Chapter 4
Relationships Between History of Mathematics and History of Art

Clara Silvia Roero

During the course of centuries mathematics has interacted in many ways with culture and human activities, and among these a place of privilege has been reserved for art and architecture. Numerous artists, architects and historians of mathematics have made these relationships evident, such as Piero della Francesca, Leonardo, Albrecht Dürer, Maurits Cornelis Escher, Le Corbusier, Felix Klein, G. David Birkhoff, Andreas Speiser and Federigo Enriques, to mention only the most celebrated.

In this chapter I will show several examples of the existence of three levels of interaction between mathematics and art: the presence of a mathematical substrate in various archaeological and artistic relics from antiquity, the conscious or unconscious application by artists of mathematical principles whose theories had not yet been fully developed, and finally the relationship established by some mathematicians with artists and art theorists that permitted an awareness and acquisition of mathematical knowledge and rules that were then applied to artistic creations. The development of these three levels of interactions between mathematics and art can be a valid aid to the creation of a unified vision of the history of culture of peoples and civilizations, indicating various kinds of influence: technical-practical, theoretical-scientific, mystical-sacred, principles and customs, etc.

Indeed, in the wake of a long-term historiographic approach, new research perspectives have emerged recently that have been favourably received by art historians and critics. In particular, I wish to refer to some of the studies of Tullio Viola (1904–1985), who in the latter years of his life was partial to interdisciplinary

First published as: Clara Silvia Roero, "Relationships between History of Mathematics and History of Art". Pp. 105–110 in *Nexus VI: Architecture and Mathematics,* Sylvie Duvernoy and Orietta Pedemonte, eds. Turin: Kim Williams Books, 2006.

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K. Williams and M.J. Ostwald (eds.), *Architecture and Mathematics from Antiquity to the Future*, DOI 10.1007/978-3-319-00137-1_4,

investigations connected to archaeology, art and technology, especially of the most remote antiquity.

In 1984 Viola wrote:

> Every geometric property found in a figurative work is in some way and some measure an index of geometric sensitivity that cannot but manifest itself or be manifest in the works that we are used to calling ‘scientific’. The ‘geometry of the deep’, which physicist Wolfgang Pauli calls “*Urintuition*”[1] is like the submerged mass of the iceberg, of which we can see and know only the tip that rises above the surface. The rational systemizations of geometric theories has their psychological, and therefore historical, roots in man’s irrationality. Constructions logically demonstrated are based on spontaneous, unconscious intuitions, of which aesthetic feeling is a trustworthy guide (Viola 1986: 314).

Under Viola’s guidance, since 1979 Livia Giacardi and I have studied a Sumerian game in which a serpent that bites its own tail moves with extraordinary regularity through a certain number of compartments, and if during the course there is a succession of natural numbers, there emerges a magic square, that is a square in which the sum of the elements of each row, column and diagonal is constant (Giacardi and Roero 1979) (Fig. 4.1).

Through an examination of other relics (with intertwined serpents, ornaments of the checkerboard of the royal tombs of Ur (Fig. 4.2) and polygonal disks with stepped sides) we arrived at interesting topological properties of such interlaced motifs and to the formulation of hypotheses as to their geometrical and arithmetical-magical meanings.[2]

Each checkerboard, however formed, can be covered in one, and only one, way, by intertwined serpents, regardless of the orientation of the intertwining, of the exchange of “underpassages” with “overpassages” in the crossing of the “doors of communication” between one compartment and the next, and of the collocation of the heads of the serpents along the corresponding paths. For rectangular checkerboards, in which the number of rows and columns are prime numbers, the braid is always composed of a single serpent, while in square checkerboards of n rows and n columns, there are exactly n serpents. In all cases, the methods of numbering of the compartments can be described so as to obtain numeric tables that present magic properties. This is an example of a mathematical substrate in an artistic creation.

In 1980 Viola studied the problem of the passage from the contemplation of ideal geometric figures of primitive man to that of the rational geometry in the work of

[1] *Jedes Verstehen ist ein lanwieriger Prozess, der lange vor der rationalen Formulierbarkeit des Bewusstseinsinhaltes durch Prozesse im Unbewussten eingeleitet wird: auf der vorbewussten Stufe der Erkenntnis sind an Stelle von klaren Begriffen Bilder mit starkem emotionalem Gehalt vorhanden, die nicht gedacht, sondern gleichsam malend geschant werden* (Every process of mental comprehension is of long duration. Much before the possibility of the conscious formulation of its content, it takes the form by means of an unconscious process. At the level of pre-consciousness, in place of clear concepts are present images of a strong emotional content. These are not only thought of, but are looked at as though painted) (Pauli 1961: 91).

[2] Viola and I presented two papers on this theme at the eleventh Congress of the Union of Italian Mathematicians in Palermo; see Giacardi et al. (1979, 1980).

