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Research paper

A behavioral model of consumer response to price information

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ABSTRACT

This paper introduces a model of choice that captures heuristics and reference dependence in consumers' responses to price information. In the model, a product's quality is positively distorted when its price is close to the reference price. The model is consistent with a kinked and upward-sloping demand curve. The price and cross-price elasticities of demand can be positive or negative, asymmetric, and product-dependent. The model provides an explanation for quality-dependent price stickiness, justifies the adoption of complex pricing strategies, and allows for the derivation of closed-form expressions for the optimal price and reference price set by a monopolist. The model is fully characterized by testable restrictions on demand data, offering a method for identifying the reference price.

1. Introduction

Extensive empirical evidence shows that consumers' response to prices is often "behavioral". Consumers rely on reference prices — internal, subjective prices used to assess the appropriateness of actual prices (e.g., Monroe, 1973; Kalyanaram and Winer, 1995; Mazumdar et al., 2005). Furthermore, consumers sometimes deviate from the standard economic intuition that higher prices *reduce* demand (e.g., Ng, 1987; Cosaert, 2018; Dusansky and Koç, 2007; Genesove and Mayer, 2001). Through the price-quality heuristic, consumers use prices as a proxy for quality leading to an upward-sloping demand curve (e.g., Scitovszky, 1944; Pollak, 1977; Gneezy et al., 2014). Understanding consumers responses to price variations is crucial for marketers who want to anticipate consumer behavior and implement effective pricing strategies. But it is also important for policymakers seeking to anticipate the effects of fiscal or monetary policies (e.g., Eichenbaum et al., 2011; Kim, 2019).

Existing "behavioral" models of consumer choice have limitations. Models of the price-quality heuristic (e.g., Gneezy et al., 2014) do not account for reference prices. Models incorporating reference prices (e.g., Winer, 1986; Lattin and Bucklin, 1989; Putler, 1992; Hardie et al., 1993; Kopalle et al., 1996; Bell and Lattin, 2000; den Boer and Keskin, 2022) assume that demand always decreases with price and that the evaluation of (relative) prices is independent of a product's quality. Moreover, as is common in models of reference-dependent behavior, these models assume an exogenous reference price, making their predictions sensitive to an external parameter.[2]

This paper introduces and characterizes axiomatically a model of random consumer choice, the Reference Price Quality (RPQ) model, which addresses these limitations and offers new predictions about consumer responses to price information. The RPQ model's key assumption is that consumers positively distort their perception of a product's quality when its price is close to the reference price.

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[2] This critique applies broadly to models of reference-dependent preferences (e.g., Kahneman and Tversky, 1979). However, empirical methods exist for estimating reference points (e.g., Baucells et al., 2011; Allen et al., 2017; Baillon et al., 2020).

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The RPQ model has several advantages. First, it is consistent with a wide range of observed consumer responses to price information that other models fail to capture. Second, the RPQ model remains tractable, as demonstrated by the closed-form characterization of the optimal price and reference price set by a monopolist in the presence of an outside option. Third, the model features a relatively simple axiomatic characterization. Notably, these axioms make it possible to uniquely determine the reference price(s) used by consumers.

The RPQ model extends the multinomial Logit demand model (e.g., Guadagni and Little, 1983). In the multinomial Logit model, the structural value of a product with quality q and price p is given by $v(q) - c(p)$, for some functions v, c. In the RPQ model, the structural value of a product with quality q and price p is $\sigma(p|p^*)v(q) - c(p)$, where p^* is the reference price and σ the *distortion function*. The key assumption is that observing a price above or *below* the reference price reduces the relevance of quality. This implies that the function σ is weakly single-peaked (i.e., single-plateaued) at the reference price.

Like other models of reference price-dependent consumer choice, the RPQ model allows for kinked demand. However, unlike these models, demand in the RPQ model can also be upward-sloping. This occurs because increasing a price that is below the reference price can have a positive effect on the perception of quality. If the positive effect outweighs the negative effect of a higher monetary cost, the product's demand increases. Furthermore, the RPQ model predicts that upward-sloping demand is more likely for high-quality products, thereby capturing a reduced form of the price-quality heuristic while refining it by incorporating reference price dependence. This property is consistent with empirical evidence showing that high-quality products exhibit stickier prices compared to low-quality ones (Kim, 2019).

In terms of price and cross-price elasticities of demand, the RPQ model predicts that they can be positive or negative, asymmetric, and dependent on product quality (see, for example, the empirical evidence in Dossche et al., 2010; Biondi et al., 2020; Iizuka and Shigeoka, 2021; Yaman and Offiaeli, 2022). Concerning the demand effects of varying the reference price, the effects are generally ambiguous. However, if the product's actual price exceeds its reference price and alternative products are priced below the reference price, increasing the reference price can lead to a higher demand for the product.

To illustrate the model's applicability, I analyze the optimal pricing decision of a monopolist facing RPQ demand in the presence of an outside option. Under general conditions, the optimal price can be expressed in closed-form using the Lambert W function (an easy-to-simulate and approximate function, see e.g., Corless et al., 1996; Aravindakshan and Ratchford, 2011). I also consider the case where the monopolist can set both the price and the reference price (e.g., in the long run). In this case, the optimal reference price is equal to the posted price and the optimal posted price takes a "logit-like" closed form. If there is no demand premium for observing a price equal to the reference price, the monopolist's short-run profit under RPQ demand is always lower than the profit obtained under Logit demand (the long-run profit). This leads to a new interpretation for the Logit model's optimal price and profit as the price and profit that emerge in the RPQ model when the monopolist has the ability to choose both the price and the reference price.

As a final application, I study a simple two-period version of the RPQ model to offer a new explanation for the widespread adoption of complex pricing strategies, such as markdown (MD) pricing, over simpler alternatives like everyday-low-price (EDLP) strategies (e.g., Özer and Zheng, 2016; Adida and Özer, 2019).

One advantage of the RPQ model is that it can be fully characterized in terms of testable restrictions on commonly available data (such as scanner data). Before characterizing the RPQ model, I characterize a general version of the Logit model, called Independent Logit, which encompasses reference price-dependent models present in the literature. In addition to some basic axioms, the defining properties of the Independent Logit model are the independence of a product's quality from its price and a downward-sloping demand for all products. In terms of observable restrictions, the independence property implies that the relative demand between two product of different qualities does not depend on prices, while the second property imposes a monotonicity condition. The characterization of the RPQ model relaxes both independence and monotonicity. I establish a testable condition on choice probabilities that uniquely identifies the reference price (or a range of reference prices). Reference prices are defined as those points at which quality differences are most relevant for the relative differences in demand. At the reference price, the distortion of quality is maximal, magnifying even small quality differences. This intuition clarifies why the interplay between quality and price is crucial for identifying reference prices. Without this interplay, reference prices cannot be determined from choice data.

To further illustrate this point, I provide the axioms that characterize the standard reference price models, where the cost function c is increasing and piecewise linear, and compare them with those characterizing the RPQ model. I demonstrate that reference prices can only be identified under specific parametric restrictions on c. Any modifications to the cost function would require testing different conditions.

Lastly, I extend the model in two directions. First, I propose a generalization of the RPQ model that allows for a price-quality interaction potentially independent of the reference price. Special cases of this model appear in the literature on price-quality interaction (e.g., Crawford et al., 2015; Li et al., 2020). In the second extension, I incorporate "context-dependent" distortion effects, where the price distorts quality depending on the other available products. This extension provides a reference price-based explanation for context effects, such as the asymmetric dominance and the compromise effects (Simonson, 1989). Similarly to the baseline model, I characterize the context-dependent model through properties of choice data. As a by-product, I show that any positive demand probabilities can be represented in a context-dependent logit-like formulation.

2. Framework and a motivating example

I consider a consumer choosing among homogeneous products (i.e., products within the same category, such as cookies). I represent each product k by a pair (q_k, p_k), where q_k denotes the quality and p_k its price. I assume that $q_k \in [q_0, q_1]$, with $q_0 < q_1$, and $p_k \in [0, \infty)$ for all products k. The consumer randomly selects a product from a finite and non-empty set of products (a choice set). I denote by $\mathbb{A}$ the family of all choice sets. The demand for a product k in a choice set A is the probability that k is selected from A, denoted by $P((q_k, p_k)|A)$.[3] The choice probabilities satisfy $P : A \times \mathbb{A} \to [0, 1]$ and $\sum_{(q_k,p_k)\in A} P((q_k, p_k)|A) = 1$ for all $A \in \mathbb{A}$. I motivate the model with a simple example.

