Dynamic inconsistency in choice and different models of dynamic choice – A review

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Abstract

In this paper the problem of inconsistent dynamic choice is discussed, as considered in the literature, both under certainty in the context of changing preferences, and under risk and uncertainty in the case of preference orderings which violate expected utility theory. The problem of inconsistent choice in a dynamic decision situation has been initially analysed in the literature in a context of certainty and related to the problem of preferences changing exogenously through time. Hammond (1976,1977) generalises the analysis, but keeps it confined to a situation without risk or uncertainty, which he introduces only later (Hammond 1988a,b;1989). Hammond overcomes the distinction between exogenously and endogenously changing tastes and concentrates the analysis on the essential aspect of the problem - that preferences get reversed over time. This implies considering dynamic choice in a general framework. The discussion on dynamic inconsistency under certainty brings about the definition of two different models of behaviour: the *myopic approach* and the *sophisticated approach*.

In a context of choice under risk and uncertainty, dynamic inconsistency occurs when preference orderings over risky or uncertain outcomes violate Expected Utility Theory, particularly through violation of the Independence Axiom. The problem of the dynamic inconsistency of non-expected utility agents is illustrated first through the arguments by Raiffa (1968). Raiffa frames the problem of inconsistent choice in a context of dynamic choice under risk, by showing that dynamic consistency is not compatible with the usual choices in an Allais paradox when this is considered as a decision problem in two stages. Then we discuss the two main models in this context, Machina (1989) and McClennen (1990), after having illustrated briefly the general theoretical debate on the justification of expected utility as a normative theory, in which the dynamic inconsistency argument and the two models are framed. Both models offer a similar - though formally different - solution to the problem of dynamic inconsistency in this context. Particular attention is given to McClennen's (1990) approach: the *resolute approach*. From the above discussion it emerges that McClennen also offers a formal and very complete model for sophisticated behaviour under risk and uncertainty. Therefore, we discuss other two approaches to this model of behaviour: Karni and Safra (1989b,1990), who elaborate a model of 'behavioural consistency' which represents a solution to the problem of dynamic inconsistency with non-expected utility preferences, extending to risk and uncertainty the sophisticated approach; and Dardanoni (1990), who frames the problem and discusses the limits of sophisticated choice in this context.

Keywords: Dynamic decision making, Myopic choice, Sophisticated choice, Resolute choice, Dynamic inconsistency

JEL classifications: D90, D81

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1. Dynamic inconsistency with changing preferences. Myopic and Sophisticated choice.

The problem of inconsistent choice through time is introduced by Strotz (1956) for the case of a consumer who must choose a plan of consumption for a future period in time so to maximise the utility of the plan evaluated at the present moment. Inconsistent behaviour occurs in this framework, when the individual does not follow the optimal plan originally selected, “even though his original expectations of future desires and means of consumption are verified” (p.165). The framework is one of certainty, and tastes are assumed to change exogenously, as they depend on time. Strotz’s work has been followed on the same line by Pollak (1968), Phelps and Pollak (1968), Blackorby, Nissen, Primont and Russell (1973), Peleg and Yaari (1973).

Two different strategies of choice are discussed by Strotz in the framework of dynamically inconsistent situations:

- “myopic” or “naive” choice;
- “Consistent planning”, later termed by Hammond (1976) “sophisticated” choice.

Myopic choice originates from the individual’s inability to recognise that the optimal plan of future behaviour formulated at a given point in time is inconsistent with his actual optimising future behaviour, so that the individual will act inconsistently with respect to his plans. Alternatively, the agent can recognise the conflict, and adopt a “strategy of consistent planning”, rejecting all plans that he anticipates he will not follow, and adopting the best plan among those that he knows he will follow.

Hammond (1976, 1977) generalises Strotz’s analysis beyond consumer theory, but keeps it confined to a situation without risk or uncertainty, which he will introduce only later (Hammond 1988a,b;1989). The distinction between exogenously and endogenously changing tastes is overcome, and the analysis concentrates on the essential problem with changing tastes, that preferences get reversed over time. This implies that dynamic choice is considered in a general framework. Furthermore, it allows to define what is meant by inconsistent dynamic choice, and what are the strategies available to the agent in this situation.

A *dynamic decision problem* is defined as a problem where an agent takes a sequence of decisions over time, responding to situations which are a function of his own previous choices, and of randomly determined events. The dynamic decision problem, which is assumed to be bounded, is represented by a *decision tree*, composed by some initial chance or choice/decision node; a set of following *choice* or *chance nodes*, which define choices to be made by the agent or by nature, represented graphically by squares and circles respectively; and a set of *terminal nodes* or *outcomes* or *consequences*.

A *plan* for a decision problem consists of a complete specification of the choices to be made at one or more future moments in time, subject to different possible chance events taking place. In case there are only choice nodes, a plan will define a unique path through the tree, from an initial node to a terminal outcome. If there are also chance nodes, the (contingency) plan will specify the choices to be made at each choice node, where the choice node the agent finds himself at is also a function of random events.

