

# Timeline tool for analyzing the relationship between students-teachers-artifacts interactions and meaning-making

Sara Bagossi<sup>1</sup> , Osama Swidan<sup>1</sup> , Ferdinando Arzarello<sup>2</sup> 

<sup>1</sup>Department of Human and Social Sciences, Ben-Gurion University of the Negev, Be'er Sheva, Israel

<sup>2</sup>Department of Mathematics, University of Turin, Turin, Italy

\*Correspondence: bagossi@post.bgu.ac.il

Received: 24 April 2022 | Revised: 12 October 2022 | Accepted: 28 October 2022 | Published Online: 31 October 2022

© The Author(s) 2022

## Abstract

Meaning-making in teaching-learning mathematical processes is a relevant issue analysed through different philosophical and educational frames. In particular, the use of digital tools in mathematics education affects the meaning-making processes. This paper discusses meaning-making from a phenomenological standpoint, in which interpretative activities are relevant. This approach requires a careful analysis of the semiotic resources' evolution, including those related to the used digital tools. The paper aims to introduce an analytical tool, the Timeline. This tool is an elaboration on previous analysis tools, like the interaction flowchart and the semiotic bundle. Such a tool allows the analysis of relationship among interactions, semiotic resources, and meaning-making. In this paper, the Timeline is used to analyze two episodes from two different learning experiments where GeoGebra and augmented reality are used. High school students from Italy and Israel participated in this study. Video recording has been used to document the entire learning experiments. The analysis provides evidence that the Timeline enables investigating the relationship between students-teacher-artifacts interactions and meaning-making. Moreover, results may give teachers ideas for using digital tools to foster students' meaning-making.

**Keywords:** Augmented Reality, Digital Artifacts, Interactions, Meaning-Making, Multimodality

**How to Cite:** Bagossi, S., Swidan, O., & Arzarello, F. (2022). Timeline tool for analyzing the relationship between students-teachers-artifacts interactions and meaning-making. *Journal on Mathematics Education*, 13(2), 357-382. <http://doi.org/10.22342/jme.v13i2.pp357-382>

Meaning-making of mathematical concepts is an essential topic in mathematics education research and one of the teacher's instruction aims, though, it "has proved to be a difficult concept" (Radford et al., 2011, p. 149), and this claim is even more valid for the notion of meaning-making, which has several different interpretations in mathematics education literature on how it may be accomplished (Seeger, 2011). These interpretations depend both on the philosophical background (e.g., cognitivism, socio-cultural perspective, or semiotics), where a researcher implicitly or explicitly embeds the term 'meaning', and on the educational approach adopted for interpreting the teaching-learning processes in the mathematics classroom. Moreover, in the last decades the issue of how digital resources influence processes of meaning-making in the classroom has been more and more investigated. For example, Drijvers et al. (2016) observed how the exploitation of the dragging option in a dynamic geometry environment helped the students to give meaning to the notion of function in terms of variation and covariation.

In this paper, an approach to meaning-making based on a phenomenological standpoint is considered and the analysis puts forward the interpretation of the semiotic aspects of teaching-learning

processes, especially when they happen in a technological context. According to the phenomenological approach, students give meaning to mathematical objects through a progressive unveiling of the mathematical ideas at stake. In recent years research in mathematics education has suggested models for analyzing the meaning-making of mathematical concepts from a phenomenological point of view (e.g., see Radford, 2010). Along this line, Swidan et al. (2020) proposed a model to examine the relationship between students' interrogative process and meaning-making. Swidan (2022) instead used the same phenomenological perspective for meaning-making to understand the relationships between students' argumentation processes and the meaning-making of a mathematical concept. Indeed, teaching-learning mathematics does not only involve questioning and verbal speech, but it is a complex process that consists of a variety of semiotic means students and teachers use through their interactions, which are intertwined at any moment (Schwarz et al., 2009). These semiotic means and interactions have proved to be essential in learning and meaning-making processes: their analysis may help us understand how verbal and non-verbal language, mathematics, visual representations, and digital tools, form a single unified system for meaning-making (e.g., Sabena et al., 2012; Radford et al., 2011, Radford, 2011).

The analysis of the meaning-making processes reveals particularly interesting when dynamic digital tools are involved in the learning process (Swidan et al., 2020). These technological tools may provide various inputs and new interactions with it. The process of meaning-making is not only driven by the interactions with the digital artifacts but also by the interactions with the other subjects involved in the learning process, e.g., other students and the teacher. Such interactions happen through various multimodal channels, which learners use to communicate and think about mathematical topics or ideas.

To understand the relationship between the students' interactions, the semiotic means they use through the interaction, and the meaning-making processes, an integration of these theoretical frameworks is needed, and a fine-grained tool of analysis is necessary to point out the intertwining of these aspects in the process of meaning-making. Starting from an analytical tool called Timeline, presented in Sabena et al. (2012), an elaboration of the Timeline tool is proposed in this paper. This elaboration, which is the main novelty of this study, enables the analysis of the meaning-making process of mathematical concepts while considering the interaction among students, teacher, artifacts, and the semiotics means that students and teachers use. In doing so, two theoretical perspectives are brought together: the phenomenological approach to meaning-making (Rota, 1991) and the multimodality approach to learning (Gallese & Lakoff, 2005).

The functioning of the Timeline is then shown through the analysis of two learning episodes involving digital tools such as GeoGebra and augmented reality. Understanding how students assign meaning to mathematical concepts through interaction in a digitally rich environment carries important theoretical, methodological, and pedagogical implications. Theoretically, the study conceptualizes the students-teacher-artifacts interaction and its relationship with meaning-making. Methodologically, the study provides an analytical tool – the Timeline – that may help mathematics educators to analyse the meaning-making process considering the students-teachers-artifacts interaction and the semiotic means that emerge from these interactions. Pedagogically, the study presents examples that may give teachers suggestions for using digital tools to foster students' meaning-making.

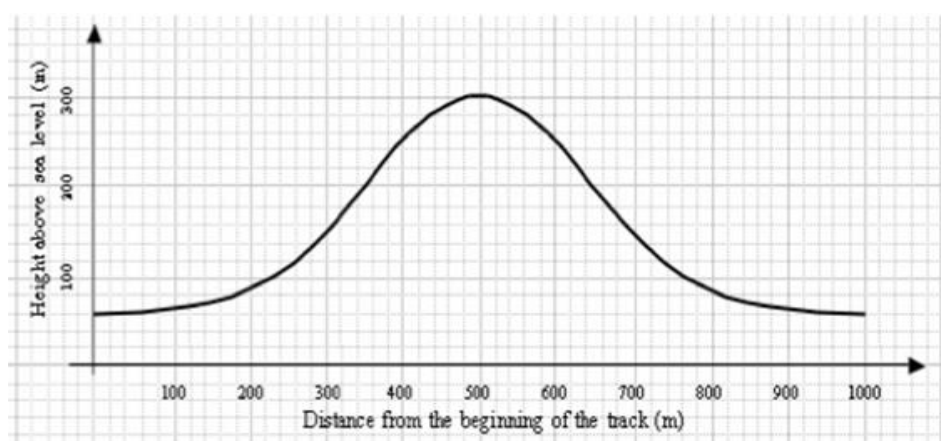
Two theoretical frameworks guide this study: the phenomenological approach to meaning-making (Rota, 1991) and the multimodality approach to learning (Arzarello & Robutti, 2010).

