Computational Management Science (2022) 19:605–626
https://doi.org/10.1007/s10287-022-00428-w

ORIGINAL PAPER

Divide and conquer: the engineering of delegation

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Received: 12 December 2021 / Accepted: 17 May 2022 / Published online: 25 June 2022

Abstract
Members of an organization have conflicting preferences. Principal-agent theory analyzes how and under which conditions this conflict can be resolved by means of incentives. In this paper we discuss an alternative to incentives: the engineering of delegation. The principal can divide the organizational decision making problem into subproblems and appropriately delegate different subproblems to different agents, letting them free to act according to their individual preferences. We introduce a formal model which analyzes whether and under which conditions the principal can in this way obtain the decisions she prefers without manipulating incentives nor using authority to overrule what agents autonomously decide.

Keywords Delegation · Conflict · Complexity · Agency theory

1 Introduction

Conflict is an ubiquitous component of all human organizations. Human beings pursue, at least partly, heterogeneous individual objectives and have diverging preferences. For instance, standard neoclassical economics assumes that human beings

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are selfish utility maximizers[1] and that they can be induced to pursue the organizational goals only by coercion or by means of (monetary) incentives which modify their preference orders. To put it very simply, consider the problem of an agent in an organization who must choose between actions A to B on behalf of the organization. The agent gets higher utility (or lower disutility) by choosing A, while B is preferable for the principal. The latter can either force by authority the agent to choose B or give him a reward for choosing B which is higher than his differential disutility for choosing B rather than A. In both cases, in addition, the organization must also be able to monitor (possibly in a way which can be verified by a third party judge) that the agent actually chooses B, which might be problematic with uncertainty and information asymmetries. Only when these conditions are met will the agent choose B and act in the interest of the principal.

Another type of conflict does not emerge from diverging objectives but from diverging views on how a common objective should be pursued. Richard Rumelt calls this type of conflict "incommensurable beliefs", i.e. "...the problem that arises when different individuals or groups hold sincere but differing beliefs about the nature of the problem and its solutions" (Rumelt 1995, p. 109). This kind of conflict has been overlooked by economic theory and in principle it looks even more difficult to manage, also because in this case the organization faces a dilemma between control and learning (Marengo and Pasquali 2012). On one side, the principal can increase control by aligning the views and beliefs of agents. This can be done through incentives, or persuasion, or simply by hiring agents whose views and believes are more similar to those of the principal. However in doing so the principal overlooks the opportunities of learning from diversity as the beliefs and views of some agents may happen to be more effective in solving the problem (Page 1996).

Whatever the source, unresolved conflict always characterizes organizations. In his seminal 1962 article "*The Business Firm as a Political Coalition*" James March states this point in the clearest way and formulates two "postulates of conflict", namely:

1. *There are consistent basic units. [...] Each elementary unit in the system can be described as having a consistent preference ordering over the possible states of the system [...] i.e. an ordering such that for any realizable subset of possible states of the system there exists at least one state as good as any other state in the subset.*
2. *There is conflict. The preference orderings of the elementary units are mutually inconsistent relative to the resources of the system. Conflict, in this sense, arises when the most preferred state of all elementary units cannot be simultaneously realized.* (March 1962, p. 663)[2].

[1] Recent developments have partly modified this assumption by admitting that individuals can hold other-regarding preferences that take into account also the utility or disutility of others (see, e.g., Fehr and Fischbacher 2002 for a survey). Reciprocal, altruistic, spiteful behaviours can result from such preferences, but the selfish idiosyncratic component remains the main argument in all utility functions.

[2] It is interesting to notice the similarities between these postulates and Simon's notion of complex systems (Simon 1981). In both cases we have a system made of several units and such units are interdepend-

In this paper we start from March's postulate and, in particular, we consider an organization made of agents (elementary units) with mutually inconsistent preference orderings over a set of interrelated policies. Thus, we assume that organizational decisions are multidimensional and complex, in the sense that decisions in one dimension usually produce effects also on other dimensions. However we do not focus on the use of incentives as a means to resolve this conflict. We acknowledge that conflict is a permanent feature of organizations and we analyze how a principal, rather than diluting it by aligning preferences with incentive, can actually exploit the multidimensionality and complexity of decisions and obtain a desired course of action also from agents who do not share her preferences. The principal can in fact act on the division and allocation of decision rights on the various policies the organization must decide on. We call this the "engineering of delegation", that is an appropriate division of the organization's decision problem into subproblems and an appropriate allocation of decision rights for each of these subproblems to specific agents. Suppose the principal knows the preferences of the agents[3], we ask whether she[4] can use delegation to obtain her most preferred outcomes from a set of agents who have preferences different from hers. The underlying idea is very simple. Suppose the organization must take decisions on a set of n interdependent policies $P = \{p_1, p_2, \dots, p_n\}$ where each policy p_i can take, for simplicity, a finite set of values $p_i = \{v_i^1, v_i^2, \dots, v_i^h\}$. The principal and the agents have complete orderings over all possible policy vectors and such orderings, in general, differ across individuals. Suppose now that the principal's most preferred policy vector is $v_1^*, v_2^*, \dots, v_n^*$ and that such a vector is not the most preferred policy by any agent. Nevertheless some agents may have preferences which are locally aligned with the principal's, for instance one agent may prefer v_i^* as an option for the i-th policy either always or at least when the other elements of the vector have some specific values. Another agent may choose v_j^*, v_k^*, v_h^* in some circumstances, and so on and so forth. Thus, it may be possible for the principal to partition[5] the n policies into subsets and delegate each subset to a specific agent in order to exploit "local" preference alignments with each of them. Using this kind of "divide and conquer" strategy the principal may obtain from the

Footnote 2 (continued)

ent in the sense that actions or changes of state in one unit may determine disruptions in other units (see also Marengo 2020 on this point). We will come back later in the paper on the relation between conflict and complexity.

[3] For simplicity and mathematical tractability we make this extreme assumption that the principal perfectly knows the agents' preferences and that the latter always act according to their individual preferences, i.e. if an agent prefers A to B he will always choose A. Thus we rule out the possibility that, alike strategic voting in social choice theory, an agent may strategically choose B although he prefers A because he realizes that by choosing B he will finally obtain a better outcome.

[4] Conventionally we will use "she" for principal and "he" for agents.

[5] For simplicity in this paper we consider only delegations which partition the set of policies, assigning each policy to one and only agent. In real organizations delegation is often non-partitional, with different agents or units taking decisions on the same policies. Actually such overlaps and ambiguities in delegation can be an additional tool that the principal may use to get favourable outcomes, but we leave this easy extension of our model to future work.

agents the policies she prefers, although none of agents fully shares her preferences.

It is worth stressing that this possibility and the properties we will discuss in this paper are strictly dependent on the assumption that the organization has to take multiple decisions. If, on the contrary, the organizational decision consisted of only one element everything we discuss in this paper would reduce to a trivial condition: either there exists one agent who has the same most preferred value for the unique policy as the principal and therefore the principal delegates the decision to him, or delegation alone will never allow the principal to obtain her most preferred outcome.

It also worth stressing that we do not put any restriction on individual preferences besides completeness and transitivity. In particular, we do not impose separability among preferences on individual policies, meaning that the preferred value of one policy may depend on the values of other policies. To be more precise, if for instance the principal's most preferred policy vector is $v_1^*, v_2^*, \ldots, v_n^*$, this does not imply that the principal must prefer $v_1, v_2, \ldots, v_i^*, v_n$ to $v_1, v_2, \ldots, v_i, v_n$ for every i and for all the values taken by the policies different from p_i. In other words, there can be interdependencies and non monotonicities in the preference orders, i.e. individual preferences may form complex and rugged landscape like Stuart Kauffman's NK fitness landscapes (Kauffman 1993; Levinthal 1997). Such interdependencies are a source of conflict when subsets of policies are allocated to different agent. Suppose that one agent has chosen his most preferred values for the policies delegated to him given the current value for the policies not delegated to him. When other agents modify the latter, his choice might no longer be optimal and he may revise it. However this revision will, in general, generate changes also in the decision of the other agents, and so on and so forth. In other words, non separabilities will induce externalities among decisions: decisions of one agent will induce changes in the decisions of the other agents. Also this source of complexity is important in our framework because it implies that, for instance, the principal may exploit this conflict in order to obtain her preferred policies.

