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Kindergarten Children and Early Learning of Number: Embodied and Material Encounters Within the Classroom

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Abstract
In this article, we draw on assemblage theory to investigate how children aged 5 engage with different material surfaces to explore ordinal and relational aspects of number. The children participate in an activity in which they first interact with a strip on the floor, then with a multi-touch iPad application, to work with numbers in expressive ways. Focus is on the physicality and materiality of the activity and the provisional ways that children, surfaces, and number come together. While the notion of assemblage helps us see how movement animates the mathematical activity, we enrich our understanding of the entanglement of children, matter, and number as sustained by coordinated movements, from which numbers emerge as relations.

Keywords Number · Ordinality · Relational · Assemblage · Coordinated movement

Introduction

The presence of symbols in the early learning of number is not very frequent in the habitual practice of schooling, especially in kindergarten. Usual approaches to number with very young children occur through the use of concrete objects and a main focus on cardinal aspects of number, even though emergent evidence in neuroscience research shows that ordinality (which centrally involves symbols) is of greater significance arithmetically than is commonly recognised (e.g., Lyons et al., 2014). Sinclair and Coles (2016) argue that current early number learning focuses much more on cardinality, in part because of its accessible, "concrete" nature in that it concerns determining the numerosity of a given set of objects (linking numbers to collections). These researchers propose that ordinal awareness of number can emerge out of work on number symbols as relations. Borrowing from this research, the article attempts to contribute to current

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discussions on the role of ordinality in the early learning of number. In doing so, we shift attention to the way that early learning of mathematics can be symbolic *and* concrete, in the sense of establishing relations.

We discuss episodes from a classroom-based intervention with a group of kindergarten children aged 5 years, who engage in activities designed to propose a *relational* approach to number sense (see the "On Relational Aspects of Number" section for a more detailed discussion). The children first interact bodily with a number line-like strip on the floor and then create and sense numbers using a multi-touch iPad application, called *TouchCounts* (Jackiw & Sinclair, 2014). The material activity of the children with the strip and the app implicates kinaesthetic and embodied, as well as symbolic engagement. We are interested in investigating the provisional and indeterminate ways in which the concept of number emerges from this engagement and the way that symbols are not just abstract representations that stand outside the body. To this aim, we use the notion of assemblage (DeLanda, 2006, 2011; Deleuze & Guattari, 1980; Deleuze & Parnet, 1987) to focus on the always-emergent conditions of the mathematical activity and the entanglements of bodies and material-discursive tasks with the surfaces at play, that is, the floor and the screen of the iPad. Because these entanglements are provisional arrangements that involve humans and other bodies moving and learning together, we use the term "learning assemblage" (de Freitas et al., 2017). We also draw on Sheets-Johnstone (2009, 2012, 2016)'s vision of kinaesthesia to understand coordinated movement as central to the changing nature of the learning assemblage. This perspective helps us to look into how bodily and material engagement can animate the concept of number as relational, while shedding light on the affective intensity of the learning assemblage. Particularly, we centre on how the concept of number grows out of the engagement of the children with the different materials, and we trace the kinaesthetic and material aspects of the activity to see the early learning of number in this context as both symbolic *and* concrete.

Theoretical Enlightenments

This section presents the theoretical elements that inform our study, namely, assemblage theory, coordinated movement, and relational aspects of number. Assemblage theory allows us to decentre the human body as the only centre of the activity and to study individuation processes as distributed among different bodies (children, tools, concepts), with a focus on the emergent relations that constitute those processes in any assemblage. We then characterise our analysis of learning assemblages through the notion of coordinated movement. Finally, some concerns about relational aspects of number and the importance of ordinal awareness for number sense are presented.

On the Emergent Nature of Assemblage

In this article, we draw on *assemblage theory* to put attention on the provisional nature of the mathematical activity of a group of children and see how the activity is both symbolic *and* concrete. To this aim, we focus on the contingency and temporality of the activity, namely, on the emergent relationships in an assemblage and their features, as we will explain in this section.

Assemblage is a concept that goes back to the work of the French philosophers Gilles Deleuze and Félix Guattari (Deleuze & Guattari, 1980). An assemblage is described as:

> a multiplicity which is made up of many heterogeneous terms and which establishes liaisons, relations between them across ages, sexes and reigns – different natures. Thus, the assemblage's only unity is that of co-functioning: it is a symbiosis, a 'sympathy'. It is never filiations which are important but alliances, alloys; these are not successions, lines of descent, but contagions, epidemics, the wind. (Deleuze & Parnet, 1987, p. 69)

According to DeLanda (2006), assemblage is a topological concept that invokes a structural generative process, along the dimensions of the temporal, the material, the relational, and the perceptual. Müller (2016) stresses that an assemblage is a mode of arranging heterogeneous entities making them work together for a specific time. For Marcus and Saka (2006), the notion is "a resource with which to address in analysis and writing the modernist problem of the heterogeneous within the ephemeral, while preserving some concept of the structural" (p. 102). The image of structure is elusive, since the time–space in which assemblage is imagined is inherently unstable and infused with movement and change, engendering puzzles of processes and relationships without deterministic understanding and overarching organising principles. Emphasis is on relations between movements, from which reality progressively emerges rather than being built or structured. Two ideas are key to this dynamic character of assemblage: temporal instability and heterogeneity, the first understood in terms of emergence and the second as a property by which systems are productive in interaction.