## The Phenomenological Approach to Meaning-Making

In this paper, the starting point is the key phenomenological assumption, pointed out by Rota, that there



is “no such thing as true seeing,” but “there is only seeing as” (Rota, 1991, p. 239). From this perspective, the meaning-making of a mathematical object is conceived as a progressive disclosure of the mathematical ideas at stake. Arzarello et al. (2011) argued in this context that “a situation may evoke different contexts and so produce a different sense-making, according to the age and the background of the students” (p. 51). For example, in the graph in Figure 1, students may disclose a mountain, a graph of a symmetric function, a normal distribution, a continuous function, an increasing and decreasing function, and so on. The realization of a didactic goal, namely that the students eventually “see” this drawing “as” the graph of a mathematical function that models track highness (but also a continuous function, an increasing and decreasing function, a concave up and down), is a delicate process. According to Rota, this process is referred to as disclosure.



**Figure 1.** Distance-height graph (adapted from Arzarello et al. (2011))

Disclosure may also happen when, using perceptual senses, one can grasp an object's functionality in a certain context or situation. This meaning-making process is anything but trivial, and students must be educated to disclose the meanings offered by a specific mathematical context (Radford, 2010). Not only, the background of students also influences the meaning-making process evoked by a particular situation. According to Rota, these contexts or situations are typically layered upon each other and may generate different meanings over time. The disclosure processes are triggered by a combination of factors belonging to the various semiotic sets that interact in the mathematics classroom environment (Swidan et al., 2020).

### **Multimodality Approach to Learning**

Multimodality has evolved within the paradigm of embodiment, which has been developed in recent years (Wilson, 2002). Embodiment is an approach in cognitive science that attributes the body to a central role in shaping the mind. The embodiment considers sensory and motor functions, as well as their importance for successful interaction with the environment. Learning thus analysed is not based on “formal abstract models, totally unrelated to the life of the body, and of the brain regions governing the body's functioning in the world” (Gallese & Lakoff, 2005, p. 455) but it also considers the multimodality of cognitive process. Multimodality uses two or more forms of communication in addition to the two primary modalities, namely auditory and visual (Loncke et al., 2006), it is deeply intertwined with perceptuo-motor activities (Arzarello & Robutti, 2010). Multimodality is also attributed to language; as Gallese and Lakoff (2005) said, “language is inherently multimodal in this sense, that is, it uses many modalities linked together—sight,

hearing, touch, motor actions, and so on" (p. 456). According to Arzarello and Robutti (2010), if human language is multimodal, human activity, which is based on language, is also multimodal. To better understand the learning activity, all the modalities to understand cognitive processes can be analysed (Arzarello & Edwards, 2005; Arzarello, 2008).

During the mathematical activities in a classroom, students produce a variety of signs such as words, gestures, gazes, interactions, and written or oral signs. Learning mathematics with artifacts is multimodal. Students' and teachers' actions are not only directed towards artifacts but also toward classroom subjects. In this context, teachers and students use spoken language, gestures, and graphic registers to communicate their thoughts. This multimodal production is vibrant when students use digital technology.

Gestures are one main component of the multimodal approach. McNeill (1992) identified four overlapping dimensions for gestures: (1) deictic, used in concrete or abstract pointing; (2) iconic or representational, arm or hand movements with a perceptual relation to the concrete object that is represented; (3) metaphoric, similar to iconic gestures but referring to abstract objects, and (4) beats, which are up and down flicks of the hand or tapping motions. Gestures may cover different functions. They have an interactive function when serving to coordinate speaking turns, regulate the rhythm of the speech, seek, or request a response, or acknowledge understanding. A narrative function is when somebody imitates the pictorial background with iconic or metaphoric gestures, such as a hand movement reproducing the trend of a curve shown on a screen. A grounding function connects verbal and gestural narratives to an object, e.g., a pointing gesture relating oral utterances to a sketch on the blackboard (Roth, 2001).

Gazes may be considered another relevant component of the multimodal approach. Cañigual and Hamilton (2019) argued that gazes that emerge during face-to-face interactions might have two functions: a sensing function when gazes aim to gain information from the environment, for example, when somebody looks at a computer screen looking for data or observing somebody performing experiment; a signaling function when gazes signal information to others, expressing a clear desire for interaction, for instance, a student looks at the teacher waiting for a response from her.

As mentioned above, interaction with digital artifacts, when it is available, is considered a central component of multimodality. The learners' interactions with different artifacts possibly contribute to producing a synergy, mainly when the use of various artifacts "can foster the integration of different and complementary meanings providing a rich support to the development of the expected mathematical meaning" (Faggiano et al., 2018, p. 1). On the other hand, a conflict between the artifacts is when the use of different artifacts to address the same situation fosters different, not converging, or even contradictory meanings for the situation.

Such an approach that emphasizes the role of signs, language, and interactions with the artifact is rooted in the socio-constructivist perspective of learning. The latter also highlights the role of social interaction between students and teacher-student interactions. According to Saxe et al. (2009), students develop understanding as they produce, coordinate, and adapt representations by interacting with others individually. Saxe and colleagues' argument is only one example that emphasizes the role of interactions in constructing knowledge and the meaning-making process of mathematical concepts and ideas.

To shed light on how the several components of the multimodal approach come into play, Arzarello (2006) introduces the term semiotic bundle to describe the multimodal semiotic activity of subjects in a holistic way as a dynamic production and transformation of various signs and their relationships. Specifically, it is suitable for a double analysis, a synchronic one in which, at a fixed time, you look at the

interactions between the different semiotic components, and diachronic analysis, focused on the evolution of a single semiotic component over time.

Although the semiotic bundle offers a global approach to looking at classroom activities, it neither explicitly addresses the students-teacher interaction nor emphasizes the relationship between the variety of semiotic means and the meaning-making process of the mathematical concept. For this reason, to answer the research question of how students-teachers-artifacts interactions contribute to the students' meaning-making of mathematical concepts, a suitable tool for analysis is required. In the following section, such a tool, the main contribution of this paper, is introduced.

## METHOD

### Timeline Tool for Analyzing Meaning-Making Process

Starting from the theoretical perspective of multimodality, a suitable tool of analysis, called Timeline, has already been presented in Arzarello et al. (2010) and Sabena et al. (2012): it arose from the need to describe in detail the didactic situations and the dense intertwining of relationships between the variables present in the classroom. It is supposed to be used as a microanalysis tool that provides a global view of the different semiotic registers: speech, body, and inscriptions. In line with the characterization of the semiotic bundle construct, two kinds of analysis may be carried out with the Timeline: a diachronic one, focusing on the evolution of the various components over time, and a synchronic one, allowing to grasp the relations of the components in a specific moment in time. The Timeline, a table with several rows and columns, is both a powerful and complex tool of analysis and deserves some words to be spent in a detailed description of its various rows dedicated to the analysis of the specific aspects of the semiotic and discursive students' and teachers' productions, including their interactions with tools. Given the density of details, the Timeline is suitable for an accurate analysis of short episodes. However, one of the advantages of this tool is that it can be easily adapted and integrated to respond to different research purposes, for example, for a macroanalysis of longer episodes (Javorski & Potari, 2009).








