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Ph.D. Thesis

MOOC's Zone Theory: creating a MOOC environment for professional learning in mathematics teaching education

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Declaration by author

This thesis *is composed of my original work, and contains* no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

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Publications during candidature

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Contents

List of Tables

List of Figures

List of Abbreviations

LD	Learning Designer
Math Edu USs	Mathematics Education Utilization Schemes
MOOC	Massive Open Online Course
MOOCs	Massive Open Online Courses
PR	Peer Review
PW	Project Work
USs	Utilization Schemes
ZFM	Zone of Free Movement
ZPA	Zone of Promoted Actions
ZPD	Zone of Proximal Development

Introduction

Massive Open Online Courses (MOOCs) are online courses aimed at unlimited participation and open access via the web. MOOCs are a recent and widely researched development in distance education, which were first introduced in 2008 and emerged as a popular mode of learning in 2012 (Pappano, 2012).

This emergence of Massive Open Online Courses, enabled by technology and social networking, has opened new educational possibilities. McAuley et al. (2010, p. 10) define a MOOC as "an online course with the option of free and open registration, a publicly shared curriculum, and open-ended outcomes". The authors put forward that "a MOOC builds on the active engagement of several hundred to several thousand students who self-organize their participation according to learning goals, prior knowledge and skills, and common interests" (ibid.).

The MOOCs have therefore become popular in universities as a way to support working students or students that are parents, and also to renew and transform both teaching and learning of various disciplines (see for example Siemens, Irvine & Code, 2013).

Despite their big success, the emergence and use of MOOCs for professional teacher development is still uncommon, especially in mathematics. In fact, although there is a wide choice of topics, covered by more than 6800 MOOCs available worldwide (in 2016)[1], when looking specifically for a MOOC aimed at mathematics teacher education the range is limited. Moreover, the specific intersection of MOOCs and professional teacher development is poorly researched (Taranto et al., 2017 a, b; Panero et al., 2017; Avineri et al., 2017).

A MOOC is a good environment to pursue two significant themes in research on mathematics teachers:

- to understand how teachers learn from this online experience;
- and if/how it influences teachers' knowledge, practices and beliefs.

These issues, however, are not widespread: at present, although there are studies in the literature on the subject, the research has not yet developed a framework sufficient to explain how changes in knowledge (understood as professional development of teachers) can possibly occur as a product of activities in these new environments totally online. The literature is scarce (Siemens, 2005; Downes, 2012 b; Ozturk, 2015; Teixeira et al., 2015) and the shortage is then almost total as regards teacher updating activities (Panero et al., 2017). In fact, at the recent 13th Four-Year Convention of the International Commission on Mathematics Education (ICME), held in Hamburg in July 2016, which collected the most significant experiences on research and teaching practices at the global level, the Topic Study Group 44, dedicated to "Distance learning, e-learning, blended learning" has shown few studies on experiences with MOOC aimed at teachers education.

On the contrary, there is a wide literature (Robutti et al., 2016) that deals with the way in which the teachers can develop their professional learning in traditional, face-to-face courses, particularly when the theme of the update concerns the relationship between education and technology.

[1] https://www.class-central.com/report/mooc-stats-2016/

Significance of the study

The present study is significant because it was conducted at a time of increasing interest in MOOCs for mathematics teacher education.

Currently, research has not yet developed a framework to explain how learning takes place in these new totally online environments. There is little literature on MOOCs (Siemens, 2005; Downes, 2012 a; Ozturk, 2015; Teixeira et al., 2015) and even less on Mathematics MOOCs for teachers' education and development (Panero et al., 2017; Avineri et al., 2017*;* Taranto et al., 2017 a, b; Aldon et al., 2017). However, as we have previously mentioned, in mathematics education there are many theories on the way teachers can develop their professional learning in face-to-face environments and on the relationship between education and technology. Based on these, a theoretical framework, called MOOC's Zone Theory, has been elaborated by the dissertation writer. It is a useful framework for research that aims at understanding the complexities of the learning trajectories of the participants in a MOOC. The protagonists that we consider are the in-service secondary school mathematics teachers enrolled in the MOOC (to whom we refer to as *trainee*) and the mathematics teacher educators involved in the MOOC design and delivering (to whom we refer to as *trainers*). With learning trajectory, we mean how these protagonists interact online, both with the platform and with each other. In particular, if and how these interactions change their knowledge, beliefs and generate perception of change in the practices. Therefore, MOOC's Zone Theory framework allows for description of the teacher's participation in the MOOC and for the analysis of their consequent professional development and possible perception of changes in teaching practices. In particular, it facilitates the study of the specific dynamics of the interactions among the teachers enrolled in the MOOCs and between these teachers and the MOOC's trainers, which occur online and in totally virtual environments. It is topical and urgent to analyse these interactions in the context of such distance learning, due to the increased interest in this approach in recent years.

Research Questions

The purpose of the present study was to capture and understand the complexity and dynamic nature of MOOCs and the influence that they have both on the trainees and on the trainers of such courses.

Three research question have guided this study.

Research Question 1:
Are there any particular structural potentialities in a MOOC that, if properly organized, can trigger suitable learning processes in the trainees (and trainers)?

Research Question 2:
Does the MOOC environment trigger and support the trainees' professional development? And if so, do these aspects lead to perception of changes in teaching knowledge, practices and beliefs?

Research Question 3:
Does the MOOC environment trigger and support the trainers' professional development relatively to design principles and strategies of trainees' assessment that the trainers have put in place?

Note that the research questions are formulated here in a generic form. They will be resumed and reformulated later, after introducing the analysis lenses.

The issues showed are all addressed using the MOOC's Zone Theory.

The first research question concerns with the investigation of the online environment designed to host teacher education, the so-called MOOC-artifact. Emphasis is given to considering the online context in which education takes place, as it holds specific characteristics, or potentialities, that are not found in a face-to-face course and that contribute to influence the possible education that follows. The research is in fact aimed at understanding how teachers' collaboration practices are carried out, how they evolve over time as a result of the received stimuli and the interaction activated on the platform and how they differ from those produced in face-to-face courses. The research is also aimed at understanding the influence exerted on the teachers from the dynamic interweaving that is established between the practices and products of the practices themselves. Namely, in terms of messages and interventions on the platform, but not only: in terms of re-elaboration of activities and educational paths, multimedia files, of software, and also in terms of designing new materials. All of these actions are included in what we call *double learning process* and show how the *MOOC-artifact* move in the *MOOC-ecosystem/instrument* for its participants.

The second research question is specific for the trainees. It is concerned with the impact that the MOOC exerts on its trainees. It is important to understand if the MOOC environment (to whom we refer to as *MOOC's ZFM/ZPA*), with its mathematical resources selected by the trainers and the interactions carried out by the trainees, allows the trainees' professional development. The research is in fact aimed at understanding if the MOOC's ZFM/ZPA trigger and support an expansion of what the Connectivism calls *network of professional knowledge*. In particular, using the networking theory strategy, this analysis can be understood as an effective shift from the actual developmental level to the potential developmental level in the *ZPD* of the trainees. In particular, analysing some case studies, the research intend to shown if these aspects lead to a perception of changes in teaching knowledge, practices, and beliefs (or *didactical praxeologies*, as they are called in the Meta-Didactical Transposition model).

The third research question is specific for the trainers, the mathematics teacher educators that have been involved in the MOOC's design, delivery and monitoring. The research question concerns with the design effort and assessment strategies that have been pursued in these online courses for mathematics education. The research is aimed to give an outline of the expansion of the *network of professional knowledge* of mathematics teacher educators in the light of the experiences lived in managed MOOCs. Therefore, it is important to understand if the *MOOCs' ZFM/ZPA* is able to trigger and support this expansion of the network of knowledge, focusing more on eventually changes made from the point of view of the design and assessment strategies in the trainers' *meta-didactical praxeologies*.

The study

The present study was conducted into two phases: a theoretical phase and an empirical phase. The theoretical phase employed an extensive review of literature to identify theoretical lenses that might contribute to capture and understand the complexity and dynamic nature of MOOCs and their influence on the protagonists involved in them (trainees and trainers). In particular, an existing framework that had been used to describe face-to-face meetings for teacher professional development, namely the Meta-Didactical Transposition (Arzarello et al., 2014), was reviewed and revised. Subsequently, specific strategies for connecting it with other theories, namely networking and hybridization, were putted in action. The considered theories are: the Instrumental Approach (Verillon and Rabardel, 1995; Trouche, 2004) and the Connectivism (Siemens, 2005; Downes, 2012 a), that were hybridized (Arzarello, 2016; 2017) with the Meta-Didactical Transposition. After, the resulting theory was networked (Bikner-Ahsbahs & Prediger, 2010; 2014) with an adaptation of Goos's Zone Theory (2005). This theoretical phase informed data collection and analysis in the subsequent empirical phase.

The empirical phase of the study was conducted over almost three years (from October 2015 to July 2017). It is focused on two Italian experience with MOOCs for mathematics teacher education: *MOOC Geometria* and *MOOC Numeri*, the first two MOOCs designed and delivered by the Math MOOC UniTo project. Precisely, the project started in the spring of 2015 and four MOOCs were designed, one for each of the main topics in the official Italian programs for secondary school: Arithmetic and Algebra, Geometry, Change and Relations, Uncertainty and Data. The first two that have been delivered are, exactly, MOOC Geometria, on geometry contents, from October 2015 to January 2016; and MOOC Numeri, on arithmetic and algebra contents, from November 2016 to January 2017.

A mixed method of research (Frechtling & Sharp, 1997), namely a methodology for conducting research that involves collecting, analysing and integrating quantitative and qualitative research, is employed and a variety of data sources contributes to the richness of the description and the overall in-depth understanding of our object of study. Moreover, the process utilised in the study provides a mechanism that could be used to guide empirical studies that investigate similar issue to the exposed research questions, thus contributing to better understanding of impact that MOOCs for mathematics education have on trainees and trainers.

Structure of the Dissertation

The dissertation consists of 7 chapters.

The purpose of Chapter 1 is to provide an overview on MOOCs and on teacher education in general.

In Chapter 2 the theoretical phase is exposed. It is the heart of the dissertation and its most delicate part. The elaboration of the theoretical framework is described with great care and attention to details. The reader is accompanied in reading and understanding all the theoretical details of the considered theories. Given the complexity of the topic being dealt with, metaphors are also used during the exposition for facilitating the reader to grasp the different constructs that are introduced. It is clearly explained why the dissertation writer has had to

resort to different theories and, finally, a theoretical framework used for the purposes of the dissertation is explained in detail and justified.
Chapter 3 provides information about the research design. First, the research is situated: the Italian educational system and the teacher education in Italy are shown. Secondly, the Math MOOC UniTo project is illustrated. In particular, a general overview on the MOOC team, on the mathematical contents proposed in MOOC Geometria and MOOC Numeri, and on the technological resources that they contain is shown; two videos, made by the dissertation writer for this purpose, are also given. In addition, general information on the trainees involved in these two distance learning experiences is presented and the differences that distinguish a face-to-face training course from an online one, such as MOOCs, are highlighted. Finally, the chapter concludes with a part dedicated to the research methodology. The position of dissertation writer as researcher, the changes made to the original research design, and the context in which the study was conducted are outlined. Detailed information about how the study was conducted and measures that were put in place to enable readers to make a judgement about the trustworthiness of findings are presented. In fact, the techniques used for data collection are explained and clarifications on the mixed method used for the analyses (i.e. quantitative and qualitative analyses) are made.
The findings of the empirical phase of the study are presented in Chapter 4, Chapter 5 and Chapter 6; namely, they are the chapters in which the analyses are exposed. In fact, each of them provides useful evidence to answer the three research questions respectively.
Chapter 7 begin with a summary of the present study. Responses to the three research questions that guide the study follow. Limitation of the study are identified and the chapter concludes with suggestions for further research.

The whole thesis has been written to the first plural person (namely, it will be usual to read "we", "our", ...). This choice was made because it was considered to be the best translation from Italian to English of the impersonal form.
However, it is important underline that what is stated in the thesis is the result of the research work conducted by the dissertation writer. Sometimes, rarely, the first person singular will be used (especially in §3). Only when specified, the "we" will refer both to the dissertation writer and to the MOOC team with whom the dissertation writer has closely collaborated during these years of research.

Chapter 1 MOOCs for mathematics teacher education

1.1 What are MOOCs?

A Massive Open Online Course (MOOC) is an online course aimed at unlimited participation and open access via the web. MOOCs are a recent and widely researched development in distance education which were first introduced in 2008 and emerged as a popular mode of learning in 2012 (Pappano, 2012). Early MOOCs often emphasized open-access features, such as open licensing of content, structure and learning goals, to promote the reuse and remixing of resources. Some later MOOCs use closed licenses for their course materials while maintaining free access for students (Cheverie, n.d.). In addition to traditional course materials such as filmed lectures, readings, and problem sets, many MOOCs provide interactive user forums to support community interactions among students, professors, and teaching assistants (Adamopoulos, 2013).

In the following we list their main features:

a) Provision and use of educational content and online activities via the Internet, through an e-learning platform such as, for example, the open-source Moodle platform. Delivering MOOCs, various international consortia (eg EdX, Coursera, iVersity, Udacity) refer to their own platforms.
b) Free access to content and training activities, with (possible) payment for the achievement of the learning certification.
c) On-line interaction between all participants in the training activities. In the more organized didactic MOOCs there is the intervention of the teachers on the basis of an established calendar. Each student can build his own personalized learning and sharing path.
d) Multimedia didactic contents (interactive videos, use of specific software, and so on). The provision of the course may include the intervention in the virtual classroom of external experts.
e) Evaluation and self-evaluation of learning. Students perform individual tasks (deliveries) that are evaluated by teachers or discussed and peer-assessed in special forums in the case of a large number of MOOC participants.

1.1.1 Different kind of MOOCs

Literature recognizes two main types of MOOCs: Connectivist Massive Open Online Courses (cMOOCs) and eXtended Massive Open Online Courses (xMOOCs).

cMOOCs are focused on the learning community and connections between members of the community across the web, rather than on course content or the instructor, and have been carried out with great success (Rodriguez, 2012). cMOOCs were the first massive open online courses, designed to test the principles of ‘connectivism’, working within a framework developed by Siemens (2005) and Downes (2008) to attempt to explain the nature of learning in highly networked environments. Early cMOOCs were designed to foster processes of "aggregation, relation, creation, and sharing" (Kop, 2011) among distributed groups communicating and collaborating online. cMOOC-type courses are structured to provide a minimum of centralised control or content, and to develop participants' ability to contribute to, and learn from, the digital network. Arguably, the 'massive' in these courses tends to refer mainly to the scale of the connections, content generation and participant activity in these courses, not their number of participants, which appears to be relatively low in comparison with first waves of xMOOCs (Bayne & Ross, 2014).

On the other hand, xMOOCs are based on scalability of provision (Coelho et al., 2015), i.e., they are focused on giving many students access to an online course within the same platform. The style of learning is also different; xMOOCs tend to privilege individual studying, while cMOOCs focus on networked learning across several web tools and services (Conole, 2013, Rodriguez, 2012, Hew & Cheung, 2014). The term xMOOC was coined to differentiate the cMOOCs from the newer, more massive, institutionally-driven and content-focused courses offered through platforms such as edX (from which the xMOOC gets its name), and Coursera (Downes, 2012 b). xMOOCs are commonly described as being driven by 'behaviourist' principles of knowledge acquisition through repetition and testing (Rodriguez, 2012). One argument for this approach is that it can scale up to cater for the numbers of people who sign up for these courses; typical enrolments reach 50,000, while the largest MOOC tracked in one study, Duke University's 'Think Again: How to Reason and Argue', had 226,652 enrolments (Jordan, 2013). This scaling up is important to proponents of these larger MOOCs, who often frame their mission as being one of opening global access to education (Knox, 2013).

As George Siemens summarizes: cMOOCs focus on knowledge creation and generation whereas xMOOCs focus on knowledge duplication (Siemens, 2012). However, recently there is a trend, especially in Europe, towards the emergence of hybrid approaches that try to mix the potential of open socially driven learning with structured learning paths. In fact, the disruptive nature of the connectivist approach to learning is quite difficult to articulate with typical institutional environments (Hernández et al., 2014). Therefore, while the cMOOC/xMOOC binary is usefully descriptive of two different trajectories of development, and is much used by those espousing a connectivist perspective to criticise xMOOCs, recent literature is beginning to move away from what is increasingly seen as a simplistic categorisation, towards more nuanced and microlevel discussion of exactly what is going on in different kinds of MOOCs. This has led some commentators to propose new forms of categorisation (Lukeš, 2012; Conole, 2013; Lane, 2012), and others to propose, for example, the notion of a 'hybrid MOOC' (Waite *et al.* 2013), or a process by which educators might 'mediat[e] the dichotomy between xMOOC and cMOOC' (Grünewald *et al.* 2013).
xMOOC pedagogy is rapidly evolving (Boyatt *et al.* 2014, p.138) and, as some researchers are beginning to note, what goes on in any given MOOC is no longer clearly determined by its 'x' or 'c' status.
Let us consider two examples taken from Bayne & Ross (2014, p. 22).

i. Gillani's (2013) analysis of patterns of participation on a business strategy MOOC on the (xMOOC) Coursera platform found that most of the 4,337 discussion forum participants in the MOOC received below a 50% score on the MOOC, suggesting that "most discussion forum participants are more interested in connecting with others to talk about issues with real-world significance and implications than they are in being formally recognized for their work" (p.43). The presence of this group of students within the broader context of an 'xMOOC' indicates that the range of types of participation in MOOCs is not as simple as the cMOOC/xMOOC binary would suggest.

ii. Neither are cMOOCs immune to these sorts of apparently contradictory participation patterns. Kop's (2011) analysis of two 2010 cMOOCs – including the Personal Learning Environments, Networks, and Knowledge (PLENK) MOOC run by key connectivist proponents Cormier, Siemens, Downes and Kop – found that while the course was explicitly designed to produce 'aggregation, relation, creation, and sharing' among participants, only a small minority of the 1,610 participants engaged in creation and distribution of digital artefacts (p.35).

The problem with over-simplistic categorisation of MOOCs is that it may do more than misrepresent what goes on in MOOCs: it may also shape and constrain future MOOC development in unhelpful ways. Clarà & Barberà's (2013) critique of connectivism from a psychological perspective urges new ways of considering MOOC pedagogy:

> recognizing... problems with the connectivist theory provides an insight into certain difficulties that learners experience in cMOOCs, difficulties that are not necessarily intrinsic to such pedagogical environments but rather a consequence of how learning in a MOOC is theoretically conceptualized... Although MOOCs were first launched by connectivists, connectivism is not intrinsic to MOOCs. (p.8)

We are starting to move away from the cMOOC/xMOOC binary toward recognition of the multiplicity of MOOC designs, purposes, topics and teaching styles. Some teachers and organisations are rejecting the MOOC acronym altogether, in favour of 'DOCCs: Distributed Open Collaborative Course' (Jaschik, 2013), 'POOCs: Participatory Open Online Course' (Daniels, 2013), 'SPOCs: Small Private Online Course' (Hashmi, 2013) and 'BOOCS: Big (or Boutique) Open Online Course' (Tattersall, 2013). Teams and institutions are reframing and reshaping the MOOC and the massive for their own purposes – for collaborations (Scholz, 2013), 'flipping' of classrooms (Bruff *et al.*, 2013), and more.
Stewart's (2013) observe that not all MOOCs are the same:

> the distinctions individual university partners and teaching faculty may make regarding their given courses needs to be kept in mind when generalizing about MOOC models.

Each MOOC is profoundly shaped by its designers, teachers, platform and participants, as we will see in this dissertation. The binary terms 'cMOOC' and 'xMOOC', which are helpful in describing the lineage of MOOCs, are limited in their usefulness for those seeking to develop a MOOC, to understand how MOOCs are actually being experienced, or to draw conclusions about good practice in MOOC design and pedagogy.

1.1.2 Strengths and weaknesses of MOOCs

Strengths[2]

[2] Retreived February 8, 2018 from https://www.crui.it/images/demo/crui_web/pubblicazioni/crui_mooc_2015.pdf

a) They are scalable to be used massively by thousands or even tens of thousands of users thanks to on-line provision and the teaching structure centred on the “student group”.
b) They are open to all and free, at least up to the level of full use of teaching materials and self-assessment tools.
c) Promote the internationalization of training and of the national university system, especially if provided in English (like the vast majority of MOOCs courses currently online).
d) They offer the possibility of experimenting with new interactive teaching methods; they can combine distance teaching with experiential teaching; they integrate with blended learning & flipped classrooms. They offer the opportunity to become part of a highly qualified international community of universities that debate methods and educational innovation.
e) Didactics is potentially organized by a real 'teaching team' (multiplicity of professional figures): e-learning designers, industry specialists, teachers, online tutors, Universities ... This multi- or even interdisciplinary interaction could develop new knowledge in the field of learning, making the lesson itself more engaging and effective.
f) On-line collaboration and peer interaction. Most of the interventions in the discussion forums take place by the students. Therefore, peer-interaction is a dialectic that enriches knowledge (think of cMOOCs based on the connectivist pedagogical approach on the web).
g) Educational paths with high flexibility, both in terms of temporal fruition and choice of training contents. There are ongoing projects to issue an electronic badge free of charge at the end of the course with the logos of the universities providing the MOOC. This badge (recognized for example by the Mozilla consortium) can be immediately shared by the student on all social networks and job search platforms with a clear image return also for the universities involved.
h) Possibility of certification of learning with possible recognition of university credits useful, in particular, in the European training context.
i) They seem to provide a positive learning environment, as also shown from the analysis conducted by the America’s Department of Education on a sample of 1000 students, which reports that the students of the MOOCs have better performances than the students in attendance (The Economist, n.d.).
j) They can reach a very diverse audience. In particular, they can be used by those who cannot afford to attend the University in standard mode or by those who work (e.g. lifelong learning).

Weaknesses[1]

a) They reduce the direct closeness between the student and the teacher and therefore the transmission – often informal – of experience that takes place during the lesson or in moments of confrontation. It is also important to remember the reduction (if not the zeroing) of the educational and experiential aspects that instead are associated with campus or university life.
b) They seem to reach mainly students who already have a high level of training and are already very motivated to higher learning (Lue, 2014; Emanuel, 2013).
c) High drop out rate of these courses (we return to this point the next paragraph).
d) Danger of monopolization of online training by large private consortia for profit that are formed around top ranking universities.

1.1.3 Tension around participation

The participant's role is hotly contested across almost all literature and debate about MOOCs. Indeed, the key dilemmas in MOOCs centre on what participation actually means, how it should be measured, and consequently, what metrics of success and quality are appropriate for these courses (Bayne & Ross, 2014). These concerns have led to a proliferation of models of participation, including Clow's (2013) 'funnel of participation'; Kizilcec et al.'s (2013) four engagement patterns of completing, auditing, disengaging and sampling (p.3); Hill's (2013) five archetypes of no-shows, observers, drop-ins, passive participants and active participants; Mak et al.'s (2010) dimensions of movement between MOOC environments; and Milligan et al.'s (2013) continuum of 'active', 'lurking' and 'passive' participation.

Part of this complexity seems to arise because there are simply so many people, doing so many different sorts of things in any given MOOC, that actual practice has to be seen as 'nuanced, strategic, dynamic and contextual' (Mak et al. 2010, p.280). This presents a challenge for researchers, educators and institutions accustomed to using 'completion' as a fairly stable measure of the success and quality of an educational offering. Formal completion rates (for MOOCs that can measure these), which rarely rise above 10% (Jordan, 2013), are increasingly thought not to be the right way to judge the quality of a MOOC or of participants' experiences. The 'outsized media attention' this statistic invariably receives is not taking sufficient account of those who may be engaging but 'do not adhere to traditional expectations, centered around regular assessment and culminating in a certificate of completion' (Kizilcec et al., 2013, p.9).

From an example taken from Bayne & Ross (2014, p. 25): in a 'Writing in the Sciences' MOOC, where participants were asked before the start of a MOOC what their intentions were with regard to the course, completion rates could be seen to be highly differentiated. For those who stated in the pre-course survey that they intended to complete the MOOC, completion rates were 24%. For the remaining course population, just 2% formally completed the MOOC (Koller et al., 2013).

The notion that people might sign up for a course not intending to complete the assessments is one that is unfamiliar to fee-charging institutions, but it is extremely common in free courses where the barrier to entry is usually as low as clicking a registration button and entering an email address. In such a context, new measures of success and quality are required, because participant behaviours and intentions are so diverse. However, we will not deal with this in this dissertation (for more details see for example Yang et al., 2013; Onah et al., 2014).

1.2 Teacher education

Since this research work has teachers as protagonists who have followed two MOOCs for mathematics teacher education, a premise on teacher education is a must.

In these few lines, we report some observations made by Jaworski (2008).

> Mathematics teaching is complex. Teachers need to know mathematics, pedagogy related to mathematics, mathematical didactics in transforming mathematics into activity for learners in classrooms, elements of educational systems in which teachers work including curriculum and assessment, and social systems and cultural settings with respect to which education is located. In addition, teachers know intimately the students with whom they work and the particularities of the schools where teaching takes place […] (p. 1)

So, there are many variables that in-service mathematics teachers must consider and know. On the other hand, mathematics teacher educators work with teachers to develop teaching. Therefore, both educators and teachers have a common aim to provide better learning opportunities for students learning mathematics. In fact,

> Educators provide courses, summer institutes, professional events of various kinds [also MOOCs] to enable practising teachers to develop knowledge in the areas indicated […]While [the educators] cannot know the students and context of each school in any depth, [they] bring a profound understanding of mathematics, didactics, pedagogy and systemic factors related to a range of settings. (Jaworski, 2008, p.1-2)

Indeed, mathematics teacher educators make their knowledge available: knowledge in terms of theories of learning and teaching, methodologies of research that inquires into learning and teaching in schools and educational systems. These aspects foster the creation of opportunity for teachers to learn and develop mathematics teaching. "Teacher education has been seen as a transfer of knowledge from educator to teacher" (Jaworski, 2008, p.3).

As we read from Jaworski, educators can offer different educational opportunities to teachers. And these can take place either in face-to-face meetings or even in online virtual environments. There are some examples of blended experiences in the literature designed for mathematics teacher education (Borba & Villareal, 2006; Owston et al., 2008; de Carvalho Borba & Llinares, 2012). What we can say about totally online educational environment, for example as MOOCs?

1.2.1 MOOCs for mathematics teacher education

Despite their big success, the emergence and use of MOOCs for professional teacher development is still uncommon, especially in mathematics. In fact, although there is a wide choice of many different topics, when looking specifically for a MOOC aimed at mathematics teacher education the range is limited (Aldon et al., 2017; Avineri et al., 2017).
Nevertheless, there is a growing interest in MOOCs involving mathematics teachers as participants, as shown by TSG44 work during the 13th ICME[3]. Therefore, MOOCs for teacher education are on the verge of gaining a foothold.

In this dissertation, we expose two Italian experiences with MOOCs for mathematics teacher education. We will enter into details of these in the Chapter 3 (§ 3.2).

[3] For more information, see http://www.icme13.org/files/tsg/TSG_44.pdf

1.2.2 The need of a suitable theoretical framework

As mentioned, MOOCs for professional teacher development is still uncommon, especially in mathematics. Moreover, the specific intersection of MOOCs and professional teacher development is poorly researched (Taranto et al., 2017, a; Panero et al., 2017; Avineri et al., 2017).

A MOOC is a good environment to pursue two significant themes in research on mathematics teachers:

- to understand how teachers learn from this online experience;
- and if/how it influences teachers' knowledge, practices and beliefs.

Currently, research has not yet developed a framework to explain how learning takes place in these new totally online environments. There is little literature on MOOCs (Siemens, 2005; Downes, 2012 b; Ozturk, 2015; Teixeira et al., 2015) and even less on Mathematics MOOCs for teachers' education and development (Panero et al., 2017; Avineri et al., 2017*;* Taranto et al., 2017 a, b; Aldon et al., 2017). However, in mathematics education there are some theories on the way teachers can develop their professional learning in face-to-face environments and on the relationship between education and technology. Based on these, the dissertation writer, working with Arzarello and Goos, elaborated a suitable theoretical framework she referred to as MOOC's Zone Theory (Taranto and Arzarello, 2017), that allows for description of the teacher's participation in the MOOC and for the analysis of their consequent professional development and possible perception of changes in teaching practices.

This new theoretical framework (MOOC's Zone Theory) facilitates the study of the specific dynamics of the interactions among trainees and between trainees and trainers, which occur online and in totally virtual environments. It is topical and urgent to analyse these interactions in the context of such distance learning due to the increased interest in this approach in recent years. Consequently, the dissertation writer reviewed and revised an existing framework that had been used to describe face-to-face meetings for teacher professional development, namely the Meta-Didactical Transposition and she put in action specific strategies for connecting it with other theories, namely networking and hybridization.

As far as it is concerned with the study of the influence that a MOOC can have on teachers' knowledge, practices and beliefs, at the heart of this approach is the notion of *productive tensions* (Goos, 2013) between teachers' thinking, actions, and professional environments, and how such tensions can become opportunities for triggering teachers' professional change. It is important to stress already here that we will not show that substantial changes have occurred in teachers' knowledge, practices or beliefs. Instead, we will see how some germs of perception of change are shown. They are the germs of a possible change, of an influence that the MOOC has exerted on them and that this should be further investigated.

In the Chapter 2, we explain all the steps that we have taken to develop the MOOC's Zone Theory theoretical framework.

Before going into the exposition of the theoretical framework, which serves as a pillar to the dissertation, we refer to some literature that describes what is meant by teacher change.

1.2.3 Teacher change

High-quality professional development is a central component in nearly every modern proposal for improving education. […] Professional development programs are systematic efforts to bring about change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of students (Guskey, 2002, p. 381).

Although teachers are generally required to take part in professional development by certification or contractual agreements, most report that they engage in these activities because they want to become better teachers. They see professional development programs as among the most promising and most readily available routes to growth on the job (Fullan, 1991; 1993), as a pathway to increased competence and greater professional satisfaction (Huberman, 1995).
It is important to note that, for the vast majority of teachers, becoming a better teacher means enhancing student learning outcomes.

Learning outcomes are defined by Guskey (1985) as follow:

> "[they] include not only cognitive and achievement indexes, but also the wide range of student affective variables, such as, involvement in class sessions; motivation for learning; and students' attitudes toward school, toward the class, and toward themselves. In other words, learning outcomes include whatever evidence teachers use to judge the effectiveness of their teaching" (p. 58)

What attracts teachers to professional development, therefore, is their belief that it will expand their knowledge and skills, contribute to their growth, and enhance their effectiveness with students. According to this, when teachers see that a new program or innovation enhances the learning outcomes of their students, then, is significant change in their beliefs and attitudes likely to occur (Guskey, 1985).
But teachers also tend to be quite pragmatic. What they hope to gain through professional development are specific, concrete, and practical ideas that directly relate to the day-to-day operation of their classrooms (Fullan & Miles, 1992).

> Another important factor to consider is that professional development activities frequently are designed to *initiate* change in teachers' attitudes, beliefs, and perceptions. Professional development leaders, for example, often attempt to change teachers' beliefs about certain aspects of teaching or the desirability of a particular curriculum or instructional innovation. They presume that such changes in teachers' attitudes and beliefs will lead to specific changes in their classroom behaviors and practices, which in turn will result in improved student learning (Guskey, 2002, p.382).

In Guskey (1985; 2002) there are three important principles to consider when planning and implementing effective development programs.

1. *Change is a slow, difficult, and gradual process for teachers*.

Programs or innovations that are dramatically different from current practices or that require teachers to make major revisions in the way they presently teach are unlikely to be implemented well (Doyle and Ponder, 1977). To be successful, development must clearly illustrate how the new practices can be implemented without too much disruption or extra work (Sparks, 1983). Changes required of teachers should be organized and presented in small, incremental steps, and they should be described clearly and explicitly with emphasis on efficiency and practicality. Furthermore, it is best to begin with changes that are relatively modest but that can result in demonstrable student improvements in a fairly short period of time.

2. *Teachers need to receive regular feedback on student lean1ing outcomes.*

Practices that are new and unfamiliar will be readily abandoned. Procedures by which teachers can receive evidence of their efforts can be planned.

Stallings (1980) found that providing teachers with regular and precise feedback on student involvement during class sessions can also be powerful in facilitating new instructional practices. Thus, it is critically important that change efforts include some procedure for giving teachers regular feedback on learning outcomes. When teachers see that a new program or innovation works well in their classrooms, change in their beliefs and attitudes can and will follow.

3. *Continued support and follow-up are necessary after initial training*.

In most cases, some time and experimentation are necessary for teachers to fit the new practices to their unique classroom conditions (Joyce & Showers, 1982). Support during this period of trial and experimentation is critical. Teachers need continuous guidance and direction in order to make adaptations while maintaining program fidelity. New programs and innovations have been found to be most successful when teachers have regular opportunities to meet to discuss their experiences in an atmosphere of collegiality and experimentation (Little, 1981). For most teachers, having a chance to share perspectives and seek solutions to common problems is extremely beneficial. In fact, what teachers like best about in-service workshops is the opportunity to share ideas with other teachers. (Holly, 1982).

After making all these references to the literature on MOOCs, on teacher education and on teacher change, we can move on to the exposition of the theoretical framework. We remember that it represents a first result of the developed research, which will be illustrated here.

Chapter 2 Theoretical framework

2.1 Strategies for connecting theories

Mathematics education is established worldwide as a major area of study, with numerous dedicated journals and conferences serving national and international communities of scholars. Research in mathematics education is theoretically orientated: "vigorous new perspectives are pervading it from disciplines and fields as diverse as psychology, philosophy, logic, sociology, anthropology, history, feminism, cognitive science, semiotics, hermeneutics, post-structuralism and post-modernism" (Von Glasersfeld, 1995). As Von Glasersfeld declare,

> "[the] theory is the practitioner's most powerful tool in understanding and changing practice. Whether the practice is mathematics teaching, teacher education, or educational research, the [theory] will offer new perspectives to assist in clarifying and posing problems and to stimulate debate" (*ibid*).

In this order of ideas, it becomes important to study how different theories can be successfully linked, respecting the conceptual and methodological hypotheses that underlie each one of them. We must also take into account the fact that the theories in mathematics education are different and the generation of a new theory can allow a better understanding of the complexity of the processes of learning and teaching (Bikner-Ahsbahs & Prediger, 2010).To date, there are at least two strategies that allow connections between theories: the Networking Theories and the Hybridization of Theories.

The theoretical framework used in this dissertation (and that is itself one of the results of the dissertation writer's research) emerged precisely by making use of these strategies for connecting theories.

Therefore, we will pause for a moment in order to explain briefly what they consist of.

2.1.1 Networking of theories as a research practice

In the research in Mathematics Education, many different theories and theoretical frameworks have been developed. To deal with the diversity of theories, at CERME4 (Congress of European Research in Mathematics Education) in 2005, the Networking Theories Group was initiated. It gathered members from France, Germany, Israel, Italy, UK, and Spain and Angelika Bikner-Ahsbahs from Germany coordinated it. They started from the shared assumption that the existence of different theories is a resource for the research in Mathematics Education. Thus, rejecting the idea of merging all into one big theory, they considered the possibilities of connecting theories. The group has grown and networking theories has become a real research practice. We make reference to the volume edited by Bikner-Ahsbahs and Prediger (2014) to give the following definition:

"By networking, we mean research practices that aim at creating a dialogue and establishing relationships between parts of theoretical approaches while respecting the identity of the different approaches" (Bikner-Ahsbahs & Prediger, 2014, p.118).

They present a scale of networking strategies, according to the degree of integration (Figure 2.1). The extreme positions are ignoring other theories and unifying globally, which are not considered as strategies. Indeed, the former would mean that theories exist as isolated entities, without the possibility of learning from each other; whereas the latter would lead to have a whole huge theory, in contrast with the aim of maintaining the peculiarities of each component theory.

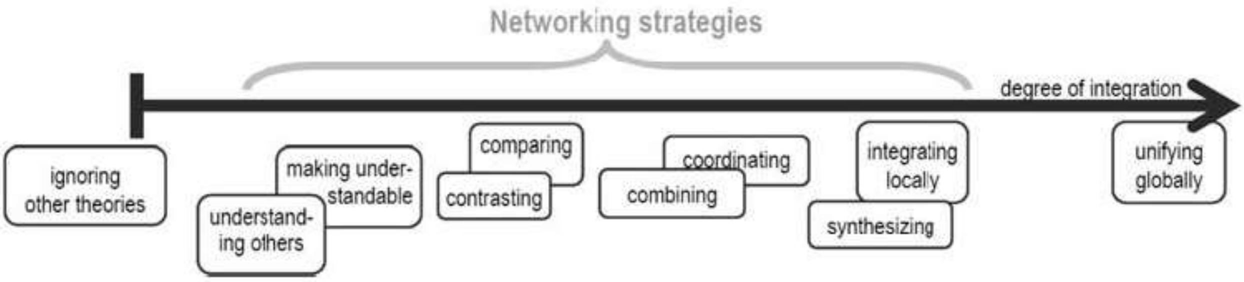

Figure 2.1: A landscape of strategies for connecting theoretical approaches (Bikner-Ahsbahs & Prediger, 2014, p.119).

Thus, networking strategies are defined as

"[...] the connecting strategies that respect on the one hand the pluralism and/or modularity of autonomous theoretical approaches but are on the other hand concerned with reducing the unconnected multiplicity of theoretical approaches in the scientific discipline" (Bikner-Ahsbahs & Prediger, 2014, p.119).

Among all possible networking strategies, we want to stress those that *combine* and *coordinate* theoretical approaches as well as the *locally integration* and *synthesis* for a networked understanding of an empirical phenomenon or a piece of data.
Combining and coordinating means looking at the same phenomenon from different theoretical perspectives as a way for going in-depth the phenomenon under analysis. In particular, the strategy of coordinating is used "when a conceptual framework is built by fitting together elements from different theories for making sense of an empirical phenomenon" (Bikner-Ahsbahs & Prediger, 2014, p.120).
Whereas the strategies of combining and coordinating mainly aim at deeper insights into an empirical phenomenon, the strategies of synthesizing and integrating locally are focused on the development of theories by putting together a small number of theoretical approaches into a new framework.

"We make a gradual distinction between the two related strategies which […] refers to the degree of symmetry of the involved theoretical approaches. The notion synthesizing is used when two (or more) equally stable theories are connected in such a way that a new piece of theory emerges. But often, the theories' scope and degree of development is not symmetric, and there are only some constructs or aspects of one theory integrated into an already more elaborate theory or converted and elaborated into another one. This integration should not be mistaken as unifying totally, which is why we emphasized the "locally" in the strategy's name integrating locally. We call a local integration symmetric if a concept at the border of two

> theories is worked out and integrated into both theoretical approaches. The latter may be further developed and result in synthesizing” (Bikner-Ahsbahs & Prediger, 2014, p.120).

For example, in the years it has been expressed the need to explain what a theoretical framework suitable for framing didactical phenomena from a theoretical point of view. The research of Luis Radford has given an important contribution to this issue.
Radford (2008) explains that a theory T (or theoretical framework) can be considered as a way of producing understanding and schemes action based on:
• a P system, of fundamental principles [expressed in L language];
• an M methodology, which includes techniques of collection and data interpretation;
• a Q set, of the paradigmatic research questions.

In particular, Radford uses the notion of semiosphere elaborated by Lotman (1990), to introduce a strategy of connection among theories. According to Lotman, the semiosphere is “an uneven multi-cultural space of meaning-making processes and understandings generated by individuals as they come to know and interact with each other” (Radford, 2008, p. 318). The fundamental characteristic of the semiosphere is to be a global semiotic space, which in its unity makes meaningful every “sign” act (text, fragment of language, etc.). It is a large single space, capable of manifesting a semiotic homogeneity. In this context, the role of a meta-language connecting two or more theories is not to erase them through uniform assimilation. Rather, it is to ensure possible forms of connecting different heterogeneous elements.

> “The challenge in the constitution of the semiosphere’s meta-language is precisely this: it should be general enough so that the theories are genuinely objects of discourse, without, at the same time, being too abstract and losing sight of the particularities of its theories. Success in the constitution of the semiosphere may reside, I want to suggest, in the dialectical tension between its plots of identity and integration” (Radford, 2008, p. 321).

The semiosphere is therefore the space in which the process that allows the generation of a new theory takes place, basing on some theories already known.

2.1.2 The phenomenon of hybridization and its methodological consequences

A variant of the networking model that highlights a similar process, but not completely included in networking, is the hybridization model. It was conceived by Ferdinando Arzarello who presented it to the Xth seminar of the young researchers of the ARDM (Xème séminaire des jeunes chercheurs de l'ARDM) in Lyon on May 7-8, 2016 and to the Seminar for the 65th birthday of A. Bikner-Ahsbahs in Bremen on September 2, 2017.
As we have seen, networking starts from the assumption that different Mathematics Education theories are used to study the same problem, possibly producing different levels of combination / integration of theories (Bikner-Ahsbahs & Prediger, 2014).
Moreover, these theories examine the same teaching-learning episode with the aim of:
(i) investigating the complementarity that may result from the study of the same set of data from different points of view;
(ii) verifying if and how the fundamental aspects, identified by a theory and a set of data, can be preserved in another theory.

Instead, hybridization occurs when a possibly even not very narrow fragment of a theory is introduced in a *coherent*, *operative* and *productive* way within the framework of another theory (this three terms in italics will be explained later in the following). Thus, an hybrid theory is obtained.
The term "hybridization" is taken from the agronomy, and refers to the intersection of populations, races or botanical varieties within the same species. In this case, hybrids are commonly produced and artificially selected, because they have appropriate characteristics and are not present, or present in a very limited way in the generation of pro-parents.
It is emphasized, therefore, that through the term "hybridization" we do not want to give any negative meaning: on the contrary, we want to make us think of the fact that it represents a richness from which the different theories involved can benefit.

More precisely, in the hybridization typically there is a basic T_0 theory in which the researcher works, and a T_1 theory, from which he extracts a bigger or smaller fragment F_1. This is adapted and integrated into the T_0 theory, which becomes an hybridized T_{01} theory. The T_1 theory can also be far from T_0 or weakly linked to the problems studied by T_0.

Arzarello (2016; 2017) specifies that the steps with which a hybridization is performed are (Figure 2.2): connection, interpretation and adaptation.

- **Connection**: in general, a process of hybridization of a T_0 theory with an element of a T_1 theory begins by establishing a connection between the two theories, which depends on at least two parameters: (1) the structure of the two theories; (2) the goal of the connection.
- **Interpretation**: an F_1 fragment of the T_1 theory expressed in the L_1 language, is interpreted into the T_0 theory (expressed in the L_0 language). The L_0 language of T_0 is thus extended (or modified) in a L'_0 language: the interpreted fragment becomes F'_1 (expressed in L'_0).
- **Adaptation**: it is a modification of the T_0 theory in order to link the interpreted F_1 fragment to the different components of the T_0 theory. A new theory is obtained, that is the T_{01} hybridized theory. The interpreted fragment may consist of a more or less important modification, possibly including the elimination of old T_0 components. The issue of consistency rules the adaptation and it is a necessary condition that allows the hybridization process.

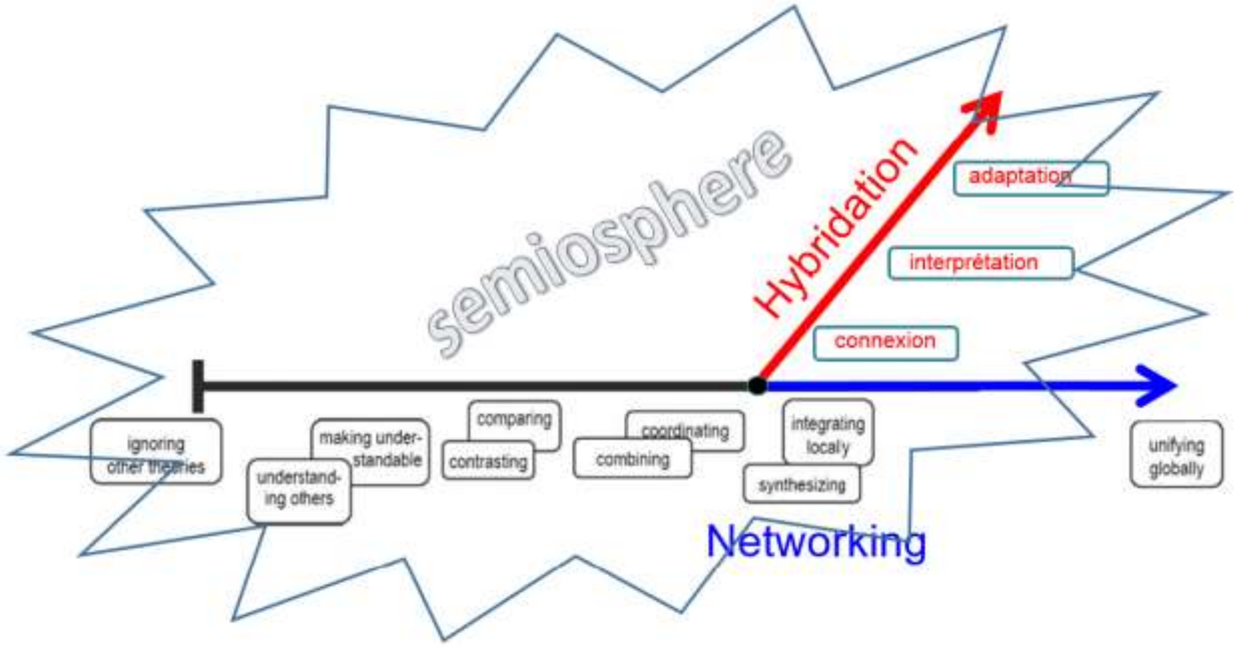

Figure 2.2: Hybridization strategies by Arzarello

We remember that the hybridization model is a variant of the networking model that highlights a similar process, but not completely included in it. In fact, a result of this connection (which would still fit into the "coordinating/combining" category of Bikner-Ahsbahs & Prediger (2014) for the Networking) may serve at the beginning for a better understanding of one's own theory. However, the follow up of this "understanding" can be different and genre two types of narration:

- *The narration of unveiled identity*, proper of networking, that takes place when one becomes aware of the fact that in the starting theory there exists a part, an aspect, which had not become conscious before and which now appears as unveiled (or "extradite", as Radford (2008) says).
- *The narrative of the change of identity*, proper of hybridization, that takes place when one realize that his theory gives an answer that is partially satisfying to the research question and therefore he can import from another theory some new fragments that modify the principles and methodology of his theory in an operative, (more) satisfactory but also coherent way (Arzarello, 2016; 2017).

As said at the beginning, there is an hybridization, instead, when a possibly even not very narrow fragment of a theory is introduced in a *coherent*, *operative* and *productive* way within the framework of another theory.
The first two terms are explained by Radford (2008, pp. 321-322) as follows: *operability* means that the methodology must be able to produce and deal with the data in such a way that ''satisfactory'' answers to the research questions are provided. ''Satisfactory'' answers may rest on e.g. statistical methods, interviews, discourse analyses, classroom episodes, etc. *Coherence* means that the rhetoric of the argumentation of the methodology (be it statistical, discursive or other) is consistent with, and rests on, the chosen principles. The research questions must be clearly stated within the conceptual apparatus of the theory.
Arzarello (2016; 2017) uses the word *productive* basing on the work of Wertheimer (1945). Namely, one gets an hybridization when not only the added fragment is coherent and

guarantees concrete operability, but also if it allows further insights and breakthroughs to the problems that the old theory was not able to focus properly: the hybridized new theory allows a true understanding that the old theory was not capable to give.

Hybridization generates a new theory, where the principles and methodology (both or only one) have changed and the results of the analysis give more satisfactory answers to research questions (which do not change).

2.1.3 Is it possible to apply networking and hybridization strategies at the same time?

In a very simplistic way, to briefly compare the two strategies described above, we can say that:

- Networking considers *a theory* "*as it is*" and interweave it at different levels with another, by elaborating a new theoretical framework with a suitable language;
- Hybridization considers *a particular component of a theory*. This is "implanted" in another theory that, for this reason, will be hybridized: its old theoretical framework is so enriched and the language as well.

The theoretical framework presented in this dissertation, the ***MOOC's Zone Theory*** framework, is the result of successive combined hybridization and networking processes. In the next section, we will explain in detail all the theoretical lenses considered and the reasons for these choices. However, we are already starting to mention them, showing operationally how the theoretical framework has been developed.
The theories that have been taken into consideration are: the Meta-Didactical Transposition or MDT (Arzarello et al., 2014), the Instrumental Approach (Verillon & Rabardel, 1995), the Connectivism (Siemens, 2005) and the Zone Theory (Valsiner, 1997; Goos, 2005, 2013).

In the following we explain schematically how the hybridization and networking steps took place. We also recommend looking at Figure 2.3 to get a clearer idea. The due and specific theoretical details, as well as the theoretical justifications have led us to consider one theory rather than another are referred to the next section.

Drawing on the practice of the hybridization of theories, it is assumed as T_0 the theory of the *Meta-Didactical Transposition* (or MDT). The first F_1 fragment is the process of *instrumental genesis* of the Instrumental Approach, grafted into MDT because, considered in its original elaboration, this theory alone seems inadequate to describe the MOOC. Thus, a T_{01} theory is obtained, but it is not yet satisfactory for describing the typical dynamics of the MOOC. A second moment of hybridization on the already hybridized T_{01} theory is then performed. This time a new F_2 fragment is considered, the concept of *network of knowledge*, taken from Connectivism. This new graft gave birth to the theory T_{012}, which we called ***MOOC-MDT***. Within it, a meta-language has been developed not for globally unifying the theories, but for connecting them (we will see in the following of this chapter).
At this stage, two moments of *narration of the change of identity* took place. Yet this obtained theory was still lacking in something. In fact, it is not able to frame all the facets that a MOOC brings with it. Something is still missing.

Reading the works of Goos (2005, 2013), which uses the socio-cultural framework of the Zone Theory, had the effect of a *disclosure*[4] (in the sense of Rota) on the dissertation writer. She felt the need to try to interpret the phenomenon also considering this further theoretical lens. Finally, all the aspects that we wanted to analyse were covered.
Therefore, a networking between the MOOC-MDT and the Zone Theory was developed, applying coordinating, combining and integrating locally strategies (Bikner-Ahsbahs & Prediger, 2014). A *narration of the unveiled identity* has thus occurred. Analysing the data with both theories, as we will see, gives complementary results: with a lens, we will be able to see a piece; with the other lens the other piece.
The following figure 2.3, as previously mentioned, schematizes the strategies for connecting theories that are put in place.

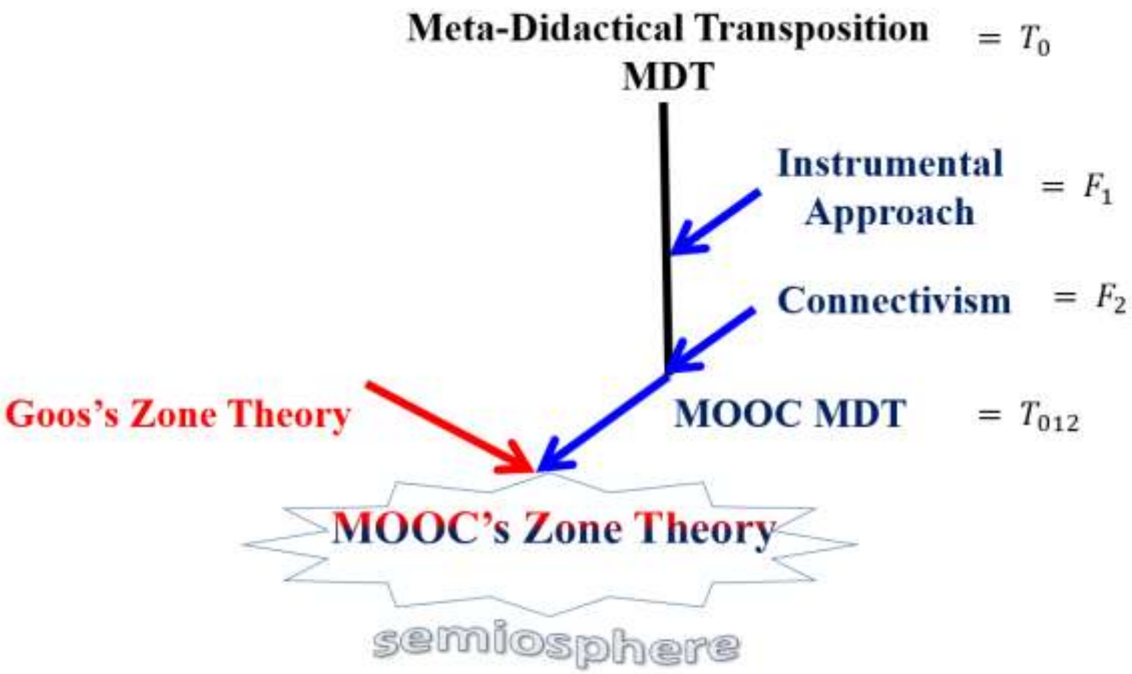

Figure 2.3: The hybridization and networking strategies that originate MOOC's Zone Theory

In the next section, we present each of these theoretical tools and justify their connection. The integration of all of these theoretical elements give us the possibility to identify and to describe the dynamics in which are involved both trainees and trainers in the dynamic and complex MOOC environment.

[4] I take this concept from the phenomenology framework. For example Rota, who adapted the phenomenological lens to mathematics, rise that it means "seeing and being able to see in mathematics". Namely, to give reason of mathematical understanding. It indicates the process by which people make sense of the world and of the situations in context to which they are exposed. For further details, see Appendix A.

2.2 MOOC's Zone Theory theoretical framework: the step made to create it

In this section, we will explain the points we have considered for framing in a theoretical way a MOOC for mathematics teacher education and the consequent rationale of our new theoretical framework. Therefore, we present each of the theoretical frameworks/lenses we have taken into consideration, justifying their connection. The integration of all of these theoretical elements give us the possibility to identify and describe the dynamics in which are involved both trainees and trainers in the complex-dynamic MOOC environment.

2.2.1 Theoretical framework/lenses to know

The MOOC's Zone Theory framework has been outlined through a delicate and painstaking work of hybridization and networking between different educational theories, some of which specifically of mathematics education.

The theoretical frameworks/lenses to which we refer are:

1. Meta-Didactical Transposition or MDT (Arzarello et al., 2014; Aldon et al., 2013);
2. Instrumental Approach (Verillon & Rabardel, 1995; Trouche, 2004);
3. Connectivism (Siemens, 2005; Downes, 2012 a);
4. The Zone Theory of Valsiner (Valsiner, 1997) that was adapted by Goos (Goos, 2005).

In the following, we will focus on all of them, but first here there is a remind of how the new framework was originated.

Two **hybridization** occurs: first between MDT and Instrumental Approach, after between these and Connectivism giving rise to the so-called MOOC-MDT.
Later, a **networking** regards the MOOC-MDT with the Zone Theory, creating a new theoretical framework that we will call the MOOC's Zone Theory.

2.2.2 Considered theories and their adaptation to comprise the MOOC environment

In this section, we will focus to recall the four theories listed above.

We are about to enter a very dense chapter of theories. Reading will be guided. The description of each theoretical element will be followed by observations (sometimes in *italic* other times with a specific subchapter) to allow us to grasp how that is an aspect present or not in the MOOC environment. These will be, therefore, the justifications that led us to adapt that specific exposed theoretical element to comprise the MOOC environment. There will be several moments when we "summarise the discourse" to allow the reader to better internalize what has been explained up to that moment. A series of metaphors will also accompany the reading, both to take a breath between one concept and another, and also to allow a better understanding of them.

We will start with the Meta-Didactical Transposition, explaining this theoretical framework in detail. It is our starting point, as well as the theory on which the two concatenated hybridizations are made. We will go faster on the Instrumental Approach and Connectivism, because we will take into consideration only some theoretical fragments of these theories. Precisely, in the Instrumental Approach, we will consider the *instrumental genesis* that implies the concepts of artifact and instrument. In the Connectivism, we will consider the *connections* from which the *network of knowledge* derives, and the *chaos* and the *self-organization* to whom is linked the connectivist meaning of *learning*. All of these fragments are useful for making the hybridizations effective. Thus, we will show the MOOC-MDT theoretical framework. Subsequently we will explain in detail the Valsiner's Zone Theory, adapted by Goos. This latter will be networked with the MOOC-MDT giving life to the new theoretical framework: the MOOC's Zone Theory.

2.3 Meta-Didactical Transposition

Meta-Didactical Transposition or, here and after, MDT is a theoretical model conceived to describe and analyse the relationship and reciprocal influence between two communities - the community of researchers and the community of teachers – involved in a traditional course in mathematics education for professional development, with respect to their professional practices.

With the adjective "traditional" we mean frontal or blended lessons, limited to a specific group of people. This course could be an educational programme of mathematics, or technology integrated in teaching mathematics, or other at national or local level.

The MDT framework was introduced first in Italy (Arzarello et al. 2012), after in the international community (Arzarello et al. 2014) and then spread out on various occasions (PME, cross-countries seminars, ICME, CIEAEM).

It is based on the Chevallard's Anthropological Theory of Didactics (Chevallard, 1985; 1992; 1999; Bosch & Gascón, 2006), which is focused on the teaching of mathematics at institution, and extends it to the context of teacher education, usually fully situated within and constrained by the institutions, because of:

- The constraints imposed by the institutions (including Schools, Universities, policy makers, teachers' associations, Mathematics Society, and Ministry of Education) that promote teacher education in view of some specific goals (e.g. promoting teachers' knowledge of new curricula, new teaching practices, or new technologies to be integrated in the teaching of mathematics);
- The complexity of mathematics teachers' professional development situated in the institutions and involving teachers' and researchers' communities, and the dialectics between the two communities;
- The professional development considered in a dynamic way, taking into account the process and giving sense of evolutions in the practices of members of the communities (the researchers and the teachers).

In fact, with reference to this last point, it is important to underline that the MDT is able to

> "captur[e] the professional development phenomena in a dynamic way, as they occur in the process and not only giving some pictures of it at certain moments. Teachers approaching a new didactical paradigm during an educational programme are not the same as they were at the beginning, nor at the end of the programme. They simply evolve, changing ideas, viewpoints, practices, or only awareness on some contents they encounter in the programme. On the other side, also researchers may evolve and change some of their practices and awareness. The event of encountering of the two communities is not neutral: it gives effects in

> both the communities. A framework that renders important this evolution is a framework that takes into account the professional development as a process, not only as a product, and describes it in a dynamical way (as a movie, not as a picture)" (Robutti, in press).

Despite the fact that this is just an introduction to the MDT theoretical framework, it is useful to consider in the analysis of a MOOC for teacher education. In fact, in a MOOC there are two communities: the trainers' community (researchers and teacher-researchers that collaborate to the design and manage the online course) and the trainees' community (teachers that are enrolled in the MOOC) – Figure 2.4. These two communities are involved in a professional development process that, as we will see, has some effects on both of them. The main and biggest difference that emerges immediately is that the MDT is a framework designed to describe teacher education in face-to-face courses. To be able to use it in the analysis of a MOOC, it will be necessary to make some adjustments.

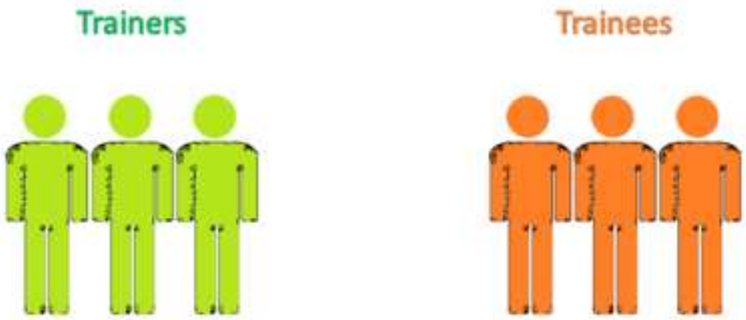

Figure 2.4: The two communities involved in a MOOC: trainers and trainees

It might seem like a minor thing, but we ask you to pay attention to the fact that green has been chosen to identify the community of trainers and orange for the community of trainees. Keep these colours in mind because they will accompany us throughout the dissertation.

2.3.1 The MDT framework adapted to MOOC

The MDT involves these intertwined features:

1. The institutional aspects,
2. The meta-didactical praxeologies,
3. The dynamics between internal and external components,
4. The role of the broker,
5. The double dialectics.

Now, we explain below these 5 main points of the framework and simultaneously explain the changes made to them in relation to the MOOC. In fact, at the end of each of them, we will make – sometimes in *italic* other times with a specific subchapter – some considerations to understand how a certain characteristic is maintained or cannot be maintained when one consider as a program for professional development no longer a traditional course, but a MOOC[5].

[5] See in §3.2.3.6 (Chapter 3) the difference between a traditional course and a MOOC for teacher education.

2.3.1.1 The institutional aspects

The Chevallard's Anthropological Theory of the Didactic, shorted in ATD from now, focuses on the institutional dimension of mathematical activity: "La Théorie Anthropologique du Didactique situe l'activité mathématique, et donc l'activité d'étude en mathématiques, dans l'ensemble des activités humaines et des institutions sociales" (Chevallard, 1999, p. 223). Hence, it places mathematical learning and teaching in the human activities related to it and in the context of social institutions.

In the MDT framework, the teachers and researchers' communities involved in the MDT process, are subjects within a certain institution. Teachers belong to the actual schools where they teach, and researchers refer to the School as a higher institution that decides curricula, has particular teaching traditions, produces textbooks, and so on. Indeed, when the researchers in Mathematics Education come into contact with the teachers' community hold simultaneously two different positions. They belong to the university or the department where they work, but in that particular occasion they act as teachers' educators.

The trainers and the trainees involved in a MOOC for teacher education are also subject to the same institutional aspects. The MOOC itself is designed according to institutional aspects: it lasts a certain number of weeks, has deliveries in terms of tasks and deadlines that must be respected.

However, the trainees - in addition to the presence of the trainers who generally are vigilant but not invasive - are not subjected to the same institutional weight that the school or the headmaster instead have for them.

In fact, we observe that the MOOC is open. It means it is open (and free) to anyone who wants to enrol. The number of members is massive. The participants never meet in person each other, they do not have space or time constraints: they can access to the MOOC whenever they want and wherever they are. The enrolled teachers are not required to finish the course.

So, inside the MOOC, the trainees' community feels the institutions as something distance, which intervenes only a little in the MOOC dynamics. For these reasons the trainees' community feels that it must not give explanations for its actions and feels itself also free to express in the space for communication (in the Chapter 4, devote to the analysis, this sentences will be clarified).

The notion of didactical transposition

At the core of ATD are the notions of didactical transposition and praxeology. We will concentrate on the first in this paragraph; we will stress the second one in the next paragraph.

A piece of knowledge, as stated by Chevallard (1992), is a particular category of objects which can be learnt, can be taught, can be used, but first of all has to be produced. For each piece of knowledge ***S*** (that stands for *savoir*) Chevallard (1992) considers the associated institution ***P(S)*** which produces ***S***. Therefore, a certain piece of knowledge initially lives in its natural habitat that is ***P(S)***. Its presence in some other institution ***I*** presupposes that a sort of "transport" from ***P(S)*** to ***I*** has occurred. Chevallard calls it *institutional transposition* of ***P(S)*** in ***I***. If we consider the school associated with ***I***, then the process of the school reconstruction of ***S***, starting from what ***P(S)*** has produced is called *didactical transposition*. Reflecting upon it, Bosch & Gascón (2006) state that

> "[the didactical transposition] formulates the need to consider that what is being taught at school (contents or knowledge) is, in a certain way, an exogenous production, something generated outside school that is moved – "transposed" - to school out of a social need of education and diffusion. For this purpose, it needs to go through a series of adapting

transformations to be able to "live" in the new environment that school offers [...] The process of didactical transposition then starts far away from school, in the choice of the bodies of knowledge that have to be transmitted. Then a clearly creative type of work - not a mere "transference", adaptation or simplification - follows, namely a process of de-construction and rebuilding of the different elements of the knowledge, with the aim of making it "teachable" while keeping its power and functional character" (p.53).

Therefore, there is "original" or "scholarly" mathematical knowledge as it is produced by mathematicians or other producers (Figure 2.5). Then, it is transformed in the knowledge "to be taught" as it is officially designed by curricula. The responsible for the first step of the transpositive work (from scholarly knowledge to knowledge to be taught) are the agents composing the "noosphere"[6], which organize and disseminate the knowledge to be taught through the production of official programmes, textbooks, recommendations to teachers, didactic materials, etc. Afterwards, there is the mathematical knowledge as it is actually taught by teachers in their classrooms and the mathematical knowledge as it is actually learnt by students. Mathematical knowledge in each of these steps is subjected to a transposition, operated firstly by the noosphere, then by the teachers, and finally by the students.

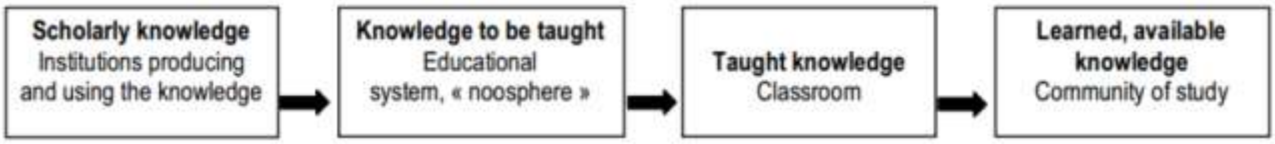

Figure 2.5: The didactical transposition process (in Bosch & Gascón, 2006, p.56)

The meta-didactical transposition

The Meta-Didactical Transposition (Arzarello et al. 2012; Aldon et al. 2013; Arzarello et al. 2014) framework places mathematical and professional learning - of teachers working together - in the human activities related to it and in the context of social institutions. The framework of MDT is useful to describe a process – analogous to the didactical transposition – that occurs when a *community of researchers* work with a *community of teachers* in a professional development activity:

- The researchers design and coach the educational programmes, as a task commissioned by institutional authorities (e.g., School administration, Ministry of Education, teachers' associations), or as a course planned by other institutional authorities (University, Research centre, Mathematical association, international project, or others). The programme can be configured - for example - as a teachers' professional development only, or as a research project where to collect and analyse data, or a dissemination of a research project.
- The teachers participate into the programme, either on a voluntary basis or because of an official duty.

As previously stated, these both communities are in relationship with the school: the actual schools where the teachers teach, and the School as an institution with its curricula, its teaching traditions, the textbooks used, etc.

[6] The noosphere is defined as the "sphere of those who think about education" (Bosch & Gascón, 2006, p.52) or more precisely "a plurality of agents [...] including politicians, mathematicians ('scholars') and members of the teaching system (teachers in particular)" (Bosch & Gascón, 2006, p.53).

The MDT is useful in describing and analysing the evolution of teachers' and researchers' practices over time. The term "meta-didactical" refers to the fact that important issues related to the didactical transposition of knowledge are faced at a meta-level.

Having clarified what is meant by didactical and meta-didactic transposition, even in a MOOC the purpose of developing such a meta-didactic transposition remains unchanged. In other words, the trainers have the objective of transposing a certain piece of knowledge, related to the teaching and learning of mathematics, to favour the professional development of the trainees, according to the reference institutions (national curricula, textbooks, ...). This trainers' knowledge to be transposed is the result of their research work, enriched by the comparison with the international research community in mathematics education, or by working in collaboration with teacher-researchers.

2.3.1.2 The meta-didactical praxeologies

Before focusing on the meta-didactical praxeologies, we take a moment back to the notion conceived by Chevallard.

Chevallard's notion of praxeology

The main theoretical tool of the ATD (Chevallard, 1999) is the notion of *praxeology*, a neologism made of two words derived by the Greek terms *praxis* and *logos*. *Praxis* as the "know how", includes different kinds of problems to be studied as well as techniques available to solve them. *Logos* as the "knowledge" or the "know why", includes the "discourses" that describe, explain and justify the techniques used and even produce new techniques (García et al., 2006).

According to Chevallard, a praxeology consists of four interrelated components: task, technique, technology (used to mean justification) and theory. The given task and the technique used to solve the task are the practical counterpart of the praxeology (the *praxis*), while the technology (in the sense of justification) and the theory are the theoretical counterpart that validates the use of that technique (the *logos*). In a mathematics classroom, we can identify the mathematical type of task (for example, *T*: calculating the value of a function *f* in a given point – Table 2.1; or *T*: solving a given second-degree equation – Table 2.2) that the students have to solve, the employed technique and the more or less explicit justification for using it, all within a specific mathematical theory. These components constitute the *mathematical praxeology*.

Task	Calculating the value of the function $f(x)=x^2-x+1$ in the point $x_0=2$.
Technique	Substitute x with 2 in the analytic expression of f, obtaining $f(2)=2^2-2+1= 3$
Technology	The point $(x_0; y_0)$ lies on the curve of equation $f(x)$ if $f(x_0) = y_0$.
Theory	Properties of the algebraic curves.

Table 2.1: Example of mathematical praxeology

Task	Solving the second-degree equation $x^2-4=0$.
Techniques	Factor the equation, obtaining $(x+2)(x-2)=0$. Set each factor that has an x term equal to zero, obtaining $x+2 = 0$ and $x-2=0$. Solve each equation for x, obtaining $x=-2$, $x=2$.
Technology	The zero product rule: if you have an equation with two first degree terms whose product (factors) equal zero then either the first factor is equal to zero or

	the second factor is equal to zero.
Theory	Field theory.

Table 2.2: Example of mathematical praxeology

At the same time, there are the teacher's questions and actions to build such a mathematical praxeology with her students, which gives birth to a *didactical praxeology*. What may occur is:

- The teacher introduces her students to a type of *task T* (didactical type of task);
- The teacher has to manage *how to* organise such an approach (didactical technique);
- The teacher has to know *why she has to* organise it like that (didactical technology – in the sense of justification);
- The teacher has to explain *why she knows that she has to* organise it like that (didactical theory).

The notion of meta-didactical praxeology

In the Meta-Didactical Transposition framework there are the *meta-didactical praxeologies*, which consist of the tasks, techniques, and justifying discourses that develop during the process of teacher professional development. As in the previous case, what may occur is:

- The researchers – as trainers – introduce the teachers – engaged in the professional development activity – to the type of *task T* (meta-didactical type of task);
- The researchers have to manage *how to* organise such an approach (meta-didactical technique);
- The researchers have to know *why they have to* organise it like that (meta-didactical technology – in the sense of justification);
- The researchers have to explain *why they know that they have to* organise it like that (meta-didactical theory).

For a schematic view of the mathematical, didactic and meta-didactic praxeologies, see the Figure 2.6.

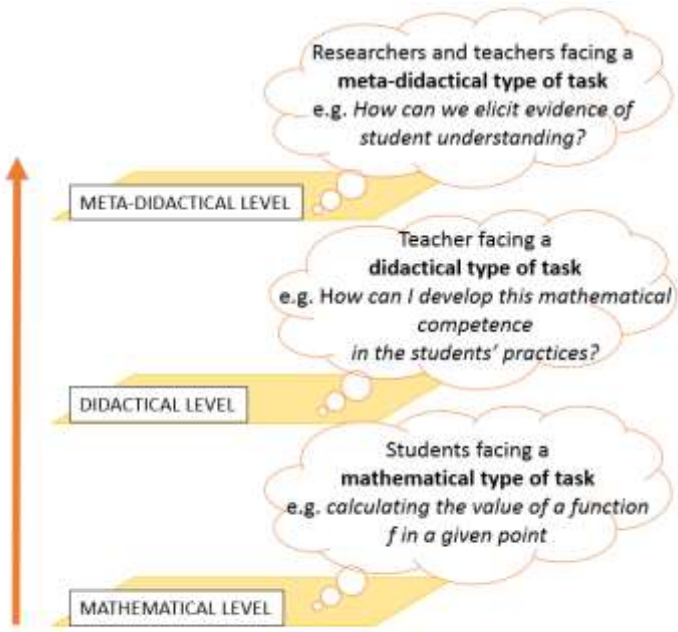

Figure 2.6: Mathematical, didactical and meta-didactical praxeologies

Note that there is a thin line that differentiates the concept of technology from theory, as conceived by Chevallard. In this dissertation we will refer to this concepts as a unique one, referring to them as *logos* or justification, in the same way Arzarello et al. (2014, p. 353) do it.

Consider the following example, taken from Arzarello et al. (2014):

> "in the teacher training course described by Sullivan (2008, p. 3), there is the question "which is bigger: 2/3 or 201/301?", in order to prompt teachers for ideas that might be used as the basis of a lesson. The discussion with the teachers made evident at least three points of view, according to which one can answer the question: the mathematics knowledge, the knowledge specific for teaching and the pedagogical knowledge. According to such knowledge, specific interventions could be designed to introduce the students to the task, e.g. to think of baseball statistics: if a player passes from 200/300 to 201/301 his score increases. This can be considered as an example of a *meta-didactical praxeology* in that the task is stimulating the teachers' reflection, and the meta-didactical techniques are those that Sullivan used in the course to promote discussion. During this discussion, it is possible that the two communities of mathematics educators and teachers, respectively, shared a common theoretical framework, which would justify the techniques being discussed. For example, based on one's professional experience, the teachers might discuss why the initial question presents difficulties for many students and why the baseball example makes sense in a classroom and thus help overcome these difficulties. Moreover, the teachers may scaffold their arguments within specific pedagogical discourses: e.g. stressing the necessity to foster the transition from every day to scientific and formal concepts, using constructivist approach, according to a Vygotskian frame. The theoretical side of the *meta-didactical praxeology* also includes the reflection made by Sullivan on the possible reasons why the activity was a good illustration of the way teachers can become aware of Mathematical Knowledge for Teaching (Sullivan, 2008), an aspect that may have been highlighted within Sullivan's exposition" (p. 353-354).

Therefore, the task can be stimulating the teachers' reflection, and the relative technique can be the collective discussion. In so doing, it is possible that the two communities of educators and teachers come to share a common theoretical framework that justifies the mathematical tasks, techniques and argumentation under scrutiny. The discussion is about the didactical praxeologies that different teachers can adopt dealing with a specific mathematical problem. For this reason, the described praxeology occurring at a meta level is called a meta-didactical praxeology. As Arzarello et al. (2014) observe:

> "[A meta-didactical praxeology is] the result of the interaction between the reflections of the community of researchers about the didactic praxeologies previously designed and developed, and the concrete practices used by the teachers in their professional activities" (p.354).

If in the *didactical transposition* the focus is on the activities to be solved in the classroom by the students, in the *meta-didactical transposition* the focus is on the activities developed by teachers in their professional development, working together and using practices and the corresponding theoretical reflections. Of course, they are the result of the interaction between the reflections of the community of researchers – as trainers - about the didactical praxeologies previously designed and developed, and the concrete practices used by the teachers in their professional activities (referring here to in-service teachers).

Different kind of meta-didactical praxeology

At the beginning of a meta-didactical transposition process, as we have seen, two distinguished communities are identifiable: that of researchers (in the role of designers and educators), and that of the teachers (in the role of teachers-students). Within these two communities, in the MDT framework, there is the distinction between two kinds of praxeologies: the *researcher praxeologies*[7] and the *teacher praxeologies*. These two kinds of praxeologies, typical of the two communities, at the beginning of a professional development programme a priori can be very different.

These programmes generally aim – with the engagement of researchers as trainers – at developing teachers' existing praxeologies, transforming them into new ones, for example targeted to the introduction of new technologies, or teaching practices, or theoretical frames by research in mathematics education, or new curricula, and so on, according to the aims of the programme.

For example, from the discussion about different techniques to address a problem, the teachers can acquire new ones, with a suitable theoretical justification, thus replacing/integrating old techniques and their theoretical support. This evolution in the praxeologies is the result of an interaction between the community of researchers and that of teachers.

If the teachers' praxeology evolves towards a new one, closer to a researchers' praxeology, it is called s*hared praxeology* (Arzarello et al., 2014). A typical example is when the new praxeology is developed in response to changes in the official curriculum or in external assessment expectations for students.

A shared praxeology can have effect also on the initial praxeologies of the two communities: the teachers can return to their classroom with new teachers' praxeologies and the researchers can come back to think about the training phases and redesign them, developing new researchers' praxeologies. In fact, speaking of evolution over time of teachers' praxeologies does not mean that the researchers' praxeologies cannot evolve: they may change, because of the interaction with the teachers, and also reflecting upon the experience of the educational programme, particularly on the nature of, and reasons for, the changes produced by the teachers' programme. This means that both the teachers' praxeologies and the researchers' praxeologies may evolve in new ones, converging (to a shared praxeology) or diverging to different ones.

It is important stress what follow, as underlined by Robutti (in press):

> "Evolution in the praxeologies does not mean that all the teachers, involved in the educational programme, evolve in the same way with the same transformation of components: in fact, different teachers may evolve in different ways, with respect to their history and experience. Therefore, further research is necessary to investigate the factors that influence these different trajectories in the praxeologies"[8].

The meta-didactical praxeologies of the two communities change within the institutional environment in which they reside. As a consequence, there could be teachers' development of both a new awareness (on the cultural level) and new competencies (on the methodological-

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[7] Of course, there may be more than one praxeology referring to researchers, as well as referring to teachers: in the text we will use either singular or plural (researchers praxeologies; teachers praxeologies). In particular the researchers have their own praxeologies as researchers, which concern the praxis and the logos of their researches; but they have also their praxeologies as teachers' educators, where the praxis and the logos concern the concrete way they coach these activities, because of their theories about teachers' educational processes (Arzarello et al., 2014, p. 354).

[8] In this dissertation, the networking between the MOOC-MDT and the Goos's Zone Theory goes in this direction. Through the analysis of some case studies, we will try to better understand what evolutions and why can concern a teacher who follows a MOOC.

didactical level, i.e. that of teaching practice), which lead them to activate, in their classrooms, a didactical transposition in line with the meta-didactical transposition.

The Figure 2.7 schematically illustrates the actors and the dynamics in the MDT process.

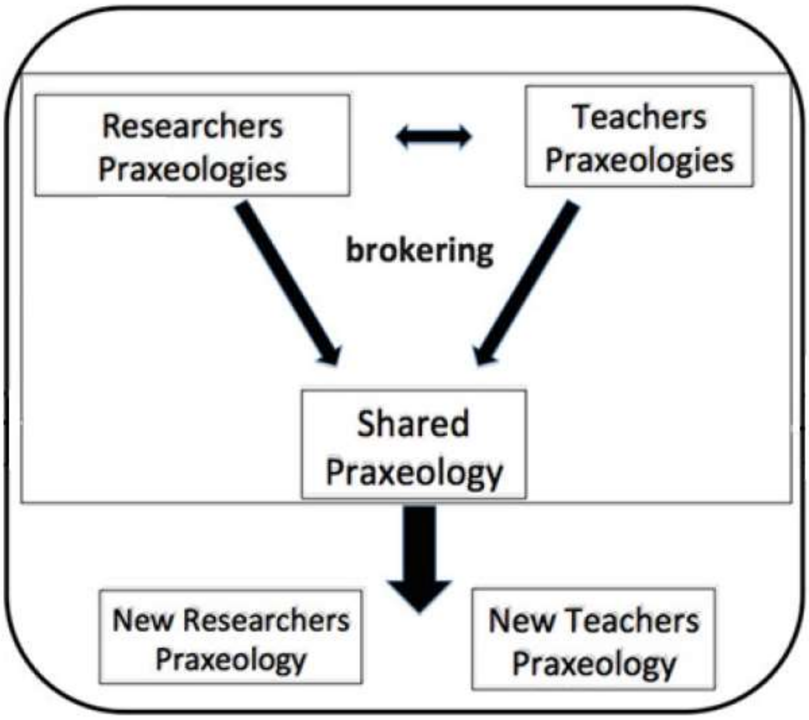


Figure 2.7: The Meta-Didactical Transposition model (in Arzarello et al., 2014, p. 355)

Let us reflect on the praxeologies in the MOOC environment

We have explained in detail what the didactical praxeologies are and that these belong to each teacher. We have explained, in parallel, what the meta-didactical praxeologies are and that are put in place by the researcher who holds the training course, in order to stimulate a change in the teacher's didactical praxeologies. Thus, during a training course, both researchers and teachers participate with their respective praxeologies.

These theoretical concepts also apply to trainers and trainees in a MOOC for teacher education. However, some clarifications must be made.

2.3.1.3 Complexity to consider

The heterogeneity of the trainees

Let us start by considering the trainees, namely the teachers who have enrolled in the MOOC, on a voluntary basis. A general consideration can be made (which also applies in particular to our MOOC experiences): each trainee will have his own praxeologies, which we will call *initial praxologies* (because they are the ones with which they enter the MOOC). All the trainees have different geographical origins, they teach in different educational levels[9];

[9] unless the MOOC declares that the target of teachers to whom it is addressed must teach at a specific education system (primary school, lower or high secondary school). Anyway, this is not a rule: remember that the MOOCs are open. For example, in our case, although it was specified that the MOOC was aimed at secondary school teachers, primary teachers also took part in it.

therefore in general they have a *different professional background*. Moreover, although the MOOC starting date is the same for everyone, *it does not necessarily mean that all teachers, at the time they participate, are at the same point in the program*. They may have already covered some of the topics seen in the MOOC, they may still have to deal with them.
In a face-to-face training course, it is less likely to deal with this *heterogeneity of the trainees' community*. Additionally, any differences in the development of the classroom program are smoothed out because all the teachers (that of course are not massive in number) are followed in attendance and they all work on the same activity that must then be effectively reported in the classroom, to continue to discuss it all together.

A priori preparation of materials
In a MOOC, on the other hand, trainers must take charge of this a priori heterogeneity. The design phase is indeed very demanding: first because *the materials and contents must be prepared and defined all beforehand*, before the MOOC begins. Hardly with fast times like those of the MOOC, they could be arranged from time to time. Second, trainers will never be able to personally compare themselves with the trainees. *Everything happens online*, in more asynchronous that synchronous way.

Uncontrollability of ongoing learning dynamics
The trainers' aim is transposing some ideals praxeologies to the trainees. Therefore, trainers have to implement meta-didactical praxeologies that invite the trainees to reflect and therefore to consider or not to appropriate the praxeology that the trainers would like to transpose. Hence, they must carefully design the MOOC and introduce in it tools that stimulate discussion and debate. Finally, they should suggest to the participants to experiment in their classes the proposed material.
Indeed, trainers can only suggest experimenting the activities that the trainees see in the MOOC. As anticipated at the beginning, there are no institutional constraints to respect. Participation is free and voluntary. *The trainees can take advantage of the materials and deepen them with their times and their ways*.
We still remember, as seen, that the didactical praxeologies aim to model the mathematical activity when solving a didactical task, such as "to teach a particular mathematical topic". The meta-didactical praxeologies concern meta-didactical tasks, such as "to reflect on possible praxeologies for teaching that particular concept".
In a face-to-face course the trainers follow a sequential path to expose the topics they want to cover, so the trainees are guided to accomplish a meta-didactical type of task (e.g. how can we overcome the misconception[10] that the students have about height in triangles? – Table 2.3).

Task	Overcoming the misconception that the students have about height in triangles.
Techniques	Show some slide to introduce the problematic. Read together a specific mathematical activity on height. Collective discussion (*). Each teacher experiment the activity in her classroom. In the next meeting, collective discussion on the classroom experience (*). (* N.B. the collective discussion is *always* orchestrated by the trainers).
Logos	Elements of the research (some specific theoretical framework or theoretical tools), which however remain in the background for the teachers.

Table 2.3: Example of meta-didactical praxeology

[10] in terms of difference between perpendicular and vertical

In the MOOC, the materials are put in sequential order (Table 2.4). However, the trainees do not necessarily follow this order of exposure. For example: we suppose that in the first module, to introduce activities useful to trainees to explain a mathematical concept (e.g. the height in triangles) to their students, the trainers have arranged first a text (A), then a video (B), then another video (C), then a sway (see Table 3.3 in Chapter 3) showing activities (D), then a communication message board in which to intervene (F), ... Any trainee could start viewing a video (B), go to read the comments on the communication message board (F) and then go back to read the activity (D), comment him (F), see the other video (C), ...
All the other trainees, in turn, will make *permutations* of this sequence of actions, going back and forth among the resources of that specific module (Table 2.5). Let us not forget that generally a MOOC has from 4 to 6 modules, so if we return to reason with the permutations, the possibilities that every trainee has to view the materials are really innumerable.

Task	Overcoming the misconception that the students have about height in triangles.
Techniques	A B C D E F
Logos	arrangement of materials that are designed by trainers in the design phase, to facilitate a clear and orderly consultation, according to the chosen design model(s)

Table 2.4: Example of trainer's praxeology

	A trainee's praxeology	Another trainee's praxeology
Task	Overcoming the misconception that the students have about height in triangles.	
Techniques	B F D F C …	A D B C F …
Logos	Before B because he prefers to see and after to read. Then F because he wants to read what other trainees in the MOOC are thinking about those materials. After that D because he wants to have a clear vision of what the others are discussing. Later F again, but this time is his turn to write what is thinking, …	Before A and D because she prefers to read and understand alone, to grasp some new ideas. After B and C to see other further explanation that can confirm the ideas she is forming. Then F so she can share something in the message boards, …

Table 2.5: Examples of trainees's praxeologies

Techniques vs multi-techniques
Within the MOOC, the definition given by Chevallard is no longer the only one that holds true.
If we consider the trainees' praxeologies, there is no longer a single technique to be satisfied and then you can move on to the next technique, until the task is satisfied. The order of execution is completely irrelevant: to perform first the technique B and after the A, does not affect the accomplishment of the task. Moreover, the techniques can all be done or not, but

you can still accomplish the task. For example, a trainee may not be able to see a video, because that day his connection does not work. Reading the materials or the interactions among the other participants, it could still "recover" what he did not see. We could say that the trainees put into practice a multitude of techniques.
In the same way, trainers who are designing a MOOC will have to meet precise requirements, namely to solve specific type of task (e.g. to encourage interaction between the trainees; … We will analyse them better in chapter 6). Taking into consideration the proposed example, possible ways to accomplish the task could be: to provide a suitable space for making the remote communication possible; to initiate discussions on forums with a prompting question; to reduce trainers' interventions; …. Also in this case there is no longer a single technique to be satisfied and then move on to the next technique until the task is satisfied; nor there is an execution order to be respected; nor it is said that what one considers to implement as techniques covers all the possible techniques. Even trainers put a multitude of techniques into practice.

Chevallard (1999) calls technique a "way of doing – which […] allows one to carry out […] some of the tasks of the given type". For example, if the type of task is "to solve a second degree equation", a possible technique is to factor the polynomial (Table 2.6). Another technique could be to use the square-root method (Table 2.7); or to calculate the discriminant (Table 2.8); or even to solve the equation graphically (Table 2.9). All these techniques, taken individually, allow solving the task at stake.

Task	Solving the second-degree equation $x^2-4=0$.
Techniques	Factor the equation, obtaining $(x+2)(x-2)=0$. Set each factor that has an x term equal to zero, obtaining $x+2=0$ and $x-2=0$. Solve each equation for x, obtaining $x=-2, x=2$.
Logos	The zero product rule: if you have an equation with two first degree terms whose product (factors) equal zero then either the first factor is equal to zero or the second factor is equal to zero.

Table 2.6: Solving the second-degree equation factoring the polynomial

Task	Solving the second-degree equation $x^2-4=0$.
Techniques	Solve for the $x^2 = d$, obtaining $x^2 = 4$. Then, $x = \pm\sqrt{d}$ are the roots. In fact, $x = \pm\sqrt{4} = \pm 2$
Logos	It is possible to use the square-root if there is no x-term.

Table 2.7: Solving the second-degree equation using the square-root method

Task	Solving the second-degree equation $x^2-4=0$.
Techniques	Use the formula $x = -b \pm \frac{\sqrt{b^2-4ac}}{2a}$, obtaining $x = \pm\frac{\sqrt{16}}{2} \Rightarrow x=-2, x=2$
Logos	In the quadratic formula b^2-4ac is called the discriminant (Δ) because its value indicates what type of roots there are (distinct: $\Delta > 0$, identical: $\Delta = 0$, no real: $\Delta < 0$ roots).

Table 2. 8: Solving the second-degree equation calculating the discriminant

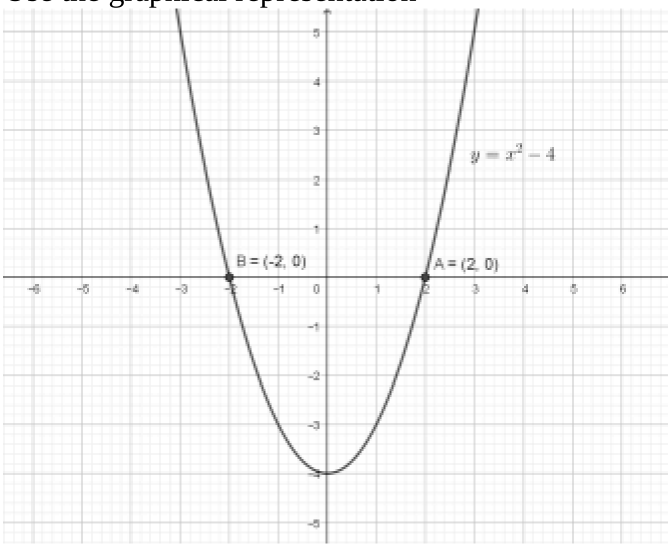
Task	Solving the second-degree equation *x^2-4=0*.
Techniques	Use the graphical representation  The parabola cross the x-axis at *x = -2* and *x = 2*. These are the roots of the quadratic equation.
Logos	• A quadratic equation has two roots if its graph has two x-intercepts • A quadratic equation has one root it its graph has one x-intercept • A quadratic equation has no real solutions if its graph has no x-intercepts.

Table 2.9: Solving the second-degree equation graphically

In the example of the Tables 2.6, 2.7, 2.8, 2.9, it was possible choose a distinct solution method (or factoring the polynomial, or using the square-root method, or calculating the discriminat, or considering the graph), perform precise steps related to the chosen method and solve the task.

Adapting the MDT model to MOOCs, we notice that in these online environments trainers and trainees are led to solve tasks using multiple procedures or multi-procedures, which we call – with the intent of extending the ATD language - **multi-techniques** (Aldon et al., in press). They are multiple procedures because if one considers only one of them individually, the task cannot be solved in a satisfactory manner.
In fact, in the MOOC if you limit your choice just to one technique, for example watching a video, it is not made by consecutive steps as those involved in applying a mathematical formula and which lead to accomplish the task. To continue the example, the video give you some partial information about the meta-didactical task that is required to accomplish it. It could be an introductory video, in which the trainer is introducing you the mathematical topic concerning the module and nothing else. Just watching the video, you cannot solve the task. The next technique that you should consider do not follow a mandatory or suggested order (in Tables 2.6, 2.7, 2.8, 2.9, depending on the chosen resolutive technique, the subsequent ones were logical consequences of the previous one). You can continue choosing what you prefer: another video, the reading of the interaction in the communication message boards, … Therefore, *there is no longer a single technique to be satisfied and then move on to the next consecutive technique until the task is accomplished*. Moreover, *there is not an execution order to be respected*: the participants are autonomous in deciding how to use the available resources. Finally, just because these techniques are multiple, *they can all be done or not, without the accomplishment of the task being compromised*. In the event that only one is

performed, it is clearly not possible to fully satisfy the task. With two, you could already grasp more. It is even probable that there is a minimum number of techniques to be applied to accomplish the task with minimum effort! However, we will not go into these details.
As we will see in the analysis section, a fair number of them need to be considered. Note, however, that – specifically about trainers – it may be inaccurate to talk about techniques since what follows in the analysis section (Chapter 6) will be a list of suggested procedures that we want to share with other potential MOOC trainers in teacher education. The procedures will become techniques once they are institutionally recognized (Chevallard, 1999) by the research community; as well as the techniques used for the mathematical problems are universally shared.

So far, we have examined two of the five characteristics of MDT. We made a long comment on the trainers and trainees' praxeologies to understand how these have been readapted to the MOOC. We refer to the comments on the shared praxeology afterwards. Let us move on to expose the third and fourth features of the MDT.

2.3.1.4 The dynamics between internal and external components

Not only a praxeology can evolve and change during professional development, also one of its component can do that (for example the technique, or the justification part). The praxeological components can be considered as internal or external to a community. They are considered internal to a community when commonly shared and used by the members of the community, and external to a community when the members of the communities do not typically use it. The components can also be internal or external to one or some members of a community, not to all the members. The idea of external and internal is taken by Clark & Hollingsworth (2002), who distinguish an external domain, located outside the teacher's personal world, from the internal domains, which "constitute the individual teacher's professional world of practice, encompassing the teacher's professional actions, the inferred consequences of those actions, and the knowledge and beliefs that prompted and responded to those actions" (Clark & Hollingsworth 2002 p. 951).
Of course, the goal of a teachers' professional development programme is to transform praxeological components that are initially external to the teachers' community into internal ones (e.g., activities using new technologies, such as new GeoGebra tools, or new pedagogical techniques, such as student-centred teaching approaches). Not all teachers' praxeological components evolve in the same manner: praxeological components for different teachers may evolve differently, due to contextual factors, or to institutional influences, or attitudes towards teaching and mathematics, believes, and so on.
Furthermore, the researchers participating into a teachers' professional development programme may benefit from transforming external praxeological components for their community into internal ones.
To exemplify, a community of teachers starts a professional development programme in which, due to some institutional situation (e.g. curriculum changes), a community of researchers introduces a specific ICT tool (e.g. a dynamic geometry software). At the end of the programme, the initial techniques (and their theoretical counterpart) have become a new a set of shared techniques, as a result of the actions taken by the researchers and teachers.
Therefore, typically, researchers' and teachers' praxeologies can initially differ: some of their components can be internal to one community but external to the other. Thanks to the interactions of the two communities they can evolve from external to internal (*internalization* process, Arzarello et al. 2014, p. 355-356).

2.3.1.5 The role of the broker

The MDT framework uses the notion of *broker* as a professional who belongs to more than one community and makes possible exchanges between them: “Brokers […] are able to make new connections across communities of practice, enable coordination, and – if they are good brokers – open new possibilities for meaning” (Rasmussen et al. 2009, p.109).

In this way, brokers can facilitate the transition of mathematical concepts from one community to the other, which is accomplished thanks to *boundary objects.* More precisely, the notion of boundary object has been adapted from sociology (Star & Griesemer, 1989) to the teaching of mathematics by Rasmussen & Keene (2015), which define the boundary objects in this way: “they are mathematical symbols, technologies, documents, software or other elements that allow people to connect different communities and work together” (p.282). In particular, “boundary objects are those objects that both inhabit several communities of practice and satisfy the informational requirements of each of them” (Bowker & Star, 1999, p. 297). In fact, for example, a teacher belongs to the community of mathematics experts, to that of her colleagues in the school, and to her classroom community.

In the Italian community of academics in mathematics education, the role of broker is often played by a so-called teacher-researcher – who is part of the communities of researchers and of teachers. Otherwise, it can be played also by a researcher, a PhD student, or a master student. The role of broker is fundamental in the exchange of information, techniques, justifications, theories, namely, all about praxeologies and their components. In fact, the role of the researchers is to manage a research project in which the educational programme is inserted, then to design the programme with its activities and actions. The role of the teacher-researchers is to collaborate in these phases, and to participate also in the professional development programme as trainers, where the role of the teachers involved is to be learners in communities with colleagues. Participating simultaneously to the researchers' community and to the teachers' community, the teacher-researcher acts as a broker between the two communities.

2.3.1.6 To summarize what has been observed so far

We quickly summarize what has been treated so far (see Figure 2.8).

Within the research community, a P_{R0}[11] didactical praxeology is elaborated, namely a connected, coherent and theoretically stable system of techniques and theoretical justifications concerning the teaching/ learning of mathematics.

From the interaction between P_{R0} and the didactical praxeologies of the communities of teachers to be trained, which we indicate with P_{T0}[12], we develop a meta-didactical transposition process, which has its own trajectory and development. The transposition is centred on specific “brokering” actions between the two involved communities. The meta-didactical transposition is the phenomenon of transition, while the brokering is what makes this transition possible.

During the training process, a community consisting of researchers and teachers tends to be formed. It has its own meta-didactical praxeology P_1. It is originated from a more or less conflictual interaction with the initial didactical praxeologies possessed by the teachers at the

[11] Didactical praxeology of researchers at the initial stage of the process, R in fact stands for researchers

[12] T stand for teachers

beginning of the training process (P_{T0}). In addition, it is more or less close to the "ideal" praxeology (P_{R0}), whose acquisition is the ideal aim of the training course.
This community reflects on the meta-didactical transposition process, not only to define a better educational trajectory for the future, but also to review and integrate P_{R0}, in the light of the experiences made. In this way a possible evolution of P_{R0} is obtained, which we will indicate with P_{R1}.
Teachers' didactical praxeologies can also evolve as a result of meta-didactical transposition: we will indicate with P_{T1} the consequent evolution of P_{T0}.

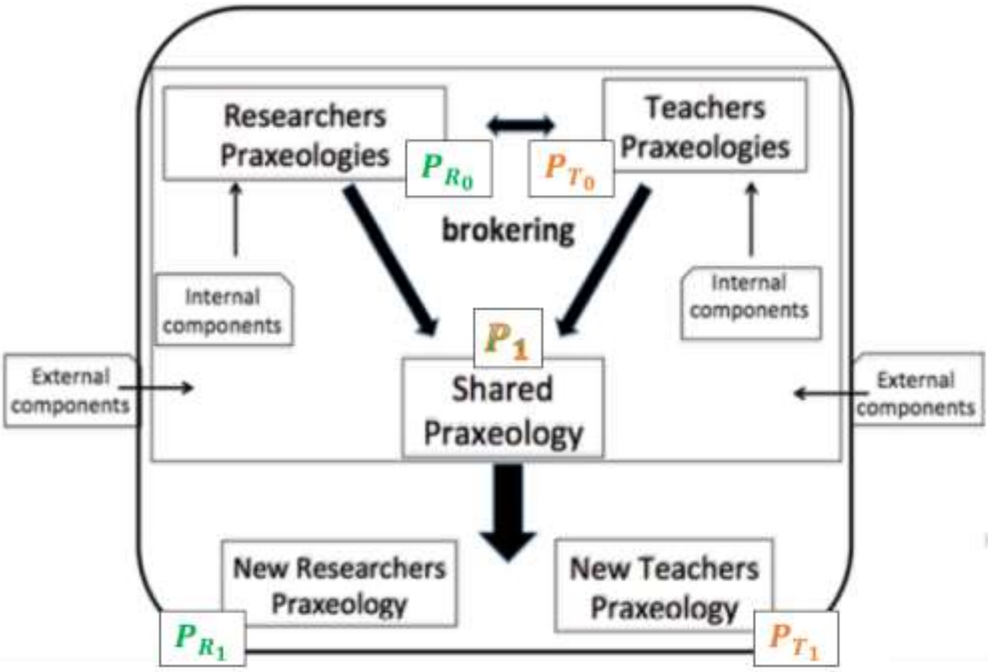


Figure 2.8: The Meta-Didactical Transposition model with explicit praxeologies

2.3.1.7 Remarks on the evolution of the praxeologies in the MOOC environment

We have seen that if the teachers' praxeology (or some components of it) evolves towards a new one, closer to a researchers' praxeology, it is called s*hared praxeology* (Arzarello et al., 2014).
Clearly, a shared praxeology can have effect also on the initial praxeologies of the two communities: the teachers can return to their classroom with new teachers' praxeologies and the researchers can come back to think about the training phases and redesign them, developing new researchers' praxeologies. This means that both the teachers' praxeologies and the researchers' praxeologies may evolve in new ones, converging (to a shared praxeology) or diverging to different ones.

Is the shared praxeology missing in a MOOC environment?
We are increasingly seeing the situation with your own eyes: the MOOC – as opposed to a face-to-face course – is a decidedly complex environment.
Once the MOOC is designed and ready, the trainers let the trainees live it. Generally, in MOOCs for teacher education (like ours or that of Panero et al., 2017) trainers do not have an invasive presence. They are vigilant, behind the scenes, but do not constantly interact with the trainees; nor they want to orchestrate the trainees' discussions on communication boards.

How could they, after all? The number of the trainees is massive, the discussions flow like a flooded river on several communication boards at the same time. In addition, the trainees have the freedom to access whenever they want (morning, afternoon, evening) and the interactions are mostly asynchronous.
In the MOOC there is not a time when the two communities confront each other in a synchronous way and converge to a shared meta-didactical praxeology. Let us see why.

In a face-to-face course, researchers and teachers see each other at the same place, at a specific time. Everyone hears what the others are saying; everyone can reflect and agree/disagree at the same time during the discussion.
Researchers show and explain the educational materials to everyone. There are collective discussions, generally orchestrated by the trainers. Researchers and teachers, all together, compare each other to these materials. Some teachers may find it necessary to modify something to better adapt the proposal to their school context. All together, they reflect on this: suggestions, clarifications (if required) are given. A community consisting of researchers and teachers tends to be formed. Then, the researchers invite the teachers to use these new resources in their classrooms. The teachers that have received guidelines, suggestions and so on to conduct the activity, experiment it in their classroom and, in the next meeting, they return to the educational course with something to share with the others. The teachers tell how the experimentation have gone to classroom, if there were any negative or positive unexpected events. The comparison is always done all together.
At the end of the course, a single message is drawn up. The initial praxeologies of researchers and teachers evolve towards a new one, possibly converging to a shared praxeology, or diverging to different ones.

Actually, in a MOOC, *the evolution of trainees' praxeologies is chaotic. The trainers do not know what a trainee really viewed in the MOOC. In addition, they do not know if the trainee has understood deeply the proposed materials*. There is not a moment of confrontation in unison that clarifies doubts for everyone or that gives suggestions to reinforce ideas. There are the materials, with everything wrote: suggestions, clarifications. However, it is not said that these are read by everyone or shared by everyone. There are communication message boards where the trainees can confront mostly with each other, but not all at the same time. Moreover, it is not said that every trainee really wants to compare himself to the others to an online platform.
The trainees then self-manage themselves. Some trainees, who have favourable conditions in their scholastic context, can tacitly share the ideal praxeology and maybe they immediately experiment in their classes. Others trainees read just the resources available, because the school context does not allow them to be used as such; or because they do not share some of the choices made by the trainers. Others trainees are absent-mindedly following the MOOC and perhaps have not yet grasped what message the trainers would like to pass. Do not happen the peculiarity step of the Meta-Didactical Transposition model: a community consisting of trainers and trainees do not tends to be formed because they can not confront each other on the same materials at the same times. Several ideas can come out in the mind of the trainees: sometimes they share them on the communication message boards and find other trainees that agree with them. However, in general, the trainers do not know what the massive number of trainees is really thinking about their prepared and shared educational materials.
For all of these reasons, a moment in which ALL the participants (trainers and trainees) find themselves discussing in a collective way is missing in a MOOC. Then, *you do not locate a time when trainers and trainees share the same praxeologies.*

Who is the broker in a MOOC?

The meta-didactical transposition is centred on specific "brokering" actions between the two communities involved. We can say that meta-didactical transposition is the phenomenon of transition, while brokering is what makes this transition possible.

By definition, we have seen that "Brokers […] are able to make new connections across communities of practice, enable coordination, and – if they are good brokers – open new possibilities for meaning" (Rasmussen et al. 2009, p.109).

What is able to create new connections between the trainers and trainees' community is precisely the MOOC! *The MOOC is* therefore *the broker*. It facilitates the transition of mathematical concepts from the trainers' community to the trainees' one another. And the *boundary objects* are all its resources: "mathematical symbols, technologies, documents, software or other elements that allow people to connect different communities and work together" – exactly as for the definition given by Rasmussen & Keene (2015, p.282).

There is another aspect that should not be underestimated: *even the trainees, sometimes, play the role of broker within the MOOC*.

As specified, the trainees communicate with each other through appropriate communication message boards predisposed by the trainers. We consider an example taken from Taranto et al. (2017, b). Let ***A*** and ***B*** two generic trainees.

A trainee ***A***, stimulated by some component of the starting situation, produces something: for example, he makes reflections on the communication message boards, sharing his own ideas or any experiments conducted in the classroom; possibly also shares its own materials.

A trainee ***B*** benefits from the observations/shares of ***A***.

The production of trainee ***A***, who reacts to what is proposed by the MOOC, is a boundary object, so to speak, standard for ***A***. ***A*** gives meaning to the interpretation of the MOOC team of that object and uses it in the classroom. For ***B***, on the other hand, the situation is different: ***A***'s experience does not generally fall within its praxeologies. ***B*** can instead be moved by ***A***'s experience (which was accomplished freely and not "imposed" by the trainers) and has an additional source of learning (what he sees in the MOOC, what he sees done by another thanks to the MOOC).

Therefore, a didactical praxeology of a trainee becomes something that can potentially become part of the meta-didactical (and non-didactical) praxeology of some other teacher.

Note that we say "meta" because if it came into the teaching practice, it means that the trainee who reads must incorporate and put into practice what he has read. We cannot have proof or certainty of this, unless he explicitly declares it on some communication message board. In general, it is not said that any trainee has the opportunity or the interest to immediately implement a teaching practice proper to another trainee.

There is no explicit shared praxeology, however …

As we said, in a MOOC there is not a moment in which we can identify a shared praxeology. On the other hand, there is certainly an evolution. Internalization processes take place, i.e. the passage of components from external to internal domain. Think about the moment when trainees enter the MOOC for the first time and have to internalize the structure of this new online environment; or when they relate with a tool chosen by the trainers who did not know before accessing the MOOC; or even when they interact with each other and exchange ideas, opinions, materials.

In a broad sense, at the end of the MOOC there is an evolution of praxeology (both for trainees and for trainers). Therefore, evidently, there is a shared praxeology, although we cannot grasp the exact moment when this sharing takes shape.

In a face-to-face course a trainer work with a small group, with which he can easily compare and check if there is a shared praxeology. In a MOOC, instead, he cannot say that everyone

has shared a praxeology. The discourse is valid only for some, those for whom there is evolution. However, concretely it is possible realize this only at the end of the course.
Indeed, a trainer can deduce if there are evolutions (even minimal) from the final activity of the MOOC, which generally ends with a final production. Usually, the final task required by the trainers is the design of an educational activity using the material learned in the MOOC. Here you can understand how deep the trainee has understood the news illustrated and if she has made them her own. Of course, each participant will have her attitudes and her ability to design. Not everyone is able to externalize competence and quality, even if they understood and used the proposed meta-didactical praxeologies.
To give value to the possible deductions with the final task there are, then, subsequent analyses: carefully reading the trainees' comments on the communication message boards, their answers to questionnaires and possible interviews.

In any case, what has been pointed out in the paragraph "*Different kind of meta-didactical praxeology*" is valid also in a MOOC:

> "Evolution in the praxeologies does not mean that all the teachers, involved in the educational programme, evolve in the same way with the same transformation of components: in fact, different teachers may evolve in different ways, with respect to their history and experience" Robutti (in press).

The meta-didactical praxeologies of the two communities change within the institutional environment in which they reside. As a consequence, there could be teachers' development of both a new awareness (on the cultural level) and new competencies (on the methodological-didactical level, i.e. that of teaching practice), which lead them to activate, in their classrooms, a didactical transposition in line with the meta-didactical transposition.

Therefore, at the end of the MOOC, a comparison can be made between the initial praxeologies – those with which each participants (trainers or trainees) started the MOOC – and the final praxeologies – those that occur after the MOOC has been completed. They can be equal to the initials, if no evolution (or internalization process) has occurred; or be (hopefully) different.

We go back to the fifth and last feature of the MDT and then move on to the final considerations regarding this theoretical framework.

2.3.1.8 The double dialectics

In the MDT framework, the *double dialectics* (Figure 2.9) represents a product of the interactions between the two communities of researchers and teachers involved in the professional development programme.
The first dialectic is at the *didactical level* and takes place in the classroom, involving the personal meanings that students have constructed when they are engaged in a mathematical activity and its scientific sense.
The second dialectic is at the *meta-didactical level*, and lies in the personal interpretation that the teacher gives to the first dialectic, as an effect of her praxeologies in the classroom, and the interpretation of the first dialectic according to the researchers' community, who help the teacher in reflecting upon it. The second dialectic corresponds to the scientific shared meaning of the first dialectic.

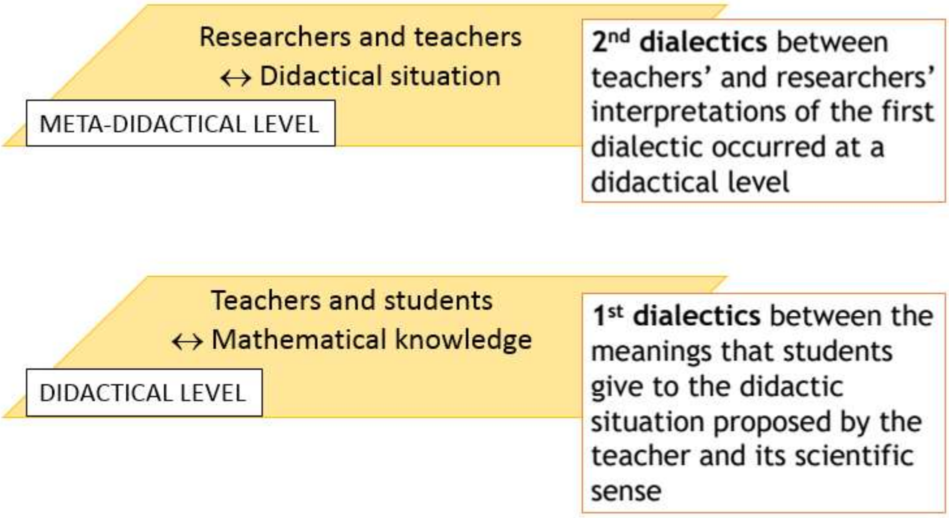


Figure 2.9: The double dialectics

It is through this double dialectic that teachers' praxeologies can change over time, during the professional development or after it, and align with the praxeologies of the researchers, and this process may cause a significant evolution of the teacher professional competences.

This last step, the double dialectic, does not happen in the MOOC. Or rather, the first dialectic occurs when the trainees experience MOOC activities in the classroom. However, the trainers never see them: they do not know when and how the trainees do the experiments - unless they are explicitly asked to fill in and return a logbook.

Of the second dialectic, the trainers sometimes see only one piece (the interpretation that the trainees make of what they see as acquired by their students). In fact, the trainers have general information about this on communication message boards. Or always from the logbooks, which must explicitly request. Since there is no face-to-face comparison, the interpretation that the researcher might have of certain attitudes, remains to the researcher. Unless she decides to personally contact the trainees.

2.3.1.9 To distil the previously presented argument

In a course aimed at teachers' professional development, like a MOOC, the researchers want to transpose the ideals praxeologies in order to generate a (possible) evolution of the didactical praxeologies of the teachers' community.
In traditional courses, trainers alternate a DO and DO REFLECT on what has been done. Instead in a MOOC you do not learn only because you are doing something that the MOOC ask you, but because other peers as your one are doing something as well. The enrolled teachers benefit from the experiences of others teachers and sharing opinions, as well as (hopefully) trying to experiment.
A MOOC could influence the didactical praxeologies of the teachers. Evolution is not deterministic process as happened in the MDT, but is more like a stochastic process. MDT it is not able to grasp this complexity. Other lenses are necessary.

We have reported in detail the picture of the MDT because this is the framework from which we started and which supports the processes of hybridization. We have seen what the “weaknesses” of this theoretical model are in relation to the MOOC:

- the different institutional weight perceived by trainees in the MOOC environment than in a face-to-face course;
- the presence of multi-techniques vs techniques;
- the missing of a shared praxeology;
- the difficult to consider the double dialectic;

and what adaptations have been made. However, as it is, this theoretical model fails to grasp the dynamism and complexity that are inherent in the interactions implemented by the participants (trainers and trainees) and that influence their possible evolutions.
Therefore, we consider the following metaphor, which is a prelude to why we resorted to the hybridization of MDT with the Instrumental Approach and the Connectivism.

The metaphor of the angler
In analysing the impact that a MOOC for teacher education has on trainees and trainers, I am like a river angler going into the sea.

Figure 2. 10: An angler in the sea

When I analyse teacher education in a traditional course I use the MDT and I am a river angler. The river is a “circumscribed” environment: I know its depth; the fishes that inhabit it; which bait to use for fishing; .... With the MOOC, emerging phenomenon that is still little studied, it is as if I went fishing in the sea. If I am a river angler it does not mean that I do not know how to fish in the sea, but I have to understand how to take advantage of the fishing knowledge I possess to be able to fish in this new environment, definitely more complex than the river. So fishing in the sea leads me to invent a device (a new theoretical framework), to better understand the behavior of fishes and catch them (i.e. identify the facets at a professional level that the sea, or the MOOC, gives its inhabitants).

2.4 The hybridization of the MDT

In previous discourses, we have given importance to two communities and to the MOOC, almost as if it were an entity to be considered. We have said that the MOOC has within it institutional aspects of different weight, less rigid, compared to the known institutions. We almost personified it with the name of broker. MOOC is certainly not a traditional course. We could almost say that it is another small world in which the protagonists live virtually. A bit like Pandora[13] in the James Cameron’s film *Avatar* (2009). And this world nourishes,

[13] ***Pandora*** is a primordial and decidedly spectacular world, covered by rainforests with tall trees even up to three hundred meters, and is inhabited by various creatures, among them the ***Na’vi***, a species of 10-foot tall (3.0 m), blue -skinned, sapient humanoids that live in harmony with nature.
To explore Pandora’s biosphere, ***scientists*** use Na’vi-human hybrids called “***avatars***”, operated by genetically matched humans. In fact, each avatar can be used and controlled only by the human being whose ***DNA*** was used

influences, "*expands*" who lives it. In return, even those who live in the MOOC feeds it, influences it and expands it; and so on. This relationship that involve subjects and objects, recalls a process known in mathematics education, the instrumental genesis of the Instrumental Approach (Verillon & Rabardel, 1995). We can therefore certainly formalize this relationship by calling into question this theoretical fragment, making an appropriate hybridization.

Therefore, we are taking the first step of hybridization: the ***connection*** *(§2.1.2). A process of hybridization of the T_0 theory, the MDT, with an element of a T_1 theory, the Instrumental Approach, which allows establishing a connection between the two theories.*

2.5 Fragment from the Instrumental Approach

In the theoretical framework of the Instrumental Approach (Verillon & Rabardel, 1995), peculiar is the difference between artifact and instrument.

2.5.1. Artifact

An artifact is an object made to pursue specific purposes. Its form is justified by the performance that is targeted and materializes the existing plans from which it originated. An artifact presupposes a project/purpose and consequently intelligence able of creative activity that can incorporate some knowledges inside the artifact (Verillon & Rabardel, 1995).
Leontiev (1976) refers the artifact to a tool and to all the object of material culture to which an infant has access during his development. Apart from the physical properties of the tool, what is important is "its operating method, elaborated socially during collective work and attached to it" (p. 74). Therefore, its appropriation process implies both transmission of the tool and the reproduction by the user of "the practical or cognitive activity adequate to the human purpose it embodies" (p. 263). We underline this Leontiev's perspective because when a subject is interfacing herself with an artifact some actions or operation cannot evolve under the influence of the object itself within a solipsistic process: they are transmitted to the subject and, therefore, imply social mediation.

Utilization schemes
An artifact imposes on the subject a set of constraints that he has to manage in the course of his actions. For example, in a task involving the assembly of a technical object, the subject must respect constraints (concerning its structure and performance) which are different from those entailed by having to operate the same object.
The reorganization and recomposition of activity due to the introduction of instruments does not only depend on the different types of possible constraints. It also results from the new

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to compose it. The connection is made thanks to a special technological capsule, where the subject falls into a sort of coma, functional to a transfer of consciousness and soul into the avatar.
In this parallel, the ***MOOC*** is Pandora. It is full of technological resources harmoniously placed by the ***trainers*** (the Na'vi). The ***teachers*** (the scientists) who want to explore the MOOC must create a ***personal account*** (as an avatar), which is used and controlled by the person who has set ***his/her credentials*** (as the DNA used in the film to create the avatar). Of course, teachers do not go into a coma when they access the MOOC, indeed they are very active, both in the virtual world and in the real world.

possibilities of action that are afforded to the user. In this sense, "the use of an instrument increases the assimilatory capacities of the subject and contributes to expanding the field of his possible actions" (Verillon &Rabarde, 1995, p. 86).
Reorganization of activity leads to the emergence of instrument utilization schemes (US) (Rabardel & Verillon, 1985; Rabardel, 1991). These could be defined as the structured set of the generalizable characteristics of artifact utilization activities. They enable the subject to develop the activity necessary to perform the functions he expects from the association of the artifact with his action. They thus form a stable basis for his activity. The USs may be considered as operative invariants. The operative aspects pertain in particular to goals, elementary or composite operations, procedures, to the organization, planning and management development, etc.
The USs have assimilatory capacities enabling repeatability of action; they have accommodating capacities enabling their application to different classes of object and situations. They confer signification to the situations in which they are mobilized.
USs have a "private" dimension in the sense that they are the schemes of a singular subject. But they also have an essential "social" dimension. This is due to the fact that their emergence results in part from a collective process to which not only the users, but also the designers of the artifacts, contribute. It also results from the fact that they are the object of social transmission processes (through operating instructions or technical training, for instance). More fundamentally, it is due to the fact that USs concern the coordination of action, not only within the subject, but also inter-subjects in collective activities (everyday life, in training or in work). That is why the USs should not only be considered in their private dimension, but also as social utilization schemes, particularly important in an educational perspective.

2.5.2 Instrument

A **instrument** is the artifact joined to the utilization schemes interpreted by a user on the basis of his culture and experience […] (Verillon & Rabardel, 1995).
According to Mounoud (1970), an instrument is any object that the subject associates with his action in order to perform a task. It prolongs and/or modifies this action and presents characteristics that simultaneously are associated with the operations of the subject and with the objects (and the context of the task) to which it is applied. As such, the instrument constitutes a sort of intermediate universe between subject and object: it is both a content concerning the subject's actions and a form concerning the objects to which it is applied. It is important to stress the difference between two concepts: the artifact, as a man-made material object, and the instrument, as a psychological construct. The point is that no instrument exists in itself. A machine or a technical system does not immediately constitute an instrument for the subject. Even explicitly constructed as a tool, it is not, as such, an instrument for the subject. It becomes so when the subject has been able to appropriate it for himself – he has been able to subordinate it as a means to his ends - and, in this respect, he has integrated it with his activity. Thus, an instrument results from the establishment, by the subject, of an instrumental relation with an artifact, whether material or not, whether produced by other or by himself.

2.3.5 Instrumental genesis

The process that leads from artifact to instrument is called *instrumental genesis* (Rabardel & Verillon, 1985; Verillon & Rabardel, 1995).
We report a well-known example, used by Verillon & Rabardel (1995), to explain how the transition from artifact to instrument happens:

> "[…] a baby learning to use a spoon. Not only does he have to elaborate efficient schemes in order to grasp and manipulate the spoon (subject-instrument interaction), but he has to learn to keep some of the milk in the spoon on the way to his mouth (instrument-object interaction). In the process of this, he acquires some knowledge about the behavior of liquids as opposed, say, to mashed potatoes (subject-object interaction mediated by the instrument). Eventually this knowledge may lead him to use his spoon differently from milk and mashed potatoes (modifying previous forms of subject-instrument interaction)" (p. 85).

The instrument is the result of a process, the instrumental genesis, through which the individual constructs utilization schemes of the artifact. These are activated and develop according to the task that the individual has to perform.
In particular, the instrumental genesis is divided in two directions:

- Instrumentation (from the artifact to the subject): it leads to the development or appropriation of utilization schemes which progressively constitute into techniques which allow us to solve given tasks efficiently;
- Instrumentalization (from the subject to the artifact): it progressively transforms the artifact for specific uses with the related utilization schemes. So it is the adaptation of the artifact to your own cognitive structures.

2.5.4. MOOC: is it artifact that became an instrument or something more?

As explained, the instrumental genesis, or the process that leads from artifact to instrument, is perceived as something that involves an individual in the first person. The individual, depending on the utilization schemes put into practice, either for his culture or for social mediation, is able to give a new form to a tool, attributing to it some functionalities.
The MOOC is clearly the subject of our discourses, so the tool to be examined.
As we pointed out above however, a MOOC is inhabited first by the trainers, who are the designers. Subsequently the MOOC is opened and inhabited by the trainees. In general, since we are dealing with large numbers, it is not easy to observe the relationship object-subject referred to every single participant. Of course, it can be done if you are interested in the study of individuals; but our intent is to describe the phenomenon in its generality, without excluding or favouring anyone. And above all to describe it in its dynamic becoming.

2.5.4.1 MOOC-artifact

We begin by observing that a MOOC is created to pursue specific purposes, in this case to accomplish a teacher education. In it the trainers have inserted various digital resources: mathematical activities, specific software, spaces to allow distance communication. Digital resources replace the voice and explanations of the trainers that are usually done in face-to-face courses. Thanks to videos, images, interactive texts, trainers are able to communicate their training intentions at distance, to share research results, methodologies and teaching strategies that can be used in class with students. Hence the MOOC presupposes a

project/purpose and consequently intelligence able of creative activity that can grasp the knowledge that trainers want to transpose by incorporating it inside the MOOC.
We can definitely say that a MOOC can be understood as an artifact, and we will indicate it as ***MOOC-artifact***, when it is inert, namely when it is just an online platform where the trainers loaded the mathematical activities and the technological resources.

If we want to be particularly precise, we could also say that the MOOC in its entirety is an artifact and all the resources within it are in turn artifacts. In fact, they are always designed by designers for a certain purpose, and they will be used and interpreted by the trainees according to their culture, knowledge and experience. In this case we can speak of a collection of artifacts that are in synergy with each other (Faggiano et al., 2017).

It is appropriate to open a quick bracket: precisely because a MOOC takes place entirely online, so there is no possibility of synchronous interaction of the trainees with the designers, it is not unlikely that what Verillon & Rabardel (1995) have defined as *catachresis* occurs, namely an artifact is used to do something it was not conceived for. In the analyses (§4.7.4, Chapter 4) we will see examples in this regard.

2.5.4.2 MOOC-instrument

Returning to us. Once the MOOC is opened to the trainees, they start to explore it, select some resources, interact with others. It becomes an instrument for each of them, namely a ***MOOC-instrument***.

The Math Edu USs

Following the theorization that makes the instrumental genesis, every trainee puts in place some USs that progressively constitute into techniques that allow him to solve given tasks efficiently. This is in line with what was observed in the Meta-Didactical Transposition regarding praxeology. In fact, in the MOOC environment, the trainers' USs are exactly the techniques (or multi-techniques) put in place in order to carry out the transposition of the ideal meta-didactical praxeologies (thus organizing the MOOC spaces appropriately, loading specific materials, ...). The trainees' USs are, instead, the techniques put in place to evaluate the resources made available (by trainers) in the MOOC, deciding whether to make them their own or not. In doing so, the trainees will arrive or not to acquire new praxeologies (the ideal ones).
In the following, when we meet the words USs, we will mean the techniques (in the sense of MDT) or multi-techniques (in the sense of Aldon et al., in press) which constitute the trainers or trainees' praxeologies. In the following, we will also indicate them as Math Education Utilization Schemes or *Math Edu USs*.
Therefore, if we consider "Adapting the MOOC-artifact to your own cognitive structures" as a task, the Math Edu USs that each trainees adopt are the techniques that allow accomplishing this task.