Motivating example. In the price-quality heuristic (e.g., Scitovszky, 1944; Pollak, 1977; Gneezy et al., 2014), demand can be upward sloping, at least over a range of prices. In terms of choice probabilities, this implies: $\partial P((q_k, p_k)|A)/\partial p_k > 0$ for some price p_k. Suppose demand is modeled using the standard (linear) Logit model of Guadagni and Little (1983),

$$P_{Logit}((q_k, p_k)|A) = \frac{e^{v(q_k)-\beta p_k}}{\sum_{(q_l,p_l)\in A} e^{v(q_l)-\beta p_l}}, \tag{Lin. Logit}$$

for some $v : [q_0, q_1] \to \mathbb{R}$ and $\beta > 0$. It is well-known that:

$$\frac{\partial P_{Logit}((q_k, p_k)|A)}{\partial p_k} = -\beta P_{Logit}((q_k, p_k)|A)(1 - P_{Logit}((q_k, p_k)|A)) < 0,$$

because $\beta > 0$ and Logit probabilities are strictly positive. Thus, the Logit model predicts a strictly downward-sloping demand curve at any price, which is inconsistent with the price-quality heuristic. More sophisticated models of consumer choice incorporate reference prices (Winer, 1986; Lattin and Bucklin, 1989; Hardie et al., 1993; Bell and Lattin, 2000). In these models, the probability of choosing a product is given by:

$$P_{RD}((q_k, p_k)|A) = \frac{e^{u_{RD}(q_k,p_k)}}{\sum_{(q_l,p_l)\in A} e^{u_{RD}(q_l,p_l)}}, \tag{RD}$$

where

$$u_{RD}(q_k, p_k) = \begin{cases} v(q_k) + \eta^+(p^* - p_k) - \beta p_k & \text{if } p_k \le p^*, \\ v(q_k) - \eta^-(p_k - p^*) - \beta p & \text{if } p_k > p^*, \end{cases}$$

for some $\eta^+, \eta^-, \beta \ge 0$, and where p^* is the reference price. Similarly to the (linear) Logit model, the (RD) model predicts a downward-sloping demand for all products (see Section 4.2):

$$\frac{\partial P_{RD}((q_k, p_k)|A)}{\partial p_k} \le 0.$$

Thus, even reference-dependent price models fails to account for upward-sloping demand curves.

3. The model

Definition 1. The choice probabilities have a Reference Price Quality (RPQ) model representation if there exist $p^* \in [0, \infty)$, a function $\sigma(\cdot|p^*) : [0, \infty) \to [0, \infty)$ and weakly increasing functions $v : [q_0, q_1] \to [0, \infty)$, $c : [0, \infty) \to [0, \infty)$ such that:

$$P_{RPQ}((q_k, p_k)|A) = \frac{e^{\sigma(p_k|p^*)v(q_k)-c(p_k)}}{\sum_{(q_l,p_l)\in A} e^{\sigma(p_l|p^*)v(q_l)-c(p_l)}}, \tag{RPQ}$$

for all $A \in \mathbb{A}$. Moreover, $\sigma(p|p^*) \ge \sigma(p'|p^*)$ if $p^* \ge p \ge p'$ or $p' \ge p \ge p^*$.

The RPQ model assumes a logit-like functional form for the choice probabilities. The structural value of a product with quality q_k and price p_k is:

$$u(q_k, p_k) = \sigma(p_k|p^*)v(q_k) - c(p_k).$$

The function v measures the perceived quality, and the function c the perceived monetary cost of a product. The function σ distorts the product's perceived quality as a function of the actual price relative to a menu-independent reference price p^*. The distortion function satisfies a weak form of *single-peakedness*—called single-plateauedness (see e.g., Moulin, 1984)—at p^*: the closer a price is to the reference price, the (weakly) more relevant quality becomes.

The RPQ model generalizes the Linear Logit demand model defined in Eq. (Lin. Logit), which corresponds to a RPQ model with $\sigma(p|p^*) = 1$ and a linear cost $c(p) = \beta p$ for some $\beta > 0$. The RPQ model also generalizes the reference price-dependent models defined in Eq. (RD). This is a particular case of the RPQ model with $\sigma(p|p^*) = 1$ for all prices and a cost function $c(p) = (\beta + \eta^+)p - \eta^+ p^*$ if $p \le p^*$ and $c(p) = (\beta + \eta^-)p - \eta^- p^*$ otherwise.

[3] This is a standard interpretation of random demand generated by a single consumer. It reflects unobservable changing tastes due, for example, to neural computational constraints (Webb, 2018) or to variations in attention, experimentation or perception (e.g., Guo, 2016; He, 2024). Alternatively, random demand may arise from a population of individuals with deterministic but unobservable preferences.

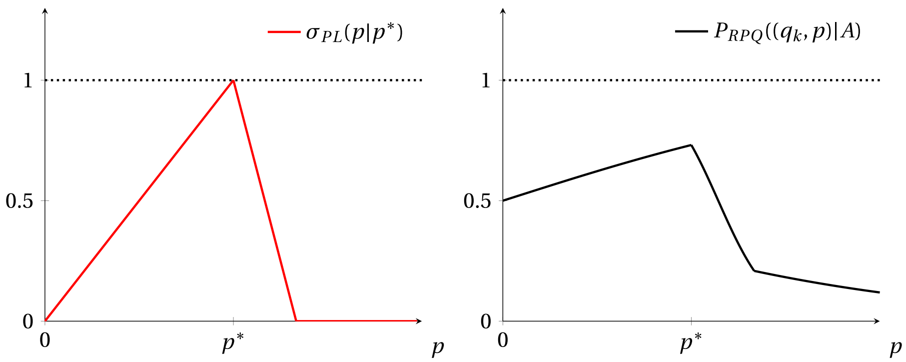

Fig. 1. Left panel: a Piecewise linear distortion with $\zeta = 1$ and loss aversion. Right panel: the probability of selecting (q_k, p) in $A = \{(q_k, p), (q_l, p_l)\}$, as a function of p.

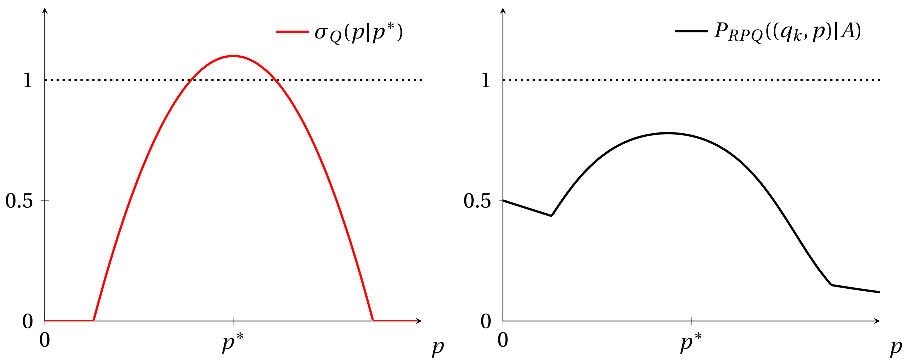

Fig. 2. Left panel: a Quadratic distortion with $\xi = 1.1$ and $\kappa = 1$. Right panel: the probability of selecting (q_k, p) in $A = \{(q_k, p), (q_l, p_l)\}$, as a function of p.

The following are some examples of the distortion function. The first, called Piecewise Linear, is defined as $\sigma_{PL}(p|p^*) = \max\{0, \hat{\sigma}_{PL}(p|p^*)\}$ where:

$$\hat{\sigma}_{PL}(p|p^*) = \begin{cases} \zeta - \eta(p - p^*) & \text{if } \ p > p^* \\ \zeta - \gamma(p^* - p) & \text{if } \ p \le p^*, \end{cases} \tag{PL}$$

for some $\zeta \ge 0$, $0 \le \gamma \le \eta$. The inequality $\gamma \le \eta$ reflects "loss aversion" (see Kalyanaram and Winer, 1995): consumers are more sensitive to losses (observed prices above the reference price) than gains (observed prices below the reference price). The parameter ζ represents the "premium" of observing a price equal to the reference price. Fig. 1 shows a possible specification of the piecewise linear distortion and the corresponding demand.

A related function with similar interpretations of the parameters is the Quadratic distortion, defined as the positive part of the function:

$$\sigma_Q(p|p^*) = \xi - \frac{\kappa}{2}(p^* - p)^2, \tag{Q}$$

for some $\xi, \kappa \ge 0$. Fig. 2 shows a possible specification of the quadratic distortion and the corresponding demand.

A third example is the "Acceptable Price Range" distortion (e.g., Monroe, 1971; Janiszewski and Lichtenstein, 1999):

$$\sigma_{APR}(p|p^*) = \begin{cases} 1 & \text{if } \ p \in [p^* - \delta_1, p^* + \delta_2] \\ 0 & \text{if } \ p \notin [p^* - \delta_1, p^* + \delta_2], \end{cases} \tag{APR}$$

for some $\delta_2 \ge 0$ and $0 \le \delta_1 \le p^*$. When the actual price is close to the reference price, it is acceptable, and the product's quality matters. If the price is "unacceptable", quality becomes irrelevant, and demand is solely driven by the monetary cost of the product (see Fig. 3).