The elaborated version of the Timeline allows us to represent the dynamic flow of the various episodes in a condensed form. With respect to the original Timeline, which focused on analyzing the specific aspects of the students and teacher semiotic productions, this elaborated Timeline contains an extra row devoted to the disclosed layers of meaning and the interaction flowchart collecting students-teacher-artifacts interactions. The interaction flowchart was primarily elaborated by Sfard and Kieran (2001) to evaluate the real interest of the interlocutors in creating a dialogue with their partners. This flowchart represents the interactions between the different subjects involved in communication as arrows. The analysis of the arrows allows one to understand whether the subjects aim at communicating with others or with themselves. Several variations of this interaction flowchart exist in the literature. For instance, Liljedahl and Andrà (2014) further developed the interactivity flowchart including gazes produced in social interactions, specifically their direction and intensity (denoted by the different thickness of the arrows). Smedlund et al. (2018) joined their flowchart with the transcripts so that the reader could follow the situation clearly and adopted a color coding of the arrows in the flowchart according to the different types of emerging mathematical content. The elaboration here proposed includes non-verbal interactions such as gestures and relevant gazes produced by the subjects involved in the discussion, and also subjects' interactions with the artifacts at stake. This elaboration reflects the theoretical assumption that learning is a multimodal activity and that even gestures and artifacts interactions shed light on the learning process. Bagossi already initiated this approach in her Ph.D. research (2022). Below





the elaborated Timeline is described in detail. [Table 1](#) collects the symbols used in the Timeline analysis.

- i. Interaction flowchart - The first row of the Timeline is dedicated to the flowchart of students-teacher interactions. The interactions, represented by arrows, can be of three kinds: reactive (diagonally back arrow), proactive (diagonally forward arrow), and reactive and proactive (both diagonally back and forward arrow). The following symbology is adopted concerning multimodal interactions: gestures are denoted as blue arrows when addressed to someone or an artifact. A different symbol is used for those gestures performed for intrapersonal purposes. Relevant gazes are denoted with dotted blue arrows. Black arrows are used for oral interactions; dashed arrows indicate questions, and double arrows underline revoiced utterances. Eventually, orange arrows denote an interaction of the students or the teacher with the different artifacts at stake.
- ii. Utterances - This section collects the oral utterances. It is divided into two rows devoted to the teacher and the students.
- iii. Gestures - Gestures are analysed according to the taxonomy of McNeill (1992). Hence, four dimensions of gestures are identified: deictic, iconic, metaphoric, and beats. The main function of gestures, narrative, interactive, or grounding, is reported within blue boxes. The features and functions of relevant gazes, sensing or signaling, are noted within blue dotted boxes.
- iv. Inscriptions - This row shows the inscriptions, sketches, and writings produced by the teacher or students on the blackboard, the interactive whiteboard, or worksheets.
- v. Artifacts - Episodes of conflict ( $\neq$ ) or synergy ( $\S$ ) between artifacts are highlighted. The involved artifacts are denoted with a capital letter.
- vi. Layers of meaning - According to the phenomenological perspective, in this row, the disclosed layers of meaning emerging in the analysed episode are outlined.

**Table 1.** Symbology used for the Timeline analysis

Section of the Timeline	Symbol	Meaning	Comments
<b>INTERACTION FLOWCHART</b>		Sentences	Short arrows denote utterances not addressed to a specific person.
		Questions	
		Revoiced utterances	
		Gestures addressed to someone or something	Short arrows denote gestures addressed to the whole classroom
		Gestures for intrapersonal discourse	
		Gazes	
	 X	Interaction with artifact X	

<b>GESTURES</b>		Nature and function of gestures	Gestures are classified as: deictic, iconic, metaphoric, or beats. Gestures may assume three different functions: narrative, interactive, or grounding.
		Nature and function of gazes	Some relevant gazes are considered, underlining their features and functions (sensing or signaling).
<b>ARTIFACTS</b>	$X \neq Y$	Conflict between artifacts X and Y	
	$X \S Y$	Synergy between artifacts X and Y	

This elaborated Timeline allows us to point out in great detail:

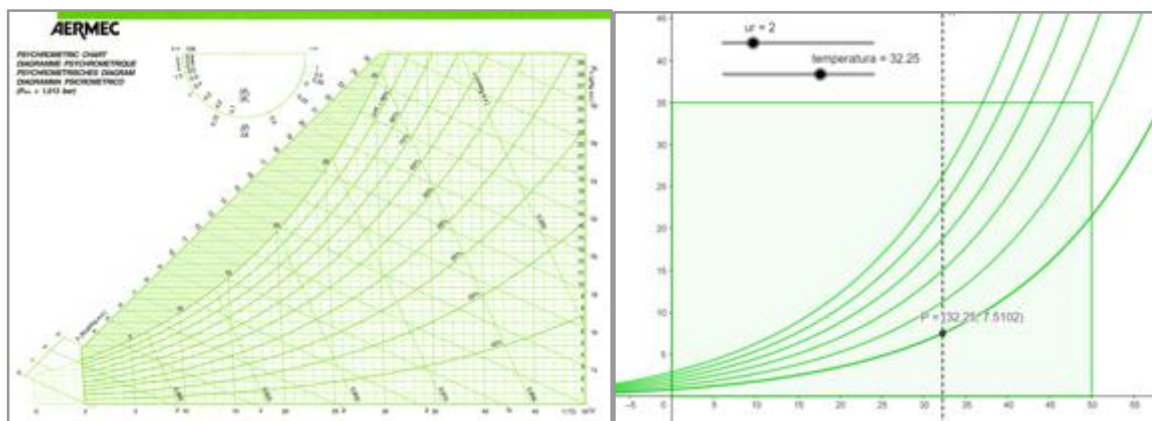
1. how the complex interactions between the learners or the teacher and the students, suitably represented in the interaction flowchart, intertwine and enhance the exploration and learning of mathematical concepts;
2. how artifacts interactions characterize the students' exploration of the mathematical concepts within the communicational environment of the classroom discussion and their evolution over time;
3. how the multimodal interactions contribute to the students' learning process and the disclosure of new layers of meaning.

### The Context of the Learning Experiments

The two learning experiments discussed in this paper were designed to engage students in covariational reasoning (Thompson & Carlson, 2017). The first experiment discusses the temperature-humidity relation, and the second experiment discusses the distance-time relation of a cube rolling along an inclined plane. These learning experiments were conducted using several artifacts and digital tools. In the first one, a GeoGebra applet reproducing a simplified psychrometric chart, the chart describing the relationship between temperature, absolute humidity, and relative humidity, was used to identify the dew point of the thermodynamic conditions of condensation. In the second experiment instead, an augmented reality (AR) device displayed the discrete distance-time graph associated with the distance that the cube travelled. The diversity of the artifacts involved is a reasoned choice to show how the findings and the analysis method are not limited to specific tools. Moreover, an added value lies in the different educational systems to which the students belong, i.e., the Italian and Israeli educational system.

