Baltic Astronomy, vol. 8, 239–251, 1999.

PROSPECTS FOR ASTROMETRIC MEASUREMENTS OF GRAVITATIONAL WAVES FROM STELLAR SOURCES WITH GAIA

M. Crosta, M. G. Lattanzi and A. Spagna

Osservatorio Astronomico di Torino, Pino Torinese, Italy

Received February 9, 1999.

Abstract. Direct observation of gravitational waves has yet to be accomplished in experimental gravitation, and such a discovery would open a new chapter in the study of the nature of gravity. Astrometric (indirect) observations alone would make it possible to test general relativity with respect to other alternative theories. Astrometric light deflection of photons crossing the buffer zone of a gravitational source belongs to this case. Among the astrophysical objects in our Galaxy we have searched for those that can produce deflections within the precision limit of the next generation space astrometric projects. We present a simple model of a source of gravitational waves which is compatible with current astrophysical scenarios and whose buffer (non radiative) zone would bend light sufficiently to be detecteable with a *Gaia*-like satellite reaching the microarcsecond accuracy limit.

Key words: cosmology: gravitational lensing, gravitational waves – orbiting observatories: Gaia

1. INTRODUCTION TO RELATIVISTIC DEFLECTION OF LIGHT BY A SOURCE OF GRAVITATIONAL WAVES

The detection of gravitational wave deflection of light by astrometric (indirect) measurements (Fakir 1994) is important because this deflection happens near the gravitational source. This is of great advantage to the gravitational signal expected on Earth or near it.

The treatment below resembles the derivations described in Fakir (1994). Very recently, Damour & Esposito-Farese (1998) and

Kopeikin et al. (1998) have shown that Fakir's approach is inadequate for a proper description of the physical phenomenon at work near the source of gravitational radiation. The main practical consequence is that gravitational wave light deflection is diminished by two orders of magnitude. In spite of this, Fakir's framework can still be useful for at least three good reasons:

(a) it provides a relatively simple way for a first evaluation of the light deflection effect at the source that can easily be scaled down to the right order of magnitude;

(b) it leads to an observational model which permits to characterize the astrophysical properties of astronomical (stellar) sources emitting gravitational radiation, and to evaluate *Gaia*'s capabilities in detecting line-of-sight wobbles of angularly close objects induced by such sources;

(c) it might give the right amplitude of the effect in the event if gravitation deviates from general relativity (Kopeikin et al. 1998).

Let SL be a source of light whose photons cross the buffer zone of an isolated gravitational wave source (briefly SG). Therefore we can linearize the Einstein field equations and use quadrupolar approximation. The buffer zone is a region of space-time where there are only perturbations generated by SG, sufficiently far away, its strong gravitational field ($r \gg 2M_{\rm SG}$, for $G = c = 1$) to be considered weak, but sufficiently near to be at a distance $r \ll \Lambda_g$ from SG, where Λ_g is the wavelength of the gravitational radiation emitted. Let us assume also that SG is stationary, monochromatic and with each polarized component of the form

$$h = \frac{H}{r} \exp\left[i\Omega(r-t)\right]$$

where H is a constant obtainable from the quadrupolar approximation and $\Omega = 2\pi/\Lambda_g$ is the angular frequency. Our spherical coordinate system is centered on the SG. Also, we consider a configuration where $\theta = \pi/2$, SL, SG and the line-of-sight are coplanar, and we take just the polarized component h_+, whose effects lie on such a plane, i.e., the SG's equatorial plane. Then, we can formulate the photon's trajectory through the non-radiative (buffer) zone as due to the perturbation of a refractive-like medium, i.e., the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowskian tensor and $h_{\mu\nu}$ is a small perturbation.

Inside the buffer zone we use for h its maximum value and we can replace r and t with their zero order (flat) approximations.