Fig. 4.1

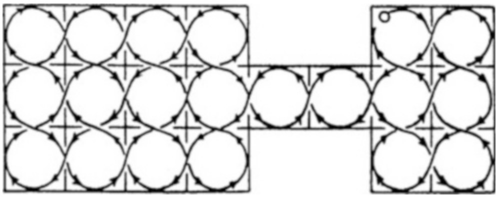


Fig. 4.2

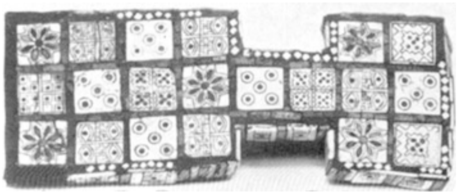
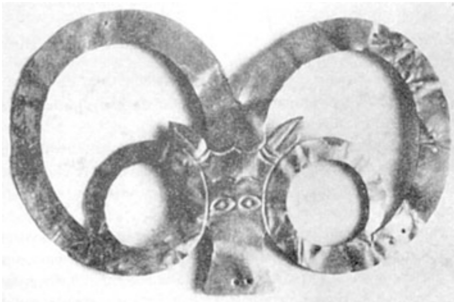


Fig. 4.3



Thales of Mileto (Rizzi and Viola 1980). In order to demonstrate this, Viola chose an ornament in gold leaf that was found in Iran and is datable to the second millennium B.C., representing the head of a ram whose long horns curl in the form of a volute, each of which has the forms of four ovals that are noticeably elliptical (Figs. 4.3 and 4.4).

Having performed an accurate graphical analysis, Viola underlined the extraordinary approximation of an ellipse of the external oval of one of the horns and concluded:

> It seems evident that the Iranian artist who created this jewel let himself be guided by exclusively aesthetic requirements to reproduce a geometric figure (the ellipse) that he contemplated in his own mind at the very moment that he was working. But simultaneous and complementary contemplation and artistic creation are not in themselves sufficient to permit the birth of the mathematical concept: for this, contemplation had to be enriched by rational needs, and this effectively occurs a millennium and a half later, in a faraway land, by another people, in an extremely complex, refined and philosophically profound cultural context. . . [T]he geometry of Thales was not yet a rational geometry, in the way in which we think of that, . . . in that, the contemplation of figures was no longer exclusively of an aesthetic nature but was already enriched by the attempt at deductive justification, going in search of the 'reason' behind certain properties (Rizzi and Viola 1980).

