of units is $K(h^t)$ and the belief is $\beta(h^t) \geq \hat{\beta}$. Assume that the buyer offers a total payment p in exchange for $k \leq K(h^t)$ remaining units. Then,*

   $(i)$ *If $p \geq (c/m)k$, then the low-type seller accepts the offer with probability one.*

   $(ii)$ *If $p < (c/m)k$ and $p < \delta\mathcal{V}_L(K(h^t),\beta)$ for all $\beta \geq \beta(h^t)$, then the low-type seller rejects the offer with probability one.*

   $(iii)$ *If $p < (c/m)k$ and there exists $\beta \geq \beta(h^t)$ with $p \geq \delta\mathcal{V}_L(K(h^t),\beta)$, then the low-type seller randomizes so that $\beta' = \max\{\beta : \delta\mathcal{V}_L(K(h^t),\beta) \leq p\}$ is the next-period posterior after rejection.*

The function $\delta\mathcal{V}_L(K,\cdot)$ acts as a stationary supply when there are $K$ units left. First, it acts as a supply function because when the buyer offers a higher price $p$, he induces a (weakly) higher posterior $\beta'$ after rejection. Therefore, the probability of acceptance of the low-type seller is (weakly) increasing in the price offered by the buyer. Second, the function $\delta\mathcal{V}_L(K,\cdot)$ acts as a *stationary* supply because the price that the buyer needs to pay to induce a posterior belief $\beta' \geq \beta(h^t)$ is independent of the current belief $\beta(h^t)$.[11]

The concept of stationary PBE (henceforth, equilibrium), together with Lemma 1, allows for a characterization of the offers that can occur with positive probability in equilibrium. In particular, consider the family of *partial offers*. The buyer makes a partial offer when he requests less than the total number of remaining units and offers a payment that does not cover the costs of the high-type seller. These offers cannot be made and accepted with positive probability in equilibrium. The intuitive reason behind this is simple. A high-type seller never accepts a partial offer, since, by definition, a partial offer does not cover his costs. Then, only the low-type seller may accept partial offers with positive probability. The acceptance of a partial offersreveals that the seller is of low type, so remaining units are traded immediately, and the low-type seller gets no payoff from that trade. Instead of making a partial offer, the buyer could offer to buy *all remaining units* at the same (total) price. The low-type seller would get the same payoff from this alternative offer, so he would accept it, and with the same probability.[12] Trade would then speed up, with the buyer obtaining the additional surplus. Thus, the buyer could obtain a strictly higher payoff by making this alternative offer, i.e. asking for all remaining units, and offering the same payment. Lemma 2 formalizes this.

[11] Our definition of stationary PBE extends the notion of stationary equilibrium (see Gul and Sonnenschein 1988; Ausubel and Deneckere 1992; Deneckere and Liang 2006; Fuchs and Skrzypacz 2010) to our setup. We conjecture generic uniqueness of PBE outcomes. However, we have not been able to show this.

[12] Our definition of stationary PBE implies that the randomization probability of the low-type seller depends on the number of units remaining, but not on the number of units requested by the buyer. This assumption is without loss of generality. In an earlier version of this paper we allow $\mathcal{V}_L$ to depend also on the number of units requested by the buyer. For generic values of the parameters, we obtain the same equilibrium outcome as with our definition of stationary PBE.

LEMMA 2 (No Partial Offers): *Fix an equilibrium. Take any history* $h^t$ *with* $K(h^t) > 1$. *Trades* $(k,p)$ *with* $k < K(h^t)$ *and* $p < (c/m)k$ *occur with zero probability.*

See online Appendix A.2 for the proof.

Consider the two remaining families of offers in the following definition.

DEFINITION (Universal and Screening Offers): *The buyer makes a universal offer for* $k \leq K(h^t)$ *units when he offers a payment* $p = (c/m)k$. *Universal offers are then of the form* $(k,(c/m)k)$ *and both types accept them. The buyer makes a screening offer for all remaining units* $K(h^t)$ *when he offers a payment* $p < (c/m)K(h^t)$. *Screening offers are then of the form* $(K(h^t),p)$ *and the high-type seller never accepts them.*

It is without loss of generality to restrict attention only to universal and screening offers. To see this, suppose that in equilibrium, at history $h^t$, the buyer makes a partial offer $(k,p)$, which the seller rejects (by Lemma 2). Replace this offer with the screening offer $(K(h^t),p)$. Stationarity implies that this offer is also rejected by the seller. By replacing all partial offers this way, we obtain an outcome equivalent equilibrium in which no partial offer is ever made. In this sense, there is no equilibrium with partial offers.[13]

Before presenting our main result, we perform a convenient change of variables. We work with the transformed beliefs $q(\beta) : [\hat{\beta},1] \rightarrow [0,1-\hat{\beta}]$ given by the continuous and strictly increasing mapping

$$q(\beta) = 1 - \frac{\hat{\beta}}{\beta}.$$

For convenience, we write $\hat{q} = 1 - \hat{\beta}$ and, with a slight abuse of notation, we let $q(h^t) = q(\beta(h^t))$. A transformed belief equal to $\hat{q}$ means that the buyer assigns probability one to the seller being of high type. This transformation allows for a simple expression for the probability that the low-type seller accepts screening offers. Assume that after the rejection of a screening offer, the buyer updates his transformed belief from $q$ to $q'$. This means that the low-type seller accepts such offer with probability $(q'-q)/(\hat{q}-q)$. Moreover, as we show in online Appendix A.3, the buyer's value function is linear in transformed beliefs $q(h^t)$.[14]

## II. Main Results

We characterize the *limit equilibrium outcome*, that is, the pattern of trade when bargaining frictions vanish and the good becomes arbitrarily divisible. We first let

[13] In fact, a stronger result holds; for generic values of the parameters, partial offers are never made in equilibrium.

[14] This change of variables is also explored in several papers in bargaining with incomplete information. Some readers may find useful the following interpretation for the variable $q$. Assume that the sellers' type $q$ is uniformly distributed in the unit interval. Whenever $q \in [0,\hat{q})$ then the seller is of low type. If instead $q \in [\hat{q},1]$, the seller is of high type. Under this interpretation for $q$, the function $P(K,\cdot)$ that we introduce in online Appendix A.3 represents the reservation price $P(K,q)$ for type $q \in [0,\hat{q})$.

the time between offers shrink to zero and we then let the number of units grow to infinity. With this order of limits we can use an inductive argument on the number of remaining units and develop an algorithm that characterizes the limit equilibrium outcome as bargaining frictions vanish. In addition, our order of limits captures the following features of most real-world environments. On the one side, parties can trade essentially at any point in time. On the other side, there is typically a lower bound on the size of each unit of the good. This is true even for goods that can be divided into a large number of units, like financial assets.[15]

### A. *The Limit Equilibrium Outcome*

In our main result (Theorem 1) we show that in the limit equilibrium outcome the high-quality good is traded smoothly over time, the buyer's payoff converges to zero, and the low-type seller's payoff converges to $\left(\int_0^1 \lambda(z)dz\right) v_L$. In order to convey our main message swiftly, we introduce in this section only the minimum necessary notions to state Theorem 1. In this section we also take advantage of some intermediate results (such as the characterization of the equilibrium for fixed $\Delta$ and fixed $m$) that we present formally and discuss at length later, in Section III.

We next provide a formal definition of the notion of limit equilibrium outcome. Consider the environment with a fixed time between offers $\Delta > 0$ and a fixed number of units $m$. We show in Section IIIA that the buyer's equilibrium behavior is deterministic; at each period $t$ he either makes a universal or a screening offer. Both types of seller always accept universal offers. While a seller of high type never accepts a screening offer, a low-type seller randomizes between accepting and rejecting them. We can then describe the equilibrium outcome in a simple way. Consider the history where all screening offers are rejected. For any period $t = 0, 1, \dots,$ we let $\tilde{K}_m^{\Delta}(t)$ and $\tilde{q}_m^{\Delta}(t)$ denote respectively the number of remaining units and the buyer's transformed belief along that history. Whenever the buyer makes a universal offer, $\tilde{K}_m^{\Delta}(\cdot)$ decreases between two consecutive periods while $\tilde{q}_m^{\Delta}(\cdot)$ remains unchanged. In contrast, when the buyer makes a screening offer, $\tilde{K}_m^{\Delta}(\cdot)$ remains unchanged while $\tilde{q}_m^{\Delta}(\cdot)$ increases.[16] The functions $\tilde{K}_m^{\Delta}(\cdot)$ and $\tilde{q}_m^{\Delta}(\cdot)$ thus summarize the equilibrium outcome.

We first let the time between offers $\Delta$ converge to zero, so that the discount factor $\delta = e^{-r\Delta}$ converges to one. To make meaningful comparisons between games with different period lengths $\Delta$, we express the number of remaining units and the transformed belief as functions of time elapsed $\tau \in \mathbb{R}_+$. We show that these functions converge pointwise as $\Delta$ shrinks to zero and we let the functions $K_m : \mathbb{R}_+ \to [0, m]$ and $q_m : \mathbb{R}_+ \to [0,1]$ denote their limits. We define the fraction of the good left for trade $z_m$: $\mathbb{R}_+ \to [0,1]$ by setting $z_m(\tau) = K_m(\tau)/m$. Finally, we define the *limit equilibrium outcome* as the limit of the functions $z_m(\cdot)$ and $q_m(\cdot)$ as $m$ grows large.