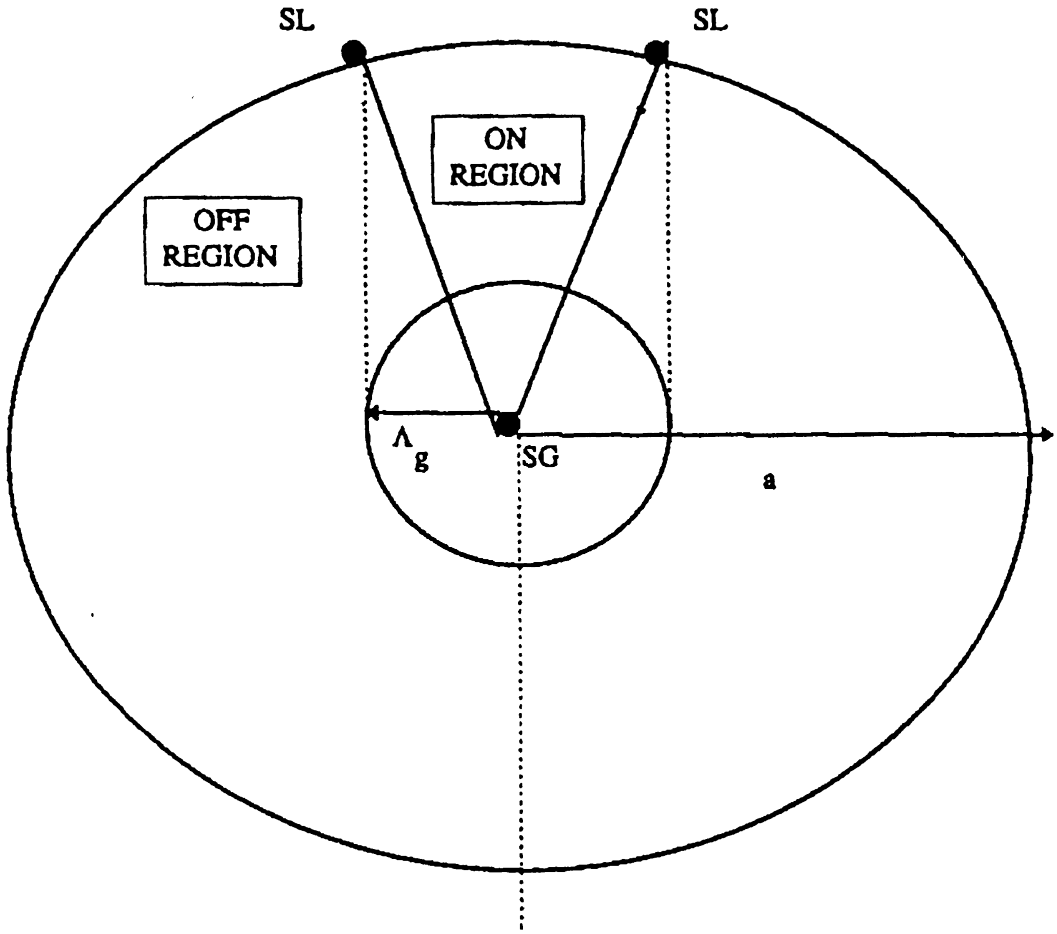

Fig. 1. $\phi_{\rm off} - \phi_{\rm on}$ ("on region") for an arbitrary orbit around the near zone generated by SG. The near zone extends to Λ_g and its lower bound is at a distance of about the Schwarzschild radius $(2M/c^2)$ of the source.

The differential equation for the pertubated photon's trajectory $u_1(\phi)$ is:

$$u_1'' + u_1 = \frac{h}{b\sin\phi}\left[1 - \frac{3}{2}\sin\phi^2 - i\Omega b\left(\frac{\cos\phi + 1}{\sin\phi} - \frac{\sin\phi}{2}\right)\right] \quad (1)$$

with b being the impact parameter.