In this paper we introduce a graph based mathematical model that formalizes this idea and derives the main conditions under which the divide and conquer strategy enables the principal to obtain her preferred outcomes. We model a principal and a population of agents holding heterogeneous preferences over the set of vectors on n policies. We only require that such preferences be complete and transitive and put no other restriction. The principal does not take any decision directly but can only partition the vector of policies and delegate decisions on each subset to a specific agent. Once this delegation structure has been put in place, the principal can no longer intervene and agents are free to decide on the policies allocated to them following their own individual preferences. Decisions by agents on the policies assigned to them can, given non-separabilities and externalities, determine an organizational equilibrium, i.e. a policy vector which is not modified by any agent because no agent can obtain a vector he prefers by changing the values of the policies allocated to him, or a cycle, i.e. a situation in which externalities among agent results in an endless repetition of a sequence of policy vectors. If they exist, organizational equilibria can be either global, if there exists a unique equilibrium which can be reached from any initial policy vector, or local, if there are multiple equilibria and the initial conditions determine which one(s) can be be reached.

We call "organizational structure" a partition of the set of policies together with an assignment of each subset to a specific agent. In Theorem 3.1 we give a necessary and sufficient condition for the existence of an organizational structure that makes a given policy vector, e.g. the one most preferred by the principal, an equilibrium for the organization. Then we introduce an algorithm, based on Theorem 3.1, which computes the probability for a local equilibrium both to exist and to be the principal's optimum and we provide numerical results which show how the probability that this equilibrium exists and coincides with the principal's most preferred outcome depends on the number of policies, the number of alternatives for each policy, and the number of agents.

Finally, we discuss more in details the role of diversity among the principal and the agents. We introduce a notion of distance between two preference orderings and we suppose that the principal can hire agents who are "similar enough" to herself, i.e. whose preferences are within a maximum distance from hers.

Our model assumes that the principal is an optimal delegation engineer and has perfect knowledge of the preferences of agents. We do realize that such knowledge and rationality requirements are unrealistic and real decision makers are characterized on the contrary by limited rationality and limited knowledge. However, on the one hand, our assumption is no more at odds with reality than the perfect rationality assumption of for instance the standard principal-agent model, where the design of an optimal incentive compatible contract requires full knowledge of the agent's utility function. On the other hand, we consider this paper a first step which provides some limit properties of optimal delegation structures. In future papers we plan to investigate how boundedly rational principals can adaptively develop effective, if not optimal, delegation structures.

The paper is organized as follows. In Sect. 2 we introduce our notation and describe the model. In Sect. 3 we provide our main results in the form of Theorem 3.1 and, in Sect. 4 we summarize the numerical results produced by an algorithm based on such a theorem. Finally in Sect. 5 we introduce a definition of distance, and we give numerical results for the case in which the distance between principal and agents is fixed, then we compare the results of the fixed distance case with those of Sect. 4 with random distance. Finally, in Sect. 6 we discuss some limitations of our approach and directions for future research.

2 Definitions and structure of the model

2.1 Preliminaries

In this subsection we introduce our notation and graph theoretic representation of individual preferences and organizational decisions.

Let $P = \{p_1, \dots, p_n\}$ be a bundle of *policies* on which an organization has to take decisions. The i-th policy can take m_i values[6], i.e. $p_i \in \{0, 1, 2, \dots, m_i - 1\}$ with $i = 1, \dots, n$. A *policy vector* is an n-uple $x = v_1 \cdots v_n$ of values such that $0 \leqslant v_i < m_i$. The set of all policy vectors will be denoted by X and its cardinality, $\prod_{i=1}^{n} m_i$, will be denoted by M.

Consider an organization composed by a principal Π and a set $\mathcal{A} = \{a_i\}_{1 \le i \le h}$ of h *agents*. Each agent $a_i \in \mathcal{A}$ is associated with a (possibly empty) subset $d_i \in \mathcal{P}(P) = \{d \mid d \subseteq P\}$ of policies under his control. The principal instead does not control any policy. We call *organizational structure* a partition of the set policies into disjoint subsets together with an assignment of the decision rights on each such subset to a specific agent. More formally, an organizational structure $\mathbf{O}$ is a function

$$\mathbf{O} : \mathcal{A} \longrightarrow \mathcal{P}(P)$$

such that, if we denote by $d_i := \mathbf{O}(a_i)$ then

$$\bigcup_{i=1}^{h} d_i = P \text{ with } d_i \cap d_j = \emptyset, \forall i \neq j. \tag{1}$$

In general we assume that $h \geq n$, i.e. that there are at least as many agents as policies, and therefore all organizational structure, including the one with the finest partition into n singletons, are feasible.

We assume that the principal Π and each agent a_i have complete, transitive and strict[7] preference orders $\succ_\Pi$ and $\succ_i$, $i = 1, \dots, h$ on the set of policy vectors X and we put no additional restriction on such orders. In particular, we focus on principal and agents having different and conflicting orders. We will also assume that everybody's choices are fully determined by her/his own preferences, i.e. that everybody acts selfishly but not strategically, therefore ruling out the possibility of choosing now a less preferred option in order to get a better outcome later, like in strategic voting in social choice theory. Thus, when asked to choose between two options, an agent will choose the one that ranks higher in his preference order.

We model preference orderings by means of graphs as follows. A total order $\succ$ on the set X defines a graph $\mathcal{Y}_\succ = (\mathcal{Y}_{0,\succ}, \mathcal{Y}_{1,\succ})$ such that the set of nodes is $\mathcal{Y}_{0,\succ} = X$ and the set of edges or arcs is

$$\mathcal{Y}_{1,\succ} = \{(y, x) \in X \times X \mid x \neq y \text{ and } x \succ y\} \subset X \times X \quad .$$

The couple $(y, x) \in \mathcal{Y}_{1,\succ}$ means that the edge or arc is orientated from y to x. For the sake of simplicity, we will use the same symbol x for the nodes of $\mathcal{Y}_\succ$ and (x, y) for its arcs. A cycle

[6] These values do not have to be interpreted necessarily as numbers, but, more generally, as elements of a set of m_i alternatives. For instance a set of possible suppliers from which a component can be bought, a set of alternative R &D projects which could be undertaken, etc.

[7] For the sake of simplicity, we consider only strict preferences, i.e. we consider only total orders on the set of policy vectors. This restriction is almost always unnecessary, but simplifies our presentation.