*Learning experiment 1* – This learning experiment was held in an 11<sup>th</sup>-grade classroom of 21 students in a scientific-oriented school in Italy. It was conducted at the beginning of the school year 2020/2021 in a mixed modality due to the Covid pandemic. This experimentation had the primary purpose of elaborating mathematically on the relationship between temperature and humidity adopting multiple representations

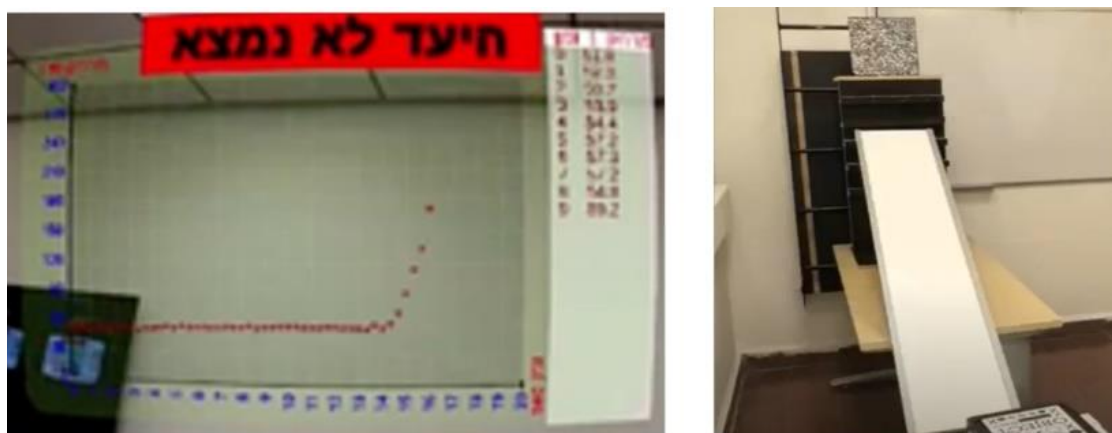
and various digital supports, e.g., GeoGebra applets. One of the first activities consisted of a classroom experiment: in a metal pot with some water at room temperature, a student gradually added ice until droplets of water appeared on the outside of the pot. At regular intervals of time, the students recorded the time spent and the temperature of the water in the pot itself and noted the temperature when the dew drops appeared (the dew point). During a working group session, students worked on understanding an actual psychrometric chart (Figure 2a), the diagram showing the exponential-like relation between temperature and absolute humidity in which each curve corresponds to a different percentage of relative humidity. Then, the teacher provided students with a slightly simplified psychrometric diagram created with GeoGebra (Figure 2b), where, above all the parameters of possible interest, the focus gets narrower on the temperature (horizontal axis), the absolute humidity (vertical axis), and the relative humidity (the parameter). The applet contains two sliders: one is associated with relative humidity, and moving it, it generates the curve corresponding to that specific value of humidity; the second slider is associated with temperature, and it enables one to move along the same curve. The first episode analysed in the Results and Discussion section is an excerpt from the 1-hour discussion in presence led by the teacher after the working group session on the psychrometric chart.



**Figure 2.** a) An actual psychrometric chart; b) A GeoGebra applet reproducing the chart with points P and Q

*Learning experiments 2* – This learning experiment was conducted with three 11<sup>th</sup>-grade Israeli students as they were observing a cube moving on an inclined plane through augmented reality headsets. The students Noam (N), Sagey (S), and Alex (A) worked on the conceptualization of the distance-time relation of the cube. The AR headsets display a distance-time graph and a table of values (Figure 3a). The distance represented in the graph and the table of values is that of a falling cube with respect to a fixed reference cube located on the top of the plane (Figure 3b). The students can interact with the physical model by releasing the cube and changing the inclined plane's angle.





**Figure 3.** a) Distance-time graph and a table of values shown through the AR headset; b) Real inclined plane with the fixed reference cube located on the top

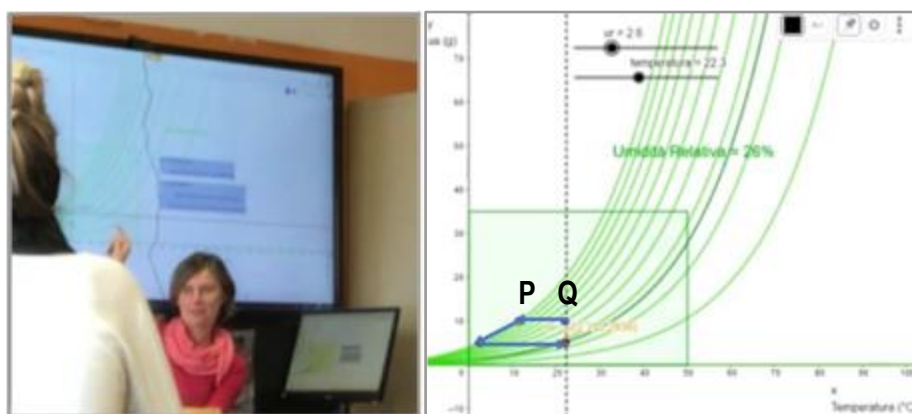
### Data Collection and Analysis

The data present in this paper is a small part of an ongoing research project. In the learning experiments, all the classroom activities were video recorded and then transcribed for a deeper qualitative analysis. The two episodes analysed in this paper were chosen because they address relations between the several semiotic components involved in the mathematical learning process and the subjects involved in the mathematical activities. The Timeline tool enabled us to perform both a synchronic and a diachronic analysis that highlighted the relationships between the students-teachers-artifacts interactions and the emerging layers of meaning.

## RESULTS AND DISCUSSION

### Episode 1: The Psychrometric Chart

In this episode, it is illustrated the use of the Timeline tool to analyse the students' meaning-making when they interpret the psychrometric chart. Special attention is given to the interactions between the teacher and the students. The teacher (T) guides the students to retrace the steps of the previously performed pot experiment (E) on the psychrometric diagram created in the GeoGebra environment (G) (Figure 4a).



**Figure 4.** a) The teacher pointing at the chart shown on the interactive whiteboard; b) Steps of the pot experiment reproduced on the chart in GeoGebra

The applet reproducing the psychrometric chart is shown on the interactive whiteboard (IW): students have already identified point P representing the point on the saturation curve (100% of relative humidity) in which the temperature coincides with the temperature of the dew point. Point Q instead has the same ordinate as P and, as abscissa, the ambient temperature (Figure 4b - arrows are added for the reader's convenience). In this episode, students elaborate on the above horizontal trait and the curved one.

- 1 T: On the graph, how can I read these passages? We said this is the starting point because we said we do not go from P to Q, but we start from Q. Starting from Q, where did we go?
- 2 G: We decreased the temperature hence we moved to the left.
- 3 T: We decreased the temperature hence we moved to the left. In which way? Did you just decrease the temperature or not? We are during the moment in which you continued to pour and pour [the ice].
- 4 E: Only the temperature decreases.
- 5 T: Only the temperature decreases. And so, on the graph, how do you move?
- 6 E: Horizontally.
- 7 T: Horizontally. We have point Q and we move horizontally to decrease the temperature. Until when do we move horizontally?
- 8 E: Until the dew point.
- 9 T: Until the dew point, that is until when we find on which of these green curves?
- 10 E: Until that of 100%.
- 11 T: [...] And then? What did we do after we reached the saturation of 100%? Did we stop immediately? [...]
- 12 G: No, we waited until it condensed well, and, in the meanwhile, we continued to add ice.
- 13 T: So, what did you do?
- 14 G: I continued to decrease the temperature.
- 15 T: Hence on the graph, where do you move?
- 16 G: To the left.
- 17 T: To the left. Horizontally?
- 18 G: No... [not really convinced]. If you have reached the dew point, yes... only the temperature changes.
- [...]
- 19 E: If we already have the dew point, humidity is decreasing.
- 20 T: If we already have the dew point, humidity is decreasing. And so?
- 21 V: It tends toward the x-axis
- 22 T: Not only the temperature decreases, and it tends toward the y-axis but also toward the x-axis. In which way do we move on this graph?
- 23 V: Following the curve.