We observe that we are in the step of hybridization called ***interpretation*** *(§2.1.2). The F_1 fragment of the T_1 theory (*the instrumental genesis of the Instrumental Approach*) expressed in L_1 language (*the Utilization Schemes or USs*) is interpreted into the T_0 theory (*the MTD*) expressed in L_0 language (*the techniques or multi-techniques*). The L_0 language of T_0 is thus modified into an L'_0 language. In fact, we are starting to talk about* Math Edu USs, *in the T_{01} theory (T_0 + F_1). The* ***adaptation*** *step (§2.1.2) that gives substance to this started*

*hybridization will be realized later. It is still necessary to introduce other concepts. Precisely, we put in place a second hybridization considering another F_2 fragment (*the network of knowledge*) from another T_2 theory, the* Connectivism*. In this way, we can better modify the T_{01} theory integrating in it the new fragment. We then have to interpret the language of the new fragment into the one of the currently achieved hybridized theory (the instrumental genesis within MDT). We have to refine them realizing a further adaptation step.*
*First of all, we discuss the reasons why the F_1 fragment (*the instrumental genesis*) is not enough for describing the transition from MOOC-artifact to MOOC-instrument within T_0 + F_1.*

2.5.4.3 The transition from MOOC-artifact to MOOC-instrument

Concretely, how does this evolution from MOOC-artifact to MOOC-instrument take place for each trainee?
We have previously stressed that it is true that everyone follows the MOOC in total autonomy, wherever and whenever he wants. This assumes that a trainee, physically, has no one beside him among the other trainees or trainers. Yet the Math Edu USs that it puts in place are not dictated only by its culture or its experience. Also what other virtual colleagues do on the platform has a weight on how he interfaces and makes use of the MOOC. So, some Math Edu USs are just the result of social mediation (remember the observation made about the trainees as broker in the paragraph "*Who is the broker in a MOOC?*").
An observation made by other authors remains undeniable: mathematics teachers' interaction with different resources has been theorized in various ways (e.g. Gueudet & Trouche, 2009; Pepin, Gueudet & Trouche, 2013; Remillard, 2005). What is clear from these theoretical frames is that such interaction is a participatory two-way process, of mutual adaptation, in which teachers are influenced by the resources (given that each resource has different affordances and constraints), and at the same time the design and use of the resources is influenced by the teachers.
How to account for this two-way process in a MOOC? We need a theoretical concept that must take into account the interactions that individuals make individually with the MOOC. However, several individuals use the MOOC at the same time. We must therefore take into account the fact that individuals are not only influenced by MOOC-artifact, but also by how many other individuals are interacting with it at the same time. The goal is to describe a process that is certainly a two-way process, but also iterated and intertwined in a very dynamic way. The instrumental approach seems to struggle a bit. Therefore, there is the need to add another fragment from another theory. A second moment of hybridization on the initiated hybridization emerges. The fragment that we have considered is the *network of knowledge* that comes from the Connectivism. Now we will see what it is and how it is able to help us in the theorization of the dynamics we are considering. Therefore, the instrumental genesis will be interpreted differently by taking inspiration from the new fragment that we will expose in the following.

2.5.5 Graphical representations related to the MOOC

Before continuing, let us pause for a moment on a schematic representation of what we have observed. We can consider a MOOC as an oval shape (Fig. 2.11).

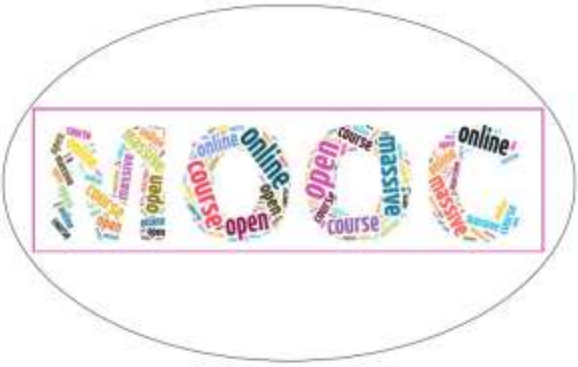


Figure 2.11: Schematic representation of a MOOC

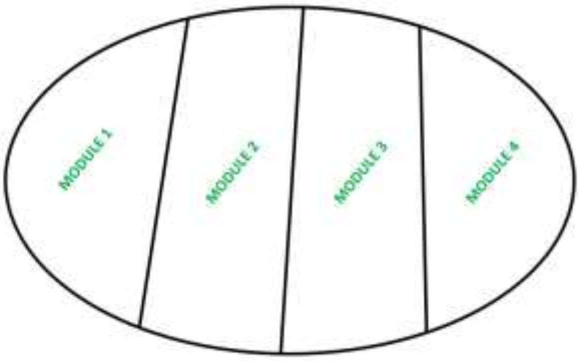


Figure 2.12: MOOC modules in schematic form

The MOOC is first inhabited by trainers who, as you will recall from §2.3 (and Figure 2.4), are represented with green. They begin to divide this oval into as many parts as the MOOC modules are (Figure 2.12).
We can think of each module in turn divided into two parts. One part is coloured in green, because it represents the design and transposition operations that have put in place the trainers, when the MOOC is still inhabited only by them. The other part initially remains white and it will be the space reserved for the trainees.
The MOOC at the inert state, i.e. the MOOC-artifact will be represented by Figure 2.13.

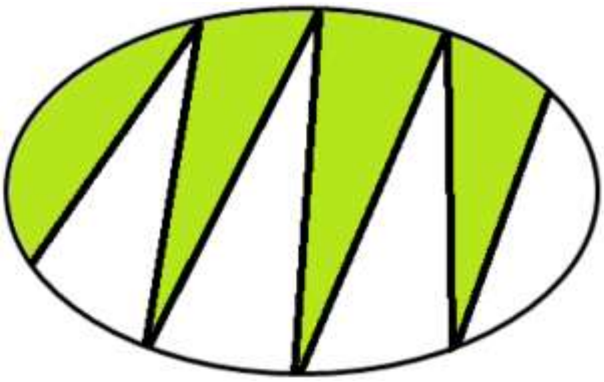


Figure 2.13: The MOOC-artifact

The white parts, those that will be inhabited by the trainees, will colour from week to week of orange (colour that has been chosen for the trainees, as you will recall from §2.3 and Figure 2.4). This happens, little by little, when the trainees access the modules and implement some Math Edu USs, so that the MOOC from artifact becomes an instrument for each of them.
When the MOOC is concluded, it will appear as in Figure 2.13. That is, the oval has been enriched as much by the contribution of the trainers as by the contribution of the trainees.

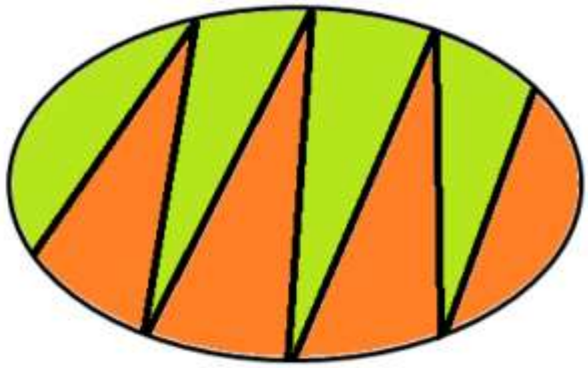


Figure 2.14: The MOOC when it is finished

We therefore explain how the trainees "colour" the parts of the modules dedicated to them.

Note that this representation of the MOOC (Figure 2.13, 2.14, – and how we will see with the Figure 2.17, 2.18) is not a juxtaposition. Rather, it is a more or less deep interpretation of the dynamic characteristics that feature the MOOC environment. For reasons of exposure, it is pictured in this "static" way, but we will concretely realize that the MOOC is not so.

2.6 Fragment from Connectivism

The fathers of Connectivism are George Siemens and Stephen Downes of the University of Manitoba (Canada). Moreover, the first MOOC, entitled *Connectivism and Connectivity Knowledge*, was held in 2008 by them. This course is actually hybrid, and has 25 participants in classroom and 2300 participants via e-learning. Connectivism is therefore a theory that we could not afford not to consider! Of course, we will not use the theory in its entirety, but will develop a hybridization, focusing our attention on the concepts of *connections*, *network of knowledge*, *learning*, *chaos* and *self-organization*.
Connectivism has at its base a fundamental assumption that serves as a pillar to the whole theory: *knowledge is a network and learning is a process of exploring this network*.

The fragment that we will consider to put in place the hybridization is the *network of knowledge*. It is a vaster fragment than the instrumental genesis. In fact, while the instrumental genesis grasp in it the concept of artifact and instrument; the network of knowledge grasp in it all the previous mentioned concepts. In fact, as we will see, the network of knowledge is made of connections; the process of exploring it is the learning and the learning happens self-organizing the informative chaos that reach every one of us.

2.6.1 Connections and network of knowledge

"A network can simply be defined as connections between entities" (Siemens, 2005, p.4). These entities are nodes and they consist of concepts, ideas and thoughts. Therefore, connections or relationships (these two are synonyms) are a links between two nodes.
"Knowledge may reside in non-human appliances" (Siemens, 2006, p. 31) like books, websites, databases connected by internet, MOOCs as well, and so on. The information,

events and experiences flow through one's ideas, thoughts and concepts in the process of thinking, dreaming, imagining and even while living and experiencing the real life. The flow of information that passes through these ideas and concepts strengthen them, while the ideas and thoughts that are rarely visited by surrounding events, experiences and information slowly lose their connections to other nodes, and eventually are removed or forgotten.
Consider this example from AlDahdouh et al. (2015, p. 10):

> "Consider […] a student who perfectly masters the Pythagorean Theorem of rectum triangle. By the time of studying the course, the student is fully aware of all details of this theory. She/he can calculate any given examples and can connect this theory to other broader areas and topics, such as circle's area and cone's volume. Over time, and if she/he doesn't face these concepts [everyday] in real life, the first thing that may be forgotten is the inner connections of the Pythagorean Theorem, the circle area and cone volume concepts. Therefore, they become ambiguous entities. Finally, the connections between the broader areas may, gradually, be lost".

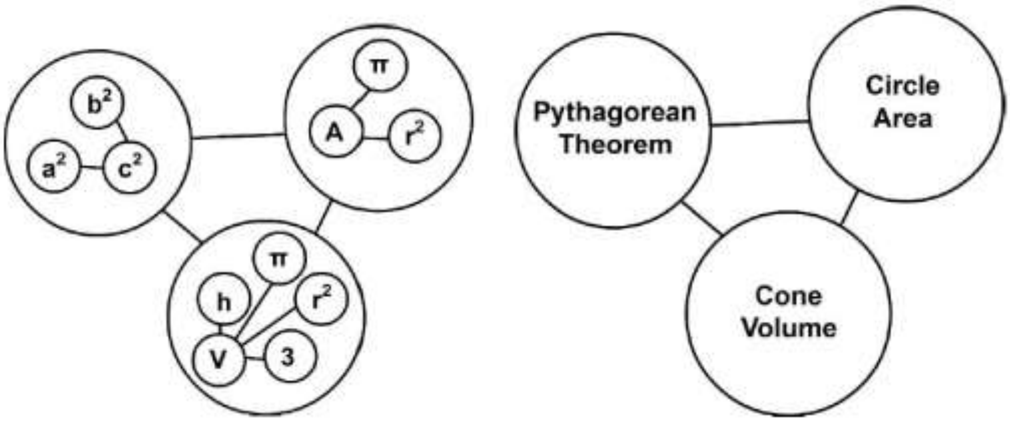


Figure 2.15: Example of network related to the Pythagorean Theorem (in AlDahdouh, et al., 2015, p.10)

On the other hand, "a learner, who continually encounters new information and events, will dynamically update and rewrite his/her network of learning and beliefs" (Siemens, 2006, p. 30).

Alive and moving network of knowledge

All previous theories (behaviourism, cognitivism and constructivism) recognize knowledge as an object or state to be acquired or built in the learner's mind. Connectivism, in contrast, conceives it as a process, which is alive and moving, a shifting reality. Connectivism sees both nodes and the connections as moving objects. In one moment some nodes appear, others disappear; some connections are strengthened, others are weakened. Siemens (2005, p.4) says: "While there is a right answer now, it may be wrong tomorrow due to alterations in the information climate affecting the decision". The node itself may change its position by changing its connections and give place to another reality.
To clarify this idea, we report an example by AlDahdouh, et al. (2015, p.12):

> "consider a car driver who was driving a car in high speed. The road appeared to be empty and all coming information made him satisfied about his decision. Suddenly, a pedestrian jumped into the roadside and crossed the street. The car, at that moment, was about 50 meters away from the pedestrian. The right decision would to press the brake pedal. Unfortunately, the

> driver didn’t take that decision. Two seconds later, the car was 10 meters away. The right decision, at that moment, was to steer the wheel. Pressing the brake pedal is no longer the right decision. In this example, there were three decisions (high speed, pressing the brake pedal and steering the wheel); each one was representing the best decision in a specific time fraction. However, when the incoming information is changing quickly, the best decision can suddenly become the worst”.

Therefore, network of a person resists new ideas unless that person has allies of concepts in his/her mind. The concept grows gradually by connecting to other concepts. The concept itself has its autonomy to “accept” or to “reject” the connections with other concepts. According to Connectivism, people have different ideas, opinions and reasoning; connection between two concepts is simply the path for reasoning.

Does knowledge already exist or is it something “in the open” and growing?
Connectivism uses the network topology to represent the knowledge structure. One of Connectivism’s questions is: “does knowledge already exist – so the learner just has to explore, discover and aggregate it – or is it something 'in the open' and growing – so the learner can invent in addition?” (AlDahdouh, et al., 2015, p.11). Connectivism recognizes both scenarios but it concentrates more on known knowledge. Describing knowledge as abundant and easy to access, connectivists hold that adding something new to the existing knowledge is very complicated and requires the effort of others. Therefore, according to the connectivist’s view, aggregating, exploring and discovering the known knowledge is more important than inventing new knowledge.

2.6.2 Learning from chaos and self-organization

In Connectivism, the structure of the knowledge is described as a network. The network is a set of nodes connected to each other. The network is not static; it is dynamic and may change over time. *Learning*, according to Connectivism, is a continuous process of network exploration. Siemens (2005) states:

> “Learning (defined as actionable knowledge) can reside outside of ourselves (within an organization or a database), is focused on connecting specialized information sets, and the connections that enable us to learn more are more important than our current state of knowing” (p. 5).

Learning is perceived beyond memorizing and knowledge transfer. In fact, “[…] memory is adaptive, it is not (necessarily) representative, and […] learning […] isn’t transferred, but grown anew by each learner” (Downes, 2012 a, p. 31).
The individual’s knowledge is like an evolving network. The knowledge that you have at a given time corresponds to a precise shape of the network. Instead, learning (i.e. increase knowledge) corresponds to the process with which the network expands itself.
There is a difference compared to constructivism. Constructivism explains that the individual builds knowledge. Connectivism explains that the individual creates connections between nodes of knowledge.

> “Our mind is a structure that creates and self-organizes connections. We not always build, but constantly connect [...] we do not live our lives in active cognition, we spend a lot of time in containers that we created. Instead of thinking, we command and select” (Siemens, 2005).

We derive our knowledge/skills from the connections we possess or of which we are a part. Let us not forget that the today's environment are permeated by technology. We are exposed daily to a myriad of information, coming from the technological tools that surround us (television, cell phone, computer, internet, ...). This innumerable information has degrees of importance or relevance that are different for each individual.

> "The need to evaluate the worthiness of learning something is a metaskill that is applied before learning itself begins. When knowledge is subject to paucity, the process of assessing worthiness is assumed to be intrinsic to learning. When knowledge is abundant, the rapid evaluation of knowledge is important. Additional concerns arise from the rapid increase in information. In today's environment, action is often needed without personal learning – that is, we need to act by drawing information outside of our primary knowledge. The ability to synthesize and recognize connections and patterns is a valuable skill" (Siemens, 2005, p. 3).

The complexity and multiplicity of connections can easily be perceived as *chaos*, information overload in which it is difficult to find meaning or coherence in information. Siemens talks about chaos, but not with a negative meaning. In fact, it gives a definition of ScienceWeek (2004) that quotes Nigel Calder's definition about chaos as "a cryptic form of order". Moreover, Siemens (2005) states: "Unlike constructivism, which states that learners attempt to foster understanding by meaning making tasks, chaos states that the meaning exists – the learner's challenge is to recognize the patterns which appear to be hidden". Chaos becomes a new reality in the people's learning process.

The Connectivism's need is to organize, within our network of knowledge, existing information outside of ourselves, with which we measure ourselves everyday: the authors define this as *self-organization*. In other words, self-organization is the spontaneous formation of structures, schemes or behaviors well organized compared to the randomness of the initial conditions.

Learning, as a self-organizing process requires that the personal system

> "be informationally open, that is, for it to be able to classify its own interaction with an environment. […] Self-organization on a personal level is a micro-process of the larger self-organizing knowledge constructs created within corporate or institutional environments. The capacity to form connections between sources of information, and thereby create useful information patterns, is required to learn in our knowledge economy" (Siemens, 2005, p.4).

Therefore, for Siemens (2005) learning is a continuous process of construction, development, self-organization of knowledge (as a network), so learning is not only to add new nodes, but especially it is to connect existing nodes with each other and make sense of these connections.

Knowledge development: the individual and the specialized learner

The starting point of connectivism is the individual. Personal knowledge is comprised of a network, which feeds into organizations and institutions, which in turn feed back into the network, and then continue to provide learning to individual. This cycle of knowledge development – *very similar with which we have observed in the instrumental genesis, especially in the observations of the §2.5.4.3)* – allows learners to remain current in their field through the connections they have formed.

As we have seen, learners are autonomous in the exploration of a network and they are different from each other in their aims and, therefore, in the way they use contents. *Note how this concept is very similar to what we have observed about the evolution of didactical*

praxeologies of the trainees, especially in the final part of the paragraph "Different kind of meta-didactical praxeologies".
The role of a specialized learner – researcher, teacher at school, or someone who has already connected to a very good network in the field – should be to help new learners to plant themselves in the network, to be connected to its nodes and to be part of it. *Note how this concept is very similar to what we have observed about the meta-didactical transposition made by trainers, see the paragraph "The meta-didactical transposition".*

2.6.3 Our theoretical assumptions in light of connectivism

We begin by pulling the strings of what has been exposed and underline what will be our assumptions for the sequel.
According to the Connectivism, *knowledge* is a particular type of network, whose nodes are "any entity that can be connected with another node": information, data, images, ideas, and feelings. Instead, *learning* is the process of exploration of this network. In particular, learning is a continuous process of construction, development, self-organization of knowledge (as a network), so learning is not only to add new nodes, but especially it is to connect existing nodes with each other and make sense of these connections.

What about learning for teachers?
It might seem that there is an "abuse" of language with respect to the use of the term "learning" for teachers. In fact, it is not the learning process that typically takes place in a classroom with students.
Learning is understood in a connective sense: it is not a "literal" learning of new things, rather it means to be able to see different concepts that were already known (reflect, think again, integrate them under a different perspective). This is just *an expansion of own network of knowledge*, which is possible through the sharing of practices and didactical theories. This is in line with what was observed in the MDT. In fact, the expansion of the network of knowledge corresponds to the trainees and trainers' praxeologies that can evolve.

It is starting from here the step of hybridization called ***interpretation*** *(§2.1.2). The F_2 fragment of the T_2 theory (*the network of knowledge of the Connectivism*) expressed in L_2 language is interpreted into the T_{01} theory (*the $T_0 + F_1$*) expressed in L'_0 (= L_{01}) language. The L_{01} language of T_{01} is thus extended in a L'_{01} language. In fact, we are starting to talk about* evolution of praxeologies *in terms of* expansion of network of knowledge. *In the following we will see how the* ***adaptation*** *step (§2.1.2) will be concretised.*

Network of knowledge for whom?
The actors of our discourses are the trainees, the trainers and the MOOC. Each of them has its own network of knowledge. For individuals (trainees and trainers) the concept is immediate; for the MOOC we should not be surprised. Connectivism, as we have observed, says "knowledge may reside in non-human appliances". The nodes of the MOOC's network of knowledge are, at first, all the mathematical activities and technological resources inserted in it by the trainers.
As stated by the Connectivism, the network of knowledge it is not static, but develops dynamically. How those networks can expand themselves? Among the features that favour the genesis of new connections within the networks of the individuals (trainees and trainers) we find: the use of technology inserted in the MOOC and in particular the MOOC itself; being

part of a community (trainees or trainers' one); and acting in a specific context (the MOOC and their daily environment). Instead, the features that favour the genesis of new connections within the networks of the MOOC are the interaction put in place by these individuals (trainees and trainers) in it.
It is interesting, therefore, to focus on the study of the interactions that take place within the MOOC.

Let us do a parallel with the Earth science
Ecology (from Greek: οἶκος, "house", or "environment"; -λογία, "study of") is the scientific analysis and study of interactions among organisms and their environment. Topics of interest to ecologists include the diversity, distribution, amount (biomass), and number (population) of particular organisms, as well as cooperation and competition between organisms, both within and among ecosystems. *Ecosystems* (Figure 2.16) are composed of dynamically interacting parts including organisms, the communities they make up, and the non-living components of their environment.

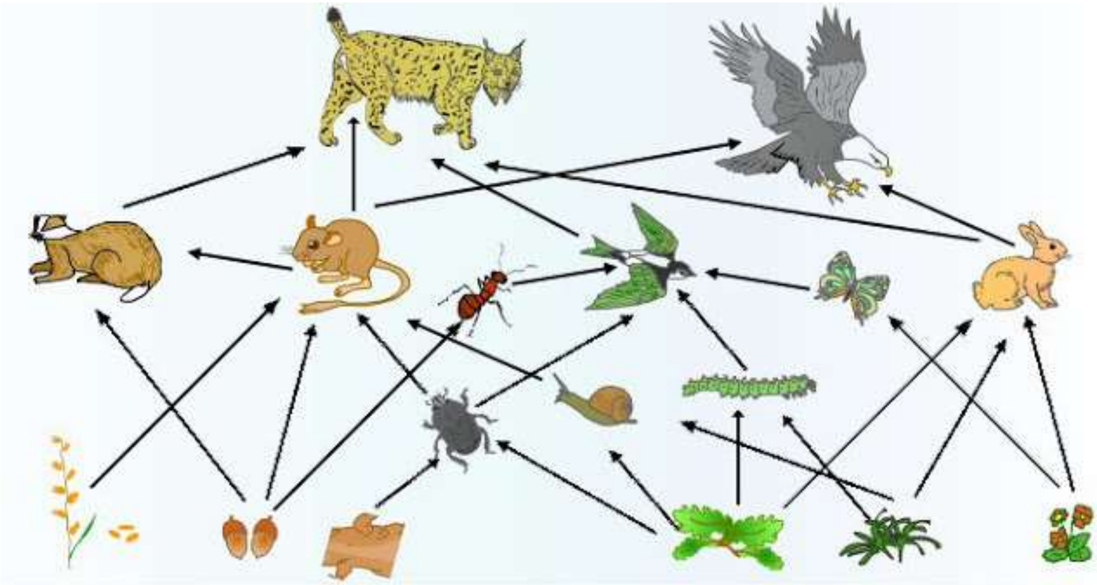


Figure 2.16: Example of ecosystem in Earth Science (picture retrieved from: https://brainly.lat/tarea/7255747)

2.6.3.1 MOOC-ecosystem

When new connections are generated in a MOOC environment by trainers and trainees, we can say that learning occurs within an ***ecosystem*** that – take inspiration from the Ecology – we define as follow:
"*all the relations (exchange of materials, experiences and personal ideas/point of view) put in place by participants of an online community, thanks to the technological tools through which they interact with each other, establishing connections within a given context*" (Taranto et al., 2017 a, p. 2481).
In fact, Siemens (2005) says that to encourage a connectivist learning, Fordist-Taylorist hierarchical structures must be replaced by dynamic structures - such as an ecosystem it is- because these are the most appropriate to facilitate the flow of knowledge.
If the first were stable and reassuring, the latter are free, informal, dynamic, chaotic, always changing.

The MOOC-ecosystem becomes a learning space, that is a place where the individual interacts with others, creates connections and expands his network of knowledge. Individuals move from passive consumers to active contributors.
Secondly, the personal knowledge in turn feeds the ecosystem. The ecosystem feeds back other people. And so on. A development cycle of knowledge is generated and it allows learners to stay updated and develop more and more their network of knowledge.
The learning goes beyond the storage and transfer of knowledge: the focus is on the individual's ability to build and crossing networks [of the ecosystem] (Downes, 2012 a).

We specify here that when the MOOC is identified as an ecosystem, it also incorporates the meaning of brokers. We had seen (in the paragraph "*Who is the broker in a MOOC?*") that, according to the language of the MDT, the MOOC is a broker and that even the traineers sometimes play this role. Understanding the MOOC as an ecosystem brings with it the ability to create new connections (or relationships) between the participants among themselves and between the participants and the MOOC itself. In fact, the ecosystem as such facilitates the transition of the *boundary objects*: "mathematical symbols, technologies, documents, software or other elements that allow people to connect different communities and work together" (Rasmussen & Keene, 2015, p.282).

It is starting from here the step of hybridization called ***adaptation*** *(§2.1.2), that is the modification of the* T_{01} *theory (*the $T_0 + F_1$*) in order to link the interpreted* F_1 *and* F_2 *fragments of the* T_1 *and* T_2 *theory (*the instrumental genesis of the Instrumental approach and the network of knowledge of the Connectivism, respectively*). The interpreted fragments consist of important modification of the* T_{01} *old components. First, in parallel with the concept of MOOC-artifact and MOOC-instrument, that come from the first interpretation step, thanks to the Connectivism contribution, the MOOC-ecosystem is originated considering the connections (relations) put in place by the participants of the MOOC (trainers and trainees). Second, the language of the* T_{01} *theory is modifying again. The concept of broker recognized both for the MOOC and (sometimes) for the trainees, is now incorporated by the new term of MOOC-ecosystem.*
It is time to describe the T_{012} *(=* $(T_0 + F_1) + F_2$*) theory, the new one that is obtained, called MOOC-MDT.*

2.7 How are the two hybridizations performed?

A MOOC at inert state (online course, implemented on a platform) is – using the language of Instrumental Approach – an artifact created by trainers that want to transpose the ideals praxeologies in order to generate a (possible) evolution of the didactical praxeologies of the trainees' community. We refers to it as ***MOOC-artifact*** and, in this sense, it is a static object.
Using the language of Connectivism, we then characterize the *MOOC-artifact* with its own *network of knowledge* (Fig. 2.17): its nodes are the contents, the ideas, the images and videos inserted by trainers; the connections are the links (created by trainers) between their node pairs.

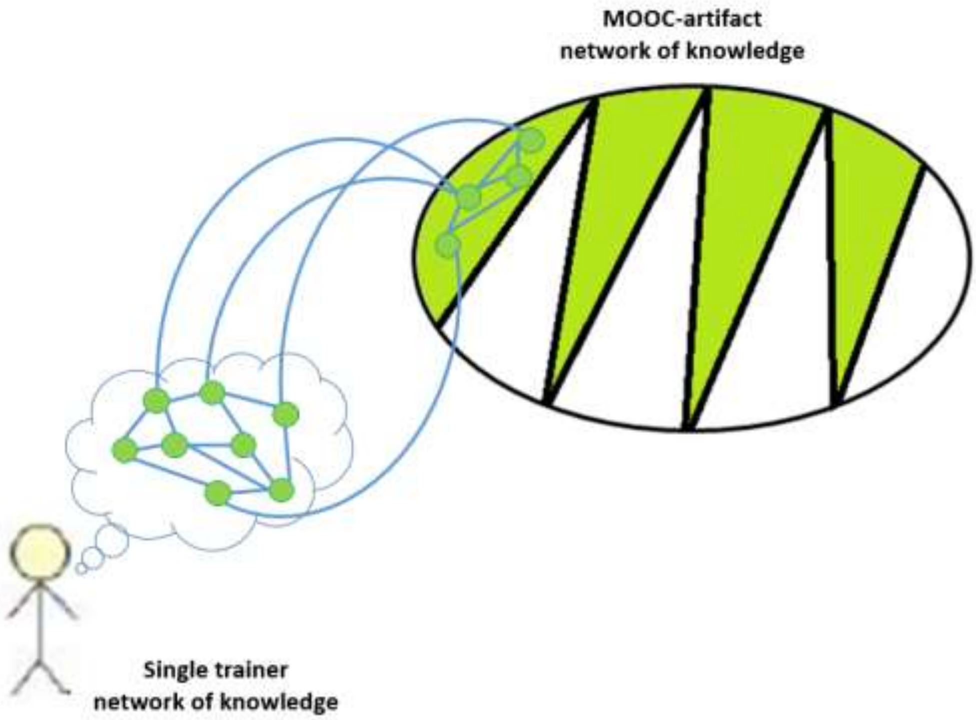


Figure 2.17: Schematic idea of the MOOC-artifact network of knowledge

This built environment is mostly lived by the designer-trainers. They organized the resources, then wait for the enrolled people start to access and live in it. It is important to underline that, when the trainees enter in the MOOC, also for them it is a *MOOC-artifact* at the beginning. Only until when they look its aspect[14]. As soon as the trainees begin even simply to think of considering one resource rather than another, the MOOC is subject to an evolution. As happen in the Instrumental Approach: if a subject apply proper USs, the artifact become an instrument for him. Here is the same, but it is more complex and we will describe it in what follow.

When the MOOC starts, it comes to life because inhabited by interactions triggered by its participants (both trainers and in particular trainees). A large number of people participate, edit and broadens the knowledge embedded in the MOOC, thanks to sharing tools. So, when a MOOC teaching module is activated, it is configured according to a complex structure: the *ecosystem* (Taranto et al., 2017 a, p. 2481).

The network of knowledge of ***MOOC-ecosystem*** (Fig. 2.18) is composed not only of the MOOC-artifact network, but also of the contribution that each participant (remember that their number is massive) gives to this network, exchanging and sharing of material, ideas, thoughts, experiences. This network is not absolutely static, but develops in a dynamic, difficult to control in a timely manner and apparently chaotic way. Moreover, when the individual user *learns* (actually modifies his network of knowledge drawing on ecosystem network), the MOOC becomes an instrument for each of them.

The ***MOOC-instrument*** exists not only in the perspective of the use of MOOC-artifact by the individual (when he explores the materials), but especially when the individual is part of MOOC-ecosystem (when he takes advantage of comments from other individuals or when he shares something of his own). Here lies the concept of self-organization of own network of knowledge.

[14] Note that "have a look on MOOC" and "participate in the MOOC" are two different aspects. Participation comes up later, when you become familiar with the context in which you are.

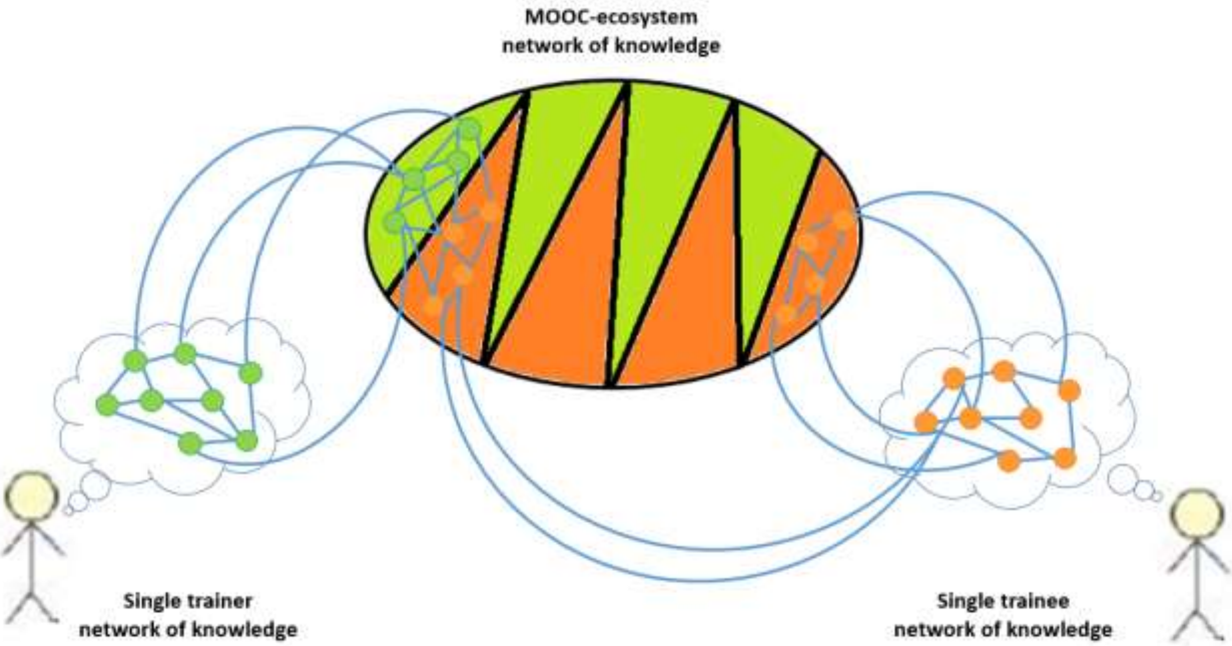


Figure 2.18: Schematic idea of the MOOC-ecosystem network of knowledge

Within the chaos of the MOOC-ecosystem, each participant creates connections between its nodes and those of the MOOC network of knowledge. Each participant appropriates the resources made available in the MOOC, through self-organization processes. Creation of connections and self-organization processes are respectively the "recomposition and reorganization activities that lead to the emergence of utilization schemes" (Rabardel & Verillon, 1985; see the paragraph "*Utilization schemes*").
However, due to the massiveness of which a MOOC is bearer, it is not possible to immediately identify the Math Edu USs put in place by individuals. Everything happens in a fraction of seconds with tens and tens of people at the same time. For this reason, it is more reasonable to consider the totality of the Math Edu USs put in place, looking at what we call the MOOC-ecosystem.

The relationships that trigger the ecosystem are linked to specific activities that the trainees must carry out within the MOOC, according to the instructions given by the trainers. Therefore, using the language that is inspired by the Meta-Didactical Transposition, inside the MOOC, every week (so for every module), the trainees have to solve some *tasks*, through *multi-techniques*, properly justified. They must look at the proposed materials, share their thoughts through interactive sharing tools, and possibly experience some of the proposed activities. Importantly, these tasks are not predetermined, because a trainee is free to choose the times, ways and depth with which to address them. The multi-techniques (or, as previously clarified, the Math Edu USs) are therefore the ways in which the trainees extend and modify their network of knowledge, drawing on the ecosystem's one, and influencing it in turn (thus affecting all other trainees). These trainees' actions (their meta-didactical praxeologies) trigger what we call "***double learning process***" (Taranto et al., 2017 a, p. 2482).

The double learning process is the heart of the hybridization put in place between Instrumental Approach and Connectivism. It follows the principles of instrumental genesis, but under a new light, that is the complexity that the MOOC brings with it. In fact, the

transformation from artifact to instrument (Verillon and Rabardel, 1995) is here redefined as an evolution from artifact to ecosystem/instrument, creating a more complex system.
We will proceed by illustrating this process, then we will make the necessary explanations that will “unveil” how the hybridization has been completed.
We anticipate in the meantime that it is important to note how the double learning process is linked to the two aforementioned theories also from a terminology point of view: the term process is taken from the Instrumental Approach; while the term learning from Connectivism.

2.7.1 The double learning process

On the one hand, the MOOC-ecosystem is a specific learning tool for the individual; on the other hand the use of MOOC-instrument by the individual generates learning for the whole ecosystem. The dynamic process has the following intertwined components (Figure 2.19):

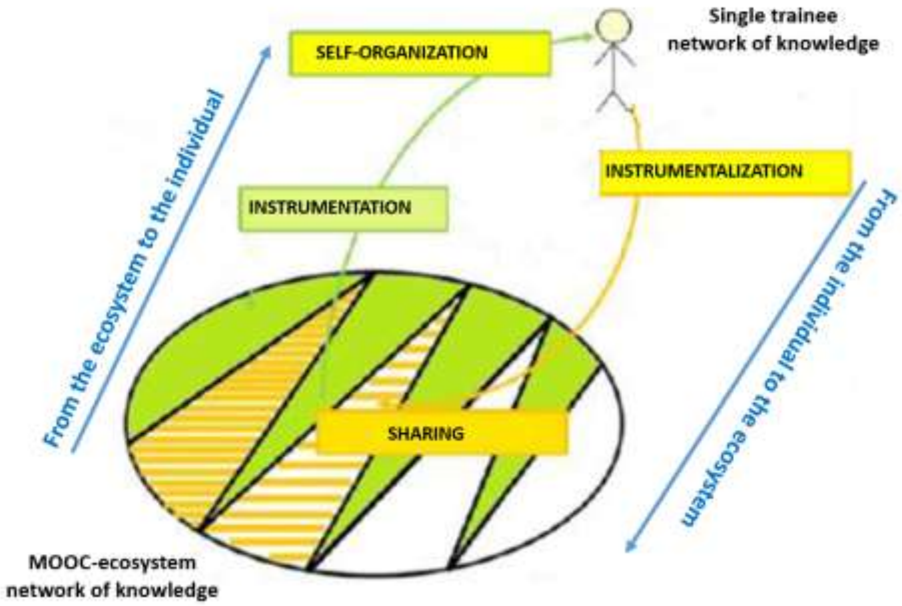


Figure 2.19: The double learning process

- Instrumentation/Self-organization (from the ecosystem to the individual) is the process by which the network of MOOC-ecosystem expands the individual’s network of knowledge. In the *Instrumentation* (Verillon & Rabardel, 1995), the chaos (Siemens, 2005) of the ecosystem network reaches the individual. The opinion and experiences showed in the MOOC module(s) – by trainers and/or trainees - make sure that the individual compares himself with new Math Edu USs. The individual then engages in a process of *self-organization* (Siemens, 2005): he selects which Math Edu USs proposed by the MOOC are valuable and which are not, she/he can extend and modify his/her own network of knowledge based on them.
- Instrumentalization/Sharing (from the individual to the ecosystem) is the process by which the individual’s network of knowledge expands the network of MOOC-ecosystem. In the *Instrumentalization* (Verillon & Rabardel, 1995), the individual, with her/his renewed network of knowledge, thinks and builds new connections independently. She is stimulated by a task requested by MOOC and address the

ecosystem to transform according to her/his own (new) Math Edu USs. She seeks to integrate it with her own cognitive structures. In the *sharing* the MOOC welcomes the contribution of the individual participant and makes it available to all: the information goes towards all members of the ecosystem that may be affected by the new content on the MOOC.

The network of knowledge of the *MOOC-ecosystem* is dynamic: it evolves basing on the MOOC-artifact's network thanks to the contribution of the participants to the network. Therefore, the network of knowledge of MOOC's individual users evolves as an interpreted and personal self-organization (Siemens, 2005, p. 4) of the MOOC-ecosystem's network of knowledge. As previously mentioned, the transformation from artifact to instrument (Verillon & Rabardel, 1995) is then redefined as an evolution from artifact to ecosystem/instrument, creating a more complex system.

The following tables (2.10 and 2.11) allow us to better grasp the meaning of our discourses. It is a reading of the instrumental genesis (§2.5.3) with the lenses of MDT and Connectivism.

Node at the beginning in my network of knowledge	Cup: an artifact for drinking	
Task	To drink from the cup	
	DESIGNER	**INDIVIDUAL**
Techniques or utilization schemes (USs)	- Create a handle - Give it a structure that is not too bulky - Make a border not too thick	- Hold the handle - Bring it close to the mouth - Place your mouth on the border
Logos	Cognitive ergonomics	Satisfy needs of primary necessity
Node at the end in my network of knowledge	Cup: an instrument that can allow someone drinking	Cup: an instrument that makes me drink

Table 2.10: From cup as artifact to cup as instrument

Node at the beginning in my network of knowledge	MOOC: artifact to transpose new praxeologies	
Task	To overcome the misconception that the students have about height in triangles	
	TRAINERS	**TRAINEES**
Multi-techniques or Math Education utilization schemes (Math Edu USs)	- Videos - Activities with sway - Communication message boards	- View the resources made available - Read the comments of the other trainees
Logos	Meta-Didactical Transposition	Double learning process
Node at the end in my network of knowledge	MOOC: an instrument to allow some teachers to acquire new praxeologies	MOOC: ecosystem/instrument to acquire new praxeologies

Table 2.11: From MOOC-artifact to MOOC-ecosystem/instrument

Let us start to consider the Table 2.10. In the network of knowledge of both designer and individual, a node is emerging "Cup: an artifact for drinking".

First, we focus on the designer's network of knowledge. As we have seen in §2.5.1, he wants to realize an artifact that allows drinking, specifically a cup. In fact, his task is to make sure that, given the artifact (the cup), it can be used for drinking. In this way, he must give a specific purpose to the artifact he wants to realize. The artifact form has to justify the performance that is targeted and materialize the existing plan from which it will be originated (drinking). Therefore, the designer puts in place some consecutive techniques (those that you see in the second column of Table Y10) that are justified by the cognitive ergonomics. In his network of knowledge, the starting node it is become “Cup: an instrument that can allow you drinking”. As we have seen in §2.5.1, an artifact presupposes a purpose and consequently intelligence able of creative activity that can incorporate knowledges inside the artifact. So, let us move on individual’s network of knowledge. His task is drinking from the cup. However, the cup imposes on him a set of constraints/affordances that he has to manage in the course of his action. The reorganization and recomposition of activity that the individual has to do for drinking leads to the emergence of some techniques (or utilization schemes), those that you see in the third column of Table Y10. The artifact joined to the USs by the individual on the basis of his culture and experience become an instrument. So, in his network of knowledge, the starting node it is become “Cup: an instrument that makes you drinking”.

In a symmetrical way, we explain Table 2.11. In the network of knowledge of both trainers and trainees, a node is emerging “MOOC: an artifact to transpose new praxeologies”.
First, we focus on the trainers’ networks of knowledge. As we have seen in §2.5.1, they want to realize an artifact that allows to transpose some meta-didactical praxeologies. To remain in the example we have previously discussed (see the paragraph “Uncontrollability of ongoing learning dynamics”), we can consider as task that of overcome the misconception that the students have about height in triangles. In this way, they must give a specific purpose to the artifact he wants to realize. The MOOC-artifact contents have to justify the performance that is targeted and materialize the existing plan from which it will be originated (overcoming the students’ misconception about triangle height). Therefore, the trainers put in place some multi-techniques (those that you see in the second column of Table 2.11) that are justified by the MDT theoretical framework. In each of their network of knowledge, the starting node it is become “MOOC: an instrument that allow some teachers to acquire new praxeologies”. As we have seen in §2.5.1, an artifact presupposes a purpose and consequently intelligence able of creative activity that can incorporate knowledges inside the artifact. So, let us move on trainees’ networks of knowledge. The trainers give the task for the trainees. However, it could also coincide with their real need to receive training on that given mathematical concept or didactic aspect. In fact, we recall that the MOOC in question are MOOCs for teacher education, so whoever enrol is aware that he will receive updates, teaching suggestions/methodologies/strategies that can be spent in teaching practices.
To remain in the example we have previously discussed, the task given by trainers to trainees is again overcoming the students’ misconception about triangle height. However, the MOOC imposes on trainees a set of constraints/affordances that they has to manage in the course of their actions. The reorganization and recomposition of activity that each trainee has to do for accomplish the task leads to the emergence of some multi-techniques (or Math Edu USs), those that you see in the third column of Table 2.11. The MOOC-artifact joined to the Math Edu USs by each trainee on the basis of their culture and experience become a MOOC-instrument for each of them and, in particular, a MOOC-ecosystem for all of them. So, in the trainees’ networks of knowledge, the starting node it is become “MOOC: an ecosystem/instrument to acquire new praxeologies”.

2.8 The MOOC-MDT

Finally, we have obtained the so-called MOOC-MDT (Figure 2.20), namely the Meta-Didactical Transposition revised for analyse MOOC environment thanks to the hybridization process, one time considering a fragment, the *instrumental genesis*, from the Instrumental Approach and a second time considering a fragment, the *network of knowledge*, from the Connectivism.
The fragment of *network of knowledge* is implanted in the Instrumental Approach, which is so adapted to MOOC's own dynamics, where the community of participants becomes subject and object of a new, more complex kind of instrumental genesis: the double learning process. In fact, it maintains the structure of the instrumental genesis, with directions from the subject to the object and vice versa, but it is also enriched with the Connectivism standpoint. If in the MDT, the trainers shape their proposals according to the practices they think appropriate, and so they can estimate how much the trainees learn such proposals, in the MOOC-MDT the process of training appears to be more difficult to control. The trainers do not know "what" the user has really looked at among the presented materials, nor they can know how (s)he interpreted them. Equally, the trainees benefit from material provided by trainers and other trainees sharing their materials and ideas using the communication boards. The process evolves stochastically: a determining role is played by each trainee feeling part of an interacting community.
MOOC-MDT facilitates the study of the specific dynamics of the interactions among trainees and between trainees and trainers, which occur online in virtual environments. Moreover, it allows perceiving possible evolution in the praxeologies of the community of trainers and trainees.

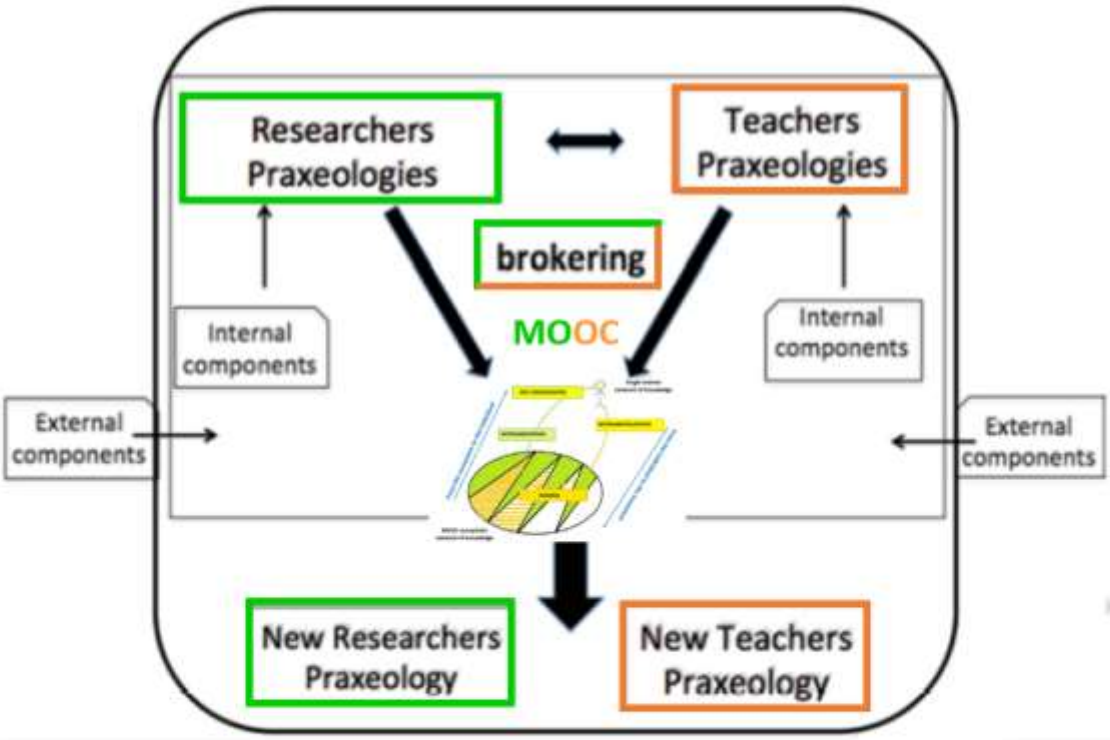


Figure 2.20: The MOOC-MDT theoretical framework

2.8.1 Are we satisfied with this theoretical result?

If the intent is to analyse the interactions in the MOOC, the MOOC-MDT framework is a valuable tool. However, if we are interested – as we are – also in analysing the possible evolution of trainees and trainers' praxeologies, MOOC-MDT it is not the best one. We intend to say that finer analyses can be conducted if another theoretical lens is "attached" to this. We are talking about the Valsiner's Zone Theory, which has been used by M. Goos to analyse teacher education phenomena. It will be the theory we will use to do networking with the MOOC-MDT.

Why is there this need for networking?
Beyond the theoretical advantages that networking brings with it (as seen in §2.1.1); analysing with the MOOC-MDT leaves points at a superficial level that can be investigated.
We have observed that every individual (trainer or trainee) has a network of knowledge, but how exactly is it done? Furthermore, the interactions that are experienced within the MOOC are dictated:
(i) by own personal culture; (ii) by the influence exerted by seeing how others interact in the MOOC; (iii) by the interactions that are established also in the daily contexts in which the individual are immersed and that influence their knowledge and the interaction within the MOOC as well.
Taking these aspects into consideration also allows us to make more accurate analyses in the small virtual MOOC world, and also outside, in the real world, allowing us to study what impact a MOOC has on the professional development of both trainees and trainers.

After that, can we be satisfied with what this theoretical result will be? Yes, finally ... at least for the purposes of this dissertation!

2.9 Valsiner's Zone Theory

In order to make it easier to understand the new theory which we are networking with achieved hybridized one (the MOOC-MDT), we will now sketchily illustrate it.

The theoretical framework elaborated by Valsiner (1997) explains children's development in the context of their relationships with their physical environment and other human beings.
Valsiner took Vygotsky's (1978) zone of proximal development (ZPD):

> "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86).

Valsiner redefined this zone as the collection of ways that an individual could develop as a result of interactions with their environment and the people in it. Consequently, this zone depends on the physical attributes, knowledge, and skills that an individual brings to a situation and includes developmental possibilities that may not occur. Therefore, Valsiner extended Vygotsky's (1978) concept of Zone of Proximal Development (ZPD) to incorporate the social setting and the goals and actions of participants. Valsiner (1997) described two additional zones: the Zone of Free Movement (ZFM), which structures learners' interactions within the learning environment, and the Zone of Promoted Action (ZPA), representing the actions of a more experienced or knowledgeable person to promote specific types of learning.

The Zone of Free Movement (ZFM)
The ZFM structures

> "(a) the child's access to different areas in the environment, (b) the availability of different objects within an accessible area, and (c) the child's ways of acting with the available objects in the accessible area. As a result of development, the child learns to set up a ZFM in his or her personal thinking and feeling – the ZFM becomes internalized". (Valsiner, 1997, p. 188)

The ZFM has a number of properties, as follow (Valsiner, 1997, pp. 189-190).

- The ZFM is always based on the child's relationship with the structure of the given environment setting. At any time in life, the child's (adult's) access to some areas in the environment is seemingly unlimited, whereas access to some other areas is blocked. The notion of *area* can also be applied beyond the geographically organized space, and can include different objects in places and different actions with the same object. In addition, area can be extended to thoughts and feelings.
- The ZFM is based on the meanings of different aspects of the environment for the social other (parents, siblings, schoolteacher …) who is the leading organizer (but not the sole determiner) of the ZFM for the child. Either the social other or the child may make the first move in structuring the ZFM.

  > "However, the child's caregiver is the gatekeeper of the ZFM as it is constructed – and reconstructed from time to time. For example, when a 2-year-old boy and his mother enter a new environment (e.g., during a visit to a friend's home), the child (who goes to a precious vase in the living room and tries to push it onto the floor) or the mother (who told the child immediately, when they entered the room, not to touch the vase) may start the construction of a particular ZFM" (Valsiner, 1997, p.190).

  The construction of the ZFM may involve both proactive and reactive child-control techniques. Whether the adult or the child starts the construction, different routes can construct the resulting ZFM. At one extreme, the caregiver may play an overwhelmingly dominant role in its construction, leaving the child with the option of conforming to the ZFM as it has been unilaterally set up. At the other extreme are occasions where the social other of the child participates minimally in the construction of a ZFM, and the major role end up being played by the child.
- The ZFM is often set up on the basis of the parent's understanding of what the child can do in the given setting, in conjunction with what the child is doing or has done in the past. Orientation towards future possible actions is thus a part of the construction of ZFM.
- The ZFM is reconstructed when the adult and the child enter a novel environment. The adult analyses the new setting on the basis of his or her knowledge of the former action of the child, and the potential future action afforded to the child by the new environment. That analysis – based on cognitive simulation of scenarios of possible events – leads to the basic understanding of how the ZFM could be constructed. Beyond that, the actual behaviour of the child may lead to further refinement, or change, of the simulated ZFM.

The Zone of Promoted Action (ZPA)

The ZPA is "a set of activities, objects, or areas of the environment, in respect of which the person's actions are promoted" (Valsiner, 1997, p. 192). Parents may get involved in special efforts to promote their child's actions with an object that they consider important for the child's development. The child may, but need not, be interested in acting with that object. The parents, however, may try to do whatever they consider feasible to promote the child's action with that particular object.

> "The ways in which ZPA functions in everyday lives of families are easily observable at any age of children. For example, during a session of "free paly" of the parents and their toddler in the living rooms at home, the parents may try to get (and keep) the child interested in reading a children's book, so that understanding of words and pictures, and knowledge of the alphabet, can promoted. The children, however, may be captivated by the book reading for only a short while, and will soon move on to other activities. The parents may try to get the child to continue with book regarding, but if many other activities are available within the ZFM for the child, parent's efforts may be to no avail" (Valsiner, 1997, p.192)

The important characteristic of ZPA is its *nonbinding nature*. When a ZPA is set up but the child does not follow the lead of the parents' promotional effort but acts with other objects (in other way) within the ZFM, there is no way in which the child can be made to act within the ZPA (unless the ZPA is turned into a ZFM).

Remarks

The ZFM included both external and internal constraints that influence how the individual is able to act; therefore, the ZFM determines what development is allowed under the existing conditions. The environment imposes the external constraints, whereas internal constraints are the result of socialisation and include beliefs and expected ways of acting. Valsiner (1997) argued that although internal constraints are not evident they could be inferred from an individual's actions and their reflections about these actions. He defined the ZPA as "a set of activities, objects, or areas of the environment, in respect of which the person's actions are promoted" (p. 192). A key feature of this zone is that, except where the ZPA and ZFM are identical, the individual is free to accept or reject the actions that are promoted. Consequently, while the ZFM can be seen as inhibiting development, the ZPA promotes development and includes an active role for the individual.

The ZFM and ZPA are dynamic and interrelated, forming a ZFM/ZPA complex that is constantly being re-organized by adults in interactions with children. Valsiner argued that development could be directed (canalised) by structuring successive ZFM/ZPA complexes for the learner. Eventually, especially in the case of child development, the ZFM/ZPA complex becomes internalised and the individual becomes self-regulating.

According to Valsiner's (1997) theoretical framework, development can only occur if the constraints that exist in an individual's ZFM *allow* the actions that are being *promoted* in the ZPA; in other words, there must be overlap between the individual's ZFM and ZPA with the ideal situation, assuming that the promoted actions are desirable, being that all promoted actions are allowed. These allowed and promoted actions must also be within the individual's *set of possibilities* for development (i.e., within the individual's ZPD); therefore, there must be some overlap when the individual's ZPD is mapped onto their ZFM/ZPA complex.

Even though much of Valsiner's (1997) work was on child development, he argued that his theory had broader application and could be used to understand human development in general. He identified education, where teachers structure the development of students and the

ZFM/ZPA complex remains observable, as one area where this theoretical framework could be utilized.

2.9.1 A zone theory approach to understanding learning

Employing Valsiner's (1997) zone theory to provide insights into teacher learning requires consideration of how the ZFM/ZPA complex experienced by the teacher interacts with the teacher's ZPD. The ZPD involves internal processes thus making it difficult to identify contributing factors through empirical research.
In Goos (2005) you find examples of how this construct could be employed in empirical research. Goos (2005) described the ZPD as a "symbolic space" (p. 37) and mapped factors known to influence teachers' use of technology onto the ZPD, ZFM, and ZPA of participating teachers (Table 2.12). She saw pedagogical knowledge and beliefs along with the experience and existing skills of the teacher as contributing to the ZPD. Then, she interprets the other two zones as follows: the ZFM represents environmental constraints that limit freedom of action and thought. So, the ZFM suggests what teaching actions are possible in the educational environment; while the ZPA represents the efforts of a teacher educator, a supervising teacher, or a more experienced teaching colleague, to promote particular teaching skills or approaches. The goal of her study was to understand how these teachers developed identities as users of technology as they moved from pre-service to beginning teachers.

Valsiner's zones	Goos's interpretation
ZPD: Zone of Proximal Development (possibilities for developing new teacher knowledge, beliefs, goals, practices)	• Mathematical Knowledge • Pedagogical content knowledge • Skill/experience in working with technology • Beliefs about mathematics, teaching and learning
ZFM: Zone of Free Movement (structures teacher's access to different areas of the environment, availability of different objects within an accessible area, ways the teacher is permitted or enabled to act with accessible objects in accessible areas)	• Perceptions of students • Access to resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures
ZPA: Zone of Promoted Action (people, objects, or areas in the environment in respect of which the teacher's actions are promoted)	• Pre-service teacher education • Professional development • Informal interaction with teaching colleagues

Table 2.10: Goos's Zone Theory (taking inspiration from Bennison & Goos, 2010, p.34)

2.9.2 Using a zone theory approach to understand professional development

"The Zone Theory framework offers a dynamic way of theorizing teacher learning […]" (Goos, 2005, p.35).
Goos (2013) interpreted the zone of proximal development (ZPD) as the set of possible ways in which a teacher might develop, the zone of free movement (ZFM) as the constraints and affordances provided by the teacher's professional context, and the zone of promoted action (ZPA) as activities that the teacher can be involved in that promote certain ways of teaching.

She claimed that such an approach enables the complexity of teacher learning and development to be analysed, while still allowing for the influence of the teacher to direct their own learning by seeking out professional development or modifying their environment (i.e., by reorganising elements of their ZPA and ZFM, respectively).

In addition to understanding teacher learning, Goos (2013) argued that Valsiner's (1997) zone theory could be used to design interventions that *change* teachers' knowledge, beliefs, and practices by utilising a teacher's current zone configuration to identify barriers to development. She cautioned that, although this approach could direct (or canalise) development in a number of possible ways, the outcome depended on how the teacher interpreted and responded to the intervention.

The teacher generally creates the ZFM in a classroom, giving students little freedom to change this zone. It could be argued that teachers have greater freedom to restructure their own ZFMs (e.g., Goos, 2013) but are only likely to do so if they can see the benefits of making these changes. This is analogous to teachers changing their practices as a result of professional learning if they regard the changes as worthwhile (Guskey, 2002; Gresalfi & Cobb, 2011). However, even if promoted actions are allowed (i.e., there is overlap between the ZFM and ZPA), these actions must also be within the developmental possibilities of learners; in other words, there must also be overlap between the ZPD and ZFM/ZPA complex. By extension, it could be argued that those designing professional development interventions for teachers (*as a MOOC for teacher education*) need to ensure that the activities that are being promoted are allowed within the teachers' professional context and that the participants can develop the necessary knowledge and affective attributes to access the ideas that are presented.

2.10 MOOC's Zone Theory: a theoretical framework for mathematics teacher education through MOOCs

2.10.1 Adapt the Goos's Zone Theory to the MOOC

Lerman (2001) claims that teachers' learning is better understood as an increasing participation in sociocultural practices that develop their identities as teachers. As a result, a variety of studies has applied sociocultural theories to teacher learning. Based on Goos's analysis (Goos 2005; 2013), who has adapted the Valsiner's (1997) Zone Theory to the factors that influence the teachers' use of technology, we propose a further adaptation to involve all MOOC's actors (trainers and trainees) and their dynamics.

2.10.1.1 ZPD… at the beginning

In this adaptation, the Goos's (2005) ZPD definition maintain its general sense, but it is adapted in the view of the MOOC-MDT. We underline better this point subsequently, after that we will have explain the networking (in § 3.5). For the moment, it is important notice that we must consider ZPD for both trainers and trainees. In particular, we remember that when trainers start to think about the design of an online training mathematics course to a large number of users, they must take great care to select the mathematical, pedagogical and technological features to be offered to trainees, as ideal didactical praxeologies. The trainers do not know in advance what is the professional level of those who enrol in the MOOC. They can only make a conjecture and hypothesize a typical teacher as a target.

Therefore, about the trainers, the ZPD comes into play when they are thinking about the ideal didactical praxeologies that they want to transpose to trainee teachers who will follow the MOOC. Hence, trainers assume a certain level of prior knowledge (ZPD) of the trainees' community (not of the individual teacher since they are forced to consider mean values).
They prepare and administer certain activities (possibly using innovative methodologies, e.g. the mathematics laboratory) in order to help the trainees' community to move from the current level (their present didactical praxeologies) to the potential one (the ideal didactical praxeologies). The ZPD allows observing the phenomenon (at the beginning) from a static point of view: it is a look towards the MOOC's trainees and trainers themselves. Therefore, some minor changes concern the ZPD that is almost the same as in Goos (2005; 2013): the terminology changes just adding the concept of (meta-)didactical praxeologies, and a ZPD both for trainees and trainers is distinguished (Table 2.13; Table 2.14).
It will be noted that it has been specified that "at the beginning", the ZPD allows observing the phenomenon from a static point of view.
We will return later on the ZPD making the necessary clarifications.

Taranto's re-elaboration/interpretation for trainers	
Goos's zones in Taranto's interpretation	**Goos and Taranto's interpretation**
ZPD: Zone of Proximal Development (possibilities for expand the own network of knowledge)	• Mathematical Knowledge • Pedagogical content knowledge • Skill/experience in working with technology • Meta-didactical praxeologies that include beliefs about mathematics, teaching and learning

Table 2.13: Table 2: Trainers' ZPD

Taranto's re-elaboration/interpretation for trainee	
Goos's zones in Taranto's interpretation	**Goos and Taranto's interpretation**
ZPD: Zone of Proximal Development (possibilities for expand the own network of knowledge)	• Mathematical Knowledge • Pedagogical content knowledge • Skill/experience in working with technology • (Meta-)didactical praxeologies that include beliefs about mathematics, teaching and learning

Table 2.11: Trainees' ZPD

2.10.1.2 The meaning of environment

Before going into details of the ZFM/ZPA complex and explaining what adaptations the dissertation writer has made, it is better to make a clarification. *Environment*, in the Goos's Zone Theory, means the place where the teacher lives, moves, teaches and relates. In particular, the ZFM is the classroom and school context at the organizational structure level. On the other hand, the ZPA is the place where the teacher relates to her/his peers (colleagues from the same school, colleagues from another school, headmaster, ...) or more experienced people (trainers, educators, …). Therefore, briefly, the ZPA denotes the space that promotes professional development. For those familiar with Zone Theory, a MOOC's association with the ZPA may be immediate. However, it is clear to those who are familiar with MOOCs that this association has limits. Among the resources to be seen and the technological tools, with which to become familiar, the MOOC presents itself to peers with different backgrounds and

from different places (Italy in our case, but it could be from all over the world) as a small world in which the teachers live. In its development, there is therefore an alternation between ZFM and ZPA or better, as cleverly and far-sightedly Valsiner and Goos say, it is a ZFM/ZPA complex. But it is definitely different from what the teacher lives in the school context. In fact, the school context is tangible, among the school desks and the faces of colleagues; the MOOC is online, but not unrealistic. Certainly, there is no human contact in the course, but there are interactions in a peer community, supported by experts, who overcome this lack.
Let us see how and what are the changes to adapt the Goos's Zone Theory to the MOOC.

2.10.1.3 ZFM/ZPA complex

The ZFM/ZPA complex, compared with Goos adaptation, is split in two levels: one internal and two external to the MOOC. The level that refers to the MOOC is called ***MOOC's ZFM/ZPA*** (Table 2.15), instead the levels external at the MOOC are: almost exactly the Goos's adaptation for the MOOC's trainees, that we call ***school's ZFM/ZPA*** (Table 2.16) and ***research environment's ZFM/ZPA*** for the trainers team (Table 2.17).
Now, we will explain these three distinct ZFM/ZPA complex carefully.

Taranto's re-elaboration/interpretation		
Valsiner/Goos's zones	**MOOC's ZFM/ZPA**	
	Trainers	**Trainees**
ZFM: Zone of Free Movement (structures trainers/trainees' access to different modules of the MOOC, availability of different resources within an accessible module of the MOOC, ways the trainers/trainees are permitted or enabled to act with accessible resources in accessible modules of the MOOC)	• Design of the platform that hosts the MOOC and perception of this new environment • Design and digital transposition of mathematical resources • Communicational resources to foster communication among trainees and between trainers themselves with trainees • Technical support • Curriculum and assessment requirements • Organisational structures and cultures	• Access to the platform that hosts the MOOC and perception of this new environment • Access to mathematical resources • Communicational resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures
ZPA: Zone of Promoted Action (virtual people, resources, or modules in the MOOC in respect of which the trainers/trainees' actions are promoted)	• Interaction with trainees reading (and sometimes answering) their post. • Professional development	• Informal interactions with enrolled trainees • Interaction with trainers • Professional development

Table 2.15: MOOC's ZFM/ZPA (trainees and trainers)

Taranto's re-elaboration/interpretation	
Valsiner/Goos's zones	**School's ZFM/ZPA (~ Goos's interpretation)**
ZFM: Zone of Free Movement (structures teacher's access to different areas of the environment, availability of different objects within an accessible area, ways the teacher is permitted or enabled to act with accessible objects in accessible areas)	• Perceptions of students • Access to resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures
ZPA: Zone of Promoted Action (people, objects, or areas in the environment in respect of which the teacher's actions are promoted)	• In-service teacher education • Experimentation in classroom of some activities see in the MOOC • Professional development • Informal interaction with teaching colleagues

Table 2.12: School's ZFM/ZPA (trainees only)

Taranto's re-elaboration/interpretation	
Valsiner/Goos's zones	**Research environment's ZFM/ZPA**
ZFM: Zone of Free Movement (structures trainers' access to different areas of the environment (University, Department of Mathematics, national/international conferences, …), availability of different objects within an accessible area, ways the trainers is permitted or enabled to act with accessible objects in accessible areas)	• Perceptions of teachers • Access to resources (literature, proceedings of conferences, teaching experiment, data obtained via questionnaires or interview, …) • Technical support (print centre, classroom booking, …) • Italian curriculum and assessment requirements • Organisational structures and cultures
ZPA: Zone of Promoted Action (people, objects, or areas in the environment in respect of which the trainers' actions are promoted)	• Participation in national/international conferences • Involvement in actions to foster teachers professional development (face-to-face meetings, MOOCs, …) • Professional development • Informal interaction with their peer and researching colleagues

Table 2.13: Research environment's ZFM/ZPA (trainers only)

2.10.2 The internal level to the MOOC

2.10.2.1. MOOC's ZFM/ZPA complex

We start considering separately the MOOC's ZFM and the MOOC's ZPA. In reality, these two zones are not always clearly distinct (the same it was observed in Valsiner and Goos's description, § 2.9). It is however necessary to consider them, a moment, such as, in order to be able to grasp their theoretical description.

2.10.2.2 MOOC's ZFM

Specifically, for the **MOOC's ZFM** the definition given by Goos (Figure a), in its general sense is valid, but it must be slightly modified for the sake of its specificity. Note that, in this case (Figure b), instead of *environment*, we must replace with MOOC as well as *area(s)* change(s) in module(s) of the MOOC, *objects* are intended as resources and the *teachers* are the MOOC's trainers/trainees.

Valsiner/Goos's zones
ZFM: Zone of Free Movement (structures teacher's access to different areas of the environment, availability of different objects within an accessible area, ways the teacher is permitted or enabled to act with accessible objects in accessible areas)

Figure a: first column of the Table 2.12

Taranto's re-elaboration/interpretation
ZFM: Zone of Free Movement (structures trainers/trainees' access to different modules of the MOOC, availability of different resources within an accessible module of the MOOC, ways the trainers/trainees are permitted or enabled to act with accessible resources in accessible modules of the MOOC)

Figure b: first column of the Table 2.15

Regarding the second part of the Table 2.15, we have distinguished two columns: one for the trainers (second column) and one for the trainees (third column), in order to consider all the participants in a MOOC. For both participants, the MOOC's ZFM maintains the general sense of the Goos's interpretation with appropriate adaptations.

Goos's interpretation
• Perceptions of students • Access to resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures

Figure c: second row and second column of the Table 2.12

Taranto's re-elaboration/interpretation	
Trainers	**Trainees**
• Design of the platform that hosts the MOOC and perception of this new environment • Design and digital transposition of mathematical resources • Communicational resources to foster communication among trainees and between trainers themselves with trainees • Technical support • Curriculum and assessment requirements • Organisational structures and cultures	• Access to the platform that hosts the MOOC and perception of this new environment • Access to mathematical resources • Communicational resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures

Figure d: second column of the Table 2.15

To explain the adaptation we list the terms of Goos's interpretation (Figure c) in italic and explain for each of them how we change them (Figure d).

Despite of *perception of students*, for trainers we consider the design of the platform that hosts the MOOC and perception of this new environment; instead, for trainees we consider the access to the platform that hosts the MOOC and perception of this new environment (organized by the trainers). As *access to resources*, for trainers we consider the design and digital transposition of mathematical resources, useful to allow the teacher education that they want to accomplish, taking into account the *curriculum and assessment requirements*. There could be mathematical activities (innovative methodologies and strategies); specific mathematical software; link to deepening materials; video lectures. For trainees, instead, we consider the access to these mathematical MOOC's resources that surely they interpret and use according to the *curriculum and assessment requirements*.

Moreover, we add what we call *communicational resources* (in parallel with the mathematical resources) for both trainer and trainees. About trainers, these are the spaces designed to encourage communication among trainees and between trainers and trainees. About trainees, they can interact with each other and with the trainers too in these spaces.

About *technical support*, the trainees can receive it asking help directly to trainers or e-tutors. In fact, in a MOOC there is always a space or an address where you can write if you need technical support. If the trainers, instead, need technical support they can address themselves to the webmaster they work with during the MOOC design.

Then, as *organisational structures and cultures*, we mean the structure designed for the MOOC that, of course, is the same for trainers and trainees. Therefore, in our case and also generally, it means weekly modules, contents according to the national curriculum, tasks to accomplish, questionnaires.

2.10.2.3 MOOC's ZPA

Specifically, for the **MOOC's ZPA** the definition given by Goos (Figure e), in its general sense, is valid, but it must be modified for the sake of its specificity. Note that, in this case (Figure f), instead of *people*, the trainees interact with virtual people, because they never see each other face-to-face (at least, in general, the vast majority of them[15]). As shown for MOOC's ZFM, also here, *environment* must replace with MOOC as well as *area(s)* change(s) in module(s) of the MOOC, *objects* are intended as resources and, of course, the *teachers* are the MOOC's trainers/trainees.

Valsiner/Goos's zones
ZPA: Zone of Promoted Action (people, objects, or areas in the environment in respect of which the teacher's actions are promoted)

Figure e: first column of the Table 2.12

Taranto's re-elaboration/interpretation
ZPA: Zone of Promoted Action (virtual people, resources, or modules in the MOOC in respect of which the trainers/trainees' actions are promoted)

Figure f: first column of the Table 2.15

Regarding the second part of the Table 4, we have distinguished again two columns: one for the trainers (second column) and one for the trainees (third column), in order to consider all the participants in a MOOC. For both participants, the MOOC's ZPA maintains the general sense of the Goos's interpretation with appropriate adaptations.

[15] In our case, a very small part declares that it has registered with "a" school colleague.

Goos's interpretation
• Pre-service teacher education • Professional development • Informal interaction with teaching colleagues

Figure g: second column of the Table 2.12

Taranto's re-elaboration/interpretation	
Trainers	**Trainees**
• Interaction with trainees reading (and sometimes answering) their post. • Professional development	• Informal interactions with enrolled trainees • Interaction with trainers • Professional development

Figure h: third column of the Table 2.15

To explain the adaptation we list the terms of Goos's interpretation (Figure g) in italic and explain for each of them how we change them.

As far as the trainers are concerned, we change *informal interaction with teaching colleague* with interaction with trainees reading and answering their post on the communication message boards. About trainees, we consider informal interactions with the other trainees thanks to the communication message boards. So they can reflect on the mathematical proposed activities and share ideas, opinions and materials. And also, interaction with trainers seeing their videos; writing for ask technical support.

Professional development is maintained because both for trainers and for trainees the MOOC is a moment of professional development (we will have chance to deepen this statement in Chapter 5 and Chapter 6).

We do not mention *pre-service teacher education*. Goos's intended that the ZPA includes moment of education for a teacher; in the case of a MOOC, the MOOC itself is the opportunity of education for an in-service teacher. However, we have made a clarification about the MOOC environment (§ 2.10.1.2). It would be simplistic to understand the MOOC as the ZPA. We have stated that the MOOC itself is a mixture of ZFM and ZPA. For these reasons it is embedded in the ZFM/ZPA complex, that is what we have just described precisely here.

2.10.3. The two external levels to the MOOC

2.10.3.1 School's ZFM/ZPA

Specifically, for the **school's ZFM/ZPA** the definition given by Goos (Figure i) is kept almost identical. In fact, this model is used to describe the development of pre-service teachers' pedagogical identities as user of technology, in face-to-face environment. In our case, we want to consider the trainees in their daily "real and not virtual" environment, that is the school. Therefore, the Goos' interpretation fits well with our intent. In particular, Goos consider pre-service teacher; the trainees that follow MOOCs for their professional development are in-service teacher. It is important that it is so (at least in our case, although there are other MOOC experiences for teacher education (Panero et al., 2017), where the same incoming requirement is required) because there are some activities in the MOOC that should be experiment in a real classroom. As mentioned before (see the paragraph "*Uncontrollability of ongoing learning*"), the experimentation it is not mandatory, anyway if a trainee is an in-service teacher, with at least one year of experience among the desk classroom, she can actively participate in the discussion on the communication message boards. So, in the Figure j, there are slight changes in relation to the ZPA row.

Goos's interpretation
• Perceptions of students • Access to resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures
• Pre-service teacher education • Professional development • Informal interaction with teaching colleagues

Figure i: second column of the Table 2.12

Taranto's re-elaboration/interpretation
• Perceptions of students • Access to resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures
• In-service teacher education • Experimentation in classroom of some activities see in the MOOC • Professional development • Informal interaction with teaching colleagues

Figure j: second column of the Table 2.16

2.10.3.2 Research environment's ZFM/ZPA

The **research environment's ZFM/ZPA** is an external part of the MOOC, so it is observable by everyone who is interested in the analysis of the mathematics teacher educators (or MTE). For the purposes of this dissertation, we are clearly more oriented towards the analysis of trainers who are involved in the design and delivering of MOOC for teacher education. Anyway, the model that will be described below can be understood and used also for other experiences where trainers are involved in more general mathematics teacher education experiences.

2.10.3.3 Research environment's ZFM

Specifically, for the **research environment's ZFM** the definition given by Goos (Figure k), in its general sense, is valid, but it must be modified for the sake of its specificity. Note that, in this case (Figure l), instead of *teachers* you must consider trainers. They could be researchers and possibly also teacher-researchers interested in teacher education and professional development on Mathematics Education. The *environment*, in this case, is represented by the space lived by the trainers: University, Department of Mathematics, national/international conferences, …. In addition, also the school, if in the trainers' set there are teacher-researchers.

Valsiner/Goos's zones
ZFM: Zone of Free Movement (structures teacher's access to different areas of the environment, availability of different objects within an accessible area, ways the teacher is permitted or enabled to act with accessible objects in accessible areas)

Figure k: first column of the Table 2.12

Taranto's re-elaboration/interpretation
ZFM: Zone of Free Movement (structures trainers' access to different areas of the environment (University, Department of Mathematics, national/international conferences, …), availability of different objects within an accessible area, ways the trainers is permitted or enabled to act with accessible objects in accessible areas)

Figure l: first column of the Table 2.16

Regarding the second column, for the research environment's ZFM, the general sense of the Goos's interpretation is maintained with appropriate adaptations.

Goos's interpretation
• Perceptions of students • Access to resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures

Figure m: second column of the Table 2.12

Taranto's re-elaboration/interpretation
• Perceptions of teachers • Access to resources (literature, proceedings of conferences, teaching experiment, data obtained via questionnaires or interview, …) • Technical support (print centre, classroom booking, …) • Italian curriculum and assessment requirements • Organisational structures and cultures

Figure n: second column of the Table 2.16

To explain the adaptation we list the terms of Goos's interpretation (Figure m) in italic and explain for each of them how we change them (Figure n).
Despite of *perception of students*, we consider perception of teachers, which is the trainees that the trainers imagine will follow the course (the MOOC in particular in our case) and then actually follow it. As *access to resources* we consider as resources the existing literature to which a researcher can access (thanks to the departmental library, looking on internet, ...); the possibility offered by the registration in a conference, so the proceedings and other deepening materials. As well as data that come from teaching experiment (like video, students' papers, teachers' logbooks, …), questionnaires, interviews. As *technical support* we mean the possibility for a researcher to use the print centre of her department, to booking a/some classroom (in the department, in a school, …) to organize meetings with her colleagues or for teachers education purposes. Asking help to the computer technician of the department for access to the resources and so on. As *curriculum and assessment requirements*, in a sense, is exactly as Goos intends them. The teachers should take into account her national curriculum and assessment requirements. For a researcher involved in teacher education it is surely mandatory take into consideration the same points. Then, as *organisational structures and cultures*, we consider the environment created to allow sharing and comparing among researchers interested in the same topic and with the same culture.

2.10.3.4 Research environment's ZPA

Specifically, for the **research environment's ZPA** the definition given by Goos (Figure o), in its general sense, is valid, but it must be slightly modified for the sake of its specificity. In fact, in this case (Figure p), instead of *teachers* we are considering the MOOC's trainers.

Valsiner/Goos's zones
ZPA: Zone of Promoted Action (people, objects, or areas in the environment in respect of which the teacher's actions are promoted)

Figure o: first column of the Table 2.12

Taranto's re-elaboration/interpretation
ZPA: Zone of Promoted Action (people, objects, or areas in the environment in respect of which the trainers' actions are promoted)

Figure p: first column of the Table 2.16

Regarding the second column, for the research environment's ZPA, the general sense of the Goos's interpretation is maintained with appropriate adaptations.

Goos's interpretation
• Pre-service teacher education • Professional development • Informal interaction with teaching colleagues

Figure q: second column of the Table 2.12

Taranto's re-elaboration/interpretation
• Participation in national/international conferences • Involvement in actions to foster teachers professional development (face-to-face meetings, MOOCs, …) • Professional development • Informal interaction with their peer and researching colleagues

Figure r: second column of the Table 2.16

To explain the adaptation we list the terms of Goos's interpretation (Figure q) in italic and explain for each of them how we change them (Figure r).
Despite of *pre-service education*, that it certainly does not concern researchers, we consider the events from which a mathematics teacher educators benefit in term of personal training. For example, participation in national/international conferences as speaker or among the audience; or as an alternative when she is involved in actions for teacher professional development, like a MOOC. The comparison with teachers is also a wealth for a trainer (we will have a chance to deepen this statement in Chapter 6). In the light of this, we can maintain the term *professional development* also for researchers and *informal interaction with teaching colleagues* is replaced by informal interaction with peers research colleagues.

2.10.4 The re-elaborated Zone Theory

The Figure 2.21 clearly represents what we have previously described.
So far, we have explained all the adaptations made on the Zone Theory to describe a MOOC for teachers education. This theorization, as it is now, can on the one hand grasp the whole socio-cultural context in which the protagonists of a MOOC (i.e. trainers and trainees) move. On the other hand, it fails to fully describe the dynamics that follow one another within the MOOC itself and which would allow better grasping the facets that lead to possible evolutions of the ZPDs (of trainers and trainees). For this reason, it makes sense to consider a networking of theories between this and the MOOC-MDT, thus obtaining what we call MOOC's Zone Theory.

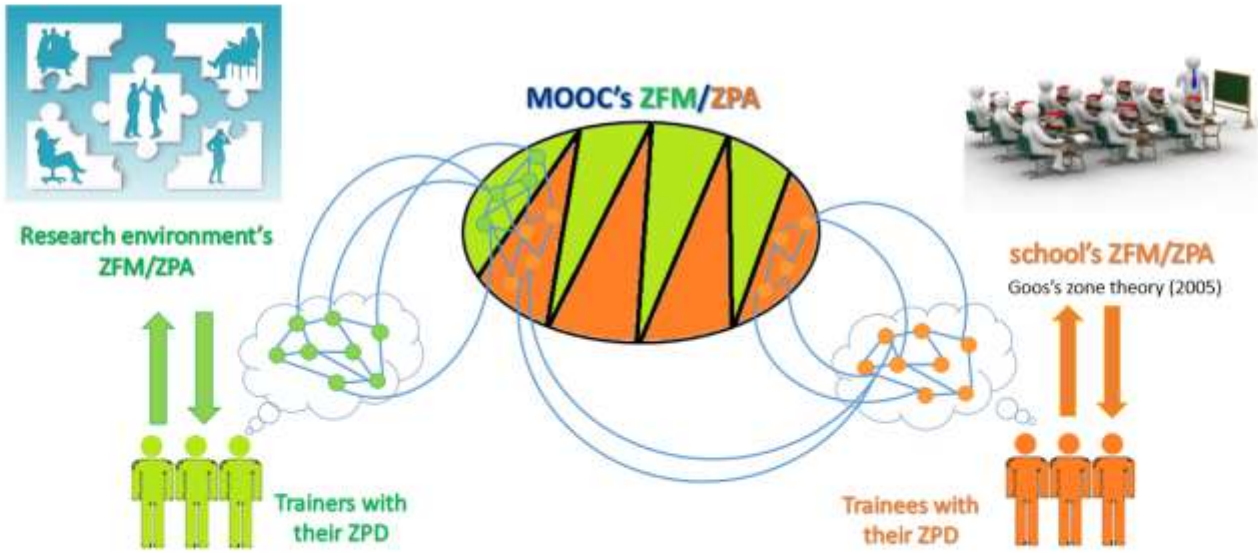


Figure 2.21: The Zone Theory re-elaborated by Taranto for a MOOC and its participants

2.10.5 The Networking between MOOC-MDT and the re-elaborated Zone Theory

The subject of our speeches is always a MOOC for teacher education.

Let us leave out for the moment the ZPD, on which we will return at the end, and concentrate on the ZFM/ZPA.

As already underlined several times, even before, to expose the re-elaboration of the Zone Theory to the MOOC, we have shown that the MOOC's participants can be collected in two distinct communities: the trainers one and the trainees one. Moreover, the re-elaborated Zone Theory, as shown, works on two levels compared to a MOOC: one internal to the MOOC (MOOC's ZFM/ZPA) and two external to the MOOC (research environment/school's ZFM/ZPA).

The external levels to the MOOC belong in a distinct and exclusive way to the trainers and the trainees respectively. This is in accordance with what is seen in the MOOC-MDT. We were talking about a process of internalization, which saw the passage of external components that became internal. As we saw in §2.3.1.4, the idea of external and internal is taken by Clark & Hollingsworth (2002). They distinguish an external domain, located outside the teacher's personal world, from the internal domains, which "constitute the individual teacher's professional world of practice, encompassing the teacher's professional actions, the inferred consequences of those actions, and the knowledge and beliefs that prompted and responded to those actions" (Clark & Hollingsworth 2002 p. 951). We must not limit ourselves to teachers only, rather referring to both trainees and trainers. Now it seems immediate the association of the external domain with school/research environment's ZFM/ZPA and of the internal domain with the trainees/trainers' ZPD.

The moments in which these two communities are in a common space are those that they live by accessing the MOOC; therefore living a virtual space.

So let us focus on the internal MOOC level, which we called MOOC's ZFM/ZPA.

Let us once again consider this complex area, distinguishing the MOOC's ZFM and the MOOC's ZPA. We return to say that, these two zones are not always clearly distinct (the same it was observed in Valsiner and Goos's description, §2.9). It is however necessary to consider them, a moment, such as, in order to be able to better understand the successive theoretical description. Anyway, we will soon touch with hand, as effectively these two zones form a truly complex one.

In the MOOC's ZFM the trainers are faced with affordances and presumed or real limits that they could meet during the design. They consider some e-learning design models and they have to transpose into digital format all the content they want to convey to trainee teachers. They have to understand what is the best way to present the course contents and choose the best web tools to stimulate communication between the participants.
This is in parallel with what we have described in the MOOC-MDT as "transposition of some ideal praxeologies", namely the techniques (or multi-techniques) considered by trainers to accomplish their educational task.
All the resources inserted in the platform by the trainers are artifacts (in the sense of Verillon & Rabardel, 1995). These artifacts are meant by trainers who have in mind how the trainees should make use of them. More precisely they are a collection of artifact that are, all together, in synergy (Faggiano et al., 2017).
The MOOC, intended as the collection of all these artifacts, is itself an artifact. At this stage, when the MOOC is inert and inhabited by trainers only, we have referred to it as ***MOOC-artifact*** and, in this sense, it is a static object. Using the language of Connectivism (Siemens, 2005; Downes, 2012 a), we have then characterized the *MOOC-artifact* with its own *network of knowledge*: its nodes are the content, the ideas, the images and videos inserted by trainers; the connections are the links (created by trainers) between their node pairs.
Once the MOOC starts, the trainees enter in it and also for them it is a *MOOC-artifact* at the beginning, when they look its aspect. Hence, the MOOC's ZFM allows observing the phenomenon from a static point of view: it is a look at the MOOC's resources and materials.
The MOOC's ZPA comes into play when the MOOC starts, namely when its first module is showed to the enrolled trainees. ZPA is the most dynamic part: it allows for observation of the MOOC coming to life because inhabited by interactions triggered by its participants (both trainers and in particular trainees). We have observed in the MOOC-MDT that when a MOOC teaching module is activated, it is configured according to a complex structure that Taranto et al. (2017 a) define as an *ecosystem*:

> "all the relationships (exchange of materials, experiences and personal ideas/point of view) put in place by participants of an online community. They are made effective by the technological tools through which the participants interact each other. They can so establish connections within a given context" (p. 2481).

Also in this case, using the language of Connectivism, we can say that the *MOOC-ecosystem* has its *network of knowledge* where its nodes are the content, the ideas, the images and videos previously inserted by the trainers. Other nodes are added to these. They are represented by the ideas, the points of view, the opinions, the possible materials shared by the trainees that explore the MOOC-artifact. The connections are the links between all of this node pairs. It is important remember that also every individual (trainers or trainees) has his own network of knowledge that fits exactly with his ZPD. This is a more delicate point on which we will focus later. We need to anticipate it now to understand that, according to his/her ZPD, each individual applies the Mathematics Education utilization schemes (Math Edu USs) to the MOOC-artifact that therefore becomes a MOOC-instrument for the individual. However, as specified in the MOOC-MDT framework, due to the massiveness of which a MOOC is bearer, it is not possible to immediately identify the Math Edu USs put in place by individuals. Everything happens in a fraction of seconds with tens and tens of people at the

same time. For this reason, it is more reasonable to consider the totality of the Math Edu USs put in place, looking at what we call the MOOC-ecosystem.
We remember that every week (so for every module), the trainees have to solve *tasks*, through *multi-techniques*, properly justified. In addition, they are free to choose the times, ways and depth with which to address them. These trainees' actions (their meta-didactical praxeologies) trigger what we have called "*double learning process*".

2.10.5.1 MOOC in far and wide

We remember an important clarification. The trainees discover the MOOC weekly, moving far and wide in it. "Far" because they will wait for the next module opening to see new materials and "wide" because every week a new module is opened, but the previous ones are kept open until the end of the course. Therefore, the trainees can discover the novelty of the new module, but at the same time may come back and see in depth the previous material and resources. They can so read in more depth some activities that they had first read quickly; they can dwell more on the comments of others and, at the same time, leave their comment that they did not leave at the beginning because of haste or shyness. Sometimes, as happened in our case, they can share their own materials with the other trainees in the communication message boards, expanding the early MOOC structure.

2.10.5.2 A stochastic phenomenon

The double learning process therefore is repeated for all the modules, either vertically, i.e. at each weekly opening, or horizontally, i.e. if the trainees decide to go back in the modules already seen.
A delicate observation therefore emerges.
It is important underline that MOOC's ZFM/ZPA is a really complex structure. When the MOOC is opened to the trainees, the ecosystem comes to life, but there is not an immediate transition from ZFM to ZPA. Every week, in a MOOC, new contents are showed, so we imagine a ZFM_i, with the index i varying from the first module to the last one. ZFM slowly expands itself and at the same time it becomes ZPA.
The transition from ZFM to ZPA is stochastic and not deterministic, because it is not a separate and distinct transition from one zone to another. As mentioned in the previous paragraph, every week a new module is opened in a MOOC and, at the same time, the previous ones remain open too. In this way the previous modules are fully lived by MOOC's participants, while the new one, when it is just opened, is part of MOOC's ZFM. In the moment in which just only one trainee starts to open some resources or write in it, that module enters in the ecosystem, so it becomes a part of the MOOC's ZPA. However, when will some of the massive number of trainees start living the new module? Will only one to do it, more than one at a time? Even a single comment can make a difference compared to what the MOOC's initial design had been and what it is delivering. We then classify this phenomenon as stochastic due to the multitude of uncontrollable events that can be concentrated in a small amount of time.

2.10.5.3 To distil again the previously presented argument

The heart, as well as the driving force, of the theoretical framework MOOC's Zone Theory has been shown here. Let us stop for a moment to understand better. We are working inside the MOOC, precisely in what we have defined MOOC's ZFM/ZPA thanks to the adaptation made to the zone theory. We then started the networking with this and the MOOC-MDT. In fact, we talked about MOOC-artifact, ecosystem/instrument and we also observed that it is possible to match a network of knowledge to each of them. We have also specified that each individual of the MOOC (trainer or trainee) possesses a network of knowledge as well. All networks are made up of nodes (MOOC materials, reflections/opinions written by the trainees and trainers in the MOOC or specific to each individual) and these nodes are connected to each other thanks to the meanings that each actor of the MOOC has built or hooked on these nodes. In the MOOC-artifact the trainers connected a certain mathematical conceptual node (the height of triangle – to resume the example that accompanies us from the beginning of our discourses) to a certain (or more) mathematical activity. In the MOOC-ecosystem, the trainees explore this initial MOOC-artifact network: simultaneously they start making connections within their personal network, giving a certain value of importance to that resource they are exploring, putting in place their Math Edu USs (alias their (multi-techniques). For example, a connection can be triggered by the following reasons: if I am a teacher who is currently facing the concept of height in the triangles with the students; if a few weeks before I checked what my students remember about the height in the triangle with a test and it went wrong; if I teach the last year of a technical institute and I realize that actually my "old" students really have a misconception about the difference between height and perpendicular; ....

At that moment, the MOOC-artifact becomes a MOOC-instrument for each individual trainee. And the individual trainee is also invited by sharing activities in the MOOC to express himself on the communication message boards sharing his agreement/disagreement, interest/indifference to the teaching proposal presented. This phenomenon of connecting the individual's network with the MOOC's network and the consequent act that makes this connection visible (the intervention of the trainee on a communication board) is made by many at the same time. Therefore, the researcher who looks at the phenomenon from the outside (the dissertation writer in this case) witnesses a rapid, sudden, almost imperceptible, and why not, chaotic, change of the initial connections of the MOOC-artifact. The existing connections are modified, because more complex ones are created, originating the MOOC-ecosystem. We have theorized this complex phenomenon with the double learning process. In addition, it should not be forgotten that this occurs every time in every module of the MOOC and, in a more extended form, for the whole MOOC in its entirety (explored in long and wide, as we have already observed).

2.10.5.4 The rationale behind the networking

Why make a networking with the Zones?

Vygotsky (1964) talks about the transition from *inter-personal* to *intra-personal*.

Inter-personal means from the internal to the external, i.e. from me to outside, so me among other people. Intra-personal means from the external to the internal, so a process of interiorization.

So with intra-personal we can consider to have an artifact and the passage from artifact to instrument: it is similar to the **ZFM**. With inter-personal, we can consider to have an individual who interacts with a community, that is, the individual and the social: it is similar to the **ZPA**. However, differently from Vygotsky, who sees these two steps as separate, in a MOOC these two aspects grow and evolve together because of the connectivistic features.

In addition, we need to understand how connections change when passing from an artifact to an ecosystem. The transition from artifact to instrument entails a phenomenological aspect. I, as a researcher, see this phenomenon from a meta-level. A "practical" step is necessary to explain exactly how this transition takes place (which is, actually, a change of connections). It

is exactly the transition from ZFM to ZPA that explains in a clearer way (or justifies) the passage from artifact to ecosystem/instrument.
In fact, in the Instrumental Approach, the transition from an artifact to an instrument takes place in a not too long period: the individual reorganizes and recomposes his activity, implementing some USs and converting the artifact into an instrument.
In the MOOC there is a longer-term aspect that is not so evident in Verillon & Rabardel (1995). We can say that the MOOC-artifact coincides with the MOOC's ZFM because, for definition, the ZFM is the zone that hold all the constraints and affordances relative to the context (so, the MOOC-artifact in our case). However, the MOOC modules are opened from week to week and the trainees enter in them progressively, moving in far and wide in them. They create new connections gradually and put in place some Math Edu USs. The MOOC's ZFM, from week to week, open up and gives way to the MOOC's ZPA. Everything happens dynamically, chaotically. The MOOC transforms itself by showing new contents, but it is also transformed by the interpretation that the trainees give to its contents. In fact, again, the MOOC is enriched by the presence of participants who connect their personal network of knowledge to the MOOC's one (they read, comment, ...). At the same time they can add something new (i.e., upload their material, share the results of a trial made in their classroom using the showed materials, ...). They then add new nodes that in turn give rise to new connections.
Therefore, to explain the passage or transition from artifact to ecosystem/instrument it is not enough to refer to the Instrumental Approach. It is not just a question of Math Edu USs implemented. It strongly affects by its evolving nature and by the participants' connections that are involved in the double learning process. So, the transition from MOOC-artifact to MOOC-ecosystem/instrument is best justified by resorting to the transition from ZFM to ZPA.
As stated in paragraph 2.9, zone theory is a dynamic theory. It allows to grasp well the dynamism and the complexity that the MOOC brings with it. We specify once again that the transition from artifact to ecosystem/instrument takes place in a very short time, almost imperceptible if we look at the phenomenon in its entirety. That is why it makes sense to talk about ZFM/ZPA complex as the zone that best captures the complexity of this dynamic phenomenon.

2.10.5.5 The last steps: the external levels and the ZPD

Last step is now to resume the other external levels and make the connection with the ZPD, explaining appropriately the networking made.

What we have so far specified and described concerns the actions that take place within the MOOC.
Recall that a MOOC is a totally remote environment, which can be accessed whenever and wherever you want. Its participants (trainers and trainees) always interact online. And the general duration of a MOOC varies from 6 to 8 weeks. Clearly, during this time, participants do not live an exclusive and binding relationship in the MOOC, but continue to conduct their daily routines normally, in school and research environments respectively.
This is why it is important to define and consider two other domains that we have defined as the school's ZFM / ZPA and research environment's ZFM / ZPA.

- The school's ZFM/ZPA for trainees is external to the MOOC, but internal to each teacher that follow the MOOC and that, in a sense, is conditioned by this complex zone, while participating to the course.

- The research environment’s ZFM/ZPA for trainers is, in a symmetric way, external to the MOOC, but internal to each trainer that design and follow the MOOC and that, in a sense, is conditioned by this complex zone, while participating to the course.

Let us understand better what it means that trainers and trainees are conditioned by their complex zones, during the period in which they attend the MOOC.
Certainly all the participants, before designing the MOOC (if trainers) or enrolling in the MOOC (if trainees), have a certain ZPD. The trainers have a certain ZPD with respect to how they have acted in the design stages to create a MOOC for mathematics teacher education and so to transpose their ideal didactical praxeologies. The trainees have a certain ZPD with respect to their current level of professional development that could potentially evolve thanks to what they find in the MOOC.
We have already given the definition of ZPD, exactly as conceived by Vygostky (1978), and then taken up by Valsiner. Goos (2005) is the first that talks about ZPD for teachers: interacting with peers or more experienced colleagues, a teacher can have “possibilities for developing new teacher knowledge, beliefs, goals, practices”. And within these possibilities she lists “Mathematical Knowledge; Pedagogical content knowledge; Skill/experience in working with technology; Beliefs about mathematics, teaching and learning” (Table 2.12).
On this false line, I have extended the concept also to the trainers. In addition, we have preferred networking with the MOOC-MDT to have a finer theoretical description (and consequent a finer analysis).

The ZPD can be identified with the own network of knowledge. We have repeatedly said that each individual has his own network of knowledge. When the individual is in particular involved in a training action, as can be a MOOC for teacher education, with respect to the whole network of knowledge, the part called into question will be the one that is relative and most suitable to the knowledge related to the teacher education (we can refer to it as *network of professional knowledge*). Then specific pipe (as called by Siemens, 2005, p. 6) are activated, which contain specific nodes, connected to each other by specific personal motivations. The motivations that gave life to the connections can be closely linked to the mathematical knowledge; to the pedagogical content knowledge; to the skill/experience in working with technology; or to the beliefs about mathematics, teaching and learning.

The definition given by Goos (2005) for the ZPD (Figure s): “possibilities for developing new teacher knowledge, beliefs, goals, practices”, because of the networking with the MOOC-MDT, it becomes (Figure t): “possibilities for expand the own network of knowledge”.

Goos’s zones
ZPD: Zone of Proximal Development (possibilities for developing new teacher knowledge, beliefs, goals, practices)

Figure s: first column of the Table 2.12

Taranto’s re-elaboration/interpretation
ZPD: Zone of Proximal Development (possibilities for expand the own network of knowledge)

Figure t: first column of the Table 2.13-2.14

Regarding the second column of the Table 2.12 for Goos’s ZPD (Figure u), the general sense of the Goos’s interpretation is maintained. Now we mean the points listed in the Figure u as the nodes of the individual’s network of knowledge and among these, we also include the praxeologies, respectively for trainers and trainees. They include the last point listed by Goos, namely “beliefs about mathematics, teaching and learning” (Figure v, w). These can also be understood as justifications that explain why I choose to use certain techniques when I have to accomplish a task.

Goos's interpretation
• Mathematical Knowledge • Pedagogical content knowledge • Skill/experience in working with technology • Beliefs about mathematics, teaching and learning

Figure u: second column of the Table 2.12

Taranto's re-elaboration/interpretation for trainers	Taranto's re-elaboration/interpretation for trainees
• Mathematical Knowledge • Pedagogical content knowledge • Skill/experience in working with technology • Meta-didactical praxeologies that include beliefs about mathematics, teaching and learning	• Mathematical Knowledge • Pedagogical content knowledge • Skill/experience in working with technology • (Meta-)didactical praxeologies that include beliefs about mathematics, teaching and learning

Figure v: second column of the Table 2.13

Figure w: second column of the Table 2.14

Without necessarily distinguishing between trainers and trainees, we can make these examples.
If I have a certain mathematical knowledge I will present that certain MOOC activity in a certain way (if I am a MOOC trainer I will introduce it to the trainees in a certain way; if I am a teacher I will introduce it to my students in class in another certain way). Idem if I have a certain pedagogical content knowledge. I will use that ICT tool of the MOOC or I will not use it depending on my skills/experience in working with technology (so, as a designer I choose the instrument I know and which I believe can allow me to obtain that certain result; as trainee I can choose to post or not on that communication board because I find myself more comfortable in using it). Beliefs about mathematics, teaching and learning then heavily influence the way in which the MOOC is structured (trainers) or the way in which the MOOC is used (trainees).
Moreover, the features that characterize the school's ZFM/ZPA (Table 5) influence trainees' didactical praxeologies. At the same, the features that characterize the researcher environment's ZFM/ZPA (Table 6) influence trainers' meta-didactical praxeologies.
As also Valsiner and Goos have noted, the three zones are interrelated and mutually influencing each other.

It can therefore be said that at the beginning of the MOOC, trainers and trainees have an actual developmental level of ZPD (or initial ZPD). In particular, the trainers organize the MOOC in such a way that the trainees can reach a potential developmental level of ZPD, what can be reach after attending the course, based on how trainers have developed the course or through interactions with other trainees. So, in this sense, at the end of the MOOC a final ZPD can be observed in the trainees. It can coincide with the initial ZPD (if there have been no changes) or reach the ideal level for trainers.
Vice versa, in following the MOOC, monitoring the trainees and pulling the ranks of the course (also thanks to feedbacks that the trainees leave via questionnaires), the trainers' ZPD can also be subject to change.

2.10.6 Considerations on the MOOC's Zone Theory

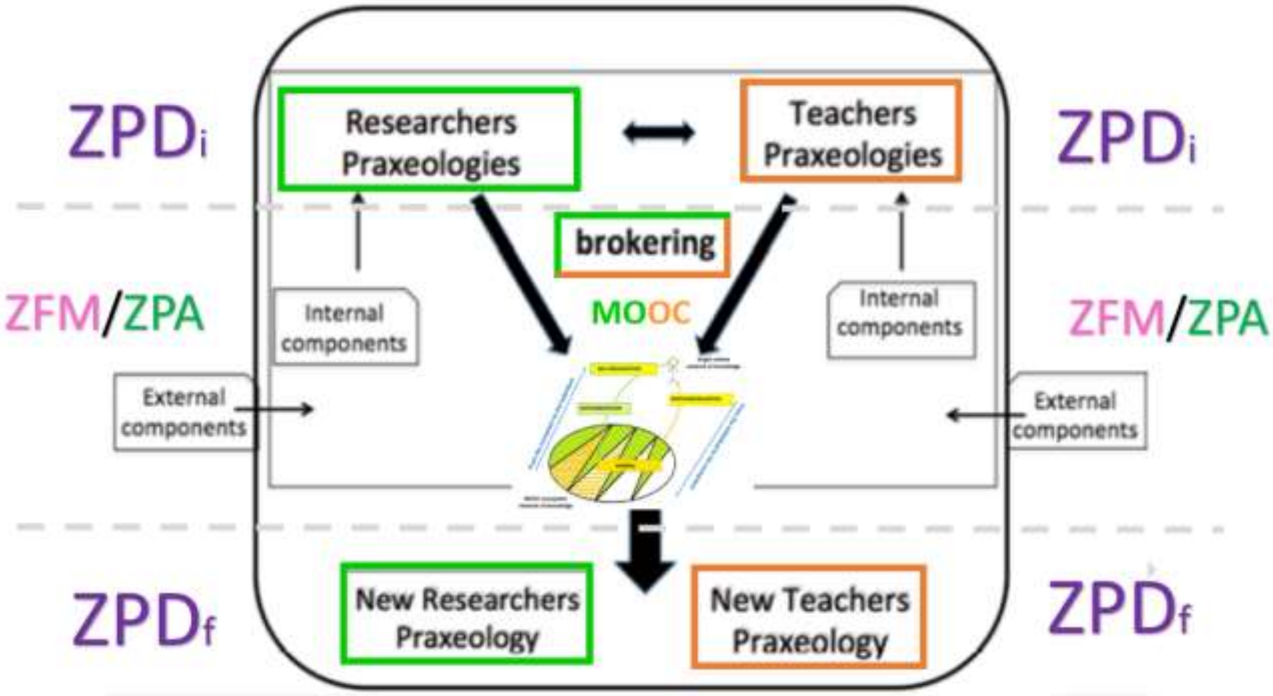


Figure 2.22: The MOOC's Zone Theory

MOOC's Zone Theory (Figure 2.22) offers a useful framework for research that aims at understanding the complexities of trainers/trainees' learning trajectories in a MOOC.
With learning trajectories, we mean how these protagonists interact online, both with the platform and with each other. So in a connective sense, as they learn. In particular, if and how these interactions change their knowledge, beliefs, and arise perception of change in the practices. In other words, what the consequences of this participation in the MOOC are.

In this dissertation we will analyse in more detail the consequences and professional development of the trainees, also because the course under investigation is a MOOC for teacher education. However, there is also a brief analysis on the consequences that trainers feel, more from the point of view of design and assessment.

Regarding the analysis of the trainees, we consider important to introduce the following theoretical lens. The notion of ***productive tensions*** (Goos 2013) is crucial for grasping teachers' changes from a zone theory perspective:
Tensions arise from dissatisfactions that teachers experience when their ZPD does not map onto the ZFM/ZPA complex in ways that promote desired development: this can be thought of as a misalignment within the zone system. The tension is productive if it triggers change that aims to bring the zones into alignment, for example, by modifying the environment (ZFM) or seeking out professional learning opportunities (ZPA) (Goos 2013, p. 523).

We should underline that even if teachers do gain knowledge or reconsider their beliefs, they might still regard the new teaching practices promoted by these interventions as not feasible to be implemented within their school environments (Goos 2013). Examples might include:

teachers' beliefs about the capacities of low achievers to benefit from changes in teaching practice; different visions of teaching between colleagues; a lack of teaching resources; and so on. However, if some productive tension impels a teacher to look beyond her boundaries (school's ZFM/ZPA), then what she has learned in the outside environment (MOOC's ZFM/ZPA) might support her in modifying aspects of her school's ZFM/ZPA. In this way, she is brought to try persevering in that direction, possibly producing slow, but profitable, changes in her professional development.

The notion of productive tension is what makes it possible to have observables in the analyses on the trainees' behaviours. So, if we take up the metaphor of the river angler who goes fishing in the sea (§2.3.1.9): what kind of device has he to invent in order to make a rich fishing? Outside the metaphor, that is definitely the MOOC's Zone Theory and the baits that the angler can use are the double learning process and the productive tension.

2.10.7 A conclusive metaphor: a cactus flower

We conclude this long and complex theoretical treatment with a metaphor that allows us to summarize what has been expounded effectively.
Consider a cactus flower. The *Echinopsis* cacti bloom at night, and their flowers last only one day: they even reach the peak of their beauty for an hour or two at most.
A computer consultant, Greg Krehel, of Jacksonville (Florida) with a passion for these plants, has decided to stop the moments of life of the flowers with the technique of time-lapse. You can see them at the following link (and you are invited to see the video before continuing to read):
http://www.nationalgeographic.it/multimedia/2015/04/17/news/timelapse_fiori_di_cactus-2571876/
The following figures (2.23) give only a marginal idea of the flowering phenomenon.

Figure 2.23: Flower buds of cactus and their flowering

We can compare the cactus plant to an artifact. The trainer-designers (university professors, teacher-researchers, PhD student) instead are the gardener who has chosen a vase, taken some ground and inserting in it the cactus plant. He has exposed the plant to the sun, fertilized and watered. The gardener hopes that the care he has for the plant will be repaid with the bud of a flower. The flower bud is the MOOC-artifact. In fact, once the labours of design are finished, the trainers consider the artifact obtained as a rough diamond. They have invested a lot in terms of fatigue, time and resources; but they cannot know a priori whether this diamond will shine with its own light. Or, to stay on the metaphor of the flower, if this flower will indeed bloom and show its rare beauty.

Actually there is no guarantee that the cactus flower blooms, although the bud has formed. Flowering is linked to the sap inside the cactus (which we can understand as the trainees). The flow of the sap is linked to both internal factors of the plant and to external conditions, including climatic conditions.

In the case of the MOOC, its flowering, that is the becoming ecosystem (transition from MOOC's ZFM to MOOC's ZPA) thanks to the use of the trainees, is also linked to the boundary conditions: on the one hand the internal design of the MOOC, with the chosen ICT tools and the tutorials available prepared by the trainers (MOOC's ZFM); on the other hand, factors outside the MOOC to which the trainees are exposed (school's ZFM/ZPA).

The cactus flower blossoms and shows its beauty at night, allowing itself to be admired for a few hours, closing in exactly one day. The timing of the MOOC is also quite short, if compared to school or university duration. The MOOC lasts a few weeks and at its end, all the dynamism and vitality that the trainees confer on it fade. And if there is no researcher who is there to grasp its facets, like Krehel with the flowering of the cactus, it would be difficult to tell/analyse it.

The cactus flower has a large crown, smooth and bright petals, various pistils. It is certainly noted more than the bud. The same goes for the MOOC: interacting with the materials available and with each other, exchanging ideas, reflections, opinions, materials, the trainees give colour to the initially inert environment of the MOOC. They make it alive, they enrich it. Even the vigilant support of the trainers means that the trainees do not feel abandoned in this educational experience. The trainees feed on the sap present in the MOOC and in turn, give it new sap. The MOOC-ecosystem that you admire is variegated, changeable and dynamic.

Chapter 3 Research Design

3.1 The research context

In this dissertation we expose two Italian experiences with MOOCs for mathematics teacher education. We will enter into details of these in the following (§3.2).
In particular, the present study was conducted in Turin, Italy; but it involved teachers from all over the country. While we will describe in details the involved participants in §3.2.2 and §3.2.3.4, here it is important underline the context in which these educational offers are inserted.

3.1.1. Italian context

Education in Italy is compulsory from 6 to 16 years of age[16], and is divided into five stages: kindergarten (*scuola dell'infanzia*), primary school (*scuola primaria* or *scuola elementare*), lower secondary school (*scuola secondaria di primo grado* or *scuola media inferiore*), upper secondary school (*scuola secondaria di secondo grado* or *scuola media superiore*) and university (*università*).

Since our MOOCs are aimed to lower and higher secondary school teachers, we concentrate our attention only on them in the following.

Secondary education in Italy lasts 8 years and is divided in two stages: *Scuola secondaria di primo grado* (Lower secondary school), also broadly known as Scuola media, which corresponds to the Middle School grades (grade 6-8), and *Scuola secondaria di secondo grado* (Higher secondary school), also broadly known as Scuola superiore, which corresponds to the high-school level (grade 9-13).
The lower secondary school lasts three years (roughly from age 11 to 14). The higher secondary school lasts five years (roughly from age 14 to 19). Every tier involves an exam at the end of the final year; in particular, in the higher secondary school it is called *esame di maturità*. It is required to gain a degree and have access to further university education.

For historical reasons, there are three types of higher secondary school:

- *Liceo* (lyceum), the education received in a Liceo is mostly theoretical, with a specialization in a specific field of studies (humanities, science, or art);
- *Istituto tecnico* (technical institute), the education given in a technical institute offers both a wide theoretical education and a specialization in a specific field of studies (e.g.: economy, humanities, administration, law, technology, tourism), often integrated with a three/six months internship in a company, association or university, during the fifth and last year of study;

[16] “comma 622”. Camera.it. 2006-12-27. Retrieved 2018-01-10.

- *Istituto professionale* (professional institute). This type of school offers a form of secondary education oriented towards practical subjects (engineering, agriculture, gastronomy, technical assistance, handicrafts), and enables the students to start searching for a job as soon as they have completed their studies.

Currently all of the higher secondary schools in Italy have most of the structure and subjects in common for the first two years (primo biennio) - such as Italian grammar, history and mathematics. In the last three years (secondo biennio and quinto anno, or triennio) most subjects are peculiar to a particular type of course (i.e. ancient Greek in the Liceo Classico, business economics in the Istituto tecnico economico or scenography in the Liceo Artistico) but subjects like Italian, English and mathematics are still taught.

3.1.2 Italian curriculum

The national curriculum is recognized in the *Indicazioni Nazionali*[17], translatable into English as "National Indications". They highlight for each discipline the fundamental learning goals that students have to achieve at the end of each cycle of instruction. The Indicazioni Nazionali have the character of general didactic guidelines and defer to teachers the responsibility of choosing and linking the specific mathematical contents to be developed in the classroom in order to reach the established learning goals. Therefore, the Italian one is not a rigid curriculum that a teacher is obliged to follow. We can say it is the lighthouse to which every teacher must pay attention when she is engaged in her teaching. She knows there is that certain program to be acquired by the students, but she has a great deal of freedom to cover the topics in the times and in the ways she considers most suitable. She can first explain a concept that is listed as last; she can use different methodologies (laboratory teaching, working group, ...) or tools (software, ...).

3.1.3 Teacher education in Italy

In-service teacher education is a right/duty of the teacher (Ficara, 2014). It is a teacher's right since she accesses freely at the educational offers provided by the institution, and it is a teacher's duty since it is an integral part of his role: "users of the public service, [i.e.] pupils and parents, are entitled to a quality service which depends decisively on improving the quality of teachers" (Liceo Politi, POF, nd). In fact, they must be able to design the training action by managing the new conditions of flexibility, modularity and methodological discretion, but at the same time guarantee the achievement of the pre-established standards, assess the results and promote improvement actions.

In-service teacher education has undergone a significant change in October 2016 with the National Plan for in-service teacher education[18], which focuses on the problem of teacher education, together with some fundamental points that enhance it, such as collaboration

[17] Link to the Italian curriculum: http://www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/_decreto_indicazioni_nazionali.pdf

[18] Piano Nazionale per la formazione in servizio dei docenti 2016-2019: http://www.istruzione.it/piano_docenti/

between teachers and at the institutional level, the quality of the educational courses, continuous innovation and professional development of the teacher. The attention of institutions for teacher education, understood in a modern sense and shared in international research, is a qualifying point of the initiative.

Teacher education must not be merely training, but rather must be aimed at acquiring high-profile professional knowledge and skills. It becomes strategic to set up the learning process both on vertical interaction with experts and specialists in the sector and on the horizontal one among the participants in the training event, in order to allow professional development based on the exchange of experiences and good practices.
With the MOOCs delivered by the Department of Mathematics "G. Peano" of the University of Turin, we wanted to promote the social dimension of learning also in e-learning: "the network is not just a vehicle for the distribution of e-content but, above all, it is a resource able to fostering distance interaction between all the actors of the training process" (Renzi, 2009).

Our MOOC project (that we will describe in the §3.2), as a formative module for in-service teachers dedicated to mathematics education, is placed in this context underlined by the institutions. In particular, it promotes first of all the collaboration between institutions (Universities and Schools) involved in the training project. Therefore, it supports and stimulates the collaboration between teachers, both at a distance, if far away logistically, and in presence if they are part of the same school or community that have the opportunity to meet. In particular, as we will see in Chapter 4, creating a virtual community on the web, it gives the opportunity to all those teachers who live in decentralized positions not to feel isolated, but to share objectives, activities, methodologies with close and distant colleagues.

Also in the Plan for in-service teacher education we read (p. 63): "The teacher education that leaves its mark is based on the comparison between peers and on the critical re-elaboration of didactic experiences, but also requires the introduction of cultural stimuli, of different looks, of perspectives that can go beyond their own community of belonging. This is the sense of the opening of the system to university structures".
Our MOOCs are, therefore, a training opportunity for teachers, but also an opportunity for researchers to conduct a study on teachers who work and learn in collaboration.
The dissertation writer's research is in fact aimed at understanding how teachers' collaboration practices are carried out, how they evolve over time as a result of the received stimuli and the interaction activated on the platform and how they differ from those produced in face-to-face courses. The research is also aimed at understanding the influence exerted on the teachers from the dynamic interweaving that is established between the practices and products of the practices themselves. Namely, in terms of messages and interventions on the platform, but not only: in terms of re-elaboration of activities and educational paths, multimedia files, of software, and also in terms of designing new materials.

In the following, we then expose our experiences of teacher training through MOOC, with a focus on some examples related to the strategies and methodologies of mathematics teaching that have been shared and deepened with the enrolled teachers.

3.2 Mathematics MOOCs for teacher education: design experiment

In the design-based research paradigm (Wang & Hannafin, 2005), the design experiments manifest both scientific and educational values through the active involvement of researchers in learning and teaching procedures and through "scientific processes of discovery, exploration, confirmation, and dissemination" (Kelly, 2003, p. 3). In this sense, the mathematics MOOCs for teacher education we discuss in this dissertation can be considered as a design experiment. In fact, the trainers involved in their design and delivering, thanks to the monitoring stages and the feedback received from the trainees of these MOOCs, are able to properly modifies them to allow better professional development experiences to their trainees.
This issue will be further developed and discussed in the analysis of the third research question (Chapter 6). Meanwhile, we begin tidily to tell the story of these our MOOCs and their protagonists.

3.2.1 Math MOOC UniTo project

Trainers in Mathematics Education, a second level master course, took place at the Department of Mathematics "G. Peano" of the University of Turin from September 2013 to June 2015. Professor F. Arzarello and O. Robutti held the master course and it addressed in-service mathematics secondary school teachers.
The participants were trained in Mathematics Education and also on innovation through the didactical material of the m@t.abel project (https://goo.gl/Q30Dn0), a plurennial National Program that promoted innovation in mathematics teaching, basing on concrete activities proposed to teachers and discussed with them in suitable professional learning programs.
At the end of the master course, the following needs had been identified by the teachers of the master (F. Arzarello and O. Robutti) and by the master learners: awareness of the need to support teaching activities with teacher education; willingness to develop best practices of innovation using software; reconsidering in terms of learning the sharing practices of social media most used by the students. Hence, it was decided to offer the opportunity of an authentic professional development experience designed for a larger group of teachers: this idea generated the *Math MOOC UniTo* project (Labasin et al., 2017), namely MOOCs for mathematics teacher education.

The project started in the spring of 2015 and four MOOCs were designed, one for each of the main topics in the official Italian programs for secondary school (*Indicazioni Nazionali*[19] or National Indication in English, remember what we have seen in §3.1.2): Arithmetic and Algebra, Geometry, Change and Relations, Uncertainty and Data. So far, the first two have been delivered: ***MOOC Geometria***, on geometry contents, from October 2015 to January 2016 (Alberti et al., in press); ***MOOC Numeri***, on arithmetic and algebra contents, from November 2016 to January 2017 (Labasin et al., in press). The third one, MOOC Relazioni e Funzioni, on change and relations contents, is started in January 2018 and will end in March 2018.

[19] Link to the Italian curriculum:
http://www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/_decreto_indicazioni_nazionali.pdf

In this dissertation, we take into consideration the concluded experiences: MOOC Geometria and MOOC Numeri.
These MOOCs are open, free, and available online for teachers on DI.FI.MA. (Didactics of Physics and Mathematics[20]), a Moodle platform. It is important to underline that each MOOC is weekly based and from 6 to 8 weeks long in total. The participation is totally and univocally online. One can access to the MOOC wherever and whenever he wants, using internet connection.

3.2.2 MOOCs team

Our courses are MOOCs for teachers designed by teachers in collaboration with university researchers. The mathematical resources inside them (didactical activities, methodologies and strategies exposed, GeoGebra[21] files, …) are made by a group of experienced secondary school teachers that attended the previous mentioned master course, in collaboration with math educators of Turin University.
From spring 2015, that is, when the Math MOOC UniTo project started to take shape, a MOOC team was formed. The MOOCs team is composed by two university professors, a group of experienced secondary school teachers (they were 9 in MOOC Geometria and 20 in MOOC Numeri) and a PhD student (the dissertation writer). All of them are involved in the design, the course delivery and monitoring its evolution in terms of interaction among participants and educational resources made available. In particular, two of the experienced teachers (V. Alberti and S. Labasin) are particularly engaged in the design of the MOOCs, while the others are reviewers engaged in the monitoring activities. In particular, as mentioned, the experienced teachers also create the activities delivered in the MOOCs, adapted from m@tabel project and revised by the university professors. Moreover, the MOOCs team helps MOOC learners to solve technical problems, to make tutorials, to recall the tasks to be done week by week with weekly emails. There is also a computer technician of the department, T. Armano, which helps us manage the platform.

During the design phase, there were several face-to-face meetings, at the Department of Mathematics "G. Peano", in which specific decisions were made. Precisely: who should be the recipients of the MOOC, what topics should have been treated, how to encourage interactions between us and the participants and among the participants themselves, how to encourage collaboration among participants, what evaluation strategies to consider for this online training experience (these issues will be explored in Chapter 6, which focuses on RQ3). The dissertation writer kept track of these meetings by taking notes and sometimes even recording.
The monitoring activities were carried out remotely, i.e. each member of the MOOC team could connect himself to the platform when and where he could. There has been an accurate and agreed division of labour, as we will see shortly in § 3.3.2.1.

20 http://difima.i-learn.unito.it/
21 GeoGebra is an open source DGS (Dynamic Geometry System)

3.2.3 An overview on MOOC Geometria and MOOC Numeri: the didactical and methodological choices of the MOOC designers

3.2.3.1 Trainers and trainees

As you could guess, within MOOC Geometria and MOOC Numeri, as well as in general within MOOCs for teacher education, two communities can be distinguished. The MOOCs team (that we have described above) and the MOOCs learners (that we will describe in §3.2.3.4). However, note that, from here and after we identify them as ***trainers*** and ***trainees*** respectively, in line with what we have seen in the MOOC's Zone Theory theoretical framework.

3.2.3.2 Mathematical contents in our MOOCs

The activities presented in our MOOCs provide teachers with suggestions to support their teaching. They aim to improve the teaching of mathematics in Italian School and are, in effect, a tool for professional development and education.
The mathematics curricula, to which the activities refer, are in line with the UMI[22] proposals (Mathematics for the Citizens; Mathematics 2001 and Mathematics 2003), as well as in line with the Italian curriculum. The activities do not exhaust all the topics of the curricula, but have the ambition to provide detailed methodological indications on how to deal with some conceptual nodes of particular importance for the mathematical education of the students. With the term "conceptual nodes", we intend to refer to central thematic concepts in an educational path, to epistemological obstacles or to non-trivial cognitive difficulties that students usually encounter.
The activities, both for their type and how they are built, stimulate the motivation and involvement of all students, even the less interested ones in the subject. They also indicate possible sub-path of consolidation, aimed at "weaker" students, or of in-depth studies, suitable for students with better results. In particular, in some of the activities presented, specific indications analyse the most common difficulties that the students might encounter during their execution and some suggestions are proposed, which the teacher will choose how to apply or integrate according to the situation of her class.
The activities are clearly proposals: they must not be considered as prescriptive indications. Each teacher is able to compare the indications provided with her curricular programming or with the shared one of her institute.

The proposals of the MOOC offer concrete examples of activities to be carried out in the classroom through a laboratory-based methodology (Anichini et al., 2004), and technologies as well. The laboratory-based methodology is a teaching and learning strategy in which the student appropriates knowledge in the context of its use. This is in contrast to conventional teaching in which knowledge is offered to students in isolation from every use and for its general characteristics (Marconato, n.d.). Therefore, the laboratory-based methodology is useful for the construction of mathematical meanings; it is a valid means of building

[22] UMI is a permanent commission of the Italian union of mathematics (Unione Matematica Italiana: UMI): http://www.umi-ciim.it/

knowledge through collaborative learning and facilitates positive interactions between people, reinforcing group identity.

All activities propose a teaching-learning of mathematics in which three fundamental aspects are intertwined:

- the disciplinary contents (mathematical knowledge);
- the situations (the contexts) in which the problems are posed, which are used as sources of stimulation for the students as they are linked to reality;
- the processes (skills) that the student must activate to link the problematic situation that he is dealing with appropriate mathematical contents.

In all the proposals there is a conception of mathematical competences as a complex of processes based both on mathematization and modelling of real situations; both on the exchange with others, on the interface between the individual and on the collective experience.

We will give below only a schematic presentation of the MOOCs activities. Refer to Chapter 4 for more information on some of them.

Mathematical contents of MOOC Geometria

In MOOC Geometria five modules on geometric contents were created on three main topics: practical manipulative activities, problem solving and proof, assessment (Table 3.1).

Macro themes	Conceptual/methodological issues	Module	N. of weeks
Manipulative practices and technological activities	Approaching distance with laboratory activity, using Geogebra as well	Module 1: "Ramps, sails, park and folding paper"	1
	Approaching angle with laboratory activity, using Geogebra as well	Module 2: "From watches, pinwheels, skaters to the Christmas show"	2
Problem solving, proof and technology	Arguing, conjecturing, proving, using Geogebra as well	Module 3: "Heritage, a Polya's problem and what demonstration?"	2
Assessment	Evaluating different skills	Module 4: "Assessment&INVALSI"	1
	Recognizing different representations with the same meaning	Module 5: "MERLO methodology"	1
Production	Designing a teaching situation with a specific web-based tool and reviewing an activity designed by another colleague	Module 6: "Project Work and Peer Review"	2

Table 3.1: The modules of the *MOOC Geometria*

After an introductory module (the first week), the activities were offered weekly and the duration of each section varied from 1 to 2 weeks (depending on the topics treated). The last module, Module 6, is related to the final task of the MOOC (we will explain better it in the next section §3.2.3.3 and in §3.3.1.4). The MOOC Geometria was 8 weeks long (plus 2 weeks to complete the final task), from October 2015 to January 2016. It should be noted that the

MOOC has been ended in all its phases in January 2016. However, only for the participants who have registered in it, the materials remain available to allow future consultation. The communication spaces are instead closed.
We will have the occasion to deepen some of these modules in the analysis section (Chapter 4).