A last example of distortion function is the "Salience" distortion:

$$\sigma_S(p|p^*) = \begin{cases} 1 + \theta & \text{if } \ p = p^* \\ 1 & \text{if } \ p \ne p^*, \end{cases} \tag{Salience}$$

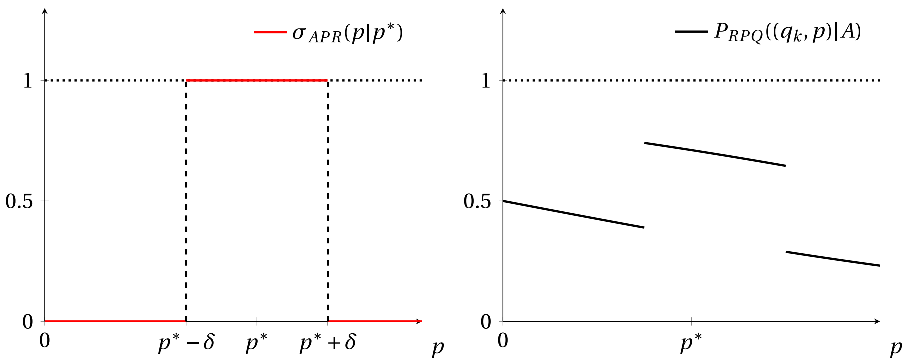

Fig. 3. Left panel: a symmetric (i.e., $\delta_1 = \delta_2 = \delta$) Acceptable Price Range distortion. Right panel: the probability of selecting (q_k, p) in $A = \{(q_k, p), (q_l, p_l)\}$ as a function of p.

for some $\theta \geq 0$. When the actual price is equal to the reference price, it attracts the consumers' attention, and the perceived value of quality is amplified. All other prices are non-distortive.

The example in Section 2 shows that both the Linear Logit and the reference price-dependent models predict downward-sloping demand. In the RPQ model instead, the demand can be *upward-sloping* (see Figs. 1–3) and, in addition, price and quality are intertwined. To illustrate these properties, consider two products $A = \{(q_k, p_k), (q_l, p_l)\}$ and assume that $\sigma_{APR}(p_l|p^*) = \sigma_{APR}(p_k|p^*) = 0$ and $c(p) = \beta p$. Since both prices p_k and p_l are unacceptable (e.g., too low), the products' quality becomes irrelevant. Thus, the demand for product k depends only on prices:

$$P_{RPQ}((q_k, p_k)|A) = \frac{e^{-\beta p_k}}{e^{-\beta p_k} + e^{-\beta p_l}}.$$

Suppose that the price of k increases by $\Delta \geq 0$, making it acceptable such that $\sigma_{APR}(p_k + \Delta|p^*) = 1$. The demand for $(q_k, p_k + \Delta)$ in A^Δ, where $A^\Delta = \{(q_k, p_k + \Delta), (q_l, p_l)\}$, becomes:

$$P_{RPQ}((q_k, p_k + \Delta)|A^\Delta) = \frac{e^{v(q_k)-\beta(p_k+\Delta)}}{e^{v(q_k)-\beta(p_k+\Delta)} + e^{-\beta p_l}}.$$

If $v(q_k) \geq \beta\Delta$, the RPQ model predicts a *higher* demand for product k after its price increased, i.e., $P_{RPQ}((q_k, p_k)|A) \leq P_{RPQ}((q_k, p_k + \Delta)|A^\Delta)$. The interpretation is that a higher price reduces the structural value of k by $\beta\Delta$, leading to a negative effect on demand. However, if the higher price is acceptable, it has a positive "signaling value" that increases the value of k by $v(q_k)$. If the positive effect outweighs the negative effect, the demand for product k will *increase*. Moreover, this conclusion depends on the quality of the product. Specifically, it is possible that increasing the price by Δ may boost the demand for a high-quality product (if $v(q_k) \geq \beta\Delta$) but decrease the demand for a low-quality product (if $v(q_l) \leq \beta\Delta$). Thus, an upward-sloping demand curve is more likely to occur for high-quality products. This conclusion is generally true in the RPQ model (see Section 4).

Remark 1. The RPQ choice probabilities can be derived from an Additive Random Utility Model (ARUM) as follows:

$$P_{RPQ}((q_k, p_k)|A) = \mathbb{P}(\sigma(p_k|p^*)v(q_k) - c(p_k) + \varepsilon_k \geq \max_{(q_l,p_l)\in A} \sigma(p_l|p^*)v(q_l) - c(p_l) + \varepsilon_l), \tag{1}$$

where the error terms ε are i.i.d. and distributed according to a Gumbel distribution (see, for example, Train, 2009). The ARUM formulation helps to understand the price-quality "inference" in the RPQ model. A higher $\sigma(p_k|p^*)$ increases the weight of quality in the structural value of product k. Since the noise terms are i.i.d., they are unaffected by a higher σ. Thus, a higher distortion reduces choice variability and increases the probability of selecting k. In this sense, the price acts as a signal of quality. When all products have the same price, the RPQ model can be interpreted as a psychophysical model of stimuli discrimination, where σ measures "randomness" in the choice of quality. The higher σ, the closer the choices are to deterministic utility maximization (see Online Appendix C).

4. The shape of demand: elasticities

Changes in demand are typically measured using price and cross-price elasticities. In this section, I explore these elasticities within the RPQ model, as well as the impact of variations in the reference price. For simplicity, I assume that both σ and c are differentiable functions (or at least they possess one-sided derivatives).

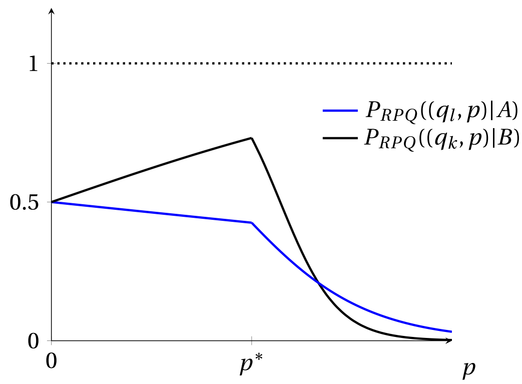

Fig. 4. Blue line: the probability of selecting (q_l, p) as a function of p in $A = \{(q_l, p), (q_m, p_m)\}$, with $c(p) = \beta p$, $\sigma_{PL}(p|p^*)$ and $\gamma v(q_l) < \beta$. Black line: the probability of selecting (q_k, p) as a function of p in $B = \{(q_k, p), (q_m, p_m)\}$, with $c(p) = \beta p$, $\sigma_{PL}(p|p^*)$ and $\gamma v(q_k) > \beta$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4.1. Price elasticity

Consider the derivative of the demand for product $k = (q_k, p_k) \in A$ with respect to its own price:[4]

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p_k} = P_{RPQ}((q_k, p_k)|A)\left[1 - P_{RPQ}((q_k, p_k)|A)\right]\left[\sigma_{p_k}(p_k|p^*)v(q_k) - c_{p_k}(p_k)\right],$$

where σ_{p_k} and c_{p_k} are the derivatives of σ and c in p_k, respectively. Since probabilities are strictly positive:

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p_k} \geq 0 \iff \sigma_{p_k}(p_k|p^*)v(q_k) \geq c_{p_k}(p_k). \tag{2}$$

The condition (2) involves all the relevant components of the RPQ model: price, quality and the reference price of product k. A higher price increases demand if the marginal cost $c_{p_k}(p_k)$ is smaller than the marginal benefit $\sigma_{p_k}(p_k|p^*)v(q_k)$. Because the distortion function is weakly single-peaked around the reference price, demand cannot be strictly upward-sloping if the actual price is larger than the reference price. In this case, $\sigma_{p_k} \leq 0$, which implies $\sigma_{p_k}(p_k|p^*)v(q_k) \leq 0 \leq c_{p_k}(p_k)$. However, a strictly upward-sloping demand can occur when the actual price is below the reference price.

Furthermore, *ceteris paribus*, the inequality (2) becomes harder to satisfy when $v(q_k)$ is small. This means that for (perceived) lower-quality products, a price increase is less likely to boost demand compared to higher-quality products. This property is consistent with empirical evidence that price stickiness is quality-dependent (Kim, 2019). In particular, high-quality products tend to have stickier prices than low-quality ones. In the RPQ model, higher quality products are more likely to display an upward-sloping demand curve if the price is below the reference price.

To further illustrate condition (2), consider the Piecewise Linear distortion function $\sigma_{PL}(p|p^*)$ of Eq. (PL), and assume $c(p) = \beta p$. In this case, the condition becomes:

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p_k} > 0 \iff \begin{cases} -\eta v(q_k) > \beta & \text{if } \; p_k > p^* \\ \gamma v(q_k) > \beta & \text{if } \; p_k < p^*. \end{cases}$$

Suppose that the price of product k is below the reference price p^*. A small increase of p_k could result in higher demand for k if the marginal cost β is smaller than the "marginal value" $\gamma v(q_k)$ of bringing the price closer to the reference p^*. However, if the actual price is above the reference price p^*, a larger price p_k will never result in higher demand for k. As mentioned earlier, the change in demand depends on the product's quality. The inequality $\gamma v(q_k) > \beta$ is more likely to hold when $v(q_k)$ is high. Fig. 4 illustrates this dynamic: the demand for a low-quality product is kinked but downward-sloping (blue line), while the demand for a high-quality product (black line) is upward-sloping when the price is below p^* and downward-sloping when it is above p^*.