Although the students do not directly interact with the pot experiment because they conducted it in a previous lesson, it strongly affects their meaning-making processes. Indeed, the first layer of meaning relates the real phenomenon to the mathematical model presented in the GeoGebra psychrometric diagram shown on the IW (L1). This layer of meaning can be recognized at [3] ("Did you just decrease the temperature or not? We are during the moment in which you continued to pour and pour [the ice]") and at [11] ("What did we do after we reached the saturation of 100%? Did we stop immediately?"). Synergic use of the experiment and the GeoGebra applet (artifacts row - Figure 5 and Figure 6) supports

this layer of meaning. Lines [3] and [11] show that the teacher herself often initiates L1. When she observes that the students are not able to progress on their own, even if they have reached a higher layer of meaning, the teacher brings them back to the real experiment (“you continued to pour and pour [the ice]” [3]; “Did we stop immediately?” [11]). When drawing attention to the experiment, she also introduces some iconic gestures with a narrative function that recall the physical actions performed during the experiment ([3], gesture row – [Figure 5](#)); this behavior is also reflected in students’ gesturing, such as at [12] when Giorgia reproduces the pouring of the ice (gesture row – [Figure 7](#)).

The second layer of meaning emerges when the students disclose the variation of a specific quantity (L2). This happens in [2,14,18] when they disclose the variation of the temperature: “We decreased the temperature hence we moved to the left” [2], a statement accompanied by the metaphoric gesture performed by Giorgia with her index to remark the direction of the movement (gesture row – [Figure 5](#)) “I continued to decrease the temperature” [14], “If you have reached the dew point, yes... only the temperature changes” [18]. Giorgia mainly interacts with the teacher in a reactive way, answering her questions (interaction flowchart – [Figure 5](#) and [Figure 7](#)).

The third layer of meaning emerged when the students disclosed the relationship between two quantities and more (L3). The shift to the third layer of meaning happens when Emanuele associates the dew point and temperature (lines for [7] to [10]). [Figure 8](#) shows that the students also engaged in the third layer of meaning when they associated the dew point with humidity ([19] to [23]). In the third layer, students figure out that the movement on the chart is not horizontal anymore but is along the green curve ([21] and [23]). The passage from L2 to L3 seems to be particularly triggered by the teacher who, through her questioning (dashed arrows in the interaction flowchart), invites students to interact with the GeoGebra applet. This kind of initiation can be observed in the interaction flowchart (orange arrows). Moreover, the teacher sometimes introduces gestures with a grounding function ([1] and [7] - gesture row – [Figure 5](#) and [Figure 6](#)) to reinforce students’ interaction with the diagram.

		[1] 00:39:54	[2] 00:40:25	[3] 00:40:28	[4] 00:40:46	[5] 00:40:50
		Teacher	Giorgia	Teacher	Emanuele	Teacher
INTERACTION FLOWCHART	T					
	G					
	E					
	V					
	Artifact					
UTTERANCES	Teacher	On the graph, how can I read these passages? We said this is the starting point because we said we do not go from P to Q but we start from Q. Starting from Q, where did we go?		We decreased the temperature hence we moved to the left. In which way? Did you just decrease the temperature or not? We are during the moment in which you continued to pour and pour [the ice].		Only the temperature decreases. And so, on the graph, how do you move?
	Students		We decreased the temperature hence we moved to the left.		Only the temperature decreases.	
GESTURES	Teacher	T reproduces with a finger the cycle on the graph and then points at point Q on the IW.  metaphoric gesture, narrative function pointing gesture, grounding function		Hand gesture reproducing the horizontal movement and then a gesture reproducing the pouring of ice into the pot.  metaphoric gesture, narrative function iconic gesture, narrative function		
	Students		Finger gesture of Giorgia denoting the movement toward the left.  metaphoric gesture, narrative function			
INSCRIPTIONS		Applet shown on the IW. 				
ARTIFACTS				E § G		
LAYERS OF MEANING			L2	L1	L1	T triggers the passage to L2

Figure 5. Timeline of episode 1 (part I)

		[6] 00:40:55	[7] 00:40:56	[8] 00:41:11	[9] 00:41:13	[10] 00:41:24	[11] 00:41:39
		Emanuele	Teacher	Emanuele	Teacher	Emanuele	Teacher
INTERACTION FLOWCHART	T						
	G						
UTTERANCES	Teacher		Horizontally. We have point Q and we move horizontally to decrease the temperature. Until when do we move horizontally?		Until the dew point that is until when we find on which of these green curves?		[...] And then? What did we do after we reached the saturation of 100%? Did we stop immediately? [...]
	Students	Horizontally.		Until the dew point.		Until that of 100%.	
GESTURES	Teacher		T moves her finger horizontally on the graph from Q to P.  metaphoric and pointing gesture, grounding function		Hand gesture reproducing the trend of the green curves.  iconic gesture, narrative function		 gaze, signaling function
	Students						
INSCRIPTIONS			Applet shown on the IW. 		Applet shown on the IW. 		
ARTIFACTS							E § G
LAYERS OF MEANING		L2	T triggers the passage to L3	L3	L3	L3	L1

Figure 6. Timeline of episode 1 (part II)




		[12] 00:42:05	[13] 00:42:13	[14] 00:42:14	[15] 00:42:18	[16] 00:42:19	[17] 00:42:20	[18] 00:42:31
		Giorgia	Teacher	Giorgia	Teacher	Giorgia	Teacher	Giorgia
INTERACTION FLOWCHART	T							
	G							
	E							
	V							
	Artifact				G	G	G	
UTTERANCES	Teacher		So, what did you do?		You continued to decrease the temperature. Hence on the graph, where do you move?		To the left. [...] Horizontally?	
	Students	No, we waited until it condensed well, and, in the meanwhile, we continued to add ice.		I continued to decrease the temperature.		To the left...		No... [not really convinced]. If you have reached the dew point yes... only the temperature changes.
GESTURES	Teacher							
	Students		Hand gesture simulating the pouring of the water-ice mixture.  iconic gesture, narrative function					
INSCRIPTIONS								
ARTIFACTS		Reference to E						
LAYERS OF MEANING		L1		L2	L2	L2	T pushes towards L3	L2

Figure 7. Timeline of episode 1 (part III)

		[19] 00:42:50	[20] 00:42:56	[21] 00:43:03	[22] 00:43:05	[23] 00:43:23
		Emanuele	Teacher	Valeria	Teacher	Valeria
INTERACTION FLOWCHART	T					
	G					
	E					
	V					
	Artifact					
UTTERANCES	Teacher		If we already have the dew point, humidity is decreasing. And so?		Not only the temperature decreases, and it tends toward the y-axis but also toward the x-axis. In which way do we move on this graph?	
	Students	If we already have the dew point, humidity is decreasing.		It tends toward the x-axis...		Following the curve.
GESTURES	Teacher				Hand gesture reproducing the trend of the movement.  metaphoric gesture, narrative function	
	Students					Hand gesture reproducing the trend of the curve.  metaphoric gesture, narrative function
INSCRIPTIONS					Applet shown on the IW. 	
ARTIFACTS						
LAYERS OF MEANING		L3	L3	L3	L3	L3

Figure 8. Timeline of episode 1 (part IV)

### Episode 2: Augmented Reality

This episode is an excerpt from a session in an augmented reality laboratory during which three students, Noam (N), Sagey (S), and Alex (A), worked on the conceptualization of the distance-time relation of a

cube rolling along an inclined plane. With the AR device, students can visualize a discrete distance-time graph while trying to conceptualize that relation. In this episode, three main artifacts are considered: (1) the virtual discrete distance-time graph (G) shown by the AR headset, (2) the TV in which headsets are mirrored (T) displaying the distance-time graph and the numerical table of values, and (3) the real inclined plane with the two cubes (P).