We next describe two simple functions that, as we show in Theorem 1, characterize the limit equilibrium outcome. The function $z^* : \mathbb{R}_+ \to [0,1]$ describes the fraction of the good left for trade and the function $q^* : \mathbb{R}_+ \to [0, \hat{q}]$ describes the

[15] We discuss the implications of inverting the order of limits in the conclusion.

[16] The rejection of a screening offer makes the buyer more optimistic about the quality of the good since only the low-type seller may accept a screening offer.

evolution of beliefs. In order to describe these functions, we let $\bar{q}(z)$ denote the belief that makes the buyer break even when he makes a universal offer for the infinitesimal unit $z$:

$$\left[\hat{q} - \bar{q}(z)\right]\left[\lambda(z)\, v_L - c\right] + (1 - \hat{q})\left[\lambda(z)\, v_H - c\right] \;=\; 0.$$

The function $\bar{q} : [0,1] \to [0,\hat{q}]$ is strictly decreasing. We let $\psi : \left[\bar{q}(1), \bar{q}(0)\right] \to [0,1]$ denote its inverse.

The construction of the functions $q^*(\cdot)$ and $z^*(\cdot)$ is simple, and can be better understood through the following artificial pattern of trade. At time elapsed $\tau = 0$, the buyer makes a screening offer and breaks even. The low-type seller accepts this offer with probability $\bar{q}(1)/\hat{q}$, so the belief at time $\tau = 0$ satisfies $q^*(0) = \bar{q}(1)$. From that point on, the buyer continuously makes both screening offers and universal offers for infinitesimal units. At any point in time $\tau \in \mathbb{R}_+$, the buyer breaks even with either type of offer. Finally, the low-type seller is indifferent between accepting and rejecting any screening offer. The functions $q^*(\cdot)$ and $z^*(\cdot)$ are the results of this artificial pattern of trade.

In the artificial pattern of trade, the buyer breaks even every time he makes a universal offer for the infinitesimal unit $z$. Thus, at any point in time $\tau \in \mathbb{R}_+$, the belief $q^*(\tau)$ and the fraction of remaining units $z^*(\tau)$ must satisfy $q^*(\tau) = \bar{q}\left(z^*(\tau)\right)$. Furthermore, since the buyer also breaks even whenever he makes a screening offer, at any point in time $\tau \in \mathbb{R}_+$ he offers to purchase the fraction $z^*(\tau)$ at the price $v_L \int_0^{z^*(\tau)} \lambda(z)dz$. Finally, the low-type seller is indifferent between accepting a screening offer at time $\tau$ or mimicking the high-type seller's behavior from $\tau$ to $\tau + \Delta\tau$ and then accepting a screening offer at time $\tau + \Delta\tau$:

$$(2) \quad v_L \int_0^{z^*(\tau)} \lambda(z)dz \;=\; \int_\tau^{\tau+\Delta\tau} e^{-r(s-\tau)} c\left(-z^{*\prime}(s)\right)ds + e^{-r\Delta\tau} v_L \int_0^{z^*(\tau+\Delta\tau)} \lambda(z)dz.$$

We next let $\Delta\tau \to 0$ and, through a first order approximation of the right hand side of equation (2), obtain that $z^{*\prime}(\tau)\left[v_L \lambda\left(z^*(\tau)\right) - c\right] = r v_L \int_0^{z^*(\tau)} \lambda(z)dz$. Together with the fact that $q^*(\tau) = \bar{q}\left(z^*(\tau)\right)$, this implies that

$$(3a) \qquad q^{*\prime}(\tau) \;=\; \frac{r v_L \int_0^{\psi(q^*(\tau))} \lambda(z)dz}{\psi'(q^*(\tau))q^{*\prime}(\tau)\left[v_L \lambda\left(\psi\left(q^*(\tau)\right)\right) - c\right]} \quad \text{and}$$

$$(3b) \qquad z^{*\prime}(\tau) \;=\; \psi'(q^*(\tau))q^{*\prime}(\tau) \;=\; \frac{r v_L \int_0^{z^*(\tau)} \lambda(z)dz}{v_L \lambda\left(z^*(\tau)\right) - c}.$$

This, together with the initial conditions $q^*(0) = \bar{q}(1)$ and $z^*(0) = 1$ pins down the functions $q^*(\cdot)$ and $z^*(\cdot)$. Conditions (3a) and (3b) guarantee that the functions $q^*(\cdot)$ and $z^*(\cdot)$ are smooth and that $q^*(\cdot)$ converges to $\bar{q}(0)$ and $z^*(\cdot)$ shrinks to zero as $\tau \to \infty$.

THEOREM 1 (Limit Equilibrium Outcome): *The sequence* $\left\{\left(z_m(\cdot), q_m(\cdot)\right)\right\}_{m=1}^\infty$ *converges pointwise to* $\left(z^*(\cdot), q^*(\cdot)\right)$. *Thus, in the limit*

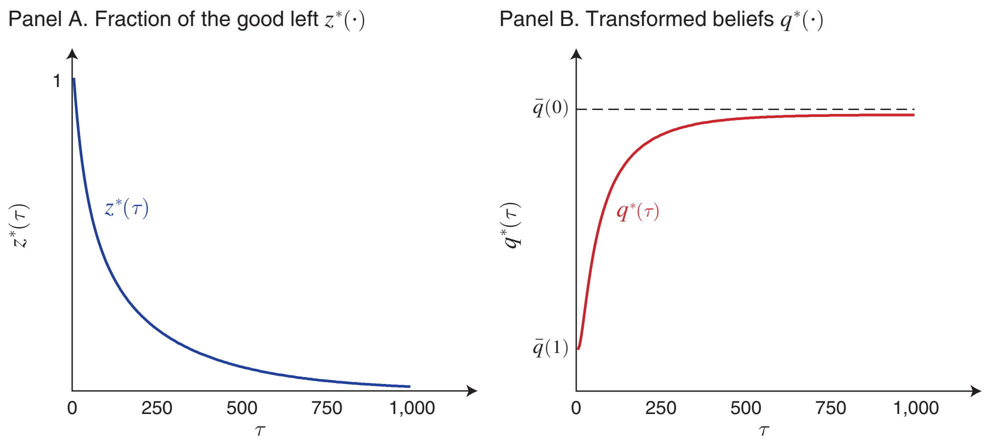


Figure 2. Limit Equilibrium Outcome $(z^*(\cdot), q^*(\cdot))$: Pattern of Trade as Bargaining Frictions Vanish and the Good Becomes Arbitrarily Divisible

*Notes:* These panels depict the limit equilibrium outcome for the following primitives: $v_H = 35$, $v_L = 1$, $c = 30$, $r = 0.1$, and $\hat{q} = 0.9$. Finally, $\lambda(z) = 1 + 0.1z + 15z^2 - 10z^3$ (this is the function shown in Figure 1).

*equilibrium outcome, the high-quality good is traded smoothly over time, the low-type seller's payoff is* $\left(\int_0^1 \lambda(z)dz\right) v_L$ *and the buyer's payoff is zero.*

Figure 2 illustrates the limit equilibrium outcome $(z^*(\cdot), q^*(\cdot))$. The buyer's belief evolves smoothly and the high-quality good is sold gradually over time. At any point in time, a positive fraction of the good is left for trade and bargaining continues forever. In Section IB, we provide an alternative interpretation of the model that includes an exogenous probability of a macroeconomic shock. In this case, Figure 2 illustrates the pattern of trade conditional on no shock ever taking place.

The proof of Theorem 1 consists of two parts. In the first one (Proposition 3) we fix the number of units $m$ and characterize the equilibrium outcome as bargaining frictions vanish ($\Delta \to 0$). We show that the equilibrium outcome takes a simple form: phases of fast trade alternate with impasses. In the phases of fast trade, parties trade without delay. The buyer purchases chunks of the good from both seller types (through universal offers) and, with positive probability, he also purchases all remaining units from the low-type seller (through screening offers). Instead, the market freezes during an impasse. The buyer screens the seller with delay, as in DL. At each impasse the buyer's continuation payoff is zero, as he would have an incentive to speed up trade otherwise. In Proposition 3 we construct an algorithm that pins down the entire sequence of phases of fast trade and impasses.

In the second part of the proof of Theorem 1 (Proposition 4) we let the number of units $m$ grow to infinity. We study the limit of the equilibrium outcome uncovered by the algorithm in Proposition 3. We show that the number of impasses grows to infinity and the length of each impasse goes to zero. The fraction of the good traded in each phase of fast trade through universal offers also goes to zero. Proposition 4 leads directly to the pattern of trade described in Theorem 1. Divisibility, together with decreasing gains from trade, introduces a new source of temptation for the buyer, which is reminiscent of the Coase conjecture. As we highlight in Section IIIC, the buyer is tempted to purchase the most valuable (small) fractions of the good. This is the driving force behind the gradual sale of the high quality good.

