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The philosophical coming of age of science. Euler's role in Cassirer's early philosophy of space and time

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ABSTRACT

Cassirer's early philosophy of space and time, overshadowed by his later work on relativity, has been scarcely explored in the literature. This paper aims to bridge this gap. It argues that understanding Cassirer's point of view requires acknowledging the pivotal role he attributed to the work of Leonhard Euler in the philosophical 'coming of age' of modern science. Against the Leibniz-Berkeley *philosophical* plea for the relativity of all motion, Euler objected that if Newton's absolute space and time did not exist, the principle of inertia would become meaningless and with it a *scientific* theory of motion. According to Cassirer, Kant took a step beyond Euler by shifting the focus from the *existence* of space and time as 'things' to their *function* as necessary 'conditions' of the possibility of mechanics. In the nineteenth century, it became clear that Newton's absolute space and time entail more structure than necessary. Nevertheless, according to Cassirer, the Euler-Kant insight still holds: a geometric structure serving as an inertial structure is the *condicio sine qua non* of a coherent theory of motion, including general relativity. This paper concludes that Cassirer came close to defending a sort of 'inertial functionalism' dressed in neo-Kantian garb.

Introduction

Cassirer's philosophy of space and time has usually been discussed through the lens of his writings on Einstein's relativity theory published in the 1920s (Cassirer, 1921; see Ryckman, 2005, Chap. 2). However, to my knowledge, it has not been noted that Cassirer's interest in this topic goes back at least two decades earlier (Cassirer to Natorp, Nov. 26,1901; Cassirer, 1995, Vol. 18, Doc. 43). This oversight likely stems from the fact that Cassirer's early remarks on the topic are somewhat 'buried' beneath the sheer mass of his extensive historical (Cassirer, 1906a, 1907) and theoretical work (Cassirer, 1910) from the late 1900s, which had a much broader scope. However, while Cassirer's early contributions to the history and philosophy of physical spacetime theories might go unnoticed on cursory reading, more careful scrutiny unveils an original and profound insight that not only sheds new light on his subsequent related work, but, to some extent, is more compelling than the latter.

This paper contends that to appreciate Cassirer's viewpoint, it is useful to dwell on the pivotal role he assigned to the work of Leonhard Euler in a lengthy section of *Das Erkenntnisproblem* (Cassirer, 1907, Chap. 7.2.II.2b). Euler's *scientific* contributions to the history of mechanics were well known during Cassirer's time, although they were often treated as mere 'footnotes' to Newton's work (Mach, 1883; see Truesdell, 1968, Chap. 2). Cassirer deserves recognition for having emphasized, I believe for the first time, the *philosophical* impact of Euler's work on eighteenth-century thought. In particular, Cassirer interpreted Euler's reflections on space and time as the philosophical 'coming of age' (*philosophische Mündigkeitserklärung*) of the new mathematical science of nature (Cassirer, 1907, 477). By defending Newton's concept of absolute space and time against the 'relativistic' objections of Leibniz and Berkeley, Euler established science over philosophy as the touchstone of objective 'knowledge'. Philosophy does not have authority over the concepts created by mathematical physics; instead, it must recognize these concepts as the fundamental 'fact' that serves as the starting point for its own investigations (Cassirer, 1919, 26f.). Thus, Cassirer goes so far as to present Euler's stance toward philosophy as the embryonic form of what would later evolve into Kant's 'transcendental method'—that is, for a neo-Kantian philosopher, the 'method of philosophy' *tout court*: the analysis of scientific knowledge as an objective historical 'fact' to search for the conditions of its possibility (Cohen, 1885, 66–79).

To counter Berkeley's and Leibniz's philosophical defense of the principle of relativity of all motions, Euler argued that the *existence* of Newton's absolute space and absolute time is proved by the fact that, without them, the principle of inertia and consequently the entire system of scientific mechanics would become meaningless. In Cassirer's reading, Kant's philosophy of space and time can be regarded

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as an attempt to translate Euler's *ontological* concern into a purely *methodological* one. If Euler asked whether space and time *exist* in themselves, Kant shifted the question to the *function*[1] that they play within a viable theory of motion. As Cassirer put it with one of his favorite puns, Kant transformed space and time from 'things' (*Dingen*) that exist into conditions (*Bedingungen*) necessary for the possibility of Newtonian mechanics. It is because they play this *function* that they should be regarded as *a priori*. Cassirer showed that in the nineteenth century, especially with the work of Ludwig Lange (Cassirer, 1910, Chap. 10.VI), it became clear that Newton's absolute space and time entailed more structure than was strictly required by Newton's theory. However, according to Cassirer, the Euler-Kant insight that a certain objective geometrical structure is required for the formulation of a coherent theory of motion could not be questioned even by the theory of relativity.

Cassirer was not an 'analytic philosopher'—a technical 'philosopher of physics' in modern terms. Such a professional figure, paradigmatically embodied by Hans Reichenbach, was only starting to emerge around that time. Cassirer, like most neo-Kantian philosophers of that generation, was a 'synthetic philosopher'[2], whose reflections on physics were ultimately supposed to be part of a larger project aimed at investigating other cultural forms, such as religion, myth, art, etc. (Cassirer, 1923–1929). However, with the wisdom of hindsight (after Stein, 1967), Cassirer's historical-critical analysis of the evolution of the scientific theory of motion seems to be more 'modern' than Reichenbach's logical-mathematical analysis[3]. In particular, contrary to Reichenbach (1924), Cassirer sees quite lucidly that Einstein was the heir of Newton and Euler rather than Leibniz and Berkeley.

Cassirer seems to have realized that no theory of motion, including general relativity, is possible without an 'inertial structure' that provides a standard of non-acceleration. For Cassirer, the choice of a spatiotemporal structure sufficient for this purpose is neither empirical nor conventional; it is ultimately determined by the *function* it plays within the mathematical theory of motion as a whole. In this sense, this paper concludes that Cassirer appears to have come close to defending a sort of 'inertial functionalism' (Baker, 2020) dressed in neo-Kantian garb[4]. Following his student at Yale, Arthur Pap (1943), one might attribute to Cassirer a 'functional-pragmatic interpretation' of the *a priori*[5]. The *a priori* is necessary in the hypothetical sense of the expression: as the history of spacetime theories shows, an 'inertial structure' is necessary *if* a coherent dynamic theory of motion is to be possible. *That* this is the case is a contingent 'historical fact' in which philosophy has no say.