From the marginal variation, I can derive the price elasticity of demand, which is defined as $E^{p_k}_{(q_k,p_k),A} = \frac{\partial P((q_k,p_k)|A)}{\partial p_k} \cdot \frac{p_k}{P((q_k,p_k)|A)}$. In the RPQ model, the price elasticity is given by:

$$E^{p_k}_{(q_k,p_k),A} = p_k\left[1 - P_{RPQ}((q_k, p_k)|A)\right]\left[\sigma_{p_k}(p_k|p^*)v(q_k) - c_{p_k}(p_k)\right],$$

which has the same sign as the marginal variation. Thus, the price elasticity can be positive or negative, depends on product quality, and asymmetric around the reference price. Empirical evidence of such asymmetric price elasticity can be found in Dossche et al. (2010), Biondi et al. (2020), Iizuka and Shigeoka (2021), Yaman and Offiaeli (2022), for example.

[4] For choice probabilities of the form $P((q_k, p_k)|A) = \frac{e^{u(q_k,p_k)}}{\sum_{(q_l,p_l)\in A} e^{u(q_l,p_l)}}$, the partial derivative with respect to a product's own price is given by $\frac{\partial P((q_k,p_k)|A)}{\partial p_k} = P((q_k, p_k)|A)\left[1 - P((q_k, p_k)|A)\right]\frac{\partial u(q_k,p_k)}{\partial p_k}$. In the RPQ model, $u(q_k, p_k) = \sigma(p_k|p^*)v(q_k) - c(p_k)$, so that the result follows.

4.2. Relation with models of reference price

Before discussing cross-price elasticity, I compare demand variations in the RPQ model with those of the reference price-dependent demand model of Eq. (RD). The latter predicts a kinked and decreasing demand function, similar to the blue line in Fig. 4. Indeed, the structural value u_{RD} can be rewritten as:

$$u_{RD}(q_k, p_k) = \begin{cases} v(q_k) + \eta^+ p^* - p_k(\beta + \eta^+) & \text{if } p_k \le p^* \\ v(q_k) + \eta^- p^* - p_k(\beta + \eta^-) & \text{if } p_k > p^*. \end{cases}$$

It is immediate to observe that demand in this model is always downward-sloping. Indeed, the condition (2) for an upward-sloping demand becomes $c_{p_k}(p_k) \le 0$ in the case of u_{RD}. However, $c_{p_k}(p_k) = \beta + \eta^+$ if $p_k < p^*$ and $c_{p_k}(p_k) = \beta + \eta^-$ if $p_k > p^*$, both of which are positive.

Now consider the structural utility of a product in the RPQ model with σ_{PL}, assuming $\sigma_{PL} > 0$ for simplicity and $\zeta = 1$:

$$u(q_k, p_k) = \begin{cases} v(q_k) + v(q_k)\gamma p^* - p_k(\beta - v(q_k)\gamma) & \text{if } p_k \le p^* \\ v(q_k) + v(q_k)\eta p^* - p_k(\beta + v(q_k)\eta) & \text{if } p_k > p^*. \end{cases}$$

This version of the RPQ model is qualitatively similar to the (RD) model when $\beta \ge v(q_k)\gamma$. In this case, higher prices always reduce demand (see the blue line in Fig. 4). However, the RPQ model is more general: it allows for upward-sloping demand when the actual price is below the reference price, and it incorporates the interaction between quality and price. In the Online Appendix A, I provide the axioms characterizing the (RD) model, and show that unlike the RPQ model, these axioms require strong assumptions about observable data.

4.3. Cross-price elasticity

A small change in the price of a product $l = (q_l, p_l) \in A$ affects the demand for $k = (q_k, p_k) \in A$ as follows:

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p_l} = -P_{RPQ}((q_k, p_k)|A) P_{RPQ}((q_l, p_l)|A) \left[\sigma_{p_l}(p_l|p^*)v(q_l) - c_{p_l}(p_l)\right].$$

As previously discussed, the condition $\sigma_{p_l}(p_l|p^*)v(q_l) \ge c_{p_l}(p_l)$ implies that the demand of l will increase. Due to the logit-like properties of the RPQ model, a higher demand for l leads to a reduction in the demand for k. Similar to the own price-elasticity, I can define the cross-price elasticity by normalizing the derivative of the demand:

$$E^{p_l}_{(q_k, p_k), A} = -p_l P_{RPQ}((q_l, p_l)|A) \left[\sigma_{p_l}(p_l|p^*)v(q_l) - c_{p_l}(p_l)\right].$$

The cross-price elasticity has the same sign as the cross-price variation, making it potentially positive or negative, asymmetric, and dependent on the products' quality. If $\sigma(p|p^*) = 1$ for all prices p, as in the Linear Logit model or in the (RD) model, then $E^{p_l}_{(q_k, p_k), A} = p_l P_{RPQ}((q_l, p_l)|A) c_{p_l}(p_l)$, which is always positive because $c_{p_l} \ge 0$.

4.4. Marginal changes in the reference price

I analyze the impact of a marginal change in the reference price on demand. The derivative of demand with respect to the reference price is given by:

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p^*} = P_{RPQ}((q_k, p_k)|A) Z(k, A), \tag{3}$$

where

$$Z(k, A) = \left[\sigma_{p^*}(p_k|p^*)v(q_k) - \sum_{(q_l, p_l) \in A} P_{RPQ}((q_l, p_l)|A)\sigma_{p^*}(p_l|p^*)v(q_l)\right].$$

Thus, the condition for demand to increase with p^* is

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p^*} \ge 0 \iff \sigma_{p^*}(p_k|p^*)v(q_k) \ge \sum_{(q_l, p_l) \in A} P_{RPQ}((q_l, p_l)|A)\sigma_{p^*}(p_l|p^*)v(q_l). \tag{4}$$

The right-hand side inequality implies that if the marginal value of a higher reference price for product k is "above average", the demand for k will increase. By the weak single-peakedness property of σ, the derivative $\sigma_{p^*}(p_k|p^*)$ is positive (negative) if the posted price p_k is above (below) the reference price. A sufficient condition for inequality (4) to hold is that p_k is above the p^* while all other prices p_l are below p^*. In the case of two products, $A = \{(q_k, p_k), (q_l, p_l)\}$, condition (4) simplifies to[5] $\sigma_{p^*}(p_k|p^*)v(q_k) \ge \sigma_{p^*}(p_l|p^*)v(q_l)$.

[5] This follows from $\sigma_{p^*}(p_k|p^*)v(q_k) \ge P((q_k, p_k)|A)\sigma_{p^*}(p_k|p^*)v(q_k) + P((q_l, p_l)|A)\sigma_{p^*}(p_l|p^*)v(q_l)$, which becomes $(1 - P((q_k, p_k)|A))\sigma_{p^*}(p_k|p^*)v(q_k) \ge P((q_l, p_l)|A)\sigma_{p^*}(p_l|p^*)v(q_l)$, but $(1 - P((q_k, p_k)|A)) = P((q_l, p_l)|A)$.

5. Applications

To illustrate the applicability of the RPQ model, I first analyze the pricing strategy of a monopolist selling a product when the consumer has an outside option. Then, I apply the RPQ model to offer a new explanation for the prevalence of complex pricing strategies, such as the Markdown (MD) pricing strategy, over simpler alternatives like the Everyday-low-price (EDLP) strategy (see Özer and Zheng, 2016; Adida and Özer, 2019).

5.1. Optimal price for a monopolist

A monopolist sells a product (q, p) and the consumer can either buy it or choose an outside option (with utility normalized to 0). The demand for (q, p) is:

$$P_{RPQ}((q,p)|A) = \frac{e^{\sigma(p|p^*)v(q)-\beta p}}{1+e^{\sigma(p|p^*)v(q)-\beta p}},$$

where $c(p) = \beta p$ for simplicity. I begin by analyzing the case in which the reference price is fixed (e.g., in the short-run), so that the monopolist can only choose the price to maximize profit Π:

$$\max_p \Pi = \max_p (p-C)P_{RPQ}((q,p)|A),$$

where C is the marginal cost. While explicit solutions are typically hard to find because of the interaction between linear and exponential terms (see, e.g., Aravindakshan and Ratchford, 2011), the following result demonstrates that, under fairly general conditions, the structural value of the product at the optimal price $\hat{p}$ can be expressed independently of $\hat{p}$ through the Lambert W function.[6] Consequently, it can be possible to obtain a closed-form expression for the monopolist's optimal price by isolating it from the structural value.