DeLanda's (2006, 2011) view offers us further insights to understand the *complex* nature of assemblage. DeLanda in fact contends that an assemblage consists of relations of *exteriority*: that is, the component parts of an assemblage (people, objects, etc.) keep a certain autonomy from the relations between them, and their properties can never explain, or be reduced to, these relations. The concept of structure, instead, implies that components have no independent existence apart from the relations in which they exist (relations of *interiority*). While relations of interiority are defined by necessity, relations of exteriority are only contingently obligatory. Assemblages then establish territories as they emerge, hold together, and stabilise themselves ("territorialisation"), but also constantly mutate, change, and break down, even causing an entirely new assemblage to emerge ("de-territorialisation"). In other words, the temporality of an assemblage is always emergent and does not involve new forms, but "forms that are shifting, in formation, or at stake" (Ong & Collier, 2004, p. 12).

In this paper, we use the notion of *learning assemblage* to better account for the relational movement that makes activity develop, and the provisional ways in which children encounter numbers while using a specific tool, so that children, number, and tools move together and learn together. In this view, meaning does not pertain to the individual nor does it spring out of the tool but is always provisionally emerging out of the mobile arrangement of the different bodies that constitute an assemblage. We see learning here as occurring in complex interaction with the surfaces and tools at play, across provisional arrangements of bodies, rather than as the product of the activity.

We therefore shift attention from studying the activity in terms of a child's intention or a tool's feedback, mental schemas that are used by the individual or activated by the tool, to accounting for the relational ways in which concepts arise from activity and are not separated from the body. Instead of looking at how children come to an accommodation or interiorisation—learning—of number (in a Piagetian or Vygotskian perspective), we are more interested in seeing what emerges from the interconnections that sustain the activity of the child with the tool (in line with post-humanist approaches that challenge the representationalism of mainstream qualitative inquiry; e.g., Maclure, 2013). To do so, we focus on the unstable and heterogeneous connections that constitute the learning assemblage over time and posit emphasis on instabilities and junctures in the activity.

The idea of assemblage helps us better capture the always-emergent conditions of the mathematical activity, shedding light not only on its embodied aspects but also on its material dimension. It helps us attune differently to the spaces in which learners move, the objects that touch them, and the forces that affect them. As an example, we might look at the rhythmic voicing of numbers or a rising pitch as sound modulations that bring into being order or discreteness of numbers or a force that destabilises the activity giving rise to a new pattern. In addition, it helps us account for the affective dimension of the activity, in which relations are not deterministic but mutually constitutive. Drawing from Sheets-Johnstone (2009), *affectivity* is the fundamental "responsivity" of life; therefore, here the affective dimension of the activity pertains to the responsive nature of bodies, the power of things to affect (and be affected by) each other in any entanglement. So, engaging of bodies in the activity depends on their capacity to be affected, which provisionally sustains the assembling.

Our interest in assemblage theory exactly rests on its tendency towards novel, non-classical discourses that invest in the emergent and the heterogeneous, calling attention to the socio-material forces at play in the mathematics classroom and for new ways of conceptualising power and space. This does not mean we put aside the expressive—content, meaning, language, sign—but that we return to a concern for materiality that allows for a better understanding of the co-constitution between humans and non-humans (tools, concepts, surfaces) and of how the material contributes to the expressive, operating alongside and within discourse. Rather than following conventional approaches to educational inquiry, which often privilege the contribution of the individual, we attempt to account for the expressive as a practice that is *inseparable* from all human and non-human elements in an assemblage. We seek a research practice that engages the materiality of the expressive and its entanglements in bodies and matter—like MacLure (2013) and Mazzei and Jackson (2016) do in rethinking voice.

On the Primacy of Movement

As mentioned above, assemblage theory allows us to account for the relational unstable and heterogeneous character of the engagement of children with material surfaces and the concept of number. However, while bringing forth attention to the provisional and the contingent within assemblages, assemblage theory does *not* emphasise how to speak about them. We see relations as constitutive of processes of assembling, but *how* can we describe and trace them while giving justice to the central role of movement in assembling? In this article, we entangle assemblage theory with the phenomenological vision of Sheets-Johnstone to shift towards a sense of *coordinated movement* as central to our analysis of classroom activity. In previous work, we focussed on a sense of coordinated movement as an attempt to study the affective intensity of learning assemblages (see de Freitas et al., 2017, 2019). Following Kelso and Engstrøm (2006), we want to overcome a view that considers coordinated movement as a mechanistic coordination of a set of components, because it would disfigure the very idea of coordinated dynamics. To avoid such a simplistic way of thinking about movement, we need to consider that movement is always kinaesthetically informed (Sheets-Johnstone, 2012, 2016).

