Doing Mathematics Outdoor: The Role of Bodily Experiences in Modelling Tasks

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Doing mathematics outdoor: the role of bodily experiences in modelling tasks

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Abstract
This paper delves into the significance of bodily experiences in the context of outdoor STEAM activities. It features a case study involving three tenth-grade students who engage in outdoor tasks–which are part of an outdoor STEAM trail–using the MathCityMap app. The research focuses on the students' emerging strategies when dealing with two modelling tasks by integrating the modelling cycle phases with a perspective on embodiment that encompasses learning as navigating mathematical places. By intertwining these theoretical stances, the analysis uncovers the students' strategies and seeks to describe their orientations during the experience. Despite the students do not reach the expected solutions, they bodily engage with the modelling tasks in meaningful ways, employing an array of various strategies. Trying to overcome the separation between domains, classically emphasised in the modelling context, we show how bodily engagement operates at their intersection and discuss the relevance of our approach for STEAM activities.

Keywords
Bodily experiences, MathCityMap, mathematical place, outdoor STEAM trail, modelling tasks

Declaration

Ethical statement
Liceo Scientifico "Gobetti" - Torino granted approval for this research.

Data availability statements
The data that support the findings of this study are available from the corresponding author upon request.

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Conflict of interest
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1. Introduction and literature review

When you are walking in the streets of your city, do you happen to look at the surroundings and see mathematical relationships, shapes, and structures? Without mathematics, very little of what is around us could have been accomplished, much less imagined or built. Nevertheless, it is not very common to think mathematically of what surrounds us. Outdoor mathematical activities open room for entering the world of mathematics by providing us with the opportunity to deal with mathematical questions that are entangled with the world we inhabit. More generally, outdoor activities provide direct experiences with the real world (Moffett, 2012) and any structured learning activity outside the classroom can create occasion for outdoor STEAM learning (Kendall et al., 2006).

In this paper, we propose a STEAM approach where mathematics outdoor modelling activities are designed and presented through a digital app, MathCityMap. According to Psycharis (2018), in a STEAM approach, methods and contents from the different STEAM domains are connected through transdisciplinarity (Kim & Bastani, 2017), similar to real-world situations, objects or problems (e.g., architecture needs art, mathematics, engineering, science and technology). Additionally, showing example of outdoor modeling and teaching how to model outdoors can enhance learners' understanding of mathematics, STEAM disciplines, their real-world applications, and foster motivation for studying these subjects (Blum et al., 2007; Maass et al., 2019). In particular, we examine the role of bodily experiences in such activities by presenting and analysing a case study involving a group of grade-10 students. To introduce this aspect, we engage the reader in recalling a well-known experience. Imagine you are walking by the streets that you travel each day to school or to your workplace: you probably do not have to think where to turn left, or where is the entrance of the building you are trying to reach. You also probably know how much force to apply to the door to open it, even though you do not know its exact weight, or you would be unable to describe it. And that is completely fine since your only interest is to leverage it enough to open it. But what if you were asked how tall the door is, or how many windows the building has in the north façade? You will need to inhabit that place in new ways and engage differently with it to be able to answer. Probably you would be comparing your height with that of the door, and you would be looking for reference points and regularities to guess the number of windows. This example aims at raising some ideas that are central in our paper. First, our orientation in the world we inhabit changes, and is always shifting, as we are challenged to think about it through mathematical problems or scientific questions. Second, ~~second,~~ inhabiting the world engages our body as a whole, including perceptions, affective sensations, previous knowledge, and experiences.

This study addresses the scarcity of research concerning the role of the body in outdoor STEAM activities. It specifically explores how students engage with mathematical concepts in outdoor settings and shows how focusing on their bodily experiences enhances our understanding of outdoor mathematical modelling. In the following section, we introduce the theoretical framework underpinning our research, combining a perspective on outdoor modelling with the concept of *mathematical place*, which informs our understanding of embodied learning. Section 3 outlines the research context and methodology we employed. In particular, we explain the technology used for outdoor STEAM activities, the specific tasks considered, and how we conducted the teaching experimentation, including participant details, methodological choices, and data collection. Our STEAM approach places a strong emphasis on mathematical modeling and technology use as prerequisites for participating in our outdoor STEAM activities. Additionally, these activities encourage a unique form of embodied learning as students engage in diverse environments, distinct from the typical classroom setting. In Section 4 we focus on a case study that has involved a group of grade-10 students in outdoor modelling activities, presenting selected episodes and their qualitative analysis. In Section 5 we outline our discussion, and, in the final section, we provide our conclusions and preview future work.

2. Theoretical framework

2.1 Mathematical modelling outdoors

Outdoor mathematics is an active learning methodology for which a set of mathematical activities are performed in an outdoor environment (Kennard, 2007). Such methodology emphasizes experiential learning, and the integration of mathematical concepts and their practical applications within real-world contexts. Students are no longer confined to the classroom, sitting at a desk, rather they find themselves moving, walking and looking around, in a less familiar (and restrictive) context. One possibility that allows students to be placed in this situation is the *math trail* (Blane & Clark, 1984). This is a mathematical walk through the environment in which mathematical problems are discovered and solved using real objects along the way (Jesberg & Ludwig, 2012). Usually, a map shows the places where such mathematical problems (or tasks) can be discovered, discussed and solved. Math trail tasks can be solved exclusively on the spot, through active mathematical actions, e.g., measuring or counting (Ludwig & Jablonski, 2019). Solving math tasks outdoors requires students to tap into their mathematical modelling competence, defined as

> “the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation, as well as the ability to analyse or compare given models by investigating the assumptions being made, checking properties and scope of a given model etc.” (Blum et al., 2007; p. 12).