Integrating Eq. (1) along the photon's path from $\phi \approx \phi_i$ (SL) to $\phi \approx \pi$ (observer), we get:

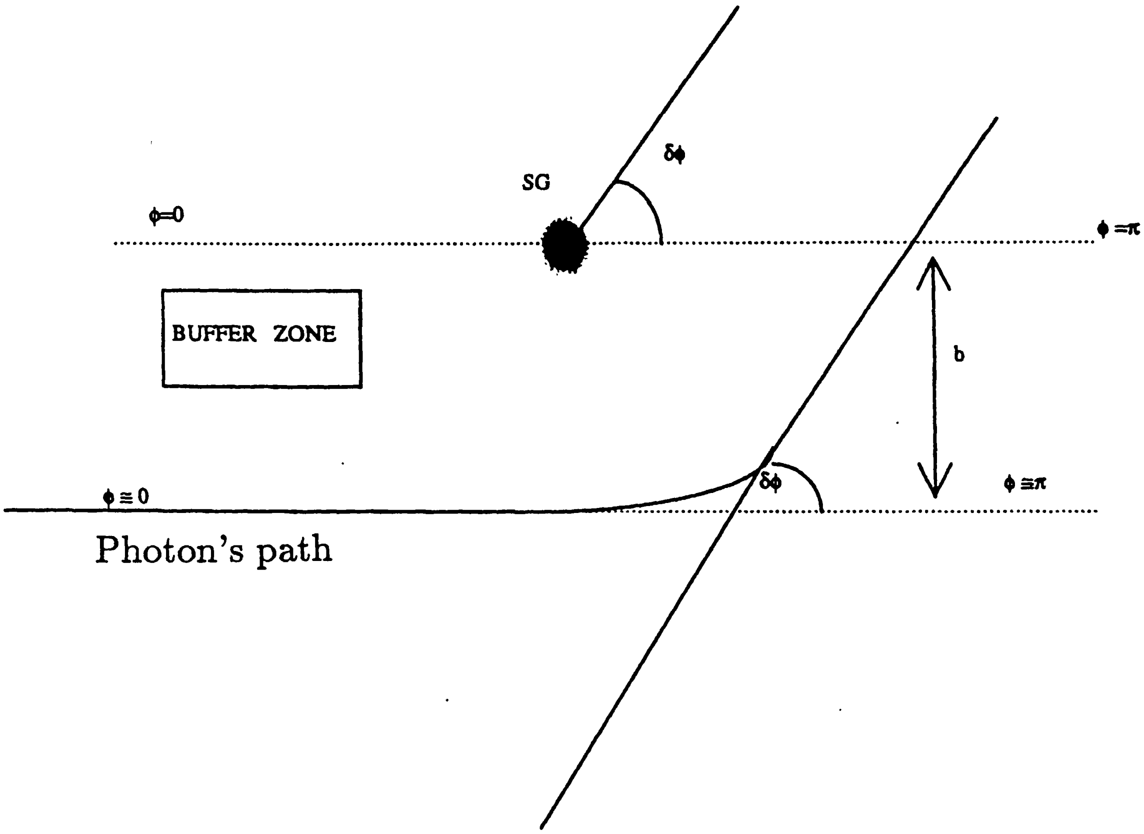

Fig. 2. Geometry (greatly exaggerated) of the photon deflection by the buffer zone when $\phi_i \approx 0$.

$$
\begin{aligned}
&- u_1'(\phi_i)\sin\phi_i + u_1(\pi) + u_1(\phi_i)\cos\phi_i = \\
&- \frac{H}{2b^2}\cos\phi_i \sin\phi_i^2 - \frac{3i\Omega H}{4b}(\pi - \phi_i) + \frac{i\Omega H}{b}\sin\phi\left(1 + \frac{\cos\phi_i}{2}\right).
\end{aligned}
$$

The first term can be estimated asymptotically to obtain the deflection angle $\delta\phi$ produced by a gravitational perturbation ($u_1 \propto b\,\delta\phi$):

$$
\begin{aligned}
\delta\phi &= -\frac{H}{2b}\cos\phi_i - \frac{H}{2b}\coth\phi_i \ln\left[\frac{1+\tan(\phi_i/2)}{1-\tan(\phi_i/2)}\right] + \\
&\quad + i\left[\frac{\Omega H}{2}\left(\pi - \phi_i + \frac{2}{\sin\phi_i}\right) - \Omega H \coth\phi_i \ln(\cos\phi_i)\right]. \quad (2)
\end{aligned}
$$

In the simple case of $\phi_i \approx 0$, we produce the same deflection obtained by Fakir (1994):

$$|\delta\phi| \approx \frac{3\Omega H\pi}{4}.$$

The buffer zone has a lower limit given by the gravitational radius of SG and an upper limit of the order of the gravitational wavelength. The range for the impact parameter is then

$$2M_{\rm SG} < b \leq \Lambda_g,$$

and we assume that the effect is on when photons from SL are emitted toward the observer through the region where $b > 2M_{\rm SG}$; this position is named $\phi_{\rm on}$. Vice versa, the effect is off when the photons "graze" the SG at distance $b \sim \Lambda_g$, corresponding to the position $\phi_{\rm off}$ (Fig. 1).

