Life-Cycle Risk-Taking with Personal Disaster Risk*

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Abstract

Inspired by a growing body of empirical work, this paper models a non-linear labor income process allowing for a personal disaster, such as long-term unemployment or disability, during working years. Such a disaster entails an uncertain but potentially large permanent shock to earnings. Personal disaster risk allows to match moderate risk-taking of young investors and a flat investment profile in age, observed in the United States, when the calibration of both the disaster probability and the expected permanent loss in the disaster state is conservative.

Keywords: disaster risk, beta distribution, life-cycle portfolio choice, non-linear income process, unemployment risk, disability risk.

JEL classification: D15, E21, G11

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1 Introduction

What happens if a person's future ability to work is permanently reduced? Insurance against permanent shocks, such as disability and long-term unemployment, is well known to be incomplete (Guvenen and Smith, 2014; Low, Meghir and Pistaferri, 2010; Low and Pistaferri, 2015). Therefore, young households make provisions to cushion against a personal disaster, even if the possibility of its occurrence is quite rare. Against this background, the current paper examines the pattern of self-insurance in financial markets over the life-cycle when there is the possibility of a rare personal disaster during working years.

The findings show that personal disaster risk can alter lifetime *ex-ante* investment choices for the average worker, even if *ex-post* most workers will not experience a disaster. Additionally, this paper reveals that uncertainty about the size of uninsured human capital losses, which characterizes rare disasters, enhances the precautionary behavior of young workers. This behavior will result in lower risk-taking at the beginning of working life, with respect to a comparable deterministic human capital loss. Uncertainty about the extent of losses is crucial in order to closely match the observed age profiles of US investors from 1992 to 2016, based on the methods of Ameriks and Zeldes (2004), when the calibrations are conservative.

We contribute to the household finance literature by linking risk-taking in financial markets to the *ex-ante* uncertain, but potentially extreme, permanent impact of income shocks. In contrast to this literature, however, we go beyond the positive probability of zero labor income implied by the linear income process proposed by Cocco, Gomes and Maenhout (2005) and later widely adopted in the field. Inspired by a growing body of empirical work showing that earnings dynamics display non-linearities (Karahan and Ozkan, 2013; Arellano, Blundell and Bonhomme, 2017; De Nardi, Fella and Paz-Pardo, 2020; Inkmann, 2020; Sanchez and Wellschmied, 2020; Guvenen, Karahan, Ozkan and Song, 2021; Shen, 2021; Catherine, 2021; Galvez and Paz-Pardo, 2023), we model the occurrence of a disaster that brings about a permanent income reduction of uncertain proportion.

Specifically, the fraction of human capital lost follows a *Beta* distribution.[1] The flexibility of such distribution allows us to concentrate a large probability mass on small values of proportional human capital reduction while leaving open the possibility of extremely unlikely but devastating realizations. This feature of the model is intended to capture the substantial heterogeneity of permanent effects following the occurrence of adverse occupational and/or health shocks documented by Guvenen, Karahan, Ozkan and Song (2021) and Gregory, Menzio and Wiczer (2021).

Importantly, when careers are calibrated to broadly match observed US labor market features, optimal investment in the risky asset remains flat over the whole working life, in line with early evidence on US portfolios (Ameriks and Zeldes, 2004), which we update to 2016. This situation occurs even when we account for the large insurance coverage of permanent income shocks, as in Guvenen and Smith (2014) and Guvenen, Karahan, Ozkan and Song (2021), which may ultimately reduce the expected human capital losses due to long-term unemployment. Without disaster risk, the implied optimal stock holding still counter-factually decreases with age before retirement, unless the long-term unemployment rate is as high as

[1] This distribution may also characterize the damage caused by natural disasters (see Bhattacharjee, 2004; Lallemant and Kiremidjian, 2015).

that observed in the post-Great Recession and protracted inactivity causes a certain and relatively large human capital drop for all long-term unemployed, as in Bagliano, Fugazza and Nicodano (2019).

With respect to the latter paper, we calibrate long-term unemployment risk in a more realistic fashion, and model permanent earnings losses due to long unemployment spells as stochastic, rather than deterministic. This new setting allows to see clearly the true driving force of the flat risky investment age profile: it is not the mere prospect of a permanent human capital loss, but the possibility that, though only in extremely rare cases, that loss amounts to a substantial fraction of future permanent earnings (the "rare disaster" scenario), an occurrence which is absent in the existing literature.

Our results highlight the role of non-linear income shocks in flattening the age profile of risk taking. With a linear income process, prior models resort to using additional features to explain reduced risk-taking in financial markets (Cocco, 2004; Munk and Sorensen, 2010; Kraft and Munk, 2011; Bagliano, Fugazza and Nicodano, 2014; Hubener, Maurer and Mitchell, 2016; Chang, Hong and Karabarbounis, 2018; Branger, Larsen and Munk, 2019). For instance, in Bagliano, Fugazza and Nicodano (2014), a positive correlation between (highly volatile) permanent income shocks and stock returns leads to lower optimal risk-taking when young. Branger, Larsen and Munk (2019) obtain reduced early-life holdings by setting the probability of losing a job and experiencing a subsequent human capital loss as decreasing in the worker's age and in the state of the economy. Finally, in Chang, Hong and Karabarbounis (2018) the driver of low risk-taking when young is related to uncertainty which resolves over time thanks to agents' learning about their income volatility. The resolution of uncertainty due to this learning process explains why the young bear more labor income risk, an intuition pioneered by Viceira (2001) and Benzoni, Colling-Dufresne and Goldstein (2007).

In our paper, uncertainty resolves because time passes without the occurrence of disasters. More precisely, we model working life careers as a three-state Markov chain driving the transitions between employment, short-term unemployment and personal disaster states. Uncertain permanent earning losses that occur in the disaster state, represent productivity loss due to long-term unemployment (Arulampalam, 2001; Schmieder, von Wachter and Bender, 2016), disability or both (Low and Pistaferri, 2015).

This model nests the traditional life-cycle framework within the household finance literature. Indeed, when the disaster probability is zero and/or human capital erosion is compensated by full insurance, the agents optimally reduce exposure to risky assets as they approach retirement. This pattern obtains since human capital provides a hedge against shocks to stock returns, which makes bearing financial risk generally acceptable. Investment in stocks should therefore be relatively high at the beginning of working careers, when human capital is large relative to accumulated financial wealth. Risky investment then gradually declines until retirement, as human capital decreases relative to financial wealth.

When personal disaster risk is instead only partially insured, the above effect is moderated by the resolution of uncertainty concerning labor and pension income as the worker safely approaches retirement age.[2] Since the risk of a personal disaster declines as an individual approaches retirement, the resolution of uncertainty compensates for the hedge effect and

[2]We do not model the option to change labor supply to buffer income shocks, as in Bodie, Merton and Samuelson (1992) and Gomes, Kotlikoff and Viceira (2008). This option is open to those who find a new job, while what drives our results is the *ex ante* possibility of a large loss in the disaster state.

the optimal investment in stocks is relatively flat over the life cycle.

Our model delivers additional implications concerning life-cycle choices in the context of incomplete insurance against personal disaster risk. First, the distribution of optimal consumption growth becomes negatively skewed, due to disasters, in line with evidence on durable consumption growth (Yang, 2011). Second, personal disaster risk changes the age profile of savings thereby shrinking the heterogeneity of optimal portfolio choices across agents characterized by different career histories. Young workers increase early precautionary savings to buffer against possible, albeit rare, future disasters. Optimal consumption consequently declines during the early years but increases during both late working years and retirement. Third, the average implied savings to income ratio increases, as in other life-cycle models highlighting the role of earnings shocks for solving life-cycle portfolio choice puzzles (Bagliano, Fugazza and Nicodano, 2014; Chang, Hong and Karabarbounis, 2018). While the implied savings to income ratio may appear counter-factually high, our model does not incorporate the effects of means-tested welfare programs that lead to zero optimal precautionary saving for poor households (Hubbard, Skinner and Zeldes, 1995).

This model does not address non-participation in the stock market. It should otherwise allow for correlation between stock returns and labor income shocks (see Bagliano, Fugazza and Nicodano (2014) for additional conditions and Bonaparte, Korniotis and Kumar (2014) for empirical results) or correlation between stock returns and the skewness of labor income shocks (Catherine, Sodini and Zhang, 2020). Likewise, prominent papers study consumption and labor market choices with permanent income shocks (Low, Meghir and Pistaferri, 2010; Low and Pistaferri, 2015), focusing on the design of social insurance against employment and productivity risk without allowing for investments in risky assets.

Our paper belongs to the household finance tradition that allows for risky investments but overlooks both moral hazard stemming from social insurance programs and the associated difference between productivity and employment risk. Finally, the personal disaster risk modelled in this paper differs from both the individual stock market disaster in Fagereng, Gottlieb and Guiso (2017) and the aggregate economic collapse in the macro-finance literature (Barro, 2006), although disasters may be correlated. As Arellano, Blundell and Bonhomme (2017) point out, macroeconomic disasters are statistically elusive events, while disasters at the micro level happen all the time.

The rest of the paper is organized as follows. In Section 2 we provide evidence on life-cycle portfolio holdings and institutional details on long-term unemployment, disability and social insurance for the United States. Section 3 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 4 and discuss our main results in Section 5, where the ability of the model to match the stock-holdings observed in real data is also assessed. Section 6 concludes the paper. A Supplementary Appendix provides additional robustness results.

2 Households portfolios and personal disasters

This section presents the main stylized facts concerning financial risk-taking and personal disaster risk in the United States. The first subsection builds on the method of Ameriks and Zeldes (2004) to examine the empirical relationship between age and conditional risky shares, i.e. the fraction of financial wealth held in risky assets conditional on participation in the stock market. These life-cycle investment profiles in US data will later be matched with the model-implied profiles. Since such profiles are calibrated to disaster risk, the second subsection summarizes some relevant features of disability and long-term unemployment.

2.1 Life-cycle profiles of households portfolios

We pool data from the independent cross-sectional surveys in the Survey of Consumer Finances (SCF), covering the years from 1992 to 2016. The SCF is nationally representative of households in the United States and collects detailed information on their characteristics and investment decisions. Following Chang, Hong and Karabarbounis (2018), we classify the households' financial assets into two categories, *safe* and *risky*. Safe assets include: checking accounts, savings accounts, money market accounts, certificates of deposit, the cash value of life insurance, US government and state bonds, mutual funds invested in tax-free bonds and government-backed bonds, and trusts and annuities invested in bonds and money market accounts. Risky assets include: stocks, stock brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stocks, trusts and annuities invested in stocks or real estate, and pension plans that are thrift, profit-sharing, or stock purchase plans. In Table 1, we report the summary statistics concerning both the households' financial assets composition and their main characteristics. We restrict the sample to households with positive financial assets and with a head of household aged between 21 and 70.

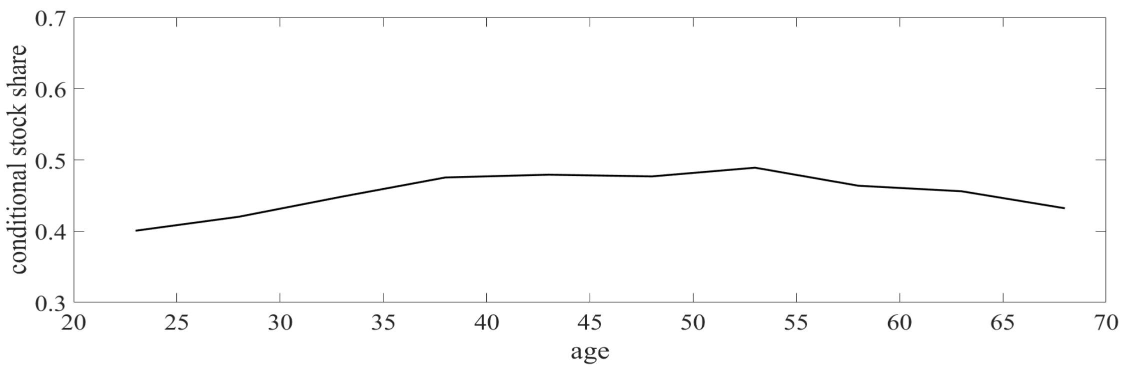
Table 1: Descriptive statistics: SCF data 1992-2016

Wave	1992	1995	1998	2001	2004	2007	2010	2013	2016
Financial assets									
Amount (2015 US $)									
Safe	126,323	135,264	138,320	148,852	139,953	137,447	143,926	126,739	141,793
Risky	70,842	91,448	167,039	202,997	161,592	162,939	129,381	137,308	159,180
Total (Safe+Risky)	197,166	226,712	305,359	351,849	301,544	300,386	273,307	264,046	300,973
Conditional Share									
Safe	64.1%	59.7%	45.3%	42.3%	46.4%	45.8%	52.7%	48.0%	47.1%
Risky	35.9%	40.3%	54.7%	57.7%	53.6%	54.2%	47.3%	52.0%	52.9%
Men	78.6%	76.8%	76.9%	77.0%	75.7%	76.5%	76.6%	75.1%	74.2%
Age	45.6	46.2	46.5	46.5	47.5	48.2	47.6	48.2	48.9
No high school	12.5%	11.5%	10.6%	10.5%	9.4%	9.1%	8.8%	7.6%	11.3%
High school	30.1%	32.6%	31.6%	31.3%	29.9%	31.4%	30.7%	29.2%	24.9%
Some college	24.2%	27.0%	27.2%	26.0%	26.3%	26.3%	26.7%	27.2%	28.4%
College	33.2%	28.8%	30.6%	32.3%	34.4%	33.2%	33.9%	36.1%	35.4%
N (households)	3906	4302	4326	4475	4526	4423	6555	6026	6261

The table reports the average composition of households financial assets and demographic characteristics across various SCF waves (1992 – 2016). The sample is restricted to households with heads aged between 21 and 70 years and with a positive amount of financial assets.