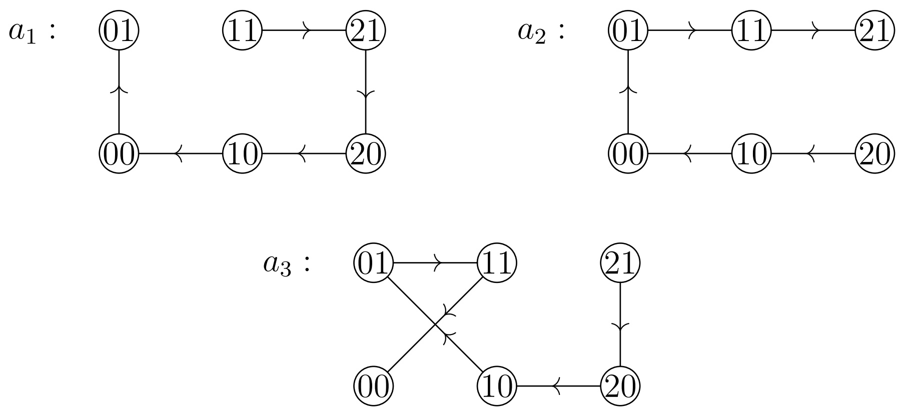


Fig. 1 Graphs of transitive preferences for agents a_1, a_2 and a_3

$$(x_2, x_1), (x_3, x_2), \dots, (x_1, x_h)$$

in the graph $\mathcal{Y}_{\succ}$ corresponds to a sequence of preferences

$$x_1 \succ x_2 \succ \cdots \succ x_h \succ x_1.$$

Note that the completeness and transitiveness assumptions on preferences imply that the graph $\mathcal{Y}_{\succ}$ is connected, without cycles and each pair of nodes $\{x, y\}$ is connected by at most one arc (see, for instance, Fig. 1 where the missing arcs can be obtained by transitivity).

Given an initial policy vector $x \in X$, an organizational structure $\mathbf{O}$ defines a transitive (i.e. without cycles) sub-graph $\mathcal{Y}_{(x,d_i)}$ of the whole graph $\mathcal{Y}_{\succ_i}$ of a_i's preferences for each agent a_i with $d_i = \mathbf{O}(a_i)$. More precisely, the set of nodes $\mathcal{Y}_{0,(x,d_i)}$ of this sub-graph contains x and all the policy vectors $y \in X$ that differ from x only in the policies in d_i, i.e. if $x = w_1 \cdots w_n$ then

$$y = v_1 \cdots v_n \in \mathcal{Y}_{0,(x,d_i)} \quad \text{iff} \quad v_j = w_j \forall j \text{ such that } p_j \notin d_i \quad .$$

An arc (x, y) is in $\mathcal{Y}_{1,(x,d_i)}$ if and only if it is an arc in $\mathcal{Y}_{\succ_i}$ with nodes in $\mathcal{Y}_{0,(x,d_i)}$.

Example 2.1 Let us consider an easy example given by a set of two policies $P = \{p_1, p_2\}$ in which the first policy can take three values $p_1 \in \{0, 1, 2\}$ and the second one only two values $p_2 \in \{0, 1\}$. The organization is composed of a principal Π and three agents $\{a_i\}_{1 \le i \le 3}$ with the following complete and transitive strict preferences $\succ_i$:

$$\begin{aligned}
\succ_1 &: 01 \succ_1 00 \succ_1 10 \succ_1 20 \succ_1 21 \succ_1 11, \\
\succ_2 &: 21 \succ_2 11 \succ_2 01 \succ_2 00 \succ_2 10 \succ_2 20, \\
\succ_3 &: 00 \succ_3 11 \succ_3 01 \succ_3 10 \succ_3 20 \succ_3 21.
\end{aligned}$$

The associated graphs $\{\mathcal{Y}_{\succ_i}\}_{1 \le i \le 3}$ are reported in Fig. 1, where, for the sake of simplicity, we draw only the necessary arcs since all missing ones can be easily obtained

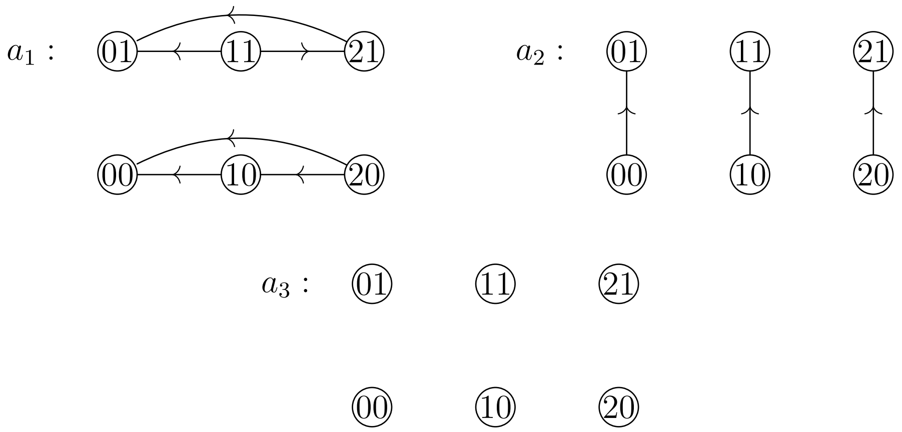


Fig. 2 The sub-graphs corresponding to the agents a_1, a_2 and a_3, if $d_1 = \{p_1\}$, $d_2 = \{p_2\}$ and $d_3 = \emptyset$

by transitivity. For example, there is an arc, which we do not draw, connecting vector 21 to 01 in the graph $\mathcal{Y}_{\succ_1}$, which is implied by the transitivity of $\succ_1$.

Now, for example, the delegation structure $d_1 = \{p_1\}$, $d_2 = \{p_2\}$ and $d_3 = \emptyset$ corresponds to considering the (disconnected) horizontal subgraph of a_1, i.e. the subgraph obtained by deleting for a_1 all vertical arcs, because the delegation structure allows a_1 to act only on the first policy item and therefore to move only horizontally. On the contrary, a_2 can act only on the second policy, therefore he can operate on his subgraph including only the vertical arcs. Finally, since a_3 is not delegated any decision, his graph is fully disconnected (see Fig. 2).

Let $x \in X$ be a policy vector, $\mathbf{O}$ an organizational structure and a_i an agent who controls the policies $d_i = \mathbf{O}(a_i)$. Hence by the transitivity of the graph $\mathcal{Y}_{0,(x,d_i)}$, subgraph of the transitive graph $\mathcal{Y}_{\succ_i}$, it follows that there is one and only one node $w \in \mathcal{Y}_{0,(x,d_i)}$ such that $(y, w) \in \mathcal{Y}_{1,(x,d_i)}$, i.e. $w \succ_i y$, for any $y \in \mathcal{Y}_{0,(x,d_i)}, y \neq w$.[8] The node w will be called the *preferred neighbor* of x for the agent a_i and the organizational structure $\mathbf{O}$ and it will be denoted by $p_i(x, \mathbf{O})$. Notice that the case $p_i(x, \mathbf{O}) = x$ is possible and, in particular, we assume that $p_i(x, \mathbf{O}) = x$ whenever $d_i = \emptyset$.

Example 2.2 Let us consider the same agents $\{a_1, a_2, a_3\}$ as in example 2.1 with the organizational structure $\mathbf{O}$ defined by $d_1 = \{p_1\}$, $d_2 = \{p_2\}$, $d_3 = \emptyset$. We get (see Fig. 2):

$$
\begin{aligned}
&p_1(01, \mathbf{O}) = 01, p_1(11, \mathbf{O}) = p_1(21, \mathbf{O}) = 01 \\
&p_2(01, \mathbf{O}) = 01, p_2(11, \mathbf{O}) = 11, p_2(21, \mathbf{O}) = 21 \\
&p_3(x, \mathbf{O}) = x \quad \forall x \in X
\end{aligned}
$$

[8] Such a node is usually called sink in graph theory.