The problem of "essential inconsistency" is illustrated by Hammond (1976) in the following simple example, in which no chance event occurs, so that the situations are only a function of the agent's choices.

Suppose that an individual is considering whether to start taking an addictive drug. The individual would prefer at most to take the drug without consequences. However, he is certain that if he starts, he will become an addict, with serious consequences for his health. Of course, he can refuse to take the drug at the first place. This agent is facing a simple dynamic decision problem with the following structure:

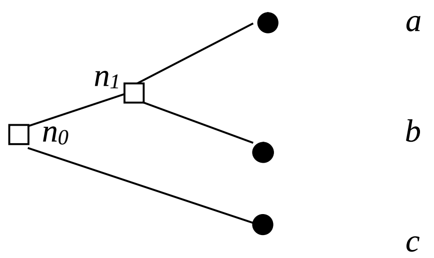

Figure 1 – The potential addict problem

where three options are available to him, which represent plans to adopt, and lead to the following outcomes:

- take the drug till it is harmless, then stop, which leads to outcome *a*
- become an addict, which leads to outcome *b*
- not take the drug, which leads to outcome *c*.

At the initial decision node n_0 the agent has to decide whether to take the drug or not, and his preferences are $a \succ b$, $a \succ c$ and $c \succ b$.

At choice node n_1 he has become an addict, and therefore the only relevant preferences are those concerning *a* and *b*, and addiction itself means that $b \succ a$. Thus, at n_1 his initial preferences between *a* and *b* get reversed. The agent will choose *b* inconsistently with his previous choice of reaching for *a*.

The problem of inconsistency with (exogenously or endogenously) changing preferences, comes from the fact that preferences get "reversed over time, and had the reversal been anticipated, quite a different option (such as *c*) might have been chosen" (Hammond 1976, page 162).

The requirement of *dynamic consistency* is a requirement of consistency between planned choice and actual choice. It requires that the rational agent implements the plan originally adopted, "particularly when the choice of a plan is based on some systematic evaluation of alternative plans" (McClennen 1990, page 116), and in the absence of new information or involuntary error. Dynamic consistency may therefore be defined in terms of the disparity which may occur between the plan originally adopted by the agent and his subsequent choice. Dynamic consistency requires coincidence between what the agent is to choose at a certain node by the strategies judged acceptable at the beginning, and the continuations of the strategies from that point on, which are judged acceptable at that node. Thus, it formalises the intuition that changes in plans, not due to new information, induce the agent in a form of dynamic inconsistency[1].

In the example, the agent decides initially to take the drug, then stop - *a*, but chooses later not to stop - *b*. The choice of the agent at node n_1 on the basis of the strategies acceptable at n_0 - *a* - does not coincide with the continuation of the strategy at n_1 which is acceptable at n_1 - *b*.

Hammond considers *myopic* and *sophisticated choice* as two possibilities available to an agent in such a situation of dynamic inconsistency, and the two strategies have the same characteristics anticipated by Strotz[2].

When acting according to a *myopic approach*, the agent treats choice at each decision point as unconnected with anything he could project on the choices he will make in the future. He selects at each point those strategies, or strategy continuations, which he judges acceptable from the perspective of that point. In this way, myopic choice involves a disparity between ex ante evaluations of ex post options, and ex post evaluations of the same options.

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[1] This notion of consistency, which is common to the literature on dynamic inconsistency and changing tastes considered above (Strotz, 1956, and others), has to be distinguished by another possible notion of consistency, which is present in Hammond (1982a,b), and does not impose restrictions on the agent's behaviour as the definition considered here. At a given node in the decision tree, each choice by the agent is consistent with some particular plan. For any decision tree, it is possible to consider a truncated part of the tree starting from a given decision node. A truncated plan is that plan which is available to the agent from that decision point on. As for every truncated plan there exists a corresponding plan, each truncated plan is the continuation of a plan whose implementation has not been ruled out by the agent's choice, so that the decision to implement that plan at the next node is consistent with the choices previously made. Then, any sequential choices will be consistent with each other, as they both are steps in the implementation of some plan. Therefore, an agent's choice is dynamically consistent *however* the agent moves in the tree**.**

[2] However, it must be noted that the focus of Hammond's analysis is different. According to Hammond's definitions, both strategies result in *consistent* dynamic choice, while they are always *incoherent*. Only in the case of no inconsistency, they coincide, and are coherent.

In Hammond's example, the myopic agent ignores that his tastes are changing, and chooses at each stage the option he considers as best at that moment. Therefore, he will choose option *a* at n_0, but change his mind and choose *b* at n_1. His final choice will be *b*.

When acting according to a *sophisticated approach*, the agent takes into account that the feasibility of any plan he can adopt in the future is conditioned by his projection of what options he will evaluate as acceptable at that point and will reject as not feasible those plans which imply a choice, he anticipates he will not make. By doing so, he always ends up choosing ex post according to his ex-ante plans and avoids violating dynamic consistency.

The sophisticated agent anticipates his future choice and chooses the best plan among those he is ready to follow to the end. In the example, at n_0 he will forecast that by taking the drug he will become an addict, and realises that his only options are *b* and *c*. Therefore, he will choose the most preferred option between the two, that is, *c*.

