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Learning with the logic of inquiry: game-activities inside Dynamic Geometry Environments

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The paper describes a game-activity proposed to 7th grade students with the goal to make them discover the geometric property concerning the mutual relationship between two circles. The activity, called "the game of the two circles", is composed of a strategic game that students play in GeoGebra and an investigative task which requires conjecturing and generalization. The aim of the activity is to trigger an approach to mathematics based on the logic of inquiry. We analyse students' dialogues and actions paying particular attention to the additional values the game confers to the more traditional exploratory activities with dynamic geometry software.

Keyword: DGE game-activity, played-game, reflected-game, inquiring and justifying processes.

Introduction

Many studies in mathematics education have documented the importance of making students explore mathematical situations before asking them to construct proofs (Boero et al. 1996, Pedemonte 2007). The exploration triggers the formulation and the checking of conjectures, introducing students into logical ways of reasoning. As pointed out by Dewey, all forms of logics, included the deductive logic, are consequence of inquiry processes:

> all logical forms (with their characteristic properties) arise within the operation of inquiry and are concerned with control of inquiry so that it may yield warranted assertions. This conception implies much more than that logical forms are disclosed or come to light when we reflect upon processes of inquiry that are in use. Of course it means that; but it also means that the forms originate in operations of inquiry. (Dewey, 1938, p.3,4)

Boero et al. (1996) have observed the possibility of a *cognitive continuity* between the processes of discovering and justifying. It occurs when the students, in the construction of the proof, exploit the argumentations employed for producing the conjecture. Pedemonte (2007) has distinguished between a cognitive unity in the *referencial system* and in its *structure*. The first occurs if some expressions, drawings, or theorems used in the proof have already been used in the argumentation for supporting the conjecture. The second occurs if the inferences produced in the argumentation and in the proof are connected through the same structure (abduction, induction, deduction). Hintikka (1999), an eminent Finnish logician, analyzing Sherlock Holmes way of reasoning in inquiry processes, showed that the clever deductions he made are obtained by reversing the abductions (Peirce, 1960) produced while investigating. His works demonstrates the existence of an *epistemic unity* between inquiring and justifying processes.

In this study, we favor and emphasize the possibility of connections between the discovery and justifying processes by introducing *strategic games* within Dynamic Geometry Environments (DGEs): these are games in which players have to make strategic choices meant for setting up and coordinating actions aimed at the achievement of a goal. As it is known from the literature (Arzarello et al. 2002, Baccaglini-Frank & Mariotti, 2010), DGEs are particularly apt for triggering inquiring processes. Our conjecture is that in virtue of the game, the conjectures and abductions produced inside the DGE are not left isolated but can be connected together and reorganized in logical chains. In fact, for making a strategic choice within a game situation, players reflect backward on the moves made and forward on the possible moves to make. These reflections can support the construction of logical links through which reorganize the geometrical invariants observed during the players' moves. In fact, since the moves are made on dynamic figures and involve geometric elements, we wish that the strategic reflections made on the moves could affect also the geometric elements involved in the moves. For this reason, *DGE game-activities* can promote a kind of thinking which is different from the one triggered by more traditional explorative activities with DGS.

Theoretical framework

The interrogative logic or logic of inquiry, introduced by Jaako Hintikka (1999), proposes a back to the origin consideration of the discipline. According to Hintikka, the modern logic switched from the study of excellence in reasoning to the study of infallibility in reasoning: "preserving one's logical virtue becomes a more important concern than developing virtuosity in drawing logical inferences" (Hintikka, 1999, p.28). The rules of inference are definitory rules, which inform us about the possible inferences, but do not say anything about which inferences are appropriate in the current moment, which are not so and which ones are better than others. These types of considerations are the concern of strategic principles.

Hintikka conceives the process of seeking new knowledge as an *interrogative game*, which is a two-player game between an *inquirer*, who asks the questions and an *oracle*, who answers him. Observations can be thought of as answers put to an environment, a controlled experiments, a database stored in the memory of a computer, a diagnostic handbook, a witness in a court of law, or one's own tacit knowledge partly based on one's memory can be considered as questions asked to nature. "Strategies of questioning play a central role in interrogative games, these include strategies of information seeking by means of different choices of questions to be asked and of the order in which they are asked." (Hintikka, 1999, p.34).

