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Networking Hybridizations within the Semiosphere:

a research trajectory for the Cultural Transposition of the Chinese Lesson Study within a Western context

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Abstract

Purpose: The aim of this paper is to describe teachers’ professional development in Lesson Study (LS) as processes situated in Semiosphere and generated by the unevenness due to different cultural traditions. We characterize teachers’ professional development in two LS experiments as processes generating new knowledge to point out their products, i.e. new professional frame teachers produce after these experiments. We use Hybridization, a particular form of Networking (NWT).

Design/methodology/approach: We collected video-registration of several Italian LS meetings. We analyse two LSs, where time emerged as a conflictual aspect. Through Hybridization components, we show how teachers make sense of LS and how they revise their professional frame.

Theoretical framework: In NWT different theories are deployed to study the same problem. To grasp the issue of unevenness in our LS, we use *Hybridization of a Theory*, a form of NWT characterized by a structural asymmetry. It is given when a construct *c* is introduced *coherently*, *operatively*, and *productively* into a theory T, obtaining a hybridized *theory T’*.

Findings: Hybridization lens allows to describe potentialities and limits of LS as a tool for teachers’ development. The two analysed LS are different: in one of them Hybridization process produced a

new theory T', whilst in the second one it was limited to the awareness of a gap between LS and initial teachers' professional frame.

Practical implications: Identifying links between different Hybridization components is a helpful tool for teacher educators/researchers to foster the shift from one another. For instance, through this tool, teacher educators could keep track of what happens in successive LS and mirror the dialogue between teachers, moving towards group-shared metareflections. This initiates the advancement of a new theory T', where asymmetries are interpreted.

Originality/value: In this approach teachers are protagonists of the construction of a new professional frame. LS is a tool for teachers' professional development, allowing teachers to question their own educational intentionalities. Hybridization components provide a tool to analyse such a process.

Keywords: Chinese Lesson Study, Semiosphere, Hybridization, Time, Cultural Transposition.

1. Introduction. The necessity of theorising: a typical Western necessity

An emerging trend in literature points out the need to model how work on Lesson Study (LS) changes from the original context to a new culture. For example, Winsløw *et al.* (2018) used Networking of Theories (NWT: Bikner-Ahsbals & Prediger, 2014) as “a *theoretical framework* (with explicitly defined categories and terms) [...] to move the analysis of mechanisms and principles of lesson study” (*ibid.*, p. 126) and elaborated *paradidactic infrastructures* to model Japanese LS within a Western context. In general, it is the need for a suitable Cultural Transposition (Mellone *et al.*, 2019) that raises this necessity of theorising instead of producing ready to use manuals about how to do LS in Western countries.

In this paper we base on some courses grounded on Chinese LS (Huang *et al.*, 2017; Chen, 2017; Li & Huang, 2013) we developed in Italy with mathematics teachers of different school grades in 2018-2021 (Manolino, 2021a; 2021b). The courses were aimed at improving teachers' professional development. A main feature we observed in teachers' meetings (see Data Analysis) was a first general sensation of “disorientation” (Ramploud *et al.*, 2022). It arose since teachers felt the need to penetrate Chinese thought behind LS, but were stuck by the distance between how its rationale can be interpreted according to a Western framework and what the authentic one might be: they were exposed to a new language and it was sometimes difficult to find semantic correspondences in it with that of their usual didactical framework. The gap between the two thoughts drove teachers to a dramatic and intense interpretative activity, which many times consisted of a continuous local translation of the LS into a fresh possibly partial didactic framework within their professional context.

Jullien (2015) has introduced the word *gap* (*écart*) to feature this distance, distinguishing it from what he calls difference (*différence*): “they have in common a separation (*séparation*), the difference *highlights a distinction*, while the gap *opens a distance*. [...] the gap is an exploring figure that I would define *heuristic*” (p. 382, italics in original; our translation). The distance, perceived as a gap, promotes “a perspective that is no longer identification but is exploration, that envisions the attainable limits of diverse possibilities and the junctures discernible by thought” (Jullien, 2012, p. 495), that is “a philosophical dialogue in which every thought, when coming towards the other, questions itself about its own *unthought*” (Jullien, 2006, p. 8).

