Extending Fault Trees with Continuous System Variables

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Fault Tree Analysis (FTA) is a widely adopted methodology where events are modeling the working/failure dichotomy of components and subsystems. However, system variables are often of continuous nature, and in some cases measured through a monitoring process. In this paper, we present an approach aimed at introducing continuous variables in a standard static fault tree (FT) formalism. We show how continuous variables can be tied to basic events in a FT, how to model probabilistic linear dependencies among them, and how influences of contextual information on system variables can be captured and modeled. We called the resulting formalism c-FT, and we propose a conversion of a c-FT into Hybrid Bayesian Networks (HBN); this allows us to exploit HBN inference algorithms, in order to perform the analyses of interest on the modeled system. As an experimental framework, we consider a model for a waste incinerator, and we present the results of specific analyses (from system reliability, to posterior probability of faulty situations) implemented through conversion of a c-FT into an HBN and by exploiting the MATLAB BNT Toolbox for inference.

Keywords: Fault tree, Hybrid Bayesian Network, Reliability analysis, Continuous system variables

1. Introduction

Recent studies on system dependability focus on hybrid stochastic processes due to the presence in real-world systems of both discrete and continuous variables such as, temperature, pressure, particle emission etc… Moreover, in real-world situations dependencies among the components of the system are present and cannot be neglected. A quite common approach to address the above issues is to resort to Probabilistic Graphical Models (PGM), and in particular to Bayesian Networks (BN) Bobbio et al. (2001). However, a direct use of PGMs in dependability and reliability studies is not always the best choice, since reliability engineers are more familiar with traditional tools as reliability diagrams, fault trees (FT), dynamic fault trees (DFT) among the others Portinale and Raiteri (2015). In particular, FT are widely adopted, because of their simplicity and intuitive interpretation, as well as their suitability to model several practical situations where the boolean failure logic of the system is clear Ruijters and Stoelinga (2015). Fault Tree Analysis (FTA) assumes that a suitable boolean abstraction of the system variables is provided, in such a way that events are modeling the working/failure dichotomy of components and subsystems. However, system variables are often of continuous nature, and in some cases measured through a monitoring process. This continuous information cannot be directly exploited in FTA, and hard thresholding mechanisms are often needed to cast it into the final FT model. For instance, if we need to model an overheating event, a given threshold on the continuous variable temperature is determined, and the event is considered true (i.e., to occur) if and only if the value of the variable temperature exceeds the given threshold.

In this paper we present an approach aimed at extending a standard FT (i.e., with only static gates and no dynamic dependencies as in DFT) with the introduction of continuous system variables directly tied to basic events. In addition to the boolean failure logic of the FT, the formalism, allows the modeling of probabilistic linear dependencies among the variables, as well as the influence of contextual information on them. We call the resulting formalism c-FT (continuous variables Fault Tree). In particular, we focus on continuous variables following a normal (Gaussian) distribution.

To perform the analysis, we propose to convert the c-FT into an Hybrid Bayesian Networks (HBN) Salmerón et al. (2018), that is a BN that includes a mixture of continuous and discrete variables. Discrete variables follow conditional multinomial distributions, while continuous variables are assumed to follow conditional normal distributions. In order to demonstrate the modeling and analysis capabilities of the c-FT approach we consider an example inspired from the case study described in Lauritzen (1992), concerning part of the burning process in a waste incinera-

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tor. We show how to compute specific reliability measures (as the probability of the top event or the probability of specific failures) as well as the prediction of specific parameters (e.g., the value of dust emissions from the incinerator in specific burning conditions).

The paper is organized as follows: in Section 2 we introduce the c-FT formalism, together with an example concerning the reliability analysis of a waste incinerator that we will use as a case study. We then briefly review basic notions about HBN (see Section 3), and we discuss how to convert a c-FT into an HBN (see Section 4); in Section 5 we quantify the incinerator case study, and in Section 6 we reports on the results obtained on analysing it. Finally, conclusions are drawn in Section 7.