Proposition 1. *Suppose that $\hat{p} \geq C$ is the optimal price and $\sigma(p|p^*)$ is differentiable at $\hat{p}$. If $\sigma(\hat{p}|p^*)v(q)+\sigma_p(\hat{p}|p^*)v(q)(C-\hat{p}) = f(C, v(q), p^*)$ for some function f (i.e., the expression $\sigma(\hat{p}|p^*)v(q)+\sigma_p(\hat{p}|p^*)v(q)(C-\hat{p})$ is independent of $\hat{p}$), then*

$$\sigma(\hat{p}|p^*)v(q) - \beta\hat{p} = -\beta C - 1 + f(C, v(q), p^*) - W\left(e^{f(C,v(q),p^*)-1-\beta C}\right), \tag{5}$$

where W is Lambert W function.

All proofs are in the Online Appendix. Eq. (5) can be rewritten as $\sigma_p(\hat{p}|p^*)v(q)(\hat{p}-C) - \beta\hat{p} = -\beta C - 1 - W\left(e^{f(C,v(q),p^*)-1-\beta C}\right)$. If $\sigma_p(\hat{p}|p^*)$ is independent of $\hat{p}$, the optimal price admits the following closed-form expression:

$$\hat{p} = C + \frac{1+W\left(e^{f(C,v(q),p^*)-1-\beta C}\right)}{\beta - \sigma_p v(q)}. \tag{6}$$

The condition $\sigma(\hat{p}|p^*)v(q)+\sigma_p(\hat{p}|p^*)v(q)(C-\hat{p}) = f(C, v(q), p^*)$ is always satisfied if σ is linear around $\hat{p}$. In this case, $\sigma_p(\hat{p}|p^*)$ will be independent of $\hat{p}$, so that Eq. (6) holds.[7] The condition is satisfied almost everywhere when $\sigma = \sigma_{PL}$ (or when $\sigma = \sigma_S$ or $\sigma = \sigma_{APR}$). Therefore, I can find closed-form expressions for the monopolist's optimal price, as shown in the following corollary:

Corollary 1. *Suppose that $\hat{p} \geq C$ is the optimal price and $\sigma_{PL}(p|p^*)$ is differentiable at $\hat{p}$, then $\gamma v(q) < \beta$ and:*

a. if $\sigma_{PL}(\hat{p}|p^*) = 0$,

$$\hat{p} = C + \frac{1+W\left(e^{-1-\beta C}\right)}{\beta},$$

b. if $\sigma_{PL}(\hat{p}|p^*) > 0$,

$$\hat{p} = \begin{cases} C + \frac{1+W\left(e^{\zeta v(q)-\eta v(q)(p^*-C)-1-\beta C}\right)}{\beta+\eta v(q)} & \text{if} \quad \hat{p} \geq p^* \\ C + \frac{1+W\left(e^{\zeta v(q)-\gamma v(q)(p^*-C)-1-\beta C}\right)}{\beta-\gamma v(q)} & \text{if} \quad \hat{p} < p^*, \end{cases}$$

where W is Lambert W function.

The Logit optimal price is obtained if $\zeta = 1$ and $\eta = \gamma = 0$, resulting in $\hat{p} = C + \frac{1+W\left(e^{v(q)-1-\beta C}\right)}{\beta}$, as shown in Aravindakshan and Ratchford (2011). The monopolist's margin in the RPQ model is always positive but may be either strictly larger or strictly smaller than the margin under the Logit demand. For example, assuming $\zeta = 1$ and that the marginal cost is smaller then the reference

[6] The Lambert W function is implicitly defined as $W(x)e^{W(x)} = x$ (see Corless et al., 1996; Aravindakshan and Ratchford, 2011). It is positive and increasing for $x \geq 0$ and it has the property that $\ln W(x) = \ln x - W(x)$.

[7] Indeed, suppose that $\sigma(p|p^*) = mp + k$ for all p in a interval around $\hat{p}$. Then, $\sigma(\hat{p}|p^*)v(q) + \sigma_p(\hat{p}|p^*)v(q)(C-p) = (m\hat{p}+k)v(q) + mv(q)(C-\hat{p})$, which simplifies to $kv(q) + mv(q)C = f(C, v(q), p^*)$ and is independent of $\hat{p}$.

price, if the optimal price is higher than the reference price, the monopolist's margin in the RPQ model is strictly smaller than in the Logit case (the next section shows that this is always true when $\zeta = 1$). This result follows because $W\left(e^{v(q)-\eta v(q)(p^*-C)-1-\beta C}\right) \leq W\left(e^{v(q)-1-\beta C}\right)$ and $\beta + \eta v(q) \geq \beta$. Similarly, the monopolist's profit under the RPQ demand can be smaller or larger than the profit under Logit demand.

5.2. Optimal reference and posted prices

Suppose the monopolist can set both the posted price and the reference price (e.g., in the long-run). For a fixed posted price, the optimal reference price is equal to the posted price. Indeed, σ is (weakly) single-peaked at p^*, so for a fixed price $\bar{p}$, the optimal reference price is:

$$\bar{p} \in \arg\max_{p} (\bar{p} - C) \frac{e^{\sigma(\bar{p}|p)v(q)-\beta\bar{p}}}{1 + e^{\sigma(\bar{p}|p)v(q)-\beta\bar{p}}}.$$

As a consequence, the optimization problem reduces to finding the optimal posted price that maximizes the expression:

$$\max_{p} (p - C) \frac{e^{\sigma(p|p)v(q)-\beta p}}{1 + e^{\sigma(p|p)v(q)-\beta p}}.$$

If $\sigma(p|p)$ is constant across reference prices (i.e., $\sigma(p|p) = \sigma(p'|p') = \bar{\sigma}$ for all prices p, p'), the solution to this problem is:

$$\bar{p} = C + \frac{1 + W(e^{\bar{\sigma}v(q)-1-\beta C})}{\beta},$$

which corresponds to the optimal price in a modified Logit model where the structural value of the product is $\bar{\sigma}v(q) - \beta p$ rather than $v(q) - \beta p$. It follows that the monopolist's short-run profit (i.e., when the reference price is fixed) is bounded by the long-run profit:

$$\Pi \leq (\bar{p} - C) \frac{e^{\bar{\sigma}v(q)-\beta\bar{p}}}{1 + e^{\bar{\sigma}v(q)-\beta\bar{p}}} = \frac{W\left(e^{\bar{\sigma}v(q)-1-\beta C}\right)}{\beta},$$

where the equality derives from $\frac{e^{\bar{\sigma}v(q)-\beta\bar{p}}}{1+e^{\bar{\sigma}v(q)-\beta\bar{p}}} = \frac{W\left(e^{\bar{\sigma}v(q)-1-\beta C}\right)}{1+W\left(e^{\bar{\sigma}v(q)-1-\beta C}\right)}$, as shown in Aravindakshan and Ratchford (2011). Lastly, if $\bar{\sigma} = 1$, which indicates that there is no demand premium for observing a price equal to the reference price, the monopolist's profit under the Logit model (i.e., the long-run profit) will always be greater than the profit under the RPQ demand (i.e., the short-run profit). This provides a new interpretation of the Logit optimal price and profit as the price and profit of a monopolist facing the RPQ demand when the monopolist can choose both the price and the reference price.

5.3. Optimality of complex pricing strategies

Complex pricing strategies, such as frequent price changes, are more common than simpler strategies in which prices are kept constant over time (e.g., Özer and Zheng, 2016; Adida and Özer, 2019). These strategies my be justified by pricing discrimination (e.g., Su, 2007) or by exploiting behavioral aspects of consumer choice, such as regret (Özer and Zheng, 2016; Adida and Özer, 2019). I propose a different explanation based on reference prices.