Figure 1 shows the life-cycle age profile of the average conditional portfolio share invested in risky assets, displaying five-year averages from age group 21-25 to age group 66-70. The conditional risky share is fairly flat over the life cycle, ranging from 40% to 49%.

Figure 1: Conditional Risky Share - SCF data



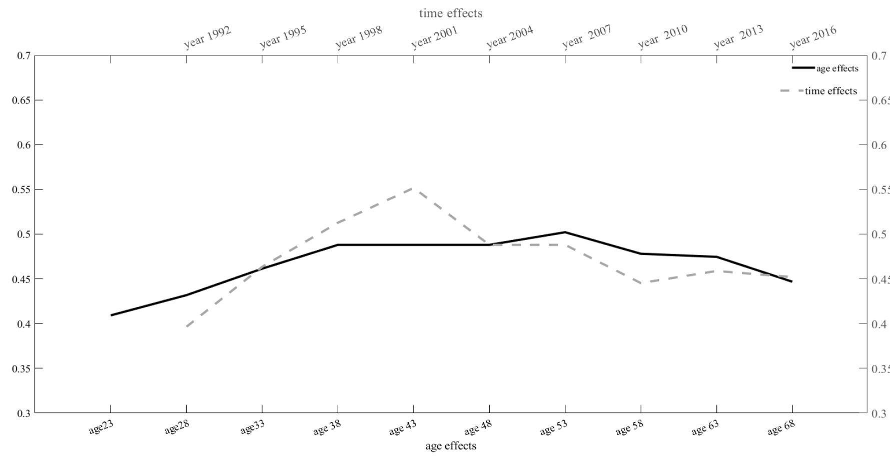
This figure displays the life cycle profile of conditional risky share of financial assets held by U.S. households grouped by five-year age classes (21-25, ..., 66-70).

Ideally, we should distinguish the impact of age on household risk taking from that of both calendar years and birth cohorts. However, the three effects cannot be separately identified. We therefore estimate three regression models in which we hold constant one effect at a time against the other two, following Ameriks and Zeldes (2004). The age dummies are constructed on the basis of five-year age groups, from 21 to 70, the reference group being aged between 46 and 50. Similarly, the birth year cohort dummies refer to five birth-year groups (from $1924-1928$ to $1989-1993$), with the cohort $1953-1958$ being the reference group. Finally, the time effects refer to the years in which the surveys are collected, and 2004 represents the reference year. In Panel (a) of Figure 2, we report the OLS regression estimates of time and age effects with cohort effects excluded (dashed and solid lines, respectively); Panel (b) displays the estimated time and cohort effects with age effects excluded (dashed and solid lines, respectively); finally, Panel (c) plots the estimated age and cohort effects with time effects excluded (dashed and solid lines, respectively).[3] The conditional risky share is remarkably flat across ages and cohorts in all specifications, with time effects showing an increase during the 1990s and a relative slowdown after year 2000. Overall, our results confirm, and extend through 2016, the patterns originally unveiled by Ameriks and Zeldes (2004).[4]

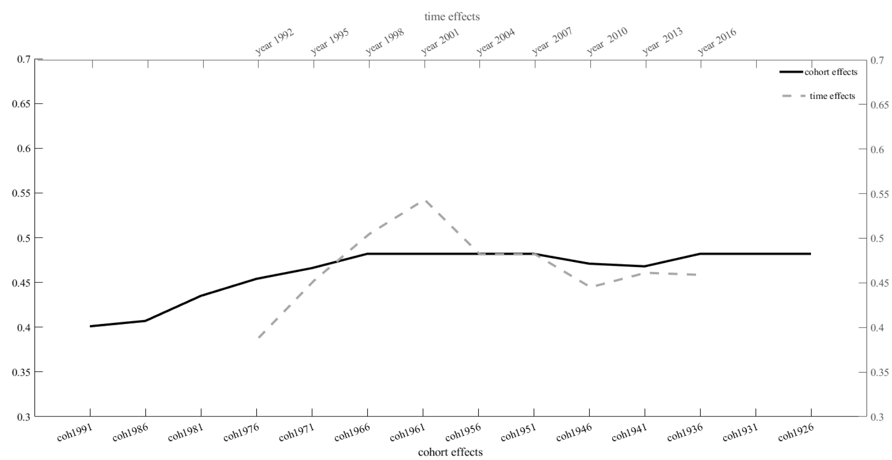
[3]We set to zero all the coefficients that are not statistically significant from zero at the 5% level.
[4]Such patterns are also robust across education levels (results unreported here).

Figure 2: Age, time and cohort effects on conditional risky share

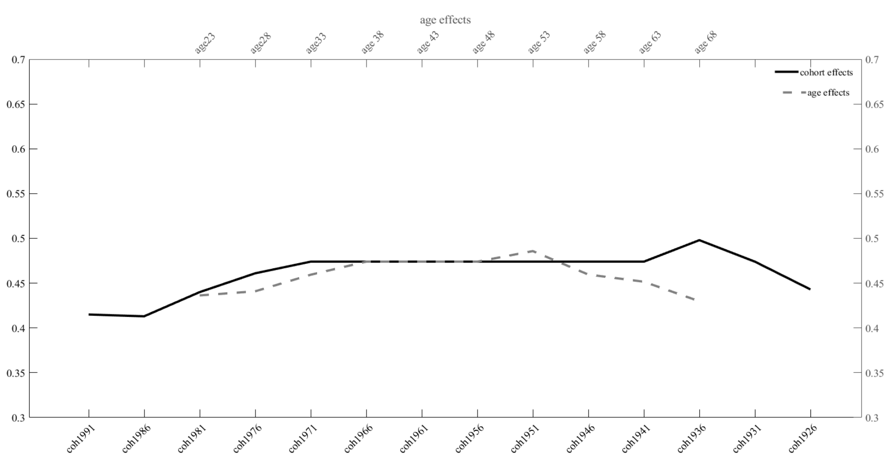
(a) Age and time effects



(b) Time and cohort effects



(c) Age and cohort effects



This figure displays the estimated age, cohort and time effects on conditional risky share under different model specifications. Panel (a): the cohort effect is assumed to be constant across ages and periods; panel (b): the age effect is assumed to be constant across cohorts and periods; panel (c): the time effect is assumed to be the same across ages and cohorts. SCF data from 1992 to 2016 on households with heads aged between 21 and 70 years are used. Coefficients that are not statistically significant at the 5% level are set to zero.

2.2 Uninsured personal disasters

This section provides an assessment of human capital losses caused by personal disasters, such as layoffs or disability. It also sheds light on estimates of the fraction of permanent income shocks that remains uninsured, touching upon heterogeneity across households and over the sample years. In Section 4 we will calibrate the labor income process and the insurance parameters of the life-cycle model against this background.

Unemployment may lead to persistent earnings losses that increase with the duration of unemployment spells, because of skill deterioration and diminishing chances of finding a new occupation. In the United States the share of unemployed workers who were jobless for more than one year, while historically low, doubled during the Great Recession, reaching 24% of total unemployment in 2014 and hitting all education groups (Kroft, Lange, Notowidigdo and Katz, 2016).[5] The chances of finding a job decrease, together with unemployment benefits, as the duration of unemployment increases.[6]

Early estimates of persistent earnings losses due to long-term unemployment (Jacobson Lalonde and Sullivan, 1993a) are around 25% of average earnings six years after separation, relative to workers with similar characteristics that stayed with the same employer during the same episode. Guvenen, Karahan, Ozkan and Song (2017) measure the effects of a full-year non-employment across workers with heterogeneous histories in a more recent

[5]For instance, in 2013, the share of US unemployed workers with a high school (college) education who had been looking for work for two or more years was 12.8% (13.5%) (Mayer, 2014).

[6]Krueger, Cramer and Cho (2014) and Kroft, Lange, Notowidigdo and Katz (2016) show that the re-employability of the long-term unemployed progressively declines over time, to the extent that they are more likely to exit the labor force than to become re-employed. The presence of more job openings does not lead to increased employment among individuals who are jobless for more than six months, and this pattern holds across all ages, industries and education levels (Ghayad and Dickens, 2012).

sample (1978-2010). Earnings losses are in the 35%-40% range after 10 years, due to both lower chances of future employment and lower income in case of re-employment. Gregory, Menzio and Wiczer (2021), using the Longitudinal Employer-Household Dynamics dataset from 1997 to 2014, find that after 5 years, around half of displaced workers still suffers from an earnings loss between 25% and 50% of pre-displacement income. Given our focus on equity investment, it is important to stress that earnings losses are large not only for workers with low earnings but also for those in the top 5% of the past earnings distribution.[7]

Large negative shocks associated with health are another important form of personal disasters. Mental health problems have an especially large impact on labor market outcomes, possibly because they also affect prime-age workers. The onset of mental illness initially reduces earnings by as much as 24%, and negative effects can last several years. Moreover, disorders reduce the probability of employment by about 14% (Currie and Madrian, 1999).

Whether personal disaster risk arises from layoffs or individual productivity declines, it is subject to incomplete insurance. Layoffs are usually partially insured by the US unemployment insurance system, but long-term unemployment is not. Personal productivity shocks are rarely insured by social welfare programs, except from major observable health problems, because of moral hazard. When awarded, disability benefits are more generous than unemployment benefits, offering a replacement rate of about 42% to the average worker (Gruber, 2000). The replacement rates are higher for low income people and for those who do not have employer-provided health insurance (Low and Pistaferri, 2015).

[7] Jung and Kuhn (2019) find that a shock at the top of the earnings distribution, such as the loss of a particularly good job, is a relevant source of persistent earnings losses.

Of course, informal insurance mechanisms, including family support, may also exist. Guvenen and Smith (2014) infer the extent of overall partial insurance from a dynamic model of consumption and linear labor income shocks where agents learn about their income growth rates. The partial insurance parameter is estimated to be 0.45, implying that almost one-half of both permanent and transitory income shocks are smoothed away through mechanisms different from savings. Blundell, Pistaferri, and Preston (2008) also estimate the extent of partial insurance. While it varies across cohorts, their estimate on the whole sample is that about 36% of permanent shocks (and almost 95% of transitory shocks) are insured.

Last but not least, the extent of the coverage is *ex-ante* uncertain, adding to the uncertainty of the losses experienced in a personal disaster state. For instance, the structure of disability insurance has an initial claim stage and an appeal process, with fluctuations over time in the award rates. Importantly, such screening may be subject to error. According to Low and Pistaferri (2015), the probability of being rejected while having a severe work limitation exceeds 0.5. Against this varied background, our model allows for residual uninsured losses in the personal disaster state that, though small on average, are uncertain as to their actual size. Calibrations will focus on the case of an expected human capital erosion as low as 20% of the permanent labor income component.

In summary, the empirical evidence presented in this section serves two purposes. First, it shows that the relatively flat age profile of the (conditional) risky portfolio share is a strong feature of US households, confirmed also on our more recent cross-sectional data. We focus on this as the main stylized fact that a satisfactory model of life-cycle investment decisions should adequately match (see Section 5 below). Second, the available evidence on the main characteristics of human capital losses caused by long unemployment spells and disability, offers valuable insights on how to calibrate the size and the amount of uncertainty surrounding such earnings losses, that we exploit in the model's quantitative evaluation (Section 4).

3 A life-cycle model with personal disaster risk

Our model extends the life-cycle framework of Bagliano, Fugazza and Nicodano (2019) to allow for a stochastic size of the loss in earnings prospects in case of a long unemployment spell. The labor income process is designed to yield a disastrous outcome for the unemployed worker only in rare circumstances.

The investor maximizes the expected discounted utility of consumption over working life, starting at age t_0, and retirement, which begins with certainty at age $t_0 + K$; she also wishes to leave a bequest. Life lasts at most T periods, and is governed by age-dependent life expectancy: at each date t, the survival probability of being alive at date $t+1$ is p_t (with $p_{t_0-1} = 1$). Individual preferences are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=-1}^{j-2} p_{t_0+k}\right)\left(p_{t_0+j-1}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1-p_{t_0+j-1})\, b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right] \tag{1}$$

where C_{it} is the level of consumption at time t, X_{it} is the amount of wealth the investor leaves as a bequest if death occurs, $b \geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter.

3.1 Labor and retirement income

During working life individuals supply labor inelastically and receive exogenous stochastic earnings. We introduce personal disaster risk by modelling working life careers as a Markov chain with three possible states: employment (e), short-term unemployment (u_1) and a disaster state characterized by long-term unemployment (u_2). Individual labor market dynamics are driven by the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix} = \begin{pmatrix} \pi_{ee} & 1-\pi_{ee} & 0 \\ \pi_{u_1e} & 0 & 1-\pi_{u_1e} \\ \pi_{u_2e} & 0 & 1-\pi_{u_2e} \end{pmatrix} \tag{2}$$

where $\pi_{nm} = \text{Prob}\,(s_{t+1} = m|s_t = n)$ with $n, m = e, u_1, u_2$ are the transition probabilities across states. A worker employed at t ($s_t = e$) can continue her employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, or can enter short-term unemployment ($s_{t+1} = u_1$) with probability $\pi_{eu_1} = 1 - \pi_{ee}$. If short-term unemployed at t ($s_t = u_1$), she exits unemployment ($s_{t+1} = e$) with probability π_{u_1e} or becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1u_2} = 1 - \pi_{u_1e}$. Finally, if the worker is long-term unemployed at t ($s_t = u_2$), she is re-employed in the following period ($s_{t+1} = e$) with probability π_{u_2e}, or remains in the disaster state with probability $\pi_{u_2u_2} = 1 - \pi_{u_2e}$.