1. Cassirer on the reception of Newton's work in the eighteenth century

In the 'Preface' to his first 1902 book on Leibniz, Cassirer announced that he was working on a more ambitious project on the 'prehistory of criticism' in the mathematical science of nature of the eighteenth century (IX). The aim was to present Kant's philosophy as the outcome of an historical process shaped by figures like Galileo, Leibniz, Descartes, and Newton (Ferrari, 2015). Cassirer appeared to have already concluded the parts on space and time that he initially planned to publish in a journal (Cassirer to Natorp, Nov. 26, 1901; see also Cassirer, 1904, 1:108–111). However, the project quickly expanded beyond his initial expectations. It was only by 1905 that Cassirer could announce the completion of the first volume of *Das Erkenntnisproblem* (Cassirer, 1906a), which covered the period from the Renaissance to Descartes (Cassirer to Natorp, Jul. 31,1905; Cassirer, 1995, Vol. 18, Doc. 70). A second volume, covering the period from Bacon to Kant, was already in preparation and was published in 1906. By that time, Cassirer had obtained the *venia legendi* at the University of Berlin, with a *Probevorlesung*, "Substanzbegriff und Funktionsbegriff", which outlined the systematic framework of his historical scholarship (see Ferrari, 1988).

Cassirer's *Das Erkenntnisproblem* aimed to illustrate the intricate and sometimes conflicting interplay between the history of the 'philosophical problem of knowledge' and the history of the 'mathematical science of nature'. In Cassirer's view, this dynamic initially appeared "to have reached a secure conclusion in Newtonian science" (Cassirer, 1907, 401). In fact, for his followers, Newton was not primarily the discoverer of the 'law of gravitation', but above all the founder of a new method of research, the 'method of induction': "His work represents for them a philosophical act, insofar as in it the inductive method not only achieved its highest results, but also reached its first logical articulation and establishment" (Cassirer, 1907, 401). The Newtonians elevated 'induction' to the sole source of physical certainty, advocating for a 'physics without hypotheses'.

To be sure, Cassirer pointed out, Newton aimed not to abolish metaphysics but to carefully demarcate it from the mathematical science of nature. However, "[o]nly at one point does Newton depart from this critical restraint: His scientific doctrine of space and time" (Cassirer, 1907, 442). Newton conceded that, from the perspective of pure observation, all knowledge of spatial and temporal determinations dissolves entirely into *relations*. The question of the existence of space and time in themselves, beyond these observable relationships of bodies, seems forbidden. And yet, Cassirer continued, Newton's *Scholium* to the Definitions of space, time, and motion in the *Principia* famously placed precisely these concepts at the forefront and continuously asserted their necessity for the foundation of mathematical physics (Cassirer, 1907, 463f.). How does Newton justify his claim that freely moving bodies travel in straight lines relative to absolute space, even though he concedes if absolute space is unobservable? How is Newton certain that the oscillations of a perfect pendulum occur at consistent intervals in relation to absolute time, if the flow of time is unobservable? If our experiences are confined to relative spaces and times, and if experience sets the limits of our knowledge, then we should not be allowed to postulate the existence of absolute space and time (Cassirer, 1907, 463f.).

It seems that an obscure metaphysical concept, borrowed from Henry More's speculative Platonism (Cassirer, 1907, 442–446)[6], is embedded in the foundations of mathematical physics itself: "But with that, the power of pure induction, as Newton understood and proclaimed it, would already be broken" (Cassirer, 1907, 464). Among the methodological rules Newton presents for research, the first demands that, in explaining phenomena, one is not allowed to resort to any other than 'true causes' that can be directly observed. "The existence of absolute space and absolute time, however, is not a '*vera causa*' in this sense" (Cassirer, 1907, 465). It is, as Newton himself emphasizes, not given directly by experience, since all our empirical observations are restricted solely to relative spaces and times. However, according to Newton, not only does mechanics require the concept of absolute

[1] Cassirer's use of the term 'function' is somewhat ambiguous. (a) In most cases, he uses 'function' in a mathematical sense, as when he famously contrasts 'function-concepts' with 'substance-concepts' (Cassirer, 1910). (b) However, on several occasions, he uses 'function' to refer to the 'role' that a particular concept plays within a specific theoretical structure. On this point, see Heis (2014, 253). In the following, I refer exclusively to meaning (b)

[2] For this distinction, see Schliesser (2019)

[3] Cassirer attributes to Euler a stance that modern commentators tend to ascribe directly to Newton (see DiSalle, 2006).

[4] Baker (2020) refers to Knox's (2013) space–time functionalism, according to which what counts as 'spatiotemporal structure' is determined by its *function* in defining the 'inertial structure' presupposed by the laws of motion. See Cassirer's use of the term 'function' in fn. 1.

[5] See Stump, 2020.

[6] Cassirer's source is possibly Lange (1886a, §4 and 47f.).

motion, but it also enables us, in empirical cases, to provide criteria to recognize it and indirectly demonstrate its effects. As is well known, in the case of rotational motion, the appearance of centrifugal forces seems to be completely independent of whether the body rotates with respect to bodies immediately surrounding it. Therefore, centrifugal forces provide us with a reliable indicator that allows us to distinguish between 'apparent' rotation with respect to other observable bodies and 'true' rotation with respect to an unobservable absolute space (Cassirer, 1904, 1:110).

Ultimately, Newton could not avoid postulating the existence of an unobservable entity to explain observable phenomena. According to Cassirer, "[i]n this conflict lies the internal crisis of the Newtonian theory of experience" (Cassirer, 1907, 465). 'Physicists' began to question this assumption, albeit cautiously at first due to the overwhelming authority of Newton. However, 'philosophers' such as Berkeley and Leibniz engaged in a frontal attack. Although starting from very different premises, both contested Newton's doctrine of the existence of absolute space and time:

Leibniz. Against Newton's reification and metaphysical hypostatization of space, Leibniz famously emphasized that space and time are "ideal orders of appearances" (Cassirer, 1907, 187). Due to the homogeneity of Euclidean space, absolute differences in absolute place are not real differences (Cassirer, 1902, 246–250). If a body is displaced, there is no way to distinguish between the original and the displaced situations without a reference body that does not participate in the displacement. Indeed, if we were to move the entire universe in an otherwise empty space—as Leibniz's celebrated argument in the correspondence with Clarke goes[7]—this motion would change nothing. It cannot, in principle, be observed. Since only the relative positions of bodies have empirical significance, Leibniz concluded that, in principle, all states of motion are equivalent (equivalence of hypotheses; see Cassirer, 1904, 1:109). If Clarke objected here that the fact of change is not conditioned by its observation, then Leibniz responded by introducing his *principe de l'observabilité*: When there is no *possible* observable change, there is no change at all (Cassirer, 1902, 249).