As researchers, we can frame it theoretically within Lotman's (1989) Semiosphere, that is "an uneven multi-cultural space of meaning-making processes and understandings generated by individuals as they come to know and interact with each other" (Radford, 2008, p. 318). Semiosphere is "the conceptual space where the theories and their connections become objects of discourse and research" (Radford, 2008, p. 317). As pointed out by Manolino (2021b), teachers' continuous 'reshuffling' of their didactic frame because of LS activities causes what Lotman calls *explosions*, "i.e. creative phenomena, dynamic tensions between highly conflicting moments" (Manolino, 2021b, p. 41). Semiosphere is the conceptual space where *unevenness* generates teachers' dynamic interpretative activities. Our research aims at highlighting their main features, investigating them as processes that generate new knowledge and pointing out their products, that is the new professional frame they produce.

We use a particular form of NWT, which we call Hybridization, adapted from Taranto & Arzarello (2020) and particularly apt to describe what happened in our courses.

In this paper, we first describe Hybridization within Networking frame, then we sketch our research methodology and analyse some data using our theoretical lens. Finally, we discuss how Hybridization allows to properly describe some of the main features of the way Chinese LS was interpreted by our teachers and point out some future streams of research.

2. Theoretical Framework

Hybridization of a Theory

In NWT different theories of Mathematics Education are deployed to study the same problem, possibly producing different levels of combination/integration of them (Bikner-Ahsbals & Prediger, 2014). As such they appear completely symmetric and complementary. Sometimes not only theories in the field of Education are considered, but also disciplines close to Mathematics Education: General Pedagogy, Psychology, Sociology, etc. In this case, networking process can generate theoretical frameworks where some more or less strong *unevenness* between its constituents appears.

As sketched in the Introduction, Semiosphere is the proper conceptual environment where we can frame what happens in our courses: in fact, unevenness (неравномерность) or *asymmetry* (Lotman, 1990, p. 127; Manolino, 2021b, pp. 40-42) is one of the structural features of the Semiosphere. It is the generator of new knowledge in our situation: the basic feature of how teachers try to enter the rationale of Chinese LS. To properly grasp the issue of unevenness we use a form of NWT, where its asymmetry is structurally present in the construct: it is the *Hybridization of a Theory* (HT). HT is our lens for our data analysis, but it is also the actual product of our teachers because of their exposure to LS and their activities and interactions when they tried to realise it in their schools. Because of this, we have two dual ways of using HT: as a theoretical frame to interpret our data and as a process developing in a concrete context because of one or more agents (for the notion of *agent/agency* in Mathematics Education, see Anderson & Norén, 2016).

One main difference with standard NWT is that in NWT different theories are used by researchers to interpret data 'from the top'; in our case HT is instead the result of an elaboration produced by our teachers 'from the bottom'. They themselves feel the need of hybridizing their professional framework because of the unevenness they sense. Researchers in a way collect and organise it within a frame, whose partial and local bricks have been built by teachers themselves, who as agents contribute in this way to theoretical elaboration.

Bottom-up development and local aspects feature HT as a process. On the other hand, HT can also be seen as a product, that is a top-down theory organised by researchers, which can have more or less general applications. Because of this, we give two (consistent) definitions of HT, one as a *static* product, the other as a *dynamic* process¹.

Static Definition. A hybridization of a theory T is given when a more or less extensive construct *c* is introduced *coherently, operatively* and *productively* into the theory T (Taranto & Arzarello, 2020). A hybridized *theory T'* is thus obtained.

Dynamic definition. A hybridization of a theory T is a process produced when an agent, fond of T, introduces in it a construct *c* heterogeneous to T and jointly elaborates at least locally the two in a *coherent, operative, and productive way*, to produce a new theoretical frame T'.

An example of HT static definition is given by the introduction of the *obstacle* construct from Bachelard within Theory of Didactic Situation (Brousseau, 1997). An example of dynamic definition is the introduction of *paradidactic praxeologies* within Anthropological Theory of Didactics (Chevallard, 1985) by Winsløw and colleagues (2018). Another example will be our interpretation of our teachers' elaboration of Chinese LS. In our research we use the dynamic definition since, as shown in Data Analysis, teachers' role (as agents of the process) is essential.