Stochastic labor income is driven by permanent and transitory shocks. In each working period, labor income Y_{it} is generated by the following process:

$$Y_{it} = H_{it} N_{it} \qquad\qquad t_0 \leq t \leq t_0 + K \tag{3}$$

where N_{it} captures a stochastic transitory element, and $H_{it} = F\left(t, \mathbf{Z}_{it}\right) P_{it}$ represents the permanent income component. In particular, $F\left(t, \mathbf{Z}_{it}\right) \equiv F_{it}$ denotes a deterministic trend that depends on age (t) and a vector of individual characteristics ($\mathbf{Z}_{it}$) such as gender, marital status, household composition and education. The stochastic permanent component is modelled as a logarithmic random walk process: $P_{it} = P_{it-1} U_{it}$. We assume that $\omega_{it} = log(U_{it})$ and $\varepsilon_{it} = log(N_{it})$ are independent and identically normally distributed with variances σ_ω^2 and σ_ε^2 respectively.[8]

Labor income received by the employed individual at time t depends on her past working history since we allow unemployment and its duration to affect the permanent component of labor income. Thus, after one-period unemployment the permanent component H_{it} is equal to H_{it-1} eroded by a fraction Ψ_1, and after a two-period unemployment spell the permanent component, H_{it-1}, is eroded by a fraction Ψ_2, with $\Psi_2 > \Psi_1$. This introduces non-linearity into the expected permanent labor income, capturing the fact that the longer the unemployment spell, the larger is the worker's human capital depreciation (Schmieder, von Wachter and Bender, 2016; Guvenen, Karahan, Ozkan and Song, 2021). In compact

[8]We abstract from possible age-dependence in the perisistence of earnings shocks as in Karahan and Ozkhan (2013) and Sanchez and Wellschmied (2020). However, in Section D of the Supplementary Appendix, we allow for an age-dependent probability of entering long-term unemployment, thereby reducing the risk of personal disasters for younger workers.

form, the permanent component of labor income H_{it} evolves according to

$$H_{it} = \begin{cases} F\left(t, \mathbf{Z}_{it}\right) P_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1-\Psi_1)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1-\Psi_2)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{4}$$

We model the human capital erosion parameters, Ψ_1 and Ψ_2, as random variables that follow standard *Beta* distributions with shape parameters (a_j, b_j): thus, $\Psi_j \sim Beta(a_j, b_j)$.[9] This distribution allows to represent outcomes, such as proportions, being defined on the interval (0,1), yielding the probability density of Ψ_j, with $j = 1, 2$, as:

$$f(\Psi_j : a_j, b_j) = \frac{\Psi_j^{a_j-1}(1-\Psi_j)^{b_j-1}}{B(a_j, b_j)} \tag{5}$$

where $B(a_j, b_j)$ is a normalization constant to ensure that the total probability is 1. The expected value and the variance of Ψ_j, with $j = 1, 2$, are then equal to:

$$E(\Psi_j) = \frac{a_j}{a_j + b_j}\, , \, Var(\Psi_j) = \frac{a_j b_j}{(a_j + b_j)^2(a_j + b_j + 1)}\, . \tag{6}$$

During unemployment, the worker receives unemployment benefits as a fixed proportion (ξ_1 and ξ_2 in the case of short-term and long-term unemployment, respectively) of her last

[9]This modelling compactly represents the uncertainties surrounding possible future negative earnings shocks. These may include the award process of disability insurance or the differential personal impact of times of crisis, such as the Great Recession or the Covid pandemic, and of ordinary business cycle contractions.

working year labor income. Thus, the income received during unemployment spells is:

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } s_t = u_1 \\ \\ \xi_2 H_{it-2} & \text{if } s_t = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{7}$$

Finally, as in the standard life-cycle model, retirement income is certain and equal to a fraction λ of the permanent labor income earned in the last working year:

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+l}}\right) P_{it_{0+l}} \tag{8}$$

where $t_0 + l$ is the last working period and λ is level of the replacement rate.

3.2 Investment opportunities

During both working life and retirement, savings can be invested in a short-term riskless asset, yielding a constant gross real return R^f, and one risky asset ("stocks") yielding stochastic gross real returns R_t^s in each period. The excess return of stocks is modelled as:

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{9}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017).

At the beginning of each period, financial resources available to the individual for consumption and saving are given by the sum of accumulated financial wealth W_{it} and current labor income Y_{it}, which we call cash on hand $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by

$$X_{it+1} = (X_{it} - C_{it})R_{it}^{P} + Y_{it+1} \tag{10}$$

where R_{it}^{P} is the investor's portfolio return:

$$R_{it}^{P} = \alpha_{it}^{s} R_{t}^{s} + (1 - \alpha_{it}^{s})\, R^{f} \tag{11}$$

with α_{it}^{s} and $(1 - \alpha_{it}^{s})$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and we assume that the investor is liquidity constrained. Consequently, the amounts invested in stocks and in the riskless asset are non-negative in all periods.

3.3 Solving the life-cycle problem

In this intertemporal optimization framework, the investor maximizes the expected discounted utility over her whole life span, by choosing the consumption and the portfolio rules given uncertain labor income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T},\{\alpha_{it}^{s}\}_{t_0}^{T}}\left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma}+E_{t_0}\left[\sum_{j=1}^{T}\beta^{j}\left(\prod_{k=-1}^{j-2}p_{t_0+k}\right)\left(p_{t_0+j-1}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma}+\right.\right.\right.$$
$$\left.\left.\left.+\left(1-p_{t_0+j-1}\right)b\frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma}\right)\right]\right) \tag{12}$$

$$s.t. \quad X_{it+1}=\left(X_{it}-C_{it}\right)\left(\alpha_{it}^{s}R_{t}^{s}+\left(1-\alpha_{it}^{s}\right)R^{f}\right)+Y_{it+1} \tag{13}$$

with the labor income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it},P_{it},s_{it}\right)=\max_{C_{it},\alpha_{it}^{s}}\left(\frac{C_{it}^{1-\gamma}}{1-\gamma}+\beta E_{t}\left[p_{t}V_{it+1}\left(X_{it+1},P_{it+1},s_{it+1}\right)\right.\right.$$
$$\left.\left.+\left(1-p_{t}\right)b\frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right) \tag{14}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_{it}), the stochastic permanent component of income at beginning of t (P_{it}), and the labor market state ($s_{it}=e$, u_1, u_2). The Bellman equation can be written by making the expectation over the employment state at $t+1$ explicit:

$$V_{it}\left(X_{it,}P_{it},s_{it}\right)=\max_{C_{it},\alpha_{it}^{s}}\left(\frac{C_{it}^{1-\gamma}}{1-\gamma}\right.$$
$$+\beta\left[p_{t}\sum_{s_{it+1}=e,u_{1},u_{2}}\pi\left(s_{it+1}|s_{it}\right)\widetilde{E_{t}V}_{it+1}\left(X_{it+1,}P_{it+1},s_{it+1}\right)\right.$$
$$\left.\left.+\left(1-p_{t}\right)b\sum_{s_{it+1}=e,u_{1},u_{2}}\pi\left(s_{it+1}|s_{it}\right)\frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right) \tag{15}$$

where $\widetilde{E_{t}V}_{it+1}$ denotes the expectation operator taken with respect to the stochastic variables ω_{it+1}, ε_{it+1}, and ν_{it+1}^{s}. The history dependence that we introduce in our set-up by making unemployment affect subsequent labor income prospects prevents having to rely on the standard normalization of the problem with respect to the level of P_t. To highlight how the evolution of the permanent component of labor income depends on previous individual labor market dynamics we write the value function at t in each possible state as (dropping the term involving the bequest motive):

$$V_{it}(X_{it},P_{it},e)=u(C_{it})+\beta p_{t}\begin{cases}\begin{cases}V_{it+1}(X_{it+1},P_{it+1},e) & \text{with prob. } \pi_{e,e}\\ \text{with } \ P_{it+1}=P_{it}e^{\omega_{it+1}} \quad \text{and}\\ X_{it+1}=(X_{it}-C_{it})R_{it}^{p}+F_{it+1}P_{it+1}e^{\varepsilon_{it+1}}\end{cases}\\ \\ \begin{cases}V_{it+1}(X_{it+1},P_{it+1},u_{1}) & \text{with prob. } 1-\pi_{e,e}\\ \text{with } \ P_{it+1}=(1-\Psi_{1})P_{it} \quad \text{and}\\ X_{it+1}=(X_{it}-C_{it})R_{it}^{p}+\xi_{1}F_{it}P_{it}\end{cases}\end{cases}$$

$$V_{it}(X_{it},P_{it},u_1) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1},P_{it+1},e) \qquad \text{with prob. } \pi_{u_1,e} \\ \text{with } \; P_{it+1} = (1-\Psi_1)P_{it-1}\, e^{\omega_{it+1}} = P_{it}\; e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + F_{it-1}P_{it+1}e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1},P_{it+1},u_2) \qquad \text{with prob. } 1-\pi_{u_1,e} \\ \text{with } \; P_{it+1} = (1-\Psi_2)(1-\Psi_1)P_{it-1} = (1-\Psi_2)P_{it} \text{ and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + \xi_2 F_{it-2}P_{it-2} \end{cases} \end{cases}$$

$$V_{it}(X_{it},P_{it},u_2) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1},P_{it+1},e) \qquad \text{with prob. } \pi_{u_2,e} \\ \text{with } \; P_{it+1} = P_{it}e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + F_{it-2}P_{it+1}e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1},P_{it+1},u_2) \qquad \text{with prob. } 1-\pi_{u_2,e} \\ \text{with } \; P_{it+1} = (1-\Psi_2)P_{it} \text{ and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + \xi_2 F_{it-2}P_{it-2} \end{cases} \end{cases} \tag{16}$$

This problem has no closed form solution, and we obtain the optimal values for consumption and portfolio shares by means of numerical techniques. To this aim, we apply a backward induction procedure starting from the last possible period of life T and computing optimal consumption and portfolio share policy rules for each possible value of the continuous state variables (X_{it} and P_{it}) by means of the standard grid search method.[10] Going backwards, for every period $t = T-1, T-2, ..., t_0$, we use the Bellman equation (15) to obtain optimal rules for consumption and portfolio shares.

[10]The problem is solved over a grid of values covering the space of both the state variables and the controls in order to ensure that the obtained solution is a global optimum.

4 Calibration

Calibration of the model requires choosing parameters of the investor's preferences, labor and retirement incomes, and the moments of stock returns. For reference, we initially solve the model by abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005), a standard benchmark in the literature. Then, we introduce unemployment risk and consider two scenarios: (i) unemployment spells cause permanent deterministic income losses; and (ii) unemployment implies a disaster risk, since it has permanent but uncertain and possibly large consequences on the worker's earnings ability.

Across all scenarios, the agent begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US National Center for Health Statistics. We set the utility discount factor $\beta = 0.96$, and the parameter capturing the strength of the bequest motive $b = 2.5$.[11] Finally, the benchmark value for the relative risk aversion is $\gamma = 5$. This value is relatively standard in the literature (Gomes and Michaelides, 2005; Gomes, Kotlikoff and Viceira, 2008; Chang, Hong and Karabarbounis, 2018) and captures an intermediate degree of risk aversion. However, Cocco, Gomes and Maenhout (2005) choose a value as high as 10 in their benchmark setting. The riskless interest rate is set at 0.02, with an expected equity premium μ^s=0.04 with standard deviation $\sigma_s = 0.157$, as in Cocco, Gomes and Maenhout (2005). Finally, we impose a

[11]This parameter bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for.

zero correlation between stock return innovations and permanent labor income disturbances. Table 2 summarizes the benchmark values of relevant parameters.

Table 2: Calibration parameters

Description	Parameter	Value
Working life (max)	T	20 -65
Retirement (max)	$t_0 + K$	65 -100
Discount factor	β	0.96
Risk aversion	γ	5
Replacement ratio	λ	0.68
Variance of permanent shocks to labor income	σ^2_ω	0.0106
Variance of transitory shocks to labor income	σ^2_ϵ	0.0738
Riskless rate	r	0.02
Excess returns on stocks	μ^s	0.04
Variance of stock returns innovations	σ^2_s	0.025

	Unemployment no disaster	**Unemployment with disaster risk**
Unemployment benefits		
Short-term unemployed (ξ_1)	0.3	0.3
Long-term unemployed (ξ_2)	0.1	0.1
Human capital erosion		
Short-term unemployed (Ψ_1)	0	0
Long-term unemployed (Ψ_2)	0.20	(expected) 0.20
—*Beta* distribution a_2	-	0.01
—*Beta* distribution b_2	-	0.04

This table reports benchmark values of relevant parameters.

4.1 Labor income and unemployment risk

The labor income process is calibrated using the estimated parameters for US households with high school education (but not a college degree) in Cocco Gomes and Maenhout (2005). For the high school group, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are equal to $\sigma_{\omega}^2 = 0.0106$ and $\sigma_{\varepsilon}^2 = 0.0738$. After retirement, income is a constant proportion λ of the final (permanent) labor income, with $\lambda = 0.68$. The parameter values assumed above are maintained across all scenarios. Section C of the Supplementary Appendix confirms our results for different education levels (no high school degree, and college degree).