2. CANDIDATE SOURCES AND A GAIA-LIKE MISSION

Every candidate SG will produce a gravitational wave with a specific Λ_g and will have a buffer zone with a radius not smaller than Λ_g.

The deflection produced by each candidate can be expressed as:

$$\delta\phi \geq \frac{3\pi^2 H}{2\Lambda_g} \approx \frac{3\pi^2}{2} h_{\rm max}, \qquad (3)$$

where each $h_{\rm max}$ depends on the emission mechanism of the gravitational wave.

A mission like *Gaia* repeatedly samples, like Hipparcos, the same "slices" of the sky during the mission. It could then monitor periodic sources as binary systems and rotating stars.

Burst signals are too short to be detected as the shortest time scale available for relative astrometry is ~3 h (corresponding to one complete revolution around the satellite spin axis). For example, the birth of a neutron star, coalescing compact binary systems and other events involving objects of 1 to 10 $M_\odot$, give rise to gravitational bursts which have a lifetime of $10^{-2} - 10^{-5}$ s. Also beyond detection are those events having rare occurence like pulsars in the Galaxy, which have an estimated rate of ~ 0.16 sources year^{-1}. Therefore we can dismiss burst objects and concentrate on those sources of the

Galaxy gravitational sky, for which the quadrupolar approximation holds, like periodic sources.

In case of a binary system, $h_{\rm max}$ in Eq. (3) can be estimated by relating gravitational luminosity (Peters & Mathews 1963) and its flux (Press & Thorne 1972) to the perturbation h itself, obtaining

$$|\delta\phi| \approx (7.3 \times 10^{-4}) \left(\frac{10^4 \rm pc}{\Lambda_g}\right) \frac{m_1 m_2}{(m_1 + m_2)^{1/3}} \left(\frac{f_g}{1 \rm Hz}\right)^{2/3} \text{arcsec}, \quad (4)$$

where f_g is the gravitational frequency emitted, and masses are expressed in unit of solar mass.

At $f_g \leq 10$ Hz we find unevolved binary systems (UBs). In the narrower band from $10^{-4.6}$ Hz to $10^{-3.9}$ Hz are the contact binaries of the W UMa type and from $10^{-4.9}$ to $10^{-3.4}$ Hz – the cataclismic binaries (CBs) (Hils et al. 1990). For these three types of binaries, we know their spatial density and that they are very stable (the loss of energy due to gravitational radiation is negligible).

Other binary systems are double compact stars, whose spatial density is uncertain: neutron star binaries (systems with frequency above 300 μHz), systems with a black hole and a neutron star companion (we do not observe any in the Galaxy), compact white dwarf binaries (CWDB), with frequencies from 100 μHz to 100 mHz. Contrary to before, the evolution of this second group and the frequency emitted is dominated principally by loss of angular momentum driven by gravitational radiation.

A first evaluation of Eq. (4) (it is dimensionally proportional to $M^{5/3} f_g^{5/3}$) tells us that we can exclude sources with frequencies from 10^{-7} to 10^{-5} Hz: even for masses of about 100 $M_\odot$, the effect is not in the μas range that we require for *Gaia*. Therefore we have considered $f_g \geq 10^{-4}$ Hz and orbital periods shorter than 5.5 hours. There are no known UBs, W UMa stars or CBs that satisfy this criterion. For example, the primary in V Pup has 16 $M_\odot$, but a frequency which is too low. To the other extreme, the frequences of known CBs are not sufficient to cover low masses: a deflection of ~ 1 μas would require 10–20 solar masses, which does not appear to be possible, as CBs are usually composed of a white dwarf and a regular dwarf of spectral types G, K and M.