2.2 The decision process

We can now describe the organizational decision process. A *domination path* $DP(x, y, \mathbf{O})$ in an organizational structure $\mathbf{O}$ is a sequence of policy vectors

$$x = x_0 \to x_1 \to \cdots \to x_s = y$$

such that $x_i = p_{k_i}(x_{i-1}, \mathbf{O})$ for $i = 1, \dots, s$. In terms of graph theory a domination path $DP(x, y, \mathbf{O})$ lives in the graph $\mathcal{Y}_{(x,y,\mathbf{O})}$ obtained as the union of the graphs

$$\mathcal{Y}_{(x,y,\mathbf{O})} = \mathcal{Y}_{\left(x_0, d_{k_1}\right)} \cup \mathcal{Y}_{\left(x_1, d_{k_2}\right)} \cup \cdots \cup \mathcal{Y}_{\left(x_{s-1}, d_{k_s}\right)}, \tag{2}$$

where the union of graphs is the graph whose nodes are given by the union of the sets of nodes and whose arcs are given by the union of the sets of arcs. Of course the graph $\mathcal{Y}_{(x,y,\mathbf{O})}$ is, in general, neither transitive nor complete, even when all graphs forming the union are both transitive and complete.

A policy vector y is said to be *reachable* from another vector x in the organizational structure $\mathbf{O}$ if there exists a domination path $DP(x, y, \mathbf{O})$.

The domination path $x_0 \to x_1 \to \cdots \to x_s$ is said to *end in x_s* if, given the organizational structure, no agent will decide to modify any policy item in x_s. It is said, instead, to reach a *cycle* if it enters a sequence

$$x_1 \to x_2 \to \cdots \to x_h \to x_1 \quad .$$

An *agenda α* for an organizational structure $\mathbf{O}$ is the order in which the agents are called to decide upon the policy items under their control, that is an ordered t-uple of indices $(k_1, \dots, k_t)$ with $t \geq h$ such that $\{k_1, \dots, k_t\} = \{1, \dots, h\}$. In other words,

1. All policy items must be considered, that is the set $\{k_1, \dots, k_t\}$ has to be equal to $\{1, \dots, h\}$, and
2. The same policy item could be considered more than once, that is $t \geq h$.

By definition of organizational structure, to state the order in which the agents decide is equivalent to order their policy items. The ordered t-uple of policy items $\alpha = (d_{k_1}, \dots, d_{k_t})$ is denoted by $\mathbf{O}_\alpha$. Note that repetitions of the agenda, in general, are allowed.

Let $\alpha = (k_1, \dots, k_t)$ be an agenda of an organizational structure $\mathbf{O}$. A domination path in $\mathbf{O}$

$$x = x_0 \to x_1 \to \cdots \to x_s = y$$

is said to be *ordered along α* if the order of the agents is given by α, i.e. if $x_{i-1} \to x_i$ for the agent a_{k_q+1} where k_q is the remainder of the division of $i - 1$ by t. Such a domination path will be denoted by $DP(x, y, \mathbf{O}_\alpha)$.

Example 2.3 Let us consider three agents $\{a_1, a_2, a_3\}$ with preferences as in Example 2.1 and the organizational structure $\mathbf{O}'$ defined by $d_1 = \{p_2\}$, $d_2 = \{p_1\}$, $d_3 = \emptyset$ (see Fig. 3). The domination path starting from the policy vector $x = 21$ ends in a cycle

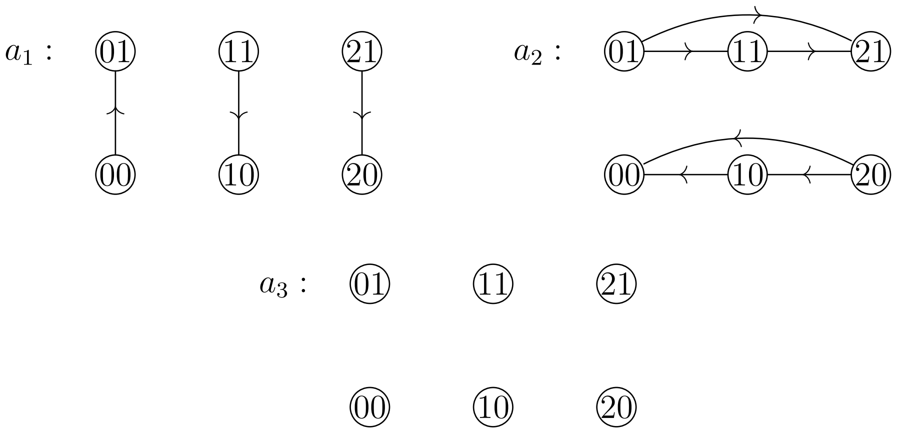


Fig. 3 The sub-graphs corresponding to the agents a_1, a_2 and a_3, if $d_1 = \{p_2\}$, $d_2 = \{p_1\}$, $d_3 = \emptyset$

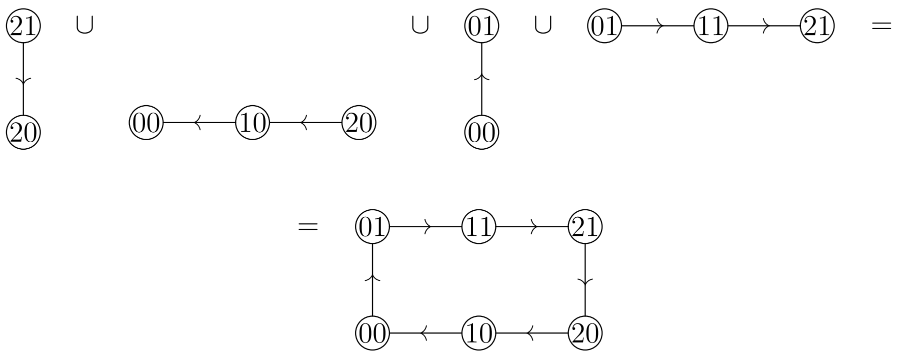


Fig. 4 Graph corresponding to the union $\mathcal{Y}_{(21,d_1)} \cup \mathcal{Y}_{(20,d_2)} \cup \mathcal{Y}_{(00,d_1)} \cup \mathcal{Y}_{(01,d_2)}$ for the organizational structure $\mathbf{O}'''$

$$21 \rightarrow 20 \rightarrow 00 \rightarrow 01 \rightarrow 21$$

independently from the chosen agenda α. It is easy to prove that all domination paths starting from 21 contain the cycle

$$\mathcal{Y}_{(21,d_1)} \cup \mathcal{Y}_{(20,d_2)} \cup \mathcal{Y}_{(00,d_1)} \cup \mathcal{Y}_{(01,d_2)}$$

shown in Fig. 4. If we consider instead the organizational structure $\mathbf{O}$ of Example 2.2 and we start from the policy vector $x = 01$, we have $p_i(01, \mathbf{O}) = 01$ for $i = 1, 2, 3$, and hence we get a domination path that ends in 01.

Actually, all domination paths in $\mathbf{O}$ starting from $y = 21$ end up in $x = 01$, while all domination paths in $\mathbf{O}'$ starting from $y = 21$ reach a cycle.

We can then give the following key definition. A *local equilibrium* for an organizational structure $\mathbf{O}$ is a policy vector $x \in X$ such that at least one domination path in **O** ends in it, i.e. $p_i(x, \mathbf{O}) = x$ for each agent a_i with $i = 1, \dots, h$. In general, more than one domination path ends in a local equilibrium and there may be more than one local equilibrium.

Example 2.4 The policy vector $x = 01$ in Example 2.2 is a local equilibrium for the organizational structure **O**. Indeed a domination path starting from $x = 01$ ends up in it. In Example 2.2 it is easy to verify that 01 is the only local equilibrium for **O**. On the contrary, the organizational structure $\mathbf{O}'$ in Example 2.3 has no local equilibria.

Remark 2.5 Consider a domination path ordered along an agenda of an organizational structure. If the agenda is repeated over and over again, then the ordered domination path ends either in a local equilibrium or in a cycle. This is a consequence of the fact that the agenda α and the set of policy vectors X are both finite sets. Indeed a domination path is given, at any step, by a couple (x_i, α_{k_j}) which uniquely determines the subsequent node x_{i+1}. Since both X and α are finte sets, $X \times \alpha$ is finite, and this implies that if the domination path doesn't "stop" in a local equilibrium, it will eventually enter the same couple (x_i, α_{k_j}) again, giving rise to a cycle.