We use data from the Current Population Survey (CPS) to calibrate the transition probabilities from employment to unemployment to reflect the risk of entering unemployment along with the observed average unemployment rates at different durations. According to the evidence based on CPS reported in Kroft, Lange, Notowidigdo and Katz (2016), the annual transition probability from employment to unemployment is 4%. Given the duration dependence and the steady decline in the annual outflow rate from unemployment to employment during the first year of unemployment (Kroft, Lange, Notowidigdo and Katz, 2016), we set the probability of leaving unemployment after the first year at 85%. This calibration appears quite conservative, considering that the measured chance of being employed 15 months later for those who had been unemployed 27 weeks or more is only 36% (Krueger, Cramer and Cho, 2014). Finally, we set the probability of persisting in a state of long-term unemployment at 15%, to be consistent with the average U.S. long-term unemployment rate.

Thus, individual labor market dynamics are driven by the following transition matrix:[12]

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.85 & 0 & 0.15 \\ 0.85 & 0 & 0.15 \end{pmatrix} \tag{17}$$

Indeed, the assumed transition matrix (17) yields an unconditional probability of being short-run unemployed equal to 3.8% and a probability of being long-run unemployed as low as 0.7%, in line with the historical evidence on the short- and long-term unemployment rates in the U.S. This calibration represents a crucial difference from Bagliano, Fugazza and Nicodano (2019), where transition probabilities were chosen to yield a much higher long-run unemployment rate (1.72%), observed in the U.S. only in the exceptional and short-lived period following the 2008-9 Great Recession.

The available empirical evidence on job displacement shows that job losses affect earnings far beyond the unemployment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement may range from 30% (Couch and Placzek, 2010) to 40% of earnings (Jacobson, Lalond and Sullivan, 1993b). Earnings losses are shown to be persistent in a range from 15% (Couch and Placzek, 2010) to about 25% (Jacobson, LaLonde and Sullivan, 1993a) of their pre-displacement levels. These estimates abstract from the effect of unemployment duration, while Cooper (2013) finds that earnings losses are larger the longer unemployment lasts. Also, based on administrative data,

[12] In Section D of the Supplementary Appendix, we assess the robustness of our results to assuming age-dependent transition probabilities, whereby young workers are less likely to enter long-term unemployment than middle-aged workers.

Jacobson, LaLonde and Sullivan (2005) estimate that average earnings losses for displaced workers amount to 43-66% of their pre-displacement wage.

This body of evidence, combined with a probability of finding a job after being unemployed for 24 months as low as 40% (Kroft, Lange, Notowidigdo and Katz, 2016), leads us to calibrate an expected drop in human capital, following a long-term unemployment spell, of about 20% leaving open the possibility of rare but much larger losses. Thus, while Ψ_1 is kept equal to 0 with certainty, Ψ_2 follows a *Beta* distribution with expected value of 20% and a standard deviation close to 40%. This calibration for the distribution Ψ_2 delivers a median value for the proportional human capital erosion close to 0% and a 75th percentile lower than 1.5%. Only a small fraction of the long-term unemployed suffers more sizable losses, even reaching 100% of the permanent income component.

Thus, the long-term consequences of not working for a long time are modest for the majority of workers but possibly very large in rare situations. In fact, in our benchmark calibration, if a worker is currently employed, she will enter long-run unemployment and suffer a earnings loss larger than 1.5% (therefore experiencing a "personal disaster") only with a 0.15% probability; even if the worker is already unemployed, the probability of being hit by a personal disaster is as small as 3.75%. This is consistent with several studies exploring the effects of unemployment on future labor income and separation rates: for example, Guvenen, Karahan, Ozkan and Song (2017) find that income losses after long-term unemployment may be substantial and are highly heterogeneous, as recently confirmed by Guvenen, Karahan, Ozkan and Song (2021) and Gregory, Menzio and Wiczer (2021).

Finally, unemployment benefits are calibrated according to the US unemployment insurance system. In particular, considering that the replacement rate with respect to last labor income is on average low and state benefits are paid for a maximum of 26 weeks,[13] we set $\xi_1 = 0.3$ in case of short-term unemployment spells as in Branger, Larsen and Munk (2019), and $\xi_2 = 0.1$ for the long-term unemployed. [14]

For comparison, we consider a calibration of the model without unemployment risk. This "*no unemployment risk*" scenario corresponds to the standard life-cycle set up with $\pi_{ee} = 1$ and all other entries equal to zero in the transition probability matrix (17). In addition, to highlight the effects of permanent consequences of unemployment on future earnings prospects, we consider a third calibration by adding the unemployment risk embedded in matrix (17) with deterministic human capital erosion. In this "*unemployment without disaster risk*" scenario, long-term unemployment has deterministic permanent consequences on future earnings implying a human capital loss of 20% (i.e. $\Psi_1 = 0$ and $\Psi_2 = 0.20$), and closely matching Bagliano, Fugazza and Nicodano (2019), where the human capital loss is set at 25% for all long-term unemployed.

[13]No additional weeks of federal benefits are available in any state: the temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.

[14]Our results go through also when we assume a more generous long-run unemployment replacement rate of 0.5. Low, Meghir and Pistaferri (2010) acknowledge that layoffs are partially insured by the unemployment insurance system, while individual productivity shocks, other than major observable health shocks, are rarely insured in any formal way. As for other welfare programs, we do not model basic consumption needs and therefore do not consider basic consumption insurance.

5 Results

In this section we present our main results on the optimal life-cycle portfolio allocation pattern in the presence of personal disaster risk and assess how close they match major observed empirical regularities. We start with a brief description of optimal investment policies.

5.1 Optimal policies

Figure 3 compares investors' optimal stock shares in the case of *no unemployment* and in the case of unemployment with deterministic human capital erosion (i.e. *unemployment without disaster risk*), jointly displayed in panel (a), to optimal shares implied by our preferred scenario of unemployment with uncertain human capital erosion (i.e. *unemployment with disaster risk*), shown in panel (b). The figure plots the optimal stock share as a function of cash on hand at three different ages (20, 40, and 70).

In the cases of no unemployment and unemployment without disaster risk, we obtain standard life-cycle results. Labor income acts as an implicit risk-free asset and affects the optimal portfolio composition depending on an investor's age and wealth. For example, at age 20 the sizable implicit holding of the risk-free asset (through human capital) makes it optimal for less-wealthy investors to tilt their portfolio towards the risky financial asset. Indeed, for a wide range of wealth levels, agents optimally choose to be fully invested in stocks. The optimal stock holding decreases with financial wealth because of the relatively lower implicit investment in (risk-free) human capital.[15]

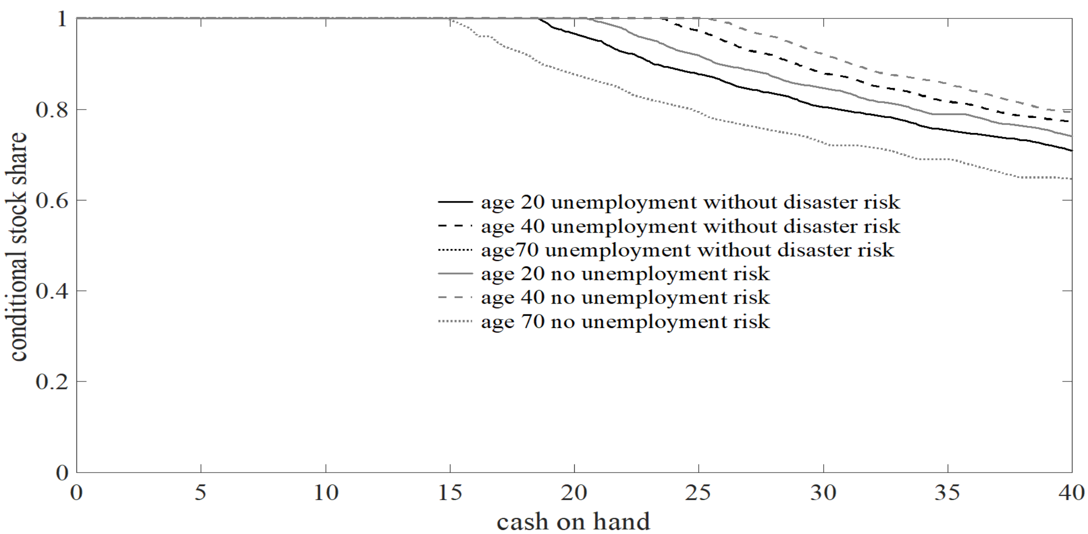
[15] The optimal policies at age 70 coincide in the three scenarios, since retired workers do not face any

When the model is extended to allow for uncertain permanent effects of unemployment spells on labor income prospects (*unemployment with disaster risk*), with the parameters governing the proportional erosion of permanent labor income set at $\Psi_1 = 0$ after one year of unemployment and at an expected $\Psi_2 = 0.2$ after 2 years, the resulting policy functions are shifted abruptly leftward. The optimal stock share still declines with financial wealth but a 100% share of investment in stocks is optimal only at very low levels of wealth. In this case, long-term unemployment implies an uncertain loss of future labor income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the case of no disaster risk.

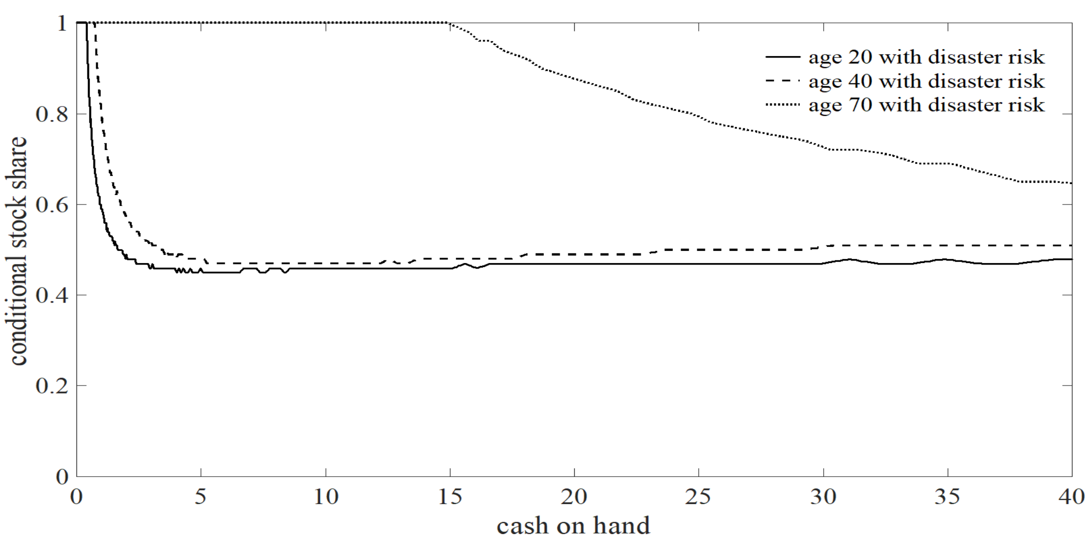
form of unemployment risk. They are nevertheless shown in Figure 3 for comparison with policies that are optimal during working life (at age 20 and 40).

Figure 3: Policy functions

(a) No unemployment risk - Unemployment without disaster risk



(b) Unemployment with disaster risk



This figure shows the portfolio rules for stocks as a function of cash on hand for an average level of the stochastic permanent labor income component. The policies refer to selected ages: 20, 40, and 70. The two panels refer to the cases of no unemployment and of unemployment without disaster risk (panel a), and of unemployment with disaster risk (panel b), respectively. In the case of unemployment without disaster risk, the human capital loss is 20% with certainty; in the case of unemployment with disaster risk, the expected human capital loss is 20%. Cash on hand is expressed in ten thousands of U.S. dollars.

5.2 Life-Cycle profiles

On the basis of the optimal policy functions, we simulate the whole life-cycle consumption and investment decisions of 100,000 agents. Figure 4, panel (a), shows the average optimal equity portfolio shares plotted against age. In the case of no unemployment risk (dotted line), the well-known downward sloping pattern emerges. Over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and decreases with age to reach around 80% at retirement. When we consider unemployment risk with deterministic human capital erosion of 20% (dashed line), the optimal portfolio share of stocks still declines with age, though being only slightly lower at all ages, with a 100% optimal stock share only for very young investors.

However, when we account for disaster risk (solid line), the optimal stock investment is reduced at any age and almost flat, at around 45-50%. The risk of potentially losing a substantial portion of future labor incomes reduces the level of human capital and increases its riskiness. Since this effect is more relevant for younger workers, it induces a lower optimal stock investment conditional on financial wealth especially when young (see Figure 3, panel b). Consequently, the age profile remains flat over the whole working life. In the subsequent retirement years, the presence of an operative bequest motive determines the still relatively low level of risky investment.[16] Allowing for state-dependent labor market transition prob-

[16] Importantly, our results are robust to the adjustment of the income diffusion volatility and drift rate to match the mean and variance of the income process in the standard calibration of Cocco, Gomes and Maenhout (2005). Details of this robustness analysis are available from the authors.