Sheets-Johnstone (2012) stresses that even the mirroring neuronal system depends on our own kinesthetically experienced capacities and possibilities of movement, and these are the gateway to our understanding of the world. Through kinaesthesia we spontaneously experience, or feel, directly distinctive qualitative dynamics of movement. Any and all movement creates its own distinctive space, time, and force, and when we turn attention to our own coordinated dynamics, we recognise *kinesthetic melodies* (Sheets-Johnstone, 2012). We can experience movement as a purely objective happening, a distinctive world phenomenon, like “when we turn attention to where or how we ourselves are moving or where or how an object other than ourselves is moving. In such instances, we perceive movement as a kinetic happening in a *three-dimensional, time-clocked* world.” (Sheets-Johnstone, 2016, p. 17, *emphasis in the original*). But we also coordinate movement in ways in which, as objects in motion, we move in harmony with each other: “Combining kinesthetic with kinetic-visual and/or kinetic-tactile awarenesses, we move in sensitively attuned ways with others.” (Sheets-Johnstone, 2016, p. 18). What matters are the “qualitative kinesthetic dynamics that are definitive of movement; definitive because they specify the basic experiential character of movement” (Sheets-Johnstone, 2016, p. 52).

Briefly speaking, movement is the dynamic attunement of the body to the world. Drawing on Sheets-Johnstone, we account for the responsive nature of bodies, how they turn away or lean in, and at the same time how they join with other bodies in coordinated movements (see de Freitas et al., 2017, 2019 for a detailed discussion of coordinated movement as involving affective bonds between components and assemblages as sustained by the dynamic force of affect). Therefore, we can move in concert and in disharmony with others as well, we can come together with things or split apart, assembling or disassembling into larger overt coordinated responses between bodies. Movement is the force that animates matter in the assembling and stresses the potentiality of the body. As the dynamics of assemblages are woven into the material environment, coordinated movements across a wide array of bodies are sustained. In this article, we are interested in the way in which coordinated movements might inform the learning event and unfold relational arrangements of moving parts from which specific number sense arises. We will look at how things move, move away, or come together, to inform what happens in the learning assemblage and, specifically, to grasp the emerging concept of number.

On Relational Aspects of Number

In the introductory section, we have sketched that the activity we discuss in this article has primarily to do with *relational* aspects of number. To explain what relational is intended in the context of number sense, we specifically draw on recent work on early number and symbolic awareness (e.g., Coles, 2014; Sinclair & Coles, 2016). A first distinction between ordinal and cardinal aspects of number is needed. As Sinclair and Coles (2016) point out, while ordinality typically refers to the capacity to place numbers in sequence, for example, to know that 4 comes before 5 and after 3 in the sequence of natural numbers, cardinality refers to the capacity to link numbers to collections, for example, to know that "4" is the correct representation to denote a group of four objects. They argue that the current emphasis on cardinal awareness in the early learning of number might be misplaced (see also Coles, 2014).

This position is sustained by recent neuro-scientific evidence, which has looked at how children understand and manipulate number symbols (numerals, like 3, 4, and 5) as a particularly crucial building block (Lyons et al., 2014). Lyons and colleagues argue that how flexible a child is in reasoning about direct relations between number symbols (like in investigating whether three numbers are in order of size, that is, in articulating relative order or in sequencing) is one of the strongest predictors of skills such as mental arithmetic and crucial to further mathematical success. According to Lyons and Beilock (2013), qualitatively distinct areas of the brain are active during ordinal tasks with number symbols, compared to tasks involving collections of objects. Coles (2014) stresses that "one clear hypothesis to emerge is that students' awareness of ordinality may be distinct from awareness of cardinality and, in terms of developing skills needed for success in mathematics, that ordinality is the more significant" (p. 331). In this paper, we share with Sinclair and Coles (2016) an interest in working on symbol-symbol relationships and in examining how ordinal awareness might arise out of this kind of work. In fact, in an ordinal approach to number, as it was conceived of by the educator Gattegno (1974), who introduced number as a relation, "the focus shifts from linking numbers with the concrete (collections of objects) onto linking numbers with each other", so that "number skills and awareness can be developed from a structure" (Coles, 2014, p. 331).

This position challenges current habits of work within the mathematics classroom, in our country as well as regarding other curricula all over the world (in Canada and the UK, for example), where the emphasis in the first years of schooling is primarily on linking number symbols to collections of objects.

The research study that we propose in this article aims at exploring the potential of taking a more ordinal approach to the early learning of number and, therefore, at contributing to the open discussion on the role of ordinality in the development of number sense. Looking at the concept of number that emerges from the entanglement of the bodily and material dimensions of the children's activity, we can see how the concept differs so significantly from the cardinal, a-symbolic meaning that currently dominates in early learning of mathematics, and how the latter can be symbolic *and* concrete at the same time. In the next section, we draw out the method and participants of the study, before reporting on the results of the empirical research.

Methodology

In line with our theoretical commitments, we will look at data in terms of movement rather than in terms of what children are thinking or a tool or surface is doing. We see the assemblage of children bodies, things, and the concept of number within the classroom as the unit of analysis for our investigation. Our interests will specifically be on the complex ways in which bodies and things move (anyhow they do), in order to speak about the learning assemblage and how the concept of number is involved in and evolved by the activity.

Context and Participants

The study discussed in this article is part of a classroom-based intervention in a kindergarten school in Northern Italy and involves a group of 25 children 5-year-olds (7 girls and 18 boys) and one of their teachers in eight 3-h sessions over the second half of the school year. The intervention was designed by the first author with two objectives: (1) to engage children in explorations of different aspects of number, including the symbolic, and (2) to work with integer numbers by means of rhythmic patterns and embodied interactions. The teacher led all but one meeting in the main hall of the school, which provides space for big group work and collective bodily interactions that involve the use of different materials. In this article, we focus on specific work with two kinds of material: a long paper strip made of differently coloured sections to represent a number line (see Fig. 1a); a multi-touch application for iPad, *TouchCounts*, to explore number in expressive and relational ways.