We now describe the four components of dynamic HT: they are not to be intended as separate successive steps, but as components that are part of a *continuum* (i.e., a continuous process of teachers' interpretation of the asymmetry):

- a) Connecting
- b) Interpreting
- c) Adapting
- d) Metareflecting

Components can also be modified to get a formulation for the static definition.

a) **Connecting**

In general, a hybridization process of a theory T with a heterogeneous, possibly conflicting, construct *c* begins by establishing a connection between the two, which depends on at least two parameters:

- the structure of T and *c*;
- the goal of the connection.

b) **Interpreting**

¹ For the definition of a Theory we refer to Radford (2008, p. 320), who writes that “a theory can be seen as a way of producing understandings and ways of action based on:

- A system, P, of basic principles, which includes implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective.
- A methodology, M, which includes techniques of data collection and data-interpretation as supported by P
- A set, Q, of paradigmatic research questions (templates or schemas that generate specific questions as new interpretations arise or as the principles are deepened, expanded or modified).”

The language L of T is extended to embrace c and the old sentences of T are expressed in the new language $L(c)$; as well new sentences involving c are produced in $L(c)$.

c) **Adapting**

Modification of theory T : new sentences involving c and connecting it to theory T are produced in a consistent way in $L(c)$, according to the properties of c , suitably adapted to T . The extended language allows to elaborate a new (perhaps local) fragment of theory, T' . The interpreted fragment can consist of a more or less important modification of T , with possible elimination of “old” components of previous theory T .

d) **Metareflecting**

Two aspects:

- issue of consistency;
- critical reflection on the new theory T' : changes allowed by it with respect to the old one T .

A result of this critical reflection produces a fresh, deeper ‘understanding’ of the old theory T through theory T' .

Hybridization vs/ Networking of a Theory

A main difference between NWT and HT is that in NWT two or more theories examine the same data with the objective of:

- (i) indicating the complementarity that can result from the study of the same set of data from different theoretical points of view: for example, when two or more theories are *compared* or *contrasted* (Bikner-Ahsbahs & Prediger, 2014, p. 119);
- (ii) checking whether and how the fundamental aspects identified by one theory in one set of data can be retained or differently interpreted in another theory: for example, when two or more theories are *combined* or *coordinated* (*ibid.*).

In dynamic HT, as we will see below, teachers as agents first try to interpret LS according to their usual didactic theory T , but it does not seem to give a consistent interpretation because some construct c of LS (time issue in our case) appears in conflict with T . Hence teachers try to elaborate a hybridization of T with c , possibly generating a new theoretical fragment of frame, T' , where some principles and some methodological aspects (both or one of the two) have changed. T' is a local consistent adaptation of T with respect to construct c . Hence, dynamic HT is within the stream of what in NWT is called *integrating locally* (Bikner-Ahsbahs & Prediger, 2014, p. 120): the main difference is that in HT construct c is not a fragment of a theory that teachers already know but something that possibly comes from a theory in any case unknown to them: what is apparent is that construct c creates a conflict with their own theory T . The result of this process will possibly, but not necessarily, be a new teachers’ framework T' , which now contains construct c in a coherent way. T' can be considered as the professional framework, according to which teachers, because of their LS experience, now design didactical activities for their students.

3. Research Question

In search of a framework to investigate teachers’ professional development, assuming the perspective of Western studies outlined in Introduction, our research question is:

How can HT, as a particular form of NWT, be used to describe (and to improve) critical reflection processes of Italian teachers involved in LS as professional development?

To answer this question, we use Hybridization as a theoretical lens to read the professional development that teachers participating in LS can develop in terms of production of a new theory T'. We use HT components to describe how teachers cope with conflicting constructs coming from the unevenness created by cultural difference. Several types of construct *c* could emerge. In this article, for reasons of space, we focus on the conflictual construct (*c*) of time, which has been highlighted in other studies as a problematic element of LS adaptations, not only in Italy (Bartolini *et al.*, 2017; Winsløw *et al.*, 2018; Bartolini Bussi, Funghi & Ramploud, 2020).

4. Methodology

To investigate our research problem, we collected recordings of various meetings of LS cycles and of LS dissemination meetings, during the years 2018-2021 (Manolino, 2021). Each meeting has been transcribed, so teachers' sentences constituted the data to be analysed.