Hintikka models the processes of verification and falsifications through a *semantical game* (Hintikka, 1998), which is a two-player game between a verifier, whose goal is to show the truth of a mathematical formula or statement and a falsifier, whose goal is to confute it. In order to establish the truth of the mathematical formula $\forall x \; \exists y \mid S[x, y]$ it is possible to imagine a game in which the falsifier choses a value x_0 "in the most unfavorable way as far as the interests of the verifier are concerned" and the verifier should find a value y_0 for y such that $S[x_0, y_0]$ is true. The formula is true if there exists a winning strategy for the verifier of the game, while it is false if there exists a winning strategy for the falsifier of the game.

In our study, taking inspiration from Hintikka's semantical game, we designed DGE game-activities in order to aid students in their discovery of geometric properties, through the game-play and the guiding questions. Analysing students' actions, we distinguish between two ways of using the game: the played-game and the reflected-game (Soldano & Arzarello, 2016). In the *played-game*, the students' aim is to win against their opponent. To reach this goal they activate strategic principles which help them to select the best move to make in a given situation. In the *reflected-game* students play the game in order to answer the questionnaire and to communicate with each other. They play the game in a fictitious way: the game helps students to formulate the correct answer. In the reflected game we distinguish between the two main cognitive processes that characterizes dragging practices (Saada-Robert, 1989; Arzarello, 2002; Olivero, 1999): ascending and descending processes. We recognize ascending processes when students use the game in order to explore the situation and formulate a conjecture and descending processes when they use the game to check it. We have integrated this analytical tool with a new cognitive modality: the detached modality, in which students refer with words to the dynamic observed in the game, but they do not use it concretely.

The game of the two circles

The game-activity presented in this paper is based on the relationship between the distance between the centres of tangent circles and the sum/difference between their radii. Students play the game on the GeoGebra file shown in Figure 1. The GeoGebra window is divided in two parts: on the right there is the numerical window with sliders and variables, in the left the graphic window here is a graphic representation of the geometric objects.

Sliders *a, b,* and *c* control respectively the distance between the centres, the radius of the circle with centre O and the radius of the circle with centre O'. The variables *d*, *e*, *f* are respectively the absolute value of the difference between the radii (d=|b-c|), the distance between the centres and the sum of the radii (f=b+c). When students drag sliders *b* or *c*, they can observe the synchronic variation of the values of *d* and *f* and of the length of one circumference.

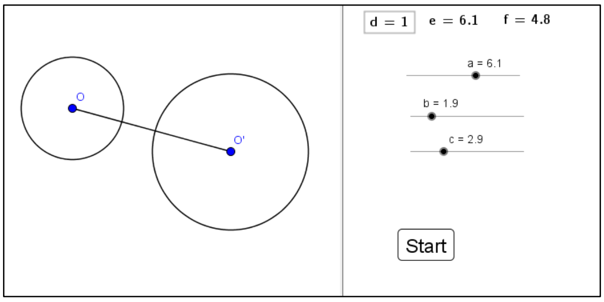


Figure 1: Game-activity

The game develops as follows: player B, the verifier, controls slider *b*, player C, the falsifier, controls slider *c* while player A, the referee, controls slider *a* and the hourglass. The goal of player B is to make e=d or e=f, the goal of player C is to make e≠d and e≠f. At the beginning of each match, the referee chooses the value of *a* and turns the hourglass over. Each time a player reaches his goal, the referee turns the hourglass over and the turn moves to the opponent. If the player cannot reach the aim within the time on his/her hands, he/she loses. The dynamic described is that of a semantical game played on the following statement: for every value of *c* there exists a value of *b* such that the circles are internally or externally tangent.

Each time that player B reaches his goal he produces an example of internally or externally tangent circles (look at Figure 2 a, b). Contrastingly, each time player C reaches his goal he produces an example of non-tangent circles (look at Figure 2 c, d, e). Since the interval of the sliders can take values from 0 to 10, players can produce also degenerate configurations (look at Figure 2 f, g). Player B can win also in this situation (look at Figure 2 h).