Mathematical contents of MOOC Numeri

In MOOC Numeri five modules on arithmetic and algebra contents were created on three main topics: practical manipulative activities, problem solving and proof, assessment (Table 3.2).

Macro themes	Conceptual/methodological issues	Module	N. of weeks
Manipulative practices and technological activities	Approaching order of size and number sense with laboratory activity, using Geogebra as well	Module 1: "Meteorites, bacteria and rice grains: the numbers and their meaning"	1
Assessment	Recognizing different representations with the same meaning	Module 2: "MERLO methodology"	1
	Evaluating different skills	Module 3: "Assessment&INVALSI"	1
Problem solving, proof and technology	Approaching the concept of induction/recursion, using Geogebra as well	Module 4: "Climbing stairs"	1
Manipulative practices and technological activities	Approaching arithmetic and algebraic language and meaning of the symbols, using Geogebra as well	Module 5: "Arithmetic, Algebra and Mathematical Languages"	1
Production	Designing a teaching situation with a specific web-based tool and reviewing an activity designed by another colleague	Module 6: "Project Work and Peer Review"	3
Experimentation	Choosing to experiment in one of your classroom either **an activity** of **MOOC Numbers**; or **your own Project Work**. Completing a suitable logbook and sending possible attachments (images, videos, files made for or by pupils …) via mail.	Optional module: "Experimentation"	From the second week of MOOC and until March 31st

Table 3.2: The modules of the *MOOC Numeri*

After an introductory module (the first week), the activities were offered weekly and the duration of each section was 1 week. The Module 6 is related to the final task of the MOOC (we will explain better it in the next section §3.2.3.3 and in §3.3.1.4) and here an optional module was inserted relative to the "Experimentation" (we will explain better it §3.3.1.4). The MOOC Numeri was 6 weeks long (plus 3 weeks to complete the final task), from November 2016 to January 2017. The time to accomplish the optional module was by the end of March. It should be noted that the MOOC has been ended in all its phases in March 2017. However, only for the participants who have registered in it, the materials remain available to allow future consultation. The communication spaces are instead closed.
We will have the occasion to deepen some of these modules in the analysis section (Chapter 4).

3.2.3.3 Technological resources of MOOC Geometria and MOOC Numeri

The technological resources implemented are the same in both MOOCs and are well descripted in the following table (Table 3.3). All of them are open source tools and they were chosen on purpose, enabling teacher-trainees to easily fit with them in their teaching practices.

Where it was used?	Tool name	What does it allow to do?
Module introduction	Powtoon	It allows you to create online presentations and engaging video with the help of nice animations.
	YouTube	Short videos to present the conceptual nodes examined.
Contents presentation	Sway	It allows you to support the "message" of teacher designers during the "narrative" of educational courses; it supports the viewing and has a very clear graphics. It incorporates office documents and Geogebra files that are directly accessible and downloadable.
	Soundcloud	For voice narration (introduction, commentary, conclusion, testimony) in training courses.
	GeoGebra	It is an interactive geometry, algebra, statistics and calculus application, intended for learning and teaching mathematics and science from primary school to university level.
Interaction	Forum	Communication tool used to express opinion on the macro themes of the modules.
	Padlet	Collector of ideas that leads to sharing and get used to a different talk mode, approaching the participatory mode.
	Tricider	It helps in decision making, crowdsourcing and idea generation. It is useful for easy brainstorming and voting.
	Big Blue Button	It is an open source web conferencing system. It supports multiple audio and video sharing, presentations with extended whiteboard capabilities - such as a pointer, zooming and drawing - public and private chat, desktop sharing, and support for presentation of PDF documents and Microsoft Office documents.
Final task design	Learning Designer	It directs the design of educational courses according to the "teaching as design" model. It facilitates the pedagogical

		approach for sharing and systematization, adjusting teaching towards the process.

Table 3.3: Technological resources inside our MOOCs

We add some other clarifications. Forum, Padlet and Tricider are what we will call *communication message boards* in this dissertation. Big Blue Button instead is the tool that the MOOC team used to arrange the *webinars*. These are educational online events, or online meetings in which an expert – generally the university professors – (seen through a camera) shares with the trainees (who can only interact via chat) some issues about the research in mathematics education and focuses on some questions that could be raised during the previous weeks in the MOOC. Precisely, the experts discuss relevant topics, share personal experiences and valued resources, and suggest strategies for implementing knowledge gained from research in everyday classrooms. In both MOOC Geometria and MOOC Numeri there were three webinars and they were organized for creating occasions of synchronous contact with the trainees.

At the end of the whole MOOC, each trainee was asked to *design a teaching activity* (Project Work, PW) on the contents of the MOOC (so geometry for MOOC Geometria and arithmetic or algebra for MOOC Numeri) and to review another teaching activity prepared by a MOOC colleague (Peer Review, PR), (Taranto et al., 2016). The design of the teaching activity was done using a specific software, Learning Designer (Laurillard & Masterman, 2009). It allows designing a own lesson that is structured as a succession of activities. In fact, the toolkit offers a list of types of TLA (Teaching Learning Activities) from which you can be inspired to "compose the lesson". In this way, it allows not only building an explicit design, but also supports the design process. In addition, it displays with a pie chart (Figure 3.1) the presence of the different types of TLA in the project, in order to understand if you have done a balanced use of them. The use of the tool does not require any technical competence.
PWs once realized and delivered, can be consulted by all the trainees. This means that the PW become a shared asset among all the MOOC members.

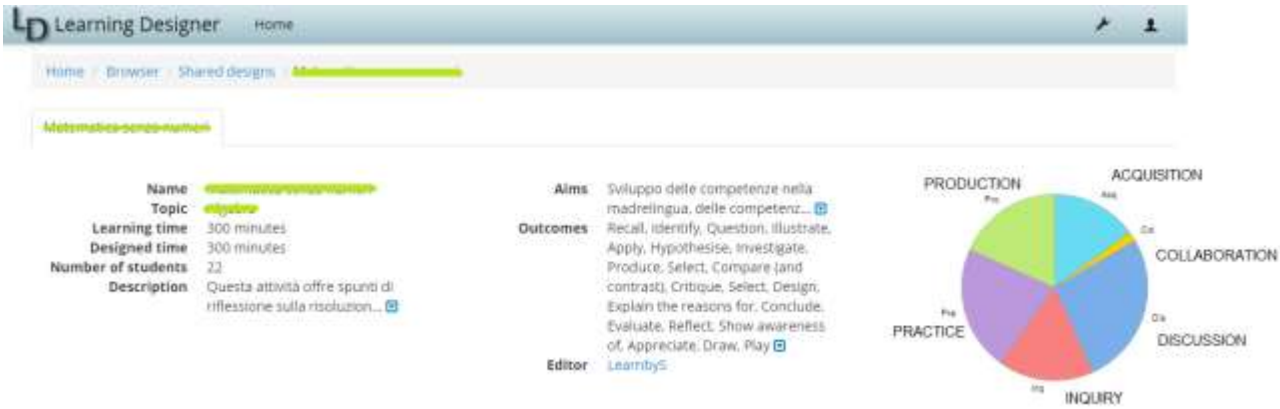


Figure 3.1: Example of the screen relative to a design of a PW

In each course, multiple types of resources were incorporated, such as classroom-ready materials (e.g., lesson plans, tasks, content instructional videos) and thought-provoking materials for trainees to reflect on their practice and deepen their content and pedagogy knowledge for teaching. These resources were often provided with multiple media, such as animated videos or podcasts to support different paths of learning activities. Participants were also often given choices to explore materials designed for different levels of understandings or grade levels (lower or higher secondary school, as we are going to specify in the following).

3.2.3.4 MOOCs target

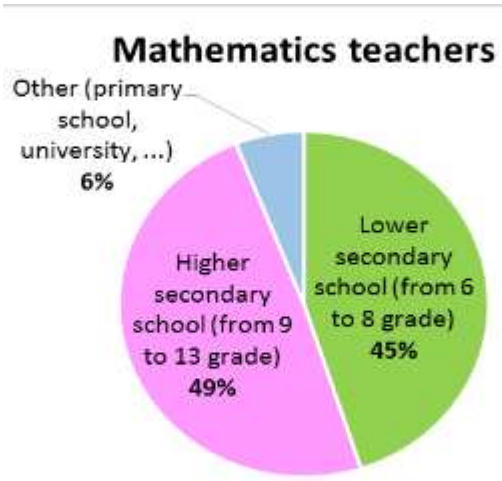
Our MOOCs target, namely the people to whom the MOOCs are addressed, are in-service mathematics secondary school teachers (from 6 to 13 grade). However, as the first O of the MOOC acronym underline, the MOOCs are *open*. In fact, everyone who wants, can enrol in them, even simply curious people. So, despite the fact that the target is clearly stated, the MOOCs doors remain open to every mathematics teachers: maybe those who are beginners can also find valid ideas, but clearly there are those who can benefit more consistently. Some primary school teachers also have decided to enrol in our MOOCs. They have adapted themselves and continued to enrol from the first edition. So, in the third edition, we have also thought to include them in the target, but this goes beyond the purpose of this dissertation.

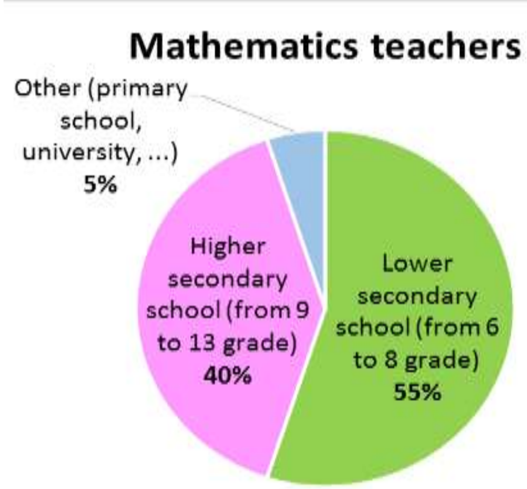
The trainees of MOOC Geometria and MOOC Numeri

In the Table 3.4 there are some significant data to describe the trainees of both MOOCs.

<table>
<tr><th></th><th>Trainees of MOOC Geometria
(October 2015 - January 2016)</th><th>Trainees of MOOC Numeri
(November 2016 - January 2017)</th></tr>
<tr><td># trainees</td><td>424</td><td>278</td></tr>
<tr><td>Gender</td><td>Women: 82%; Men:18%</td><td>Women: 86%; Men:14%</td></tr>
<tr><td>Geographic origin (all Italians)</td><td>


</td><td>


</td></tr>
<tr><td>Educational levels</td><td>


</td><td>


</td></tr>
<tr><td>Trainees had never</td><td>88%</td><td>31%
(50% of the MOOC Numeri trainees had</td></tr>
</table>

attended a MOOC		been enrolled in the MOOC Geometria)
Completion rate	36%	42%

Table 3.4: The trainees of MOOC Geometria and MOOC Numeri

Every week the trainees worked individually to become familiar with different approaches. In our MOOCs these activities included: watching videos where an expert introduced the conceptual knot of the week; reading the mathematics laboratory-based methodology activities (and the option to experiment with these in their classroom). Trainees were invited to share thoughts and comments about the activities and their contextualization within their personal experience, using specific communication message boards (Forum, Padlet, Tricider). Doing that they could so collect their weekly badges, which acknowledged their different kinds of participation.
In fact, once all the module requests were accomplished, the platform released a badge (an example in Figure 3.2). In this way, it is quite easy for the MOOCs team to monitor the progress of the trainees, knowing the amount of badges they had collected.

Figure 3.2: Badge of Module 4 in the MOOC Geometria

At the end of the whole MOOC, as previously mentioned, each trainee was asked to design a teaching activity (Project Work) and to review another activity prepared by a colleague (Peer Review). For all those who completed the course in all its stages, a participation certificate was issued. Unlike what happens in general with MOOCs (Chapter 1), the certification that releases our MOOCs is totally free.

3.2.3.5 A video overview of MOOC Geometria and MOOC Numeri

To summarize what we have described, at the following links:
MOOC Geometria: https://goo.gl/ztnmHV
MOOC Numeri: https://goo.gl/myGxp6
you will find two short videos. They are two overviews of our MOOCs, created by the dissertation writer, which allow having a clearer idea of how MOOC Geometry and MOOC Numbers are structured. We invite you to look at them before proceeding with the reading of the dissertation.

3.2.3.6 General remarks

We propose below some general remarks and fuel for thought that will be gradually taken up during this dissertation.

Completion rate
One of the major recurring issues raised in both academic literature and the popular press is the consistently high dropout rate of MOOC learners (Onah et al., 2014). Although many thousands of participants enrol on these courses, the completion rate for most courses is below 13% (see: http://www.katyjordan.com/MOOCproject.html). In particular, MOOCs for mathematics teacher education has a completion rate of 12% (Panero et al., 2017). Even with our MOOCs we could not avoid this, although our completion rates are very different from those reported in the literature: 36% for MOOC Geometria and 42% for MOOC Numeri (see the last row of the Table 3.4).
Some consideration on the motivations we have identified to explain this our "success" will be addressed in analysis section, in Chapter 4.

Differences between face-to-face and MOOCs for teacher education
There are certainly substantial differences between face-to-face and online education.
We do not want to have the claim to highlight them all here. General remarks will be done both from the trainer's point of view and from the trainee's point of view. These are based on the personal experience of the dissertation writer who was both a trainer of some MOOCs on general topic and also a member of the Math MOOCs UniTo team.

From the MOOC trainers' point of view

- **Numerosity:** it is massive, of the order of hundreds or thousands. The trainer can address the same lessons to a much wider audience than in a face-to-face educational course[23].
- **The affordances of technology:** compared to a face-to-face course, in a MOOC there are no problems of timing, audio or visibility. The trainer is not in a hurry to explain something, because he does not have to follow a certain schedule or to respect by the time of education; rather there are repeatable audios and videos.
- **Present a teaching activity:** in a MOOC the trainer will never see his interlocutors or his audience in person. How should you prepare an activity to make it usable? The format must be easily accessible, not too static. There must be images, colours, links to relevant and dynamic resources. Clarity of exposition - precise and global language (Kvilekval, n.d.) is required so that no perplexity arises. You should try to anticipate possible questions. The text must not be too long otherwise the attention threshold decreases[24].

From the MOOC trainees' point of view

- **Time:** a trainee can connect himself on the MOOC whenever he wants, organizing his free time as he wishes.
- **Space:** a trainee connects himself wherever he wants. He covers geographical distances more easily: if he was Sicilian (South Italy) and there was an educational course in Piedmont (North Italy), he would risk losing an opportunity.

[23] There are educational face-to-face courses which are numerus clausus. The classrooms in which the lessons are held, at most, can accommodate a hundred people. It is not necessary to indicate the numerical interval that considers the minimum or maximum number of teachers in a training course. It is immediately clear that it hardly exceeds hundreds.

[24] The attention threshold in presence it is 45 min (Panetto, n.d.) and at distance ranges from 12 to 8 seconds (Tom's Hardware, n.d.).

- **Pace of learning:** a trainee is autonomous. He can study how many hours he wants. He may have the will to go deep into the contents or not.
- **The affordances of technology:** Thanks to repeatable audios and videos, a trainee in a MOOC does not risk losing "pieces". He can see and listen to them as often as he wants, he can try and try again to replicate without slowing down the learning rhythms of his colleagues.
- **Different institutional weight:** enrolment in a MOOC is voluntary, not imposed by anyone. Completing the MOOC is always a voluntary choice. The meetings in synchronous with the trainers are mostly non-existent. In our case, there are webinars, but always with voluntary participation. There are only a few undelayable deadlines, for example the final activity (Project Work) and the end of the course. In general, there is maximum freedom.
- **Multiculturalism:** it is not specifically our case, because our trainees are all Italians. However, in our MOOCs teachers from different educational system (lower secondary school, higher secondary school) meet together. They would not have had the opportunity to meet each other without MOOC (note that they have never seen each other in presence, they know each other virtually). How we will see in the analysis section (Chapter 4), our trainees have had the possibility to enrich each other. They have had the opportunity to read what another peer is doing and maybe get ideas.

The heterogeneity of trainees

As you can see from the picture of the fourth and fifth row of Table 3.4, all the trainees have different geographical origins and teach in different educational levels[25]; therefore in general they have a *different professional background*.

To this, we must add a typical Italian facet that we have previously and briefly mentioned in §3.1.2.: the fact that the national curriculum is recognized in the *Indicazioni Nazionali* ("National Indications" in English). So, although the MOOC start date is the same for everyone, *it does not necessarily mean that all teachers, at the time they participate, are at the same point in the program*. They may have already covered some of the topics seen in the MOOC, they may still have to deal with them. They could decide to not treat them.

In a face-to-face training course, it is less likely to deal with this *heterogeneity of the trainees community*. Additionally, any differences in the development of the classroom program are smoothed out because all the teachers (that of course are not massive) are followed in attendance and they all work on the same activity that must then be effectively reported in the classroom, to continue to discuss it all together.

[25] Unless the MOOC declares that the target of teachers to whom it is addressed must teach at a specific education system (primary, lower secondary or high secondary school). In our case, although it was specified that the MOOC was addressed secondary school teachers, primary teachers also took part in it.

The complexity of a MOOC
We illustrate here a final remark. It acts as a prelude to subsequent discourses and we will interface ourselves with it during the dissertation.

A MOOC is a very complex environment in which several protagonists alternate their roles. In fact, in the design phase, when the MOOC starts to take shape, it is inhabited by designers and trainers who propose the materials and/or the resources to be included in it. When the MOOC is ready, it is still at an inert state; then it is opened in order to accommodate the entry of new inhabitants: the trainees. They will discover it weekly, moving far and wide. “Far” because they will wait for the next module opening to see new materials and “wide” because every week a new module is opened, but the previous ones are kept open until the end of the course. So, the trainees can discover the novelty of the new module, but at the same time may come back and see in depth the previous material and resources. They can so read in more depth some activities that they had first read quickly; they can dwell more on the comments of others and, at the same time, leave their comment that they did not leave at the beginning because of haste or shyness. Sometimes, as happened in our case, they can share their own materials with the other trainees in the communication message boards, creating a real community of practice (Wenger, 1998) and expanding the early MOOC structure.

3.3 Methodology

MOOC’s Zone Theory offers a useful framework for research that aims at understanding the complexities of trainers/trainees’ learning trajectories in a MOOC. Therefore, what are the consequences of their participation in a MOOC for mathematics teacher education?
More precisely, we intend learning in a connective sense (as explained in §2.6.3). In fact, we are interested in how these protagonists interact online, both with the platform and with each other. In particular, if and how these interactions could be occasion of expansion of their own network of knowledge, or in other words if there are evolution in their praxeologies and so, if their ZPD move from the actual developmental level to the potential one.

In this dissertation, the courses under investigation are MOOCs for mathematics teacher education (MOOC Geometria and MOOC Numeri, whose general structure has been exhibited in §3.2.3.2, §3.2.3.3). For this reason, we analyse in more detail the trainees’ learning trajectories and we will refer to them simply in terms of professional development (Chapter 5). However, there is also a brief analysis on the trainers’ professional development, more from the point of view of design and assessment (Chapter 6).

The MOOC’s Zone Theory theoretical framework enable us to formulate the research problem into the following research questions:

Research Question 1 (or RQ1):
Are there any particular potentialities in a MOOC-artifact that, if properly organized, trigger the double learning process and therefore the transition to the MOOC-ecosystem/instrument?

Research Question 2 (or RQ2):
It is specific to the trainees:

Does the MOOC's ZFM/ZPA trigger and support an effective shift from the actual developmental level to the potential developmental level in the ZPD of the trainees? And if so, which kind of expansion of the network of professional knowledge this shift brings with it?

Research Question 3 (or RQ3):
It is specific to the trainers:
Does the MOOC's ZFM/ZPA trigger and support an expansion of the network of professional knowledge of the trainers relatively to design principles and strategies of trainees' assessment that the trainers have put in place?

3.3.1 Empirical Phase of the study

The empirical phase of the study was conducted over almost a three-years period (2015-2016-2017).
The analyses that have been conducted are both qualitative and quantitative. The manner in which these were conducted follows below.
There are many different approaches to do qualitative and quantitative research; however, the role of the researcher and the emergent nature of the research design are two characteristics shared across these approaches (Creswell, 2013). So, we start to address these aspects of the present study.

3.3.1.1 Positioning the researcher

The researcher is the key instrument in collecting data in qualitative research (Creswell, 2013), so must position herself within the study. Throughout the research process the researcher brings a particular perspective that determines how the study is conducted, who participates in the study, what questions are asked, what is observed, what documents are reviewed, what is analysed, and what is reported (Merriam, 1998). These prior understandings also influence the interpretations that arise as meaning is created in the interactions between the researcher and the participants (Denzin & Lincoln, 2003). Consequently, this section was written in the first person, as it is about my background and role in the present study.
I began my doctoral study after my master degree with a thesis on "Teaching/learning of geometric loci in the mathematics laboratory with DGS[26]" based on a project where teachers were involved. Through this experience, I develop an interest in the professional learning of teachers and, in particular, the factors that influence how teachers transpose professional learning experiences into their classroom practice. Therefore, when – in the spring of 2015 – Prof. Ferdinando Arzarello involved me in the project of Math MOOC UniTo, which was still a primitive idea, I accept it with curiosity and interest.
During my doctoral study, I had the dual roles of researchers for my study and member of the team for MOOC design. It was essential to "get my hands dirty" to better understand the phenomenon I was studying. I have good computer and editing skills, so I actively participated in the digitization of the contents and their arrangement on the Moodle platform, which hosts our MOOCs.
I knew the structure of the MOOC and all its materials like my pockets and this was a precious advantage that enabled me to follow the developed interactions within it.

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[26] Dynamic Geometry System

My role during MOOC observations was that of a ‘participant observer’, that is, immersed in the setting but also at a respectful distance from the teachers (Schwandt, 2007) and without direct involvement in MOOC activities. Of course, participants know my name (written as signature in some e-mails sent to collect data) and sometimes they saw me on the webinars; but after these my presence had little impact on MOOC activities.
MOOC is a complex environment, as we have seen, that goes on very fast. Moodle platform keeps track of all the virtual actions of the participants (when they log themselves, what they see, where they write their comments, …), but they are hundreds and it is impossible to observe everything that is happening, so it is necessary to focus on what is relevant to the purpose of the study.
MOOC observations, questionnaires, Skype or written interviews and collection of logbook were primary methods of data collection of my doctoral study. Case study methodology is also employed in the present study.
Although there are advantages in having the researcher as the primary instrument of data collection, as the situation in case study research (Merriam, 1998), there are also disadvantages. On one hand, I was attuned to the purpose of the study and could focus on collecting data relevant to the study (e.g., asking follow-up questions in Skype interviews to seek clarification of information related to the research questions); on the other hand, the quality of the study was highly dependent on my skills as a researcher. For example, Kvale and Brinkman (2009) described interviewing as a craft that is dependent “on the practical skills and the personal judgments” (p. 17) of the interviewer that can only be developed through experience. My same skills in this area developed through experience in this research project.

3.3.1.2 Re-imagining the research design

The responsiveness of the research design to the circumstances of the study is the second characteristic shared by different approaches to qualitative research (Creswell, 2013). Generally, adjustments are made to the fieldwork components of a study and may include – as it happened to me - changing interview/questionnaires questions; or incorporating additional modules in the MOOC to expand data collection. We will enter later into these details.
The absence of a framework for studying the processes that are triggered by the MOOC or that can be developed within it meant that much greater emphasis was placed on theoretical aspects. In fact, a suitable theoretical framework was created, as presented in Chapter 2. Therefore, the study was conducted in a theoretical phase (previously described in Chapter 2) and in an empirical phase.
In the following some more explanations that allow the reader to understand how we arrive to conduct the study in this way.
The original research design was understanding how teacher’ teaching practices change after his attendance in a MOOC. However, we had not yet realized the phenomenon we were going against. First of all, the research had not yet developed a theoretical framework apt to analyse the teachers’ use of and interaction within MOOCs. The literature review on this topic was meagre (Ozturk, 2015; Teixeira et al., 2015) and the shortage was almost total with regard to the teacher education activities. On the contrary, as we already pointed out, a great deal of literature exists about how teachers can develop their professional development in traditional face-to-face courses, especially when the topic concerns the relationship between education and technology. Second, there is no possibility of going to see what happens in the class of a teacher who attends the MOOC. This is because every teacher follows the course with his times and his ways. Moreover, the enrolled teachers were scattered throughout Italy.

However, little by little, we realized that we noticed some aspects that had not been initially envisaged: the interactions in the new online environment; the impact on the professional development of trainers and of trainees. Hence the research design changed significantly.
For this reason, the first change made to the research design was to include a theoretical phase to develop a suitable theoretical framework. Thus, the study was conducted in a theoretical phase (previously described in Chapter 2) and in an empirical phase (Chapters 4, 5, 6).

3.3.1.3 Situating the Research

The present study was conducted in Turin, Italy; but it involved teachers from all over the country.
Clarifications on the Italian context and curriculum have already been done in §3.1.

Participants

The participants in the present study are all the participants involved in the MOOCs Geometria and Numeri. That is means the community of trainers that design, deliver and monitor the MOOCs and the community of trainees that enrolled in them.

About the first community, we distinguish the two MOOCs experiences.
For MOOC Geometria the team was composed by 13 people: 3 researchers of the Department of Mathematics "G. Peano" at University of Turin (Prof. Arzarello, Prof. Robutti, myself as Ph.D. student), 1 master student and 9 experienced teachers. Two of them (Alberti and Labasin) were particularly engaged in the design of the MOOC, while the others were reviewers engaged in the monitoring activities. For MOOC Numeri there were no master students and the number of experienced teachers has increased to 20.
All of the experienced teachers are experienced because – as mentioned in §3.2.1 – they had attended a second level Master "Trainers in Mathematics Education" that had taken place at the same Department from September 2013 to June 2015. They were trained in Mathematics Education and also on innovation through the didactical materials of the m@t.abel project (https://goo.gl/Q30Dn0), a plurennial National Program that promoted innovation in mathematics teaching, basing on concrete activities proposed to teachers and discussed with them in suitable professional learning programs. The experienced teachers are the authors of the activities proposed in the MOOCs modules and revised by the university professors.
We concentrate our attention on this community in the last part of the dissertation (Chapter 6), because the RQ3 is focused on them.

As regards the second community, the trainees one, a premise is necessary.
In §1.1.3, we have observed that one of the major recurring issues raised in both academic literature and the popular press is the consistently high dropout rate of MOOC learners. Although many thousands of participants enrol on these courses, the completion rate for most courses is below 13%. In particular, MOOCs for mathematics teacher education has a completion rate of 12% (Panero et al., 2017). Even with our MOOCs we could not avoid this, although our completion rates are very different from those reported in the literature: 36% for MOOC Geometria and 42% for MOOC Numeri. For this reason, in this dissertation I decide to consider the totality of enrolled and active trainees when I analyse their interactions in the MOOC platform; but for the analysis of specific data sources (as questionnaires or interviews) I will focus only on the trainees that have totally completed the MOOCs in all their stages.

With respect to those who have abandoned, I will analyse secondly the answers given on why such abandonment has occurred.
Anyway, to present the trainees in general, we can however specify that they are all Italian and all mathematics teachers in lower or higher secondary school. We will make clarifications that are more specific when we analyse them in RQ1 (Chapter 4) and RQ2 (Chapter 5).

3.3.2 Data collection

Using a variety of data sources contributes to the richness of the descriptions that can be developed about a case (Creswell, 2013; Denzin & Lincoln, 2003). Table 3.5 show all the methods of data collection used in this research. They provide different perspectives; thus contributing to an overall in-depth understanding of our object of study. A description of these data sources follows (see Appendix B for questionnaires forms, interview protocols and logbook: they are shown in the original, i.e. in Italian).

	2015			**2016**				**2017**			
	Oct-Nov	Nov-Dec	Dec-Jan	Mar	Apr	Nov-Dec	Dec-Jan	Jan-Feb	Mar	Apr	Jun-Jul
	MOOC Geometria					***MOOC Numeri***					
MOOC observations	X	X	X			X	X	X			
Questionnaires for trainees	Initial	Intermediate	Final			Initial	Intermediate	Final			
Written interview for trainees		First	Second	Third							X
Logbook									X		
Skype interviewing for trainees											X
Questionnaires for trainers					X					X	

Table 3.5: Timetable of data source relative to my research

3.3.2.1 MOOC observations

During the MOOCs delivering there was an intense monitoring activity of the platform made by the trainers team.
On the one hand, to ensure a profitable trend of the training course, the trainers team sent weekly e-mails to tell the course progress and communicate important notices (i.e. the webinars). They provided technical support (account issues, clarification of some doubts, ...) with personal e-mails or answering on the predisposed forum, called “technical forum” (Forum tecnico, in Italian). They have made additional tutorials to clarify how certain technological tools work.

No tutorials have been done for *communication message boards*: Forum, Padlet, Tricider. We have introduce them in Table 3.3 (§3.2.3.3). The Table 3.6 shows their peculiar characteristics and the reason why we chose to use them. In the communication message boards the trainers adopted a technique to initiate discussions with a prompting question in order to accompany trainees in reading the materials and identifying their focus. So, in each of them there were specific question to be answered or a title that serves as talking point. In particular, the trainers' team chose to limit interventions in the communication message boards as much as possible, in order to support the birth of an interactive trainees-only community.

Tool	Affordances	Reason of the choice	In which module was it there?
Forum	For public discussion, where everyone can read and answer to messages, using nested replies.	To give teachers a friendly and known tool for discussion.	Geometria 1,2,3 Numeri 1,2,3,4,5
Padlet	Board of collaboration/sharing material (images, videos, documents, text) on common tasks.	To give a talking mode different from the forum, for supporting teachers in participatory methods.	Geometria 1,2,3,4,5,6 Numeri 1,4
Tricider	For easy brainstorming and voting. For decision making, crowdsourcing and idea generation.	To facilitate decision making after any discussion, with request of voting.	Geometria 2,3 Numeri 3,5

Table 3.6: The communication message boards in MOOC Geometria and MOOC Numeri

On the other hand, since I was interested to investigate the interactions of participants (*first research question* or RQ1[27]), it was essential to monitor the platform in a certain way. Moodle platform keeps track of all the virtual actions of the participants (when they log themselves, what they see, where they write their comments, …), but also human observations were fundamental.
The experienced teachers were divided into small groups, as many as each MOOC module, and each group took care of the module for which they had prepared its activities.
In MOOC Geometria, during a team meeting, a structure of the table that would be used to collect the trainees' posts on communication message boards was decide (see Appendix B, pp. 348-349). I then added a classification (Table 3.7) to identify the types of interventions in order to quantify them by categories.

A = explicit answer to the questions
B = considerations
C = sharing of materials
D = sharing of experiences
E = experimentation
F = other

Table 3.7: Categories to classify the types of intervention of the trainees on the communication boards

In MOOC Numeri, the experienced teachers continued to use the tables and, in addition, I have prepared some sheets to collect accurate data from the communication boards, called "module X – monitoring sheets" (with X specific for each module: 1 if the first one, and so

[27] In fact, as we will see in the analysis of the RQ1, the possibility of creating interactions between the participants is one of the potentials that a MOOC-artifact possesses

on). In the Appendix B (pp. 346-347) there is an example of their structure (partial English translation in *italic*).
In general, the sheets contained similar questions (there are slight differences depending on the specific communication board contained in the module that should be monitored).
Each group uploaded the monitoring sheets on One Drive[28] within two weeks after the end of the monitored module. However, as trainees had the freedom to go back to examine again the materials and interacting with modules throughout MOOC, each group had to continue to monitor its assigned module and eventually update the sheet until the end of the MOOC.
Moreover, the trainers' team met regularly and, at the end of each module, they shared what they had observed during that specific module. In particular, the most significant trainees' interventions or sharing actions were discussed.

To make the analysis of the trainees' posts in the communication message boards, I have examined the file filled by the monitoring groups, but I have re-elaborated these data into another table and I will use this one in the analysis section (§4.5; §4.7) to show the reader the discussion/intervention.
To summarize the terms of the double learning process, I will use the following codes in the table:

- **Instrumentation/self-organization** (from the ecosystem to the individual) = E→I
- **Instrumentalization/sharing** (from the individual to the ecosystem) = E←I

It is important to note which verbs are used by the trainees. For instrumentation/self-organization the verbs are at the future form (I will do it, I will re-propose, I will test it, I will use it, ...) or there are verbs or adjectives to express your own judgment (I have noticed, I really appreciated, nice idea, …). For instrumentalization/sharing instead the verbs refer to their own self (I reflect, I know, I thought, …) when they are creating new connection stimulated by the MOOC-ecosystem; while the verbs are at the present form when they share their didactical praxeologies (I do this, I use that, …).
All the intervention are written in a normal type. If you find bold, underlined, italic word, those will be "signs" inserted by me to accomplish the analysis.

Something different happens in the last module of both MOOC Geometria and MOOC Numeri. The final activity of trainees consists in the production and delivery of an activity designed by them (one for each trainee) and in the peer review of an activity produced by another trainee (who teach in their same school order). Trainees must so carry out a didactic design (Project Work, PW) on an activity seen in the MOOC or relating to the subject of the MOOC (Geometry or Numbers respectively). This design was done using a specific software, Learning Designer (Laurillard et al., 2009), previously explained in §3.2.3.3. For trainees was also available a revision greed with some guidelines for reviewing. Let us point out that this was the practice adopted for MOOC Numeri. In MOOC Geometria, the revision greed was not immediately available for the trainees, because we did not intend to constrain their creativity in the PW design. The trainees, however, have pointed out that they would have preferred to consult it immediately. For this reason, the methodology has changed in MOOC Numeri.
In this last module, two experienced teachers (Alberti and Labasin) check that PW deliveries take place within the established date (MOOC's unique undelayable delivery) and check that all PWs have a reviewer.

[28] It is a cloud storage and backup service offered by Microsoft

Subsequently, when the MOOC is finished, these experienced teachers and I read all the productions and reviews, but not with the intention of assessing them, rather to know the contents of them and produce an informative statistic to be shared with the MOOC team (for more information see Taranto et al., 2016).

When the MOOC is finished, its completion level is also reviewed. Thanks to the Moodle platform that keeps all the logs, one is able to know how many readings there are for each activity and how many writings in each message board. In particular, it is possible to trace the profile of each trainee (what he has read, where and how many times he has written, how many badges he has collected, …). So, this will let us know who has done what. It is a general overview useful for issuing final certificates too.

3.3.2.2 Questionnaires and written interviews for trainees

In order to answer the other questions (*second and third research question* or RQ2 and RQ3) it was necessary to prepare some questionnaires and make interviews. Table 3.8 summarized them.

MOOC Geometria	MOOC Numeri
• 3 questionnaires to all the MOOC's trainees • 3 written interviews to a sample of MOOC population	• 3 questionnaires to all the MOOC's trainees • Skype interviewing or written interviews to deepen some case studies

Table 3.8: Questionnaires and interviews in MOOC Geoemtria e MOOC Numeri

There are some differences between the data collection tools that I have used in the two MOOCs. This is why MOOC Geometria was the first experience. So, with MOOC Numeri, I have refined these tools. Let us proceed in order explain all of them in details.

In MOOC Geometria, three ***questionnaires*** were administered to analyse both trainees' satisfaction level and the effects that the *MOOC Geometria* has had on them (in the sense of professional development). One questionnaire was administered during the first week of the MOOC, another at half-course and the third during the final week. All participants were required to answer (fill in the questionnaire was one of the necessary and sufficient conditions in order to receive the badge of the week in which it was inserted). However, physiologically, the number of participants of a MOOC tends to drastically decrease relative to the number of actual subscribers (in our case: 424). So, the number of respondents to each questionnaire has gradually decreased; nevertheless, it always included the entire population of active participants in the MOOC at that time. The intermediate survey had 189 responses and the final one 152, a number that coincides with the participants who actually completed the MOOC in all its steps. As mentioned in §3.3.1.3, I will consider for each of these questionnaire the answers of the 152 trainees that ended the MOOC Geometria in all its stages.
The three questionnaires were produced using Google Modules, an open source application for online surveys, and uploaded on the Moodle course platform. All were divided into different sections and contained open-ended, semi-open, closed, and Likert scale questions (you can see them in Appendix B; they are in their original language, Italian: initial questionnaire pp. 350-355; intermediate one pp. 356-359; final one pp. 360-370). The analysis of responses was performed with Excel.

In MOOC Numeri the administration of questionnaires took place in the same modality and also here the descending number of respondents was occurred (277, 128, 116). Also in this case, as mentioned in §3.3.1.3, I will consider for each of these questionnaire the answers of the 116 trainees that ended the MOOC Numeri in all its stages (you can see them in Appendix B; they are in their original language, Italian: initial questionnaire pp. 371-378; intermediate one pp. 379-389; final one pp. 390-403). The same questions of MOOC Geometria have been addressed, but also more. These new questions also include questions that had been submitted through interviews in MOOC Geometria. These, instead, were not replicated in MOOC Numeri and now let us see why.

Three ***written interviews*** were conducted with a sample of teachers enrolled in the MOOC Geometria. Precisely, we asked the 424 members of the MOOC if they were available to be interviewed in a written form (as we will clarify in a moment). Despite the fact that 243 (57% of 424) trainees said they were available, only 67 (16% of 424) of them actually answered our questions in the first interview.
The choice to consider a sample on voluntary basis[29] was made because the dissertation writer did not want to burden MOOC's weekly commitments, forcing everyone to find time to respond[30]. Unfortunately, however, the number of respondents has drastically decreased from the first interview to the second: from 67 it passed to 30. A possible reason for that is that the teachers were perhaps overwhelmed by school engagements in that period. So, for the third contact the dissertation writer decided to do an "interview" to all the MOOC Geometria trainees when the MOOC was finished, sending an online questionnaire, including questions also addressed in previous interviews.

About the term "written interview" it is important to clarify what we mean. These interviews were not conducted face-to-face, but in a written form. One might therefore think that these are always questionnaires. However, there is a clarification to be made. The questionnaires (initial, intermediate and final) previously illustrated, required a maximum of 20 minutes to be filled. The open questions they contained were very rare and were for the most part such as "motivate the previous answer" (For example, the options of answer were: Yes, No. Then explain why). Instead, the questions contained in what we call written interviews were for the most part all open. The interview required at least 30 minutes of writing. In the first interview it was also clarified that one could respond with a written production that contained a detailed explanation of all the questions addressed. Almost as if the trainee was telling his answers in a face-to-face dialogue[31]. For the second interview we asked to answer each question individually (the reason of that is contained in the footnote 31). Anyway, due to this exposed different nature of the setting in the question, we will continue to call these as written interviews, distinguishing them from the questionnaires.

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[29] You might observe that we could have chosen a sample representative from the questionnaires or from the observations made by the platform. However, this could not be possible. The MOOC proceeded very quickly, it was not possible to extract precise information as it went on. The questions of the written interviews had to be done hot and not too long after the MOOC.

[30] I preferred to receive answers from those who had the pleasure and interest to do so. I would not have been satisfied with superficial answers.

[31] We immediately realized that it was not a good idea because this had lengthened the phases of interpretation and analysis of the data, since the answers given in the form of a theme did not follow the order of exposure of the written questions.

Note that the number of interviewees was high (both the hypothetical – 242 – and the actual – 67) and generally the dissertation writer was the only person who managed data collection and analysis. To get help in managing what should have been a large amount of work, a master student (S. Gaido), who was writing her dissertation on MOOC Geometria, has collaborated on these data collection and analysis with this dissertation writer. The only contact we had with these trainees was online. Therefore, for the first two interviews, we prepared a Word file with questions and sent it by e-mail, asking the interviewees to send their responses back to us in the same way. For the third one, we set it in a Google Module (as we have made with the questionnaires). You can see all of them in the Appendix B (they are in their original language, Italian: first interview p. 404; second one pp. 405-408; final one pp. 409-437).
Despite the fact that the third contact was a questionnaire, we continue to call it written interview, to distinguish it from the other questionnaires sent during the MOOC Geometria. In particular, it is so because it was semi-structured, in the sense that, depending the answer given, different kind of questions could be addressed to respondents (instead the previous questionnaires were structured: they start and end in the same way for all the respondents).

The first interview (November 2015) invited teachers to talk about their school contexts and professional learning experiences in relation to technology.
The second (December 2015) was like the previous, but the teachers had to elaborate further on their reasons for using technology in mathematics lessons, their perception of constraints or opportunities within their teaching environment. They also had to talk about what proposals in the MOOC were judged useful for overcoming the difficulties of the students in Geometry; and if some activities of the MOOC had already been tried out in their classroom.
The third interview (March 2016), consisting of several closed, semi-open and open questions, invited teachers to reflect on possible changes between the period before the MOOC and that after it. We discussed about perception of changes both in their teaching practices and in their beliefs about their students. In particular, as the third interview was addressed at all those who had enrolled in the MOOC Geometria, the first questions were a barrage. In other words, in the hypothesis that trainees who did not finish the MOOC could answer, in the first part – after the general information – we asked if they had finished the whole MOOC. If the answer was *yes*, the trainee could continue to proceed with the interview. If instead the answer was *partially* or *no*, we asked to explain why they had not completed this experience. Here the number of respondents was greater: 120. To avoid such complex situation, in MOOC Numeri it was decided not to select a sample, but to include the questions of the interview directly in the questionnaires, to get answers from all the trainees.

3.3.2.3 Logbook and Skype interviewing or written interviews for MOOC Numeri trainees

An optional module, not present in MOOC Geometria, has been inserted into MOOC Numbers. It is the “Experimention module” (or Modulo Sperimentazione, in Italian). Its introduction was an idea of mine, gladly accepted by the trainers’ team. It allowed me to better investigate possible perception of changes in the trainees’ didactical practices.
This module was opened from the second week of MOOC Numeri, until March 31 (two months after MOOC’s conclusion). In it there was a video (Figure 3.3) in which I explained the purpose of the module and also how to perform the required task. The transcription of the video was also present, so that everyone could easily consult it.

Figure 3. 3: Video with instructions to carry out the Experimentation Module

If the trainees were available to accomplish this optional module they should do the following.
During MOOC, trainees certainly were proposing to their class the activities that they will see in the MOOC's modules: all/only part of them because they like a particular aspect and share their experiences in the communication message boards. Anyway, when they were invited to experiment, I required a different commitment. So, they should experiment in one of their classroom:

– either an activity that they could freely choose among those that were presented in MOOC Numeri modules;
– or their Project Work, that is their final activity of the MOOC Numeri.

During the experimentation, they had to make sure to complete a given logbook in a complete and detailed way. It was made using some other examples of logbook used by the Turin mathematics researchers and uploaded in a digital format using Google Drive (you can see it in Appendix B; the language is Italian, pp. 440-444). Logbook had to be filled in online format; any attachments should be scanned and sent via email at the predisposed address: moocdidattica.dm@unito.it.
In particular, in the logbook, the trainee should describe the classroom context; how much time he devotes to the experimentation; how he had carried out it (if identical to the one proposed in the MOOC or modified and in this case how); some considerations about his students (level of attention, involvement, possible contingencies both positive and negative); if in general he was satisfied with the experimentation.

We have collected 35 logbook, made by a voluntarily sample of MOOC population. Of all these logbook arrived, some (both of lower and higher secondary school) were used to respond in depth to the RQ2. Precisely, through personal emails, we asked for these trainees' availability to be interviewed and 9 of them accepted (5 of these had already been trainees of MOOC Geometria).
The interviews took place between the end of June and the beginning of July 2017. Being teachers from all over Italy, the tool chosen to conduct the interviews was Skype and all video calls were videotaped with the consent of the respondents (you can find examples of addressed questions – in Appendix B, always in Italian, pp. 438-439 – and consent form – in Appendix C, always in Italian, p. 464).
The interviews - on Skype or paper - lasted about 20-30 minutes and were semi-structured. The questions wanted to deepen and to have a full overview of their experience in one or both

MOOCs. Therefore, the interviewees investigated what memory was left of the online training experience; how the interactions with others were experienced; if some changes were perceived in the of teaching some topics of geometry or arithmetic and algebra; if the MOOC had helped to design something different (in reference to the PW); comments on the experience of experimentation conducted in the classroom.
When the Skype meeting could not be carried out due to the incompatibility of the proposed dates, it was decided to send a written interview containing the questions we have prepared imaging to interview them on Skype. The written interview was sent via e-mail, with the request to send it back duly completed and also with a short video in which the teacher told the final design experience, conducted in the MOOC Geometria and/or Numeri.

3.3.2.4 Questionnaires for trainers

As stated before, the trainers' team met regularly and, at the end of each module, they shared what they had observed during that specific module. In particular, the most significant trainees' interventions or sharing actions were discusses, in order to make a fruitful exchange of ideas on the progress of the course and its becoming. During meeting I took notes and, as I was doing (and I do) part of the trainers' team, I also recorded what was said. All monitoring phases (including delivery of participation certificates) have been entered in April 2016 for MOOC Geometria and April 2017 for MOOC Numeri. Subsequently to these respective periods, to answer the last research question (RQ3), I have administered two questionnaires to the senior trainers' team (of both MOOC Geometria and MOOC Numeri), to evaluate the impact of MOOCs on them.
These questionnaires were produced using Google Module and send by e-mail to the members of the team. It contained open-ended, semi-open and closed questions. Of course, I had the occasion to speak face-to-face with them, so we have deepened some answers. In this latter case, I have taken note of their answers.

3.3.3 Mixed Method Research

In this dissertation, I make use of the mixed method research. It is a methodology for conducting research that involves collecting, analysing and integrating quantitative and qualitative research (Frechtling & Sharp, 1997). This approach to research is used because it provides a better understanding of the research problem than either of each alone. In fact, as Frechtling & Sharp (1997) said, by mixing both quantitative and qualitative research and data, the researcher gains in breadth and depth of understanding and corroboration, while offsetting the weaknesses inherent to using each approach by itself. We explain below how the quantitative and qualitative analyses employed in the dissertation are carried out.

First of all, we specify that the analyses will be conducted in parallel on both MOOC Geometria and MOOC Numeri. However, it should not be surprising if sometimes, using the terminology of the theoretical framework, we will refer to the two MOOCs as a single one. In other words, in making general considerations, we will use the term MOOC-artifact and/or MOOC-ecosystem/instrument, or MOOC's ZFM/ZPA meaning considerations that can be extended to both the studied MOOCs (Geometria e Numeri). Explicit distinctions will be made when comparisons are to be made on different aspects of the two MOOCs.

Secondly, we analyse only the answers of the trainees who started and finished the MOOC in all its stages. Among them some distinction are necessary. We distinguish:

- the trainees that completed MOOC Geometria in all its stages will be indicated, from here and after, as "*Geometria trainees*". They are 152;
- the trainees that completed MOOC Numeri, that we can call "*Numeri trainees*", are 116. In particular they are distinguished in "*new entry trainees*" (the trainees that were enrolled only in MOOC Numeri, 51 trainees) and in "*former trainees*" (the trainees that were also enrolled in MOOC Geometria, 65 trainees).

3.3.3.1 Quantitative analysis

Quantitative research methods are used to answer questions on relationships within measurable variables with an intention to explain, predict and control phenomena (Leedy 1993).
The quantitative data that we will show have been gathered through questionnaires and written interviews that were carefully developed and structured to provide numerical data that have been explored statistically and yield a result that can be generalized to some larger population.
The quantitative method we have followed began with data collection based on the hypothesis of answer we expected from the research questions, so in particular, it was based on the MOOC's Zone Theory framework. Then, it is followed with the application of descriptive statistics (Spiegel, 1976; Rossman et al., 2008).

Quantitative research methods fall under the broad heading of descriptive research (Rossman et al., 2008). This type of research corresponds to identifying the characteristics of an observed phenomenon, or exploring correlations between two or more entities. The types of descriptive research we considered in this dissertation are: observation studies and survey research.

Observation studies are involved in both quantitative and qualitative research methods. However, in quantitative methods, the focus of observation studies is on a particular factor of behavior and it is quantified. In this type of design, a researcher will try to maintain objectivity in assessing the behaviour being studied (Rossman et al., 2008). Among the strategies used in observation studies, in our questionnaires we made use of rating scale (e.g. Likert-scale) to evaluate the behaviour of trainees or trainers in terms of specific factor or reasons.
This kind of descriptive research is used to answer all three research questions. The questionnaires and the third written interview for the MOOC Geometria trainees were plenty of Likert-scale questions, as well as of closed, semi-open and open questions.

Kerlinger (1973) defined **survey research** as a study on large and small populations by selecting samples chosen from the desired population and to discover relative incidence, distribution and interrelations. The ultimate goal of survey research is to learn about a large population by surveying a sample of the population; thus we may also call it descriptive survey. In this method, a researcher poses a series of questions to the respondents, summarises their responses in percentages, frequency distribution and some other statistical approaches.

This kind of descriptive research is used also to answer all three research questions. In particular, as we have specified at the beginning of this paragraph, we do not analyse the entire population of MOOC members, but only a sample of it, namely those who have concluded the MOOC experience (both MOOC Geometria and MOOC Numeri) in all its phases. We employ skype interviews and the common approach using questionnaires.

Moreover, in survey research there are two types of survey: cross-sectional survey and longitudinal survey. Normally, the type of survey method used depends on the scope of the research work.
In *cross-sectional survey*, a researcher collects information from a sample drawn from a population. It involves collecting data at one point of time. The period of data collection can vary and it depends on the study weightage (Kerlinger, 1973). When we analyse the experiences of the two MOOCs, considered them individually, we can talk about a cross-sectional survey.
In *longitudinal surveys*, data collection is done at different points of time to observe the changes (Kerlinger, 1973). The types of longitudinal surveys employed in this dissertation was the Panel Studies. In Panel Studies, a researcher can identify a sample from the beginning and follow the respondents over a specified period of time to observe changes in specific respondents and highlight the reasons why these respondents have changed. An analysis in this sense will be done in chapters 4 and 5. We can not speak of a very detailed analysis, because, as announced, some questions have been changed from one MOOC experience to another. Where however there is a similarity in the data, comparison analysis on the evolution of the students who attended both MOOCs is proposed.

All quantitative analyses were performed using the Microsoft Excel software.

3.3.3.2 Qualitative analysis

The aim of qualitative research may vary with the disciplinary background, such as a psychologist seeking to gather an in-depth understanding of human behavior and the reasons that govern such behavior. Qualitative methods examine the *why* and *how* of decision making, not just what, where, when, or "who", and have a strong basis in the field of sociology to understand government and social programs. Qualitative research is popular among political science, social work, and special education and education researchers (Bogdan & Biklen, 1982).

Various analysis tools have been used to carry out the analysis from the qualitative point of view. Precisely: the common approach using questionnaires that contains closed, semi-open and open question. With regard to the latter, the analysis was also carried out by identifying categories based on the answers given by the interviewees. An identification of categories was also made to analyse the posts written by the trainees on the communication boards, as mentioned in § 3.3.2.1.
Then we also consider the answers obtained from the trainees' written interviews and the trainers' questionnaires. To respond, in particular to RQ2, we have also considered the logbooks (delivered by the Numeri trainees), the interviews on skype and some case studies (on this point we return to the next paragraph). Therefore, in addition to the posts in the communication message boards and to the questionnaire responses we had transcripts of teacher interviews and a qualitative analysis of these was conducted. We wished to

understand why these teachers were interested in attending an online course for teacher education and development, as well as in identifying their level of prior knowledge (indicative of their ZPD). We looked also for responses that revealed a state of *tension* that reflects the definition of Goos (2013). Referring to the theoretical framework, we then identified the phrases that explained the daily school context of the teacher (school's ZFM / ZPA) and those which instead explained how the online environment was lived by trainees (MOOC's ZFM / ZPA), using ICT tools, interacting with colleagues, and creating new didactical ideas basing on the activities proposed by the MOOC and from the comments made by other trainees (ZPD).

Furthermore, in the analysis of RQ1 and RQ3, it was useful to produce graphs showing the expansion of the network of knowledge. We do not fall into the study of graphs specifically, but some illustrations that follow are made with a specific software, yEd Graph Editor[32], used in the study of graphs and turn out to be really enlightened for our purposes.

Using case study methodology

A case study of a single teacher, who could be regarded as a "specific, unique, bounded system" (Stake, 2003, p. 136), would enable some understanding of this particular teacher's professional development in the context of MOOC for mathematics teacher education. However, such an approach would provide almost no opportunity to evaluate the affordances offer in general by the MOOC, because it pays no attention to potential differences that may exist due to the unique experiences of individual teachers. On the other hand, a collective case study (Stake, 2003) that includes several cases of teachers with differing disciplinary backgrounds and professional contexts has potential to provide a greater understanding of what professional development can be reach in a MOOC. The present study was limited to a small number of cases because data collection for case studies is time consuming, especially when there are multiple cases (Yin, 1994).

The selection of a case to be studied is important and involves purposive sampling so that the case has characteristics that reflect the purpose of the study (Merriam, 1998). For a collective case study, this may mean selecting cases because they are similar or cases that are expected to produce predictable but contrasting findings (Yin, 1994). While agreeing with Merriam (1998) and Yin (1994) about the need to select cases carefully, Stake (2003) argued that the most important reason for choosing a particular case is that it provides an opportunity to learn about the phenomenon of interest. The cases (teacher-trainees) were chosen for the present study because they provided an opportunity to learn about how teachers experiment professional development in a MOOC environment.

Three distinct case studies will be presented. The first will be the case of Lucy, a Geometria trainee. Then there will be the case study of Stephen, a Geometria and Numeri trainee. Finally, a negative case study, related to Ester, a Geometria trainee.

Ethical consideration

Gaining informed consent form participants, maintaining the confidentiality of data, and considering the consequences of publishing research findings were ethical issued identified in the empirical phase of the present study (Kvale & Brinkman, 2009).

[32] yEd Graph Editor is an open source software that can be used to quickly and effectively generate high-quality diagrams (see: http://www.yworks.com/products/yed).

Each questionnaire reported a sentence on the processing of personal data and respect for privacy. In the mails sent to ask for availability to be interviewed in paper format or on skype, teacher were supplied with details of the study and asked to provide acknowledgement of their understanding of what the study entailed and agreement to participate by signing a consent form. See Appendix C for samples of the information mails and consent forms provided to the teacher-trainees.
To safeguard the anonymity of participants, in this dissertation and other publications, pseudonyms were used for all participants.

3.3.3.3 Summarizing the analysis research method

The data collection and its analysis is done in the following way, as shown in Table 3.9:

Data	Emerging from trainees in spontaneous and/or non spontaneous form	Relating to only some trainees to deepen their learning trajectories	Relating to the trainers' team
Data sources	Posts in the communication message boards Questionnaires	Questionnaires Interviews Logbook	Notes and recording during meetings Questionnaires Interviews
Data analysis	Quantitative and qualitative analysis	Quantitative analysis and qualitative analysis by case studies	Quantitative and qualitative analysis

Table 3.9: Data of the research study

Chapter 4 Analysis of the potentialities within a MOOC

The findings presented in this chapter contribute to addressing the first research question: *Are there any particular potentialities in a MOOC-artifact that, if properly organized, trigger the double learning process and therefore the transition to the MOOC-ecosystem/instrument?*

The analysis we are going to show is all in the MOOC's ZFM / ZPA. We reproduce below the Table 4.1, shown in Chapter 2, which briefly describes the aspects with which the community of trainers and the community of the trainees compare themselves.

Taranto's re-elaboration/interpretation		
Valsiner/Goos's zones	MOOC's ZFM/ZPA	
	Trainers	Trainees
ZFM: Zone of Free Movement (structures trainers/trainees' access to different modules of the MOOC, availability of different resources within an accessible module of the MOOC, ways the trainers/trainees are permitted or enabled to act with accessible resources in accessible modules of the MOOC)	• Design of the platform that hosts the MOOC and perception of this new environment • Design and digital transposition of mathematical resources • Communicational resources to foster communication among trainees and between trainers themselves with trainees • Technical support • Curriculum and assessment requirements • Organisational structures and cultures	• Access to the platform that hosts the MOOC and perception of this new environment • Access to mathematical resources • Communicational resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures
ZPA: Zone of Promoted Action (virtual people, resources, or modules in the MOOC in respect of which the trainers/trainees' actions are promoted)	• Interaction with trainees reading (and sometimes answering) their post. • Professional development	• Informal interactions with enrolled trainees • Interaction with trainers • Professional development

Table 4.1: MOOC's ZFM/ZPA (trainees and trainers)

In the research question does not appear the terminology of the zones, i.e. MOOC's ZFM / ZPA, but in the theoretical framework, we have underlined that this theoretical construct justifies the transition from MOOC-artifact to MOOC-ecosystem/instrument (see §2.10.5.4).

We specify that the analyses will be conducted in parallel on both MOOC Geometria and MOOC Numeri. However, it should not be surprising if sometimes, using the terminology of the theoretical framework, we will refer to the two MOOCs as a single one. In other words, in making general considerations, we will use the term MOOC-artifact and/or MOOC-ecosystem/instrument, or MOOC's ZFM/ZPA meaning considerations that can be extended to both the studied MOOCs (Geometria e Numeri). Explicit distinctions will be made when comparisons are to be made on different aspects of the two MOOCs.

We have been able to show, generally speaking, the MOOC-artifact (or the MOOC's ZFM trainers – Table 4.1) when in Chapter 3 we have exposed the mathematical contents and the technological tools contained in MOOC Geometria and MOOC Numeri; the two videos describing the MOOCs also aimed for this purpose (§3.2.3.5). So, in Chapter 3, from 3.2.2 to §3.2.3.5, there is the analysis of the MOOC's ZFM following the voice listed in Table 4.1.
Here, we will go into the details of some modules, exploring their contents better, as designed by trainers. The goal is to highlight the potential that a MOOC-artifact has for its trainees. Therefore, we will consider the entry of the trainees in these modules and we will see how the transition from MOOC's ZFM to MOOC's ZPA takes place. In other words, we will see how the trainees begin to live these new spaces, implementing the double learning process. It will allow the MOOC-artifact to evolve to MOOC-ecosystem, then perceived by each of the trainees as MOOC-instrument.

4.1 The importance to consider the MOOC context

The first research question investigates the online environment designed to host teacher education, the MOOC-artifact. In a face-to-face teacher education it would mean focusing on the training course classroom[33].
It must be said that if one is analysing face-to-face courses for teacher education, one does not take into particular consideration how teachers are interacting with the surrounding environment. No particular attention is paid to the size of the room, its acoustics, the position of the desks, or to the fact that the students can converse with each other, in a low voice, while the trainer is explaining. It may be that those whispers are exchanging illuminating information, which we will never know about. In a MOOC, instead, every smallest detail has an impact, albeit minimal, on the involvement of those who are living the training module. It also makes the difference whether a web page opens in a new window or directly on the same page. In the first case, you can easily switch from the new topic to the previous one; you can quickly go back if you need to do it. In the second case, the page – depending on the content it contains and the connection you have – needs some time to load and so to go back and sometimes, this wait is not always pleasant!
Interactions have a different weight: we do not see each other, we are not face-to-face. If I agree with what I read/see and nod, nobody sees me. It is not necessarily that I want to show myself, but if I participate, a minimum, I will also want others notice me!
There is no trainer who, dressed in her dress, is standing in front of you, holding her lesson. In place of her there is a video, that she shot some time ago, aware (and hopeful) that many trainees would have seen it long after it.
There is no longer the classmate to ask "What did she say? I was distracted!".
It is therefore important to know if there have been interactions between trainers and trainees and even among the trainees; if they have been effective; how they actually happened.

[33] Remember that in §3.2.3.6 we had already mentioned some difference between face-to-face and online teacher education

Voice interactions are only those through video, but the only voice that speaks is that of the trainer.
All the interactions made by trainees happen between the typing of a key and another, in suitable communication boards. It is not a question of thoughts aloud in freedom, but of written text, in which there is more time to reflect. In which it is necessary to press the return button to ensure that the thought of one is spread to many. It almost seems that these are the constraints of communication, yet the careful eye sees a treasure trove of information. You do not have to dig too deep to realize that interactions are the precious gems of a MOOC for teacher education. More valuable than those that could ever occur in a face-to-face course. And we will see it soon.

4.2 A quantitative overview of the MOOC-artifact

Let us start with an objective overview of the MOOC-artifact and what the trainees think of it. The data that will be exposed refer to the questionnaires (initial, intermediate and final) administered to the trainees of MOOC Geometria and Numeri. As anticipated in the methodology section (Chapter 3), we analysed only the answers of the trainees who started and finished the MOOC in all its stages.

4.2.1 MOOC Geometria

MOOC Geometria was held from October 2015 to January 2016.
The trainees that complete MOOC Geometria in all its stages are 152 (women: 83%; men: 17%).
All of them are Italian in-service mathematics teachers. In Figure 4.1 you can see where they come from and in Figure 4.2 in which educational level they teach. Just over the half of them (51%) come from Piedmont, the region in which there is Turin – the city where there is our Department that deliver the MOOC. Sicily (10%), Puglia (9%), Lombardia (8%) follow. Seven of twenty regions have no representative (at the beginning of MOOC Geometria, instead, there was at least one representative for each region). Even with the educational level the situation is a bit different from the beginning (see Table 3.4 in Chapter 3). There are more trainees that teach in the lower secondary school (51%) than in the higher one (45%). As anticipated in Chapter 3, it remained a small percentage representing primary school teachers (4%) who actually completed the MOOC.
The average age of the trainees is 44 (min 23 < 44 < max 63). It is lower if compared to the national average age (49 years)[34]. Moreover, from this data, we can already grasp the heterogeneity of the participants. In fact, if the minimum age is 23 and the maximum is 63, it means that among them there are beginners teachers and also experienced ones. However, we cannot have further confirmation of this, because in the question of the initial questionnaire: "*How long have you teach?*", 27% of the trainees did not respond (Figure 4.3). Anyway, among the respondent the situation seems quite varied.

[34] Italian school teachers are the oldest in the OECD countries: even 49 years of average, compared to about 43 of other nations; with 50% over 50, and 11% over the 60 year threshold (retrieved February 6, 2018 from: https://goo.gl/FFyR77).

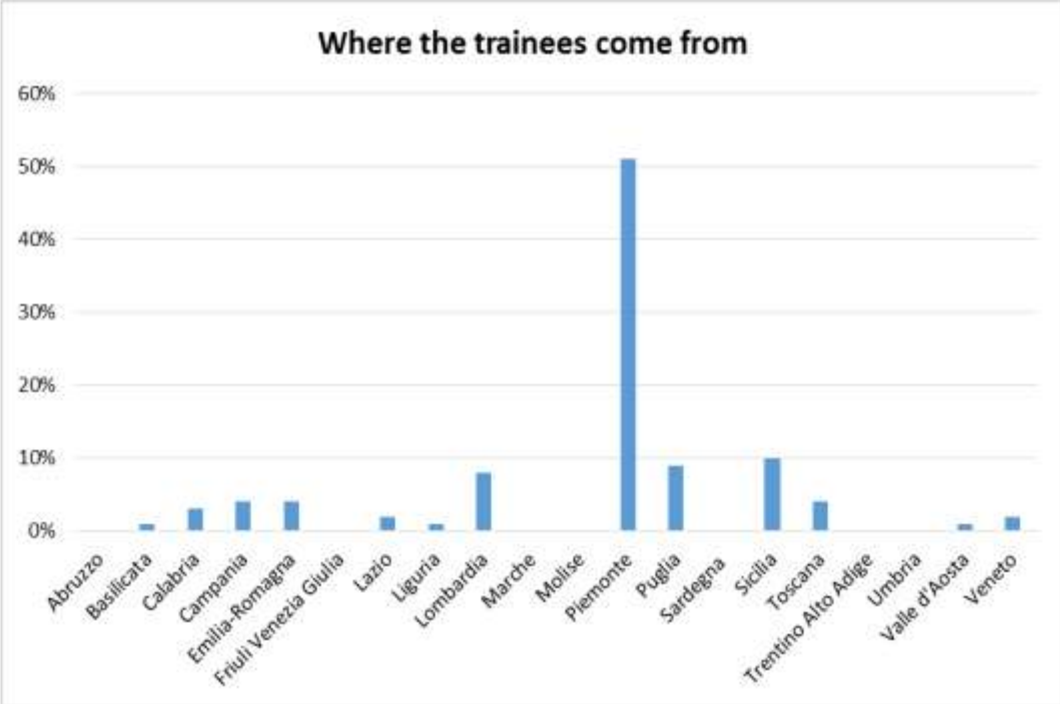


Figure 4.1: Where the trainees of MOOC Geometria come from

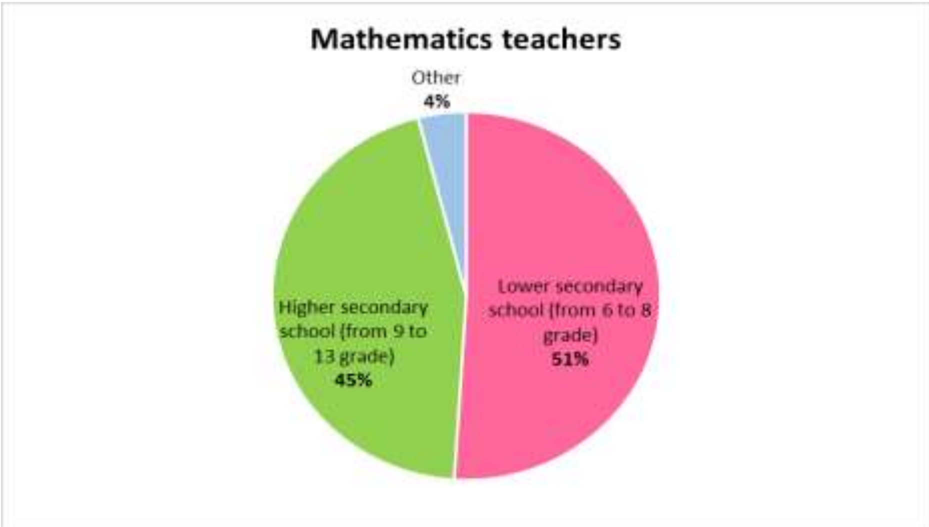


Figure 4.2: Educational level where the trainees of MOOC Geometria teach

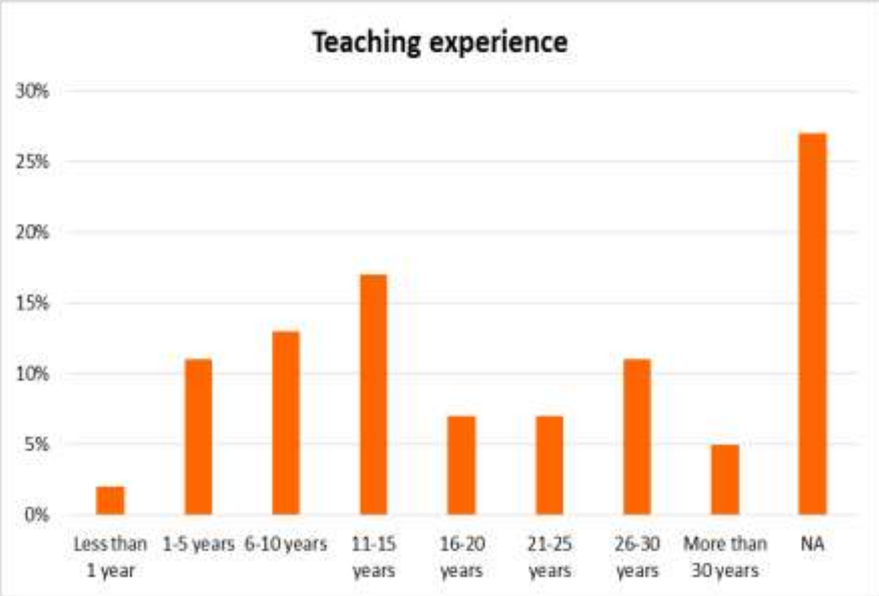


Figure 4.3: MOOC Geometria trainees' teaching experience

We have investigated their experience with MOOCs. In fact, in the initial questionnaire, we asked if, before MOOC Geometria, they had attended other MOOCs. As you can see from Figure 4.4, almost all of them (92%) had never had a similar experience before. It is also fair to say that MOOC Geometria was the first MOOC in Italian addressing mathematics teachers. In the intermediate questionnaire, with the question "*Compared with other MOOCs you attended, or if however this is the first that you follow, do you feel that it is easy to use?*", we wanted to check if the MOOC was easy to use for them, as the trainers had structured it (Figure 4.5). 84% of them answered "*Definitely yes*". This means that for the majority of them there were no constraints in implementing new usage schemes (attention, not Math Edu USs, but precisely those referring to the use of the platform intended as an artifact in the sense of Verillon & Rabardel, 1995). The 11% instead is part of the "*More no than yes*" and "*Definitely no*" respondent. Despite the fact that they seem to perceive the *chaos* as information overload (in the connective sense of Siemens, 2005) of the MOOC, due to entry into a technological environment never explored before, and need more time than others to *self-organize* (Siemens, 2005) themselves on the platform, they then finished the course.

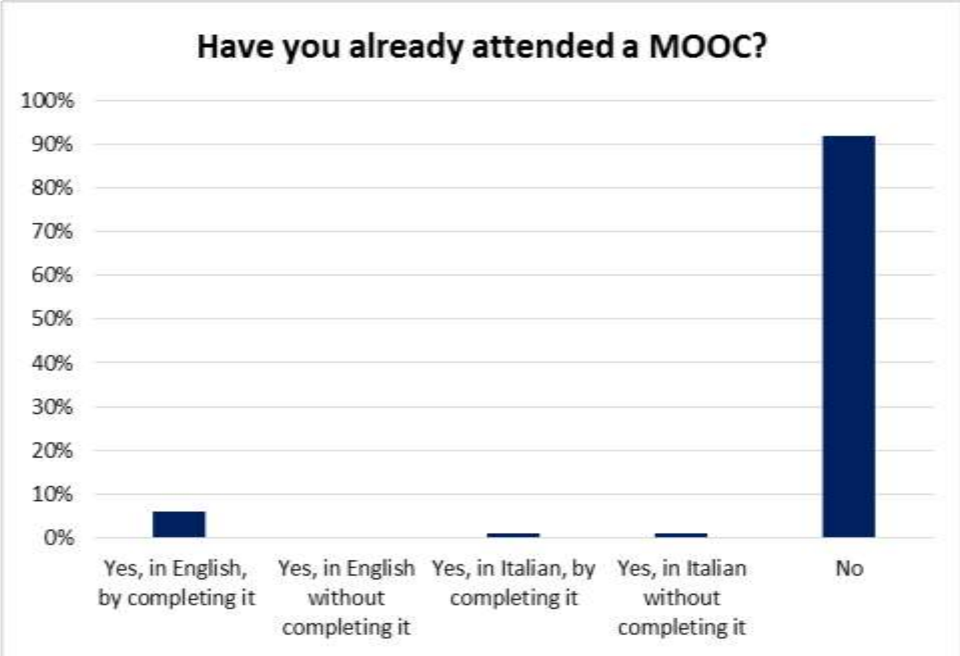


Figure 4.4: MOOC experiences before MOOC Geometria

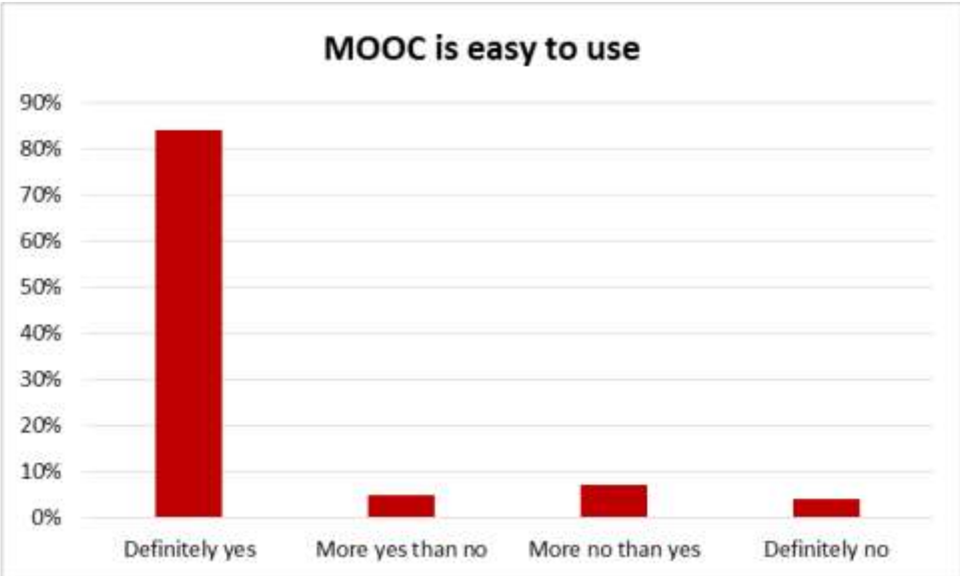


Figure 4.5: MOOC Geometria trainees' opinion on ease of using the MOOC

Then, always in the intermediate questionnaire, to understand how the trainees *self-organize* themselves, we asked with a Likert-scale question if the MOOC was flexible in terms of (Figure 4.6; a score of 1 represents totally disagreement and a score of 5totally agreement):

- *Space*, with the following meaning: the lessons are accessible anywhere via laptops and mobile devices, and not necessarily in a given classroom;
- *Time*, with the following meaning: following the course when you want to, planning the most of your free time;
- *Pace of learning*, with the following meaning: defining your own learning pace.

The opinion of the trainees is for the most part positive. If we consider the answers of those who have indicated a score of 4 and 5, we have that the trainees believe that the MOOC is an environment that can be accessed when they prefer (Time: 84%); wherever they are (Space: 83%); and which allows them to independently manage their own pace of learning (80%).

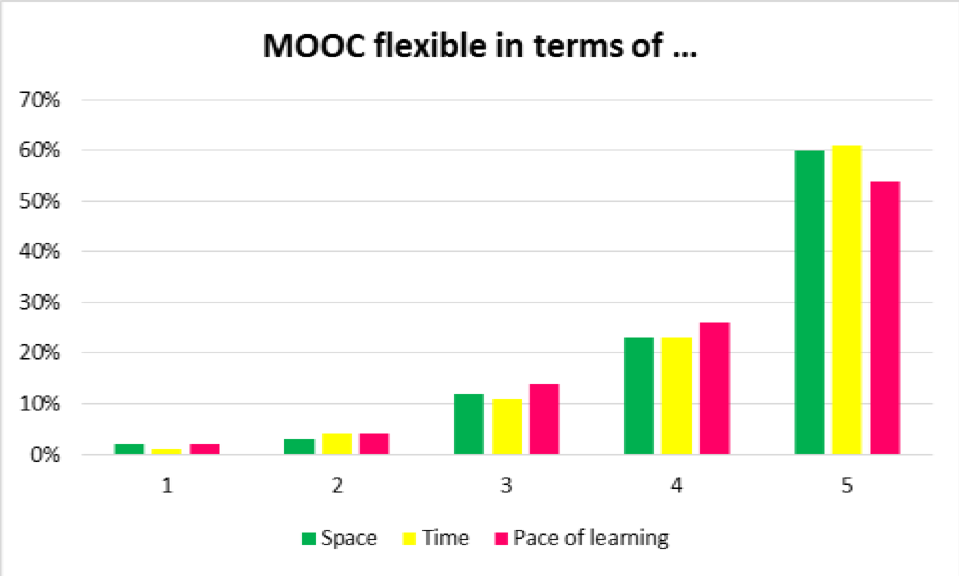


Figure 4.6: Flexibility of MOOC Geometria in terms of space, time and pace of learning

4.2.2 MOOC Numeri

MOOC Numeri was held from November 2016 to January 2017.
The trainees that complete MOOC Numeri in all its stages are 116 (women: 87%; men: 13%). Of these, 65 (56%) were trainees in MOOC Geometria.
Also all of these trainees are Italian in-service mathematics teachers. In Figure 4.7 you can see where they come from and in Figure 4.8 in which educational system they teach. The percentage of geographic distribution was maintained compared to the MOOC Geometria, in fact the majority of them (45%) come from Piedmont; Sicily (19%), Puglia (8%), Lombardia (6%) follow. Even this time, seven on twenty region have no representative.

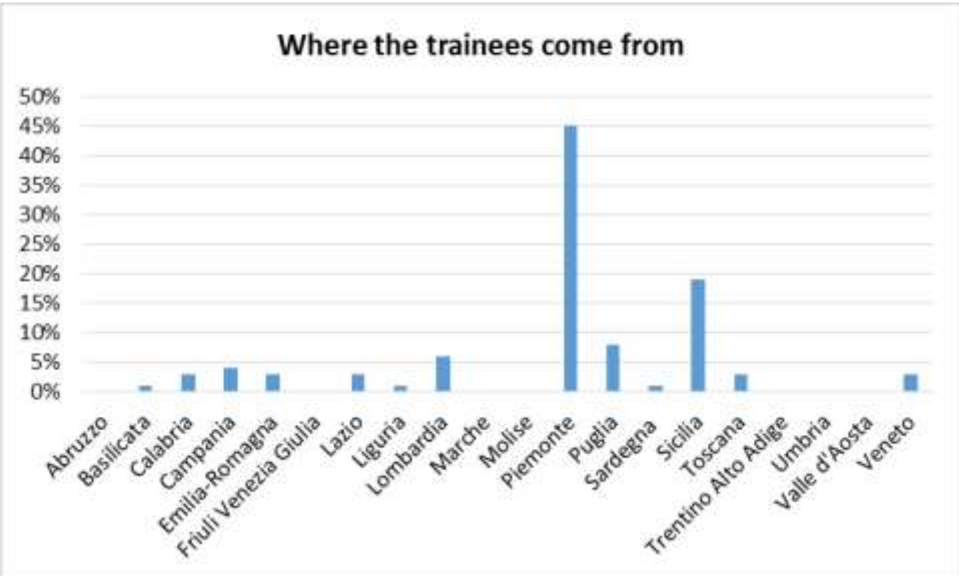


Figure 4.7: Where the trainees of MOOC Numeri come from

With the educational system the situation is different from the beginning (see Table 3.4 in Chapter 3): the number of lower secondary school teachers decrease from 55% to 44%, instead increase the one of higher secondary school from 40% to 55%. In particular, this is the opposite situation of MOOC Geometria. Also in MOOC Numeri remained a very small percentage representing primary school teachers (1%) who actually completed the MOOC.
The average age of the trainees is 46 (min 27 < 46 < max 64). Also in this case, it is lower if compared to the national average age (49 years) and two point more than MOOC Geometria average age. From this data, we can already grasp the heterogeneity of the participants. In fact, if the minimum age is 27 and the maximum is 64, it means that among them there are beginner teachers and also experienced ones. A further confirmation of this come from a question of the initial questionnaire: "*How long have you teach?*". In Figure 4.9 is reported the variety of teaching experience of the MOOC Numeri trainees.

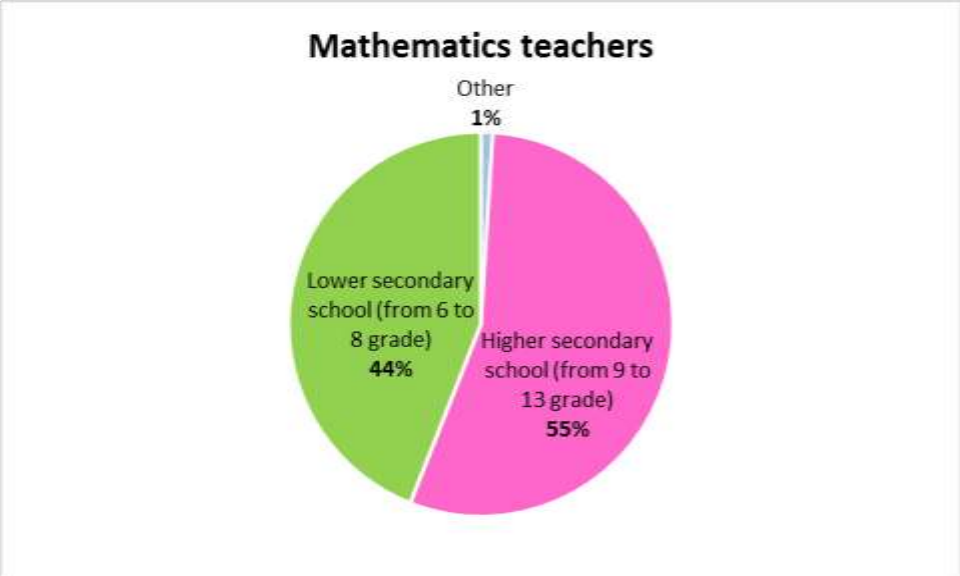


Figure 4.8: Educational level where the trainees of MOOC Numeri teach

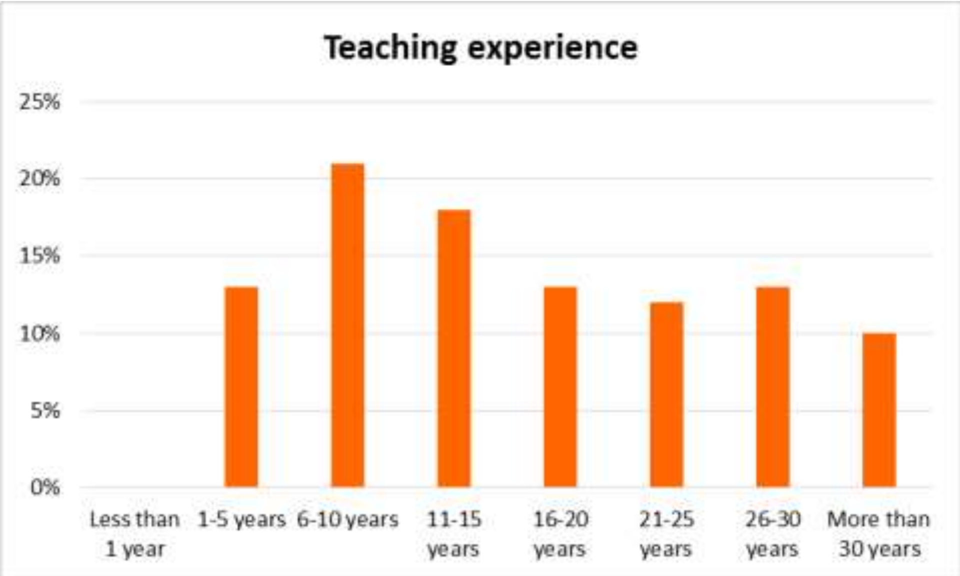


Figure 4.9: MOOC Numeri trainees' teaching experience

In this case, we have investigated again the experience of trainees with MOOCs. As we can image, this time the situation is different (Figure 4.10). Considering that 56% of respondents were trainees of MOOC Geometria, it is not surprising that now the percentage of those who have never attended a MOOC has decreased (28%), while the percentage of trainee who have attended an Italian MOOC by completing it is clearly increase from 1% (Figure 4.4) to 59%[35] (Figure 4.10).

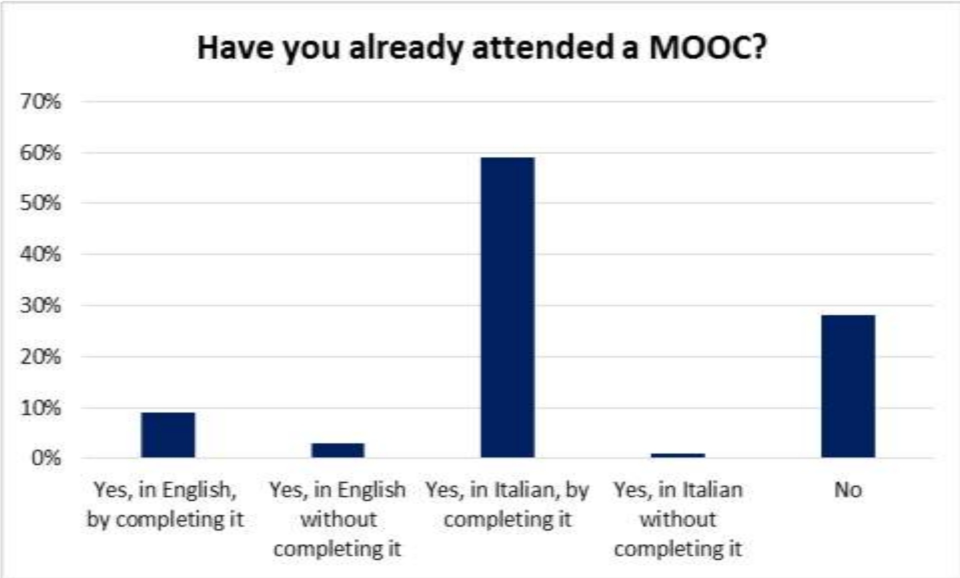


Figure 4.10: MOOC experiences before MOOC Numeri

In the intermediate questionnaire, with the question "*Compared with other MOOCs you attended, or if however this is the first that you follow, do you feel that it is easy to use?*" to check if the MOOC was easy to use, we observed that all the 65 former trainees, of course, declare "*Definitely yes*". It is obvious: they know the platform, how the trainers organized the resource into it. In fact, only to them we asked in the final questionnaire, with a semi-open question (response options: Yes, No; then why): "*Do you think that the familiarity with the MOOC environment acquired with MOOC Geometria has facilitated you in managing your pace of learning in MOOC Numeri?*". The totality of them answer "*Yes*". We report an answer of a trainee that should justify his previous response. It summarizes all the justifications received for this question.

> "Knowing already the structure of the MOOC, being very similar to that of Geometry, it was very easy for me to start and follow all the proposed activities with a good pace of learning right from the start".

It is evident as the feeling of disorientation or chaos experimented the first time was absent this time. Therefore, we concentrate ourselves on the 51 trainees that are new entry in the MOOC environment (Figure 4.11).
The 94% of the respondent is part of the answer "*Definitely yes*" and "*More yes than no*". This time nobody complain answering "*Definitely no*". It is surely due to the fact that in MOOC Numeri, thanks to the previous experience and taking into account the received feedback of MOOC Geometria trainees, the trainers organized in a more friendly way the

[35] They are 76 people and 65 of them are the former trainees of MOOC Geometria.

platform. As you probably notice in the MOOCs video description of §3.2.3.5, while MOOC Geometria extended itself on a vertical structure, MOOC Numeri was organized with a grid structure. An icon represented all the modules. You can open them in separate window just clicking on them. In addition, a blue table was inserted in each module (an example is Table 4.2). It summarized the contents and deliveries provided in the module. These edits avoid the *chaos* feeling that many trainees have experimented in MOOC Geometria.

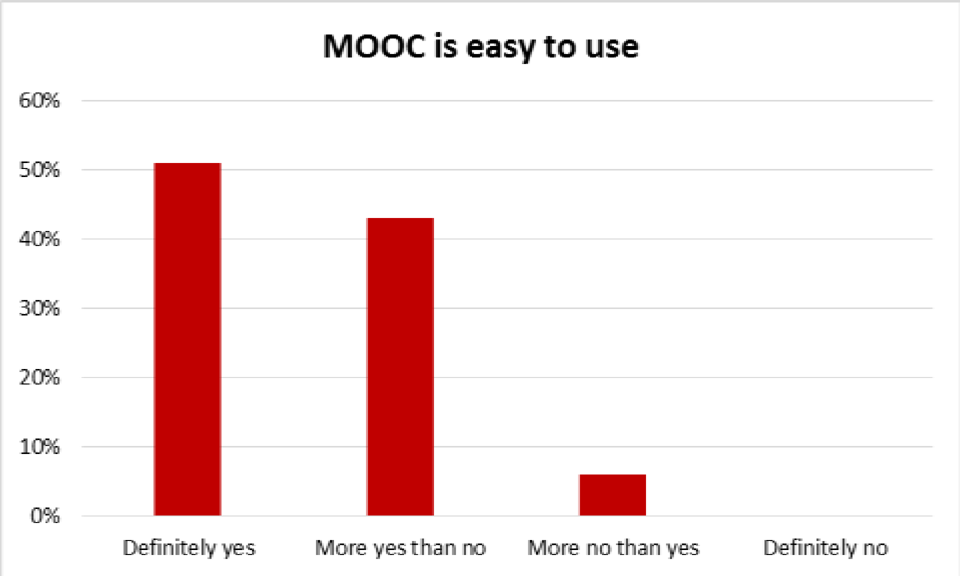


Figure 4.11: MOOC Numeri trainees' opinion on ease of using the MOOC

In the intermediate questionnaire, to understand how the trainees *self-organize* themselves, we asked with a Likert-scale question if the MOOC was flexible in terms of *Space*, *Time* and *Pace of learning* – with the same meaning clarified above (Figure 4.12; a score of 1 represents totally disagreement and a score of 5 totally agreement). We analyze separately the answer given by the "new entry" trainees (the ones that were enrolled only in MOOC Numeri) and by the "former" trainees (the ones that were enrolled both in MOOC Geometria and MOOC Numeri).

The opinion of the trainees is for the most part positive. If we consider the answers of those who have indicated a score of 4 and 5, we have that the "new entry" believe that the MOOC is an environment that can be accessed when they prefer (Time: 90%); wherever they are (Space: 88%); and which allows them to independently manage their own pace of learning (66%). Instead, the "former" declare the following percentages for Time: 76%; Space: 91%; Pace of learning: 63%. It surprise a bit that the former have lower percentage in time and pace of learning than the new entry. The following is a possible reason of that. The former trainees know the structure of the MOOC, in terms of resources disposition, badge obtaining and deadlines. This has led them to slow down their entry into the MOOC, namely they accessed less frequently than they did in MOOC Geometria and perhaps they underestimate the effort required by the new activities, considering the MOOC not able to support their pace of learning. It is the contradiction of the MOOCs: the freedom to organize yourself as you wish, becomes your greatest bond.

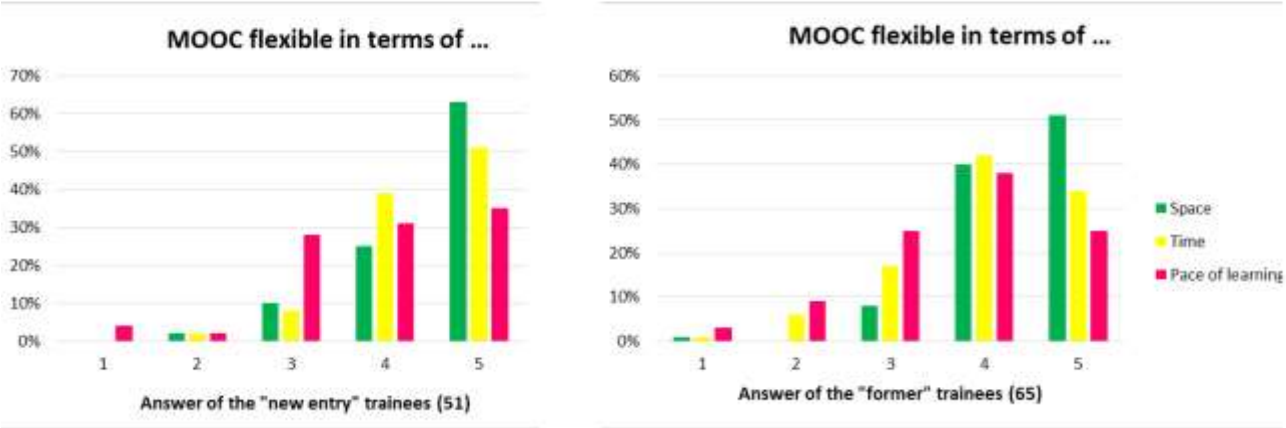


Figure 4.12: Flexibility of MOOC Numeri in terms of space, time and pace of learning

4.2.3 MOOC Geometria and MOOC Numeri

We have seen what the trainees have thought about the MOOC-artifact in general: *access into the platform* (MOOC's ZFM of trainees, Table 4.1), *self-organization*. Let us see now their general opinion about the mathematical contents and the technological resources inserted in the MOOCs.

At the beginning of each module there were resources designed to guide the trainees in the access to its mathematical contents or communicational resources.

PowToon and blue table

In each MOOC module there is a video made with PowToon (see Table 3.3 in Chapter 3), a presentation tool that allows making animated cartoon videos. These MOOC videos contain guidelines for addressing the deliveries of the week. At the following link there is an example related to the fifth MOOC Numeri module (the video is in Italian, anyway it is friendly-looking and easy to understand in its general message):
https://www.youtube.com/watch?v=iH6G6n7hozU&feature=youtu.be
In MOOC Numeri, the PowToon video message was strengthened in the written version with the presence of the blue table (Table 4.2).

Italian			English	
UNITS	**MODALITÀ di FRUIZIONE**		**UNITS**	**HOW MAKE USE OF THEM**
1) Presentazione del modulo Riflessione sulla dimostrazione in campo numeico	Video Video		1) Presentation of the module Reflection on the proof in the numerical field	Video Video
2) Attività m@t.abel: "L'aritmetica aiuta l'algebra, l'algebra aiuta l'aritmetica"	Consultare l'attività Partecipare al forum		2) m@t.abel activity: "Arithmetic helps algebra, algebra helps arithmetic"	Consult the activity Participate in the forum
3) Attività	1 attività per secondaria di I grado 1 attività per secondaria di II grado		3) Activities	1 activities for lower secondary school 1 activities for higher secondary school
Proposta per utilizzo delle tecnologie	2 attività		Proposal for use of technologies	2 activities
Attività di approfondimento sul Metodo della Ricerca Variata (MRV)	1 attività		In-depth study on the Variable Research Method (VRM)	1 activity
4) Sperimentazione	se possibile		4) Experimentation	if possible
5) Compito	Condividi degli esempi, vota e commenta su tricider		5) Task	Share examples, vote and comment on tricider

Table 4.2: Blue table inserted in first MOOC Numeri module

Video of an expert trainer
Then there is a video in which an expert talks (usually a university professor): she or he briefly (from 3 to 5 minutes) illustrates the conceptual node that are addressed in the module.

We asked trainees to express themselves about the effectiveness/utility of PowToon (Figure 4.13), blue table (in MOOC Numeri; Figure 4.14) and video of the expert (Figure 4.15) with a Likert-scale question (score of 1 represents totally ineffective/useless and a score of 5 totally effective/useful). Note that we analyze three distinct sample: the trainees that completed MOOC Geometria in all its stages, indicated from here and after in the diagrams as "*Geometria*"; the trainees that completed MOOC Numeri in all its stages, distinguished as follow. From here and after, in the diagrams, we refer to the trainees that were also enrolled in MOOC Geometria as "*former trainees*" and the trainees that were enrolled only in MOOC Numeri as "*new entry trainees*".

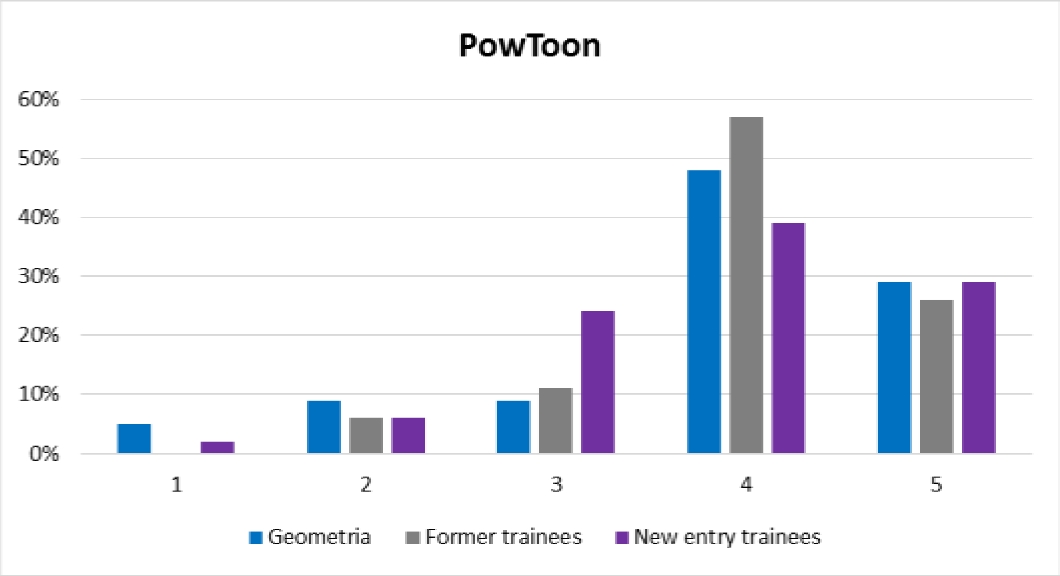


Figure 4.13: Effectiveness/utility of PowToon according to the trainees

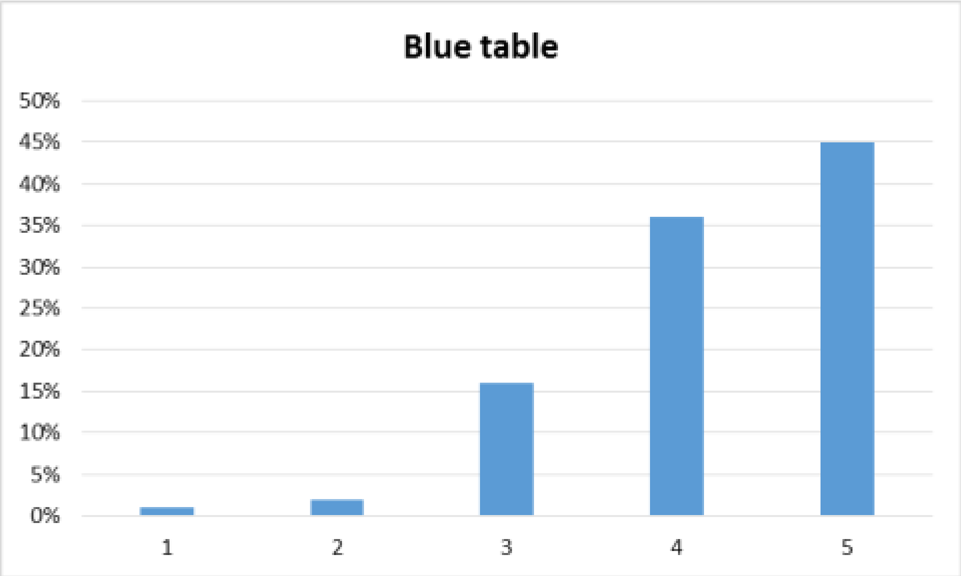


Figure 4.14: Effectiveness/utility of blue table according to the trainees of MOOC Numeri

As it can be seen from the histograms, the trainees appreciated these resources made available. If we consider the answers of those who have indicated a score of 4 and 5 (Figure 4.13), we have that for the trainees enrolled in both MOOC Geometria and Numeri the percentage of satisfaction increase (77% for *Geometria* and 83% for *former trainees*); about the *new entry* the 68% have judged effective/useful PowToon videos. Similarly for videos of the experts (Figure 4.15): for the trainees enrolled in both MOOC Geometria and Numeri the percentage of satisfaction is equal to 95% for *Geometria* and 94% for *former trainees*; 78% for the *new entry*. The videos were much appreciated.
As for the blue table, we have not distinguished between the three samples, but considered only all the trainees who finished MOOC Numeri. In fact, in MOOC Geometria there was not the blu table, so it is a novelty for all the trainees. Considering the answers of those who have indicated a score of 4 and 5 (Figure 4.14), it was appreciated by the 81%.

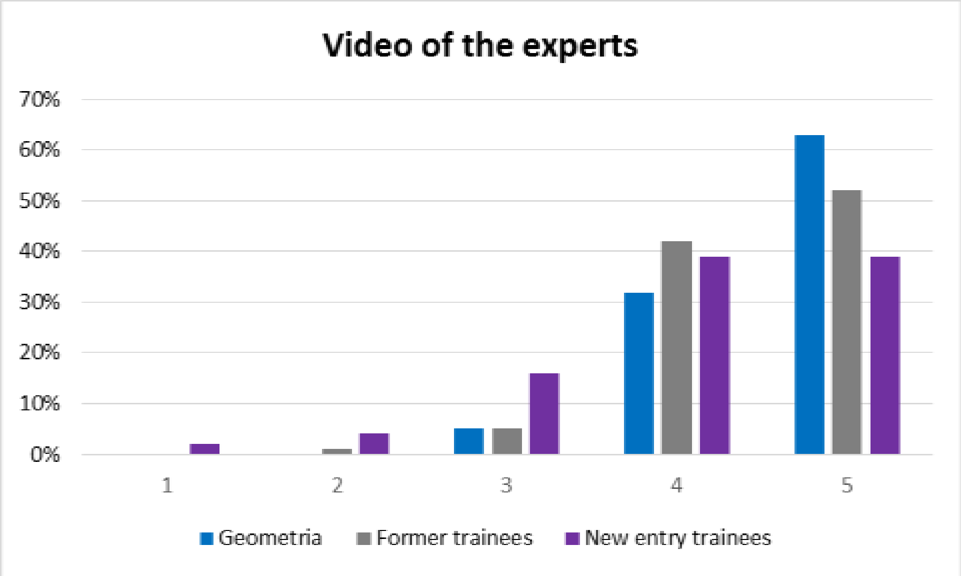


Figure 4.15: Effectiveness/utility of the video of the experts according to the trainees

4.3 MOOC as repository

Focusing only on the level of design, the MOOC-artifact, i.e. the place where only trainers (university professors and teacher-researchers) have access, is the container of specific products, namely materials rich in innovative teaching methods and specific tools technology (as seen in the descriptions of MOOC Geometria and Numeri in §3.2.3.2 and §3.2.3.3). We can therefore understand it as a repository from which teachers can draw inspiration.
In the following, we will show two modules, one of MOOC Geometria (Module 1: *Ramps, sails, park and folding paper*) and one of MOOC Numeri (Module 5: *Arithmetic, Algebra and Mathematical Languages*). The wealth of activities and methodologies offered to trainees will be touched by hand.
Note that we are going to move into the MOOC's ZFM for the trainers, considering the design and digital transposition of mathematical resources.

The choice of showing these two modules is not linked to particular reasons, but two clarifications must be made. For now, we have excluded the modules dedicated to the final activities, on which we will return later. We have instead avoided to dwell on modules 4, 5 of MOOC Geometria and 2, 3 of MOOC Numeri (see Tables 3.1 and 3.2) because they refer respectively to the MERLO methodology and to the INVALSI tests, typical Italian training assessment methodology, on which we do not want to go into details in this dissertation.

As explain in §3.2.3.2, the MOOCs activities are inspired by reality and are centered on laboratory-based methodology (Anichini et al., 2004). They offer incentives to the trainees to enrich their own teaching in the laboratory sense, also using the technologies, and they are a support to develop the skills of the students as required by the Italian curriculum (National Indications).
As you have noticed, all the titles of the MOOC modules are captivating. They take inspiration from the objects of reality, on which the MOOC activities are based.

4.4 Module 1 in MOOC Geometria: Ramps, sails, park and folding paper

The module presents itself with a nice and clear layout (Figure 4.16, 4.17, 4.24). The following is a narration of the contents, referring to what is highlighted by the coloured squares of the following figures.

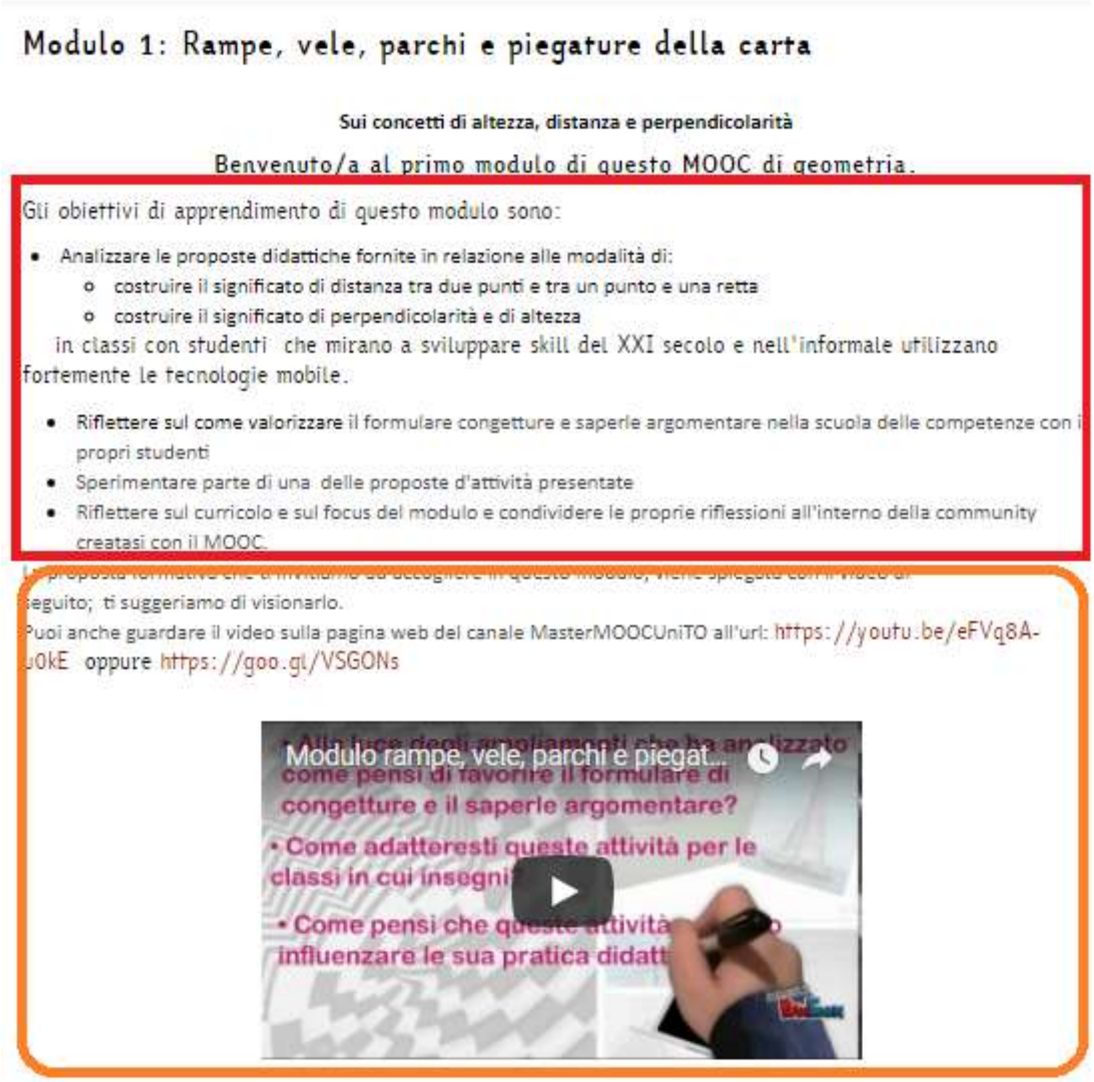


Figure 4.16: Module 1 of MOOC Geometria (part 1)

Aims

The red box (Figure 4.16) lists the aims that the activities of the module intend to pursue. They are:

- To construct the meaning of distance between two points and between a point and a line;
- To construct the meaning of perpendicular and height;
- To formulate conjectures and know how to argue;
- To experiment, if possible, at least part of one of the activities presented;
- To reflect on the conceptual node of the module and share its reflections on the communication message boards.

PowToon video
The orang box (Figure 4.16) is referred to the PowToon video. In 1:20 minutes, it explains how the form is set up, what you need to do to get the badge and a warm invitation to use forums and padlet to share experiences and ideas.

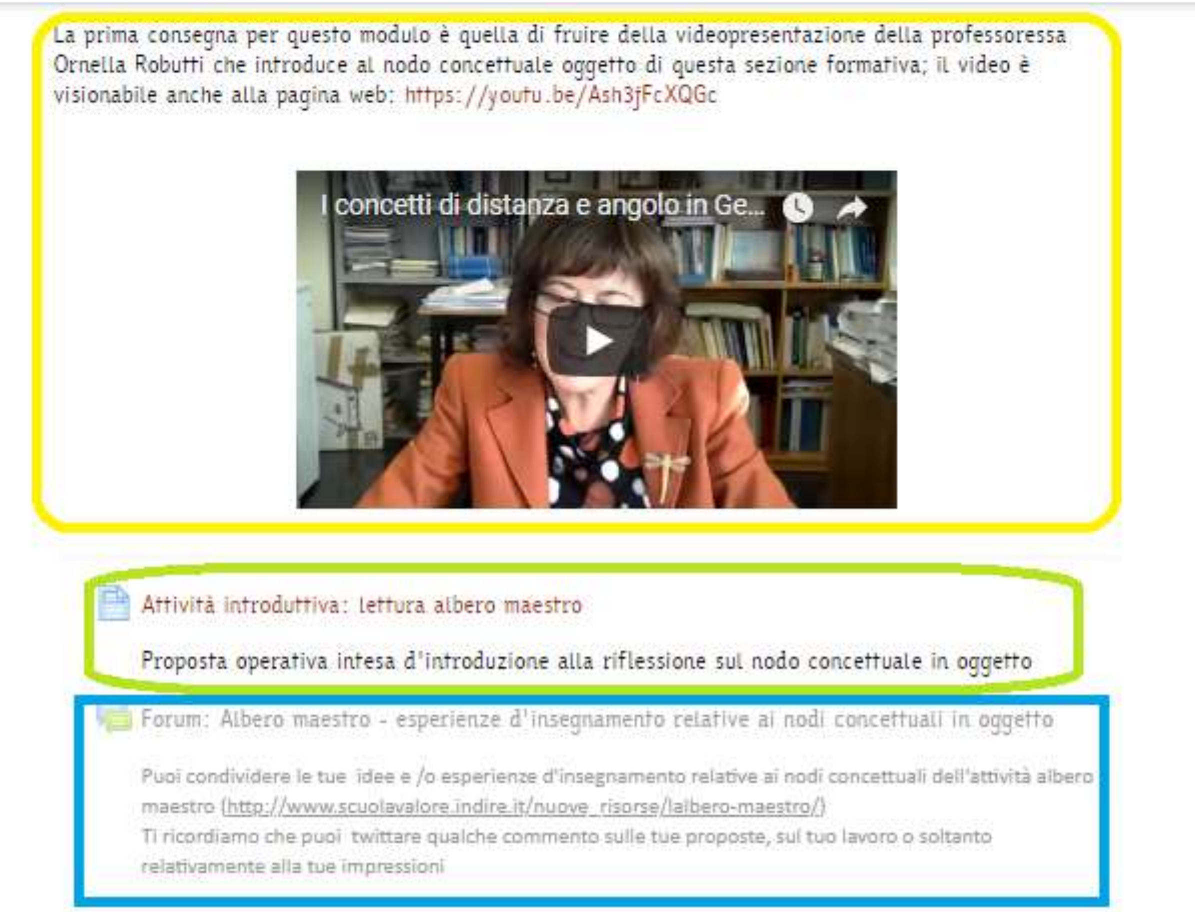


Figure 4.17: Module 1 of MOOC Geometria (part 2)

Prof. Robutti's video
After, in the yellow box (Figure 4.17), there is a video (3:22 minutes) in which Prof. Robutti introduces the conceptual node that is dealt with in this module: the distance. She begins by specifying that speaking of distance we can think of that between two points, or between a point and a line. The first one is of immediate comprehension for the students. The second hides some trap, because it is linked to the concept of perpendicularity, of height within a triangle. The ideal would be to deal with it in activities of manipulation, discovery, conjecture such as that of the mainmast - it is the first activity the trainees meet in this module - where students are asked to determine the distance of a point from a line in contexts also untied by mathematics. Among the trap, there is the one in which the students confuse the perpendicular with the vertical. For example, if the line with respect to which the distance of an external point is located is not horizontal, a student may be tempted to trace the vertical instead of the perpendicular. You can still have problems if we work with an obtuse triangle, where it can happen that the height falls outside the triangle and then outside the opposite side. The student who has not internalized the concept of straight-line distance could do everything to make this height fall inside the triangle. Working well on these conceptual nodes makes sure that the students internalize them well, avoiding the formation of misconceptions.

The mainmast

In the green box (Figure 4.17) there is the link to the first MOOC activity: “The mainmast”, for lower secondary school (grades 6-8). It comes from m@t.abel project[36] and it is an activity that has already been tested in classroom previously. So its exposition will be full of suggestions for the teacher (as we said in §3.2.3.2). It refers to the conceptual node of the *distance between a point and a line*, which – as we have previously underline – is the cause of many misconceptions among students, often linked to stereotypical line situations and horizontal or vertical segments.

Stage 1

It starts from a concrete situation: the teacher gives each student a white circular sheet with a sketch of a boat on a sea wave (Figure 4.18).

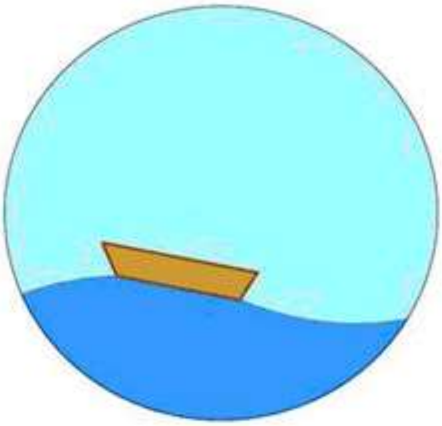


Figure 4.18: Boat on round sheet

The request for the pupils is to draw on the boat its mainmast, of about the same length as the boat. The request to draw on a round and not a squared sheet is made to prevent the horizontal and vertical references, induced by rectangular sheets. The lack of reference helps students to think only about the relationship between the boat and the mainmast. Then, the teacher can pass to the normal sheets of notebook paper making students aware of avoiding the mismatch between perpendicular and vertical lines in problems of this type. The activity continues with the observation of the various drawings and with the discussion of the various solutions found. The teacher can intervene directly with the request: “Explain why”.

This activity should bring out the discussion around the two concepts: (i) *vertical* (physical concept linked to the gravitational field); (ii) *perpendicular* (geometric concept linked to the right angle).

Stage 2

Then, the activities continue with other four stages. The teacher procures strips of paper (or cloth) of different heights. The strips must be of a certain length and with the ends torn off, so as not to overlap with the mental image of the rectangle (Figure 4.19).

Figure 4.19: Strips of paper

The request is to measure the height of the strips. Students will implement different strategies, including that of folding the strip on itself and measuring bending. Students discuss and work in pairs. A phase of collective discussion in the classroom follows with regard to the elaborated solutions. The concept of the height of the strip as

[36] For more details see: http://www.scuolavalore.indire.it/nuove_risorse/lalbero-maestro/

a distance between the straight lines forming the two edges of the strip is therefore born spontaneously.

Stage 3

Subsequently the activity continues with a work in pairs. The teacher gives each couple a round white sheet with the drawing of a segment r and a point P out of it (Figure 4.20).

The request for students is as follows:

"*Find the shortest path that, in your opinion, goes from point P to segment r, draw it and then measure its length. Then write down the procedure you have followed. Now draw other straight roads, measure them and compare the measurements with the one you have traced as the shortest. Write down your comments*".

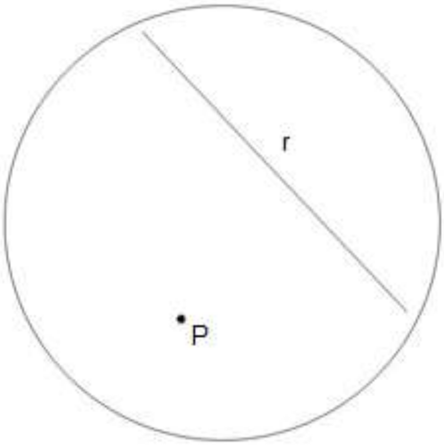


Figure 4.20: Distance of a point from a line

The boys discuss and work in pairs. There follows a phase of collective discussion in the class regarding elaborate solutions.

This should help to reinforce the concept of distance of a point from a straight line as a shorter path from the point to the line itself and the concept of perpendicularity.

Stage 4

The teacher can ask her students: "*What comes to mind when you hear the word height?*"

From the following discussion we must make an aspect that often remains implied, namely the height of particular things (a tree, a candle, a person, ...) is somehow "intrinsic", that is not linked to the position of things which are considered; while the height of the geometric figures depends on the base chosen from time to time.

Stage 5

Finally, each students receives an acute triangle scalene. The students can also work in pairs. It is asked to trace the height of the triangle at least in two different ways, using the most appropriate tools (on the desk there will be available set square, plumb line, ruler and compass). It must follow a discussion from which the following points should emerge:

1) The triangle has three heights.

2) The height of a triangle can be identified in different ways.

After the pupils have sufficiently mastered the different techniques for identifying the height of a triangle, the teacher can propose the exploration of the different types of triangles.

How are the heights of an equilateral triangle? And those of an isosceles triangle? And those of a scalene rectangle triangle? And those of a right triangle? And of an obtuse triangle?

These are the contents of this first MOOC activity. Along with the question of the point-line distance, the students deal with other conceptual nodes related to it, such as the perpendicularity and heights of a triangle, in non-stereotypical situation, such as when there are no horizontal or vertical sides, or when the triangle is obtuse and the height is outside the opposite side. The expectation is that at the end of this activity the student should be capable of correctly tracing the heights in a triangle (but also in parallelogram and a trapezoid), knowing the meaning of point/line distance, perpendicularity and height.

Forum

After “The mainmast” activity, a Forum follows (in the blue box, Figure 4.17). The trainers have inserted a delivery in order to stimulate the discussion among the trainees:

“Share your ideas and/or teaching experiences related to the conceptual nodes of the mainmast activity”

The forum collects 24 discussions, each of them with from 0 to 62 response replicas (Figure 4.21). In the following, we just give a taste. We will talk about this more thoroughly later in the section dedicated to the interactions.

Here are some trainees’ initial thoughts on the matter. These are testimonies of the implementation of the first phase of the **double learning process**: instrumentation/self-organization (we recall the picture used in the theoretical framework, Figure 4.22).

Discussion	Started by	Reply	Not read	Last intervention
Discussione	Iniziato da	Repliche	Non letto	Ultimo intervento
utilizzo del materiale dell'Albero Maestro	[redacted]	62	0	Maria Grazia Sereno sab, 2 gen 2016, 19:19
Adattare l'attività 'Albero maestro' alla prima nella secondaria di secondo grado.	[redacted]	6	0	Lucia Rapella dom, 22 nov 2015, 15:19
google, mappe e triangoli	[redacted]	23	0	Stefano Barbieri mar, 17 nov 2015, 21:21
Introduzione a GeoGebra	[redacted]	21	0	Chiara Tallone lun, 16 nov 2015, 12:00
Per gli studenti con BES	[redacted]	0	0	Stefania Galizia dom, 15 nov 2015, 21:23
Albero Maestro	[redacted]	0	0	ALFREDO SMIGLIO gio, 12 nov 2015, 19:51
sulle altezze del triangolo	[redacted]	31	0	antonella console sab, 7 nov 2015, 19:44
albero maestro associato alla scatola dei triangoli costruttori montessoriana	[redacted]	1	0	Silvia Nigro sab, 7 nov 2015, 11:41
Albero maestro esperienze di insegnamento	[redacted]	4	0	Maria Antonietta Conte gio, 5 nov 2015, 11:21
Richiesta aiuto per Tricider	[redacted]	0	0	Ornella Myriam Millerba mer, 4 nov 2015, 20:34
Geometria "in piastra"	[redacted]	1	0	Francesca Sartogo mar, 3 nov 2015, 23:18
Caccia al tesoro (l'asse del segmento come luogo dei punti equidistanti)	[redacted]	1	0	chiara barraco mar, 3 nov 2015, 17:01
Spunti per un simpatico approfondimento disciplinare	[redacted]	3	0	laura bracco lun, 2 nov 2015, 19:32
Albero maestro - Integrazione nelle mie esperienze in classe	[redacted]	1	0	barbara finato lun, 2 nov 2015, 16:20
raccolta di attività: rampe, vele, parchi ...	[redacted]	6	0	Cinzia Soldera lun, 2 nov 2015, 16:00
fogli tondi	[redacted]	2	0	chiara modotti lun, 2 nov 2015, 14:32
Adattamento attività "Albero maestro" ad altre classi di scuola secondaria I grado	[redacted]	2	0	donato roggero dom, 1 nov 2015, 19:46
Perpendicolare e minima distanza	[redacted]	3	0	Sara Colombera dom, 1 nov 2015, 11:31
la tomba del faraone	[redacted]	5	0	Sara Colombera dom, 1 nov 2015, 09:07
Inizio sperimentazione Albero maestro	[redacted]	2	0	Luisa Gervasio gio, 29 ott 2015, 22:31
albero maestro esperienze di insegnamento	[redacted]	3	0	Lucia Manfredotti gio, 29 ott 2015, 18:29
rette perpendicolari	[redacted]	3	0	Anna Abrile gio, 29 ott 2015, 13:28
Albero maestro....e altro	[redacted]	3	0	Maria Paola Ungari mer, 28 ott 2015, 20:09
Albero maestro	[redacted]	0	0	maria beatrice mar, 27 ott 2015, 01:48

Figure 4.21: Interventions in forum of Module 1 – MOOC Geometria

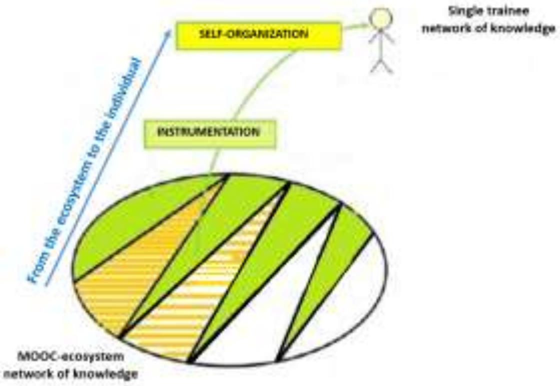


Figure 4.22: Instrumentation/self-organization

M.B. "*Hello everyone! I teach in a lower secondary school, [...] At the end of the week I plan to propose this work to my new class first ..... then I will let you know*".
M.C. "*Hi, I'm M., I teach in a lower secondary school. I noticed it too: for many students the triangle has only one height .... and only one base! I tried to make it clear that it depends on the 'reference system', for example being standing or lying down ... [...] just think of a square put on a vertex, many say that it is a rhombus! Surely the [round] sheet without horizontal and vertical references helps to overcome misconceptions*".

All those who started to participate in the forum experimented an **instrumentation** phase. It started at the same time they clicked the link that directed them to the mainmast activity. The **self-organization** phase follows spontaneously: in the moment when the trainees read the information and consider it an added value for their teaching practices, they are starting to automatically expanding their network of knowledge.
On the one hand, each one formulates his own judgment, putting in place an instrumentation phase; on the other hand someone also begins to respond to comments from other colleagues, already putting in place a phase of **instrumentalization**, i.e. begins to build new connections autonomously using the MOOC-instrument, while expanding the network of the MOOC-ecosystem.

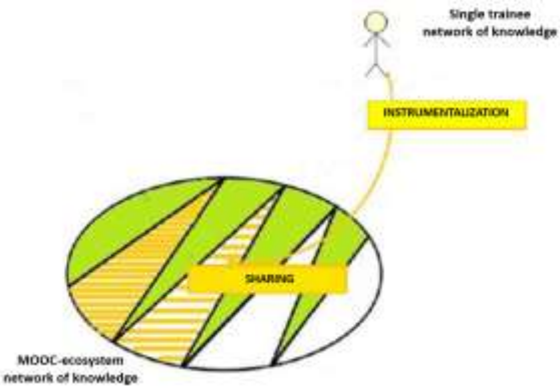


Figure 4.23: Instrumentalization/sharing

N.G. "*I think the same: the round sheets are very useful ... they can initially surprise the students, but then 'positively' oblige them to ask themselves questions, to 'see' things differently*".
C.M. "*I think it is even better to use 'torn off' and not cut out sheets*"

We have sketched out here a first use of the double learning process. We will see better this aspect in the section dedicated to the interactions on communication message boards.

Adaptations and insights of the activities on Sway tool
Next, in the violet box (Figure 4.24), there are a series of activities organized in four Sways. They are adaptations of the mainmast activity and insights related to the conceptual node of distance. As anticipated in §3.2.2, these activities were created by the experienced teachers who are part of the MOOCs' trainers.