Berkeley. If Leibniz raised 'logical' objections against Newton's absolute space, Berkeley (1721), according to Cassirer, sought to attack the latter using the tools of 'psychological' criticism. His critique of the doctrine of space is rooted "in polemics against abstract concepts" (Cassirer, 1907, 465). We are not allowed to isolate a simple property observed in specific circumstances and treat it as an independent content (Cassirer, 1907, 465f.). Even the supreme laws of mechanics, such as the principle of inertia, "cannot contain in themselves any moment that is not indirectly or directly rooted in experience and can be substantiated in it" (Cassirer, 1907, 467). The affirmation that every body left to itself persists in its state of rest or of uniform rectilinear movement "loses none of its value if, instead of referring the continuation of the movement of the body to absolute space, we commensurate it with the position of the body with respect to the sky of the fixed stars" (Cassirer, 1907, 467).

Leibniz and Berkeley seem then to achieve the same *ontological* conclusion: "The rejection of the absolute reality of space and time" (Cassirer, 1907, 467). However, they started from opposite *methodological* premises: "The 'abstraction,' which for Berkeley is the source of error, represents for Leibniz the foundation of all rational and scientific understanding" (Cassirer, 1907, 469). Berkeley's sensualist theory excludes absolute space and absolute time because both are not given in *actual* observation, the sole warrant of reality; for Leibniz's rationalist approach, given the perfect uniformity of mathematical space and time, differences in absolute positions in an empty uniform space cannot be the object of *possible* observation.

[7] Cassirer just edited the German translation, Leibniz, 1904, 120-241.

According to Cassirer, the objections raised by Leibniz and Berkeley against Newton's doctrine of space and time, precisely because of the disparity between their philosophical premises, defined the boundaries of the 18th-century debate (Cassirer, 1907, 470). Physicists, even those of Leibnizian persuasion, ultimately sided with Newton; philosophers of all schools tended to agree with his opponents (Cassirer, 1907, 470). In Cassirer's view, the struggle between Leibniz and Berkeley on the one hand, and Newton on the other reveals more than a disagreement on technical matters; it was the manifestation of a deeper tension between philosophy and science. For the philosophers, the principle of relativity of all motions was the inevitable conclusion of both the logical and psychological analysis (Cassirer, 1904, 1:110). However, the entire historical development of mechanics seemed to directly contradict such a premise. It turned out to be impossible to build a coherent theory of motion while maintaining a relativistic view (Cassirer, 1904, 1:110). The resolution of this divergence, Cassirer argues, could only occur when the real crux of the dispute was clearly identified: "It is Leonhard Euler who accomplishes this feat, thereby ushering the general problem into a new phase" (Cassirer, 1907, 472).

Cassirer notes that, in Euler's view, in disputing the Newtonian concepts of space and time, Berkeley seems to agree with Leibniz on all counts; "this apparent unity extends so far", that he "in his defense of the 'mathematician' Newton against the 'philosophers,' juxtaposes the two philosophical lines of thought indiscriminately" (Cassirer, 1902, 261). According to Cassirer, this was not simply the consequence of Euler's alleged philosophical naivety. On the contrary, Euler lucidly understood that the philosophical quarrel between 'rationalism' and 'empiricism' did not grasp the issue at stake (Cassirer, 1904, 1:112). Indeed, representatives of both schools, although starting from different premises, pursued the same strategy. They evaluated Newton's 'mathematical' theory of space and time against a purely 'philosophical' standard (*Maßstab*). However, according to Euler, the proper standard cannot be taken either from pure reason or from pure experience. It can only be derived "from the pure dynamic laws; they are the content with which space and time must be filled in order to become natural realities" (Cassirer, 1902, 461).

2. Cassirer on Euler's defense of Newton's absolute space and time

According to Cassirer, Euler's first comprehensive work on mechanics, published in 1736, already posited the decisive problem without being able to offer a solution. 'Motion', as it appears in the initial naive observations, simply presents itself as a process of change in the 'relative place'—a finite portion of space with respect to which we decide on the state of motion or rest of bodies. However, 'place' as such can only be determined as 'absolute place'—a part of the boundless or infinite space in which the material world is contained (Euler, 1736, Vol. 1, Chap. 1, Sect. 8, p. 14.). However, Cassirer noted that Euler realized that the logical and ontological *status* of this boundless and infinite space was hard to grasp. On the one hand, Euler asserts that pure mathematical space is nothing more than an 'idea'; it does not concern us whether it really exists (Cassirer, 1907, 407). On the other hand, it becomes clear that we cannot fully grasp this 'idea' with everything that the senses and imagination present to us (Euler, 1736, Vol. 1, Chap. 1, Sect. 77, p. 32). According to Cassirer, Euler seems to be at his wits' end: "If we consider absolute space in light of the ordinary metaphysical fundamental distinction between physical or psychic being, we immediately see it placed in an untenable middle position: The sphere of the 'subject' as well as that of the 'object' seems to exclude it in equal measure" (Cassirer, 1907, 474).

Euler addressed the issue head-on only over a decade later in his 'Réflexions sur l'espace et le tems', presented to the Berlin Academy in 1748 (Euler, 1750). According to Cassirer, for the first time, Euler makes it clear that neither logical-psychological nor ontological-metaphysical considerations are adequate to solve this problem: "The touchstone [...] can lie nowhere else than in the principles of *scientific*

mechanics and in the *laws of motion* that mechanics places at the forefront. These laws are so firmly established and of such incontrovertible certainty that they must serve as the sole foundation for all our judgments about the world of bodies: And they maintain this value, regardless of whether it is possible to derive them from allegedly higher principles of metaphysics or not" (Cassirer, 1907, 475). Regarding the nature of space and time, we can only gain a firm guideline when we consider these concepts not in isolation but rather in the "relational context [*Verhältnisstellung*] and connection they enter into with each other in the principle of inertia [*Prinzip der Beharrung*]" (Cassirer, 1907, 475), whose truth is removed from all philosophical disputes of the schools.