Hammond's example of dynamic inconsistent choice allows also to introduce another possible model of behaviour, which is formalised only later in the literature by McClennen in the context of dynamic inconsistency and non-expected utility preferences and is going to be discussed extensively below - *Resolute Choice*.

Resolute choice stands as an alternative to sophisticated choice in tackling the problem of dynamic inconsistency. The resolute agent anticipates that the evaluation he is going to make at some future node in the decision tree is going to constrain the feasibility of a plan he judges as best at present, and then decides to "treat evaluation at the initial decision point as dispositive of evaluation in the future" (McClennen 1990, page 157). The agent resolves to act according to a plan judged best from an ex-ante perspective, and intentionally acts on that resolve when the plan imposes on him ex post to make a choice, he disprefers. By so choosing he manages to act in a dynamically consistent manner.

In Hammond's example, at n_0 a resolute agent would have resolved to act according to the plan leading to the most preferred outcome *a* – take the drug till it is harmless, then stop; and he would have acted on that resolve when at n_1 the plan imposed on him to choose the less preferred option - going for *a* and not for *b*.

2. Dynamic inconsistency with non-expected utility preferences

The problem of dynamic inconsistency as examined in the literature on changing tastes occurs in situations of certainty. In a context of choice under risk or uncertainty, dynamically inconsistent choices may occur when the preference orderings over risky (uncertain) outcomes are nonlinear in the probabilities, that is, violate Expected Utility Theory, through violation of the Independence

Axiom[3]. When applied to sequential decisions, non-linearity may cause optimal strategies to be dynamically inconsistent.

This can be seen by considering the link between independence and linearity, and how violation of independence can cause dynamic inconsistency.

Many formulations of the independence axiom are given in the literature. It is sufficient to consider here this version by Samuelson (1952a,b)[4] corresponding to '*strong independence*', which considers both the cases of strong preference and indifference:

> *Lottery A is (as good or) better than lottery B, if and only if the compound lottery [A, p; Q, 1-p] is (as good or) better than the compound lottery [B, p; Q, 1-p] for any positive probability p and lotteries A, B and Q.*

That is, if lottery *A* is (as good or) better than lottery *B*, then any probability mixture of getting *A* or *Q* is (as good or) better than any probability mixture of getting *B* or *Q*, for any positive value of the probability *p*. In Samuelson's words, "using the same probability to combine each of the two prizes with a third prize should have no 'contaminating' effects upon the ordering of those two original prizes" (1952b, p.133).

As noted by Fishburn (1988), the Independence Axiom is known as the linearity assumption, and is associated with similar axioms referred to as substitution principles, cancellation conditions, additivity axioms and sure-thing principles. The contribution of the Independence Axiom to the demonstration of the existence of a utility function and of its linearity, a property which is referred to also as the expected utility hypothesis (the utility of a lottery is equal to the expected utility of its prizes) has been shown by many authors (Luce and Raiffa, 1957, Marschak, 1950; Kreps, 1988; Fishburn, 1988; Samuelson, 1952b). Machina (1989) notes that independence is equivalent to the property that the individual preference function takes the expected utility form, and according to Anand (1993) the notion of independence is "used in a sense that amounts to requiring that utility is linear with respect to probability".

Without providing a formal demonstration of the equivalence between linearity and independence, it will suffice to consider here the behavioural interpretation of linearity given by Machina (1987, 1991), which provides a clear example of the link between the two properties.

The property of linearity in the probabilities can be considered as a restriction on an agent's preferences concerning the probability mixtures of lotteries. Consider two probability distributions P and P* over a common set of outcomes, x. The probability mixture (α; 1-α) of the two lotteries P

[3] The problem of dynamic inconsistency may occur also when the preference orderings violate expected utility through violation of the Weak Ordering condition, as will be mentioned later. In the course of the following work, attention will be on the problem of inconsistency when independence is violated.

[4] This version of independence corresponds to the definition in Samuelson (1952a), modified as in Samuelson (1952b).

and P* will be equal to the compound lottery given by $\alpha P+(1-\alpha)P^* = \alpha p_1+(1-\alpha)p_1^*,\ldots,\alpha p_n+(1-\alpha)p_n^*$. This can be viewed as a two-stage lottery, giving an α chance of P and a $(1-\alpha)$ chance of P*.

As linearity implies that the utility of this two-stage lottery is equal to the sum of the expected utilities of the lotteries multiplied by the probabilities which mix them, the property of independence follows. That is, as

$\Sigma U(x_i)(\alpha p_i+(1-\alpha)p_i^*) = \alpha\Sigma U(x_i)p_i+(1-\alpha)\Sigma U(x_i)p_i^*$,

for the agent who maximises expected utility, if lottery P is preferred to lottery P*, then the probability mixture $\alpha P+(1-\alpha)P^{**}$ will be preferred to the mixture $\alpha P^*+(1-\alpha)P^{**}$ for any positive α and P**.