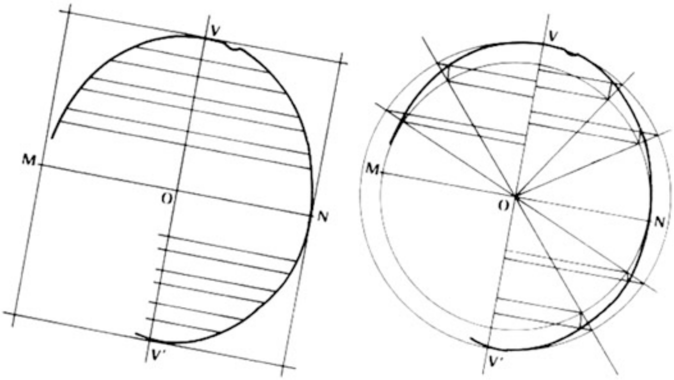


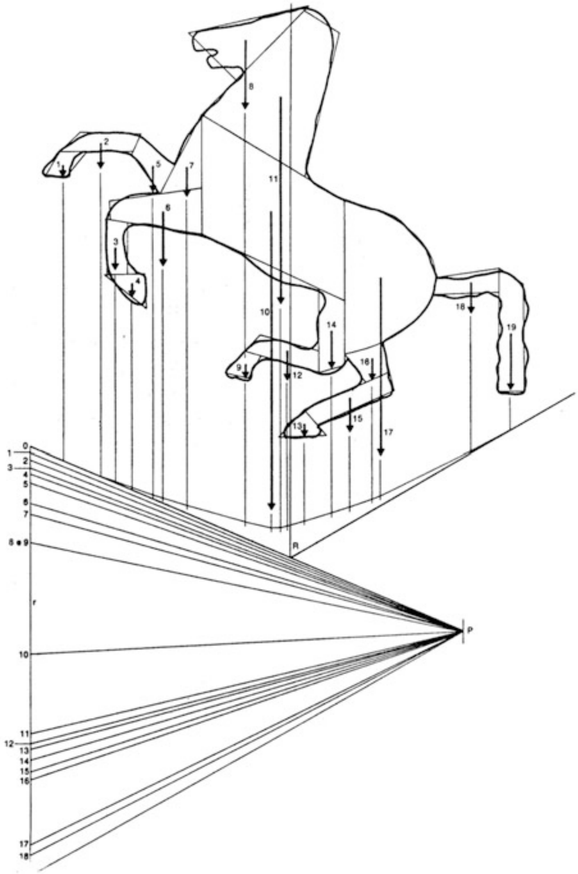
Fig. 4.4

Fig. 4.5

In the same year, Viola suggested that I study a bas-relief of a metope of the Parthenon, the results of which enjoyed a considerable degree of success in the international arena (Roero 1981, 1982; Ragghianti 1982). It was in fact possible to demonstrate how a great artist such as Phideas intuited and visualized admirably in his frieze the physical principle of equilibrium, two centuries before that would be explicitly formulated by Archimedes (Figs. 4.5 and 4.6).

The rearing horse with only one hoof touching the ground is in static equilibrium. Analogous research was undertaken by Viola in collaboration with Maria Teresa Navale and Silvia Mazzoni (Manzoni and Navale 1980; Manzoni et al. 1980), in which they were able to identify the possible construction technique of the tunnel of the island of Samos in the fourth century B.C. by Eupalino of Megara with the aid of triangulation (Manzoni et al. 1980, 1985).

Fig. 4.6



Together with Navale, Viola was then able to define the geometric form of the profile of the Narmer Palette, a celebrated Egyptian masterpiece in slate of 3000 B. C., demonstrating that it is an excellent approximation of the catenary curve (Navale and Viola 1985, 1986) (Fig. 4.7).

They also found very interesting results in a long and complex mathematical-historical analysis of some Ionic volutes of temples in Greece and Italy (Navale and Viola 1980, 1982). Together with Silvio Curto, then director of Turin's Egyptian Museum, Viola studied the measurements of some Egyptian colossi (Curto and Viola 1980) and conjectured as to the construction of the pyramid of Cheops, without however arriving at definitive conclusions. His historic approach was further stimulated by the research of Lina Mancini Proia and Marta Menghini on the evolution of the shape of cupolas in churches, from circular to ovals and only in the seventeenth century to a form that was decidedly elliptical (Mancini Proia and Menghini 1984). These authors

Fig. 4.7

maintain, for valid reasons, that the three Roman architects of the Baroque Gianlorenzo Bernini, Giacomo Berrettini da Cortona, and above all, Francesco Borromini, to whom we owe the first elliptical cupola, that of S. Carlo alle Quattro Fontane were inspired by astronomical research and the fascinating findings of Galileo.

The ellipse of the Iranian ram, the rearing horse of the Parthenon frieze and the curve of the Narmer Palette are all examples of conscious or unconscious application of mathematical principles whose formal theories would be fully developed only much later.

Finally, to illustrate the third level of interaction and exchange of knowledge between mathematicians and artists we can recall the historic studies conducted on the proportional models to represent the beauty of the human body in classical Greece and the Renaissance (Roero 1999, 2000), on the geometry of the fixed compass from the Medieval to the 1900s (Roero 2006), and the symmetry of Guarino Guarini (Roero 2005).

Translated from the Italian by Kim Williams

Biography Clara Silvia Roero is full professor of History of Mathematics at the University of Torino. She is a member of the editorial board of several journals, including *Bollettino di Storia delle Scienze Matematiche*, *Revue d'histoire des mathematiques, Lettera Matematica Pristem,* and *Il Maurolico.* She is on the Scientific Committee for the collected scientific papers of the mathematicians and physicists of the Bernoulli family, for the National Edition of R. G. Boscovich's works, and for the papers of M. G. Agnesi. She was President of the Italian Society of History of Mathematics (SISM) from 2000 to 2008, and a member of the International Commission of History of Mathematics. She is currently Director of the Torino Research Group on History of Mathematics.

References

Curto, S. and Viola, T. 1980. Per un computo ponderale di alcuni manufatti litici egizi. *Atti dell'Accademia delle Scienze di Torino*, Cl. Scienze Morali, **114**: pp. 155-171. Turin: Accademia delle Scienze.

Giacardi, L. and Roero, C. S. 1979. *La matematica delle civiltà arcaiche*, Turin: Stampatori.

Giacardi, L., Roero, C. S. and Viola, T. 1979. Ipotesi sull'esistenza di una matematica magico-sacrale presso gli antichi Sumeri. Pp. 143-160 in *Arithmos-Arrythmos, Skizzen aus der Wissenschaftsgeschichte, Festschrift für J.O. Fleckenstein zum 65 Geburtstag*. Munich: Minerva Publikation.

Giacardi, L., Roero, C. S. and Viola, T. 1980. Proposte d'interpretazione di alcuni reperti sumerici risalenti al III millennio a.C. (Ipotesi sull'esistenza di una matematica magico-sacrale). *Quaderni di Matematica della Università di Torino*, **10**: pp. 1-64. Turin: Università degli Studi di Torino.