- 24 N: As if it was already our initial conjecture... but what Sagey said about the distance...
- 25 S: Here...Look at the graph. Here, you see that from one point to another point, the distance increases.
- 26 N: Yes.
- 27 S: As the slope increases, the distance increases.
- 28 N: That indicates that it accelerated. Am I right?
- 29 S: What does that indicate?
- 30 N: It is acceleration
- 31 S: Is it possible? [laughing]...Mmm.
- 32 N: Because, as if... when the cubes... the points are further from each other, they move away as if the distance between them is greater and the ratio of time is even closer...say, in less time it simply... It seems to me because it's a drop. (Removing the AR device). Are you listening to me? Or looking at them?
- 33 A: We are looking at the graph.
- 34 S: Yes.
- 35 N: It seems to me because it was...mmm...because it's going downhill, that is an inclined plane, so... as time went on, it (the cube) also travelled a greater distance.
- 36 A: Yes.
- 37 N: Because it really accelerated, also because of the distance of the points. Are you looking at yours [headset]? So...is it correct that the last two points are more distant than the first two ones, for example, when it [graph] begins to ascend?
- 38 S: [With AR headset] What a strange thing! From second 8 to second 9, the distance in second 8 is 54 and in second 9 is 89, compared to the first second, which is 51.8... the first second it's 52, so the distance increases as time...mmm...
- 39 N: As I said!
- 40 S: Yes.

In this episode, students are working on conceptualizing the relationship between time and distance traversed by the cube sliding along the inclined plane. Students have already performed the real experiment, and now they are focusing on the graph displayed on the AR headset and mirrored on the TV screen. While Alex does not intervene much in the discussion (see only [33] and [36]), Noam and Sagey deeply engage in conceptualizing the situation proposed, but their approach is entirely different. Noam, who has already grasped the graph's meaning, looks for Sagey's confirmation to progress in her learning process ([32] and [37]). Sagey has not conceptualized the situation and instead of listening to Noam, he looks at the graph on the TV screen, searching for answers to his doubts, as the sensing gazes at [28] and [32] manifest (gesture row – [Figure 9](#) and [Figure 10](#)). These different approaches can be visually recognized by looking at the interaction flowchart: the direction of the arrows reveals her proactive attitude of Noam towards Sagey. Noam's efforts to explain to Sagey what she has already cleared in her mind go empty because Sagey is not listening to her but searching for answers on his own. Noam

perceives not being heard [32], and many times throughout the dialogue, she remarks that Sagey is saying things that she already outlined before [39]. The different interactions emerging in the learning environment correspond to different layers of meaning disclosed by Noam and Sagey in different moments. Sagey initially discloses the variation of the cube's distance (L2), and only towards the end of the episode also the relationship between distance and time (L3) due to the table of numerical values shown on the TV screen. Noam instead succeeds first in disclosing the distance-time relationship (L3) as in [32], and sometimes she incorrectly condenses this relationship in the term acceleration [28, 30]. Noam desires to interact with Sagey, which is also manifested by signaling gazes at [32] and [37] (gesture row – Figure 10 and Figure 12). To share her understanding of the situation with him, Noam first refers to the real phenomenon, i.e., the cube and the inclined plane [35], using it in a synergic approach with the graph shown on the TV screen (artifacts row – Figure 11). Then to support her argument about acceleration, she addresses the distance of the points (L2) accompanying her utterances with metaphoric gestures with a narrative function ([37], gesture row – Figure 12). Even if Sagey cannot see them because he is wearing the headset, these help Noam express her thoughts.

		[24] 00:18:55	[25] 00:19:06	[26] 00:19:10	[27] 00:19:14	[28] 00:19:19
		Noam	Sagey	Noam	Sagey	Noam
INTERACTION FLOWCHART	N					
	S					
	A					
	Artifact		G	G	G	T
UTTERANCES		As if it was our initial conjecture...but what Sagey said about the distance...	Here... Look at the graph. Here, you see that from one point to another point, the distance increases.	Yes.	As the slope increases, the distance increases.	That indicates that it accelerated. Am I right?
GESTURES			Sagey looks at Noam while speaking. 			Sagey looks at the TV 
INSCRIPTIONS			Noam headset 	Noam headset 	Noam headset 	TV screen 
ARTIFACTS			Reference to G		Reference to G	
LAYERS OF MEANING			L2	L2	L2	L3

Figure 9. Timeline of episode 2 (part I)

		[29] 00:19:30	[30] 00:19:31	[31] 00:19:36	[32] 00:19:43
		Sagey	Noam	Sagey	Noam
INTERACTION FLOWCHART	N				
	S				
	A				
	Artifact				
UTTERANCES		What does that indicate?	That it is acceleration.	Is it possible? [laughing]... Mmm.	Because, as if... when the cubes... the points are further from each other, they move away as if the distance between them is greater and the ratio of time is even closer... say, in less time it simply... it seems to me because it's a drop. [Removing the AR device]. Are you listening to me? Or looking at them?
GESTURES		Sagey looks at Alex  gaze, signaling function	 metaphoric gesture, narrative function	Noam's fingers gesture reproducing distance between the cubes. Sagey is looking at TV and eventually, after removing the AR headset, Noam looks at Sagey.  gaze, sensing function	 gaze, signaling function
INSCRIPTIONS				  Noam headset and the TV screen	
ARTIFACTS				Reference to G and P	
LAYERS OF MEANING		L3	L3		L3

Figure 10. Timeline of episode 2 (part II)



		[33] 00:20:03	[34] 00:20:05	[35] 00:20:06	[36] 00:20:22
		Alex	Sagey	Noam	Alex
INTERACTION FLOWCHART	N				
	S				
	A				
	Artifact				
UTTERANCES		We are looking at the graph.	Yes.	It seems to me because it was... Mmm... because it's going downhill, that is inclined plane, so... as time went on, it [the cube] also travelled greater distance.	Yes.
GESTURES		Alex looks at the graph on TV. 		Noam looks at the graph on TV screen. 	
INSCRIPTIONS					
ARTIFACTS		Reference to the graph shown on TV screen		P § T	
LAYERS OF MEANING				L1	

Figure 11. Timeline of episode 2 (part III)