2. Introducing continuous variables in Fault Tree

2.1. *The c-FT model*

We introduce the **c-FT** model (FT with continuous system variables), as a diagrammatic extension to standard FT formalism. We consider four types of nodes (see Fig. 1):

Fig. 1. Graphical representation of events

(i) basic events: they represent boolean random variables following a discrete probability distribution;
(ii) internal events: they represent boolean random variables whose true value is determined by the structure of the FT; a special internal event is the *top event* (TE), the boolean variable representing the catastrophic event at the root of the FT;
(iii) continuous system variables: they represent continuous variables assuming values in a given range, following a given distribution (e.g., a normal distribution with a specific mean and variance). We focus here on normal distribution;
(iv) context events: events representing contextual situations influencing the status of continuous variables. They are multistate variables influencing the values of continuous variables.
We consider the following gates (see Fig. 2):
(i) boolean gates (AND/OR): as in standard FT. Input can be basic or internal events, output can be only internal events;
(ii) linear dependency gate (Σ-gate): it represents a linear dependency between the input and output continuous variables.

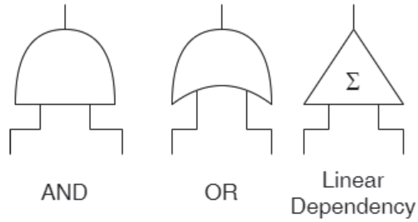


Fig. 2. Graphical representation of nodes

Concerning Σ-gates, we consider here a normal (Gaussian) distribution for each continuous variables. If a continuous variable c_o is in the output of a Σ-gate, then the mean of c_o is a linear combination of the values of the input variables. In other words the mean of c_o is determined through a linear regression on the input variables. We finally consider specific connection types between nodes in addition to gates (see Fig. 3):
(i) *c-b edge* (continuous to basic): connecting a continuous variable to a basic event;
(ii) *c-c edge* (context to continuous): connecting a context event to a continuous variable.
The first kind of edge ties the values of a variable

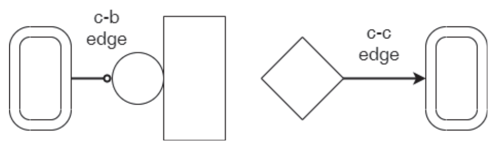


Fig. 3. Node connections

to the occurrence of a basic event trough a soft threshold mechanism; the second kind of edge introduce a dependency of a specific variable from a given contextual situation. Details are presented next.

Let E be a generic event in the model, and e a possible occurrence state; in case of boolean events we use e to model the event occurrence (true state) and $\overline{e}$ to model the non-occurrence (false state). A c-FT model is quantified through the following parameters:
(i) basic event E with no connection to continuous variables: a probability of occurrence $P(e) = 1 - P(\overline{e})$[a];
(ii) basic event E connected to a continuous variables C: the parameters w_0, w_1 of a logistic

[a] Any parametric specification from which the probability of occurrence can be computed is a viable option; e.g., by assuming an exponential distribution, we can parametrize via an occurrence rate λ and an analysis time t ($P(e) = 1 - \exp(-\lambda t)$),

regression model on C such that[b]

$$P(e) = \frac{1}{1 + \exp(-w_0 - w_1 c)} \qquad (1)$$

(iii) context event: a discrete probability distribution over possible context states;
(iv) continuous variables with no connection to context event and not in the output of a Σ-gate: a normal distribution $\mathcal{N}(\mu, \sigma^2)$ with mean μ and variance σ^2.
(v) continuous variable not in the output of a Σ-gate, but connected to some contexts $C_1, \dots C_k$: for each k-tuple $j = (c_1, \dots c_k)$, a normal distribution $\mathcal{N}(\mu_j, {\sigma_j}^2)$
(vi) continuous variable connected to some contexts $C_1, \dots C_k$ and in the output of a Σ-gate with continuous variables $E_1, \dots E_m$ in the input: for each k-tuple $j = (c_1, \dots c_k)$, a normal distribution $\mathcal{N}(\mu_j, {\sigma_j}^2)$ where μ_j is a linear regression model with parameters $w_0, w_1, \dots \dots w_m$ on events $E_1, \dots E_m$ (each w_i $(1 \leq i \leq m)$ corresponding to E_i)