In particular, as students solve math tasks outdoors using real-world physical objects, they are engaged in the process of transferring the external context into the mathematical world, and of integrating practices of scientific inquiry, technological and engineering design (Johnson, 2013, p. 367). Besides, depending on the task, students need to apply creative thinking to identify and connect real-world information to in-class learnings (Land, 2013). Therefore, they can experience a STEAM approach. In fact, modelling is a cyclical process in which, starting from a real problem, the student structures the problem according to mathematical concepts and solves it, and then translates the mathematical solution in relation to the real situation. It is therefore an extra-mathematical domain, a mathematical domain and a mapping between them that leads to results that are translated into the extra-mathematical domain (Borromeo Ferri, 2006).

Among the most comprehensive schematic descriptions of the modelling cycle is the one introduced by Blum and Leiß (2007). Figure 1 shows a modified version of the modelling cycle that emphasises these transfer processes specifically in the context of an outdoor math task. Students must first engage in understanding and structuring/simplifying the outdoor context. Then they must mathematise to move from the real object to the mathematical model. After using the necessary procedures to solve the mathematics, the results must be transferred back to reality, i.e. the interpretation of the results. These then have to be validated.

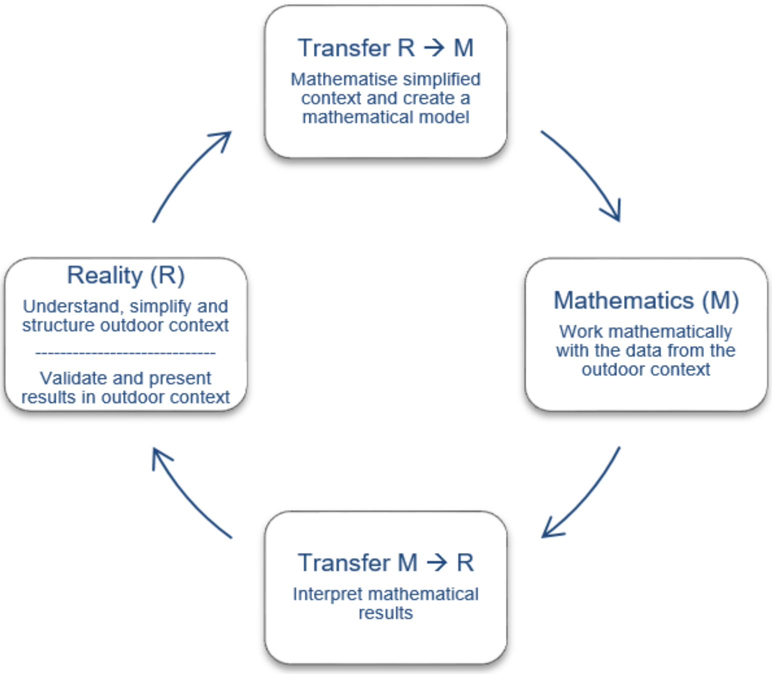


Figure 1: Modelling cycle for the outdoor context (from Jablonski & Bakos, 2022, p. 4)

While this framework mainly refers to mathematical modelling, it is true that the process of modelling is relevant in many different disciplines and part of STEAM curricula (González-Martín et al., 2021). We then take the cycle developed by Jablonsky and Bakos (2022) as our main reference for the purposes of this paper in the understanding that, in a STEAM approach, all the processes involved in the cycle pertain to the different disciplines involved and are infused with the specificities of each of those.
A peculiar aspect of modelling activities that are really integrated into the environment which they refer to, like in the case of math trails, is that the students can explore the modelling context also with their bodies. While in paper and pencil tasks proposed in the classroom, such context might be only evoked, imagined or observed through images, in the case of math trails it is the setting in which mathematical activities unfold. As Kelton and Ma (2018) state, "mathematics learning environments strongly shape–and are meaningfully transformed by–the activities of learners" (p. 177-178). We agree with this scholars' perspective, and we are interested in the processes of transformations that are at play in an outdoor context, where–we argue–the students are engaged in learning much differently than inside the mathematics classroom. In the following, we thus elaborate on the idea of learning environments in relation to embodiment theories in the context of mathematics education.

2.2 Mathematical places and embodied experiences

In making sense of graphical and symbolic representations created and used by children in modelling motion activities, Nemirovsky (2005) proposes that "(u)sing mathematical representations is not a matter of holding correspondence between an outside and an inside, but that of inhabiting symbolic places that embrace both the symbol-user and the world in which he or she lives" (p. 45). Places are distinct from locations in that they are shaped by the bodies that inhabit them: "one is oriented toward something while at the same time one is being oriented by that something" (Nemirovksy, 2005, p. 50). To navigate the notion of "mathematical place", for the purposes of this paper, the concept of “orientation” (Nemirovksy, 2005) is crucial, for that each time we are orienting ourselves, the place as whole matters, and our entire body (not just our eyes, or our ears) are guided by and responsive to the environment. We use the idea of “mathematical place” to account for mathematical activities happening in a different location than the mathematics classroom, which are incorporating the user and the world that is explored in mathematically significant experiences. We propose that this perspective also allows us to reconsider the modelling cycle along the lines of experiential happenings, beyond the theoretical model, and in relation to locations and environments as they acquire meaning for students in their activity and transform with learners. As we inhabit places with our bodies by moving and being moved in shifting orientations, the role of perceptuo-motor experiences in the teaching and learning of mathematics is a central aspect. Several researchers have investigated the role of gestures in communicating mathematical ideas (e.g., Alibali & Nathan, 2012), the multimodality of learning (Ferrara, 2014) and the relationships between movement and thinking (e.g., Roth & Maheux, 2015).

Abrahamson and Bakker (2016) “are particularly interested in understanding relations between physical actions and conceptual learning as these relations bear on theoretical and pragmatic problems in the research field of mathematics education and perhaps beyond into other STEM domains”. Their work is oriented towards the development of design principles informed by embodiment theories, focusing specifically on examining the sensorimotor schemes that emerge in the analysis of students’ movements and their interactions with tools. We share with these authors a serious concern for individual and interactional mechanisms that are at play in embodied STEM learning, especially those activities that include interactive technologies.