### B. *Implications of Divisibility*

Before diving into the explanation of Propositions 3 and 4, we take advantage of the characterization in Theorem 1 to shed light on the pattern of trade of an arbitrarily divisible good.[17]

How does the pattern of trade of securities look like? Theorem 1 shows that high-quality securities are traded in dribs and drabs. In contrast, when securities are of low quality, the parties either trade small portions or make a final transaction for all remaining securities.

We provide novel and testable predictions on market efficiency. Our model highlights a rationale for markets of divisible goods (like markets for securities) to be more efficient than markets for indivisible goods (like real estate markets). We also show that these markets differ sharply on the split of the gains from trade. Divisibility is detrimental to buyers (who are uninformed and have bargaining power) and beneficial to sellers of lemons. Finally, markets of securities and real estate markets share some features. We show that when adverse selection worsens, the speed of trade of high quality mortgages becomes slower.

We now describe in detail the implications of divisibility presented in the previous paragraphs. We first discuss the efficiency of the limit equilibrium outcome. We compare the (expected) gains from trade in the limit equilibrium outcome to those (i) under the buyer's optimal mechanism with commitment, (ii) under the most efficient mechanism with commitment, and (iii) in the limit, as bargaining frictions vanish, of the model *without* divisibility (DL).[18]

The gains from trade are equal to $\hat{q}\left(\int_0^1 \lambda(z)dz\right)v_L$ in the limit equilibrium outcome. This follows from Theorem 1, as the low-type seller's payoff is equal to $\left(\int_0^1 \lambda(z)dz\right)v_L$, both the high-type seller and the buyer obtain a payoff of zero and the seller is of low type with probability $\hat{q}$.

The limit equilibrium outcome is as efficient as the buyer's optimal mechanism with commitment. Under this mechanism, the buyer purchases the whole good immediately from the low-type seller, and pays him a price of zero. The buyer and the high-type seller do not trade.[19] Thus, the buyer extracts all the surplus. So although the gains from trade are equal in these two environments, the split of the surplus between the parties is starkly different.

Samuelson (1984) shows that the buyer's optimal mechanism does not achieve the second best: the gains from trade under the buyer's optimal mechanism are lower

[17] The reader more interested in the explanation of the driving force behind our main result may safely skip what remains of this section and proceed directly to Section III.

[18] The gains from trade under both mechanisms with commitment are independent of the number of units $m$ and the period length $\Delta$.

[19] To see why the buyer cannot improve upon this mechanism, note that for the buyer to purchase a marginal unit from the high-type seller, he must pay the marginal cost $c$ to both types. Equation (1) implies that it is not profitable for the buyer to do so.

than those under the most efficient mechanism with commitment.[20] Thus the limit equilibrium outcome is bounded away from the second best.

The gains from trade in our limit equilibrium outcome are higher than those in the model without divisibility (DL).[21] Without divisibility, both the low-type seller and the buyer obtain a positive payoff (the high-type seller obtains a zero payoff). Thus, although divisibility improves overall efficiency, it is detrimental for the buyer. Furthermore, while divisibility also increases the gains from trade conditional on the good being of high quality, this is not necessarily the case if the good is instead of low quality. Finally, we compare the speed of trade of the high-quality good with and without divisibility. We show that when the good is divisible, the high-quality good is traded faster.[22]

We next show how the primitives of the model affect the speed of trade for both the high-quality and the low-quality good. We start with a configuration of the primitives $\left(\hat{q}, \lambda(\cdot), c, v_L, v_H, r\right)$, modify one of them (resulting in a new configuration that also satisfies the assumptions of our model) and compare the resulting limit equilibrium outcomes.

PROPOSITION 1 (Speed of Trade of the High-quality Good): *Let* $\left(z^*(\cdot), q^*(\cdot)\right)$ *denote the limit equilibrium outcome associated to the primitives* $\left(\hat{q}, \lambda(\cdot), c, v_L, v_H, r\right)$. *Consider next an alternative configuration of primitives with associated limit equilibrium outcome* $\left(\tilde{z}^*(\cdot), \tilde{q}^*(\cdot)\right)$. *For any of the following alternative configurations of primitives, the high-quality good is traded faster, i.e.,* $\tilde{z}^*(\tau) < z^*(\tau)$ *for every* $\tau > 0$:

*(i)* $\left(\hat{q}, \tilde{\lambda}(\cdot), c, v_L, v_H, r\right)$ *with* $\tilde{\lambda}(z) > \lambda(z)$ *for all* $z \in (0,1]$;

*(ii)* $\left(\hat{q}, \lambda(\cdot), \tilde{c}, v_L, v_H, r\right)$ *with* $\tilde{c} < c$;

*(iii)* $\left(\hat{q}, \lambda(\cdot), c, \tilde{v}_L, v_H, r\right)$ *with* $\tilde{v}_L > v_L$;

*(iv)* $\left(\hat{q}, \lambda(\cdot), c, v_L, v_H, \tilde{r}\right)$ *with* $\tilde{r} > r$.

*Finally, the parameters* $v_H$ *and* $\hat{q}$ *do not affect the speed of trade of the high-quality good.*

See online Appendix A.7 for the proof.

The intuition behind Proposition 1 is simple. The speed of trade of the high-quality good is such that the low-type seller is always indifferent between accepting the current screening offer or rejecting all screening offers and obtaining the discounted value of future universal offers. An increase in either $v_L$ or in the function $\lambda(\cdot)$ makes

[20] Both the high-type seller and the buyer obtain a payoff equal to zero under the most efficient mechanism with commitment.

[21] The outcome without divisibility (DL) is similar to the one when only one unit remains in our model. We describe this in detail in footnote 30, on page 30.

[22] Formally, $-\int_0^\infty e^{-r\tau} z^{*\prime}(\tau)\, d\tau > e^{-rT_{DL}}$, where $T_{DL}$ represents the time at which the high-quality good is traded in DL as bargaining frictions vanish. This inequality follows from the expressions of the low-type seller's payoffs and from the fact that the low-type seller is indifferent in equilibrium (both with and without divisibility).

each screening offer more attractive. Similarly, a decrease in $c$ or an increase in $r$ lower the value of future universal offers. In all these four cases the high-quality good must be traded faster to keep the low-type seller indifferent.

Unlike the high-quality good, the low-quality good is not always traded smoothly. Trade occurs smoothly while the low-type seller mimics the high-type seller's behavior. However, the buyer purchases the whole remaining fraction of the good as soon as the low-type seller accepts a screening offer. Therefore, the fraction of the low-quality good remaining at time elapsed $\tau$ is a random variable that takes a value of zero with probability $q^*(\tau)/\hat{q}$ and a value of $z^*(\tau)$ with the remaining probability. Then $g^*(\tau) = \left[\left(\hat{q} - q^*(\tau)\right)/\hat{q}\right] z^*(\tau)$ is the expected remaining fraction of the low-quality good at time elapsed $\tau$ and reflects the speed of trade of the low-quality good.

The following corollary, which follows directly from Proposition 1 and from the fact that $q^*(\tau) = \bar{q}\left(z^*(\tau)\right)$, describes how changes in the parameters $r$, $v_H$, and $\hat{q}$ affect the speed of trade of the low-quality good.

COROLLARY 1 (Speed of Trade of the Low-quality Good): *Whenever either $r$ increases, or $v_H$ decreases, or $\hat{q}$ increases, then the low-quality good is traded faster, i.e., $g^*(\tau)$ decreases for every $\tau > 0$.*

The remaining primitives ($\lambda(\cdot)$, $v_L$, and $c$) have ambiguous effects on the speed of trade of the low-quality good. It is easy to construct examples where changes in these primitives can either increase or decrease $g^*(\tau)$ for some $\tau$.

## III. Mechanism Behind the Limit Equilibrium Outcome

We now turn back to the explanation of our main result, Theorem 1. We first study the environment with a fixed time between offers $\Delta > 0$ and a fixed number of units $m$. We discuss equilibrium existence (Proposition 2) and describe in detail the pattern of trade. We then describe the pattern of trade as bargaining frictions vanish (Proposition 3). We finally let the number of units grow to infinity and present Proposition 4, which directly leads to Theorem 1.

### A. *Equilibrium Existence*

In this subsection we study the bargaining game when the good is divided into a fixed number of units equal to $m$ and the period length is fixed and equal to $\Delta$.

PROPOSITION 2 (Existence): *An equilibrium exists.*

See online Appendix A.3 for the proof.

We show equilibrium existence by construction. Within our construction, we introduce the function $P_m^\Delta(K,q) : \{1,\ldots,m\} \times [0,\hat{q}] \rightarrow \mathbb{R}$, which plays a key role in the description and the analysis of the equilibrium. We derive this function from $\mathcal{V}^L(\cdot,\cdot)$ (see Section ID), and show that $P_m^\Delta(K,\cdot)$ is an increasing and left-continuous step function for every $K \in \{1,\ldots,m\}$. The function $P_m^\Delta(\cdot,\cdot)$ describes the relevant screening offers available to the buyer in equilibrium. Its interpretation is as follows.

Suppose that there are $K$ units left and that the current belief is $q \in [0,\hat{q}]$. Consider any discontinuity point $q'$ of the function $P_m^{\Delta}(K,\cdot)$ with $q' \geq q$. Then, if the buyer makes a screening offer $\left(K, P_m^{\Delta}(K,q')\right)$ and it is rejected, his posterior belief is $q'$.

