

## Drag reduction without boundaries: the viscoelastic turbulent Kolmogorov flow

G. Boffetta<sup>1</sup>, A. Celani<sup>2</sup>, A. Mazzino<sup>3</sup>, A. Puliafito<sup>2</sup>

<sup>1</sup>Dipartimento di Fisica Generale and INFN  
Università degli Studi di Torino, Via Giuria 1, 10125, Torino, Italy

<sup>2</sup>Institut Nonlinéaire de Nice, CNRS  
1361 Route des Lucioles, 06560 Valbonne, France

<sup>3</sup>Department of Physics and INFN  
Università di Genova, Via Dodecaneso 33, 16146 Genova

Contact address: *antonio.celani@inln.cnrs.fr*

### 1 Introduction

In 1949 the British chemist Toms reported that the turbulent drag could be reduced by up to 80% through the addition of minute amounts (few tenths of p.p.m. in weight) of long-chain soluble polymers to water. This observation triggered an enormous experimental activity to characterize this phenomenon (see, e.g., [1][2][3][4][5]). In spite of these efforts, no fully satisfactory theory of drag reduction is available yet. However, a recent breakthrough has been the observation of drag reduction in numerical simulations of the turbulent channel flow of viscoelastic fluids [6]. Most of the features of experimental flows of dilute polymer solutions are successfully reproduced by these models, even at the quantitative level [7][8].

Here we present the results of an extensive numerical investigation of the viscoelastic turbulent Kolmogorov flow. This flow is realized by driving the fluid through a parallel force with a sinusoidal profile. We will show that drag reduction takes place notwithstanding the absence of material boundaries.

### 2 The viscoelastic turbulent Kolmogorov flow

To describe the dynamics of a dilute polymer solution we first consider the linear viscoelastic model (Oldroyd-B) [9]

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nu_0 \Delta \mathbf{u} + \frac{2\eta\nu_0}{\tau} \nabla \cdot \boldsymbol{\sigma} + \mathbf{F} \quad (1)$$

$$\partial_t \boldsymbol{\sigma} + (\mathbf{u} \cdot \nabla) \boldsymbol{\sigma} = (\nabla \mathbf{u})^T \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot (\nabla \mathbf{u}) - 2 \frac{\boldsymbol{\sigma} - \mathbf{1}}{\tau} + \kappa \Delta \boldsymbol{\sigma}, \quad (2)$$

where  $\boldsymbol{\sigma}$  is the conformation tensor of polymers  $\sigma_{ij} = \langle R_i R_j \rangle / R_0^2$ , being  $\mathbf{R}$  the end-to-end separation and  $R_0$  the equilibrium gyration radius of the polymer molecule. The parameter  $\tau$  is the (slowest) polymer relaxation time.  $(\nabla \mathbf{u})_{ij} =$