2.2. *Example*

To illustrate the modeling features of the c-FT formalism, we consider the following example inspired from Lauritzen (1992) related to a waste incinerator plant. Main parts of the plant are a firebox where the waste is burned, an electro-filter to filter the emissions of the produced dust, and a chimney from which dust is emitted out. The emissions from a waste incinerator differ because of compositional differences in incoming waste. The filter efficiency is a variable that depends on the technical state of the electro-filter and the composition of the waste. Heavy metals may be emitted depending on both the original concentration of such metals in the incoming waste and the emission of dust particulates in general. The emission of dust as well as the concentration of metals in incoming waste both depend on the type of burned waste. A c-FT diagram for this example is reported in Fig. 4. We notice that the c-FT diagram can be naturally decomposed into two different parts: a standard FT (top part) leading to the TE of interest; a schematic representation of the interactions between variables and contexts (the bottom part identified with the cloud contour). The connection between the two parts is provided by the c-b edges introduced above. We consider the following characterization (see also Lauritzen (1992); Hansen and Dalager (1985)).

[b]Depending on what is easier to elicit, we can apply the logistic function to either $P(e)$ or $P(\overline{e})$, and consequently to compute the dual probability by taking into account that $P(e) = 1 - P(\overline{e})$.

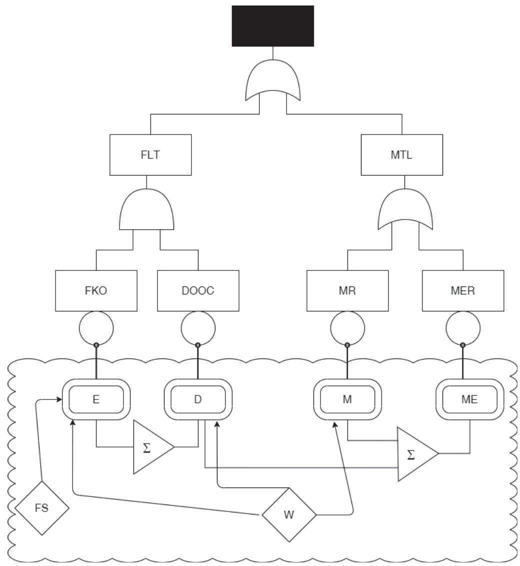


Fig. 4. c-FT for the Waste Incinerator Example

W: it represents the type of waste and has possible states `industrial, household`.
FS: the status of the filter with possible states `intact, defective`.
E: continuous variable measuring the filter (in)efficiency on a logarithmic scale; we assume efficiency to be the percentage of out-dust (out) removed from in-dust (in), that is $out = in - \delta$ where $\delta = in \times \rho$ and $0 \leq \rho \leq 1$ is the so called efficiency coefficient (measuring the filter efficiency from 0% to 100%). This results in the equation $out = in \times (1 - \rho)$. On a logarithmic scale this becomes $\log(out) = \log(in) + \log(1 - \rho)$. We finally consider the (in)efficiency E measured as $e = \log(1 - \rho)$.
D: continuous variable representing the dust emission level and measured on a logarithmic scale; it depends from W and E, thus the mean value of D is a linear function of E for each possible context W. Thus, on a logarithmic scale $\log(d) = w_0 + w_1 e$. If we know the expected value d of D in each situation, we can compute $w_0 = \log(d) - w_1 e$ by setting a suitable value for w_1 (e.g., $w_1 = 1$), or alternatively $w_1 = \frac{1}{e}(\log(d) - w_0)$ by setting a suitable value for w_0 (e.g., $w_0 = 0$). Of course if a data-set of observations is available, a linear regression model can be fitted to such data, resulting in the suitable values for w_0 and w_1. Similar considerations hold for all the continuous variables described below.
M: it represents the metal concentration in the original waste, measured as a percentage on a logarithmic scale; we assume a maximum value of 1 thus if $p \in (0, 1)$ is the percentage of metal present in the waste, then $d = \log(p) + 1$. It