We solve the buyer's dynamic optimization problem. For any state $(K,q)$, we let $W_m^{\Delta}(K,q) : \{1,\ldots,m\} \times [0,\hat{q}] \rightarrow \mathbb{R}$ denote the (normalized) buyer's continuation payoff.[23] When it is optimal for the buyer to make a screening offer $\left(K, P_m^{\Delta}(K,q')\right)$ for some discontinuity point $q'$, the low-type seller accepts it with probability $(q'-q)/(\hat{q}-q)$. The buyer's continuation payoff satisfies

$$W_m^{\Delta}(K,q) = (q'-q)\left(\sum_{s=1}^{K}\Lambda_s^m v_L - P_m^{\Delta}(K,q')\right) + \delta W_m^{\Delta}(K,q').$$

If instead it is optimal for the buyer to make a universal offer $\left(k,(c/m)k\right)$, the buyer's continuation payoff satisfies

$$\begin{aligned} W_m^{\Delta}(K,q) = {} & \left(\sum_{s=K-k+1}^{K}\Lambda_s^m\right)\left[(\hat{q}-q)\,v_L + (1-\hat{q})\,v_H\right] \\ & - (1-q)\frac{c}{m}k + \delta W_m^{\Delta}(K-k,q). \end{aligned}$$

We show that the low-type seller is indifferent between accepting and rejecting all screening offers that he receives in equilibrium (see online Appendix A.3). Assume that in equilibrium the buyer makes a screening offer $\left(K, P_m^{\Delta}(K,q)\right)$. If the low-type seller accepts it, he obtains a continuation payoff of $P_m^{\Delta}(K,q)$. If he instead rejects it, the number of units left stays at $K$ and the buyer's posterior is $q$. The buyer's subsequent offer can be either screening or universal. If the buyer makes a screening offer $\left(K, P_m^{\Delta}(K,q')\right)$, then the low-type seller's indifference requires that the prices of these consecutive screening offers be linked: $P_m^{\Delta}(K,q) = \delta P_m^{\Delta}(K,q')$. Assume instead that the buyer makes a universal offer $\left(k,(c/m)k\right)$ after the rejection of the screening offer $\left(K, P_m^{\Delta}(K,q)\right)$. This universal offer must be followed by a screening offer $\left(K-k, P_m^{\Delta}(K-k,q'')\right)$.[24] The low-type seller's indifference then requires that $P_m^{\Delta}(K,q) = \delta(c/m)k + \delta^2 P_m^{\Delta}(K-k,q'')$.

The focus on the equilibrium that we construct in Proposition 2 is without loss of generality as, for generic values of the parameters, the equilibrium outcome is unique.[25]

We show that the game ends after finitely many periods in the proof of Proposition 2. Let $h_m^{*\Delta}$ denote the on-path history along which the seller rejects all screening offers. The history $h_m^{*\Delta}$ is the longest on-path history. We let $T_m^{*\Delta}$ denote its length. As described in Section II, we let $\tilde{q}_m^{\Delta}(t)$ denote the transformed beliefs at the beginning of period $t$ along the history $h_m^{*\Delta}$, for any $t \leq T_m^{*\Delta}$. Similarly, $\tilde{K}_m^{\Delta}(t)$ denotes the number of units left at the beginning of period $t$ along the history $h_m^{*\Delta}$, for any $t \leq T_m^{*\Delta}$. Together with $P_m^{\Delta}(\cdot,\cdot)$, the functions $\tilde{K}_m^{\Delta}(\cdot)$ and $\tilde{q}_m^{\Delta}(\cdot)$ completely characterize the equilibrium pattern of trade.

[23] $W_m^{\Delta}(K,q)$ is normalized in the sense that we multiply the buyer's continuation payoff by $1-q$.

[24] We show this result in the proof of Proposition 2. The intuition behind it is simple. In equilibrium, the buyer's continuation payoff is positive at every state. Thus, he has an incentive to combine any two consecutive universal offers.

[25] An earlier version of this paper contains the result of generic uniqueness of the equilibrium outcome.

### B. *Limit Equilibrium Outcome as Bargaining Frictions Vanish*

In this subsection we fix the number of units $m$ and let the time between offers $\Delta$ converge to zero. To do so, we first characterize the equilibrium outcome as a function of time elapsed $\tau \in \mathbb{R}_+$. In a game with period-length $\Delta$, the time elapsed $\tau$ after $t$ periods is $\tau = t\Delta$. We express the number of remaining units and the transformed beliefs as functions of time elapsed $\tau$:

$$K_m^{\Delta}(\tau) = \tilde{K}_m^{\Delta}\left(\min\left\{\lfloor \tau/\Delta \rfloor, T_m^{*\Delta}\right\}\right)$$

$$q_m^{\Delta}(\tau) = \tilde{q}_m^{\Delta}\left(\min\left\{\lfloor \tau/\Delta \rfloor, T_m^{*\Delta}\right\}\right)$$

To examine the limit equilibrium outcome as bargaining frictions vanish, we take a sequence $\{\Delta_n\}_{n=1}^{\infty} \to 0$ and study the limit of its associated sequence $\left\{\left(K_m^{\Delta_n}(\cdot), q_m^{\Delta_n}(\cdot)\right)\right\}_{n=1}^{\infty}$. In Lemma 3 (online Appendix A.4) we show that for any $\{\Delta_n\}_{n=1}^{\infty} \to 0$, the associated sequence $\left\{\left(K_m^{\Delta_n}(\cdot), q_m^{\Delta_n}(\cdot)\right)\right\}_{n=1}^{\infty}$ converges pointwise to the same limit functions $\left(K_m(\cdot), q_m(\cdot)\right)$. Similarly, for any $\{\Delta_n\}_{n=1}^{\infty} \to 0$, the associated sequence $\left\{\left(P_m^{\Delta_n}(K,\cdot), W_m^{\Delta_n}(K,\cdot)\right)\right\}_{n=1}^{\infty}$, with $K \in \{1, \ldots, m\}$, converges pointwise to the same limit functions $\left(P_m(K,\cdot), W_m(K,\cdot)\right)$. The functions $\left(K_m(\cdot), q_m(\cdot)\right)$ describe the limit equilibrium outcome as bargaining frictions vanish.

The pattern of trade that emerges as bargaining frictions vanish is simple: there is a sequence of phases of fast trade, mediated by impasses. We show this in Proposition 3 but first we provide a formal definition of this pattern of trade.

DEFINITION (Phases of Fast Trade and Impasses): *We say that the limit equilibrium outcome as bargaining frictions vanish consists of a sequence of phases of fast trade, mediated by impasses whenever* $K_m(\cdot)$ *and* $q_m(\cdot)$ *are (left-continuous) step functions that are discontinuous at the same points in time. Moreover, we say that the collection of quantities and beliefs* $\left\{(k_j, q_j)\right\}_{j=1}^{J}$ *characterizes this limit equilibrium outcome as bargaining frictions vanish whenever there exist times* $\tau_1 > \ldots > \tau_{J+1} = 0$ *such that*

$$\left(K_m(\tau), q_m(\tau)\right) = \begin{cases} (m, 0) & \textit{if} \quad \tau = 0 \\ (k_j, q_j) & \textit{if} \quad \tau \in (\tau_{j+1}, \tau_j] \quad \textit{for} \quad j \in \{1, \ldots, J\} \\ (0, \hat{q}) & \textit{if} \quad \tau > \tau_1 \end{cases}.$$

The phases of fast trade correspond to jumps in $K_m(\cdot)$ and $q_m(\cdot)$, while $K_m(\cdot)$ and $q_m(\cdot)$ are constant during each impasse. Each pair $(k_j, q_j)$ describes quantities and beliefs during an impasse. The total number of impasses is $J \leq m$. We index impasses in reverse order, so $j = 1$ corresponds to the last impasse $(k_1, q_1)$, while $j = J$ corresponds to the impasse $(k_J, q_J)$ that occurs first. Therefore, $k_{j+1} > k_j$ and $q_{j+1} < q_j$ for all $j$.