Figure 4.24: Module 1 of MOOC Geometria (part 3)

We choose to illustrate just one more of all the activities in the Sways. Since the opening activity of the module was designed for the lower secondary school, we present one designed for higher secondary school (grades 9-10), called “the triangular park”. The authors are L. Giustino, P. Laiolo, G. Trinchero, F. Turiano (all experienced teachers that attended the master, see §3.2.1).

The triangular park

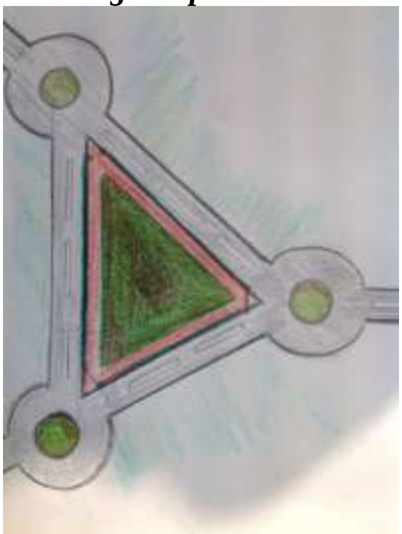


Figure 4.25: The triangular park

The activity starts from a real problem that leads to the use of a triangle center and is divided into two stage. The work mode chosen is work in pairs. It takes place in the classroom and then you can move to the laboratory to explore the solution with GeoGebra.

Stage 1

Three long routes with a high traffic car delimit a large triangular park. The green area is surrounded by a cycle path (it is shown in red in Figure 4.25). We want to create new cycle paths that lead from each entrance (located at the vertexes) to the opposite side. With a limited budget, it is agreed that each of the new paths is as short as possible. At the meeting point we want to place a “rent a bike” station reachable through the new cycle paths.

1) *Determines where to place the station and represents the situation graphically. Argue your choices*.

2) *Write down your solution considerations by exploring other possible triangular shapes*.

Methodological indications: You can think of proceeding like follow.

a) Understanding of the real problem. It is necessary to think about the minimum distance that connects an entrance of the park to the cycle path located in the opposite course. If these paths meet, the meeting point will be the place to locate the bike rental station inside the park.
b) Transforming the real problem into a mathematical problem. The park can be represented as a triangle. The entrances are the vertexes, the existing cycle paths the sides of the triangle. The problem is reformulated to “determine the heights of a triangle and determine their meeting point” (Figure 4.25).
c) To translate the mathematical solution into the real situation. The solution found is applied to the actual situation of the park. We need to think about the solution and recognize that if one of the three angles is obtuse or rectangle the solution would not be appropriate since the cycle paths and the rental station should be located outside the park.

Stage 2

We want to create the minimum pedestrian path that connects the three sides of the park. A section of the route is already under construction and coincides with the FG segment (Figure 4.26). Design, motivating your choice, the entire route.

Stimulus questions:

– Will the FG choice be random?
– Is it possible to find another "inscribed" triangle in the park with a smaller perimeter? Explore the situation with GeoGebra.

Figure 4.26: The shortest way

By working with the dynamic geometry software, you can discover and verify the properties of the orthic triangle (Figure 4.27). The orthic triangle is the triangle of minimum perimeter inscribed in the acute triangle. The triangular park heights are bisectors of this minimum triangle (so the orthocenter of the triangle coincides with the incentre of the orthic triangle).

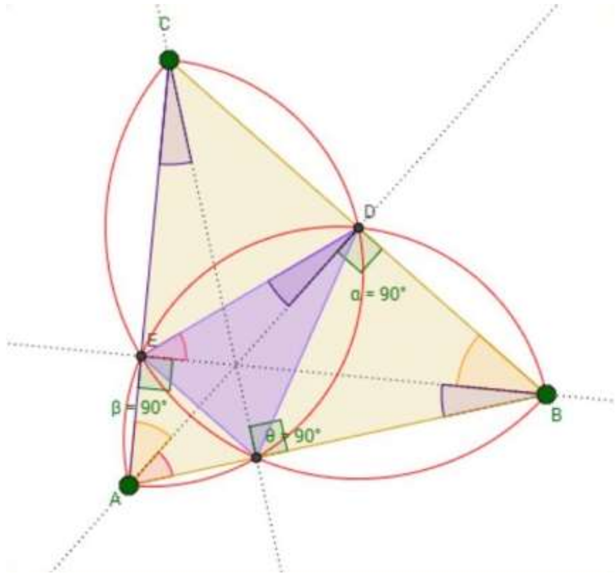


Figure 4.27: The orthic triangle with GeoGebra

Task

The module ended with a final task (pink box, Figure 4.24): share on the padlet board your opinions, ideas and/or productions about the following questions/ideas for the discussion on the introduction of the concept of distance.

1. How would you adapt these activities to the classes in which you teach (specify the classes you are referring to)?
2. How do you think these activities can influence your teaching practice (with regard to the way you present and conduct the situation in the classroom)? If you choose individual activities, group activities, assign homework, …
3. In light of the adaptations you have seen, how do you think to foster in your students the formulation of conjectures and the skills to argue with them?
4. With respect to the previous question, do you think that the students' argumentative productions are a consequence of the proposed situation or do you plan to solicit them after giving some examples to your students?

Figure 4.28: Padlet in module 1 – MOOC Geometria

Figure 4.28 represents a reduced view of the padlet. Padlet is like a wall in which all the trainees can attach their own post. We will enter into details about that in the section dedicated to the interaction on communication message boards. However, here you can have an idea of how many people interact with it to accomplish the given task for collect the first module badge.

We conclude the presentation of module 1 of MOOC Geometria showing the network of knowledge of the MOOC-artifact (Figure 4.29)[37] related to this module 1.

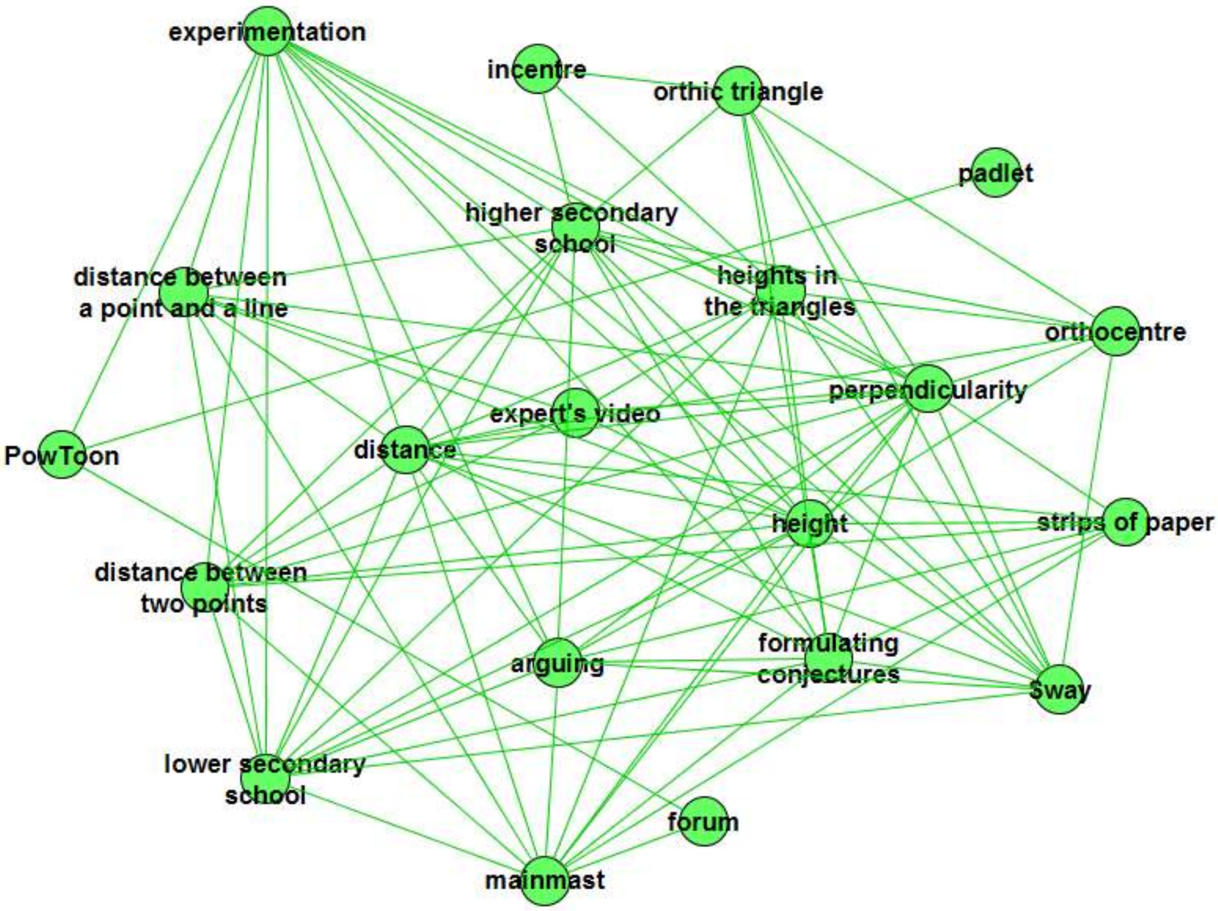


Figure 4.29: The network of knowledge of the MOOC-artifact referred to the module1

As you can see, it seems that chaos is embedded in it. And, surely, it is the same feeling that experimented the trainees the first times that they were entering and discovering this module, and in particular the new (for them) MOOC environment.

However, the network of knowledge here showed is relating only to what we have presented in this analysis of the module 1. This means that all the connections that could have come out were missing, if we had also taken into consideration the activities of the other sways.

It is important understand that all the nodes and the arcs in this network are green because it represents the design actions and digital transposition made by the trainers (remember the remarks made in §2.3 about the colour to identify trainers and trainees).

[37] The graphs were created with yEd Graph Editor, an open source software that can be used to quickly and effectively generate high-quality diagrams (see: https://www.yworks.com/products/yed).

We could say that, it is the image that represents the ideal praxeologies that the trainers would like to transpose to the trainees in the first module. In fact, all the connections between the nodes are the result of the meta-didactical praxeologies of the trainers. For example, pay attention to what is connected with lower secondary school or higher secondary school (in light of what we have outlined above).
Although we have shown some comments of the participants in the forum, at the moment these connections have not been reported in the network.

4.4.1 A quantitative overview of the MOOC Geometria as repository

We have examined in detail the contents proposed by module 1 of MOOC Geometria. We have also specified (in §3.2.3) that the modules have more or less the same structure. We will not go into the details of all the other MOOC Geometria modules; rather we will show the judgment expressed by the trainees by making an analysis of the answers that come from the final questionnaire. We remember that it registered 152 answers, in particular, as anticipated in the methodology section (Chapter 3), these answers are relative to the trainees who started and finished the MOOC in all its stages).

With a series of closed and semi-open questions with multiple answer options, which we will now see, we asked trainees to express their judgment on the MOOC Geometria materials.
First, we asked if the contents of the MOOC corresponded to their expectations. The answer options were: *Yes*, *Partially*, *No*, *Other*. In Figure 4.30 we can see the answers distribution. The expectations of the majority (75%) have been met. Moreover, the 4% who declared *Other* is made up of 6 people who have all expressed themselves in the same way. We repeat one of the sentences they wrote: "*Beyond expectations*". So, we could add the percentage declared in *Other*, to the percentage of *Yes* (ranging from 75% to 79%). Only 1 trainee answered *No*. We will make some deeper consideration on this trainee in the Chapter 5.

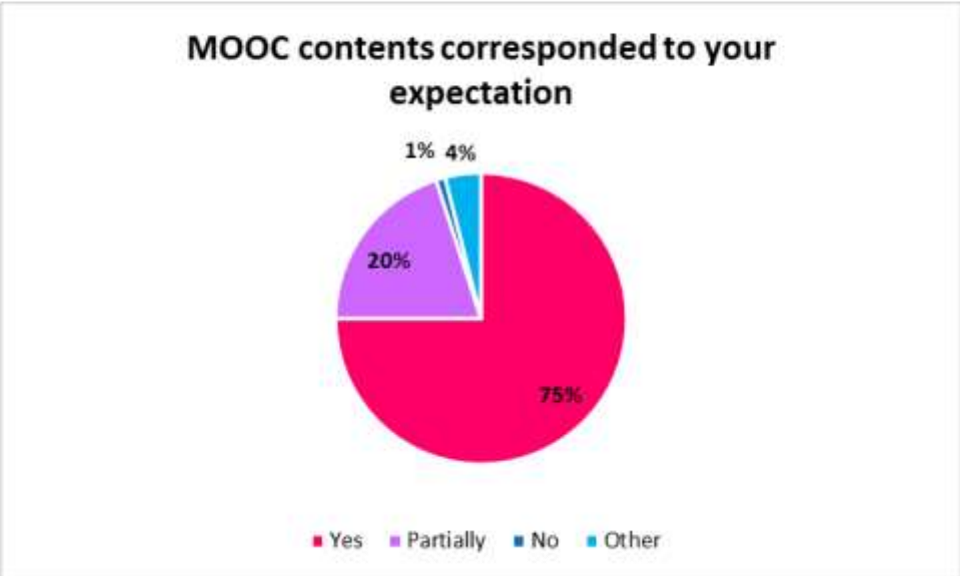


Figure 4.30: Did the contents of the MOOC Geometria correspond to your expectations?

Then, we asked how the trainees judged the duration of the course with respect to the topics discussed. It was a closed question with the following answer options: *Insufficient* (24%), *Good* (76%), *Too long* (0%). None of the trainees who have completed the MOOC considered the duration of it to be too long, indeed the majority states it was a good duration compared to the topics that were treated.

Later we asked how the trainees evaluated the teaching materials provided (videos, activity proposals, Geogebra files, ...). There were three response options that recorded the following percentages: 99% *useful*, 1% *fairly useful*[38], 0% *useless*. This testifies an (almost) total appreciation of the resources provided to the trainees. Moreover, with a multiple-choice question, we wanted to ask the trainees how much they had made use of the resources made available in the MOOC. Making use of the materials, in this question, means having explored the resources made available on the platform, having reflected on them, possibly having them also used in the classroom. In Figure 4.31 we can see that only 29% of them said they had used up to 40% of the MOOC materials. The majority declares instead to have widely used them, denoting an active participation in the course.

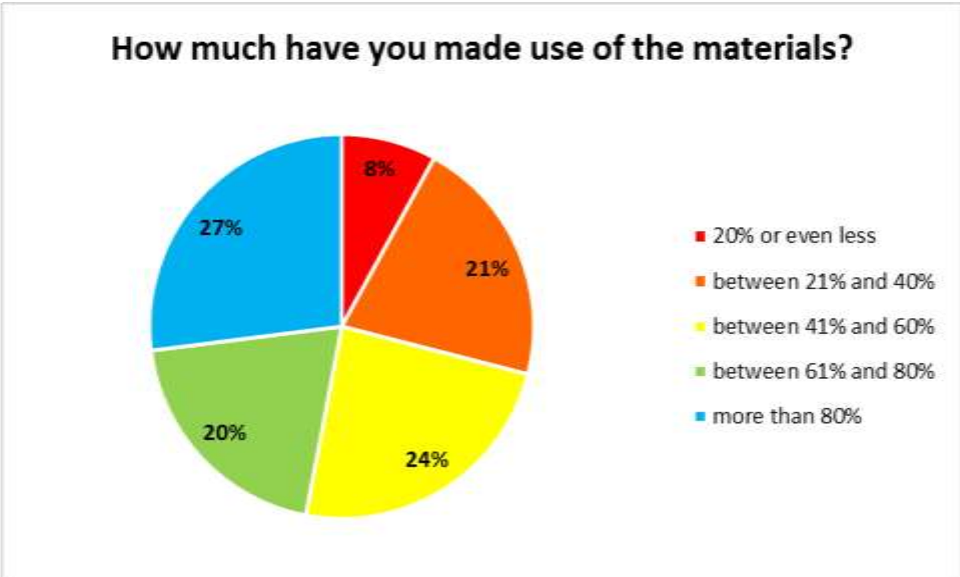


Figure 4.31: How much have you made use of the materials (e.g., videos, activity proposals, software, ...) provided in MOOC Geometria?

In particular, to the more specific question: "*Have you already transposed, at least in part, in your teaching practices what it has been learned in the MOOC?*" – closed question with answer options *Yes*, *No* – the 78% answered *Yes*. This underlines once again an active participation, an appreciation of the materials and a willingness of the trainees to try experimenting new (or in any case different from their usual) didactical practices.

We have not asked the trainees if they consider that the MOOC is a repository, but it is the conclusion to which we can easily arrive from the analysis of these responses. Moreover, as specified in §3.2.3, even if the activities of delivery of the MOOC are concluded, its materials can continue to be consulted. So, actually, having positively judged the materials contained in it, both ongoing and after its duration, the MOOC is for the trainees exactly a repository. Namely, a place where you can find interesting activities, discover new didactical methodologies and be able to propose them to your students in the class.

[38] It is always a single trainee, the one on which we will make some deeper consideration in the Chapter 5

4.5 Interactions on the communication boards in module 1 of MOOC Geometria

The communication message boards inserted into the module 1 in MOOC Geometria, as we have seen, are forum and padlet.
We propose here an analysis of the interactions that took place on them using the lens of the double learning process.

4.5.1 Forum in module 1 – MOOC Geometria

As mentioned before, the forum collected 24 discussions, each of them with from 0 to 62 response replicas (Figure 4.21), for 207 post in total. The trainers have inserted a delivery in order to stimulate the discussion among the trainees:

"Share your ideas and/or teaching experiences related to the conceptual nodes of the mainmast activity"

The Table 4.3, show what types of interventions contains the forum, according to what we have explain in the methodology section (§3.3.2.1). As is easily understood, each intervention is not necessarily classifiable within a single category.

A = explicit answer to the questions/observations	59
B = considerations	134
C = sharing of materials	12
D = sharing of experiences	44
E = experimentation	11
F = other	1

Table 4.3: Types of intervention of the trainees on the forum in module 1

The forum also keeps track of the date and time when the post was published. It is interesting to note that the trainees were writing at any time of the day or night, in a range from 5:51 am (in the early morning) to the 01:48 late at night.

Let us see in detail some of the most interesting discussions or interventions. To make the analysis the dissertation writer has used the file filled by the monitoring groups (§3.3.2.1), but she has re-elaborated these data into another table and she uses this one to show the reader the discussion/intervention.
To summarize the terms of the double learning process, we will use the following codes in the table:

- **Instrumentation/self-organization** (from the ecosystem to the individual) = E→I
- **Instrumentalization/sharing** (from the individual to the ecosystem) = E←I

It is important to note which verbs are used by the trainees. For instrumentation/self-organization the verbs are at the future form (I will do it, I will re-propose, I will test it, I will use it, ...) or there are verbs or adjectives to express your own judgment (I have noticed, I

really appreciated, nice idea, …). For instrumentalization/sharing instead the verbs refer to their own self (I reflect, I know, I thought, …) when they are creating new connection stimulated by the MOOC-ecosystem; while the verbs are at the present form when they share their didactical praxeologies (I do this, I use that, …).
All the intervention are written in a normal type. If you find bold, underlined, italic word, those are "signs" inserted by the dissertation writer to accomplish the analysis.

Discussion (started by X; on gg/mm/aa; at hh:mm)	# reply	Reply (by X; on gg/mm/aa; at hh:mm)	Category	Intervention	Double learning process
MAINMAST (by M.B., 27/10/15; 01:48)	0		B	Hello everyone! I teach in a lower secondary school, [...] At the end of the week **I plan to propose this work to my new first class** .... then I will let you know	E→I

Table 4.4: Trainees' interventions on the forum in module 1

M.B. (in Table 4.4) is the first one to comment in the forum (at 01:48 am!). She is experimenting the instrumentalizaion/self-organization process. She entered into the *chaos* (information overload) of the MOOC-artifact and she was invaded by new activity proposals to propose to her students to address the conceptual node of distance. In organizing the information she has received, she activates the comparison with her Math Edu USs[39]. From her comment emerges that she has positively evaluated the Math Edu USs proposed by the mainmast activity. In fact, she writes that she wants to try it in her classroom. So, based on her praxeologies, she has self-organized this new information in her network.

Let us consider another discussion that was opened in the forum (Table 4.5).

Discussion (started by X; on gg/mm/aa; at hh:mm)	# reply	Reply (by X; on gg/mm/aa; at hh:mm)	Category	Intervention	Double learning process
MAINMAST… AND MORE (by M.C., 28/10/15; 16:04)	3		B, C	Hi, I'm M., I teach in a lower secondary school. **I noticed it** too: for many students the triangle has only one height .... and only one base! I tried to make it clear that it depends on the 'reference system', for example being standing or lying down ... [...] just think of a square put on a vertex, many say that it is a rhombus! Surely the [round] sheet without horizontal and vertical references helps to overcome misconceptions. The plumb line can be misleading [...] can confuse the concepts of perpendicular and vertical, better to go gradually [...]	E→I E←I
		A.R.; 28/10/15; 18:49	A, D	**I use** the plumb line in an initial phase ... that is when they do not understand that in a triangle we can consider each side as a base. In this step I rotate the triangle so that it rests on a different base each time	E←I

[39] We remember that the techniques of the praxeologies of MDT, in hybridization with the instrumental approach, are read as Mathematics Education Usage Schemes (Math Edu USs).

				... in this way we see that the line marks the height. In a second step, I test the students by placing the base on an inclined plane ... at this point the plumb line is definitely deceptive ... here starts the reflection on the concept of perpendicularity.	
		B.P.; 28/10/15; 19:29	B	The use of plumb line is actually **very useful** ... **I will try to use** it in the classroom too. Your ideas are very exceptional, I teach recently and I really need it :) thanks!	E→I
		M.P.U.; 28/10/15; 20:09	B	Hello, definitely plumb line and round sheets are **very useful**	E→I

Table 4.5: Trainees' interventions on the forum in module 1

The trainees are making considerations on the usefulness of the plumb line and exchange experiences of activities conducted in the classroom.
M.C. hooks a node in the MOOC-ecosystem network (difficulty in drawing the heights in the triangles) with her personal experience in the classroom, like a few others (in just previous posts) had done before her.
Note that we referred to the MOOC-ecosystem and not to the MOOC-artifact.
It is not artifact, because the artifact (precisely in this example) was composed of two nodes: height of the triangles, difficulty in tracing the heights. Trainers who, thanks their experience, had already found that this connection exists in classroom practices connected these two nodes. The trainers then transposed a meta-didactical praxeology into the MOOC-artifact: offering strategies and methodologies to avoid the formation and/or the settling of these misconceptions (difference between perpendicular and vertical).
The few trainees that precede the intervention of M.C. begin to comment and agree with these trainers' observations because they have also seen them in their classroom dynamics (as it is also the case of M.C.). The unanimous agreement of the trainees makes that connection, initially thought by the trainers, becomes a connection also of the community of the trainees. This is why we say that it is a node of the MOOC-ecosystem.
Therefore, M.C. has put in place a connection between the node of the MOOC and its node. Then she shares her didactical praxeologies related to the task "overcoming the misconceptions related to the height of the triangles". So, the strategies she shares are none other than her Math Edu USs, triggering the second process of the double learning process.
A few hours later, A.R. responds. Here, the first step of the process (E→I), is intended as an implicit. Certainly, A.R. has experienced the phase of instrumentation/self-organization, but responding to the colleague she is going to connect further with another node that has emerged, the plumb line. So, she shares her praxeologies in merit to show she does not totally agree with the previous comment. In fact, she explains how, according to her, the plumb line wire can be used.
An hour later, B.P. explains that she is still a beginner teacher. She is invaded by the *chaos* of the MOOC and of one that is shared by the other trainees. She is trying to self-organize her network of knowledge that seems to be benefiting from it. She says "[the] plumb line is actually very useful", but she has never used it in person. She deduces it from the comments of the others. Siemens (2005) said "we can no longer experience and acquire learning that we need to act. We derive our competence from forming connections" (p. 3 - 4).
A few time later, the last intervention in this comment is made by M.P.U. She is simply in agreement with what is written, so she limits herself to externalizing it. She does not add

nodes either to her network, or to that of the MOOC-ecosystem. Simply connects, self-organizes the information overload.

Another interesting discussion is the following (Table 4.6).

Discussion (started by X; on gg/mm/aa; at hh:mm)	**# reply**	**Reply (by X; on gg/mm/aa; at hh:mm)**	**Category**	**Intervention**	**Double learning process**
PERPENDICULAR STRAIGHT LINES (by G.M.; 26/10/15; 19:10)	3		B, D	Hi, I have been teaching in a lower secondary school for several years. Some of my students are wrong or have doubts when they have to trace perpendicular segments to straight lines that have different inclinations and obviously they cannot trace the heights of the rectangle triangles and the obtuse triangles. For some years now **I have been trying to draw** straight lines on the ground. **I make** a student lie down on one of them and **I ask** another one to arrange himself perpendicular to the lied down schoolmate. With the chalk, **I trace** the shape and then we see if it forms a 90 ° angle.	E→I E←I
		B.B.; 27/10/15; 16:23	A, B, D	Hi G.M., I'm B., I've been teaching in lower secondary school for several years too and **I have also realized** this problem for the students. Yours seems like a **good idea** ... TOUGH, I would say. Does it seem to work in your opinion? **I work** a lot **with** the lim[40], making draw horizontal segments to the students. Then, **they trace** the perpendicular; after **they rotate** the same segments of different angles and **try** to redraw the perpendicular. When **we return** to the initial position, **they notice** the differences, so they have learned to rotate their worksheet.	E←I
		A.B.; 28/10/15; 16:23	D	I [...] **use** a sheet of tracing paper (from now on, **I will use** it round !!!!) on which they must trace the distance between a point and an oblique line. At the end they have to rotate it, so that the straight line is horizontal and they can verify the correctness of their drawing ... if necessary, using the squared notebook sheet as a reference. The ship is **really pretty** ... as an initial activity it is **very captivating**, **I will adopt it** from next year	E→I E←I
		F.D.B.; 28/10/15; 17:47	D	Hello .... **I use** their ruler .... it already has the right angle at its ends ... just match one side of the ruler with one side of the triangle and slide it to the opposite vertex ... it works!	E←I

Table 4.6: Trainees' interventions on the forum in module 1

[40] LIM in Italian stand for lavagna interattiva multimediale, that is interactive whiteboards.

As M.C. which opened the previous discussion, also G.M. connects herself to the MOOC-ecosystem network of knowledge, showing agreement (instrumentation/self-organization). Then she shares her didactical praxeologies by explaining her related Math Edu USs (note the use of the verb in the present form: "*I make …*; *I ask …*; *I trace …*"). She illustrates a technique that had not been presented among those of the MOOC.
As A.R. who had replied to the colleague in the previous discussion, also B.B. hooks to two nodes: to the fact that she teaches in a lower secondary school as G.M. and to the fact that she has also noticed these discussed difficulties in her students. The idea of G.M. intrigues B.B. "*good idea, TOUGH*" (note that she use the capital letter to express her positive judgment), but it does not convince totally her. In fact, she calls for more confirmation "*it seems to work in your opinion?*". She also shares her didactical praxeologies, explaining how she combines the corporeality that G.M. uses with her students, with technology (the use of LIM).
The day after, A.B. joins the discussion. She also share her didactical praxeologies relate to tracing the perpendicular line (instrumentalization/sharing). In addition, we note an *internalization* (Arzarello et al., 2014, p. 355-356) of a trainer praxeology: she says "*from now on, I will use [the round sheet]*". She also appreciates the idea of the boat on which to trace the mainmast and adds "*I will adopt it from next year*". We do not know if she will really do it, but we perceive that she has internalized a component that came from the outside. This testifies to a phase of instrumentation/self-organization in which she has given value to a node (the round sheet) and has inserted it into her network of knowledge.
The discussion ends with the intervention of F.D.B. The previous process (E→I) has certainly been put in place, since she joins a started discussion in a pertinent way. In fact, she has hooked to the nodes emerged in the MOOC-ecosystem and in particular shares her didactical praxeologies (an alternative use of the ruler) with the others.

In this direction, we consider other interventions coming from another discussion (Table 4.7).

Discussion (started by X; on gg/mm/aa; at hh:mm)	**# reply**	**Reply (by X; on gg/mm/aa; at hh:mm)**	**Category**	**Intervention**	**Double learning process**
ON THE HEIGHTS OF THE TRIANGLE (by A.P.; 26/10/15; 18:22)	31	…			
		E.L.I; 26/10/15; 20:08	D	Hi, [...] I do not teach in any school at the moment [...] but I have experience of 'private' teaching. Speaking of heights and perpendicular lines [...] To make a fourteen-year-old boy understand how to draw the height of one side of a triangle, whatever nature it had, **I invite to** stretch out the thumb and index finger of one hand (usually the left) so as to position the index finger on the side in question: the position of the thumb, once intercepted the vertex opposite the side in question, identified the desired height.	E←I
		A.A.; 29/10/15; 12:47	A	**Genial**, the set square always at your fingertips! **I will immediately adopt** your suggestion	E→I
		E.G.; 31/10/15; 16:02	A, B	Dear, **I also use** this strategy ("natural" set square) for lower secondary school boys, and **I realized that** it works very well, even after some time.	E→I

Table 4.7: Trainees' interventions on the forum in module 1

If we have previously seen that B.P. thanked the most expert colleagues for the exchange of new teaching strategies (Table 4.5), now it is E.L.I who, despite being a beginner, suggests a strategy (instrumentalization/sharing) that she uses with the students to whom she gives repetitions (Table 4.7).
Two other trainees respond enthusiastically, which develop the process of instrumentation/self-organization, connecting to this node (the manual set square) their network of knowledge. In particular, E.G. confirms with her teaching experience what E.L.I says.

It is interesting to note that these introduced techniques (draw on the ground, use the LIM, use the ruler, use your fingers) are very connected to corporeity. It seems, therefore, a fairly widespread belief (and also shared) among the trainees the one to make discovering students by first-hand, with their hands and body, the mathematical properties. Without going into too much detail, we can see that the logos of these praxeologies can certainly be identified in the theories of the Embodied (Lakoff & Núñez, 2000).

On this trail of corporeity, we also analyse the next discussion (Table 4.8), which shows an experimentation conducted in classroom, carried out after the ideas received from the MOOC-ecosystem.

Discussion (started by X; on gg/mm/aa; at hh:mm)	**# reply**	**Reply (by X; on gg/mm/aa; at hh:mm)**	**Category**	**Intervention**	**Double learning process**
PERPENDICULAR AND MINIMUM DISTANCE (by D.T.; 30/10/2015; 21:37)	3		E	Good evening to everyone. I hope I do not go off the subject telling this little story. I teach in a two-year technical institute. This morning in a first **I tried a little experiment**: I asked a boy to get up, I placed him in the middle of the classroom and I turned it towards a corner. Then I pointed to him a wall and told him "reach it along the shortest way" (I preferred to avoid words like distance). I repeated it twice, with different walls and different initial positions. I always got the same result: the boy left immediately and without hesitating in a direction perpendicular to the wall. In the next small discussion I asked "Why did everyone take that direction?", "Which direction is it?". I found it interesting that many people told me that it was an instinctive action. In fact I did not succeed in that phase to receive a conscious explanation. Also, to get the word perpendicular out, it took a while. What do you think? Will there really be this intuitive (or "instinctive") idea of perpendicular as a minimum distance? Is it something that can be exploited to our advantage or is it a	E→I

				misunderstanding to be dismantled? Thanks to everyone and good evening.	
		C.G.; 31/10/15; 20:57	B	Greetings to all. I teach in a professional institute and **I agree** with the colleague that often our students have acquired some concepts with experience but then fail to formalize them. In some cases the specific terminology of mathematics strides with their daily life so much to confuse them. I believe that it is our task, as **also suggested by Prof. Robutti and the activities presented**, to start from previous experience and manipulation to bring out the contents and, in a second time, cover them with the correct terminology and the rigor of mathematics.	E←I E→I
		A.P.; 01/11/15; 09:38	B	[…] I found the idea of D. **very good**, that is to invite the students to physically research the shortest straight path inside the classroom as an intuitive approach to the concept of perpendicularity. The next step, before arriving at the formalization of the concept, **could be** to repeat the previous experience on paper, drawing the shortest route on a floor plan of the classroom.	E→I

Table 4.8: Trainees' interventions on the forum in module 1

By now, in the forum, we are already several days from the beginning of the discussions, the phase of instrumentation/self-organization has been accomplished by all those who continue to write.
In particular, here we see the sharing of an experimentation put in place by D.T., based on the ideas he had received from the MOOC.
It is interesting that he shares not only the experience and the idea, but also considerations (far from trivial) on which others are invited to express themselves.
An answer to D.T. arrives the day later by C.G. After a phase of self-organization, she accepts the questions of D.T. She interprets them in the light of her didactical praxeologies and also of those she has received and self-organized in the MOOC (in fact she says "*as also suggested by Prof. Robutti and the activities presented*"), so she shares her thought. C.G. is not denying that students can perceive the concept of perpendicularity in an intuitive way, rather she specify that they only have difficulty formalizing it from a mathematical point of view. Moreover, she underlines how this is the task to which teachers as they are called.
It is interesting the answer given by A.P. a day later. He shows to appreciate the idea of D.T. and it suggests an operative way to help students to formalize the intuitive perpendicular concept, that is to report the body experience on paper.

We conclude this analysis of the forum by reporting two more examples of discussions, in which the trainees do something that the trainers did not expect: to share their own material (Table 4.9).

Discussion (started by X; on gg/mm/aa; at hh:mm)	# reply	Reply (by X; on gg/mm/aa; at hh:mm)	Category	Intervention	Double learning process
GOOGLES, MAPS AND TRIANGLES (by A.P.; 27/10/15; 18:50)	23		C (Latina's map)	The idea is to play with the heights of the triangles and I half minded to propose it to my pupils :) This is a draft of text. 3 male friends Antonio, Bruno and Carlo are at the top of the triangle in the figure (Figure X29). 3 female friends Antonella, Barbara and Carlotta are also at the top of the triangle in the picture. Friends via whatsapp agree to find themselves in the orthocentre of the triangle while the friends will meet in the centroid of the triangle. Draw the meeting points of the two groups. PS: I used the map of Latina, my city. Didactic note: I deliberately chose an obtuse triangle and the position of the triangle is not that stereotyped by the boys.	E←I
		P.R.; 27/10/15; 23:16	B	This activity is **beautiful**: **I will propose** it next week (obviously using a map of a city closer to my boys, like Turin) to see how they have internalized the concepts of orthocentre and centroid, since they have just discovered heights and medians […]	E→I
		M.L.; 28/10/15; 10:40	B	**I really like** the proposal and I hypothesize a variant of the text: in a treasure hunt the competitor Alberto of the team is in A, Bruno in B and so Caterina in C. The next clue will be given only when all three competitors will meet in the orthocentre of the triangle and communicate the position to the director ... etc ... it could also be said that there is a tolerance of a certain amount of meters for the possible presence of buildings on the geometrically found point. Other points of discussion could arise on the comparison between the mutual positions of center of gravity and orthocenter. What do all of you think?	E→I
		A.P.; 29/10/15; 16:45	B	**I really like** the use of tolerance! [...] Thanks for the idea :)	E→I
		R.R.; 29/10/15; 18:23	B	**Beautiful** ideas, congratulations! It is just to steal ...	E→I
		R.B.; 30/10/15; 17:53	B	It is a **beautiful** idea!	E→I
		E.G.; 28/10/15; 11:48	B	**Beautiful**! I steal it ;-)	E→I
		S.C.; 29/10/15; 15:27	B	**Compliments**! This year I have no classroom of 10 or 11 grade but I have noted for the future. Thank you	E→I

		….			
		L.G.; 29/10/15; 22:35	B	**nice activity**, you could also import the image on GeoGebra	E←I
		…			
		A.M.; 16/11/2015; 18:43	C (file GeoGebra)	Hello everyone! **I created** the GoeGebra file with the map of Cuneo, if you want to give it a look I accept advice! Good job everyone!	E←I
		S.B.; 17/11/15; 21:21	B	**Very nice** the initial idea of the 'randez vous' of A.[P.] and your GeoGebra file, A.[M], with the 4 triangle centres points and the customizable City Map (changing the image). At this point, favored by the environment of dynamic geometry, why not propose to the boys to trace the line that passes through the orthocenter and the circocenter and ask them what happens to the other triangle centres points? Hello to all.	E→I E←I

Table 4.9: Trainees' interventions on the forum in module 1

The involvement of A.P. it is such that, driven by the stimuli he received by reading the materials and the posts of others (instrumentation-self-organization), he produces a material that he shares with others, spontaneously, asking for opinions and collaboration!
The trainers had asked to share their experiences. A.P.'s sharing it is an unexpected event, but it has certainly enriched the whole MOOC-ecosystem, triggering a circle of good practices as we will now see. The following figure (Figure 4.32) is the Latina's map that A.P. shares.

Figure 4.32: Latina's map shared by A.P. with the MOOC-ecosystem

P.R. congratulates with A.P. and, in self-organizing herself, she makes visible the fact that she has added a new node to her network of knowledge: "*I will propose it*", but not in an indefinite future time, rather specific "*next week*", linking it to the mathematical concepts that she has already treated with her classroom.

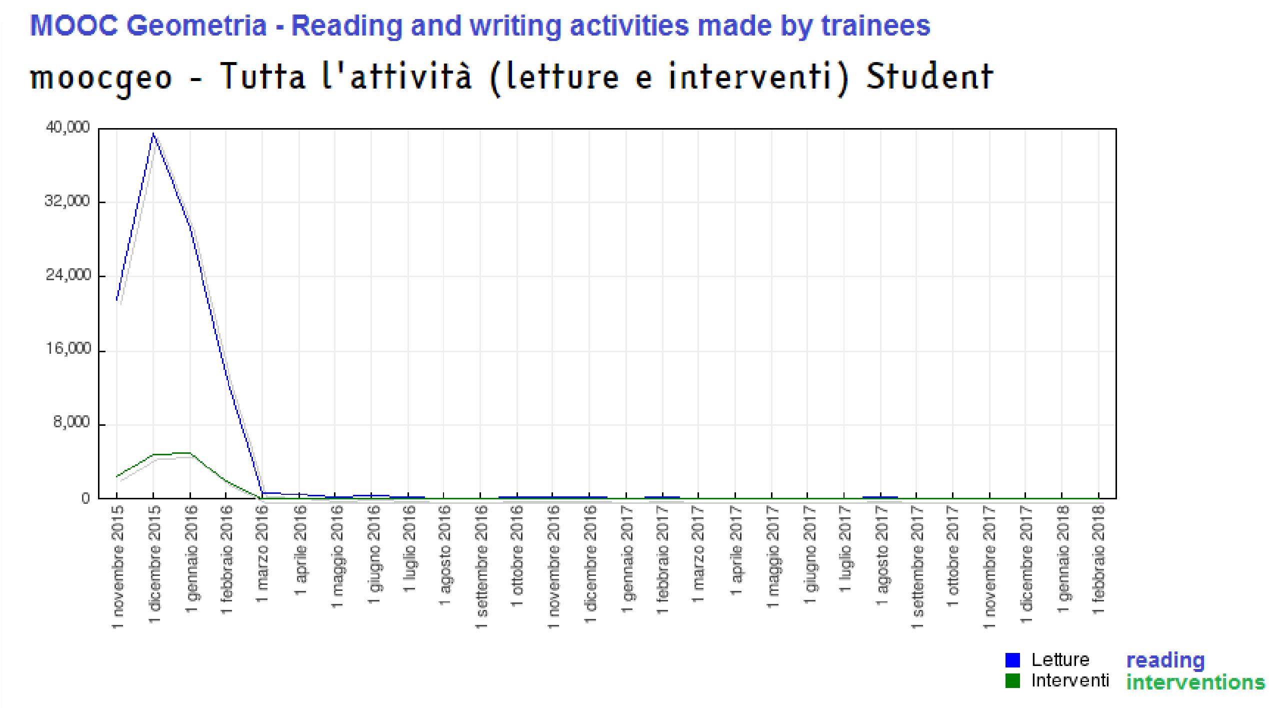
M. L. positively evaluates the idea of A.P. and, in making it her own, she hypothesizes a variant, to stimulate reflection and argumentation in the students. A.P. replies to M.L. showing appreciation for the suggestion received. He is updating his network of knowledge. Note that all of these interactions happened in three different days.
A certain number of trainees begin to compliment with A.P. for his idea (R.R., R.B., E.G, S.C., …)! All those that congratulate, have appreciated and so it is very likely that they have incorporated this new node in their network of knowledge. In particular, S.C. show clearly this: "*I have noted for the future*". As Siemens (2005) said, "the pipe is more important than the content within the pipe" (p.6). Namely, if you know that you have an information, you just remember where it is stored, and then go to fetch, deepen and use at the appropriate time.
Morevoer, L.G. leaves a suggestion "*import the image on GeoGebra*". It will be put in place eighteen days later by A.M. that shares the GeoGebra file that she has created, considering however the map of her city, Cuneo. We see then that, as anticipated, in the wake of A.P. also other trainees share their material and invite the other colleagues to express themselves on these productions, trying to put in place a collaboration.
The discussion ends nineteen days after its beginning, with the S.B.'s intervention. He appreciates the A.P. and A.M.'s ideas. In addition, he responds directly to A.M. advising her of the extensions, putting in place an obvious instrumentalization/sharing process.

The actions of A.P. allow us to make this reflection. He uses the ideas he receives from the MOOC to develop a new and original product. The other trainees have the opportunity to have an extra source of learning: what they see in the MOOC and what they see doing by another trainee thanks to MOOC. So, the others are led to trigger a process of instrumentation/self-organization and to embed in their network that product.
On the one hand, it is a boundary object (Bowker & Star 1999, p. 297), since it is a praxeology between A.P. and another trainee. On the other hand, the process in which this mediation is inserted is that of the double learning process and in this sense the object do not mediate a knowledge, but enters into a genesis of Math Edu USs produced at that time.
A.P. is not the unique trainees that put in place the sharing of own materials. However, probably the one started this phenomenon.
These same observations are valid for all other shares of experiences and observations that have occurred. All can be understood as products, arising from the processes of the double learning process.

4.5.2 Remarks

Considering in general the interactions that took place in the forum (including those that have been proposed here), we underline that there has been an extreme confidence in the exchange of ideas, experiences and materials. Note in particular that we are in the first module, or these are the very first interactions that have been put in place by the trainees. This clarification is made not because the others that follow are different, but precisely to highlight the fact that despite it was at the beginning, there were no shyness or reticence to participate.
It can certainly be said that the "Forum" tool has been accepted and managed on the basis of a growth of knowledge of the trainees, namely in terms of expanding their own network of knowledge, adding the nodes proposed by the others (if the trainee considered them valuable). In fact, adding a node to your own network means expanding it. That is to learn in a connective connotation. From this point of view, all those who have intervened have certainly given proof of a successful learning, or expansion of their own network of knowledge.

One might think that the nodes inserted by the others become nodes surely for the person inside that discussion, but for the others? We have no guarantees of this. Yet, from the statistics of the Moodle platform (Figure 4.33)[41], we observe that the readings are much more than the writings. It is not to be excluded that some lurkers[42] could have nevertheless taken tacit advantage from the reading.



Term period (Month)	Readings	Interventions	Log
30 giugno 2016	187	6	Corso Log
31 maggio 2016	250	3	Corso Log
30 aprile 2016	158	7	Corso Log
31 marzo 2016	466	1	Corso Log
29 febbraio 2016	665	6	Corso Log
31 gennaio 2016	13610	1896	Corso Log
31 dicembre 2015	29257	4961	Corso Log
30 novembre 2015	39448	4847	Corso Log
31 ottobre 2015	21485	2453	Corso Log

Figure 4.33: Reading and writing activities relative to forums in MOOC Geometria

[41] Remember that MOOC Geometria was delivered from October 2015 to January 2016.

[42] In Internet culture, a lurker is typically a member of an online community who observes, but does not participate (Dennen, 2008).

Although no specific examples have been shown, from the forum there is a tendency not to always follow the dialogue remaining within the thread, rather opening up new areas of discussion several times, even if already initiated previously by someone else. This perhaps testifies to a habit of concentrating the presences on the platform in a temporal sense, without analysing all the pre-existing interventions.
The shared materials (such as those of A.P. and A.M.) are interesting and show a tendency of the trainees to experiment with new teaching practices and a critical sense in the analysis of the results.
A moderate sharing of the meta-didactical praxeology that the trainers wanted to transpose to the trainees emerges. It is the one linked to the overcoming of the misconceptions related to the concept of height. Of course, we can not talk about shared praxeology as for MDT. In fact, sharing does not take place for everyone or even between everyone at the same time, but a slight sharing begins to take shape, at least among those who intervened in the forum.
Finally, we remember that, with the hybridization, the term broker (which could be referred to both the MOOC and possible trainees in the MOOC), is inserted into the meaning of ***ecosystem***, that we recall here for completeness, so that we now have a clearer idea, in the light of what has been analysed so far:

"*all the relations (exchange of materials, experiences and personal ideas/point of view) put in place by participants of an online community, thanks to the technological tools through which they interact with each other, establishing connections within a given context*" (Taranto et al., 2017, p. 2481).

4.5.3 Padlet in module 1 – MOOC Geometria

The padlet collected 152 posts. The trainers have inserted some question in order to stimulate the discussion among the trainees. The types of interventions contained in it the padlet are presented in Table 4.10. Of course, this time, all of them can be intended as A (explicit answer to the question proposed by the trainers). However, we made a more fine distinction in the other categories too.

A = explicit answer to the questions/observations	152
B = considerations	151
C = sharing of materials	20
D = sharing of experiences	47
E = experimentation	18
F = other	5

Table 4.10: Types of intervention of the trainees on the padlet in module 1

The padlet does not keep track of the date and time when the post was published. Each intervention provides only a space to report the author's name and surname.

Let us see in detail some of the most interesting posts and make some considerations. To make the analysis I have used the file filled by the monitoring groups (§3.3.2.1), but I have re-elaborated these data into another table and I use this one to show you the discussion/intervention.

Secondary school (lower/higher); Name	Category	Intervention	Double learning process
Higher; M.L.S.	B	I admit that these contents have **a bit floored** me. When I introduced the concept of congruence I told my student of 9 and 10 grades that if we were at primary school, I would arrive in the classroom with paper and scissors to show them how two congruent figures are superimposable and they laughed amused feeling extremely bigger than a primary school child. Now [the MOOC] suggests to me to use similar methods (those of folds of sheets, for example) for the concepts of perpendicularity. I do not want to question the validity of such methods, indeed! **Given the discomforting results** especially with regard to the heights in the triangles, it is welcome to pick up some objects that allow you to experiment with these things […]	E→I

Table 4.11: Trainees' interventions on the padlet in module 1

In M.L.S (Table 4.11), we note that among her beliefs was that of not having to use manipulative activities because they are suitable for primary school pupils. The MOOC with its proposals displaces her: "I told my students that, if we were in the primary school, I would have arrived in the classroom with paper and scissors [...] and they laughed [...] now [the MOOC] suggests to me to use similar methods".
She has read the posts of the forums (she have also commented in it). She has clearly implemented the process of instrumentation/self-organization. She has ascertained a situation that defines "discomforting" with regard to tracing the heights of the triangles. Therefore, reading the posts of others seems to re-evaluate her conviction on practical-manipulative activities. This is an example of how the MOOC-ecosystem can have an influence on teachers' beliefs.

Secondary school (lower/higher); Name	Category	Intervention	Double learning process
Higher; R.M.	B	The activity proposals seen in this course provide **stimulating ideas** for a more **engaging and accurate teaching**. In the third [11 grade] **I will propose** the problem of the park, applied in analytic geometry, with the calculation of the triangle centers points. One could also examine the relationship between axes and heights of triangles, one inscribed in the other (construction of lines parallel to the sides of the inner triangle). [...] Students should be encouraged to use reasoning to validate ideas or to refute them, in a systematic way. **The laboratory aspect present in all the proposed activities, is an incentive** for all of us teachers, to enhance the search for effective solutions through processes of verification and validation of hypotheses, typical of the hypothetical-deductive method.	E→I

Table 4.12: Trainees' interventions on the padlet in module 1

R.M. (Table 4.12) shows appreciation for the activities and methodologies proposed by the MOOC. She defines them as an "incentive". In her instrumentation / self-organization process, she is reinforcing what were her initial praxeologies, already quite in line with what the trainers want to transpose.

Secondary school (lower/higher); Name	Category	Intervention	Double learning process
Lower; V.D.	B	**The activities I have already proposed** to my classroom [the use of magnets and plumb line; the coat rack produced by the students; using the Sketchometry App] **have influenced my way of proposing the topic in class** in the sense that they have improved the class's approach to these arguments.	E→I

Table 4.13: Trainees' interventions on the padlet in module 1

V.D. (Table 4.13) has intervened several times in the forum. He also told of having experimented some MOOC activities. From this his comment shows a perception of the change of the practice. He writes: "*activities have influenced my way of proposing the topic in the classroom*" because he saw in his students a different approach to the discipline. This testifies to a germ of evolution of his didactical praxeologies.

4.5.4 Remarks

Considering in general the interactions that took place in the padlet (including those that have been proposed here), we underline that they are composed by longer and more complex interventions than the ones in the forum, precisely because the trainees want to answer the 4 questions asked by the trainers.

Several trainees prefer to write on word files and then upload the file (sometimes even up to 3 pages!). Many attach images or photos to give more strength to the experiences they are telling (Figure 4.34). They all begin by presenting themselves, also specifying the school in which they teach, as indeed trainers had requested it. The contact with the other trainees here is almost absent: they greet each other – "*hello to all colleagues*" – before starting their post, but there are no interventions nested or someone's answers to others.

Figure 4.34: Examples of post in the padlet

The process of instrumentation/self-organization emerges more from the padlet than in the forum. The trainees explain about their organizations regarding the materials offered and how the settled these in their network of knowledge in according to their didactical praxeologies. For all those who have not yet experienced, one (or more) node(s) has been formed, but still there is not a firm connection that is transformed exactly in the interiorization and therefore in the effective expansion of the network of knowledge. In only some of them it is shown an expansion germ of it.

We conclude the presentation of the interactions that occurred in Module 1 of MOOC Geometria showing the network of knowledge of the MOOC-ecosystem (Figure 4.35)[43] related to this module 1.

As you can see, it is more chaotic than the one showed for MOOC-artifact. Now, orange nodes and arcs have been added (remember the remarks made in §2.3 about the color to identify trainers and trainees). They refer to all the nodes that the trainees have emerged during the interactions in the communication boards (forum and padlet) and to the connections that they have generated between them and between the existing nodes in the MOOC-artifact network. Even in this case, the network of knowledge here showed is relating only to what we have presented in this analysis of the module 1. This means that, if we had taken into consideration also the activities of the other sway and all the interactions that occurred in the communication message boards, the network of knowledge of the MOOC-ecosystem it would have been much denser than it already is in the Figure 4.35.

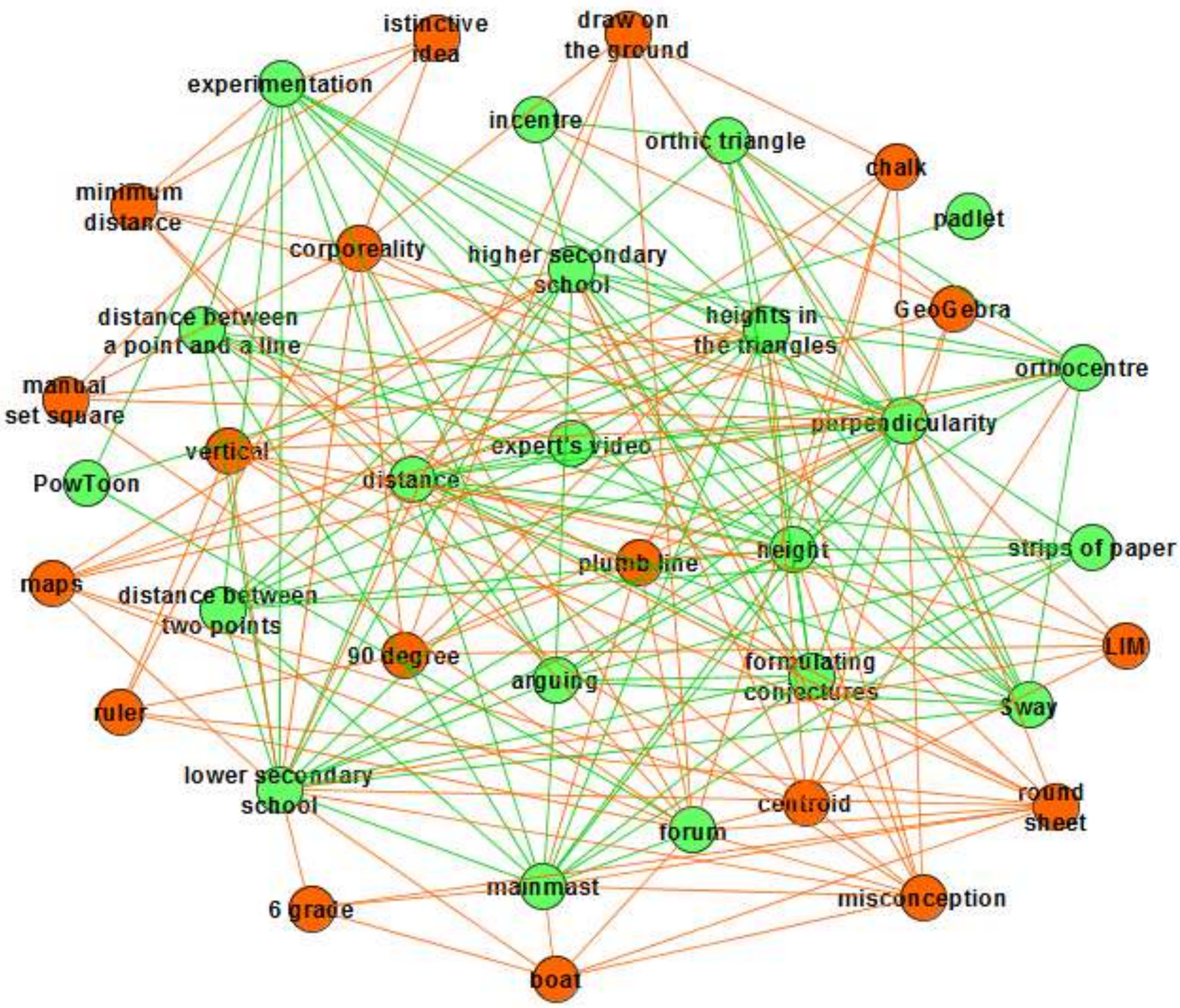


Figure 4.35: The network of knowledge of the MOOC-ecosystem referred to the module1

[43] The graphs were created with yEd Graph Editor, an open source software that can be used to quickly and effectively generate high-quality diagrams (see: https://www.yworks.com/products/yed).

4.6 Module 5 in MOOC Numeri: Arithmetic, Algebra and Mathematical Languages

The module presents itself with a nice and clear layout (Figure 4.36, 4.37, 4.38, 4.44, 4.46). The following is a narration of the contents, referring to what is highlighted by the coloured squares of the following figures.

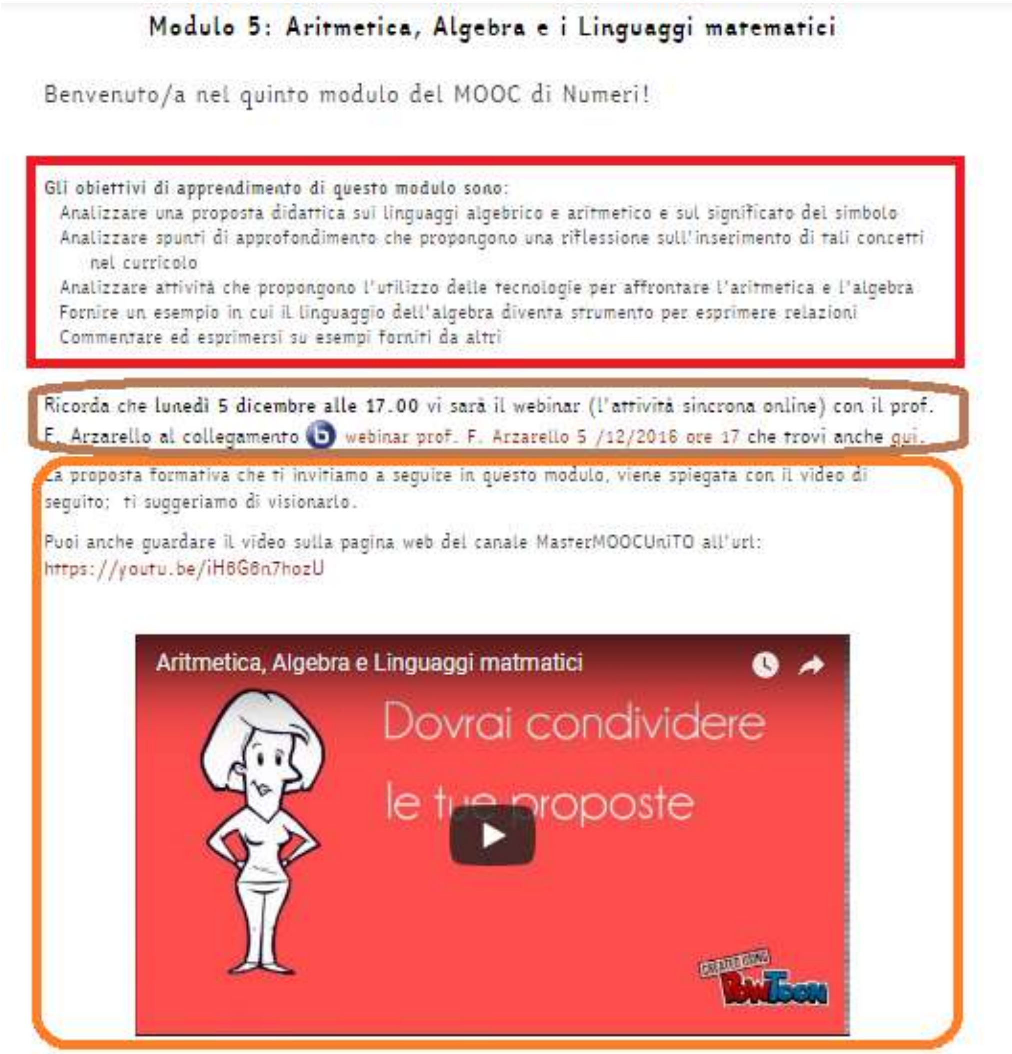


Figure 4.36: Module 5 of MOOC Numeri (part 1)

Aims

The red box (Figure 4.36) lists the aims that the activities of the module intend to pursue. They are:

- To analyse a didactic proposal on arithmetic and algebraic languages and on the meaning of the symbol;
- To analyse points of study that propose a reflection on the insertion of these concepts in the curriculum;
- To analyse activities that propose the use of technologies to deal with arithmetic and algebra;

- Provide an example in which the language of algebra becomes a tool for expressing relationships;
- Comment and express yourself on examples provided by others.

Webinar
In the brown box (Figure 4.36), there is a reminder for the trainees, to remember the webinar appointment scheduled during the week dedicated to module 5. We will talk better about this later in the section dedicated to the interactions.

PowToon video
The orang box (Figure 4.36) is referred to the PowToon video. In 1:01 minutes, it explains how the form is set up, what you need to do to get the badge and a warm invitation to use forums and tricider to share experiences and ideas.

Blue table
The PowToon video message is strengthened in the written version with the presence of a blue table (see the yellow box, Figure 4.37. It was also translated into English in the §4.2.3 – Table 4.2). The blue table summarize the contents and deliveries provided in the module.

Nello schema di seguito trovi il quadro di tutto il modulo

UNITS	MODALITA' di FRUIZIONE	
1) Presentazione del corso Riflessione sulla dimostrazione in campo numerico	Video Video	
2) Attività m@t.abel "L'aritmetica aiuta l'algebra, l'algebra aiuta l'aritmetica"	Consultazione attività Partecipazione al forum	
3) Attività	• 1 attività per secondaria di I grado	• 1 attività per secondaria di II grado
Proposte attività utilizzo tecnologie	2 attività	
Attività di approfondimento sul Metodo della Ricerca Variata (MRV)	1 attività	
4) Compito	Aggiungi degli esempi, vota e commenta su tricider	

Modulo 5 - Unit 1

La prima consegna per questo modulo è quella di fruire della videopresentazione del prof. Ferdinando Arzarello che introduce al nodo concettuale oggetto di questa sezione formativa; il video è visionabile anche alla pagina web: https://youtu.be/P8T76mI2K6M

Trasversale a tutti i moduli è il discorso sulla dimostrazione; un suggerimento che viene dato per avviare questo processo è partire da dimostrazioni semplici, ma ricche, in campo numerico.

Figure 4.37: Module 5 of MOOC Numeri (part 2)

Prof. Arzarello's videos

After, in the green box (Figure 4.37), there is a video (3:16 minutes) in which Prof. Arzarello introduces the conceptual node that is dealt with in this module: the mathematical languages. He starts underline that it is a very wide theme. In fact, students begin to face it since elementary school, even earlier as soon as they start counting. These languages gradually grow in quality and quantity: there is algebraic language, the language of functions, the language of analysis, the language of probability. Each of these refers to theoretical aspects and didactic practices that must be considered. Let us not forget the natural language, that which all of us (pupils and teachers) use to help us understand and conceptualize. In the passage from this language to the mathematical one there is some difficulty. The first difficulties are already in arithmetic, but then especially in algebra. Translating the relationships that I can explain in natural language in algebraic formulas constitutes a first difficulty. There is a famous problem that, given to young and old, gives many wrong answers. It is called "the problem of students and professors". In a school the number of students is equal to 6 times the number of professors. It is suggested to translate into formal language using the letters s and p instead of students and professors respectively. Many people translate this relationship with 6s = p. Students are understood as a predicate and not as a number of students (and the same for teachers). There is a channelling of attention, which is answered incorrectly, incorrectly translating into formal language. This happens very much in algebra and brings with it negative consequences that can arise in the continuation of the studies. Algebra is in fact at the base of the discovery of relationships, functions, analysis and so on.

Therefore, a strong attention must be given to the way in which we can translate from natural language to algebraic (or analytical, numerical) language the relations that are studied in mathematics.

Figure 4.38: Module 5 of MOOC Numeri (part 3)

In another video (bourdeaux box, Figure 4.38), immediately following the previous one, Prof. Arzarello proposes a brief reflection (2:27 min) on mathematical proof.
The proof is one of the most important activities and it is the result of a series of processes, which involve skills that must be prepared in crescendo, starting from primary school. The steps to follow are: starting from a reasoning, an argumentation made in natural language up to more rigorous mathematical forms. Above all, getting used to answering questions like "Why is that? Why is not it? How would it be if ...". These are pillars and by exercising a gym like this in the classroom, this produces the transversal skills of a logical nature that will lead to the ability to produce a proof on their own. Usually the proof is linked to the geometry, but very simple and rich demonstrations can be made also in arithmetic context: numerical properties, properties of recurrence relations, intertwining of empirical discoveries made with computers with typical properties of arithmetic. All this should be pursued over time, without forcing, and it is important to understand that it is useless to make students memorize if the meaning of the proof and/or what is being done is not understood.

Arithmetic helps algebra and algebra help arithmetic

The blue box (Figure 4.38) there is the link to the MOOC activity: "Arithmetic helps algebra and algebra help arithmetic", for higher secondary school (grades 9-10). It comes from m@t.abel project[44] and it is an activity that has already been tested in classroom previously. So its exposition will be full of suggestions for the teacher (as we said in §3.2.3.2).
Games of mathematical "magic" and challenges of mental calculation abilities are at the heart of this activity for dealing with the conceptual node *natural language* and *algebraic language*. The activity refers to the introduction of the rules of algebra and the difficulties encountered when the student must translate a problem algebraically ("put it into a formula"). Concretely, the activity wants to give meaning to algebraic calculation, to ensure that students do not interpret the algebraic formulas as pure sequences of signs. We propose problems in which the language of algebra overcomes that of arithmetic and becomes a tool for expressing relationships and generalities: a language that is useful for both understanding and providing. The main purpose of the activity is to give meaning to the calculation rules, to understand the meaning, and use algebraic calculation to solve problems. Here we are also trying to present mathematics as a thinking tool, highlighting the conceptual aspects.

Stage 1

To avoid students interpreting algebraic formulas as well as a sequence of signs, it is necessary to propose problematic situations in which the language of algebra exceeds that of arithmetic and becomes an instrument to express relationships and generalize a useful language both to understand and to demonstrate. In the light of this, the activity starts with a game-stimulus "think of a number". The teacher, addressing the entire class, proposes to each student to execute instructions in the notebook; the teacher does not know which number was chosen initially by each student.

- Think an integer
- add to it 12
- multiplies the result by 5
- subtract 4 times the number thought
- adds to the result 40

The teacher asks some students for the final result; the she subtracts 100 from this result and "guesses" the starting number. The teacher then justifies her "foresight" with the symbolic

[44] For more details see: http://www.scuolavalore.indire.it/nuove_risorse/lalbero-maestro/

calculation. Invite some students to rewrite the given operations in order of the whiteboard, without executing them, as follows:

n.	**… + 12**	**… · 5**	**… - 4 · …**	**… + 40**	**what do you get?**
7	7 + 12	(7 +12) · 5	(7 + 12) · 5 – 4 · 7	[(7 + 12) · 5 – 4 · 7] + 40	…
…					

The teacher asks, therefore, to generalize the written expression independently of the number thought:

n.	**… + 12**	**… · 5**	**… - 4 · …**	**… + 40**	**what do you get?**
a	a + 12	(a +12) · 5	(a + 12) · 5 – 4 · a	[(a + 12) · 5 – 4 · a] + 40	…

Finally, she invites to compile a table like the following to reflect on how it is possible, with appropriate calculations, to make the expressions simpler.

Before		**After**
a + 12	For now this expression can not be written differently: it is a simple expression	**a + 12**
(a +12) · 5	Here instead you can apply the **distributive property** of the product	a · 5 + 12 · 5
	Then	**5 · a + 60**
	Using the result just found, we write:	5 · a + 60 - 4 · a
(a + 12) · 5 – 4 · a	Now we change the order (*why can it be done?*)	5 · a - 4 · a + 60
	Add a parenthesis (*why can you do it?*)	(5 · a - 4 · a) + 60
	We can now apply the **distributive property** for the expression within brackets:	(5 – 4) · a + 60
	Performing the calculation we have:	1 · a + 60
	but 1 is a *neutral element for the product*, then:	**a + 60**
[(a + 12) · 5 – 4 · a] + 40	Using the result just found, we write:	a + 60 + 40
	and finally the final result will be:	**a + 100**

Now you can unveil the teacher's "trick"!
The teacher then observes that the rules of calculation are none other than the application of the rules of arithmetic; in particular she emphasizes the role of distributive property that allows us to "distribute" a product on a sum but also to "collect" a common factor, depending on how we interpret the equivalence: $a \cdot (x + y) = a \cdot x + a \cdot y$

The teacher explains to the class how this calculation rule has a simple geometric interpretation.
If we consider two rectangles, the first of sides a and x, and the second of sides a and y, as in the figure, these can be arranged so as to form a single rectangle of sides a and (x + y).
And the sum of the areas of the first two rectangles is equal to the area of the third one (Figure 4.39)[45]:

[45] The geometrical figures were created with GeoGebra, an open source dynamic geometry system (see: https://www.geogebra.org/).

$$a \cdot (x + y) = a \cdot x + a \cdot y$$

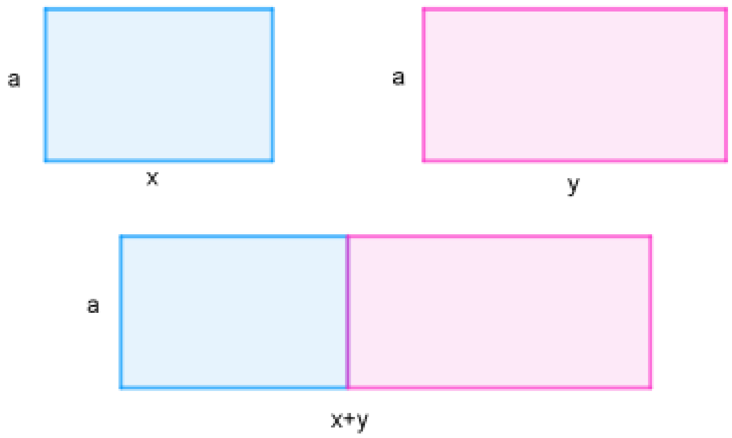


Figure 4.39: Geometric aspect of distributive property

There is now a phase of generalization and consolidation. The teacher proposes to the class the simplification of symbolic expressions in which the product is "distributed" in relation to the sum, without neglecting the inverse operation of "collecting" the common factor. It is important to point out immediately to the students that there is not a "best" writing in absolute, but that the collection or distribution is functional to the purpose that is set.
Morevoer, in the activity, we read: "The material for this phase can be found in any manual. It is recommended only not to exceed the execution of excessively complicated exercises and ends in themselves".

In the stage 2 and in the following one (stage 3) we want to introduce the special products *square of a binomial* and *difference of squares*, deducing them first from the properties of the numbers and then giving them a geometric interpretation.

Stage 2

In the following activity, the teacher addresses the whole class, and challenges it to understand how the following game works.

- Think an integer
- adds 5 to it
- elevates the result to the square
- now subtract the square of the thought number
- subtract 10 more times the number thought.

The teacher asks some students for the final result: she points out that it is independent of the starting number. How to justify this fact? To direct students to the explanation of the game, the teacher invites each of them to perform the calculations in the notebook, using the numbers chosen by some of them, as in the tables below.

First table • write the number thought • add it to 5 • elevates the result to the square and write the result					Second table • elevates the starting number to the square • adds to the result the starting number multiplied by 10 • and finally adds the new result to 25			
n.	**… + 5**	**$(\ldots)^2$**	**what do you get?**		**$(\ldots)^2$**	**$\ldots + 10 \cdot \ldots$**	**$\ldots + 25$**	**what do you get?**
7	$7 + 5$	$(7 + 5)^2$	…		7^2	$7^2 + 10 \cdot 7$	$7^2 + 10 \cdot 7 + 25$	…

As in stage 1, these tables allow to pass to a general and symbolic formulation of the calculations; however they do not lead to the justification of equivalence. Therefore, the teacher explicitly poses the problem of justifying the equivalence:

$$(a + 5)^2 = a^2 + 10a +25$$

She uses the distributive property to develop the calculation, pointing out the "bidirectionality of the equivalence" (i.e. the fact that the equivalence can be read from left to right, but also from right to left):

$$(a + 5)^2 = (a + 5) \cdot (a + 5) = (a + 5) \cdot a + (a + 5) \cdot 5 = a^2 + 5a + 5a + 25 = a^2 + 10a + 25.$$

To move into the generalization, the teacher proposes the square $(a + b)^2$ and suggests to apply also now the distributive property to develop the calculation.
Subsequently, the following work proposal is presented to the class.

Proposition 4 (Euclid's Elements, II Book):
"*If you divide a straight line [i.e. a segment] into two parts, the square of the whole line [segment] is equal to the sum of the squares of the parts and the double of the rectangle comprised by the parts themselves*".

This simple activity is intended to provide a geometric meaning to the formula of the square and also to promote proper storage of the formula itself. It also lends itself to the extension to the square of the trinomial $(a + b + c)^2$.
For the display of squares, you can refer to Figure 4.40.
Finally, the teacher consolidates what is seen using manual exercises. It is also recommended in this case to propose both exercises of "expansion" of squares, and "recognition" of squares, pointing out the bidirectionality of the equivalence.

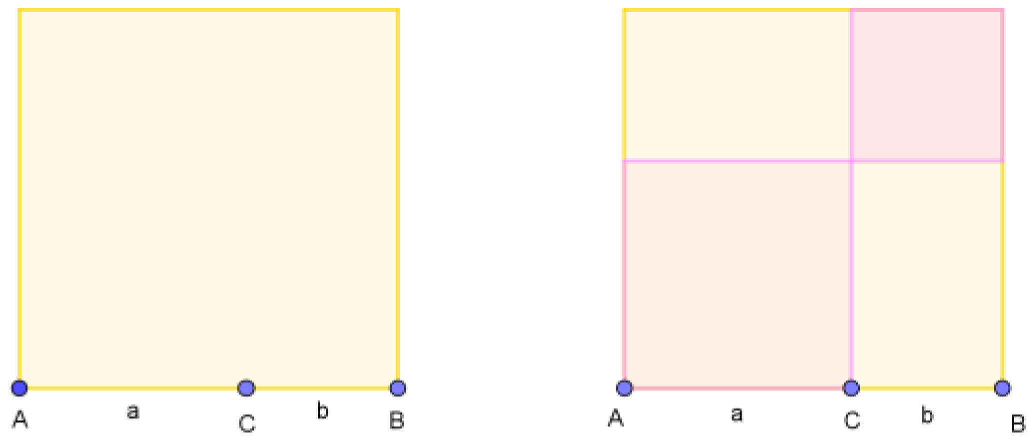


Figure 4.40: Geometric aspect of the square of a binomial

Stage 3

In this stage, the teacher immediately poses the problem of finding a formula for the product

$$(a + b) \cdot (a - b),$$

starting from a numerical case, then substituting a number with an indeterminate one, and finally passing to the general case. Again, this is the application of distributive property.

Moving into the geometric aspect, the aim is to provide a geometric meaning to the difference of squares formula.

The teacher proposes to the class to geometrically represent the special product. There are more strategies possible. For the display, refer to the Figure 4.41.

The decomposition suggested in the figure is very simple, but requires a small algebraic passage: the areas of the two rectangles on the right are $a \cdot (a - b)$ and $b \cdot (a - b)$ and, consequently, the area of the whole region indicated is $(a + b) \cdot (a - b)$.

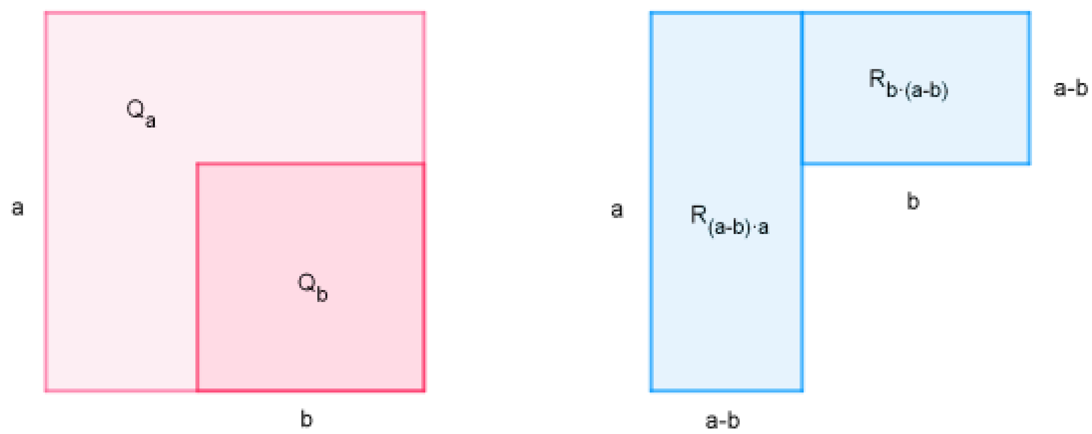


Figure 4.41: Geometric aspect of the difference of squares

Stage 4

The aim of this fourth stage is to show students that the “abstract” rules of symbolic calculation are useful in the calculations: algebra, sometimes, can help arithmetic.

Figure 4.42: Girò the wizard

The following story is presented to the class.

Girò (Figure 4.42) is a wizard that boasts of knowing how to “guess” the square of a number that ends with 5.

James puts he to the test: the square of 45.

Girò answers without hesitation: 2025.

The square of 85. And the wizard: 7225.

Increasingly difficult: the square of 115.

The wizard thinks about it a little more, but after a while he answer 13225.

James thinks to put the wizard in difficulty by asking for the 1005 square, but Girò seems to be even faster than before, answering 1010025.

Does Girò really possess magical qualities that allow him to “guess” the squares, or does he calculate the result in his mind, perhaps by applying some algebraic “trick”?

Giacomo is really very amazed, but the wizard explains to him:

“*It is easy, I multiply the number formed by the digits preceding 5 for its next, I write the product obtained and 25 to follow: so I get the square required! Try to apply this procedure to the numbers you gave me!*”.

The students are so invited to try to justify the procedure described by Girò.

The teacher can justify the procedure in various ways, in particular by remembering the remarkable product.

$a^2 – b^2 = (a + b) \cdot (a - b)$, from which $a^2 = (a + b) \cdot (a - b) + b^2$.

For example, we want to find the square of the number a = 45; as a value of b we always take 5.

Then

$$a + b = 45 + 5 = 50$$

and

$$a – b = 45 – 5 = 40$$

it results

$$452 = (a + b) \cdot (a – b) + b^2 = 50 \cdot 40 + 25$$

and therefore

$$452 = 50 \cdot 40 + 25 = 2000 + 25 = 2025.$$

The first two digits are the product of 4 by 5 (the next of 4); the last two digits are 25.

To convince students of the usefulness of algebraic calculation, one can think of an hour in which to organize a mental calculation competition, subdividing the class into teams and proposing exercises of the type described below.

Quiz 1

The wizard’s trick works when the number you want to make the square is halfway between two tens (when it ends with 5), but it does not work in other cases.

Here is how the symbolic calculation, in particular the formula of the square of a binomial, allows you to find the square also of the other numbers. This method is convenient if the numbers end with “close” to 0 digits.

Two examples:

$$21^2 = (20 + 1)^2 = 400 + 2 \cdot 20 + 1 = 441$$

$$19^2 = (20 - 1)^2 = 400 - 2 \cdot 20 + 1 = 361$$

Try, now, to quickly calculate the squares of some numbers.

Actually, the last examples are debatable because probably for some students it is faster to directly perform multiplication $21 \cdot 21$ than to resort to the binomial square. On the other hand, it is important for students to take more paths to get a result, even to activate forms of control.

Quiz 2

Now look at how you can use the difference of square formula to calculate some products:

$$19 \cdot 21 = (20 - 1) \cdot (20 + 1) = 202 - 1 = 399$$

$$14 \cdot 16 = (15 - 1) \cdot (15 + 1) = 152 - 1 = 225 - 1 = 224$$

$$48 \cdot 52 = (50 - 2) \cdot (50 + 2) = 2500 - 4 = 2496$$

$$97 \cdot 103 = \ldots$$

You have three minutes to write other products similar to those of the examples presented above.

Stage 5

This is the last stage of the activity. We want students to understand the “power” of symbols that allow us to express, communicate, generalize and solve problems. Strengthening the idea, already explained in previous activity, that symbolic calculation is not meaningless and repetitive activity, we want to underline the importance of “knowing how to transform an algebraic expression in a desired sense”[46]. In transforming a formula, new information is produced and different aspects of the situation to which the formula refers are revealed.

The price of the dress

In this first phase we can present a problem where the use of the symbols is necessary to solve it (common sense is undoubtedly useful, but sometimes leads to incorrect answers).

The following question is proposed to the class:

[46] This sentence is taken from § 2 Algebra, Mathematics Syllabus, published in 1980 by the Italian Commission for the Teaching of Mathematics (CIIM) of the Italian Union of Mathematics (UMI).

It is known that the price p of a dress has been increased by 6% and, also, has been decreased by 6%. One can not remember, however, whether one or the other of the operations took place before. What can be said about the final price of the dress?
(taken from: the Italian state examination at scientific school in 2007)

The teacher must invite the students to read the text carefully, before starting the discussion, helping them to formulate conjectures. The first thing to ask is whether to increase the price by 6% and then to reduce it by the same percentage, is the same thing as to decrease it by 6% and then increase it by 6%. Some guys will probably tend to conclude that the two proceedings lead to the same result and that the two successive transactions do not change the price of the dress (probably due to a sort of "compensation"). Others will say that the final price is conditioned by the order in which the two operations are performed: in one case the price of the dress increases and in the other it decreases. The teacher at this point, if it does not arise spontaneously, must induce the need to go to the symbols to clarify the situation.
6% increase on p:

$$p + 0.06p = 1.06p$$

and then 6% decrease on 1.06p:

$$1.06p – 0.06 \cdot 1.06p = 1.06p \cdot (1 – 0.06) = (1.06) \cdot (0.94)\ p$$

6% decrease on p:

$$p – 0.06p = 0.94p$$

and then 6% increase over 0.94p:

$$0.94p + 0.06 \cdot 0.94p = (0.94)\ (1.06)\ p$$

The two situations lead to the same result.
The passage to the symbols has shown clearly and concisely that the order in which the operations are performed is irrelevant and that there is always a decrease in the price of the dress.

<u>The pond</u>

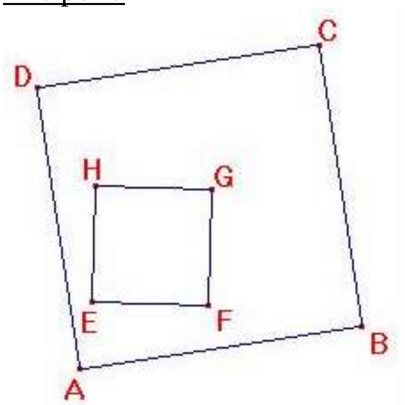


Figure 4.43: The pond

The activity goes on to underline how an appropriate algebraic manipulation allows to arrive more easily to the solution of a problem. The following situation is presented to the class:

A plot of land consists of a square (ABCD in Figure 4.43). Inside it, there is a square-shaped pond (EFGH). Both the external perimeter of the ground and of the pond are fenced with metal mesh. 360 meters of network were needed in total. In addition, the ABCD fence required 280 meters more network than the EFGH fence.What is the area of the walkable part of the plot of land?

After a classroom discussion, managed by the teacher, you should see, for example, with *a* and *b* the measures of the side of the ground and the side of the pond, and then write the following reports deduced from the data of the problem:

$$4a + 4b = 4 \cdot (a + b) = 360 \text{ and}$$
$$4a – 4b = 4 \cdot (a - b) = 280.$$

At this point the fundamental step is to write the expression of the walkable area of the ground

$a^2 - b^2$ and to exploit the remarkable product $(a + b) \cdot (a - b)$ in order to replace the values of $(a + b)$ and $(a - b)$ taken from previous reports. The result is $6300m^2$.
Of course, you can also derive the values of a, b by solving a simple system, but the problem does not require it. Another interesting thing to note is that in this case, the irregularity of the figure does not recommend the evaluation for the decomposition of areas: the equal shape of polygons is not effective.

Didactical remarks

We make some consideration on the light of our theoretical framework. This activity is the core of the module and, together with the messages shared with the videos of Prof. Arzarello, constitutes the ideal praxeologies that the trainers want to transpose to the trainees. Then, we pull the strings on all these stages of the activity we have shown, to briefly illustrate this meta-didactical praxeology.
This activity is placed at the central moment of the introduction to the symbolic calculation, when it is formalized the set of rules of calculation that constitutes the characteristic trait of algebra.
It is at this point that the fracture between symbols and meaning can occur, as Prof. Arzarello said in the video. The proposal places the symbolic calculation in the context of activities where the meaning of the rules is maintained. With this in mind, the first two stages suggest an itinerary to arrive at the basic rules of the symbolic calculation in close connection with the properties of the numerical calculation, of which they are the generalization. In the passage from arithmetic to algebra the reflection on the distributive property of the product with respect to the sum plays a fundamental role, which allows to prove the formal correctness of the special products (in general, not only those presented here). However, it should not be overlooked that the other properties of the operations are also present: this can be done in one of the phases of the arrangement of the argument. There is also an attention to the geometric interpretation of the calculation rules. This favours the memorization of the rules themselves, and facilitates the understanding of their justification.
If in the first stages, arithmetic helps the algebra, in the subsequent ones the algebra helps the arithmetic. In fact, the stages 4 and 5 are intended to illustrate the usability of the symbolic calculation, both in relation to the arithmetic calculation and in the resolution of problems. Special attention is paid to rapid mental computation, an activity often forgotten in our schools, and instead of great utility, because it allows us to maintain “confidence” with numbers and to detect unexpected properties.
With stage 4 (and other in-depth activities), it is shown how algebra allows to develop simple and powerful calculation techniques, but at the same time sophisticated. In stage 5, some problems are presented that highlight the need to use symbols to be solved (the dress) and problems in which an appropriate algebraic manipulation allows to arrive at the solution (the pond).

Forum

After “Arithmetic helps algebra and algebra help arithmetic” activity, a Forum follows (in the pink box, Figure 4.38). The trainers have inserted a delivery in order to stimulate the discussion among the trainees:
“Share your teaching experiences related to the conceptual nodes of mathematical language”
We will talk about this more thoroughly later in the section dedicated to the interactions.

Adaptations and insights of the activities on Sway tool

Next, there are a series of activities organized in five Sways (Figure 4.44). They are adaptations of the “Arithmetic helps algebra and algebra help arithmetic” activity and insights related to the conceptual node of mathematical language. However, we will not go into details of any of them. because the attention of the trainees who have intervened in the communication message boards was for the majority devoted to the proposals of the first activity, which we have illustrated in detail above.

Figure 4.44: Module 5 of MOOC Numeri (part 4)

Task

The module ended with a final task (violet box, Figure 4.45) to be accomplished using the tricider communication message boards. The trainers delivery is:

Add an example in which the language of algebra becomes a tool for expressing relationships and generalizations. Vote what you like best. In addition, you can comment on the various *proposals that you have viewed in this module, which propose ideas and paths to be included in the curriculum.*

We will talk about this more thoroughly later in the section dedicated to the interactions.

You can see in Figure 4.45 how tricider show itself. Its structure is very similar to that of the forum. There is the possibility to open discussions and make interventions nested. Contrary to the forum, the time when the post is written is not reported. Tricider is to be understood as a collector of ideas and those who read can also express agreement or disagreement leaving "like". For this reason, a quick description of Tricider could be "it is useful for easy brainstorming and voting" (as we have reported in Table 3.2 in Chapter 3). Tricider also counts how many likes each discussion receives and the most voted one is marked with a star.

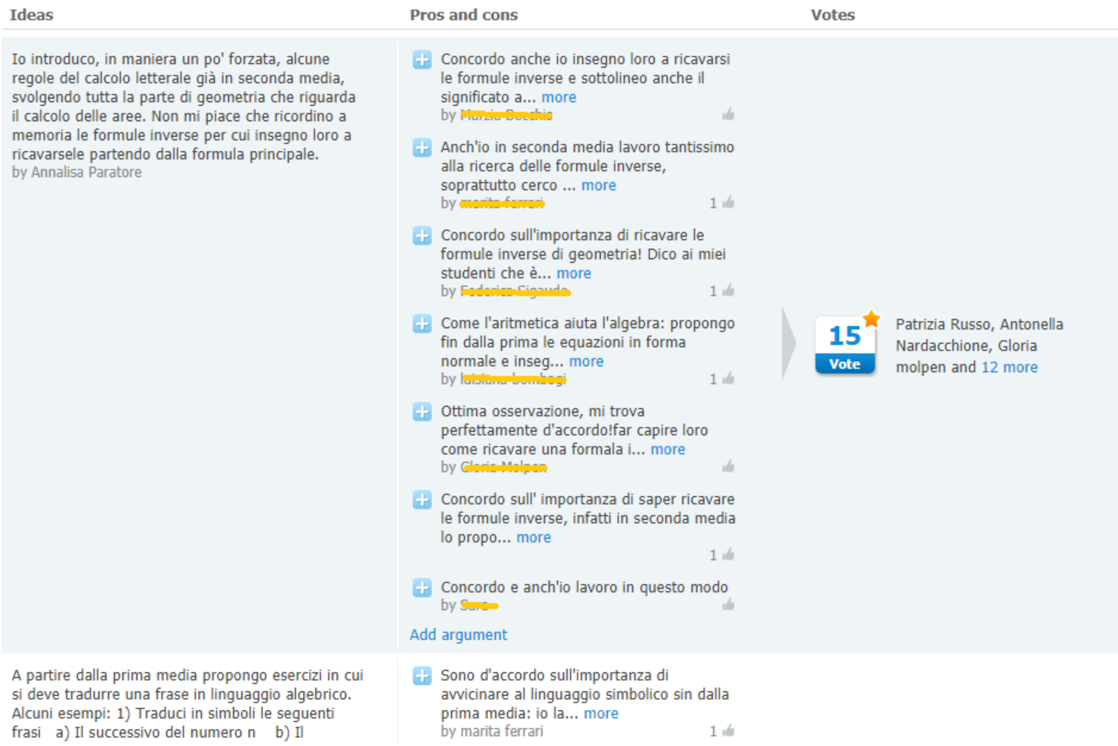


Figure 4.45: Tricider in module 5 – MOOC Numeri

Badge docente sperimentatore

Finally, there is a link (grey box of Figure 4.45) called "Badge to be an experimenting teacher". It is a reminder addressed to the trainees, to remind them to consider the possibility of participating in the experimentation of one of the activities of the MOOC or their Project Work and return a logbook prepared (as specified in §3.3.2.3). We will talk about this more thoroughly in Chapter 5.

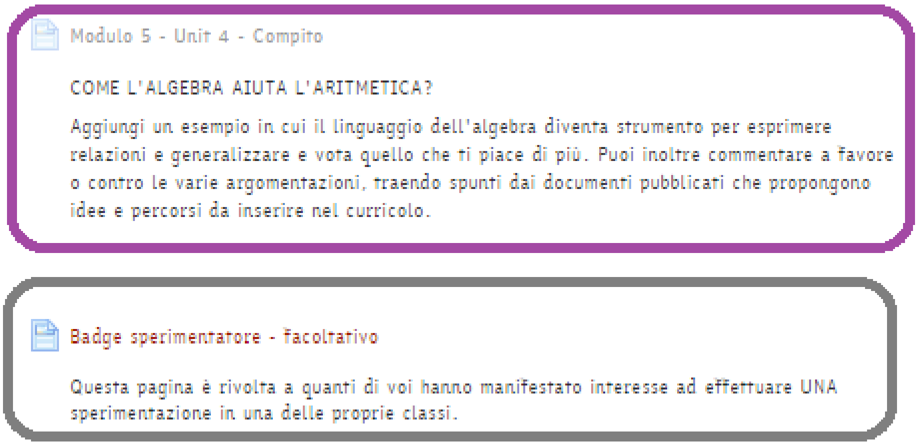


Figure 4.46: Module 5 of MOOC Numeri (part 5)

We conclude the presentation of module 5 of MOOC Numeri showing the network of knowledge of the MOOC-artifact (Figure 4.47)[47] related to what we have exposed about this module 5.

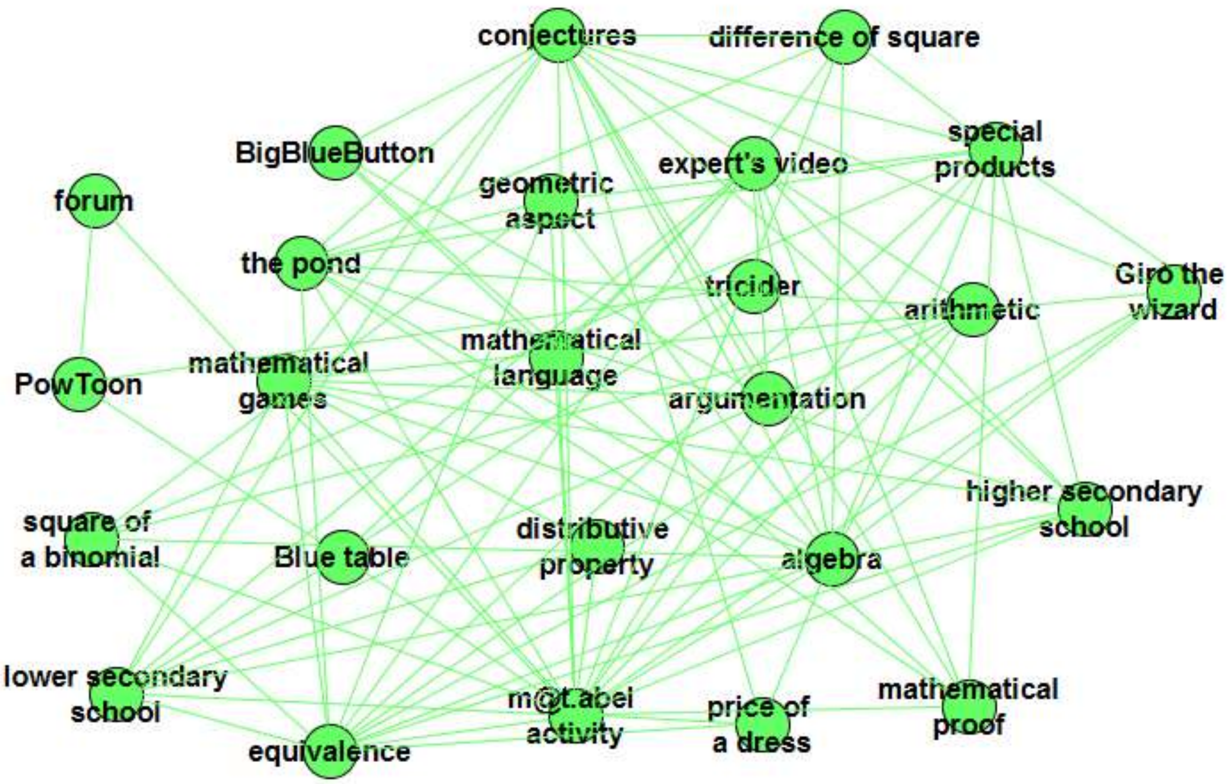


Figure 4.47: The network of knowledge of the MOOC-artifact referred to the module 5

Also in this case, it seems that chaos is embedded in it. For sure, it is not a feeling that the trainees experimented, because it is the fifth module of MOOC Numeri and the previous ones

[47] The graphs were created with yEd Graph Editor, an open source software that can be used to quickly and effectively generate high-quality diagrams (see: https://www.yworks.com/products/yed).

had the same setting. We underline again that the network of knowledge here showed is relating only to what we have presented in this analysis of the module 5. This means that all the connections that could have come out were missing, if we had also taken into consideration the activities of the sways.
Again, all the nodes and the arcs in this network are green because it represents the design actions and digital transposition made by the trainers (remember the remarks made in in Figure 2.4 in Chapter 2 about the colour to identify trainers and trainees).
We could say that, it is the image that represents the ideal praxeologies that the trainers would like to transpose to the trainees in this fifth module. In fact, all the connections between the nodes are the result of the meta-didactical praxeologies of the trainers (as you can discover if look at them).
Now, we move on to show the interaction that take place in this module, to enrich this network with orange elements.

4.6.1 A quantitative overview of the MOOC Numeri as repository

We have examined in detail the contents proposed by module 5 of MOOC Numeri. We have also specified (in §3.2.3) that the modules have more or less the same structure. We will not go into the details of all the other MOOC Numeri modules; rather we will show the judgment expressed by the trainees by making an analysis of the answers that come from the final questionnaire. We remember that it registered 116 answers, in particular, as anticipated in the methodology section (Chapter 3), these answers are relative to the trainees who started and finished the MOOC in all its stages). At this stage, we will not make the distinction between the trainees in the "*former"* one (the trainees that were also enrolled in MOOC Geometria) and "*new entry*" one (the trainees that were enrolled only in MOO Numeri).

With a series of closed and semi-open questions with multiple answer options, which we will now see, we asked trainees to express their judgment on the MOOC Numeri materials.
First, we asked if the contents of the MOOC corresponded to their expectations. The answer options were: *Yes*, *Partially*, *No*, *Other*. In Figure 4.48 we can see the answers distribution. The expectations of the majority (72%) have been met. Moreover, the 3% who declared *Other* is made up of 3 people who have all expressed themselves in the same way. We repeat one of the sentences they wrote: "*they have overcome them [the expectations]*". So, we could add the percentage declared in *Other*, to the percentage of *Yes* (ranging from 72% to 75%). No one answered *No*.

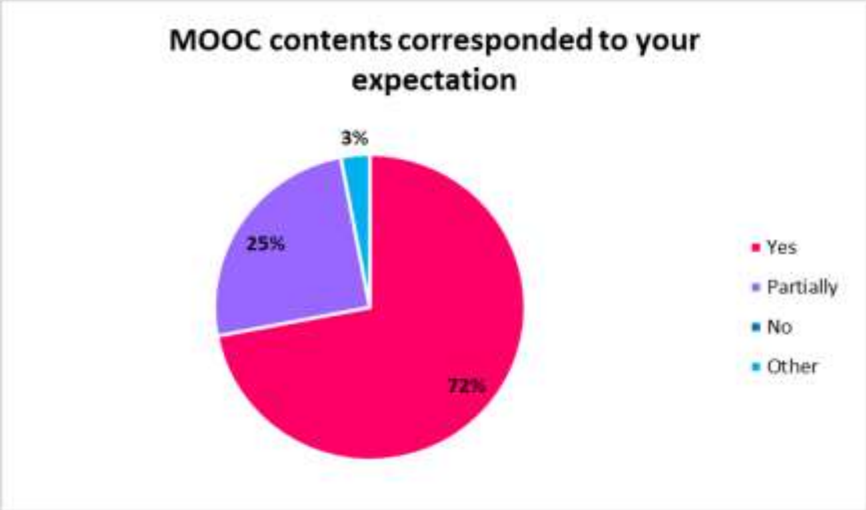


Figure 4.48: Did the contents of the MOOC Geometria correspond to your expectations?

Then, we asked how the trainees judged the duration of the course with respect to the topics discussed. It was a closed question with the following answer options: *Insufficient* (21%), *Good* (78%), *Too long* (1%). The majority states it was a good duration compared to the topics that were treated.

Later we asked how the trainees evaluated the teaching materials provided (videos, activity proposals, Geogebra files, ...). There were three response options that recorded the following percentages: 100% *useful*, 0% *fairly useful*, 0% *useless*. This testifies a total appreciation of the resources provided to the trainees. Moreover, with a multiple-choice question, we wanted to ask the trainees how much they had made use of the resources made available in the MOOC. Making use of the materials, in this question, means having explored the resources made available on the platform, having reflected on them, possibly having them also used in the classroom. In Figure 4.49 we can see that only 20% of them said they had used up to 40% of the MOOC materials. The majority declares instead to have widely used them, denoting an active participation in the course.

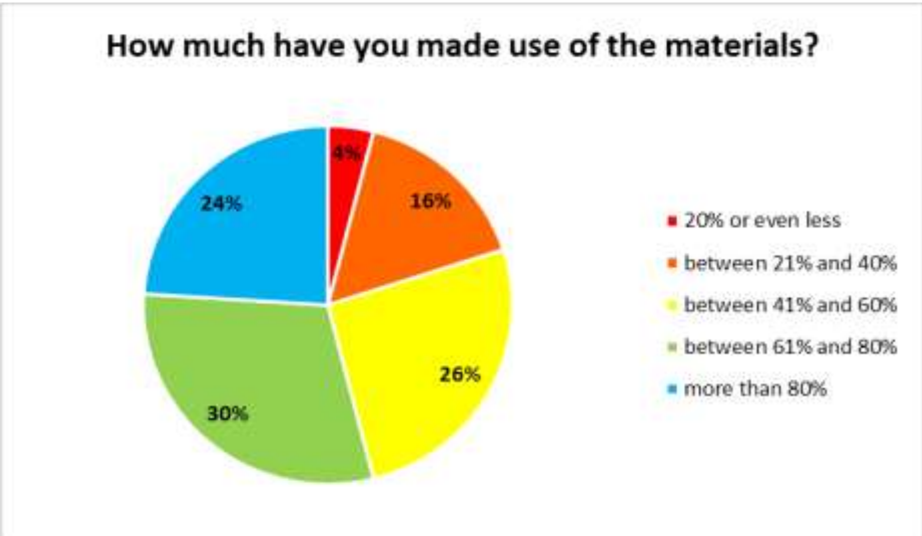


Figure 4.49: How much have you made use of the materials (e.g., videos, activity proposals, software, ...) provided in MOOC Geometria?

In particular, to the more specific question: "*Have you already transposed, at least in part, in your teaching practices what it has been learned in the MOOC?*" – closed question with answer options *Yes*, *No* – the 78% (exactly as in MOOC Geoemtria) answered *Yes*. This underlines once again an active participation, an appreciation of the materials and a willingness of the trainees to try experimenting new (or in any case different from their usual) didactical practices.

We have not asked, neither this time, the trainees if they consider that the MOOC is a repository, but it is the conclusion to which we can easily arrive from the analysis of these responses. Moreover, as specified in §3.2.3, even if the activities of delivery of the MOOC are concluded, its materials can continue to be consulted. So, actually, having positively judged the materials contained in it, both ongoing and after its duration, the MOOC is for the trainees exactly a repository. Namely, a place where you can find interesting activities, discover new didactical methodologies and be able to propose them to your students in the class.

4.7 Interactions in module 5 of MOOC Numeri

There were three different opportunity to establish interaction among trainers and trainees in the module 5 of MOOC Numeri.
During the week dedicated to this module, the trainers have planned a webinar. Moreover, the communication message boards inserted into the module 5, as we have seen, are forum and tricider.
We make some considerations about the webinar and then we propose an analysis of the interactions that took place on the communication message boards using the lens of the double learning process. Note that in this step we are moving progressively from MOOC's ZFM to MOOC's ZPA (see Table 4.1).

Webinar
Since in this module there was planned a webinar, we take advantage of this to report here some data and some comments on these experiences in general.
As we had occasion to mention (in §3.2.3.3), the webinars are online meetings in which an expert – seen through a camera – shares with the trainees – who can only interact via chat – some issues about the research in mathematics education and focuses on some questions that could be raised during the previous weeks in the MOOC. In particular, the experts discuss relevant topics, share personal experiences and valued resources, and suggest strategies for implementing knowledge gained from research in everyday classrooms.
Big Blue Button is the tool that allow these educational online events. In both MOOC Geometria and MOOC Numeri there were three webinars and they were organized for creating occasions of synchronous contact with the trainees. Participation in these events was felt.
The Table 4.14 summarizes the maximum number of presences that have been reached during each of these meetings.

	MOOC Geometria				MOOC Numeri		
When	11/11/15 Prof. Robutti	24/11/15 Prof. Arzarello	10/12/15 Dr. Coviello		15/11/16 Prof. Robutti	25/11/16 Dr. Coviello	5/12/16 Prof. Arzarello
What	Focus on Module 2	Focus on Module 3 – The proof in mathematics	MERLO methodology		Presentation and introduction	MERLO methodology	Focus on Module 4 - The recursion
How many participants (%[48])	90 (21%)	70 (17%)	50 (12%)		52 (19%)	36 (13%)	37 (13%)

Table 4.14: Webinars in MOOC Geometria and MOOC Numeri

They may seem small numbers compared to those registered in the MOOC, but it should be noted that these were afternoon meetings (from 5 pm to 6:30 pm) and took place on a weekday day. Sometimes the trainee-teachers could not take part in them because they were involved with school boards. We report, for example, some comments of trainees of MOOC Geometria that for other commitments could not participate in synchronous to the first webinar.
"*For problems that have occurred, I will not be able to be at the webinar tomorrow. What should I do?*"
"*Good morning. Wednesday, November 11th there is the webinar. I am on duty at school. I can take a permit if necessary*".
To meet the needs of everyone, webinars were video-registered. In this way, anyone who could not follow in synchronous, could have recovered asynchronously. So, the answer to the previous comments was not to worry because there would be video recording. However, these comments make smile among the trainers. They testify to the interest in participating in these initiatives that were then consolidated also in the edition of MOOC Numeri.

4.7.2 Forum in module 5 – MOOC Numeri

The trainers have inserted a delivery in order to stimulate the discussion among the trainees:
"Share your teaching experiences related to the conceptual nodes of mathematical language"
The forum collects a unique discussion, in which there are 55 interventions.
The Table 4.15, show what types of interventions contains the forum, according to what we have explain in the methodology section (§3.3.2.1). As is easily understood, each intervention is not necessarily classifiable within a single category.

A = explicit answer to the questions/observations	3
B = considerations	42
C = sharing of materials	5

[48] Compared to the total number of MOOC participants: 424 in MOOC Geometria; 278 in MOOC Numeri. In reality, it does not make much sense to consider the total number of MOOC participants, if not in the first webinar of each MOOC. This is so because the number of actual active trainees is never corresponding to the number of participants, due to the physiological drop out rate that characterizes all MOOCs in general.

D = sharing of experiences	3
E = experimentation	2
F = other	0

Table 4.15: Types of intervention of the trainees on the forum in module 5

The forum also keeps track of the date and time when the post was published. It is interesting to note that the trainees were writing at any time of the day or night, in a range from 07:54 am (in the early morning) to the 01:09 late at night.

Let us see in detail some of the most interesting discussions or interventions. The previous methodological clarification (made on Forum in module 1 – MOOC Geometria) are valid.

Intervention (by X; on gg/mm/aa; at hh:mm)	**Category**	**Intervention**	**Double learning process**
A.B.; 5/12/16; 22:33	B	I teach in a lower secondary school and, although it is clear that this is an activity [Arithmetic helps algebra and algebra help arithmetic] to be performed in a higher secondary school, I find the first two [stages] **really stimulating**. **I like** the problematic introduction and the enrichment with the geometric appearance, which **I already use** for the literal calculation in the third.	E→I
G.P.; 6/12/16; 23:47	B	Indeed, the proposed activities are a bridge between lower and higher secondary school. However the stage 1 and in part the stage 2 can also be proposed in the 6 grade when, by treating the four operations and their properties, the mental calculation is dealt with. It is true that the stage 2 can be fully included in the 8 grade where you could try to treat the stage 3. Even if you 'lose' a lesson maybe you give someone the chance to have an extra tool or it could be a way to enhance excellence.	E→I E←I
A.A.; 12/12/16; 12:04	B	**I agree**, but **I think** the activity on the price of the dress is feasible and **very interesting** (to solve the obstacles related to the %) [...] Dedicating some time to the geometric interpretation of the difference of squares, I think you could also try Girò the wizard and the pond .... **funny algebra**!	E→I
S.B.; 11/12/15; 18:32	B	Yes, **I think** it is essential not to wait for the third year [8 grade] to do algebra and literal calculation ... Already from the first one [6 grade] (and from the primary) they are exposed to the letters ..., formulas of the perimeters and areas or simply when the properties of the operations are resumed and generalized. [...] **I advise** you to sow already in the first [6 grade] so then in the second [7 grade] they do not memorize all the inverse formulas of the areas, but get them ... :-) P.S. **beautiful** games "guess number" [stage 1] and the development of notable products [stage 2]	E→I E←I

Table 4.16: Trainees' interventions on the forum in module 5

In Table 4.16, the considered trainees teach in a lower secondary school. The activity "Arithmetic helps algebra and algebra help arithmetic" is presented as an activity for 9-10 grade. However, as was hoped, the trainees were not only positively affected "*I like …*; *the activity is […] very interesting*; *funny algebra*", but they also think they can readjust it into their classes.
There is therefore an **instrumentation/self-organization** process in them, which is leading them to reflect on a possible use of the activity in their own classes. In particular, G.P. show a more organized network of knowledge than A.B. and A.A., because he has already come to think about which classes to propose what. He has already made estimations: "*Even if you*

'lose' a lesson maybe you give someone the chance to have an extra tool or it could be a way to enhance excellence". He then put in place a process of **instrumentalization** when he completed his reflections and **sharing** in the moment in which he shared them with the rest of the MOOC-ecosystem.
Interesting is also the comment of S.B. He **share** a his own thought, that is do not wait the 8 grade to start to use letters or to talk about algebra. He underline that the students, even at primary school, are exposed to them. So he urges his colleagues not to be reticent, and he does so with a smile!

Intervention (by X; on gg/mm/aa; at hh:mm)	**Category**	**Intervention**	**Double learning process**
D.B.; 6/12/16; 19:22	D	**I looked at the material** and today [I started to explain the passage] from the numbers to the letters [...] through classic magic games and also the one suggested by the MOOC team. The eighth graders were fascinated. I also mentioned the concept of generalization and demonstration. The students, even if not all, from the questions they have posed, seem to have understood the message. **I will continue the activity in phases**: in the first one you will probably learn the algebraic calculation without asking too many questions about "what you are doing and why". In the second phase I would like to develop a more articulated path, highlighting the need for a correct translation from the common language to the mathematical language and vice versa. You will go through a geometric representation of the concept, until you get to simple demonstrations on algebraic properties that the boys will have to discover through cooperative methodology [...] fortunately on Friday I have two hours of consecutive mathematics [...]	E←I

Table 4.17: Trainees' interventions on the forum in module 5

In Table 4.17, D.B. has already implemented the **instrumentation/self-organization** process; in fact, she has already seen the materials, evaluated them according to her Math Edu USs, selected which ones to use and implemented them in the classroom. We do not know how exactly she managed the lesson, or what the questions the students have done, to the point of making her believe they understood. However, she is satisfied with this result and **shares** with the MOOC-ecosystem her didactical praxeologies, organized in the light of a process of *internalization* of the MOOC activities (in fact she stresses that she wants her students to observe the passage from arithmetic to algebra and vice versa, enhancing also the geometric aspect). She knows that this activity will take her time, but she has already organized her network of knowledge in this sense. Indeed, she is heartened by the fact that Friday has two consecutive hours!

Another interesting comment in this direction is the one of S.L.C. (Table 4.18).

Intervention (by X; on gg/mm/aa; at hh:mm)	Category	Intervention	Double learning process
S.L.C.; 8/12/16; 10:37	D	**I proposed** yesterday in a 8 grade the first part of the activity 2 (the square of a binomial special product) [...] great participation in the 'game' phase but the first hitch: not all of them have achieved the expected meaning (calculation errors). Second hitch: some have not been able to correctly translate the arithmetic into algebraic language. Third hitch: many have shown great difficulty in 'finding the words' to justify the equality or have not even tried. **My expectations have not been met by reality**. **Despite this** or, indeed, for this reason, I will continue on this path	E←I

Table 4.18: Trainees' interventions on the forum in module 5

As D.B. (Table 4.17), also S.L.C. (Table 4.18) have already put in place the instrumentation/self-organization process, in fact she write about an experimentation conducted in her class, using MOOC materials. She clashes with what she calls '*hitch'*, or unexpected events, events that she had not considered. In fact, she had very different expectations: "*My expectations have not been met by reality*". However, she is not discouraged by this. They are one more reason for her to continue to insist on these topics. There is, therefore, a reflection on the reflection: if after consulting the materials in the MOOC, she had planned to handle the discussion of the topics in one way, now she has to reflect again to implement different strategies that can help her students to correctly internalize these mathematical meanings.

4.7.3 Remarks

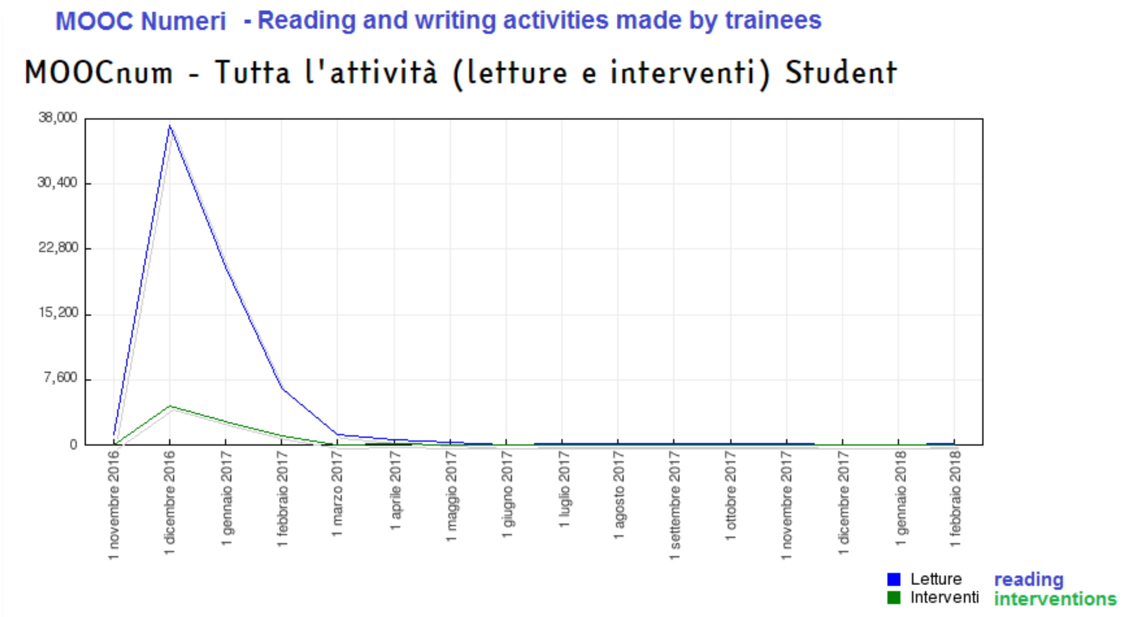
Considering in general the interactions that took place in the forum (including those that have been proposed here), we underline that this time (compared to the forum fo Module 1 in MOOC Geometria) there are not many nested interventions. The comments were mostly timely. It should also be noted that the MOOC Numeri was coming to an end and that the Christmas period was upon us, with all the school deadlines it brings. However, also in this case, it can be said that the "Forum" tool has been accepted and managed on the basis of a growth of knowledge of the trainees, namely in terms of expanding their own network of knowledge, adding the nodes proposed by the others (if the trainee considered them valuable). Also for the forums of MOOC Numeri we can consider the statistics generated by Moodle platform (Figure 4.50)[49]. Therefore, again examining click logs, we found there were also many more discussion views than postings. Some discussion views were done by participants who were active posters; however, other views were done by non-posters. Thus, many saw discussion forums as an opportunity for learning, even for just reading the posts of others. Participants who engage in such "lurking" are present, but not visible; thus, exactly why they read discussions and what they have learned from them is unknown.

About the other comments in the forum of module 5, in general they express agreement to the fact that "*the passage from arithmetic to algebra should be done gradually but consistently*"

[49] Remember that MOOC Numeri was delivered from November 2016 to January 2017.

(D.R.; 18/12/15; 08:49). The comment of S.C. (08/12/15; 17:58) provides a centered synthesis "*I am very happy to see that we agree that it makes no sense to favor the formal study of language without having acquired those interpretative and expressive skills, essentially linked to the mathematization and problem solving activities, which are at the base of 'doing mathematics' . In my opinion, the surprise, the paradox or the unexpected result are stimulating elements for the cognitive activity, they are 'the prestigious game of which we seek the trick'. When a student solves a problem or a game becomes a protagonist as an inventor or discoverer of the solution; his being an active subject positively influences his attention, the quality of learning and his motivation*".

It seems that a sharing of the meta-didactical praxeology that the trainers wanted to transpose to the trainees emerges. As previously observed, we cannot talk about shared praxeology as for MDT. In fact, sharing does not take place for everyone or even between everyone at the same time, but a slight sharing begins to take shape, at least among those who intervened in the forum.



Term period (Month)	Readings	Interventions	Log
30 giugno 2017	141	0	Corso Log
31 maggio 2017	53	0	Corso Log
30 aprile 2017	244	2	Corso Log
31 marzo 2017	611	121	Corso Log
28 febbraio 2017	1239	48	Corso Log
31 gennaio 2017	6655	1018	Corso Log
31 dicembre 2016	20795	2655	Corso Log
30 novembre 2016	37219	4561	Corso Log
31 ottobre 2016	1210	0	Corso Log

Figure 4.50: Reading and writing activities relative to forums in MOOC Numeri

4.7.4 Tricider in module 5 – MOOC Numeri

Tricider collected 131 discussion, each of them with from 0 to 15 response replicas (Figure 4.51), for 247 post in total. The trainers have inserted a delivery in it in order to stimulate the discussion among the trainees.

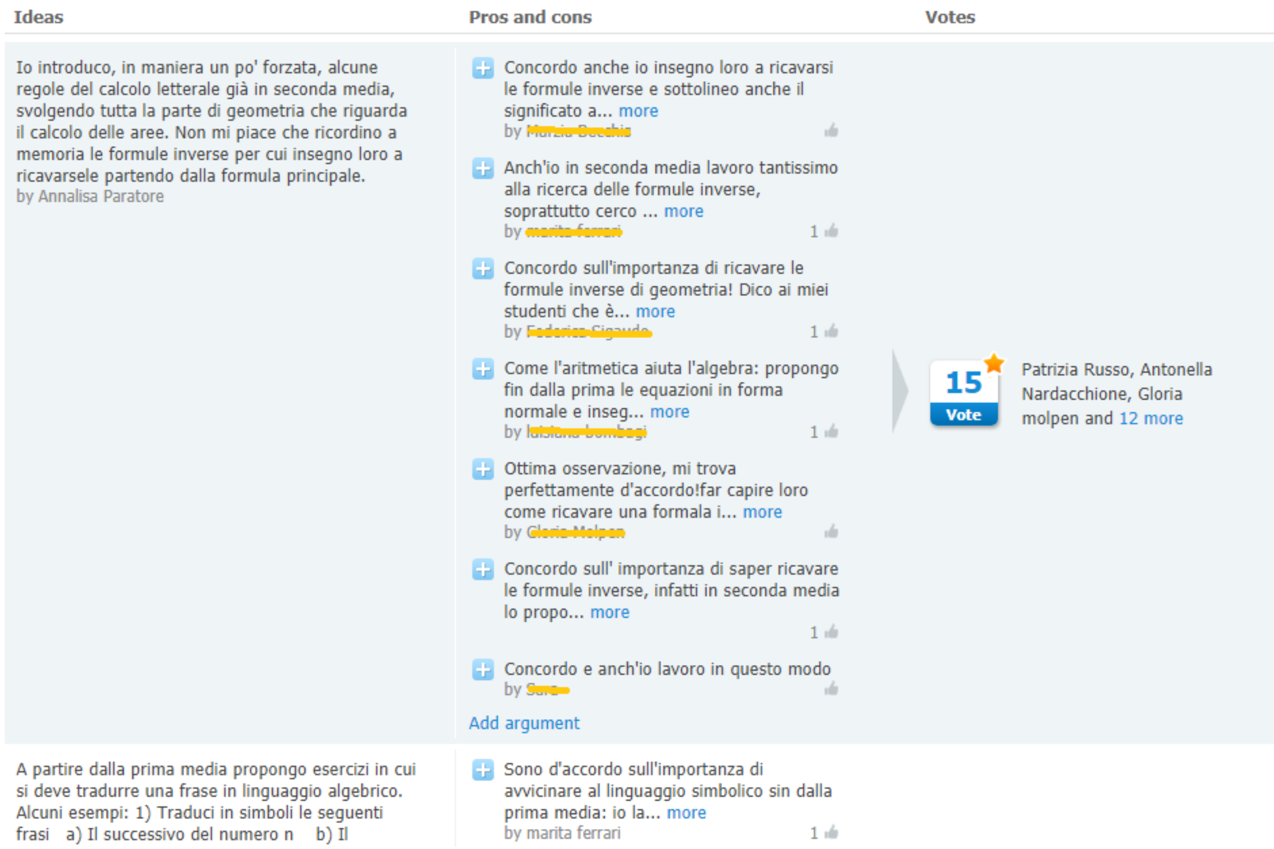


Figure 4.51: Interventions in tricider of Module 5 – MOOC Numeri

We must make a clarification. As described above, when we presented the tricider, it is a tool that should facilitate brainstorming and voting processes. Tricider had already been used in MOOC Geometria and makes its first appearance in Module 2[50], with the following delivery:

Which paths [among those proposed] use to build the meaning of angle/arc?
You can intervene by contributing and voting for the idea of the colleagues you share.

Therefore, for trainers, the tricider had the goal of triggering simple threads, most of all confined to the approval or not of ideas, by voting through “likes”. However, the trainees used it more for collecting ideas and comparing their didactical experiences – as a forum – rather than for the expected use. Practically, the trainees realized a *catachresis* (Verillon & Rabardel, 1995): an artifact is used to do something it was not conceived for. Due to the fact

[50] We have not entered the details of this form. However, it focused on the conceptual angle node and suggested strategies to overcome a widespread misconception among students: the difference between arc and angle.

that they explored the tool for the first time, and also because they usually need to explain and to go in depth when they express an idea, so the simple vote would not have let them satisfied. For these reason, the trainers have changed the way of making deliveries, accepting the fact that the trainees would not be limited to writing little. In fact, the delivery of this Tricider is more detailed the one inserted, for example, in module 2 of MOOC Geometria.
We report it here for the reader's convenience.

Add an example in which the language of algebra becomes a tool for expressing relationships and generalizations. Vote what you like best. In addition, you can comment on the various proposals that you have viewed in this module, which propose ideas and paths to be included in the curriculum.

The types of interventions contained in it the trickier are presented in Table 4.19. All of them can be intended as A (explicit answer to the question/observations proposed by the trainers or trainees). However, we made a more fine distinction in the other categories too.

A = explicit answer to the questions/observations	131
B = considerations	90
C = sharing of materials	27
D = sharing of experiences	75
E = experimentation	0
F = other	0

Table 4.19: Types of intervention of the trainees on the Tricider in module 5

Let us see in detail some of the most interesting posts and make some considerations. Since the trainees are invited to satisfy the delivery of the trainers, all of them have already putted in place the process of **instrumentation/self-organization**. Namely, they have reviewed the proposed materials. They evaluated them according to their own Math Edu USs. The request of the trainers substantially invites the trainees to share their Math Edu USs, to see how much assonance there is with the proposals they presented. That is, to understand what the trainees think when they have to take their students to the delicate passage from the arithmetic to algebra.

To make the analysis I have used the file filled by the monitoring groups (§3.3.2.1), but I have re-elaborated these data into another table and I use this one to show you the discussion/intervention.
We anticipate immediately that the most voted interventions were the first in chronological order. In the next phase the option has been little used. Therefore, in the analysis that will follow, the most voted interventions will not the one that we will necessarily expose. First we expose them, and after we make some considerations.

Discussion (started by X; secondary school)	**# reply**	**Reply (by X)**	**Category**	**Like received**	**Intervention**	**Double learning process**
MONKEYS AND MICE (by L.G.; lower)	4		C	8	[...] Algebra can be learned from the first year of lower secondary school using geometric designs or shapes instead of letters. I use the method of the squares and the dots to show, for	E←I

					example, that the sum of three consecutive numbers is divisible by three. [...] I attach an example [Figure X46].	
		S.M.; lower	B		**Very nice** this example: I will definitely use it!	E←I
		…				
		M.S.; lower	B		**Nice example**, **very clear**. Thanks for sharing.	E←I

Table 4.20: Trainees' interventions on the tricider in module 5

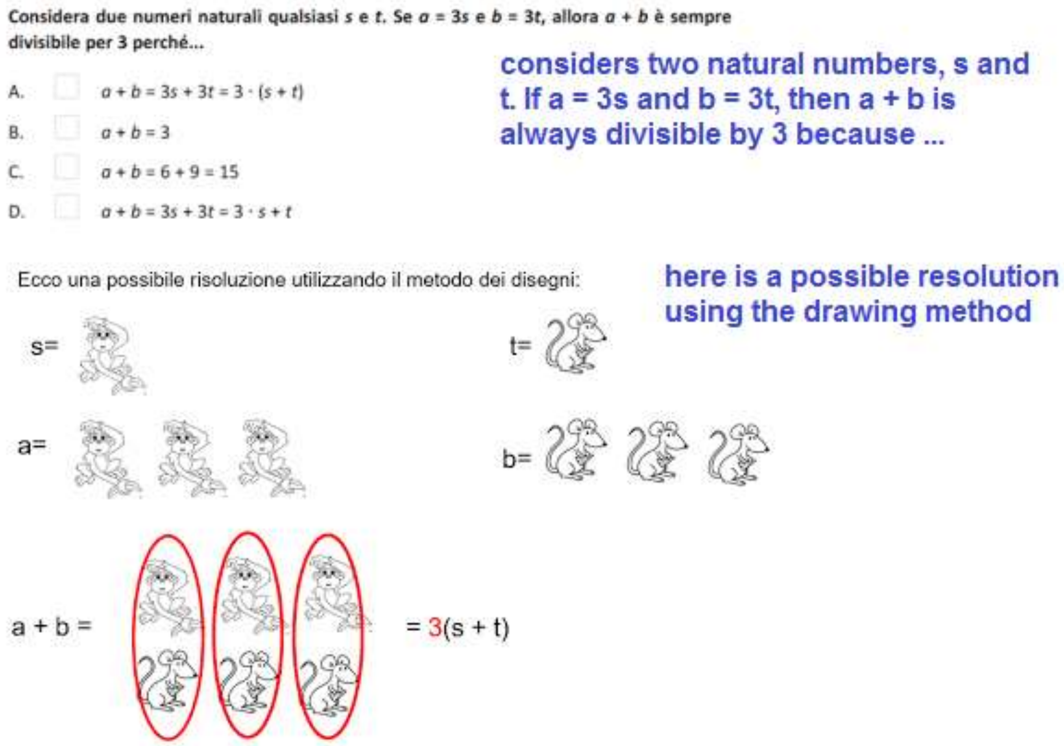


Figure 4.52: Example of activity attached by L.G.