depends on the type of burned waste, since in industrial waste the presence of metals is usually larger.
ME: variable measuring the total concentration of emitted metals (metal emissions) on a logarithmic scale; of course it is a linear function of the metal concentration in the emitted dust D (i.e., metals produced by the burning process), and the original metal concentration in the waste M.
FKO: basic event representing a failure of the filter function and based on continuous system variable E; in particular, the probability of such a failure is larger for lower values of the efficiency coefficient.
$DOOC$: basic event representing the dust emission out-of-control and depending on D; larger is the amount of emitted dust, larger is the probability of this event.
MR: basic event representing the occurrence of a metal risk (excessive heavy metal concentration in the original waste) and depending from values of M; the larger the metal concentration, the larger the event's probability.
MER: basic event representing the occurrence of a metal emission risk, the actual risk due to the excessive metal concentration in the emitted dust; it depends from ME and the larger the total emission of metals, the larger the event's probability.
FLT internal event representing the occurrence of failures related to filter subsystem;
MTL: internal event representing the occurrence of an excessive metal concentration in the whole burning process.
The TE of the diagram is the occurrence of either a filter failure or an excessive metal concentration.

3. Hybrid Bayesian Networks

In this section we outline basic notions about HBN.

Definition 3.1. A Bayesian Network (BN) is a pair $\langle G, P\rangle$ where: $G = \langle V, E\rangle$ is a DAG whose vertices $V = \{X_1, ...X_n\}$ represent (discrete) random variables and any edge $(X_i \rightarrow X_j) \in E$ represents a direct influence of X_i over X_j; P is a probability distribution over the variables represented by V, such that;

$$P(X_1, ...X_n) = \prod_{i=1}^{n} P(X_i|pa(X_i))$$

Definition 3.2. A Linear Gaussian Bayesian network (LGBN) is a pair $\langle G, P\rangle$ where: $G = \langle V, E\rangle$ is a DAG whose vertices $V = \{X_1, ...X_n\}$ represent continuous random variables with Gaussian distribution and any edge $(X_i \rightarrow X_j) \in E$ represents a linear dependency of X_i over X_j; P is a probability distribution over the variables represented by V.

A root node X follows a normal probability distribution $\mathcal{N}(\mu, \sigma^2)$ with mean μ and variance σ^2.

Let Y be a child node with parents $X = \{X_1, ...X_k\}$: the conditional distribution of Y given X is given by the following expression with parameters: b: offset, $W^T = [w_1, ..., w_k]$: vector of weights, σ^2: variance.

$$(Y|X = x) \sim \mathcal{N}(b + W^T x; \sigma^2) \qquad (2)$$

The expected value (mean) of Y is then given by

$$\mu_y = \sum_{i=1}^{k} w_i x_i + b$$

When discrete variables are allowed as parents of continuous variables, but no discrete variable is a child of a continuous one, the model is called Conditional Linear Gaussian Bayesian Network (CLGBN). The parametrization is the same as in the case of standard BN and LGBN, except that the distribution of a continuous node is actually a set of distributions as in "Eq. 2", one for each configuration of the discrete parents.

Finally, if also the restriction concerning the presence of discrete children for continuous nodes is removed, we get the most general class of HBN (properly called Hybrid Bayesian Networks). In this case, a discrete child having only discrete parents is specified as in a standard BN; in case continuous parents are present, a soft threshold model like probit or logit Aldrich and Nelson (1984) can be adopted. If the variable D is binary, the logit model corresponds to the logistic function in "Eq. 1". Of course, if discrete nodes are present as parents of D, a different specification has to be provided, for each configuration of discrete parents.

4. Conversion to Hybrid Bayesian Network

In this section we show how to convert a c-FT diagram into a general HBN. The HBN structure is obtained as follows.
For each context event, create a discrete variable with no parents; states of the variable are identified with alternative values of the context.
For each basic or internal event, create a discrete binary variable.
For each continuous system variable C, create a Gaussian variable. Connect such a variable in the following way: if C is connected to a context CXT with a c-c edge, then add CXT as a parent of C; if C is in the input of a Σ-gate, add C as a parent of C' where C' is the output of the Σ-gate; if C is connected to a basic event E through a c-b edge, add C as a parent of E.
For each logical gate having event E in the output, connect E with all the discrete variables that are in the input of the gate.