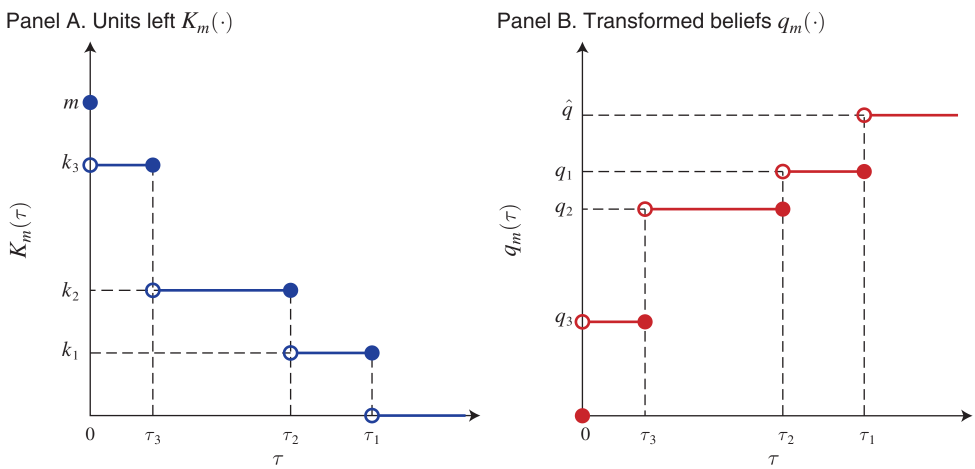


Figure 3. Pattern of Trade $(K_m(\cdot), q_m(\cdot))$ as Bargaining Frictions Vanish

Figure 3 depicts an example of the limit equilibrium outcome as bargaining frictions vanish. At the beginning of the game, there is a phase of fast trade. The transformed belief $q_m(\cdot)$ jumps to $q_3$ at time elapsed $\tau = 0$, which reflects that the buyer makes (a sequence of) screening offers. The low-type seller accepts with total probability $q_3/\hat{q}$. The number of units left $K_m(\cdot)$ jumps to $k_3$ at time elapsed $\tau = 0$, which reflects that the buyer makes a universal offer for $m - k_3$ units. Although for any given $\Delta > 0$ these offers occur in different periods, as $\Delta \to 0$ the total time it takes to jump to $k_3$ and $q_3$ converges to zero.

After the first phase of fast trade, an impasse follows. Intuitively, an impasse is an interval of time elapsed in which no trade occurs. The first impasse depicted in Figure 3 takes place in the interval $(0, \tau_3]$. Within this interval, $K_m(\cdot)$ remains constant at $k_3$ and $q_m(\cdot)$ remains constant at $q_3$. First, the fact that the number of units left is constant reflects that, in the limit, the buyer makes a sequence of screening offers after the first universal offer. As $\Delta \to 0$ the total number of such screening offers goes to infinity. Crucially, it does so sufficiently fast so that the total time elapsed while making these offers converges to $\tau_3 > 0$. Second, the fact that the belief $q_m(\cdot)$ is constant reflects that, in the limit, the low-type seller accepts these screening offers with total probability zero. This is possible because as $\Delta \to 0$, the probability of acceptance of each screening offer goes to zero fast enough to overcome that the total number of screening offers goes to infinity. Finally, Figure 3 illustrates that after the first impasse, there are three phases of fast trade, mediated by impasses.

We construct an algorithm that pins down the phases of fast trade and the impasses that take place as bargaining frictions vanish. This algorithm also identifies some key properties of the limit functions $P_m(\cdot,\cdot)$ and $W_m(\cdot,\cdot)$. In order to state these properties, we define the belief $\bar{q}_m(K)$ for any $K \in \{1,\ldots,m\}$ as follows. Assume that the buyer makes an offer $\varphi = (1,(c/m))$ when there are $K$ units left; that is, he offers to pay the high-type's cost in exchange of one unit. Then, $\bar{q}_m(K) \in (0,\hat{q})$ is the transformed belief that makes the buyer break even:[26]

$$\left[\hat{q} - q'_m(K)\right]\left(\Lambda_K^m v_L - \frac{c}{m}\right) + (1-\hat{q})\left(\Lambda_K^m v_H - \frac{c}{m}\right) = 0.$$

Note that $\bar{q}_m(m) < \ldots < \bar{q}_m(1)$, as gains from trade are decreasing. Finally, for any $(K,q)$ let $P_m^-(K,q) = lim_{q'\uparrow q} P_m(K,q')$ and $P_m^+(K,q) = \lim_{q'\downarrow q} P_m(K,q')$.

PROPOSITION 3 (Equilibrium Outcome as Bargaining Frictions Vanish): *Fix m. The limit equilibrium outcome as bargaining frictions vanish consists of a sequence of phases of fast trade and impasses characterized by a collection of quantities and beliefs* $\left\{(k_j,q_j)\right\}_{j=1}^J$ *with* $1 \leq J \leq m$, $(k_1,q_1) = \left(1,\bar{q}_m(1)\right)$ *and* $\bar{q}_m(k_j+1) < q_j < \bar{q}_m(k_j)$ *for all* $j > 1$.

*Moreover,* $W_m(k_j,q_j) = 0$ *for every* $j \in \{1,\ldots J\}$. *Finally,*

$$P_m^+(k_1,q_1) = P_m^+\left(1,\bar{q}_m(1)\right) = \frac{c}{m}, \tag{4a}$$

$$P_m^-(k_j,q_j) = \left(\frac{v_L \sum_{s=1}^{k_j} \Lambda_s^m}{P_m^+(k_j,q_j)}\right)^2 P_m^+(k_j,q_j) \qquad \forall j \in \{1, \ldots ,J\}, \quad \textit{and} \tag{4b}$$

$$P_m^+(k_{j+1},q_{j+1}) = (k_{j+1}-k_j)\frac{c}{m} + P_m^-(k_j,q_j) \qquad \forall j \in \{1, \ldots ,J-1\}. \tag{4c}$$

See online Appendix A.5 for the proof.

Proposition 3 shows that there is at least one impasse. The last impasse always occurs at $\left(1,\bar{q}_m(1)\right)$, that is, when only one unit remains and the belief is $\bar{q}_m(1)$. The buyer's continuation payoff is zero at all impasses. Finally, Proposition 3 describes the limit functions $P_m(\cdot,\cdot)$ around each impasse $(k_j,q_j)$.

Equation (4) links the limit functions $P_m(\cdot,\cdot)$ between two consecutive impasses. After the impasse $(k_{j+1},q_{j+1})$ is resolved, the state shifts without delay to $(k_j,q_j)$. To fix ideas, suppose that the shift consists of one universal offer for $k_{j+1}-k_j$ units followed by a screening offer $\left(k_j,P_m^-(k_j,q_j)\right)$.[27] The low-type seller obtains a continuation payoff $(k_{j+1}-k_j)(c/m) + P_m^-(k_j,q_j)$, which must be equal to the price $P_m^+(k_{j+1},q_{j+1})$ that the buyer has to pay in the limit to induce a belief $q > q_{j+1}$ close to $q_{j+1}$. Equation (4a) follows the same logic as equation (4c): after the last impasse is resolved the buyer purchases without delay the last unit at the price $c/m$.

Equation (4b) shows that the limit function $P_m(k_j,\cdot)$ is discontinuous at $q_j$. The jump between $P_m^-(k_j,q_j)$ and $P_m^+(k_j,q_j)$ pins down the length of the impasse $(k_j,q_j)$. Let $\tilde{\tau}$ be the necessary time elapsed for the buyer's valuation for the

[26] We define $\bar{q}_m(K)$ in an analogous way to $\bar{q}(z)$ from Section III. While $\bar{q}(z)$ depends on the infinitesimal unit $z$, $\bar{q}_m(K)$ depends on the number of remaining units $K$. For convenience, we set $\bar{q}_m(m+1) = 0$.

[27] Equation (4c) holds regardless of the particular sequence of offers that characterizes the shift from $(k_{j+1},q_{j+1})$ to $(k_j,q_j)$.

low-quality good $v_L \sum_{s=1}^{k_j} \Lambda_s^m$ to be equal to the discounted value of $P_m^+(k_j, q_j)$: $v_L \sum_{s=1}^{k_j} \Lambda_s^m = e^{-r\tilde{\tau}} P_m^+(k_j, q_j)$. Equation (4b) shows that the delay is of length $2\tilde{\tau}$:

$$P_m^-(k_j, q_j) = e^{-2r\tilde{\tau}} P_m^+(k_j, q_j) = \left(\frac{v_L \sum_{s=1}^{k_j} \Lambda_s^m}{P_m^+(k_j, q_j)}\right)^2 P_m^+(k_j, q_j).$$

This finding is in line with DL's double delay result, which characterizes the length of each impasse.[28] We extend this result to the case of a divisible good.

*Description of the Algorithm.*—In what follows we describe the construction of the algorithm that characterizes the limit equilibrium outcome as bargaining frictions vanish. The algorithm follows an inductive approach. In the base step, we identify the last impasse. We show that it occurs when only one unit remains, and pin down both the length of the impasse, and the belief at which it occurs. In the inductive step we take an impasse and construct the previous one. Throughout this explanation, we focus directly on the "limit game," in the sense that the low-type seller's behavior is given by the limit function $P_m(\cdot,\cdot)$.[29]

When only one unit remains and the belief is higher than $\bar{q}_m(1)$, the buyer can guarantee a positive continuation payoff by making a universal offer for the last unit. Since his continuation payoff is strictly positive, the usual Coasean forces imply that the buyer has an incentive to speed up trade. Thus, the buyer purchases the remaining unit without delay. In equilibrium, the low-type seller is indifferent between accepting and rejecting the buyer's screening offers. As the price of a screening offer represents the low-type seller's continuation payoff, then $P_m(1, q) = c/m$ for all states $(1, q)$ with $q > \bar{q}_m(1)$.