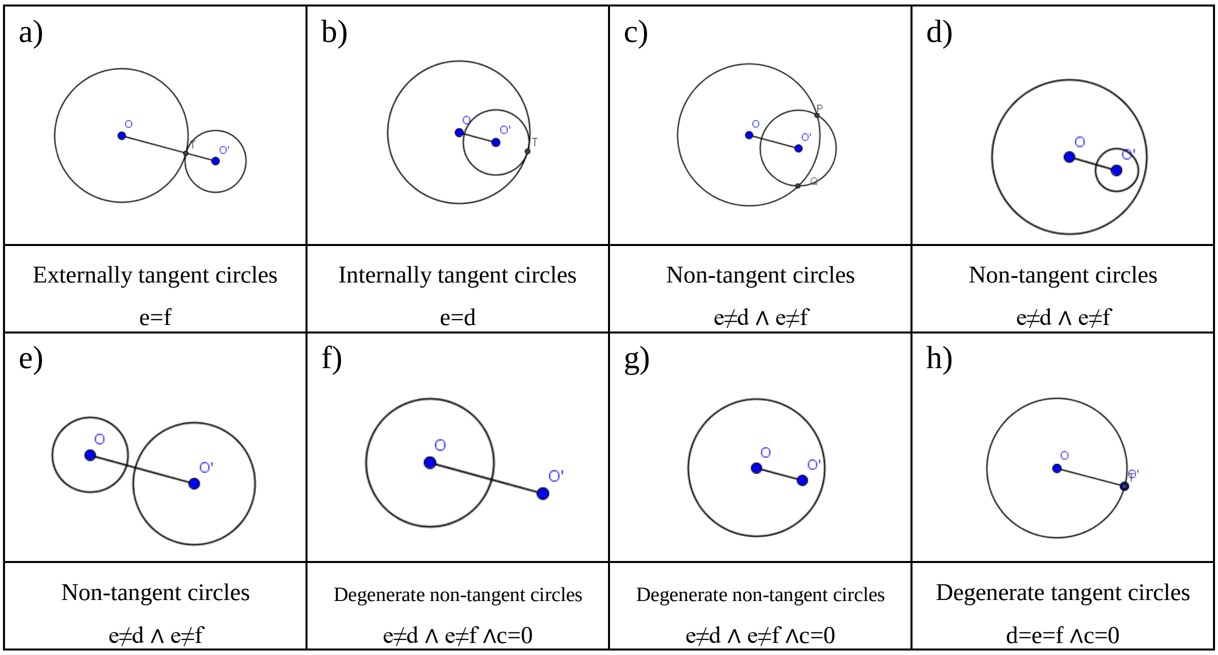


Figure 2: Example space associate with “the game of the two circles”

Theoretically it is always possible for players to reach their goals. Therefor the outcome of the game is determined by the time limit.

After playing the game, students are required to answer to the following questions:

1. Which are the mutual positions between the two circles each time player B reaches his aim?
2. Which are the mutual positions between the two circles each time player C reaches his aim?
3. What do the sliders a, b and c represent?
4. What do the value of d, e and f represent?

The questionnaire is intended to help students shift their frame of reference from the game to the geometric theory. In particular, the first two questions are intended to change the focus of attention from the numerical values of variables *d*, *e* and *f* to the mutual positions between circles. In this way students discover the geometric invariants which characterizes player B’s moves: each time the verifier reaches the goal the circles are tangent. Question number three is intended to link the values of the sliders to the length of the radii and the distance between the centres. Finally, question number four is intended to link the values of the parameters to the sum and difference of the radii. In this way students can discover another invariant which characterized player B’s moves: each time the verifier reaches the goal the distance between the centres is equal to the sum or difference between the radii

Methodology and data collection

The study reported in this paper involves one classroom of 7th grade Italian students. The game of the two circles is the first of a group of four game-activities related to the geometry topic of circles. Note that the properties on which the game are designed are not part of the classroom knowledge: the goal of the activity is to guide students in their discovery. Each activity lasts almost two hours: in the first hour and half students are divided into groups of three students and they play the game and answer a questionnaire using a computer or a tablet. In the last half an hour the teacher revisits students discoveries and systematizes the mathematical knowledge. The data for the analysis includes the transcript of students’ dialogue and the GeoGebra diagrams explored during the game and the questionnaire. We videotaped two groups and the final class discussion but, for space reason, we will represent only one group's work.