We can now describe the decision process, which proceeds along the following steps:

- Start from a *status quo* policy vector, i.e. a given $x_0 \in X$.
- Take the first set d_{k_1} of policy items in the agenda α, and look for the preferred neighbor of x_0 for the agent a_{k_1}:
  - If there is one, "move" to it;
  - Otherwise "stay" in x_0.
- Repeat for all policy items in the agenda.
- Repeat until either a local equilibrium or a cycle are encountered.

Hence a decision process gives rise to a domination path ordered along an agenda α, starting from a status quo x_0 and ending either in a cycle or in a local equilibrium.

3 Local equilibria

In this section we will give necessary and sufficient conditions for a policy vector to be a local equilibrium for at least one organizational structure when a set of agents is given. The question we are addressing is whether and under which conditions a principal can obtain her own preferred policy vector by delegating decisions to agents holding any preference profile. In a sense we formalize a "divide and conquer" process where, by properly dividing the decision making labour and properly allocating decisions to a set of agent, the principal can obtain her preferred outcome without using authority nor incentives, but letting agents free to decide on the policies allocated to each of them.

The following theorem states that x is a local equilibrium if and only if there exists a partition of the set of policies such that, for each subset of policies assigned to agent a_i, x is a preferred neighbour for a_i.

3.1 Main theorem

In order to state the theorem, we first need to define, for each agent a_i, $i = 1, \dots, h$, the following set of *best policy assignments*:

$$\mathcal{A}_i = \{(x, d) \in X \times \mathcal{P}(P) \mid p_i(x, \mathbf{O}) = x \text{ for all } \mathbf{O} \text{ s.t. } \mathbf{O}(a_i) = d\}. \tag{3}$$

Of course this set will depend on the specific policy vector x under consideration.

Theorem 3.1 *A policy vector $x \in X$ is a local equilibrium for at least one organizational structure $\mathbf{O}$, if and only if there exists a set $D = \{d_1, \dots, d_h\}$ such that $(x, d_i) \in \mathcal{A}_i$ for $i = 1, \dots, h$ and*

$$\bigcup_{i=1}^{h} d_i = P \text{ with } d_i \cap d_j = \emptyset, \forall i \neq j.$$

Proof If there exists a set $D = \{d_1, \dots, d_h\}$ with $(x, d_i) \in \mathcal{A}_i$ for $i = 1, \dots, h$ such that

$$\bigcup_{i=1}^{h} d_i = P \text{ with } d_i \cap d_j = \emptyset, \forall i \neq j,$$

then we consider the organizational structure $\mathbf{O}$ such that $\mathbf{O}(a_i) = d_i$ for $i = 1, \dots, h$. With respect to $\mathbf{O}$ the domination path (which does not depend on the agenda) starting in x remains in x, which is therefore a local equilibrium.

Vice versa, if x is a local equilibrium for an organizational structure $\mathbf{O}$ (such that $\mathbf{O}(a_i) = d_i$ for $i = 1, \dots, h$) then $p_i(x, \mathbf{O}) = x$ for $i = 1, \dots, h$, that is $D = \{d_1, \dots, d_h\}$ is the required set. In fact, $(x, d_i) \in \mathcal{A}_i$ for $i = 1, \dots, h$ and

$$\bigcup_{i=1}^{h} d_i = P \text{ with } d_i \cap d_j = \emptyset, \forall i \neq j,$$

□

Example 3.2 In Example 2.1 the power set $\mathcal{P}(P)$ of the set of policies P is given by $\mathcal{P}(P) = \{\emptyset, \{p_1\}, \{p_2\}, \{p_1, p_2\}\}$. So, for example, by definition of best policy pairs, we get that the set of the best policy pairs for the agent a_1 will be given by

$$\mathcal{A}_1 = \{(x, d) \in X \times \mathcal{P}(P) \mid p_1(x, \mathbf{O}) = x \text{ for all } \mathbf{O} \text{ s.t. } \mathbf{O}(a_1) = d\}.$$

Hence, in particular, if $\mathbf{O}(a_1) = d = \emptyset$ then $p_1(x, \mathbf{O}) = x$ for any $x \in X$ by definition. If $\mathbf{O}(a_1) = d = \{p_1\}$ then $x = 00$ and $x = 01$ are the only two nodes which satisfy $p_1(x, \mathbf{O}) = x$. This can be seen by looking at the upper left graph (the one related to a_1) in Fig. 2. Analogously, looking at the upper left graph in Fig. 3 corresponding to the case $\mathbf{O}(a_1) = d = \{p_2\}$, we get that $x = 01, x = 10$ and $x = 20$ are the nodes which satisfy $p_1(x, \mathbf{O}) = x$. Finally if $\mathbf{O}(a_1) = d = \{p_1, p_2\}$ then $x = 01$ is the only node such that $p_1(x, \mathbf{O}) = x$ and we get:

$$\begin{aligned}\mathcal{A}_1 = \{&(00, \emptyset), (00, \{p_1\}), (01, \emptyset), (01, \{p_1\}), (01, \{p_2\}), (01, \{p_1, p_2\}),\\ &(10, \emptyset), (10, \{p_2\}), (11, \emptyset), (20, \emptyset), (20, \{p_2\}), (21, \emptyset)\}.\end{aligned}$$

Analogously for the agents a_2 and a_3 we get the following sets of best policy pairs:

$$\begin{aligned}\mathcal{A}_2 = \{&(00, \emptyset), (00, \{p_1\}), (01, \emptyset), (01, \{p_2\}), (10, \emptyset), (11, \emptyset),\\ &(11, \{p_2\}), (20, \emptyset), (21, \emptyset), (21, \{p_1\}), (21, \{p_2\}), (21, \{p_1, p_2\})\},\\ \mathcal{A}_3 = \{&(00, \emptyset), (00, \{p_1\}), (00, \{p_2\}), (00, \{p_1, p_2\}), (01, \emptyset), (10, \emptyset),\\ &(11, \emptyset), (11, \{p_1\}), (11, \{p_2\}), (20, \emptyset), (20, \{p_2\}), (21, \emptyset)\}.\end{aligned}$$

Hence there are four policy vectors – 00, 01, 11, and 21 – that can be local equilibria for some organizational structure. In particular:

- 01 is a local equilibrium for the organizational structures $d_1 = \{p_1, p_2\}$, $d_2 = d_3 = \emptyset$ and $d_1 = \{p_1\}, d_2 = \{p_2\}, d_3 = \emptyset$;
- 21 is a local equilibrium for $d_2 = \{p_1, p_2\}, d_1 = d_3 = \emptyset$;
- 00 is a local equilibrium for $d_3 = \{p_1, p_2\}$, $d_1 = d_2 = \emptyset$ and $d_1 = \emptyset$, $d_2 = \{p_1\}$, $d_3 = \{p_2\}$;
- 11 is a local equilibrium for $d_1 = \emptyset, d_2 = \{p_2\}, d_3 = \{p_1\}$.

It is also worth pointing out that 11 is a policy vector that is a local equilibrium but is not the preferred choice of any agent.

3.2 The algorithm

We now describe an algorithm which finds all local equilibria for a given set of agents $\{a_1, \dots, a_h\}$ and for different organizational structures. This algorithm is a straightforward application of Theorem 3.1. It goes through two main steps:

Step A For $i = 1, \dots, h$, construct the set $\mathcal{A}_i$.