The strip was unrolled on the floor when used, and the children could interact with it essentially through placement and order, while the tasks with the iPad engaged them differently, in group work or in pair interviews that required tapping. In short, the first three sessions focused on collective discussions on preliminary knowledge of the number, digit, and role of zero, while in the fourth session, the strip was introduced. The iPad was first used during the fifth session, when the first author (the researcher, in the following) visited the classroom and led the activity. The children were divided into three groups, and the researcher used *TouchCounts* with each group in different moments during the day.

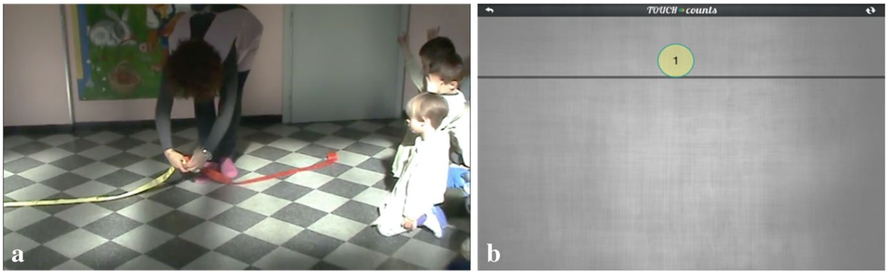


Fig. 1 **a**, **b** The long strip on the floor; screenshot of shelf mode in the Enumerating World

TouchCounts and the Enumerating World

TouchCounts (Jackiw & Sinclair, 2014) is a free multi-touch application that is primarily for counting but is also used for subtracting and adding. It essentially offers two different worlds for exploring number: an Enumerating World and an Operating World, which provide respectively ordinal and cardinal models of number. In the study, the children used only the first world. Every time one finger touches the screen, a yellow disc is produced, labelled with a numeral, and the numeral is simultaneously said aloud. The tactile, the aural, and the symbolic thus come together in the application. Each consecutive touch makes a new yellow disc appear with the consecutive numeral on it, until the screen is reset, and a new session starts. The Enumerating World might be explored with or without the gravity mode on. If gravity is turned off, then the discs stay in the position where the finger has tapped the screen. If gravity is turned on, the discs fall off the screen, unless the shelf is also turned on. The shelf is a line that divides the screen into two areas (Fig. 1b). Tapping below the shelf makes discs fall off, while tapping above leaves a disc on the shelf. Since the application is multi-touch, it is responsive to the simultaneous touch of more than one finger. It is possible, for example, to show only one number—a 6—on the shelf, by either first using a single five-finger tap or five individual taps below the shelf, causing the first 5 numbers to fall off the screen, followed by a sixth tap above the shelf. Therefore, in the Enumerating World, emphasis is put on ordinal aspects of number, even though the multi-touch, which allows recognising both consecutive and simultaneous taps, also implicates cardinality. These potential configurations open room for work on sequences of non-consecutive numbers, which we believe might engender interesting occasions to develop relational awareness of number.

Method and Data

In this article, we draw attention first to a short period of an activity led by the teacher that involved the whole class in interacting with the strip (day 4) and then to a particular moment in which a group of seven children was engaged by the researcher in working with the iPad (day 5). We have chosen these two activities as they share specific focus on odd and even number sequences, as it will be detailed in the next section, which we believe will illustrate the relational vision of number we are proposing in this paper. Our aim is to trace out the ways that different coordinated movements unfold aspects of number sense and inform the learning event, while bringing forth relationships between the two activities and how they characterise the concept of number that gets to be assembled. The data for the analysis consists of videotaping of the classroom activities through a mobile camera and written notes. We will provide accounts of what occurred based on the transcripts from the video data, taking assemblage as our unit of analysis in the two different moments of the study. Therefore, we will centre on the becoming of the learning assemblage paying attention to the coordinated movements of the children with concepts and surfaces.

Embodied and Material Encounters: Findings

The Coordinated Movement of Standing-Laying Down

The children already worked with the ten digits 0 to 9 and used the strip as a number line to position themselves along it. They played the role of specific numbers they had previously chosen, wearing red cardboards with corresponding numerals, and moving inside coloured hoops placed on the floor, so that numbers were ordered at the end (Fig. 2a).

In the first episode we consider, the teacher proposes the new task that when a numeral is uttered, the child who wears the corresponding cardboard jumps on the other side of the strip and lays down on the floor but face up (so the numeral is still visible) (Fig. 2b). The ten numerals are repeated by other children in the room right after the teacher pronounces them. The task is repeated twice: in the first time, the teacher follows a non-ordered sequence, and in the second time, she chooses the specific, regular pattern that will leave only the odd or even children-numerals standing at the end.

This is not a trivial task: some children need hints from other classmates or to check their cardboard numeral before jumping. In the end, once the sequence of 0, 2, 4, 6, and 8 is actualised by the children laying down (Fig. 2c and d), a child is asked to first pronounce these five numerals aloud and then the numerals of the standing children (odd numerals).