The limit price $P_m(1,\cdot)$ must be discontinuous at $\bar{q}_m(1)$, with $P_m(1,q) \leq \Lambda_1^m v_L$ for beliefs $q < \bar{q}_m(1)$. If this were not the case, the buyer's continuation payoff at $q < \bar{q}_m(1)$ would be negative, as $W_m(1, \bar{q}_m(1)) = 0$. The discrete jump in $P_m(1,\cdot)$ at $\bar{q}_m(1)$ implies that there must be delay. Deneckere and Liang show that the length of the impasse is twice the necessary time elapsed for the buyer's valuation for the low-quality good $v_L \Lambda_1^m$ to be equal to the discounted value of the price of a screening offer after the impasse $(1, \bar{q}(1))$ is resolved. Thus,

$$P_m^-(1, q'_m(1)) = \left(\frac{v_L \Lambda_1^m}{c/m}\right)^2 c/m = \left(\frac{v_L \Lambda_1^m}{c/m}\right) v_L \Lambda_1^m < v_L \Lambda_1^m.$$

The inequality above implies that the buyer's continuation payoff is strictly positive at any state $(1, q)$ with $q < \bar{q}_m(1)$. Intuitively, the buyer can make a screening offer with a price $P_m^-(1, \bar{q}_m(1))$ that the low-type seller accepts with strictly positive probability. Since the buyer has a positive continuation payoff, the usual Coasean forces kick in, and the state $(1, \bar{q}_m(1))$ is reached without delay. Therefore $P_m(1, q) = P_m^-(1, \bar{q}_m(1))$ for any $q < \bar{q}_m(1)$.[30]

[28] The key feature behind DL's double delay result is the symmetry of the steps of the function $P_m^\Delta(1,\cdot)$ around the buyer's valuation (Deneckere and Liang 2006, p. 1323).

[29] By continuity, all results of the "limit game" hold for $\Delta$ sufficiently close to zero.

[30] The arguments presented in these paragraphs also apply (with minor modifications) to the model with an indivisible good (DL). In DL's environment the buyer trades immediately with the low-type seller at a price $\left(v_L \int_0^1 \lambda(z) dz / c\right)^2 c$ with positive probability. If there is no immediate trade, an impasse follows. After the impasse, the buyer purchases the whole good from both types at a price $c$ and breaks even.

In our model, there must be at least one impasse. If there were none, then the buyer would buy all units without delay, pay $c/m$ for each of them, and obtain a negative payoff. Moreover, the last impasse must occur at state $\left(1, \bar{q}_m(1)\right)$. Assume instead that the last impasse occurs at state $(K, q)$, with $K > 1$. First, notice that $q$ must be strictly smaller than $\bar{q}_m(1)$. This is because for any $q \geq \bar{q}_m(1)$ the buyer can obtain a strictly positive continuation payoff by making a universal offer for all remaining units, and so there cannot be delay. Second, after the impasse $(K, q)$ is resolved, the buyer purchases all remaining units without delay and therefore pays $c/m$ for each of them. However, because of divisibility, there exists an alternative course of action that gives the buyer a higher continuation payoff. The buyer can instead first make a universal offer for $K-1$ units. Then, he can make a screening offer $\left(1, P_m^-\left(1, q'_m(1)\right)\right)$, which is accepted by the low-type seller with probability $\left(\bar{q}_m(1) - q\right)/(\hat{q} - q) > 0$. If instead the offer is rejected, the buyer pays $c/m$ for the remaining unit. Thus, divisibility allows the buyer to take advantage of the positive profits from the screening offer before the impasse $\left(1, \bar{q}_m(1)\right)$, and so he has a profitable deviation.

We now describe the inductive step of our algorithm. We explain how it identifies the penultimate impasse. At the end of this section, we discuss how the argument generalizes to arbitrary impasses. To identify the penultimate impasse, we consider a simple course of action that allows the buyer to take advantage of the positive profits from screening offers before the last impasse. This course of action brings the buyer from any state $(K, q)$ with $K > 1$ and $q < \bar{q}_m(1)$ to the last impasse $\left(1, \bar{q}_m(1)\right)$, where the buyer's continuation payoff is zero. The buyer first makes the universal offer $\left(K-1, (c/m)(K-1)\right)$ and then the screening offer $\left(1, P_m^-\left(1, \bar{q}_m(1)\right)\right)$. We let $\check{q}(K)$ be the threshold belief that makes the buyer break even when he follows this course of action:[31]

$$\left[\left(\hat{q} - \check{q}(K)\right) v_L + (1 - \hat{q}) v_H\right] \sum_{s=2}^{K} \Lambda_s^m - \left(1 - \check{q}(K)\right)(K-1)\frac{c}{m} + \left(\bar{q}_m(1) - \check{q}(K)\right)\left[\Lambda_1^m v_L - P_m^-\left(1, \bar{q}_m(1)\right)\right] = 0.$$

A buyer who follows this simple course of action from state $(K, q)$ obtains a negative payoff if $q < \check{q}(K)$ and a positive payoff if $q > \check{q}(K)$.

The penultimate impasse $(k_2, q_2)$ occurs at the quantity $k_2 = \operatorname{argmax}_{K \geq 2}\left\{\check{q}(K)\right\}$ and belief $q_2 = \check{q}(k_2)$. This result relies on the shape of $\check{q}(K)$: it first increases, then reaches a maximum at $\check{q}(k_2)$ and after that it decreases. We show this and the following string of inequalities, in online Appendix A.5:[32]

$$\bar{q}_m(k_2 + 1) < q_2 < \bar{q}_m(k_2). \tag{5}$$

[31] To ease the exposition, we drop the dependence of $\check{q}(K)$ on $m$. We also set $\check{q}(K) = 0$ whenever the simple course of action leads to a positive payoff for every belief $q \in \left[0, \bar{q}_m(1)\right)$. If $\check{q}(K) = 0$ for all $K \geq 2$, then there is no penultimate impasse. In what follows, we assume that $\check{q}(K) > 0$ for some $K \geq 2$.

[32] To see the link between $\check{q}(K+1)$ and $\check{q}(K)$, notice that a simple course of action starting at state $(K+1, q)$ can be decomposed into a universal offer for one unit and a simple course of action from state $(K, q)$. While the threshold belief for the simple course of action from $(K, q)$ is $\check{q}(K)$, the belief that makes the buyer break even with a universal offer is $\bar{q}(K+1)$. The inequalities in (5) are strict because we focus on generic values of the parameters (for details, see Remark 1 on page 20 of the online Appendix).

We explain why the penultimate impasse occurs at $(k_2, q_2)$ through an example with $m = 6$ and $k_2 = 4$. Starting at any state $(K, q)$ with $K \in \{2,3,4\}$ and $\check{q}(K) < q \leq \bar{q}_m(1)$, the buyer must reach $(1, \bar{q}_m(1))$ without delay. This is because the simple course of action yields a positive payoff to the buyer and $\check{q}(2) < \check{q}(3) < \check{q}(4)$.[33] As the price of a screening offer represents the low-type seller's continuation payoff, the limit equilibrium price is

$$P_m(K,q) = (K-1)\frac{c}{m} + P_m^{-}(1, \bar{q}_m(1)). \tag{6}$$

When instead $q < \check{q}(K)$, the limit equilibrium price is (we show this in online Appendix A.5):

$$P_m(K,q) = \left(\frac{v_L \sum_{s=1}^{K} \Lambda_s^m}{(K-1)\frac{c}{m} + P_m^{-}(1, \bar{q}_m(1))}\right)^2 \left((K-1)\frac{c}{m} + P_m^{-}(1, \bar{q}_m(1))\right) \tag{7}$$

Equations (6) and (7) characterize the limit equilibrium price for $K \in \{2,3,4\}$. The discontinuity point at $(K, \check{q}(K))$ reflects that there is a (potentially off-path) impasse at $(K, \check{q}(K))$. The fact that $P_m(K,q)$ is constant for $q < \check{q}(K)$ reflects that starting at any state $(K,q)$ with $q < \check{q}(K)$, the buyer's optimal course of action is to reach the impasse $(K, \check{q}(K))$ without delay.

Why is there a (potentially off-path) impasse at $(K, \check{q}(K))$ for $K \in \{2,3,4\}$? Consider a buyer at a (potentially off-path) state $(K,q)$ with $K \in \{2,3,4\}$ and $q = \check{q}(K) - \varepsilon$. The buyer cannot reach the state $(1, \bar{q}_m(1))$ immediately, as this would yield a negative payoff. Moreover, since $\check{q}(2) < \check{q}(3) < \check{q}(4)$, the buyer would also obtain a negative payoff from any universal offer. Consequently, the delay must arise when $K$ units remain. This argument holds for any small $\varepsilon$, so the (potentially off-path) impasse must take place at $(K, \check{q}(K))$. Equations (6) and (7) highlight that DL's double delay result extends to arbitrary impasses: the delay is twice the time necessary to make the low-type seller indifferent between the price after the impasse (equation (6)) and the buyer's valuation for the remaining units of the low-quality good ($v_L \sum_{s=1}^{K} \Lambda_s^m$).

There is double delay at all (potentially off-path) impasses $(K, \check{q}(K))$ for $K \in \{2,3,4\}$. Thus, as the buyer's belief approaches $\check{q}(K)$ from the left, the limit equilibrium price is lower than the buyer's valuation for the remaining units of the low-quality good. Therefore, the buyer has a course of action that guarantees a strictly positive payoff whenever $q < \check{q}(K)$. This in turn implies that some impasse must be reached without delay. Using equations (6) and (7) and some straightforward calculations, we show that the best course of action for a buyer in state $(K,q)$ with $K \in \{2,3,4\}$ and $q < \check{q}(K)$ is to reach the impasse $(K, \check{q}(K))$ immediately. This is why $P_m(K,q)$ is constant for $q < \check{q}(K)$.