At this point, the property of independence can be given the following interpretation, which will turn useful below when the argument for dynamic inconsistency is introduced. The choice between $\alpha P+(1-\alpha)P^{**}$ and $\alpha P^*+(1-\alpha)P^{**}$ is for the agent like tossing a coin which has a $(1-\alpha)$ chance of tail, and in this case the agent will get lottery P**, and having to decide before the coin is tossed whether, in case head turns out (with probability α), lottery P or P* is preferred. If the coin lands tail, the agent gets lottery P**; if it lands head, the agent is back to the choice between P and P* and is supposed to rationally make the same choice as before.

The argument for the dynamic inconsistency of non-expected utility preferences is illustrated first by Raiffa (1968). Raiffa frames the problem of inconsistent choice in a context of dynamic choice under risk, by showing that dynamic consistency is not compatible with the usual choices in an Allais paradox when this is considered as a decision problem in two stages. Raiffa reports of a similar example given by Schlaifer (1969). For this argument see also Markowitz (1959)[5].

Before introducing Raiffa's argument, it is useful to illustrate the Allais paradox and show how it implies violation of expected utility and independence.

2.1 An example of the Allais paradox

Consider an agent who has to make a choice between the following couples of prospects, corresponding to the Allais paradox problem:

a_1=(1 chance of \$1million) or

a_2=(.10 chance of \$5million; .89 chance of \$1million; .01 chance of \$0)

and

a_3=(.10 chance of \$5million; .90 chance of \$0) or

a_4=(.11 chance of \$1million; .89 chance of \$0).

[5] For a discussion of the plausibility of this argument see Anand (1993), which refers to McClennen (1988a).

The typical (most common) preference pattern chosen by subjects in this problem (see Machina (1989) for references on the experimental evidence) is a_1 over a_2 and a_3 over a_4. This pattern of choice can be shown to violate the expected utility hypothesis.

According to expected utility, the preference for $a_1 \succ a_2$ implies that

u($1M) $\succ$ .10u($5M) + .01u(0) + .89 u($1M), which can be rewritten as

.11u($1M) + .89u(0) $\succ$ .10u($5M)) + .90u(0).

Therefore, expected utility implies $a_4 \succ a_3$ and not $a_3 \succ a_4$.

It is also possible to show how the pattern of choice considered violates the independence axiom. To show this, however, it is necessary to assume that the reduction of compound lotteries axiom (RCLA) holds.

According to RCLA, any compound lottery, which has another lottery as one or more of its prizes, can be reduced to a simple lottery of the more basic prizes by operating with the probabilities. Consider the following formal definition of reduction formulated by Luce and Raiffa (1957):

> Reduction (RD). Any compound gamble is indifferent (I) to a simple gamble with outcomes $o_1,...,o_r$, their probabilities being computed according to the ordinary probability calculus.
> In particular, if $g^{(i)} = (o_1, p_1^{(i)}; o_2, p_2^{(i)};...;o_r, pr^{(i)})$, for i = 1,...s,
> then, $(g^{(1)}, q1; g^{(2)}, q2;...;g^{(s)}, qs)$ I $(o_1, p_1; o_2, p_2;...;o_r, p_r)$,
> where $p_1 = q_1\, p_i^{(1)} + q_2\, p_i^{(2)} +...+q_s\, p_i^{(s)}$.

Given the RCLA, it is possible to rewrite the Allais prospects as

a_1 = (A, .11; $1M, .89)

a_2 =(B, .11; $1M, .89)

a_3 =(B, .11; $0, .89)

a_4 =(A, .11; $0, .89),

where A ≡ ($1M, 1) and B ≡ ($5M,10/11; $0,1/11).