I consider a two-period ($t = 1, 2$) version of the RPQ model and only one product k. The consumer's choice is either to buy k or nothing n (with the value of n normalized to 0). There are two possible prices for product k, $p_l < p_h$. I denote by p_1, p_2 the prices of k in period one and two, respectively. A pricing strategy is called everyday-low-price (EDLP) if $p_1 = p_2 = p_l$, while the Markdown (MD) strategy is such that $p_1 = p_h > p_l = p_2$. Assuming that the reference price is the first period price, the overall demand under the EDLP strategy is then given by:

$$P_1^{EDLP}(q, p_l) + P_2^{EDLP}(q, p_l) = \frac{e^{\sigma(p_l|p_l)v(q)-c(p_l)}}{1 + e^{\sigma(p_l|p_l)v(q)-c(p_l)}} + \frac{e^{\sigma(p_l|p_l)v(q)-c(p_l)}}{1 + e^{\sigma(p_l|p_l)v(q)-c(p_l)}}.$$

Under the MD strategy, the overall demand is given by:

$$P_1^{MD}(q, p_h) + P_2^{MD}(q, p_l) = \frac{e^{\sigma(p_h|p_h)v(q)-c(p_h)}}{1 + e^{\sigma(p_h|p_h)v(q)-c(p_h)}} + \frac{e^{\sigma(p_l|p_h)v(q)-c(p_l)}}{1 + e^{\sigma(p_l|p_h)v(q)-c(p_l)}}.$$

For simplicity, I normalize $\sigma(p_l|p_l) = 1$. I can state the following result:

Proposition 2. *There exists $k \geq 0$ such that if $\sigma(p_l|p_h) \geq k$, the MD strategy leads to higher demand than EDLP strategy, i.e., $P_1^{MD}(q, p_h) + P_2^{MD}(q, p_l) \geq P_1^{EDLP}(q, p_l) + P_2^{EDLP}(q, p_l)$.*

The MD pricing strategy involves two contrasting forces. On the one hand, a high initial price reduces demand due to the higher monetary cost of acquiring the product. On the other hand, this high initial price can also boost demand through the signaling effect of observing a high price. If the latter effect outweighs the former, the overall demand under the MD strategy exceeds that under the EDLP strategy. The condition in Proposition 2 requires $\sigma(p_l|p_h) \geq \sigma(p_l|p_l)$, meaning that observing an initial high price followed by a low price must be more distorting than observing the low price only.

Importantly, both the Logit model and the reference price-dependent model are incompatible with a higher demand for the MD strategy. In both models, σ is equal to 1 and c is increasing. Consequently, these models predict that the overall demand under the EDLP strategy is always larger than that under the MD strategy.

6. Axiomatic characterization

In this section, I introduce the properties of demand data that characterize the RPQ model, making it falsifiable. Before discussing the RPQ model, I begin by characterizing a general version of the multinomial Logit demand model, called Independent Logit. This model encompasses both the Linear Logit and the reference price-dependent model described in Eq. (RD). There are two reasons for studying the Independent Logit. Firstly, the behavioral restrictions of both the Linear Logit and the reference price-dependent models are unknown. Since the Independent Logit generalizes both, understanding its behavioral characterization is a necessary step towards understanding the behavioral characterization of the Linear Logit and the reference price-dependent models. Secondly, the behavioral restrictions characterizing the Independent Logit illustrate key properties of the RPQ model.

I assume that the analyst can observe the consumer's choice among finitely many products. Therefore, there are finitely many observed quality levels, denoted by $\mathcal{Q} \subset [q_0, q_1]$ with $|\mathcal{Q}| \geq 2$. Additionally, I assume that $q_0 \in \mathcal{Q}$. There are finitely many observed prices, denoted by $\mathcal{P} \subset [0, \infty)$ with $|\mathcal{P}| \geq 2$. I denote by $\mathcal{A}$ the family of all choice sets in the dataset. The first three assumptions are standard:

Axiom (*Positivity - P*)**.** *For all $A \in \mathcal{A}$ and $(q_k, p_k) \in A$, $P((q_k, p_k)|A) > 0$.*

This assumption is rather weak; it cannot be rejected in any finite dataset since, empirically, a small but strictly positive probability is indistinguishable from zero. The second property is the standard Independence of Irrelevant Alternatives (IIA):

Axiom (*IIA*)**.** *For all $A, B \in \mathcal{A}$ and $(q_k, p_k), (q_l, p_l) \in A \cap B$,*

$$\frac{P((q_k, p_k)|A)}{P((q_l, p_l)|A)} = \frac{P((q_k, p_k)|B)}{P((q_l, p_l)|B)}.$$

The last assumption states that, for a fixed price, demand increases with quality:

Axiom (*Quality Monotonicity - QM*)**.** *For all $p \in \mathcal{P}$, all $q, q' \in \mathcal{Q}$ with $q \leq q'$ and all choice sets A such that $(q, p), (q', p) \in A$, $P((q, p)|A) \leq P((q', p)|A)$.*

The choice probabilities satisfy the *Basic Axioms* if they satisfy Positivity, IIA and Quality Monotonicity. I have the following immediate result.

Theorem 1 (*Logit*)**.** *The choice probabilities satisfy the Basic Axioms if and only if there is a function $u : \mathcal{Q} \times \mathcal{P} \to \mathbb{R}$ increasing in its first argument such that:*

$$P((q_k, p_k)|A) = \frac{e^{u(q_k,p_k)}}{\sum_{(q_l,p_l)\in A} e^{u(q_l,p_l)}},$$

for all $(q_k, p_k) \in A$ and $A \in \mathcal{A}$. The function u is unique up to location (i.e., up to translation by a constant).

6.1. Independent logit

In the Independent Logit, the structural value of a product (q, p) is $u(q, p) = v(q) - c(p)$. This form implies a separation between quality and price (v and c are independent), and a downward-sloping demand for all products (because c is increasing). The following two axioms capture these two properties:

Axiom (*Odds Independence - OI*)**.** *For all $q_k, q_l \in \mathcal{Q}$, all $p_k, p_l \in \mathcal{P}$, and all $A \in \mathcal{A}$ with $(q_k, p_k), (q_k, p_l), (q_l, p_l), (q_l, p_k) \in A$:*

$$\frac{P((q_k, p_k)|A)}{P((q_l, p_k)|A)} = \frac{P((q_k, p_l)|A)}{P((q_l, p_l)|A)}.$$

Odds Independence means that *relative* quality preferences are independent of the price. The next condition requires that, *ceteris paribus*, a higher price decreases the likelihood of selecting a product:

Axiom (*Monotonicity - M*)**.** *For all $q_k, q_l \in \mathcal{Q}$ and all $p_k, p_l \in \mathcal{P}$ with $p_k \geq p_l$, $P((q_k, p_k)| \{(q_l, p_l), (q_k, p_k)\}) \leq P((q_k, p_l)| \{(q_k, p_l), (q_l, p_l)\})$*

The previous axioms characterize the Independent Logit demand:

Theorem 2 (*Independent Logit*)**.***The choice probabilities satisfy the Basic Axioms, Odds-Independence and Monotonicity if and only if there are weakly increasing functions $v : \mathcal{Q} \to \mathbb{R}$ and $c : \mathcal{P} \to \mathbb{R}_+$ such that:*

$$P((q_k, p_k)|A) = \frac{e^{v(q_k)-c(p_k)}}{\sum_{(q_l,p_l)\in A} e^{v(q_l)-c(p_l)}},$$

for all $(q_k, p_k) \in A$ and $A \in \mathcal{A}$. The functions v and c are unique up to location (i.e., up to translation by a constant).

In the Online Appendix A, I will characterize the Linear Logit (Proposition 7) and the reference price-dependent model (RD) (Proposition 6).

6.2. RPQ model

In the RPQ model, the choice probabilities are strictly positive, IIA holds, and higher quality increases demand. However, price and quality are intertwined and the demand for some products may increase with price. Therefore, both Odds Independence and Monotonicity need to be relaxed. Let $L_{q_k,q_l}(p, A)$ denote the log-odds of selecting q_k and q_l in a choice set A when they have the same price p:

$$L_{q_k,q_l}(p, A) = \ln \frac{P((q_k, p)|A)}{P((q_l, p)|A)}.$$

Log-odds represent the relative demand between two quality levels, q_k and q_l, at a given price p. The following axiom employs log-odds to establish the type of "independence" that holds in the RPQ model:

Axiom (*Log-odds Independence - LOI*)**.** *For all $p, p' \in \mathcal{P}$ and all $A, B \in \mathcal{A}$:*

$$\frac{L_{q_k,q_m}(p, A)}{L_{q_k,q_m}(p', B)} = \frac{L_{q_l,q_n}(p, A)}{L_{q_l,q_n}(p', B)}, \tag{7}$$

for all $q_k, q_l, q_m, q_n \in \mathcal{Q}$ such that either ratio is well defined.

Regarding its interpretation, Log-odds Independence implies that *relative changes* in demand between pairs of quality levels are independent of price. Suppose that all log-odds are non-zero. Then Log-odds Independence allows me to express the relative demand between two quality levels at price p as a quality-independent function of the relative demand at price p', $\frac{P((q_k,p)|A)}{P((q_m,p)|A)} = \left(\frac{P((q_k,p')|B)}{P((q_m,p')|B)}\right)^{\psi(p,p')}$, where $\psi(p, p') = L_{q_l,q_n}(p, A)/L_{q_l,q_n}(p', B)$ is independent of q_l and q_n.