So finally, why is it that, on-path, the penultimate impasse must be at $(4, \check{q}(4))$? Consider a buyer at a state $(K,q)$ with $K > 4$ and $q < \check{q}(4)$ who is contemplating making a universal offer for at least $K - 4$ units. Any such offer can be decomposed

[33] For example, starting at $(3,q)$ with $q > \check{q}(3)$, the buyer can never reach a (potentially off-path) impasse with two units, since $q > \check{q}(3) > \check{q}(2)$.

into two; first, an offer for exactly $K - 4$ units, and second, an offer for some extra units. The offer for $K - 4$ units takes the buyer to a state $(4, q)$ with $q < \check{q}(4)$. As mentioned above, from that state, it is optimal for the buyer to reach the impasse $(4, \check{q}(4))$ immediately. Therefore, of all universal offers for at least $K - 4$ units, the optimal one is for *exactly* $K - 4$ units. Then, starting at a state $(K, q)$ with $K \geq 4$ and $q < \check{q}(4)$, the buyer never reaches a state $(K', q')$ with $K' \in \{1,2,3\}$ and $q' < \check{q}(4)$. This shows two things. First, the penultimate impasse cannot be at $(2, \check{q}(2))$ or $(3, \check{q}(3))$, since $\check{q}(2) < \check{q}(3) < \check{q}(4)$. Second, there must be a penultimate impasse and this impasse cannot arise when five or six units remain. Otherwise, there would be a state $(K, q)$ with $K \in \{5,6\}$ and $q < \check{q}(4)$ from which the buyer immediately takes advantage of the low price associated with the last impasse, instead of first taking advantage of the low price associated with the impasse $(4, \check{q}(4))$.

Our algorithm proceeds by induction by taking the impasse $(k_j, q_j)$ and identifying the previous impasse $(k_{j+1}, q_{j+1})$. To do this, we construct a simple course of action, analogous to the one before, where the buyer takes advantage of the positive profits from screening offers before the impasse $(k_j, q_j)$. This course of action brings the buyer from any state $(K, q)$ with $K > k_j$ and $q < q_j$ to the impasse $(k_j, q_j)$. As before, we define for each $K > k_j$ the threshold belief such that the buyer breaks even following this simple course of action. The previous impasse occurs at $(k_{j+1}, q_{j+1})$, where $k_{j+1}$ is the number of units that maximizes the threshold belief, and $q_{j+1}$ is the threshold belief when $k_{j+1}$ units remain. The algorithm ends in finitely many steps and there are at most $m$ impasses.

## C. *Arbitrarily Divisible Good*

In this subsection we describe the limit of the equilibrium outcome identified in Proposition 3 as the number of units $m$ grows to infinity. In order to keep track of the number of units, we add the index $m$ to the collection of quantities and beliefs $\left\{\left(k_j^m, q_j^m\right)\right\}_{j=1}^{J_m}$ that characterize impasses and let $J_m$ denote the number of impasses when the good is divided into $m$ units. We also let $z_j^m = k_j^m/m$ represent the fraction of the good left for trade at impasse $j$. Thus, we denote impasses by $\left\{\left(z_j^m, q_j^m\right)\right\}_{j=1}^{J_m}$ in this subsection.

PROPOSITION 4 (Impasses for an Arbitrarily Divisible Good): *The limit equilibrium outcome satisfies*

$$\lim_{m\to\infty}\left(\max\left\{z_j^m - z_{j-1}^m\right\}_{j=2}^{J_m}\right) = 0 \tag{8a}$$

$$\lim_{m\to\infty}\left(\max\left\{q_{j-1}^m - q_j^m\right\}_{j=2}^{J_m}\right) = 0 \tag{8b}$$

$$\lim_{m\to\infty} z_{J_m}^m = 1 \tag{8c}$$

$$\lim_{m\to\infty}\left(\max\left\{\left|q_j^m - \bar{q}\left(z_j^m\right)\right|\right\}_{j=1}^{J_m}\right) = 0 \tag{8d}$$

$$\lim_{m\to\infty}\left(\max\left\{\left|P_m^-\left(mz_j^m, q_j^m\right) - v_L\int_0^{z_j^m}\lambda(z)dz\right|\right\}_{j=1}^{J_m}\right) = 0 \tag{8e}$$

$$\lim_{m\to\infty}\left(\max\left\{\left|P_m^+\left(mz_j^m,q_j^m\right)-v_L\int_0^{z_j^m}\lambda(z)dz\right|\right\}_{j=1}^{J_m}\right) = 0. \tag{8f}$$

See online Appendix A.6 for the proof.

Proposition 4 directly leads to Theorem 1. As the good becomes arbitrarily divisible, the number of impasses goes to infinity. The fraction that the buyer purchases through each universal offer shrinks to zero (equation (8a)). The change in the buyer's belief between two consecutive impasses also shrinks to zero (equation (8b)). Thus, in the limit, the high-quality good is traded smoothly over time, and beliefs also evolve continuously. The first impasse takes place with the whole good left for trade (equation (8c)). Let $z(\tau) = \lim_{m\to\infty} z_m(\tau)$ and $q(\tau) = \lim_{m\to\infty} q_m(\tau)$ denote respectively the limit fraction of the good left and the limit belief at time elapsed $\tau$. Equation (8d) shows that these functions are linked: $q(\tau) = \bar{q}(z(\tau))$. Furthermore, at any screening offer for a fraction $z(\tau)$ of the good, the buyer offers a price $v_L\int_0^{z(\tau)}\lambda(z)dz$, and so he breaks even (equations (8e) and (8f)).

The limit equilibrium outcome then coincides with the artificial pattern of trade described in Section II. At time zero the buyer makes a screening offer for the whole good at price $v_L\int_0^1\lambda(z)dz$. The low-type seller accepts this offer with probability $\bar{q}_m(1)/\hat{q}$. Then, the buyer continuously makes both universal and screening offers. The low-type seller's indifference between accepting different screening offers implies that the fraction $z(\tau)$ must satisfy equation (2). This pins down the pattern of trade in the limit: $z(\tau) = z^*(\tau)$ and $q(\tau) = q^*(\tau)$, as stated in Theorem 1.

The proof of Proposition 4 relies on the key properties identified in Proposition 3. Equation (8d) directly results from the condition $\bar{q}_m(k_j^m+1) < q_j^m < \bar{q}_m(k_j^m)$ in Proposition 3. As $m$ goes to infinity, $(k_j^m+1)/m \to k_j^m/m = z_j^m$. We next explain, through a unified argument, why equations (8a), (8b), (8e) and (8f) hold true.

We say that an impasse $(z_j^m, q_j^m)$ is short whenever $P_m^-(mz_j^m, q_j^m)$ and $P_m^+(mz_j^m, q_j^m)$ are close (and so both are close to the valuation $v_L\int_0^{z_j^m}\lambda(z)dz$).[34] The buyer makes a profit with a screening offer before each impasse. Whenever an impasse is short, this profit is low. The driving force behind Proposition 4 is that whenever $m$ is large, if an impasse $(z_j^m, q_j^m)$ is short, then the previous impasse $(z_{j+1}^m, q_{j+1}^m)$ must also be short. Moreover, the fraction $z_{j+1}^m - z_j^m$ that the buyer purchases between these two impasses must be small. To show this, we link two consecutive impasses $(z_{j+1}^m, q_{j+1}^m)$ and $(z_j^m, q_j^m)$. The buyer obtains a zero continuation payoff at every impasse. Thus, the difference $W_m(mz_{j+1}^m, q_{j+1}^m) - W_m(mz_j^m, q_j^m)$, which we express in equation (9), is also zero:

$$\overbrace{(\hat{q}-q_{j+1}^m)\left[\int_{z_j^m}^{z_{j+1}^m}\left[\lambda(z)\,v_L - c\right]dz\right] + (1-\hat{q})\left[\int_{z_j^m}^{z_{j+1}^m}\left[\lambda(z)\,v_H - c\right]dz\right]}^{(*)}$$
$$+\underbrace{(q_j^m - q_{j+1}^m)\left[\int_0^{z_j^m}\lambda(z)\,v_L dz - P_m^-(mz_j^m, q_j^m)\right]}_{(**)} = 0. \tag{9}$$

[34] Proposition 3 guarantees that $P_m^-(mz_j^m, q_j^m) < v_L\int_0^{z_j^m}\lambda(z)dz < P_m^+(mz_j^m, q_j^m)$ for every impasse $(z_j^m, q_j^m)$. Whenever $P_m^-(mz_j^m, q_j^m)$ and $P_m^+(mz_j^m, q_j^m)$ are close and different from zero, their ratio is close to one. The low-type seller must be indifferent between accepting and rejecting screening offers, so it takes a short time for the price to go from $P_m^-(mz_j^m, q_j^m)$ to $P_m^+(mz_j^m, q_j^m)$. In this sense the impasse is short.