Discussion

The activity involves different material objects: the numbered red cardboards, the long strip made by many pieces along the floor, and the hoops on one side of the strip. In this assemblage, numerals are associated with the children and are represented on the red cardboards. Numbers as relations, though, emerge out of the movements of the ten children and their cardboards, as they place themselves inside specific hoops when they are made present through the teacher's utterances. At the very beginning, numerals are assembled with the strip and the children as they position

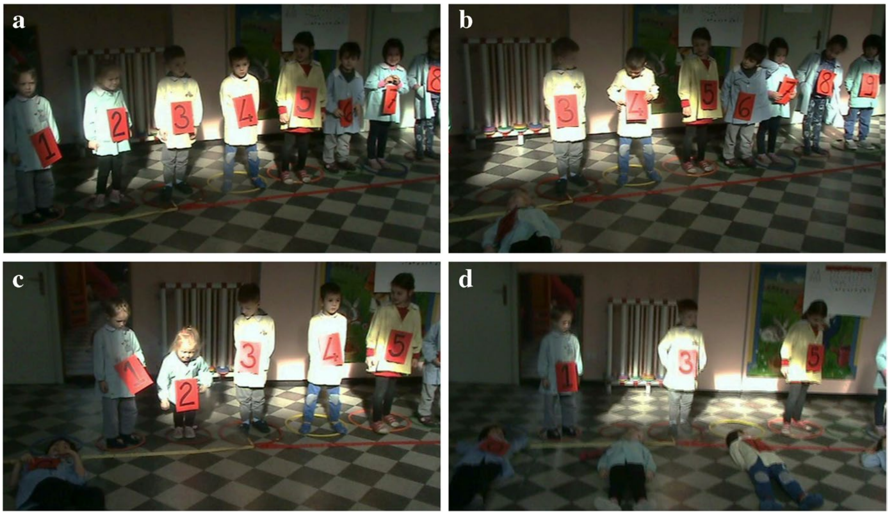


Fig. 2 **a–d** (left to right) The children positioning and playing patterns

inside the hoops, but they change to new configurations once the children play the movement of one child standing, one child jumping and laying down. Note that the numerals emerging are not cardinal quantities, rather they are instantiated as positions along a number line, positions that establish relations, among children, among numbers, and among children and numbers.

The entanglement of children and number is initially one for which children are the numerals they choose, the spatial locations on the floor, and the specific sounds coming with specific symbols on cardboards. The concept of number does not come from orally counting the fingers one by one or the objects in a collection but from the specific material arrangement of the bodies, the cardboards, and the floor surface delimited by hoops and strip. Number names are uttered, but it is a movement that dominates. Number is that which can be placed and emerges in and through interaction with the strip-number line. Ordinal awareness of number is constituted of physically moving and positioning in a certain way. The directionality of numbers along the line (given by the strip on the floor), the somehow fixed space intermediate to two consecutive numbers (due to the hoops' fixed diameter), and the privileged way of envisioning the ten digits (written on red cardboard) partake of this movement on the floor, along the strip, inside the hoops. In the assemblage, the number 6, for example, is instantiated as the individual body in a particular placement, on the one side of the strip, after or before another child, after and before other sounds, and capable of moving back and forth off the line as well as of standing up or laying down. It is relational in nature. It is implicated in a collective coordinated movement of assembling with the number line, once each child is brought near to, or apart from, another child by the teacher's voicing and attention drawn to the little circular space carved out by hoops. Relationships between numbers come to matter in terms of relationships between children.

When the new task starts with the teacher and one child speaking out loud some number names in random order, the arrangement changes to a new one that requires coordinated movements involving the children's jumps and stillness and the teacher and child's voice, even though every time a sound is uttered, only one jump is eventually performed. When the task is played again, another coordinated movement emerges from half of the children standing on one side of the strip (the ones who do not move) and the other half jumping on the other side (the ones who do move). In both cases, the spatial character of the coordination is captured by the physical placements on one side or the other of the strip, placement being the expressive dimension of the coupling voice-cardboard. The temporal character instead develops through the unfolding of the dynamic bonds between the children, as they are in turn partaking in the collective movement. In particular, the slowness with which the movement occurs is important because it shows how temporality is meaningful in the assembling. Hesitation is an integral part of movement, with some children looking at their number-cardboard or those of the closest mates before deciding whether to stay or to jump. The entanglement of body, voice, jumps, and number line is now enriched by the eye to move forward.

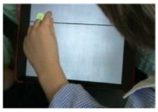
Once only alternate number names are spoken out loud in specific order, the very first odds and evens start emerging. The cut in the floor surface actualised by the long strip also partakes of this process through which the children come to encounter the two sequences. Odd and even numbers come from the material arrangement of bodies (standing or laying down), numerals, and the surfaces delimited by the strip (one or the other side). Odd numbers are what can be placed on the one side, even numbers on the other, and they emerge as two different but similar choreographies on the floor, along the strip, from one side to the other–one choreography made of standing bodies, the other one of laying bodies. Staying or jumping, together with proximity to standing bodies or laying bodies, is that which comes to constitute initial ordinal and structural awareness of one or the other kind of number.