The decision trees corresponding to the above prospects are

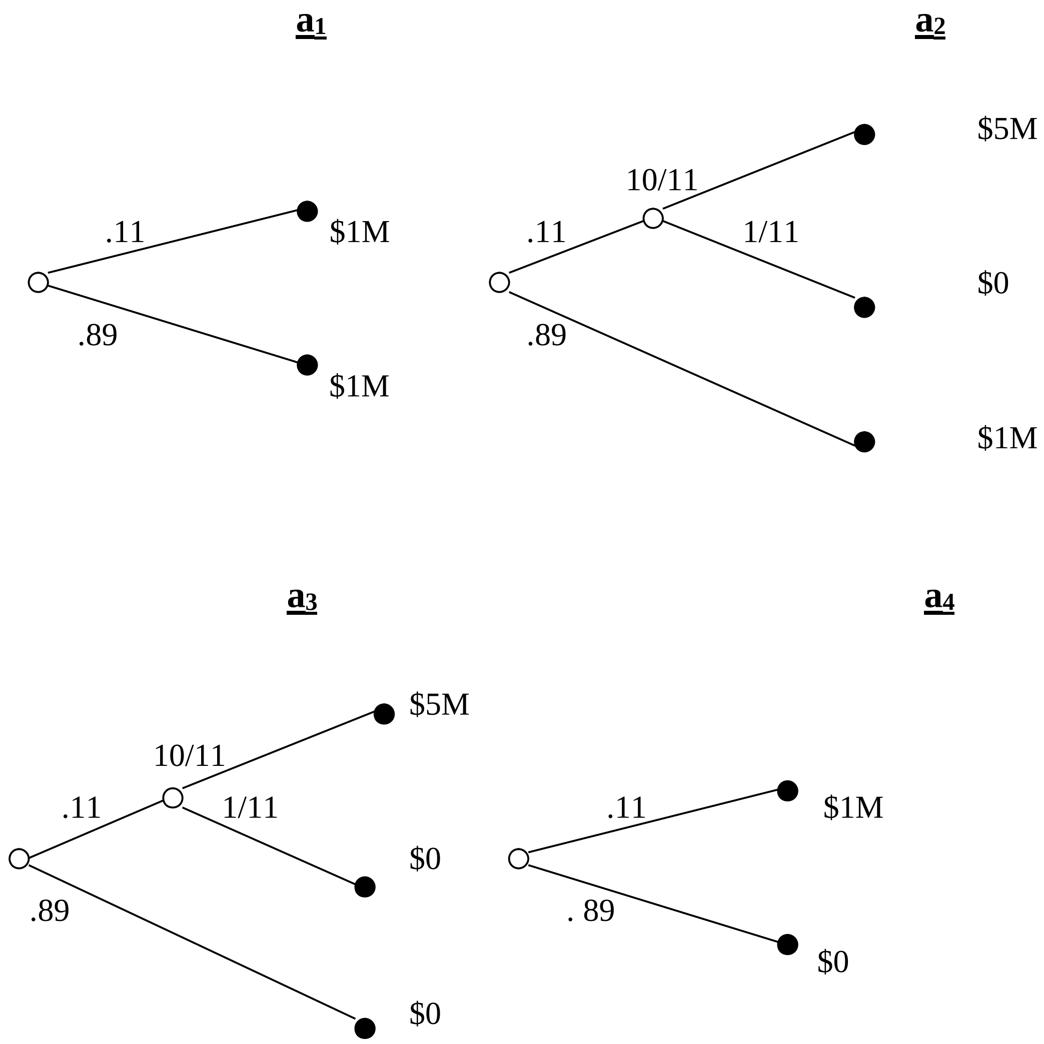


Figure 2 – Tree diagrams of the Allais paradox prospects with reduction

The Samuelson definition of independence given above, implies the condition:

$A \succ B \;\Leftrightarrow\; [A, p; Q, 1\text{-}p] \succ [B, p; Q, 1\text{-}p] \qquad \forall Q, p.$

Having defined A ≡ ($1M, 1) and B ≡ ($5M,10/11; $0,1/11), it results that the condition above implies

$a_1 \succ a_2 \;\Leftrightarrow\; A \succ B \;\Leftrightarrow\; a_4 \succ a_3$

contrary to the usual Allais choices.

The condition considered above is a condition of static and counterfactual consistency. It states that if the agent *had been* offered a choice between A and B instead of the actual choice between [A, p; Q, 1-p] and [B, p; Q, 1-p], then his preference would be consistent as stated.

The independence condition can be interpreted in terms of dynamic, instead of static, consistency in the manner illustrated by Raiffa.

2.2 The dynamic inconsistency of non-expected utility agents

Consider an urn containing 89 orange balls and 11 white balls. An agent has to draw from this urn one ball at random. If the ball is orange, the agent will receive a prize Q. If the ball is white, the agent will be given the choice between $1 million for sure (alternative A), or a lottery giving $5 million with probability 10/11 and $0 with probability 1/11 (alternative B).

At 8.55 in the morning the agent is asked to announce his choice between alternatives A and B, in case a white ball is drawn. That is, he is asked to choose *ex ante* between [A, p; Q, 1-p] and [B, p; Q, 1-p], with p = probability that a white ball is drawn.

At 9 am the agent draws a white ball.

At 9.05 the choice between A and B has to be made. That is, ex post the agent is asked to choose between A and B. There are two crucial questions here:

- If a white ball is drawn, would the choice between A and B depend on the detailed description of prize Q?
- If at 8.55 the alternative chosen has to be announced, in case a white ball is drawn, would the choice differ from the choice to be made at 9.05?

If at 9.05 the agent is offered the choice between A and B, and at 8.55 he has declared he wants A, it would be *dynamically inconsistent* to change decision. Thus,

(ex-ante) [A, p; Q, 1-p] $\succ$ [B, p; Q, 1-p] $\Leftrightarrow$ (ex-post) A $\succ$ B

as required by independence. If the agent is dynamically consistent in this way for any Q, it also follows that his ex-ante choice must be independent of Q.

In addition, if the agent's choice between the sure payoff and the lottery at 9.05 is independent of what prize Q is going to be, then there is no reason to postpone declaring of preferences at 8.55, and choice at this time should not differ from choice at 9.05.

Therefore, the answer to the above questions should be negative, and negative answers are consistent with the independence assumption.