Data analysis has been carried out in this way: we consider T – Italian teachers' theory – as including a child-centred, socio-constructivist and inclusive approach to teaching-learning, with a major focus on long-term goals, due to institutional context (see Bartolini Bussi, Funghi & Ramploud, 2020). We identify the language L of T with teachers' sentences referring to aspects of theory T. Instead, we seek for conflictual constructs *c* in sentences where teachers express a *gap* between Chinese LS and their cultural context. So, constructs *c* emerge from data analysis: they are not established *a priori* nor refer to any theoretical frame, but have an empirical origin rooted in teachers' varied experiences with LS.

To answer the research question of this paper, we present two LS experiments both involving *time* as construct *c* – we name them LS1 and LS2. Specifically, LS1 is an example of effective hybridization of construct *c* within theory T, whereas LS2 is an example of a hybridization process still in progress.

Both have been realised in schools in Northern Italy and were facilitated by one of the authors. LS1 was focused on the introduction of arithmetic operations and of relational meaning of equal sign (Sun & Bartolini Bussi, 2018, p. 62 and ff.) in 1st and 3rd grade. It involved 5 primary teachers (Enzo, Noemi, Michele, Marcella, Vanessa – pseudonyms are used); they all work in the same school and two of them are used to work with Mathematics Education research groups (Enzo, Noemi). The planned lesson was experimented 4 times, 3 times in different 1st grade classes, and the last time in a 3rd grade class. In relation to this experiment, we analyse Enzo's speech during a LS dissemination meeting occurred more than one year after the 4th iteration, reporting reflections shared by the group about LS.

LS2 was focused on ratio concept in 3rd grade and 5th grade. It involved 3 primary teachers (Clea, Martina, Diana) and 2 teacher students (Eleonora, Francesca). They are from different schools and do not usually work together; two of them are used to work with mathematics education research groups (Martina, Diana). The planned lesson was experimented 2 times, the first time in a 5th grade class (in presence) and then in a 3rd grade class (in distance education, because of COVID-19 pandemic). Also in this case, we initially analysed the group speech during LS dissemination meeting occurred after the 2nd iteration, reporting shared reflections. However, since in this case time construct *c* did not explicitly emerge in teachers' words but we knew that it was one of the gaps the

group handled, we searched for evidence of time construct in previous meetings transcriptions. Here we analyse excerpts taken from 4 of their LS meetings.

5. Data Analysis

LS1

The analytical trajectory we follow here is linked to teachers' considerations regarding the time issue, presented by Enzo during a LS dissemination meeting, after illustrating the experiment he participated in. Some excerpts of his speech are analysed using Hybridization components.

a) CONNECTING

We are not used [...] to have a defined lesson time. [...] Usually, lesson time in primary school is not a defined one. Documents on LS say: "Time constraint is a peculiar feature of LS". [...] "Teachers are invited to plan a lesson to be developed in a definite time". This is a difficult task. [...]. Initially it seemed almost impossible for us to think of concentrating on an activity in an hour, when we are not even used to think about how much time we need to spend on it.

This excerpt shows the connection and the gap in encountering the Other (Jullien, 2012), a different cultural tradition, signalled by the expression "*we are not used to...*", connoting Enzo's belonging to a culture different from LS original one. We can see here a gap between LS and the didactic theory (T) Italian teachers usually refer to (cfr. *logos* in Chevallard, 1985), with a focus on inclusiveness. In particular, Enzo identifies an aspect concerning both teaching planning and teaching practice: lesson time. The gap between "*Defined lesson time*" in LS and "*lesson time in primary school*", an (important) fragment of teachers' theoretical framework T, constitutes the connecting process. Time emerges as an element of an uneven context, placing the teaching research group in a situation of disorientation, and giving birth to a Hybridization process to insert time construct (*c*) within T. Enzo's words, in this case, show an attempt to make sense of strict timings planning within a context that is mostly focused on long-term processes, which can be interpreted as the goal of the connection.

b) INTERPRETING

The second step of hybridization, interpreting, is articulated in two different excerpts which we distinguish as *Interpreting 1* and *Interpreting 2*.

Interpreting 1

LS requires, for example, to go to the essentials. It requires to "slim down" the lesson: what is called *labor limae*, I mean, polishing [...] [LS requires] To discuss the "weight" that each action has in terms of minutes. [...] We are not used to say: "It takes five minutes to introduce the lesson...". When did any of us ever think of "putting on a watch" and say: "Now I dedicate five minutes to the introduction"? We usually initiate an introduction, but then if we have to wait five minutes more because of children's interventions, it is absolutely not a problem for us.