The HBN corresponding to the c-FT diagram of Fig. 4 is reported in Fig. 5 (continuous nodes are double circled). Conditional probability distributions (CPD) of the HBN are directly obtained from the quantification of the c-FT; CPDs of discrete nodes corresponding to internal events are obviously boolean deterministic distributions (AND/OR depending on the gate).

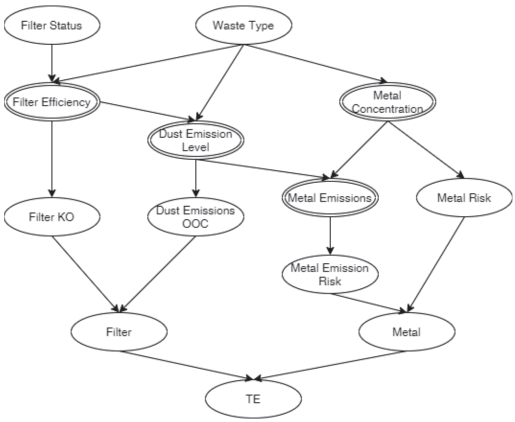


Fig. 5. HBN for the Incinerator Example

5. The Waste Incinerator Case Study

Focusing on the model described in Section 2.2, we consider the following parameterization inspired from Lauritzen (1992).

W: $P(W = \texttt{industrial}) = P(W = \texttt{household}) = 0.5$.

FS: $P(FS = \texttt{intact}) = 0.95$; $P(FS = \texttt{defective}) = 0.05$.

E: we consider the following efficiency coefficients: $\rho_1 = 0.96$ for an intact filter in a household plant; $\rho_2 = 0.39$ for a defective filter in a household plant; $\rho_3 = 0.98$ for an intact filter in an industrial plant, and $\rho_4 = 0.33$ for a defective filter in an industrial plant. By considering $e_i = \log(1 - \rho_i)$ we get the following values that can be considered the mean values of a normal distribution:
$e_1 = -3.2, e_2 = -0.5, e_3 = -3.9, e_4 = -0.4$.
We finally assume some variances reflecting the uncertainty about the actual filter efficiency, resulting in the following distributions:
$P(e|\texttt{intact}, \texttt{household}) = \mathcal{N}(-3.2, 0.08)$
$P(e|\texttt{defect}, \texttt{household}) = \mathcal{N}(-0.5, 0.15)$
$P(e|\texttt{intact}, \texttt{industrial}) = \mathcal{N}(-3.9, 0.08)$
$P(e|\texttt{defect}, \texttt{industrial}) = \mathcal{N}(-0.4, 0.15)$

D: we know that $d = \log(d') = w_0 + w_1 e$ and we assume for simplicity $w_1 = 1$; suppose we know that a good filter produces a dust concentration $d' = 16.4\ mg/Nm^3$ in a household plant[c], and a concentration $d' = 13.5\ mg/Nm^3$ in an industrial plant. This results in the following distributions (assuming a suitable variance):
$P(d|e, \texttt{household}) = \mathcal{N}(6 + e, 0.4)$
$P(d|e, \texttt{industrial}) = \mathcal{N}(6.5 + e, 0.3)$
Indeed, a good household filter has a mean value $e = -3.2$, thus $w_0 = \log(16.4) + 3.2 = 6$. Similarly for an industrial plant($w_0 = \log(13.5) + 3.9 = 6.5$); of course such a parameterization implies a dust concentration $d' = exp(6 - 0.5) = 244.69\ mg/Nm^3$ in a defective household filter, and $d' = exp(6.5 - 0.4) = 445.85\ mg/Nm^3$ in a defective industrial filter. It should be clear that if measured concentrations differ from the ones above, it would mean that dust emission level will depends not only on filter efficiency, but also directly on filter status. In the model of Fig. 4 this is not the case, since the continuous variable D has no entry arc from FS.