Gestures studies, on the other hand, highlight the cognitive functions of specific gestures in mathematical discourse (e.g., Arzarello et al., 2009). In the context of math trails, which are of interest for this paper, Jablonski (2021) has discussed these functions in relation to steps of the aforementioned modelling cycle. She has investigated how specific functions attributed to gestures (i.e., activating, manipulating, structuring and exploring) can be related to the *simplifying* or *structuring* steps in modelling processes. This research is one of the few that addresses the role of the body in outdoor mathematical activities and highlights very well the relevance of considering an embodied approach to mathematical modelling activities outdoors. Nevertheless, it still is somehow limited in considering solely gestures, that is, a small part of the human body that is actually recruited in such activities. Moreover, it seems to account for how different students use gestures to support their learning process, thus posing an emphasis on the individual, rather than studying how these gestures propagate or transform in the interaction. As a last point, it analyses the role of gestures in a way that emphasises the body-mind distinction, taking them as instantiations of cognitive or epistemic actions.

For the purpose of our paper, we align with a more non-dualist standpoint, which allows us to consider the mathematical activity as multimodal and strive to conceptualise the doing of mathematics as fundamentally entailing perceptuo-motor activities and focus on whole-body collaborative interactions (e.g., Kelton & Ma, 2018). We especially share with Kelton & Ma (2018) an interest in attending to bodies as dynamic, heterogeneous multiplicities, and considering embodied experiences in the specific context of outdoor activities.

To summarise, there is little research focusing on the role of the body in outdoor mathematical activities, and this study aims at contributing to this gap. We investigate the learning of mathematics in the outdoor context of a math-trails, and we ask: *How do students inhabit mathematical places in the context of modelling activities outdoors? How do their bodily strategies enable us to enrich our understanding of outdoor modelling?*

3. Research context and methodology

3.1 Technology: MathCityMap

MathCityMap[1] is a technology that enables teachers to implement smartphone supported mathematical trails in mathematics classes. It is a two-component system: a web portal – the teacher's workspace where tasks (and trails) can be created; a smartphone application, which allows students to work on a math trail. The MathCityMap app uses GPS to display the tasks' position on a map and presents the task through the picture of an object (e.g., a statue). Each task is designed to be solved only in that position (e.g., by taking measures on the spot). The MathCityMap app offers up to three suggestions, which help the user to undertake a solving strategy for the task. Moreover, it gives automatic (right/wrong) feedback on the solutions students provide. For our study, we used MathCityMap to design a math trail consisting of 10 activities located in the city centre of Turin[2], designed for grade-10 students. The mathematical topics touched on by the tasks included in the trail concern: Numbers (percentages, proportions, sequences, modular arithmetic); Geometry (quadrilaterals, ellipses, cylinders, area, volume, slope, angular coefficient, central symmetry, midpoint, similarity, Thales' Theorem); Combinatorial Calculus and Descriptive Statistics (permutations, frequency, mode). Each tasks require to model a situation where a building, architecture or real-world object is involved. The tools required to carry out the trail are: paper, pen, measuring tape, calculator, empty plastic water bottle and chronometer.

3.2 Task analysis

In line with Haas and colleagues (2021), we have designed an outdoor STEAM trail. In fact, the methods required to solve the tasks are scientific-mathematical; the concepts involved are mainly mathematical ones; but the objects on which the tasks are designed are real-world objects, with artistic-architectural facets that have to be examined; finally, using the technology (the MathCityMap app) is a requirement to access these tasks and walk the trail. Out of the 10 tasks that the trail is composed of, we focus on two that concern the estimation of direct and indirect measurements.

The "The tip of the sword" (https://mathcitymap.eu/en/portal-en/#!/task/2967187) asks the students to estimate at what distance from the ground is the tip of the sword held by the statue of Carlo Alberto (see here for the sample task solution: https://urly.it/3vd08). The first provided link allows the reader to see the English translation of the task the students faced. Moreover, Table 1 shows the hints given inside the app for the "The tip of the sword" task. For example, the second hint, given as an image, suggests comparing specific heights (one which is unknown, one that is measurable) while looking at them from a distance (Table 1).

Table 1. The hints provided inside "The tip of the sword" task.

Hint 1	**Hint 2**	**Hint 3**
You should be able to compare the height of the statue to another length/height that you		The observed heights that you can measure are in proportion to the real heights.

[1] See: https://mathcitymap.eu/en/ ; project developed at the Goethe University of Frankfurt (Germany).

[2] The math trail "Matematica in Centro" (*Mathematics in the city centre*) can be found (in Italian) on the MathCityMap app using the code 1810212.

know or can measure. For example, the height of one of your peers.		

We point out that the result that is given in the solution example is what we, the designers of the task, obtained with our measurements. For this kind of task, MathCityMap's standard is to set the answer as a range of values (e.g., an interval as in Figure 2), distinguished between closer to the designer's solution (deviating by ±3% from all the measurements found by the designer, displayed in green in Figure 2) and permissible (deviating by ±7% from all the measurements found by the designer, displayed in orange in Figure 2). This is done because the user does not necessarily have the same precision as the designer in taking the measurement, nor does she use the same tool. Therefore, upstream errors must be considered, and the solution cannot be an exact number but is given as an interval. Figure 2 represents the range of values we have set for this task.

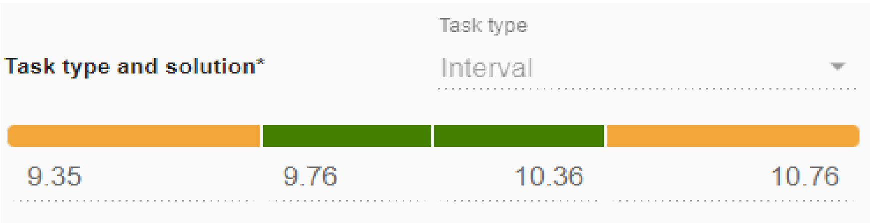


Figure 2. The solution interval of the "The tip of the sword" task

The second task "Estimating the number of cobblestones" (https://mathcitymap.eu/en/portal-en/#!/task/3467190) requires, as suggested by its title, estimating the number of cobblestones in a given area of a square (see here for the sample task solution: https://urly.it/3v2kd).