This excerpt highlights a "resonance" between *labor limae* (*lit.*, work of file, finishing touches) – a concept typical of Western culture, specifically Latin – with the need to "go to the essentials" of Chinese LS. We can see that Enzo puts in close connection the effort of removing superfluous elements from the lesson (indicated by expressions such as "*go to the essentials...*" or "*slim down*")

with a work of *labor limae*, of continuous improvement. These considerations are highly resonant with the LS case considered by Fernandez and Yoshida (2004), where the teaching research group discusses very in detail what to remove in task design and lesson structure (pp. 49-66).

This excerpt also shows an attempt to interpret LS constraints in a constructive attitude, namely, in a way allowing Enzo to see these LS characteristics not only as limitations but also as potentialities. In fact, on one hand Enzo underlines a gap with socio-constructivist perspective on teaching – he refers exactly to children’s interventions (L of T); on the other hand, time constraints are linked to *labor limae* (the expressions “*slim down*”, “*polishing*”, “*discuss the “weight” that each action has in terms of minutes*”, “*it takes five minutes*”, “*putting on a watch*”, extend L to embrace *c*, namely L(*c*)). Therefore, elaborating on time becomes an element of reflection, and its conflict with a child-centred teaching approach is explained with an effective example: since Italian teachers’ have to pay attention to each child, they are allowed/are forced to extend lesson time as much as they need.

Interpreting 2

Why should a teacher spend so many resources for planning lesson timings? First, in our experience this meant realising a design that is more realistic and respectful of students’ attention spans. Often teachers say [...] attention spans are limited for some children. A lesson that does not take this into account can lead to a greater attention deficit. Moreover, we interpreted the request for pre-set timings as an opportunity to reflect about task suitability for the classroom, and possible changes to make groupwork inclusive.

The gap produced by an “Other” way of seeing time in a lesson, allows Enzo to interpret the practice of defining strict lesson timings within a socio-constructivist perspective on teaching (producing L(*c*)). The ‘time’ that matters for him is that related to students’ attention spans, and therefore, related to the attention teachers have to pay to their students (indicated by expressions as “*more realistic and respectful of students’ attention spans*”, “*teachers say [...] attention spans are limited for some children*”, belonging to L of T), accordingly with a child-centred perspective on teaching. Moreover, in this excerpt also the issue of inclusiveness comes to the fore, which is an important theoretical component of Italian didactical-pedagogical context. The need to pay attention to every child’s educational needs is “translated” in a greater attention to time planning and tasks’ choice (the expression “*pre-set timings as an opportunity to reflect about task suitability for the classroom, and possible changes to make groupwork inclusive*” belongs to L(*c*)).

c) ADAPTING

In the following excerpt, Enzo expands his reflection to adapt LS with a part of theory T and linked to the time issue. This is very important because it shows the development of Hybridization as a *continuum* leading from the interpreting step to a first attempt of adapting step.

Research in Mathematics Education tells us that, to restore students’ negative attitudes, particularly towards mathematics, “it is extremely important to adopt a teaching methodology that values the role of time in mathematical activities, that contributes to undermine the idea that success in mathematics consists in quickly giving the correct answer”. This sentence, taken from an article by Zan and Baccaglioni-Frank, very clearly emphasises the attention the teacher must give to the time s/he provides to students. So, in recent years there has been a growth in teachers’ awareness of the need for distended times. On the other hand, we have seen LS as a tool for developing intentionality in didactic practice [...] That is, realising that

every didactic action should be planned based on conscious reasons. Then, these two aspects, which are both important, can also be combined in the experience of LS, where distended times should not always be understood as extended times, but as adequate times to achieve a well-defined goal.

In this excerpt, the gap opened by time construct in Chinese LS leads to a new reading of the same construct within the usual theory T (expressed in the quotation of Zan & Baccaglioni-Frank (2017), a renowned book for the Italian mathematics teachers professional development, which belongs to L of T): intentionality in selecting short term-goals is conjugated, in LS experience, with distended times for achieving long-term goals (indicated especially by the expression “*distended times should not always be understood as extended times, but as adequate times to achieve a well-defined goal*”, belonging to L(c)). In this perspective, “*adequate times to achieve a well-defined goal*” in LS allows to elaborate a new (local) fragment, T’, summed up by the expression “*distended times should not always be understood as extended times*”, where these two meanings of time are gathered by intentionality issue in didactic practice.

d) METAREFLECTING

There is an aspect that we take a little for granted, or we do not even always consider, that is the relationship between a teaching activity and National Guidelines. When we realise a teaching activity, we should ask ourselves “What goal for competence development do I want to achieve?”. This is an interesting aspect, the fact that [through LS] we get used to designing an activity relating it with a wider perspective, like a piece of a path, of a more articulated process. [...] We also thought that *limited* time is not a *limiting* time. And that the shift in thinking about time... from time in the classroom to time in a-priori lesson design is an important shift.