		[37] 00:20:23	[38] 00:20:34	[39] 00:21:00	[40] 00:21:02
		Noam	Sagey	Noam	Sagey
INTERACTION FLOWCHART	N				
	S				
	A				
	Artifact	G	G		
UTTERANCES	Because it really accelerated, also because of the distance of the points. Are you looking at yours [headset]? So... is it correct that the last two points are more distant than the first two ones, for example, when it [graph] begins to ascend?	[With AR device] What a strange thing! From second 8 to second 9, the distance in second 8 is 54 and in second 9 is 89, compared to the first second, which is 51.8... the first second it's 52, so the distance increases as time... mmm...	As I said!	Yes.	
GESTURES	Noam looks at Sagey while speaking. Her finger gesture denotes distance between points.  metaphoric gesture, narrative function gaze, signaling function				
INSCRIPTIONS	 Sagey headset	Sagey focuses on the table of numerical values in his headset 			
ARTIFACTS	Reference to G	Reference to the table of numerical values on TV screen			
LAYERS OF MEANING	L2	L3			

Figure 12. Timeline of episode 2 (part IV)

This study examines the relationship between students-teacher-artifacts interactions and the process of students' meaning-making of mathematical concepts. The multimodality perspective of learning, which guided this study, considers the interaction between subjects in a broad sense, including verbal and non-verbal interactions and interactions with artifacts at stake. Hence, an analysis tool is designed to pursue the research aim. This analytical tool, which is the main contribution of this paper, enables us to unravel possible relations between the layers of meaning disclosed by students in their learning processes and the multimodal interactions.

Analytical tools that address the multimodality of learning and interaction can be found in the literature. For example, the Timeline tool (Sabena et al., 2012) is a table collecting all the multimodal resources and activities interlacing in the mathematics classroom in several rows. The semiotic sets addressed in this Timeline are those outlined in the analytical model of the semiotic bundle, and this tool offers an overview of their relationships and evolution over time. Another tool, which seems helpful for this research, is the interaction flowchart (Sfard & Kieran, 2001), a diagram representing as arrows the oral interactions between subjects involved. As described in the Method section, some elaboration of this flowchart has been introduced in different studies, for example integrating the transcripts (Smedlund et al., 2018) or gazes between learners (Liljedahl & Andrà, 2014). Though these tools address multimodality and interaction, they do not treat the relationship between interaction, multimodality, and meaning-making processes. This reflection led us to produce a further elaboration of these existing tools.

The new version of the Timeline, presented in detail in the Method section, has three points of strengths. First, the elaborated flowchart enables one to see all the multimodal interactions at once: communicative channels are created through oral communications and gestures and gazes, and interactions with artifacts strongly affect the learning process. This new flowchart reflects the assumption that learning is a multimodal activity. Second, the integration of the flowchart with the original Timeline tool, whose rows describe the semiotic components, allows the integration of the interactions already revealed by the flowchart with some details about them, i.e., their nature and function. Eventually, the integration of the flowchart, the Timeline, and a specific row for layers of meaning reveals connections between the multimodal interactions and the emerging layers in the meaning-making process.

The elaborated Timeline has suitable analytical features to answer the research question. This section contains two examples of how this tool works and how it is useful for this investigation. The two selected episodes, considering the common topic of enhancing covariational reasoning in mathematical modeling, were mainly chosen for the diversity of features they address. While episode 1 consists of an excerpt of a classroom discussion in which the role of the teacher proves essential, episode 2 is focused only on interactions between students, and the mathematical expert was not active. Moreover, the artifacts involved are different: in episode 1, the class works with a GeoGebra applet, while in episode 2 students are engaged with augmented reality. A common aspect is the influence of experiments reproducing the real phenomenon. In the first case, it is the pot experiment conducted in a previous lesson and so not physically present in the classroom, in the second case it is the inclined plane with the cubes present in the laboratory. In the episode analysed in this paper, the students do not directly interact with it. Students disclosed three layers of meaning in the two episodes, revealing an increase in mastery of the mathematical concepts: L1) relating the real phenomenon to the mathematical model; L2) variation of a specific quantity, and L3) covariational relationships between quantities. As the Timelines reveal, the layers are neither hierarchical nor ordered. They are intertwined throughout the episodes and may be disclosed by the students at different moments.

The main result of episode 1 highlights that the passage from L1 to L2 is mainly triggered by a reference to physical actions related to the pot experiments, and the teacher often initiates it. The passage from L2 to L3 is supported by the interactions with the GeoGebra applet and triggered by the teacher's questions. Drawing students' attention to relevant artifacts, the pot experiment or the GeoGebra applet in this study, is a strategy that fosters responsive guidance of students (Swidan et al., 2022). This paper argues that this strategy enables students to disclose how quantities vary or covary. Episode 2 presents an entirely different scenario in which the learners are three students and it shows how two of them, one remains in the background, disclosed layers of meaning with different timings and interactions, as the

flowchart reveals. While Noam has already reached L3 and desires to interact with Sagey searching for confirmation, Sagey still struggles to disclose how distance and time are covarying. The interactions between Sagey and Noam are not successful for the meaning-making process because the two students disclose different layers of meaning at different times and draw their attention to the information provided by various artifacts struggling to interpret them coherently.

Some other comments on the semiotic means involved in the meaning-making process follow. In episode 1, the teacher and the students introduce several iconic and metaphoric gestures (McNeill, 1992) with a narrative function that support them in the description of the movements to be performed on the psychometric chart or to recall the actions of the pot experiment. Moreover, the teacher adopts some pointing gestures with a grounding function (Roth, 2001) to specifically address the applet shown on the interactive whiteboard. In episode 2, instead, the gazes are those assuming greater relevance. Sagey's sensing gazes address the graph the TV screen suggests and express his struggle in looking for his answers; Noam's signaling gazes (Cañigüeral & Hamilton, 2019) instead clearly manifest her desire to interact and confront Sagey. Both the excerpts offer examples of a synergic use of the artifacts at stake which contributes to a progression in the disclosure of various layers of meaning. All these details provide complementary information which proves to be necessary for a full understanding of the learning experiments.

## CONCLUSION

As mentioned in the beginning of this paper, the notion of meaning-making has several meanings in the mathematics education literature. The model here introduced adopts the phenomenological perspective of meaning-making, which considers meaning-making as a progressive disclosure of mathematical ideas (Rota, 1991). Of course, Rota's perspective is not the only approach for meaning-making in mathematics education, so the row in the Timeline intended for meaning-making can be replaced with any other approach to meaning-making mentioned in Seeger (2011). Changing the meaning-making perspective may result in a further direction of research examining the validity of the proposed tools. All in all, this paper provides evidence that the suggested model works in two different examples. However, more research is still needed to examine the stability of the proposed model for understanding the relationship between interactions and meaning-making.

## Declarations

- Author Contribution : SB: Conceptualization, Introduction, Theoretical Framework, Methodology, Data analysis, Discussion, Writing - Original Draft;  
OS: Conceptualization, Introduction, Theoretical framework, Methodology, Data analysis, Discussion, Writing - Review & Editing;  
FA: Conceptualization, Introduction, Validation and Supervision, Writing - Review.
- Funding Statement : The AR study in this paper was supported by the Israel Science Foundation (Grants No. 1089/18).
- Conflict of Interest : The authors declare no conflict of interest.



