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# A Mathematica interface to NNPDFs

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### Abstract

We present a *Mathematica* interface for handling the parton distribution functions of the NNDPF Collaboration, available from the NNPDF hepforge website http://nnpdf.hepforge.org/. As a case study we briefly summarise the first PDF set which includes all relevant LHC data, NNPDF2.3, and demonstrate the use of our new *Mathematica* interface.

Keywords: Parton distribution functions, NNPDF, Mathematica

We have developed a *Mathematica* package which provides direct interactive access to any NNPDF parton set. This package reads the NNPDF LHgrid files as available from the LHAPDF library and performs their interpolation in  $(x,Q^2)$  space by means of proper *Mathematica* built-in functions. The user can therefore make use of PDF ensembles within a *Mathematica* notebook in order to compute observables, evaluate PDF central values and variances and plot PDFs instantaneously. In this note we shall present a brief motivation and summary of the key features present in the new NNPDF2.3 [1] PDF set, and demonstrate the NNPDF *Mathematica* package by examining this new determination.

In recent years the volume of collider data of application in the accurate determination of parton distribution functions has significantly increased. Previous PDF determinations based solely upon high energy collider data collected by Tevatron and HERA collaborations were poorly constrained in comparison to global fits including data from fixed target deep inelastic scattering and Drell-Yan experiments. These collider only determinations are of particular interest in that they are free from the potential issues arising from nuclear corrections and higher twist effects present in the low energy data of global fits. Studying the impact of LHC data upon PDF determinations is therefore crucial not only for further constraint upon global determinations, but also for their potential utility in providing competitive collider only fits.

The NNPDF2.2 parton set [2] was the first determination to investigate the constraining power of LHC mea-

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surements upon parton distributions, specifically the *W* lepton charge asymmetry measurements of the AT-LAS [3] and CMS [4] collaborations in addition to the D0 collaboration measurements [5, 6]. This data was included by taking advantage of the Monte Carlo uncertainty estimation in the NNPDF methodology. NNPDF sets provide a probability distribution in the space of PDFs allowing for the use of a Bayesian reweighting technique [7, 8] and the rapid inclusion of new datasets into a prior set. However, this method is limited to the inclusion of a relatively small dataset.

In order to include an updated and more comprehensive LHC dataset into an NNPDF fit, an extremely fast method of collider observable computation has been developed. The FastKernel method for hadronic observables [1] combines the Monte Carlo weight grids from fast NLO software packages such as APPLgrid [9] and FastNLO [10], with FastKernel PDF evolution tables [11]. The resulting FastKernel table may then be combined with the initial scale parton distributions to produce the required observable. For a given hadronic observable  $\sigma$  the operation required to compute the theoretical prediction for such an observable is simply,

$$\sigma_I = \sum_{i,j}^{N_{\text{pdf}}} \sum_{\alpha\beta}^{N_x} \Sigma^I_{\alpha\beta ij} N^0_{\alpha i} N^0_{\beta j} , \qquad (1)$$

where  $\Sigma$  is the final FastKernel table and the *N* are the initial scale, DGLAP evolution basis parton distributions. The indices *i*, *j* run over the  $N_x$  points in the fitting scale *x*-grid, and  $\alpha$ ,  $\beta$  run over the reduced flavour basis of  $N_{\text{pdf}}$  light PDFs at this initial scale. This operation

is computationally very simple and allows for a much accelerated fit.

With this method we are able to add a relatively large LHC dataset to the existing data in the NNPDF 2.1 family to produce a set of fits denoted NNPDF 2.3. The NNPDF2.3 LHC dataset includes ATLAS collaboration  $35 \text{ pb}^{-1}$  measurements of the inclusive jet cross section [12] and electroweak vector boson rapidity distributions [13]. The CMS collaboration's 840 pb<sup>-1</sup> measurement of the W electron asymmetry [14] and the LHCb W boson rapidity distribution data [15] are included also. The expanded kinematic reach of the NNPDF2.3 dataset is described in Fig. 1. In addition to the inclusion of the LHC data, the NNPDF2.3 family of fits features a number of methodological improvements.

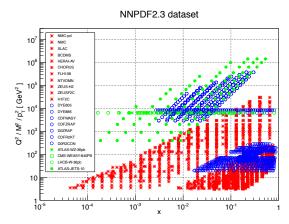


Figure 1: Kinematic range of the NNPDF2.3 dataset.

In order to assess the impact of the included LHC data, we compare the NLO NNPDF2.3 [1] and NLO NNPDF2.1 [16] singlet sector PDFs at  $Q_0^2 = 2 \text{ GeV}^2$  (see Fig 2). We have used our new *Mathematica* interface to draw these plots. From Fig. 2, it is clear that PDFs from the two sets differ by less, and usually much less, than one sigma.

The main advantages of our interface come from the possibility to combine the use of any NNPDF set with all the *Mathematica* features. In particular, this allows to perform a variety of manipulations on PDFs in a straightforward and interactive way, as we briefly demonstrate in the following discussion.