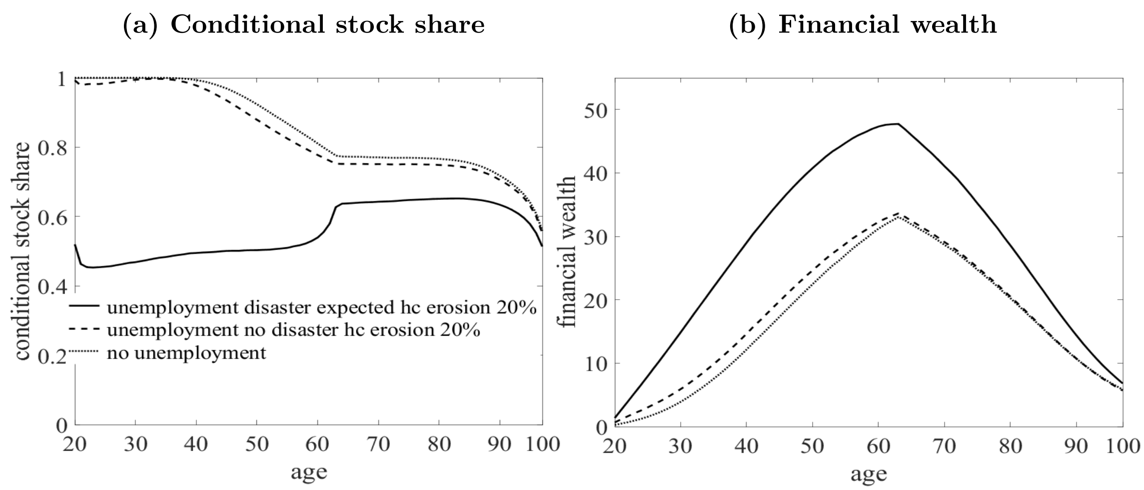
abilities, as in Branger, Larsen and Munk (2019), would strengthen our results.[17]

These findings highlight that it is the remote possibility of incurring potentially large negative shocks to human capital, albeit small in expectation, that dampens the incentive to invest in stocks. They portray the effects on risk-taking of the "unusual" negative shocks that explain the consumption dynamics of US households in Arellano, Blundell and Bonhomme (2017) across the earnings distribution. The reduction in the optimal portfolio share allocated to stocks is due to higher wealth accumulation, in turn induced by larger precautionary savings.

Panel (b) of Figure 4 displays the average financial wealth accumulated over the life cycle for the three scenarios considered. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during their working life to buffer possible disastrous labor market outcomes. Optimal consumption when young consequently falls, but it is much higher during both late working years and retirement years.

[17] In Branger, Larsen and Munk (2019), the main driver of the reduction in the optimal risk-taking by young workers is the decline in earnings due to unemployment (increasing in unemployment duration) and the state-dependence of unemployment and re-employment probabilities.

Figure 4: Life-cycle average profiles



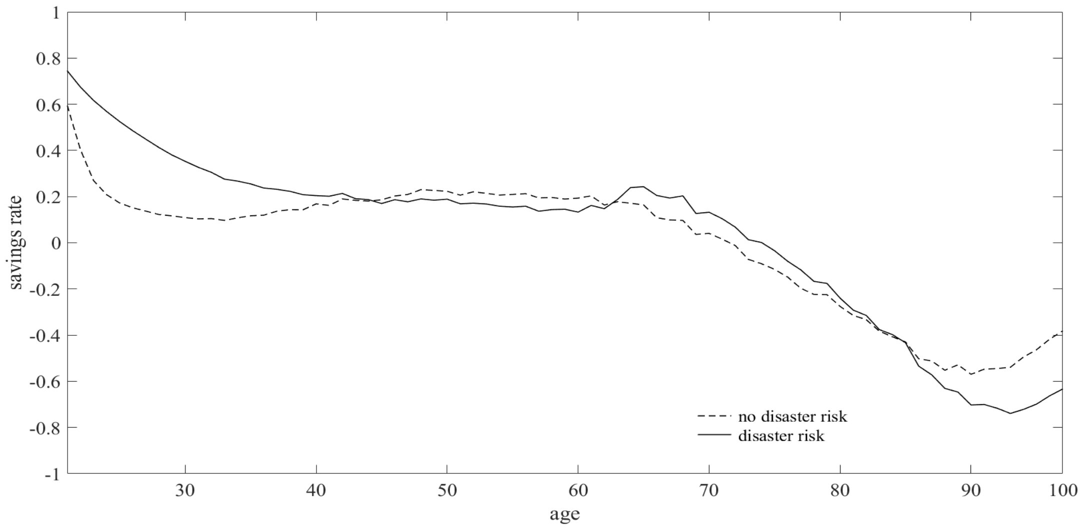
The figure displays the mean simulated stock investment, and financial wealth accumulation life-cycle profiles in panel (a) and (b), respectively. Age ranges from 20 to 100, with retirement occurring at the age of 65. The three lines correspond to no unemployment risk (dotted line); unemployment without disaster risk (dashed line); unemployment with disaster risk (solid line). In the case of unemployment without disaster risk, the human capital loss is 20% with certainty; in the case of unemployment with disaster risk, the expected human capital loss is 20%. Financial wealth is expressed in ten thousands of U.S. dollars.

Figure 5 displays the life-cycle profile of the ratio between savings and total (financial plus labor) income[18]. When the worker faces disaster risk, her average propensity to save is higher than in the case of unemployment without disaster risk throughout the first two decades of working life. Such propensity monotonically decreases in age, converging to the known pattern when the worker is in her mid-forties. The figure clearly shows the impact on savings of the resolution of uncertainty as individuals age.[19]

[18]In the case of no unemployment risk (not shown), the optimal savings age profile is very close to the case of unemployment with no disaster risk (dotted line).

[19]Data on Norwegian households show that they engage in additional saving, shifting portfolio composition towards safe assets, in the years prior to unemployment. There is depletion of savings after the job loss (Basten, Fagereng and Telle, 2016).

Figure 5: Life-cycle profiles of savings rate



This figure displays the savings rate dynamics for individuals of age 20 to 100, relative to total income (i.e. labor income plus financial income). The two lines correspond to unemployment without disaster risk (dotted line) and unemployment with disaster risk (solid line). In the case of unemployment without disaster risk, the human capital loss is 20% with certainty. In the case of unemployment with disaster risk, the expected human capital loss is 20%.

These results suggest that the prospect of a higher benefit, cushioning disastrous outcomes, could mitigate the adverse impact of long-term unemployment on human capital, reducing the need for cautious investing and saving during early working life. The variation of institutions across countries may thus generate different life-cycle patterns in equity investing. In this light, the decreasing stock holdings in Norwegian data detected by Fagereng, Gottlieb and Guiso (2017), may be a consequence of higher long-term unemployment benefits with respect to the US.

Higher savings obviously implies lower consumption when young. What is less obvious is whether higher accumulated wealth shields consumption from the skewness of labor income shocks. Table 3 reports the mean and the standard deviation of the skewness of labor income shocks and of consumption growth rates across individuals during working life. Without disaster risk, the average skewness of labor income shocks is negligible (-0.007), while with disaster risk is sizably negative (-2.817), in line with values found in the literature on PSID data, ranging from 0 to -4 (De Nardi, Fella and Paz-Pardo, 2020).

The labor income process that we consider is consistent with empirical earnings dynamics displaying substantial deviations from lognormality. For instance, Guvenen, Karahan, Ozkan and Song (2021) find that negative shocks to earnings, while small on average, may be large in rare cases. In our model, large negative shocks are associated with the rare event that, after experiencing long-term unemployment, the worker cannot restore her previous earnings capacity.

Moreover, with disaster risk the average skewness of consumption growth rates is negative (-0.32), close to the estimate (-0.31) obtained by Bekaert and Engstrom (2017) on quarterly growth rates. Overall, we view our results as broadly consistent with the evidence of partial insurance against permanent income shocks (Arellano, Blundell and Bonhomme, 2017), and with models that attribute the negative skewness of consumption growth rates to permanent shock to labor income prospects (as in Bekaert and Engstrom, 2017) rather than to large transitory shocks (as in Gabaix, 2012).

However, our adopted income process does not fully reflect the non-linearities in income dynamics recently documented by the empirical literature. Hence, in Section A of the Supplementary Appendix we investigate whether our results are robust to different specifications of the (non-linear) labor income dynamics, along the lines of Arellano, Blundell and Bonhomme (2017) and Guvenen, Karahan, Ozkan and Song (2021).

Table 3: Skewness of labor income shocks and consumption growth rates

	Labor income shocks	Consumption growth
disaster risk	-2.817	-0.32
	(0.814)	(0.19)
no disaster risk	-0.007	0.03
	(0.026)	(0.27)

The table reports the mean and the standard deviation of the skewness of labor income shocks (permanent plus transitory) and consumption growth rates (between age t and age $t-1$) faced by $100,000$ simulated investors.

5.2.1 Heterogeneity

The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker. In the face of a rare but large human capital depreciation, workers on average invest about 50% of their financial wealth in stocks. However, this average pattern may hide considerable differences across agents. The present section investigates the distribution across agents of both conditional optimal stock share and accumulated wealth.

Panel (a) of Figure 6 shows the 25^{th}, 50^{th} and 75^{th} percentiles of the distributions in the case of unemployment without disaster risk.[20] Both the optimal stock share and the stock

[20]With no unemployment risk, all distributions are very close to the case of unemployment with no disaster.

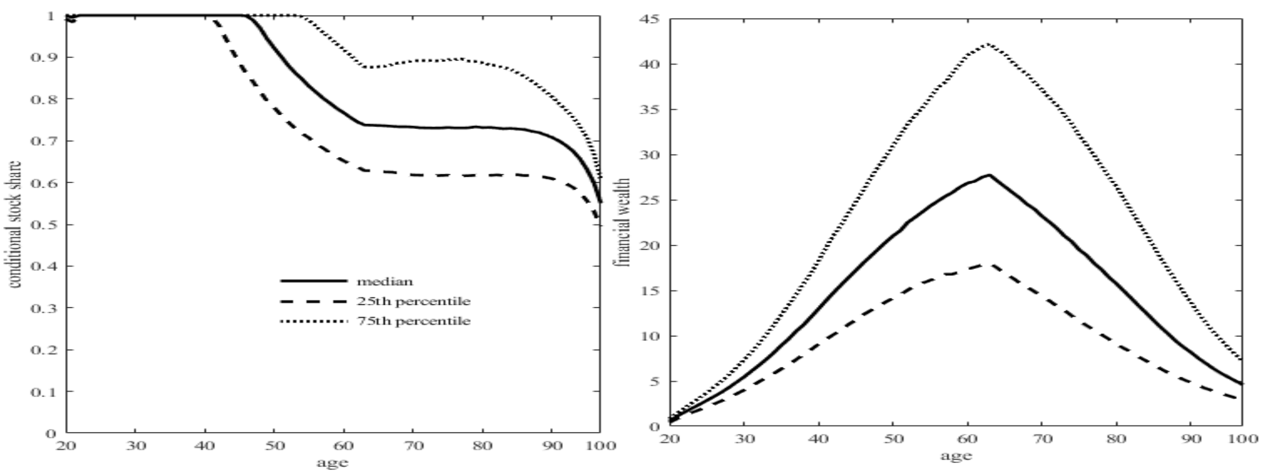
of accumulated financial wealth are highly heterogeneous across workers as well as retirees. The exception is young workers as they tilt their entire portfolio towards stocks given the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions displayed in Figure 3, relating optimal stock shares to the amount of available cash on hand, and on the level of cash on hand itself. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices. At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for all workers to be fully invested in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation. This situation pushes investors on the steeper portion of their policy functions and determines a gradual increase in the heterogeneity of optimal risky portfolio shares during their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions; as a consequence, the dispersion of optimal shares tends to persist.

Panel (b) of Figure 6 displays the life-cycle distribution of stock shares and financial wealth when workers face the prospect of unemployment with disaster risk. Compared with the previous case, optimal stock shares are much less heterogeneous. Heterogeneity shrinks during working life even for young workers, given the high human capital risk they bear at the beginning of their careers. Indeed, policy functions are relatively flat when long-term unemployment is uninsured (as shown in panel (b) of Figure 3) implying that even large differences in the level of accumulated wealth result in homogeneous asset allocation choices.

Figure 6: Life-cycle percentile profiles

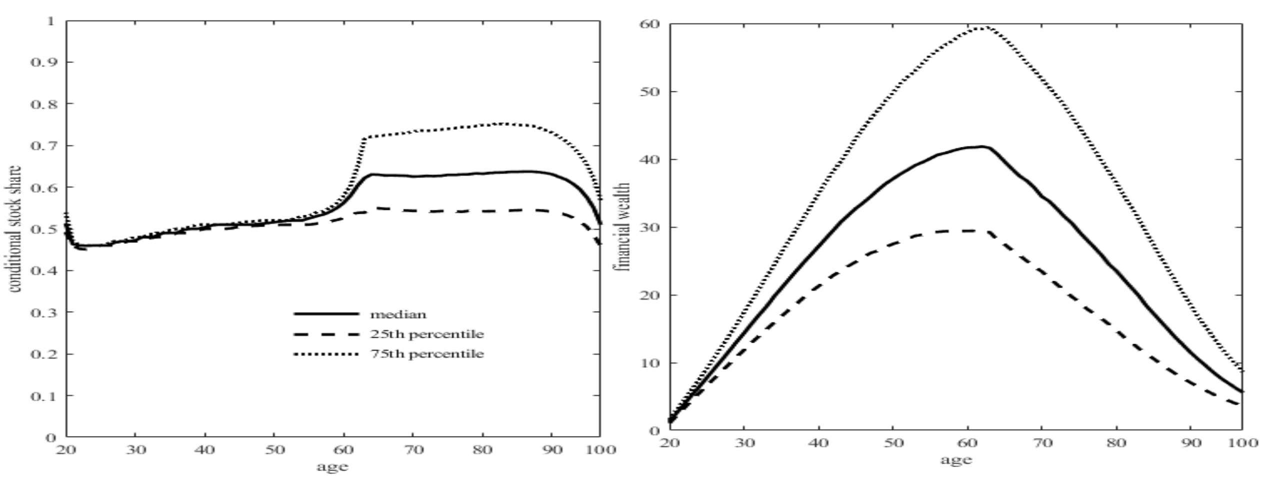
(a) Unemployment without disaster risk

Conditional stock share **Financial wealth**



(b) Unemployment with disaster risk

Conditional stock share **Financial wealth**



This figure displays the distribution of simulated equity share investment and financial wealth accumulation for individuals of age 20 to 100 in the case of unemployment without disaster risk, panel (a), and unemployment with disaster risk, panel (b). In the case of unemployment without disaster risk, the human capital loss is 20% with certainty. In the case of unemployment with disaster risk, the expected human capital loss is 20%. Financial wealth is expressed in ten thousands of U.S. dollars.