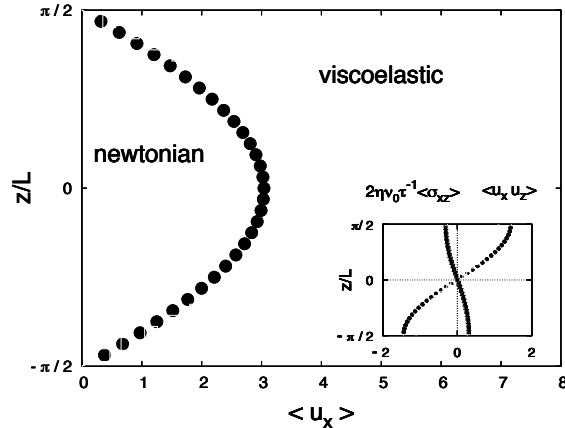


Figure 1: Mean velocity profiles for a Newtonian ( $\eta = 0$ ) and a viscoelastic simulation ( $\eta = 0.3$ ,  $El = 0.019$ ) at given forcing amplitude  $F = 1.5$ . The measured profiles are undistinguishable from  $\langle u_x \rangle = U \cos(z/L)$  (full lines) in both cases. The effect of elasticity is to increase the peak value  $U$  with respect to the Newtonian case: in the present case this corresponds to a reduction of the drag coefficient, defined in eq. (3), of about 40%. In the inset, the profiles of the Reynolds stress  $\langle u_x u_z \rangle = S \sin(z/L)$  and the mean polymer stress  $2\nu_0\eta\tau^{-1}\langle\sigma_{xz}\rangle = -T \sin(z/L)$ . In this case the Reynolds stress is reduced upon polymer addition to approximately 70% of its Newtonian value, consistently with experimental results at comparable drag reduction. The "missing" turbulent shear stress is compensated by the contribution of the polymer stress: the sum of  $S$  and  $T$  is equal to  $F$  in both the Newtonian and viscoelastic case. Data result from the numerical integration of eqs. (1) and (2) in a periodic cube of side  $2\pi$  by means of a fully dealiased pseudospectral code with  $64^3$  collocation points. The mean flow lengthscale is  $L = 1$  and the viscosity is  $\nu = 0.015625$ . Starting from an initial configuration with a small amount of energy on the smallest modes, after the system evolved into a statistically stationary state, time averages over 100 to 1000 eddy-turnover times have been performed to obtain the mean velocity profiles.

$\partial_i u_j$  and  $\mathbf{1}$  is the unit tensor. The solvent viscosity is denoted by  $\nu_0$  and  $\eta$  is the zero-shear contribution of polymers to the total solution viscosity  $\nu = \nu_0(1 + \eta)$ . The diffusive term  $\kappa\Delta\sigma$  is added to prevent numerical instabilities [10]. The forcing  $\mathbf{F}$  maintains the system in a statistically stationary state and has the form  $F_x = F \cos(z/L)$ ,  $F_y = F_z = 0$ .

As shown in Fig. 1 the mean velocity profile measured in numerical experiments is  $\langle u_x \rangle = U \cos(z/L)$ ,  $\langle u_y \rangle = \langle u_z \rangle = 0$ , where  $\langle \dots \rangle$  denotes the time average. The drag coefficient is thus defined in terms of the centerline mean

velocity as

$$f = \frac{FL}{U^2}. \quad (3)$$

As shown in Fig. 2 the viscoelastic flow is characterized by drag reduction for several values of fluid parameters (for further details see Ref. [11])

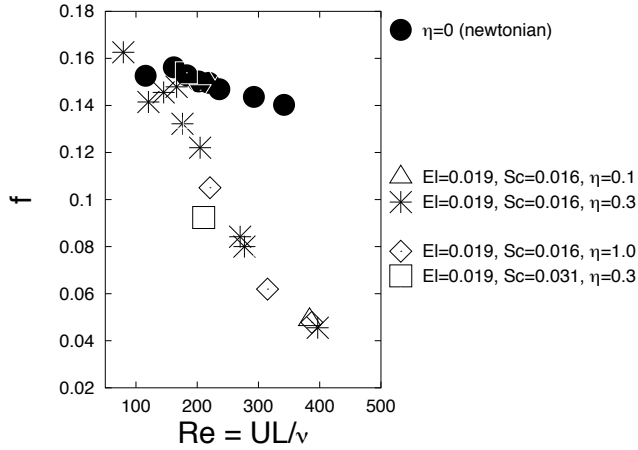


Figure 2: The drag coefficient for different viscoelastic fluid parameters.

### 3 The limit of strong polymer elongation

An even simpler model of viscoelastic flow is obtained by taking the limit of strong polymer elongation, or, equivalently, of vanishingly small equilibrium gyration radius [12]. In this case the governing equations take the following form:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nu_0 \Delta \mathbf{u} + \frac{2\eta\nu_0}{\tau} \mathbf{R} \cdot \nabla \mathbf{R} + \mathbf{F} \quad (4)$$

$$\partial_t \mathbf{R} + (\mathbf{u} \cdot \nabla) \mathbf{R} = \mathbf{R} \cdot \nabla \mathbf{u} - \frac{\mathbf{R}}{\tau} + \kappa \Delta \mathbf{R} \quad (5)$$

where  $\mathbf{R}$  denotes the typical end-to-end separation of a polymer molecule.