The various factions, both rationalists and empiricists, did not contest the content of the principle of inertia; instead, they sought to explain it through their respective *philosophical* concepts of space, time and motion (Cassirer, 1907, 476). Cassirer suggests that for Euler this was the crux of the issue: our philosophical concepts should be evaluated against the principle of inertia, the only legitimate *scientific* standard: "The decisive question is not what space and time *are* [*sind*] in and of themselves, but rather what they are *used* [*gebraucht*] for in the enunciation and formulation of the law of inertia" (Cassirer, 1907, 476). If the consideration of relative positions and relative motions suffices to make the content of the principle of inertia understandable, then we could forgo the notions of absolute space and time; however, if it turns out conversely that the principle of inertia is meaningful only if we admit an absolute space and an absolute time, then the necessity of these concepts is established (Cassirer, 1907, 476). The objection that we are illegitimately hypostasizing our own 'ideas' becomes moot. Indeed, Euler (1750, 382) could simply declare as absurd the notion that a pure fiction of our imagination could serve as the basis for the principles of mechanics and account for their success (Cassirer, 1907, 476). According to Cassirer, in this manner, Euler introduced "a new path" (Cassirer, 1907, 476) between rationalism and empiricism, putting the entire debate on the nature of space and time on a new footing:

> The true nature of space and time is not revealed to us by immediate sensory observation—but also the psychological analysis of ideas cannot lead us to the goal. The essence of both is rather to be determined solely by the *function* they fulfill in the system of mathematical physics. The set of the mechanical principles, since it constitutes the prerequisite for all exact explanations of phenomena, forms the *Archimedean* point of our knowledge. Vague speculative endeavors are met here with a solid fact that cannot be pushed aside or interpreted away. Euler's doctrine is the philosophical declaration of maturity [*Mündigkeitserklärung*] of the new mathematical science, which henceforth undertakes to establish from within itself the true standard of 'objectivity,' instead of allowing it to be imposed by any foreign interest. Philosophy, as now sharply and unequivocally stated, should not master experience, but merely understand it and lay bare its foundations (Cassirer, 1907, 476f.)

The principle of inertia states that a body, as long as no external forces act upon it, remains in a state of rest or uniform rectilinear motion. But rest or uniform motion—relative to what? Perhaps to the fixed stars, Euler suggests, possibly alluding to Berkeley (Cassirer, 1907, 479). However, we cannot exclude the possibility, or even the likelihood, that the fixed stars are only apparently at rest with respect to one another. Ultimately, the principle of inertia is not strictly valid for any empirically given reference body; yet this law must have absolute truth, because without it the entire science of dynamics would collapse, including Newton's laws of motion and the law of gravitation. Anyone who acknowledges these laws must also acknowledge the reference systems for which they are valid; and no empirically given body suffices, but only the 'absolute space' (Cassirer, 1907, 479).

According to Cassirer, Euler's line of argument implicitly reshaped the relations between philosophy and the exact sciences. If our 'philosophical' concepts prove insufficient to grasp the content provided by physical science, then the inadequacy lies solely with those concepts. Therefore, we must refine them until they become fully compatible with the secure content of physical science: "In this view, Euler only fixes the general ideal that had constantly hovered over the exact research of the time" (Cassirer, 1907, 477). By upholding this general criterion, the objections of Leibniz and Berkeley can be seen in a new light:

- The *rationalists* who, in the name of principle of relativity of all motions, deny the concepts of absolute space and absolute time, must simply try to use their own definitions to see to what extent they are able to construct with them a coherent theory of motion compatible with the principle of inertia: "However, one only needs to attempt this experiment once to be immediately convinced of its impracticability" (Cassirer, 1907, 478). A fully 'relativistic' theory of motion lacks the notion of constancy of direction over time without which principle of inertia is meaningless.
- A reference system must be tacitly presupposed when we attribute uniform velocity and direction to a body moving on its own. However, no reference system can meet the *empiricist*'s demand, as no observed body is entirely at rest (Cassirer, 1907, 479). Any attempt to present the principle of inertia as an empirical proposition describing the relations among individual bodies, like moving bodies and fixed stars, would threaten the status of the principle of inertia as a universal principle: "Indeed, we are not dealing with the establishment of a single fact in it, but with an ideal norm by which we judge all natural phenomena" (Cassirer, 1907, 479)

Thus, according to Cassirer, Euler always returns to the same point: the very possibility of mechanics requires that we postulate the existence of absolute space and absolute time: "Both concepts possess undeniable reality" not because they are validated solely by pure experience or pure reason, "but because – a fact that carries greater importance – they are indispensable to the entirety of our scientific understanding of the world" (Cassirer, 1907, 479). The principle of inertia, as a universal principle, is meaningful only if one postulates the existence of absolute space and time.

According to Euler, 'philosophers' – whether empiricists like Berkeley or rationalists, like the representatives of the Leibniz-Wolffian School – have misunderstood this point. They treat the concepts of space and time as "mere abstracts, thereby denying them their true concrete [*gegenständlichen*] content" (Cassirer, 1907, 480). However, their conclusion is ultimately rooted in the ambiguous nature of the very notion of 'abstraction'. Of course, an intellectual effort is required to elevate oneself to the idea of pure mathematical space and time. Yet, the method of 'reflection' involved in this endeavor is fundamentally different from how we typically form general concepts of genus and species. (Cassirer, 1907, 480). The concept of 'place' cannot be obtained in the same way as, for example, the concept of 'yellow' as the property common to lemons, daffodils, canaries, etc. In fact, the 'place' where a thing is located is not a 'property' that belongs to the thing alongside its other characteristics (Cassirer, 1907, 480). In contrast, the idea of 'place' results when one thinks of the body as a whole as removed, so that place cannot have been a 'property' of the body (Cassirer, 1907, 480)[8].

However, as Cassirer pointed out, Euler now faces a fundamental philosophical difficulty. If one wishes to obtain the correct 'psychological' correlate for the space of mathematical physics, then we must "insert a new intermediate link between perception and concept, between concrete sensation and abstract thought, as interpreted by the

[8] See Euler (1750, §XV).

school tradition in logic" (Cassirer, 1907, 481). The conventional separation proves inadequate when compared to the concepts of exact science: the analysis of objective scientific knowledge urges a reorganization of our *philosophical* categories. Euler could not find the conceptual tools apt to grasp the nature *sui generis* of space and time. He could simply conclude that, since pure space and pure time are necessary for the formulation of the laws of motion, they must exist as separate things: "It is Euler's self-evident and unquestioned assumption that for the objective significance of the principles, a substantive correlate in absolute being is to be assumed and demanded" (Cassirer, 1907, 481).