3.3 Data analysis and participants

In June 2022, during the last week of the Italian school year, we experimented with this trail involving 30 grade-10 students from a high school in Turin. These students constituted a class doing extra mathematics hours, in which they are engaged in laboratorial and interdisciplinary STEAM activities. They were divided into 10 groups of 3 members each and worked in cooperative learning (Johnson & Johnson, 1999). In fact, each member of the group had a specific role: a measurer, whom is responsible for measurements; a calculator, whom notes down the measurements taken and performs the calculations; a technologist, whom manages the app (i.e. using GPS, s/he leads the group to the location of the object; s/he evaluates, together with the group, whether to consult the hints; s/he enters the answers in the app). The students were given 90 minutes to run the trail. The MathCityMap app distributed the tasks among the different group, so that two groups did not work on the same task at the same time. The students were asked to spend no more than 15 minutes solving each task, so that they could try all the tasks on the trail.

The groups were formed by the class teacher, at our request to make them as heterogeneous as possible with respect to the school's level of performance in mathematics. Following the teacher's suggestion, we identified a group consisting of three upper-middle level students, whom we videotaped as they went through the trail. For this group, the tasks described in the previous section are the second and the last tasks they encountered while running the math trail. We choose these two tasks, which are presented respectively in the first and second episode in the analysis section, for two reasons. First, because we are interested in analysing cooperation within the group and bodily engagement in different moments of the trails. Second, because the two tasks request to measure a height of an object the students cannot physically reach and to estimate an area by indirect calculation: we assumed that both would involve rich (and different) embodied strategies.

Transcripts of the video recordings of the group activity were made. We systematically reviewed all videos and transcripts for each episode and broke them into sections, wherever there seemed to be a natural change in focus or strategy; for these, we developed summaries and identified key moments. In the following, extracts of dialogues together with the descriptions of students' multimodal engagement and our analysis in the form of commentaries, are presented. We draw on micro-ethnographic studies (Streek & Mehus, 2005) to present the data coming from the experiment, providing transcripts of the interactions among the members of the focus group and detailed descriptions of their activity. Focus is on bodily movements that are expressions of specific strategies for dealing with the tasks and how these matter in the students' meaning making. In line with Nemirovsky (2011), our aim is "to generate rich, evocative descriptions of lived experiences" to grasp the nature of the participants' experience in the activity. For this paper, we addressed the issue of validity by adopting a methodology of data collection that allowed capturing rich data and then analysing them through methodologies that are coherent with the rationale of the study. Trying to overcome the separation between the mathematical and the reality domains that are classically emphasised in the modelling context, the analysis seeks to show how bodily engagement operates at the intersection, and rather than becoming a bridge from one world to another, is the compenetrating of the two. In doing so, we try to underline the specificity of the outdoor context, the unexpected and destabilising events that the students come to inhabit.

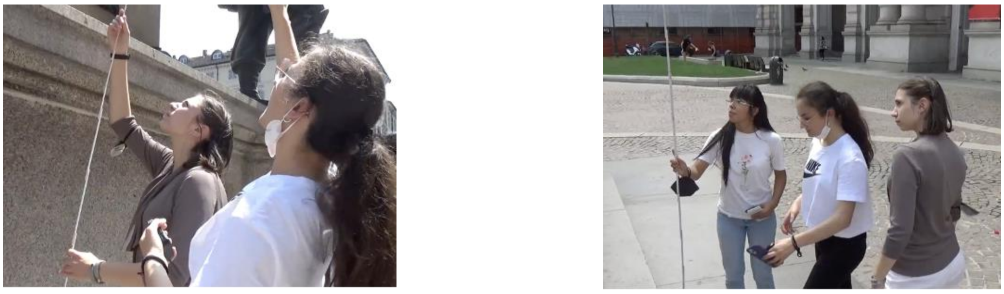
4. Selected episodes

In this section, we present two episodes that allow us to examine the embodied dimension of outdoor modelling activities. The group is composed of three girls: Ari, Bea and Carla (these are pseudonyms). Ari is playing the role of the secretary, Carla is the measurer, and Bea is the navigator. One researcher (the first author, indicated as R in the transcripts) is following them during the trail with a camera and is video recording their interactions in solving the tasks.

4.1 Episode 1- The tip of the sword

Excerpt 1

The group is solving the "The tip of the sword" task (see section 3.2 for the details). After reading the task, the group starts approaching the problem by discussing how to measure something they cannot reach. They look for information all around the base of the statue, and Carla starts using the measuring tape along the stairs. Bea and Ari decide to open the first hint inside the app, which suggests they could confront the height they wish to measure to the height of a person or object they can measure. Bea wishes to measure Carla's height but her two mates are not convinced and move closer to the statue, opening the metre so that it is now like a stick of 2-metres in length. Carla, then Ari, uses the metre vertically, close to the base of the statue, trying to use it to measure some parts (Fig. 3a and 3b).



a b

Figure 3. a) Carla and b) Ari holding the metre vertically in relation to the statue

Bea is instead more interested in exploring the comparison with a mate's height, as suggested by the first hint. She tries to gather her group mates' attention and asks:

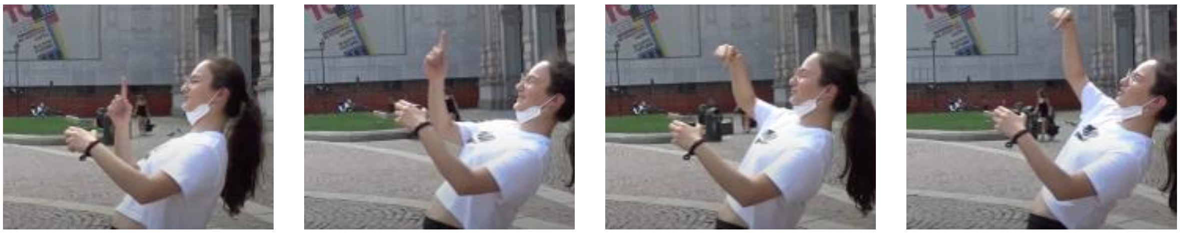
Bea: You know painters, who do that thing with their finger, with proportion?