## REFERENCES

- Arzarello, F. (2006). Semiosis as a multimodal process. *Revista Latinoamericana de Investigación en Matemática Educativa RELIME*, 9(Extraordinario 1), 267–299. Retrieved from <https://revistas.ucr.ac.cr/index.php/cifem/article/view/39888/40429>
- Arzarello, F. (2008). Mathematical landscapes and their inhabitants: Perceptions, languages, theories. In: Niss, M. (Editor), *Proceedings of 10th International Congress on Mathematical Education*, Plenary lecture, 158–181.
- Arzarello, F., Ascari, M., Baldovino, C., & Sabena, C. (2011). The teacher's activity under a phenomenological lens. In: U. Behiye, *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*. Vol. 2, pp. 49–56.
- Arzarello, F., & Robutti, O. (2010). Framing the embodied mind approach within a multimodal paradigm. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), *Handbook of international research in mathematics education* (pp. 730-759).
- Arzarello, F., Bazzini, L., Politano, L., & Sabena, C. (2010). Multimodal processes in teaching and learning mathematics: A case study in primary school. In G. Pérez-Bustamante, K. Phusavat, F. Ferreira (Eds.) *Proceedings of the IASK International Conference* (pp. 286–292).
- Arzarello, F., & Edwards, L. (2005). Gesture and the construction of mathematical meaning. In H. Chick & J. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 123-154). Retrieved from <https://www.emis.de/proceedings/PME29/PME29ResearchForums/PME29RFArzarelloEdwards.pdf>
- Bagossi, S. (2022). Second-order covariation: an analysis of students' reasonings and teacher's interventions when modelling real phenomena. *Unpublished Ph.D. Thesis*. University of Modena and Reggio Emilia in agreement with the University of Ferrara and the University of Parma, Italy.
- Cañigüeral, R., & Hamilton, A. F. de C. (2019). The role of eye gaze during natural social interactions in typical and autistic people, *Frontiers in Psychology*, 10, 560. <https://doi.org/10.3389/fpsyg.2019.00560>
- Drijvers, P., Ball, L., Barzel, B., Heid, M.K., Cao, Y., & Maschietto, M. (2016). *Uses of Technology in Lower Secondary Mathematics Education: A Concise Topical Survey*. Springer Open. <https://doi.org/10.1007/978-3-319-33666-4>
- Faggiano, E., Montone, A., & Mariotti, M. A. (2018). Synergy between manipulative and digital artefacts: a teaching experiment on axial symmetry at primary school. *International Journal of Mathematical Education in Science and Technology*, 49(8), 1165–1180. <https://doi.org/10.1080/0020739X.2018.1449908>
- Gallese, V., & Lakoff, G. (2005). The brain's concepts: The role of the sensory-motor system in conceptual knowledge. *Cognitive Neuropsychology*, 22(3-4), 455–479. <https://doi.org/10.1080/02643290442000310>
- Javorski, B., & Potari, D. (2009). Bridging the macro- and micro-divide: Using an activity Theory model to capture sociocultural complexity in mathematics teaching and its development. *Educational Studies in Mathematics*, 72(2), 219–236. <https://doi.org/10.1007/s10649-009-9190-4>



- Liljedahl, P., & Andrà, C. (2014). Students' gazes: new insights into student interactions. In: *Views and beliefs in mathematics education - contributions of the 19th MAVI conference*, 213–226. Ed: Springer.
- Loncke, F. T., Campbell, J., England, A. M., & Haley, T. (2006). Multimodality: A basis for augmentative and alternative communication—psycholinguistic, cognitive, and clinical/educational aspects. *Disability and Rehabilitation*, 28(3), 169–174. <https://doi.org/10.1080/09638280500384168>
- McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. University of Chicago press.
- Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities. *For the Learning of Mathematics*, 30(2), 2–7. <https://doi.org/10.2307/20749442>
- Radford, L. (2011). Book Review: Classroom Interaction: Why is it Good, Really? Baruch Schwarz, Tommy Dreyfus and Rina Hershkowitz (Eds.) (2009) Transformation of knowledge through classroom interaction. *Educational Studies in Mathematics*, 76(1), 101–115. <https://doi.org/10.1007/s10649-010-9271-4>
- Radford, L., Schubring, G., & Seeger, F. (2011). Signifying and meaning-making in mathematics thinking, teaching and learning: Semiotic perspectives. *Educational Studies in Mathematics*, 77(2-3). 149–156. <https://doi.org/10.1007/s10649-011-9322-5>
- Rota, G. C. (1991). The End of Objectivity: The Legacy of Phenomenology. Lectures at MIT.
- Roth, W.-M. (2001). Gestures: Their role in teaching and learning. *Review of Educational Research*, 71, 365–392. <https://doi.org/10.3102%2F00346543071003365>
- Sabena, C., Robutti, O., Ferrara, F., & Arzarello, F. (2012). The development of a semiotic framework to analyze teaching and learning processes: Examples in pre- and post-algebraic contexts, *Recherches en Didactique des Mathématiques, Enseignement de l'algèbre élémentaire*, Numéro spécial, 237–251.
- Saxe, G.B., Gearhart, M., Shaughnessy, M., Earnest, D., Cremer, S., Sitabkhan, Y., Platas, L., & Young, A. (2009). A methodological framework and empirical techniques for studying the travel of ideas in classroom communities. In B. Schwarz, T. Dreyfus, & R. Hershkowitz (Eds.) *Transformation of Knowledge Through Classroom Interaction*. Routledge: London, 2009; 211–230.
- Schwarz, B., Dreyfus, T., & Hershkowitz, R. (Eds.) (2009). *Transformation of knowledge through classroom interaction*. London: Routledge.
- Seeger, F. (2011). On meaning making in mathematics education: Social, emotional, semiotic. *Educational Studies in Mathematics*, 77(2-3), 207–226. <https://doi.org/10.1007/s10649-010-9279-9>
- Sfard, A., & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture and Activity*, 8(1), 42–76. [https://doi.org/10.1207/S15327884MCA0801\\_04](https://doi.org/10.1207/S15327884MCA0801_04)
- Smedlund, J., Hemmi, K., & Røj-Lindberg, A. S. (2018). Structuring students' mathematical conversations with flowcharts and intention analysis—affordances and constraints. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.). *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 203-210). Umeå, Sweden: PME.

- 
- Swidan, O. (2022). Meaning making through collective argumentation: The role of students' argumentative discourse in their exploration of the graphic relationship between a function and its anti-derivative, *Teaching Mathematics and its Applications: An International Journal of the IMA*. <https://doi.org/10.1093/teamat/hrab034>
- Swidan, O., Sabena, C., & Arzarello, F. (2020). Disclosure of mathematical relationships with a digital tool: A three layer-model of meaning. *Educational Studies in Mathematics*, 103(1), 83–101. <https://doi.org/10.1007/s10649-019-09926-2>
- Swidan, O., Bagossi, S., Beltramo, S., & Arzarello, F. (2022). Adaptive instruction strategies to foster covariational reasoning in a digitally rich environment. *The Journal of Mathematical Behavior*, 66, 100961. <https://doi.org/10.1016/j.jmathb.2022.100961>
- Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for Research in Mathematics Education*, Reston, VA: National Council of Teachers of Mathematics, 421–456.
- Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, 9(4), 625–636. <https://doi.org/10.3758/BF03196322>