5.3 Uncertainty on the size of human capital loss

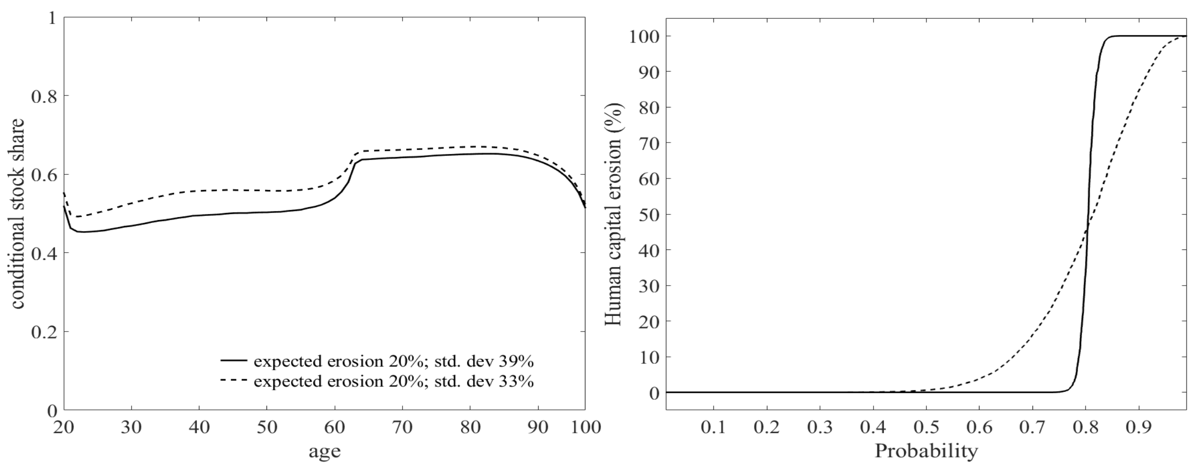
We now check the sensitivity of optimal life-cycle profiles to the amount of uncertainty surrounding the size of the human capital erosion caused by long-term unemployment. This is governed by the two shape parameters of the Beta distribution (a_2 and b_2), that we vary to allow for different degrees of uncertainty around an unchanged expected loss equal to 20% of the permanent income component.

Figure 7 compares the benchmark case, in which human capital erosion has a standard deviation of 39%, with an alternative in which the standard deviation is reduced to 33%. Panel (b) of the figure displays the (inverse) cumulative distribution function of the human capital loss in the two cases. Compared with the benchmark, a reduction in the dispersion of the loss size implies that a lower fraction of the long-term unemployed may suffer an extremely large drop in earnings potential, making the occurrence of a disastrous outcome less likely. The associated life-cycle profile of the stock share, shown in panel (b), confirms the results of the benchmark case. Although the average stock share is slightly higher, being in the 50-55% range over the working life, the age profile is remarkably flat, as in the benchmark case. This outcome supports the conclusion that rare but potentially disastrous labor income shocks may be relevant to understand cautiousness by young investors and their limited risk-taking in the stock market. [21]

[21] As shown in section F of the Supplementary Appendix, our results are confirmed also when we consider a transitory human capital loss, assuming that the earnings loss due to long-term unemployment is partially recovered when the worker finds a new job opportunity.

Figure 7: Uncertainty on the size of human capital loss

(a) Conditional stock share **(b) Human capital loss**



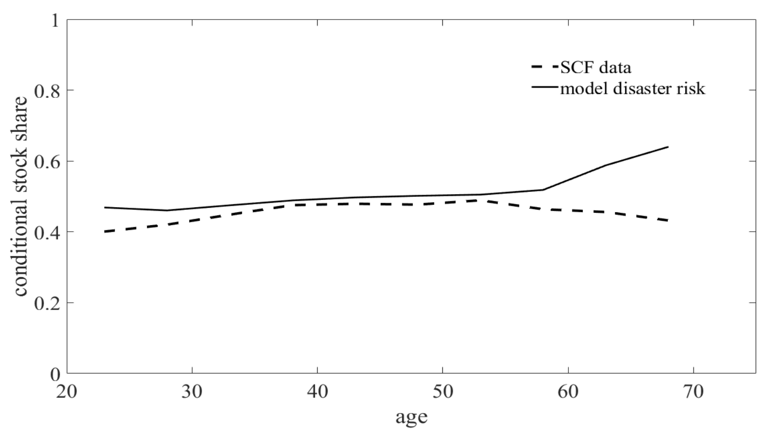
Panel (a) displays the life-cycle profile of optimal conditional stock holdings for two different parameterizations of the distribution of human capital erosion. Expected erosion is 20% in both cases, whereas the standard deviation is 39% in the benchmark case (solid line), and 33% in the alternative (dashed line). Panel (b) shows the two corresponding inverse cumulative distribution functions of the human capital erosion.

5.4 Matching empirical regularities

The key implication of our model is that optimal investment profiles are almost flat over the life cycle. In this section, we finally compare our results with conditional stockholdings for U.S. male investors observed in the Survey of Consumer Finances data (waves from 1992 to 2016).

Figure 8 compares the stock portfolio shares of stock market participants for different age classes obtained from our model with the corresponding US SCF data. The matching is close during working years, which are the focus of this paper since we are concerned with personal disasters related to long-term unemployment. In fact, the model yields an average stock share over the working life of 49%, compared with the observed average share of 45.6%. Health disasters past retirement age, unrelated to unemployment spells, might improve the matching in the decumulation phase.

Figure 8: Life-cycle conditional stock share profiles



This figure displays the life-cycle profiles of conditional stock holdings of investors from age 20 to 70 observed in SCF data with that obtained from the benchmark model with disaster risk.

A related important result of the model with personal disaster risk, is that agents are induced to increase wealth accumulation over working years to cushion the prospect of a sizable human capital loss. Under this respect, the matching between the model and the data is less satisfactory.

In fact, the baseline calibration of our model delivers high average wealth-to-income ratios: 7.5 across the whole population (i.e. at all ages), with a peak of 10.7 at the retirement age of 65. Instead, the US Survey of Consumer Finances yields wealth-to-income ratios in the

5-7 range. This result is mainly attributable to our choice of the utility discount factor (β), that we calibrated at 0.96, a standard value in the literature (see e.g. Cocco, Gomes and Maenhoud, 2005; Fagereng, Gottlieb and Guiso, 2017).

Thus, we identified a value of the discount factor that is able to match more closely the observed wealth-to-income ratios. When β=0.85, our model with personal disaster risk delivers average wealth-to-income ratios of 4.3 across all ages and of 6.5 at retirement age, much closer to the evidence, as shown in Table 4. This lower value of the discount factor is in line with recent findings: e.g. Fagereng, Gottlieb and Guiso (2017) match the observed wealth-to-income ratio in Norwegian data with a discount factor lower than 0.80. As shown in detail in Section B of the Supplementary Appendix, our main result of a flat age profile of the risky investment share is fully confirmed also with a lower utility discount factor. [22]

Table 4: Wealth to income ratios

(a) all ages			(b) age 65		
Discount factor β	Model	SCF Data	Discount factor β	Model	SCF Data
0.96	7.5	4.8	0.96	10.7	6.7
0.85	4.3		0.85	6.5	

The table shows the wealth-to-income ratios obtained from the simulated model and derived from the 1992-2016 waves of the US Survey of Consumer Finances, both at all ages and at the retirement age of 65. Model simulated ratios are obtained with two different values of the utility discount factor β: 0.96, as in the benchmark calibration, and 0.85.

[22] Additional robustness checks, exploring the effects of stock market participation costs (section G) and of a stronger bequest motive (section H) are provided in the Supplementary Appendix.

6 Conclusions

This paper shows that even a small probability of experiencing human capital erosion of an uncertain, but potentially extreme, size generates optimal conditional stock shares in line with those observed in US data, along with a skewed consumption growth distribution. Non-linear income shocks, which have recently become essential in consumption studies, appear to play a first-order role in choices on risk-taking. Because of the remote possibility of a future personal disaster, younger workers face higher uncertainty concerning future income than older workers and optimally invest a higher portfolio share in the risk-free asset. These results are based on a methodological innovation in the way we model human capital erosion conditional on the occurrence of a rare disaster.

Our analysis has implications for the design of pension plans in the United States, that increasingly offer Target Date Funds (TDF) to plan members. While there is considerable dispersion in TDF returns, a common feature of their design tilts the composition of optimal portfolios towards equity investments when young (Balduzzi and Reuter, 2019). On the contrary, our results point out that a flatter design should fit workers with different career histories in the face of limited and uncertain protection against a future personal disaster. More generally, the pattern of risk-taking at different ages in TDF should be related to the share of uninsured disability and long-term unemployment risk.

The model also suggests that the observed variation in the age pattern of stock investing across countries may depend on the insurance coverage against personal disasters. Moreover, it points to the possibility that compound lotteries, such as the disasters modelled in this paper, potentially play a role in explaining both the low consumption levels of the elderly and the equilibrium risk premium. We leave the systematic investigation of these insights for future research.

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Life-Cycle Risk-Taking with Personal Disaster Risk

Supplementary Online Appendix

September 2023

This Appendix provides additional robustness results to the paper "Life-cicle risk taking with personal disaster risk". Several dimensions of robustness are considered: (i) alternative specifications of non-linear labor income dynamics; (ii) different values of the utility discount factor; (iii) different education levels; and (iv) age-dependent disaster risk; (v) state-dependent risk aversion; (vi) transitory human capital loss; (vii) the presence of stock market participation costs; (viii) a stronger bequest motive.

A. Non-linear income dynamics

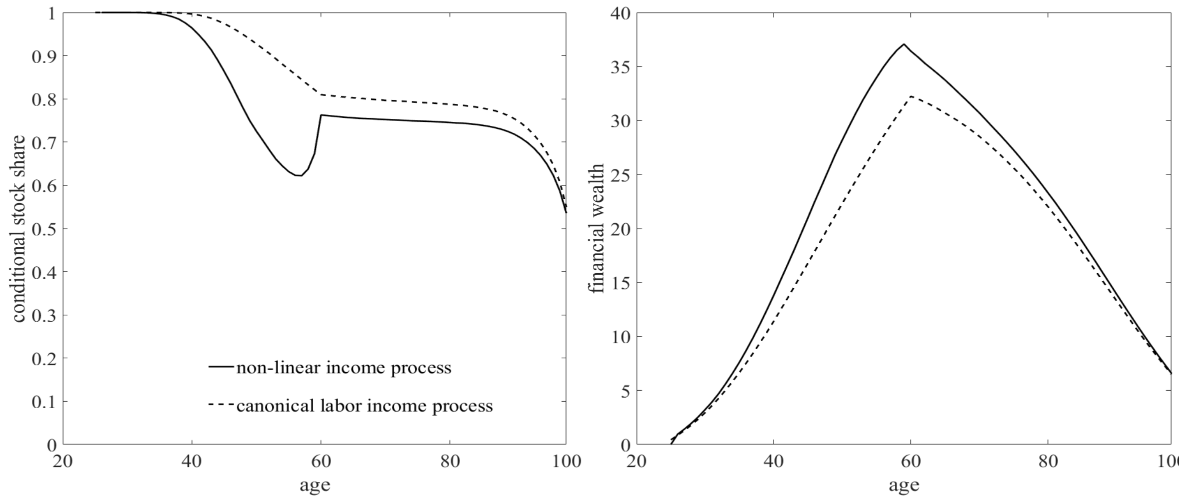
In the paper, we extend the standard life-cycle model of Cocco, Gomes and Maenhout (CGM, 2005), based on a linear earnings process with normally distributed shocks, to allow for rare personal disasters, which imply that permanent earnings shocks are no longer normal but negatively skewed. Our model shows that the negative skewness induced by the (rare) occurrence of large earnings losses predicts an optimal stock investing that broadly matches the available evidence. However, our adopted process does not fully reflect the non-linearities in income dynamics recently documented by the literature, in particular by Arellano, Blundell and Bonhomme (ABB, 2017) and Guvenen, Karahan, Ozkan and Song (GKOS, 2021).

In this Appendix, we assess whether our results are robust to different specifications of the (non-linear) income dynamics, and show that: (i) modelling labor income as in ABB, *in the absence of rare disaster shocks*, yields risky investment profiles that are still conterfactually decreasing in age, as in the standard CGM model; and (ii) when we extend our set-up by making younger, low-income workers face a larger unemployment risk as in GKOS, our main result of a flat age profile of risky investment is confirmed.