The Coordinated Movements of Tapping Above-Below the Shelf

In day 5, the class is divided into three groups of six or seven children. The groups work in turn with the researcher and one iPad. The researcher sits on the floor keeping the iPad on her knees, while a group of children sits around her to work with *TouchCounts*. The researcher shows how to create numbers and then encourages the children to use finger tapping and the shelf to explore simple patterns of numbers, like odds and evens. In this episode, we focus on the work of one group composed by seven children. The researcher (R below) starts by recalling the previous activity with the line strip to introduce the shelf and then invites the children to work on a new task:

> R: Do you see that here [*pointing to the shelf*] there's a line? I know that you played a task with the strip: you were one on one side, another on the other side, weren't you? Can we do the same here? Let's pretend that we touch on one side or the other as if we were putting a child on one side or the other of the strip!

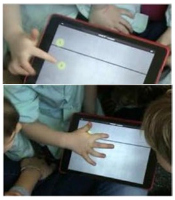
Table 1 Giovanni creates the first five odd numbers

#	Spoken	Gesture	Image	*TouchCounts*
1.		*Giovanni ^ with grasping index and thumb*		One (*1 above the shelf*)
2.	Matteo: And one here	*Matteo points to the area below the shelf*		
3.	R: And one ...?			
4.		*Giovanni _*		Two (*2 falls off*)
5.	R: Below, ok. Then?			
6.		*Giovanni ^*		Three (*3 on the shelf*)
7.	R: Then one above, one below			
8.		*Giovanni _^_^_^ ^ with the same grasping gesture and back and forth movement with his head while tapping*		Four, five, six, eight, nine, ten (*5, 7, 9 and 10 on the shelf*)
9.	R: Why did you put it here? Shouldn't we put it on the other side?	*R points to 10 on the far-right side of the shelf, then quickly to the area below the shelf*		
The children stay silent for almost 6 seconds.				
10.	Giovanni: And why do these ones have to stay here?	*Giovanni points to the iPad and ^ on the left side of the shelf ^*		Eleven, twelve *(11 and 12 created on the shelf then slightly dragged along the shelf)*
11.	R: Can we try to do it again?	*R resets the screen*		
12.	Giovanni: So does only 10 have to stay below?	*Giovanni points to the area below the shelf*		

The researcher invites one of the children, Giovanni (who is seated on her right side), to try playing the task (^: touches above the shelf; _: touches below the shelf; Table 1).

After Giovanni's question, the researcher keeps asking questions about which numbers stayed and which fell away (like "Did only 10 stay below?", accompanied by the right-hand finger moving along the shelf), and Sara, who sits in front of the

Table 2 Sara recreates the sequence of odds

#	Spoken	Gesture	Image	*TouchCounts*
21.	Sara: First they were put here, and then we jumped below	*Sara points to the area above and then to the area below the shelf, towards R's right-hand side*		
22.	Matteo: Can Sara try? She's very good	*Matteo moves closer to the iPad, Sara and R*		
23.	R: Come on, Sara, try			
24.		*Sara* ^_^_^_^_^_ *with focused and slow taps*		One, two three, four, five, six, seven, eight, nine, ten (*1, 3, 5, 7, 9 on the shelf*)
25.	R: Can we go on?	*Encouragingly*		
26.	Matteo: Yes, here	*Matteo points to the far-right side of the screen*		
27.	R: Come on!			
28.		*Sara* ^_^_		Eleven, twelve, thirteen, fourteen (*11 and 13 on the far-right side of the screen*)
29.	R: Can we go on?	*With challenging tone*		
30.	Giovanni: There is no more space			

researcher and looks at the screen from the opposite side, opens her eyes widely, moves towards the researcher, looks at the iPad, and speaks (Table 2).

Suddenly, Giovanni corrects himself saying that it is still possible to use empty space, scrolling his finger close to the screen along the shelf. This discussion introduces discourse about odd numbers in the children's activity. Later, changing the rhythm of above-below tapping to a below-above tapping will move the children to notice new occurrences and numbers that fall off the screen, actualising only the multiples of 2 on the shelf.

Discussion

In this episode, we see how the division of the screen into two areas, one in which numbers remain on the screen (above the shelf) and one in which numbers fall off the screen (below the shelf) echoes the cut of the strip on the floor from which the rhythm of standing-laying down emerged. The researcher's proposal to focus on the shelf as if it was the strip allows to extend the coordinated movement previously explored on the floor to the new material surface, by finger tapping one time above and one time below the shelf and so on. This at first makes the evens appear on the shelf.

We focus here on the creation of odd numbers as the children experienced it in *TouchCounts*, but it is crucial for our analysis to notice that the shift in movement just described is pivotal in actualising both odd and even numbers: both sequences are about alternating the tapping gesture above-below but arise from a different starting point. This observation points to a more relational vision of number than the classical cardinal conception.