Nevertheless, Cassirer continued, Euler himself seems to have been uncomfortable about the status of this presupposition. Indeed, according to Cassirer, Euler's last comprehensive presentation of mechanics, the *Theoria motus* from the year 1765, shows that his 'philosophical' scruples could not be permanently appeased. After nearly two decades, Euler returned to the problem anew and presents both opposing views, the relativistic and absolutist, side by side without initially taking stance (Cassirer, 1907, 484). In the first part of the book, motion's relativity is shown as the inevitable conclusion when considering motion solely through direct sensory perception. However, in the second part of the book Euler insists again that this conclusion is clearly incompatible with "the highest principles of mechanics" (Cassirer, 1907, 484). Euler's presentation is not simply inconsistent, as some interpreters contended (Streintz, 1883, 45). Rather, it places us in front of the fundamental paradox that what cannot be an object of experience, absolute space and time, is required for the construction of a theory of motion compatible with experience: "The 'abstraction,' which was just rejected, must therefore be reinstated and restored to its rights" (Cassirer, 1907, 484).

In Cassirer's reading of this conundrum, one observes "[t]he motive that dominates Euler's entire intellectual development" (Cassirer, 1907, 484). To make sense of scientific experience, he is continually compelled to move from *sensory* perception back to pure *conceptual* constructions like absolute space and absolute time that have no direct empirical counterpart. However, once he is forced to admit the *existence* of such unobservable entities, "the boundary between exact science and metaphysics seems once again abolished" (Cassirer, 1907, 484). The concepts of absolute space and absolute time cannot be mere fictions of our imagination: They are implicitly contained in Newton's laws of motion, whose success is undeniable. However, their particular manner of objective reality cannot be compared to that which we ascribe to physical bodies; indeed, we assume that they continue to subsist even if the latter are moved elsewhere or even completely removed. Euler's analysis of notions of space and time deals a fatal blow to both traditional rationalist and empiricist theories of 'concept formation': "[t]he 'categories' [*Klassen*] of metaphysics had to be first broken in order to grant the new reality – as required by exact science and its laws – its rightful place within the overall system of knowledge" (Cassirer, 1907, 485). Here, the destructive aspect of Euler's argument is taken to its logical conclusion. However, according to Cassirer, Euler ultimately failed to articulate a constructive proposal, namely, to provide a new set of conceptual tools capable of expressing the peculiar status of space and time.

As the reader might already anticipate, in Cassirer's reading, it was Kant who was able to provide the *pars construens*, or at least to indicate a way toward it. In his 1768 treatise on space, Kant explicitly aims to take one step further back along Euler's line of argumentation (AA 2:378): Not only does the 'fact of mechanics', but already the 'fact of geometry' requires space to exist independently of relations among specific bodies (Cassirer, 1907, 689)[9]. Once again, however, one faces the paradox that space is something that is neither a body nor a property or relation of bodies; yet, it is difficult to determine in what sense it 'exists' as an unobservable empty container. In Cassirer's interpretation, Kant identified the key to resolving the paradox during the critical period. The problem of the *ontological status* that space and time possess with respect to material bodies is transformed into that of the *methodological function* that they play within geometry and mechanics. Space and time are empirically real – not in the sense that they exist as independent objects – but because they are *a priori* conditions[10] of the possibility for all our objectively valid empirical knowledge of the disposition and motion of bodies : "To pure space and pure time belongs the objectivity of the condition [*Bedingung*], while the objectivity of the thing [*Ding*] remains precluded" (Cassirer, 1907, 700f.).

3. Cassirer and the problem of the 'reference frame'

Early on, Cassirer planned to integrate the first two volumes of *Das Erkenntnisproblem* with the third, a systematic volume (Cassirer to Natorp, Jul. 31,1905; Cassirer, 1995, Vol. 18, Doc. 70), that ultimately became his monograph *Substanzbegriff und Funktionsbegriff* in 1910. The sections of *Das Erkenntnisproblem* that describe the historical evolution of the 18th-century debate about the nature of space and time from Newton to Euler (Cassirer, 1907, Chap. 7.2) have their theoretical counterpart in the section of *Substanzbegriff und Funktionsbegriff* dedicated to the late 19th-century German-speaking debate on the meaning of the principle of inertia from Neumann to Lange (Cassirer, 1910, chap 4.VI). Although many readers today might be aware of the latter debate (see DiSalle, 1988), it is worth noting that, as far as I can see, Cassirer was the first and possibly the only philosopher to have attempted to provide an overview and a philosophical appreciation of it.

As Cassirer pointed out, in the 18th century, the controversy about the nature of space and time focused exclusively on the contrast between 'absolute' and 'relative' spatial and temporal determinations. However, no physicist, Cassirer argued, has ever taken that the concept of 'absolute' space would literally exclude consideration of any reference frame whatsoever: "The conflict is only with regard to the kind of system of reference, only with regard to whether it is to be taken as material or as immaterial, as empirically given or as an ideal construction" (Cassirer, 1910, 229; tr. 1923, 172f.). However, it was only in the late nineteenth century that the 'problem of absolute space' explicitly became the 'problem of the reference-system': relative to what system of reference is the motion of a free body rectilinear; relative to what time-scale is it uniform?

In Cassirer's reconstruction, Carl Neumann (1870) and Ernst Mach (1872, 47–50) were the first to reformulate the issue in these terms, setting the agenda for the debate. As Neumann pointed out, the principle of inertia is indeterminate without an explicit statement of the reference body or reference system with respect to which it is applicable: For instance, a trajectory that appears straight when observed from Earth may appear curved when viewed from the perspective of the Sun. Since no observable body can be found that serves the purpose, Neumann introduced an unobservable one, which he famously labeled the 'Alpha Body'. The content of the principle of inertia then becomes that a material point left to itself moves in a straight line with respect to the Alpha Body (Cassirer, 1910, 238–240; tr. 1923, 180–181). Mach agreed with Neumann's assessment of the problem; however, he denied the need to postulate the existence of an unobservable reference body. Sensible experience itself unambiguously imposes a suitable reference

[9] Kant's 'incongruent counterparts' are, so to speak, the geometrical equivalent of Euler's principle of inertia.

[10] Cassirer emphasized the Eulerian roots of Kant's Metaphysical Exposition of space (B37–40): in particular, the claim that space is not an abstract genus concept. However, space is also not a concrete individual thing that can be given in sensible intuition. In Cassirer's reading, Kant's notion of 'pure intuition' was meant to provide the philosophical 'class' that Euler was missing. See also, Natorp (1910, 273)

body on us. Without mentioning Berkeley, Mach suggests that when we observe a body in inertial motion, we find that it moves linearly and uniformly with respect to the fixed stars. This empirically observable fact is all that we truly know from experience about the behavior of moving bodies (Cassirer, 1910, 230–234; tr. 1923, 173–177).