The next condition weakens Monotonicity and states that the demand for the product of lowest quality decreases when its price increases:

Axiom (*Worst Monotonicity - WM*)**.** *If $p \geq p'$, then $P((q_0, p)| \{(q_0, p), (q_l, p')\}) \leq P((q_0, p')| \{(q_0, p'), (q_l, p')\})$.*

The last condition is needed to identify the reference price(s) and to impose weak single-peakedness of the distortion function. Note that IIA implies that log-odds are independent of the choice set A. Hence, I can write $L_{q_k,q_l}(p)$. Consider $q_k, q_l \in \mathcal{Q}$ and $\bar{p} \in \mathcal{P}$ such that $L_{q_k,q_l}(\bar{p}) > 0$ (which exists unless all probabilities are uniform). I define P^* to be the (set of) price(s) given by:

$$P^* = \underset{p\in\mathcal{P}}{\operatorname{argmax}} \frac{L_{q_k,q_l}(p)}{L_{q_k,q_l}(\bar{p})}.$$

The normalization only serves to show that, by Log-Odds Independence, these ratios are independent of q_k, q_l. The following condition requires that log-odds have a peak at (the potentially multiple) $p^* \in P^*$:

Axiom (*Weak Single-Peakedness - WSP*)**.** *For all $p^* \in P^*$, if $p \geq p' \geq p^*$ or $p^* \geq p' \geq p$ then $L_{q_k,q_l}(p') \geq L_{q_k,q_l}(p)$.*

Intuitively, the prices in P^* are the *revealed* reference prices. The WSP condition ensures that the further a price is from the reference price, the lower is the relative likelihood of choosing one quality over the other. Reference prices are those at which quality differences are most relevant for the relative difference in demand. It is crucial to highlight that, in models where quality and price are independent, such as the Independent Logit (thus the Linear Logit and the (RD) models), all prices are revealed reference prices (i.e., $P^* = \mathcal{P}$) because the log-odds are independent of the price. I can now state the main result of this section:

Theorem 3 (*Reference-Price-Quality*)**.** *The choice probabilities satisfy the Basic Axioms, Log-odds Independence, Worst Monotonicity, and Weak Single-Peakedness if and only if there is $p^* \in \mathcal{P}$ and there are weakly increasing functions $v : \mathcal{Q} \to \mathbb{R}_+$ with $v(q_0) = 0$, $c : \mathcal{P} \to \mathbb{R}_+$, and a function $\sigma(\cdot|p^*) : \mathcal{P} \to \mathbb{R}_+$ such that:*

$$P((q_k, p_k)|A) = \frac{e^{\sigma(p_k|p^*)v(q_k)-c(p_k)}}{\sum_{(q_l,p_l)\in A} e^{\sigma(p_l|p^*)v(q_l)-c(p_l)}},$$

for all $(q_k, p_k) \in A$ and all $A \in \mathcal{A}$. The function $\sigma(\cdot|p^*)$ satisfies $\sigma(p'|p^*) \geq \sigma(p|p^*)$ if $p^* \geq p' \geq p$ or $p \geq p' \geq p^*$.

If $P^* = p^*$, then p^* is the unique reference price. However, consider the acceptable price range function σ_{APR} defined in Eq. (APR). In this case, all prices in $[p^* - \delta_1, p^* + \delta_2]$ are revealed reference prices and belong to P^*. For example, suppose that $p^* = 2$ and $\delta_1 = \delta_2 = 2$, then $P^* = \mathcal{P} \cap [0, 4]$. However, $p_1^* = 1$, $\delta_1' = 1$, and $\delta_2 = 3$ determine the same acceptable price range $P^* = \mathcal{P} \cap [0, 4]$. Thus, in some cases, it is not possible to identify a unique reference price from choice data. In any case, the Weak Single-Peakedness property ensures that P^* is an interval within $\mathcal{P}$. I conclude by stating the uniqueness properties of the RPQ model.

Proposition 3 (*Uniqueness*)**.** *The set P^* is unique. If $\bar{\sigma}, \bar{v}, \bar{c}$ also represent the choice probabilities, and these probabilities are not uniform, then there are $a, b, d \in \mathbb{R}$ such that $\bar{\sigma} = a\sigma$, $\bar{v} = \frac{1}{a}v + b$ and $\bar{c} = c + ab\sigma + d$.*

7. Extensions

In this section, I study two extensions of the RPQ model. In the first extension, there is a price-quality interaction, but there is not necessarily a reference price (e.g., Crawford et al., 2015; Li et al., 2020). In the second extension, the quality distortion is context-dependent.

7.1. General price-quality interaction

In the first generalization of the RPQ, called the General PQ model, the structural value of a product of quality q and price p is given by $\sigma(p)v(q) - c(p)$, where σ is a positive function. This model is characterized by excluding the WSP axiom from the axioms used to characterize the RPQ model in Theorem 3:

Proposition 4 (*Price-Quality Interaction*)**.** *The choice probabilities satisfy the Basic Axioms, Log-odds Independence, and Worst Monotonicity if and only if there is there are weakly increasing functions $v : \mathcal{Q} \to \mathbb{R}_+$ (with $v(q_0) = 0$), $c : \mathcal{P} \to \mathbb{R}_+$, and a function $\sigma : \mathcal{P} \to \mathbb{R}_+$ such that:*

$$P((q_k, p_k)|A) = \frac{e^{\sigma(p_k)v(q_k) - c(p_k)}}{\sum_{(q_l, p_l) \in A} e^{\sigma(p_l)v(q_l) - c(p_l)}}$$

for all $(q_k, p_k) \in A$ and all $A \in \mathcal{A}$.

Special cases of the General PQ model have appeared in the literature studying price-quality interaction. For instance, Crawford et al. (2015) and Li et al. (2020) propose that the value of a product is given by $u(q, p) = \alpha_0 + \alpha_1 q - \beta p + \alpha_2 qp$. This corresponds to a General PQ model with a distortion $\sigma(p) = \alpha_1 + \alpha_2 p$ for some $\alpha_1, \alpha_2 \in \mathbb{R}$, a linear cost function $-c(p) = -\alpha_0 - \beta p$ for some $\alpha_0 \in \mathbb{R}$ and $\beta \geq 0$, and $v(q) = q$. In this model, the demand for a product is upward-sloping if $\alpha_2 q \geq \beta$ (see condition (2)).

7.2. Context effects

The second extension of the RPQ model relaxes its logit-like functional form. It is well-known that the Logit model is not suitable for modeling context-dependent behaviors, such as the asymmetric dominance effect and the compromise effect (Simonson, 1989). These phenomena violate a property called *regularity*, which states that the demand of a product cannot increase if a new product is added to the choice set. Formally:

Definition 2 (*Regularity*)**.** The demand P satisfies regularity if $P((q_k, p_k)|A) \geq P((q_k, p_k)|B)$, for all products (q_k, p_k) and all choice sets $A \subseteq B$.

To address violations of Regularity, I will consider the following context-dependent generalization of the PQ model:

$$P((q_k, p_k)|A) = \frac{e^{\sigma_A(p_k)v(q_k) - c(p_k)}}{\sum_{(q_l, p_l) \in A} e^{\sigma_A(p_l)v(q_l) - c(p_l)}}, \tag{8}$$

where $\sigma_A : \mathcal{P} \to \mathbb{R}$ is a choice set-dependent distortion and c is a cost function. For example, a choice set-dependent distortion can derive from a choice set-dependent reference price, i.e., $\sigma_A(p_k) = \sigma(p_k | p_A^*)$. One possible interpretation is that p_A^* is the average price in A. In the Online Appendix B (Theorem 4), I provide the axiomatic characterization of a general context-dependent model that includes as special case the model in Eq. (8).

Suppose that the initial choice set A contains only two products, $k = (q_k, p_k)$ and $l = (q_l, p_l)$. A violation of Regularity occurs if adding product m to A increases the probability of selecting k (and/or l). Simple algebra shows that:

Proposition 5. *The context-dependent model of Eq. (8) violates Regularity if and only if*

$$e^{v(q_k)\left(\sigma_{A\cup m}(p_k) - \sigma_A(p_k)\right)} > e^{v(q_l)\left(\sigma_{A\cup m}(p_l) - \sigma_A(p_l)\right)} + e^{\sigma_{A\cup m}(p_m)v(q_m) - c(p_m) - \sigma_A(p_l)v(q_l) + c(p_l)}.$$

To observe a violation of Regularity, the additional distortion of the quality of $k = (q_k, p_k)$ when m is added to A (the left-hand side of the inequality) must be larger than the additional distortion of the quality of l plus the relative value of m over l (the right-hand side of the inequality). If Regularity is violated, it must be the case that:

$$v(q_k)\left(\sigma_{A\cup m}(p_k) - \sigma_A(p_k)\right) > v(q_l)\left(\sigma_{A\cup m}(p_l) - \sigma_A(p_l)\right).$$

If the distortion of the quality of l is unaffected by the presence of m (i.e., $\sigma_{A\cup m}(p_l) = \sigma_A(p_l)$), the necessary condition simplifies to:

$$\sigma_{A\cup m}(p_k) > \sigma_A(p_k), \tag{9}$$

which states that the distortive effect of p_k is larger when m is present.