Step B For each $x \in X$, consider the subsets

$$\mathcal{A}_i(x) = \{(x, d) \in X \times \mathcal{P}(P) \mid p_i(x, \mathbf{O}) = x \text{ for all } \mathbf{O} \text{ s.t. } \mathbf{O}(a_i) = d\} \subset \mathcal{A}_i$$

for $i = 1, \dots, h$, and search for h best policies assignment $(x, d_1), (x, d_2), \dots, (x, d_h)$ such that

- $(x, d_i) \in \mathcal{A}_i(x)$ for all $i = 1, \dots, h$,

- $\{d_1, d_2, \dots, d_h\}$ satisfies (1).

If we find such h best policies assignment, the policy vector x is a local equilibrium for the organizational structure $\mathbf{O}$ such that $\mathbf{O}(a_i) = d_i$; otherwise, x is not a local equilibrium for any organizational structure (Theorem 3.1).

Let us describe the two steps in details. Step A consists of the following operations, which are repeated for $i = 1, \dots, h$:

Step A1 List all pairs $(x, d) \in X \times \mathcal{P}(P)$ and construct a first version of $\mathcal{A}_i$.

Step A2 For each couple $(x, y) \in X \times X$ such that $y \succ_i x$ remove all pairs (x, d) from $\mathcal{A}_i$ such that d contains all the policy items by which the policy vectors x and y differ.

Step A3 For each $x \in X$, the pairs (x, d) that remain after Step A2 are the elements of $\mathcal{A}_i(x)$.

Step B is recursive and, for each $x \in X$, repeatedly fixes one best policy assignment (x, d_i) for each agent, finally ending up with $\{d_1, d_2, \dots, d_h\}$. Let us describe the recursive step that depends on an integer i indexing the agents:

Step B-Rec(1) For each pair $(x, d_1) \in \mathcal{A}_1(x)$ we fix it and go to the recursive Step B-Rec(2).

Step B-Rec(i) In the recursive step at level i (with $1 < i < h$), we suppose we have found $(x, d_j) \in \mathcal{A}_j(x)$ for $j = 1, \dots, i-1$ such that $d_j \cap d_{j'} = \emptyset$ if $j \neq j'$. For each pair $(x, d_i) \in \mathcal{A}_i(x)$ such that $d_i \cap d_j = \emptyset$ for $j = 1, \dots, i-1$ we fix it and go to the recursive Step B-Rec($i+1$).

Step B-Rec(h) We suppose we have found $(x, d_j) \in \mathcal{A}_j(x)$ for $j = 1, \dots, h-1$ such that $d_j \cap d_{j'} = \emptyset$ if $j \neq j'$. For each pair $(x, d_h) \in \mathcal{A}_h(x)$ such that $d_h \cap d_j = \emptyset$ for $j = 1, \dots, h-1$ we fix it.

For each $x \in X$, we start from Step B-Rec(1), and we can end in two possible ways:

- If a set $\{d_1, d_2, \dots, d_h\}$ such that $\bigcup_{i=1}^{h} d_i = P$ is found, the policy vector x is a local equilibrium for the organizational structure $\mathbf{O}$ such that $\mathbf{O}(a_i) = d_i$ (Theorem 3.1);
- If Step B-Rec(1) ends without finding any set $\{d_1, d_2, \dots, d_h\}$ such that $\bigcup_{i=1}^{h} d_i = P$, x is not a local equilibrium for any organizational structure (Theorem 3.1).

The time complexity of the algorithm is at most $O(M^{O(h)}h!)$. For the sake of simplicity, we have bound the complexity with lazy computations, but sharper ones would not improve the result. In order to compute the complexity let us start from Step A, which is the fastest one. The three Steps A1, A2 and A2 take $O(M2^n)$, $O(M^2 n)$ and $O(1)$ time, respectively, which add up to $O(M^2 \log M)$ time (because $2^n \leqslant M$). Since the three Steps must be repeated h times, Step A takes $O(hM^2 \log M)$ time. Instead, Step B is recursive. Step B-Rec(1) takes $O(2^{3n})$ time, because $\mathcal{A}_i(x)$ may have at most 2^n elements, it calls Step B-Rec(2) at most 2^n times, and must be repeated 2^n times (once for each $x \in X$). Step B-Rec(i) for $2 \leqslant i \leqslant h-1$ takes $O(2^{2n} n(i-1))$ time because the check $d_i \cap d_j = \emptyset$ for $j = 1, \dots, i-1$ takes $O(n(i-1))$ time, for each $(x, d_i) \in \mathcal{A}_i(x)$, and it calls Step B-Rec($i+1$) at most 2^n times. Step B-Rec(h) takes $O(2^n n(h-1))$ time because the check $d_h \cap d_j = \emptyset$ for $j = 1, \dots, h-1$ takes $O(n(h-1))$ time, for each $(x, d_h) \in \mathcal{A}_h(x)$. The times of Steps B-Rec($*$) multiply to $O(M^{2h}(\log_2 M)^{h-1}(h-1)!)$, indeed we have

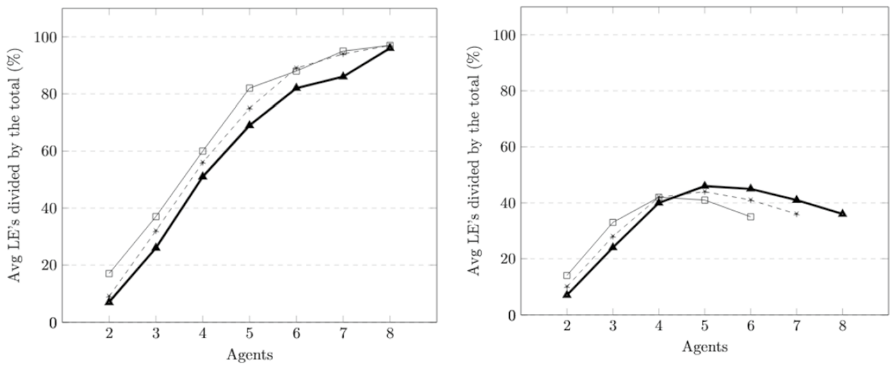
$$2^{3n}\left(\prod_{i=2}^{h-1} 2^{2n} n(i-1)\right) 2^n n(h-1) \leqslant M^{2h}(\log_2 M)^{h-1}(h-1)!$$

The algorithm can be useful in real life only when very few agents and policy vectors are involved in the decision. Indeed, apart from the factorial time which makes the algorithm useless when many agents are involved, the algorithm is useless also when many policy vectors are involved in the decision, because, in order to use the algorithm, it is necessary to know the preferences of all agents (i.e., the whole $\succ_i$ for every $i = 1, \dots, h$)—which is also unusual in real life.

4 Numerical results

In this section we present some numerical results obtained with the computer program `FLEOSStat`, written by the first author of this paper, which implements the algorithm described in the previous section. The computer code and a more detailed description of its functionalities, together with instructions on how to run the program are freely available.[9] The program takes as inputs the number of values of

[9] The reader can find the computer code along with a detailed description in the web page: http://www.dm.unipi.it/~amendola/files/software/fleosstat/.



(a) **(b)**

Fig. 5 Average number of local equilibria when we have 6(thin, square), 7(dashed, star) or 8(thick, triangle) binary policies. Some agents may be idle in **a** whereas all agent must be active in **b**

each policy, the number of random agents, and the number of repetitions, and it works as follows:

- It repeatedly
    - Creates a random set of agents,
    - Computes the number of local equilibria;
- It collects and outputs the following results:
    - The number of local equilibria,
    - The percentage of cases where a local equilibrium is equal to the principal's most preferred policy vector.

In the following subsections we summarize the main numerical results for binary and non binary cases.

4.1 Binary policies

We start by comparing two cases where we increase the number of agents from 2 to 8, observing the behaviour of the results listed above when we have 6, 7 and 8 binary policies. We consider two different scenarios: in the first scenario the principal is free to choose any organizational structure, including those in which some agent are left idle and not delegated any decision. Results on this scenario are reported in Figs. 5a and 6a. In the second scenario instead the principal must delegate at least one policy to each agent and none of the latter may be left idle. Results on this second scenario are reported in Figs. 5b and 6b.