The other candidates, i.e. isolated neutron stars, can still be suitable for astrometric detections. Pulsar's rotation, diverging from

Table 1. $\delta\phi$ calculated from Eq. (6) for neutron stars at a distance of about 200 pc.

PSR	P(s)	Λ_g(km)	$\delta\phi$(arcsec)
J 0437-4715	0.0057	852	2.7×10^{-5}
J 1730-2304	0.008	1210	9.6×10^{-6}

the axial symmetry, generates $h_{\rm max}$ (Thorne 1987) that produces the deflection:

$$|\delta\phi| \approx 2.5\times10^{-21}\left(\frac{100\,\mathrm{pc}}{\Lambda_g}\right)\left(\frac{f_g}{10\,\mathrm{Hz}}\right)\ \mathrm{arcsec}. \qquad (5)$$

A less rigorous estimate of $|\delta\phi|$ can be obtained by substituting $H \approx MR^2\Omega^2$ (in quadrupolar approximation) to Eq. (3):

$$|\delta\phi| \approx \frac{3}{4}\pi MR^2\Omega^3 \approx (1.2\times10^6)\pi^4 MR^2 f_g^3 \quad \mathrm{arcsec}, \qquad (6)$$

where R is the dimension of the SG.

2.1. First scenario

First we consider neutron stars both as single sources (e.g. Geminga, see Table 2) and companions in binary systems (0021-72A, Table 3).

The mutual proper motion of the SG and SL would increase the possibility that SL's photons cross the buffer zone, especially during the five years of the *Gaia* mission. Here, we have the extra requirement of good alignment of the SG with an appropriate (dense) star field. Geminga, for example, has a relatively large proper motion (about 0.15 arcsec/year), therefore the probability that a background SL could graze its buffer zone is increased. However, the angular size of the buffer zone still depends on the observer's distance to the SG. As we can see from Table 1, the effect is important for rotational periods less or equal to about 0.02 s; this means that the buffer zone is very small (about 3000 km); even if we estimate the effect through Eq. (6) (Table 2), the buffer zone is still quite small (about 30 000 km). The same discussion is also valid for the binary 0021-72A (Table 3), whose distance of 2500 pc is just too large.

Table 2. $\delta\phi$ calculated from Eq. (5) for neutron stars at a distance of about 200 pc.

PSR	P(s)	Λ_g(km)	$\delta\phi$(arcsec)
Geminga	0.237	3.4×10^4	6×10^{-4}
1451-68	0.227	3.4×10^4	6.8×10^{-4}
J 0633+1746	0.237	3.5×10^4	6×10^{-4}
B 0950+08	0.253	3.8×10^4	4.9×10^{-4}
B 0809+74	1.29	1.9×10^5	3.7×10^{-6}
B 0959-54	1.43	2.1×10^5	1.73×10^{-6}
B 1133+16	1.18	1.7×10^5	4.8×10^{-6}
B 1929+10	0.226	3.4×10^4	6.92×10^{-4}

Table 3. Neutron star binaries with $P_{\rm orb} \leq$5.5 hours.

CWDBs	f_g(Hz)	$M_1/M_\odot$	$M_2/M_\odot$	Λ_g(km)	$\delta\phi$(arcsec)
0024-72J	1.92×10^{-4}	1.4	1.4	1.56×10^9	6.7×10^{-8}
0021-72A	1.04×10^3	1.4	1.4	2.9×10^8	1.12×10^{-6}
1721-1936	8.9×10^{-5}	1.4	1.4	3.36×10^9	1.9×10^{-8}
1748-2446A	3.1×10^{-4}	1.4	1.4	9.6×10^8	1.4×10^{-7}

2.2. Second scenario

Our second, more promising scenario, focuses on white dwarfs. Binary systems like CWDBs (Table 3) with periods < 0.5 h provide encouraging numbers. From the studies of Iben & Tutukov (1984) on the evolution of close binary systems leading to the formation of Type I supernovae and CO white dwarfs, two interesting scenarios, A and B, can be inferred. These include close CWDBs with $P_{\rm orb}$ of 8–12 min (case A) and with $P_{\rm orb}$ of $\sim$ 30 min (case B). Both scenarios consider separations in the range 1.7–6 $R_\odot$ and masses of $\sim$ 3–9$M_\odot$.