(i) First, we solve the standard life-cycle model (CGM) assuming that the labor income process is non-linear as in ABB. In their model, the persistence of past earnings shocks varies according to the size and sign of the current disturbance. This labor income process is able to capture also the ARCH effects found by Meghir and Pistaferri (2004). We use the state spaces and transition matrices associated with the ABB non-linear process estimated by De Nardi, Fella and Paz-Pardo (2020). Following ABB, they estimate a flexible, non-linear and non-normal labor income process on PSID data for pre-tax earnings of male workers aged from 25 to 60. The estimated process is then used to simulate a large sample of histories for both the transitory and the persistent earnings components from which they obtain the state spaces and transition matrices (made available on the authors' website). We employ those state spaces and matrices to approximate the non-linear income process. In this new calibration the life cycle spans from age 25 to 100, retirement occurs at 60, and all other parameters are set as in CGM, apart from the risk aversion parameter, that we keep set at 5. Figure A1 shows the age profiles of the average conditional stock share and financial wealth accumulation delivered by a non-linear ABB-type labor income process (solid lines), compared with those obtained from the canonical CGM process (dashed lines).

Allowing for non-linearities and non-normalities in earnings dynamics lowers the optimal stock share compared with the standard model, but still delivers a counterfactual, downward sloping age profile of stock investing over the working life. Therefore, an ABB-type income process is not able to generate the relatively flat age profile of the stock share, which is the main implication of our model.

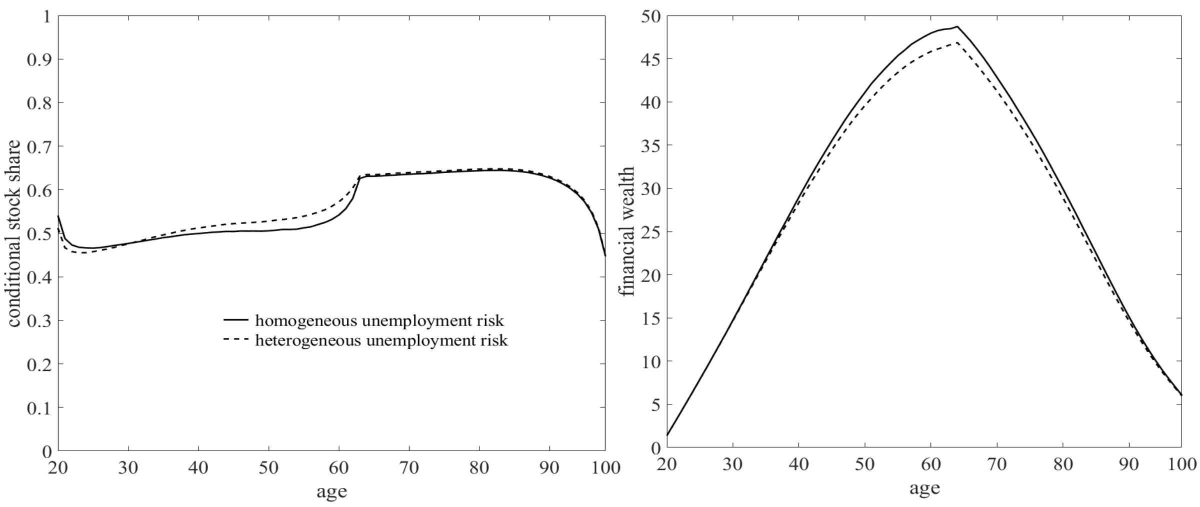
Figure A1. Life-cycle profiles: the effect of a non-linear labor income process



The figure displays the life-cycle profiles of the mean simulated stock investment share and financial wealth accumulation Age ranges from 25 to 100, with retirement occurring at the age of 60. Solid lines are obtained assuming a non-linear labor income process as in Arellano, Blundell and Bohnomme (2017); dashed lines refer to the canonical labor income process of Cocco, Gomes and Maenhout (2005). Risk aversion is set at 5. Financial wealth is expressed in ten thousands of U.S. dollars.

(ii) Second, we address the issue of the heterogeneity of unemployment risk, with the poorer workers facing larger risks, as documented by GKOS. To this aim, we re-calibrate the model allowing for lower unemployment risk for higher quantiles of the permanent income distribution. In particular, we solve the model assuming a negligible risk of long-term unemployment for investors whose permanent income is higher than the 75^{th} percentile at age 20. Figure A2 displays the age patterns of the stock share and the financial wealth obtained by allowing for heterogeneous unemployment risk, compared with those delivered by our benchmark model (with homogeneous unemployment risk), showing that our main results are robust to this alternative calibration.

Figure A2. Life-cycle profiles: the effect of heterogeneous unemployment risk



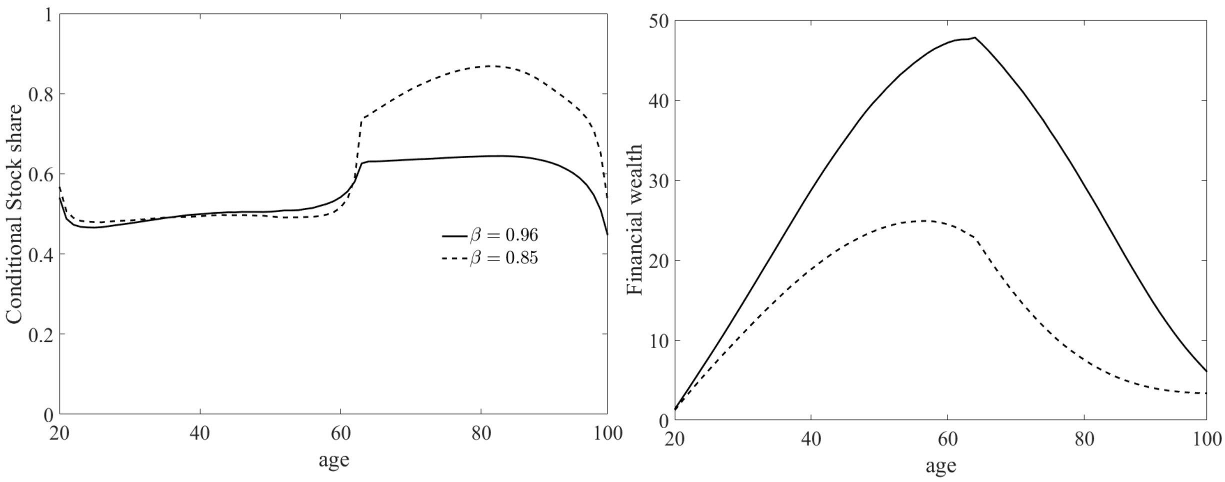
The figure displays the life-cycle profiles of the mean simulated stock investment share and financial wealth accumulation Age ranges from 20 to 100, with retirement occurring at the age of 65. Solid lines refer to the benchmark case where the long-term unemployment risk is equal across workers; dashed lines refer to the case where the long-term unemployment risk is negligible if permanent income is larger than the 75^{th} percentile at age 20. Financial wealth is expressed in ten thousands of U.S. dollars.

B. Robustness to different discount factors

In our simulations, the utility discount factor (β) has been calibrated to 0.96, a standard value in the literature (see e.g. Cocco, Gomes and Maenhout, 2005; Fagereng, Gottlieb and Guiso, 2017). This choice affects the worker's wealth accumulation, delivering high average wealth-to-income ratios: 7.5 across the whole population (i.e. at all ages), with a peak of 10.7 at the retirement age of 65. Instead, the US Survey of Consumer Finances yields wealth-to-income ratios in the $5-7$ range. Thus, as explained in sub-section 5.4 of the main text, we identified a value of the discount factor that is able to match more closely the observed wealth-to-income ratios. When $\beta = 0.85$, our model with personal disaster risk delivers average wealth-to-income ratios of 4.3 across all ages and of 6.5 at retirement age, much closer to the evidence, as shown in Table 4. This lower value of the discount factor is in line with recent findings: e.g. Fagereng, Gottlieb and Guiso (2017) match the observed wealth-to-income ratio in Norwegian data with a discount factor lower than 0.80. Here, we check that our main result of a flat age profile of the optimal risky portfolio share is robust to lower utility discount factors. Figure A3 shows the average stock share and wealth accumulation life-cycle profiles for two values of β:

0.96 and 0.85. Wealth accumulation is more moderate in the latter case, and the flat age profile of risky investment over working life is fully confirmed.

Figure A3. Life-cycle profiles: robustness to the discount factor



The figure displays the life-cycle profiles of the mean simulated stock investment share and financial wealth accumulation. Age ranges from 20 to 100, with retirement occurring at the age of 65. Solid lines refer to the benchmark case where the utiity discount factor $\beta = 0.96$; dashed lines refer to the case where $\beta = 0.85$. Financial wealth is expressed in ten thousands of U.S. dollars.

C. Robustness to different education levels

In the paper, we follow Cocco, Gomes and Maenhout (CGM, 2005) and calibrate the labor income process using the parameters estimated for US households with high school education (but not a college degree), which is their (and our) benchmark case. Here, we assess the robustness of our main results to different education levels and solve the model with long-term unemployment risk also for workers with no high school degree and for college graduates, using the parameter calibration in CGM. In Table A1 below we report the estimated variances of labor income shocks and the replacement ratios used in the simulations.

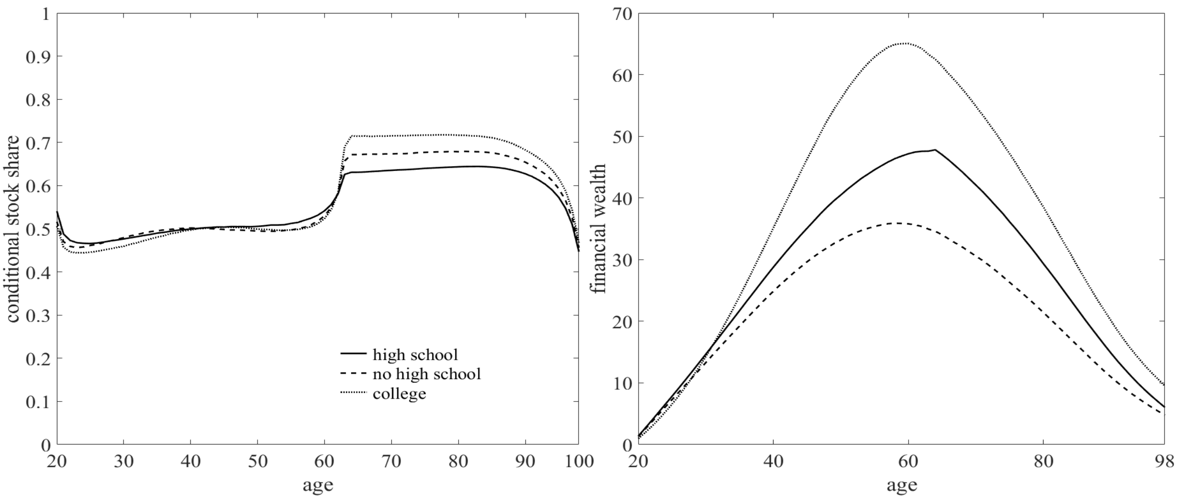
Table A1. Labor income process parameters by education level

Parameter		**No high school**	**High school**	**College**
Variance of permanent shocks	σ_ω^2	0.106	0.074	0.058
Variance of transitory shocks	σ_ε^2	0.011	0.011	0.017
Replacement ratio	λ	0.89	0.68	0.94

Source: Cocco, Gomes and Maenhout (2005, Tables 2 and 3)

Figure A4 shows the life-cycle profiles of risky investment for workers with different educational attainments. The flattening of conditional stockholding over working life is robust across education groups. The larger stock share during retirement for workers with no high school degree and with a college degree is due to the higher replacement ratio of pension income (0.89 and 0.94, respectively, compared with 0.68 of high school workers), which implies a larger proportion of total wealth invested in human capital.

Figure A4. Life-cycle profiles: the effect of education levels



The figure displays the life-cycle profiles of the mean simulated stock investment share and financial wealth accumulation Age ranges from 20 to 100, with retirement occurring at the age of 65. Solid lines refer to the benchmark case where the long-term unemployment risk is equal across workers; dashed lines refer to the case where the long-term unemployment risk is negligible if permanent income is larger than the 75^{th} percentile at age 20. Financial wealth is expressed in ten thousands of U.S. dollars.