M: we expect a mean metal concentration $p = 22\%$ in household waste, and a mean concentration $p = 60\%$ in industrial waste. Moreover, metal concentration in industrial waste has larger variability. Since $m = \log(p) + 1$, this results in the following distributions:
$P(m|\texttt{household}) = \mathcal{N}(-0.5, 0.05)$
$P(m|\texttt{industrial} = \mathcal{N}(0.5, 0.1)$

ME: since the plant emits an amount of metals given by the sum of the metals in the original waste and the metals produced by the burning process we assume the following distribution:
$P(me|d, m) = \mathcal{N}(d + m, 0.02)$

Concerning the probability distribution on basic events, we assume a logistic function based on the values of the corresponding continuous system variable as follows:

FKO: we assume a limiting filter efficiency $\rho_l = 0.3$ (corresponding to $e_l = \log(1 - \rho_l) = -0.36$) such that with an efficiency $\rho \ll \rho_l$ we have a significant probability of occurrence of FKO, and with $\rho \gg \rho_l$ we have a significant probability of $\overline{fko}$ (i.e., $FKO = \texttt{false}$). We consider than the following logistic function $P(fko) = \frac{1}{1+exp(-50(e+0.36))}$ (see Fig. 6a). Remember that E is measured on a logarithmic scale, thus for instance, for $\rho = 0.8$ we get $e = \log(1 - 0.8) = -1.61$ and then $P(fko) = \frac{1}{1+\exp(-50(-1.61+0.36)} = 0.016$, i.e., it is almost sure that the filter is not KO[d].

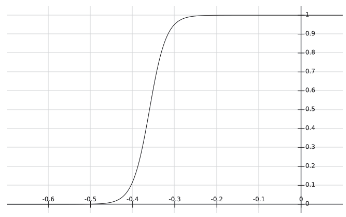
[c] Nm^3 is the *normal cube meter*, the unit measure of a gas in "normal" condition, i.e., at temperature $t = 0$ Celsius and at atmospheric pressure $p = 1.01325$ bar.

[d] Notice that with a logistic characterization the limiting filter

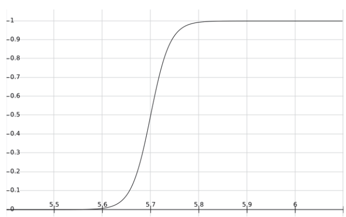
$DOOC$: we assume the dust emission limit $l = 300\ mg/Nm^3$. On a logarithmic scale this corresponds to $d = \log(l) = 5.7$, thus we consider the following quantification $P(dooc) = \frac{1}{1+exp(-50(d-5.7))}$ For example if the emission is 365 N/m^3, then $d = \log(365) = 5.9$ and $P(dooc) \approx 1$ (see Fig. 6b).

MR: we assume a metal concentration risk threshold on the waste (measured again on a logarithmic scale) $m = 0.69$. This results in $P(mr) = \frac{1}{1+exp(-100(m-0.69))}$. For example if the metal concentration in the waste is $m = 0.75$, then $P(mr) = 0.95$.

MER: by assuming the limits $d = 5.7$ on D and $m = 0.69$ on M, we get a limit on total metal emission $me = 5.7 + 0.69 = 6.39$. We then model $P(mer) = \frac{1}{1+exp(-40(me-6.39))}$. For example if total metal emission is $me = 6.45$, then $P(mer) = 0.92$.



(a) Probability distribution of FKO depending on E



(b) Probability distribution of $DOOC$ depending on D

Fig. 6. Probability Distributions for Basic Events

6. System Analysis

Given the c-FT model previously introduced and the correspondig HBN, we have then performed some analysis on the case study. All the experiments have been implemented in MATLAB using the BNT toolbox[e] and the *likelihood weighting* inference algorithm, a Monte Carlo algorithm able to estimate the desired posterior probability Schacter and Peot (1990). We first consider the reliability of the plant under the 4 possible contextual situations: (1) industrial waste with an intact filter, (2) industrial waste with a defective filter, (3) household waste with intact filter, and (4) household waste with defective filter. We set some experimental parameters as reported in the following table; in particular, a baseline filter efficiency coefficient ρ for each situation, a degradation rate λ for ρ (different in case of industrial or household waste), and an expected metal concentration p in the waste (larger in industrial waste). We have