Moreover, in case the Q prize were $1 million, choice of A would correspond to a choice of a_1, and choice of B to a choice of a_2. In case the Q prize were $0, choice of A would correspond to a choice of a_4, and choice of B to a choice of a_3. Therefore, the negative answers to the above questions, which are consistent with independence, are not compatible with choices for a_1 and a_3 (or a_2 and a_4) in the Allais paradox.

Besides, choices compatible with the Allais paradox would imply that choice at different times would differ, leading the agent to dynamically inconsistent choices. If Q is $1 million, the agent will say that in the case of a white ball, he would choose B at 9.05, but then at 9.05 he would instead choose A, being inconsistent with his previous choice, and with the negative answer to the two questions.

Raiffa's example shows how violation of expected utility may lead to dynamically inconsistent behaviour. Consider now the version of the argument given by Machina (1989, 1991).

3. The problem of dynamic inconsistency in Machina

The problem of the dynamic inconsistency of non-expected utility agents when acting in a dynamic context has been introduced in the previous paragraph through the example by Raiffa (1968), which is one of the first and best-known arguments of this kind.

Machina constructs his argument of inconsistency by considering the violation of independence indirectly through violation of the separability properties which combine into the independence axiom. Machina's argument

- uses again the Allais paradox as an example of violation of the independence axiom in a static context;
- shows how an agent with such non-expected utility preferences may incur into choices which are inconsistent when acting in a dynamic choice framework.
- outlines - differently from Raiffa's - the strategies which are available to the agent in a dynamic choice context, offering a solution to the dynamic inconsistency problem.

As mentioned earlier, the characteristic property of the expected utility preference function is to be linear in the probabilities. This property does in turn imply that expected utility preferences are *separable across mutually exclusive events.* Separability can be of two different sorts: *replacement* and *mixture* separability[6]. The properties of separability (across events and/or sublotteries) can be combined in the independence axiom.

According to *replacement separability*, an agent who prefers the lottery $(y_1,p_1;\ x_2,\ p_2;\ldots;\ x_n,p_n)$ to $(x_1,p_1;\ x_2,\ p_2;\ldots;\ x_n,p_n)$, would also prefer to replace (x_1,p_1) by (y_1,p_1) in any lottery of the form $(x_1,p_1;\ x^*_2,\ p^*_2;\ldots;\ x^*_m,p^*_m)$. This property is a direct expression of the fact that, due to the additive form of the expected utility function, the contribution to the sum of every utility/probability product does not depend on any of the other utility/probability products.

According to *mixture separability*, an agent will prefer $(y_1,p_1;\ x_2,\ p_2;\ldots;\ x_n,p_n)$ to $(x_1,p_1;\ x_2,\ p_2;\ldots;\ x_n,p_n)$ if and only if he would prefer y_1 to x_1 in a direct choice between the two outcomes.

[6] The two properties of separability also apply to mutually exclusive sublotteries of a compound lottery.

This property directly derives from the fact that each element of the expected utility sum is the product of the utility of an outcome and its probability. The probability mixture of an outcome can be preferred to the same probability mixture of another outcome only if the utility of the first outcome is bigger than the utility of the second outcome, that is, if the first outcome is preferred to the second one.

The two properties of separability (over events or sublotteries) can be combined in the independence axiom, as previously defined:

Lottery A is preferred (indifferent) to lottery B if and only if (A, p; Q, 1-p) is preferred (indifferent) to (B, p; Q, 1-p) for all lotteries Q and all positive probabilities p.

As shown in Machina, the axiom of independence implies both mixture and replacement separability.

The definition of independence implies directly mixture separability, as

$[A, p; Q, 1\text{-}p] \succ [B, p; Q, 1\text{-}p] \Leftrightarrow A \succ B$.

Applying the condition twice gives replacement separability:

$[A, p; Q, 1\text{-}p] \succ [B, p; Q, 1\text{-}p] \Leftrightarrow A \succ B \Leftrightarrow [A, p; Q^*, 1\text{-}p] \succ [B, p; Q^*, 1\text{-}p]$.

In this framework, the typical preferences in the Allais paradox can be shown to violate the independence axiom through violation of replacement separability over sublotteries.

Consider an agent who has to make a choice between the pairs of prospects of an Allais paradox decision problem described above and suppose that his preferences are for $a_1 \succ a_2$ and $a_3 \succ a_4$. Recalling the decision-tree representation of the Allais prospects when RCLA holds in Figure 2, the agent who prefers a_1 to a_2 and a_3 to a_4 is willing to replace the sublottery ($5M,10/11; $0,1/11) with $1M for sure when the lower branch yields $1M, but not when it yields $0.

That is, for this agent

($1M,.11; .89, $1M) $\succ$ (($5M,10/11; $0,1/11), .11; .89, $1M), but not

($1M, .11; .89, $0) $\succ$ (($5M,10/11; $0,1/11),.11; .89, $0),

in violation of replacement separability and of the independence axiom.

3.1 Dynamic choice with Allais-type non-expected utility preferences

Machina develops a specific example of dynamic inconsistency by using the typical non-expected utility preferences in the Allais paradox. He notes, however, that the problem arises with any kind of violation of the independence axiom, through violation of replacement or mixture separability, and in general of any violation of expected utility.