In this excerpt, we can see the beginning of a metareflecting process: the new fragment T’ produced before is considered in relation with educational goals indicated by national institutions – which relates to a consistency check of T’ with Italian institutional context. In fact, on the one hand, Enzo describes lesson planning in LS a tool for structuring short-term objectives; on the other hand, time construct is re-thought starting from the need to include these short-term objectives within a long-term process, aimed at developing competences, as prescribed by Italian National Guidelines.

Moreover, the last sentence represents a key element for point 2 of the metareflecting process. In fact, a comparison between the “old” (limited) time (in T) and the “new” (non-limiting) time (in T’) opens up. In other words, Enzo operates a possible comparison between the old theory T and the new one T’, based on changes that the time construct has undergone, both at an operational and theoretical level.

LS2

In the analysis of the second case considered here, we continue to maintain our analytical trajectory, applying Hybridization components to some excerpts taken by different LS meetings.

a) CONNECTING

The first excerpt we consider is taken from the first planning meeting, when teachers are discussing the topic to deal with in the lesson.

- Clea: I am thinking of a didactic unit linked to...
- Martina: that you can practically consider finished in 40 minutes. Then of course we will come back to it again in class. But considered to be taught in 40 minutes. And I was thinking, for grade 5, about something on the circle. Because it's, a bit, a self-contained thing. So you can do it a little earlier, a little later... I mean, of course it's related to everything else... But I mean... The work, for example, on pi, or...
- Diana: The circle no longer exists in the National Guidelines!
- Martina: I mean, in my opinion, yes ... it's important to give it some attention! That is, for example, "the circle and its features" seems to me something that can fit in 40 minutes.

In this excerpt, the connecting process is initiated by teachers' reflection about the gap between what in Italian context is considered the time frame necessary to address a mathematical concept with the class (part of theory T), and the requirement to plan a one-hour lesson (conflictual construct *c*). Or rather, the gap occurs within the very idea of planning a mathematics teaching moment. According to Clea, Martina and Diana, time construct (*c*) concerns two different aspects: (1) of scheduling, i.e. when to include the experience in the long-term educational planning of the class and this also concerns the relationship between different mathematical contents, i.e. there is content that can be worked out in class "a bit detached from the others" (indicated by expressions related to circle, as "*a bit of self-contained thing*", "*you can do it a little earlier, a little later*"); (2) of time-content correlation – there are contents that cannot be worked out in class in a short time, others that instead can (indicated by expressions related to circle, as "*something that can fit in 40 minutes*").

Furthermore, in the following excerpt the connection of *c* with T is manifest:

- Martina: In my opinion they ask us to do something in between, for instance a work on problem-solving but about a very specific content, so that you can develop it in those 40 minutes.

[Second planning meeting]

Problem-solving (element of theory T) is explicitly related to the conflictual element of pre-established and "tight" timings with respect to teachers' habits (through the expression "*for instance*", which puts the seeds for a further development of L(*c*)). Also in this case, the goal of the connection remains to understand how LS narrow constraints can be adjusted within a context and a didactic practice prioritising "distended" times.

b) INTERPRETING

To solve the time construct (*c*) issue, teachers focus on time-content correlation: their reflections continue dealing with contents considered interesting and important to address (differently from what they have discussed until this moment, more related to LS time constraints). During the discussion, then, ratio emerges as a content interesting for all teachers of the group. This shift of discussion focus – from times to content of ratio – makes the lesson temporal dimension absorbed in the relationship between short-term goals and long-term goals, and more specifically between goals of different grades.

- Diana: I really like the idea that, since we are going to make the same lesson in two different classes in two different grades, we can deal again with the content of ratio, but in a

new way! That is, in 3rd grade a first step is done [...] while in 5th grade the level of complexity can be increased but we deal with the same content!