When the children begin playing the pattern above-below the shelf, it is for them a new way of exploring numbers that they already encountered. The entanglement of children and number is in this episode one for which taps, no longer bodies, are numbers, with numerals spoken out loud by the iPad. The number comes from the material arrangement of the hand, the symbols on the shelf, the screen areas, and the iPad's voice. Again, it is the movement that dominates. Despite the rhythmic gesturing, attention initially seems to be more on ordinal awareness of number from previous experience and on the material power of the surface that limits (perceived freedom of) movement. Significant in the episode is how Giovanni and Sara's ways of tapping are slow and tend to lose the coordination of movement implicated in the patterning taps. No matter what exactly happens on the screen and which numbers stay on the shelf or fall off, what matters in directing the finger to the screen is to keep directionality along the shelf (moving left to right) and "fixed" empty space intermediate to two consecutive taps above the shelf (locating numbers above at the same distance from each other). These are clear contaminations of the body-based interaction with the strip. Instead, differently from before, a symbolic form is given by the yellow discs with numerals inside, rather than the number symbols written on cardboards. Novelties as well are the numbers that disappear even though they have been pronounced, their materiality being expressed mainly through the iPad's voice, and the fact of being able to easily produce numbers bigger than 9.

The material configuration engendered by the surface, the shelf, and the discs partakes in the process with which the children come together with even and odd numbers. It circumscribes the creation of evens on and around the shelf, through the children's movements. We observe this in Giovanni and Sara's slightly different ways of tapping (Table 1, [1]; Table 2, [23]). Giovanni is the first to play and is more hesitant (at least at the beginning) and messier than Sara in keeping control of the constraints mentioned above: Giovanni's taps resemble a number-grasping gesture, which is performed with thumb and forefinger kept together. After leaving the 9 on the shelf, however, Giovanni loses the rhythmic coordination and touches above once again, producing also the number 10 on the shelf. He suddenly pauses, before the researcher asks him why he "put" it there and whether they were not expected to "put" it on the other side, therefore drawing attention to the area of touch (the verb "to put" is significant in terms of Giovanni's way of tapping). Giovanni then accidentally touches the screen once on the left side and once on the close right and makes the unexpected 11 and 12 appear on the shelf (Table 1, [10]). While the number 10 emerged from a disruption of the rhythm above-below, the following numbers are created by an accidental pointing that is generative in terms of possibly placing numbers without following a fixed directionality. This is disappointing for the researcher in some sense, as she eventually resets the screen, but literally breaks some of the overt and covert constraints that structured the movement of the assemblage until that moment. At the beginning of the second part of the episode, Sara intervenes shifting attention back to what they did, namely, placing numbers above first and then jumping on the other side ("First they were put here, and then we jumped below"; Table 2, [21]). Sara, who looks at the screen from the opposite point of view with respect to Giovanni's, is encouraged to create her pattern. She thus very precisely produces five times the movement above-below the shelf from right to left and stops as soon as 10 falls away from the screen, no matter the empty space left. Like Giovanni did, the numbers on the shelf are created by Sara's taps preserving some space in between. Sara's gestures are focused, all her fingers are kept outstretched and tense while she uses her forefinger to tap, and unfold in the same direction of Giovanni's sequence, even though she is now looking at the iPad from a different perspective. As the researcher poses a different challenge: "Can we go on?" (Table 2, [25]), Matteo comes close to the empty space pointing with his finger and answers: "Yes, here" (Table 2, [26]). After encouragement, Sara restarts her rhythmic above-below tapping until she reaches the far-right side of the screen, adding 11 and 13 on the shelf. When the issue of going on is re-questioned (Table 2, [29]), Giovanni intervenes again, first raising up the issue of no vacancy, then, perhaps recalling his previous accidental taps, pointing to free spaces on the shelf in-between discs. Movements on the surface are potentially unbounded but materially constrained by the perception of available space on the shelf, as well as by the gravity that makes some number disappear from the screen, or by the children and researcher's utterances that suggest the possibility of more tappings.

In this episode, we have seen how the seven children interact with *TouchCounts* to produce odd numbers and how this material activity is affected by the previous activity with the strip and ordinal awareness, while unfolding coordinated movements on and around the shelf. In the next section, we briefly discuss the way that for the children, this movement becomes generative of the new sequence of even numbers and partakes of its relationships with odds.

The Coordinated Movement of Odds and Evens

The researcher shifts attention to the fact that not all the numbers up to 13 are on the shelf, to further investigate the relationship between evens and the specific coordinated movement that produced them. In the attempt to explain that 2 instead of 3 should be there after 1 (see again Table 2, [28]), Giovanni accidentally touches the screen, and 15 appears on the shelf, overlapping with 3. Right away, Giovanni speaks out loud that 15 came out, actualising the possibility of going on. At the request of explanation by the researcher, Sara emphasises that her classmate touched above the shelf. We might ask: What is the relationship between the missing numbers and the visible ones? It is at this point that Giovanni pinches over the screen several times as if he grabbed all the missing (even) numbers and speaks them aloud in sequence. In this way, he brings forth the sequence of evens as those non-visible, missing numbers, which fell away from the screen one by one and, at the same time, would fill the empty spaces on the shelf.

A new challenge, that of leaving the even numbers on the shelf, is then threw to the children, who are surprised at first. Matteo suggests turning the tablet upside-down (as if numbers above the shelf were supposed to fall off because of this movement). The researcher resets the screen and asks Sara how they could do. Sara stresses: “First, below”, before, very slowly but carefully, touching the screen three times (below and above) from right to left. Suddenly, Giovanni enters the rhythm with an additional below-above tapping gesture that eventually puts 8 on the shelf. Sara moves Giovanni’s hand over and continues the pattern with another tap that adds 10 on the shelf. Giovanni and Sara come therefore together in a single choreography on the screen that actualises the sequence of 2, 4, 6, 8, and 10 (Fig. 3a and b).