Mancini Proia, L. and Menghini, M. 1984. Dalle ovali policentriche alle ellissi, nell'architettura barocca (Una possibile derivazione concettuale dall'astronomia). *Atti dell'Accademia delle Scienze di Torino*, Cl. Sci. Fis. Mat. Nat., **118**: pp. 325-338. Turin: Accademia delle Scienze.

Manzoni, S. and Navale, M. T. Osservazioni del presentatore della nota di, su talune proprietà matematiche di un bassorilievo di Fidia. *Atti dell'Accademia delle Scienze di Torino* **114**, 1980, pp. 466-467. Turin: Accademia delle Scienze.

Manzoni, S., Navale, M. T. and Viola, T. 1980. Problemi geometrici applicati alle tecniche costruttive e rappresentative, con particolare riguardo al tunnel di Samo (Ipotesi di triangolazione topografica nel VI sec. a.C.). *Quaderni della Scuola di Disegno*, **80.3**: pp. 1-66. Turin: Università degli Studi di Torino.

———. 1985. Imago et mensura mundi. Pp. 505-514 in *Atti del IX Congresso Internazionale di storia della cartografia*. Rome: Istituto dell'Enciclopedia Italiana.

Navale, M. T. and Viola, T. 1980. Le volute joniche nei capitelli della Grecia classica: saggio di un'analisi strutturale oggettiva. *Atti dell'Accademia delle Scienze di Torino*, **114**: pp. 303-317. Turin: Accademia delle Scienze.

Navale, M. T. and Viola, T. 1982. Nuove ricerche sulle volute dei capitelli jonici della Grecia classica. *Atti e Rassegna tecnica della Società degli Ingegneri e degli Architetti in Torino*, **36**, 9: pp. 489-512. Turin: Società degli Ingegneri e degli Architetti.

———. 1985. Il profilo della Tavolozza di Narmer. *Atti dell'Accademia delle Scienze di Torino*, Cl. Scienze Morali, **119**: pp. 87- 94. Turin: Accademia delle Scienze.

———. 1986. *Memorie dell'Accademia delle Scienze di Torino*, Cl. Sci. Mor., **5-10**, pp. 3-29. Turin: Accademia delle Scienze.

Pauli, W. 1961. *Aufsätze und Vorträge über Physik und Erkentnis Theorie*. Braunschweig: Verlag Vieweg.

RAGGHIANTI, L. 1982. Recenti scoperte a proposito di arte e geometria. Quel cavallo di Fidia . . . *La Nazione*. Florence, 2 September 1982, p. 3.

RIZZI, B. and VIOLA, T. 1980. Dalla contemplazione ideale delle figure geometriche nell'uomo primitivo a quella della geometria razionale attraverso l'opera di Talete di Mileto. *Atti dell'Accademia delle Scienze di Torino*, **114**,: pp. 355-363. Turin: Accademia delle Scienze.

ROERO, C. S. 1981. La statica dall'arte alla scienza. *Le Scienze*, **150**: pp. 88-97. Roma: Gruppo Editoriale l'Espresso.

———. 1982. Statik zwischen Kunst und Wissenschaft. *Spektrum der Wissenschaft*, pp. 68-77. Stuttgart: Verlagsgruppe Georg von Holtzbrinck.

———. 1999. Mean, Proportion and Symmetry in Greek and Renaissance Art, Symmetry: Culture and Science. In *The Quarterly of the International Society for the Interdisciplinary Study of Symmetry* (ISIS-Symmetry). Special Issue: G. Darvas, ed. *Chapters from the History of Symmetry*. Vol. **9, 1-2**, pp. 17-47.

———. 2000. Media, proporzione e simmetria nella matematica e nell'arte, da Policleto a Dürer. Pp. 40-59 in L. Giacardi and C. S. Roero, eds. *Conferenze e Seminari 1999-2000*. Turin: Associazione Subalpina Mathesis.

———. 2005. Les symétries admirables de Guarino Guarini. In *Symétries, Contribution au sé minaire de Han-sur-Lesse*. Pp. 425-442 in P. Radelet de Grave, ed. *Réminisciences*, **7**. Turnhout: Brepols.

———. 2006. Il compasso nella geometria e nell'arte. In L. Giacardi and C.S. Roero, eds. *Matematica Arte e Tecnica nella storia. Atti del Convegno in memoria di T. Viola*. Foligno: Edizioni dell'Arquata.

VIOLA, T. 1986. Aspetti e problemi della matematica antica (Alcune proposte di un nuovo indirizzo di ricerca). Pp. 314-315 in *Atti del Convegno Storia degli studi sui fondamenti della matematica e connessi sviluppi interdisciplinari*, Vol. **1**. Rome: Tip. Luciani.