Two robust violations of Regularity are the *asymmetric dominance effect* and the *compromise effect* (Simonson, 1989). In the asymmetric dominance effect, a decoy product is added to a choice set to increase the demand for a targeted product. The decoy is

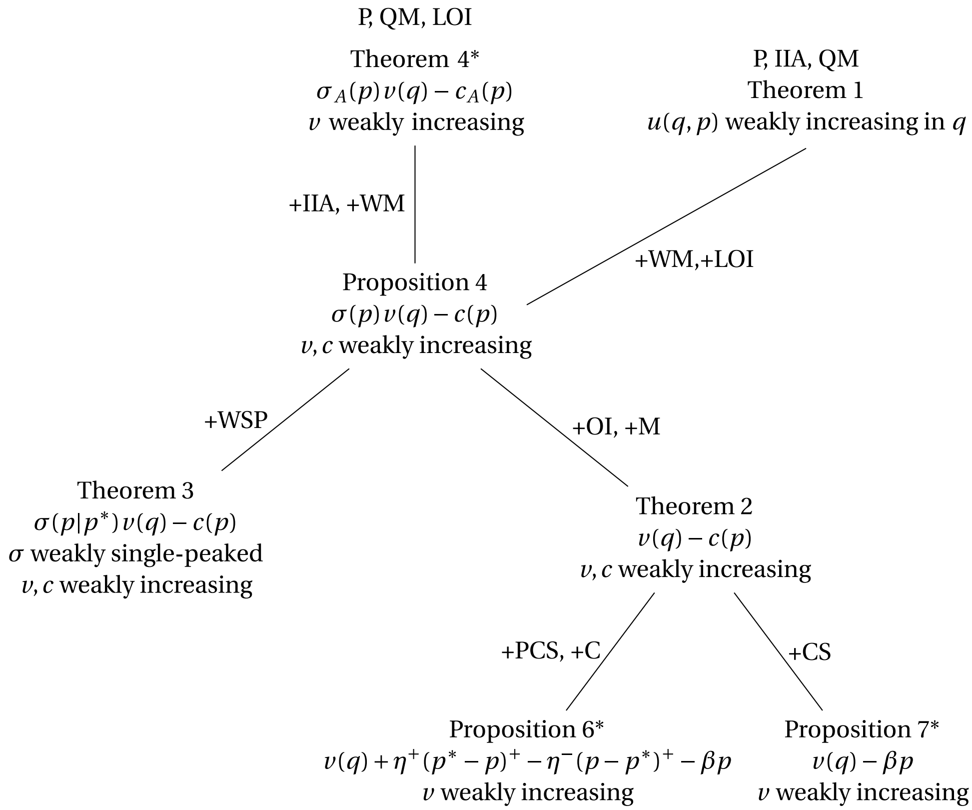

Fig. 5. Summary of the axioms and the structural utilities in the associated representation results. A star next to the result indicates the presence of technical conditions in addition to the axioms. Theorem 4, Proposition 6 and Proposition 7 are in the Online Appendix.

a product that is dominated by the targeted product (e.g., it has a higher price and lower quality than the target) but not dominated by the alternative product. The asymmetric dominance effect occurs when the probability of selecting the targeted product increases after adding the decoy. In the compromise effect, the demand for a product increases when it becomes a "compromise" between two extreme alternatives. For example, if k has higher quality and a higher price than l, adding m that has higher quality and a higher price than k increases the demand for k.

The necessary condition (9) implies that the presence of a decoy or an extreme product boosts demand of the target because the price of the latter becomes more "salient".

7.2.1. *Context-dependent "salience"*

To illustrate an additional application of the context-dependent model, consider the wine purchase example discussed by Bordalo et al. (2013). In a wine shop, a cheap but low-quality wine (l) sold at \$10 per bottle may be preferred over an expensive and high-quality wine (h) sold at \$20 per bottle. However, this preference may be reversed in a restaurant, even if the price difference remains the same. For instance, the restaurant may sell the high-quality wine at \$60 and the low-quality wine at \$50. Let $A = \{(q_l, 10), (q_h, 20)\}$ and $A' = \{(q_l, 50), (q_h, 60)\}$, and $c(p) = p$. The context-dependent PQ model is consistent with $P((q_h, 60)|A') \geq P((q_h, 20)|A)$. Indeed:

$$P((q_h, 60)|A') \geq P((q_h, 20)|A) \iff v(q_l)(\sigma_{A'}(50) - \sigma_A(10)) \leq v(q_h)(\sigma_{A'}(60) - \sigma_A(20)). \tag{10}$$

Suppose that $v(q_h) \geq v(q_l)$, then a sufficient condition for the inequality (10) to hold is $\sigma_{A'}(50) - \sigma_A(10) \leq \sigma_{A'}(60) - \sigma_{A'}(20)$. Consider the special cases in which only the reference price is context-dependent: $\sigma_A(p) = \sigma_{PL}(p|p^*_A)$ and $\sigma_{A'}(p) = \sigma_{PL}(p|p^*_{A'})$. If the reference price at the wine shop is \$10 and it is \$60 at the restaurant, I have:

$$\sigma_{PL}(50|60) - \sigma_{PL}(10|10) = -\gamma 10 \leq \eta 10 = \sigma_{PL}(60|60) - \sigma_{PL}(20|10),$$

which is always satisfied.

Fig. 5 summarizes the relationship among the representation results proved in the text.

8. Related literature

This paper contributes to the literature on random choice and behavioral consumer choice.

In the random choice literature, the axiomatic characterization of my model is one of the first to exploit the bi-dimensional nature of products. In this literature, Falmagne and Iverson (1979) studied Weber's law for random choice over bi-dimensional objects, and recently, Allen and Rehbeck (2023) extended the perturbed random utility model (e.g., Fudenberg et al., 2015) to multidimensional choice objects. In general, the results of the present paper can inform models of discrete choice over bi-dimensional objects, such as dated outcomes or two-person allocations. Although with a different scope and primitives, Cerreia-Vioglio et al. (2023) axiomatized a dynamic version of the multinomial Logit in which the noise decreases over time. In their model, the structural value of an alternative a at time t is $v(a)/\lambda(t) + \alpha(a)$. This can be viewed as a model in which time distorts the value of a, as the price distorts quality in the General PQ model of Proposition 4.

The context-dependent extension of Section 7.2 and its axiomatic characterization in the Online Appendix B contribute to the vast literature on extensions of the Logit model. Some recent advancements include the work of Kovach and Tserenjigmid (2022b), who axiomatized a generalization of the Logit model to incorporate the menu-dependent focality of certain alternatives, and Kovach and Tserenjigmid (2022a), who axiomatized the nested-logit model. Other extensions of the Logit model include Rehbeck (2022), who allows for menu-dependent variance, and Rehbeck (2024), who allows for a menu-dependent distortion to the deterministic utility of an item. Alternative approaches extending the Logit model integrate salience (Chambers et al., 2021) or reference-dependence (Kibris et al., 2024).

The literature on "behavioral" consumers' responses to price information is extensive (e.g., Monroe, 1971, 1973; Cheng and Monroe, 2013). Within this literature, the paper contributes to research on reference prices (see Briesch et al., 1997; Mazumdar et al., 2005, for reviews), as well as to research on the interdependence of price and value (e.g., Scitovszky, 1944; Cosaert, 2018; Ng, 1987; Pollak, 1977; Dodds et al., 1991; Gneezy et al., 2014). The current paper offers a first axiomatic characterization of two models that are popular in this literature: the Linear Logit model of Guadagni and Little (1983) (see Theorem 2 and Proposition 7 in the Online Appendix), and the model of choice with reference prices (Theorem 2 and Proposition 6 in the Online Appendix). Moreover, the RPQ model extends and refines these models, as existing reference price models typically disregard the interaction between prices (or reference prices) and quality, and typically assume that a higher price reduces demand. Additionally, these models often assume an exogenous reference price.[8] This paper is the first to allow for the identification of reference price(s) from choice data.

Regarding the literature on the price-quality heuristic, the present model derives from an additive random utility model in which the consumer is uncertain about the overall value of a product. The function distorting quality can be interpreted, as if, the price "signals" quality by increasing the weight of the quality over the random component, thereby reducing choice variability. This approach differs from Gneezy et al. (2014), where consumers are uncertain about the product's quality and form expectations based on prices. Models of the price-quality heuristic (e.g., Bagwell and Riordan, 1991; Wolinsky, 1983; Pollak, 1977; Gneezy et al., 2014) are independent of reference prices, so the RPQ extends these approaches by including reference prices. Moreover, I provide the first behavioral characterization of a model that allows for the interaction between price and quality (Proposition 4).

Although not the primary focus of the paper, the context-dependent extension introduced in Section 7.2 contributes to the vast literature on context-effects (e.g., Tversky, 1972; Guo, 2016; Steverson et al., 2019; Webb et al., 2021; He, 2024).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jebo.2024.106872.

Data availability

No data was used for the research described in the article.

[8] One exception is Baucells and Hwang (2017), who proposed a model of intertemporal consumption with a time-varying reference price to account for various behavioral biases.

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