Analysis

The videotaped group is composed by three students: Gu and Al are males, Bia is a female. They play the game on the computer. In the first match Bia is the referee, she chooses the value of a and she turns the hourglass. Gu is player C, the falsifier. He has to move slider c so that e≠d and e≠f. Al is player B, the verifier. He has to move slider b so that e=d or e=f. Figure 3 contains, in the first row, the diagrams produced during the first match. Below each diagram are the reported values of sliders and variables which appear in the numerical window. Finally the last row contains students' role (Falsifier (F), Verifier (V)) who produces the diagram, the type of example created and the time spent producing it. Remember that slider a controls the distance between the centres, slider b the radius of the circle with centre O and the slider c the radius of the circle with centre O'. The values of d, e and f are, instead, the respective absolute values of the difference between the radii, the distance between the centres and the sum of the radii.

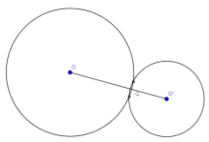
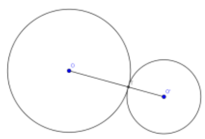
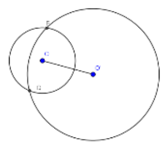
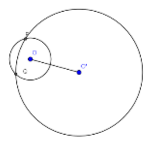
a)	b)	c)	d)
			
$a = 8 \quad b = 5.1 \quad c = 3$	$a = 8 \quad b = 5 \quad c = 3$	$a = 8 \quad b = 5 \quad c = 10$	$a = 8 \quad b = 3.3 \quad c = 10$
$d = 2.1 \quad e = 8 \quad f = 8.1$	$d = 2 \quad e = 8 \quad f = 8$	$d = 5 \quad e = 8 \quad f = 15$	$d = 7.7 \quad e = 8 \quad f = 13.3$
F, secant circle,7 sec	V, externally tangent, 6 sec	F, secant circle, 4 sec	V, secant circle, Time's up

Figure 3: First match

The match lasts short length time and it ends with the winning of the falsifier (Figure 3). In the last move the time ends before Al reaches his goal, hence Al loses even if, theoretically, he could have won. Al knows that he could have won if he had had more time, in fact he says "it should have been like this", making internally tangent circles. After Al demonstrates the winning configuration, Bia says "So B should always win". This sentence reveals the activation of the anticipatory thinking (Harel, 2001). After playing another match, students move to the first question.

Gu: In order to reach the goal they have to touch each other in only one point.

Al: On the other hand the answer to the question: 'In which mutual positions are the two circles when C reaches his/her goal?' is any position. They can touch each other in two points or nowhere.

Bia: No, they always touch each other in exactly two points. (looking at an example of secant circles).

Gu: They can touch each other in two points, but they can also not touch each other.

Bia: Ah… (moving *c* so that the circles do not intersect each other. Then she moves c back and forth for 30 seconds) Yes, that's right!

Al and Gu approach the question in a different way from Bia. They are in detached modality, they rethink what has happened in the played-game and then they answer the question. Bia, instead, uses the game in order to investigate the situation, she is in descending modality: she is using the reflected-game in order to check her schoolmates' claims. The group repeats this approach (detached versus descending) in answering the subsequent questions. When they get to the last one, the students do not agree with each other: according to Al and Bia, *d* and *f* are the radii of the circles, while Gu does not agree with them. The disagreement invokes the need of a justification.

Al: Let me prove that it's the radius (taking the mouse)! They have to coincide perfectly… (moving the centre O' on the other circumference, see Figure 4e)

Gu: It's not the radius… Because if you change the radius of a circle, you don't automatically change the radius of the other one! Both values [d and f] change! You should change just one [d or f] by moving it [b] by changing the length of one radius… you are not changing the other one!

Al: Don't you notice? Don't you see? (he moved c to 0, obtaining that e is equal to d and f, see Figure 4f). Point T just appeared and, putting this on zero. Do you notice? It's 2.9 2.9 2.9.

In order to refute Al's conjecture, Gu tries to explain the contradiction he noticed between the graphic and the numerical window. In detached modality, Gu explains why Al's claims creates a dynamic contradiction in the conversion (Duval, 2006) from the numerical to the graphic register. Al, instead, tries to provide evidence for explaining that what he claims is true. In this attempt, he uses the value of the parameter *e* (distance between the centres) in order to measure the length of the radius of one circle. His goal is to show that this value is equal to the value of *d* or *f*. If this process were to be applied on a generic example, it would have led to a contradiction, but since Al moves the value of the radius of the circle O' to 0, he produces a supporting example.