Discussion (started by X; secondary school)	# reply	Reply (by X)	Category	Like received	Intervention	Double learning process
A.R.; lower	0		B, D	2	This year in the 8th grade class **I introduced the literal calculation** starting from the problems on the segments dealt with in 6 grade: **I divided the blackboard into two parts** with an arithmetic resolution on the right and an algebraic on the left. The comparison of the same problem in which a generic value 'n' was substituted for a specific value was in my opinion very effective. In fact, the boys themselves have requested other 'geometric' problems to be solved in this new way ... Clear that 1) I have been insisting a lot with the translation of the textual language in the	E←I

					mathematical one 2) I have necessarily followed the theoretical part (not negligible formalism)	

Table 4.21: Trainees' interventions on the tricider in module 5

Discussion (started by X; secondary school)	# reply	Reply (by X)	Category	Like received	Intervention	Double learning process
E.V.; lower	0		B, D	2	**I find** it useful to reason the boys in algebraic terms already in the first steps in geometry. At 6 grade **I introduce** the measurement of the segments and their comparison with exercises such as: "one segment AB is the triple of another segment BC and their sum measures 20 cm; how much is each segment measured?". The students translate the data in algebraic form (AB = 3BC; AB + BC = 20) and little by little they get used to solving the exercise by changing the value of AB as a function of BC in the sum. Naturally, **I make them reason by using simple pieces of string** knotted together that represent the segments, with which they can realize what is indicated in the data. Then they, with the aid of the drawing, come quite easily to the resolution of the exercise. Initially, they find some difficulties, but slowly, starting so early, at 8 grade students do not have great difficulty translating real situations or geometry problems into algebraic language, when they learn to use this language in a way that is finally more conscious	E←I

Table 4.22: Trainees' interventions on the tricider in module 5

Discussion (started by X; secondary school)	# reply	Reply (by X)	Category	Like received	Intervention	Double learning process
P.S.; higher	2		B, C	1	My 10 grade students generally have a rather negative approach to algebraic calculus. They tell me that they rarely understood the meaning of what has been proposed to him except a set of rules to be memorized. So when I resume the numerical sets **I often propose exercises** like the one in the image [Figure X47]	E←I
		…				
		D.R.; higher	B		The question of nonsense is really a **common problem**. My students usually say that "when there are letters, nothing can be understood ..."	E→I

Table 4.23: Trainees' interventions on the tricider in module 5

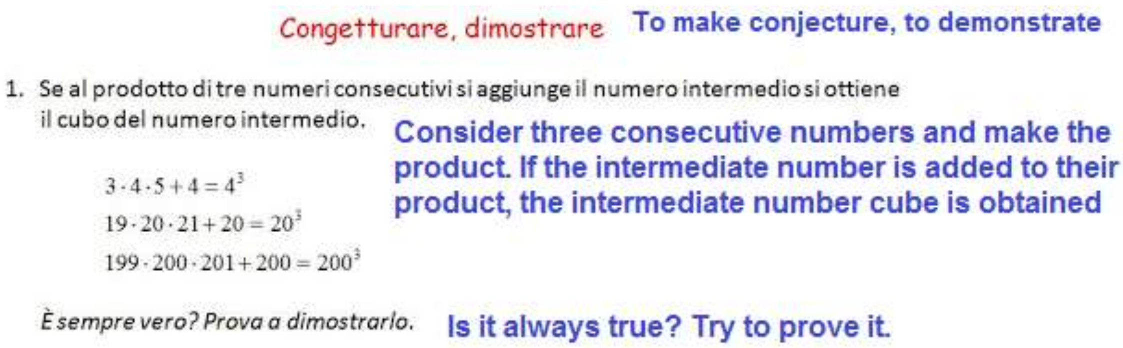


Figure 4.53: Example of activity attached by P.S.

The process that is outlined by reading the posts of the trainees is therefore that of the **instrumentation/sharing**. In fact, the trainees, with their renewed network of knowledge, think and build new connections independently. They are stimulated by a task requested by MOOC and address the ecosystem to transform according to their own (new) Math Edu USs. The MOOC welcomes the contribution of each trainee and makes it available to all: the information goes towards all members of the ecosystem that may be affected by the new content on the MOOC.

The interventions focus on the following aspects, which constitute a framework of the **didactical praxeologies** of the trainees:

- the need to avoid learning based mainly on automatisms, far from the deeper meanings of algebra (for example A.R. in Table 4.21; E.V. in Table 4.22);
- the importance of introducing the concept of variable already in the initial phase of the learning path (according to some interventions already in primary school) through a pre-algebraic language consisting of iconic-symbolic representations such as drawings, segments, flowers or animals, along a path of learning that gradually leads to abstract formalism (for example L.G. in Table 4.20);
- some trainees also suggest laboratory activities in which the role of the variable is covered by pieces of string, to give a tangible “concreteness” to the concept (for example E.V. in Table 4.22);
- motivate learning by introducing algebra as a powerful method for solving problems and constructing the first medializations represented by simple algorithms or simple equations (for example L.G in Table 4.20; P.S. in Table 4.23);
- algebra as a basic structure for the construction of functions starting from particular situations, for example in the geometric field (for example E.V. in Table 4.22);
- in the numerical field, algebra as a method to identify particular regularities in numerical sequences or to discover and demonstrate general properties (for example L.G in Table 4.20; P.S. in Table 4.23);
- on the methodological level, many of the participants' actions call attention to discovery activities in which the ludic aspect often organized according to group work is often requested (for example A.R. in Table 4.21);
- many underline the importance of the use of educational software and in particular of GeoGebra in teaching practice; in particular, activities are reported in which the algebraic development is flanked by numerical tables that maintain the link between the algebraic language and the numerical plan (for example A.R. in Table 4.21).

We conclude the presentation of the interactions that occurred in Module 5 of MOOC Numeri showing the network of knowledge of the MOOC-ecosystem (Figure 4.54) related to this module 5.
As you can see, it is more chaotic than the one showed for MOOC-artifact. Now, orange nodes and arcs have been added (remember the remarks made in §2.3 about the colour to identify trainers and trainees). They refer to all the nodes that the trainees have activated during the interactions in the communication boards (forum and tricider) and to the connections that they have generated between them and between the existing nodes in the MOOC-artifact network. Even in this case, the network of knowledge here showed is relating only to what we have presented in this analysis of the module 5. This means that, if we had taken into consideration also the activities of the other sway and all the interactions that occurred in the communication message boards, the network of knowledge of the MOOC-ecosystem it would have been much denser than it already is in the Figure 4.54.

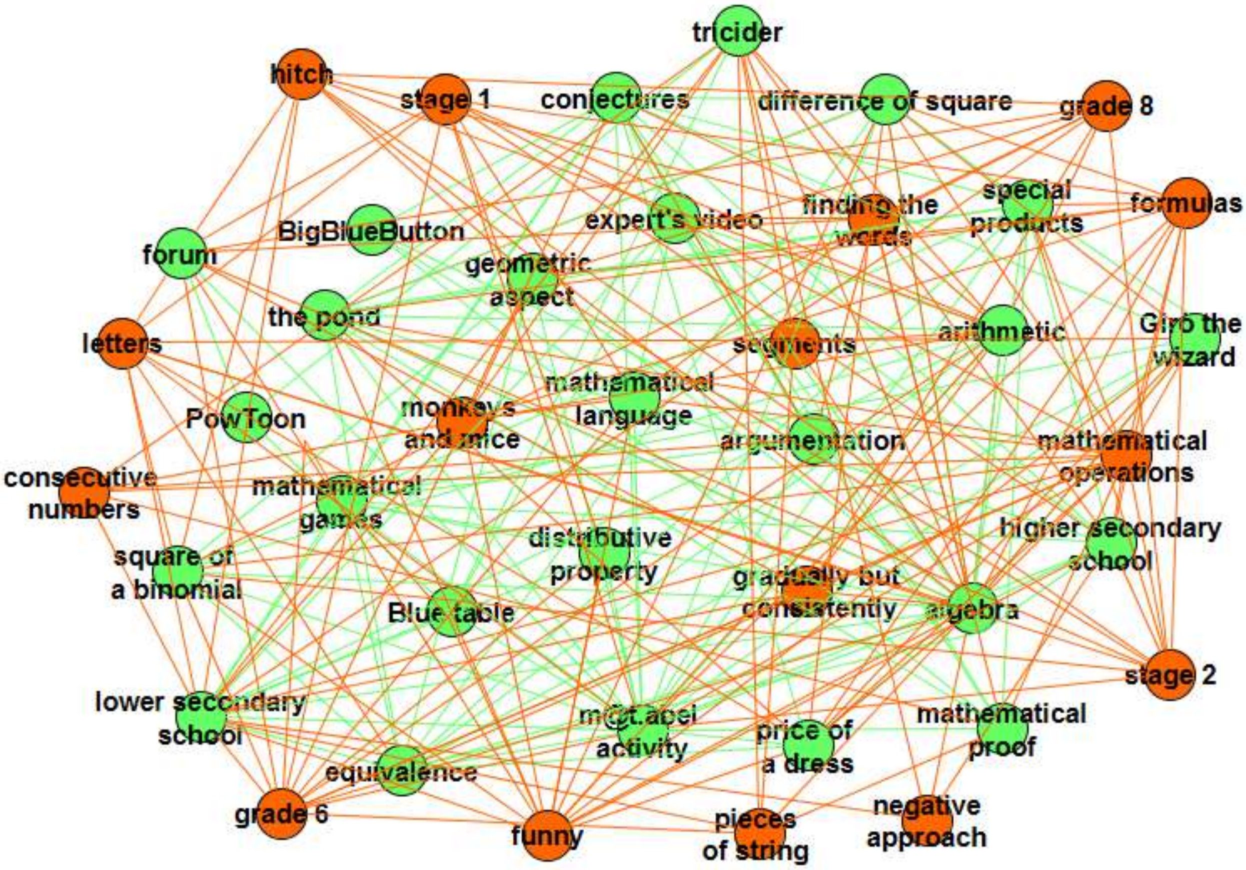


Figure 4.54: The network of knowledge of the MOOC-ecosystem referred to the module 5

4.8 A quantitative overview of the MOOC-ecosystem

After entering into the detail of some mathematical activities and interactions in communication message boards (forum, padlet, tricider), we now see the general judgment that the trainees have expressed about all the communication message boards inserted in the MOOCs.

The data that will be exposed refer to the questionnaires (intermediate and final) administered to the trainees of MOOC Geometria and Numeri. As anticipated in the methodology section (Chapter 3), we analysed only the answers of the trainees who started and finished the MOOC in all its stages.

In the intermediate questionnaire of both MOOC Geometria and MOOC Numeri, we inserted only a question relative to the communication message boards. With a Likert-scale question (a score of 1 represents totally disagreement and a score of 5 totally agreement) we wanted to understand what was the perception that the trainees had of these spaces set up for online communications. So, we ask them to express their agreement respect to the fact that the communication message boards could or could not allow them: to express own personal opinions about the course contents (Figure 4.55); to enable a comparison dialogue between colleagues (Figure 4.56); to give the opportunity to benefit from experiences/ways of thinking of the others (Figure 4.57). For each of these voices we report the data analysis referred to three distinct sample: the trainees that completed MOOC Geometria in all its stages, indicated from here and after in the diagrams as "*Geometria*"; the trainees that completed MOOC Numeri distinguished in "*former trainees*" (the trainees that were also enrolled in MOOC Geometria) and "*new entry trainees*" (the trainees that were enrolled only in MOO Numeri).

As it can be seen from the histograms (Figure 4.55, 4.56, 4.57), more than half of the trainees show agreement in the possibilitues offered by communication message boards. Precisely, if we consider the answers of those who have indicated a score of 4 and 5, we have that for the trainees enrolled in both MOOC Geometria and Numeri the percentage of agreement relative to the possibility that the communication message boards allow:

- to express their opinions about the course contents moves from 83% for *Geometria* and 75% for *former trainees*. This little decrease it may be because the participants, in their second experience (MOOC Numeri), have given more importance to the interactions with others in terms of comparison and sharing of ideas, rather than preferring to comment only on the contents of the course. In fact, the percentages referring to the 4.56 and 4.57 histograms remains almost unchanged, which therefore supports this hypothesis.
- to enable a comparison dialogue between colleagues remains constant at 77% for *Geometria* and *former trainees*.
- to have the opportunity to benefit from experiences/way of thinking of the others show a little increase from 90% for *Geometria* to 91% for *former trainees*.

About the *new entries*, they express an agreement equal to 79%, 76% and 80% for each of three answer options respectively (if we consider the answers of those who have indicated a score of 4 and 5).

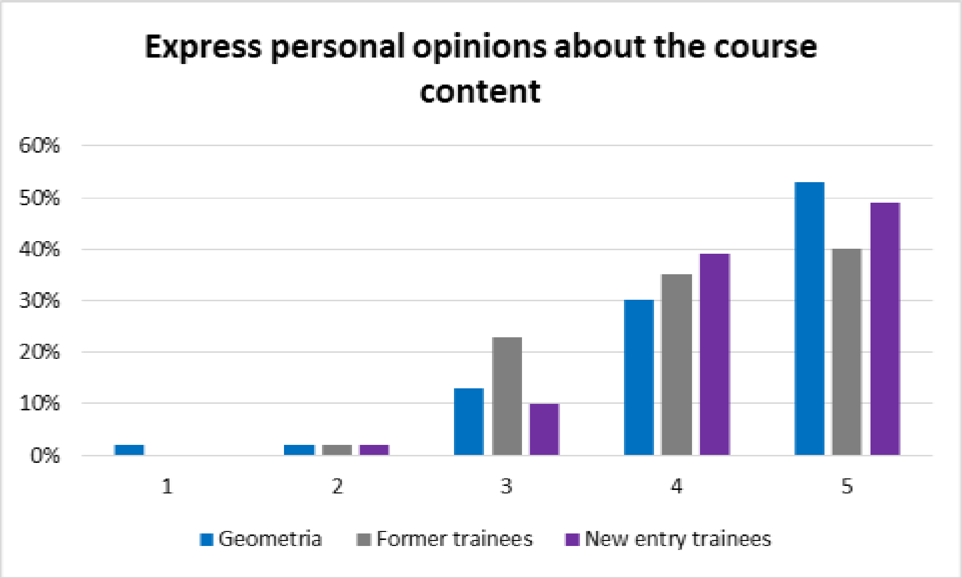


Figure 4.55: Degree of agreement expressed by the trainees about the possibility that the communication message boards allow to express their opinions about the course contents

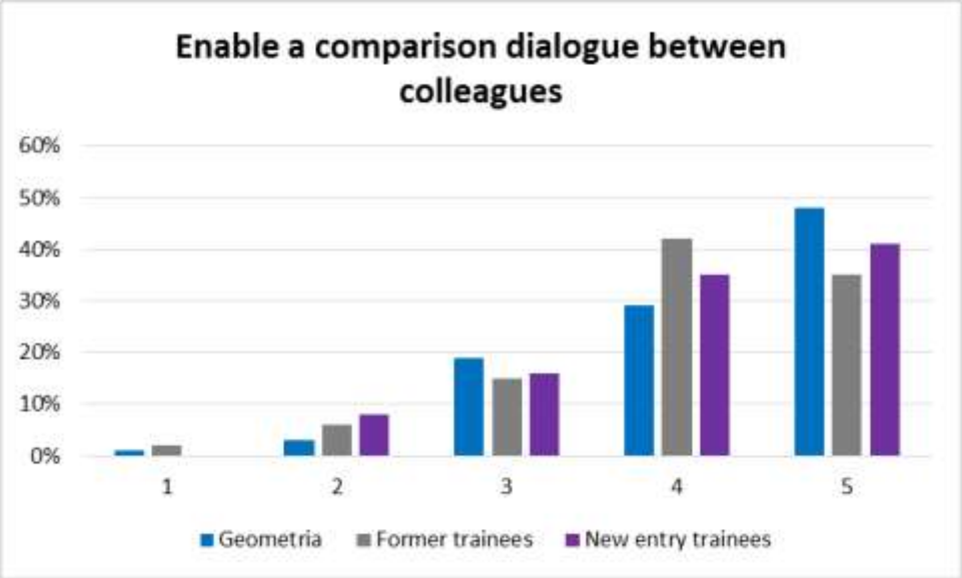


Figure 4.56: Degree of agreement expressed by the trainees about the possibility that the communication message boards allow to enable a comparison dialogue between colleagues

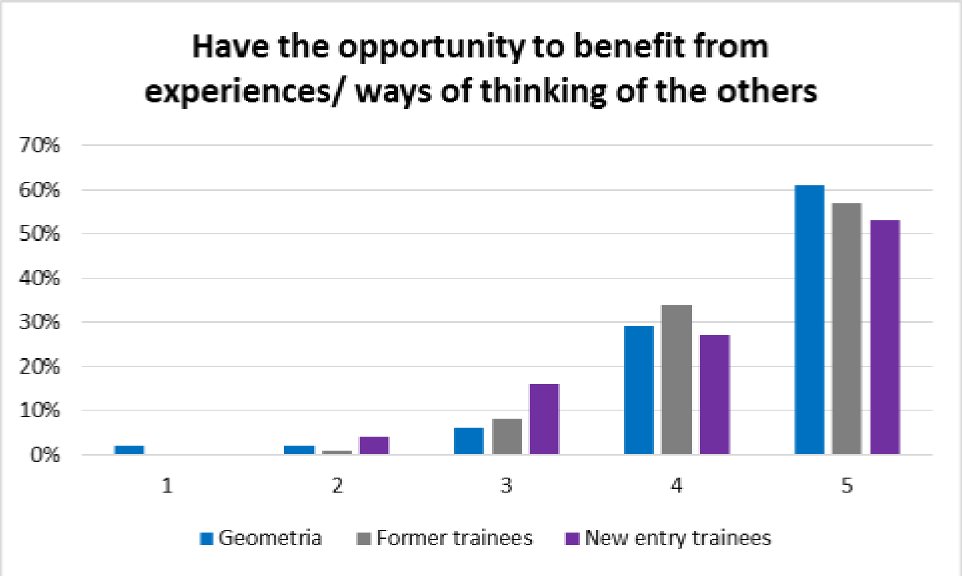


Figure 4.57: Degree of agreement expressed by the trainees about the possibility that the communication message boards allow to have the opportunity to benefit from experiences/way of thinking of the others

We again emphasize that this question was asked in the intermediate questionnaire, at halfway through the educational course. The fact that the percentages of the "*former trainees*" are confirmations of those of the previous edition suggests that their judgment on the communication message boards has not changed and therefore they hold them valid for the indicated purposes. While the "*new entry*" trainees, already halfway through, they consider them effective to pursue the goals on which an evaluation was requested.
A clarification must be made: in the intermediate questionnaire of MOOC Numeri, to these three answer options, related to the possibilities offered by the communication boards, another has been added, namely "*Develop the sense of belonging to a community*".

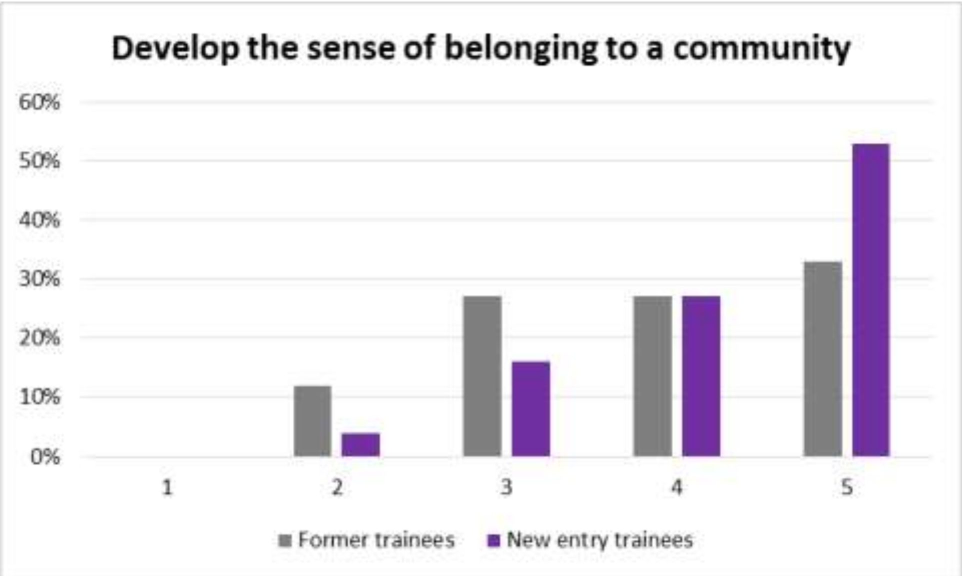


Figure 4.58: Degree of agreement expressed by the trainees about the possibility that the communication message boards allow to develop the sense of belonging to a community

It is interesting to report separately what the former and new entries think of this (Figure 4.58).
There is indeed a difference in the percentage: 74% for the *former trainees* and 60% for the *new entry trainees* (if we consider the answers of those who have indicated a score of 4 and 5).
At half MOOC Numbers, the former trainees already know that the communication message boards allow developing a sense of belonging to a community. In fact, this option was added after analysing the data from the final questionnaire of MOOC Geometria, which investigated the sense of belonging to a community. We will return to this topic in more detail in a following paragraphs (§4.9).

Now let us move on to the analysis of the answers given in the final questionnaire about the communication message boards.
Figures 4.33 and 4.50 gave us a clear overview of the readings and writings occurred on the platform. We have already seen that the trainees have made more readings than writings. The writings, in particular, can be posts written as a first intervention or they can be replies to interventions written by others. We wanted to ask the trainees the importance that these actions have had for them (reading, posting and commenting). To investigate this aspect we use a Likert-scale question (a score of 1 means very unimportant and a score of 5 very important).

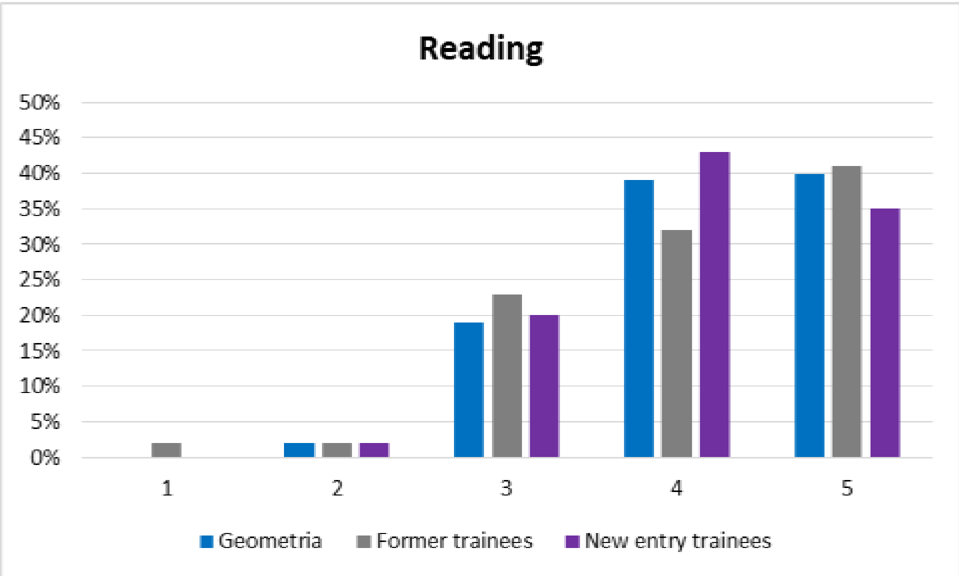


Figure 4.59: Importance of reading for trainees

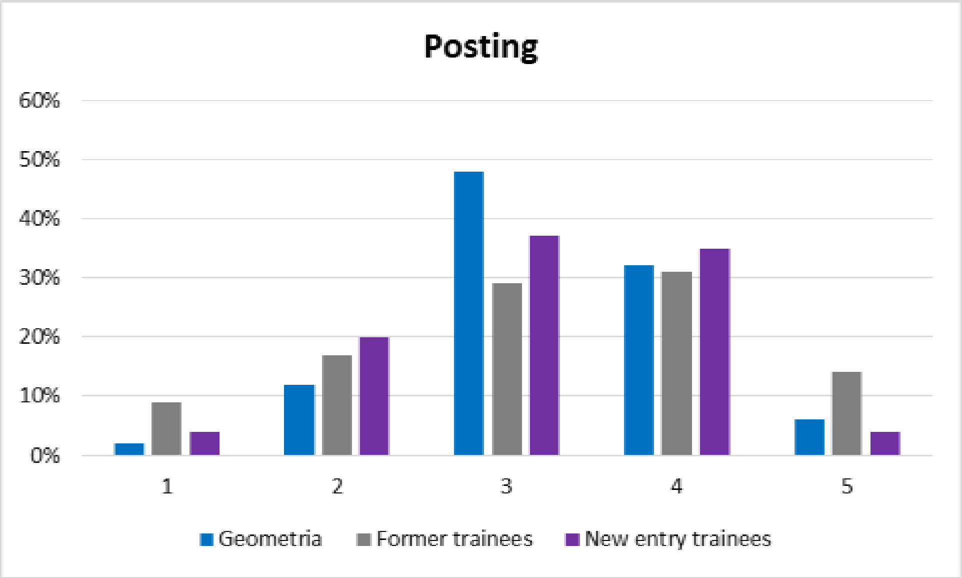


Figure 4.60: Importance of posting for trainees

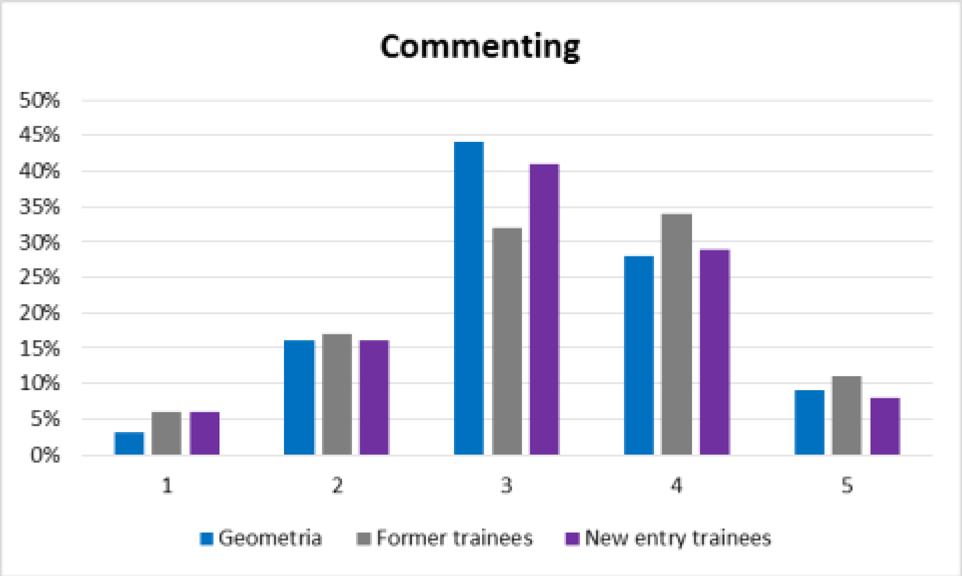


Figure 4.61: Importance of commenting for trainees

Reading is the action that the trainees consider most important to be done on communication message boards and this is in line with the statistics released by the platform. If we consider the answers to the score of 4 and 5 (Figure 4.59), we have that the importance of reading is distributed as follows: 79% for the *Geometria* trainee, 73% for the *former* trainee and 78% for the *new entry*.

For writing actions (posting and commenting) it is not surprising that the percentage of importance attributed is less. We observe in particular that for *Geometria* trainees and *new entry* trainees, posting (Figure 4.60) has importance equal to 3 for 48% and 37% respectively. In the *former* trainees this value changes and the majority settles on the value 4 (31%).

Also for commenting this distribution of frequencies is maintained (Figure 4.61): 44% of *Geometria* and 41% of *new entries* give importance equal to 3; the *former* 34%. This “evolution” of the degree of importance declared by the *former* trainee could mean that, after the first experience, the role of writing has been reevaluated. This could be confirmed if, in a future MOOC, the new entry trainees will declare a similar increase compared to the percentages currently declared.

We use also a semi-open question to ask: "*Do you think that the process of writing a post and receive or make comments*[51] *is useful for you? Why?*". The answer options to the first question were: *Yes, In part, No*. In particular, the 63% of *Geometria* trainees and 58% of *Numeri* trainees[52] consider the activities of reading and writing useful (Figure 4.62). The question was not mandatory, so there is a little percentage (19% in MOOC Geometria and 18% in MOOC Numeri) that not give an answer. The justification instead was open (and optional too). We did not want to provide response options because in explaining why doing that certain action was important or not, the trainee would have had the chance to perform some sincere self-assessment and self-reflection. However, we have identified some categories based on the answers obtained, precisely 3 categories for who have answered *Partially* and 9 for who have answered *Yes*. The justification of who have answered *Partially* or *Yes* are collected in Tables 4.24 and 4.25 respectively.

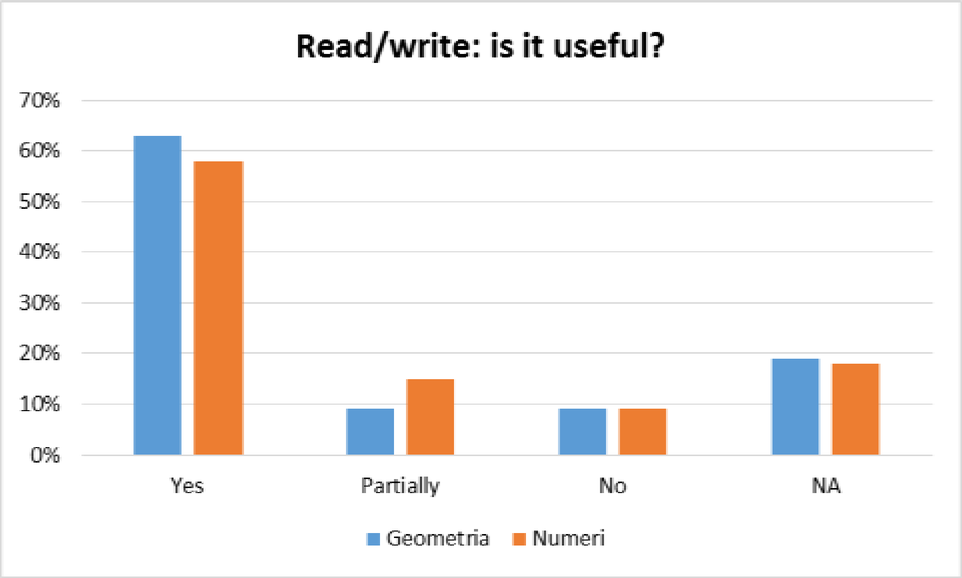


Figure 4.62: Read/write utility for trainees

If Partially (#)	Categories	MOOC Geometria (14)	MOOC Numeri (17)
a)	Even if it enriches, it takes too much time	26%	47%
b)	Very fast MOOC rhythms	28%	6%
c)	NA	36%	47%

Table 4. 24: Read/write is partially useful because …

[51] making a comment presupposes reading the post

[52] With *Numeri* trainees we consider the totality of the finalists of the MOOC Numeri, without distinguishing between the *former* and the *new entry* trainees.

The trainees that answered that the communication message boards are partially useful (14 in MOOC Geometria and 17 in MOOC Numeri) state that because even they recognized their benefit, reading or writing a post require too much time. Moreover, the MOOC goes on very fast and it means that many posts begin to accumulate and they end up not reading any or all of them.

If Yes (#)	Categories	MOOC Geometria (104)	MOOC Numeri (74)
a)	**Writing** helps to clarify and reflect	5%	9%
b)	**Writing** allows you to share	4%	3%
c)	**Writing** allows you to help other trainees	2%	0%
d)	**Writing** creates a community	2%	1%
e)	**Reading** is an instrument of reflection and comparison	49%	62%
f)	**Reading** allows you to benefit from the sharing	10%	7%
g)	**Reading** allows you to receive help from other trainees	14%	15%
h)	**Reading** makes you feel part of a community	6%	3%
i)	NA	8%	9%

Table 4.25: Read/write is useful because …

The trainees that answered that the communication message boards are useful were 95 in MOOC Geometria and 67 in MOOC Numeri. In justifying their answers, they gave importance either to reading, or to writing or to both. So, their answers, framed in the categories, become 104 in MOOC Geometria and 74 in MOOC Numeri.
We report some of the answers given by the trainees. We will indicate G if she/he is Geometria trainee, N if she/he is Numeri trainee.
About writing process, it is considered useful because it helps to clarify and reflect, so you can communicate what you consider essential [a), in Table 4.25]. For example, in G, someone answers saying:
"*For me, writing remains an important process that helps me, first of all, to give me clarity and then the synthesis stimulates me to communicate to others what I consider essential to share*";
while in N: "*Expressing your opinion through a post is useful because it forces you to stop for a moment to reflect on the topic in question*".
At the same time, reading helps to internalize the MOOC proposal and to compare yourself with the others: there is an exchange of opinions, you can discover different ways of seeing things [e), in Table 4.25]. In G we can read: "*I did not write much, but reading other posts helped me to better internalize the proposed activities*". And someone else, in G: "*[...] I have known different points of view and different approaches that have led me to reflect on my work*". And in N: "*In some cases it [reading] allowed me to have some clarifications on some topics dealt with in the various modules*"; and also: "*Reading the posts of colleagues helped me above all to question some of my teaching practices; it was a spur and enrichment*".
These aspects mean, in particular, that writing and reading foster the process of **instrumentation/self-organization**.

Moreover, writing is considered useful because allows you to share ideas and proposals, with a view to improving your education [b) in Table 4.25]. In G, someone says: "*it was useful because it helped me to improve my teaching, I actually put into practice some of the suggestions of my colleagues*" and in N: "*It allowed me to share difficulties and doubts and find and share solutions*".
The same is for the reading [f), in Table 4.25]. For example in G: "*I enjoyed reading and comparing the various comments on the posts and interacting with the other trainees through the digital communication message boards. I find it a very direct and fast way to interact and for me, it was useful to learn how to use them and find material to use with my students*". While in N: *"[it is useful] to improve my teaching*".
These aspects mean, in particular, that writing and reading foster the process of **instrumentalization/sharing**.
Writing and reading allows you to give or receive help if there are technical and/or practical difficulties [c) and g) in Table 4.25]. In fact, in G someone says: "*in many cases they have allowed me to resolve doubts or perplexities or to find confirmation of my thoughts, in others to give help to colleagues in difficulty by providing the requested information*". Someone else in N: "*It was useful because the problems I could have encountered had already been expressed by other colleagues*".
In addition, writing and reading contribute to create a community and feel part of it [d) and h) in Table 4.25]. They makes you connect with colleagues you do not know, but who share your interests. It also reduces not only geographical distances, but also the ones referred to different school orders. For example in G, you can read: "*The exchange of ideas or simple comments among colleagues is stimulating, makes you feel less alone [...] online mode also allows those who are more reserved to express their ideas and to benefit from a very interesting cultural exchange*". Always in G: "*It is a way to understand that there are also others with you, who live and share with you the same school and life problems, off the web. The virtual world is different from the physical one, even if I believe that people communicate better at a distance: there are no antipathies, rivalries, absurd principles to be asserted*". In N, instead, we read: "*I have been interacting for years on social media in dedicated groups and I am a supporter of online cultural exchange. We compare and grow professionally, sometimes more than we can do in a relationship between colleagues at school. This is because the social media (and therefore the communication boards used in this course) are generally "frequented" by people / colleagues who share the desire to learn and improve*".

4.8.1 Remarks

We can now draw the conclusion on what we have been able to observe regarding the communication message boards, the spaces where the ***double learning process*** is triggered.
The complex ecosystem structure developed as soon as the trainees begin to access the MOOC. They are asked to enter into what, at first glance, may look like *chaos*, because of the multitude of materials and available technological resources. In fact, initially the trainees may not have enough self-confidence with the situation (*instrumentation*). Gradually they implement the *self-organization* phase: appropriating the use of the MOOC's usage schemes and comparing what they explore with their Math Edu USs, they begin to use resources and materials (*instrumentalization*) and also to contribute comments to the communication boards (*sharing*).
Forum, padlet and tricider are the communication tools that have been selected and implemented in the DI.FI.MA platform on Moodle by the trainer-designers to allow

interactions within the MOOC online environment. They proved to be effective, as they supported and facilitated interactions.
In each of these, the trainers include prompting questions or specific deliveries in order to accompany the trainees in reading the materials and identifying their focus. Moreover, the trainees are also invited to share their teaching experiences with the mathematical content they are examining. We remember the methodology chosen by the team of trainers in this regards, namely they chose to limit their own interventions in these message boards to a minimum in order to support the birth of a trainees-only community.
Each communication message board allow a different interaction to take place.
The **forum** played a predominant role with respect to the other tools. Despite being an almost outdated mode (based on web 1.0), the trainees were very fond of it and used it to share their experiences of learning or of working. There was no moderator in the discussions: each trainee had the opportunity to read a diversity of opinions and experiences, and when (s)he understood how it worked, then (s)he introduced her/himself, became an author of posts, influenced other colleagues, or appreciated the idea expounded by a colleague.
If the forum was the right place for the trainers to talk about themselves, including their strengths and weaknesses, the **padlet** was the place where the trainees began to share photos, videos and, spontaneously, their own materials[53]. It is clear that the Padlet did not help to structure the exchange, but many trainees obtained inspiration from the exchange of materials in this place. For example, it was re-used and proposed by a trainee as a tool to track her training programme with the construction of a Learning Diary: "*I am reviewing all of the course materials ... Because of my age, I can hardly remember the various proposals, ideas offered in this course surely professionally enriching and among the best I've attended to! So I thought to produce a Learning Diary with Padlet. Step by step it will enrich it, even with external links, with the materials I have looked for during this course or suggested by colleagues in the forums. Can it be useful to anyone?*".
The **tricider** had the goal of triggering simple threads, most of all confined to the approval or not of ideas, by voting through "likes". However, the participants used it more for collecting ideas and comparing their didactical experiences – as a forum – rather than for the expected use. Practically, the trainees realized a *catachresis* (Verillon & Rabardel, 1995): an artifact is used to do something it was not conceived for. Due to the fact that they explored the tool for the first time, and also because they usually need to explain and to go in depth when they express an idea, so the simple vote would not have let them satisfied. The posts written in Tricider are rich of ideas for both trainees and trainers. The trainees were introduced to a new tool for them. The trainers acquired awareness about the necessity to be clear in writing the tasks, in exemplifying the use of the tools and in providing tutorials on their affordances.
Beyond some trainers' interventions in the forums, or email communications with administrative aims, the actual contact between trainees and trainers is realized through three online **webinars** (using the chamber BigBlueButton of Moodle): they supported the community with synchronous interaction. While the trainers in the webinar could use video and chat, the trainees could use only the text chat. The trainers presented themes linked to the didactics of geometry or arithmetic/algebra (it obviously depends on the reference MOOC) and from mathematics education research. In all the six webinars (three in MOOC Geometria and three in MOOC Numeri) there was a high participation of trainees, who posed questions and doubts.

[53] Actually, also on the forum or on the tricider the trainees have sometimes shared their material, but generally they were photos or Geogebra files that were small in terms of megabyte (in fact, such spaces had a maximum limit of MB to support). In the padlet, these gigabyte limits did not exist, so that – as we pointed out – some of them shared their "long" response by loading a word file.

In general, in the communication message boards, the will to establish the threads often leaks out, though it is very difficult that they take shape in a broad and articulated manner. In fact, the threads tend to split into different groups, which are formed and split locally and for a certain period of time, depending on the needs felt by the individual, but generally they contribute to give to all trainees the sense of a common participation in one unitary event, precisely the MOOC. Using a term from neuroscience, we call this property ***plasticity***, which makes it possible to adapt to various situations in different groups and times. It is true that situations and times change, but within a community that preserves its global unity. This unity consists in the collaborative sharing of what happens, even if the active participation converges on more than one local theme. The sharing processes (of materials, thoughts, ideas, experiences) in fact gives life to the ecosystem, enhancing the materials and expanding the individual's network of knowledge. Even the "contact points" with trainers via webinars contribute to this purpose. Through sharing processes the ecosystem becomes more and more structured; fragments from the history of web communication (from web 1.0 on) coexist and complement each other, and are used by the trainees. This aspect is interesting and little pointed out in the literature. It is something similar to the multimodal interactions that take place in the classroom thanks to the activation of different registers: we call it ***technological multimodality***.
Plasticity and *technological multimodality* are the two main properties distinguishing the evolution of a community in a MOOC from that in a traditional training course.

It is clear, therefore, that communication message boards have done more than allowing interaction. Already from the trainees' answers (§4.8 a quantitative overview of the MOOC-ecosystem) where we have analysed their usefulness, we have seen how the trainees have defined them as spaces that allow them to clarify what the course is proposing in terms of didactic praxeologies. The communication message boards also stimulate reflection, that is, they allow a comparison with the trainee' own Math Edu USs (to internalize the proposals) and with those of others. The communication message boards help trainees to share their ideas, their opinions, their teaching experiences (to create new connections in their own network of knowledge). Last, but not least, as it emerged spontaneously from the open answers of the trainees (Table 4.25, line d and h), they help to develop a sense of belonging to a community. We could summarize all this with one word: **collaboration**.

These collaboration processes, which we will examine in greater detail in the following paragraph, have created a community of trainees that has specific characteristics.

4.9 Community in a MOOC

We have started, from the beginning, saying that in a MOOC for teacher education two communities are identified, the trainers' community and the trainees one.
At this point, it is opportune to stop for analysing, both from a quantitative and qualitative point of view, the community of trainees that has come to delineate itself.
As we have just resumed in the previous paragraph, the trainers had made the methodological choice to intervene as little as possible on the communication message boards to encourage interaction of the trainees-only community. It is characteristic how the trainees interact with each other within the online environment: without ever having had the opportunity to meet in

person, in a spontaneous way, they exchange ideas, make their mathematics teaching experience available, share their own materials.
While the invitation to exchange teaching experiences was asked by the trainers, the exchange of their materials is an entirely spontaneous fact.
We have already seen how the shared materials are original productions that arise from the vision of what is proposed by MOOC (see A.P. and A.M. in Table 4.9), or materials, always personal, previously used in class with their students, which are in line with the mathematical concept that the MOOC module is dealing with (see L.G. in Tables 4.20; P.S. in Table 4.23). Always from these tables (4.9, 4.20, 4.23), we read not only appreciation of this practice "*[...] thanks for the idea*" (A.P., Table 4.9), but a series of compliments that the participants exchange each other "*Beautiful idea, congratulations!*" (R.R., Table 4.9); "*Very nice this example*" (S.M:, Table 4.20).
Another aspect that does not go unnoticed is that, when the trainees tell their teaching experiences, they do it by opening themselves totally. That is, they tell their own strengths, from which others can take a cue (see, for example, A.R. in Table 4.5; E.L.I in Table 4.7; E.V. in Table 4.22), but also come to confide and seek advice. Let us read this example from the forum in module 1 of MOOC Geometria.

Discussion (started by X; on gg/mm/aa; at hh:mm)	**# reply**	**Reply (by X; on gg/mm/aa; at hh:mm)**	**Category**	**Intervention**	**Double learning process**
USE OF THE MAINMAST MATERIAL (by E.G.; 13/10/15; 17:43)	62	…			
		…			
		D.B.; 26/10/15; 16:27	B	[...] I teach in a technical institute [higher secondary school]. Having not yet started to do geometry, reading the proposed activity [the mainmast], **I thought to take a cue** from this work to review what they [the students] now know from the lower secondary school and that often forget.	E→I E←I
		E.G.; 26/10/15; 17:10	A	D, some middle school students can not learn, they do not have the ability to 'see' it as a concept, although we are committed to explaining it in different ways, also in didactic laboratories ... **I am almost completely convinced that it is also a question of maturity in the ability of reasoning**, which do not reach everyone at the same time ... someone from time to time **discourages me** ...	E←I
		D.B.; 27/10/15; 08:19	A	**I know** that you, teachers at lower secondary school, work hard and that kids forget. **I myself realize** how much they change in the transition from grade 9 to grade 10 and how some topics are assimilated better in grade 10. The problem of grade 9 is also that of linking the levels of preparation of young students coming from many different lower secondary schools. These activities that are proposed seem a useful tool for this purpose in my opinion. Bye bye and good work	E←I
		E.C.; 30/10/15; 23:31	A	**I agree** with you E.[G.], pupils often have different maturation times, but surely most of them (there are also extreme and desperate cases in which nothing knows	E←I

				how to involve them) is more willing to try if we present their manipulative and laboratory activities and that they foresee a collaboration between peers. So **do not be discouraged** :) we put some seeds and we hope that sooner or later they sprout :) there are seeds that remain dry for a long time but then suddenly sprout and give seedlings full of green leaves. **Never lose hope** :)	

Table 4.26: Trainees' interventions on the forum in module 1 of MOOC Geometria

The interventions taken into consideration (Table 4.26) show a comparison between teachers of different school orders. It is very unlikely that in an environment different than this [our MOOC environment that is aimed at both lower and higher secondary school teachers], they would have the chance to do these discussions. D.B. writes, self-organizing her network of knowledge, which intends to try the mainmast activity with her grade 9 students, to see what they remember about what they did in lower secondary school. E.G., lower secondary school teacher, puts D.B. on guard. She underlines that despite the great commitment of teachers in this scholastic order, students are not always able to correctly internalize a mathematical concept, as is the case of perpendicularity versus verticality. E.G. also confesses that certain students discourage her. D.B. encourages the colleague saying that she knows well how the teachers of the lower secondary schools work hard. However, the most interesting comment is that of E.C., who not only reinforces and support the thought of E.G., but nicely advises her by making a comparison between maturing students and seedlings that sprout. This empathic gesture has occurred unexpectedly in an online environment.

This community is different from those that usually characterize the traditional training courses (the face-to-face ones): it was born spontaneously since participation in the MOOC took place voluntarily. In addition, the participants freely express themselves: there is no institutional component that wants to restrict them. Notice how the discussions proceed in a very free and spontaneous way, even with the use of emoticons (as seen in the comment by E.C. in Table 4.26).
It was, in fact, the intention of the trainers to intervene as little as possible in the interactions among the trainees, to facilitate the creation of a community. This is a remote, voluntary, free and collaborative community, not subject to institutional pressures.
Collaboration is a characteristic that can not be inferred only from communication message boards. We concentrate now on it.

Collaboration

The notion of mathematics teachers' working and learning through collaboration is not new, but gains more and more attention in educational research and practice, particularly after the report about Lesson Study in Japan from the TIMSS classroom video study (Stigler et al., 1999).
We report a quotation to better enter into this order of ideas:

> "Collaboration implies co-working and co-learning (working together and learning together); it involves teachers in joint activity, common purpose, critical dialogue and inquiry, and mutual support in addressing issues that challenge them professionally; it helps them in reflecting on their role in school and in society. Across education systems, mathematics teachers work and learn through various forms of collaboration which contribute to learning and development in differing ways. Efforts to understand what teachers do in and for improving their teaching and expertise have led to ever-increased interest in exploring and

> examining different activities, processes, and the nature of differing collaborations through which mathematics teachers work and learn" (p. 1).
> (Jaworski et al., 2016).

The words of Jaworski et al. (2016) are found exactly in the experiences conducted by the trainees in the MOOC. The trainees voluntarily joined the MOOC for teacher education with the aim of improving their mathematics teaching competencies so that they share a common purpose. They are involved in joint activities. In particular, they are invited to reflect on the activities proposed by the MOOC, to evaluate weather using them in their classrooms, to consider the potentialities of the methodologies or strategies shared with them. In doing that, they generate critical dialogue and inquiry in the communication message boards. Even in these spaces, they confide in each other, share ideas, opinions, their own materials. Some also give themselves mutual support. This is surprising, since they have never seen each other in person!
The final activity of the MOOC, in particular, require their collaboration (remember the explanation made in §3.2.3.3; §3.3.2.1). Individually, each trainee must design a teaching activity (Project Work, or PW). Then, this will be subsequently reviewed by another trainee (Peer Review, or PR) and then these PWs become a shared asset among all the MOOC trainees.

4.9.1 A quantitative and qualitative overview on the sense of belonging to a community

Compared to what can be deductions made by the general vision of the MOOC environment, we thought it interesting to analyse as much as possible what the trainees thought. As anticipated in the methodology section (Chapter 3), we analysed only the answers of the trainees who started and finished the MOOC in all its stages. In particular, as previously made, we analyse distinguishing the answer given by the trainers enrolled in MOOC Geometria that we will denote with *Geometria* trainees; the *former* trainees, the ones that were enrolled both in MOOC Geometria and MOOC Numeri; and the *new entry* trainees, the ones that were enrolled only in MOOC Numeri.

4.9.1.1 Community in the communication message boards

We have already seen how trainees stated that the communication message boards help them to develop the sense of belonging to a community (Figure 4.58) and in particular in which way the action of writing and reading foster this aspect (Table 4.52).

4.9.1.2 Community in doing the Peer Review

We also analyse what has produced the collaboration implemented by the Peer Review (PR) activity. In the final questionnaire of both the MOOC Geometria and the MOOC Numeri, we asked with an optional open question "*To what extent and why make and receive the peer review was important to me?*". We did not want to provide response options because in explaining why doing that certain action was important or not, the trainee would have had the chance to perform some sincere self-assessment and self-reflection. However, we have identified some categories based on the obtained answers. Each sentence was split in two parts, the one referred to receive the PR, for which we identify 4 categories and the other one referred to make the PR, for which we identify 4 categories. They are exposed in the following Tables 4.27 and 4.28 respectively.

Receive the PR (#)	Categories	Geometria (152)	Former (65)	New entry (51)
a)	Peer comparison	56%	66%	75%
b)	Exchange of knowledge	9%	11%	4%
c)	Stimulate reflection	25%	12%	12%
d)	NA	10%	11%	9%

Table 4.27: Importance to make the PR

We report some of the answers given by the trainees. We will indicate G if she/he is *Geometria* trainee; while G&N if she/he is *former* trainee and N if she/he is *new entry* trainee.
Receiving the peer review was considered a peer comparison in the sense that you can benefit from another way of seeing and thinking. Aspects that you not considered are brought to light and the feedback comes from an external eye, but of the trade [a), in Table 4.27]. For example, in G someone writes:
"*It is very important to be peer assessed because colleagues can see defects in your work that you yourself do not perceive [...]*";
Always in G someone else writes:
"*I really liked the peer review because it gave me the opportunity to hear the opinion of another competent teacher without having the filtering of mutual knowledge, which can sometimes compromise the real objectivity*".
In N a trainee says:
"*It allows you to have a different point of view from your own and to question ideas "built" over time and / or preconceptions about certain behaviors and / or answers of the students to certain topics*".
While in N&G we can read:
"*The peer evaluation allows to have a judgment from someone who knows well the scholastic reality, knows how the students reason, knows their possible difficulties and understands the actual feasibility of a path. So having an opinion from someone who knows the reality we are talking about is very useful*".

Receiving the PR was also considered a means to benefit of knowledge exchange. Reading the received review you can get ideas and suggestions to apply [b), in Table 4.27].
For example, in G someone writes:
"*[...] the colleague can have some improvements or new ideas to introduce [...]*";
And in N&G someone else writes:
"*The peer review of my work was done in a very thorough and gave me suggestions to improve the activity*".

Last but not least, receiving a revision from another trainee may stimulate reflection on what to change. It is as a self-criticism that come as a consequence of the comparison [c), in Table 4.27].
In G we read: "*The exchange of ideas is always important and is a useful tool not only for knowledge but also for evaluation. In my opinion it also leads to self-assessment and reflection of what we do, how we do it and why we do it that way. Last but not least, can we do it better?*".

This comment is very interesting because the trainees, without knowing, is making a reflection from the praxeologies point of view in terms of task, technique and justification (as we have explained in Chapter 2).
In N someone writes:
"*Allows you to realize any shortcomings in your work*".
And in G&N:
"*It was important because it allowed me to compare myself with a colleague, but at the same time it was also a self-analysis*".

We can observe that in general receiving the PR from another trainee is perceived as a learning moment and its richness lies on the fact that it is a peer that reads the work of another teacher. The trainees do not feel the weight of an institution that is evaluating them, rather they live it as a collaborative moment that acts as a glue to being part of a community.

Now let us see what emerged considering the task of making the PR.

Make the PR (#)	**Categories**	**Geometria (152)**	**Former (65)**	**New entry (51)**
a)	Constructive exchange	27%	19%	16%
b)	Self-reflection and new ideas	32%	23%	22%
c)	Difficult task	5%	3%	0%
d)	NA	36%	55%	62%

Table 4.28: Importance to make the PR

We report some of the answers given by the trainees. Again, we will indicate G if she/he is *Geometria* trainee; while G&N if she/he is *former* trainee and N if she/he is *new entry* trainee.
We note that there was a high percentage of people who did not respond [e), in Table 4.27]. This may have been dictated by the fact that the question to speak about the received PR and the made PR was unique. Some in fact responded to the importance of receiving it, but they did not continue to explain why it was important for them to do it.
For those, who have answered to this aspect of the question, making the PR is viewed as a constructive exchange. In fact, you can have the possibility to get a different vision and approach from your own with respect to a mathematical topic. A new point of view is then discovered [a), in Table 4.28].
For example, in G someone writes:
"*It also allows you to appreciate the different work approaches*";
"*I liked to evaluate the work of a colleague of mine was how to enter the classroom with her to teach*";
In G&N we see:
"*It was very important because it allowed me to take a critical view of a wonderful activity*";
And in N:
"*It gave me the opportunity to go into the details of the colleague's activity and to appreciate creativity and simplification in some theoretical parts and in proof*".
The exchange then is certainly constructive if it continues in a deeper collaboration, as we read from this commentary in G
"*I evaluated the work of a colleague with whom I started an email correspondence; we exchanged opinions and materials and began a fruitful collaboration*".

Making the PR is also a way to advise the other, making her/him perceive your own point of view. In particular, it allows the reviewer to make a self-reflection asking himself "how do I deal with that concept in the classroom?" and, at the same time, allow the reviewer to have new idea as a consequence of the self-reflection.
For example in G someone writes:
"*It allows you to give advice and encouragement*";
"I*t allowed me to reflect on the different possibilities of designing a teaching unit by reviewing the work of the colleague*";
In G&N we see:
"*From the evaluation of the peer review of the colleague I learned to use applications that I did not know and ideas to use in the classroom*".

Making the PR for some trainees was a challenging task, sometimes also difficult and "embarrassing". In fact, in G we read:
"*A bit of initial embarrassment in front of this 'adventure' has certainly been there. First of all because evaluating a colleague's work without knowing the context is certainly difficult. It was important to understand how a colleague, unknown, sets his work on certain issues*".
"*Honestly, at the beginning, I thought it was a simple thing, but to fully understand the goals and objectives achievable, in my opinion, with the activity reviewed, I had to reread several times and interpret what was written by the colleague. It is not always easy to concentrate what you want to say in a few lines*".
In G&N instead we read:
"*It created me a lot of difficulties because I found a way of working very different from mine*".
"*Very sincerely it seemed to me embarrassing to review the work of a colleague without knowing the reasons that led her to make that design*".

Despite a minority that found difficult to accomplish the task of making the review of a project designed by another peer, we can observe that in general making the PR is perceived also as a learning moment, because critically reading the work of another teacher encourages self-reflection on one's teaching practices. These, in light of new ideas, can also be reconsidered as a collaborative result of the participation in a community made (for the most part) of peers.

4.9.1.3 Community to whom be part of

To specifically analyse the sense of belonging to the community, in the final questionnaire of both the MOOC Geometria and the MOOC Numeri, we asked two specific questions. First of all, the closed question: "*Do you think that the MOOC trainees have contributed to your education?*", with answer options *Yes* and *No* (Figure 4.63).

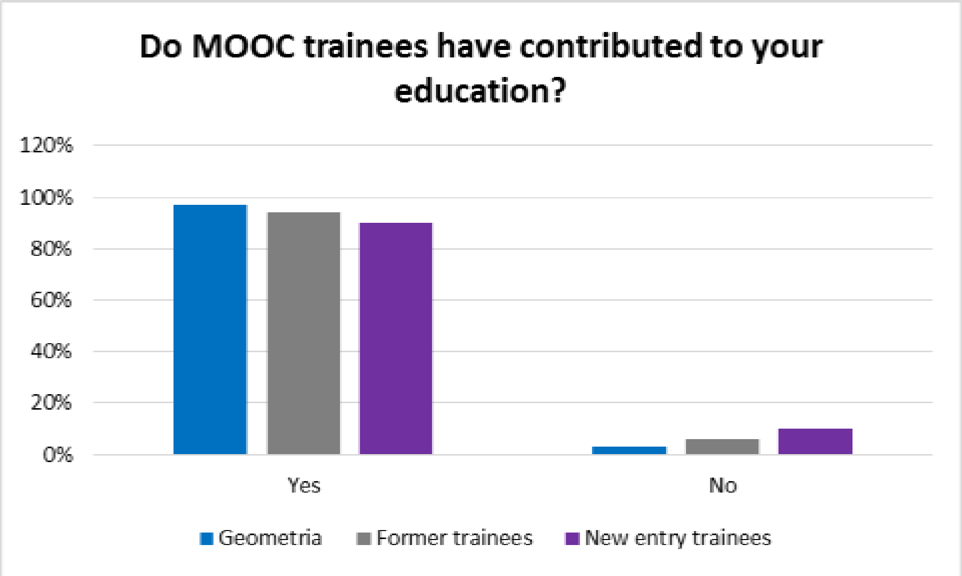


Figure 4.63: Do you think that the MOOC trainees have contributed to your education?

As the graph reads, the trainees (97% Geometry, 94% former, 90% new entry) confirm that the other trainees have contributed to the training opportunity obtained thanks to MOOC. This is not surprising because it is in line with what we have already observed from the previous analysis.
The question "*As MOOC trainees, to what extent did you feel part of a community?*" provided "*Not much*, *Quite a lot*, *A lot*" as response options. And then we asked to justify the choice made with "*Why?*". The situation, as shown in Figure 4.64, is very varied.

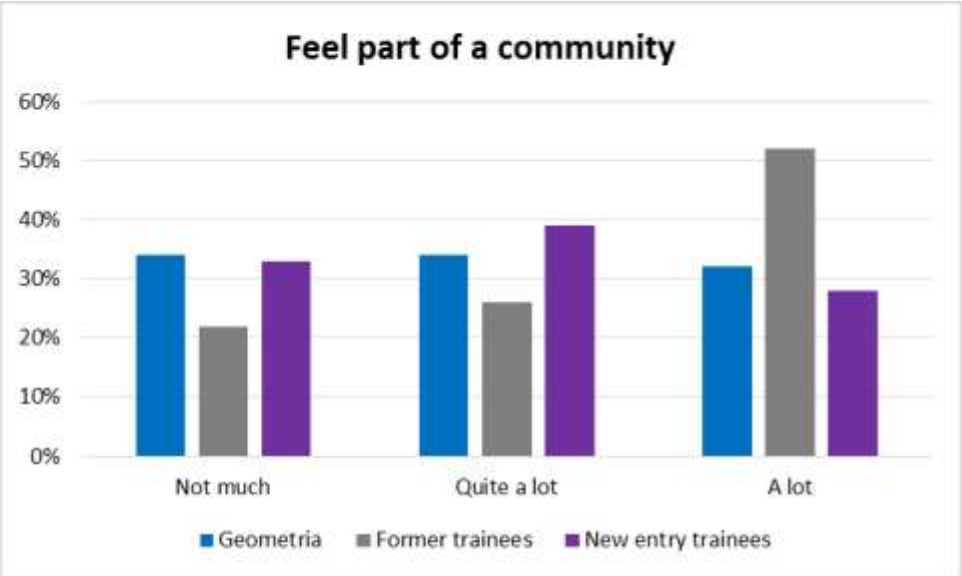


Figure 4.64: As MOOC trainees, to what extent did you feel part of a community?

We begin to analyse the MOOC Geometria trainees (Figure 4.64). Their answer have the following percentage: 34% not much; 34% quite a lot; 32% a lot feel part of a community. It is interesting reading the justification that they give to these their answers.
Those who responded to feel ***not much*** or ***quite a lot*** part of the community writes:

"*If the course had been proposed in the second part of the scholastic year, surely my level of involvement would have been greater and I would have been able to deal with the modules in real time [...]. Unfortunately, I was involved in the preparation [of a series of informative documents of a bureaucratic nature for my school]: the deadlines are all in this first segment of the school year, so I have not always been able to face this beautiful formative moment with tranquility and dedication that I wanted*".

"*Time did not allow me to feel part of a community. In my opinion, an only online community does not have the same effectiveness as a community that also shares moments of real encounter. For example, it would be interesting to propose local meetings with the participants*".

"*In some moments I did not really feel part of the community because of the lack of time. The forums were updated too quickly and often I did not keep up with the others*".

"*My participation was not that of a community for personal reasons. All the material I had access to was used and I am inserting it into my lessons. I realize, however, that I have participated in a rather selfish way, not comparing myself much with the community*".

"*Being an extremely large community it was very difficult to feel truly an integral part. I am sure that my posts have been read by the participants, but given the high participation many times it was difficult for me to really realize the community of which I was part*".

On the one hand, we understand that personal or scholastic commitments, the fact of preferring a face-to-face comparison rather than online, the extreme dynamism that characterizes the flow of MOOC are the aspects that led the trainees not to feel fully part of a community.
On the other hand, those who responded to feel ***a lot*** of part of the community write:

"*A lot both for what concerns the trainees, but also for what concerns the trainers. I felt in a great 'class'* ".

"*The fact of sharing on communication message boards and in forums already makes you feel part of a community; to check if someone has commented on my post, did the review to my project, wrote an email ... all this makes me feel part of a community. I felt a lot of this belonging during the webinars, when we as trainees chatted with the teachers and among us in real time*".

"*I liked to share ideas, opinions, frustrations and satisfactions with colleagues*".

"*Being part of something that brings together people from all over Italy has a certain effect*"

From these testimonies, which in any case constitute the minority, a strong sense of belonging to a community emerges instead. A community of which one is proud of belonging, a community with which one can compare and confide in, a community that unites teachers from all over Italy.

Let us move on to consider the answers given by the *new entry* trainees, that is, those who have been enrolled in MOOC Numeri (Figure 4.64). These trainees can be compared exactly

to the *Geometria* trainees, because for them the MOOC Numeri is the first massive online educational experience.
From the point of view of the distribution of the percentage frequencies, the situation is slightly and partially better than the one outlined in MOOC Geometria. In fact, 33% say they feel not much part of a community; while in MOOC Geometria they were 34%. 39% say they feel quite part of it; while in MOOC Geometria they were 34%. Instead, there is a decrease in feeling very belonging to a community, in fact it is 28% compared to 32% of MOOC Geometria. We read some testimony of those who answered ***not much*** or ***quite a lot***:

"*I know I have an introverted character so communication was not immediate*".

"*Enough. Surely I knew I was working with other colleagues around Italy with the same goals. Padlet made me feel more part of the MoocNumeri community, even if there were no real discussions, there were everyone's ideas*".

"*I have had too much lack of time*".

We note that also in this case the lack of time or personal predispositions did not facilitate the belonging to a community. While those who responded to ***a lot*** writes:

"*I had the feeling of being part of a big family*"

"*Because of my character, I struggle to relate with people I do not see or do not know. But the vitality of this community has involved me since the first moment. A special thanks to the team*"

Even if it is always the minority, a strong sense of belonging to the community emerges. There are those who even feel like in a family and who, thanks to these interactions, have overcome personal inhibitions.

Now let us see what the *former* trainees think, that is, those who have been enrolled both MOOC Geometria and MOOC Numeri (Figure 4.64). Just looking at the histogram already a different trend from those previously observed emerges. In fact, we have the following frequency distribution: 22% not much; 26% quite a lot; 52% a lot feel part of a community. Repeating the experience of massive online education has strengthened this sense of belonging. Now it is the majority that claims to be a member of a community. We read again what the trainees write in their justifications. In the following the testimonies of those who answered ***not much*** or ***quite a lot***:

"*The community is very large and the time available to read, comment, participate was too little. So I did not feel particularly involved*".

"*Not much, because of me. I have read enough (also learning new things, innovative methods and practices, videos and useful links), but I did not intervene, because the time available to me was little and when I read, often the answers had already been many and well formulated*".

"*Not much because I can interact very little through social platforms*".

Certainly, there is no lack of those who feel little about belonging to the community. It is perfectly understandable that not everyone feels comfortable integrating into virtual environments. However, what is lost in the human relationship seems to be compensated by participating in a global and intergenerational community. In fact, let us see what the majority or those who feel ***a lot*** part of the community write.

"*It is the second MOOC to which I participate, this idea of sharing at a distance has, in both cases, made me feel really part of a community of teachers who are willing to improve their teaching practices*".

"*I participated last year at the MOOC Geometria that allowed me to know, even if not in person, new colleagues. I found many of them again this year at the MOOC Numeri and so it is as if I knew them. I therefore felt part of this community*".

"*I feel part of a community that learns at any time of day or night. It made me happy and made me smile*".

"*I felt very involved with a cross (at national level) and vertical (for different school orders) community*".

"*A lot. Unfortunately, I find more math colleagues with a teaching approach similar to mine in the MOOC that not physically at school. But this gives me the certainty of not being an eccentric white fly, but that the kind of activity I do in the classroom is well supported by solid theoretical principles and it is really effective in training students' mathematical and social competence*".

These phrases thus testify how a repetition of online training experiences can be important not only from the professional point of view, but also from the communicative point of view, because they create virtual bonds that, as Jaworski et al. (2016) say, "help them in reflecting on their role in school and in society. Across education systems, mathematics teachers work and learn through various forms of collaboration which contribute to learning and development in differing ways".

4.9.1.10 Remarks

These aspects examined and discussed can be framed from a theoretical point of view.
One of the first researchers who analysed the idea of community is Wenger (1998), who introduced the theoretical construct of ***community of practice*** to indicate groups of individuals who share an interest or passion for something they do and learn to do better when they interact with each other regularly, through shared practices (Wenger, 1998). The members of these communities are, in fact, involved in common activities, through the use of family resources and the sharing of what has been learned. Wenger (1998) distinguishes three different forms of participation in a community of practice:

- *Engagement*, namely doing things together, producing artifacts, confronting each other;
- *Imagination*, namely building an image of oneself and of own community in order to understand how to reflect on situations, explore possibilities and direct one's actions;
- *Alignment*, namely coordinating perspectives, interpretations and actions in order to achieve higher goals.

Another very widespread theoretical construct is that of ***community of inquiry*** (Jaworski, 2006), introduced, starting from the community of practice, to indicate the communities of teachers involved in training programs and research projects. Jaworski (2006) highlighted the problematic of the *alignment* mechanism aimed exclusively at preserving the norms and expectations of a community and the perpetuation of shared practices (Cusi & Robutti, 2017). It therefore introduces the idea of *critical alignment*, as the desirable process by which the members of a community shared norms and practices, asking questions and analysing these practices critically with the aim of developing and improving them. The questioning here does not mean to give a negative connotation to the experience, but to analyse it critically, to integrate it with other ideas, experiences proposals, reports of experimented activities, personal materials used with own students.
Our MOOC experience is particularly in tune with the two theoretical constructs presented by Wenger and Jaworski, and presents particularly interesting examples of critical alignment. We can, therefore, speak of the MOOC community of teacher-trainees as a community not only of practice but also more precisely of inquiry. This is because the sharing of practices among teachers in educational context plays a decisive role, both for participation and for production, but also and above all, because of belonging to a group of training teachers who, together, are trying to improve themselves. This search for improvement, through practices of critical alignment, was realized in the activities on the platform, realized as "co-working" and "co-learning" (Robutti et al., 2016).
In fact, as we have seen in the previous paragraphs, there are not a few testimonies of participants describing how they have experienced some of the activities of the MOOC. On the other hand, how they have spontaneously decided to share their materials with other trainees, both in response to problems exposed by others, and as a means to receive ideas for improvement on these materials of their personal experience. In addition, worthy of note is the work accomplished as a final activity of design of a teaching activity and peer review. Most of them understood these as moments of professional growth, to appropriately rethink their didactical praxeologies with their students.

As for the trainers, there is no doubt that they are a community of practice. They respond to the three requirements listed by Wenger when they deal with creating the MOOC-artifact. In addition, they are also a community of inquiry in the sense of Jaworski, when they are engaged in the monitoring and reflection of the feedbacks that come from the observations on the platform and the analysis of the questionnaire administered to the trainees. In fact, they pose questions to themselves and analyse their practices critically, with the aim of developing them. We will focus more on this in Chapter 6 dedicated to them.

4.10 Final conclusions

The interactions that take place on the communication boards, the collaboration that arises among the trainees and being part of a community of practice (and sometimes even of inquiry), are all evolutions that denote the transition from MOOC ZFM to MOOC ZPA. In light of what we have examined, we could enrich the Table 4.1 with specific details to our MOOCs (Table 4.29).

<table>
<tr><th colspan="3">Taranto's re-elaboration/interpretation</th></tr>
<tr><th rowspan="2">Valsiner/Goos's zones</th><th colspan="2">MOOC's ZFM/ZPA</th></tr>
<tr><th>Trainers</th><th>Trainees</th></tr>
<tr><td>ZFM: Zone of Free Movement
(structures trainers/trainees' access to different modules of the MOOC, availability of different resources within an accessible module of the MOOC, ways the trainers/trainees are permitted or enabled to act with accessible resources in accessible modules of the MOOC)</td><td>• Design of the platform that hosts the MOOC and perception of this new environment
• Design and digital transposition of mathematical resources: Sway with mathematical activity (innovative methodologies and strategies); specific mathematical software like GeoGebra; link to deepening materials; video lectures
• Communicational resources to foster communication among trainees and trainers themselves with trainees: communication message boards to interact with peers (Forum, Padlet, Tricider), webinar chamber, forum for technical support
• Technical support
• Curriculum and assessment requirements
• Organisational structures and cultures: weekly modules, contents according to the Italian curriculum, questionnaires</td><td>• Access to the platform that hosts the MOOC and perception of this new environment
• Access to mathematical resources: Sway with mathematical activity (innovative methodologies and strategies); specific mathematical software like GeoGebra; link to deepening materials; video lectures
• Communicational resources: communication message boards to interact with peers (Forum, Padlet, Tricider), webinar chamber, forum for technical support
• Technical support
• Curriculum and assessment requirements
• Organisational structures and cultures: weekly modules, contents according to the Italian curriculum, questionnaires</td></tr>
<tr><td>ZPA: Zone of Promoted Action
(virtual people, resources, or modules in the MOOC in respect of which the trainers/trainees' actions are promoted)</td><td>• Interaction with trainees reading (and sometimes answering) their post on the communication message boards, on webinars (to reflect in depth on methodological aspects), on technical forum, via mail.
• Professional development</td><td>• Informal interactions with enrolled trainees (that come from all over Italy and are teaching or in the lower or in the upper secondary school) thanks to the communication message boards. So they can reflect on the mathematical proposed activities and share ideas, opinions and materials.
• Interaction with trainers seeing their videos, on webinars (to reflect in depth on methodological aspects), on technical forum, via mail.
• Professional development</td></tr>
</table>

Table 4. 29: "MOOC's ZFM/ZPA" in light of the analyses made in this chapter

The MOOC in its dynamic becoming fosters the expansion of the *ecosystem*: the materials take color thanks to the comments and reflections made by the trainees; the inert communication message boards start to liven up. The trainees intervene without interruption, “in all hours of the day and night”. They start to know each other virtually better. They self-organize the information they receive from the MOOC-artifact and evaluate them according to their own Math Edu USs. Sometimes they internalize activities, methodologies, strategies that are proposed by the MOOC-ecosystem. This leads them to make new reflections, to share with others the expansion of their network of knowledge. They are pleased to tell themselves because they are sure there will be someone who will confront them.
We have therefore able to answer the first research question:

Are there any particular potentialities in a MOOC-artifact that, if properly organized, trigger the double learning process and therefore the transition to the MOOC-ecosystem/instrument?

We had the opportunity to touch with hand, during the analysis we have here exposed, that in a MOOC-artifact there are some particular potentialities that trigger the double learning process and therefore the transition to the MOOC-ecosystem/instrument. In the following, we remember and list them.

First of all, focusing only on the level of design, the MOOC-artifact is the container of specific products, that is, materials rich in innovative teaching methods and specific technological tools. We can therefore understand it as a ***repository*** from which teachers can draw inspiration.
Once the MOOC is opened, it is inhabited by the teacher-trainees, who set the ecosystem in motion with those that are the processes applied to the products. In fact, thanks to the communication message boards included in the MOOC, the trainees – inhabitants of different geographical realities and of different scholastic order – find themselves having the rare opportunity to reflect together to appropriately orientate strategies, processes and materials, linked not only to the students’ age, type of school and social/individual situation but also to the contents of the discipline that present specific cognitive obstacles. The richness of their interactions means that everyone has the possibility to extend their own network of knowledge, but at the same time, it is the same MOOC that is enriched with their contributions, in an intertwined and iterated alternation of MOOC-ecosystem/instrument.

A second potentiality of a MOOC is indeed the presence of special spaces that promote communication at a distance, such as ***communication message boards***. Without the presence of these tools, the MOOC would be just a website to consult. Precisely the communication message boards that allow interaction give the richness of this online environment. It is precisely within these spaces, where *plasticity* and *technological multimodality* alternate with each other, that the double learning process is triggered and reified. This is the process by which we can observe the expansion of the MOOC-ecosystem and also identify germs of possible changes in the didactic praxeologies of the trainees, which seem to converge towards the meta-didactical praxeologies that trainers would like to transpose to them.

The trainers’ methodological choice (decided during the design phases) to limit their interventions as much as possible in the communication message boards has certainly helped to make the trainees cohesive. In fact, the third and last (not for importance) potentiality that we identify in the MOOC is the presence of the ***community of trainees***. This is not a completely spontaneous aspect in a MOOC, because it is strictly connected with the collaboration that cannot be considered as a spontaneous way of working, especially within such remote contexts. Designers have to make it possible through specific techniques. In fact, it is necessary to consider precise methodological choices and to support them. A choice of the trainers is certainly the one repeatedly discussed of limiting their presence on the communication message boards, even though they are always vigilant behind the scenes. Another is exactly the vigilant presence on the platform. Make the trainees understand that the trainers are available and they are participating when needed. Moreover, supporting and encouraging the link between the trainees with activities that see them as protagonists. For example, the webinars, periodic meetings where the trainers talk about of mathematics education but underline also what the trainees are discussing during the course. And the activities of PW, but above all of PR, which allow to develop a greater sense of belonging to a peer community that pursues the same educational purpose. By implementing the double learning process, the trainees constitute a community made by peers, which is a community of practice (Wenger, 1998) and, sometimes, also of inquiry (Jaworski, 2006), supported by the vigilant, but not intrusive presence of the trainers. In this order of ideas, the trainees construct meanings that are not only related to mathematics (and therefore to its epistemological value), but also linked to the didactic and methodological value that mathematics has.

It is worth noting that these discussed potentialities have emerged thanks to the design and methodological choices that have been made upstream and in progress by trainers. Precisely, the choice of contents to be proposed, the preparation of communication spaces, the way in which the birth of a community made only by trainees was favoured. For this reason, the research question highlighted the fact that MOOC-artifact should be properly organized if similar results want to be reached. Further exploration of the role played by the trainers will be carried out in Chapter 6.

All these aspects are the bases from which to get the result for which the trainers have worked and the trainees have enrolled in the MOOC, that is to benefit from professional development. And this is what we will deal with in the next chapter (Chapter 5).

4.11 Considerations and consequences

Before moving on to the next chapter, in which we are going to analyze the data that will allow us to answer the second research question, we want to dwell on some considerations that derive from the analysis carried out in this chapter.

4.11.1 Differences between face-to-face and MOOCs for teacher education

In §3.2.3.6, we have listed some differences between face-to-face and online teacher education. We have done some remarks that are valid for MOOCs in general, distinguishing between trainer's point of view and trainee's point of view.
After the analysis exposed until here, we can add other two points relative to the trainee's point of view and specific of our MOOCs experiences.

- **More comparison possibilities:** it could seems similar to the last bullet in the previous list ("Multiculturalism"). However, here we want to emphasize that the "small groups" are not formed. In a face-to-face course, they generally born because you go there with some people that you previously know; or because you sit down always in the same place and are surrounding by the same people. In a MOOC everyone talks to everyone, without privileged recipients. Moreover, the *plasticity*, previously described in §4.8.1, is the feature that characterizes the interactions.

- **Get compliments and share your own material:** these attitudes was a typical facet of our MOOCs. It is possible that it starts as a way to make one's presence felt, but it has become a feature consolidated over time (it has also continued in MOOC Numeri, in which we remember that 50% of subscribers came from MOOC Geometria). In a face-to-face course, you are more reluctant. You hardly say to a colleague, in front of everyone "good job, that is a beautiful idea"; and never ever would you share what is yours with everyone else. That's your job, you have a certain judgment about him. One often think "what belongs to others is mine and what is mine is mine!".

4.11.2 Success and quality of our MOOCs

We have seen in Chapter 1 that the participant's role is hotly contested across almost all literature and debate about MOOCs. Indeed, the key dilemmas in MOOCs center on what participation actually means, how it should be measured, and consequently, what metrics of success and quality are appropriate for these courses (Bayne & Ross, 2014).
Part of this complexity seems to arise because there are simply so many people, doing so many different sorts of things in any given MOOC. This presents a challenge for researchers, educators and institutions accustomed to using 'completion' as a fairly stable measure of the success and quality of an educational offering.
Formal completion rates for MOOCs for teacher education is 12% (Panero et al., 2017). We agree that probably it is not the right way to judge the quality of a MOOC or of participants' experiences, because this statistic is not taking sufficient account of those who may be engaging but 'do not adhere to traditional expectations, centered around regular assessment and culminating in a certificate of completion' (Kizilcec et al., 2013, p.9).
As stated in Chapter 1, we will not go into these matters. We will only make some considerations about what our completion rates have been. Although the literature reports that they are not the most appropriate yardstick for assessing the success or quality of a MOOC, our completion rates – as mentioned in Chapter 3 - are anyway higher than those that generally relate to the completion of MOOC for teacher education: 36% for MOOC Geometria and 42% for MOOC Numeri.

The considerations will be preceded by some data that emerges from the so called "third interview" (see §3.3.2.2 in the methodology section) that we have administered to all the trainees enrolled in MOOC Geometria and from the final questionnaire administered both in MOOC Geometria and in MOOC Numeri.

The third interview (March 2016), as mentioned, was addressed at all those who had enrolled in the MOOC Geometria and its first questions were a barrage. In other words, in the hypothesis that trainees who did not finish the MOOC could answer, in the first part – after the general information – we asked if they had finished the whole MOOC. If the answer was *yes*, the trainee could continue to proceed with the interview. If instead the answer was *partially* or *no*, we asked to explain why they had not completed this experience. So, in the

following we will show the analysis relative to the people that do not finish the MOOC Geometria in all its stages.
120 trainees (on 424 enrolled in MOOC Geometria) answered this "interview". About them, 45 are the trainees that do not finish the MOOC Geometria. Precisely, 7 of them answer they have *partially* finish the MOOC (they made more than 40% but less than 100% of the total activities required) and 38 answer they have *not* finish the MOOC (they made less than 40% of the total activities required).
To those who had partially finished we asked to explain what they had not done and why. The following table (Table 4.30) summarize the answers.

Who have partially finished the MOOC Geometria			
What you did not do?	# trainees	**Why you did not do?**	# trainees
Final tasks (PW and PR)	5	Overlapping with school commitments	1
From module 5 (the penultimate) onwards	1	Overlap with personal commitments	4
All parts of social interaction (some of which were necessary and sufficient condition to obtain the badge)	1	Both previous ones	1
		NA	1

Table 4.30: Trainees that have partially finished the MOOC Geometria

To those who had not finished we asked to explain why. The question was open, so we identified categories based on the answers obtained. The Table 4.31 summarize their answers.

Who have not finished the MOOC Geometria		
Why you did not do?	# trainees	Frequency in percentage
Overlapping with school commitments	3	8%
Overlap with personal commitments	18	47%
Both previous ones	2	5%
I could not keep up	4	11%
I would have preferred a course aimed only at my school order	4	11%
Little confidence with technology	2	5%
Sometimes complex requests	1	3%
Too many materials to examine	1	3%
I found it dispersive	2	5%
Personal internet connection problems	1	2%

Table 4.31: Trainees that have not finished the MOOC Geometria

We notice that who partially finished the MOOC has been hampered by personal or scholastic commitments. While, among those who have not finished there are trainees (the minority) that were not at ease in this online environment either because it was too quick in its progress, or because it was aimed at both lower and higher secondary school.
It is interesting read some of the answer given by the majority of them, which is the trainees that cannot finish the MOOC for overlap with personal commitments:

"*The numerous school assignments [...] and family work did not allow me to follow the MOOC. However, I found the material made available that I consider to be more relevant during the summer*[54]"

"*I was doing other courses (as a trainer) and I had no more time. Unfortunately I live at 80km from the school and at the end I returned home 4 days a week at 8.00 pm completely out of use. I was very disappointed because I thought it was very interesting*".

From these statements it seems that the trainees were interested and had positively evaluated the materials of the course and the formative possibility that it intended to offer.

Of all these 45 trainees, only one was enrolled in the MOOC Numeri.

In §4.4.1 and §4.61, we considered the analysis of some questions – in the final questionnaire – that expressed the level of appreciation of the trainees on the MOOC they had attended. The analysis is limited only to those who have completed the experience (so it means that the analysis refers to the students who identify the rate of completion of the MOOC).

We have seen that the **contents of the MOOC** corresponded to their expectations (75% in MOOC Geometria and 72% in MOOC Numeri – Figure 4.30 and 4.48). The **teaching materials provided** (videos, activity proposals, Geogebra files, ...) were judged *useful* by the 99% in MOOC Geometria and 100% in MOOC Numeri. The **duration of the course** with respect to the topics discussed was considered *good* by the 76% in MOOC Geometria and 78% in MOOC Numeri.

Moreover, with a closed question, we asked: "*If you were offered the chance to participate in another MOOC, always on a mathematical nucleus, will you do again this experience (will you enroll again)?*". The answer option were *Yes* and *No*.

In MOOC Geometria on 152 trainees, 149 answered *Yes*. About the 3 trainees who answered *No* we can say that: one actually is no longer enrolled and she is the trainee that we will analyze as a negative case at the end of Chapter 5. Instead, about the other 2 trainees, even if they have declared *No*, they have then enrolled in MOOC Numeri. One of the two has also completed MOOC Numeri in all its stages.

At the same question, in MOOC Numeri, out of 116 finalists, 100% answered *Yes*.

Moreover, it is important to note that the completion rate of Geometria was 36%, but among the trainees enrolled in MOOC Numeri, the 50% of them had been enrolled in MOOC Geometria.

If on the one hand, it is true that this description concerns only the finalists of the MOOCs, on the other hand, they cover a not indifferent slice of trainees in our MOOCs.

In light of the data presented above and those analysed in this Chapter, we believe there are 4 reasons why we recorded this educational success.

1. As evidenced by the data shown, the training opportunity that each of our MOOCs offered was valid both in terms of time and content.
2. The trainees have constituted more than just an online community, in fact, we have talked about communities of practice and sometimes even inquiry.
3. The vigilant presence of the tutors has certainly "reassured" the teacher-trainees who did not feel abandoned in an online environment.
4. Last, but not least, as we have seen in Chapter 3, teacher education is a right and a duty for Italian teachers. What better chance if you do not take advantage of a MOOC: online learning space (accessible wherever and whenever) and free!

[54] It should be remembered that the materials of the MOOC remain available for consultation to all its members

Chapter 5 Analysis of the MOOC’s trainees

The findings presented in this chapter contribute to addressing the second research question that is specific to the trainees:
Does the MOOC’s ZFM/ZPA trigger and support an effective shift from the actual developmental level to the potential developmental level in the ZPD of the trainees? And if so, which kind of expansion of the network of professional knowledge this shift brings with it?

This is a very delicate part of the dissertation. Quantitative and qualitative analyses will alternate to give a more complete view of the study conducted to respond to this complex research question.
We remember that the trainers involved in a MOOC for teacher education start to think about the design of the MOOC for a huge number of users. They do not know, in advance, what is the professional level of those who enrol in the MOOC. The trainers think about the ideal didactical praxeologies they want to transpose and assume a certain level of prior knowledge (ZPD) of the trainees. The trainers prepare and administer certain activities in order to help the trainees to move from the current level to the potential one.
It is, therefore, necessary to use tools that allow understanding the starting point of the trainees and tools to understand and/or identify possible developments with respect to this starting point.

It is important to make the following observation. Thanks to the networking between the re-elaborated Zone Theory and the MOOC-MDT (§2.10), the shift from the current level to the potential level of the trainees' ZPD can also be understood as an expansion of the network of professional knowledge. This brings with it two possible consequences:

- on the one hand, it is interesting to understand if there is an *evolution in the meta-didactical praxeologies* of trainers, so a real professional development;
- on the other hand, it is interesting to understand if there is a *perception of evolution in the didactical praxeologies*. More precisely, if there is a perception of changes in teaching knowledge, practices, and beliefs.

In fact, if the meta-didactical praxeologies evolve, new nodes are added and new connections are created within the own network of professional knowledge. While perceiving changes in own didactical praxeologies leads to making these new connections stronger and more stable since they are actually put into practice.

The chapter begins with a quantitative overview of the impact that the MOOC’s ZFM/ZPA has had on the trainees, analysing data that come from questionnaires administered to the trainees or from their compilation of written interviews (more details and clarifications will be provided below). Afterwards, two case studies follow to deepen the aspects linked to the shift from the actual developmental level to the potential developmental level in the ZPD of the trainees that is an expansion of the network of professional knowledge. The first is related to Lucy, a trainee who followed the MOOC Geometria. The second is related to Stephen, a trainee who followed both the MOOC Geometria and the MOOC Numeri. The chapter then concludes with the analysis of a negative case study, related to Ester, a trainee who followed the MOOC Geometria.

Note that we are talking about ***perceptions*** of changes in the didactical praxeologies of the trainees because the analyses were all done at distance. The interviews, as we will see in the case studies, were also conducted in writing or in virtual presence, on Skype. There has never been a time when the dissertation writer met the trainees personally or went to the classrooms to follow their lessons.

This choice is linked to a twofold reason.

On the one hand, for economic reasons. In fact, choosing one or more trainees and following them in their classroom requires moving frequently from the research centre (Turin, in this case) to the place where these trainees live, which is not said to coincide with the city in which the research is conducted[55]. From the analysis of RQ1 (the first research question) it was shown that the trainees are all Italians and most of those who have completed the MOOC live in Piedmont (the region where Turin is located). However, precisely because Turin is an active centre from the point of view of teacher training, (the Department of Mathematics "G. Peano" boasts at least a forty-year experience: see the Appendix D), many of the Piedmont trainees are used to work according to the innovative methodologies proposed by the MOOCs. In these cases, it would not have been possible to speak properly of evolution or change, but of consolidation of good practices already implemented since before the MOOC. Instead, trainees who live in geographic areas far from Turin could have had a greater advantage. Therefore, the interest was mainly addressed to teachers who lived far from Turin and Piedmont.

On the other hand, the choice of not having contacts in presence with trainees was also wanted. The intention of the research was to see what can be collected, deduced and identified by analysing only the declarations made online by the participants. Then, to analyse what impact the MOOC had on them compared to what they say, possibly even via Skype. Going to the classroom, in a sense, would condition the trainee to show something he saw in the MOOC and maybe does it in a certain way, which might not be his usual way, just to satisfy the researcher's expectations.

A different analysis strategy that could be followed for future researches will be presented in the conclusions chapter (Chapter 7).

For these reasons, the analyses presented here have been made taking into consideration both the comments on the communication message boards and the answers given on the questionnaires and, based on these, some targeted questions have been addressed to specific trainees identified as interesting case studies, both requesting compilations of questionnaires/interviews, and interviewing them via Skype.

[55] Remember that our MOOCs were designed in Turin and delivered by the DI.FI.MA platform managed by the Department of Mathematics "G. Peano" at the University of Turin. While the trainees are teachers who live throughout Italy.

5.1 A quantitative overview of the impact of MOOC's ZFM/ZPA on the trainees

5.1.1 Important clarification

Before starting with the analysis, let us make some important clarifications.

First of all, we specify that the analyses will be conducted in parallel on both MOOC Geometria and MOOC Numeri. However, it should not be surprising if sometimes, using the terminology of the theoretical framework, we will refer to the two MOOCs as a single one. In other words, in making general considerations, we will use the term MOOC-artifact and/or MOOC-ecosystem/instrument, or MOOC's ZFM/ZPA meaning considerations that can be extended to both the studied MOOCs (Geometria e Numeri). Explicit distinctions will be made when comparisons are to be made on different aspects of the two MOOCs.

Secondly, we analyse only the answers of the trainees who started and finished the MOOC in all its stages. Among them some distinction are necessary.
As we have seen in the previous chapter, during the analysis of the first research question (Chapter 4), we distinguish:

- the trainees that completed MOOC Geometria in all its stages will be indicated, from here and after, as "*Geometria trainees*". They are 152;
- the trainees that completed MOOC Numeri are 116 and they are distinguished in "*new entry trainees*" (the trainees that were enrolled only in MOOC Numeri, 51 trainees) and in "*former trainees*" (the trainees that were also enrolled in MOOC Geometria, 65 trainees).

Third, as mentioned before and also in the chapter devoted to the methodology (Chapter 3), in order to answer the second research question it was necessary to prepare some questionnaires and make interviews. Table 5.1 summarized them.

MOOC Geometria	**MOOC Numeri**
• 3 questionnaires to all the MOOC's trainees • 3 written interviews to a sample of MOOC population	• 3 questionnaires to all the MOOC's trainees • Skype interviewing or written interviews to deepen some case studies

Table 5.1: Questionnaires and interviews in MOOC Geoemtria e MOOC Numeri

There are some differences between the data collection tools that we have used in the two MOOCs. This is why MOOC Geometria was the first experience and, thanks to that experience, the analysis tools have been refined from the second one, the MOOC Numeri. We have explain all of them in details in the Chapter 3. Briefly, we remember that for the analysis of this chapter, we will mainly refer to the final questionnaires administered at the end of each MOOCs. In particular, the same questions of MOOC Geometria have been addressed in MOOC Numeri, but also more. These new questions also include questions that had been submitted through interviews in MOOC Geometria. These interviews were not replicated in MOOC Numeri exactly why we refine the analysis tools (see §3.3.2.2).

Remember that with written interview we means a specific kind of interview: these interviews were not conducted face-to-face, but in a written form. However, they cannot be considered as questionnaires because their setting is different (see §3.3.2.2).
For the purpose of the quantitative analysis that we want to accomplish, we will consider only the final interview (the third one) made to the MOOC Geometria's trainees. It was addressed to all the MOOC Geometria trainees when the MOOC was finished (March 2016). It consists of several closed, semi-open and open questions, invited teachers to reflect on possible changes between the period before the MOOC and that after it. We discussed about perception of changes both in their teaching knowledge, practices and beliefs.
75 trainees that filled this interview were also finalist of MOOC Geometria. Only 38 of them were also finalists in MOOC Numeri. We indicate them with this label: "*the former 38*". So, we will analyses some questions to consider this sample of trainees into account. See the following Figure 5.1 to have a clearer idea.

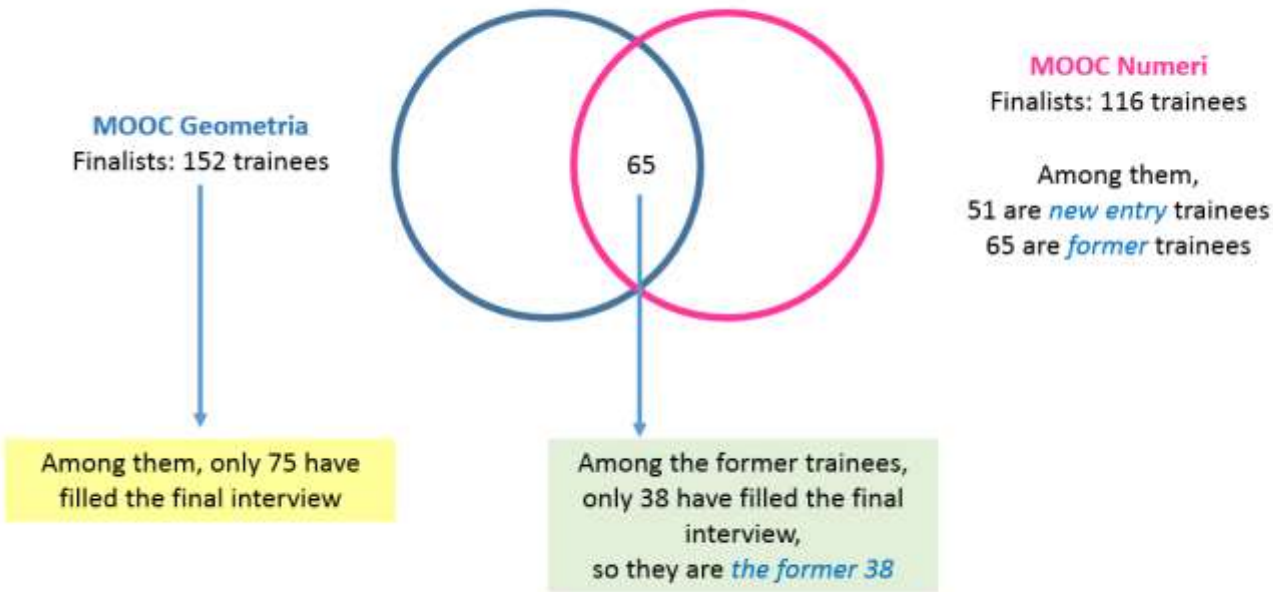


Figure 5.1: The distribution of the finalists[56] in the MOOCs

5.1.2 Quantitative analysis

The aim of this quantitative analysis is to underline some possible evolutions between the period before the MOOCs and that after them. Precisely, considering only the trainees that completed the MOOCs in all their stages, we consider what they declare before the MOOC Geometria and after its end. Later, we do the same for MOOC Numeri, but we distinguish among the *new entry trainees* (that can be considered as the *Geometria trainees*, because for them, MOOC Numeri is their first MOOC for mathematics education experience) and the *former trainees*, that are ending their second experience with MOOCs for mathematics education.
We discuss about professional development and perception of changes in both the trainees' teaching knowledge, practices and beliefs.

[56] Trainees that have finished the MOOC in all its stages

5.1.3 Why did you enrol in the MOOC?

Let us start analysing why the trainees decided to enrol themselves in such MOOCs.
A semi-open question, "*Why did you enroll in the MOOC?*", was asked in the final questionnaire of MOOC Geometria (filled by the 152 finalists). The Figure 5.2 summarized the answers. The majority (83%) declare that feel some need for education and development; the 14% instead answer that wants to have the experience of a distance course (and remember that MOOC Geometria was the first Italian MOOC for mathematics teacher education). Only five of them (3%) answer 'other' and specifying that they joined the MOOC because they wanted to be in contact with the university (in fact, remember that MOOC Geometria was delivered by the Department of Mathematics "G. Peano" at University of Turin).

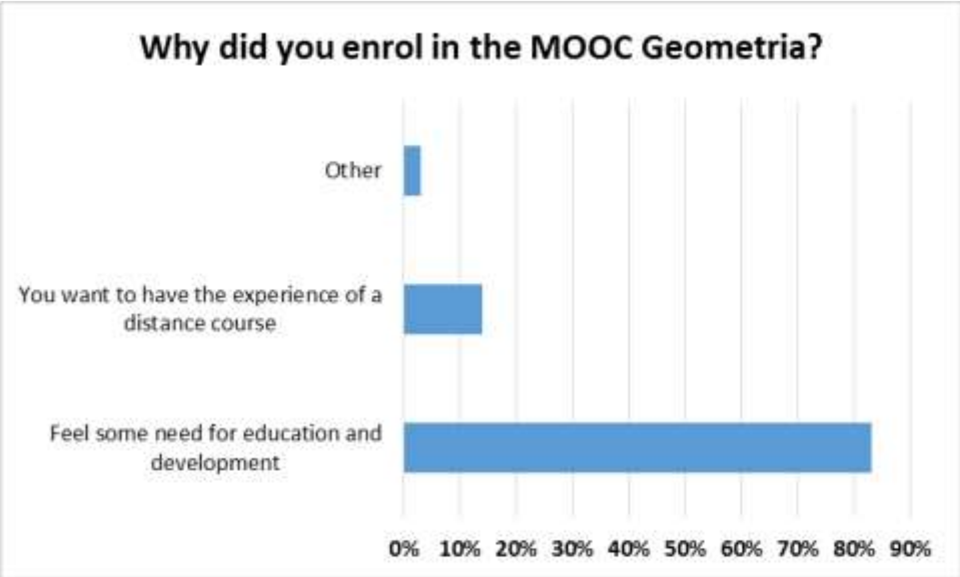


Figure 5.2: The reasons why the finalist trainees enrol in MOOC Geometria

This same question was instead asked in the initial questionnaire of MOOC Numeri. In the following analysis we take into account only the answers given by the 116 finalists. The Figure 5.3 summarized the answers. The majority of respondent (58%) was previously trainee in MOOC Geometria, so that means that they considered the proposal to be valid and they wanted to enrol in this new training offer. The 30% declare that feel some need for education and development; the 9% instead answer that wants to have the experience of a distance course. Again, a little part (3%) answer 'other', saying that they joined the MOOC because they wanted to be in contact with the university.

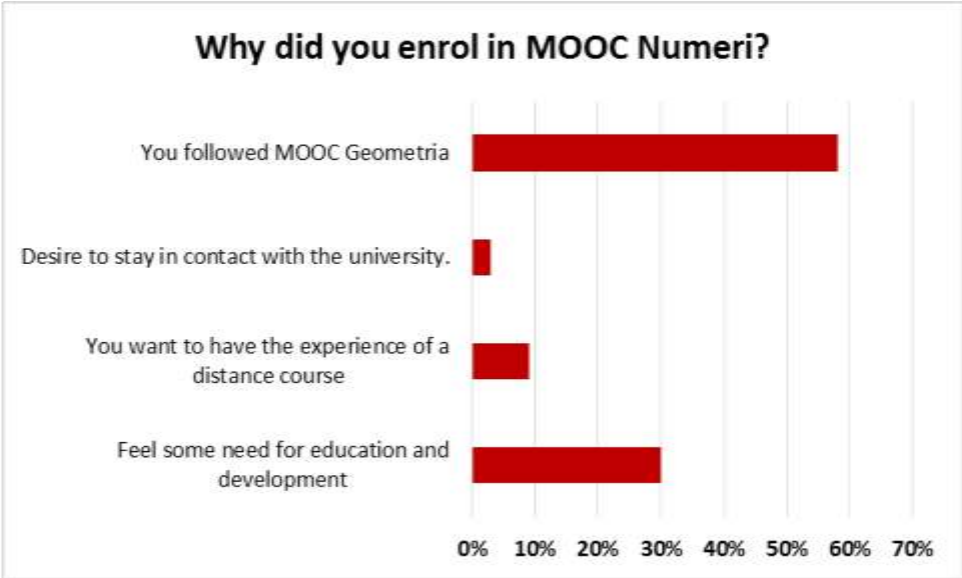


Figure 5.3: The reasons why the finalist trainees enrol in MOOC Geometria

It is important to note that in MOOC Geometria, the question of why enrolling in the MOOC was placed at the end of the distance learning experience. This “forced” the trainees to reflect on why, months before, they had decided to start this educational course. Recall that we are analysing the answers of those who have completed all the MOOC. The fact that the majority responds by admitting that they feel the need for training explains how effectively the trainees have given some confidence to the possibilities of formation that could have been gained from such a long-distance experience. The fact that they affirm it at the end of the experience makes us realize that the MOOC has had (positive) repercussions on their education. In other words, the finalists explain that they signed up because they needed training; they arrived at the end of the MOOC completing all its phases and to the question of why enrolling in a MOOC, they “confess” they were looking for ideas for education and development. Evidently, for completing the course in all its steps, the MOOC will have had a positive impact on them. The subsequent analyses will confirm this assumption even more.
In the MOOC Numeri, on the other hand, we preferred to address the same question at the beginning, so that the trainees immediately made an awareness. Also in this case, apart from the majority that comes from MOOC Geometria experience, the second high percentage reports the need for education and development. In addition, those who have been *Geometria trainees* may have subscribed again both because the experience with the MOOC Geometria enriched them or in some way has been positive, both because they still need education and development on another topic. Recall that the first MOOC focused on the topic of Geometry, while the MOOC Numeri was focused on the topic of Arithmetic and Algebra.
Confirmations of these assumptions do not emerge from the questionnaires. Only by Skype interviews has it been possible to have the certainty of this, in fact, we report some testimonies at the question: “*How did you find out about the MOOC Numeri?* – and only for the former trainees – *Why did you decide to continue to attend this our other training offer?*”.

M.A.A. (new entry trainee): […] I decided to join [the MOOC Numeri] because I was driven by the desire to research innovative teaching methods

S.L.C. (former trainee): […] The main motivations for registration were the strong training needs [...] (I teach only from 2014 [...]) and the need to understand more, to find paths and teaching strategies to be able to do better my job [...] I decided to continue because the experience of the former [MOOC Geometria] has been positive in many ways: new approach to discipline, participatory and comparison activities, constructive criticism, discovery of new IT tools (padlet, powtoon,. ..).

R.R. (former trainee): […] It was a great pleasure, both last year and this year, to follow your courses because they are very stimulating [...] you learn by doing, your proposals are always interesting, give the possibility to give new ideas to us as teachers. [...] I am really excited [to have participated].

We have had occasion in the chapter devoted to the theoretical framework (Chapter 2) to introduce the theoretical lens of notion of ***productive tensions*** (Goos 2013). It is crucial for grasping teachers' changes from a zone theory perspective. So, we recall its definition, because we will use it in the following:

> Tensions arise from dissatisfactions that teachers experience when their ZPD does not map onto the ZFM/ZPA complex in ways that promote desired development: this can be thought of as a misalignment within the zone system. The tension is productive if it triggers change that aims to bring the zones into alignment, for example, by modifying the environment (ZFM) or seeking out professional learning opportunities (ZPA) (Goos 2013, p. 523).

In both the MOOCs experiences, as we have observed, a wide part of trainees feel some need for education and development. We can interpret this as a feeling of certain tension that has pushed the trainees to attend the MOOC; a mild misalignment of the zones system. In other word, these trainees felt that their ZPD (their prior knowledge relatively to mathematical content, pedagogy, technology) could be improved to fit better in their school's ZFM/ZPA (see Table 5.2). The fact that these trainees have completed the MOOC(s) in all its (them) stages – with active participation, satisfying all the tasks required – invite us to assume that the initial tension has evolved into a productive one. The following data will reinforce this sentences.

Taranto's re-elaboration/interpretation	
Valsiner/Goos's zones	**School's ZFM/ZPA (~ Goos's interpretation)**
ZFM: Zone of Free Movement (structures teacher's access to different areas of the environment, availability of different objects within an accessible area, ways the teacher is permitted or enabled to act with accessible objects in accessible areas)	• Perceptions of students • Access to resources • Technical support • Curriculum and assessment requirements • Organisational structures and cultures
ZPA: Zone of Promoted Action (people, objects, or areas in the environment in respect of which the teacher's actions are promoted)	• In-service teacher education • Experimentation in classroom of some activities see in the MOOC • Professional development • Informal interaction with teaching colleagues

Table 5.2: School's ZFM/ZPA (trainees only)

5.1.4 Teaching methodologies and technologies

In the theoretical framework we have identified the ZPD with the own network of knowledge. We have said that each individual has his own network of knowledge. When the individual is in particular involved in a training action, such as a MOOC for teacher education, with respect to the whole network of knowledge, the part called into question will be the one that is relative and most suitable to the knowledge related to the teacher education (we have referred to it as *network of professional knowledge*). So, we have considered as elements that compose the ZPD that the trainees activated attending the MOOC the one listed in the Table 5.3.

Taranto's re-elaboration/interpretation for trainees	
Goos's zones in Taranto's interpretation	**Goos and Taranto's interpretation**
ZPD: Zone of Proximal Development (possibilities for expand the own network of knowledge)	• Mathematical Knowledge • Pedagogical content knowledge • Skill/experience in working with technology • (Meta-)didactical praxeologies that include beliefs about mathematics, teaching and learning

Table 5.3: Trainees' ZPD

To know the initial ZPD that characterize each trainees, or to know their actual level of prior knowledge, we decided to pose some questions in the initial questionnaire, administered at the beginning of each MOOC. The same questions were posed at the end of each MOOC (in the final questionnaire, or also in the final interview in the case of MOOC Geometria - see Table 5.1 and the Table 3.5 in the Chapter 3) to understand if some expansion of network knowledge happened.

We did not examine the ***mathematical knowledge*** of the trainees. All trainees are in-service mathematics teachers. We have assumed the fact that their knowledge of mathematical contents was certainly of a medium-high level, as well as all the activities proposed in the MOOCs. The trainees have never complained, during the MOOCs, difficulties in understanding the mathematical activities. So, it is enough to confirm their mathematical knowledge. Furthermore, MOOCs were not intended to teach new mathematical content, rather the ideal praxeologies that the trainers wanted to transpose concerned innovative methodologies and new teaching strategies, also using the technologies. In fact, as we observed in Chapter 3, the proposals of the MOOCs offer concrete examples of laboratory-based methodology activities (Anichini et al., 2004) to be carried out in the classroom, and with technologies as well.

5.1.5 Before and after the MOOC Geometria

Regarding ***pedagogical content knowledge*** and ***skill/experience in working with technology***, some questions are posed.
Let us start considering the *Geometria trainees*. The data that we show come from the analysis of the first and final questionnaire and the final written interview. The goal is to

analyse whether there have been changes between the period before the MOOC and the one after it. The data relating to the period following the MOOC are those relating to those who completed the final written interview. We have only 75 responses from trainees who are also finalists of the MOOC. For this reason, in the following analyses the data refer only to these 75 *Geometria trainees*.

In the initial questionnaire, about ***pedagogical content knowledge*** we ask the following questions.
First of all, the closed question: "*Do you implement laboratory-based methodology activities in your classes?*". The Figure 5.4 shows the distribution of responses. Most of the trainees (89%) say they implement this methodology in the classroom, compared to a 7% who, despite knowing it, does not use it. While 4% say they do not know laboratory teaching. This might be a little surprising, but let us remember that the trainees are a very heterogeneous group of teachers. From the analysis of RQ1 we have seen how the ages of the trainees are very varied as well as the years of experience in teaching mathematics (see Figure 4.3).

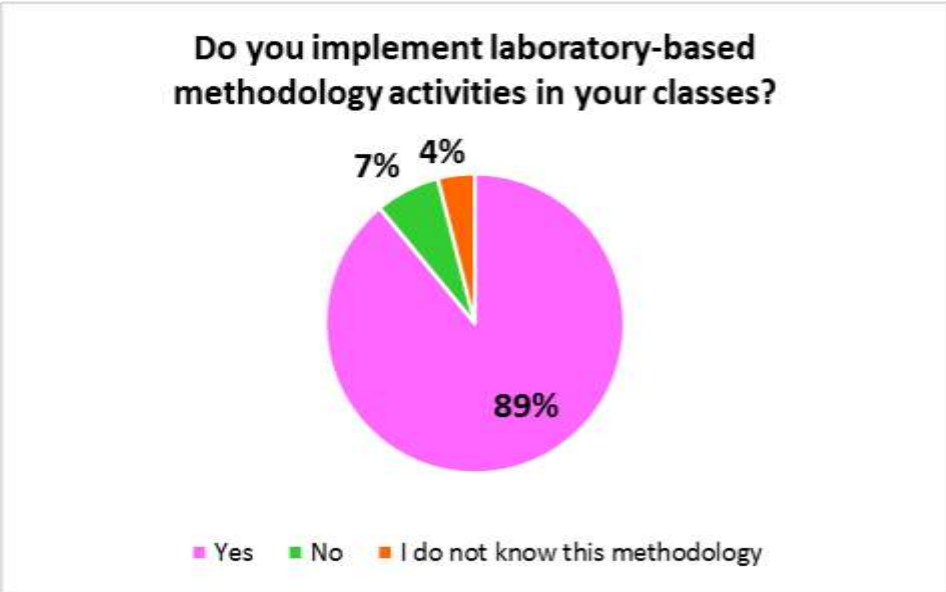


Figure 5.4: Teaching laboratory implementation before attending the MOOC Geometria

After, we asked the closed question: "*Do you know practical-manipulative activities to use for presenting math concepts to your students?*". As you can see in the Figure 5.5, 87% answer *Yes*. Therefore, we asked to who answer *Yes* if, since they know it, they use it in the classroom, with a closed question: "*Did you use practical-manipulative activities in your classes?*". The 20% of them answer *No*. To them, with a semi-open question, we asked to explain why they do not use them, despite the fact that they know them. The answers given by the trainers are varied (Figure 5.7): 31% of them say that they did not fully understand how to use them. The majority (38%) explain that it did not know anyone that use them to compare itself with them about their use in classroom. Another 31% that they thought the practical-manipulative activities were more suitable for the primary school.

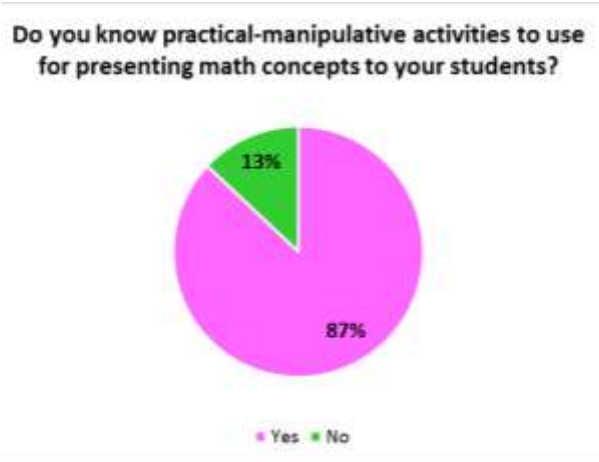


Figure 5.5: Practical-manipulative activities knowledge before attending the MOOC Geometria

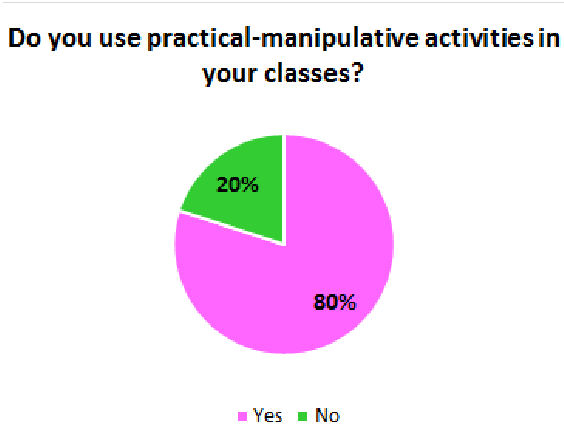


Figure 5.6: Practical-manipulative activities use before attending the MOOC Geometria

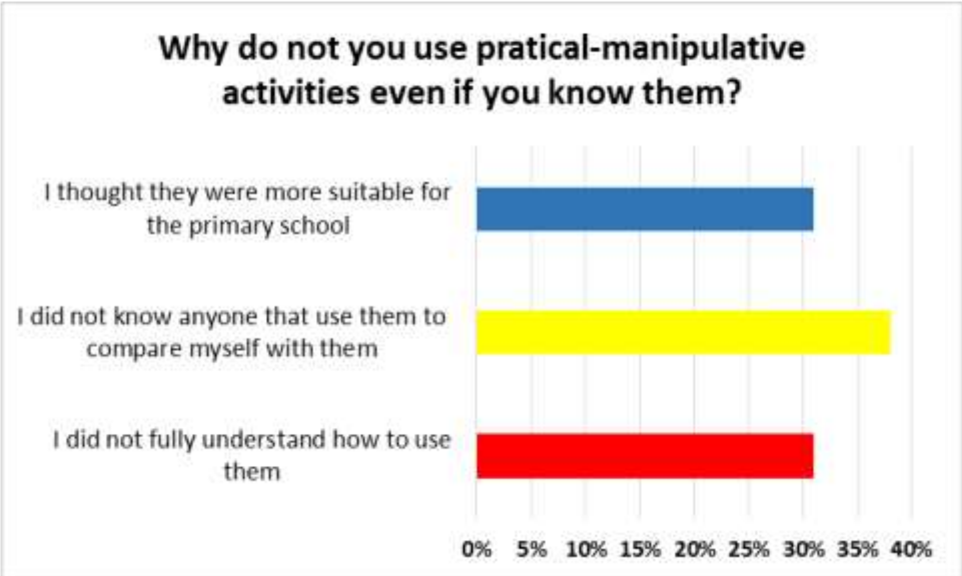


Figure 5.7: Reason for which practical-manipulative activities are not used even though they are known, before attending the MOOC Geometria

We continue to ask the following closed question: "*About the problem solving you make in classroom, do they take on real situations?*". The majority of respondent (56%) answer that they do not use problem solving that take inspiration from real situations, as you can see from Figure 5.8.

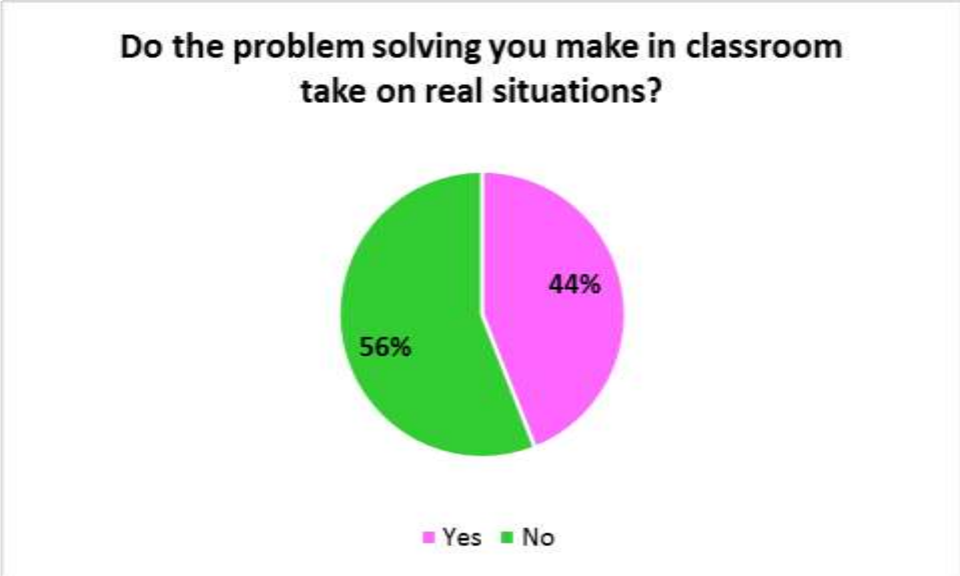


Figure 5.8: Problem solving that take inspiration from real situations before attending the MOOC Geometria

To have an idea about ***skill/experience in working with technology***, we asked to the trainees, always in the initial questionnaire and with a closed question: "*Do you use technology in your teaching practices?*". Only 2% of them not much use technology in own teaching practices, while 39% and 59% use it quite a lot and a lot respectively (Figure 5.9).

Figure 5.9: Use of technology in teaching practices before attending the MOOC Geometria

This question was then joined by an open one, which asked: "*What are, in your opinion, the advantages and benefits of using technology in mathematics hours?*". Its answers are categorized as follow in Figure 5.10. The 21% of the trainees did not answer to this question. However, the answers received are very varied. In descending order, we have that: 23% think that the technology contributes to increasing motivation/attention; it is followed by 22% that responds by saying that it allows to observe dynamism in geometrical figures and helps in graphical visualization. The other percentages are smaller, but we report them anyway: 9%

say that technology stimulates different forms of learning such as creativity; 8% think that it allows a greater understanding of the concepts; 5% believe that it helps to get closer to the world of digital natives; 3% argues that it helps to present the topics differently and 2% sees technology as an inclusive tool. 7%, on the other hand, answers 'Other'.

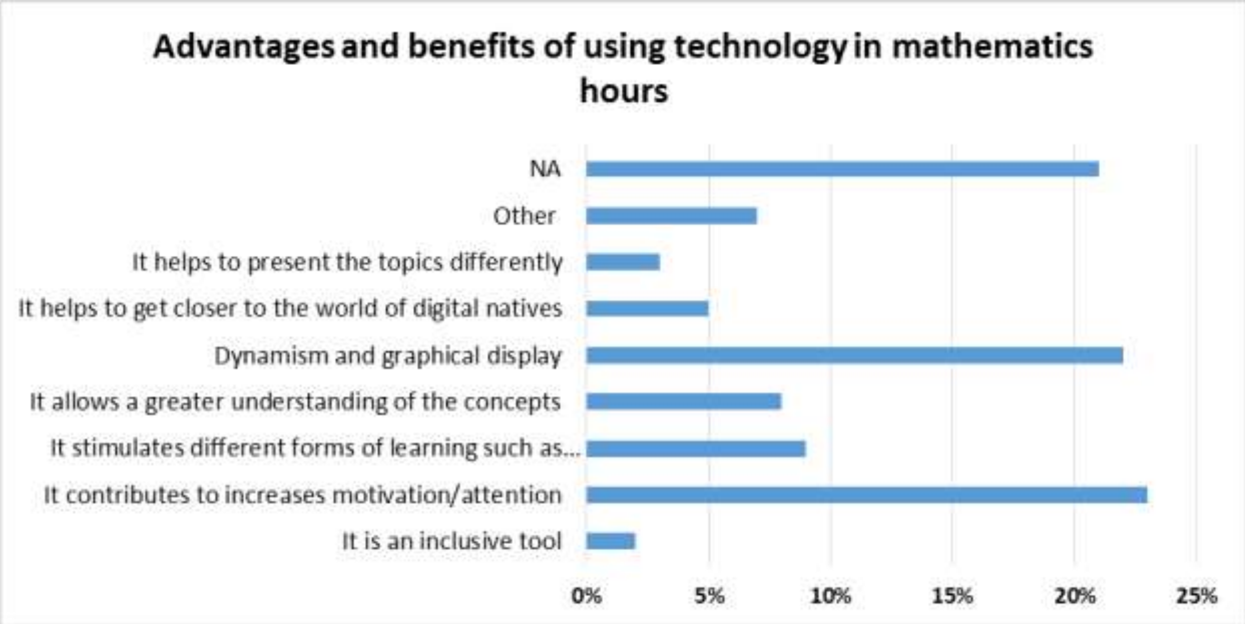


Figure 5.10: Advantages and benefits of using technology before attending the MOOC Geometria

This starting analysis allow us to have an idea of the actual level of *Geometria trainees'* ZPD. The questions asked above all refer to methodologies that the trainees have then found in the MOOC. Consider, for example, the description of the activities illustrated in Module 1 of MOOC Geometria (Chapter 4). To overcome the misconceptions related to the concept of the height of the triangles, we start with situations that are inspired by reality (the mainmast of the boat or the triangular park). It is favoured the laboratory methodology, which requires students to work in groups, formulate conjectures, also make use of practical manipulative activities (think of the strips of paper whose height has to be determined). There are also activities that invite the trainees to make use of the technology. The purely mathematical technologies suggested are the spreadsheet and GeoGebra. But, between the lines, the invitation is also to consider the technologies used by designers, which we remember are all open source. For example, the Padlet, the Tricider and also the Sway are technologies that teachers could decide to integrate into their teaching practices (see Table 3.3 in the Chapter 3).
Therefore, we now see what happened to the *Geometria trainees*, considering the answer given by the 75 *Geometria trainees* to the final written interview. Then the theoretical justifications of what we will observe follow.
The question asked are the same previously analysed. At the closed question "*Do you implement laboratory-based methodology activities in your classes?*", all the 75 trainees answer *Yes*. It means that not only the 4% that before to attend the MOOC Geometria did not know that methodology (Figure 5.4), after the MOOC has understood what it is; but also that this 4% together with the 7% that despite the fact that it knew teaching laboratory it did not use it (Figure 5.4), after the MOOC implement this methodology in their teaching practices.

About practical-manipulative activities we did not ask if the trainees knew them, because after the MOOC the answer would be of course *Yes*. We directly asked "*Did you use practical-manipulative activities in your classes?*". Before attending the MOOC Geometria, only 52 on 75 trainees had answered that they used them in their classes (Figure 5.6); after the MOOC 59 on 75 trainees (Figure 5.11).

Figure 5. 11: Practical-manipulative activities use after attending the MOOC Geometria

Moreover, we asked to explain, with a semi-open question, "*What are the advantages you get from using them? (you can select a maximum of 3 answer options)*" (Figure 5.12). The 29% answer that using practical-manipulative activities enhancing the phase of discovery of properties. The 23% say that their use allow avoiding formation of misconceptions. For the 20% they allow stimulating argumentation and for the 11% they enhance the formulation of conjectures. The 17% think that learning with the body allow you remembering more. Anyone add other advantages to these one that were provided as answer options.

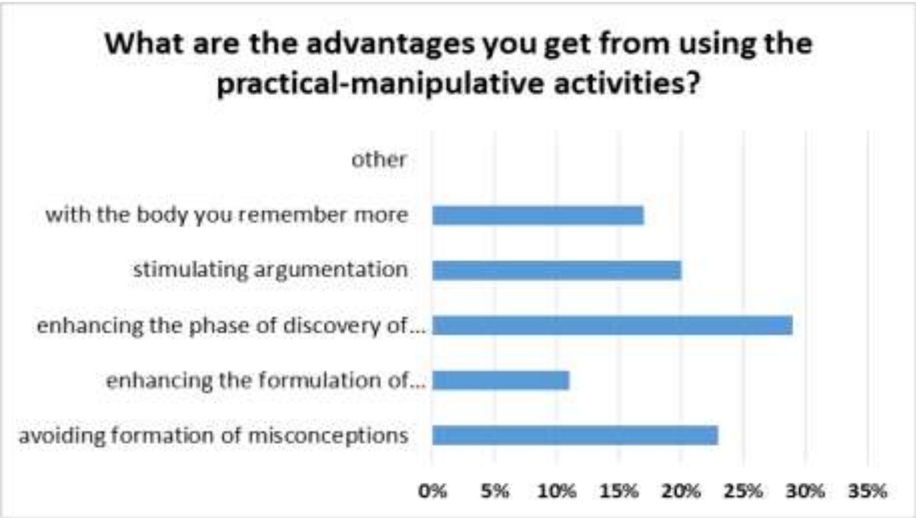


Figure 5.12: Advantages from using practical-manipulative activities after attending the MOOC Geometria

A significant increase in percentage regards the question "*About the problem solving you make in classroom, do they take on real situations?*". Before attending the MOOC only 44% answered *Yes* (Figure 5.8). After attending the MOOC the situation is showed in Figure 5.13.