The children will begin to verbally express the strict relationships between the two tapping rhythms they had experienced as soon as Giovanni says: “it always jumps one” (referring to the movement above/below the shelf).

Discussion

We can see that even and odd numbers are provisionally arranged as above and below numbers, and vice versa, emerging out of hand-based symbol-symbol relationships. Coordinated movement above-below/below-above the shelf comes to characterise new ordinal and structural awareness, for which now what makes a difference is that numbers can be placed one after the other in relation to the shelf provided by *TouchCounts*. Coordination is not always a matter of being together in harmony. This is apparent in the choreographic movement that involves Sara and Giovanni. Their apparently conflictual tap on the screen (as Giovanni steps in and Sara pushes his hand away) concurs to create the rhythmic alternation of above and below that characterises the actualised sequences.

In this episode, assemblage theory also helps us recognise the power of the iPad surface in the activity. As Matteo asks to turn the tablet upside-down, the non-visible numbers that fell off the screen are imagined to be present again by changing the screen orientation, as if the iPad was nothing else but a box, inside which gravity

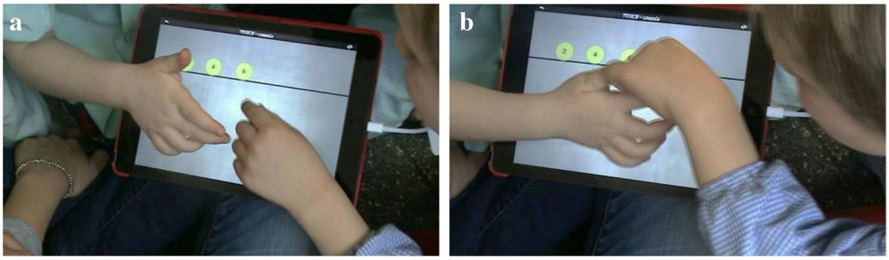


Fig. 3 **a**, **b** Giovanni and Sara's coordinated movement with evens

manifests. Reversing orientation implies reversing the situation on the shelf, showing again falling numbers but this time the other ones (which were on the shelf!). This reveals the way that the tool has an agential role in the activity and imaginatively extends its use to a feature that might be interesting to implement in the application, in addition to gravity.

We see that the meanings of odd/even numbers are established by the ways they are made, the tapping of above-below or below-above, the material cut created by the shelf, and the gravity in *TouchCounts* and come to co-exist temporally and spatially in the learning assemblage.

Conclusion

In this article, we have drawn on assemblage theory to study the contingent and always-emergent conditions of doing mathematics by shedding light on its embodied and material dimensions, in the context of kindergarten children working on early learning of number. We believe that assemblage theory has the potential to foster new approaches to mathematical concepts and learning events, which can inspire new research directions and methodologies (de Freitas & Walshaw, 2016).

In our case, the notion of assemblage helped us focus on the relational movement that constitutes the mathematical activity and the provisional ways in which children and number are entangled. Putting emphasis on relations between movements, we explored these relations to understand how young children engage in early learning of number, while doing justice to a powerful idea of movement as the force that animates matter and the body. To this aim, we used the notion of coordinated movement, which we borrowed from Sheets-Johnstone and her appreciation of the qualitative kinaesthetic dynamics that are definitive of movement. A sense of coordinated movement is central to an investigation of mathematical activity that accounts for the responsive nature of bodies and informs the becoming of the learning assemblage.

By analysing the children's activity in terms of assemblage theory, we studied both the dynamics of the components of the assemblage in interaction and the movement of the learning assemblage. The various components of the assemblage—the surface (floor or iPad), the tool (strip or shelf), the cardboards/numerals or odds and evens, classmates, order and patterns, and voices (of the teacher or the iPad)—came to be visible at different times over the course of the activity, bringing forth number as a relational entity. The episodes showed how coordinated movements sustain ordinal and structural awareness of number within the learning assemblage. The entanglement of children, matter, and concept was one for which bodies, in the first episode, and taps, in the second episode, were numbers. The meaning of number always arose from specific arrangements of bodies, materials, and surfaces, namely, from provisional configurations in which the spatial, the temporal, and the expressive were all dimensions of movement.

Attention to how coordinated movements moved the learning assemblage in specific directions, with its instabilities and junctures, allowed us to better look at the concept of number that emerged out of material activity and at the unexpected and productive ways in which this happened. Relations were mutually constitutive of the changing assemblage as well as of the emerging concepts. The material and embodied interactions of the children with the surfaces, and strip and shelf, did not only reveal the force of the surfaces in the activity, but rather they did establish numbers as relations. Learning of odds and evens was both symbolic *and* concrete, concerning the numerals on cardboards or yellow discs and the movements of staying-jumping the strip or tapping above-below/below-above the shelf. Odds and evens came into being in terms of relational difference, and the ones were created in relation to the others. Children standing were eventually children who did not lay down and vice versa. Discs staying were discs which did not fall off and vice versa. It is this relational difference which makes number vibrant and sustains an ordinal understanding of it in the activity. We can ultimately emphasise how activities like the ones we describe in this article might be nourishing a sense of number as that which can not only be counted, but especially produce dynamics of ordering and patterning.

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Declarations

Conflict of Interest The authors declare no competing interests.

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