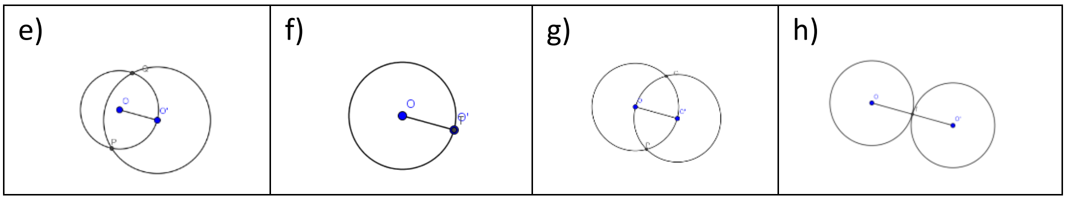


Figure 4: Reflected-game

After exploring silently the situation, Bia who at first supports Al's claim, changes her mind.

Bia: Anyway he's right… If you put them like this (each centre belongs to the other circle, see Figure 4g), the radius is the same thing, isn't it? I mean, it's the same here and here (pointing at the two circles), but here they are different (pointing at the values of sliders d, f)… Then it must be another thing, do you get it?

Adopting Al's strategy, she uses the distance between the centres to create two circles with the same radii in the graphic window. She notices that the two parameters which are supposed to be the radii are not equal in the numerical window. In contrast to Gu, who creates a dynamic counterexample, Bia exhibits a static counterexample, but this one also fails to convince Al that he is mistaken. The discussion continues with Gu who repeats his dynamic example, Bia who produces other static counterexample and Al who moves back to his supporting special example and cannot understand why he is wrong. Finally, observing the value d=0 when circles are externally tangent (Figure 4h) and with the same radii, Al formulate a new conjecture: d is the difference between the radii of the circles. This discovery allows him to unlock the situation and to explain his special example.

Discussion and conclusion

Game-activities can operationalize a functional approach to geometry. Within these activities, students deal with *soft* tangent circles, namely dynamic circles in which some constructive steps that make the circles *robustly* tangent (the tangency is preserved by dragging) are voluntarily not performed (Healy, 2000). A constructive step creates a functional dependence between the geometric elements, which is hidden in the robust construction of the figure. In DGE game-activities, this functional dependence is made explicit through the verifier/falsifier's dialectic. More precisely, when the verifier has observed the invariant tangent configuration produced by his/her moves, he/she can create a cause-effect link between his/her goal and the invariant produced.

We describe the verifier's dragging as follows: $b \xrightarrow{values\ of\ the\ sliders} tangent\ circles$ which indicates that the verifier, by moving the slider b, can observe the invariant tangent configuration as the effect of making sliders values coincide. Once discovered the invariant, the verifier can accomplish the move with the goal of building tangent circles. In this case, he observes the coincidence of the values of the slider as the effect of making tangent circles. This time the verifier's dragging is described in this way: $b \xrightarrow{tangent\ circles} values\ of\ the\ slider$. By switching the focus of attention of the move, the DGE game-activities, create a sort of frame, which helps students to appreciate the "if and only if" relationship between tangent circles and the fact that the distance between the centres is equal to the sum or absolute values of the difference of the radii.

DGE game-activities enrich the exploration supporting the in-depth investigation of situations: the presence of the falsifier, who tries to create trouble to the verifier, exposes the verifier to different initial situations triggering the exploration of both standard and non-prototypical examples of tangent circles. In this way, the game-activities enlarge students' *accessible example space* (Goldenberg & Mason, 2008) associated with tangent circle configuration. This is a very important aspect for the construction of mathematical concepts: proposing students only standard configurations can be source of mathematical misconceptions.

Finally, the game tool enriches and supports students' arguing abilities and the coordination of numerical and graphic information. In order to communicate their claim, students activate a versatile use of the game: not only for formulating and checking conjectures but also for supporting their claim, confuting different opinions and explaining ones' point of view. The game assumes a fundamental role in promoting mathematical ways of reasoning. Al, for example, uses the game to show evidences of the truth of his claim, hence uses the game for constructing a supporting example, while Bia and Gu use it to show that he is wrong, hence for constructing counterexamples to what has been claimed by Al. In producing these arguments, students make conversion between numerical and graphic registers. Concluding, the game instruments help students not to assume the absolute truth of external opinions, but to establish a dialectic approach to them.

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