In Fig. 5 we report the number of local organizational equilibria as the number of agents increases. While in the case in which each agent must be assigned at least one policy (panel b) we have an inverted U-shaped curve, in the case in which the

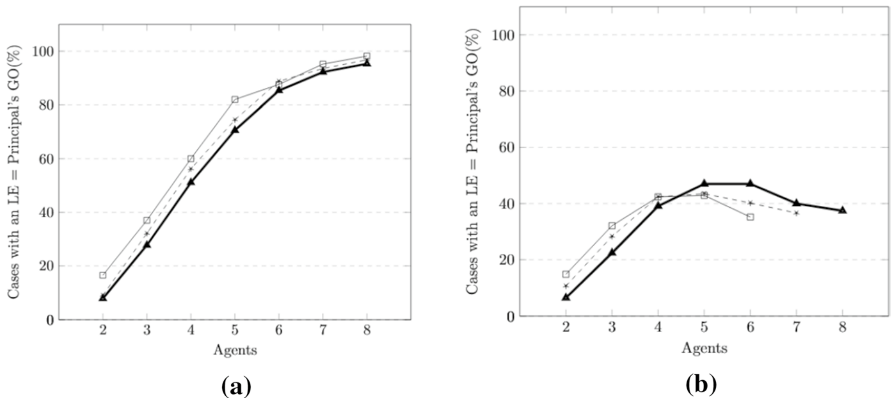


Fig. 6 Percentage of cases in which the principal can obtain her own preferred policy vector. Cases with 6(thin, square), 7(dashed, star) or 8(thick, triangle) binary policies. Some agents may be idle in **a** whereas all agent must be active in **b**

Principal has the possibility to leave some agents idle, we observe, as expected, a uniform increase in the number of local equilibria as we increase the number of agents.

Let us now study the probability with which the principal may get by delegation her most preferred policy vector. Results are reported in Fig. 6a for the case in which some agents can remain idle and in Fig. 6b when instead all agents must be assigned at least one policy. In the former case we observe a steep increase of such a probability. In the latter case instead we observe an inverted U-shaped curve, where the percentage rises until the number of agents is slightly over half the number of policies and then decreases as we approach the maximum number of agents. We also see that, in the case of Fig. 6a, the principal will be able to find a local equilibrium equal to her preferred choice almost 100% of the times provided she has a large enough number of agents. This is not true in the case of Fig. 6b, where her best chances lie at most in the 40–50% range with a number of agents close to half of the number of policies to be assigned.

4.2 Probability in the binary case

Similarity between Figs. 5 and 6 suggests that the probability for the Principal to get her preferred outcome only depends on the number of local equilibria. Indeed the probability for the principal's preferred policy vector to be a local equilibrium for the organization equals the probability that a given policy vector x – the principal's best – is a local equilibrium for some organizational structure $\mathbf{O}_x$. Let us denote by x_i the vector which differs from x only for the policy i, i.e. if $x = v_1 \dots v_i \dots v_n$ then $x_i = v_1 \dots \overline{v_i} \dots v_n$ where $\overline{v_i} = 0$ if $v_i = 1$, or $\overline{v_i} = 1$ if $v_i = 0$. In order for x to be a local equilibrium for an organization, there must exist at least one agent who prefers x over x_i for each policy p_i. Let L_x^i denote the event that there exists an agent $a \in \mathcal{A}$ that prefers x to x_i, we are interested in computing

$P(L_x^i)$. In order to do this, let us note that the probability that x is preferred to x_i by one agent is $\frac{1}{2}$. This means that the probability of the complement, x_i is preferred to x, is also $\frac{1}{2}$. It follows that the probability that x_i is preferred to x for all agents is $(\frac{1}{2})^h$ since we have h agents whose preferences are independent. It follows that

$$P\left(L_x^i\right) = 1 - \frac{1}{2^h} \quad .$$

Hence, if L_x is the event in which for every p_i there exists an agent who prefers x over x_i, then

$$P(L_x) = \prod_{i=1}^{n} \left(1 - \frac{1}{2^h}\right) = \left(1 - \frac{1}{2^h}\right)^n$$

which increases as the number of agents h increases, rapidly approaching 1. By the previous remark, we assume that the probability that x is a local equilibrium when we have n policies and h agents is compatible with the probability $P(L_x)$ conditioned on the fact that there exists at least one organization satisfying Theorem 3.1 among the ones which assign policy p_i to one of the agents for whom x is preferred to x_i. That is, if $\mathbf{O}_x$ is the event that such an organization exists, if some agents may not be assigned a policy and the number of agents h is sufficiently large, then $P(\mathbf{O}_x) \simeq P(L_x)$. Indeed an organization $\mathbf{O}_x$ such that $d_i = \{p_i\}$ and $(x, d_i) \in \mathcal{A}_i$, $i = 1, \dots, n$ satisfies the conditions of Theorem 3.1 and it exists with probability $\prod_{i=0}^{n-1}(1 - \frac{1}{2^{h-i}})$ which goes rapidly to 1 if we increase the number of agents h and keep the number of policies *n fixed*.

If instead all agents must be assigned at least one policy then only organizations $\mathbf{O}_x$ for which $\mathbf{O}_x(a_i) \neq \emptyset$ for any agent a_i in $\mathcal{A}$ are admissible. Since there are $\binom{n}{h}$ ways to assign n policies to the h agents, that is $\binom{n}{h}$ admissible organizations among n^h possible organizations, then $P(\mathbf{O}_x) \sim \frac{\binom{n}{h}}{n^h} \cdot P(L_x)$ and the final probability follows $\frac{\binom{n}{h}}{n^h}$ when n grows since $P(L_x)$ rapidly goes to 1.

4.3 Non binary polices

In this case, we look at the effect that increasing the number of alternatives for each policy has on the results. Let us for instance consider the case with four agents. Figure 7 shows that, regardless of whether agents must be assigned a policy or not, increasing the number of alternatives determines a fall in the percentage of cases with a local equilibrium equal to the principal's optimum. We also see that, in line with the previous results, imposing that all agents must be assigned at least one policy sharply decreases this percentage.

To conclude, the principal's possibilities to obtain her best preferred policy vector are higher when the number (and therefore the diversity) of agents increases, the number of alternatives for each policy is small, and some agents can be left idle without any decision assigned to them.

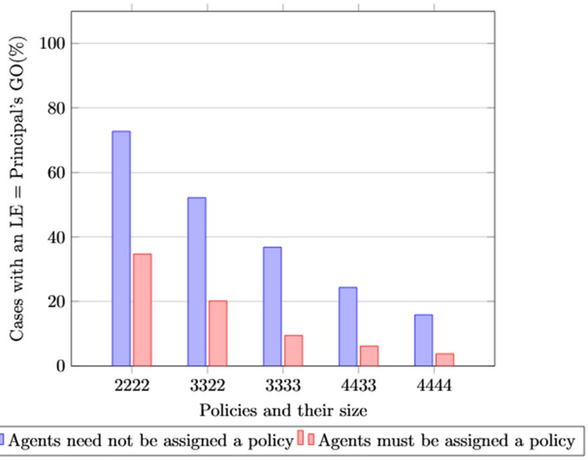


Fig. 7 The effect on the percentage of cases with a local equilibrium equal to the principal's optimum as we change policy size with 4 agents