The existence of multiple stellar systems with white dwarfs and cataclysmic binaries, such as the quadruple system G 107-69/70 (an unevolved binary with $P_{\rm orb}$ = 0.94 years and a CWDB with $P_{\rm orb}$= 20.5 years, Harrington et al. 1981), PU Vul (an M giant orbiting

Table 4. Examples of candidate CWDBs. None of these systems fit the proposed *Gaia* detection model; however, they provide actual cases of candidates yet to be confirmed as suitable sources for an astrometric search of gravitational waves.

CWDBs	f_g(Hz)	$M_1/M_\odot$	$M_2/M_\odot$	Λ_g(km)	$\delta\phi$(arcsec)
G61-29	7.12×10^{-4}	0.6	0.6	4.2×10^{8}	1.5×10^{-7}
L870-2	1.5×10^{-5}	0.7	0.7	2×10^{10}	0.3×10^{-9}
PG 1101+364	1.6×10^{-4}	0.5	0.5	1.7×10^{9}	8.91×10^{-9}
WD 0957–666	2×10^{-5}	0.7	0.7	1.5×10^{10}	4.9×10^{-10}
1713+332	2×10^{-5}	0.5	0.5	1.5×10^{10}	2.8×10^{-10}
1241–010	7×10^{-6}	0.5	0.5	4.3×10^{10}	4.8×10^{-11}
1317+453	5×10^{-6}	0.5	0.5	6×10^{10}	2.76×10^{-11}
2331+290	7×10^{-5}	0.5	0.5	4.3×10^{9}	2.25×10^{-9}
A	2.7×10^{-3}	0.7	0.7	1.1×10^{8}	1.7×10^{-6}
A′	2.7×10^{-4}	0.8	0.6	1.1×10^{9}	4.2×10^{-8}
B	4.2×10^{-3}	0.4	0.4	7.1×10^{7}	1.4×10^{-6}
C	2.7×10^{-4}	0.9	0.25	1.1×10^{9}	1.8×10^{-8}

a WD–WD binary, with $P_{\rm orb}$ = 13.45 years, Kolotilov et al. 1995), CH Cyg (a triple system made of a binary with a giant primary, a WD secondary and orbital period $P_{\rm orb}$ = 756 d and a G–K dwarf orbiting with $P_{\rm orb}$ = 14.5 years, Munari et al. 1996), allow us to sketch a realistic detection model for *Gaia*.

3. A DETECTION MODEL FOR GAIA

In our model we consider a triple star made of a CWDB (the SG) with $P_{\rm orb} \le 0.5$ h (i.e. $\Lambda_g \le 1.8$ AU) and, for example, a red giant companion or some other star (the SL) sufficiently brighter than the CWDB to give *Gaia* the ability to make precise measurements of the SL, even if the system is at significant distance from the Sun.

We point out that white dwarfs are difficult to observe beyond 200 pc, and the situation becomes increasingly complex when trying to detect their binary nature. Nevertheless, it would certainly be worthwhile initiating a specialized observing campaign devoted to the discovery of CWDBs in multiple systems. As for the spatial density of CWDBs, we adopt the estimation by Robinson & Shafter (1987) of $N_{\rm CWDB} = 3 \times 10^{-5}$ pc^{-3}.

What we are trying to detect are anomalies in the SL motion which can be attributed to the SL orbiting a CWDB, the source of gravitational waves (the SG). The critical constraints are with the choice of the SL: (a) it must be possible to go to a considerable distance to ensure that enough volume is probed around the Sun to yield finite chances of discovery, * (b) it must be of sufficient brightness for *Gaia* to make most accurate measurements, and (c) it must have an orbital period compatible with the way the satellite surveys the sky (observability).