[Second planning meeting]

Diana: We have found, let's call it "essential knowledge", that there are some fundamental contents, both in 3rd grade and 5th grade. [...] multiplication, division, decimal numbers, etc., fractions, circles... they are all linked by this thing [i.e. the content of ratio] [...] It will be better to start from the 5th grade goals and then go back, as is often done in designing curricula for longitudinal grades, starting from above [i.e., the ultimate goals of the higher grade considered, here 5th grade] and then "crumbling them backwards" [i.e., dividing them into smaller goals to be achieved throughout the previous grades]. In 5th grade, students must develop these competences: how can this development be scaffolded by going backwards?

[Third planning meeting]

The last excerpt shows how teachers' reflection has returned completely to context (there is no longer any word recalling LS or its constraints, but just expressions belonging to L of T) and the temporal dimension (*c*) can only be intuitively grasped in the background of the expression "*start from above*" [i.e., from long-term goals to be achieved in 5th grade] and "*crumbling them backwards*", where especially the term "crumbling" raises the issue of how to connect short-term goals to long-term ones. Also the issue of eliminating what is superfluous (detectable in the expression "*essential knowledge*") – which in LS1 was one of the elements used to construct a new interpretation of time – in this case remains more loosely linked to the issue of choosing a content, but it is not used explicitly to give a new reading of LS constraints in a constructive key within theory T. Although the gap initiated a first connection of time construct (*c*) with theory T, the interpreting process does not develop explicitly. In fact, the language L used by teachers remains within T and it does not produce (at least not explicitly) new sentences capable of embracing the conflictual construct *c*, as seen in LS1, even if there are some elements that potentially could have supported that development. From teachers' words it emerges instead as an adaptation process understood in a local and pragmatic perspective, which does not concern L or change in theory T, but only a temporary adjustment of their point of view to cope with LS as a particular episode.

Once the LS cycle was concluded, during a meeting for the creation of a presentation to illustrate their experience to others, the teachers show that they still perceive an element of conflict in the temporal dimension, which somehow remains disconnected from the issue of relationships between long-term goals and short-term goals. The hybridization process is interrupted because the adapting process of the time construct is not rationalized in a coherent, operational, and productive way.

Eleonora: But if we consider timings, the trouble they gave us... I would state that it [i.e., the lesson plan] must be so detailed and so in time, and that this doesn't belong to our way of teaching. It seemed to me that you were more... freer, I mean, that your thought is "we follow students' contributions" and... they [i.e., Chinese teachers] structure the lesson more than us perhaps; instead, in this case we have to go against our natural tendency and try to plan everything in pre-established times. We then saw how much they are actually respected because it is difficult to anticipate everything [...]

Francesca: Perhaps in a perfect LS you also must anticipate students' times, you have to anticipate their interventions, you have to also anticipate what is unpredictable. [...] It doesn't belong to our way of teaching anyway, for sure.

[...]

Martina: Therefore, it would be a critical point, critical point: time.

[...]

Eleonora: Actually, the timings... I noticed the minutes spent in each phase: apart from the technical problems, planned timings were perfect! [...] Expected times were good.

[...]

Martina: In my opinion [...] the worry for respecting timings becomes a priority over students' learning process. [...] Indeed, LS didn't tell you the teaching methodology, we are the ones who put our own [teaching methodology] into it... [...] In my opinion that is Cultural Transposition, I mean, this box [i.e. LS] establishing... for example, to be detailed in the design, to realise it together [...] and then we filled this box with our idea of doing mathematics [...] a "mathematical laboratory".

Even in this *a posteriori* meeting, teachers' conversation is completely focused on several critical aspects of time, even if timings anticipation worked out well, as Eleonora points out. Time, therefore, returns to be a conflictual element, even though the "seeds" of the interpreting process were present in teachers' words in previous meetings. Even if the hybridization process began, it has not led to the production of a new complete theory T', but only to a local adaptation aimed to solve the problem of unevenness in the specific case of that lesson, not in general. The effort of the group seems to shift instead to other aspects of LS, such as teaching methodology and a posteriori reflection, which are reinterpreted in terms of *mathematical laboratory* construct (Bartolini Bussi & Martignone, 2013, p. 3-4).