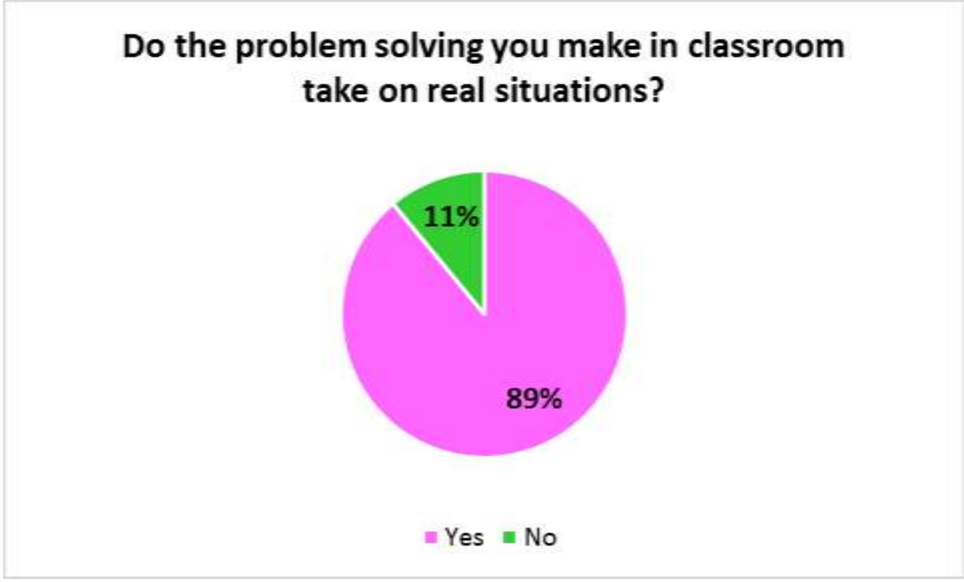


Figure 5.13: Problem solving that take inspiration from real situations after attending the MOOC Geometria

As far as the use of technology is concerned, it was no longer asked how much it was used in the classroom, but the question about the advantages and benefits that are attributed to its use in the classroom was asked again. Table 5.4 compares the percentage frequencies of the before and after the frequency of the MOOC. However, no significant changes are recorded.

What are, in your opinion, the advantages and benefits of using technology in mathematics hours?		
	Before attending the MOOC Geometria	After attending the MOOC Geometria
It is an inclusive tool	2%	1%
It contributes to increases motivation/attention	23%	22%
It stimulates different forms of learning such as creativity	9%	11%
It allows a greater understanding of the concepts	8%	11%
Dynamism and graphical display	22%	27%
It helps to get closer to the world of digital natives	5%	7%
It helps to present the topics differently	3%	1%
Other	7%	7%
NA	21%	13%

Table 5.4: Advantages and benefits of using technology in mathematics hours after attending the MOOC Geometria

The analysed data suggest that, compared to methodologies and technologies, the ZPD of the *Geometria trainees* has undergone a positive change. After attending the MOOC Geometria the percentages show an increase. This means that the professional network of knowledge has expanded, i.e. it has moved from a current level to a potential level.
The expansion of the network of knowledge is linked to the **double learning process** that the trainee puts in place (and of which we have extensively discussed in the previous chapter analysing the RQ1). Remember that in the double learning process it is important both the interaction that the trainee establishes with the resources made available by the trainers in the MOOC, and the interactions that are generated on the communication message boards used by the trainees in exchanging reflections, opinions, ideas and also materials. Remember also that learning is understood in a connectivistic sense: it is not a "literal" learning of new things, rather it means to be able to see different concepts that were already known (reflect, think again, integrate them under a different perspective).
For example, about laboratory methodology, we have observed that before MOOC Geometria there were both 4% of the trainees that instead did not know this methodology, and a 7% of the trainees that instead knew this methodology but did not use it in their teaching practices. After attending the MOOC Geometria the 100% of the trainees declare to use laboratory methodology in their classes. Therefore, the first group (the 4%) add a new node in their professional network of knowledge; while the second one (the 7%) see under a new light this own knowledge about teaching laboratory. In such a way they have experimented learning in connectivist way.

5.1.6 Before and after the MOOC Numeri

Regarding ***pedagogical content knowledge*** and ***skill/experience in working with technology***, some questions are posed.
Let us start considering the *Numeri trainees*. Here we can distinguish three different groups of trainees (see Figure 5.1):

1. the *new entry trainees* (51 trainees), the trainees that were enrolled only in MOOC Numeri and that we can consider as the *Geometria trainees*, as they are following for the first time a MOOC for mathematics teacher education;
2. the *former trainees* (65 trainees), the trainees that were also enrolled in MOOC Geometria, that did not necessarily complete the MOOC Geometria in all its phases.;
3. the *former 38 trainees* (38 trainees), a subset of the former trainees. They are the trainees who had enrolled in the MOOC Geometria completing it in all its phases and also compiling the last written interview, from which we have been able to obtain data to observe the expansion of their network of knowledge after the experience of the MOOC Geometria. Recall that 75 *Geometria trainee* had compiled the last written interview, of these only 38 were also finalists in MOOC Numeri.

The data that we show come from the analysis of the first and final questionnaire. The goal is to analyse whether there have been changes between the period before the MOOC Numeri and the one after it. The questions posed to analyse pedagogical and technological knowledge are the same ones that have turned to the *Geometria trainees*.

In the initial questionnaire, about ***pedagogical content knowledge*** we ask the following questions.
First of all, the closed question: "*Do you implement laboratory-based methodology activities in your classes?*". The Figure 5.14 shows the distribution of responses relative to the three different groups of trainees.

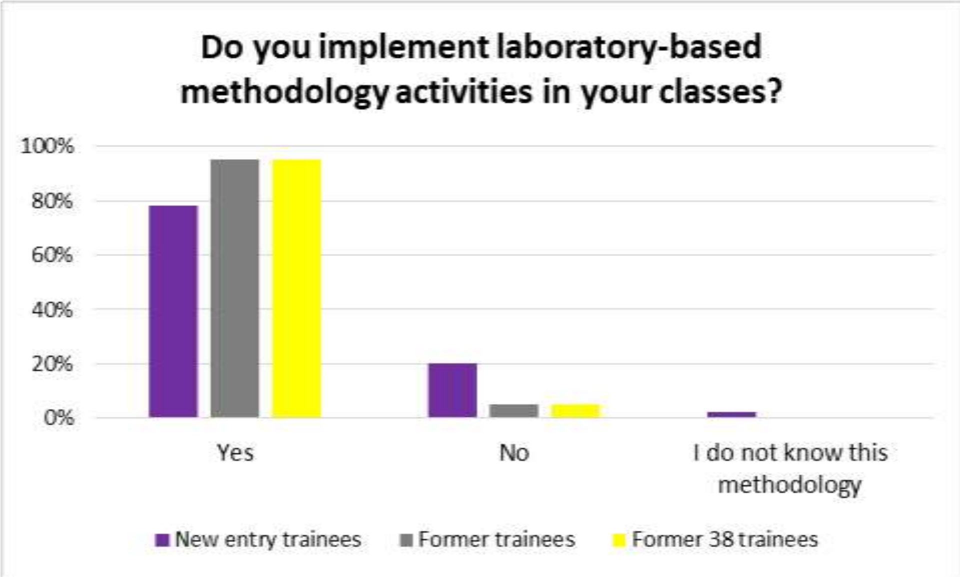


Figure 5.14: Teaching laboratory implementation before attending the MOOC Numeri

In general, most of the participants implements this methodology in the classroom (78% *new entry trainees*, 95% *former trainees*, 95% *former 38 trainees*). It is not surprising that almost all the former trainees and the *former 38 trainees* make use of it and, above all, that none of them declare that they do not know it. Yet among them, there is a 5%, respectively, that despite knowing it continues to not use it. This is slightly contrary to the statements of 100% of the *former 38 trainees* who, at the end of the MOOC Geo, had declared to use them all in their own classes. Instead, among the *new entry trainees*, we repeat what we observed with the *Geometria trainees*. A small percentage (2%) does not know the laboratory-based methodology activities and the 20% knowing it, does not use it.

After, we asked the closed question: "*Did you use practical-manipulative activities in your classes?*" (Figure 5.15). As for the *new entry trainees*, the situation is not different from what was observed for the Geometria *trainees*. In fact, most (63%) use practical-manipulative activities. This time we do not ask why they do not use practical-manipulative activities; instead we directly asked why they consider this activities advantageous in their teaching practices, with a multiple-choice question: "*What are the advantages you get from using the practical-manipulative activities?*". The *new entry trainees* give the following reasons (Figure 5.16): the 31% answer that using practical-manipulative activities enhancing the phase of discovery of properties. Follow the 27% that think that learning with the body allow you remembering more. Then, for the 16% they allow stimulating argumentation and for the 12% they enhance the formulation of conjectures. The 7% say that their use allow avoiding formation of misconceptions and another 7% answer with 'Other'.

Figure 5.15: Practical-manipulative activities use before attending the MOOC Numeri

Figure 5.16: Advantages from using practical-manipulative activities before attending the MOOC Numeri

As for the former *trainees*, we observe that the percentage of those who use the practical-manipulative activities (85%) is higher than that of *new entry trainees* (Figure 5.15). After all, this does not surprise us: on the use of practical-manipulative activities, enough emphasis was placed on MOOC Geometria. It is interesting to observe the statements of the *former 38 trainees* because we can compare them with the statements of the final interview made after the end of MOOC Geometria. Remember that (as you can see from the Table 3.5) the final interview was done in March 2016, while the initial questionnaire of MOOC Numeri in November 2016.

In particular, if we compare their previous answer with this new one (Figure 5.17), we can see that after 8 months (from March to November) there were no changes in their statements on the use of practical-manipulative activities in the classroom, in fact the percentages remained constant.

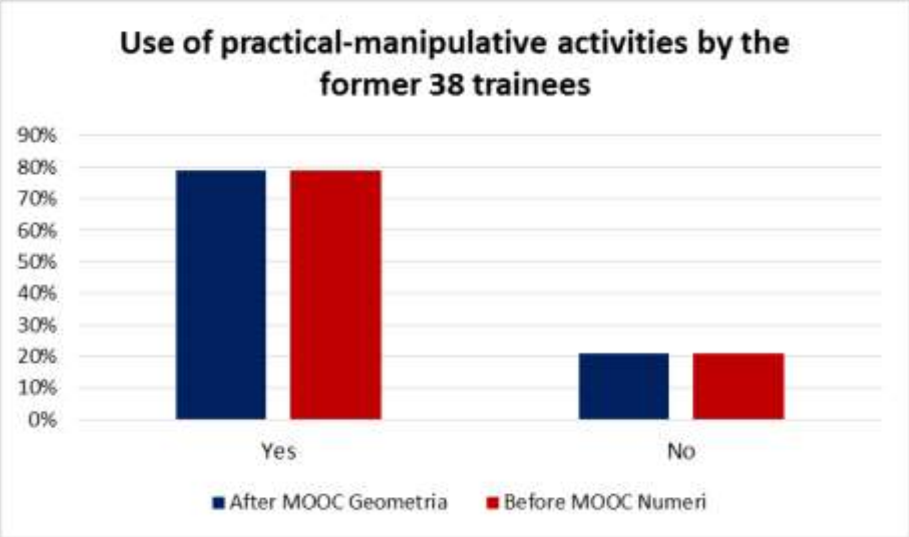


Figure 5.17: Use of practical-manipulative activities by the former 38 trainees after attending the MOOC Geometria and before attending the MOOC Numeri

If we want, in particular, compare what they thought and think about the advantages in the use of this kind of activities in classroom, we can consider the Figure 5.18.

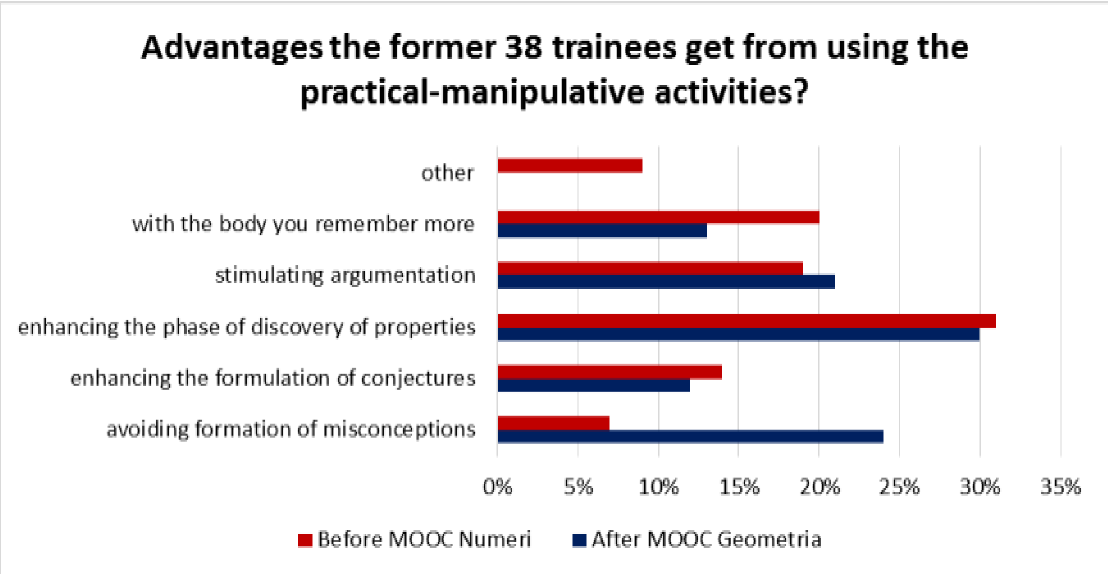


Figure 5.18: Advantages in the use of practical-manipulative activities by the former 38 trainees after attending the MOOC Geometria and before attending the MOOC Numeri

After 8 months, there is a shift in their opinion: the majority are mildly shift, as showed in figure 5.18. However, more emphasis is given to the fact that these activities involved the body and thank to this you remember more (from 13% to 20%); while there is a decrees in the consideration of the fact that these activities allow avoiding formation of misconception (from 24% to 7%). This is surprising because in the MOOC Geometria the practical-manipulative activities had been presented above all to help overcome the formation of misconceptions (think of the activity described for Module 1 of MOOC Geometria on the misconception between perpendicular and vertical). Therefore, if at the end of the MOOC Geometria these trainees had given some weight to the advantage that these kinds of activities gave in terms of avoid misconception because the experience and the examples of the MOOC were strong within them, now it seems that after 8 months this opinion has changed. In terms of the network of knowledge, it means that the former 38 trainees give a weight of minor importance to the node that connects practical-manipulative activities with overcoming the misconceptions.

We continue to ask the following closed question: "*About the problem solving you make in classroom, do they take on real situations?*". It is interesting notice from the Figure 5.19 that the *new entry trainees* are in a more positive trend compared to the *Geometria trainees* that answered the initial questionnaire of MOOC Geometria (Figure 5.8). In fact, in that questionnaire, more than half of Geometria trainees have answered *No* to this question. It is also true that the Italian curriculum aims a lot in its indications to propose to the students problematic situations that are inspired by reality. The fact that *new entry trainees* are already operational on this front is positive for their professional role.

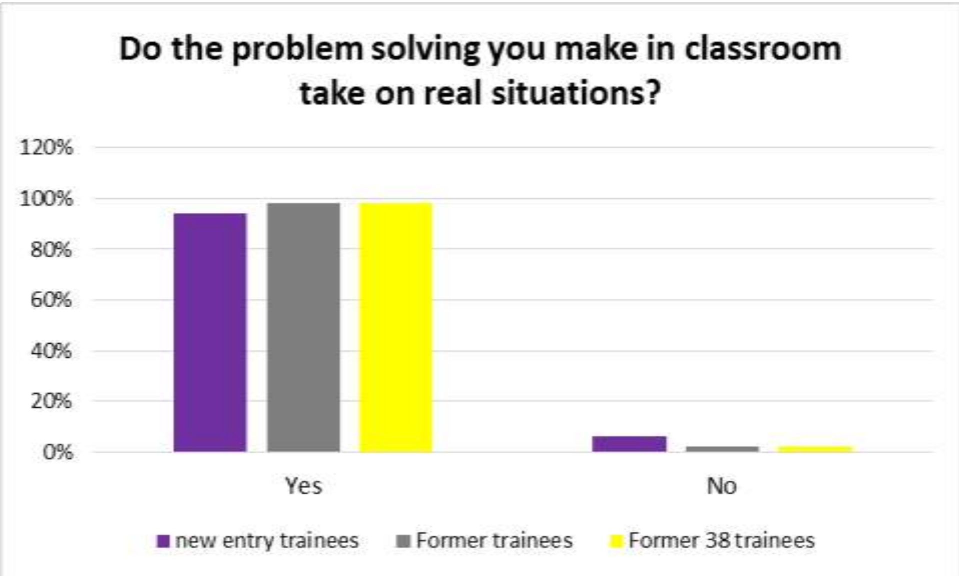


Figure 5.19: Problem solving that take inspiration from real situations before attending the MOOC Numeri

As for the *former 38 trainees*, we compare this response again with that given in the final MOOC Geometria interview (Figures 5.20).

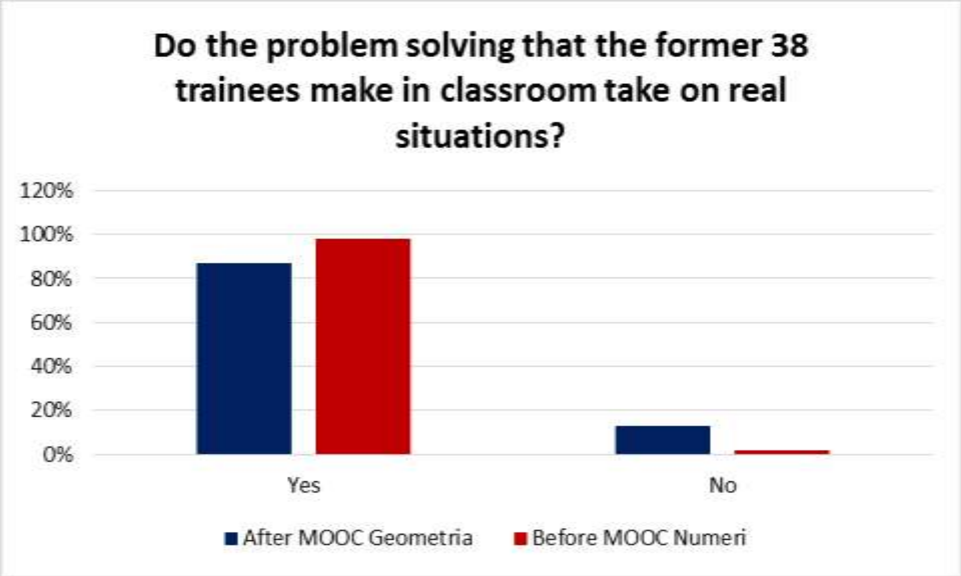


Figure 5.20: In which percentage the problem solving that the former 38 trainees make in classroom take on real situation after attending the MOOC Geometria and before attending the MOOC Numeri

We can observe that there is a positive shift (from to 87% a 98%): 8 months after the end of the MOOC Geometria, these trainees have continued to propose problematic situations in the classroom that are inspired by real situations.

To have an idea about ***skill/experience in working with technology***, we asked to the trainees, always in the initial questionnaire and with a closed question: "*Do you use technology in your teaching practices?*".

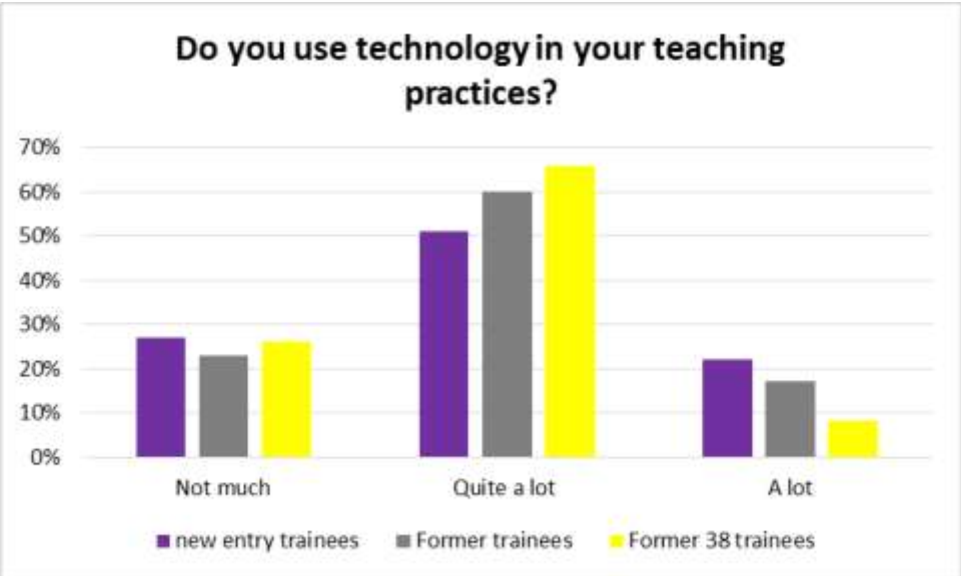


Figure 5.21: Use of technology in teaching practices before attending the MOOC Numeri

The situation showed in Figure 5.21 is quite varied: the trainees in general use quite a lot technology in their teaching practices. Anyway, also the extreme position are covered: *not* *much* is the answer of 27%, 23%, 26% of *new entry*, *former* and *former 38 trainees* respectively; *a lot* is the answer of 22%, 17%, 8% of *new entry*, *former* and *former 38 trainees* respectively. This situation is very different compared to the answer given by the *Geometria trainees* (Figure 5.9), where the majority declared that they used a lot of technology in their teaching practices and only the 2% said not much. It seems that the *Numeri trainees* are more cautious and reflective in giving answers.

This question was then joined by a semi-open one, which asked: "*What are, in your opinion, the advantages and benefits of using technology in mathematics hours?*". This time the question was mandatory and semi-open, not open as in the initial questionnaire of MOOC Geometria and in the option of answer we propose the determined categorize in the previous analysis that show an elevate percentages (Figure 5.22).
The *new entry trainees* give the following reasons (Figure 5.22): the 31% answer that using technologies contributes to increases motivation/attention in the students. Follow the 30% that think that it allows a greater understanding of the concepts. For the 27% the technology stimulates different forms of learning such as creativity and only the 7% consider it as an inclusive tool. Another 5% answer with 'Other'.

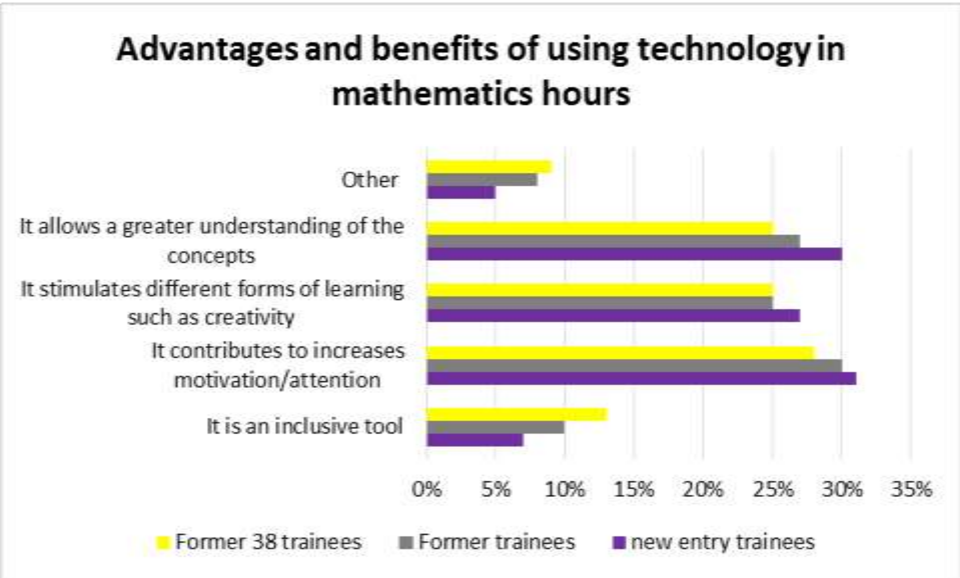


Figure 5.22: Advantages and benefits of using technology before attending the MOOC Numeri

Regarding the statements of the *former 38 trainees* we can not compare the answer given to the question "*Do you use technology in your teaching practices?*" with something that they answered in the last interview, because in the last interview we knew that they made use of technology and we did not ask how much they continued to use it. We instead asked the advantages that they think the use of technology can give. However, it is not very significant compare these answers because, as specified, in the last interview in MOOC Geometria, the question was open, while here in the initial questionnaire of MOOC Numeri it was semi-open, with the answer option that corresponded to some elevate percentage of the previous analysis. Anyway, the Figure 5.23 show the comparison of the answers given in these two-distinct moments by the former 38 trainees.

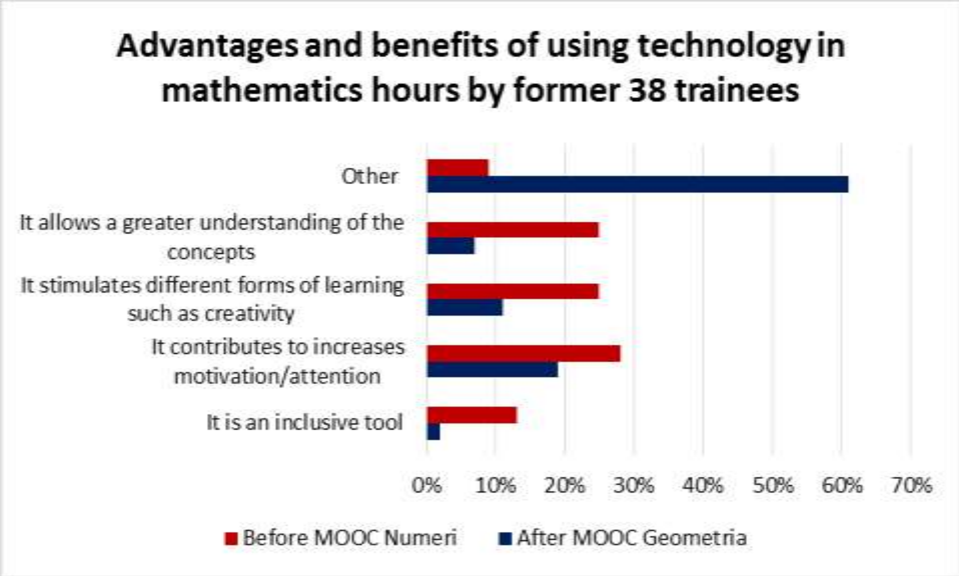


Figure 5.23: Advantages and benefits of using technology in mathematics hours by the former 38 trainees after attending the MOOC Geometria and before attending the MOOC Numeri

There is a high percentage in *other* that is not redistributed in the other answers of the *former 38 trainees*. This high level is due to the fact that we also included in *other* the category "the technology helped in the drawings and in the graphic visualization" that had been declared by 30% of the *former 38 trainees* who compiled the final interview of MOOC Geometria (Figure 5.10). Since it seemed a foregone answer, we did not propose it among the answer options, believing that it would be declared in *other* even by future respondents. However, it has not been so. Those who answered *other* almost never referred to this possibility offered by technology. Therefore, the trainees, in general, preferred to choose between the proposed options, rather than "trying" to insert others.

This starting analysis allows us to have an idea of the actual level of *Numeri trainees'* ZPD.
The questions asked above all refer to methodologies that the trainees have then found in the MOOC. Consider, for example, the description of the activities illustrated in Module 5 of MOOC Numeri (Chapter 4). To become familiar with the algebraic language, MOOC proposed activities concerning mathematics game and challenges of mental calculation abilities, since these laboratory activities favour learning by discovery and the collaboration among students. Then, to ensure that students do not interpret algebraic formulas as pure sequence of sign, each formula is associate with a geometric interpretation that can be produced using the GeoGebra technological software. Moreover, problems in which the language of algebra overcomes that of arithmetic and becomes a tool for expressing relationships and generalities were proposed taking inspiration from real life situations (for instance**,** the problems of the price of the dress or the problem of the pond).
Therefore, we now see what happened to the *Numeri trainees*, considering the answer given by the trainees to the final questionnaire. Then the theoretical justifications of what we will observe follow.

The asked questions are very similar to the ones analysed previously: the formulation of the question and the answer options change slightly, as we will see below.

We start with this semi-open question: "*Now that the MOOC is over, will you do laboratory-based methodology activities in your classroom?*". The answer options are: "*Yes*; *No*; *I did it before*; *Other (specify)*" and are distributed as shown in Figure X23.

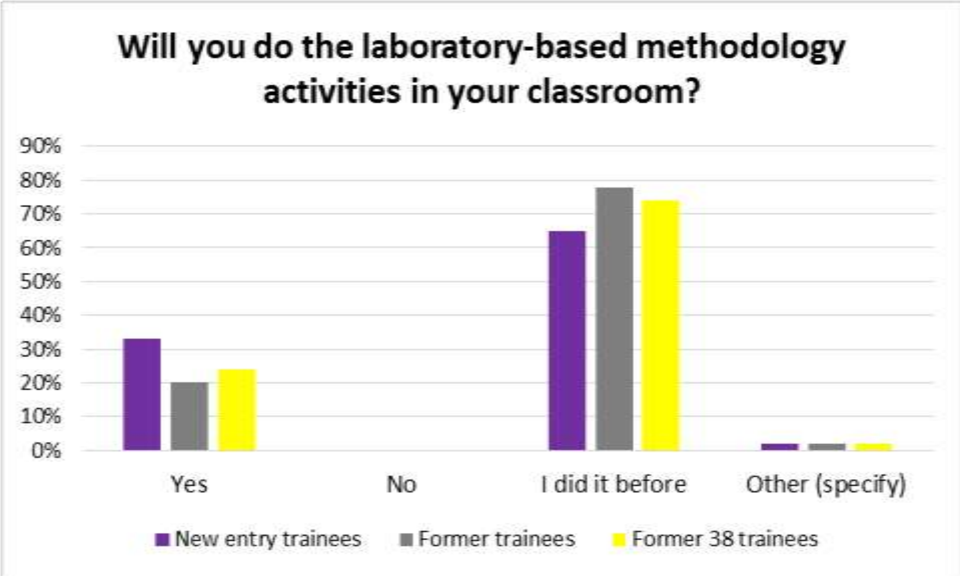


Figure 5.24: Teaching laboratory implementation after attending the MOOC Numeri

If we compare this graph with that of Figure 5.14, we observe that the percentages of those who answered "*I did it before*" of this question in the final questionnaire and the percentages of those who answered "*Yes*" (that stands for "I use teaching laboratory in my classes") of the question in the initial questionnaire do not coincide as one would have expected. However, the goal of the MOOC Numeri was to avoid that there were trainees who, after the examples of activities discussed and shown, could think that the teaching laboratory methodology might not be valid or not worthy to be used. Since no trainee answers "*No*", the transposition of this meta-didactical praxeology by the trainers to the trainees seems to have been successfully accomplished.
The MOOC Numeri did not insist much on the proposal of practical manipulative activities, so in the final questionnaire the question that analysed their use in didactic practices was not repeated.

About the use of problem solving based on real situation we posed the following semi-open question: "*Now that the MOOC is over, will the problems solving you make in class base on real situations?*". The answer options are the same exposed before and the trainees' answers are shown in Figure 5.25. As far as concern the *new entry trainees*, if we compare the Figure 5.25 with the Figure 5.19, relative to the initial questionnaire, we can notice that the aim was to reduce the answer relative to the voice "*No*". This aim was successfully achieved: in fact, the percentage of those who answered "*No*" shifts from 6% to 0%. It means that not only the meta-didactical praxeologies that the trainers want to transpose to the trainees have been effectively transposed, but also that the *new entry trainees* have the occasion to experiment learning in a connectivist way: they see under a new light the importance to propose problem solving that take inspiration from real situation to their students.

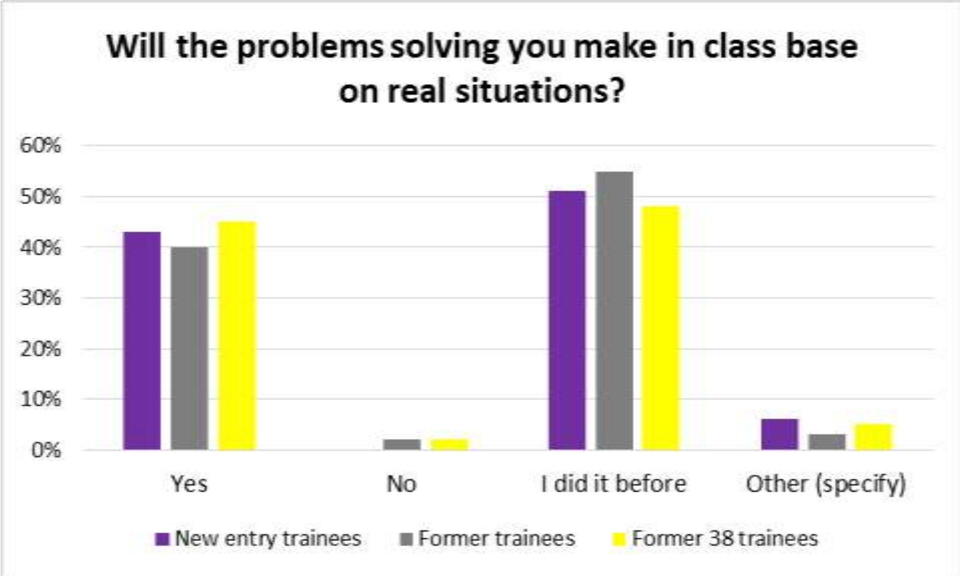


Figure 5.25: Problem solving that take inspiration from real situations after attending the MOOC Numeri

The situation is not really changed for the *former trainees* and the *former 38 trainees*. From Figure 5.19 we can see that the 98% of both of them is used to propose problem solving based on real situation and only the 2% is not used to do it. The situation remain unchanged after the MOOC Numeri, in fact the percentage in Figure X24 remained constant: the 98% of respondent is distributed on the answers "*Yes*; *I did it before*; *Other*", while the entry "*No*" still registers 2% of replies. The same considerations are valid for the *former 38 trainees*.
It is important to note, however, that the attitude towards proposing problems that are inspired by real situations, compared to the *former 38 trainees*, had already undergone a huge change after attending MOOC Geometria (see Figure 5.8, 5.13, 5.20). Note in particular that 2% corresponds to a single person who in addition to being part of the former trainees is in particular also one of the former 38 trainees. It means that the MOOC Numeri has not particularly affected the didactic convictions of this trainee with regard to problem solving proposals based on real situations.

Regarding the analysis of the use of technology and of its advantages and benefits, the final questionnaire of MOOC Numeri focuses on investigating how the use of GeoGebra dynamic geometry software has changed, thanks to the ideas received from the MOOC. However, this is something that we are not particularly interested in studying in this thesis. So we will not dwell on the analysis of these answers.
The analysed data suggest that, compared to methodologies and technologies, the ZPD of the trainees has undergone some changes. The changes are more evident in the *new entry trainees*, precisely for what concerns the teaching laboratory methodology and the use of problem solving base on real situation. On the other hand, the change is almost imperceptible to the former trainees (including the former 38 trainees). After all, we could not expect otherwise. The distance that separates a MOOC from the other is less than a year. In MOOC Geometria we recorded more evident changes because the former trainees had been exposed to innovative methodologies and new technological tools for the first time exactly on the occasion of MOOC Geometria. In MOOC Numeri, it is true that the proposed activities change, because the thematic core of reference changes (from geometry to arithmetic and algebra), but the methodologies that are proposed do not change.

5.2 Professional development and perception of change in teaching practices

5.2.1 The influence of school's ZFM/ZPA

Before analysing the answers in order to observe if there was professional development and perception of change in teaching practices, let us make the following remark.

When the MOOC's modules are opened, i.e. the MOOC's ZFM_i (with *i* from 1 to the last MOOC's module) follow each other, these are explored and filled with active interactions of the trainees, giving rise to the complex and dynamic interweaving of MOOC's ZFM/ZPA. We have seen, in the analysis of RQ1 (Chapter 4), that the MOOC starts as MOOC-artifact. When a teaching module is activated, thanks to the interactions of all the participants, it is configured according to a complex structure called ecosystem by Taranto et al. (2017 a). In the MOOC-ecosystem the trainees explore the MOOC-artifact network (where the trainers connected a certain mathematical conceptual node to a certain mathematical activity). So, the trainees start making connections within their personal network, giving a certain value of importance to that resources that they are exploring, putting in place their Mathematics Education Utilization Schemes (Math Edu USs). At that moment, the MOOC-artifact becomes a MOOC-instrument for each individual trainee and the double learning process continue to iterate itself dynamically, as we have seen in the analysis of the RQ1 (Chapter 4). It is in the repetition of the phases of the double learning process that the trainees can benefit from an expansion of their network of knowledge.

Moreover, a MOOC is a totally remote environment and its participants always interact online. During the duration of the MOOC, the trainees do not live an exclusive and binding relationship in the MOOC, but continue to conduct their daily routines normally, in school environment. So, the school's ZFM/ZPA (Table 5.2) is external to the MOOC, but internal to each trainee that follow the MOOC and that, in a sense, is conditioned by this complex zone, while participating to the MOOC. We underline this here, because the trainees, in their "real and not virtual environment", that is the school, can experiment some activities see in the MOOC and also this practice can influence the discussion on the communication message boards, as well as, the expansion of their professional network of knowledge.

5.2.2 After the MOOC Geometria

From the analysis of the data we have shown in the previous paragraphs related to the MOOC Geometria, it emerged that there were some changes in the ZPD of the trainees between the period before the MOOC and the following one. However, these are deductions that emerge from the analysis of two questionnaires (initial and final) and the final written interview that took place at different times. In order to have a safer base in affirming that the MOOC's ZFM/ZPA could have an impact on the professional development of the trainees, it was decided to ask them if they thought they had effectively benefited.

In the final written interview, the following question was asked: "*From 1 to 10, how has the MOOC been useful for your professional development?*". It was a scale-Likert question where a score of 1 represent lowest usefulness and a score of 10 highest usefulness. The Figure 5.26 shows the answer given by the *Geometria trainees*.

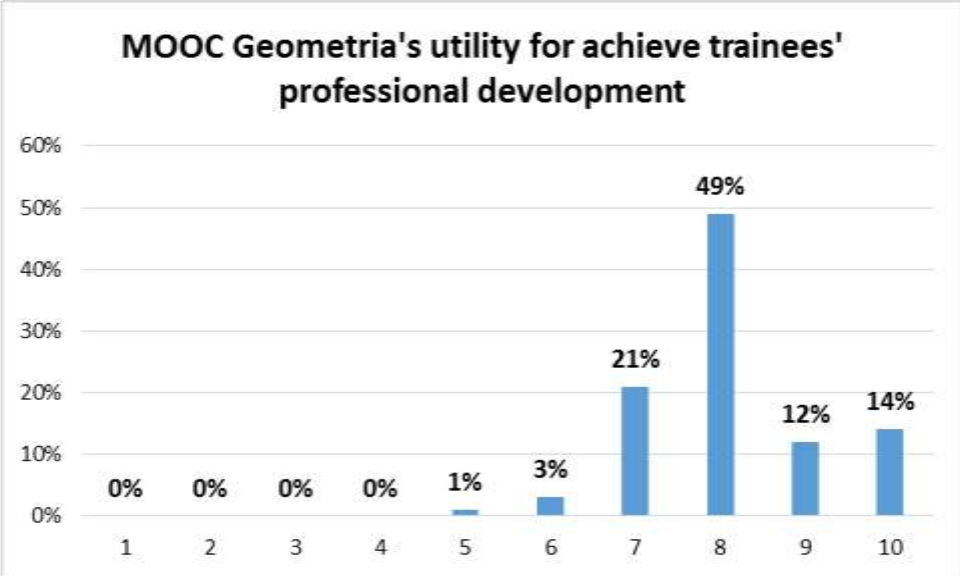


Figure 5.26: From 1 to 10, how the MOOC Geometria has been useful for the PD of its trainees

If we consider that starting from 6 (in the ascending verse) to the MOOC we can attribute the recognition of a usefulness for one's own professional development, then we can say that almost all the trainees believe the MOOC Geometria has been useful in this sense. In particular, almost half of the *Geometria trainees* (49%) give it a vote equal to 8 and there is a 14% who does not hold back its judgments and gives a vote equal to 10!
This question has been accompanied by another one that tries to better understand the meaning of the vote that the trainees have attributed to the MOOC. We reproduce it and the responses received in Table 5.5.

This professional development took place thanks to:				
	For nothing	*Few*	*Quite*	*Very Much*
The activities proposed in the MOOC	0%	5%	45%	50%
The fact that you have designed a teaching activity (Project Work)	3%	28%	37%	32%
The fact of having revised the activity designed by another colleague (Peer Review)	4%	20%	55%	21%
The fact of having to reflect and write your own thought	0%	27%	44%	29%
The fact of sharing (giving and receiving) your own teaching experience with other colleagues	0%	12%	40%	48%
The fact of having had contacts with experts in mathematics education (videos, webinars)	0%	9%	35%	56%
The fact of having been part of a community	4%	21%	43%	32%
The fact of being in contact with people who experiment these new ways of teaching	1%	11%	44%	44%

Table 5.5: Reasons why the MOOC Geometria was useful for the trainees' professional development

It is immediately clear, at a glance, that the answers that have the highest percentages are all between *quite* and the *very much*. This is certainly not only an indication of appreciation of

the training offer but also a confirmation of the usefulness from the professional point of view that it has exercised on the trainees.
Let us now comment the table entries. We consider the answers of those have indicated *quite* and *very much* together.
Undoubtedly the ***activities proposed in the MOOC*** have been incisive, in fact the 95% of the trainees attribute to these the merit of their professional development. We have described in detail only Module 1 of MOOC Geometria, but the others – as mentioned – are not less in the quality of the contents and in the setting of the same (see Chapter 3 and 4).
Here a testimony that we find as an answer to the optional open question "*Do you want to refer to significant episodes related to your participation in the MOOC?*" placed in the final questionnaire of MOOC Geometria.

> Many times I have really exalted myself for some ideas I have read, such as paper folding or some contextualized problems that had a correct and profound mathematical sense […]

The **activities of Project Work and Peer Review** (that we have discussed in Chapter 4) also had a positive impact on the professional development of the trainees. They are judged useful for their professional development by 69% and 76% of the trainees, respectively.
Here some testimonies that we find as an answer to the optional open question "Do you want to refer to significant episodes related to your participation in the MOOC?" placed in the final questionnaire of MOOC Geometria.

> [Doing the PW with LD] helped me reflect on how to organize lesson time by leaving nothing to chance or improvisation

> I liked a lot the peer review because it gave me the opportunity to hear the opinion of another competent teacher without having the filtering of mutual knowledge, which can sometimes compromise the real objectivity.

The fact of having to **reflect and write about their own thought**, considered useful for their own professional development by 73% of the trainees, together with the fact of **sharing** (giving and receiving) **about their own teaching experience with other colleagues**, considered useful for their own professional development by 88% of the trainees, are the phases that identify the ***double learning process*** put in place by the trainees. We have been able to appreciate the power of the double learning process in the analysis of RQ1 (Chapter 4). The *self-organization* phase allows reflection and meditation on the proposals, allowing to reorganize the own network of knowledge. With the *instrumentation*, the trainee also gets to form a new node or to see an existing one under a different light. The phase of *instrumentalization* generates in the individual the will to externalize the reorganization of ideas that have been completed and this is accomplished with the *sharing* phase that enriches not only the individual trainees but the whole MOOC-ecosystem.

For 91% of the trainees their professional development was also affected by the **contacts with experts in mathematics education** through the videos inserted in each MOOC modules and the organized webinars (Chapter 3 and 4).
Here a testimony that we find as an answer to the optional open question "*Do you want to refer to significant episodes related to your participation in the MOOC?*" placed in the final questionnaire of MOOC Geometria.

> I really enjoyed participating in webinars with the ability to interact with both the students and the organizers / speakers of the course.

Furthermore, **being part of a community** and **being in contact with people who experience the same innovative activities** during the same period has affected their professional development for 75% and 88% of the trainees, respectively. We have in fact been able to see, always in the analysis of RQ1 (Chapter 4), how strong the sense of belonging to the MOOC community was for the trainees and how much they considered as enriching the possibility of comparing among peers.

A more delicate issue concerns the perception of a change in the trainees' teaching practices as a result of the experience lived thanks to the MOOC Geometria. Trainees were then explicitly asked if they had made or notice changes in their practice as a result of participation, with the following closed question in the final questionnaire: "*As a result of your participation in the MOOC, did you make/notice changes in your teaching practices?*". 81% indicate "*Yes*" (Figure 5.27).

Figure 5.27: Perception of change in the teaching practices after attending MOOC Geometria

When asked to describe how they were applying what they learned to their professional practice, participants typically responded by citing one of the following (Figure 5.28): i) integrating new tools and strategies (31%), ii) implementing MOOC activities in my classes (64%), and iii) using course content for instructional coaching or professional development with colleagues (2%).

Figure 5.28: How the Geometria trainees apply what they learned to their professional practice

More in-depth findings were obtained thanks to the interviews conducted via Skype. In the following some examples that come from the transcription of the Skype interview. The question posed during the interview was: "*At the level of geometry teaching, what did the MOOC do? Something changed in your way of teaching geometry or teaching geometry in general (for example, rethinking which important contents, ...)?*". The relevant parts are transcribed in bold or italic.

> S.L.C.: I teach in a lower secondary school. What ***I have understood*** is that *I have to leave more space for discovery, manual construction of models or experimentation with the use of ICT*, ***I have to allow*** the kids to *make conjecture not anticipating their response* and *encouraging argumentation, discussion, support of their theses*. Aiming at understanding meaning, understanding the meaning of what is being studied was something I was already aiming for, *the MOOC offered strategies and activities to try and succeed*. Another fundamental point is that ***I understood I have to*** *verify whether the learners understood well or not*. I was struck by the false beliefs on the [...] heights and distances (confusion between orthogonal and vertical), [...] Thanks to the MOOC's spur ***I tried to use*** both *Geogebra and spreadsheets as a tool to facilitate understanding of concepts and solving math problems*: initially using the plexus computer lab and then the student's own equipment (BYOD[57]). *The students greatly appreciated* both the experimental manipulative activities, the group work [...] and the use of the computer, because they were considered less boring, less 'generating anxiety', more stimulating, more fun [...]

> P.M. I state that my life is very complicated (4 children and a distant job) and time management is essential. But ***I am happy*** to be able *to find the time to connect* and follow the activities and webinars (2 out of 3). *The community* created *was active* and ***I was pleased to share with colleagues*** interested in their own training and not bored slackers of their work and above all ***be guided in the work by a group of experts*** very good at proposing training activities. [...] I would like to share with the colleagues of my school what I learned in a sort of conference [...] ***I have tested the activities*** of part 1 and 2 of the MOOC and *the guys have*

[57] Bring Your Own Device

> *appreciated* […] *the proposals were based on skills and they activated the brain*, there was no a rule to apply, they were all very well structured

The first testimony is more structured than the second one. S.L.C. indicates how she has already changed, or will change, her practices with her own students. She describes how her participation supported her refined attention to and understanding of their students' thinking and their own personal improvement in knowledge of mathematics. She several times repeats "I have understood, … I have to do …". This means that she has interiorized the formative messages that the MOOC Geometria wanted to transpose to the trainees. However, we have no other evidence that she really act has she described or that she will act has she as seems to be trying to do.
The second testimony is a bit fuzzy in its intent. P.L. does not center exactly the point of the question. She underlines her appreciation of the MOOC: her pleased to share with colleagues and to be guided by a group of experts that testimony how she fells sustained in her professional development. Only in the last part her comments address changes to her approach to teaching. In fact she say that she tested some MOOC activities and she appreciate them because there was an increased focus on concepts as opposed to algorithms.

As we have announced at the beginning, we chose not to have contacts in presence with trainees. The intention of the research was to see what could be collected, deduced and identified by analysing only the declarations made online by the participants. Clearly, this implies that the data on which to base oneself are the declarations released in written form on communication message boards, questionnaires or verbally through interviews via Skype. As dissertation writer, I am conscious about the fact that see an evidence in class is different than reading it o listen to it (if the interview is conducted via Skype). However, the interviews were conducted in a time following the end of the MOOCs and this gave the trainees time to reflect on the innovations explored in the MOOC and eventually start to integrate them into their teaching practices. From the Chapter 3 on the methodology, it will be recalled that the interviews were conducted both for *Geometria* and *Numeri trainees* in the summer of 2017. So, for MOOC Geometria means more than a year after its end.
In the extracts of the proposed interviews, we observe that in the words of S.L.C. there is a clear awareness of the impact that the MOOC Geometria has had on her professional development and her teaching practices. Among the words of P.M. instead, the memories related to the positive experience that she has experienced in the MOOC Geometria and the appreciation of the activities that do not provide to mechanically use formulas to be resolved, but to "activate the brain" emerge.

So far, we may have reason to believe that, in general, MOOC Geometria allows for effective professional development and seems to lead perception of changes in the teaching practices of its trainees. In the following, we will analyze the case study of Lucy, a Geomertia trainees, and we will return to deepen aspects related to professional development and the perception of change in teaching practices.

5.2.3 After the MOOC Numeri

From the analysis of the data we have shown in the previous paragraphs related to the MOOC Numeri, it emerged that there were some changes in the ZPD of the trainees between the period before the MOOC and the following one. In order to have a greater security in

affirming that the MOOC's ZFM/ZPA could have an impact on the professional development of the trainees, it was decided to ask them if they thought they had effectively benefited.
This time, unlike what was done with MOOC Geometria, it was not decided to send an interview after the end of the MOOC, because as we have seen, in what we call final interview, on 152 finalists, only 75 of them answered. It was therefore decided to "beat the iron as long as it was warm" and specific questions were integrated into the final questionnaire. Their formulation is different from those in the final written interview of MOOC Geometria, but the intent is the same. We will analyse this data below and, for some questions, we will always make a distinction among the three groups of trainees: *new entry*, *former* and *former 38 trainees*.

In the final questionnaire, we asked the trainees to express how much some statements were true for them. Therefore, with a scale-Likert question: "*Please rate from 1 to 5 the following statements (1 = absolutely false, ..., 5 = absolutely true)*" we ask the trainees to express how much they judge true some statements for them.
We report in the following table (Table 5.6) some of the phrases that we have proposed to the trainees, those that are useful for conducting the analyses of our interest.

	1	*2*	*3*	*4*	*5*
I have proposed to my classes some of the activities (even partially) seen in the MOOC Numeri	4%	9%	33%	26%	28%
I shared my teaching practices with other participants to the MOOC (I told them something on myself as teacher)	10%	22%	42%	21%	5%
I shared with other participants to the MOOC some materials I use in my classes	21%	29%	29%	16%	5%

Table 5.6: How much the MOOC Numeri trainees judge true this statements

For these statements, it is not necessary to look at the different answers by type of trainees, because they are practices that are independent of the previous experience with MOOC Geometria.
Compared to what had emerged from MOOC Geometria, the practices related to the use of MOOC materials in the classroom, sharing their practices with the other trainees and sharing some of their materials with the other trainees were considered useful to favor the professional development of the trainees (Table 5.5).
The question placed in MOOC Numeri is different. The trainees are not asked to judge how much to put into practice a certain behavior in the MOOC has had repercussions on their professional development. Rather, one is asking how true it is that that particular attitude has been put into effect. We note here that the percentages of those who have implemented the practices described in the Table 5.6, if we add the scores of those who answered 4 or 5, do not even reach half of the trainees. In fact, about proposing some of the MOOC activities in classroom was an attitude put in place by the 54% of the trainees; sharing own teaching practices with the other trainees was done by the 24% of them; and sharing some of own teaching materials was done by the 21% of them.
However, if we analyze the truth that the trainees attribute to the fact that the MOOC Numeri has allowed their professional development, we will observe that this has happened for most of them, although most of the trainees did not put into practice the practices of the Table 5.6 that in MOOC Geometria seemed important. Let us move on to the analysis of this question.

We consider the following statement: “*I think the MOOC Numbers has helped my professional development of math teacher*” (Figure 5.29). We make a distinction among the three groups of trainees. The opinion of the trainees, for the most part, confirms that they think the MOOC Numeri has contributed to their professional development. If we consider the answer of those who have indicated a score of 4 and 5, we have that the 72% of the *new entry trainees*, the 88% of the *former trainees* and the 89% of the *former 38 trainees* attribute veracity to this sentence.
The fact that the students give a recognition in terms of truth to this sentence means that the educational offer of the MOOC Numeri was valid and above all, for most of them, was significant for their professional development. The distinction between *new entry* and *former trainees* is interesting for two reasons. On the one hand we note the judgement of the *new entry trainees*: for 10% the MOOC Numeri has not contributed to their professional development, while for 18% the impact on their professional development has been indifferent (they are the trainees who answer judging the sentence true with score 3). This does not surprise us, indeed it is in line with the percentages of the new entry trainees who, in the analysis of the RQ1, declared that they had difficulty managing the MOOC in terms of ease of use (Figure 4.11 in Chapter 4); flexibility in terms of space, time and pace of learning (Figure 4.12 in Chapter 4). On the other hand, none of the *former* and the *former 38 trainees* states that they do not consider the MOOC Numeri useful for their professional development. Certainly the proposals of mathematical activities and methodologies have been appreciated (Figure 4.48, in RQ1), but also the confidence with the MOOC online environment, acquired thanks to the previous experience with MOOC Geometria, has contributed to a better use of its resources and take benefit for the own professional development.

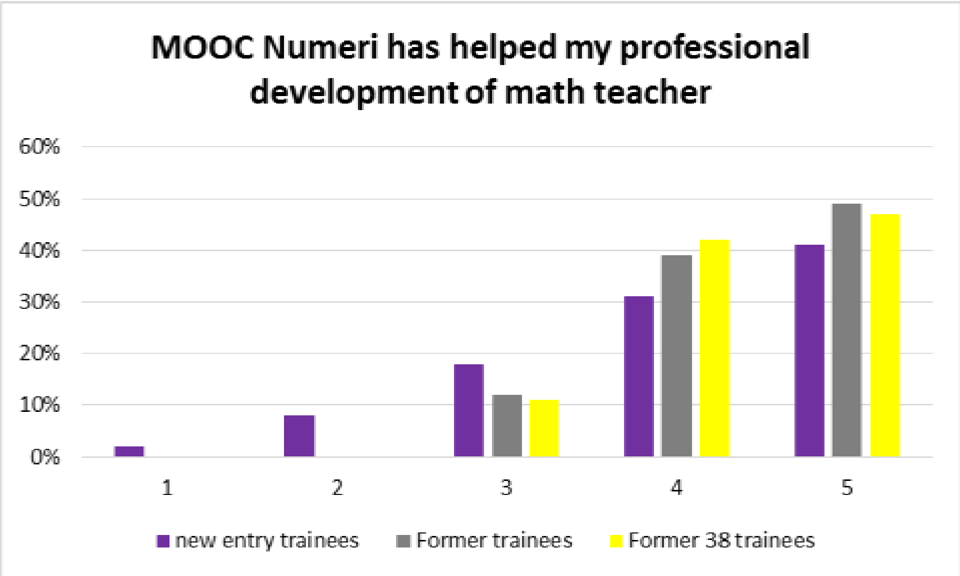


Figure 5.29: How much the MOOC Numeri trainees think that it contributes to their professional development

In support of these observations, in the final questionnaire, a closed question (with *Yes* or *No* answer options) asked respondents to say whether they had followed the MOOC Geometria earlier. To those who answered *Yes*, i.e. the former trainees, this question was addressed: “*As you may have noticed, the setting of the MOOC Numeri modules is similar to that of MOOC Geometria. Do you think that the familiarity with the MOOC environment acquired with Geometria has facilitated you in managing your learning pace in Numeri?*”. The 100% of the former trainees answered *Yes*. Then, an open question asked them to explain why. With more or less similar words, everyone agrees that the motivation lies in the advantage of knowing the philosophy of MOOC, that is, its setting, its general structure. Here are some of the answers given by the trainees.

> Knowing already the structure of the MOOC, being very similar to that of Geometria, it was very easy for me to start and follow all the proposed activities with a good rhythm right from the start.

> Last year I had to understand how the platform worked, the forum, the use of the materials available on the course platform, LD .... This year I felt at home and I focused on the proposed content, with less effort.

It is beyond doubt that the feeling of being part of a community and the interactions with the trainers during the webinars contributed to the professional development. Of these aspects we had analysed the usefulness and importance for the trainees in the previous Chapter 4. Also remember all the considerations made on the usefulness of the communication boards (Figures 4.55, 4.56, 4.57, 4.58).
Even though not everyone has shared their practices, those who did provide food for thought for others. These ideas have certainly been enriching, in fact from Figures 4.56 and 4.57 respectively, we know that 77% of the trainees has benefited from a dialogue with their colleagues and 91% had the opportunity to benefit from experiences/way of thinking of the others.

To analyze what kind of professional development the MOOC Numeri gave the trainees, the following open question was also addressed: "*In which way the role of mathematics, present with its various representations in the MOOC (concepts, formulas, graphics, software, ...), has contributed/benefited your training on the Numbers issue?*". The question is a profound question and it is not easy to make a category of your answers. This is why, in particular, because some answers are a bit superficial and do not go in depth as we would have expected. For example, someone answers "Very much", or "The course has made me clear the sense of doing mathematics. THANK YOU". However, these answers do not explain how the MOOC Numeri actually was useful for their training. Surely, this is a limitation of the analysis tool. In fact, the question was placed towards the end of the final questionnaire that was full of questions and probably the trainees wanted to finish the compilation. We propose in the following some of the most significant testimonies and anticipate that a similar question was asked the trainees that were interviewed via Skype.

> Participating in this MOOC allowed me to focus on some aspects that up to now I had neglected, such as the relationship between arithmetic and literal calculation and the importance of presenting the same conceptual node with different representations.

> I really liked the continuous connection with the geometry or in any case with the graphic representation of the numbers. I think it is very important to make students understand that math is always all connected […].

> [The MOOC] contributed a lot in the sense that I feel even more encouraged to deal with all the topics in the program by making a connection with real aspects, even when I was more reluctant to do so

> I have certainly acquired/improved knowledge on some tools (sway, geogebra, etc.) but the overall improvement has not been on my training on the subject, rather on my teaching on the topic

From these statements, it is clear how thanks to the MOOC Numeri the participants paid more attention to aspects of the discipline that were previously neglected. The first statement confirms that that teacher had neglected the link between arithmetic and algebra that now seems to reconsider. From the third one, we see how the message to propose activities inspired by reality was clearly transposed. The last sentence shows that the trainee notes improvements regarding the acquisition of new knowledge from the technological point of view. Moreover, other improvement are relative not to mathematical concepts, rather to the methodology with which to deal with them. And this is precisely the goal at which the MOOC Numeri aimed. We have already stated that the MOOC did not offer activities that had to give new knowledge of content, but proposed innovative methodologies and different strategies for teaching-learning mathematics, compared to the arithmetic and algebra core. Let us dwell again on another testimony.

> For a more effective growth, a need for more dilated times there would be in order to have time to reflect calmly, to rework, to share more with the other trainees. Instead the urgency of the course that was going on, to be followed by a commitment work and another, has led me to do everything a little too quickly. I believe I have acquired the main structural aspects, but I miss the reworking, "how I do insert it in my teaching?"

This sentence clearly shows how the trainee actually experienced the first phase of the ***double learning process*** (*instrumentation/self-organization*). He reorganized his network of knowledge in light of the ideas he received from the MOOC and/or interactions with other students. However, he has not fully realized the next phase of the process, or *instrumentalization/sharing*. He has certainly commented on communication message boards (he in fact writes: "to share *more* with the other trainees"), but he feels the need of more time than the one offered by the MOOC[58] to consolidate its network of knowledge. There are now new nodes in his network, but the links between these and his previous knowledge must still be created or strengthened more firmly.

A more delicate issue concerns the perception of a change in the trainees' teaching practices as a result of the experience lived thanks to the MOOC Numeri. Something seems to emerge from the phrases that we have proposed before, for example the trainee that says to pay more attention to the proposal of activities that are inspired by real situations. However, as noted for MOOC Geometria, we do not have sufficient evidence to affirm that the testimonies of the trainees translate into real changes in their teaching practices. Moreover, as evidenced by the last testimony, the changes can be realized in the long term, if you can have the time to internalize the proposals and if you start to experiment in class, including them in your teaching practices.
Speaking of change of practices at the end of the MOOC Numeri, that is while completing the final questionnaire, seems a bit premature. However, in the wake of what we had done in the final interview of MOOC Geometria, we also asked the MOOC Numeri trainees, in the final questionnaire, whether they thought they would regain changes in their practices as a result of

[58] We recall that MOOC Numeri lasted 6 weeks, plus 4 weeks for deliveries of final activities (from November 2016 to February 2017).

their participation in the MOOC Numeri. A scale-Likert question like the previous was used to accomplish this aim: "*Please rate from 1 to 5 the following statements (1 = absolutely false, ..., 5 = absolutely true): After this training experience I feel that something has changed in my teaching practices*" (Figure 5.30).

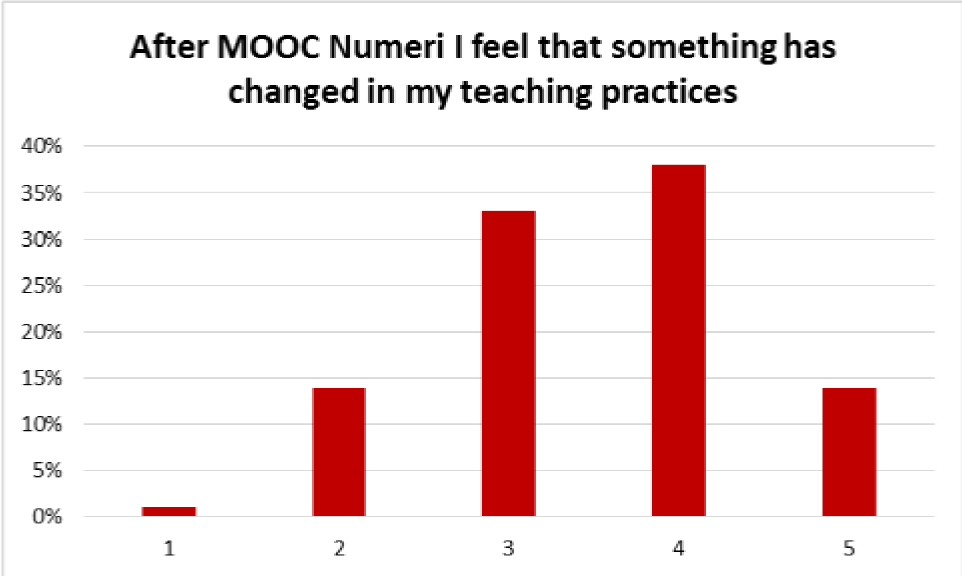


Figure 5.30: Perception of change in the teaching practices after attending MOOC Numeri

If we consider the answers given by those who answered with a score of 4 or 5, we observe that 52% believe that after the training experience in the MOOC Numeri something has changed in their own teaching practices. With the analysis of the first paragraphs on MOOC Numeri in this chapter, we saw that something had changed with respect to the use of new methodologies (Figures 5.24 and 5.25). But let us underline that these answers do not give us concrete evidence of changes in teachers' practices.

On the understanding that the investigation was based on the choice to not meet the trainees personally, to have more evidence on how the trainees could conduct class activities using the materials of the MOOC, an optional module has been included in the MOOC Numeri, the Experimentation module (which we have described in the methodology, Chapter 3). We remember that this module was opened from the second week of MOOC Numeri, until March 31 (two months after MOOC's conclusion). If the trainees were available to accomplish this optional module they should do the following in one of their classroom:

– either an activity that they could freely choose among those that were presented in MOOC Numeri modules;
– or their Project Work, that is their final activity of the MOOC Numeri.

During the experimentation, they had to make sure to complete a given logbook in a complete and detailed way. There, the trainee should describe the classroom context; how much time he devotes to the experimentation; how he had carried out it (if identical to the one proposed in the MOOC or modified and in this case how); some considerations about his students (level of attention, involvement, possible contingencies both positive and negative); if in general he was satisfied with the experimentation.
We have collected 35 logbook, made then by a voluntarily sample of MOOC population. Of all these logbook arrived, some (both of lower and higher secondary school) will be selected to conduct case study analyses and thus to respond in depth to the RQ2.

We will not describe all the experimentation made by the Numeri trainees. We will analyse only one of them as a case study in the following: Stephen case study.

So far, we have reasons to claim that, in general, MOOC Numeri allows for effective professional development and seems to lead perception of changes in the teaching practices of its trainees. In the following, we will analyze the case study of Lucy, a Geomertia trainees; the case study of Stephen, a Numeri trainees and we will end the chapter with a last negative case study relative to Ester, a Geometria trainee.

5.2.4 Remarks

Before proceeding with the subsequent analyses, we want to reflect on the tools used to collect data for the quantitative analyses presented so far.
The survey methodologies used to respond to this second research question have been in some respects similar and different. In both MOOC the choice to submit three questionnaires was maintained and it is a valid way to collect data. About the difference, there are the choices to use written interview in MOOC Geometria that was abandoned in MOOC Numeri. While the time to administer these interview was good because especially for the final (the unique that we considered until now) the trainees have had sufficient time to reflect about the concluded experience of MOOC Geometria; the number of responding, however, was not the same of the finalists. So we have obtained data only relative to a sample of MOOC finalists. For this reason, we have decided to include some of the questions of the final written interview of MOOC Geometria directly in the final questionnaire of MOOC Numeri, so to obtain data from all the finalists. This lead to having, on the one hand, a more refined tool, but on the other hand it was too long than the final questionnaire of MOOC Geometria and in some answers, the trainees did not go so in depth as we expected. Other interviews to deep some aspects were anyway necessary. Skype interview and Experimentation module are the analysis tool used to accomplish this task after MOOC Numeri experience. Anyway, being the MOOCs a field of research still in progress, we can say that these used tools have been "tests" to understand what gives more information and how it is convenient to proceed in the future.

5.3 Case studies

Our research is based on the MOOC's Zone Theory in order to analyse trainees' learning (in a connectivist sense) in the MOOCs (Geometria and Numeri) and their consequent professional evolution. Until now, the quantitative analysis showed the effects that the MOOC's ZFM/ZPA had on the trainees. However, the trainees do not live in an exclusive relationship with the MOOC, every day they are immersed in their school contexts. They relate to colleagues in attendance, they relate with their students to whom they can also propose the activities they view on the MOOC. It is therefore important not to neglect the influence that the school's ZFM/ZPA has on them. The individual and social relationships that occur in this real context, as opposed to the MOOC, have a certain impact on the way in which the trainees continue to attend the MOOC and therefore on their professional development.
The influences of the school's ZFM/ZPA do not emerge distinctly in the MOOC. However, they can be traced by monitoring and interviewing individual trainees appropriately.

A more detailed analysis, therefore, emerges considering the case studies. Focusing on specific trainees, we can better understand all the facets that both the MOOC's ZFM/ZPA and the school's ZFM/ZPA exert on the trainee in the period in which he attends the MOOC.

5.3.1 A case study for MOOC Geometria

Among MOOC Geometria trainees, we chose to report the case study of a teacher we call Lucy. Lucy's case has been selected because it illustrates how a tension (Goos, 2013) between her ZPD (Lucy's beliefs about mathematics teaching-learning) and her school's ZFM/ZPA (Lucy's professional environment and interaction with teaching colleagues) became a productive tension thanks to her participation in the MOOC, that is also reflected in her professional development with a perception of a small but significant change of teaching practices.

The data that we will show in the following come from Lucy's:

- intervention on the communication message boards;
- answers given to the three questionnaires administered during the MOOC Geometria;
- answers given to the so-called three written interviews (in the Chapter 3 you can find a detailed description about them).

Introducing Lucy

Lucy has been teaching mathematics for 15 years. The school in which she teaches is a technical institute in a small town in Lombardy (region in Northern Italy). In such schools students have from 3 to 2 mathematics hours per week. During the 2015-2016 school year, she had four classes (grades 9, 11, 12, 13). Lucy willingly uses various kinds of technology with her students. In her classrooms she uses PCs and video projector. For many years, she has been using Geogebra (DGS, dynamic geometry system). In October 2015 she enrolled in MOOC Geometria. Lucy declares, in the final questionnaire, that she enrolled in the MOOC because she felt a need for training and also "Wished to share teaching experiences, desire to receive ideas related to the teaching of geometry". She actively participated in the MOOC, contributing to the discussions in the communication message boards (she posts on the forum for the most part) and experiencing in her class some of the activities proposed in the MOOC. She has regularly concluded all the MOOC activities in January 2016.

Lucy and her general experience within the MOOC's ZFM/ZPA

From the intermediate questionnaire, Lucy declares that she considers the MOOC easy to use. She considers it flexible in terms of space, time and pace of learning (giving the score 5 to all the items - remember that the question was a scale of Likert with score 1 representing total disagreement and score 5 that represents total agreement). Also on the communication message boards she expresses very positive judgments (always a score of 5), considering that they allow her to express her personal opinions regarding the contents of the course; to allow her a dialogue with other colleagues; to have the opportunity to benefit from the experiences of others; and to develop a sense of belonging to a community.
In the second written interview, Lucy states in particular that padlet and tricider did not particularly like her and that she really appreciated the forum. From the final questionnaire, we observe that Lucy has read a number of posts greater than 80% of the total, while the answers given to comments from other trainees fall between 1 and 20%. So she has more read than written. In particular, on the advantage of reading/writing post declares: "The exchange

of ideas is always advantageous, the use of the Moodle forum, compared to other tools where posts are used, makes the discussion more organized".
She believes that she has worked hard in the MOOC and having participated in the MOOC was useful for her because: "I had a lot of food for thought about geometry, I have already used some ideas in teaching practice". From the third interview and from her postings on the forum we observe that she paid more attention to the activity of the mainmast, described in Module 1 of MOOC Geometria (Chapter 4, but in the following we will remember it briefly).

A snapshot of Lucy's practice
In the first module of MOOC Geometria the proposed activities aimed to eliminate the misconception that many students have between perpendicular and vertical lines. We concentrate our attention on the first activity, "The mainmast". It starts from a concrete situation: the teacher gives each student a white circular sheet with a sketch of a boat on a sea wave (Figure 5.31). The request for the pupils is to draw on the boat its mainmast, of about the same length as the boat. The request to draw on a round and not a squared sheet is made to prevent the horizontal and vertical references, induced by rectangular sheets. The lack of reference helps students to think only about the relationship between the boat and the mainmast. Then, the teacher can pass to the normal sheets of notebook paper making students aware of avoiding the mismatch between perpendicular and vertical lines in problems of this type.
After a few days of MOOC Geometria, Lucy intervenes in the forum of this module. She starts showing up at MOOC's community and talks about her grade 9 class:

> "[...] You could say that in the first year of upper secondary school, such concepts are well known, especially in a school for surveyors like this, where students make technical drawings. In reality, it is not so: I realize that there are still problems to solve when I request my students to draw the heights in a triangle [...] the problem is the confusion between the concept of perpendicular and vertical. [...] I found ingenious the use of the circle [round sheet]".

Figure 5.31: Boat on round sheet

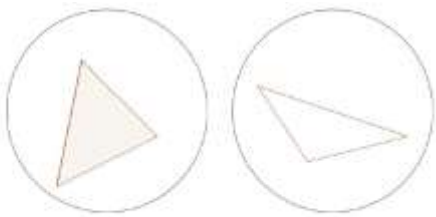


Figure 5.32: Round sheet without squares

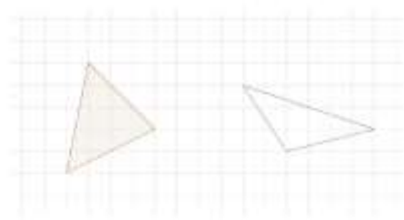


Figure 5.33: Sheet with squares

She explains how she intends to propose the activity to her students. She plans to propose to her grade 9 class the task in this form: a) distributing to students two drawings: one with an acute-angled triangle and one with an obtuse-angled triangle. Both will be on a round not squared sheet (Figure 5.32); b) asking students to draw the three heights in each triangle; c) asking the students to paste their drawings in their notebook, provided she is satisfied their work; d) in a further lesson distributing the same drawings with the same task, but on a squared rectangular sheet (Figure 5.33).
Lucy explains that she prefers to use the triangles instead of the boat, because "sometimes it seems counterproductive to propose childish drawings in upper secondary school. Students sometimes feel treated as children and underestimate the proposed experience [...]". In the end, she comments: "[...] I find this simple idea really useful to 'eradicate' the confusion

between the two concepts, although I fear that my students will still recreate the same mistakes because they were only superficially engaged on the task request".

A few days later she also shares, always on the forum, a PDF file in which we see how she has actually organized her previous ideas (see Appendix E: the file is the original one created by Lucy, so it is in Italian). We are witnessing a reorganization of its network of knowledge with respect to the nodes "perpendicular vs vertical line" and "practical-manipulative activities". She plans that this activity will be divided into 6 phases. We can translate this her PDF file interpreting this work in terms of her didactic praxeologies.

Task 1	to correctly trace the perpendicular on a round sheet
Techniques	- give everyone a round sheet in which a segment is drawn - the perpendicular is traced by folding the sheet - one must understand why it is the perpendicular
Task 2	to identify the orthocentre of the triangle
Techniques	- give everyone an acute triangle (drawn on a sheet and already cut out) - bend the triangle to trace the three heights
Task 3	to reflect on height as a path of minimum distance
Techniques	- propose the problem of the park (explained in Chapter 4) - students must find a strategy to trace the three paths and find their meeting point
Task 4	to identify the heights of an octagonal triangle drawn on a round sheet
Techniques	- give everyone an octagonal triangle (drawn on a round sheet) - draw the three heights - to point out that two of the three heights are external to the triangle
Task 5	to identify the heights of an octagonal triangle drawn on a squared sheet
Techniques	- give each one a rectangular and squared sheet in which the same octagonal triangle of the previous phase is drawn - draw the three heights - compare this design with the previous one
Task 6	to identify the heights of the triangles using a dynamic geometry software
Techniques	- use GeoGebra and draw the three heights of a triangle - see how heights vary when the figure varies dynamically
Logos	Encourage student'' learning by proposing practical-manipulative activities and activities inspired by reality

Table 5.7: The Lucy's didactical praxeology related to the overcome of the misconception related to the heights of triangles

It is important to note how Lucy inserted in her praxeologies an element that was previously external, that is the round sheet. She herself states on the forum: "I found ingenious the use of the round sheet". This means that her network of knowledge has taken on a new knot and she is self-organizing it. Note also that the PDF file (Appendix E), which seems to put her ideas on paper, is still a non-definitive design. In fact, under the table in which she describe the 6 phases of her activity, Lucy makes some didactic considerations in anticipation of the experimentation she is going to do. The words

> [In the classroom] I have an hour particularly difficult to manage, that of Saturday at the last hour. I **could propose** each of these manipulations with the card the last ten minutes, in which the attention threshold is reduced to zero. With a playful manipulative activity **I could get to occupy** this time in a profitable way, I resume reflective discourses during the next lesson. In this classroom **I would add** the activity (7) related to the construction of parallelograms and trapezoids starting from a strip of paper [...]

The verbs that have been reported in bold denoting that this is still an untested design.

A month later, Lucy returns to the first module forum and shares with the MOOC's community the results of the experience she has with her students. After explaining the concept of height, Lucy delivers to each student the material of Figure 5.32 (round not squared sheet). She says that in the case of the acute-angled triangle the students have drawn the heights correctly. In the obtuse triangle case instead, she found some "psychological resistance" - so says Lucy - in using "the space outside the triangle". She checks that all students have traced the heights properly and then they pasted the two drawings in their notebook. In the next lesson, Lucy proposes again the same triangles on a single rectangular squared sheet and asks students to draw the heights, stating that it is not a test for assessment. Out of 26 pupils: 16 have successfully completed the activity; 2 did not complete successfully; the remaining ones (8) have drawn properly the heights in the acute-angled triangle, but not in the obtuse-angled one. Lucy writes:

> "I do not think it is a comforting result; although the previous lesson I 'lost time' to make sure that everyone had traced the right heights, I have helped and re-explained personally to all those who did not succeed, the next lesson already 10 people were wrong [...] the methodology is important, ideas which help not to create false truths are nice [...], but if there is no the interest in learning, [...] nothing will remain in the learner's head!".

A few days after this discomforting comment by Lucy, another trainees commented on the forum in response to Lucy.

> Dear L.,
> the understanding of many mathematics concepts is difficult for the kids and it would be too good if after the first activity the kids managed to extricate themselves inside them. I think it takes a lot of work and dedication. I also believe that it is difficult for us to propose a new activity (thought by others) in a new way. We must not give up! I tell it to every obstacle ... and there are so many obstacles! for everyone! Then sometimes the boys amaze us!
> Greetings M.

Lucy's evolving ZPD/ZFM/ZPA system

The MOOC's Zone Theory perspective allows for picturing Lucy's profile from the data collected through the three interviews and the final questionnaire. In the first interview Lucy describes her school's ZFM. She declares that in school she deals with students who "[...] are mostly passive users [of technology], they comply with instructions to carry out the task, but they are not proactive. This is an attitude that they have not only with new technology. The indifference is a generalized behavior in my technical institute classes". She feels limited in the choice of the activities that she can propose and she does not believe that her students are able to perform their assigned tasks properly. In her 15 years of experience as a teacher she

has acquired confidence in her own ability (her actual level of ZPD), and she is convinced that most of her students do not really learn mathematics. She is pervaded by a sense of skepticism in the potential of her students. Even before carrying out the activity of the triangles heights, she already spoke of “superficially engaged students who after the lesson still make the same mistakes”. Actually, it was only the minority of the class who made mistakes. However, the disappointment of the fact that not all students have succeeded in carrying out the task correctly, makes prevail in her a sense of frustration, rather than a positivity one for the achievement of the goal by most of her students.
Notwithstanding this, Lucy teaches with passion and tries to develop her lessons in a captivating way, also using technology. Showing a part of her actual level of ZPD, she declares in the first interview: “I am deeply convinced that integrating the lessons with the opportunities offered by new technologies (e-learning, social networks, software) is not only useful but also vital to the future of education in a modern world”. Hence Lucy shows she is willing to integrate technology into her teaching practices. Lucy’s colleagues did not seem to share these thoughts. In fact, in the second interview: “Despite my enthusiasm in experimenting with new educational solutions and the use of technology for innovative education solutions [...], I find that among colleagues these initiatives are not appreciated (envious? fear of getting involved?); I do find also that the efforts for modernizing the school are not evaluated with the appropriate weight by the headmaster, who has, in my opinion, a biased vision of the use of technology in the school”.

The negative beliefs about her students and the equally negative consideration that she has of her colleagues and headmaster constitute the school’s ZFM/ZPA complex of Lucy. Her ZPD, understood as the set of emerging personal meanings that could be constructed from this person-environment relationship, is so circumscribed. There is a misalignment between her ZPD and the school’s ZFM/ZPA that generates tension; tension that can be resolved, for example, by looking for a new environment: the MOOC. In the second interview, Lucy declares: “In general the participation in courses for teacher education allows you to get in touch with motivated colleagues (MOOC’s ZPA), who share their own experiences. So teacher education is a time when the teacher batteries recharge. In particular, in this MOOC we have spoken of geometry (MOOC’s ZFM), which is one of my favorite parts [...]”. Lucy then was living a productive tension that pushed her to join the MOOC Geometria and fitted into the MOOC’s ZFM/ZPA.

In the final interview, she declares that before attending the MOOC, despite the fact of knowing the practical-manipulative activities, she did not use them since she considered them unimportant. After the MOOC, she reconsiders them saying that they allow for enhancing the discovering of geometric properties. She declare:

> “Handling with paper has made the transmission of knowledge about primitive axioms and entities more pleasant. **Now I pay more attention** to the practical aspect and to the manipulation”.

In addition to those proposed in the MOOC, she continued to give other similar activities to her grade 9 classroom and she has noticed her students are *quite* involved. We observe thus a change in her attitudes with respect to her students (school’s ZFM), and we perceive a change, albeit minimal, in her teaching practices (ZPD): hence we find that Lucy benefits of professional development. We can have a confirm about this by the following question of the final questionnaire: “*From 1 to 10, how has the MOOC been useful for your professional development?*”. Lucy answered with 7 (where a score of 1 denotes lowest usefulness and a score of 10 highest usefulness). She additionally specified that: the MOOC has influenced her *enough* for its proposed activities, *very much* for sharing (giving and receiving) one's own teaching experience and, feeling part of a community that experienced the activities of the MOOC pushed her *enough* to do likewise. (The italicized words here represent the response options chosen by Lucy in answering the questionnaire). These changes were initiated and triggered by the tension that Lucy experienced through the misalignment between her evolving ZPD and the school's ZFM/ZPA she had experienced at school. Although the MOOC was a short experience, it could nevertheless influence Lucy's professional development in a small but significant way.

5.3.2 Discussion

We have discussed a case study where we trace the evolution of a higher secondary school teacher's beliefs about her teaching practices and about her students' attitudes towards mathematics as an effect of her participation in a MOOC for mathematics teacher education.

The MOOC's ZFM/ZPA in a sense mimics for Lucy the same dynamics of the school's ZFM/ZPA: this dynamic in fact is triggered by the potential content of the MOOC-artifact and develops because of the actions and interactions of the participants in the MOOC-ecosystem/instrument. The two configure as a specific ZMF/ZPA zone for the MOOC activities: they structure a specific level of actions and productions where the story of the MOOC can be described. What happens within the MOOC has important consequences for many components in school's ZFM/ZPA, which is a structure external to the MOOC, but is internalized by Lucy and it is also conditioned as a specific effect of her active participation to the MOOC.

From a MOOC's Zone Theory perspective, Lucy changes her interpretation of two aspects of school's ZFM/ZPA – her beliefs on the mathematical abilities of her students (that seem *quite* involved in doing practical-manipulative activities) and on her professional development (the attention to the practical aspect and to the manipulation) – thanks to the new person-environment relationship offered by the MOOC Geometria. These changes were triggered by a tension that arises from a misalignment within the zone system. Actually, Lucy experienced dissatisfaction because her ZPD did not map onto the school's ZFM/ZPA complex in ways that promoted desired development. Attending the MOOC Geometria this tension became productive. In fact, because there was no ZPA within her school, which could map onto her ZPD, Lucy looked outside (precisely towards the MOOC) to advance her personal goal of thinking differently about mathematics education. The MOOC's ZFM/ZPA needed to be brought within the school's ZFM/ZPA so that the actions promoted were seen to be permitted rather than forbidden. Some elements of school's ZPA did not seem to afford a change in teaching practices – the colleagues and the headmaster seem not to appreciate the efforts to modernize the school. However, in Lucy's last interview, when the MOOC had ended some time previously, she declares: "if you are convinced that what you are doing makes sense, you persevere", witnessing her desire to continue in this direction. As a personal conjecture, a certain impact on her decision to continue in this direction may have been given by the comment made on the forum by the other trainee as an answer to the discouraging comment related to the experimentation that Lucy reports on the forum.

Anyway, in her classes Lucy proposes more than one of the activities seen in the MOOC and she is able to create a set of possibilities for developing new beliefs, knowledges, goals and practices (she has reached the potential level of ZPD that the MOOC's trainers had planned). To implement the different teaching approach promoted by her involvement in the MOOC

(MOOC's ZFM/ZPA), Lucy had to change her environment (school's ZFM). In fact, she has used the MOOC's activity adapting them to its classes to be consistent with her goal of improving students learning (i.e. she had created the image of the triangles rather than using the boat one).

From the presented analysis, as far as Lucy is concerned, we can answer to the second research question with a positive response. We asked ourselves: *Does the MOOC's ZFM/ZPA trigger and support an effective shift from the actual developmental level to the potential developmental level in the ZPD of the trainees? And if so, which kind of expansion of the network of professional knowledge this shift brings with it?*

The MOOC Geometria allowed for an effective professional development experience. It has triggered and supported changes in what Lucy thought about her way of teaching geometry using also practical-manipulative activities. Moreover, exactly the activities proposed in the first module (part of MOOC's ZFM) and the actions triggered in the MOOC's ZPA, that is the comparison with the other trainees in the platform, led Lucy to reconsider the practical-manipulative activities. She actually stated that she very much appreciated sharing teaching experiences and feeling part of a community that experienced the MOOC activities: they were able to push her enough to do the same. As a result, Lucy began to make more use of these activities in her classroom, noting a different interest and attention in her students. This can be considered a small, but significant perception of change in her teaching practices.

5.3.3 A case study for MOOC Numeri

Among MOOC Numeri trainees we chose to report the case study of a teacher we call Stephen.

Stephen's case has been selected because his Project Work, or the final design activity of the MOOC Numeri, was a reworking of an activity proposed in the MOOC (precisely "*Arithmetic helps algebra and algebra helps arithmetic*", which was described in detail in Chapter 3) in the light of his experience as a teacher and of the experience lived in the MOOC. He also experimented with this activity in the classroom, documenting it with the delivery of the logbook. From these data we can see how the MOOC's ZFM/ZPA contributed to the Stephen's professional development (understood as the development of his ZPD) and consolidation of his didactical praxeologies.

The data that we will show in the following come from Stephen's:

- intervention on the communication message boards;
- answers given to the three questionnaires administered during the MOOC Numeri;
- logbook compiled during the experiment conducted in the classroom;
- answers given to the Skype interview.

Introducing Stephen

Stephen has been teaching mathematics for 12 years. The school in which he teaches is a lower secondary school in a town in Emilia Romagna (region in Northern Italy). He has three classes (grade 6, 7, 8) and in such school students have 6 mathematics hours per week. Stephen willingly uses various kinds of technology with his students. In his classrooms he uses PCs and LIM; and as mathematics software he uses Geogebra (DGS, dynamic geometry system) and Excel.

In October 2015 he enrolled in MOOC Geometria. Stephen declares, in the final questionnaire of MOOC Geometria, that he enrolled in the MOOC because he felt

> "willingness to train; desire for comparison/ sharing/ideas; gratuity; topics covered (geometry is often more difficult for the students); prestige of the proposing university; possibility of working asynchronously from home".

He actively participated in the MOOC Geometria, contributing to the discussions in the communication message boards and experiencing in his class some of the activities proposed in the MOOC. He has regularly concluded all the MOOC Geometria activities in January 2016.

In November 2016 he enrolled in MOOC Numeri. In its initial questionnaire, he declares that he enrolled in this MOOC because

> "I had excellent reminiscences of last year, a course at Km 0 and with customizable times. I wanted to compare myself with colleagues and to analyse materials selected and structured by experts on the issue of numbers. Since 2009 I have been working with mathematical machines and I have developed laboratory-based methodology activities especially in geometry".

Stephen was therefore looking for new ideas and laboratory-based methodology activities on the core of numbers.

Also this time, he actively participated in the MOOC Numeri, contributing to the discussions in the communication message boards and experiencing in his class some of the activities proposed in the MOOC. He has regularly concluded all the MOOC Numeri activities in February 2017 with a Project Work that took inspiration from an activity of Module 5 of MOOC Numbers "*Arithmetic helps algebra and algebra helps arithmetic*" (this activity and its phases were described in detail in Chapter X). Stephen also participated to the facultative module on the experimentation. So, in March 2017 he delivered the logbook that documented the experimentation he had carried out in the classroom and which in particular was based on the Project Work he had designed as the final activity of the MOOC Numeri. He practically implemented his design and, together with the logbook, he sent us the materials he had created for his students (activity sheets).

In the following we will focus more on Stephen's experience in the MOOC Numeri.

Stephen and his general experience within the MOOC's ZFM/ZPA

Stephen found himself at ease in both MOOCs. From the intermediate questionnaire of MOOC Geometria, Stephen declares that he considers the MOOC easy to use. He considers it flexible in terms of space, time and pace of learning (giving the score 5 to all the items - remember that the question was a scale of Likert with score 1 representing total disagreement and score 5 that represents total agreement). Also on the communication message boards he expresses very positive judgments, considering that they allow her to express her personal opinions regarding the contents of the course (with a score of 5); to allow her a dialogue with other colleagues (with a score of 4); to have the opportunity to benefit from the experiences of others (with a score of 5). From the final questionnaire of MOOC Geometria, we observe that Stephen has read a number of posts between 60-80% of the total, while the answers given to comments from other trainees fall between 1 and 20%. So he has more read than written. In particular, on the advantage of reading/writing post declares: "it is an asynchronous way of proposing, asking, discussing, sharing, helping, being helped in a serene way [...]". He believes that he has worked hard in the MOOC and having participated in the MOOC was useful for him because

> "I have strengthened my spirit of approach to the construction of meanings, I received suggestions (but also confirmations) to conduct laboratory experiments, the MOOC has unified me to other colleagues in the national territory"

He said he felt part of the community born in the MOOC Geometria at 101% and he would re-enroll with a mathematical MOOC, as he actually did.
In the interview conducted via Skype, to the question: "*Why did you decide to continue to attend our training offer?*" he answers:

> "The course on Geometry was interesting: environment, materials, stimuli, proposed activities, etc ... so I also thought about attending the MOOC Numeri. Over the years I have focused on strategies and laboratory-based methodology activities related to geometry: I could not lose the comparison and the opportunity for new stimuli on the world of Numbers"

In MOOC Numeri, from the intermediate questionnaire, Stephen said he knew how to move perfectly on the platform thanks to previous experience. He self-organized himself in terms of timing, methods and in-depth study of materials. In the Skype interview, we asked: "*How did you manage your access to the MOOC: how many times and how long did you connect?*". He answers as follow:

> "Although the MOOC is stimulating, well-balanced and keeps the level of curiosity and activities up to date high and balanced, I often could not maintain the rhythm of the various modules because I am committed to the school where I teach ... [...] so I could not connect every day. Actually, there is no need for a daily connection, and if I stayed behind some activity related to the unit in progress, it is positive that I could finish it the following week or the one after [...]
> Very positive is the fact that I could cut out the time to devote to it in a very free and flexible way [...] I always participated from home, in the days and times I could to dedicate all the time it deserved [...]"

Also in MOOC Numeri, from the final questionnaire, we observe how Stephen intervened on the communication message boards. The readings were kept at a constant rate between 1% and 20%, while the number of writings decreased, between 1% and 20%. This is because, as he then clarified during the interview on Skype, he had many school commitments. However, his sense of belonging to the community remains strong. On the final questionnaire, to the question "*As MOOC trainees, to what extent did you feel part of a community?*" he writes: "100% is an active environment, full of stimuli". And he still confirms his position during the interview on Skype, where we ask him to explain to us because he believes that the other students contributed to his training. He answers:

> "The MOOC allows a critical re-reading of one's own work and that of others, allows sharing experiences, opinions, methodologies, teaching strategies, but also simple ideas to develop. You can find yourself in continuity with your work or in net disagreement with personal experiences: therefore a rich environment/experience".

In the final questionnaire of MOOC Numeri, as you will recall from the previous analyses in this chapter, with a scale-Likert question, we asked the trainees to express their judgment about the truth of certain statements. A score of 1 indicated that the sentence was not considered true at all, while a score of 5 that the sentence was considered absolutely true. Responding to the fact that the MOOC was useful for the professional development, Stephen

responded with a score of 5; while compared to the fact that the MOOC led to the perception of a change in the teaching practices, he responded with a score of 3.
This is in agreement with his statements and his answers, both in the questionnaires and in the Skype interview. He repeatedly stressed how he was looking for cues for laboratory activities related to the core of numbers. The fact that he attributed a score of 5 to the MOOC as a tool for his professional development testifies how he evidently found what he was looking for. Also, in the intermediate questionnaire, to the question: "*Have you started to transpose some of the activities of the MOOC into your classes? If yes, with what innovations compared to your previous teaching practices?*" Stephen had replied:

> "[...] the activities presented here [...] are in line with an experiential approach to laboratory-based methodology that I carry out in the classroom. I see a kind of continuity of my actions and an openness to different experiences from mine".

As illustrated in Chapter 3, the activities carried out by the MOOC focus a lot on the laboratory-based methodology. Since Stephen, as he affirms, is already a constant user of this kind of methodology, it is not surprising that he gave a score of 3 to the question he asked if the MOOC was deemed to have changed its teaching practices. Responding with score 3 means remaining neutral, and indeed, as he himself says, there is continuity between the MOOC's proposals and its teaching practices.
Furthermore, we asked a similar question in the Skype interview: "*At the level of teaching the Numbers core, what did the MOOC do? Has anything changed in your teaching of arithmetic/algebra or in the teaching of arithmetic/algebra in general (for example, to rethink what are the important contents, ...)?*" Stephen replied:

> "The MOOC Numeri has allowed me to fully agree with the 'early algebra' that I have been practicing for years [...], so it has strengthened the 'mathematical life project' that I follow and evolve for the students of the first level secondary school where I teach. In particular, through the project work that the MOOC requires to do and the related experimentation to which I have joined, I have tried to build a mix on activities proposed by the MOOC and my practices".

A snapshot of Stephen's practice
In the fifth module of MOOC Numeri the proposed activities aimed to deal with the conceptual node *natural language* and *algebraic language*. The activities refer to the introduction of the rules of algebra and the difficulties encountered when the students must translate a problem algebraically. The activities want to give meaning to algebraic calculation, to ensure the students do not interpret the algebraic formulas as pure sequences of sign. Five different activities are proposed. Precisely, they are referred to a unique activity called "*Arithmetic helps algebra and algebra helps arithmetic*" and are showed as five stages. We suggest to see the stage from 1 to 3, because Stephen was referred especially to them in his Project Work that we are going to illustrate.

After a few days of the opening of this module, Stephen intervenes in its forum. He takes part of a discussion we have proposed in Table 4.16 of Chapter 4 (Stephen's intervention is that of the last line). There, he was sharing his own thought with the MOOC-ecosystem, that does not wait for the 8 grade to start to use letters or to talk about algebra. He ends his comments with this sentence: "P.S. beautiful game 'guess the number' [stage 1 of the Module 5 activity] and the development of notable products [stage 2 and 3]". And in fact, the Project Work, which he then designed about two weeks after that comment, took its cue from those activities.

Stephen's Project Work

The trainees had to realize the Project Work (here and after PW) through the use of a specific software, Learning Designer (we have discussed this in §3.2.3.3). The PW of each student can be consulted online. Stephen's PW, entitled "*Mathematics without numbers*", can be consulted at the following link: https://v.gd/OYK0v8. The PW is written in a very organized way and it is easy to consult, thanks also to the Learning Designer software, which leads to organize the project in schematic phases. Reading the PW you can see the organization of the teacher's network of knowledge about the mathematical topic he wants to deal with. We can translate this his PW interpreting this work in terms of his didactic praxeologies.

Task 1	Discovering the distributive property
Techniques	- The teacher offers the class a 'magic game' asking everyone to think of a number. The teacher guesses the number thought by the students; - The students are divided into pairs and must reconstruct (with the representation they prefer: numerical, graphics, ...) the path of the chosen number and discuss the validity of the result; - The students must reflect on the question: "Does the result depend on the number chosen?" - The teacher proposes another 'magic game' to the class - The students are divided into pairs and must reconstruct (with the representation they prefer: numerical, graphics, ...) the path of the chosen number and discuss the validity of the result. They must always reflect on the question: "Does the result depend on the number chosen?" - Student pairs must rebuild the number sequence on a spreadsheet. The validity of the calculation is so generalized on numbers of other pairs of students.
Task 2	Applying the distributive property
Techniques	- The following problem is proposed: "I have to paint two contiguous walls of my room and I know that the painters ask for a fee based on the surface to be coloured, regardless of the presence of doors and windows. Suggest plausible measures of your room and a method to calculate the measurement of the surface of all the walls". - Students work in pairs and propose resolutive strategies. - The different strategies are compared with the whole class on the LIM. - In pairs, the students reconstruct the two members of the distributive property: $a \cdot (b + c)$ and $ab + bc$. - Student pairs must develop a graphical (dynamic) visualization of the distributive property with GeoGebra. - Some results are analysed and discussed
Task 3	Discovering the square of a binomial
Techniques	- The challenge of "Think of a number" introducing the binomial square is proposed - The students, in pairs, reconstruct the numerical sequence with a spreadsheet.

	- Students must analyse the scriptures *$(a + 5)^2$* and *$a^2 + 10a + 25$* emphasizing the meaning of binomial square and its development through the distributive property. - The whole class, together with the teacher, discusses the generalization of *$(a + b)^2$* and the teacher proposes a graphic resolution through GeoGebra. - The students, with GeoGebra, must construct a square on the side *$(a + b) = constant$* and varying *a* or *b* to display the areas of the decomposed figure
Task 4	Discovering the difference of square
Techniques	- The class must analyse the meaning of (a + b) (a-b). - Students in pairs can use the spreadsheet and/or GeoGebra. - The whole class analyses and discusses the various approaches to formalize the development of the outstanding product assigned.
Logos	Encourage student'' learning by proposing collaborative activities, also inspired by reality and with the use of the ICT

Table 5.8: The Stephen's didactical praxeology related to the discovery and managing the algebraic language

Each task proposed in Stephen's design requires 1 hour of time.
In **Taks 1**, Stephen refers to Stage 1 of Module 5 activity, which involved making students discover the distributive property. Stephen proposes two games of magic and specifies the respective references for each game. The first game is found in a textbook, the second is just what was proposed in stage 1 of the MOOC's activity. It is interesting to note how he made the activity versatile, introducing the use of the spreadsheet. The activity that the MOOC proposed in fact provided for a first phase to be carried out only with paper and pen and only after having identified the distributive property, was the teacher who showed the class its geometric representation (possibly using GeoGebra).
In **Task 2**, Stephen continues to make stage 1 of the MOOC's activity versatile. In fact, he proposes to his students a problem inspired by a real situation "to determine the surface of two adjoining walls of their bedroom". In planning, he never neglects moments of work as a couple and moments of collective discussion. Furthermore, it proposes an alternative use of GeoGebra with respect to the activity proposed in the MOOC. In the MOOC, as mentioned, it is the teacher who shows a GeoGebra file to the students to underline the link between algebraic property and its geometric meaning. Instead, Stephen requires that the students create a GeoGebra file. This certainly means that students not only know the software, but they use it regularly.
In **Task 3** Stephen takes much into consideration the stage 2 of the MOOC activity that leads to discovering the binomial square. He adds again the use of the spreadsheet. Furthermore, to show the geometric meaning of the binomial square, Stephen proposes the use of GeoGebra by the teacher, as suggested by the MOOC activity. In addition, it requires students to perform a dynamic construction based on that shown by the teacher.
In **Task 4**, Stephen begins as proposed by stage 3 of the MOOC's activity for the discovery of the difference of squares. In addition, he suggests to the students the use of the spreadsheet or Geogebra, without specifying to have to create a file to show the geometric meaning of this property. This instead was a clear step of the activity proposed by the MOOC and it was always the teacher who had to show it with the help of a software, possibly GeoGebra.

Stephen's Experimentation

What described above corresponds only to the design of the teaching activity that Stephen presented as PW for the final activity of the MOOC Numeri.

Subsequently, he experiments his design in the classroom. The intent is to adhere to the optional experimentation proposed in the MOOC Numbers.

The logbook that documents the trial in the classroom is shown in Appendix F (it is in original as delivered online by Stephen, then in Italian).

From the logbook, we read that the target class of the activity is its grade 8 class. The class consists of 22 pupils and the work climate is collaborative and purposeful. Stephen declares that he started the experimentation on March 17, 2017, and ended it on March 31, 2017, for a total of 5 hours. This time corresponds with what he had foreseen in the PW. In the logbook, in addition to a request to briefly describe the proposed activity, we asked to specify if it has been performed exactly as it was designed or if further details have been added. Stephen claims to have used it by adding insights and specification. In a comment accompanying the question, he specifies: "I prepared some worksheets for the students that could better guide the path and structure it in a more autonomous way".

Stephen shared these worksheets with us. They are listed in Appendix F. These are 5 worksheets, one for each task that has been designed. They were created by him and reflect both an orderly way of working and give deliver to students, and also a passion for his work. They are in fact very well done: coloured, with images. They have a tabular structure. On the left side, there is a space dedicated to deliveries or questions on which students must reflect. On the right side, there is a space organized to allow students to bring back reflections and answers. Some activities of the worksheets incorporate the same tables that had been proposed by the activity of the MOOC (see for example the worksheet 2 with the tables presented in stage 1 of the MOOC's activity), but the general organization of the worksheets is clearly the result of the creativity of the teacher.

From the logbook we read that the students participated in the laboratory-based methodology activity in a "sparkling" way, so Stephen writes. In fact, he tells of an unexpected event: "in the use of the PC a couple of pupils did not correctly use the instrument available and did not behave properly and I was forced to stop the activity for everyone". He also explains: "[that day] I finished the activity (there were no longer any educational and interest extremes) and then I take it back another day".

In general, however, Stephen reports that in carrying out the activity the students seemed *very* attentive, *very* interested and *fairly* involved. The level of difficulty he perceived in his students, from 1 to 5 (with a score of 1 = very difficult activity and 5 = very easy activity) was 3. And he adds: "Some have intuitions, but they can not support them or are not able to argue them in an appropriate way". The level of satisfaction perceived by Stephen at the end of the experimentation, from 1 to 5 (with a score of 1 = totally dissatisfied and 5 = totally satisfied) was 4.

During the Skype interview, we also asked questions about experimentation. In the following, we report in particular two questions asked and Stephen's answers.

> Researcher: "Is there any particular reason why you chose this activity?"
>
> Stephen: "I chose this subject because I do not consider algebra as a "program for the grade 8", but as a continuum that accompanies students on all occasions (geometric, arithmetic, science) of generalization: towards a mathematics without numbers"
>
> Researcher: "Did you say that the experimentation was part of the MOOC project? What do your students think (if they have commented) that you followed the MOOC?"

> Stephen: “I always tell to the pupils of my training courses, experiments or laboratory-based methodology activities that I intend to do with them and they are always very enthusiastic: they feel included in the experimentation process that involves them directly and they realize that their teacher does not promote static activities, but constantly evolving where the experiment and the learning are mixed. Often they are activities of a working group or couple, the argumentation and the comparison between peers is always promoted with a collective revision of the meanings built by the class itself; in "mathematics without numbers" [the PW] everything is always surrounded by a somewhat mysterious that motivates them and makes them curious.”

The interview is a confirmation of statements that Stephen had already issued in writing, for this we do not report all the questions asked, but only those that give more information than those already presented. From the answer to the first question, we note how the choice of the activity on algebra is made because, as he had indicated on the MOOC forum, he believes that algebra is a generalization tool that should not necessarily be acquired starting from grade 8.
From the answer to the second question, it is interesting to note how he always involves his pupils in his educational experiences. He is pleased to tell them and, with a certain pride, says that “[the students] realize that their teacher does not promote static activities”. He wants to give a professional image of himself to his students, of a teacher who “cares” to do his job well. To involve his students also in the telling of his extracurricular activities concerning his training seems to mean for him as make students aware that his updating is finalized to create activities that are more engaging for them.

5.3.4 Discussion

The MOOC’s Zone Theory perspective allows for picturing Stephen’s profile from the data collected through the MOOC’s questionnaire, the logbook and the Skype interview.
Stephen describes his school’s ZFM / ZPA as a positive context. In the logbook, he talks about a collaborative and active class. It also tells of an unexpected event that occurred during the experimentation, which prevents the normal development of the lesson, but this does not have negative repercussions neither on the activity, which in fact is resumed the next day, nor on the general climate of the class. Stephen continues to talk about very attentive, interested and fairly involved students. He has a good relationship even with his colleagues. We deduce this from the interview on Skype, where he tells us: “a colleague of mine knew that I was attending the MOOC: she had enrolled in the one of Geometry, but then she was not able to attend for numerous school commitments”. Stephen talks a lot with his colleagues as well as with the students. In fact, he tells the students to follow the MOOC and makes them active protagonists in experimentation. We have seen how it emerges from his proposals and from the materials he shared with us, how much care and how much passion he has for the teaching of mathematics. He is careful to dose the methodologies well in his lessons. He prefers, as he has repeatedly stated, the laboratory-based methodology activities. He alternates moments of collaborative work, collective discussion, and reasoned use of technology. His students seem to be used to work with Excel and GeoGebra. Therefore, no tension emerges in Stephen. There is no misalignment of its zones system. We could say that, since his ZPD and his school’s ZFM/ZPA are aligned, he is already in a phase of productive tension. There is a balance between his knowledge and his peaceful relationship with colleagues and pupils. These positive factors act as stimuli for him to always tend to give more and to do his best as a teacher.

In the final questionnaire of MOOC Geometria, to the question: “Do you look for specific material on communication boards?” he had given this answer:

> “I am a hunter of ideas and an experimenter (but I can not directly apply an idea / project of another: I have to make it mine, cut it and reassemble it according to my style of teaching)”

He joined the MOOC Numeri to look for cues of teaching on laboratory-based methodology activities relative to the numbers core. It clearly states that he is “used to doing laboratory activities with the collective construction of meanings starting from problematic situations to which the students must find the solutions”. However, he confesses that he does not have a vast repertoire on activities involving the numbers core. The MOOC’s ZFM/ZPA for him is a rich environment, full of stimuli and colleagues to deal with to reflect together and exchange ideas and strategies. To remain in his metaphor, in which he described himself as a hunter, the MOOC’s ZFM/ZPA is for him a forest to explore where the prey is represented by the activities that arouse his interest. The prey, therefore, should not be attacked, but hunt in a reflective way: it must be studied, thought out, included in its teaching practices. The shot that allows you to capture the prey is its internalization in the network of knowledge, consciously connected with its nodes, revised in light of new knowledge in input from the MOOC’s ZFM/ZPA. This new knowledge is then experimented with creativity and desire in the school’s ZFM / ZPA, with satisfactory results both for Stephen and for his students, who actively participate in his lessons. The decision to experiment with a laboratory-based methodology activity on the issue of numbers confirms the fact that Stephen's ZPD has moved from the current level to the potential level. Thanks to the comparison with the peers within the MOOC's ZFM / ZPA and to the comparison with the trainers that was realized with the fruition of the proposed activities, Stephen was able to enrich his repertoire of laboratory-based methodology activities on numbers core. In this way, he realizes the wish he had when he joined the MOOC Numbers. He was therefore able to benefit of professional development.

5.3.5 Not everything turns out as it should: a negative case study for MOOC Geometria

During the period in which the analyses on the final questionnaire of the MOOC Geometria were conducted, a singular situation jumped to the eye. There were always 1% negative replies to the questions concerning the appreciation of the course, while 99% gave positive answers (some mention about it was made in Chapter 4). It did not take long to realize that that very small negative percentage always referred to the same person. This intrigued the dissertation writer and for this reason, she wanted to analyse in detail her profile.
Reading in detail all her answers, the dissertation writer felt the need to address other questions to her. The dissertation writer tried to get in touch with her in October 2017. She wrote her an email and sent her a word file with the questions she wanted to ask her, with the request to reply directly to the word file. It had been a long time since the end of the MOOC Geometria. In fact, the MOOC Geometria was finished in January 2016. Therefore, the dissertation writer was not confident that this trainee would answer. Instead she has answered all the questions that have been addressed to her.
Then, a negative case study, which refers to a trainee of MOOC Geometria that we will call Ester is proposed.

The data that we will show in the following come from Ester’s:

- answers given to the final questionnaire of MOOC Geometria;
- answers given to the questions posed in the online contact happened in December 2017.

Introducing Ester

Ester has been teaching mathematics for 22 years. The school in which she teaches is a higher secondary school in a town in Valle d'Aosta (region in Northern Italy). In October 2015 she enrolled in MOOC. Ester declares, in the final questionnaire, that she enrolled in the MOOC because: "I thought that the course would have stimulated me more to solve new problems of mathematics to be then addressed to my students". She participated in the MOOC in a non-active way, as we are going to clarify in the next session. However, she got all the badges of the modules and carried out the final activities, bringing the course to the conclusion in January 2016.

Ester and her general experience within the MOOC's ZFM/ZPA

The following data emerge from the analysis of the final questionnaire (the underline words represent the response option chosen by Ester in answering the questionnaire). Ester claims to have <u>never</u> read and commented on the communication message boards. She took advantage of <u>20% or less</u> of the material made available to the MOOC. As a participant of the MOOC, she felt <u>little</u> about being part of a community. To the open question: "*What similarities or differences do you see on yourself in relation to the other trainees?*", Ester replies:

> "similarities: interest in the subject [mathematics] - differences: lack of passion for teaching by myself".

When she is asked to express a judgment on Learning Designer (here and after LD), the software used to do the Project Work, she says that it considers it <u>useless</u> because:

> "I can not follow a written trace of the lessons, depending on the classes sometimes I need to dwell more on the theory other times on the exercises, other times on ways to intrigue the students. If I clearly have in mind the arguments to be explained in class I find it is a waste of time to further formalize them. Rather than use the learning designer, I prefer to spend my time to look for other paper or online material about ideas for explanations or exercises".

On the Peer Review, on the other hand, she is expressed as follow:

> "[I find it] very useful to compare the way others see my work".

Let us move on to the part where Ester had to make a judgment on the MOOC and its materials.

She writes that the contents of the MOOC <u>did not meet</u> her expectations. She judges the duration of the course compared to the subjects treated <u>good</u>, but she finds <u>insufficient</u> the teaching methods used in the course. The provided teaching materials are <u>useful</u>. She also says that the content of such materials is <u>sufficient</u> and the way the contents are expressed is <u>poor</u>. However, it is worth noting that there is a contradiction with what she said before because the contents did not meet her expectations and she used - by her own declaration - only 20%, or even less, of the materials proposed. She believes that she has engaged in the MOOC <u>neither very nor very much</u>.

Being involved in this MOOC was useful because: "[I] found useful material" (which it is always contradictory, because she has criticized and considered it poor).

Being involved in this MOOC was useless because: "I wanted to work more on mathematical problems to refresh my mathematical knowledge". An observation stands out: criticizing LD, she said that she considers the design practice as useless because *she has the arguments clear in her mind*, yet here she admits that she wanted to refresh her mathematical knowledge.
To the open question: "*Do you want to refer to significant events that occurred in the classroom following your participation in the MOOC?*" Ester responds like this:

> "using the [MOOC's] material [...] on trigonometry I made my students be less afraid of solving problems and I got bored less than usual from my own lessons".

Two (provocative) observations stand out: she does not have a high reputation of herself as a teacher, why? In addition, if she knows her lessons are boring, why does she not try to change her methodology?

Contacting Ester for some clarification
There are a number of answers given by Ester that do not go unnoticed. The dissertation writer felt it necessary to compare herself with her. The comparison was not in attendance, both because it would be difficult to organize a meeting in person, or because the dissertation writer believed that Ester would not want to answer our questions. Timidly, the dissertation writer tried to contact her and ask questions via email, sending her a word file.
Here we present the 5 questions we have decided to ask her and the answers she gave. Each question addressed to Ester was preceded by the question to which she had replied to the final questionnaire and the answer she had given. In what follow, we reproduce the answer file that has arrived and we will make specific comments immediately after the new answers that Ester has given to our questions. Our questions (those of the final questionnaire and new ones) are shown in **bold**. Ester's answers to the questions in the final questionnaire, which we have already explained in the previous section, will be reported in *italics*. The new Ester's answers will be reported with a higher indentation than the text indentation.

1. **Why did you sign up for MOOC Geometria?**
   "I thought that the course would have stimulated me more to solve new problems of maths to be then addressed to my students".
   What types of mathematical problems did you expect? Could you give us an example?

   > Problems such as trigonometry and also Euclidean geometry, well organized by topics. The INVALSI questions were useful but, if I remember correctly, it was a problem to download them and it would be useful to collect them for topics or educational problems such as the concept of perpendicular. Problems related to reality on limits.

The MOOC modules were distinguished by topics (as we will recall from Table 3.1) and for each topic there were a series of proposed activities, whose structure is similar to the one shown in Chapter 4 on Module 1 of MOOC Geometria. Trigonometry problems have been proposed in Module 2 of the MOOC Geometria, which was dedicated to overcoming the misconception between arc and angle. The activities of the MOOC Geometria, as the name of the MOOC itself suggests, were all based on Euclidean geometry. There were no problems on the limits because this topic falls within the calculus and not in geometry. We nevertheless remember that Ester has declared to have viewed the 20%, or less, of the proposed material.

2. **Being involved in this MOOC was useless because:** *“I would have liked to work more on mathematical problems to refresh my mathematical knowledge”*.
   Can you explain to us what kind of problems would you find useful, in order to allow a review of mathematical knowledge?

   > I mean problems related to real situations also to be used as evidence for skills. To review, I would like to face problems like those of the state exam.

As we have repeatedly stressed in previous analyses, the MOOC has proposed many problems related to real situations to be used as evidence for skills. Proposing problems that start from real situations and that develop students’ skills were some of the methodologies on which the trainers focused most, i.e. some of the ideal praxeologies they wanted to transpose to the trainees.
The problems proposed for the state examination, in particular, concern precisely these aspects, so those of the MOOC were in line with these characteristics.
We once again recall that Ester has declared that she has viewed about 20% of the MOOC material.

3. **The MOOC Geometry, in addition to providing examples of activities that can be spent in the classroom, has been very focused on suggesting working methods to be used in the classroom. You had expressed yourself in this way:**
   She judges the duration of the course compared to the topics covered good, but she finds insufficient the teaching methods used in the course.
   Could you give an example of an efficient teaching method, perhaps suggesting one of those used to use it in class?

   > To face the peer review I went in confusion because during the course many technological ideas have been proposed that have not been in some way deepened or explained and therefore I would not feel to use them in the classroom.
   > Instead, following the DMT webinar, I learned to use a little Maple and used it to show the geometric transformations of the goniometric functions. Maple is very complex, but if you learn a small part, functional to a small part of the program and immediately applied to address a topic. it remains more in mind.

Ester’s answer does not centre the question that was asked of her. Compared to what she wrote, having judged insufficient teaching methods used in the course seems a bit strict if the problem lies only in having had confusion in doing the peer review.
Surely she for Peer Review intends Project Work (remember that it has been more than a year between the end of the MOOC and these questions). However, the purpose of Project Work was not to use technological cues as the one seen in the MOOC. The technological cues in MOOC Geometria were:

- *communication message boards*: which she had declared she has never used. She writes that they have not been explained. This is true, but the use of communication message boards was friendly, it would have been enough to try to access them. Furthermore, in the MOOC there was a technical forum on which you could write doubts and email addresses to contact the MOOC team in case of need for clarification.
- *Geogebra*: every time it was recalled in some activity, we put the file to which reference was made. Of course, the construction of the file was not explained, but this was not the purpose of the MOOC.

It is likely that Ester did not grasp the sense of MOOC because she had other expectations.

Before continuing with the analysis, some observations regarding the Ester's statements about Project Work and Peer Review, showed in the previous section.
Ester, compared to what she said on Peer Review, seems willing to understand what others think of her work. One wonders then why she is not so willing to explain her work using the structure of LD. One can agree that not all classes are the same and that arguments can not always be explained in the same way. However, the practice of designing is not so much to have clear topics to be treated (this is the essential basis from which a teacher must always start), rather to test own ability to know how to deal with the chosen topic in the way it was decided to do, trying to manage and anticipate the possible unexpected events that the class or topic carries with it. LD "forces" to make working hypotheses in terms of time, to focus on the methodologies that will be used in class (it is not mandatory that there are more methodologies in a lesson, but if we want attentive pupils and above all to stimulate their interest, the same methodology is not the strategy to follow).
Moreover, putting on paper the prediction of what will be done is a way to allow the teacher to reflect on his processes: "*have I kept the schedule I had done? If not, why? Something unexpected event has emerged! I take note of it, it may re-emerge in the future and I already know how to deal with it*". Perhaps for a teacher who has been teaching for many years, it may seem like a useless practice, but knowing how to do a self-analysis of one's teaching is a virtue to be cultivated.
Furthermore, LD's idea is to create a personal portfolio, in which the materials are already organized in order to be able to reuse them the following year, for example, and eventually to update them; and - last but not least - sharing, if you want, your work with others: to get new ideas and maybe benefit from constructive criticism (always if you are willing to get involved).

4. **We were interested in your declaration: Do you want to refer to significant events in the classroom following your participation in the MOOC?**
   "Using the [MOOC's] material of the trigonometry I made my students less afraid of solving problems and I got bored less than usual from my own lessons".
   Can you better explain this ironic self-evaluation?

   > Trigonometry in textbooks is addressed with definition-example-exercise and repeating this pattern for years becomes boring especially for the teacher. Unfortunately, it is not easy to find alternatives that do not occupy too many hours of the school year, but which at the same time make it easier to clarify the concept of what the definition-example-exercise scheme does.

This is a tension (in the sense of Goos) that arises in Ester as a teacher. She would like something that does not degrade the teaching of certain concepts, something different from the mechanically exercise made by both the teacher and the high school students.
It was a pity that Ester did not have the curiosity to delve into the rest of the materials of the MOOC, because she would change her mind and find many other materials like the one she cites, which would surely brighten up her lessons. This is not a thought of "parent" of the MOOC, but a thought that all the other enthusiastic participants shared, as shown in the previous analyses.

5. **What similarities or differences do you see in relation to other trainees?**
   "Similarities: interest in the subject [mathematics] - differences: lack of passion for teaching by myself".

Indicate two or three behaviours/attitudes that help us to understand how you perceive this lack of passion, contrasting it with concrete examples of behaviours/attitudes that illustrate a passion for other aspects of the discipline.

> The challenge in solving problems fascinates me, I still feel like a student. If the teaching provides me with insights that intrigue my students, it also intrigues me. I find the teaching that teaches me to use hours and hours of lesson to demonstrate the three criteria of congruence of the triangles, instead of offering exercises models in which students are told that the method of research of the demonstration starts from the thesis of a problem and that with this objective in mind the figure and the triangles that compose it are analysed, is not adequate to my style of teaching.

Ester criticizes these methods of mechanical teaching, but it also seems that she does not know others. She recognizes that she is more interested in a problem when she feels challenged to find the solution. The same is undoubtedly true for his students. It would be enough to start from simple things: do not explain the rule, let it be discovered.
Of course, if she is never been used to teach like that, she can not get there alone.
Again, it is a pity that she did not grasp the potential that the MOOC could have given her, if only she had given it the opportunity it deserved.

5.3.6 Final remarks on the case studies

Three very different case studies have been illustrated.
The first is on Lucy, a trainee of MOOC Geometria. We have shown how she sees in the MOOC's ZFM/ZPA a means to escape from the reality of her school's ZFM/ZPA and take refuge in a stimulating world where she feels understood.
The second is on Stephen, a trainee of both MOOC Geometria and MOOC Numeri. He lives a decidedly idyllic situation in his school's ZFM/ZPA compared to Lucy. We have seen how, for Stephen, the MOOC is almost the Eden Garden, a perfect place to draw juicy fruits to increase its repository of activities on laboratory teaching.
For both of these trainees, the MOOC's ZFM/ZPA allow for an effective shift from the actual developmental level to the potential developmental level in the ZPD of both of them. The MOOC's ZFM/ZPA trigger and support an expansion of the network of professional knowledge; namely, it allows their professional development.
These aspects lead to the perception of changes in Lucy's teaching knowledge, practices and beliefs, because she has been reconsidering the practical-manipulative activities and her students seem to be involved with a different attitude during these hours of class. So, we can talk about a beginning of an evolution of Lucy's didactical praxeologies.
We can not speak of perception of changes in Stephen's teaching knowledge, practices and beliefs. He is a teacher who proves to be updated: he uses different methodologies in his lectures and also integrates technological tools. MOOCs do not change his didactical praxeologies, but confirm that he is working in the right direction.
Finally, the case of Ester, a negative case related to a trainee of MOOC Geometria. Ester does not spend much time on MOOC and its materials. There seems to be tension between her ZPD and her school's ZFM/ZPA. This tension, however, is not resolved nor by the activities that the MOOC proposes, because Ester consults only a small part (those on trigonometry that appreciates), nor from the interactions of the other trainees, because she does not use the communication message boards. In light of the quantitative analyses that showed a general appreciation of the MOOC in terms of professional development, we can speak for Ester about a missed opportunity.

Chapter 6 Analysis on the MOOC's trainers

The findings presented in this chapter contribute to addressing the third research question that is specific to the trainers:
Does the MOOC's ZFM/ZPA trigger and support an expansion of the network of professional knowledge of the trainers relatively to design principles and strategies of trainees' assessment that the trainers have put in place?

The following clarification is important. All dissertation is focused on the MOOCs for mathematics teacher education and more attention has been paid to the trainees, that is, to the recipients of these MOOCs.
However, in the course of the discussion, there has been an opportunity to remember those who have been behind the scenes of MOOCs, that is, the trainers who have been involved in their design, delivery and monitoring.
In this chapter, which concludes the dissertation analyses, we will focus on trainers. The analyses will not be as detailed as those made for the trainees, but they would like to underline the design efforts and assessment strategies that have been pursued in these online courses for mathematics teacher education by the mathematics teacher educators.

6.1 The trainers of MOOC Geometria and MOOC Numeri

The MOOCs have been described quite extensively in this dissertation. A hypothetical designer could certainly take these descriptions as examples. Nevertheless, we still give some other suggestions to possible readers who will be convinced of the potential of MOOCs, if managed in a certain way.

As we had seen, in the chapter on the description of our MOOCs (§3.2), we can understand a MOOC as a design experiment, based on the design-based research paradigm (Wang & Hannafin, 2005). In this view, the design experiments manifest both scientific and educational values through the active involvement of researchers in learning and teaching procedures and through "scientific processes of discovery, exploration, confirmation, and dissemination" (Kelly, 2003, p. 3). In design-based research researchers manage research processes in collaboration with participants, design and implement interventions systematically to refine and improve initial designs, and ultimately seek to advance both pragmatic and theoretical aims affecting practice (Wang & Hannafin, 2005).
Already at the beginning of the analysis, we had mentioned how the trainers involved in the MOOCs design and delivering, thanks to the monitoring stages and the feedback received from the trainees, were able to properly modify them to allow better professional development experiences to their trainees. Think, for example, what we have seen in Chapter 4 on the grid structure given to MOOC Numeri, the blue table, …. Without the pretension of entering the field of design-based research, which is delicate and articulated, in this chapter we will limit ourselves to give an outline of the evolution of the meta-didactical praxeologies of researchers

in the light of the experiences lived in the managed MOOCs. The analyses will be more focused on changes made from the point of view of design and assessment strategies. The analyses will be conducted always referring to the MOOC's Zone Theory.

Active involvement and process of discovery, exploration, confirmation, and dissemination are the features that have distinguished the work of the trainers involved in these distance learning experiences. We have described how the MOOCs team is composed in §3.2.2. We remember them briefly also here. The MOOCs team is composed by two university professors, a group of experienced secondary school teachers (they were 9 in MOOC Geometria and 20 in MOOC Numeri) and a PhD student (the dissertation writer). All of them are involved in the design, the course delivery and monitoring its evolution in terms of interaction among participants and educational resources made available. In particular, two of the experienced teachers (V. Alberti and S. Labasin) are particularly engaged in the design of the MOOCs, while the others are reviewers engaged in the monitoring activities. Moreover, the experienced teachers also create the activities delivered in the MOOCs, adapted from m@t.abel project and revised by the university professors. In addition, the MOOCs team help MOOC learners to solve technical problems, make tutorials, and recall the tasks to be done week by week with weekly emails.
As observed in the methodology paragraph (§3.3), the trainers' team met regularly, both during the design of the MOOCs and at the end of each activated module, sharing what they had observed during that specific module. In fact, the most significant trainees' interventions or sharing actions were discusses, in order to make a fruitful exchange of ideas on the progress of the course and its becoming.