D. Age-dependent disaster risk

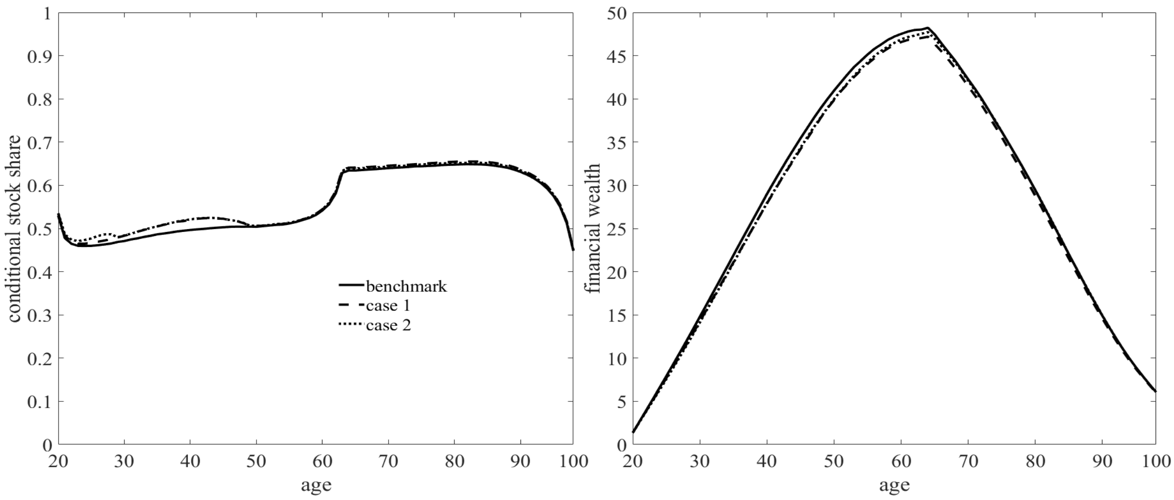
Here we allow for both age-dependent and smaller probabilities of a personal disaster caused by long-term unemployment in the transition probabilities matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 1-\pi_{u_1u_2} & 0 & \pi_{u_1u_2} \\ 0.85 & 0 & 0.15 \end{pmatrix}$$

In our baseline calibration in (17), $\pi_{u_1u_2} = 0.15$ irrespective of the worker's age. Now we consider two cases. In "*case 1*" the probability of entering long-term unemployment is reduced from 0.15 to 0.10 only for workers younger than 50 years old. In "*case 2*", we further reduce this probability for very young workers, setting $\pi_{u_1u_2} = 0.075$ for individuals less than 30 years old. In both scenarios, transition probabilities imply steady-state long-term unemployment rates lower than the value obtained in the baseline case (0.7%), being 0.45% and 0.35% in case 1 and 2, respectively.

Figure A5 displays life-cycle profiles of optimal conditional stock holding and financial wealth accumulation when long-term unemployment risk is age-dependent. The age profile of stock investment is very close to the baseline case with time-invariant probabilities. Lower long-term unemployment risk for younger workers implies only a slightly higher stock share until the age of around 45, with no effect in late working life and during retirement. In addition, the pattern of wealth accumulation is not affected. Overall, the main result of a flat age profile of risky investment is fully confirmed even with age-dependent disaster risk.

Figure A5. Life-cycle profiles: the effect of age-dependent long-term unemployment risk

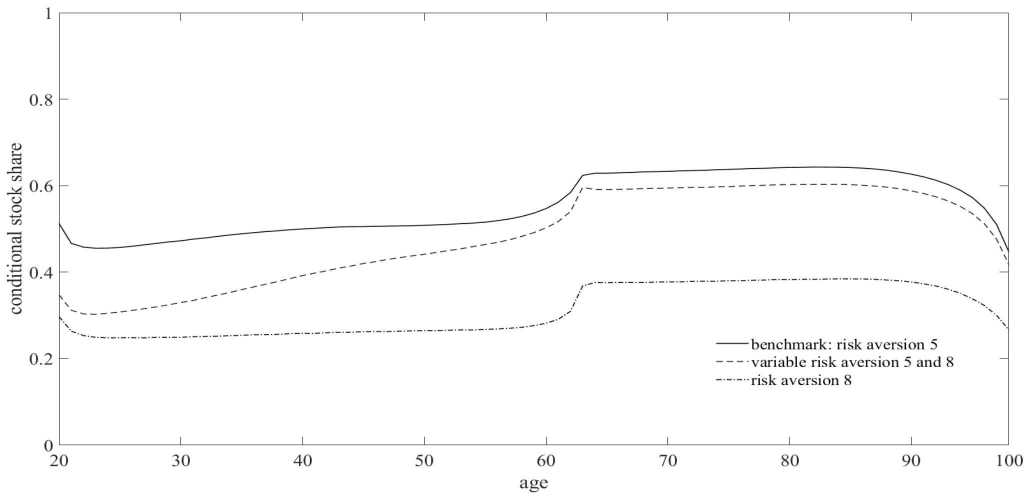


The figure displays the life-cycle profiles of the mean simulated stock investment share and financial wealth accumulation for individuals of age 20 to 100, with retirement occurring at the age of 65. Solid lines refer to the benchmark case where the long-term unemployment risk is age-invariant, with the probability of entering long-term unemployment for an already unemployed worker being 0.15. Dashed lines refer to "case 1", where this probability is reduced to 0.10 for workers younger than 50. Dotted lines refer to "case 2", where such probability is further reduced to 0.075 for workers younger than 30. Financial wealth is expressed in ten thousands of U.S. dollars.

E. State-dependent risk aversion

In the benchmark model, risk aversion is captured by a constant parameter, independent of the investor's working history. We explore here the possibility that the investor's risk aversion depends on the occurrence of a personal disaster event. In particular, we assume that if the individual is either employed or short-term unemployed, her risk aversion parameter is $\gamma = 5$ (the baseline value in our calibration), and increases to 8 in the case of long-term unemployment, remaining at this higher level even if re-employment occurs. The resulting life-cycle risky portfolio share is shown in Figure A6, compared with two cases in which risk aversion is state-invariant ($\gamma = 5$ and $\gamma = 8$).

Figure A6. Life-cycle profiles: state-dependent risk aversion



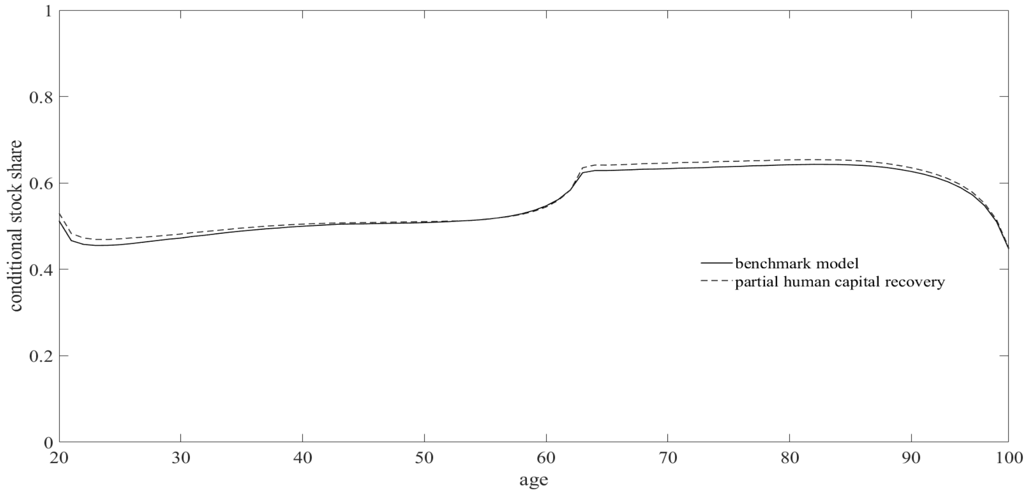
The figure displays mean simulated stock investment life-cycle profiles. Age ranges from 20 to 100, with retirement occurring at the age of 65. The solid line refers to the benchmark case with $\gamma = 5$, the dashed line refers to the case in which risk aversion increases to 8 once a personal disaster event has occurred; the dashed-dotted line refers to the case with $\gamma = 8$.

As expected, this change induces the investor to be more conservative, reducing her risky portfolio share for all ages; however, it implies also a counterfactual increasing pattern of stock investment during working life, instead of the flatter profile obtained with a constant risk aversion, which matches more closely the available empirical evidence, as shown in Section 5.

F. Transitory human capital loss

Personal disasters may decrease workers' earnings ability only temporarily instead of permanently, as assumed in our benchmark model. We address this possibility by exploring a scenario in which the reduction in human capital due to long-term unemployment is partially recovered when the worker finds a new job opportunity. Specifically, we assume that in the event of re-employment, the worker recovers 50% of the earnings loss experienced during the (long-term) unemployment spell. Figure A7 below shows that our main result (i.e. the relative flat life-cycle pattern of the risky investment share) is robust to this change in the persistence over time of human capital losses.

Figure A7. Life-cycle profiles: transitory human capital loss

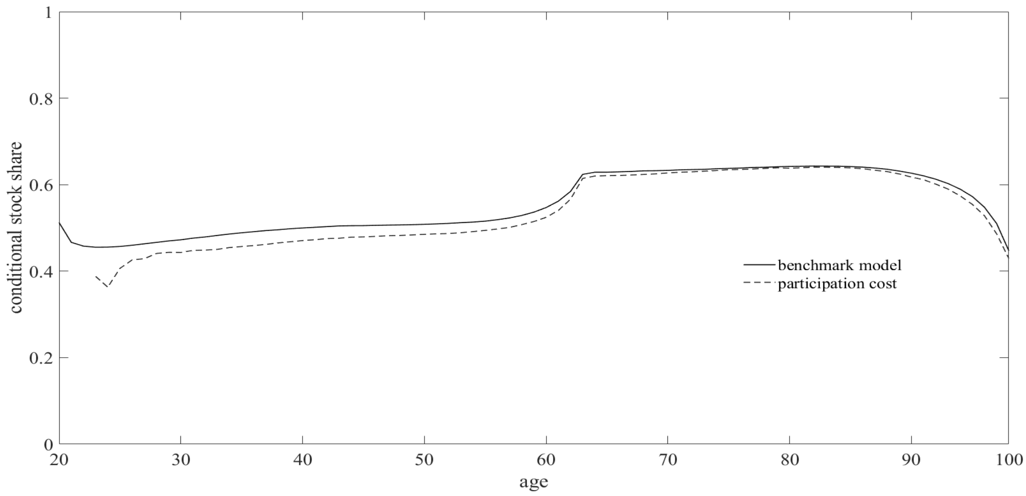


The figure displays mean simulated stock investment life-cycle profiles. Age ranges from 20 to 100, with retirement occurring at the age of 65. The solid line refers to the benchmark case with permanent human capital loss in the case of long-tern unemployment; the dashed line refers to the case of partial human capital recovery when re-employment occurs following a long-term unemployment spell.

G. Stock market participation costs

In our benchmark model, we assume that individuals have access to frictionless financial markets and can invest in the risky asset without paying any transaction costs. Although our model does not aim to address the non-participation issue in the stock market, we checked the robustness of our results when financial frictions are introduced in the (standard) form of a fixed stock market participation cost, calibrated as a small but non-negligible fraction (around 5%) of a young worker's labor income, amounting to around 800 US$. As shown in Figure A8 below, where the age pattern of the risky portfolio share conditional on participation is plotted, investment in the stock market is slightly reduced throughout life, but the flat age profile is retained for workers between the age of 25 and retirement, confirming our main result. Now only the youngest individuals, aged 20-25, do not participate in the stock market.

Figure A8. Life-cycle profiles: the effect of participation costs

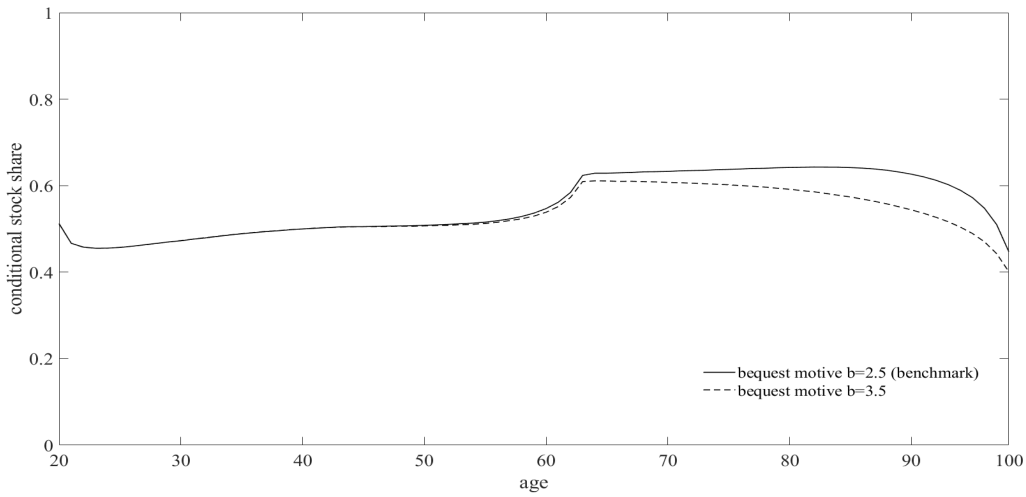


The figure displays mean simulated stock investment life-cycle profiles. Age ranges from 20 to 100, with retirement occurring at the age of 65. The solid line refers to the benchmark case with no stock market participation costs; the dashed line refers to the case in which participation in the stock market implies a fixed per-period cost of 800US$.

H. Stronger bequest motive

In the life-cycle model used in the paper, an operative bequest motive is incorporated into the investor's utility function, equation (1). To explore the effect of a stronger bequest motive on her life-cycle asset allocation, we perform a new calibration exercise: we increase the parameter (b) governing the willingness to leave a bequest (bearing the interpretation of the number of years of her descendants' consumption that the investor intends to save for) from 2.5 (a standard value in the literature) to 3.5. Figure A9 fully confirms the robustness of our main result to this extension, showing a remarkably flat age profile of the conditional risky asset portfolio share.

Figure A9. Life-cycle profiles: bequest motive



The figure displays mean simulated stock investment life-cycle profiles. Age ranges from 20 to 100, with retirement occurring at the age of 65. The solid line refers to the benchmark case with a bequest motive parameterized by b=2.5 (standard value); the dashed line refers to the case with a stronger bequest motive (b=3.5).

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