According to Cassirer, Neumann's and Mach's seemingly opposing proposals ultimately share the same shortcoming: the principle of inertia has meaning only with reference to an *individual* existing body, whether it is unobservable or observable. However, the reference to individual bodies in the formulation of the principle of inertia deprives it of the status of a fundamental principle[11]. According to Cassirer, it was Neumann's former student, the Austrian physicist Heinrich Streintz (1883), who first attempted to address this issue. Streintz substituted the single 'body of reference' with respect to which the principle of inertia holds with a *class* of bodies of reference that satisfy certain empirical criteria: to perform no rotational motion and to be subjected to no external force. Streintz designates a body thus characterized as a 'Fundamental Body' (*Fundamental-Körper*) (*FK*). For Streintz, the principle of inertia was nothing more than an inductive generalization about how bodies happen to behave relative to the *FK*'s. However, Cassirer objected that Streintz's proposal is ultimately based on a vicious circle. It is only because the we assume the validity of the principle of inertia that we chose rotation- and force-free bodies as *FK*'s.

For Cassirer, the early phase of the debate certainly had the merit of shifting the focus from the problem of absolute space to that of the 'reference frame'. However, in Cassirer's view, these early attempts to 'materialize the reference frame' were ultimately based on an epistemological fallacy: "It is not the *existence*" of a certain material frame (the Alpha Body, the fixed stars, the *FK*'s, etc.), "but the *assumption* of this existence, on which the validity of our mechanical concepts depends" (Cassirer, 1910, 240; tr. 1923, 181). Whether or not this is the case does not add or subtract anything from the abstract formulation of the principle of inertia. In Cassirer's historical account, it was Ludwig Lange (1885, 1886b) who brought this point to light by attempting to address the problem of the reference frame without 'existential hypotheses' (*Existenzhypothese*) (Lange, 1886b, 275).

As is well-known, Lange calls a system in which the principle of inertia holds an 'inertial system' (*Inertialsystem*): a reference system in which at least three points chosen *by convention* project non-collinearly from a point and move in a straight line. In this way, as already suggested by Neumann (1870, 16f.), we can also obtain an inertial measure for time, an 'inertial time scale' (*Inertialzeitskala*): the inertial unit of time is the time in which a free mass point that we choose *by convention* travels a certain distance in the inertial reference system. If the principle of inertia has to hold, then it must always be possible to construct a system in which arbitrarily many ($n > 3$) free particles move uniformly (Lange, 1886b, 273f.). In this way, Lange stripped the Newton–Euler absolute space and absolute time of their superfluous elements. Newton and Euler implicitly held that, while inertial frames are empirically indistinguishable, they are not theoretically equivalent. They move with various, although unknown, uniform velocities relative to 'absolute space'. However, this assumption is not necessary for the construction of Newtonian mechanics.

In place of one absolute space and one absolute time, Lange showed that it is sufficient to introduce an equivalent class of possible inertial systems (moving at constant velocity relative to each other) with a global time scale. The problem of the *existence* of two metaphysical entities is transformed into the *role* that two conceptual constructions play within the entirety of classical mechanics:

> Thus there is no hypostatization of absolute space and absolute time into transcendent *things*; but at the same time both remain as pure *functions*, by means of which an exact *knowledge* of empirical reality is possible. The fixity, that we must ascribe to the original and unitary system of reference, is not a sensuous but a logical property; it means that we have established it as a *concept*, in order to regard it as identical and unchanging through all the transformations of calculation. [...] Only experience can ultimately decide whether this schema is applicable to the reality of physical thing and processes. Here also it is never possible to isolate the fundamental hypotheses and to point them out as valid individually in concrete perceptions; but we can always only justify them indirectly in the total system of connection that they effect among phenomena. We develop the determination of the 'inertial system' and the mathematical consequences connected with it purely in theory. In so far as any empirically given body seems to conform to these determinations, [...] we affirm that a material point left to itself must move uniformly in a straight line with reference to that body (Cassirer, 1910, 242; tr. 1923, 182)

Whether indeed such an inertial system exists cannot be decided *a priori*. However, that a system of bodies can serve as an inertial system (for example, the system of the fixed stars) does not signify "a *fact* that can be directly established by perception or measurement, but means that a *paradigm* is found here in the world of bodies for certain principles of pure mechanics, in which they can be, as it were, visibly demonstrated and represented" (Cassirer, 1910, 242f.; tr. 1923, 183). For Cassirer, Lange's merit was to have clearly distinguished between the 'practical construction' of an *actual* reference system and the 'ideal construction' as the 'prototype' of all *possible* practical constructions (Lange, 1902, 36).

Physical reference frames are for Lange only more or less successful concrete exhibitions of an ideal model, the 'inertial system', whose possibility is *required* by the principle of inertia. As a matter of fact, we know that "this requirement is never exactly fulfilled in experience, but always only with a certain approximation" (Cassirer, 1910, 242; tr. 1923, 182). Just as there is no real straight line that fulfills all the properties of the pure geometric concept, there is no real body that corresponds in all respects to the mechanical definition of the inertial system. Therefore, the possibility always remains open to establish, "by the choice of a new point of reference, a closer and more exact agreement between the system of observations and the system of deductions" (Cassirer, 1910, 242; tr. 1923, 183). Galileo initially took an Earth-fixed reference system as an inertial system in a first approximation. However, a solar-centered astronomical reference system proved superior because the fixed stars exhibit negligible motion relative to it. Yet, this system might ultimately prove inadequate and so on. Similar considerations apply to the measurement of time: "This relativity is indeed unavoidable; for it lies in the very concept of the object of experience. It is the expression of the necessary difference that remains between the exact conceptual laws we formulate and their empirical realization" (Cassirer, 1910, 450; tr. 1923, 183).