6.2 Some methodological clarifications

Starting from the methodological choices of the trainers, we propose some reflections about design principles of MOOCs for mathematics teacher education. We focus on the trainers' ZPD (their networks of professional knowledge). As we have seen in §2.5.1, they want to realize an artifact that allows to transpose some meta-didactical praxeologies to a large number of users. Moreover, trainers' practices aim to foster participation and collaboration among trainees and to assess efficiently this kind of engagement. In fact, as we have occasion to notice in the previous Chapters of this dissertation, MOOC Geometria and MOOC Numeri aspire at creating collaborative contexts for teachers' work, where they can learn through sharing their practices and working collaboratively on joint tasks. Such aims are related to the interest in the design and the implementation of teacher professional development programmes to include the role of teachers working and learning in communities (Wenger, 1998; Jaworski & Goodchild, 2006). Therefore, the trainers put in place some multi-techniques (or Mathematics Education Utilization Schemes, Math Edu USs) that are justified by the MOOC-MDT theoretical framework and also taking into account the influence exercised by the research environment's ZFM/ZPA.

The originality of this research resides in those design principles that are relevant and useful to mediate teachers' professional development courses with technology, and in the assessment of the impact of such distance courses on mathematics teachers' engagement. A specific attention is paid on trainers and their role in supporting interactions and learning communities that emerged during the MOOC. Trainers' techniques and their evolution are presented and analysed in order to highlight and discuss their methodological and theoretical justifications. In this way, the reader will have opportunity to benefit from our expertise with online educational environments such as MOOCs.

We point out some essential meta-didactical types of tasks that, according to our experiences, any trainer of a MOOC for mathematics teacher education should address. Precisely, we consider four topics related to the design principles:

- Target;
- Theme;
- Trainers' interaction with trainees;
- Collaboration among trainees.

Moreover, we take into account three topics related to the assessment strategies:

- Test;
- Project Work;
- Peer Review.

For each topic, we describe the related meta-didactical praxeologies, as described in §2.3.1.2. In fact, we identify the related meta-didactical types of tasks, the (multi-)techniques (or Math Edu USs) adopted by trainers to solve such tasks, as well as the related justification (logos). For the logos, we particularly wondered how the chosen techniques were justified and supported by theories in Mathematics Education or more generally in the educational field.
The identification of these meta-didactical praxeologies has been possible by reflecting on the design phases in which the trainers were involved both in the first and the second season of our MOOCs; but also on the massive assessment phases. In particular, we analyzed the evolution of the trainers' meta-didactical praxeologies to design the subsequent season of MOOC Geometria that is exactly the MOOC Numeri. The reasons for this evolution (intended as an improvement of the professional development programme) came from the trainers' self-analysis of the respective experiences but also from some trainees' comments (via questionnaires or posts in communication message boards). In the following section, we focus on these aspects. Moreover, during this analysis we also take into consideration some answers given by trainers to the questionnaires that the dissertation writer administers to them when all monitoring phases (including delivery of participation certificates) have been entered for MOOC Geometria and for MOOC Numeri (see §3.3).

6.3 Design principles in MOOC Geometria and MOOC Numeri

6.3.1 Target

Our MOOCs aim at professional learning and raising awareness of the possibilities for technology use in schools. Given this aim, it is important to identify a hypothetical target trainee: who could be the teachers that can benefit from this educational massive open online course? However, MOOC designers cannot know in advance the teachers who will decide to enrol in the MOOC and they will never meet them in person. For these reasons, as trainers, they are forced to hypothesize a *mean* Zone of Proximal Development (ZPD) of their future trainees. The ZPD (Vygotsky, 1978) concerns an internal level and comes into play when the trainers think about the ideal didactical praxeologies that they want to transpose to trainee teachers who will follow the MOOC. Therefore, the trainers assume a certain level of prior knowledge (ZPD) of the trainees' community (not of the individual teacher since they are forced to consider mean values). They prepare and administer certain activities in order to help the trainees' community to move from the current level (their present didactical praxeologies) to the potential level (the ideal didactical praxeologies).

As we have shown in Chapter 5, the current level of the trainees' community can be recognized in their ZPD. Precisely, it includes the professional development level of the trainee-teachers in terms of: mathematical knowledge; pedagogical content knowledge; skill/experience in working with technology; as well as, beliefs about mathematics, teaching and learning (Goos, 2013, p. 524). Such current level of professional development could evolve thanks to the contents the trainees find in the MOOC. The MOOC contents are carefully designed and implemented by trainers and they are related to specific mathematics topics or important themes of the curricula.

Regarding the topic "Target" (Table 6.1) there has not been any evolution from a MOOC season to another. The target was clearly stated and, since the enrolled participants proved to be in line with trainers' expectations, no changes were needed.

(1)	**Target**
Task	to identify a hypothetical target trainee (lower and higher secondary school teachers)
Technique	to choose activities of a specific school level (according to the target), related to specific mathematics topics
Logos	to hypothesize a mean ZPD of the future trainees
Evolution	None

Table 6.1: The meta-didactical praxeology related to "Target"

Note that, however, in MOOC Numeri primary school teachers were enrolled (it had already happened in MOOC Geometria, but the trainers did not believe they would re-present themselves in MOOC Numeri). This led to considerations in terms of change in the choice of content and also in the design for the next MOOC (Relations and Functions). However, we will not go into such details in this dissertation.

From the questionnaire sent to the MOOC Geometria team (11 trainers interviewed: 2 university professors and 9 experienced teacher-trainers), at the open question "*What was your reaction to the proposal to collaborate in the realization of a MOOC for teacher education?*" all trainers declare to have begun with enthusiasm and curiosity the experience of preparing a course designed for distance education. The 82% of respondents did not expect the massive participation that there has been, but all agree that their expectations about MOOC progress were met.

This question was not proposed in the questionnaire sent to the MOOC Numeri team, because the enthusiasm of the first trainers was the incentive that then involved other experienced trainers in the next design (in fact, the number of experienced teachers involved in the design of MOOC Numeri went from 9 to 20). This is because it was also understood that it needed more collaborative work to be able to better manage such as phenomenon of massive scale.

6.3.2 Theme

Another essential aspect of a MOOC design for mathematics teacher education is the "Theme" (Table 6.2). To this purpose, the trainers face two types of tasks and for each they can adopt different multi-techniques.

The choice of the theme is naturally related to the identified target and to institutional purposes of the professional development programme. Both MOOCs aim to respond to specific teachers' needs identified in the institutional and social contexts, referring to national curriculum for teacher professional development.

Designers have to evaluate essentially two possibilities, according to their long-term educational aim: to keep the same theme and deliver the same content, considered as crucial in the professional development, in every season; or to change the MOOC theme from season to season trying to cover one by one different crucial aspects and educational objectives. Such a decision influences the potential MOOC audience. Indeed, with the former choice, as in the Panero et al. (2017) case, for example, the opportunity of professional development is offered to an increasing group of teachers (including those who have not completed the previous season). With the latter choice, that is the one taken by our MOOC trainers, the same group of teachers can enroll into every season of the MOOC to pursue their professional development. In fact, we saw in Chapter 4 how being familiar with the MOOC environment reduces the cognitive effort that is involved in the phases of self-organization: the mathematical contents of the MOOC change, but its structure remains unchanged in terms of general timing (weekly modules) and tasks to be performed.

Once the theme and its possible evolution from season to season are decided, designers have to consider the time variable. There could be two possible approaches: decide how much time has to be devoted to each module of the MOOC or how much material is possible to read and to work on in a module that has a fixed duration (e.g. one week). The trainers team chose the first approach and, according to the theme, they decided to devote one week or two to the same content or methodology because of its complexity or of the material profusion. In fact, the required techniques are those listed in Table 6.2. It is important to note that designers need to make an average of the estimated learning times of the target (Carroll, 1963). After the first season of the Geometria MOOC, the trainers' team decided to reduce the quantity of the provided material and to pay a greater attention to differencing the material for different school levels.

(2)	**Theme**
Task 1	to identify the main theme to address in the MOOC
Techniques	to focus every season on a different core part of the curriculum (Geometry, Number) and to choose activities around specific topics according to the theme
Logos	to innovate methodology and strategies of teaching mathematics as highlighted in the *Piano nazionale per la formazione docenti* and the Italian curriculum (*Indicazioni nazionali*[59]).
Evolution	The first season was devoted to Geometry, while the second one to Numbers. Once a topic is covered, the professional development programme moves on to another one, with the

59 Link to the Italian curriculum: http://www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/_decreto_indicazioni_nazionali.pdf

	long-term aim of deepening the professional development of the same group of teachers. 50% of enrolled in the second season came from the previous one.
	Time
Task 2	to decide how much time is devoted to each module of the MOOC
Techniques	to estimate the time necessary to acquire the treated topic, taking into account an estimated engagement of 4h per week: - if necessary, to divide theoretical and practical parts; - if the material is too dense, to devote two weeks to the same topic;
Logos	average of estimated learning times of the target
Evolution	to reduce the quantity of the material provided; greater attention to differentiating the material for different school levels.

Table 6.2: The meta-didactical praxeology related to "Theme"

In the questionnaire sent to the MOOC Numeri team (20 trainers interviewed, all of them experienced teacher-trainers), we asked: "*How do you judge the way in which your resources have been made available to trainees (choosing a video, choosing a text file, choosing an audio file, ...), in the light of what has been the educational experience that your materials have allowed? Motivate your answer?*". All 20 were in agreement that the methods selected for sharing the resources and make them available to the trainees were optimal and effective. Let us read some comments:

> "The resources were generally much appreciated, the use of sway, audio recordings and the insertion of sample files seems to me very interested in the students."
>
> "I consider them good. I think that a seed has been left, it is then up to the teacher to decide to cultivate it or not".
>
> "This year we have taken better care of the presentation of the contents and the accompaniment in the use of the resources provided and this has been positive because a higher percentage has completed the training experience. [...]".

From this last comment, we can see how the experience of the previous MOOC was put to good use. It translates into greater awareness of both the organization of materials and the digitization of content.

6.3.3 Trainers' interaction with trainees

To make the online interaction with the trainees possible, the trainers are called to put into action some multi-techniques (Table 6.3). The first kind of interaction in which the trainees are involved within MOOC is the reading of available materials and didactical resources. Digital resources replace the voice and explanations of the trainers that are usually done in face-to-face courses: so the trainees interact with videos, images, interactive texts. In this way trainers are able to communicate their training intentions at distance, share research results, methodologies and teaching strategies that can be used in class with students.

The activities have been transposed into a digital format according to the E-tivity framework (Salmon, 2013). The E-tivities are designed before opening the MOOC to participants. They support learners in achieving the learning outcomes: in fact, they promote a learner-centred task and problem-based approach to online learning.
Based on the 7Cs of learning design (Conole, 2014), and in particular "Capture[60]" and "Communication[61]", the trainers created institutional mail addresses to send e-mails periodically in order to have moments of direct contact with the group and/or with the individual trainees. Precisely, weekly mails were sent to all members to remember the content and required activities, and private emails or specific forums were set up for technical issues. In addition, the trainers preferred to alternate the platform control moments, managed by groups of teacher-designers per module (as specified in §3.3), with synchronous contact moments through webinars (as specified in Chapter 4). Remember that webinars are online meetings in which an expert (seen through a camera) shares with the trainees (who can only interact via chat) some issues about the research in mathematics education and focuses on some questions that could be raised during the previous weeks in the MOOC.
As specified in the section dedicated to the methodology (§3.3), questionnaires were administered for a feedback on the degree of appreciation of the educational offer. The Italian team has administered three questionnaires (at the beginning, at halfway of the course, at the end). From the feedback they received, the trainers understood how to better refine some questions to get clearer information.

(3)	**Trainers' interaction with trainees**
Task	to make the interaction WITH the trainees possible
Techniques	- to transpose in a digital format materials and didactical resources for teacher education - to create institutional e-mail addresses for sending periodic e-mails (e.g. weekly e-mail, private e-mail for technical problems) - to open forums for technical and didactical issues - to organize webinars for creating occasions of synchronous contact - to prepare and administrate questionnaires
Logos	E-tivity framework for digital transposition; "Capture" and "Communicate" from the 7Cs
Evolution	some questions in the questionnaires have been changed

Table 6.3: The meta-didactical praxeology related to "Trainers' interaction with trainees"

In the questionnaire sent to the MOOC Geometria team, three of the eleven respondent were speakers of webinars, unique moments of synchronous comparison between trainers and trainees. Two out of three defined this experience "exciting and educational", even if different from a seminar in the presence. Here is a significant comment of one of them:

60 in terms of capturing resources to be used: What resources are being used and what other resources need to be developed? (Conole, 2014, pp. 1, 3)
61 mechanisms to foster communication: How are the learners interacting with each other and their tutors? (Conole, 2014, pp. 3-4)

> "[…] you speak to an audience from which you may have only written responses via chat. These make the interaction different from that of a seminar in the presence: all embodied aspects are missing and this makes the answers more impersonal [...]".

6.3.4 Collaboration among trainees

Fostering collaboration among trainees (Table 6.4) is not a peculiarity of all MOOCs, but it is a fundamental aspect of our MOOCs, distinguishing them from other kinds of online courses where the trainee alone has to watch videos and accomplish activities. In §1.1, we have specified how in the literature it is not significant to distinguish between cMOOCs and xMOOCs. As we conceive our MOOCs as authentic collaborative experiences, our MOOCs have features that would classify them more as cMOOCs than xMOOCs.

However, collaboration cannot be considered as spontaneous way of working, especially within such remote contexts. Designers have to make it possible through specific multi-techniques. The trainers praxeologies related to this task constitute some effective examples of how to solve it.

The trainers' teams ground their choices on "Collaborate" in the sense of Conole (2014). They used forums provided by DI.FI.MA. platform, where the courses were delivered, and decided to add some collaborative tools such as Padlet and Tricider from the outside. The trainers felt the need to augment the "official" platform with additional tools to properly foster collaboration. This fact is relevant for us and can be interpreted as the current lack of remote platforms for online courses, which can fully support collaboration among participants.

In the forums, the trainers adopted a technique to initiate discussions with a prompting question in order to accompany trainees in reading the materials and identifying their focus. We can identify the influence of a technique used for interacting with the trainees, that is how and how much to intervene in the trainees' work. The trainers' team is focused on global collaboration, fostering it within the entire community of the MOOC and aiming at the creation of a global community of practice made only by trainees. In other MOOCs for mathematics teacher education, such as Panero et al. (2017), the trainers can focused on local collaboration, fostering it within small groups of the MOOC community and aiming at the creation of small local communities of practice around a common project, where the trainer intervenes and acts as a tutor before and as an evaluator in the end (Panero et al. 2017).

(4)	**Collaboration among trainees**
Task	to make the interaction AMONG trainees possible
Techniques	- to provide a suitable space for making the remote communication possible (communication message boards such as forum, padlet, tricider) - to initiate discussions on forums with a prompting question (in order to accompany trainees in reading the materials and identifying their focus) - to reduce trainers' interventions, monitoring behind the scene
Logos	"Collaborate" in the 7Cs; to foster the birth of a community of practice
Evolution	to provide more tutorials to allow trainees to move autonomously in the collaborative space and to use collaborative tools as efficiently as possible.

Table 6.4: The meta-didactical praxeology related to "Collaboration among trainees"

The analysis of the comments on communication boards was a task done by 8 of the 11 respondents of the trainers of MOOC Geometria. For the 75% of them to carry out this task was not difficult, although for all it has been onerous in terms of time and classification of interventions. They declare that to efficiently carry out this work it is necessary to know the materials inside the MOOC (the created MOOC's ZFM or the MOOC-artifact) and "*a constant presence in the platform not to lose the train of thought*" (the MOOC's ecosystem originated by the trainees or the MOOC's ZFM/ZPA). Again, this information was shared with the new members of the MOOC Numeri team. For this reason the same question was not repeated in the questionnaire for the trainers of the MOOC Numeri.

As we have seen in Chapter 4, the trainees did not use the communication message boards exactly in the way the trainers were expecting. For example, the Tricider had the goal of triggering simple threads, most of which should confine to the approval or not of mathematics teaching ideas, by voting through "like". Instead, the participants used it more for collecting ideas and comparing their didactical experiences – as a forum – rather than for the expected use. The trainees realized a *catachresis* (Verillon & Rabardel, 1995): an artifact is used to do something it was not conceived for. The trainees were introduced to a new tool for them. The trainers acquired awareness about the necessity to be clear in writing the tasks, in exemplifying the use of the tools and in providing tutorials on their affordances. In any case, all the trainers who have monitored the communications boards – both in MOOC Geometria and in MOOC Numeri – declare that this task has been enriching:

> "Exchanges with [trainees] has enriched me […]: coming from different experiences and schools each of them give his special contribution, from each of them you can learn something".

6.4 Assessment strategies in MOOC Geometria and MOOC Numeri

6.4.1 Test

The massiveness makes it very difficult to personally follow every participant. In Table 6.3 we stated that the trainers were always vigilant, with private emails, through more trainers per module to follow the development of the work from within. But, how one might get immediate understanding of the progress that each trainee does? In MOOC Geometria, the trainers had introduced weekly test to understand the trainees' appropriation of the video content and module activities. The tests consisted in MCQs (Table 6.5) allowing up to 3 attempts: in fact, we gave feedback about the given answer (Velan et al., 2008). The trainees could go through reviewing the resources and trying to find the correct answer. Correct answers indicated that the resources had been explored in depth and not superficially. Additionally, granting multiple attempts was a guarantee of success for trainees.
In the Geometria MOOC, the trainees did not share the same opinion about the tests: they saw the test as an overload of work besides the commitment already required by the MOOC on a weekly basis. For example, we can read a comment left by a trainee on the forum dedicated to didactical issue:

> "[… There is] a question I would like to share: the validity and usefulness of the test. I have assumed that if a teacher enrols herself on a platform like DI.FI.MA., she attends professional development courses because she is motivated to grow professionally and because she is

> willing to get involved on multiple levels by implementing sharing skills as collaboration, comparison, learning by doing. She does this by trying to cut off time, among the thousands of daily and work-related commitments. She does it with pleasure and passion. Is it necessary a test at the end of the module? "Perhaps yes". It is the answer I have given thinking about the research/action activity that is carried out at higher levels, even if after graduating and having supported more competitions I do not think it is so 'rewarding' to respond to a cross-test [...] Mine does not want to be a provocative question but simply a free thought to think about. Thank you".

As a result of little positive feedback from trainees like this one, the tests were present in the first edition (MOOC Geometria), but removed in the second ones (MOOC Numeri).

A technique that, instead, remained unchanged is the end module badge. It was obtained if the trainee self-declared to have seen some specific resources, if she wrote on the communication message boards when required and if she uploaded specific materials when asked. Once all the module requests were accomplished (test included), the platform released the badge (see Figure 3.2 in Chapter 3). In this way, it was quite easy for the trainers monitoring the progress of the trainees, knowing the amount of badges they had collected.

(5)	**Test**
Task	to assess the degree of participation of the trainees weekly
Techniques	- Multiple Choice Test with up to 3 attempts related to the video content and module activities - release of the badge (the test was a necessary and sufficient condition for its release)
Logos	Choosing MCT because MOOCs are massive
Evolution	Test was present in the first season, but removed from the second one

Table 6.5: The meta-didactical praxeology related to "Test"

6.4.2 Project Work

The trainers chose a project-based methodology (Bender, 2012) to assess the trainees' engagement, that turned out to be efficient (Table 6.6).
The project consisted in designing a classroom activity (as observed in §3.2): by describing and analysing a priori its potential for the learning of mathematics, trainees had to demonstrate acquired teaching competencies and expertise. In fact, the project-based methodology has been chosen to give trainees the opportunity to get involved in the MOOC activities in terms of methodology, creativity, and with the aim of sharing and discussing them in the community.
The trainees had to produce an individual Project Work (PW). They were free to choose the theme of their project, in line with the theme of the MOOC: so, geometry for MOOC Geometria; arithmetic or algebra for MOOC Numeri. The trainers gave a lot of freedom to trainees: trainers did not want neither to influence them nor to restrain their creativity. Trainees had to use a web-based tool, the Learning Designer (hereafter LD) designed by Laurillard (2016). LD – as seen in §3.2 and Chapter 4 – is a software that guides and encourages planning of a lesson: it is characterized by a standard format that allows to integrate technologies, to have an overview of the teaching/learning dynamics centred on the

student and to share online what the teacher has produced. Trainers' techniques include the creation of videos and pdf tutorials in order to familiarize the trainees with LD. In the Geometria MOOC, these tutorials were available two weeks before the opening of the last module.
Furthermore, deadlines for accomplishing the PW were announced as sharp because trainers wanted to allow everyone to do a Peer Review (see Table 6.7). However, some trainees expressed the need to have more time to accomplish their PW. In the following some their comments left in the forum for technical issue:

> "Pity that the delivery was in this very hot period: I did not have time to produce something new and I arranged something already experienced .... arranged, in fact, not even reviewed in light of the interesting ideas offered in the course. Sigh".

> "The time to prepare this activity [PW] is short. I hope to succeed as soon as possible".

Thus, in both subsequent seasons of the MOOCs the deadline was extended by 2 weeks.

(6)	**Project work**
Task 1	to assess the competences acquired through the MOOC
Techniques	- trainees are asked to carry out an individual project - recommendation to use the LD software - trainees can choose the content to address in their project according to the theme of the MOOC
Logos	project-based learning
	Time
Task 2	to decide how much time is devoted to the individual project work
Techniques	to estimate the time necessary to carry out the individual project (one week); to give instructions/tutorials about LD starting from the week before
Logos	the time for appropriation of an artefact as LD
Evolution	the deadline to carry out the project work was extended by 2 weeks

Table 6.6: The meta-didactical praxeology related to "Project work"

6.4.3 Peer Review

To stimulate collaboration among trainees and to foster formative assessment among peers (Black & Wiliam, 2009), the trainers team proposed a Peer Review (PR) activity (Table 6.7). As for the PW, for the PR the trainers have to face two tasks and for each they can adopt different multi-techniques.
It was a 1 by 1 peer review: each trainee had to review a colleague's PW from an educational point of view, without any marking intention. The teachers were divided, thanks to an excel table, taking into account their school level. In the excel table each trainee found the PWs title and links to LD to facilitate spotting of the PW to review. The instructions for the PR were given in a more specific way compared with the PW. In the week dedicated to the PR, a revision grid containing the review criteria was given: attention to the main aspects of each educational intervention and to a conscious use of digital software. The grid provides 5 categories: Connections to the real world; Creativity; Collaboration; Use of technology;

General considerations. For each of these categories, some features are indicated. They are to be evaluated by using a scale from 1 (= little present aspect) to 5 (= highly present aspect). The final request was to leave a comment highlighting the strengths of the project, the parts that could be improved and possible reviewer's curiosities. The Italian team gave one week to accomplish this task, considering this as a suitable time for internalizing (Arzarello et al., 2014, p. 9-10) the criteria of assessment.
Also in this case, in the forum dedicated to technical problems, some trainees expressed the need of having more time available for accomplishing their PR and also to receive in advance the criteria to better accomplish the design task of the PW. We can read the following comments for example:

> "I kindly ask the tutors [...] can you please help us? the situation [doing PR ...] takes much longer than expected"

In the subsequent season of the MOOC, the deadline for accomplishing the PR was extended from one to two weeks. Moreover, the revision grid was given at the beginning of the two weeks of PW (i.e., two weeks before the start of the revision process). In addition, the project to be reviewed was assigned by the trainers to each trainee taking into account the school level. This choice was done because in the previous season more than one trainee selected the same PW and some PW remained without a reviewer. In both seasons the PRs were delivered on the platform and made available to each trainee.

(7)	**Peer Review**
Task 1	to review the PW
Techniques	- trainees are asked to do a peer review (1-1) of a project they choose at the same school level - an excel table is provided to organize the finalized PWs (with links to LD) to facilitate the choice of the potential reviewers - revision grid - PRs (sent as a task on moodle) shared with all the participants on the platform
Logos	stimulate collaboration, peer assessment (formative assessment), deal with the massive nature of MOOC
	Time
Task 2	to decide how much time is devoted to the peer review
Techniques	- to estimate the time necessary for reviewing one colleague's project (one week) - provide the revision grid, in the week of the PR (last module)
Logos	time for internalising the criteria of assessment (MDT)
Evolution	the deadline to accomplish the PR was extended and the revision grid was given at the beginning of the two weeks of PW. The project to be reviewed was assigned by the trainers to each trainee taking into account the school level.

Table 6.7: The meta-didactical praxeology related to "Peer Review"

6.5 Similarities and differences in the two online educational experiences from the trainers' point of view

In the questionnaire administered to the trainers of MOOC Numeri, we asked the following open question: "*If you have also been involved in the MOOC Geometry, what do you think there has been SIMILY and/or DIFFERENT in MOOC numbers? Have you had some denials or confirmations?*". As it is easy to guess, this question received 11 answers, as many as the trainers involved in the MOOC Geometria. They all agree that MOOC Numeri has been organized more consciously and therefore the result has been better than MOOC Geometria. Here some of their comments:

> "I perceived a greater 'organization' in the MOOC Numeri, probably due to the fact that it was no longer the first experience"

> "[…] the MOOC Numeri was a confirmation of the appreciation of the works proposed in the last MOOC edition and of the fact that online training is an excellent methodology for the dissemination of skills/knowledge"

> "definitely confirmed in the fact that the experience gained in the first MOOC could allow to improve, prevent more easily, drive, detect/improve network behavior, good practices and inspire innovation"

From this last comment, in particular, an awareness of own role as a trainer emerges: it increases own ability to focus on the needs of the trainees in the platform, but also on the training needs by providing innovative ideas.

In both the questionnaires addressed to the trainers (both from MOOC Geometria and from MOOC Numeri), the last open question asked: "*What did this experience leave you with?*".
The answers are all positive comments. The experience of design and monitoring is perceived by the trainers as an exciting opportunity and an occasion for professional enrichment. Let us see how someone expressed herself. From MOOC Geometria we can read:

> "I learned new methodologies, techniques, practices, I reflected ... in short, I feel enriched and I feel I have contributed to enrich other people"

While from MOOC Numeri we can read:

> "It left me very enthusiastic to see many teachers work productively; a definitely positive experience that I would do again"

> "Interest, emotion, effort, satisfaction […]"

Trainers do not deny that clearly following a massive open online course requires effort, but it is a fulfilling experience.
In particular, from the answers to this last question of both questionnaires, it emerged that in MOOC Geometry 4 out of 11 trainers speak of having benefited thanks to this experience of professional development; while in MOOC Numeri this is explicitly said by 6 trainers out of 20. We can read just some comments. From MOOC Geometria:

"I had the opportunity to create my first training course organizing content and using technologies for its transmission [...] it was not usual for me until then. These skills have moved naturally in my teaching practice as well as in the training"

"Reasoning on the teaching proposals to be presented in the MOOC also urged me to deepen my reflection on the same objectives that the MOOC aimed to: methodological choices, conceptual nodes, use of technology. The work in the MOOC therefore also helped me to improve my teaching practices and to broaden my knowledge and experience."

6.6 Remarks and discussion

The trainers' meta-didactical praxeologies that we have examined constitute the network of the MOOC-artifact.
It is apparent that the double learning process that brings the MOOC-ecosystem to life is experienced by the trainees who live the MOOC in the first person, not by the trainers who are the designers of it. However, the trainers, who observe the phenomenon from the external, visually notice how the transition from MOOC-artifact to MOOC-ecosystem/instrument takes place. In other words, they realize how the MOOC's ZFM they designed, from the beginning inert space gradually becomes dynamic. The ZFMi follow each other from week to week and the actions and interactions of the trainees make way for the MOOC's ZPA, generating the MOOC's ZFM/ZPA complex. In seeing this, or in completing the monitoring phases, trainers have the opportunity to consolidate or modify the methodological choices that led to the creation of the MOOC-artifact, depending on the experience that the trainees are experimenting.

We note how in identifying the target of teachers to whom the MOOC should be addressed (Table 6.1), as well as in the choice of the theme to be treated (Table 6.2), the trainers are conditioned by ***research environment's ZFM*** (Table 6.8). They have a certain *perception of the teachers* that they image will follow the MOOC, making some hypothesis on their mean ZPD. Identifying the target is then closely connected not so much to the choice of the theme itself, but to how to decline this theme taking into consideration the scholastic levels to which the training is addressed. Specially, the *Italian curriculum and assessment requirements* are points that can not be ignored. Furthermore, the choice of the theme is closely linked to the task of time, to which it corresponds, in the research environment' ZFM, the consideration of the *organizational structures and cultures*. Another aspect that has not been overlooked by designers has been to compare with other researchers involved in the design and delivering of MOOCs for mathematics teacher education. *Accessing to resources* such as literature, proceedings of conference, but also – as declined in ***research environment's ZPA*** – *participating in national/international conference* and entertaining an *informal interaction with research colleagues*, the trainers were able to confirm the methodological choices made, such as for example to decide to change the theme from time to time in each MOOC season, or decide how much time is devoted to each module of the MOOC (Table 2).
About the trainers' interactions with/among trainees (Table 6.3, 6.4) is evident the conditioning that the ***research environment's ZPA*** have exerted in terms of *involvement in actions to foster teacher professional development*. The trainers have done the methodological choice to limit their interventions as much as possible in the communication message boards;

however, they were vigilant behind the scenes and also organized some webinars to have occasion of synchronous contact with trainees.
For the strategies of assessment, the dynamic that characterized the ***MOOC's ZFM/ZPA*** in its evolution, has influenced the choice made by the trainers. Think for example to the negative judgment that the trainees have done about the test (Table 6.5); or even their request to receive more time to accomplish the final task (Table 6.6); or even more the decision to anticipate the consultation of the revision grid and the assignment of the PW made directly by the trainers to avoid that some PW could not be chosen (Table 6.7). Especially for this latter point and the will to remove the test, a not indifferent weight was exerted by the *perception of trainees* from the trainers' point of view, in line with the ***research environment's ZFM*** (Table 6.8).

Taranto's re-elaboration/interpretation	
Valsiner/Goos's zones	**Research environment's ZFM/ZPA**
ZFM: Zone of Free Movement (structures trainers' access to different areas of the environment (University, Department of Mathematics, national/international conferences, …), availability of different objects within an accessible area, ways the trainers is permitted or enabled to act with accessible objects in accessible areas)	• Perceptions of teachers • Access to resources (literature, proceedings of conferences, teaching experiment, data obtained via questionnaires or interview, …) • Technical support (print centre, classroom booking, …) • Italian curriculum and assessment requirements • Organisational structures and cultures
ZPA: Zone of Promoted Action (people, objects, or areas in the environment in respect of which the trainers' actions are promoted)	• Participation in national/international conferences • Involvement in actions to foster teachers professional development (face-to-face meetings, MOOCs, …) • Professional development • Informal interaction with their peer and researching colleagues

Table 6.8: Research environment's ZFM/ZPA (trainers only)

Therefore, during this chapter, we have analysed the trainers' praxeologies that can be considered as meta-didactical in the sense that they deal with a discourse about didactical issues. We chose the meta-didactical praxeologies by selecting the tasks that are essential for the design of a MOOC. These tasks concern both the design principles and the assessment strategies. Through the analysis of the praxeologies associated to these tasks, we have tried to catch several essential topics regarding MOOCs. Firstly, the relationship between design principles and professional development that can be grasped through the audience of each of the MOOC (Table 6.1), but also the theme itself which is essential from an institutional point of view (Table 6.2); moreover, the delicate question of the relationships between trainees and trainers (Table 6.3 and 6.4). Secondly, we have observed that looking carefully at the assessment strategies included in the MOOC design (Table 6.5, 6.6, 6.7) gives important clues to assess the mathematics teachers' engagement.
The methodological choice of the project-based assessment has proved to be effective. The model itself of MOOC does not allow researchers to observe directly the effect of the training courses proposed by the MOOCs and to gather feedback from observations in classes. For this reason, the trainers considered the PW as a suitable way to assess the competencies acquired

by the trainees. The PW was individual and a PR was proposed to evaluate the work. The evidence is that the connection between trainees does not go without saying and that the role of tutors as well as their scope of activities must be included in the design principles of MOOCs. Moreover, time to devote to these tasks is an important issue to consider and it was increased from a season to another.

We recall that the research question we want to address in this chapter is:
Does the MOOC's ZFM/ZPA trigger and support an expansion of the network of professional knowledge of the trainers relatively to design principles and strategies of trainees' assessment that the trainers have put in place?

The praxeological analysis of the selected types of tasks seems to give significant answers to it. In particular, the observation of the MOOC's ZFM/ZPA by the trainers actually triggers and supports an expansion of their network of professional knowledge relatively to design principles and strategies of assessment. More precisely, in the Table where in the voice "Evolution" there is written *none* (Table 6.1 – "Target"), the network of professional knowledge has not undergone an expansion. Rather, the connection between the pre-existing nodes has somehow been strengthened (Figure 6.1). The trainers have gained a greater awareness of their role and of how they can better come up against the needs of the target audience of the training offer.

Figure 6.1: The meta-didactical praxeology related to "Target" is reinforced

In the other Tables (from Table 6.2 to Table 6.7) an evolution has been shown, or in other words an expansion of the network of professional knowledge. For example, the "*PW*" node (Table 6.6 – PW) was previously linked to the "*1 week*" node, understood as sufficient time for accomplish this task. We consider now the moment that corresponds to the understanding that more time has to be given for deliveries. It is as if the "*PW*" node unties itself from the described connection and attaches itself to a new "*2 weeks*" node, which is the new time considered optimal, based on the observations of the MOOC's ZFM/ZPA and based on the feedback received from the trainees (Figure 6.2). With the addition of a new node, one can, therefore, speak of network expansion.

Figure 6.2: The meta-didactical praxeology related to “Project work” is evolved

Note that the “1 week” node is not deleted from the trainers’ network of knowledge. It remains, but with the awareness that it is not the efficient strategy, rather the one to be avoided. The new node, “2 weeks” is taken into consideration, hoping that this new methodological choice will actually prove better than the previous one.

These described evolution is clearly an evolution of the ZPD that is moved from the current level, with which the trainers have started the MOOC adventure, to the actual level the one reached after the told experiences of MOOC Geometria and MOOC Numeri.

The MOOC’s ZFM/ZPA, therefore, triggers and supports an expansion of their network of professional knowledge relatively to design principles and strategies of assessment, but it also does something more. From the last question in both questionnaires administered to the trainers of MOOC Geometria and MOOC Numeri, emerges that thanks to the involvement in the design and monitoring of MOOCs for mathematics teacher education, the trainers can benefit of *professional development*. We have not studied this aspect in detail so far. We have put more emphasis on the evolution of the trainers’ ZPD in terms of the evolution of design to accomplish the task to transpose the ideal didactical praxeologies and the evolution of the assessment strategies to understand the impact of such distance courses on mathematics teachers’ engagement. The evolution of the ZPD in terms of professional development of the mathematics teacher educators is certainly an aspect that deserves to be investigated further.

A last consideration. A main result is that collaboration cannot be considered as a spontaneous way of working, especially within such remote contexts. Designers have to make it possible through specific techniques. The showed trainers’ praxeologies related to this task constitute some effective examples of how to solve it. In particular, in the way the trainers team used forums provided by DI.FI.MA. platform, but also some collaborative tools such as Padlet and Tricider from the outside. Our analysis shows that a real involvement of trainees in collaborative work needs to be triggered and supported by suitable tools added to the platform. The availability in the platform of tools consonant with the social networks used in everyday life increases the triggering of what Manlove et al. (2006) call co-regulated learning, in the sense that the trainees themselves regulate their tasks and collaboration. Our analysis leaves open the question of which devices are the best for improving active collaboration among the trainees: possibly further research and concrete experimentations will be able to give a more definitive contribution to this crucial issue. What is interesting here is that our analysis centred on collaboration processes through the adaptation of the meta-didactical lens has made possible to grasp this important problem in a clear way. This suggests that the way of research we have undertaken is promising and fruitful for further results along this stream.

Chapter 7 Discussion and Conclusion

The present study sought to capture and explain the complexity and dynamic nature of a MOOC for in-service secondary school mathematics teacher education and the influence that an online course like that has both on mathematics teachers in education within it and on mathematics teacher educators that are involved in its design, delivery, and monitoring.

The study was conducted through a theoretical phase (presented in Chapter 2) and an empirical one (presented in Chapter 4, Chapter 5, and Chapter 6) in order to address three research questions. We will begin with considerations on the MOOC's Zone Theory theoretical framework, underlining how it has embodied the properties of operability, coherence and productivity that were foreseen by networking and hybridization strategies.
Drawing on the findings of both phases, the theoretical and the empirical ones, responses are given to each of the three research questions in turn in the second, third and fourth section of this chapter. Followed by examination of the limitation of the study, the chapter concludes with suggestions for further research.

7.1 Consideration on the MOOC's Zone Theory

The MOOC's Zone Theory framework has been outlined through a delicate and painstaking work of hybridization and networking between different educational theories, some of which specifically of mathematics education.

The theoretical frameworks/lenses to which we refer are:

1. Meta-Didactical Transposition or MDT (Arzarello et al., 2014);
2. Instrumental Approach (Verillon & Rabardel, 1995);
3. Connectivism (Siemens, 2005);
4. The Zone Theory of Valsiner (Valsiner, 1997) that was adapted by Goos (Goos, 2005).

We propose again the Figure 7.1 that schematizes the strategies for connecting theories that were put in place. Remember that two **hybridization** occurs: first between MDT and Instrumental Approach, after between these and Connectivism giving rise to the so-called MOOC-MDT.
Later, a **networking** regards the MOOC-MDT with the Zone Theory, creating a new theoretical framework that we will call the MOOC's Zone Theory.

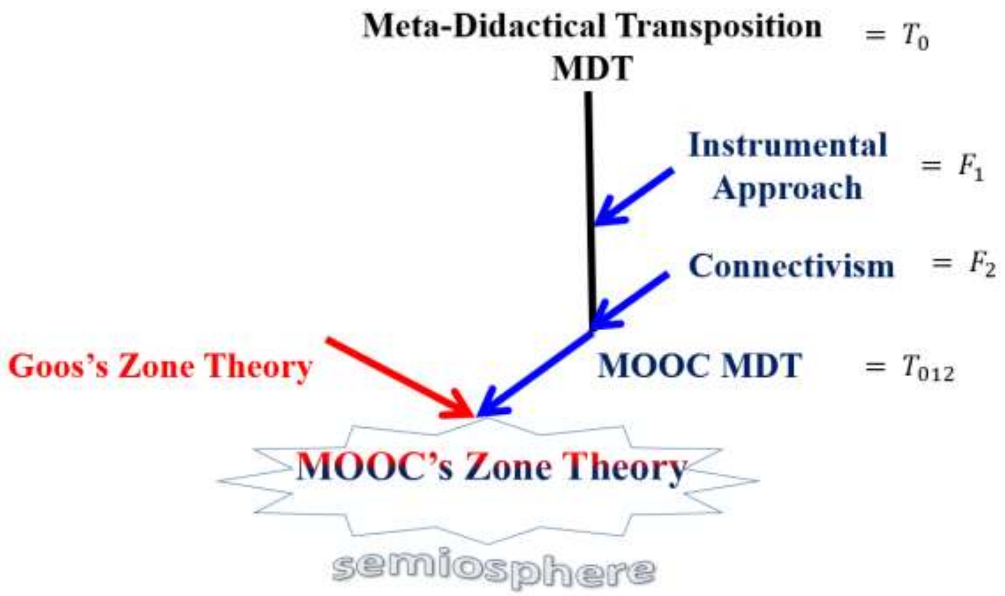


Figure 7.1: The hybridization and networking strategies that originate MOOC's Zone Theory

The MOOC's Zone Theory has been shown to reflect the three characteristics that networking and hybridization bring with them, namely operability, coherence and productivity.
The productivity is typical of hybridization and therefore we must refer to the MOOC-MDT. This obtained framework is certainly productive. In fact, taking up the definition of Arzarello (2016; 2017), the fragments of the Instrumental Approach and of the Connectivism, implanted in the MDT, are not only coherent and guaranteed concrete operability, but also allow further insight and breakthroughs to the MOOCs dynamics that the MDT alone was not able to focus properly. In the exposition of the theoretical framework, we have underlined more times the limits of MDT and how the hybridization with the two fragments exceeded these limits.

The "weaknesses" of the MDT in relation to the MOOC are:

- the different institutional weight perceived by trainees in the MOOC environment than in a face-to-face course;
- the presence of multi-techniques vs techniques;
- the missing of a shared praxeology;
- the difficult to consider the double dialectic;

In particular, remember that MDT, as it is, fails to grasp the dynamism and complexity that are inherent in the interactions implemented by the MOOC's participants (trainers and trainees) and that influence their possible evolutions.

MOOC-MDT, namely the Meta-Didactical Transposition revised for analysing a MOOC environment thanks to the hybridization process, is obtained one time considering a fragment, the *instrumental genesis*, from the Instrumental Approach and a second time considering a fragment, the *network of knowledge*, from the Connectivism.
The fragment of *network of knowledge* is implanted in the Instrumental Approach, which is so adapted to MOOC's own dynamics, where the community of participants becomes subject and object of a new, more complex kind of instrumental genesis: the ***double learning process***. In fact, it maintains the structure of the instrumental genesis, with directions from the subject to the object and vice versa, but it is also enriched with the Connectivism standpoint.

The MOOC-MDT framework (Figure 7.2) facilitates the study of the specific dynamics of the interactions among trainees and between trainees and trainers, which occur online in virtual environments. Moreover, it allows perceiving possible evolution in the praxeologies of the community of trainers and trainees.

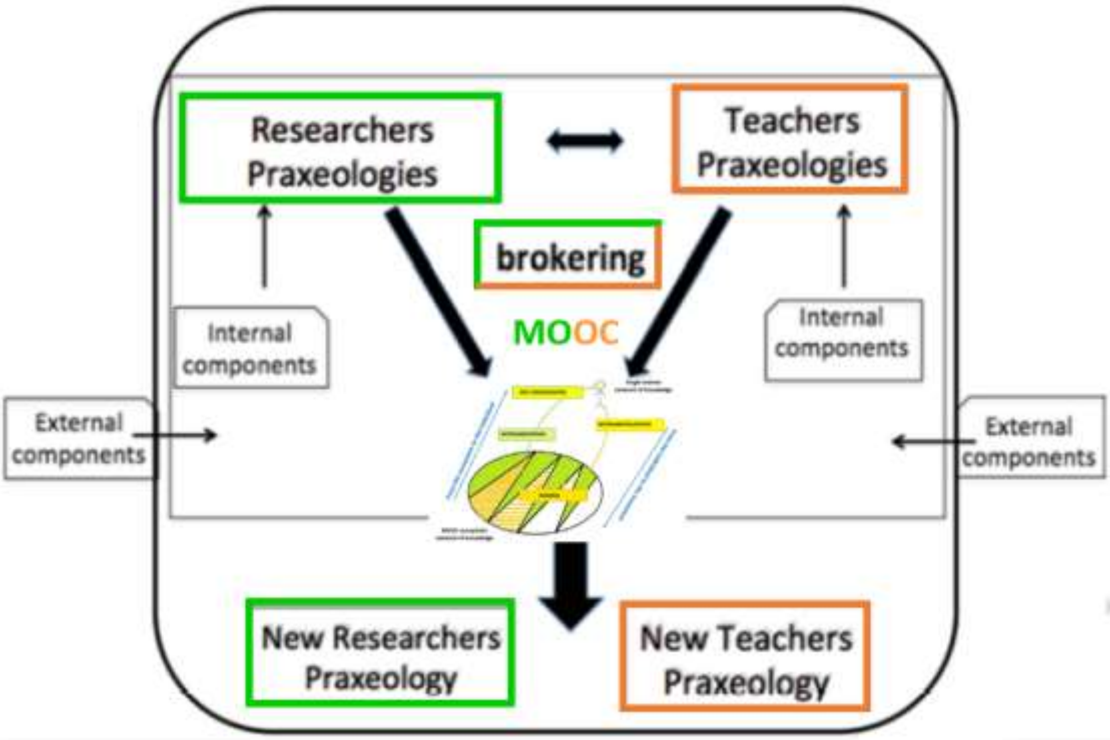


Figure 7.2: The MOOC-MDT theoretical framework

However, if one is also interested in analysing in depth the possible evolutions of trainers and trainees' praxeologies, considering the influence exercised by the own personal culture and knowledge; and by the interactions that are established also in the daily context in which the individual are immersed and that influence their knowledge as well as the interaction within the MOOC, a networking between MOOC-MDT and the Zone Theory revisited is a winning choice.

MOOC's Zone Theory (Figure 7.3), the framework resulting from the networking between the Zone Theory revisited to adapt it to the MOOC environment and the MOOC-MDT, is operative and coherent. Between MOOC's ZFM/ZPA and MOOC-MDT, a networking was carried out through coordination and combining strategies. In fact, it is really referring to the construct of MOOC's ZFM/ZPA that we can explain the phenomenological aspect of the transition from MOOC-artifact to MOOC-ecosystem/instrument. So, we use these strategies "for making sense of an empirical phenomenon" (Bikner-Ahsbahs & Prediger, 2014, p.120). Evidence of this emerged in Chapter 4, in the analysis of interactions among MOOC participants.

The strategy of integrating locally emerges in particular when "a concept at the border of two theories is worked out and integrated into both theoretical approaches" (Bikner-Ahsbahs & Prediger, 2014, p.120). Considering the concepts of praxeologies and ZPD, the concept at the border of the two lenses is the professional development. It is worked out and integrated into both the theoretical approaches, the Zone Theory and the MOOC-MDT. In fact, on the one hand, professional development means the evolution of one's own praxeologies, but on the other hand, also moving from the current level of the ZPD to the potential level. These are two sides of the same coin. Evidence of this emerged in Chapter 5, in the analysis of the professional development relative to the trainees who attended and completed education in the MOOC.

Returning to the definitions of Radford (2008, pp. 321-322), a guarantee of operability and coherence arises from the fact that: first of all, the research questions have been formulated within this theoretical framework. In particular, the methodology chosen to investigate the answers to these questions proved to be able to produce and deal with the data. The rhetoric of the argumentation of the methodology was consistent with, and rests on, the chosen principles.

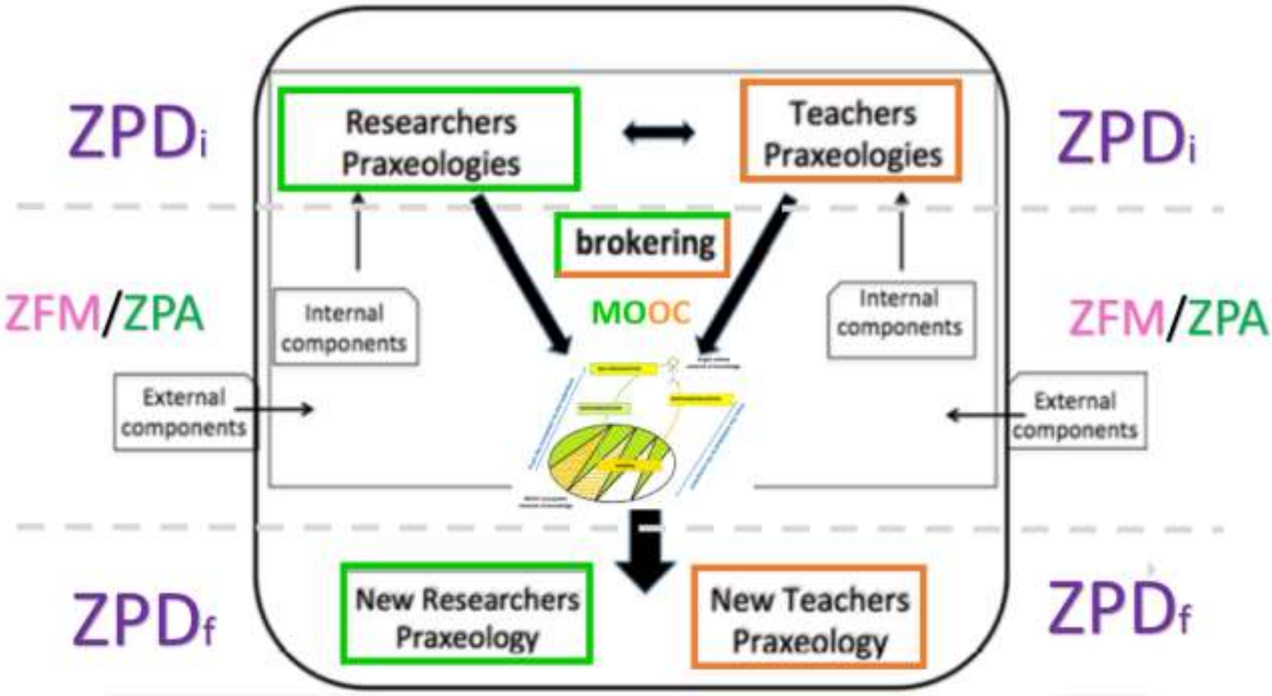


Figure 7.3: The MOOC's Zone Theory

MOOC's Zone Theory offers a useful framework for research that aims at understanding the complexities of trainers/trainees' learning trajectories in a MOOC. With learning trajectories, we mean how these protagonists interact online, both with the platform and with each other. So in a connective sense, as they learn. In particular, if and how these interactions change their knowledge, beliefs, practices. In other words, what the consequences of this participation in the MOOC are.

In particular, the notion of ***productive tensions*** (Goos 2013) is crucial for grasping perception of teachers' changes from a zone theory perspective. It makes possible to have observables in the analyses on the trainees' behaviours. So, if we take up the metaphor of the river angler who goes fishing in the sea (§2.3.1.9), the device he has to invent in order to make a rich fishing definitely the MOOC's Zone Theory and the baits that the angler can use are the double learning process and the productive tension.

The findings of empirical phase of the present study suggest that the MOOC's Zone Theory framework can provide useful insight into how to capture and explain the complexity and dynamic nature of a MOOC for mathematics teacher education and the influence that it has on trainees and trainers. Therefore, satisfactory answers, that lie on several methods (statistical methods, interviews, discourse analysis), to the research questions are provided. We review them here together.

7.2 Research Question 1:

Are there any particular potentialities in a MOOC-artifact that, if properly organized, trigger the double learning process and therefore the transition to the MOOC-ecosystem/instrument?

The first research question, addressed in Chapter 4, was concerned with the investigation of the online environment designed to host teacher education, the MOOC-artifact. In it we recognize the presence of three potentialities:

- The fact that the MOOC can be considered as a repository from which the trainees can draw inspiration for their teaching practices.
- The presence, inside the MOOC-artifact, of the communication message boards. Virtual spaces that allow the trainees' interaction. In fact, thanks to them the trainees compare each other, reflect, and exchange ideas and materials. It is in the communication message boards that the double learning process takes shape, which leads to the transition from MOOC-artifact to MOOC-ecosystem/instrument.
- The community of practice and sometimes of inquiry that, thanks to the design settings provided by the trainers on the MOOC-artifact, is formed in the MOOC-ecosystem/instrument.

We review the analyses that allowed us to deduce these results.
The analysis starts with an objective overview of the MOOC-artifact and what the trainees think of it. It starts with the MOOC Geometria and a quantitative analysis. The first data on the trainees show how they are a heterogeneous sample. They have different geographical origins (they live in different regions of Italy), a very wide age group (from 23 to 63 years) and their teaching experience is equally varied (some have been teaching for less than a year and some for more than thirty). For the vast majority, it is the first experience of online mathematics education. However, the MOOC is perceived as a flexible space in terms of space, time and pace of learning. In parallel, the same analysis is carried out considering the trainees of the MOOC Numeri. Also in this case the trainees are a heterogeneous sample. They have different geographical origins (they live in different regions of Italy), a very wide age group (from 27 to 64 years) and their teaching experience is equally varied (some have been teaching for less than a year and some for more than thirty). This time more than half of the trainees had been trainees in the MOOC Geometria, so more confidence with the online environment emerges. This overview shows us both the heterogeneity of the group of trainees, but also their way of approaching a new educational online environment. As we have seen, the trainees of MOOC Geometria and MOOC Numeri are not the same people and comparative analyses between the two groups have been shown. However, the results that we expose below are generally valid for both groups.

7.2.1 MOOCs as repository

Focusing only on the level of design, the MOOC-artifact, i.e. the place where only trainers have access, is the container of specific products, namely materials rich in innovative teaching methods and specific tools technology (as seen in the descriptions of MOOC Geometria and Numeri in §3.2.3.2, §3.2.3.3). We can therefore understand it as a **repository** from which teachers can draw inspiration. Being a repository is the first potentiality that a MOOC has. The activities of the MOOC-artifact constitute the backbone of the network of knowledge of the MOOC-artifact itself. Moreover, they provide teachers with suggestions to support their teaching. They aim to improve the teaching of mathematics in Italian School and are, in effect, a tool for professional development and education. The activities, both for their type and how they are built, stimulate the motivation and involvement of all students, even the less interested ones in the subject. They also indicate possible sub-paths of consolidation, aimed at "weaker" students, or of in-depth studies, suitable for students with better results. In particular, in some of the presented activities, specific indications analyse the most common difficulties that the students might encounter during their execution and some suggestions are proposed, which the teacher will choose how to apply or integrate according to the situation of her class. The proposals of the MOOC offer concrete examples of activities to be carried out in the classroom using laboratory-based methodology (Anichini et al., 2004). It is a teaching and learning strategy useful for the construction of mathematical meanings; it is a valid means of building knowledge through collaborative learning and facilitates positive interactions between people, reinforcing group identity. In all the proposals there is a conception of mathematical competences as a complex of processes based on both mathematization and modeling of real situations; both on the exchange with others, on the interface between the individual and on the collective experience. This is a potentiality because on the one side the trainees have available high-level materials that they can always consult, even after the end of the MOOC (because the materials remain available to trainees even after the end of the educational experience). On the other hand, the wealth of these materials trigger both the trainees' interest to continue the educational path and trainees' reflections, new ideas, and sharing of experiences on teaching practices that are similar or different from what these activities propose. In fact, remember that the quantitative analysis showed that the MOOC contents correspond to the trainees' expectation (Figures 4.30, 4.48) and the majority of them used the MOOC's material in his classrooms testifying an active participation, an appreciation of the materials and a willingness to try experiment new (or in any case different from the usual) didactical practices. Therefore, the MOOCs are for the trainees exactly a repository. Namely, a place where they can find interesting activities, discover new didactical methodologies and be able to propose them to their students in the class.

7.2.2 The communication message boards

The **communication message boards** inserted in the MOOC-artifact are the forum, the padlet and the tricider. Without the presence of these tools, the MOOC would be just a website to consult. The trainers, usually, insert on them a delivery or prompting questions, in order to stimulate the discussion among trainees. However, a methodological choice of the trainers is the one of limiting their presence on the communication message boards, even though they are always vigilant behind the scenes, to foster the birth of an only-trainees community. The analysis of the interactions that took place on them use the lens of the double learning process.

The complex ecosystem structure developed as soon as the trainees begin to access the MOOC. They are asked to enter into what, at first glance, may look like *chaos*, because of the multitude of materials and available technological resources. In fact, initially the trainees may not have enough self-confidence with the situation (*instrumentation*). Gradually they implement the *self-organization* phase: appropriating the use of the MOOC's usage schemes and comparing what they explore with their Math Edu USs, they begin to use resources and materials (*instrumentalization*) and also to contribute comments to the communication boards (*sharing*). Each communication message board allowed a different interaction to take place.

Forum

The **forum** played a predominant role with respect to the other tools. Despite being an almost outdated mode (based on web 1.0), the trainees were very fond of it and used it to share their experiences of learning or of working. There was no moderator in the discussions: each trainee had the opportunity to read a diversity of opinions and experiences, and when she understood how it worked, then she introduced herself, became an author of posts, influenced other colleagues, or appreciated the idea expounded by a colleague.

Padlet

If the forum was the right place for the trainers to talk about themselves, including their strengths and weaknesses, the **padlet** was the place where the trainees began to share photos, videos and, spontaneously, their own materials. It is clear that the Padlet did not help to structure the exchange, but many trainees obtained inspiration from the exchange of materials in this place.

The trainees' actions that spontaneously share their own materials, created thanks to the stimuli received from the MOOC, on the communication message boards allow us to make this reflection. Let ***A*** a single trainee that spontaneously share an own new generated material. He uses the ideas he receives from the MOOC to develop a new and original product. The other trainees have the opportunity to have an extra source of learning: what they see in the MOOC and what they see doing by another trainee thanks to MOOC. So, the others are led to trigger a process of instrumentation/self-organization and to embed in their network that product. On the one hand, it is a boundary object (Bowker & Star 1999, p. 297), since it is a praxeology between ***A*** and another trainee. On the other hand, the process in which this mediation is inserted is that of the double learning process and in this sense the object do not mediate a knowledge, but enters into a genesis of Math Edu USs produced at that time. The sharing of experiences and observations that have occurred in the MOOC can be understood as products, arising from the processes of the double learning process.

Tricider

The **tricider** had the goal of triggering simple threads, most of all confined to the approval or not of ideas, by voting through "likes". However, the participants used it more for collecting ideas and comparing their didactical experiences – as a forum – rather than for the expected use. Practically, the trainees realized a *catachresis* (Verillon & Rabardel, 1995): an artifact is used to do something it was not conceived for. Due to the fact that they explored the tool for the first time, and also because they usually need to explain and to go in depth when they express an idea, so the simple vote would not have let them satisfied. The posts written in Tricider are rich of ideas for both trainees and trainers. The trainees were introduced to a new tool for them. The trainers acquired awareness about the necessity to be clear in writing the tasks, in exemplifying the use of the tools and in providing tutorials on their affordances.

7.2.3 Plasticity and technological multimodality

In general, in the communication message boards, the will to establish the threads often leaks out, though it is very difficult that they take shape in a broad and articulated manner. In fact, the threads tend to split into different groups, which are formed and split locally and for a certain period of time, depending on the needs felt by the individual, but generally they contribute to give to all trainees the sense of a common participation in one unitary event, precisely the MOOC. Using a term from neuroscience, we call this property ***plasticity***, which makes it possible to adapt to various situations in different groups and times. It is true that situations and times change, but within a community that preserves its global unity. This unity consists in the collaborative sharing of what happens, even if the active participation converges on more than one local theme. The sharing processes (of materials, thoughts, ideas, experiences) in fact gives life to the ecosystem, enhancing the materials and expanding the individual's network of knowledge. Even the "contact points" with trainers via webinars contribute to this purpose. Through sharing processes the ecosystem becomes more and more structured; fragments from the history of web communication (from web 1.0 on) coexist and complement each other, and are used by the trainees. This aspect is interesting and little pointed out in the literature. It is something similar to the multimodal interactions that take place in the classroom thanks to the activation of different registers: we call it ***technological multimodality***.

Plasticity and *technological multimodality* are the two main properties distinguishing the evolution of a community in a MOOC from that in a traditional training course.

It is clear, therefore, that communication message boards have done more than allowing interaction. Already from the trainees' answers (§4.8 "A quantitative overview of the MOOC-ecosystem") where we have analysed their usefulness, we have seen how the trainees have defined them as spaces that allow them to clarify what the course is proposing in terms of didactic praxeologies. The communication message boards also stimulate reflection, that is, they allow a comparison with the trainee' own Math Edu USs (to internalize the proposals) and with those of others. The communication message boards help trainees to share their ideas, their opinions, their teaching experiences (to create new connections in their own network of knowledge). Last, but not least, as it emerged spontaneously from the open answers of the trainees (Table 4.25, line d and h), they help to develop a sense of belonging to a community.

7.2.4 The community of trainee

The trainers' methodological choice (decided during the design phases) to limit their interventions as much as possible in the communication message boards has certainly helped to make the trainees cohesive. In fact, the third and last (not for importance) potentiality that we identify in the MOOC is the presence of the ***community of trainees***. This community is different from those that usually characterize the traditional training courses (the face-to-face ones): it was born spontaneously since participation in the MOOC took place voluntarily. In addition, the participants freely express themselves: there is no institutional component that wants to restrict them. Notice how the discussions proceed in a very free and spontaneous

way, even with the use of emoticons (as seen in the comment by E.C. in Table X25). This is a remote, voluntary, free and collaborative community, not subject to institutional pressures.
The trainees voluntarily joined the MOOC for teacher education with the aim of improving their mathematics teaching competencies so that they share a common purpose. They are involved in joint activities. In particular, they are invited to reflect on the activities proposed by the MOOC, to evaluate weather using them in their classrooms, to consider the potentialities of the methodologies or strategies shared with them. In doing that, they generate critical dialogue and inquiry in the communication message boards. Even in these spaces, they confide in each other, share ideas, opinions, their own materials. Some also give themselves mutual support. This is surprising, since they have never seen each other in person!
The birth of such as community is not a completely spontaneous aspect in a MOOC, because it is strictly connected with the collaboration that cannot be considered as a spontaneous way of working, especially within such remote contexts. Designers have to make it possible through specific techniques. In fact, it is necessary to consider precise methodological choices and to support them. A choice of the trainers is certainly the one repeatedly discussed of limiting their presence on the communication message boards, even though they are always vigilant behind the scenes. Another is exactly the vigilant presence on the platform. Make the trainees understand that the trainers are available and they are participating when needed. Moreover, supporting and encouraging the link between the trainees with activities that see them as protagonists. For example, the webinars, periodic meetings where the trainers talk about of mathematics education but underline also what the trainees are discussing during the course. And the activities of Project Work, but above all of Peer Review, which allow to develop a greater sense of belonging to a peer community that pursues the same educational purpose. By implementing the double learning process, the trainees constitute a community made by peers, which is a community of practice (Wenger, 1998) and, sometimes, also of inquiry (Jaworski, 2006), supported by the vigilant, but not intrusive presence of the trainers. In this order of ideas, the trainees construct meanings that are not only related to mathematics (and therefore to its epistemological value), but also linked to the didactic and methodological value that mathematics has.

All these aspects are the bases from which to get the result for which the trainers have worked and the trainees have enrolled in the MOOC, that is to benefit from professional development. And this is what is addressed by the following research question.

7.3 Research Question 2:

Does the MOOC's ZFM/ZPA trigger and support an effective shift from the actual developmental level to the potential developmental level in the ZPD of the trainees? And if so, which kind of expansion of the network of professional knowledge this shift brings with it?

The second research question, addressed in Chapter 5, specific for the trainees, was concerned with the impact that the MOOC exerts on its trainees. Precisely it was shown that the MOOC's ZFM/ZPA allows for an effective shift from the actual developmental level to the potential developmental level in the ZPD of the trainees. It is possible because the MOOC's ZFM/ZPA trigger and support an expansion of the trainees' network of professional knowledge. In particular, analysing some case studies, it was also shown how these aspects lead to a perception of changes in teaching knowledge, practices, and beliefs.

We review the analyses that led to these results.

7.3.1 Quantitative analysis on MOOCs' trainees

The analysis starts with a quantitative overview of the impact that the MOOC's ZFM/ZPA has had on the trainees, to underline some possible changes between the period before the MOOCs and that after them. Also in this case, considering only the trainees that completed the MOOCs in all their stages, we consider what they declare before the MOOC Geometria and after its end. Later, we do the same for MOOC Numeri, but we distinguish among the *new entry trainees* (that can be considered as the *Geometria trainees*, because for them, MOOC Numeri is their first MOOC for mathematics education experience) and the *former trainees*, who are ending their second experience with MOOCs for mathematics education. However, the results that we expose below, unless otherwise stated, are generally valid for all of them.

In both the MOOCs experiences a wide part of trainees feel some need for education and development. This can be interpreted as a feeling of a certain tension (Goos, 2013) that has pushed the trainees to attend the MOOC; a mild misalignment of the zones system. In other word, these trainees felt that their ZPD (their prior knowledge relatively to mathematical content, pedagogy, technology) could be improved to fit better in their school's ZFM/ZPA. The fact that these trainees have completed the MOOCs in all their stages – with active participation, satisfying all the tasks required – invite to assume that the initial tension has evolved into a productive one.
To know the initial ZPD that characterize each trainee, or to know their actual level of prior knowledge, some questions were posed in the initial questionnaire, administered at the beginning of each MOOC. The ***mathematical knowledge*** of the trainees was not examined. MOOCs were not intended to teach new mathematical content, rather the ideal praxeologies that the trainers wanted to transpose concerned innovative methodologies and new teaching strategies, also using the technologies. Regarding ***pedagogical content knowledge*** and ***skill/experience in working with technology***, was analysed:

- the use of laboratory-based methodology activities;
- the use of practical-manipulative activities;
- if the problem-solving proposed to the students take inspiration from real situations;
- the use of technology in the teaching practices.

The analysed data suggest that, the ZPD of the *Geometria trainees* and *the new entry trainees* (the ones that enrol themselves for the first time in a MOOC for mathematics teacher education, precisely MOOC Numeri) has undergone a positive change. After attending the MOOC Geometria and the MOOC Numeri, the percentages show an increase. This means that what the trainers want to transpose (their ideal didactical praxeologies) have been effectively transposed. Therefore, the trainees' professional network of knowledge has expanded, i.e. it has moved from a current level to a potential level. The expansion of the network of knowledge is linked to the **double learning process** that the trainee puts in place. In the double learning process it is important both the interaction that the trainee establishes with the resources made available by the trainers in the MOOC, and the interactions that are generated on the communication message boards used by the trainees in exchanging reflections, opinions, ideas and also materials. Moreover, that learning is understood in a connectivistic sense: it is not a "literal" learning of new things, rather it means to be able to see different concepts that were already known (reflect, think again, integrate them under a different perspective).

As regards the former trainees, in the period following the end of the MOOC Geometria and which coincides with the start of the MOOC Numeri, in some cases there are confirmations, in other slight changes (both positive and negative). For example, 8 months after the end of MOOC Geometria, 5% of these trainees do not use laboratory-based methodology activities in their teaching practices (despite the fact that at the end of the MOOC Geo 100% of the finalists had said yes).
The use of practical-manipulative activities remains unchanged, but they change the motivations for which they are considered useful for their teaching practices. More emphasis is given to the fact that these activities involved the body and thank to this one remembers more (from 13% to 20%); while there is a decrees in the consideration of the fact that these activities allow avoiding formation of misconceptions (from 24% to 7%). Therefore, if at the end of the MOOC Geometria these trainees had given some weight to the advantage that these kinds of activities gave in terms of avoid misconception because the experience and the examples of the MOOC were strong within them, at the beginning of MOOC Numeri seems that after 8 months this opinion has changed. In terms of the network of knowledge, it means that the former trainees give a weight of minor importance to the node that connects practical-manipulative activities with overcoming the misconceptions.
As for the use of problem-solving that are inspired by real situations, there is a positive shift (from to 87% a 98%): 8 months after the end of the MOOC Geometria, these trainees have continued to propose problematic situations in the classroom that are inspired by real situations. This percentage distribution did not undergo significant changes after the end of the MOOC Num. Therefore, MOOC Numeri has not particularly affected the didactic convictions of this trainees, compared to the experience that they have done in MOOC Geometria. This is not a surprising result. The distance that separates MOOC Geometria from MOOC Numeri is less than one year. In MOOC Geometria we recorded more evident changes because the former trainees had been exposed to innovative methodologies and new technological tools for the first time exactly on the occasion of MOOC Geometria. In MOOC Numeri, it is true that the proposed activities change, because the thematic core of reference changes (from geometry to arithmetic and algebra), but the methodologies that are proposed do not change.

7.3.2 Professional development

In order to have a greater security in affirming that the MOOC's ZFM/ZPA could have an impact on the professional development of the trainees, it was decided to ask them if they thought they had effectively benefited. Two different data collection methods were followed between MOOC Geometria and MOOC Numeri. Although with MOOC Numeri it was believed to make a positive change in the data collection, the method used in MOOC Geometria was more satisfying comparing the data collected.

For the MOOC Geometria a written interview was proposed about two months after the end of the MOOC. It contained questions that investigated how much and why the MOOC Geometria was useful for the trainee's professional development. The **activities proposed in the MOOC** have been incisive, in fact the 95% of the trainees attribute to these the merit of their professional development. The **activities of Project Work and Peer Review** had also a positive impact on the professional development of the trainees. They are judged useful for their professional development by 69% and 76% of the trainees, respectively. The fact of having to **reflect and write own thought**, considered useful for its own professional development by 73% of the trainees, together with the fact of **sharing** (giving and receiving) **own teaching experience with other colleagues**, considered useful for its own professional development by 88% of the trainees, are the phases that that identify the ***double learning process*** put in place by the trainees. The *self-organization* phase allows reflection and meditation on the proposals, allowing to reorganize the own network of knowledge. With the *instrumentation*, the trainee also gets to form a new node or to see an existing one under a different light. The phase of *instrumentalization* generates in the individual the will to want to externalize the reorganization of ideas that have been completed and this is accomplished with the *sharing* phase that enriches not only the individual trainees but the whole MOOC-ecosystem. For 91% of the trainees to affect their professional development were also the **contacts with experts in mathematics education** through the videos inserted in each MOOC modules and the organized webinars. Furthermore, **being part of a community** and **being in contact with people who experience the same innovative activities** during the same period has affected their professional development for 75% and 88% of the trainees, respectively.

For MOOC Numeri, we did not wait the end of the MOOC to interview the trainees, rather, some specific questions were integrated into the final questionnaire. Their formulation is different from those in the final written interview of MOOC Geometria, but the intent was the same. The trainees are not being asked to judge how much to put into practice a certain behavior in the MOOC has had repercussions on their professional development. Rather, one is asking how true it is that that particular attitude has been put into effect. However, we could not make the same deductions emerged with the analysis of MOOC Geometria data.
The opinion of the Numeri trainees, for the most part, confirm that they think the MOOC Numeri has contributed to their professional development (72% of the *new entry trainees*, the 88% of the *former trainees* and the 89% of the *former 38 trainees* attribute veracity to this sentence). The professional development happen even if the practices related to the **use of MOOC materials in the classroom**, **sharing their practices with the other trainees** and **sharing some of their materials with the other trainees** were put in place by the 54%, 24% and 21% of the Numeri trainees respectively. These three aspects are underlined because these practices were considered useful for fostering the professional development of Geometria trainees. Evidently, it is not necessarily only from these practices that the benefit of professional development depends. Indeed, feeling of **being part of a community** and the **interactions with the trainers during the webinars** have certainly contributed for the professional development of Numeri trainees.

7.3.3 Perception of change in teaching practices

A more delicate issue concerns the perception of a change in the trainees' teaching practices as a result of the experience lived thanks to the MOOCs. Trainees were then explicitly asked if they had made or notice changes in their practice as a result of participation. Both of the two MOOCs' trainees said they noticed changes in their practices after attending MOOCs (81% of Geometria trainees, 52% of Numeri trainees). The answers given to the question that asks to explain the reason for these statements give us an idea of what these changes consist of: for someone it means integrating new tools and strategies in own teaching practices; for someone else implementing MOOC's activities in the classroom. However, these answers are not enough to have concrete evidence of changes in teachers' practices. The changes can be realized in the long term, if one can have the time to internalize the proposals and if one starts to experiment in class, including them in her own teaching practices. The case studies methodology it could give more information about it.

7.3.4 The case studies

The quantitative analysis showed the effects that the MOOC's ZFM/ZPA had on the trainees. However, the trainees do not live in an exclusive relationship with the MOOC, every day they are immersed in their school contexts. They relate to colleagues in attendance, they relate with their students to whom they can also propose the activities they view on the MOOC. It is therefore important not to neglect the influence that the school's ZFM/ZPA exercises on them. The individual and social relationships that occur in this real context, as opposed to the MOOC, have a certain impact on the way in which the trainees continue to attend the MOOC and therefore on their professional development. The influences exercised by the school's ZFM/ZPA do not emerge distinctly in the MOOC. However, they can be traced by monitoring and interviewing individual trainees appropriately. A more detailed analysis, therefore, emerges considering the case studies. Focusing on specific trainees, we can better understand all the facets that both the MOOC's ZFM/ZPA and the school's ZFM/ZPA exert on the trainee in the period in which he attends the MOOC.

7.3.4.1 Lucy

Among MOOC Geometria trainees we chose to report the case study of a teacher we call Lucy. Lucy's case has been selected because it illustrates how a tension (Goos, 2013) between her ZPD (Lucy's beliefs about mathematics teaching-learning) and her school's ZFM/ZPA (Lucy's professional environment and interaction with teaching colleagues) became a productive tension thanks to her participation in the MOOC, that is also reflected in her professional development with a perception of a small but significant change of teaching practices.

From a MOOC's Zone Theory perspective, Lucy changes her interpretation of two aspects of school's ZFM/ZPA – her beliefs on the mathematical abilities of her students (that seem *quite* involved in doing practical-manipulative activities) and on her professional development (the attention to the practical aspect and to the manipulation) – thanks to the new person-environment relationship offered by the MOOC Geometria. These changes were triggered by a tension that arises from a misalignment within the zone system. Actually, Lucy experienced dissatisfaction because her ZPD did not map onto the school's ZFM/ZPA complex in ways that promoted desired development. Attending the MOOC Geometria this tension became productive. In fact, because there was no ZPA within her school, which could map onto her ZPD, Lucy looked outside (precisely towards the MOOC) to advance her personal goal of thinking differently about mathematics education. Lucy proposes more than one of the activities seen in the MOOC and she is able to create a set of possibilities for developing new beliefs, knowledges, goals and practices (she has reached the potential level of ZPD that the MOOC's trainers had planned). To implement the different teaching approach promoted by her involvement in the MOOC (MOOC's ZFM/ZPA), Lucy had to change her environment (school's ZFM). In fact, she has used the MOOC's activity adapting them to its classes to be consistent with her goal of improving students learning. The MOOC Geometria allowed for an effective professional development experience. It has triggered and supported changes in what Lucy thought about her way of teaching geometry using also practical-manipulative activities. Moreover, exactly the activities proposed in the first module (part of MOOC's ZFM) and the actions triggered in the MOOC's ZPA, that is the comparison with the other trainees in the platform, led Lucy to reconsider the practical-manipulative activities. She actually stated that she very much appreciated sharing teaching experiences and feeling part of a community that experienced the MOOC activities: they were able to push her enough to do the same. As a result, Lucy began to make more use of these activities in her classroom, noting a different interest and attention in her students. This can be considered a small, but significant perception of change in her teaching practices.

7.3.4.2 Stephen

Among MOOC Numeri trainees we chose to report the case study of a teacher we call Stephen.

Stephen's case has been selected because his Project Work, or the final design activity of the MOOC Numeri, was a reworking of an activity proposed in the MOOC in the light of his experience as a teacher and of the experience lived in the MOOC. He also experimented with this activity in the classroom, documenting it with the delivery of the logbook. From these data we can see how the MOOC's ZFM/ZPA contributed to the Stephen's professional development (understood as the development of his ZPD) and consolidation of his didactical praxeologies.

The MOOC's Zone Theory perspective allows for picturing Stephen's profile. Stephen describes his school's ZFM / ZPA as a positive context. He talks about a collaborative and active class; he has a good relationship even with his colleagues. The care and the passion he has for the teaching of mathematics emerge from his proposals and from the materials he shared with us. He is careful to dose the methodologies well in his lessons. He prefers, as he has repeatedly stated, the laboratory-based methodology activities. He alternates moments of collaborative work, collective discussion, and reasoned use of technology. Therefore, no tension emerges in Stephen. There is no misalignment of its zones system. Since his ZPD and his school's ZFM/ZPA are aligned, he is already in a phase of productive tension. There is a balance between his knowledge and his peaceful relationship with colleagues and pupils. These positive factors act as stimuli for him to always tend to give more and to do his best as a teacher. He joined the MOOC Numeri to look for cues of teaching on laboratory-based methodology activities relative to the numbers core. It clearly states that he is "used to doing laboratory-based methodology activities with the collective construction of meanings starting from problematic situations to which the students must find the solutions". However, he confesses that he does not have a vast repertoire on activities involving the numbers core. The MOOC's ZFM/ZPA for him is a rich environment, full of stimuli and colleagues to deal with to reflect together and exchange ideas and strategies. MOOC's activities for him must be studied, thought out, included in own teaching practices. Only after that the internalization in the network of knowledge, consciously connected with his nodes, revised in light of new knowledge in input from the MOOC's ZFM/ZPA, can happen. This new knowledge is then experimented with creativity and desire in the school's ZFM / ZPA, with satisfactory results both for Stephen and for his students, who actively participate in his lessons. The decision to experiment with a laboratory-based methodology activity on the issue of numbers confirms the fact that Stephen's ZPD has moved from the current level to the potential level. Thanks to the comparison with the peers within the MOOC's ZFM/ZPA and to the comparison with the trainers that was realized with the fruition of the proposed activities, Stephen was able to enrich his repertoire of laboratory activities on numbers core. In this way, he realizes the wish he had when he joined the MOOC Numbers. He was therefore able to benefit of professional development.

7.3.4.3 Ester

Chapter 4 concludes with a negative case study. During the period in which the analyses on the final questionnaire of the MOOC Geometria were conducted, a singular situation emerged. There were always a negative reply to the questions concerning the appreciation of the course, while the other entire trainee gave positive answers. This was an intriguing thing and for this reason, the profile of this trainee was analysed. We refer to this MOOC Geometria trainee by calling her Ester.

Ester does not spend much time on MOOC and its materials. She claims to have never read and commented on the communication message boards. She took advantage of 20% or less of the material made available to the MOOC. Doing the Project Work with Learning Designer was useless for her, although she considered the Peer Review phase useful to know the opinion of a peer on her work. She writes that the contents of the MOOC did not meet her expectations, because she wanted to work more on mathematical problems to refresh her mathematical knowledge: "*problems related to real situation also to be used as evidence for skills*". The MOOC Geometria has proposed many problems related to real situations to be used as evidence for skills. Proposing problems that start from real situations and that develop students' skills were some of the methodologies on which the trainers focused most, i.e. some of the ideal praxeologies they wanted to transpose to the trainees. However, Ester has declared that she has viewed about 20% of the MOOC material.

Ester has also stated that the provided teaching materials are useful (clearly, the answer is based on what she has consulted). In particular, she said: "*Using the [MOOC's] material of the trigonometry I made my students less afraid of solving problems and I got bored less than usual from my own lessons*". When we asked clarification about this answer of hers, we understand that there was a tension (in the sense of Goos) that arises in Ester as a teacher. She would like something that does not degrade the teaching of certain concepts, something different from the mechanically exercise made by both the teacher and the high school students. Moreover, she declared that the difference that she sees on herself in relation to the other MOOC's trainees is that she has lack of passion for teaching. She justifies this by saying that she does not like to explain mathematical concepts that require mechanical exercises and rigid demonstrations to be acquired. Ester criticizes these methods of mechanical teaching, but it also seems that she does not know others.

There seems to be tension between her ZPD and her school's ZFM/ZPA. This tension, however, is not resolved nor by the activities that the MOOC proposes, possibly because Ester consults only a small part (those on trigonometry that appreciates), nor from the interactions of the other trainees, because she does not use the communication message boards. In light of the quantitative analyses that showed a general appreciation of the MOOC in terms of professional development, we can speak for Ester about a missed opportunity.

7.3.5 Remarks

Apart from the negative case of Ester, the other two case studies proposed clearly show how Lucy and Stephen benefited from professional development thanks to the MOOCs they attended. The professional development involves the evolution in the praxeologies. This does

not mean that all the teachers, involved in the educational programme, evolve in the same way with the same transformation of components (Robutti, in press). Moreover, the didactical praxeologies of a teacher change within the school's ZFM/ZPA in which she resides. As a consequence, there could be teachers' development of both a new awareness (on the cultural level) and new competencies (on the methodological-didactical level, i.e. that of teaching practice), which lead them to activate, in their classrooms, a didactical transposition in line with the meta-didactical transposition.
In the light of that, we can talk about for Lucy's case of development of new competencies, because she starts considering in a different way the practical-manipulative activities and she declares that she integrates them in her didactic praxeologies. While for Stephen's case we can talk about the development of a new awareness, because he was looking for some laboratory-based methodology activities on the number core and he has found what he was searching in the MOOC Numeri. The new awareness, therefore, lies in the fact that, even for the numbers core, he now has at his disposal some activities based on the laboratory methodology to which he can refer.

So far, we may have reason to believe that, in general, MOOC Geometria and MOOC Numeri allow for effective professional development and seems to lead perception of changes in the teaching practices of their trainees.

7.4 Research Question 3:

Does the MOOC's ZFM/ZPA trigger and support an expansion of the network of professional knowledge (namely, professional development or evolution of meta-didactical praxeologies) of the trainers relatively to design principles and strategies of trainees' assessment that the trainers have put in place?

The third research question, addressed in Chapter 6, specific for the trainees, was concerned with the design effort and assessment strategies that have been pursued in these online courses for mathematics education. The originality of this research resides in those design principles that are relevant and useful to mediate teachers' professional development courses with technology, and in the assessment of the impact of such distance courses on mathematics teachers' engagement. A specific attention is paid on trainers and their role in supporting interactions and learning communities that emerged during the MOOC. Trainers' techniques and their evolution were presented and analysed in order to highlight and discuss their methodological and theoretical justifications. In this way, the reader had opportunity to benefit from these expertise with online educational environments such as MOOCs.

We pointed out some essential meta-didactical types of tasks. Precisely, we consider four topics related to the design principles:

- Target;
- Theme;
- Trainers' interaction with trainees;
- Collaboration among trainees.

Moreover, we take into account three topics related to the assessment strategies:

- Test;
- Project Work;

- Peer Review

They constitute the network of knowledge of the MOOC-artifcat and were detailed described according to the meta-didactical praxeologies model.

It is apparent that the double learning process that brings the MOOC-ecosystem to life is experienced by the trainees who live the MOOC in the first person, not by the trainers who are the designers of it. However, the trainers, who observe the phenomenon from the external, visually notice how the transition from MOOC-artifact to MOOC-ecosystem/instrument takes place. In other words, they realize how the MOOC's ZFM they designed, from the beginning inert space gradually becomes dynamic. The ZFMi follow each other from week to week and the actions and interactions of the trainees make way for the MOOC's ZPA, generating the MOOC's ZFM/ZPA complex. In seeing this, or in completing the monitoring phases, trainers have the opportunity to consolidate or modify the methodological choices that led to the creation of the MOOC-artifact, depending on the experience that the trainees are experimenting.

Identifying the target of teachers to whom the MOOC, as well as in the choice of the theme to be treated, the trainers are conditioned by ***research environment's ZFM***. They have a certain *perception of the teachers* that they image will follow the MOOC, making some hypothesis on their mean ZPD. Identifying the target is then closely connected to how to decline this theme taking into consideration the scholastic levels to which the training is addressed. Specially, the *Italian curriculum and assessment requirements* are points that can not be ignored. Furthermore, the choice of the theme is closely linked to the task of time, to which it corresponds, in the research environment' ZFM, the consideration of the *organizational structures and cultures*. *Accessing to resources* such as literature, proceedings of conference, but also – as declined in ***research environment's ZPA*** – *participating in national/international conference* and entertaining an *informal interaction with research colleagues*, the trainers were able to confirm the methodological choices made.
About the trainers' interactions with/among trainees is evident the conditioning that the ***research environment's ZPA*** have exerted in terms of *involvement in actions to foster teacher professional development*. The trainers have done the methodological choice to limit their interventions as much as possible in the communication message boards; however, they were vigilant behind the scenes and also organized some webinars to have occasion of synchronous contact with trainees.
For the strategies of assessment, the dynamic that characterized the ***MOOC's ZFM/ZPA*** in its evolution, has influenced the choice made by the trainers. Think for example the trainees' request to receive more time to accomplish the final task.

The praxeological analysis of the selected types of tasks seems to give significant answers to the third research question. In particular, the observation of the MOOC's ZFM/ZPA by the trainers actually triggers and supports an expansion of their network of professional knowledge relatively to design principles and strategies of assessment. Through the analysis of the praxeologies associated to these tasks, we have tried to catch several essential topics regarding MOOCs. Firstly, the relationship between design principles and professional development that can be grasped through the audience of each of the MOOC, but also the theme itself which is essential from an institutional point of view. Moreover, the delicate question of the relationships between trainees and trainers. Secondly, we have observed that looking carefully at the assessment strategies included in the MOOC design gives important clues to assess the mathematics teachers' engagement.

The methodological choice of the project-based assessment has proved to be effective. The model itself of MOOC does not allow researchers to observe directly the effect of the training courses proposed by the MOOCs and to gather feedback from observations in classes. For this reason, the trainers considered the PW as a suitable way to assess the competencies acquired by the trainees. The PW was individual and a PR was proposed to evaluate the work. The evidence is that the connection between trainees does not go without saying and that the role of tutors as well as their scope of activities must be included in the design principles of MOOCs. Moreover, time to devote to these tasks is an important issue to consider and it was increased in both experiences.

A main result is that collaboration cannot be considered as a spontaneous way of working, especially within such remote contexts. Designers have to make it possible through specific techniques. The trainers praxeologies related to this task constitute some effective examples of how to solve it.

7.5 Limitation of the study

All studies are conducted within boundaries and, it could be argued, acknowledging these boundaries adds to the credibility of reported findings. Theoretical, methodological, and practical limitations that need to be taken into account when considering the findings of the present study are identified in this section.

7.5.1 Theoretical limitations

The MOOC’s Zone Theory theoretical framework is complex. And after all it is also complex the phenomenon of MOOCs that has been analyzed in this dissertation.

The theoretical framework arises from the consideration of 4 different theories: the Meta-Didactical Transposition, the Instrumental Approach, the Connectivism and the Zone Theory. However, they are not considered all four in their entirety. In fact, hybridization processes are in place between the first three theories listed and, with the theory resulting from this hybridization (the MOOC-MDT), a networking is performed.

The MOOC’s Zone Theory is very plastic and versatile. In fact, we used its specific “pieces” depending on what we wanted to analyze. In Chapter 4, where we were interested in analyzing the potential of the MOOC, we have made extensive use of the MOOC-MDT. In chapter 5, which zoomed in on the trainees, the networking between MOOC-MDT and Zone Theory was illuminating. In fact, it allowed us to consider the *school’s ZFM/ZPA*, which has a certain influence on the trainees that follow the MOOC. In chapter 6, the focus was on the trainers. Here too we have exploited the power of networking, drawing attention to the *research environment’s ZFM/ZPA*.

Without the possibility of being able to refer to these four theories, we would not have obtained the results we have outlined here.

The reader is invited to reread the cactus flower metaphor, which we had illustrated to conclude the exposition of the theoretical framework. After becoming aware of the results presented in the analyzes, it should acquire even more meaning. In fact, it highlights the role of each of the actors considered: trainees, trainers and the variegated, changeable and dynamic MOOC.

7.5.2 Methodological limitation

MOOC is a complex environment, as we have seen, that goes on very fast. Moodle platform keeps track of all the virtual actions of the participants (when they log themselves, what they see, where they write their comments, …), but they are hundreds and it is impossible to observe everything that is happening, so it is necessary to focus on what is relevant to the purpose of the study.

MOOC observations, questionnaires, Skype or written interviews and collection of logbook were primary methods of data collection of this doctoral study. Case study methodology is also employed in the present study. Although there are advantages in having the researcher as the primary instrument of data collection, as is the situation in case study research (Merriam, 1998), there are also disadvantages. On the one hand, as a dissertation writer, I was attuned to the purpose of the study and could focus on collecting data relevant to the study (e.g., asking follow-up questions in Skype interviews to seek clarification of information related to the research questions); on the other hand, the quality of the study was highly dependent on my skills as a researcher. For example, Kvale and Brinkman (2009) described interviewing as a craft that is dependent “on the practical skills and the personal judgments” (p. 17) of the interviewer that can only be developed through experience. My skills in this area have been developed through experience in this research projects. For example, regarding the video interviews, I realized that I could “insist” more on certain points, rather than “be content” with the trainee’s first answer.

There are some differences between the data collection tools that we have used in the two MOOCs. This is why MOOC Geometria was the first experience. Thanks to that experience, the analysis tools have been refined for the second one, the MOOC Numeri. However, even if the instruments have been refined, they have not always given the expected or desired results.

To save time and not to contact the trainees more than once, it was decided to avoid sending the so-called written interviews (used with MOOC Geometria trainees). So, the questions they contained were "spread" on the questionnaires administered during the MOOC frequency. In general, the sense of the questions has been maintained, even if some of them have been changed in the formulation or the typology (for example, from the closed question to Likert-scale one). Change the formulation and/or the typology of the questions was not an optimal choice. First, because comparative analyses were more difficult. Secondly, having condensed all the questions in the three questionnaires made these longer and the trainees responded in a more superficial way. Finally, especially for questions relating to professional development, we believe it is necessary to contact the trainees some time later the MOOC experience and not immediately after its end, to allow them to reflect further on their experience and make more precise evaluations.

7.5.3 Practical limitations

We talked about *perceptions* of changes in the didactical praxeologies of the trainees because the analyses were all done at a distance. The interviews, as we have seen in the case studies, were also conducted in writing or in virtual presence, on Skype. There has never been a time when the dissertation writer met the trainees personally or went to the classroom to follow their lessons.

On the one hand, it was dependent for economic reasons. In fact, choosing one or more trainees and following them in their classroom requires moving frequently from the research centre (Turin, in this case) to the place where these trainees live. On the other hand, the choice of not having contacts in presence with trainees was also wanted. The intention of the research was to see what can be collected, deduced and identified by analysing only the declarations made online by the participants. Clearly, this implies that the data on which to base oneself are the declarations released in written form on communication message boards, questionnaires or verbally through interviews via Skype. As a dissertation writer, I am conscious about the fact that see an evidence in class is different than reading it or listening to it (if the interview is conducted via Skype). However, the interviews were conducted in a time following the end of the MOOCs and this gave the trainees time to reflect on the innovations explored in the MOOC and eventually start to integrate them into their teaching practices. From the Chapter 3 on the methodology, it was be recalled that the interviews were conducted both for Geometria and Numeri trainees in the summer of 2017. So, for MOOC Geometria means more than a year after its end.

For future research that intends to investigate if a MOOC for mathematics education is able to change the teaching practices of its trainees, we suggest following the following methodology.

The MOOC registrations are opened about a month before its beginning and it is advertised in more ways: presentation to conferences, communications to the regional school offices, sending emails to well-known groups of teachers, advertising on social networks, passing word.

When the trainees voluntarily register, the researcher can choose some of them randomly. He can contact them to see if they are willing to be followed and monitored in the presence during their use of the MOOC. A preliminary analysis (with a face-to-face interview) would be necessary to understand what their starting level is, see how they generally do their lessons. In particular, it would be important choose one of the topics that will be addressed in the MOOC, to see if and how, after the ideas received from the MOOC, the way the trainee teaches that topic changes or not.

The change will not be immediate. The researcher will have to continue to follow the teacher several months after the end of the MOOC, to identify even slight changes in the practice, attributable to his training experience conducted in the MOOC.

7.6 Future research

The present study identify four areas in need of further research:

1. In the analyses that explained the evolution of the MOOC knowledge network, of the trainees and of the trainers, we showed some graphs realized with yEd (https://www.yworks.com/products/yed), a specific software also used to do studies related to graph theory. It is believed that it might be interesting to do studies that involve mathematics education and graph theory, also to see the importance and the weight that the nodes take in their own network of knowledge.

2. Our analysis shows that a real involvement of trainees in collaborative work needs to be triggered and supported by suitable tools added to the platform. The availability in the platform of tools consonant with the social networks used in everyday life increases the triggering of what Manlove et al. (2006) call co-regulated learning, in the sense that the trainees themselves regulate their tasks and collaboration. Our analysis

leaves open the question of which devices are the best for improving active collaboration among the trainees: possibly further research and concrete experimentations will be able to give a more definitive contribution to this crucial issue. What is interesting here is that our analysis centred on collaboration processes through the adaptation of the meta-didactical lens has made possible to grasp this important problem in a clear way. This suggests that the way of research we have undertaken is promising and fruitful for further results along this stream.

3. Our MOOCs are to be considered as professional development activities designed to initiate change in teacher's knowledge, beliefs, and practices. It will be interesting to continue to follow the development of the teachers who started from MOOC Geometria and will conclude the path of the 4 MOOCs that are scheduled within the Math MOOC UniTo project through a longitudinal study.

4. From the last question in both questionnaires administered to the trainers of MOOC Geometria and MOOC Numeri, emerges that thanks to the involvement in the design and monitoring of MOOCs for mathematics teacher education, the trainers can benefit of professional development. We have not studied this aspect in detail so far. We have put more emphasis on the evolution of the trainers' ZPD in terms of the evolution of design to accomplish the task to transpose the ideal didactical praxeologies and the evolution of the assessment strategies to understand the impact of such distance courses on mathematics teachers' engagement. The evolution of the ZPD in terms of professional development of the mathematics teacher educators is certainly an aspect that deserves to be investigated further.

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Appendix A Rota: The disclosure of mathematical objects

Passage from F. Arzarello, M. Ascari, C. Baldovino, C. Sabena (2011). The teacher's activity under a phenomenological lens. In: (a cura di): U. Behiye, Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education. vol. 2, p. 49-56, Ankara, Turkey: PME, ISBN: 9789754292961

We will now sketchily illustrate how a phenomenological stance can be useful to focus learning processes in mathematics; to do that we will follow the elaboration given by the outstanding mathematician and philosopher G. C. Rota (1991) to Husserl phenomenology. The starting point of the analysis is the issue of what it means "seeing and being able to see in mathematics", according to the title of a wonderful book by B. De Finetti (1937). As underlined by Radford (2010, p. 4), students must be taught to "see and recognize things according to 'efficient' cultural means" and to convert their "eye (and other human senses) into a sophisticated intellectual organ". Namely it is necessary to promote a "lengthy process of domestication" (*ibid.*) of the way they are looking at things while learning mathematics. This process is based on the key phenomenological assumption, pointed out by Rota, that there is "no such thing as true seeing", but "there is only seeing as" (Rota, 1991, p. 239). Hence, learning mathematics requires different modes of *focusing*: "just like seeing is focusing upon some functions which may be present, similarly, remembering, imagining, or visualizing are other modes of focusing" (*ibid.*). For example to see the slope features of a track from a mathematical graph representing it, as in Fig. 3. This delicate process is far from being natural: on the contrary, to be achieved it requires precise didactical interventions of the teacher.

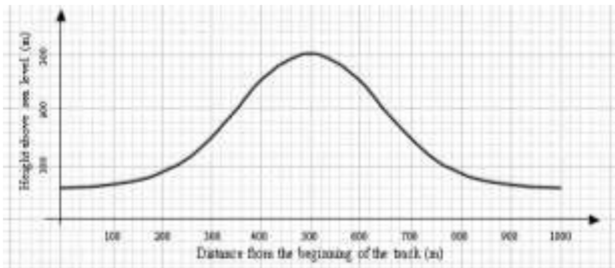

Figure A.1: Distance-height graph of a mountain track

Specifically, we refer to two related didactic techniques from the literature: *making present* absent things, and *prompting* them in order to direct students' *attention* towards them. As pointed out by Ferrara (2006) and following Husserl, we can distinguish two aspects of making present, namely *remembering*, that is "making present the past (the absent being the past)"; and *imagining*, that is "making present the not yet known (the absent being the not yet known)" (*ibid.*). Rota considers both remembering and imagining as modes of *focusing*. Prompting the students' attention to specific parts of the context, possibly enriched with recalled or imagined elements, may support them towards a progressive *disclosure* of the mathematical objects at stake.

Disclosure is a Husserlian concept further elaborated by Rota (1991) to give reason of mathematical understanding. It indicates the process by which people make sense of the world and of the situations in context to which they are exposed. The first encounter with the world is with and through *perceptual senses*:

> The world is primarily a world of sense […]. Our primary concern is with sense itself, how it originates in the world, how it functions in the world. In short how it relevates […]. The basic relationship to the world is […] our senses. (*ibid.,* p. 61).

Disclosure develops when one is able to grasp the *functionality* of the objects in the context:

> Sense-making depends ultimately on our own being-in-the-world, on the situation of our interacting, our dealing with the contextual situation in the world […]. If you deconstruct the notion of an object, what you find is pure functionality, the pure 'being good for' of that object or something. So that the world, instead of being a world of objects, will become a world of functions, of tools. (*ibid.,* pp. 156-159, *passim*).

A further key-passage in disclosure is the ability of seeing not just functions, but the *relationships* between them:

> Such functions are related to each other "by a system of references, a network of references among them. […] The world is disclosed to us not just as a system of functions, but as a network of related functions" (*ibid.,* p. 159).

In a didactical setting, students are educated by the teacher to make sense of what they see when exposed to a mathematical situation. Generally a situation may evoke different contexts and so enact different sense-makings which push towards an increasing disclosure of the object, according to the students' age and background. For example, the graph in Fig. 1 may be seen as a mountain, a graph of a symmetric function, a normal distribution, and so on. Such different contexts are not isolated; indeed they are *layered* upon each other, and these layers can generate different levels of disclosure in the flow of time:

> Side-by-side with our realization that sense is purely contextual goes the realization that contexts are not units. Contexts themselves are layered upon each other in various ways, and to be in a context is not to be in just one context. … Be-ing in a context does not in any way presume that such be-ing is be-ing in one context at a time. (*ibid.*, p. 126)

Because of the fundamental role of the layered contexts, an emotional component (*mood*) is always present along with an intellectual one (*grasp*). They are both necessary in mathematical problem-solving, though with different grades of intensity:

> There are phenomena of disclosure where the actual grasp in the context fits the major role and the mood component fits the minimal role – for example, our approach to solving a mathematical problem. This is not saying that we have to like the problem, but the minimum of mood lets us get involved in it. Unless we get really involved in it, we get nowhere. This is the mood-wise component of the mathematical problem disclosing itself. Without this component of mood, no matter how little, the problem will not be disclosed. (*ibid.,* p. 269).

Appendix B Data sources

MONITORAGGIO MODULO 1 – SENSO DEL NUMERO

Italian	*English*
I corsisti sono invitati a: • scrivere sul forum a proposito dell'attività IL LIVELLO DEL MARE; • scrivere sul padlet; • leggere gli Sway; • eventualmente sperimentare;	*Trainees are invited to:* *• write on the forum about the activity THE LEVEL OF THE SEA;* *• write on the padlet;* *• read the Sway;* *• possibly experiment;*

FORUM

- **Numero totale di interventi (sia di prima mano che repliche).**
  - *Total number of interventions (both first hand and replies).*

- **Se si mantiene la distinzione I e II grado (è stato chiesto di specificarlo), quanti sono post del I e quanti quelli del II grado?**
  - *How many lower and higher secondary school interventions are there?*

- **Quante e quali categorie di intervento si individuano? (risposta esplicita alle domande, considerazioni, condivisione di materiali, condivisione di esperienza, sperimentazione, …) e quanti interventi per ognuna di esse?**
  - *How many and what categories of intervention do you identify? (Explicit answer to questions, considerations, sharing of materials, sharing experience, experimentation, …) and how many interventions for each of them?*

- **Quanti hanno dichiarato di aver provato una o più di una attività in classe?**
  - *How many have claimed to have tried one or more activity in their classroom?*

- **Quale è stata l'attività maggiormente proposta in classe?**
  - *What was the most appreciated activity?*

- **Riportare i commenti più significativi (non solo relativi alla sperimentazione in classe), con tanto di nome, cognome e ordine di scuola**
  - *Report the most significant comments (not just for classroom experimentation), with the teacher's name, surname and order of the school.*

- **Riportare un sunto generico di tutti gli interventi.**
  - *Report a generic summary of all the interventions*

Padlet con le seguenti domande

- Quali i punti di forza dell'attività per lavorare sugli ordini di grandezza?
- Una proposta così susciterebbe l'interesse dei tuoi studenti?
- Come potresti svilupparla in classe? Pensi di farlo?
- In quale punto della tua programmazione affronti gli ordini di grandezza?
- Come attività di problem posing/problem solving possono aiutare ad affrontare i nodi concettuali dell'unità proposta?
- L'utilizzo del foglio di calcolo può aiutare a raggiungere una maggiore consapevolezza del senso del numero, supportando le fasi operative di calcolo?

- **Numero totale di interventi (sia di prima mano che repliche)?**
- **Se si mantiene la distinzione I e II grado (è stato chiesto di specificarlo), quanti sono i post del I e quanti quelli del II grado?**
- **Quante e quali categorie di intervento si individuano? (risposta esplicita alle domande, considerazioni, condivisione di materiali, condivisione di esperienza, sperimentazione, …) e quanti interventi per ognuna di esse?**
- **Quanti hanno dichiarato di aver provato una o più di una attività in classe?**
- **Quale è stata l'attività maggiormente proposta in classe?**
- **Riportare i commenti più significativi (non solo relativi alla sperimentazione in classe), con tanto di nome, cognome e ordine di scuola**
- **Riportare un sunto generico di tutti gli interventi.**

In generale

- **Fanno commenti sugli SWAY? Di più sui contenuti o c'è anche qualcuno che si esprime sullo strumento SWAY? Riporta qualche commento.**

Possibili categorie di intervento:
A = **risposta esplicita alle domande**
B = **considerazioni**
C = **condivisione di materiali**
D = **condivisione di esperienza**
E = **sperimentazione**
F = **altro**

Forum

Forum				
Repliche Sec. I grado	**Repliche Sec. II grado**	**Ha sperimentato? Quale attività?**	**Categoria di intervento**	**Sunto veloce**

Autore	**Risorsa**	**url**	**Thread**	**Post**	**Data**	**Coerenza con il thread**	**Particolarità**

Padlet

Padlet				
Sec. I grado **(Nome, Cognome)**	**Sec. II grado** **(Nome, Cognome)**	**Ha sperimentato?** **Quale attività?**	**Categoria di intervento**	**Sunto veloce**

Tricider

Tricider				
Sec. I grado **(Nome, Cognome)**	**Sec. II grado** **(Nome, Cognome)**	**Ha sperimentato?** **Quale attività?**	**Categoria di intervento**	**Sunto veloce**

Questionario iniziale MOOC di geometria

Caro Collega,
ti chiediamo di compilare questo questionario per una raccolta di informazioni relativa ai partecipanti alla prima edizione del MOOC di geometria dell'Università di Torino.
Le informazioni che ti appresti a fornire saranno del tutto anonime e verranno condivise nel primo modulo di learning sottoforma di infografica.
Ti ringraziamo per la tua adesione e la compilazione del presente.

*Campo obbligatorio

1. **Nome** *

2. **Cognome** *

3. **In quale ordine di scuola insegni?** *
   da completare al fine di fornire indicazioni per una ottimale collaborazione e condivisione
   Contrassegna solo un ovale.
   - secondaria di primo grado
   - secondaria di secondo grado biennio
   - secondaria di secondo grado triennio

4. **Da quanti anni insegni?** *
   Contrassegna solo un ovale.
   - meno di 5 anni
   - da 5 a 10 anni
   - da 11 a 15 anni
   - da 16 a 20 anni
   - da 21 a 25 anni
   - da 26 a 30 anni
   - da più di 31 anni

5. **Seleziona di seguito** *
   Contrassegna solo un ovale.
   - uomo
   - donna

La formazione

richiesta di informazioni relativamente alla formazione come docente

6. **Hai seguito la formazione m@t.abel?** *
Contrassegna solo un ovale.

- si come corsista
- si e sono diventato formatore
- no
- non l'ho seguita ma conosco i materiali

7. **Hai già seguito un "corso" di formazione online (diverso da un MOOC)?** *
seleziona la risposta che reputi rifletta la tua situazione
Contrassegna solo un ovale.

- si completando la formazione
- si ma non ho completato il corso
- no

8. **Hai già frequentato un MOOC?** *
Contrassegna solo un ovale.

- si in lingua inglese completandolo
- si in lingua inglese senza completare il corso
- si in lingua italiana completando
- si in lingua italiana non completando il corso
- no

9. **Se la risposta precedente è affermativa, indica il tipo di MOOC frequentato**
indica il titolo del MOOC, l'ente erogante e quando l'hai fruito

Pratiche didattiche

indagine relativa all'uso di tecnologia nei percorsi didattici degli iscritti al MOOC

10. **Utilizzi le tecnologie nella pratica d'insegnamento?** *
Seleziona tutte le voci applicabili.

- [ ] si spesso in classe
- [ ] si, solo talvolta
- [ ] no

11. **Se la risposta precedente è affermativa, specifica quali tecnologie di seguito**

12. **Nella tua pratica d'insegnamento, utilizzi uno spazio d'apprendimento anche online per le tue classi?** *

ad esempio come repository o altro
Contrassegna solo un ovale.

- si un blog di classe
- si un wiki di classe
- si un gruppo (di facebook, di whatsapp, google o altro,..)
- si una classe virtuale
- no
- Altro: ____________

13. **Se la risposta precedente è affermativa, specifica di seguito che tipo di spazio d'apprendimento online hai configurato**

Potresti descrivere il tipo di classe virtuale, oppure inviare link di riferimento

____

14. **Utilizzi nella tua pratica d'insegnamento materiali del piano m@t.abel?** *

Contrassegna solo un ovale.

- si spesso
- si talvolta
- no
- non so

15. **Attui didattica laboratoriale nelle tue pratiche d'insegnamento** *

Contrassegna solo un ovale.

- si
- no
- non so

16. **Utilizzi GeoGebra in classe durante le tue lezioni?** *

Contrassegna solo un ovale.

- si abitualmente
- si ma raramente
- no

17. **Se hai risposto positivamente alla domanda precedente, relativamente all'utilizzo di GeoGebra, seleziona quanto meglio rappresenta la tua situazione**

Seleziona tutte le voci applicabili.

- [ ] costruisco fogli di lavoro di GeoGebra caricandoli in GeoGebratube
- [ ] creo file ad hoc, ma li presento senza condividerli in rete
- [ ] utilizzo file preconfezionati rintracciati in GeoGebratube
- [ ] raccolgo i fogli in GeoGebrabook dedicati per ogni classe

18. **Utilizzi software disciplinari diversi da GeoGebra?** *
Seleziona tutte le voci applicabili.

- [ ] Derive
- [ ] WolframAlpha
- [ ] Desmos
- [ ] Matlab oppure Octave
- [ ] Wiris
- [ ] Cabri
- [ ] Ti-Nspire
- [ ] Altro: ____________________

Ricerca Sperimentale

Gent.le collega

questo corso MOOC al quale stai per prendere parte, viene erogato per la sua prima (e speriamo non ultima) volta! Al fine di valutarne l'impatto che potrebbe generare nei docenti che come te lo seguiranno e la sua sperabile efficacia, il gruppo di ricerca in Didattica della Matematica dell'Università di Torino avrebbe piacere di seguire con particolare cura le fasi di monitoraggio e chiedere la tua eventuale disponibilità a partecipare alla sperimentazione.

SE MI RENDO DISPONIBILE CHE COSA MI VERRÀ CHIESTO DI FARE?
A seconda di come si svolgeranno le dinamiche di formazione all'interno del MOOC, in questa nuova e accattivante veste di apprendimento a distanza, vedrai che il tuo impegno si limiterà solo a prendere seriamente la decisione di iniziare e terminare il corso in maniera attiva e assidua.

Se deciderai di volerti rendere disponibile per la sperimentazione, potrai essere contattato
DA CHI?
da uno dei docenti erogatori del corso e/o da un ricercatore che è una studentessa di dottorato
PER FARE COSA?
per eventuali interviste, che potranno essere telefoniche, su skype o in presenza, a seconda delle tue preferenze
QUANDO?
sempre previo avviso e prevediamo di fare un'intervista durante la fase intermedia del corso, una nella fase finale e almeno un'altra dopo un paio di mesi dalla fine del corso.
QUAL È IL NOSTRO OBIETTIVO?
Quello di valutare il tuo ruolo nella figura di

- Studente, durante le fasi di apprendimento, ovvero quando frequenti il corso on-line;
- Docente che viene formato e sperabilmente arricchito da tale corso, vedendo quindi come hai beneficiato del materiale offerto e di come intendi farne uso all'interno del contesto classe o quando progetti delle attività per i tuoi studenti.

PRIVACY
Nel momento in cui deciderai di compilare il sondaggio, ti verrà richiesto di indicare i tuoi dati anagrafici, un tuo indirizzo e-mail, più il nickname che utilizzerai per inviare commenti alle chat a cui prenderai parte. Il tuo nickname ci permetterà di collegare le tue risposte con la tua attività all'interno della comunità.
Tutte le informazioni fornite saranno mantenute riservate. Il ricercatore non userà le informazioni personali per scopi al di fuori di questo progetto di ricerca. Inoltre, il ricercatore non includerà il tuo nome o qualsiasi altra cosa che potrebbe identificarti nelle relazioni di studio. I dati saranno custoditi al sicuro da una password di protezione e verranno conservati per un periodo di almeno 5 anni, come richiesto dall'Università.

Si prega di stampare o salvare questo modulo di consenso.
Dichiarazione di consenso:
Ho letto le informazioni di cui sopra e sento di comprendere i motivi dello studio, tanto da poter prendere una decisione riguardo al mio coinvolgimento cliccando sul link in basso. Capisco che sto accettare i termini sopra descritti.

19. **ACCETTAZIONE** *

Desideri aderire alla sperimentazione?
Contrassegna solo un ovale.

- SI
- NO *Interrompi la compilazione del modulo.*

Ricerca Sperimentale

20. *

Nome:

21. *

Cognome:

22. *

Città:

23. Provincia:

24. *

Nazione:

25. *

Scrivi il tuo indirizzo e-mail per permetterci di metterci in contatto con te:

26. *

Conferma il tuo indirizzo e-mail:

27. Vuoi lasciarci un commento:

Ti ringraziamo sin da ora per la tua collaborazione, il gruppo di ricerca in Didattica della Matematica di Torino

Per ogni dubbio o richiesta di ulteriori informazioni, potrai contattarci al seguente indirizzo: moocdidattica.dm@unito.it

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QUESTIONARIO INTERMEDIO

Caro insegnante
ti chiediamo 10 minuti del tuo tempo per rispondere ad un breve questionario sulla tua esperienza fin qui maturata con il MOOC.
I tuoi dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy.
I risultati saranno analizzati e presentati solamente in forma aggregata e saranno resi disponibili successivamente a quanti concluderanno il MOOC.
Grazie in anticipo per la tua disponibilità!

*Campo obbligatorio

ANAGRAFICA

1. **Nome** *

2. **Cognome** *

3. **Genere** *
Contrassegna solo un ovale.
- Uomo
- Donna

4. **Età** *

5. **Città** *

6. **Regione** *

LA TUA ESPERIENZA CON MATH MOOC UNITO

7. **1. Rispetto ad altri MOOC che hai frequentato, o se comunque questo è il primo che segui, ritieni che sia di facile utilizzo?** *
Contrassegna solo un ovale.
- Si
- No
- Altro: ____________

8.
Cosa pensi riguardo alle video interviste dei Prof. Robutti e Arzarello che hai visto? Lascia un tuo commento. *

9.
Cosa pensi riguardo ai video di introduzione ai moduli realizzati con PowToon? Lascia un commento. *

10.
Conoscevi il software di presentazione PowToon? *
Contrassegna solo un ovale.

- Si
- No

11.
Pensi di voler provare ad usare PowToon nella tua didattica (per le tue lezioni, per farlo conoscere e usare agli studenti, …)? *
Contrassegna solo un ovale.

- Si
- No
- Altro: ____________

12.
Pensi che le piattaforme di interazione sociale connesse al MOOC (forum, bacheca padlet, tricider, canale twitter) ti consentono di:
(dove: 1 = totalmente in disaccordo e 5 = totalmente d'accordo)
Contrassegna solo un ovale per riga.

	1	2	3	4	5
esprimere le tue opinioni personali riguardo i contenuti del corso	◯	◯	◯	◯	◯
consentire un dialogo di confronto tra colleghi	◯	◯	◯	◯	◯
avere la possibilità di beneficiare di esperienze/modi di pensare altrui	◯	◯	◯	◯	◯
Altro (specificare sotto, al più due)	◯	◯	◯	◯	◯

13.
Altro 1:

14. **Altro 2:**

15. **Pensi che il MOOC sia flessibile in termini di** *

(dove: 1 = totalmente in disaccordo e 5 = totalmente d'accordo)
Contrassegna solo un ovale per riga.

	1	2	3	4	5
Spazio (le lezioni sono accessibili ovunque grazie a laptop e mobile device e non necessariamente in una determinata aula)	◯	◯	◯	◯	◯
Tempo (seguire il corso quando si vuole, pianificando al meglio il proprio tempo libero)	◯	◯	◯	◯	◯
Ritmo di apprendimento (poter decidere il proprio ritmo di apprendimento)	◯	◯	◯	◯	◯

16. **Hai già trasposto, almeno in parte, quanto appreso nel MOOC nelle tue pratiche didattiche?** *

Contrassegna solo un ovale.

- ◯ Si
- ◯ No *Interrompi la compilazione del modulo.*

17. **Quali argomenti hai già trasposto nelle tue pratiche didattiche?** *

Seleziona tutte le voci applicabili.

- ☐ Albero maestro
- ☐ Rampe, fili e giravolte
- ☐ Adattamento "L'albero maestro con riferimento alle indicazioni nazionali"
- ☐ Il Parco - adattamento per il biennio liceo scientifico
- ☐ Dalla piegatura della carta a GeoGebra - adattamento per il primo biennio
- ☐ L'orologio
- ☐ Muoversi con gli angoli
- ☐ Dall'astronomia alla trigonometria
- ☐ Orologi, girandole e pattinatori
- ☐ Lo spettacolo di Natale
- ☐ Ma quanto sarà ripido

18. **Con quali innovazioni rispetto alle tue pratiche precedenti?**

19. **Quali pregi e difetti trovi nelle proposte di cui hai fruito, sulla base di queste esperienze didattiche che hai fatto?**

QUESTIONARIO FINALE

Cari insegnanti
vi chiediamo 20 minuti del vostro tempo per rispondere al questionario finale del MOOC che tende a valutare non solo il livello di gradimento del corso, ma anche ad analizzare il profilo di voi primi (e speriamo non ultimi) corsisti che avete intrapreso un percorso di formazione on-line.
I dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata e saranno resi disponibili successivamente a quanti concluderanno il MOOC.
Grazie in anticipo per la vostra disponibilità e soprattutto per essere giunti sin qui!

*Campo obbligatorio

ANAGRAFICA

1. **Nome** *

2. **Cognome** *

3. **Genere** *
   Contrassegna solo un ovale.
   - Uomo
   - Donna

4. **Età** *

5. **Città** *

6. **Provincia** *

7. **Regione** *

PROJECT WORK & PEER REVIEW

8. **Prima di utilizzarlo, conoscevi lo strumento Learning Designer?** *
Contrassegna solo un ovale.

- Si
- No

9. **Ritieni che questo strumento web-based ti sia stato utile per creare e organizzare una serie di attività di insegnamento/apprendimento (TLA) e ti ha supportato nella progettazione di una serie di esperienze per portare lo studente verso gli obiettivi di apprendimento, così come è stato ideato dal London Institute of Education?** *
Contrassegna solo un ovale.

- Decisamente no
- Più no che sì
- Più sì che no
- Decisamente sì

10. **Motiva la precedente risposta.**

11. **In che misura e perché la valutazione tra pari (peer review) è stata importante per me?** *

12. **Quali sono stati gli aspetti più difficili che ho trovato nell'effettuare la peer review?** *
(puoi selezionare anche più di un'opzione)
Seleziona tutte le voci applicabili.

- Vincoli di tempo
- Mancanza di conoscenza dei contenuti espressi dal collega
- Visione diversa circa la cultura della scuola

BACHECHE DI COMUNICAZIONE

13. **Quando POSTAVI sulle bacheche di comunicazione (forum, padlet, tricider), a chi ti rivolgevi o a chi erano indirizzati i tuoi commenti di solito?** *

Seleziona tutte le voci applicabili.

- [ ] Al team del MOOC
- [ ] Ai corsisti del MOOC che conoscevi
- [ ] A tutti i corsisti del MOOC (conoscenti e non)
- [ ] Altro: ____________________

14. **Quando RISPONDEVI sulle bacheche di comunicazione (forum, padlet, tricider), a chi ti rivolgevi o a chi erano indirizzati i tuoi commenti di solito?** *

Seleziona tutte le voci applicabili.

- [ ] All’autore del messaggio iniziale
- [ ] All’audience formatosi su quella discussione
- [ ] All’intero audience del MOOC
- [ ] Al team del MOOC
- [ ] Altro: ____________________

15. **Qual è la percentuale di tutti i post che hai letto?** *

Contrassegna solo un ovale.

- ( ) Non ho letto post
- ( ) Dall’1% al 20% di tutti i post
- ( ) Dal 21% al 40% di tutti i post
- ( ) Dal 41% al 60% di tutti i post
- ( ) Dal 61% al 80% di tutti i post
- ( ) Maggiore dell’80% di tutti i post

16. **In che percentuale i tuoi post sono stati commenti fatti su post scritti da altri utenti MOOC, anche solo per far riconoscere che li avevi letti?** *

Contrassegna solo un ovale.

- ( ) Non ho mai commentato i post di nessuno
- ( ) Tra l’1% e il 20% sul totale dei miei post
- ( ) Tra il 21% e il 40% sul totale dei miei post
- ( ) Tra il 41% e il 60% sul totale dei miei post
- ( ) Tra il 61% e l’80% sul totale dei miei post
- ( ) Più dell’80% dei miei post

17. **Nelle bacheche di comunicazione (forum, padlet, tricider), secondo te, c’è stato qualche partecipante che è spiccato come leader della discussione?** *

Contrassegna solo un ovale.

- ( ) Si
- ( ) No

18. **Nelle bacheche di comunicazione (forum, padlet, tricider), ti è mai capitato di cercare messaggi di particolari persone?** *
Contrassegna solo un ovale.

- Si *Passa alla domanda 19.*
- No *Passa alla domanda 20.*

19. **Perchè?**

BACHECHE DI COMUNICAZIONE

20. **Quali sono state, a tuo parere, le discussioni più interessanti che sono emerse? (Puoi fare riferimento ai moduli del MOOC).**

21. **Ritieni che il processo di scrittura di un post e il ricevere o fare commenti è stato vantaggioso per te? In che senso?**

22. **Ti è mai capitato di scrivere un post e poi eliminarlo prima di pubblicarlo?** *
Contrassegna solo un ovale.

- Si
- No

23. **Se assumiamo che la partecipazione è correlata al numero di post presenti in una bacheca di comunicazione, secondo te nelle bacheche:** *
Contrassegna solo un ovale.

- C'erano troppi post e commenti
- C'erano una giusta quantità di post e commenti
- C'erano troppi pochi post e commenti

Non ho mai preso parte alle bacheche di comunicazione

24.
È capitato che hai scritto qualcosa e nessuno ha risposto al tuo post oppure che ci sia voluto molto tempo prima che qualcuno rispondesse al tuo post? *
Contrassegna solo un ovale.

- Si *Passa alla domanda 25.*
- No *Passa alla domanda 26.*

25.
Se si, questo ti ha portato a non postare più? *
Contrassegna solo un ovale.

- Si
- No
- Altro: ____________________

BACHECHE DI COMUNICAZIONE

26.
Quanto è importante per te che gli altri leggano i tuoi messaggi? *
Contrassegna solo un ovale.

- Davvero poco importante
- Poco importante
- Né poco né molto importante
- Molto importante
- Davvero molto importante

27.
Per ogni azione sotto riportata, riferita all'uso delle bacheche di comunicazione, indica il livello di importanza che gli assegneresti *
(dove: 1 = davvero poco importante e 5 = davvero molto importante)
Contrassegna solo un ovale per riga.

	1	2	3	4	5
Leggere					
Postare					
Commentare					

COMUNITÀ

28.
In che misura ti senti simile o diverso rispetto agli altri partecipanti che hai incontrato in questo MOOC?

____
____
____
____
____

29. **Quali somiglianze o differenze tra te e gli altri ritieni che ti abbiano spinto a frequentare questo MOOC?** *

30. **Come partecipante del MOOC, fino a che punto ti sentivi di essere parte di una comunità?** *

31. **Ritieni che i partecipanti del MOOC abbiano contribuito alla tua formazione?** *
Contrassegna solo un ovale.

- Si
- No

32. **Hai interagito con qualche partecipante del MOOC in sedi diversa dalle bacheche di comunicazione? Seleziona tutte le risposte pertinenti.** *
Seleziona tutte le voci applicabili.

- No
- Email
- Facebook
- Chat
- Skype
- Di persona
- Altro:

33. **Se venisse offerta la possibilità di partecipare a un altro MOOC, su un nucleo matematico diverso da Geometria, ripeteresti questa esperienza (ti iscriveresti nuovamente)?** *
Contrassegna solo un ovale.

- Si
- No

34. **Dopo la fine del MOOC, se ci fosse la possibilità di far continuare le discussioni intraprese con i tuoi colleghi del MOOC, pensi che ne prenderesti parte?** *
Contrassegna solo un ovale.

- Si
- No

MOOC

35.
Perché ti sei iscritto al MOOC? *

Contrassegna solo un ovale.

- Sentivi una qualche esigenza di formazione
- Pensavi che sarebbe stato un corso di aggiornamento
- Volevi provare l'esperienza di un corso a distanza
- Altro: ____________________

36.
I contenuti del MOOC hanno corrisposto alle sue aspettative? *

(puoi lasciare eventuali commenti alla voce Altro)

Contrassegna solo un ovale.

- Si
- Parzialmente
- No
- Altro: ____________________

37.
Come giudichi la durata del corso rispetto agli argomenti trattati? *

Contrassegna solo un ovale.

- Insufficiente
- Buona
- Eccessiva

38.
Qual è la tua opinione sull'efficacia dei metodi didattici impiegati nel corso? *

Contrassegna solo un ovale.

- Soddisfacenti
- Poco soddisfacenti
- Insoddisfacenti

39.
Come valuti il materiale didattico fornito? *

Contrassegna solo un ovale.

- Utile
- Poco utile
- Inutile

40.
Vorremmo una tua valutazione sul contenuto delle attività e sulla chiarezza espositiva delle stesse: *

Contrassegna solo un ovale per riga.

	Scarso	Sufficiente	Buono	Ottimo
Contenuto				
Esposizione				

41. **Quanto ritengo di essermi impegnato nella partecipazione al MOOC?** *

Contrassegna solo un ovale.

- Decisamente poco
- Né poco né molto
- Decisamente molto

42. **Essere stato coinvolto in questo corso è stato UTILE perché…**

43. **Essere stato coinvolto in questo corso è stato INUTILE perché…**

44. **Per l'esperienza che ne hai fatto, pensi che affiancare un MOOC ai metodi di insegnamento tradizionali possa avere una ricaduta positiva sulla motivazione dei tuoi studenti? (ad esempio potresti considerare una flipped classroom in cui gli studenti, prima della lezione, imparano teoria e contenuti tramite MOOCs. Durante le ore in aula tu potresti concentrarti sul risolvere incomprensioni, esercizi o discussione di case studies).** *

Contrassegna solo un ovale.

- Decisamente no
- Più no che sì
- Più sì che no
- Decisamente sì

45. **A seguito della tua partecipazione al MOOC, hai fatto/notato dei cambiamenti nelle tue pratiche didattiche?** *

Contrassegna solo un ovale.

- Si *Passa alla domanda 46.*
- No *Passa alla domanda 47.*

46. **Se si, seleziona quelle in cui ti rivedi maggiormente:** *

(oppure aggiungi un commento alla voce Altro)

Contrassegna solo un ovale.

- Interagisco con nuovi strumenti e uso nuove strategie
- Sto realizzando in classe i percorsi esaminati nel MOOC, declinati secondo la didattica laboratoriale
- Uso i contenuti del corso per uno sviluppo professionale tra colleghi
- Altro: ____________________

MOOC

47. **Vuoi riferirci episodi significativi relativi alla tua partecipazione al MOOC?**

____

____

____

____

____

48. **Vuoi riferirci episodi significativi accaduti in classe a seguito della tua partecipazione al MOOC?**

____

____

____

____

____

MATERIALI

49. **Quanto hai usufruito dei materiali (ad esempio, video, proposte di attività, software, …) forniti in questo MOOC?** *

Contrassegna solo un ovale.

- il 20% o anche meno
- tra il 21% e il 40%
- tra il 41% e il 60%
- tra il 61% e l'80%
- più dell'80%

50. **Come hai affrontato le attività presenti in ciascun modulo?** *

(puoi selezionare più di un'opzione)

Seleziona tutte le voci applicabili.