• Compute PDF central value and variance We have defined proper functions to keep the computation of PDF central value and variance very easy. These built-in functions only need x,  $Q^2$  and PDF flavour as input. The user can also specify the confidence level to which central value and variance should be computed.

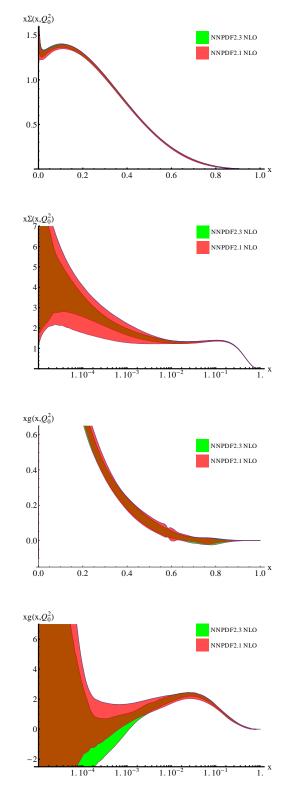


Figure 2: NLO NNPDF2.3 (green) singlet sector PDFs at  $Q_0^2 = 2 \text{ GeV}^2$ , compared to their NNPDF2.1 (red) counterparts. All error bands shown correspond to a one sigma interval. These plots are drawn with the new NNPDF *Mathematica* interface.

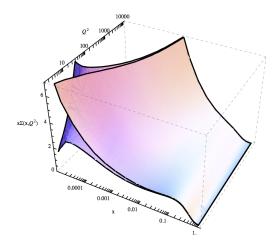


Figure 3: Simultaneous dependence from both x and  $Q^2$  for the one sigma error of the NLO NNPDF2.3 Singlet PDF. This plot is drawn with the new NNPDF *Mathematica* interface.

- Make PDF plots *Mathematica* enables a wide range of plotting options. As a few examples, we show the 3D plot and the contour plot of the Singlet PDF from the NLO NNPDF2.3 parton set (see Figs. 3 - 4 respectively).
- **Perform computations involving PDFs** PDF manipulation can be carried out straighforwardly since we have defined functions which handle either single replicas or the whole Monte Carlo ensemble. The user can then easily perform any computation which involves PDFs. For example, we show in Fig. 5 a snapshot of a typical *Mathematica* notebook in which we use our interface to check the momentum and valence sum rules from the NLO NNPDF2.3 parton set.

The new NNPDF *Mathematica* package can be downloaded from the NNPDF web site,

http://nnpdf.hepforge.org/

together with a sample notebook containing a step by step explanation of the NNPDF usage within *Mathematica* as well as a variety of examples.

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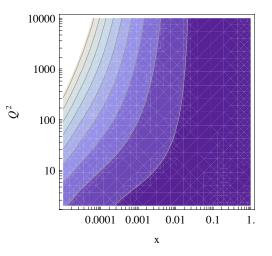


Figure 4: Contour plot of the NLO NNPDF2.3 Singlet PDF in the  $(x,Q^2)$  plane. This plot is drawn with the new NNPDF *Mathematica* interface.

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<pre>[NITLegrate[(xPDFBnsemble[x, Q20, 1][[j]]/x - xPDFBnsemble[x, Q20, -1][[j]]/x), [x, 10 ^ (-9), 1]], [j, 1, replicas]], . StandardDeviation[integral]) 25094) 25094) -d(x)]dx = 0.99999 ± 0.01251, in agreement with \$\integral{G}[d(x) - d(x)]dx = 1</pre>	$u(x) - \overline{u}(x)]dx = 2.003 \pm 0.013$ , in agreement with $\int_0^1 [u(x) - \overline{u}(x)]dx = 2$ .	
25094) $-\vec{d}(x)]\vec{a}x = 0.9999 \pm 0.01251$ , in agreement with $\int_{0}^{\infty} [\vec{d}(x) - \vec{d}(x)]\vec{a}x = 1.$	able[NIntegrate[(xPDFEnsemble[x, Q20, 1][[j]]/x-xPDFEnsemble[x, Q20, -1][[j]]/x), {x, 10^(-9), 1}], {j, 1, replicas]], ral], StandardDeviation[integral]}	
$-\overline{d}(x)]dx = 0.9999 \pm 0.01251$ , in agreement with $\int_{0}^{\infty} [d(x) - \overline{d}(x)]dx = 1.$	0.0125094}	
::::: * sumules.nb * (su ia)	$d(x) - \overline{d}(x) \left] \overline{\sigma} x = 0.99999 \pm 0.01251$ , in agreement with $\int_{0}^{1} \left[ d(x) - \overline{d}(x) \right] \overline{\sigma} x = 1.$	
sumules.nb* (su ia)		
		2 oggetti nel cestino
h	🌞 sumrules.nb * (su ja	*

Figure 5: A snapshot of the *Mathematica* notebook written with our interface for checking the momentum and valence sum rules from the NLO NNPDF2.3 parton set.