	(1)	(2)	(3)	(4)
ρ	0.98	0.33	0.96	0.39
$\lambda\ (\frac{1}{\text{hour}})$	2.5e-05	2.5e-05	7.5e-05	7.5e-05
p	60%	60%	22%	22%

then computed the plant's unreliability from time $t = 0$ until $t = 10^4$ hours and results are reported in Fig. 7. This evaluation has been performed on the HBN of Fig. 5, by computing the posterior probability of node TE given the evidence on nodes W, FS, E and M. We notice that, in case of an intact filter, the plant exhibits a reliable behavior in the considered time period (after 10000 hours there is a 99% probability of system working correctly when burning household waste, and a 93% probability when burning industrial waste). On the other hand, as expected, processing industrial waste with a defective filter is very problematic. A better situation is evidenced when processing household waste with a defective filter; however, we can notice a steep increase in plant unreliability from $t = 2000$ to $t = 5000$. This

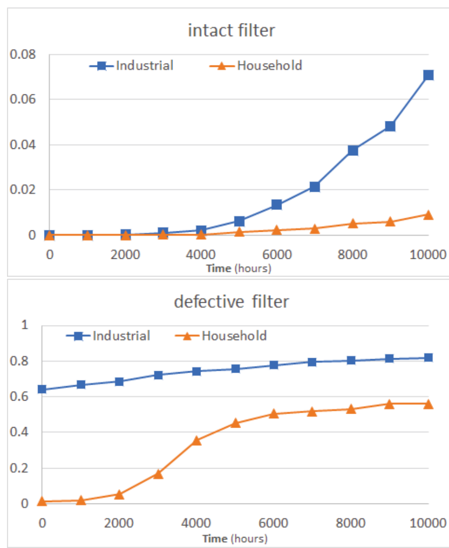


Fig. 7. Probability of TE (system unreliability)

efficiency will always corresponds to a 50% probability of event occurrence.

[e]https://github.com/bayesnet/bnt

is well explained by considering the probability of failure of the filter (basic event FKO) as reported in Fig. 8. We notice that for household waste, the same increase pattern occurs in the same period; this is because in the considered time span, the efficiency score (the value of variable E) ranges from approximately $e = -0.4$ to approximately $e = -0.3$. By looking at Fig. 6a, this is exactly the range of values where the probability of filter failure reaches a significant value. In order to have an occurerence of the TE, a knock-out of the filter is not sufficient (see c-FT in Fig. 4), since an occurrence of dust emission out-of-control is also necessary. Fig. 9 shows the trend of the dust emission level in case of defective filter. The trend is increasing with time as expected, and in the interval $[2000, 5000]$ the emission levels will correspond (on a log scale) to values for variable D from approximately $d = 5.6$ to approximately $d = 5.7$; this is again a range related to a significant increase of the probability of basic event $DOOC$ (see Fig. 6b), finally explaining the pattern evidenced in Fig. 7.

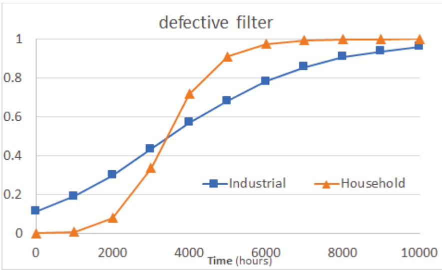


Fig. 8. Probability of filter failure (FKO)

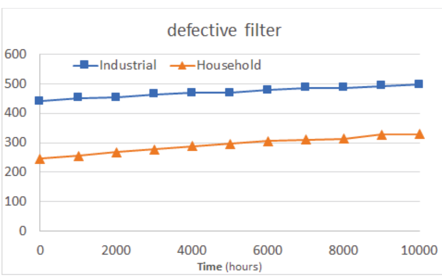
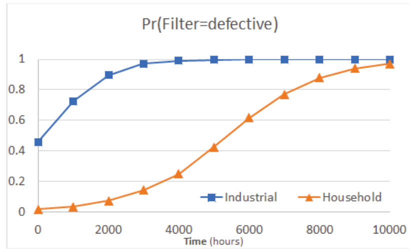
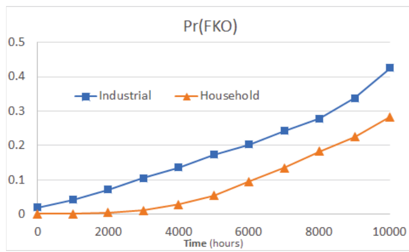