6. Discussion and conclusions

The previous section reports the analysis of two examples of LS and some excerpts from the transcripts of the group discussion in different steps of the process (the planning meetings and the final *a posteriori* reflection). In both examples the focus is on the same potentially conflictual construct (time) that emerged also in other experiments carried on in the Italian context (Bartolini *et al.*, 2017; Manolino, 2021b). At the beginning of the teaching experiments, the members of either group shared a common framework which includes a socio-constructivist and child-centred approach to teaching, with a focus on inclusiveness and on long-term goals, due to the institutional context (see Bartolini Bussi, Funghi & Ramploud, 2020). In both experiments a process of hybridization was started with the following components:

- a) Connecting
- b) Interpreting
- c) Adapting
- d) Metareflecting

The components of the two processes were exploited to analyse the different steps and to look for a possible new theory T', that considered the gap and the unevenness created by cultural difference.

However, the two processes are different. In LS1 the hybridization process is complete as the metareflection allowed a comparison between the “old” (limited) time (in T) and the “new” (non-limiting) time (in T’). In other words, Enzo operated a possible comparison between the old theory T and the new one T’, based on changes that the time construct has undergone, both at an operational and a theoretical level. In LS2, on the contrary, the hybridization process does not lead to the production of a new theory T’ but only to a local adaptation aimed to solve the problem of unevenness in the specific case of that lesson, not in general. The effort of the group seems to shift instead to other aspects of LS, such as teaching methodology and *a posteriori* reflection. Although the metareflection does not produce a new theory T’, it fostered the understanding of the gap due to cultural reasons. The different processes are likely to depend on the different experiences of LS: the LS1 teachers had already carried on a consistent number of LS cycles, whilst the LS2 teachers had carried on only two cycles. Other characteristics of the teacher group could be relevant, for instance teachers in LS1 are from the same school, whereas teachers in LS2 are from different schools, so perhaps teachers in LS1 felt more comfortable in having conversations about teaching than teachers in LS2. Also, 2 of the 5 teachers in LS1 are used to work with Mathematics Education research groups, and so they may have developed theoretical lenses in analysing and reflecting on their own teaching practice.

We can now put the two examples on the background of what is known as the Italian RME (Research in Mathematics Education, in Arzarello & Bartolini Bussi, 1998), that is a way to reconcile practice and theory: in this approach the teachers are actors (and protagonists) of the construction of a theoretical approach T’, if any. For instance, the theoretical construct of the *mathematical laboratory* was the product of this collective effort to reconcile practice and theory (Barbin & Menghini, 2014). The introduction of LS in the Italian context is a way to focus on a practice coming from a different culture, hence to exploit LS as a tool for teachers’ professional development.

Our analysis shows how hybridization gives a new lens to describe potentialities of LS as a tool for teachers’ development. The two cases are different. In LS1, the LS potentiality allows teachers to change their mind about the role of time in mathematics teaching; with our lens of HT, this is translated in the production of a new theory T’. In LS2, instead, the hybridization process was not complete and limited to the awareness of a gap between the two practices. But the analysis of the hybridization process allowed to show LS potentiality for teachers to become aware of the very gap and to find words to express it. LS practice allowed to introduce a relationship between the long-term objectives (as in National Guidelines) and short-term objectives, namely the focus of LS lesson planning.

The analysis of the two processes (LS1 and LS2) highlighted that the hybridization process is not a sequence of separate steps but rather a continuous process through which teachers take care of the detected asymmetry. For teacher educators/researchers it is very useful to identify the links between connecting, interpreting, adapting and metareflecting and to foster, whenever possible, the shift from one another. We argue that, in LS2, it would have been possible to foster the shift from adapting to metareflecting. For instance, the teacher educator in successive cycles could keep track of what has happened and is happening, mirror the dialogue between teachers through the terms of the hybridization process and, with teachers, move towards a group-shared metareflection. This initiates the advancement of a new theory T’, where asymmetries are interpreted.

Further research might investigate whether hybridization process appears in other studies and whether other conflictual constructs (beside time) are focused. For instance, “the material conditions for teachers’ work at the school” (Winsløw *et al.*, 2018, p. 129) may appear in dialogues between teachers. Another issue to be investigated could be whether it is necessary and/or beneficial if teachers participating in LS are explicitly introduced to the HT framework, or how could teacher educators and researchers help teachers go beyond the operational level to reach theoretical level.

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