Ari: Oh, yes

Bea: Here, you stay there (*to Ari, while she moves further from the statue*)

Ari: Since that is two metres… (*referring to the measuring tape she is holding*)

Bea Ok, then… (*she opens the measuring tape, then closes it again*) How is it? Like that? One... (*closes the left eye, and extends the right index finger in front of her*, Figure 4a, *while with her left hand is holding the tape*). two... three… four (*moves the index vertically downward, then upward, then downward again, each time starting from approximately the position at which the previous movement the tip of her index finger was*, Figure 4b-d). Four times!



a b c d

Figure 4. Bea uses the painter's method

Commentary

At the very beginning of the episode, according to the modelling cycle, the girls are understanding, simplifying, and structuring the outdoor context. They are changing their way of looking at the statue and their orientation concerning the task, with respect to their first interactions, which involved proximity to the statue. Bea now moves further from the statue, while Ari stays close to it, following Bea's request.

Interpreting the strategy applied by Bea, "the painter's method", from the point of view of the modelling cycle, we see that she has identified a model (applying a proportion) in the Mathematical domain and put it into action (transfer R→M). Bea first seems undecided about whether the measuring tape is actually useful, she opens it but then closes it right after and uses the length of her index finger to make sense of the ratio between the height of her mate and the unknown height that is to be measured. She seems to position her finger so that it fits Ari's height seen from a distance then replicate this measure vertically several times to get to the tip of the sword. This strategy brings in two essential features of a mathematical place: 1) we inhabit a place with our body and 2) our own past experiences create orientations to explore mathematical places. In fact, we observe that Bea is recalling a method she has probably already seen, that of the painter, and this changes her orientation towards the issue of measuring; she is no longer looking for ways to directly measure, but, following the hint, looks for ways to compare things. This includes moving far from the statue and using her body (instead of the measuring tape, which she readily discards) to compare heights. She engages in a controlled movement, where the hand moves vertically from the ground, while her torso moves back each time, resulting in her back being quite slanted at the end. While this is quite problematic from the point of view of the mathematical model she might be applying here, it raises the issue of what is relevant in this case for her in this phase: unable to see all the statue in a glimpse of an eye in the position she is standing, she accommodates with her whole body the relationships she wants to maintain using her hands. In an effort to perform the movement vertically with the hand, the back bends and she looks uncomfortable but focused on the objective. This approach is meaningful to Bea, and will propagate among the other groupmates, as we will see in the next excerpt.

Excerpt 2

Carla wishes to do it herself (she actually is the "measurer" of the group, i.e., the one in charge of measuring practices) and asks Ari to hold the measuring tape vertically, while she moves far from the statue. She engages in a movement quite similar to that of Bea, but she keeps her body quite still and moves her limbs very slowly using the span of her hand as a reference (Figure 5a-e, her thumbs and little finger are extended, while the other fingers are bent towards the hand palm, like in the *shaka gesture*).

a b c d e

Figure 5. Carla uses the painter's method

Bea first observes in silence some steps away from Carla, waiting for her to finish the process. When Carla seems done with it, Bea gets closer to her:

Bea: How many times? (*to Carla*)

Carla: (*walks closer to the statue without looking at Bea, grumbles, arms are dangling*) Five metres?

Ari: (l*ooking at them*) No, five times two metres

Carla: (*takes the measuring tape held by Ari and goes away*)

Commentary

Carla's bodily engagement (grumbling, dangling arms) suggests that she is reluctantly accepting the strategy that Bea has used, but still, she engages in first person with a similar approach. From the point of view of the modelling cycle, she seems to enter the same phase that Bea just did, using a similar bodily approach. While it is unsure whether this is done to confirm the group mate's measuring strategy or to discard it, it seems quite crucial for her to inhabit that mathematical place without mediation. Carla's movements (as evocated from Figures 5a-e) are slow and focused, and she takes her time and puts her own disposition in applying "the painter's method". Bea stays close but does not speak to Carla until she has finished. When asked about what she found out, she answers with an interrogative "Five metres?". In terms of the modelling cycle, it is Ari that tries to mediate her answer in terms of the real context and reinterpret the "five" as "how many times" she has to consider the reference length of two metres. Ari seems to be here trying to lead the group towards closing the cycle, while Carla seems to exit the shared place (both physically and metaphorically). The solution which would result from Carla's measurement (correctly re-interpreted by Ari) is the closest one to the actual solution of the task that the group will produce during the session. Nevertheless, it has been discarded by the girls, who do not insert it in the app for it to be validated by the system.

Excerpt 3

Bea then follows the first hint provided inside the app, namely that of considering a mate's (Ari's) height and relating it to that of the statue. Bea measures' with the tape Ari's height:

Bea: You are 1 and 52 [metres]

Ari: one-fifty [metre], let's approximate

Bea: Yes, let's do one-fifty [metres]. And we have to do one-fifty [metres] 5 times (*digits on the calculator*)

Ari: (*steps on the statue base and gets closer to it; moves her right hand higher and higher with repetitive gestures, as if she was imagining shifting her height towards the top of the statue*) Four and something more, four, five

Bea: (r*eading on the calculator*) So, 750 centimetres that then are… 7 metres

Ari: and a half

Bea: 7 metres and a half […] Then you were one-seventy [metres], I saw you 4 times. It was seven metres and a half, then one-seventy times 4… 6 metres and 8, instead. So more or less 7 metres

Ari: 7 metres, anyway.

Commentary

Bea goes back into the cycle to continue her reasoning from where she had stopped. In this phase, she is taking into account the relationship that Carla, with her five-times gesturing with thumb-finger extended, has brought to their investigation by saying "we have to do one-fifty [metres] 5 times"). Then, Bea goes back to her measurement experience to calculate the resulting height by multiplying the height of Carla by four. She tries to consider both perspectives jointly and enters the transfer M→R as she is trying to interpret mathematically the results that the previous measurements have produced.