In spite of the crude approximation, drag reduction is observed in this system as well, as shown in the table below. These results point to the conclusion that the basic mechanism for drag reduction does not depend neither on boundary conditions nor on the particular choice of the model.

$\tau$	$\langle u_x'^2 \rangle$	$\langle u_y'^2 \rangle$	$\langle u_z'^2 \rangle$	$U$	DR[%]	$f$	$\tilde{f}$	$-\tau_R$	$-\tau_P$	$\nu_e$	$\mu_e$
0.1	4.01	3.21	3.56	3.57	0	0.16	0.1	2.0	0	0.57	0
1	4.01	3.1	3.74	3.64	3.8	0.15	0.1	2.0	0	0.55	0
2	3.78	2.72	3.32	3.61	2.2	0.15	0.12	1.97	0.06	0.55	0.02
4	4.16	2.11	2.27	4.04	28.1	0.12	0.16	1.80	0.22	0.45	0.05
5	4.25	1.81	1.98	4.25	29.4	0.11	0.19	1.72	0.3	0.40	0.07
10	5.89	1.41	1.36	5.31	54.8	0.07	0.21	1.5	0.54	0.28	0.10
$\infty$	8.81	0.78	0.80	8.66	83	0.03	0.26	0.69	1.29	0.08	0.15

Table 1: Results from the integration of Eqs. (4) and (5) at  $Sc = 100$  and  $F = 2$ . The columns are: the polymer relaxation time  $\tau$ , the velocity variance in the three directions, the centerline mean velocity  $U$ , the drag reduction  $DR$ , the friction factor  $f$ , the ratio between energy input and velocity fluctuations  $\tilde{f} = (FU/2)/u_{rms}^3$ , the peak Reynolds stress  $\tau_R = -\langle u_x u_z \rangle$ , the peak polymer stress  $\tau_P = 2\eta\nu\tau^{-1}\langle R_x R_z \rangle$ , the eddy viscosity defined by  $\tau_R = \nu_e \partial_z \langle u_x \rangle$ , and the effective polymer viscosity defined by  $\tau_P = \mu_e \partial_z \langle u_x \rangle$ .

## References

- [1] J. Lumley, *Annu. Rev. Fluid Mech.* **1**, 367 (1969).
- [2] P. .S. Virk, *AICHE Journal* **21**, 625 (1975).
- [3] W. D. D. Mc Comb, *The physics of fluid turbulence*, Oxford University Press (1992).
- [4] R. H. Nadolink and W. W. Haigh, *ASME Appl. Mech. Rev.* **48**, 351 (1995).
- [5] K. R. Sreenivasan and C. M. White, *J. Fluid Mech.* **409**, 149 (2000).
- [6] R. Sureshkumar, A. N. Beris, and R. A. Handler, *Phys. Fluids* **9**, 743 (1997).
- [7] Ptasinski, P.K., Boersma, B.J., Nieuwstadt, F.T.M., Hulsen, M.A., van den Brule, B.H.A.A., and Hunt J.C.R. *J. Fluid Mech.* **490**, 251 (2003).
- [8] T., Min, J. Y. Yoo, H. Choi, and D. D. Joseph, *J. Fluid. Mech.* **486**, 213 (2003).
- [9] R. B. Bird, C. F. Curtiss, R. C. Armstrong, and O. Hassager, *Dynamics of polymeric fluids* Vol.2, Wiley, New York (1987).
- [10] R. Sureshkumar and A. N. Beris, *J. Non-Newtonian Fluid Mech.* **60**, 53 (1995).
- [11] G. Boffetta, A. Celani, and A. Mazzino, <http://arxiv.org/nlin.CD/0309036> (2003).
- [12] E. Balkovsky, A. Fouxon, and V. Lebedev, *Phys. Rev. E* **64**, 056301 (2001).