The choice of CWDBs as SG is justified by the fact that they have a large action radius for the effect, i.e., an angular buffer zone $\phi_{\rm on}$–$\phi_{\rm off}$ of relatively large size. $\phi_{\rm on}$ and $\phi_{\rm off}$ are the angles, counted from an arbitrary point on the SL orbit in Fig. 1, delimiting the effect *on region*. The deflection of photons from the SL starts as the source grazes the Λ_g region and continues (in the sense that keeps changing) until it reaches the boundary with the "off region". As the projection of the SL path is close to the gravitational radius of the SG ($r_{\rm grav} = 2{\rm M}/{\rm c}^2$), the weak-field approximation is not valid anymore and it becomes difficult to predict what happens to the incoming SL photons. If $M_{\rm CWDB} = 1 M_{\odot}$, then $r_{\rm grav} \simeq 2 \times 10^{-8}$ AU, and we can adopt $r_{\rm on} = 2 \times 10^{-6}$ AU as the linear distance of SL from SG at which the weak-field approximation holds again.

Let us now evaluate how the SL motion relative to SG is compatible with *Gaia*'s observations. *Gaia* will scan the sky several times during its 5 years of mission lifetime: it will go through the same object at least six times a year, a sampling time τ is about two months.

In the hypothesis that SL moves with circular uniform speed around SG and nearly tangent to our line of sight ($i \sim 90°$), we can estimate, via Kepler's third law, the conditions of observability utilizing the duration of the effect T and $\Delta\phi = \phi_{\rm off} - \phi_{\rm on}$ as a function of the orbital parameters of the tertiary component orbiting the CWD binary.

In this scenario, the SL Keplerian orbital motion projected on the celestial sphere will look like an angular harmonic shift with

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* For a distance limit of 200 pc the adopted spatial density for CWDBs yields $\sim$ 1000 candidates.

amplitude $\theta(t) = \theta_0 \cos \omega t$, where θ_0 is the maximum amplitude given as

$$\theta_0'' = \frac{a_{\rm SL}\,({\rm AU})}{d\,({\rm pc})}$$

and where $a_{\rm SL} = a - a_{\rm SG}$ is the SL-barycenter separation of the system.

With simple passages, using

$$M_{\rm SG}\, a_{\rm SG} = M_{\rm SL}\, a_{\rm SL}$$

we have

$$a = a_{\rm SL} + a_{\rm SG} = a_{\rm SL}\left(1 + \frac{M_{\rm SL}}{M_{\rm SG}}\right).$$

With the adopted assumptions ($M_{\rm SL} = 2M_\odot$ and $M_{\rm SG} = 1M_\odot$) we get $a_{SL} = a/3$. Therefore, even in the case of a very close triple system ($a \simeq 1$ AU) at a distance of 200 pc, the amplitude of the Keplerian perturbation will be $\theta_0 \simeq 1.6$ mas, above the sensitivity threshold for a single observation made by *Gaia*. We can then assume the orbital motion can be subtracted out and concentrate on the relativistic effect in Eq. (4), which will produce a modulation of ~1 μas superimposed to the orbital motion. For a given orbit, this perturbation can be observable for a period T:

$$T \simeq \frac{1}{2\pi}\left(2 \arcsin \frac{\Lambda_g}{a}\right) P, \tag{7}$$

which corresponds to the transit of the SL through the buffer zone (cf. Fig. 1). [P is the orbital period of the SL.]

Now the ratio P_G/P gives the number of orbital periods that *Gaia* will observe, being $P_G = 5$ years the mission life-time. The total duration D of the effect, i.e. during the whole mission, will be

$$D = T \cdot \frac{P_G}{P}$$

where T/P is the probability to observe this effect during a single transit.