Cassirer concedes that, when we base our statements about reality on free *ideal constructions* in this manner, it appears that a moment of 'arbitrariness' is allowed into our scientific consideration. In this sense, Lange characterizes "the concepts of the 'inertial system' and the 'inertial time-scale' [...] as mere *conventions*, which we introduce in order to survey the facts more easily, but which have no immediate objective correlate in empirical fact" (Cassirer, 1910, 247; tr. 1923, 187). Lange's choice of the term 'convention'[12] is understandable as the recognition that scientific thought does not merely behave 'receptively', but displays a peculiar 'spontaneity' (Cassirer, 1910, 248; tr. 1923, 187). Yet, Cassirer points out, "this self-activity [*Selbsttätigkeit*] is not unlimited

[11] Indeed, according to Cassirer, Mach consciously embraced this line of argument and, going well beyond Berkeley, argued that inertial motion is not simply *referred* to the fixed stars but *determined* (*bestimmt*) by them as causal factors (Mach, 1883, 217ff.).

[12] See DiSalle, 1990.

and unrestrained" (Cassirer, 1910, 248; tr. 1923, 187). Indeed, the choice of a convention considered in itself is indeed *contingent*; however, this contingency does mean arbitrariness. We prefer one definition over another, and not simply because of its 'convenience'; we favor one definition on account of its *necessity*, in achieving a certain 'goal': the notion of an inertial system is necessary for the very possibility of the principle of inertia, or better of the *leges motus* as a whole. The latter in turn are justified by the fact that serve as the basis for the formulation of a progressively more inclusive system of force laws. There is no external internal "criterion of 'objectivity'" beyond this process of convergence (Cassirer, 1910, 248; tr. 1923, 187).

With the transition from one absolute space to one class of equivalent inertial frames, Lange achieved a fundamental advancement over Euler. However, Cassirer suggests, that, by focusing on the problem of the 'reference frame', Lange unwittingly obscured Euler's fundamental point. The crucial issue is not the relation between Newton's laws of motion and the reference frame. Indeed, the main takeaway of Lange's analysis is that the validity of these laws *does not* depend on the choice of a particular inertial system. The essential concern is the relationship between Newton's laws of motion and the geometrical structure identifying inertial frames and trajectories that they presuppose[13]. The primary outcome of the nineteenth-century debate was to reveal that Newton's absolute space and time entail *more* structure than is strictly necessary for the possibility of Newtonian dynamics. However, a well-defined geometric framework is still needed for this purpose. In modern terms, the infinite collection of all inertial frames, along with their common time-scale, defines the three-plus-one chrono-geometric structure that underlies classical mechanics. For Cassirer, the entire 'ontological' dispute between relativists and substantialists concerning the 'existence' of this structure misses the fundamental 'methodological' point. What is relevant is the role that such a structure plays within the system of Newtonian dynamics.

In Cassirer's interpretation, the choice of this particular structure cannot be determined by a purely logical argument, nor is it merely a generalization from experience. However, it is also not a matter of arbitrary convention, as Lange's terminology misleadingly suggests. The choice is constrained by the *function* that such a structure plays within the system of classical mechanics as a *whole*. In Cassirer's neo-Kantian language, this geometrical structure can be termed *a priori* because it plays the role of a necessary condition for the possibility of Newton's laws of motion, which, in turn, provides the necessary constraints for possible empirical force laws. In this sense, Cassirer is not afraid to suggest a comparison between emerging American pragmatism and critical philosophy (Cassirer, 1910, 421–425; tr. 1923, 317–322). In both cases, "[t]he validity [*Geltung*] of a concept is determined by its performance [*Leistung*]" (Cassirer, 1913, 37). However, contrary to pragmatism, critical philosophy provides a definite theoretical standard against which this performance can be measured: the 'fact of science' or, more precisely, the 'fact of classical mechanics'.

Conclusion: The principle of inertia and the principle of observability

After more than a decade as a *Privatdozent* in Berlin, in June 1919 Cassirer finally received the so-called *Ruf* at the recently established University of Hamburg. On November 6 1919, he held the inaugural lecture on the relationship between philosophy and exact sciences. Relying on material he had analyzed in his *Das Erkenntnisproblem*, Cassirer once again credited Euler for having reshaped these relationships in the midst of the Age of Reason. With respect to the question of the nature of space and time, the mathematical science of nature is the only authority we can rely upon (Cassirer, 1919, 24–27). However, the trustworthiness of this authority was called into question a few days later. The London announcement of favorable results of Eddington's eclipse observation on November 8 sanctioned the replacement of Newton's theory of gravity by the theory of general relativity. The Archimedean point upon which critical philosophy relied, the 'fact' of Newtonian physics, no longer provides an absolutely firm foundation. Inevitably, Cassirer needed to take a stand (Giovanelli, 2023). A few months after assuming his Hamburg professorship, Cassirer completed a book on relativity (Cassirer, 1921) in the summer of 1920, and held a series of lectures on the topic in the following winter semester (Cassirer, 1920–1921; see also Cassirer, 1920).

Cassirer continued to insist on the importance of philosophy maintaining a close relationship with the sciences. However, the 'relativity revolution' was a clear warning that philosophers should be careful to take science as a source of new questions rather than ready-made answers. In fact, "modern science gives us the exact opposite answer to the question of the true nature of space and time than mathematical physics gave us about 100 to 150 years ago!" (Cassirer, 1920–1921, 95). For Euler, space and time possess "absolutely certain physical reality" since they were necessary for formulating the laws of motion (Cassirer, 1920–1921, 95). However, at first sight, this answer appears to have become inadequate. Indeed, Einstein could now declare that, after general relativity, "space and time have been robbed of even the last vestige of physical objectivity [*Gegenständlichkeit*]" (Cassirer, 1920–1921, 95). This transformation was, of course, not accidental but occurred under the pressure of experience. However, to this pressure there must correspond a counter-pressure from the side of critical philosophy, until a new state of equilibrium is provisionally found: "Here we have again a dialectic within empirical science itself – which must become the awakener, the Paraclete of critical thinking" (Cassirer, 1920–1921, 96).

In the writings from the 1920s, relying on the same material he had addressed a decade earlier, Cassirer presents the history of space-time theories as the interplay between the principle of inertia and the principle of observability (Cassirer, 1920–1921, 89–101). Leibniz embraced the principle of relativity of motion as a consequence of the principle of observability. From a purely kinematic point of view, any arbitrary motion is empirically indistinguishable from stasis. It was Euler who showed that it is impossible to maintain the privileged status of uniform rectilinear motion required by the principle of inertia while maintaining a relativistic view of motion. However, when Euler concluded the existence of absolute space and time, he once again violated the principle of observability, introducing theoretical differences between 'rectilinear uniform motion' and stasis which have no empirical counterpart[14]

As we have seen, in Cassirer's reconstruction, a first balance between the two principles was reached only toward the end of the nineteenth century, when Lange introduced the notion of "'inertial system' that satisfied both requirements, those of the inertia principle and those of the '*principe de l'observabilité*'" (Cassirer, 1921, 99; tr. 1923, 376). For Cassirer, special relativity—which he felt was premature to take into account in his 1910 book—fits quite well into this scheme. By abandoning the notion of absolute simultaneity, the theory extended the concept of 'inertial system' to account for the failure of 'ether-wind' experiments (Cassirer, 1920–1921, 101f.). Minkowski's (1909) four-dimensional formulation made fully perspicuous that special *relativity* actually makes a precise claim about the *absolute* geometrical structure of the 'world' (Cassirer, 1921, 121; tr. 1923, 445f.).