Quite independently from Bea, Ari gets again closer to the statue and seems to shift the group's orientation. She seems to explore another way to confirm what the other girls have found out from a distance by physically relating her height to that of the statue. With her bodily approach, Ari validates the proportional relationships which Bea and Carla had previously discovered, by stating "Four and something more, five". Bea and Ari, in the end, agree that 7 metres might be a good approximation for the solution, but they do not insert it into the app.

Excerpt 4

Slightly after, Bea proposes to replicate the measurement using her height as a reference. Ari propagates the gestural strategy, while Bea measures her height. After this new measurement, they decide to insert the answer "7" inside the system and immediately receive negative feedback. This moves them to examine also the second and third hints: they find confirmation of their previous reasoning, so they appear puzzled by the negative feedback but decide to try again. Following the third hint, Bea moves again further from the statue and measures the height of Ari (16 cm, Figure 6a) and that of the statue (50 cm, Figure 6b) observed from that distance using the measuring tape.

"Like this you are 16 centimetres" "and Carlo Alberto is 50 centimetres"

Figure 6. Bea uses a new method to compare proportional heights

Then, Bea and Ari try to formulate a proportion, but seem to think differently about how to write it down. Carla comes close to them. Bea explains:

Bea: 16 is to 50. 16 is your proportional height, 50 that of Carlo Alberto.
As 150, that is your height is to…

Carla: What is 50? I am lost

Bea: Well, I saw her first, then the whole Carlo Alberto, and then we put it in proportion

Carla: And how many times is it?

Bea: 50 [centimetres]. We don't have to do how many times she stays [inside the statue] (*repeats the vertical gesturing used by Carla to figure out the proportion, Figure 7*), we really have to measure it.

Ari solves the proportion and gets 4,5 m. The girls decide to approximate to 5 m, which again gets negative feedback from the app, as they insert this value inside the system.

Figure 7. Bea repeats the gestures used by Carla in working out the proportion

Commentary

Despite the bodily strategy gathered consensus and the girls agree that it perfectly aligns with the hints, the negative feedback creates the need for new explorations. The modelling cycle restarts again as Bea uses the measuring tape to measure the observed heights (of Ari and the statue), instead of simply estimating their ratio. Then, Bea moves in the Transfer M→R, with the intention of interpreting mathematically the new measurements. In doing so, she shifts her orientation in considering the relationships that are at play. She explains her reasoning by comparing the different numbers that are to be related using a proportion. Carla's question ("What is 50?") raises the issue for Bea of distinguishing from the previous method, which involved looking at "how many times she stays [inside the statue]" (Figure 7). Now direct measurements have to be compared. These shifts in orientation also correspond to a back-and-forth movement in the modelling cycle: we might also say that Bea is now navigating a symbolic place, orderly juxtaposing measurements which refer first to Bea and then to the statue to navigate the relationships with an equation.

At this point, the girls are running out of time, so the researcher interacts with the girls to stimulate reflection on the methods they used as well as on the plausibility of the results they obtained. Such interventions lead the girls to enter the modelling cycle a couple of times, obtaining similar results each time. They then move to the following activity of the trail.

4.2 Episode 2 - The number of cobblestones

The second episode concerns the "The number of cobblestones" task with which the girls engaged at the very end of their trail. As detailed in section 3.2, the task requires estimating the number of cobblestones in a given (approximately rectangular) area of a square. The girls at this point are quite tired but have consolidated their ways of interacting and their roles. We develop this episode by describing the different bodily strategies that the girls use in solving the task.

Excerpt 1

Looking at the image inside the task, and possibly at the first hint provided by the app, Ari quickly recognises that a rectangular shape is to be analysed: "It's a rectangle, treat it as if it were a rectangle". Then, she asks Carla to count the number of cobblestones on one side of the rectangle, while she moves towards an adjacent side.

Ari starts counting the cobblestones using her feet as a reference. She opens her feet at a hip-width distance and progressively counts the ones that are in between her feet (Figure 8a), sliding along the longest side of the selected area. Carla, instead, points with her right hand towards the cobblestones along the other side of the area, while standing still with the rest of her body, to count them (Figure 8b). Meanwhile, Bea looks disappointed and asks the researcher "Is this really how it works?". She is then encouraged to discuss her thoughts with her group mates and proposes a new strategy: measuring the length of the bigger stones along one of the sides around the delimited area (Figure 8c) and counting the number of these stones (stepping on them), then multiply these numbers to obtain the length of that side. While Bea explains her reasoning and takes the measurements, both Carla and Ari stop counting and follow her. She is looking for a quicker method, but Ari notices that these stones are quite different in length from one another, so it is not possible to apply Bea's strategy. "Damn!" says Bea, and sighs.

a b c

Figure 8. a) Ari counts the cobblestones using her feet as a reference; b) Carla points and counts cobblestones; c) Bea measures the length of pavement stones

Commentary

This is the last task the group has to face to end the trail. Between the one analysed earlier and this one, the students have already worked on 8 tasks. Ari immediately identifies the mathematical model she wants to consider when approaching the task. She is in the Transfer R $\rightarrow$M and seems to navigate with confidence the mathematical place she is now inhabiting. She quickly frames the space in which the task is immersed and decides not only how to face it but also shares her strategy with Carla, actively involving her in carrying it out. It is apparent from their bodily engagement, though, that they are navigating the mathematical place with different orientations. Ari experiences that space using a part of her body (her feet), focusing on a restricted area of the pavement. Carla stands still and starts counting silently while pointing with her index finger. They are both interrupted by Bea, who is not convinced about what her mates are doing. In asking "Is this really how it works?", Bea is questioning the validity and efficacy of the approach. In fact, while her group mates are in the mathematical domain, with respect to the modelling cycle, Bea is still in the Transfer R $\rightarrow$ M: she does not seem to find the modelling strategy chosen by her companions optimal, not because it is not correct, but because she probably finds it time-consuming. This results in a shift in the orientation of the group, which moves Carla and Ari out of the meticulous counting procedure they were engaging with. Despite Bea trying to use a more efficient strategy, Ari points out an issue that invalidates it, so they have to take a step backwards again in the modelling cycle.