I shall consider this general argument first. Then, it will be straightforward to see how the application of this general argument for inconsistency to the specific lotteries and preference patterns of the Allais paradox allows to derive Machina's specific example of inconsistency in a dynamic Allais paradox.

An agent faces a simple dynamic choice problem, where the dynamic aspect is given by the fact that the agent has to pre-commit his decision at the beginning of the tree - before any uncertainty is resolved - on how he will move if and when he reaches the decision node and will actually have to move accordingly when he is there. Here, the agent has to pre-commit his choice at the decision node between lotteries **A** and **B**, in case he reaches it with probability **p**.

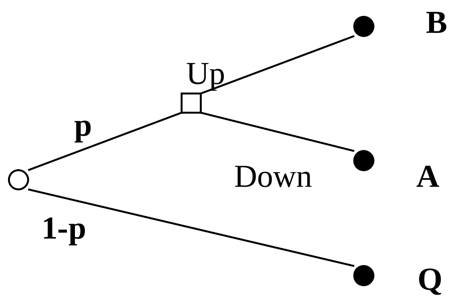


Figure 3 – Dynamic decision tree which shows the potential inconsistency for any violation of the Independence Axiom

If the agent's preferences are of the kind considered above, that is, violate independence through violation of either replacement or mixture separability, for lotteries **A**, **B** and **Q**, and probability **p**, they will be of the form

$A \succ B$ but $(B, p ; Q, 1\text{-}p) \succ (A, p ; Q, 1\text{-}p)$.

Therefore, at the beginning of the decision tree the agent will plan to go Up, to get B at the choice node, before the uncertainty is resolved; but, after the uncertainty is resolved, when at the decision node (in case he arrives there) he will actually choose Down, to get lottery A. His behaviour will be dynamically inconsistent, as his actual choice at the decision node will differ from his planned choice for that node.

Before introducing Machina's example of inconsistency in a dynamic Allais paradox decision problem, I will consider how the specific example of dynamically inconsistent choice may be obtained by applying Machina's general argument for inconsistency to the specific lotteries and preference patterns of the Allais paradox.

Suppose that

A = (1M, 1)

B = (5M, 10/11; 0, 1/11)

Q = 0

p = .11, (1-p) = .89.

Then, the reasoning above when applied to the Allais prospects will imply:

1. A $\succ$ B	that is,	(1M, 1) $\succ$ (5M,10/11; 0,1/11), but
2. (p,B; 1-p,Q) $\succ$ (p,A ; 1-p,Q)		(.11, (5M,10/11; 0,1/11); .89,0) $\succ$ (.11, (1M,1); .89,0)
	<u>and</u>	
3. (p,A; 1-p,A) $\succ$ (p,B; 1-p,A)		(.11,1M; .89,1M) $\succ$ (.11, (5M,10/11; 0,1/11); .89,1M).

The preferences which result by applying the general argument for inconsistency to the Allais prospects show violation of replacement separability - therefore independence - under (2) and (3); violation of independence (through mixture separability) and inconsistency under (1) and (2); and no violation or inconsistency under (1) and (3).

The above application of the general case of dynamically inconsistent preferences to the Allais paradox prospects allows to construct Machina's specific example for the dynamic inconsistency of non-expected utility NEU agents with Allais-type preferences.

Consider an agent who is facing the following decisions:

a) a direct choice between the two prospects: B=(5M,10/11; 0,1/11); and 1M for sure (A);

b) a choice on how to move in the two dynamic problems

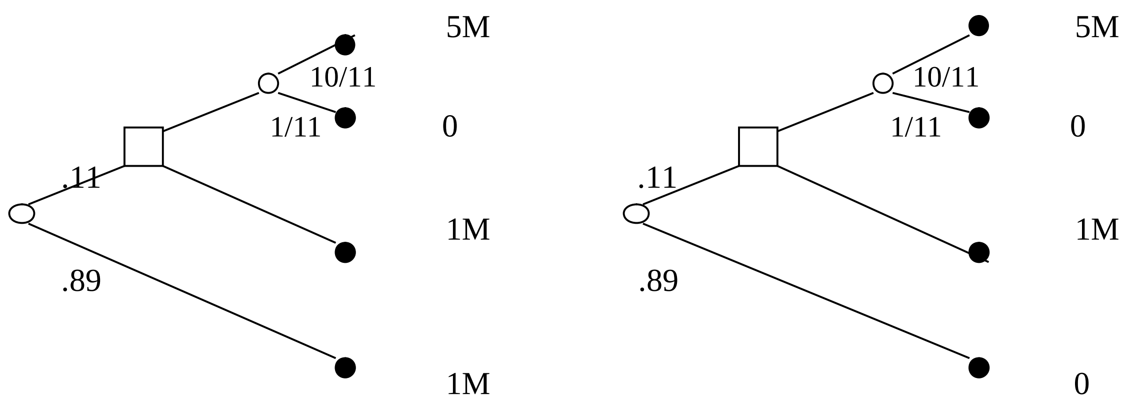


Figure 4 – Dynamic decision problems that generate the same opportunity sets as the Allais paradox problem

The different choices the agent can make in these two trees imply the Allais paradox prospects considered above.