Cassirer realized that general relativityrepresented, so to speak, a more significant *rupture épistémologique*. Due to the "*physical identity* of

[13] See, fn. 4.

[14] It is interesting that Einstein criticized Cassirer's defense of Kant's doctrine of the ideality of space and time, on the basis of the fact that classical mechanics "demands an absolute (objective) space in order to assign real significance to acceleration, which Kant does not seem to have recognized" (Einstein to Cassirer, Jun. 5, 1920; Einstein, 1987, Vol. 18, Doc. 1044). However, for Cassirer 'ideal' is not the opposite of 'objective'

phenomena of inertia and weight" (Cassirer, 1921, 66; tr. 1923, 401), it becomes impossible to empirically distinguish free-fall frames from inertial frames (Cassirer, 1920–1921, 103). By appealing to the principle of observability, Einstein elevated to the status of "a *principle*" the requirement that "for the physical description of the processes of nature, no particular reference body is to be privileged above any other" (Cassirer, 1921, 44; tr. 1923, 383). Cassirer's formulation closely follows that of Einstein. However, precisely for this reason, it suffers from the same ambiguity. Indeed, as Hermann Weyl (1920) pointed out at around the same time, it is hard to see what a theory of motion satisfying this constraint could be like. If all coordinate systems are equally 'good', one could choose a coordinate system in which all time-like world-lines (four-dimensional trajectories of particles) become vertical straight lines, that is, they are all transformed at relative rest (Weyl, 1924b). Thus, in such a theory, not only the absolute motion of a single body, but even the concept of relative motion of multiple bodies becomes meaningless (Weyl, 1924a). If one takes the principle of relativity of arbitrary motion at face value, "no solution" to the problem of motion is possible (Weyl, 1927, 74).

Like many of his contemporaries (see Reichenbach, 1924), Cassirer did not initially fully grasp this point[15]. Only some decades later, he seems to have realized that also in the case of general relativity, one should "bring the law of inertia into agreement with the principle of observability" (Cassirer, 1936, 156; tr. 1956, 126). Indeed, despite Einstein's philosophical inclinations, the final version of general relativityentails the conception of a privileged state of motion of bodies not subject to forces, just like any previous theory. As Cassirer seems to recognize, although rather in passing, Einstein's theory did not implement a 'generalized principle of relativity', but formulated a "'generalized law of inertia' which allows to combine in one expression the phenomena of inertia and gravitation. For every motion of a point under the influence of inertia and gravity the principle now holds that its world-line is always a 'geodesic' in the spacetime continuum" (Cassirer, 1936, 63; tr. 1956, 50; translation modified). Einstein's theory did not achieve the relativity of arbitrary motion, but the relativity of the inertia/gravitation split (Janssen, 2014).

If Cassirer had expanded this point, he would come close to advocating a sort of neo-Kantian variant of 'inertial functionalism' (see Baker, 2020)[16]. Spacetime geometry must be evaluated because of its *function* within a coherent dynamic analysis of motion as a whole. Space–time must have *enough* structure to single out a preferred class of motions that define a standard of 'non-acceleration' (principle of inertia). However, this structure should not incorporate *redundant* features that lack empirical counterparts (principle of observability). As we now know, this formulation of the problem becomes fully clear only in four-dimensional formalism (Stein, 1967). One might say that Leibniz's spacetime has *not enough* structure to define inertial motion, since it cannot distinguish straight and curved time-like wordlines; Newton's spacetime has *too much structure*, since it singles out absolute rest, a vertical straight wordlinewith respect to others (Ehlers, 1973, 75). It was Lange who recognized that, in modern terms, what is needed is an 'affine' four-dimensional structure whose timelike straight worldlines represent free motions (Ehlers, 1973, 75). In general relativity, the affine structure was not abolished, but, because of the empirical indistinguishability between free and free-falling motion, it became a dynamical field incorporating the effects of both gravitation and inertia on the motion of bodies (DiSalle, 2006).

Cassirer conceded that, taken *logically*, the spacetime structure is not a discovery, but a choice. However, considered *functionally*, that is considering its 'role' within the rest of the theory's dynamics, the choice is *necessary*. It is required for the possibility of a feasible theory of motion, that is, a theory constrained by the principle of inertia on the one hand and the principle of observability on the other. In this sense, Cassirer could regard the spacetime structure as *a priori*, at least in the peculiar neo-Kantian sense of the expression. As his Yale student, Arthur Pap, pointed out, "Cassirer tends to assimilate Kant's doctrine of the *a priori* to the functional-pragmatic interpretation of the *a priori*" (Pap, 1943, 455). Something is *a priori* because is hypothetically necessary, that is "functionally speaking", a necessary means for something else (Pap, 1943, 449). In our case, a sufficiently rich spacetime structure is necessary for the possibility of a dynamical theory of motion, in which there is an objective difference between free motion and motion influenced by interactions. The history of spacetime theories from Leibniz to Einstein shows that fully relativistic theories of motion repeatedly failed. In Cassirer's language (Cassirer, 1910, 355f.; tr. 1923, 269f.), one could then argue that the 'inertial structure' is a sort of 'invariant' in the historical succession of spacetime theories. As such, it can be provisionally considered a good candidate for the role of an *a priori* condition for the possibility of a theory of motion[17]. The necessity of an inertial structure is the true core of what Cassirer considered Euler's insight: "without the Newtonian concepts of an absolute space the law of inertia and, accordingly, the whole system of mechanics would become meaningless" (Cassirer, 1943, 390).

CRediT authorship contribution statement

Marco Giovanelli: Writing – review & editing, Writing – original draft, Methodology, Investigation, Conceptualization.

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[15] Weyl (1924b) raises this objection against Schlick (1919)

[16] See above, fn. 4.

[17] Should a Machian theory prove successful, Cassirer should concede that this provisional hypothesis is refuted.

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