5 Choosing agents of your own kind

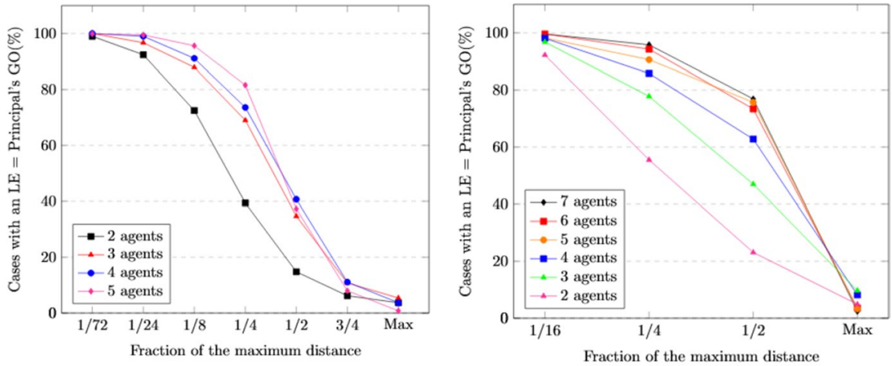
In the previous sections we have assumed that the principal must employ agents with random and therefore potentially very diverse preferences. In this section instead we investigate the case in which the principal can select agents whose preferences are relatively similar to hers. How similar do the agents' preferences have to be to the principal's, and how does this similarity affect the probability that she gets her most preferred policy implemented? In order to investigate these questions, we must first define a notion of distance between preferences.

For the sake of simplicity in this section we focus only on the case in which all agents must be assigned at least one policy, which is, as we showed in the previous sections, the case in which it is harder for the principal to have her own objectives pursued.

Given two systems $\succ_1$ and $\succ_2$ of transitive and strict preferences on X, i.e. two total orders on X, we define the distance between $\succ_1$ and $\succ_2$ as the minimum number of contiguous exchanges which must be performed on one order to make it equal to the other. For example, the two orders $1 \succ_1 2 \succ_1 3 \succ_1 4$ and $3 \succ_2 1 \succ_2 2 \succ_2 4$ differ by two contiguous exchanges,[10] and therefore the distance between them is 2. This distance is known as the Kendall τ rank distance or bubble-sort distance (named after the bubble-sort algorithm) and is widely used in statistics to measure, for instance, rank correlation.

In terms of the graphs associated to the two total orders $\succ_1$ and $\succ_2$, each exchange corresponds to inverting the direction of one arc.

[10] Starting e.g. from $1 \succ_1 2 \succ_1 3 \succ_1 4$ we can exchange 2 with 3 and obtain $1 \succ_1 3 \succ_1 2 \succ_1 4$ and then exchange 1 with 3 to obtain the same order of $\succ_2$.



(a) Case of 5 policies with sizes (2,2,3,3,4). **(b)** Case of 7 binary policies.

Fig. 8 Percentage of cases in which the organization has a local equilibrium equal to the principal's optimum as a function of the maximum distance between the principal's and the agents' orders

We chose Kendall's τ rank as our notion of distance because it seems plausible in our framework. Indeed it is reasonable to assume that a principal with preferences $1 \succ 2 \succ 3 \succ 4$ will perceive someone with preferences $3 \succ 1 \succ 2 \succ 4$ as closer than someone with preferences $4 \succ 3 \succ 2 \succ 1$. Of course this choice remains largely arbitrary and different notions of distance could be used.

If we want to consider only agents whose distance from the principal is at most m, then for each agent we take a random integer d with $0 \leqslant d \leqslant m$ and generate his preferences by performing d random contiguous exchanges on the principal's preferences (avoiding exchanges that simply undo previous ones).

Let us now look at the effect that fixing a maximum distance has on the probability for the principal to get her most preferred policy vector as a local equilibrium for the organization. An example is reported in Fig. 8a, that presents the case in which five policies must be allocated to a number of agents ranging from 2 to 5 and each agent has a maximum distance from the principal which is reported on the horizontal axis as a percentage of the maximum possible distance (which is 10296). The five policies are a mixture of binary and non-binary with 2 binary, 2 ternary and 1 of size 4. Obviously the percentage of cases in which the organization has a local equilibrium equal to the principal's optimum approaches 100 when the distance between principal and agents goes to zero and quickly drops when this distance increases, going to 0 when the distance reaches its maximum. It is however worth pointing out that this decrease is steeper the lower the number of agents. For instance if we take a distance equal to 1/4 of its maximum, the principal can get her best preferred policy vector only about 40% of the cases if she employs 2 agents and 80% of the cases if she employs 5 agents. Once again, number and therefore diversity of agents strongly increases this probability.

Another example is reported in Fig. 8b, with seven binary policies to be allocated to a number of agents ranging from 2 to 7. Here the difference between the case with only 2 agents and the one with one agent for all the policies is even sharper. With a distance of 1/2 of its maximum, the principal can get her best preferred policy vector only about 23% of the cases if she employs 2 agents and 75% of the cases if she employs 7 agents.

6 Conclusions

In this paper we have presented a graph-theoretic model of delegation in an organization that is called to take decisions on several interdependent policies. In our model the principal does not take any decision directly but can only partition the set of decisions and allocate each part to a different agent. We have discussed how and under which conditions this "divide-and-conquer" strategy may allow the principal to obtain from the agents the decisions she prefers. We have shown that having a relatively large set of diverse agents increases the possibilities for the principal to successfully apply this strategy, and showed that diversity among agents plays a key role.

This paper is only a rather preliminary investigation and our model is based on a couple of assumptions which are rather unrealistic and should be relaxed in future work. First, we suppose that the principal can be an optimal engineer of delegation, meaning that she knows the preferences of every agent and can therefore optimally allocate decisions to them. We do acknowledge that this assumption is far from being realistic and strongly limits the applicability of our approach. However we believe that our model offers a rather original perspective on the use of delegation inside organization, which could fruitfully complement existing theories and empirical works.

The standard principal-agent model takes delegation as the source of agency problems. In so far as the principal must delegate some actions or decisions to an agent and the latter cannot be perfectly monitored, incentive contracts must be designed to motivate the agent to pursue the organizational goals. However we know that in the presence of uncertainty and information asymmetries, agency costs and inefficient contracts are the norm. The perspective we offer here is that delegation could on the contrary mitigate the need of incentive contracts by partly (if not optimally as discussed in our model) aligning the agents' decisions with the principal's goals. This—we argue—is a feature which emerges when the object of delegation are sets of interdependent decisions, and the principal can delegate specific subsets thereof.

Indeed the type of optimal delegation which we investigate in this paper requires that the principal has perfect knowledge of the agents' preferences. A possible way to relax this unrealistic assumption is to assume that the principal learns the preferences of the agents by observing their behaviour and modifies adaptively the organizational structure as a consequence of learning. In such a model the organizational structure would have a double role: sampling and learning the preferences and behaviours of the agents and getting appropriate decision from them, similarly to what we find in exploration versus exploitation models (March 1991).

Second, we have assumed that the agents' preferences are fixed and that their choices strictly reflect such preferences, without any possibility of strategic behaviour and strategic misrepresentation of one's preferences. Both phenomena are indeed very common in real organizations, and indeed organizations can partly shape and modify the preferences of their members. Also in this case, without resorting to a game-theoretic model with fully rational agents, an agent-based model with boundedly rational and adaptive agents who are capable of limited and myopic strategizing could provide valuable insight.

Finally, we considered here the engineering of delegation as an alternative to incentives and developed a model in which the principal can only act on delegation but not on incentives. Indeed, delegation and incentives could complement each other and incentives could be used to support delegation and to compensate for sub-optimal delegation structures which may emerge from the principal's imperfect knowledge of the agents' preferences. Models which study the relations between delegation and incentives do already exist (Bester and Krahmer, 2008, e.g.), but our perspective could extend them to complex sets of interdependent decisions that typically characterize real organization. We plan to pursue this line of research in the near future.

Funding Open access funding provided by Università degli Studi di Torino within the CRUI-CARE Agreement.

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