We note that the two strategies are quite different: that of Ari seems safer but longer; that of Bea is shorter but fails due to material constraints. Ari's strategy moves directly towards the object whose number has to be captured. Bea's strategy accounts for the environment in search of regularities that can be easily controlled. They approach the problem benefiting from two different modelling strategies: Ari is trying to work out directly the number of cobblestones, while Bea is going to arrive at that number by measuring the sides.

Excerpt 2

Since Bea's strategy has been discarded, Ari pushes the group to go back to counting the cobblestones; Carla suggests instead using the metre and measuring the length of each side, then estimating the number of cobblestones. Bea and Carla collaborate in taking the measures (Figure 9a), but they are very tired at this point and forget several times the measurements they take and frequently lose the reference points for their measurements.

Ari seems unsure about this strategy and asks: “And how much is a cobblestone worth?”. Bea explains how they will then use the measures: “In a square metre we have to compute how many [cobblestones] are there”.
Using this strategy, they now arrive at measuring the length of one side (6,70 m) and estimate the other length (measuring a part of it, they decide to approximate it with 10 m). Once they have calculated the area of the delimited region, again Bea and Ari diverge in their reasoning, proposing two different strategies. Bea wants to follow the second hint, which suggests counting how many cobblestones are in 1 square metre: “We have to do [count the cobblestones] inside a square metre, then multiply by 67”. Ari proposes that they measure the area of one cobblestone and divide the total area by that number to get the total number of cobblestones.

a b c

Figure 9. a) Carla and Bea measure one side of the area; b) Bea uses the measuring tape to visualise 1 square metre on the floor; c) Bea measuring the side of a cobblestone

They start following Bea’s idea (she visualises the 1-square metre area by using the metre, Figure 9b) but Ari, in particular, is reluctant to count (“by hand”, as they say) the cobblestones in that area, since they are again running out of time. So they proceed to measure the length of the side of a cobblestone (8 cm; Figure 9c) and compute its area taking it as approximately squared. They gather around their smartphones to mathematically treat the numbers they got: they seem to struggle with the mental calculations, the unit of measure and the approximations. In converting the area of the single cobblestone (64 cm^2) into square metres, they divide 64 by 100, then 1000. They then obtain 1040 (approximating the result of $67 \div 0{,}064$) and insert this number in the system and get negative feedback. Then, they wonder whether to insert 1000 or 1050 in the app, considering them as possible alternatives to the rounding of the division previously carried out. In the last three minutes, they engage with the researcher in explaining their reasoning and strategies but do not converge in any way toward a shared understanding of the situation. The researcher asks: “Do you think that 1000 is plausible?”. Bea looks around and exclaims: “Of course!” and Carla adds “If you think that in a little bag there were 2000 chickpeas…”. They go back to calculating, but again confuse the measurements and units, so they give up and reach the other classmates. Bea, smiling, says “It is June 7th for everyone…”.

Commentary

After Bea’s previous strategy has not been carried out, the girls move towards the direct measurement of the sides of the rectangular shape they are considering, as suggested by Carla. As the measurer, Carla again pushes towards using the instruments to gain information. She orients herself and her group mates into gaining a sense of how big the area is. In doing that, the girls move far from one another, unrolling the measuring tape as much as possible, and spread out in the given area, regardless of other people passing by. Conversely, when they have to perform calculations they all gather around their smartphones. For space constraints, we did not describe in detail their bodily interactions, but we have presented the different scales at which the measurements recruit the girls' bodies (from petite to ample body movements). This is relevant in our analysis for discussing how inhabiting a mathematical place is oriented by being in that place as well as the mathematical strategies that are put forward. Using the modelling cycle, we might say that Bea and Ari are again entering the transfer R→ M considering relying on different modelling strategies. Once again, perceptions on which of the strategies is more efficient and the time constraint strongly influence the choice of the group. While performing the calculations, they express the difficulty of doing mental calculations and seem to push the immersive measuring experience in the background. Their bodily inhabiting the space, even the counting that Ari has already done in the first part of the activity, seems completely disconnected from the mathematical work they are now doing.

The transfer M→ R seems much more complicated for the girls, who struggle in working on the mathematical treatment of the measurements they got. Even though they bring in past experiences (like Carla, recalling the experience of counting the number of chickpeas in a small bag), this does not help them in questioning the result. Rather, they think of how their approximation might be modified.

It is apparent from the video that the girls are tired and probably most of their calculation errors are connected to rough approximations and mental calculations and are due to the different environment they are now in.

5. Discussion

In this paper, we proposed an outdoor activity with a connected STEAM approach that conjures mathematics and technology to address our research questions: *How do students inhabit mathematical places in the context of modelling activities outdoors? How do their bodily strategies enable us to enrich our understanding of outdoor modelling?*

We presented two episodes where three grade-10 girls solved two tasks of a math trail we designed in the MathCityMap web portal. The tasks ask to measure the height of a statue and to estimate the number of cobblestones that are covering a delimited area inside the same square where the statue is.

To discuss the role of bodily experience in the context of modelling activities outdoors, we focused on students' mathematical doing by carefully describing their perceptuo-motor engagement during their collaborative interactions.

Concerning the first research question, in the analysis we enriched our understanding of the girls' mathematical activity following the modelling cycle by examining their bodily engagement in solving the tasks; this helped us delve into their strategies and their orientations. In fact, following Nemirovsky (2005), we considered their mathematical experience as inhabiting a mathematical place: we accounted for how their orientations in solving the tasks are always shifting, reconfiguring at the same time the students' strategies, their interactions and the mathematical concepts they are inhabiting.