From one impasse to the next one, the buyer makes a loss with a universal offer $(*)$, and a profit with a screening offer $(**)$. This profit is close to zero since the price $P_m^-\left(m z_j^m, q_j^m\right)$ of the screening offer is close to the buyer's valuation. Therefore, the loss associated to the universal offer must also be close to zero, which can only happen if $z_j^m$ is close to $z_{j+1}^m$.[35]

We next show that the previous impasse $\left(z_{j+1}^m, q_{j+1}^m\right)$ must also be short. Equations (4b) and (4c) in Proposition 3 imply that

$$P_m^-\left(m z_{j+1}^m, q_{j+1}^m\right) = \left(\frac{v_L \int_0^{z_{j+1}^m} \lambda(z) dz}{\left(z_{j+1}^m - z_j^m\right)c + P_m^-\left(m z_j^m, q_j^m\right)}\right)^2 P_m^+\left(m z_{j+1}^m, q_{j+1}^m\right).$$

Since $z_j^m$ and $z_{j+1}^m$ are close and the price $P_m^-\left(m z_j^m, q_j^m\right)$ is close to the buyer's valuation, then the first term on the right hand side is close to one.

We argue next that the last impasse must be short as $m$ grows large. The last impasse occurs when only one unit remains. Equations (4a) and (4b) in Proposition 3 imply that $P_m^-\left(1, \bar{q}_m(1)\right)$ and $P_m^+\left(1, \bar{q}_m(1)\right)$ both converge to zero as $m$ grows large.

To complete the argument it remains to be shown that there are no cumulative effects in the sense that if one impasse is short, all previous ones must also be short. This is a technical part that we present in online Appendix A.6.

Finally, the intuition behind equation (8c) is simple. If it does not hold, then for large $m$ the buyer reaches the first impasse after purchasing a strictly positive fraction of the good though a universal offer. This offer yields a loss to the buyer. At each impasse the price of the screening offer is close to the buyer's valuation, so the buyer's profit from this offer is negligible. Therefore, if equation (8c) is violated, the buyer obtains a negative continuation payoff at the beginning, which can never happen.

## IV. Extensions

In our first extension, we study the limit equilibrium outcome when equation $(1)$ does not hold. Equation $(1)$ reflects an extreme form of adverse selection: under the prior belief, the buyer's expected valuation from any fraction of the good exceeds the high-type seller's cost. Therefore, the buyer needs to screen the seller even to purchase the most valuable fraction of the good.

We first assume that $\left[\hat{\beta} v_H + \left(1 - \hat{\beta}\right) v_L\right] \lambda(\bar{z}) = c$ for some $\bar{z} \in (0,1]$, so the buyer obtains a positive payoff if he buys any infinitesimal unit $z \in \left[\bar{z}, 1\right]$ through a universal offer. Our analysis directly extends to this case.[36] In the limit equilibrium outcome, the buyer purchases the first fraction $1 - \bar{z}$ from both types without delay, paying $c\left(1 - \bar{z}\right)$. The environment after the units $z \in \left[\bar{z}, 1\right]$ are traded resembles that from our baseline model. Theorem 1 pins down the pattern of trade for the remaining fraction $\bar{z}$. Similarly to the case when equation $(1)$ holds, divisibility is

[35] Equation (8d) implies that for $m$ large, the buyer is close to breaking even if he makes a universal offer for an arbitrarily small unit at state $\left(m z_{j+1}^m, q_{j+1}^m\right)$. Since gains from trade are decreasing, any nonnegligible universal offer would lead to a loss bounded away from zero.

[36] The proof of the characterization of the limit equilibrium outcome in this case is analogous to the proof of Theorem 1 so we omit it.

detrimental to the buyer. Although he obtains a profit from the units $z \in [\bar{z}, 1]$, he must pay the high-type seller's cost for them. He then obtains a zero profit from the remaining units. Furthermore, like in the benchmark model, the high-quality good is traded smoothly, but only for the units $z \in [0, \bar{z}]$.

We next assume that $\left[\hat{\beta} v_H + (1 - \hat{\beta}) v_L\right] \lambda(0) \geq 0$. In this case, the buyer obtains a positive payoff if he buys any fraction through a universal offer, so the standard Coasean forces apply. For any $m$, as bargaining frictions vanish, the buyer purchases the whole good from both types without delay and pays $c$.[37]

In our second extension we assume that $\lambda(\cdot)$ is either constant or strictly decreasing, which correspond, respectively, to constant gains from trade or increasing gains from trade. In either of these cases divisibility plays no role: for any $m$ the buyer only makes offers $(m, p)$ with $p \leq c$ in equilibrium. The equilibrium outcome is identical to the one when the good is indivisible. Proposition 5 formalizes this.

PROPOSITION 5 (Constant or Increasing Gains from Trade): *When gains from trade are constant or increasing, the buyer only makes screening offers in equilibrium.*

See online Appendix A.8 for the proof.

The intuition behind Proposition 5 is simple. Whenever the buyer is happy to pay the high-type seller's cost for some units, then he is also happy to pay that cost for subsequent units. As gains from trade are constant or increasing, those subsequent units are at least as valuable as the previous ones.

The limit equilibrium outcome with constant (or increasing) gains from trade differs starkly from that when gains from trade are decreasing. This difference highlights that there is a discontinuity in the shape of the gains from trade. Consider a family of strictly increasing functions $\lambda_n(\cdot)$ converging (pointwise) to a constant function $\lambda(\cdot)$. For every $n$, the high-quality good is traded smoothly, since gains from trade are decreasing (Theorem 1). Instead, when $\lambda(\cdot)$ is constant, in the limit equilibrium outcome the high-quality good is sold all at once (this follows directly from Proposition 5). Because of this discontinuity, one should be wary in applying Theorem 1 in environments where gains from trade may be close to constant. Instead, Theorem 1 provides clear predictions when there are strong economic forces that lead to decreasing gains from trade (e.g., the buyer benefits from diversifying his portfolio).

## V. Conclusion

We study bargaining over a divisible good. We characterize the limit equilibrium outcome as bargaining frictions vanish and the good becomes arbitrarily divisible. Our model generates novel and testable predictions for dynamic markets with

[37] In our model, the buyer's valuation of the last infinitesimal unit of the low quality good $\lambda(0) v_L$ exceeds the seller's cost $c_L$, which we normalize to zero. An alternative extension of our model would be to set $\lambda(0) v_L = c_L > 0$. In such environment, Proposition 3 holds for any number of units $m$. We do not know whether Proposition 4 holds in this environment (some steps in the current proof do not extend to it). However, we can show that most of the qualitative features of the limit equilibrium outcome continue to hold: as the good becomes arbitrarily divisible, the number of impasses goes to infinity and the fraction of the high quality good left for trade converges asymptotically to zero.

adverse selection. When gains from trade are constant or increasing, the pattern of trade is identical to that of parties negotiating over an indivisible good. Time on the market is the main signaling device and the buyer keeps some of his bargaining power. On the other hand, when there are decreasing gains from trade, the high-quality good is traded smoothly over time and the buyer loses all the bargaining power in the limit.

In this paper we first let the time between offers shrink to zero and we then let the number of units grow to infinity. This order of limits both reflects many real-world environments and allows for tractability. The tools developed in this paper do not allow for a complete characterization of the pattern of trade if we instead invert the order of limits. However, one of our main findings extends to that environment. If we invert the order of limits, the number of transactions of the high-quality good (and the number of impasses) must also grow without bound.[38]

Our model relies on some simplifying assumptions that make the analysis tractable. First, we assume that the quality of the good can take only two values. Our results extend to a model with finitely many types provided that the buyer's valuation for a good of any intermediate quality is sufficiently close to his valuation for the good of the highest quality. Future research can shed further light on bargaining with divisibility and many types.

Second, we focus on the benchmark case in which all learning is strategic: the buyer learns about the quality of the good only through the seller's behavior. Although this assumption is reasonable in a number of important applications, there are many markets where buyers obtain information as they purchase parts of the good.[39] This new channel of endogenous arrival of information opens many relevant paths for future research: information could be public or private, perfect or imperfect. A natural extension of our model is to allow for the buyer to receive public and imperfect information as he purchases parts of the good. We have constructed examples with a good divided into a finite number of units that suggest that some of our findings extend to such environment. In equilibrium the buyer alternates between screening and universal offers. Furthermore, while we do not have a general algorithm that characterizes the limit equilibrium outcome as bargaining frictions vanish, we can show that the pattern of trade is characterized by impasses mediated by phases of fast trade.

[38] The intuition behind this is simple. Assume instead that in the limit there is a finite number of transactions, and take the last transaction for a positive fraction of the good. Consider the last impasse before this transaction (such an impasse must exist; otherwise the buyer would pay the high-type seller's cost for the whole good and obtain a negative payoff). At this impasse, the buyer's payoff is zero and his belief is such that he breaks even if he makes a universal offer for the remaining fraction of the good. But then the buyer has a profitable deviation; because of decreasing gains from trade, he obtains a positive payoff by making a universal offer for less than the remaining fraction of the good.

[39] Buyers may also obtain exogenous information while bargaining. In their pioneering work, Daley and Green (2012, 2020) study the effects of the exogenous arrival of information over time when parties bargain over an indivisible good.

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