- [ ] Hai stampato
- [ ] Hai salvato file
- [ ] Hai preso appunti

☐ Altro: ______________

51.
Hai usato (o hai intenzione di usare) materiali che altri insegnanti hanno messo a disposizione su questo MOOC? *
Contrassegna solo un ovale.

- Si
- No

52.
Hai apprezzato l'idea lanciata dal Prof. Arzarello sulla creazione di un GeoGebrabook inedito del MOOC? Se si o se no, perché?

53.
Nelle bacheche di comunicazione ti è capitato di cercare post relativi a materiali specifici? *
Contrassegna solo un ovale.

- Si
- No

54.
Considerando le tue interazioni con i materiali e gli altri partecipanti del MOOC, quali esperienze di apprendimento ritieni che ti siano state più utili? Come mai?

55.
Saresti disposto a contribuire a questo studio partecipando a un colloquio volontario sulle interazioni dei partecipanti con i materiali e con gli altri utenti MOOC? *
Contrassegna solo un ovale.

- Si *Passa alla domanda 56.*
- No *Passa alla domanda 60.*

56.
Se sì, per favore, inserisci di seguito le tue informazioni personali e ci metteremo in contatto con te. *
NOME

57.
COGNOME *

58.
E-MAIL *

59.
CONFERMA E-MAIL *

SUGGERIMENTI

60.
Riporta eventuali suggerimenti e commenti per migliorare l'organizzazione del corso:

Grazie per aver completato il questionario. Sperando che conserverai un piacevole ricordo di questa avventura nel MOOC, ti salutiamo calorosamente! Il team del MOOC.

Questionario iniziale

Caro insegnante
ti chiediamo 10 minuti del tuo tempo per rispondere ad un breve questionario, in modo da permetterci di conoscerti meglio.
I tuoi dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata e saranno resi disponibili successivamente a quanti concluderanno il MOOC.
Grazie in anticipo per la tua disponibilità!

*Campo obbligatorio

1. **Indirizzo email** *

Anagrafica

2. **Nome** *

3. **Cognome** *

4. **Sesso** *
Contrassegna solo un ovale.
   - Maschio
   - Femmina

5. **Età (scrivere solo il numero, es: 25)** *

6. **Stato** *
Contrassegna solo un ovale.
   - Italia
   - Altro: ____

7. **Regione** *
Contrassegna solo un ovale.

- Valle d'Aosta
- Piemonte
- Liguria
- Lombardia
- Trentino-Alto Adige
- Veneto
- Friuli-Venezia Giulia
- Emilia Romagna
- Toscana
- Umbria
- Marche
- Lazio
- Abruzzo
- Molise
- Campania
- Puglia
- Basilicata
- Calabria
- Sicilia
- Sardegna

8. **Città (di attuale domicilio)**

____

9. **In quale ordine di scuola insegni?** *
Seleziona tutte le voci applicabili.

- [ ] Scuola primaria
- [ ] Scuola secondaria di primo grado
- [ ] Biennio - Scuola secondaria di secondo grado
- [ ] Triennio – Scuola secondaria di secondo grado
- [ ] Università
- [ ] Altro: _______________________________

10. **Da quanto tempo insegni?** *
Contrassegna solo un ovale.

- Meno di 1 anno
- 1-5 anni
- 6-10 anni
- 11-15 anni
- 16-20 anni
- 21-25 anni
- 26-30 anni
- Più di 30 anni
- Altro: ____________________

11. **Quante classi hai quest'anno?** *

____
____
____
____
____

Formazione online

12. **Hai già frequentato un/dei MOOC?** *
Seleziona tutte le voci applicabili.

- [ ] Si, in lingua inglese completandolo/i
- [ ] Si, in lingua inglese senza completarlo/i
- [ ] Si, in lingua italiana completandolo/i
- [ ] Si, in lingua italiana senza completarlo/i
- [ ] No

13. **Ti eri iscritto al nostro precedente "MOOC Geometria" (Ottobre 2015 - Gennaio 2016)?** *
Contrassegna solo un ovale.

- Si
- No

14. **Perché ti sei iscritto a questo "MOOC Numeri"?** *
Contrassegna solo un ovale.

- Senti esigenza di formazione
- Vuoi provare l'esperienza di un corso a distanza
- Hai seguito il "MOOC Geometria" e sei interessato/a a questa nostra nuova offerta formativa
- Altro: ____________________

Metodologie e tecnologie

15.
Attui didattica laboratoriale nelle tue classi? *
Contrassegna solo un ovale.

- Si
- No
- Non conosco questa metodologia

16.
Utilizzi la metodologia MERLO nelle tue classi? *
Contrassegna solo un ovale.

- Si
- No *Passa alla domanda 17.*
- Non conosco questa metodologia *Passa alla domanda 17.*

17.
Quali sono i vantaggi che trai nell'usare la metodologia MERLO (da parte tua e/o degli studenti)? *
Contrassegna solo un ovale.

- inserire nella pratica didattica l'uso di diverse rappresentazioni semiotiche di uno stesso concetto
- abituare gli allievi all'argomentazione, alla discussione tra pari e al confronto
- Altro: ______________________

18.
Utilizzi attività pratico-manipolative per presentare concetti matematici ai tuoi studenti? *
Contrassegna solo un ovale.

- Si
- No *Passa alla domanda 19.*

19.
Quali sono i vantaggi che trai nell'usare attività pratico-manipolative (da parte tua e/o degli studenti)? (max 3 opzioni di risposta) *
Seleziona tutte le voci applicabili.

- [ ] evitare formazione di misconcetti
- [ ] valorizzare il formulare congetture
- [ ] valorizzare la fase di scoperta di proprietà
- [ ] stimolare l'argomentazione
- [ ] stimolare la manualità degli studenti
- [ ] quando i ragazzi fanno esperienze di apprendimento usando il proprio corpo ricordano più a lungo
- [ ] Altro: ______________________

20. **I problem solving che fai fare in classe prendono spunto da situazioni legate alla realtà?** *

Contrassegna solo un ovale.

- Si
- No
- Non faccio fare problem solving

21. **Utilizzi le tecnologie nelle tue pratiche di insegnamento?** *

Contrassegna solo un ovale.

- Poco
- Abbastanza
- Molto

22. **Quali sono, secondo te, i vantaggi e i benefici dell'uso della tecnologia nelle ore di matematica? (max 3 opzioni di risposta)** *

Seleziona tutte le voci applicabili.

- [ ] Migliora l'attitudine ad aiutarsi tra compagni
- [ ] Favorisce l'interdisciplinarietà
- [ ] Accresce la motivazione
- [ ] Stimola forme di apprendimento diverse come la creatività
- [ ] Consente una maggiore comprensione dei concetti
- [ ] Altro: ______________________

23. **Riesci a risolvere (o a fare risolvere ai tuoi alunni) problemi ricorrendo a modellizzazioni create con software specifici?** *

Contrassegna solo un ovale.

- Si
- In parte
- No

24. **Le disponibilità tecnologiche del tuo istituto scolastico ti permettono di usare software specifici durante le tue ore di lezione?** *

Contrassegna solo un ovale.

- Si
- No

25. **Quali sono gli ostacoli? (puoi selezionare anche più di una risposta)** *
Seleziona tutte le voci applicabili.

- [ ] non ci sono ostacoli
- [ ] l'aula informatica non è sempre disponibile
- [ ] l'aula informatica ha attrezzature obsolete
- [ ] non è consentito far portare il tablet/laptop agli studenti
- [ ] non disponiamo della LIM in classe
- [ ] non disponiamo di adeguato collegamento internet
- [ ] Altro: ____________________

26.
Quali software specifici usi nello svolgere la tua professione di insegnante di matematica? Elenca i nomi e brevemente spiegane per ciascuno il perché. *

____
____
____
____
____

27.
Qual è la tua esperienza nell'uso di GeoGebra? *
Contrassegna solo un ovale.

- ( ) Nessuna esperienza
- ( ) Poca esperienza (ho già utilizzato una applet di GeoGebra nella mia classe)
- ( ) Utente intermedio di GeoGebra (ho già utilizzato alcune applet di GeoGebra nella mia classe)
- ( ) Utente esperto di GeoGebra (uso regolarmente le applet di GeoGebra nella mia classe)

28.
Come usi GeoGebra in classe? *
Contrassegna solo un ovale per riga.

	Mai	Meno di una volta al mese	Una volta al mese	Meno di una volta a settimana, ma più di una volta al mese	Una volta a settimana	Più di una volta a settimana	Tutti i giorni
Come uno "strumento di presentazione" - Proietto i materiali di GeoGebra in classe	( )	( )	( )	( )	( )	( )	( )
Come uno "strumento attivo nel laboratorio di informatica"- Gli studenti lavorano in modo indipendente su materiali dinamici nell'aula informatizzata	( )	( )	( )	( )	( )	( )	( )

29. **In quale misura, nella tua pratica didattica, hai già utilizzato materiali dinamici creati con GeoGebra rispetto ai seguenti aspetti? ***

Contrassegna solo un ovale per riga.

	Mai	Una volta	Tra due e cinque volte	Tra sei e dieci volte	Più spesso
Apprendimento per scoperta					
Presentare contenuti					
Visualizzazione					
Modellizzazione					
Calcoli					
Risoluzione di problemi					
Discussione di problemi					
Comprensione di concetti matematici					
Ostacolare misconcetti					
Sperimentazione					
Approfondire					
Problemi legati a contesti reali					
Dimostrazione					

30. **Intendiamo per Problema Aperto "una situazione in cui l'allievo è invitato ad esplorare, a formulare congetture su quali ipotesi implichino conseguenze interessanti nella situazione in esame (e a farne una dimostrazione)". In classe, lavori con problemi aperti? ***

Contrassegna solo un ovale.

- Si, con Geogebra
- Si, senza Geogebra
- Si, con e senza Geogebra
- No *Passa alla domanda 31.*

31. **Quali sono i vantaggi che trai nell'usarli (da parte tua e/o degli studenti)? (max 3 opzioni di risposta) ***

Seleziona tutte le voci applicabili.

- [ ] favorire l'approccio al congetturare, all'argomentare, al dimostrare
- [ ] dare ai ragazzi la possibilità di collaborare in gruppo
- [ ] dare ai ragazzi la possibilità di risolvere un compito da soli, senza la presenza "invasiva" dell'insegnante
- [ ] far ragionare i ragazzi su situazioni legate alla realtà
- [ ] l'aiuto del software favorisce la modellizzazione della realtà
- [ ] far sperimentare ai ragazzi un uso sensato della tecnologia
- [ ] Altro: ______________________

Sperimentazione

Gent.mo/a insegnante
Prima di rispondere ti chiediamo di leggere con cura (se non lo hai già fatto) l'avviso sulla sperimentazione che trovi nel modulo introduttivo.

Per ogni dubbio o richiesta di ulteriori informazioni, potrai contattarci al seguente indirizzo: moocdidattica.dm@unito.it

Ti ringraziamo sin da ora per la tua collaborazione,
il gruppo di ricerca in Didattica della Matematica di Torino

32. **Desideri aderire alla sperimentazione?** *

Contrassegna solo un ovale.

- Si
- No

33. **Desideri lasciarci un commento**

Una copia delle risposte verrà inviata via email all'indirizzo fornito

Questionario intermedio

Caro insegnante
ti chiediamo 15 minuti del tuo tempo per rispondere ad un breve questionario sulla tue esperienza fin qui maturata con il MOOC.
I tuoi dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata e saranno resi disponibili successivamente a quanti concluderanno il MOOC.
Grazie in anticipo per la tua disponibilità!

*Campo obbligatorio

1. **Indirizzo email** *

Anagrafica

2. **Nome** *

3. **Cognome** *

4. **Sesso** *

   Contrassegna solo un ovale.

   - Maschile
   - Femminile

5. **Età (scrivere solo il numero, es: 25)** *

6. **Regione** *
Contrassegna solo un ovale.

- Valle d'Aosta
- Piemonte
- Liguria
- Lombardia
- Trentino-Alto Adige
- Veneto
- Friuli-Venezia Giulia
- Emilia Romagna
- Toscana
- Umbria
- Marche
- Lazio
- Abruzzo
- Molise
- Campania
- Puglia
- Basilicata
- Calabria
- Sicilia
- Sardegna

7. **In quale ordine di scuola insegni?** *
Seleziona tutte le voci applicabili.

- Scuola primaria
- Scuola secondaria di primo grado
- Biennio - Scuola secondaria di secondo grado
- Triennio – Scuola secondaria di secondo grado
- Università
- Altro

8. **Hai già seguito o seguirai la formazione del PNSD in qualità di** *
Contrassegna solo un ovale.

- Animatore Digitale
- Uno dei 3 docenti del team dell'innovazione
- Uno dei 10 docenti individuati dal tuo istituto
- Non seguirò nessuna formazione

Il MOOC Numeri

9. **Rispetto ad altri MOOC che hai frequentato, o se comunque questo è il primo che segui, ritieni che sia di facile utilizzo?** *
Contrassegna solo un ovale.

- Decisamente sì
- Più sì che no
- Più no che sì
- Decisamente no

10. **Eventuali commenti in merito alla risposta precedente.**

Tutorial

Dai un voto (1=pessimo, ..., 5=eccellente) ai tutorial che sono stati messi a disposizione nella sezione tutorial

11. **Tutorial maker della mappa** *
Contrassegna solo un ovale.

	1	2	3	4	5	
pessimo						eccellente

12. **Tutorial Sway** *
Contrassegna solo un ovale.

	1	2	3	4	5	
pessimo						eccellente

13. **Tutorial stampare il libro contenente gli Sway** *
Contrassegna solo un ovale.

	1	2	3	4	5	
pessimo						eccellente

14. **Tutorial assegnazione badge** *
Contrassegna solo un ovale.

	1	2	3	4	5	
pessimo						eccellente

15. **Tutorial Learning Designer** *

Contrassegna solo un ovale.

	1	2	3	4	5	
pessimo	◯	◯	◯	◯	◯	eccellente

16. **Vuoi lasciare un commento in merito ai tutorial?**

____
____
____
____
____

Risorse tecnologiche

All'inizio di ogni modulo, come avrai notato, ci sono risorse pensate per guidarti nell'affrontare il modulo. Giudica la loro utilità/efficacia (1= decisamente inutile/inefficace, …, 5=decisamente utile/efficace)

17. **PowToon** *

Contrassegna solo un ovale.

	1	2	3	4	5	
decisamente inutile/inefficace	◯	◯	◯	◯	◯	decisamente utile/efficace

18. **Tabella riassuntiva** *

Contrassegna solo un ovale.

	1	2	3	4	5	
decisamente inutile/inefficace	◯	◯	◯	◯	◯	decisamente utile/efficace

19. **Video dell'esperto** *

Contrassegna solo un ovale.

	1	2	3	4	5	
decisamente inutile/inefficace	◯	◯	◯	◯	◯	decisamente utile/efficace

20. **Vuoi lasciare un commento rispetto alla domanda precedente?**

____
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____
____

21.
Conoscevi il software di presentazione PowToon? *

Contrassegna solo un ovale.

- Sì
- No

22.
Pensi di voler provare ad usare PowToon nella tua didattica (per le tue lezioni, per farlo conoscere e usare agli studenti, …)? *

Contrassegna solo un ovale.

- Lo uso già
- Sì
- No
- Altro: ____________

23.
Motiva la tua precedente risposta. *

24.
Come hai potuto notare, le attività sono state presentate utilizzando l'app di Windows, Sway. La conoscevi prima d'ora? *

Contrassegna solo un ovale.

- Sì
- No

25.
Pensi di voler provare ad usare Sway nella tua didattica (per le tue lezioni, per farlo conoscere e usare agli studenti, …)? *

Contrassegna solo un ovale.

- Lo uso già
- Sì
- No
- Altro: ____________

26.
Motiva la tua precedente risposta. *

Webinar

27. **Rispetto ai webinar che si sono tenuti fino a questo momento (relatori: Prof.ssa Robutti e Dott.ssa Coviello), li hai seguiti (puoi selezionare più di un'opzione di risposta):** *

Seleziona tutte le voci applicabili.

- [ ] Entrambi in modalità sincrona (cioè collegandoti alla camera nel giorno e ora stabiliti)
- [ ] Entrambi in modalità asincrona (cioè vedendone la registrazione)
- [ ] Uno in modalità sincrona e uno in modalità asincrona
- [ ] Nonostante li abbia seguiti in sincrono, li ho voluti rivedere in asincrono
- [ ] Non ho ancora avuto modo di prenderne visione
- [ ] Altro: ____________________

28. **Se ne hai seguito almeno uno in sincrono, ti sei trovato a tuo agio con la camera BigBlueButton?**

Contrassegna solo un ovale.

- ( ) Sì
- ( ) No

29. **Perchè?**

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30. **Se ne hai seguito almeno uno (in sincrono o asincrono), scrivi 3 aggettivi che ci permettano di capire il tuo giudizio su di essi.** *

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Bacheche di comunicazione

31. **Pensi che gli strumenti e le piattaforme di interazione sociale connesse al MOOC (forum, bachecha padlet, tricider, canale twitter) ti consentano di: (1 = totalmente in disaccordo, …, 5 = totalmente d'accordo)**
Contrassegna solo un ovale per riga.

	1	2	3	4	5
Esprimere le tue opinioni personali riguardo i contenuti del corso	◯	◯	◯	◯	◯
Consentire un dialogo di confronto tra colleghi	◯	◯	◯	◯	◯
Avere la possibilità di beneficiare di esperienze/modi di pensare altrui	◯	◯	◯	◯	◯
Sviluppano il senso di appartenenza ad una comunità	◯	◯	◯	◯	◯
ALTRO (da specificare di seguito)	◯	◯	◯	◯	◯

32.
Se hai aggiunto ALTRO, specifica qui la voce che vorresti aggiungere e il voto che ne dai da 1 a 5.

33.
Quale tra le seguenti bacheche di comunicazione è quella che preferisci? *
Contrassegna solo un ovale.

- ◯ Forum
- ◯ Padlet
- ◯ Tricider
- ◯ Twitter

34.
Motiva la tua risposta precedente. *

35.
Conoscevi la bacheca di comunicazione Padlet? *
Contrassegna solo un ovale.

- ◯ Sì
- ◯ No

36. **Pensi di voler provare ad usare Padlet nella tua didattica (per le tue lezioni, per farlo conoscere e usare agli studenti, …)?** *

Contrassegna solo un ovale.

- Lo uso già
- Si
- No
- Altro: ____________________

37. **Motiva la tua risposta precedente.** *

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38. **Conoscevi la bacheca di comunicazione Tricider?** *

Contrassegna solo un ovale.

- Sì
- No

39. **Pensi di voler provare ad usare Tricider nella tua didattica (per le tue lezioni, per farlo conoscere e usare agli studenti, …)?** *

Contrassegna solo un ovale.

- Lo uso già
- Si
- No
- Altro: ____________________

40. **Motiva la tua risposta precedente.** *

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41. **Pensi che il MOOC sia flessibile in termini di (1 = totalmente in disaccordo, …, 5 = totalmente d'accordo):** *
Contrassegna solo un ovale per riga.

	1	2	3	4	5
Spazio (le lezioni sono accessibili ovunque grazie a laptop e mobile device e non necessariamente in un determinato luogo)	◯	◯	◯	◯	◯
Tempo (seguire il corso quando si vuole, pianificando al meglio il proprio tempo libero)	◯	◯	◯	◯	◯
Ritmo di apprendimento (poter decidere il proprio ritmo di apprendimento)	◯	◯	◯	◯	◯

Con la/le tua/e classe/i

42. **Hai già trasposto, almeno in parte, quanto appreso nel MOOC nelle tua pratiche didattiche?** *
Contrassegna solo un ovale.

- ◯ Sì
- ◯ No *Passa alla domanda 52.*
- ◯ Altro: ______________________ *Passa alla domanda 52.*

Quali argomenti hai già trasposto (anche in parte) in classe?

43. **Modulo 1 (I grado)**
Seleziona tutte le voci applicabili.

- ☐ Il meteorite
- ☐ Infinitamente piccolo

44. **Modulo 1 (II grado)**
Seleziona tutte le voci applicabili.

- ☐ Il livello del mare
- ☐ Che senso hanno questi numeri?
- ☐ Tecnologie e numeri

45. **Modulo 2**
Seleziona tutte le voci applicabili.

- ☐ Schede MERLO

46. **Modulo 3**
Seleziona tutte le voci applicabili.

- ☐ Quesiti INVALSI

47. **Modulo 4 (I grado)**

Seleziona tutte le voci applicabili.

- [ ] Salire le scale
- [ ] Le diagonali di un poligono
- [ ] I multipli di 3
- [ ] Numeri quadrati, triangolati, Gauss
- [ ] Il triangolo di Sierpinski

48. **Modulo 4 (II grado)**

Seleziona tutte le voci applicabili.

- [ ] L'induzione in matematica: dagli anagrammi alla torre di Hanoi
- [ ] Gioco-problema
- [ ] Matematica e quattrini
- [ ] Approssimazione

49. **Con quali innovazioni rispetto alle tue pratiche precedenti?** *

50. **Quali pregi (+) e/o difetti (-) trovi nelle proposte che hai fruito sulla base di queste esperienze didattiche che hai fatto?** *

51. **Dopo aver riproposto (anche solo in parte) una/delle attività, hai sentito la necessità/voglia di raccontare o condividere con i tuoi colleghi del MOOC quanto verificatosi in classe?** *

Contrassegna solo un ovale.

- ( ) Sì
- ( ) No

52. **Se lo hai fatto, indica ordinatamente: Modulo X, Nome della bacheca in cui hai scritto (forum, padlet, tricider, twitter), ho ricevuto/non ho ricevuto commenti dagli altri colleghi**

Commenti e/o suggerimenti

53. **Desideri lasciare un commento/suggerimento?**

Una copia delle risposte verrà inviata via email all'indirizzo fornito

QUESTIONARIO FINALE

Cari insegnanti
vi chiediamo 20 minuti del vostro tempo per rispondere al questionario finale del MOOC che tende a valutare non solo il livello di gradimento del corso, ma anche ad analizzare il profilo di voi corsisti che avete intrapreso un percorso di formazione on-line.
I dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata e saranno resi disponibili successivamente a quanti concluderanno il MOOC.
Grazie in anticipo per la vostra disponibilità e soprattutto per essere giunti sin qui!

*Campo obbligatorio

1. **Indirizzo email** *

ANAGRAFICA

2. **Nome** *

3. **Cognome** *

4. **Sesso** *
   Contrassegna solo un ovale.
   - Uomo
   - Donna

5. **Età (scrivere solo il numero, es: 25)** *

6. **Regione** *
Contrassegna solo un ovale.

- Valle d'Aosta
- Piemonte
- Liguria
- Lombardia
- Trentino-Alto Adige
- Veneto
- Friuli-Venezia Giulia
- Emilia Romagna
- Toscana
- Umbria
- Marche
- Lazio
- Abruzzo
- Molise
- Campania
- Puglia
- Basilicata
- Calabria
- Sicilia
- Sardegna

7. **Città** *

8. **In quale ordine di scuola insegni?** *
Seleziona tutte le voci applicabili.

- [ ] Scuola primaria
- [ ] Scuola secondaria di primo grado
- [ ] Biennio - Scuola secondaria di secondo grado
- [ ] Triennio – Scuola secondaria di secondo grado
- [ ] Università
- [ ] Altro: ____________

9. **Nome della scuola di appartenenza** *

PROJECT WORK & PEER REVIEW

10. **L'argomento su cui verte il tuo Project Work ha preso spunto dal MOOC Numeri? ***
Contrassegna solo un ovale.

- Sì
- In parte
- No

11. **Se si (o in parte), a cosa hai fatto riferimento? (puoi selezionare più di un'opzione)**
Seleziona tutte le voci applicabili.

- [ ] Attività del Modulo 1 (Meteoriti, batteri, chicchi di riso...i numeri e il loro significato)
- [ ] Attività del Modulo 2 (Metodologia MERLO)
- [ ] Attività del Modulo 3 (Valutazione e INVALSI)
- [ ] Attività del Modulo 4 (Salire le scale)
- [ ] Attività del Modulo 5 (Aritmetica, Algebra e i Linguaggi matematici)
- [ ] Sway
- [ ] Padlet
- [ ] Tricider
- [ ] Altro: ______________________

12. **Prima di utilizzarlo, conoscevi lo strumento Learning Designer? ***
Contrassegna solo un ovale.

- Si
- No

13. **Ritieni che questo strumento web-based ti sia stato utile per creare e organizzare le tue attività di insegnamento/apprendimento (TLA) e ti ha supportato nella progettazione di esperienze utili a condurre lo studente verso gli obiettivi di apprendimento? ***
Contrassegna solo un ovale.

- Decisamente no
- Più no che sì
- Più sì che no
- Decisamente sì

14. **Motiva la precedente risposta.**

____

____

____

____

____

15. **Da 1 a 5, quanto ritieni che l'aver avuto a disposizione sin da subito la rubrica valutativa (linee guida per la peer review) ti abbia aiutato a realizzare il tuo Project Work?** *

Contrassegna solo un ovale.

1	2	3	4	5
◯	◯	◯	◯	◯

16. **Ritieni che la rubrica valutativa (linee guida per la peer review) consenta una efficace ed efficiente peer review?** *

Contrassegna solo un ovale.

◯ Decisamente no
◯ Più no che sì
◯ Più sì che no
◯ Decisamente sì

17. **In che misura e perché la valutazione tra pari (peer review) è stata importante per me?** *

____
____
____
____
____

18. **Quali sono stati gli aspetti più difficili che ho trovato nell'effettuare la peer review?** *

(puoi selezionare anche più di un'opzione)
Seleziona tutte le voci applicabili.

☐ Vincoli di tempo
☐ Mancanza di conoscenza dei contenuti espressi dal collega
☐ Visione diversa circa la cultura della scuola
☐ Altro: _______________________________

BACHECHE DI COMUNICAZIONE

19. **In quale delle seguenti bacheche di comunicazione hai fatto il tuo primissimo intervento (scrittura di un post, commento di risposta)?** *

Contrassegna solo un ovale.

- ( ) Mai fatto un intervento
- ( ) Forum tecnico
- ( ) Canale twitter del MOOC Numeri
- ( ) Modulo 1 (meteoriti, batteri, ….) – Forum
- ( ) Modulo 1 (meteoriti, batteri, ….) – Padlet
- ( ) Modulo 2 (MERLO) – Forum
- ( ) Modulo 3 (INVALSI) – Tricider
- ( ) Modulo 3 (INVALSI) – Forum
- ( ) Modulo 4 (Salire le scale) – Forum
- ( ) Modulo 4 (Salire le scale) – Padlet
- ( ) Modulo 5 (Artimetica, Algebra, …) – Forum
- ( ) Modulo 5 (Artimetica, Algebra, …) – Tricider

20. **Quando POSTAVI sulle bacheche di comunicazione (forum, padlet, tricider), a chi ti rivolgevi o a chi erano indirizzati i tuoi commenti di solito?** *

Seleziona tutte le voci applicabili.

- [ ] Al team del MOOC
- [ ] Ai corsisti del MOOC che conoscevi
- [ ] A tutti i corsisti del MOOC (conoscenti e non)
- [ ] Altro: ____________________

21. **Quando RISPONDEVI sulle bacheche di comunicazione (forum, padlet, tricider), a chi ti rivolgevi o a chi erano indirizzati i tuoi commenti di solito?** *

Seleziona tutte le voci applicabili.

- [ ] All’autore del messaggio iniziale
- [ ] All’audience formatosi su quella discussione
- [ ] All’intero audience del MOOC
- [ ] Al team del MOOC
- [ ] Altro: ____________________

22. **Qual è la percentuale di tutti i post che hai letto?** *

Contrassegna solo un ovale.

- ( ) Non ho letto post
- ( ) Dall’1% al 20% di tutti i post
- ( ) Dal 21% al 40% di tutti i post
- ( ) Dal 41% al 60% di tutti i post
- ( ) Dal 61% al 80% di tutti i post
- ( ) Maggiore dell’80% di tutti i post

23. **In che percentuale i tuoi post sono stati commenti fatti su post scritti da altri utenti MOOC, anche solo per far riconoscere che li avevi letti?** *
Contrassegna solo un ovale.

- Non ho mai commentato i post di nessuno
- Tra l'1% e il 20% sul totale dei miei post
- Tra il 21% e il 40% sul totale dei miei post
- Tra il 41% e il 60% sul totale dei miei post
- Tra il 61% e l'80% sul totale dei miei post
- Più dell'80% dei miei post

24. **Nelle bacheche di comunicazione (forum, padlet, tricider), ti è mai capitato di cercare messaggi di particolari persone?** *
Contrassegna solo un ovale.

- Si
- No

25. **Se si, perché?**

26. **Ritieni che il processo di scrittura di un post e il ricevere o fare commenti è stato vantaggioso per te? In che senso?**

27. **Per ogni azione sotto riportata, riferita all'uso delle bacheche di comunicazione, indica il livello di importanza che gli assegneresti** *
(dove: 1 = davvero poco importante e 5 = davvero molto importante)
Contrassegna solo un ovale per riga.

	1	2	3	4	5
Leggere					
Postare					
Commentare					

COMUNITÀ

28. **In che misura ti senti simile o diverso rispetto agli altri partecipanti che hai incontrato in questo MOOC?**

29. **Come partecipante del MOOC, fino a che punto ti sentivi di essere parte di una comunità? ***

30. **Ritieni che i partecipanti del MOOC abbiano contribuito alla tua formazione? ***
Contrassegna solo un ovale.

- Si
- No

MATERIALI

31. **Quanto hai usufruito dei materiali (ad esempio, video, proposte di attività, software, …) forniti in questo MOOC? ***
Contrassegna solo un ovale.

- il 20% o anche meno
- tra il 21% e il 40%
- tra il 41% e il 60%
- tra il 61% e l'80%
- più dell'80%

32. **Hai usato (o hai intenzione di usare) materiali che altri corsisti del MOOC hanno messo a disposizione su questo MOOC? ***
Contrassegna solo un ovale.

- Sì
- No

33. **Giustifica la tua precedente risposta.** *

34. **Nelle bacheche di comunicazione ti è capitato di cercare post relativi a materiali specifici?** *

Contrassegna solo un ovale.

- Sì
- No

35. **Considerando le tue interazioni con i materiali e gli altri partecipanti del MOOC, quali esperienze di apprendimento ritieni che ti siano state più utili? Come mai?** *

AUTOVALUTAZIONE

36. **Valuta le seguenti affermazioni (1 = assolutamente falso, … , 5 = assolutamente vero)** *
Contrassegna solo un ovale per riga.

	1	2	3	4	5
All'ingresso nel MOOC (le prime volte) non sapevo come muovermi in piattaforma					
Dopo qualche accesso in piattaforma ho preso familiarità con la struttura del MOOC					
Ho posto domande per ricevere chiarimenti tecnici					
Mi sono auto-organizzato nello svolgimento delle attività per quanto riguarda la TEMPISTCA					
Mi sono auto-organizzato nello svolgimento delle attività per quanto riguarda la MODALITÀ con cui svolgerle (es. stampavo, prendevo appunti, …)					
Mi sono auto-organizzato nello svolgimento delle attività per quanto riguarda l'APPROFONDIMENTO di ognuna di esse (es. leggere con cura o meno il materiale fornito, …)					
Ho proposto alle mie classi alcune delle attività (anche parzialmente) visionate nel MOOC					
Mi sentivo parte di una comunità che cresceva in maniera collaborativa					
Ho condiviso con gli altri corsisti le mie pratiche di insegnamento (mi sono raccontato)					
Ho condiviso con gli altri corsisti materiali che uso nelle mie classi					
Ho discusso più di matematica che di metodologie didattiche					
Piuttosto che intervenire di persona mi limitavo a leggere i commenti degli altri					
Ho apprezzato i momenti di contatto sincrono con i formatori mediante i webinar					
Dopo questa esperienza di formazione sento che è avvenuto un cambiamento nelle mie pratiche didattiche					
Ritengo che il MOOC Numeri ha giovato al mio sviluppo professionale di docente di matematica					

37. **Adesso che il MOOC è finito, attuerai la didattica laboratoriale nelle tue classi?** *
Contrassegna solo un ovale.

- Sì
- No
- Lo facevo già prima
- Altro: ____________

38. **Adesso che il MOOC è finito, attuerai la metodologia MERLO?** *

Contrassegna solo un ovale.

- Sì
- No
- Lo facevo già prima
- Altro: ______________________

39. **I Problem Solving che fai fare in classe prenderanno spunto da situazioni legate alla realtà?** *

Contrassegna solo un ovale.

- Sì
- No
- Lo facevo già prima
- Altro: ______________________

40. **Adesso riesci a risolvere (o a fare risolvere ai tuoi alunni) problemi ricorrendo a modellizzazioni create con software specifici?** *

Contrassegna solo un ovale.

- Sì
- In parte
- No
- Lo facevo già prima
- Altro: ______________________

41. **Adesso credi che cambierà il tuo uso in classe di Geogebra? Rispondi indicando quante volte credi che lo userai.** *

Contrassegna solo un ovale per riga.

	Mai	Meno di una volta al mese	Una volta al mese	Meno di una volta a settimana, ma più di una volta al mese	Una volta a settimana	Più di una volta a settimana	Tutti i giorni
Come uno "strumento di presentazione" - Proietto i materiali di GeoGebra in classe	◯	◯	◯	◯	◯	◯	◯
Come uno "strumento attivo nel laboratorio di informatica"- Gli studenti lavorano in modo indipendente su materiali dinamici nell'aula informatizzata	◯	◯	◯	◯	◯	◯	◯

42. **Spiega perché (se cambierà o se non cambierà).** *

43. **Ricordando che per Problema Aperto intendiamo "una situazione in cui l'allievo è invitato ad esplorare, a formulare congetture su quali ipotesi implichino conseguenze interessanti nella situazione in esame (e a farne una dimostrazione)". Adesso, credi che in classe lavorerai con problemi aperti?** *

Contrassegna solo un ovale.

- Si, con Geogebra
- Si, con Geogebra e lo facevo già
- Si, senza Geogebra
- Si, senza Geogebra e lo facevo già
- Si, con e senza Geogebra
- Si, con e senza Geogebra e lo facevo già
- No

44. **Vuoi riferirci episodi significativi relativi alla tua partecipazione al MOOC?**

45. **Vuoi riferirci episodi significativi accaduti in classe a seguito della tua partecipazione al MOOC?**

46. **Avevi seguito il MOOC Geometria?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 46.*
- No *Passa alla domanda 51.*

47. **Perchè ti sei voluto iscrivere anche al nostro nuovo MOOC?** *

48. **La configurazione dei moduli non tutti a cascata, ma con aperture su pagine indipendenti, credi ti abbia agevolato nella fruizione del corso?** *

Contrassegna solo un ovale.

- Sì
- No

49. **Come avrai notato, la struttura dell'impostazione dei moduli del MOOC Numeri è simile a quella del MOOC Geometria. Ritieni che la familiarità all'ambiente MOOC acquisita con Geometria ti abbia permesso con più semplicità di gestire il tuo ritmo di apprendimento qui in Numeri?** *

Contrassegna solo un ovale.

- Sì
- No

50. **Giustifica la tua risposta precedente.**

51. **Cosa hai apprezzato, di più o di meno in Numeri rispetto a Geometria? (es. i moduli che si aprono su pagine indipendenti, la tabella che riassume gli argomenti di ogni modulo, …).** *

MOOC

52. **I contenuti del MOOC hanno corrisposto alle sue aspettative?** *
(puoi lasciare eventuali commenti alla voce Altro)
Seleziona tutte le voci applicabili.

- [ ] Si
- [ ] Parzialmente
- [ ] No
- [ ] Altro: ____________________

53.
Come valuti il materiale didattico fornito? *
Contrassegna solo un ovale.

- ( ) Utile
- ( ) Poco utile
- ( ) Inutile

54.
Come giudichi la durata del corso rispetto agli argomenti trattati? *
Contrassegna solo un ovale.

- ( ) Insufficiente
- ( ) Buona
- ( ) Eccessiva

55.
Quanto ritengo di essermi impegnato nella partecipazione al MOOC? *
Contrassegna solo un ovale.

- ( ) Decisamente poco
- ( ) Né poco né molto
- ( ) Decisamente molto

56.
Se venisse offerta la possibilità di partecipare a un altro MOOC, sempre su un nucleo matematico, ripeteresti questa esperienza (ti iscriveresti nuovamente)? *
Contrassegna solo un ovale.

- ( ) Sì
- ( ) No

57.
Al di là degli aspetti pedagogici, QUANTO e IN CHE MODO ritieni che il ruolo specifico della matematica presente con le sue varie rappresentazioni nel MOOC (concetti, formule, grafici, software, …) abbia contribuito/giovato alla tua formazione sul tema Numeri? C'è qualche aspetto che pensi avrebbe dovuto essere (più) presente e che invece ritieni fosse mancante (o carente). Eventualmente fai qualche esempio. *

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58. **Che cosa – secondo te – contraddistingue questo MOOC da un MOOC che potrebbe vertere ad esempio su Filosofia?** *

SUGGERIMENTI

59. **Riporta eventuali suggerimenti e commenti per migliorare l'organizzazione del corso:**

Grazie per aver completato il questionario. Sperando che conserverai un piacevole ricordo di questa avventura nel MOOC, ti salutiamo calorosamente! Il team del MOOC.

Una copia delle risposte verrà inviata via email all'indirizzo fornito

Da rimandare in versione PDF, rinominandolo col tuo cognome, all'indirizzo: moocdidattica.dm@unito.it

Intervista scritta
Cognome:
Scuola:
Classi in cui insegni:

PRIVACY
Ricordiamo sempre che i tuoi dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata.

LA MIA CLASSE

Caro insegnante
raccontaci, in una decina di righe, della/le tua/e classe/i.
Tieni presenti i seguenti punti:

- la mia classe è composta da tot alunni, italiani/stranieri,
- clima/coinvolgimento durante le mie lezioni.

IL MIO RAPPORTO CON LA TECNOLOGIA

Il più importante cambiamento sociale degli ultimi anni è stato la comparsa delle tecnologie dell'informazione e della comunicazione digitale: effetti grafici, animazioni, comunicazione interattiva, simulazioni, giochi per computer, Internet, realtà virtuale.
Potremmo quasi dire che siamo stati "travolti" dalla tecnologia, non solo nel quotidiano, ma anche nel mondo della scuola. Si pensi alle LIM, ai fogli di calcolo elettronici, ai software di geometria dinamica, ….

E tu, che rapporto hai con la tecnologia? È una tua abitudine integrare le tecnologia nelle tue lezioni e i tuoi studenti, ne fanno effettivamente uso per raggiungere i risultati di apprendimento previsti?
Per aiutarti a scrivere questa auto-riflessione, prova a porti le seguenti domande (se possibile, inserisci anche i link dei siti a cui fai riferimento):

1. Qual è il valore aggiunto nell'usare la tecnologia nell'insegnamento/apprendimento? Che cosa effettivamente permette a me e ai miei studenti di fare?
2. Integro la tecnologia nelle mie lezioni, la collego chiaramente agli obiettivi di apprendimento? In che modo?
3. Io e i miei studenti abbiamo sufficienti abilità digitali per poter beneficiare delle potenzialità offerte dalla tecnologia? Se no, perché no e come potrei risolvere il problema?
4. La tecnologia supporta le scelte di apprendimento e personalizza l'apprendimento? Ovvero, gli studenti usano la tecnologia per gestire le loro conoscenze, scegliendo risorse appropriate o strumenti per sostenere il loro lavoro?
5. Riesco ad usare la tecnologia consentendo ai miei studenti di essere co-produttori del loro apprendimento e non solo consumatori? Cioè, il mio uso della tecnologia permette ai miei studenti di andare oltre le tradizionali competenze di base e sviluppare più abilità trasversali tipiche del 21° secolo (tipo problem solving collaborativo)?
6. Concedo abbastanza tempo ai miei studenti per lavorare con la tecnologia e che cosa potrei fare per rendere le mie attività più agevoli da eseguire?
7. Quali compiti specifici do che richiedono l'uso della tecnologia?

La composizione deve avere un minimo di 30 righe fino a un massimo di 60 righe.

Da rimandare in versione PDF, rinominandolo col tuo cognome, all'indirizzo: moocdidattica.dm@unito.it

RICERCA DIDATTICA

PRIVACY

Ricordiamo sempre che i tuoi dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata.

Cognome:					
Tipo di scuola *es. Scuola secondaria di primo/secondo grado – Liceo, Istituto tecnico*					
Classi					
Presenza LIM *Scrivi SI o NO sotto ciascuna delle tue classi*					
POSSIBILITÀ DI USARE LA LIM SPOSTANDOSI DALLE CLASSI IN CUI È ASSENTE	*(SI/NO)*				

PARTE UNO

Cari insegnanti,

Abbiamo analizzato i file "intervista scritta" che ci avete mandato a seguito della nostra richiesta.

Tornando a ringraziarvi, volevamo condividere con voi quanto è emerso dalle vostre risposte.

Molti di voi (circa il 45% delle interviste che ci sono pervenute) hanno dichiarato che gli studenti, pur essendo per definizione nativi digitali, non sembrano possedere le peculiarità della definizione stessa.

Per esempio, qualcuno di voi si esprime così:

> "*Questa generazione, che pure viene definita "nativa digitale", fa un uso molto poco consapevole e piuttosto superficiale degli strumenti informatici. "Smanettano" spesso a casaccio, utilizzando una piccolissima frazione delle potenzialità degli strumenti a loro disposizione*".

Oppure qualcun altro

> *"Quasi nessuno utilizza il pc, il tablet o lo smartphone per studiare o aumentare la propria conoscenza. È difficili abituarli a un uso sensato della tecnologia [...] fino ad ora ciascuno di loro si è avvicinato alla tecnologia esclusivamente per giochi ludici e ritiene impossibile che possa essere utilizzata per apprendere. Anzi si scocciano anche quando qualcuno cerca di andare in questa direzione"*.

Ancora,

> *"I miei studenti nativi digitali sanno fare molto meno [di me] [...] A me sembra che i ragazzi non siano interessati ad utilizzare la tecnologia per studiare."*

Riproponiamo di seguito altre due vostre frasi:

> *"[...] gli studenti [...] usano il computer e la rete prevalentemente fuori dalla scuola e, quindi, le loro competenze digitali si formano in altri ambienti, senza percorsi organizzati di apprendimento, ma su tentativi ed errori, supportati dall'aiuto dei coetanei"*.

> *"Ho notato negli anni un peggioramento nell'entusiasmo di fronte a nuove proposte e una diminuzione della capacità di apprendere nuovi programmi digitali da parte degli alunni. Alcuni anni fa [...] bastava dare il via e poi erano loro stessi ad insegnarmi come usare al meglio tutti i vari programmi. [...] Ora i ragazzi sono più attirati dagli smartphone che dai PC"*.

Queste ultime in particolare ci hanno indotto alla seguente riflessione: **non sarà mica che la scuola propone qualcosa di obsoleto e gli alunni ci paiono per questo poco capaci?**

1. Lasciate di seguito vostre osservazioni in merito.
   In particolare, riferendovi alla vostra esperienza, evidenziate – se ci sono – quelle che secondo voi sono le **maggiori difficoltà** che i vostri studenti incontrano nel relazionarsi con la tecnologia (distinguendo tra allievi di I e II biennio se di scuola secondaria di secondo grado, o per classe se di scuola secondaria di primo grado) e quali rimedi avanzereste.

2. Vorremmo inoltre capire meglio di quali strumenti tecnologici fate (o comunque intendete fare presto) uso in classe e in che modalità. In particolare siamo interessati a capire se e come questo MOOC vi ha aiutato in questo contesto. Per chiarezza espositiva, perciò, vi chiediamo di completare la seguente tabella mettendo un "SI" o un "NO" a seconda dell'uso che fate dello strumento tecnologico prima o dopo il MOOC, o spiegando a parole ove necessario.

Se fate usi diversi da quelli citati, o se desiderate condividere qualche momento significativo su come il MOOC vi ha aiutato, aggiungete un commento ai piedi della tabella.

AULA COMPUTER	
Mediamente, quante volte usi l'aula computer? Mai Almeno una volta a settimana Almeno una volta al mese Almeno una volta a quadrimestre	*Evidenzia la tua risposta, o riportala in questa cella.*
	Rispondi di seguito scrivendo SI/NO
Lo spostamento destabilizza la classe	
Computer lenti	
Computer si bloccano	
Fa perdere tempo	
I ragazzi lo prendono come un gioco	

Geogebra **(GG)**	Disegno dinamico		Componenti dinamiche specifiche di GG (Slider, trascinamento…)		Problemi aperti	
	prima	dopo	prima	dopo	prima	dopo

PIATTAFORME DIGITALI **(Moodle, Edmodo, Argo…)**	Condivisione di materiale con gli studenti		Forum di discussione		Padlet o tricider		Quiz/Esercizi interattivi	
	prima	dopo	prima	dopo	prima	dopo	prima	dopo

LIM	Proiettore		Conservare e avere memoria del lavoro svolto in classe		Esercizi interattivi		Problemi aperti (**)	
	prima	dopo	prima	dopo	prima	dopo	prima	dopo

(**) intendiamo dire se vi è già capitato di usare la LIM come un "tablet gigante" in mano alla classe? In particolare, avete mai pensato di proporre un problema aperto di Geogebra mandando uno o più ragazzi alla LIM per fare la costruzione, congetturare e successivamente dimostrare? Avete già sperimentato qualcosa del genere o ne avete intenzione? Raccontateci…

PARTE DUE

Per quanto riguarda il discorso disciplinare, tenendo presente soprattutto la **Geometria**, argomento del MOOC, ma, qualora lo riteniate utile, con eventuali riferimenti anche a **Algebra**, **Aritmetica**, e **Analisi**, evidenziate quelle che secondo voi sono le maggiori difficoltà dei vostri studenti (distinguendo tra allievi di I e II biennio se di scuola secondaria di secondo grado, o per classe se di scuola secondaria di primo grado) e quali rimedi avanzereste, cioè quali situazioni e metodologie didattiche ritenete essere opportuni rimedi.

1) In particolare, quali proposte del MOOC possono essere utili per superare/ovviare queste difficoltà? E quali altre proposte non discusse nel MOOC?
2) Avete già fatto sperimentazioni o intendete farne?
3) Se ne avete già fatte, potreste riassumercele seguendo EVENTUALMENTE il modello di un diario di bordo (giorno, alunni presenti, impostazione dell'attività, discussione) e inviarcele in un file separato? Sentitevi liberi di inserire anche immagini o foto.
4) Se intendete farne, su quali siete interessati a sperimentare?

PARTE TRE

Qual è, secondo te, il valore formativo della matematica (ed in particolare della geometria)? Cerchi di trasmetterlo ai tuoi ragazzi? Secondo te lo comprendono?
In quali momenti e modalità questo MOOC ti ha aiutato in questa tua importante missione?

Intervista rivolta a tutti i corsisti del MOOC di Geometria

*Campo obbligatorio

Intervista rivolta a tutti i corsisti del MOOC di Geometria (sia che abbiano o meno finito il MOOC) al fine di effettuare un'analisi su un eventuale cambiamento delle pratiche didattiche

Sebbene le domande che seguono possano sembrarti ripetitive (o credi di aver già dato simili risposte in questionari precedenti), ti chiediamo di dedicare 25 minuti del tuo prezioso tempo al fine di riflettere e rispondere con cura.

PRIVACY
Ricordiamo sempre che i tuoi dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata.

1.
Contrassegna solo un ovale.

- Opzione 1

ANAGRAFICA

2.
Nome *

3.
Cognome *

4.
Età *

5.
Durante quest'anno scolastico (2015-2016) hai insegnato matematica? *
Contrassegna solo un ovale.

- Sì
- No *Passa alla domanda 8.*

6. **In quale ordine di scuola insegni?** *

Seleziona tutte le voci applicabili.

- [ ] Secondaria di primo grado
- [ ] Secondaria di secondo grado (biennio)
- [ ] Secondaria di secondo grado (triennio)
- [ ] Altro: ____________________

7. **Da quanto tempo insegni?** *

____

8. **Hai finito il MOOC di Geometria?** *

Contrassegna solo un ovale.

- ( ) Si *Passa alla domanda 11.*
- ( ) No (meno del 40%) *Passa alla domanda 9.*
- ( ) Parzialmente (più del 40%) *Passa alla domanda 10.*

9. **Perchè non hai finito il MOOC?** *

____
____
____
____
____

Interrompi la compilazione del modulo.

10. **Cosa non hai fatto del MOOC e perché?** *

____
____
____
____
____

ANALISI PRIMA/DOPO MOOC:

Nel seguito ci saranno alcune domande sugli effetti che ha avuto il MOOC sulle tue pratiche didattiche. Al termine di ogni serie di domande sul modulo vengono fatte alcune domande sulle tue conoscenze sugli stessi argomenti PRIMA del MOOC, al fine di far emergere delle differenze. Ti chiediamo di rispondere con sincerità, facendo un piccolo sforzo di memoria per poter descrivere il paragone.

MODULI 1 e 2

DOPO IL MOOC

11\. **Nei Moduli 1 e 2 (Albero maestro & co; Orologio & co.) del MOOC sono state presentate attività pratico-manipolative. Le hai sperimentate nelle tue classi?** *
Contrassegna solo un ovale.

- ( ) Sì *Passa alla domanda 15.*
- ( ) No *Passa alla domanda 12.*

Moduli 1 e 2

DOPO IL MOOC

12\. **Hai pianificato di sperimentarle nelle tue classi?** *
Contrassegna solo un ovale.

- ( ) Si *Passa alla domanda 14.*
- ( ) No *Passa alla domanda 13.*

Moduli 1 e 2

DOPO IL MOOC

13\. **Perchè non hai pianificato di sperimentarle?** *

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____
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____
____

Passa alla domanda 21.

Moduli 1 e 2

DOPO IL MOOC

14\. **In quali punti del programma scolastico e perché?** *

____
____
____
____
____

Passa alla domanda 19.

Moduli 1 e 2

DOPO IL MOOC

15.
Quali attività hai sperimentato? *
Seleziona tutte le voci applicabili.

- [ ] 1. L'albero maestro
- [ ] 2. Rampe, fili e giravolte (2.1 Equilibrio di un corpo su piani con diversa inclinazione, 2.2 Srotolando un rocchetto di filo, 2.3 Simulare con Geogebra lo srotolamento di un rocchetto di filo, 2.4 Giravolte di perpendicolari)
- [ ] 3. Adattamento dell'albero maestro con riferimento alle indicazioni nazionali (3.1 La macchina telecomandata, 3.2 Ville e altro, 3.3 L'albero maestro e le vele, 3.4 Gara di nuoto (problema di minimo))
- [ ] 4. Il parco: un adattamento per il biennio liceo scientifico
- [ ] 5. Un adattamento per il primo biennio: dalla piegatura della carta a Geogebra
- [ ] 6. L'orologio
- [ ] 7. Muoversi con gli angoli
- [ ] 8. Dall'astronomia alla trigonometria (8.1 Problema ombre e angoli, 8.2 In gita a Roma)
- [ ] 9. Orologi, girandole e pattinatori
- [ ] 10. Lo spettacolo di Natale
- [ ] 11. Ma quanto sarà ripido?

16.
Se dei blocchi selezionati, non hai usato tutte le attività, indica quali NON hai usato (basta riportare il numero che precede il titolo dell'attività). *

____
____
____
____
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17.
Rispetto alle attività che hai sperimentato (puoi selezionare anche più di un'opzione): *
Seleziona tutte le voci applicabili.

- [ ] le hai usate esattamente come presentate
- [ ] le hai usate semplificandole
- [ ] le hai usate aggiungendo approfondimenti
- [ ] Altro: ________________________________

18. **Perché hai fatto queste scelte?**

MODULI 1 E 2

DOPO IL MOOC

19. **Quali sono i vantaggi che trai nell'usarle? (puoi selezionare al massimo 3 opzioni di risposta)** *

Seleziona tutte le voci applicabili.

- [ ] evitare formazione di misconcetti
- [ ] valorizzare il formulare congetture
- [ ] valorizzare la fase di scoperta di proprietà
- [ ] stimolare l'argomentazione
- [ ] quando i ragazzi fanno esperienze di apprendimento usando il proprio corpo ricordano più a lungo
- [ ] Altro: ____________________

20. **Quando i tuoi alunni usano attività pratico-manipolative ti sembrano:** *

Contrassegna solo un ovale per riga.

	Per niente	Poco	Abbastanza	Molto
Attenti	◯	◯	◯	◯
Interessati	◯	◯	◯	◯
Partecipi	◯	◯	◯	◯

MODULI 1 e 2

DOPO IL MOOC

21. **Ritieni che ci siano degli svantaggi, da parte tua o degli studenti, nell'utilizzarle?** *

Contrassegna solo un ovale.

◯ Si *Passa alla domanda 22.*

◯ No *Passa alla domanda 23.*

MODULI 1 E 2

DOPO IL MOOC

22. **Quali sono gli svantaggi (da parte tua o degli studenti)?** *

MODULI 1 E 2

DOPO IL MOOC

23.
In generale, grazie a quanto appreso tramite il MOOC, hai usato o hai pianificato di utilizzare ALTRE ATTIVITA' pratico-manipolative nelle tue classi? *
Contrassegna solo un ovale.

- Sì *Passa alla domanda 24.*
- No *Passa alla domanda 25.*

MODULI 1 E 2

DOPO IL MOOC

24.
Elenca qualche esempio brevemente. *

MODULI 1 E 2

PRIMA DEL MOOC

25.
Conoscevi attività pratico-manipolative da utilizzare per presentare concetti matematici? *
Contrassegna solo un ovale.

- Sì *Passa alla domanda 26.*
- No *Passa alla domanda 28.*

MODULI 1 E 2

PRIMA DEL MOOC

26. **Ne avevi fatto uso nelle tue classi?** *

Contrassegna solo un ovale.

- ⬭ Sì *Passa alla domanda 28.*
- ⬭ No *Passa alla domanda 27.*

MODULI 1 E 2

PRIMA DEL MOOC

27. **Perché no? (puoi selezionare anche più di una risposta)** *

Seleziona tutte le voci applicabili.

- [ ] non avevo capito a fondo come usarle
- [ ] non conoscevo nessuno che le usasse per potermi confrontare
- [ ] Altro: ______________________

MODULO 3

DOPO IL MOOC

28. **Nel Modulo 3 (Eredità & co.). sono state presentate attività di problem solving. In particolare, molte di queste sono strettamente legate all'uso di Geogebra (GG). Le hai sperimentate nelle tue classi?** *

Contrassegna solo un ovale.

- ⬭ Sì, con GG *Passa alla domanda 32.*
- ⬭ Sì, con e senza GG *Passa alla domanda 32.*
- ⬭ No *Passa alla domanda 29.*

MODULO 3

DOPO IL MOOC

29. **Hai pianificato di sperimentarle nelle tue classi?** *

Contrassegna solo un ovale.

- ⬭ Sì *Passa alla domanda 31.*
- ⬭ No *Passa alla domanda 30.*

MODULO 3

DOPO IL MOOC

30. **Perchè non hai pianificato di sperimentarle?** *

Passa alla domanda 37.

MODULO 3

DOPO IL MOOC

31. **In quali punti del programma scolastico e perché?** *

Passa alla domanda 36.

MODULO 3

DOPO IL MOOC

32. **Quali attività hai sperimentato?** *

Seleziona tutte le voci applicabili.

- [ ] 1.Esplorazione di figure piane- introduzione alla dimostrazione
- [ ] 2.Quale dimostrazione
- [ ] 3. L'eredità
- [ ] 4. Dalla congettura alla dimostrazione: il teorema di Pitagora
- [ ] 5. Angoli al centro e alla circonferenza: approccio alla dimostrazione
- [ ] 6. Un problema di Polya (6.1 Il triangolo e l'esagono regolare)

33. **Se dei blocchi selezionati, non hai usato tutte le attività, indica quali NON hai usato (basta riportare il numero che precede il titolo dell'attività).** *

34. **Rispetto alle attività che hai sperimentato (puoi selezionare anche più di un'opzione):** *
Seleziona tutte le voci applicabili.

- [ ] le hai usate esattamente come presentate
- [ ] le hai usate semplificandole
- [ ] le hai usate aggiungendo approfondimenti
- [ ] Altro: ____________________

35. **Perché hai fatto queste scelte?**

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____
____
____
____

MODULO 3

DOPO IL MOOC

36. **Quali sono i vantaggi che trai nell'usare attività di problem solving? (puoi selezionare al massimo 4 opzioni di risposta)** *
Seleziona tutte le voci applicabili.

- [ ] favorire l'approccio al congetturare, all'argomentare, al dimostrare
- [ ] dare ai ragazzi la possibilità di collaborare in gruppo
- [ ] dare ai ragazzi la possibilità di risolvere un compito da soli, senza la presenza "invasiva" dell'insegnante
- [ ] far ragionare i ragazzi su situazioni legate alla realtà
- [ ] l'aiuto del software favorisce la modellizzazione della realtà
- [ ] far sperimentare ai ragazzi un uso sensato della tecnologia
- [ ] Altro: ____________________

MODULO 3

DOPO IL MOOC

Nel MOOC sono stati trattati anche un particolare tipo di problem solving, i PROBLEMI APERTI, dove con problema aperto intendiamo "una situazione in cui l'allievo è invitato ad esplorare, a formulare congetture su quali ipotesi implichino conseguenze interessanti nella situazione in esame (e a farne una dimostrazione)". Per esempio, nel MOOC si era discusso del seguente: "Dato un quadrilatero convesso con i suoi assi, cosa accade ai punti di intersezione degli assi?" (presentato nel webinar del Modulo 3, tenuto a novembre 2015 dal Prof. Arzarello).

37. **Lavori con problemi aperti?** *

Contrassegna solo un ovale.

- Si, con GG *Passa alla domanda 38.*
- Si, senza GG *Passa alla domanda 38.*
- Si, con e senza GG *Passa alla domanda 38.*
- No *Passa alla domanda 39.*

MODULO 3

DOPO IL MOOC

38. **Quali sono i vantaggi che trai nell'usare questa particolare modalità di problem solving?** *

MODULO 3

DOPO IL MOOC

39. **In generale, grazie a quanto appresto tramite il MOOC, hai usato o hai pianificato di utilizzare ALTRE ATTIVITA' di problem solving nelle tue classi? (indica: con (%) se in particolare si tratta di problemi aperti; con (*) quelle che hai prodotto in autonomia; con (**) quelle che hai appreso grazie ad altri corsisti MOOC; con (***) quelle che hai cercato sul web; con ($) se con GG; con ($$) se senza GG.** *

40. **I problem solving che fai fare in classe prendono spunto da situazioni legate alla realtà?**

Contrassegna solo un ovale.

- Sì
- No

41. **Lavorando con PROBLEM SOLVING, rispetto alla lezione frontale, trovi che i tuoi studenti siano più:**
Contrassegna solo un ovale per riga.

	Per niente	Poco	Abbastanza	Molto
Attenti				
Interessati				
Partecipi				

42. **Lavorando con PROBLEMI APERTI su Geogebra, rispetto alla lezione frontale, trovi che i tuoi studenti siano più:**
Contrassegna solo un ovale per riga.

	Per niente	Poco	Abbastanza	Molto
Attenti				
Interessati				
Partecipi				

43. **Ritieni che ci siano degli svantaggi, da parte tua o degli studenti, nell'utilizzare simili attività?** *
Contrassegna solo un ovale.

- Sì *Passa alla domanda 44.*
- No *Passa alla domanda 45.*

MODULO 3

DOPO IL MOOC

44. **Quali sono gli svantaggi (da parte tua o degli studenti)?** *

MODULO 3

PRIMA DEL MOOC

45. **Conoscevi attività di problem solving da utilizzare per aiutare/stimolare gli allievi nelle fasi di dimostrazione e argomentazione?** *
Contrassegna solo un ovale.

- Sì *Passa alla domanda 46.*
- No *Passa alla domanda 48.*

MODULO 3

PRIMA DEL MOOC

46.
Ne avevi fatto uso nelle tue classi? *
Contrassegna solo un ovale.

- Sì *Passa alla domanda 48.*
- No *Passa alla domanda 47.*

MODULO 3

PRIMA DEL MOOC

47.
Perché non ne avevi fatto uso nelle tue classi? (puoi selezionare anche più di una risposta) *
Seleziona tutte le voci applicabili.

- [ ] non avevo capito a fondo come usarle
- [ ] non conoscevo nessuno che le usasse per potermi confrontare
- [ ] Altro: ____________

MODULO 3

PRIMA DEL MOOC

48.
Eri in grado di saper situare nella realtà proposte di percorsi didattici che favoriscono il congetturare, rendendo consapevole l'argomentare? *
Contrassegna solo un ovale.

- Sì
- No

49.
Riuscivi a risolvere (o far risolvere ai tuoi alunni) problemi ricorrendo a modellizzazioni create con software specifici? *
Contrassegna solo un ovale.

- Sì
- No

MODULO 3

In particolare sulla DIMOSTRAZIONE:

Da un'indagine rivolta a docenti di matematica sulle difficoltà dei propri studenti in geometria è emerso che la maggiore difficoltà sia costruire un pensiero logico-deduttivo, e quindi in particolare riuscire nelle dimostrazioni.

50. **Sei d'accordo con questa affermazione?** *

Contrassegna solo un ovale.

- ( ) Sì *Passa alla domanda 51.*
- ( ) No *Passa alla domanda 52.*

MODULO 3

In particolare sulla DIMOSTRAZIONE:

51. **Rispetto al problema in esame, hai tratto beneficio dai contenuti presentati nelle due settimane di problem solving e dimostrazione? E perché?** *

____
____
____
____
____

MODULO 3

In particolare sulla DIMOSTRAZIONE:

52. **Se hai sperimentato attività di dimostrazione in modalità problem solving, trovi che i tuoi alunni abbiano tratto beneficio rispetto al "costruire" un pensiero logico-deduttivo, e quindi in particolare riuscire nelle dimostrazioni"? Racconta…**

____
____
____
____
____

53. **Avevi già seguito altri corsi di formazione con lezioni specificatamente sulla dimostrazione?** *

Contrassegna solo un ovale.

- ( ) Sì
- ( ) No

MODULI 4 e 5

DOPO IL MOOC

Nei Moduli 4 e 5 (INVALSI; MERLO) sono state presentate attività in cui si è posta attenzione alla valutazione e ti sono stati presentati esempi di quesiti e/o schede da poter sottoporre ai tuoi allievi.

Rispetto al MODULO INVALSI:

54. **Hai usato i quesiti presentati nell'ebook dal titolo "Quesiti di verifica"?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 58.*
- No *Passa alla domanda 55.*

MODULI 4 E 5

INVALSI - DOPO IL MOOC

55. **Hai pianificato di utilizzarli nelle tue classi?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 57.*
- No *Passa alla domanda 56.*

MODULI 4 E 5

INVALSI - DOPO IL MOOC

56. **Perchè non hai pianificato di utilizzarli nelle tue classi?** *

____

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____

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____

Passa alla domanda 63.

MODULI 4 E 5

INVALSI - DOPO IL MOOC

57. **In quali punti del programma scolastico e perché?** *

Passa alla domanda 62.

MODULI 4 E 5

INVALSI - DOPO IL MOOC

58.
Quali hai usato? *
Seleziona tutte le voci applicabili.

- [ ] 1. Altezza e perpendicolarità
- [ ] 2. I nostri suggerimenti per la valutazione (2.1 Geometria in bicicletta, 2.2 Il nuotatore, 2.3 La casa di Marianna, 2.4 La torre di Pisa)
- [ ] 3. Suggerimenti per la valutazione della secondaria di secondo grado (3.1 Triangoli e parallelogrammi, 3.2 L'autocisterna, 3.3 L'impianto di irrigazione, 3.4 La bandiera delle Seychelles, 3.5 La bandiera dell'Eritrea, 3.6 Il logo, 3.7 La tenda, 3.8 Trapezio e triangoli)
- [ ] 4. Suggerimenti: proposte di verifica (4.1 Il problema dell'orario, 4.2 Il problema della moneta e del sole, 4.3 Robot in movimento, 4.4 La risoluzione del triangolo, 4.5 Un viaggio in nave, 4.6 Diamoci uno sguardo intorno: il baseball)

59.
Se dei blocchi selezionati, non hai usato tutte le attività, indica quali NON hai usato (basta riportare il numero che precede il titolo dell'attività). *

60.
Rispetto alle attività che hai sperimentato (puoi selezionare anche più di un'opzione): *
Seleziona tutte le voci applicabili.

- [ ] le hai usate esattamente come presentate
- [ ] le hai usate semplificandole
- [ ] le hai usate aggiungendo approfondimenti
- [ ] Altro: ____________________

61. **Perchè hai fatto queste scelte?**

MODULI 4 E 5

INVALSI - DOPO IL MOOC

62.
Quali vantaggi trai nell'usarle? (puoi selezionare anche più di una risposta) *
Seleziona tutte le voci applicabili.

- [ ] valutare le capacità degli studenti al temine di un percorso didattico
- [ ] avviare una discussione per riflettere su quale poteva essere la risposta corretta
- [ ] per farli esercitare in vista delle prove INVALSI
- [ ] Altro: ______

MODULI 4 E 5

INVALSI - DOPO IL MOOC

63.
Hai provato a creare tu dei quesiti su questa falsa riga? *
Contrassegna solo un ovale.

- ( ) Sì *Passa alla domanda 64.*
- ( ) No *Passa alla domanda 65.*

MODULI 4 E 5

INVALSI - DOPO IL MOOC

64.
Quali benefici hai tratto da questa pratica? (puoi selezionare più di una risposta) *
Seleziona tutte le voci applicabili.

- [ ] mettermi in gioco
- [ ] verificare l’apprendimento dei miei studenti tenendo il passo con gli argomenti del programma
- [ ] Altro: ______

MODULI 4 E 5

Rispetto al MODULO MERLO - DOPO IL MOOC

65. **Conosciuta questa metodologia, ne fai uso nelle tue classi?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 69.*
- No *Passa alla domanda 66.*

MODULI 4 E 5

MERLO - DOPO IL MOOC

66. **Hai pianificato di utilizzarla nelle tue classi?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 68.*
- No *Passa alla domanda 67.*

MODULI 4 E 5

MERLO - DOPO IL MOOC

67. **Perchè non hai pianificato di utilizzarla nelle tue classi?** *

Passa alla domanda 73.

MODULI 4 E 5

MERLO - DOPO IL MOOC

68. **In quali punti del programma scolastico e perché?** *

Passa alla domanda 72.

MODULI 4 E 5

MERLO - DOPO IL MOOC

69. **Quali schede suggerite dal MOOC hai usato?** *

Seleziona tutte le voci applicabili.

- [ ] Punto medio
- [ ] Traslazione
- [ ] Limiti di una funzione
- [ ] Punti di non derivabilità
- [ ] Distanza
- [ ] Spazio e figure
- [ ] Definizione di limite
- [ ] Rettangoli equivalenti
- [ ] Rettangoli isoperimetrici
- [ ] Rolle
- [ ] Simmetria assiale
- [ ] Risoluzione triangoli rettangoli
- [ ] Valor medio

70. **Rispetto alle schede che hai sperimentato (puoi selezionare anche più di un'opzione):** *

Contrassegna solo un ovale.

- ( ) le hai usate esattamente come presentate
- ( ) le hai usate semplificandole
- ( ) le hai usate aggiungendo approfondimenti
- ( ) Altro: ______________________

71. **Perchè hai fatto queste scelte?**

____

____

____

____

____

MODULI 4 E 5

MERLO - DOPO IL MOOC

72. **Quali sono i vantaggi che trai nell'usare la metodologia MERLO? (puoi selezionare anche più di una risposta)** *

Seleziona tutte le voci applicabili.

- [ ] inserire nella pratica didattica l'uso di diverse rappresentazioni semiotiche di uno stesso concetto
- [ ] abituare gli allievi all'argomentazione, alla discussione tra pari e al confronto

☐ Altro: ____________________

MODULI 4 E 5

MERLO - DOPO IL MOOC

73. **Hai provato a creare tu una scheda MERLO sulla falsa riga di quelle viste nel MOOC?** *
Contrassegna solo un ovale.

- ◯ Sì *Passa alla domanda 74.*
- ◯ No *Passa alla domanda 76.*

MODULI 4 E 5

MERLO - DOPO IL MOOC

74. **Rispetto a quale/i nodo/i concettuali?** *

____
____
____
____
____

75. **Quali benefici hai tratto da questa pratica? (puoi selezionare anche più di una risposta)** *
Seleziona tutte le voci applicabili.

☐ mettermi in gioco
☐ verificare l'apprendimento dei miei studenti tenendo il passo con gli argomenti del programma
☐ Altro: ____________________

MODULI 4 E 5

PRIMA DEL MOOC

76. **Nelle fasi di valutazione, eri in grado di creare prove che prendessero spunto da un contesto reale?** *
Contrassegna solo un ovale.

- ◯ Sì
- ◯ No

77. **Conoscevi la metodologia MERLO?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 78.*
- No *Passa alla domanda 80.*

MODULI 4 E 5

78. **Ne avevi fatto uso nelle tue classi?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 80.*
- No *Passa alla domanda 79.*

MODULI 4 E 5

79. **Perchè non ne avevi fatto uso nelle tue classi? (puoi selezionare anche più di una risposta)** *

Seleziona tutte le voci applicabili.

- [ ] non avevo capito a fondo come usarle
- [ ] non conoscevo nessuno che le usasse per potermi confrontare
- [ ] Altro: ____________________

Geogebra

In quasi tutti i Moduli del MOOC sono state presentate attività che potevano svolgersi/realizzarsi con l'ausilio del software di geometria dinamica Geogebra.

80. **Avevi seguito altri corsi di formazione/aggiornamento, diversi dal MOOC, specificatamente su Geogebra?** *

Contrassegna solo un ovale.

- Sì
- No

USO DI GEOGEBRA IN CLASSE

DOPO IL MOOC

81. **In percentuale, quanto usi tu Geogebra in classe? (% rispetto al totale delle tue ore di lezione)** *

Contrassegna solo un ovale.

- più del 50%
- tra il 50 e il 35%
- tra il 34 e il 15%
- tra il 14 e il 5%
- meno del 5%
- non lo uso

PRIMA DEL MOOC

82. **In percentuale, quanto usavi tu Geogebra in classe? (% rispetto al totale delle tue ore di lezione)** *

Contrassegna solo un ovale.

- più del 50%
- tra il 50 e il 35%
- tra il 34 e il 15%
- tra il 14 e il 5%
- meno del 5%
- non lo usavo

COMPONENTI SPECIFICHE DI GEOGEBRA

DOPO IL MOOC

83. **Quali di queste componenti specifiche di Geogebra conosci:** *

Seleziona tutte le voci applicabili.

- [ ] Trascinamento
- [ ] Slider
- [ ] Gioco di quadri
- [ ] nessuna

84. **Con i tuoi studenti, di quali tra queste componenti specifiche di Geogebra fai uso:** *

Seleziona tutte le voci applicabili.

- [ ] Trascinamento
- [ ] Slider
- [ ] Gioco di quadri
- [ ] nessuna

85. **Rispetto alle componenti che conosci, ma non usi, spiega il perché non ne fai uso (puoi selezionare anche più di una risposta):** *

Seleziona tutte le voci applicabili.

- [ ] le conosco e le uso tutte
- [ ] non ho capito a fondo come usarle
- [ ] mi costa fatica usarle
- [ ] non conosco nessuno che le usi per potermi confrontare
- [ ] Altro: ______________________

86. **Indica in quali punti del programma hai usato o hai pianificato di utilizzare le componenti di GG (trascinamento, slider, gioco di quadri):** *

____

____

____

____

____

87. **Quali sono i vantaggi che trai nell'usare queste componenti specifiche di GG (puoi selezionare al massimo 3 opzioni di risposta):** *

Seleziona tutte le voci applicabili.

- [ ] permettono la realizzazione di disegni geometrici con maggiore precisione
- [ ] permettono di far sperimentare ai ragazzi un uso sensato della tecnologia
- [ ] permettono di dare dinamicità ad una situazione didattica
- [ ] fanno comprendere meglio il significato dei parametri
- [ ] mostrano diverse rappresentazioni in registri diversi
- [ ] Altro: ______________________

88. **Rispetto al tempo che dedichi all'uso di queste componenti di Geogebra in classe, in percentuale, quanto fai usare tali componenti di GG ai tuoi studenti in classe? (% rispetto al tempo che usi GG in classe)** *

Contrassegna solo un ovale.

- ( ) SEMPRE
- ( ) più del 90%
- ( ) tra l'89 e il 70%
- ( ) tra il 69 e il 50%
- ( ) tra il 49 e il 30%
- ( ) meno del 30%
- ( ) MAI

89. **Lavorando con queste componenti specifiche di Geogebra trovi che i tuoi studenti siano più** *

Contrassegna solo un ovale per riga.

	Per niente	Poco	Abbastanza	Molto
Attenti				
Interessati				
Partecipi				

PRIMA DEL MOOC

90. **Quali di queste componenti specifiche di Geogebra conoscevi:** *

Seleziona tutte le voci applicabili.

- [ ] Trascinamento
- [ ] Slider
- [ ] Gioco di quadri
- [ ] nessuna

91. **Con i tuoi studenti, di quali tra queste componenti specifiche di Geogebra facevi uso:** *

Seleziona tutte le voci applicabili.

- [ ] Trascinamento
- [ ] Slider
- [ ] Gioco di quadri
- [ ] nessuna

92. **Rispetto alle componenti che conoscevi, ma non usavi, spiega il perché non ne facevi uso (puoi selezionare anche più di una risposta):** *

Seleziona tutte le voci applicabili.

- [ ] le conoscevo e le usavo tutte
- [ ] non avevo capito a fondo come usarle
- [ ] mi costava fatica usarle
- [ ] non conoscevo nessuno che le usasse per potermi confrontare
- [ ] Altro: ______________________

93. **Rispetto al tempo che dedicavi all'uso di queste componenti di Geogebra in classe, in percentuale, quanto facevi usare tali componenti di GG ai tuoi studenti in classe? (% rispetto al tempo che usi GG in classe)** *

Contrassegna solo un ovale.

- SEMPRE
- più del 90%
- tra l'89 e il 70%
- tra il 69 e il 50%
- tra il 49 e il 30%
- meno del 30%
- MAI

94. **Elenca, se ci sono stati, altri cambiamenti significativi dettati dalla presenza di queste componenti specifiche di Geogebra all'interno del MOOC.**

Tecnologia

95. **Ritieni che l'uso della tecnologia all'interno del MOOC sia stato appropriato?** *

Contrassegna solo un ovale.

- Sì
- No

96. **In cosa è stato molto buono?** *

97. **In cosa vorresti migliorarlo?** *

PRIMA DEL MOOC

98.
Quali erano, secondo te, i vantaggi e i benefici dell'uso della tecnologia nelle ore di matematica? *

____

____

____

____

____

DOPO IL MOOC

99.
Quali sono, secondo te, i vantaggi e i benefici dell'uso della tecnologia nelle ore di matematica? *

____

____

____

____

____

TECNOLOGIA A SCUOLA

100.
Le disponibilità tecnologiche del tuo istituto scolastico ti permettono di usare Geogebra durante le tue ore di lezione? *

Contrassegna solo un ovale.

- ( ) Sì *Passa alla domanda 102.*
- ( ) No *Passa alla domanda 101.*

TECNOLOGIA A SCUOLA

101.
Quali sono gli ostacoli? (puoi selezionare anche più di una risposta) *

Seleziona tutte le voci applicabili.

- [ ] l'aula informatica non è sempre disponibile
- [ ] l'aula informatica ha attrezzature obsolete
- [ ] non è consentito l'uso di cellulari in classe (e quindi dell'app Geogebra)
- [ ] non è consentito far portare il tablet/laptop agli studenti
- [ ] Altro: _______________________________

TECNOLOGIA A SCUOLA

102. **I tuoi studenti come reagiscono all'uso della tecnologia in classe che fai e che fai loro fare?** *

Contrassegna solo un ovale.

- malissimo
- male
- indifferenti
- bene
- benissimo

103. **Quali sono, secondo te, le ragioni di questo loro atteggiamento?** *

104. **Trovi sia cambiato il loro atteggiamento in base al tuo cambiamento professionale avvenuto in seguito alla tua partecipazione al MOOC?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 105.*
- No *Passa alla domanda 106.*

TECNOLOGIA A SCUOLA

105. **Quale è stato il cambiamento e perché.** *

Passa alla domanda 107.

TECNOLOGIA A SCUOLA

106. **Cosa avrebbe potuto offrirti il MOOC per far avvenire questo cambiamento?**

TECNOLOGIA A SCUOLA

107. **Racconta, se ci sono stati, altri eventuali cambiamenti nell'uso della tecnologia in classe (da parte tua o degli studenti) avvenuti mediante il MOOC**

____

____

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____

Il tuo sviluppo professionale

108.
Ordina i moduli dal MOOC, da 1 a 6, rispetto all'importanza/utilità che hanno avuto per te. *
Contrassegna solo un ovale per riga.

	1	2	3	4	5	6
Modulo 1 = Rampe, vele, parchi e piegature della carta	◯	◯	◯	◯	◯	◯
Modulo 2 = Orologi, girandole, pattinatori e lo spettacolo di Natale	◯	◯	◯	◯	◯	◯
Modulo 3 = Eredità, Un Problema di Polya e quale dimostrazione – esplorazione di figure piane	◯	◯	◯	◯	◯	◯
Modulo 4 = Valutazioni con Invalsi	◯	◯	◯	◯	◯	◯
Modulo 5 = MERLO	◯	◯	◯	◯	◯	◯
Modulo 6 = Project Work e Peer Review	◯	◯	◯	◯	◯	◯

109.
Da 1 a 10, quanto valuti che il MOOC sia stato utile per il tuo sviluppo professionale?
*

____

110. **Tale sviluppo è avvenuto grazie a** *

Contrassegna solo un ovale per riga.

	Per niente	Poco	Abbastanza	Molto
Attività proposte nel MOOC				
Il fatto di aver creato tu stesso un'attività nel Project Work				
Il fatto di aver letto l'attività di un collega nella Peer Review				
Il fatto di dover riflettere e scrivere il proprio pensiero				
Il fatto di condividere (dare e ricevere) la propria esperienza didattica con altri colleghi				
Il fatto di aver avuto dei contatti con esperti in didattica della matematica (Video, WEBINAR)				
Il fatto di aver fatto parte di una comunità				
Il fatto di esserti trovato a contatto con persone che sperimentano queste nuove modalità di insegnamento				
ALTRO1				
ALTRO2				

111. **ALTRO1**

112. **ALTRO2**

113. **Rispetto a quanto hai risposto nella precedente tabella, spiega per ogni voce perché.** *

114. **Il fatto di esserti trovato a contatto con persone che sperimentano queste nuove modalità di insegnamento, ti ha spinto a fare altrettanto?** *

Contrassegna solo un ovale.

- no
- per niente
- forse un poco
- sì, abbastanza
- sì, moltissimo

115. **Vuoi raccontarci meglio?**

116. **Cosa il MOOC ti ha dato IN PIU' rispetto ad una formazione in presenza?** *

117. **Cosa il MOOC ti ha dato IN MENO rispetto ad una formazione in presenza?** *

INTERVISTE SKYPE

solo Geometria

1) Come sei venuta a conoscenza del MOOC Geometria e perché hai deciso di iscriverti?
2) È stata la tua prima esperienza di formazione a distanza?
3) A distanza di quasi due anni, quali sono i tuoi ricordi dell'esperienza di formazione online con MOOC Geometria?
4) Come hai vissuto le interazioni con gli altri? Ti sentivi parte di una comunità? Perché?
5) A livello dell'insegnamento della geometria, che cosa ha apportato il MOOC? È cambiato qualcosa nel tuo modo di insegnare geometria o nell'insegnamento della geometria in generale (per esempio ripensare a quali sono i contenuti importanti, …)?
6) Nella realizzazione del Project Work, come ti sei messo in gioco? Il MOOC ti ha aiutato a progettare qualcosa di diverso? *
7) Perché non ti sei iscritto al MOOC Numeri?
8) C'è qualcos'altro che vorresti aggiungere?

* Saresti disponibile a raccontare il punto 6) facendoti un video? Previo tuo espresso consenso questo video (o parti di esso) sarà poi mostrato durante i lavori del convegno DI.FI.MA. che si terrà a Torino dal 16 al 18 ottobre 2017, a cui sei ovviamente invitata.
Se ti rendi disponibile, ti chiediamo di firmare la seguente liberatoria. [vedi sopra]

solo Numeri

1) Come sei venuta a conoscenza del MOOC Numeri e perché hai deciso di iscriverti?
2) È stata la tua prima esperienza di formazione a distanza?
3) A distanza di quasi un anno, quali sono i tuoi ricordi dell'esperienza di formazione online con MOOC Numeri?
4) Come hai vissuto le interazioni con gli altri? Ti sentivi parte di una comunità?
5) A livello dell'insegnamento del nucleo Numeri, che cosa ha apportato il MOOC? È cambiato qualcosa nel tuo modo di insegnare aritmetica/algebra o nell'insegnamento di aritmetica/algebra in generale (per esempio ripensare a quali sono i contenuti importanti, …)?
6) Nella realizzazione del Project Work, come ti sei messo in gioco? Il MOOC ti ha aiutato a progettare qualcosa di diverso? *
7) Sei tra i docenti che hanno svolto anche il modulo sulla sperimentazione:
   - Come giudichi complessivamente l'esperienza della sperimentazione in classe?
   - Sei soddisfatta o potevi fare qualcosa di più o di diverso?
   - Perché hai scelto proprio quella attività? Come l'hai introdotta? Hai detto che la sperimentazione era parte del progetto MOOC? Cosa pensano i tuoi studenti (se magari hanno fatto commenti) del fatto che tu hai seguito il MOOC?
   - Se hai scelto di sperimentare un'attività presa dal MOOC, l'hai usata così com'era, l'hai semplificata, l'hai approfondita?
   - Se invece hai sperimentato il tuo Project Work hai preso in considerazione i feedback che ti aveva mandato il tuo collega revisore? E in particolare, sperimentando il PW, hai notato differenze tra la progettazione teorica e il suo svolgimento pratico? Se sì, in cosa consistevano queste differenze e come le hai gestite?
8) C'è qualcos'altro che vorresti aggiungere?

* Saresti disponibile a raccontare il punto 6) facendoti un video? Previo tuo espresso consenso questo video (o parti di esso) sarà poi mostrato durante i lavori del convegno DI.FI.MA. che si terrà a Torino dal 16 al 18 ottobre 2017.
Se ti rendi disponibile, ti chiediamo di firmare la seguente liberatoria. [vedi sopra]

Geometria e Numeri

1) Come sei venuta a conoscenza del MOOC Geometria e perché hai deciso di iscriverti?
2) Invece, come sei poi venuta a conoscenza del MOOC Numeri? Perché hai deciso di continuare a frequentare questa nostra altra offerta formativa?
3) A distanza di quasi due anni, quali sono i tuoi ricordi dell'esperienza di formazione online con MOOC Geometria?
4) Come hai vissuto le interazioni con gli altri? Ti sentivi parte di una comunità?
5) Invece rispetto a Numeri, quali sono i tuoi ricordi?
6) In riferimento ad entrambe le esperienze che hai vissuto online:
   - A livello dell'insegnamento della geometria, che cosa ha apportato il MOOC? È cambiato qualcosa nel tuo modo di insegnare geometria o nell'insegnamento della geometria in generale (per esempio ripensare a quali sono i contenuti importanti, …)?
   - A livello dell'insegnamento del nucleo Numeri, che cosa ha apportato il MOOC? È cambiato qualcosa nel tuo modo di insegnare aritmetica/algebra o nell'insegnamento di aritmetica/algebra in generale (per esempio ripensare a quali sono i contenuti importanti, …)?

7) [domanda eventuale, a seconda di come risponde] Credi che uno dei due MOOC ti abbia lasciato di più rispetto all'altro? Perché?
8) Nella realizzazione del Project Work, come ti sei messo in gioco? Il MOOC ti ha aiutato a progettare qualcosa di diverso? *
9) Sei tra i docenti che hanno svolto anche il modulo sulla sperimentazione:
   - Come giudichi complessivamente l'esperienza della sperimentazione in classe?
   - Sei soddisfatta o potevi fare qualcosa di più o di diverso?
   - Perché hai scelto proprio quella attività? Come l'hai introdotta? Hai detto che la sperimentazione era parte del progetto MOOC? Cosa pensano i tuoi studenti (se magari hanno fatto commenti) del fatto che tu hai seguito il MOOC?
   - Se hai scelto di sperimentare un'attività presa dal MOOC, l'hai usata così com'era, l'hai semplificata, l'hai approfondita?
   - Se invece hai sperimentato il tuo Project Work hai preso in considerazione i feedback che ti aveva mandato il tuo collega revisore? E in particolare, sperimentando il PW, hai notato differenze tra la progettazione teorica e il suo svolgimento pratico? Se sì, in cosa consistevano queste differenze e come le hai gestite?
10) C'è qualcos'altro che vorresti aggiungere?

* Saresti disponibile a raccontare il punto 8) facendoti un video? Previo tuo espresso consenso questo video (o parti di esso) sarà poi mostrato durante i lavori del convegno DI.FI.MA. che si terrà a Torino dal 16 al 18 ottobre 2017.
Se ti rendi disponibile, ti chiediamo di firmare la seguente liberatoria. [vedi sopra]

Diario di bordo - MOOC Numeri

*Campo obbligatorio

1. **Indirizzo email** *

2. **Corsista MOOC (Nome e Cognome)** *

3. **ATTIVITÀ SCELTA (specificare se si tratta di un'attività del MOOC e del Project Work; indicare titolo riportato nel MOOC e obiettivi dell'attività che vuoi si perseguano)** *

4. **Eventuali osservazioni relative alla scelta dell'attività sperimentata**

5. **Scuola** *

6. **Classe destinataria dell'attività** *

7. **CONTESTO CLASSE (numero alunni, clima di lavoro, ...)** *

8. **Data inizio attività (gg/mm/aa)** *

9. **Data fine attività (gg/mm/aa)** *

10. **Durata complessiva dell'attività in ore** *
Contrassegna solo un ovale.

1	2	3	4	5	6	7	8	9	10
( )	( )	( )	( )	( )	( )	( )	( )	( )	( )

11. **BREVE DESRIZIONE DELL'ATTIVITÀ SVOLTA IN CLASSE (descrivere a grandi linee come è stata suddivisa l'attività, indicando lo scopo principale di ogni parte; la metodologia adottata; l'organizzazione dell'aula; e il tempo dedicato ad ogni parte)** *

12. **Livello di competenza/familiarità del docente con la metodologia** *
Contrassegna solo un ovale.

1	2	3	4	5
( )	( )	( )	( )	( )

13. **Rispetto all'attività sperimentata** *
Contrassegna solo un ovale.

- ( ) I'hai usata esattamente come presentata nel MOOC o come l'hai progettata per la consegna del PW
- ( ) I'hai semplificata
- ( ) I'hai usata aggiungendo approfondimenti

14. **Eventuali commenti rispetto alla risposta precedente**

15. **Elenco dei materiali usati nell'attività, specificando se ne sono stati eventualmente predisposti altri (es. cartelloni, riprese audio e/o video, LIM, ...)**

16. **Elenco dei protocolli interessanti degli allievi (spiegali qui e poi inviali all'indirizzo mail moocdidattica.dm@unito.it, dopo averli scansionati - si prega di voler inviare il tutto in un'unica mail)**

17. **È prevista una discussione o lezione dialogata al termine dell'attività?** *

Contrassegna solo un ovale.

- Sì
- No

18. **Se sì, indicare il livello di attenzione e partecipazione degli allievi alla discussione**

Contrassegna solo un ovale.

	1	2	3	4	5	
Nessuna partecipazione						Partecipazione attiva da parte di tutti

19. **Quali sono le novità nel percorso sperimentato?** *

20. **Domande degli allievi e/o imprevisti (in positivo e in negativo) eventualmente presentatisi**

21. **Come hai superato eventuali difficoltà tue o dei tuoi allievi?**

22. **Brevi osservazioni sul proprio operato e su quello degli allievi (osservazioni/valutazioni sui processi cognitivi, abilità e competenze attuate durante il percorso didattico)** *

23. **Nello svolgere questa attività i tuoi studenti ti sono sembrati** *
Contrassegna solo un ovale per riga.

	Per niente	Poco	Abbastanza	Molto
Attenti				
Interessati				
Partecipi				

24. **Livello di difficoltà dell'attività percepito dagli studenti** *
Contrassegna solo un ovale.

	1	2	3	4	5	
Attività molto difficile						Attività molto facile

25. **Livello di difficoltà dell'attività percepito dal docente** *
Contrassegna solo un ovale.

	1	2	3	4	5	
Attività molto difficile						Attività molto facile

26. **Livello di soddisfazione dell'attività percepito dal docente** *
Contrassegna solo un ovale.

	1	2	3	4	5	
Totalmente insoddisfatto						Totalmente soddisfatto

27. **Eventuali osservazioni ritenute rilevanti**

☐ Inviami una copia delle mie risposte

Intervista rivolta ai ricercatori e insegnanti-ricercatori coinvolti nel MOOC di Geometria

*Campo obbligatorio

Intervista rivolta ai ricercatori e insegnanti-ricercatori coinvolti nel MOOC di Geometriaal fine di effettuare un'analisi su un eventuale cambiamento delle pratiche meta-didattiche

Sicuramente, quando hai iniziato "l'avventura del MOOC" sarai partito/a con delle convinzioni, credenze, idee di progettazione, ipotesi di sviluppo che nel corso del MOOC si saranno o meno rafforzate.
Questa intervista vuole farti ripercorrere un po' tutta l'esperienza che ti ha visto coinvolto/a nel MOOC, per valutare se e quanto le tue praxeologie hanno subito variazioni.

Ti ringrazio sin da ora per la tua preziosa collaborazione.
Eugenia Taranto

PRIVACY
I tuoi dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata.

1. **Nome** *

____

2. **Cognome** *

____

3. **La prima volta che hai sentito parlare di MOOC e hai capito di cosa si trattava, ricordi quale è stata la tua reazione alla proposta di collaborare alla realizzazione di un MOOC che avesse la finalità di formare docenti interamente a distanza?** *
Contrassegna solo un ovale.

- ( ) pensavo che i tempi fossero maturi per poter affrontare una simile formazione
- ( ) ero incuriosito, ma al tempo stesso non credevo che avrebbe riscosso troppo successo
- ( ) sentendo che la letteratura riportava un tasso di abbandono del 95%, pensavo che sarebbe stato un fiasco
- ( ) Altro: ______________________________

4. **Ti aspettavi una partecipazione MASSIVE da parte dei docenti italiani? Ti ricordo che ci sono stati 424 docenti italiani, con almeno un rappresentante per ogni regione italiana.** *
Contrassegna solo un ovale.

- ( ) no, sono rimasto piacevolmente sorpreso
- ( ) non proprio (mi aspettavo una certa partecipazione, ma le mie aspettative erano molto ridotte)
- ( ) si, c'era stata una buona pubblicizzazione

5. **Sei stato relatore di un webinar?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 6.*
- No *Passa alla domanda 8.*

6. **Descrivi con tre parole questa esperienza.** *

7. **Rispetto alle modalità con cui ne hai fatto esperienza, perché un webinar potrebbe essere migliore o peggiore di un seminario in presenza?** *

8. **Sei stato incaricato di analizzare i forum e/o le bacheche di discussione?** *

Contrassegna solo un ovale.

- Sì *Passa alla domanda 9.*
- No *Passa alla domanda 14.*

9. **Hai ritenuto difficile analizzare i commenti lasciati dai corsisti, trattandosi di una moltitudine di interventi?** *

Contrassegna solo un ovale.

- Sì
- No

10. **Motiva la tua precedente risposta.** *

11. **Se dovessi istruire un tuo collega a ripetere un simile lavoro, come gli consiglieresti di procedere?** *

12. **Pensi di aver imparato qualcosa dalla lettura di riflessioni, opinioni e pareri dei corsisti master?** *

Contrassegna solo un ovale.

- ( ) Sì *Passa alla domanda 13.*
- ( ) No *Passa alla domanda 14.*

13. **Se si, cosa?**

Abbiamo osservato, durante le riunioni del gruppo MOOC, che i corsisti hanno un pò frainteso l'uso di Padlet e Tricider. Rispetto al primo, piuttosto che rispondere alle domande che avrebbero dovuto guidare la discussione, spesso si è assistito alla condivisione di commenti non sempre attinenti alle domande; mentre per quanto riguarda il secondo ne è stato palesemente travisato l'uso: non è stato usato come un classificatore di idee da votare, ma ancora una volta come forum.

14. **Quali, secondo te, sono aspetti positivi e quali negativi di quanto verificatosi?** *

15. **Che soluzioni proporresti per far in modo che i docenti utilizzino tali bacheche nel modo originariamente pensato dai progettisti del MOOC? (puoi selezionare più di un'opzione di risposta)** *

Seleziona tutte le voci applicabili.

- [ ] metterei a disposizione un tutorial cartaceo (cioè un file word/PDF)
- [ ] metterei a disposizione un video tutorial
- [ ] in quanto responsabile del modulo in cui è inserita la bacheca, commenterei per primo, in modo da dare l'esempio
- [ ] Altro: ____________________

16. **Sei stato coinvolto in prima persona nelle fasi di progettazione?** *

Contrassegna solo un ovale.

- ( ) Sì *Passa alla domanda 17.*
- ( ) No *Passa alla domanda 25.*

17. **Con quali aspettative hai intrapreso questa "avventura"?** *

18.
Ritieni che le tue aspettative sono state soddisfatte? *
Contrassegna solo un ovale.

- Per niente
- Poco
- Abbastanza
- Molto

19.
Con quale "categoria" di docenti ti immaginavi ti saresti trovato ad avere a che fare? *
Contrassegna solo un ovale.

- insegnanti esperienti dal un punto di vista dei contenuti
- insegnanti inesperienti da un punto di vista tecnologico
- insegnanti che non avrebbero saputo gestire e/o portare a termina una formazione interamente a distanza
- Altro: ______

20.
Ti sei sentito mai una "balia" dei corsisti MOOC? *
Contrassegna solo un ovale.

- Sì *Passa alla domanda 21.*
- No *Passa alla domanda 23.*

21.
In quale occasione?

22.
Pensi che ci sia un modo per migliorare questo aspetto?

23. **Relativamente al modulo conclusivo del MOOC, ti ritieni**
Seleziona tutte le voci applicabili.

- [ ] soddisfatto per le produzioni realizzate dai corsisti
- [ ] amareggiato per la confusione che i corsisti hanno creato col crasch di LD
- [ ] sorpreso della buona riuscita di una buona percentuale di corsisti
- [ ] Altro: ____________________

24. **Quali sono stati (e se ci sono stati), a tuo avviso, il MIGLIORE punto di forza e il PEGGIORE punto di debolezza che ha avuto il MOOC nella sua progettazione? (Motiva la tua risposta)** *

____
____
____
____
____

Gli obiettivi di progettazione del MOOC erano volti a:
1) diffondere non nuovi concetti, bensì nuove metodologie didattiche, apprese/testate/sperimentate durante i due anni di Master;
2) ripensare all'uso della tecnologia in classe per venire in contro agli studenti del XXI secolo (nativi digitali).

25. **Pensi che 1) sia stato impostato correttamente nel MOOC?** *
Contrassegna solo un ovale.

- ( ) Sì
- ( ) No

26. **Rispetto alla precedente domanda, perchè?** *

____
____
____
____
____

27. **Pensi che 1) sia stato conseguito dai corsisti MOOC?** *
Contrassegna solo un ovale.

- ( ) Sì
- ( ) No

28. **Rispetto alla precedente domanda, perchè?** *

29. **Pensi che 2) sia stato impostato correttamente nel MOOC?** *
Contrassegna solo un ovale.

- Sì
- No

30. **Rispetto alla precedente domanda, perchè?** *

31. **Pensi che 2) sia stato conseguito dai corsisti MOOC?** *
Contrassegna solo un ovale.

- Sì
- No

32. **Rispetto alla precedente domanda, perchè?** *

33. **Rispetto alla tecnologia, oltre Geogebra, quali altri software matematici sono stati presentati ai corsisti?**

34. **Degli altri software, che non erano matematici (PowToon, Padlet, Tricider, BigBLueButtom, Sway, Learning Designer), quali vantaggi pensi che i corsisti hanno/avrebbero dovuto trarre?**

Nel MOOC di Geometria non sono presenti riferimenti espliciti rispetto all'utilizzo della LIM (che invece molti docenti che hanno risposto ad interviste, hanno nominato, anche se era il suo utilizzo come proiettore che andava per la maggiore).

35. **Ritieni che ci possano essere degli usi matematicamente più efficaci che si potrebbero fare della LIM?** *

Contrassegna solo un ovale.

- Sì
- No

36. **Se si, quali?**

37. **Pensi che valga la pena condividerli con futuri corsisti MOOC?**

Contrassegna solo un ovale.

- Sì
- No
- Altro: ____________

Dalla letteratura si legge:
"La cultura partecipativa dà un forte sostegno alle attività di produzione e condivisione delle creazioni digitali e prevede una qualche forma di mentorship informale, secondo la quale i partecipanti più esperti condividono conoscenza con i principianti. All'interno di una cultura partecipativa, i soggetti sono convinti dell'importanza del loro contributo e si sentono in qualche modo connessi gli uni con gli altri" – da "Reconsidering Digital Immigrants", Confessions of an Aca-Fan, 5 dicembre 2007, Jenkins.

38. **Ritieni che quanto descritto si sia verificato all'interno del MOOC?** *

Contrassegna solo un ovale.

- Sì
- No

39. **Definiresti la comunità nata nel MOOC una comunità di pratica e/o di riflessione?** *

Contrassegna solo un ovale.

- Comunità di pratica
- Comunità di riflessione
- Comunità di pratica e di riflessione
- Altro: ______________________

40. **Motiva la tua precedente risposta.** *

____
____
____
____
____

41. **Come ricercatore/formatore in didattica della matematica, l'esperienza del MOOC Geometria ti ha** *

Contrassegna solo un ovale.

- arricchito
- lasciato indifferente
- Altro: ______________________

42. **Motiva la tua precedente risposta.** *

____
____
____
____
____

43. **Cosa ti aspetti dal MOOC Numeri?** *

Contrassegna solo un ovale.

- conferme rispetto a quanto accaduto con il MOOC Geometria
- miglioramenti sia in termini di progettazione che di fruizione
- Altro: ______________________

44. **Se sei del parere che “dal passato si può sempre imparare e migliorare”, in cosa Numeri dovrebbe migliorare e/o imparare da Geometria? (puoi selezionare più di un'opzione di risposta)** *

Seleziona tutte le voci applicabili.

- [ ] spunte per il conseguimento dei badge
- [ ] visualizzazione per moduli di Moodle
- [ ] audio dei video ai docenti
- [ ] velocità di scorrimento delle frasi nei video realizzati con PowToon
- [ ] Altro: ____________________

45. **Vuoi lasciare eventuali commenti/suggerimenti?**

____
____
____
____
____

Intervista rivolta agli insegnanti-ricercatori coinvolti nel MOOC di Numeri

*Campo obbligatorio

1. **Indirizzo email** *

Intervista rivolta agli insegnanti-ricercatori coinvolti nel MOOC di Numeri al fine di effettuare un'analisi su un eventuale cambiamento delle pratiche meta-didattiche

Sicuramente, quando hai iniziato "l'avventura del MOOC" sarai partito/a con delle convinzioni, credenze, idee di progettazione, ipotesi di sviluppo che nel corso del MOOC si saranno eventualmente rafforzate oppure saranno cambiate in tutto o in parte.
Questa intervista vuole farti ripercorrere un po' tutta l'esperienza che ti ha visto coinvolto/a nel MOOC, per valutare se e quanto le tue pratiche, i tuoi quadri teorici, le tue credenze, … hanno subito variazioni.

Ti ringrazio sin da ora per la tua preziosa collaborazione.
Eugenia Taranto

PRIVACY
I tuoi dati saranno raccolti in forma anonima e trattati nel perfetto rispetto della privacy. I risultati saranno analizzati e presentati solamente in forma aggregata. Sarà mia cura farvi pervenire i risultati aggregati dell'intervista.

2. **Nome** *

3. **Cognome** *

4. **Da un punto di vista generale, a prescindere dal MOOC, che cosa secondo te costituisce un buon programma di formazione insegnanti? Che cosa deve prevedere per essere definito tale?** *

5. **Sempre da un punto di vista generale, in un programma di formazione insegnanti, secondo te che ruolo deve ricoprire la tecnologia?** *

6. **Ritieni che, globalmente, quanto ha offerto il MOOC Numeri sia un buon programma di formazione insegnanti? Motiva la tua risposta.** *

Nel dettaglio del MODULI del MOOC Numeri

7. **Di quale modulo del MOOC Numeri ti sei occupato, sia in termini di progettazione sia di monitoraggio?** *

Seleziona tutte le voci applicabili.

- [ ] Modulo introduttivo
- [ ] Modulo 1: Meteoriti, batteri, chicchi di riso...i numeri e il loro significato
- [ ] Modulo 2: Metodologia MERLO
- [ ] Modulo 3: Valutazione e INVALSI
- [ ] Modulo 4: Salire le scale
- [ ] Modulo 5: Aritmetica, Algebra e i Linguaggi matematici
- [ ] Modulo 6: Project Work e Peer Review

Rispetto al modulo del quale ti sei occupata, in termini di PROGETTAZIONE:

8. **Hai lavorato:** *

Contrassegna solo un ovale.

- ( ) da sola
- ( ) col tuo gruppo di lavoro
- ( ) sia da sola sia col tuo gruppo di lavoro
- ( ) Altro: ____________

9. **Quale attività o aspetto della progettazione del tuo modulo hai curato?** *

10. **Quali principi hai/avete usato scegliendo di mettere nel MOOC certi task piuttosto che altri?** *

11. **Come giudichi il modo in cui sono state rese fruibili ai partecipanti le tue/vostre risorse (scelta di un video, scelta di un file di testo, scelta di un file audio, …), alla luce di quella che è poi stata l'esperienza di formazione che i tuoi/vostri materiali hanno permesso? Motiva la tua risposta.** *

12. **Il MOOC Numeri aveva due specifici obiettivi: trasmissione di metodologie didattiche diverse dalla lezione frontale e fare riflettere sulla scelta della tecnologia da usare in e con la classe.Ritieni che il modulo che hai/avete curato abbia consentito di mettere in pratica i due obiettivi che si prefiggeva il MOOC Numeri?** *

Contrassegna solo un ovale.

- Sì
- No

13. **Perché? Riporta, eventualmente, degli esempi.** *

Rispetto al modulo del quale ti sei occupata, in termini di MONITORAGGIO:

14. **In che cosa hai notato differenze rispetto a una formazione in presenza e quella del MOOC, realizzatasi a distanza?** *

15. **Pensi che si possa parlare di APPRENDIMENTO per i docenti che hanno frequentato il MOOC? Anche per coloro i quali eventualmente non lo hanno terminato? Motiva la tua risposta.** *

16. **Se hai elementi per rispondere, fino a che punto, secondo te, una simile esperienza per un insegnante può incidere sui suoi studenti?**

17. **Se sei stato anche coinvolto nel MOOC Geometria, che cosa ritieni ci sia stato di SIMILE e/o di DIVERSO in MOOC Numeri? Hai avuto smentite, conferme?**

18. **Che cosa ti ha lasciato questa esperienza?** *

19. **Eventuali commenti**

Una copia delle risposte verrà inviata via email all'indirizzo fornito

Appendix C   Sample of information mails and consent form

01 novembre 2015

Cari insegnati

La presente per ringraziarvi della vostra adesione alla sperimentazione. Siete numerosissimi a voler contribuire e questo ha superato di gran lunga le nostre aspettative al riguardo.
Come avrete avuto modo di leggere, nella sezione "RICERCA SPERIMENTALE" del questionario iniziale, il nostro obiettivo è quello di valutare il vostro ruolo sia nella figura di STUDENTE che frequenta il corso MOOC, ma anche di DOCENTE che viene formato e (speriamo) arricchito da questo corso, volendo capire come beneficerete del materiale che abbiamo messo a vostra disposizione, ovvero che uso potreste farne con i vostri studenti, integrando al meglio anche la tecnologia.
Sicuramente tra voi corsisti avrete modo di conoscervi, socializzando attraverso forum e bacheche; in questa "sede" vi chiediamo invece di rilasciarci un'intervista cartacea, ovvero raccontare qualcosa in più di voi a noi ricercatori.
Trovate in allegato il file che dovreste quindi compilare, riempire e ri-inviarci in versione PDF sempre a questo indirizzo, rinominandolo col vostro cognome.
La scadenza per l'invio di questa prima intervista non è rigida, ma vi chiediamo la gentilezza di farci avere una vostra risposta entro giorno 8 novembre.

Certi di risentirvi vi auguriamo una proficua e soddisfacente avventura nel MOOC,
Eugenia Taranto e Ferdinando Arzarello

26 gennaio 2016

Cari insegnanti,

con la presente intendiamo ringraziare quanti di voi ci hanno inviato la compilazione della seconda intervista scritta, che avevamo intitolato "Ricerca Didattica" e che riproponiamo in allegato. Questo sarebbe il nostro secondo contatto con voi che avete dato disponibilità per la sperimentazione didattica inerente al MOOC di Geometria da poco conclusosi.
L'obiettivo della nostra ricerca è anche quello di valutare l'impatto che ha avuto il MOOC sulle vostre pratiche didattiche. Vi saremo pertanto molto grati se, rinnovandoci la vostra disponibilità, terminerete la compilazione di questa seconda intervista.
Il termine di consegna dell'intervista era previsto entro il mese di Gennaio, ma intendiamo prorogarlo fino alla seconda settimana di Febbraio. Aspetteremo, pertanto, chi altri volesse consegnare e non lo ha ancora fatto fino al 15 febbraio 2016.

Grazie per la vostra preziosa collaborazione.
Un caro saluto,
E. Taranto & F. Arzarello

21 giugno 2017

Buongiorno XXX,

la contattiamo perché Lei è una delle docenti che si sono maggiormente distinte nel MOOC XXX e, previa Sua disponibilità, avremmo il piacere di farle qualche domanda.
L'intento è poi quello di condividere la sua esperienza con il nostro MOOC al Convegno DI.FI.MA (http://www.difima.unito.it/difima17/; che si terrà a Torino dal 16 al 18 ottobre e al quale Lei, se ha disponibilità di poter venire, è invitata a partecipare).
Sarebbe dunque disponibile ad essere ricontattata? Ci faccia sapere entro il 27 giugno e le daremo ulteriori istruzioni: potremmo fissare un incontro su skype di non più di 20 minuti oppure chiederle di fare un video nel quale risponderà a delle domande che le invieremo.

Sperando di sentirLa presto,
Cordiali saluti.
Il team di MathMOOCUniTo

28 giugno 2017

Gent.m* XXX

Grazie per la sua disponibilità. Siamo lieti di averLe fatto cosa gradita.
L'intervista che vorremmo rivolgerLe non durerà più di 20 minuti. Le rivolgeremo meno di 10 domande, ripercorrendo un po' il suo profilo e la sua esperienza col MOOC, con particolare attenzione alla parte sull'attività finale del MOOC, ovvero il Project Work (e la sua Sperimetazione in classe – *se l'ha fatta*!).
I dati verranno trattati nel rispetto della privacy e le inviamo già da ora in allegato una liberatoria che dovrà restituirci firmata, solo ad intervista conclusa.
L'idea di svolgere l'intervista su skype ci aiuterebbe a modulare le domande in base alle sue risposte e le date in cui potremmo fissare l'incontro sarebbero:
mercoledì 28 giugno (S), giovedì 29 giugno, venerdì 30 giugno
lunedì 10 luglio, martedì 11 luglio, mercoledì 12 luglio, giovedì 13 luglio (S)
Può rispondere indicando una o più preferenze ed anche il Suo nome skype. Comunicheremo successivamente l'orario, che tendenzialmente sarà tra le 10 e le 13 al più.
Se per qualsiasi motivo non riuscisse ad effettuare il collegamento da skype per una delle date proposte, procederemo con la seconda alternativa: le invieremo un testo con le domande che vorremmo rivolgerle e, solo per una di queste domande, le chiederemo di realizzare un video nel quale Lei risponde alla domanda (sarà poi questo il video che mostreremo al Convegno).

Nell'attesa di un Suo riscontro,
porgiamo cordiali saluti.
Il team del MOOC

Da inviare e restituire firmata all'indirizzo: moocdidattica.dm@unito.it

LIBERATORIA

Io sottoscritto ____________________________, docente che ha frequentato il percorso formativo online
[ ] MOOC Geometria e/o [ ] MOOC Numeri,

[ ] AUTORIZZO [ ] NON AUTORIZZO

L'utilizzo del materiale audio/video/fotografico rilasciato ad uso didattico, informativo e divulgativo.

Data, ___/____/______

Firma ……………………………….

Appendix D The Department of Mathematics “G. Peano”

The Department of Mathematics ‘Giuseppe Peano’ at the University of Turin has as primary mission to promote excellence in research and teaching in all areas of Mathematics. Particularly in Mathematics Education, the Department has made a long, deep and excellent work in the last forty years, among groups coordinated by Professors F. Arzarello[1] and O. Robutti[2].
About research on Mathematics Education, the Department has a leader national and international position, with exchange of visitors from all over the world.
For in-service teacher education, the Department promotes many initiatives: groups of teacher that are trained to become teacher-researchers in research projects as, for example: inclusion of students with special needs in math classes; analysis of students’ cognitive processes; task design on the use of technology in math education; MERLO items for activities and assessment. O. Robutti is the person in charge of the GeoGebra Institute of Torino (http://community.geogebra.org/it/), important reference for the training of in-service and pre-service teachers on the integration of technologies in mathematics lessons.
Important features of the Department is the DI.FI.MA. platform, a Moodle platform containing all the projects materials and the interactions of participants - more than 2500 teachers of all school levels. From October 2015, the Department is engaged in an innovative initiative: delivering of MOOCs (Massive Open Online Courses) for Italian in-service mathematics teacher education, with the use of platform DI.FI.MA.. The aims of these online courses are to cover the main topics in the official Italian programs for secondary school (Arithmetic and Algebra, Geometry, Change and Relations, Uncertainty and Data) from a mathematical, didactical, and methodological point of view, and to and give teachers an opportunity of professional development at national level. So far, the first two have been delivered, while the third one is delivering until the end of April 2018. The percentages of teachers who have completed the educational MOOC courses (36% in the MOOC Geometry and 42% in the MOOC Numbers) are a confirmation of the validity of the proposal (if compared with similar experiences at international level, with percentages of 12%).
Another innovative initiative in which is involved the Department is the experimental curriculum of Mathematics Lyceum, within the Piano Lauree Scientifiche project. It provides for the creation of a new type of secondary school (grades 9-13), with a strengthening in mathematics obtained through these features: more hours per years, laboratory activities, interdisciplinary and mathematical topics. To reach these features, the Mathematics Lyceum project uses two fundamental ways: (i) Teacher training in presence (one meeting per month), on the features listed above, (ii) Teaching experiments in their classes, and (iii) Creation of new professional figures among teachers participating to the project, namely teachers able to be part of research groups, to participate to congresses as speakers, to design mathematical tasks.

[1] Ferdinando Arzarello is Professor of Elementary Mathematics from a superior point of view. He was president of the UMI-MIUR Commission for the elaboration of a Mathematics curriculum (Mathematics for the Citizen) and scientific manager of the ministerial project m@t.abel for the teaching of mathematics; from 1998 to 2006 president of CIIM; from 2009 to 2013 president of the ERME; from 2013 to 2016 president of ICMI and he continues to belong, as the past President, to its Executive Committee. He is author of more than 150 articles and chapters of books in magazines and international volumes on various topics of mathematics teaching: algebra teaching, geometry and elementary analysis in the school, Embodiment and mathematics, Curricular design, Theoretical frameworks for teaching/learning of Mathematics.

[2] Ornella Robutti is Associate Professor in Mathematics Education (University of Turin, since 2003). She is a member of the CIIM. She is head of: GeoGebra Institute of Turin; PLS or Piano Lauree Scietifiche (Scientific Degrees Plan); DIFIMA project; Mathematics Lyceum project; MERLO project; a YouTube channel called "Didactics of Mathematics Ornella Robutti". She is author of materials for teachers and many research articles. Her research fields are mathematics learning with technologies, teacher communities, and inclusion in mathematics.

Appendix E Lucy's materials

Tra le attività proposte la piegatura della carta e il problema del parco sono quelle che mi hanno attratto di più. Rivedrei anche l'idea che ho abbozzato nel forum iniziale sull'albero maestro integrandola in queste attività.

A breve introdurrò in entrambe le mie prime (una prima liceo scientifico e una prima Costruzioni Ambiente e Territorio) i concetti di perpendicolarità e i triangoli. Pertanto distribuirei le attività in quest'ordine:

	materiale	consegna	riflessione
1	distribuzione di un foglio rotondo a testa con disegnato un segmento	piegando il foglio tracciare la perpendicolare	capire perchè si tratta della perpendicolare
2	distribuzione di un triangolo acutangolo a testa, già ritagliato	piegare il foglio tracciando le tre altezze	focalizzare l'attenzione sull'angolo retto osservare l'ortocentro
3	proporre il problema del parco	trovare come tracciare i tre sentieri e individuare il loro punto di incontro	riflettere sull'altezza come cammino minimo; introdurre il triangolo ortico
4	distribuzione di un triangolo ottusangolo in un foglio di forma rotonda	disegnare le tre altezze	far notare che due delle tre altezze cadono fuori dal triangolo
5	distrubuzione di un foglio rettangolare e quadrettato in cui è disegnato lo stesso triangolo ottusangolo precedente	disegnare le tre altezze	confrontare i due disegni per individuare eventuali confusioni tra verticale e perpendicolare
6	utilizzare geogebra	disegnare le tre altezze in un triangolo, facendo attenzione alla variazione dinamica della figura (da triangolo acutangolo a rettangolo a ottusangolo)	riflessioni conclusive sulle altezze in un triangolo; il triangolo ortico nel triangolo rettangolo e ottusangolo

Nella prima CAT ho un'ora particolarmente difficile da gestire, quella del sabato all'ultima ora. Potrei proporre ognuna di queste manipolazioni con la carta gli ultimi dieci minuti, in cui la soglia di attenzione è ridotta a zero. Con un'attività ludico-manipolativa potrei ottenere di occupare in modo proficuo anche questo tempo, riprendento i discorsi di riflessione durante la lezione successiva.
In questa classe aggiungerei (7) l'attività relativa alla costruzione di parallelogrammi e trapezi a partire da una striscia di carta, che non introdurrei invece nella prima liceo, dove la scansione del piano di lavoro concordato a livello di dipartimento prevede i quadrilateri in seconda.

Con gli studenti di prima liceo, invece,penso che approfondirò maggiormente le proprietà del triangolo ortico.

Appendix F Stephen's materials

Diario di bordo - MOOC Numeri

Indirizzo email *

stefano.barbieri@tin.it

Corsista MOOC (Nome e Cognome) *

Stefano Barbieri

ATTIVITÀ SCELTA (specificare se si tratta di un'attività del MOOC e del Project Work; indicare titolo riportato nel MOOC e obiettivi dell'attività che vuoi si perseguano) *

Sperimentazione del Project Work "Matematica senza numeri" - riflessione sulla risoluzione dei problemi aritmetici e la loro generalizzazione tramite lettere, promuovere l'approccio algebrico come risolutore di categorie di problemi e introdurre i prodotti notevoli tramite diverse rappresentazioni ed esplorazioni

Eventuali osservazioni relative alla scelta dell'attività sperimentata

Scuola *

IC "Marconi" Castelfranco Emilia (MO)

Classe destinataria dell'attività *

3B

CONTESTO CLASSE (numero alunni, clima di lavoro, ...) *

22 alunni (1H, 1 DSA, 2 Stranieri, 1 BES) clima collaborativo e propositivo

Data inizio attività (gg/mm/aa) *

17/03/17

Data fine attività (gg/mm/aa) *

31/03/17

Durata complessiva dell'attività in ore *

1	2	3	4	5	6	7	8	9	10
○	○	○	○	◉	○	○	○	○	○

BREVE DESRIZIONE DELL'ATTIVITÀ SVOLTA IN CLASSE (descrivere a grandi linee come è stata suddivisa l'attività, indicando lo scopo principale di ogni parte; la metodologia adottata; l'organizzazione dell'aula; e il tempo dedicato ad ogni parte) *

Ho seguito il project work alternando momenti di proplem solving e problem posing con lavoro individuale, di coppia e revisione collettiva della classe utilizzando la LIM, PC portatili (a coppie) con Excel e Geogebra: Ho strutturato 5 schede-alunno con un tempo previsto di 1h ciascuna

Livello di competenza/familiarità del docente con la metodologia *

1	2	3	4	5
○	○	○	○	◉

Rispetto all'attività sperimentata *

- ○ l'hai usata esattamente come presentata nel MOOC o come l'hai progettata per la consegna del PW
- ○ l'hai semplificata
- ◉ l'hai usata aggiungendo approfondimenti

Eventuali commenti rispetto alla risposta precedente

Ho preparato delle schede-alunno che potessero guidare meglio il percorso e strutturarlo in modo più autonomo

Elenco dei materiali usati nell'attività, specificando se ne sono stati eventualmente predisposti altri (es. cartelloni, riprese audio e/o video, LIM, ...)

Fotocopie, LIM, PC portatili con Excel e Geogebra

Elenco dei protocolli interessanti degli allievi (spiegali qui e poi inviali all'indirizzo mail moocdidattica.dm@unito.it, dopo averli scansionati - si prega di voler inviare il tutto in un'unica mail)

In realtà non ho finito la sperimentazione: invio solo le schede-alunno che ho preparato (poi devo raccogliere i materiali alla LIM e le schede-alunno e se posso le invio dopo...)

È prevista una discussione o lezione dialogata al termine dell'attività? *

- ◉ Sì
- ○ No

Se sì, indicare il livello di attenzione e partecipazione degli allievi alla discussione

	1	2	3	4	5	
Nessuna partecipazione	○	○	○	◉	○	Partecipazione attiva da parte di tutti

Quali sono le novità nel percorso sperimentato? *

Si presenta una situazione ludica di "Magia" per indovinare i numeri e si propone di scoprire "il trucco" passando da situazioni puntali (numeriche) alla generalizzazione (letterali)

Domande degli allievi e/o imprevisti (in positivo e in negativo) eventualmente presentatisi

(+) La partecipazione nelle attività laboratoriali è sempre frizzante.
(-) Nell'utilizzo di PC una coppia di alunni non ha usato correttamente lo strumento dato a disposizione e non ha tenuto un comportamento adeguato e sono stato costretto ad interrompere l'attività per tutti.

Come hai superato eventuali difficoltà tue o dei tuoi allievi?

Ho terminato l'attività (non c'erano più gli estremi educativi e di interesse) per poi riprenderla un altro giorno

Brevi osservazioni sul proprio operato e su quello degli allievi (osservazioni/valutazioni sui processi cognitivi, abilità e competenze attuate durante il percorso didattico) *

Alcuni hanno delle intuizioni, ma non riescono a sostenerle o ad argomentarle in modo appropriato. L'attività è in realtà un raccordo cognitivo di ciò che consapevolmente o inconsapevolmente già conoscono (l'uso delle lettere in nelle formule geometriche, la generalizzazione delle proprietà, teoremi, la scrittura formale nella risoluzione dei problemi aritmetici e geometrici, i principi di equivalenza [trattati come "equilibrio della bilancia"])

Nello svolgere questa attività i tuoi studenti ti sono sembrati *

	Per niente	Poco	Abbastanza	Molto
Attenti	○	○	○	◉
Interessati	○	○	○	◉
Partecipi	○	○	◉	○

Livello di difficoltà dell'attività percepito dagli studenti *

	1	2	3	4	5	
Attività molto difficile	○	○	◉	○	○	Attività molto facile

Livello di difficoltà dell'attività percepito dal docente *

	1	2	3	4	5	
Attività molto difficile	○	○	○	◉	○	Attività molto facile

Livello di soddisfazione dell'attività percepito dal docente *

	1	2	3	4	5	
Totalmente insoddisfatto	○	○	○	◉	○	Totalmente soddisfatto

Eventuali osservazioni ritenute rilevanti

Nella mia scuola si sono attivati troppi progetti e sperimentazioni: rimangono tempi ristretti e le attività si accavallano. Non ho ancora finito la sperimentazione con gli alunni e raccolto il materiale da loro prodotto.

MATEMATICA SENZA NUMERI

Prof. Stefano Barbieri

Sperimentazione in base al project work del MOOC - Numeri, DiFiMa - Dipartimento di Matematica - Università di Torino

SCHEDA 1 **Classe 3_____** *Alunno________________________*

Indovina il numero

<table>
<tr><td>Scrivi il testo...

____
Alunno</td><td>indovinello:
•
•
•
•
•
•
• risulta...</td></tr>
<tr><td>Esegui i calcoli →

Confronta qualche calcolo dei tuoi compagni: cosa puoi concludere?</td><td></td></tr>
<tr><td>Com'è possibile tale magia?</td><td></td></tr>
</table>

MATEMATICA SENZA NUMERI

Prof. Stefano Barbieri

Sperimentazione in base al project work del MOOC - Numeri, DiFiMa - Dipartimento di Matematica - Università di Torino

SCHEDA 2 **Classe 3_____** *Alunno*_____________________

Pensa un numero

Scrivi il testo...

____

Alunno

come sequenza di calcoli (continua nella prima riga):

n	....+ 12				risulta

prova anche qualche numero dei tuoi compagni

Generalizza il procedimento

ricopia la prima riga e continua tu:

n	....+ 12				risulta
a					

Analizziamo la situazione

Prima		*Dopo*

MATEMATICA SENZA NUMERI

Prof. Stefano Barbieri

Sperimentazione in base al project work del MOOC - Numeri, DiFiMa - Dipartimento di Matematica - Università di Torino

SCHEDA 3 **Classe 3_____** *Alunno*_____________________

Colora due pareti della tua stanza

La tua stanza...  ________________ *Alunno*	Proponi misure plausibili della tua stanza: altezza: a = lato 1: b = lato 2: c =
Proponi una strategia risolutiva	Spiega come calcolare l'area della superficie delle due pareti contigue:
Scrivi la strategia alternativa dei tuoi compagni	Ecco come si poteva anche fare:
Ricostruisci le due strategie con GeoGebra	Costruzione passo passo (in coppia con ____________________________) • • • • • • • • • • • • • • •

Sperimentazione in base al project work del MOOC - Numeri, DiFiMa - Dipartimento di Matematica - Università di Torino

SCHEDA 4 **Classe 3_____** *Alunno______________________*

Pensa un numero: la sfida

Scrivi...

____

Alunno

Scrivi il testo e… i calcoli:

• pensa a un numero	
•	
•	
•	
•	
• risulta ….	

Automatizza il calcolo e prova diversi numeri

Automatizza il calcolo con Excel

n	....+ 5				risulta

Rifletti sui due procedimenti

(automatizzali poi in Excel)

Tabella 1

n	....+ 5		risulta

Tabella 2

n^2	....+ 10·		risulta

Generalizza la situazione

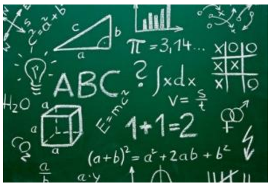
Generalizza in modo che la situazione sia indipendente dal numero che hai pensato e da 5

Proponi un modello geometrico
(anche con GeoGebra)

SCHEDA 5

***Classe 3*____** *Alunno*____________________

Una nuova sfida

Scrivi...  ______________ *Alunno*	Analizza la seguente scrittura: $$(a + b) \cdot (a - b) = ?$$
Proponi metodi investigativi	
Generalizza il risultato	$$(a + b) \cdot (a - b) =$$