In episode 1, we discussed the relevance of the "painter method" proposed by Bea, and how that method propagated through the activity, being modified and re-purposed by her groupmates. In this case, we have seen that the sharing or coordination of bodily strategies alone did not allow the students to cooperate in the solution of the task. Giving justice to the role of the body in the learning context often also means that the whole complexity of being inhabiting a place matters, including the affective relationships with others and the material engagement with the surroundings. We also have shown that the students came to inhabit a symbolic place, e.g., in discussing the proportion that can model the relationships at play in the task "The tip of the sword".

In episode 2, we discussed the diverse embodied strategies that are put forward by the group to count the cobblestones. Mathematically, these strategies might be considered equivalent, but they have recruited the bodies differently, leading to managing and organising the space, the measurements, and their relationships in very different ways. Once again, we see that the students' orientations in navigating the mathematical place are complex arrangements that are inseparable from the environment (material constraints) and pervaded by nuances of affective engagement with mathematics (beliefs about the validity of the strategy, disappointment when it fails). We also highlighted how material constraints and affective nuances influenced the students' orientations, like the tiredness experienced by the group, and the different environment that seemed to call for a different approach to calculations with respect to the one carried out in school.

Concerning the second research question, this perspective allowed us to reconsider the modelling cycle looking at the outdoor STEAM experience of students in a holistic manner. The moments that the modelling cycle indicates as Transfer from R → M or from M → R are moments of transition from the domain of reality (R) to the mathematical domain (M) and vice versa.

In episode 1, we connected a back-and-forth process in the modelling cycle to continuous shifts in orientations, which we have described through careful microanalysis. In episode 2, we have further discussed how in general the transfer M→ R seems much more complicated for the girls, while valuing the richness of strategies emerging in the phase identified as transfer R→ M.

We focused on these moments of the students' work trying to understand the complexity of inhabiting a mathematical place, paying attention to how bodies interact and engage with others and with the surroundings. The idea of mathematical place helps us to recognise the role of the body that is creating the mathematical place at that moment. In fact, as Nemirovsky (2005) states, "[p]laces come to be by way of a lived body" (p. 49). The tasks create the occasion for the students to look at the statue and the pavement differently. In doing so, it is not that mathematics is setting for them a pair of glasses through which they observe the surroundings in a different way. It rather entails an entirely new manner of dealing with it, that encompasses recollections and bodily strategies, as well as and affective relationships and beliefs, in an integrated STEAM approach.

Trying to address Reality and Mathematics, no longer as two distinct objects/sets, but as two domains inhabited by the body-mind in a holistic manner, is a point of novelty that characterises our study compared to previous research on outdoor modelling activities.

6. Conclusion

We believe that the present study has several implications for teachers, researchers and practitioners interested in outdoor STEAM activities. We elaborate on some of them in the following, ranging from more practical to more theoretical ones.

Outdoor activities should be proposed several times during the school year in order to accustom students to the management and understanding of the dynamics that such an experience brings with it.

From the perspective of STEAM approaches, it would also be interesting to have occasion to develop new activities in the classroom that are connected to the math trail, like creating a physical model of the outdoor situation to get new insights into the problem.

In the episodes, we have seen that the students did not always respect the role assignments they had agreed upon at the beginning of the trail, especially concerning the role of the measurer. Not really fulfilling the assigned role, or rather, also fulfilling the role of the measurer, we conjecture, is inherent in living a mathematical place. Inhabiting a mathematical place entails all the actions that are somehow separated in the roles usually attributed to the group components. Experiencing a mathematical place is not a matter of selecting one mode of operating in the world, rather is navigating possibilities in coordination with the world.

From a mathematical point of view, in the tasks reported in the analysis, the students do not manage to enter a correct solution in the app, despite various attempts and strategies implemented. In this respect, the following considerations must be made for each of the tasks.

With regard to the “sword” task, all the strategies adopted by the group all relate to the concept of proportion. This is an effective approach to model the situation. We conjecture that what does not lead them to achieve a permissible result is that they neither scrupulously nor 'consistently' retrieve their data. In fact, their ways of taking measurements are very rough: they often tend to approximate, missing out on significant digits and generating a measurement error propagation that heavily impacts the final result. Added to this, when they chose to make indirect measurements, they positioned too close to the statue. For example, when using the painter’s strategy (Figure 4), Bea is forced to arch her back several times in order to appreciate the statue in its entirety. While we have thoroughly discussed the relevance of this approach in the experience, we have to notice that these movements unbalance her reference system, which suffers from distortions in finding the measurement. In addition, the students did not grasp that the scene presented in the second hint is photographed at a distance from the statue exactly to avoid this problematic.

Concerning the “cobblestones” task, we notice how the students, now on the last task of the trail, collaborate in carrying out the strategies but are often dealing with more than one strategy at a time. Their ideas of approximating and measuring seem rooted in the belief that it does not require so much accuracy. From a mathematical point of view, as already observed in the analysis, a certain difficulty in estimating real-world quantities emerges, as well as validating the obtained results. We argue that outdoor modelling experiences would allow students to be more familiar with the concepts of estimation and orders of magnitude, mathematical concepts which, at least in the Italian curriculum, are little dealt with even though they constitute meaningful aspects of number sense.

While the fact that, in the episode discussed in this paper, the students did not get to the expected answer might raise some concern, we believe that a collective discussion with the class teacher, questioning the reasons why that happened, would didactically be an effective intervention to tackle the issue.

What we believe is the theoretical contribution of this paper is to show how focus on bodily experiences provide fertile ground to think about how we read the interpenetrating of the disciplines in outdoor situation, and how these practices are affected by new orientations in the outdoor context. In this paper, the analysis carried out has not pointed to what the students did wrong, but rather to highlight: how they did create meanings through their experience in this new context; how their being together in the collective experience impacted their strategies; and how their strategies mattered in the specific context analysed.

We argue that we must understand more about the complexity of students’ orientations in order to develop innovative learning environments, both indoor and outdoor, that allow students to see the relevance of STEAM disciplines in relation to real-world situations and to develop the soft skills they will need in the future. Future research might contribute to the development of this line of investigation, paying attention to the role of bodily experiences in outdoor STEAM activities.

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