Fig. 9. Dust emission levels: mg/Nm^3 (mean values)

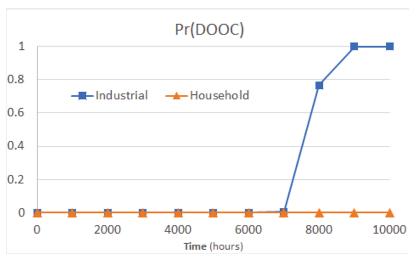
Another interesting analysis that can be performed, concerns the evaluation of specific conditions in the plant, given some measured quantities. Suppose the plant is equipped with a sensor for the detection of the dust emission level; consider a situation where the dust emissions have a 15% increase (from some baseline values) every 1000 hours. Suppose we have no other information but the type of burned waste. Fig. 10 reports the posterior probabilities on specific plant conditions (considering dust emission baseline levels as 100 and 50 mg/Nm^3 for industrial and household waste respectively). We notice that (see Fig. 10a), given the considered time series of dust emissions, the status of the filter is very likely to converge to



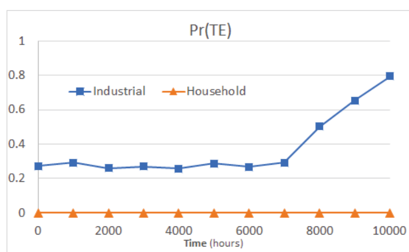
(a) Probability of FS=def.



(b) Probability of FKO



(c) Probability of $DOOC$



(d) Probability of TE

Fig. 10. Plant probabilities given dust emission level

a defective status (with a faster convergence for industrial waste). The dust emission (Fig. 10c) is going out of control at the end of the monitored period for industrial waste (it remains under control for household waste); together with the trend on FKO (Fig.10b), this results in an increase on the plant unreliability (Fig.10d) at the end of the period. Finally, Fig. 11 reports the probability of a metal emission risk (MER) on the left vertical axis, and the corresponding metal emission score (the value of continuous variable ME) on the right vertical axis. We notice that the increase of the estimated metal emission score produces a significant increase in the probability of metal risk in case of industrial waste, while the metal risk is under control for household waste (metal concentration in industrial waste is about three times than in household waste).

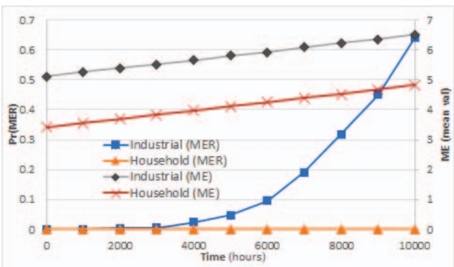


Fig. 11. Prob. of metal risk and trend in metal emissions

7. Conclusion

The introduction of continuous variables into FTA has been initially proposed in Yang and Xue (1996) and exploited in Xiaoyang et al. (2009); Ma et al. (2011). However, the proposed formalism (called *State Tree*) only allows the modeling of deterministic linear relationships and a very limited type of analysis. Moreover, basic limitations of FT (e.g., independence of components) are kept, even if this is not usually a valid assumption (Bobbio et al., 2001). Our approach is more general, since uncertainty about the behavior of continuous variables is directly addresses, as well as a well-defined connection with the standard FT model. Evidence-based analysis (i.e., posterior probability computation) can be performed as discussed in the proposed case study. A further possibility for dealing with continuous values is to resort to Fuzzy FTA Ruijters and Stoelinga (2015) where fuzzy set theory is used to models linguistic terms. However fuzzy logic is usually exploited only to address the potential vagueness of the probabilistic parameters; a noticeable exception can be found in Ren and Kong (2011), where fuzzy linguistic values are also exploited to model multi-state components, but without any clear connection to the occurrence of basic events related to continuous variables, as done in our approach. The c-FT model is a sound and interpretable way for dealing with the introduction of continuous variables in FTA, and the direct connection with HBN opens the door to the exploitation of very general analysis capabilities.

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