The total number of observations made by the satellite will be on average

$$n_G = \frac{P_G}{\tau} \simeq 30$$

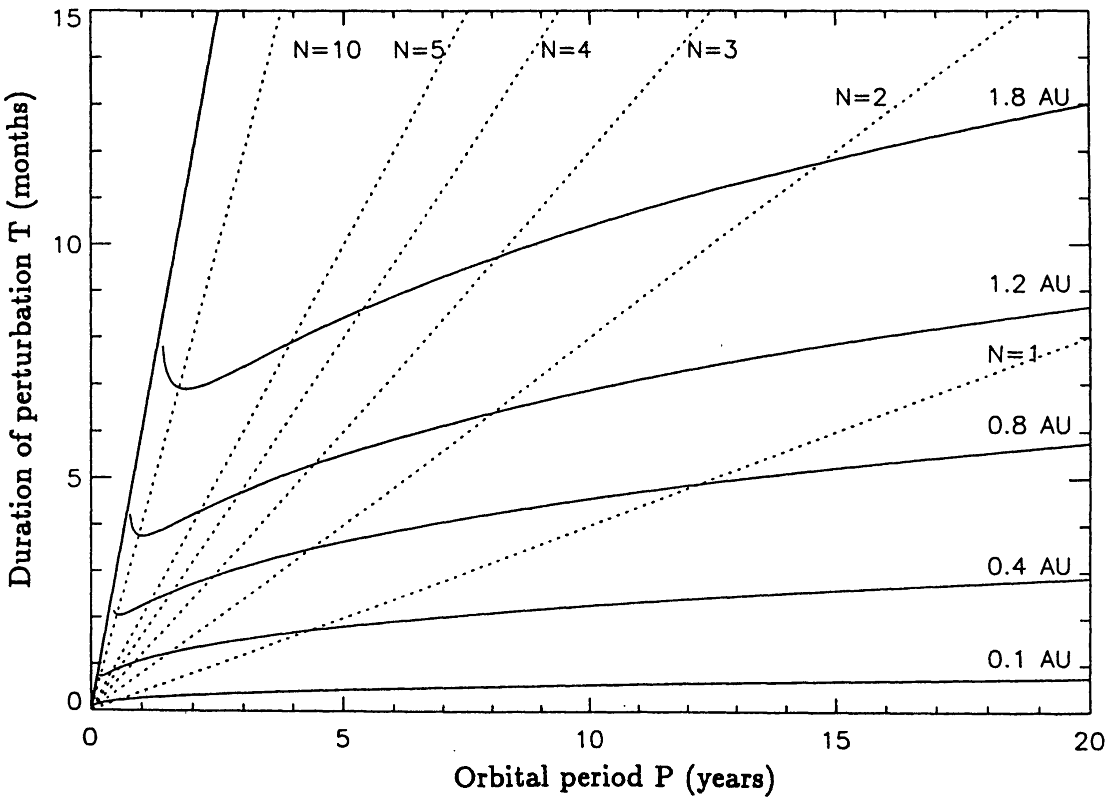

Fig. 3. Duration of perturbation as a function of SL orbital period for systems with $\Lambda_g = 0.1, 0.4, 0.8, 1.2$ and 1.8 AU (solid lines) respectively. Intersections with the dotted lines represent the longest orbital period that can produce, on average, at least N successfull detections of light deflection by the near zone associated with the SG. The region on the left of the solid straight line contains the systems having orbital radii smaller than Λ_G, so that SL is always within the buffer zone.

as the actual number depends on the ecliptic coordinates of the target.

For each observing epoch (once every 2 months), we can expect either a positive or negative outcome in a random fashion, i.e., the observation of the effect follows a binomial distribution. Therefore, the mean number of transits made by the satellite, corresponding to a favorauble observation of the effect, is *:

* This result holds if we assume that orbital phase is averaged over the total number of potential targets, i.e., approximately 1000, as estimated previously.

$$N_{\rm oss} = p_{\rm st} \cdot n_{\rm G} = T \cdot \frac{1}{\tau} \cdot \frac{P_{\rm G}}{P}. \qquad (8)$$

As shown in Fig. 3, by changing $N_{\rm oss}$ this relationship generates straight lines whose intersections with the curves corresponding to Eq. (7) indicate the *largest* orbital period which produces, on average, $N_{\rm oss}$ successful detections of the light-deflection effect.

Let us consider two extreme cases in Fig. 3: for $\Lambda_g = 0.1$ AU, *Gaia* makes at least one positive observation of the effect up to $P \leq 0.5$ year, and up to tens of years for systems with $\Lambda_g = 1.8$ UA (square); in the second case, *Gaia* should perform at least five observations for periods up to $P \approx 4$ yr.

ACKNOWLEDGMENTS. We thank J. Miller, F. De Felice and B. Bertotti for their helpful comments.

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