In the left hand-side tree, a choice of Up at the decision node, when combined with the probability at the initial chance node, implies the prospect a_2=(.10 chance of $5M; .89 chance of $1M; .01 chance of $0). A choice of Down implies the prospect a_1=(1 chance of $1M).

In the right hand-side tree, a choice of Up implies a_3=(.10 chance of $5M; .90 chance of $0). A choice of Down implies a_4=(.11 chance of $1M; .89 chance of $0).

An agent with the typical Allais non-expected utility preferences will act inconsistently.

1. If his choice has been for $B \succ 1M$ (A). In case nature goes Up in the left hand-side tree, he will face exactly B and 1M, and decide to go Up (B), but change his plan to choose Down at the decision node (1M) instead of Up ($a_1 \succ a_2$). In the right hand-side tree, the agent will not be inconsistent. He will plan to go Up for a_3 and will implement that decision when at the node.

2. If his choice has been for $1M \succ B$. In case nature goes Up in the right hand-side tree, he faces B and 1M, and decides to go Down (1M), but changes his plan to choose Up at the decision node instead of Down ($a_3 \succ a_4$). In the left hand-side tree, the agent will not be inconsistent: he will plan to go Down for a_1 and will implement that decision when at the node.

That is, the agent's behaviour *in either of the two* dynamic problems will be dynamically inconsistent, whatever his choice in the direct alternative between the lottery and the sure outcome.

3.2 The different approaches to choice available to the agent in a dynamic choice situation

The argument on the dynamic inconsistency of non-expected utility agents sketched above relies on an assumption on the way in which the agent reconsiders his choice in the middle of a decision tree. In the previous example, when the problem occurred of determining how the agent would have moved had he actually reached the decision node, it has been assumed that he would have considered the tree starting at the decision node as a new decision tree. Thus, the reconsideration of the optimal strategy to be adopted at that point would depend only on the initial preferences and the outcomes/probabilities of the new tree.

The uncertainty belonging to the rest of the tree - which has been cut off - does not matter for decision (in the example above, this uncertainty is the .89 probability of getting 1M in the left hand-side tree; and the probability of getting 0 in the right hand-side tree).

In the previous example, if this holds, decision at the choice node is determined only by the agent's preferences regarding the outright choice between the lottery and the sure outcome. It follows that dynamically inconsistent behaviour of the agent in the previous situation is a consequence of this assumption. This assumption is referred to (Machina, 1989) as *consequentialism*. The kind of agent who is non-expected utility and consequentialist ('β -people' for Machina) can be thought of being induced to dynamically inconsistent behaviour, by acting in a myopic manner, even if Machina does not refer to myopia explicitly. I shall refer to this agent as a straight inconsistent non-expected utility agent (**NEU**).

An alternative to this approach would be to consider the uncertainty which has been already borne at the decision node as "gone in the sense of (…) *consumed* (or "borne"), rather than gone in the sense of *irrelevant*" (page 1647), by keeping the entire probability distributions into the opportunity sets, so that the agent will not confront at the node a new set of opportunities, but a subset of the original ones. In this way, the opportunities which were initially preferred, and corresponded to the initial plan, will always be available to him. In the example given, the agent will face at the choice node the same prospects that he was facing at the beginning of the tree.

By so doing, he will manage to be dynamically consistent. I shall refer to this kind of agent as a Machina-dynamically consistent non-expected utility agent (**MNEU**). Machina refers to these agents as γ -people. This different approach suggested by Machina denies consequentialism as it "takes into account the risk already borne in a way consistent with the agent's original preferences".

Let us consider now how such an approach can be formalised. The key feature of its formalisation lies in the way in which nonseparable non-expected utility preferences are "appropriately" extended to dynamic choice settings.

In the previous example, the agent had the NEU Allais type preferences $a_1 \succ a_2$ and $a_3 \succ a_4$. It has been shown that, whatever his preferences over lottery B and 1M, he will act inconsistently in one of the two dynamic decision problems. Consider $a_1 \succ a_2$ and B $\succ$ 1M, where a_1 = (1M, 1) and a_2 = (5M, .10; 1M, .89; 0, .01). As B = (5M, 10/11; 0, 1/11), it is possible to write a_1 = (1M, .89;1M, .11) and a_2 = (1M, .89;B, .11).

Define V(1M, .89;1M, .11) and V(1M, .89;B, .11) as the preference functions over the two lotteries.

a_1 and a_2 are the prospects the agent faces at the beginning of the left-hand-side tree of Figure 4 in the previous section. The inconsistency derives from the fact that when at the decision node, the prospects the agent faces are not a_1 and a_2, but B and 1M.

According to Machina's approach, the appropriate extension of NEU preferences to dynamic situations has to consider that the agent with non-expected utility preferences feels the risk, which is gone as consumed, but not as irrelevant. This can be taken into account in two ways:

1. by inserting the whole probability distribution into the original preference function. At the choice node, the agent will compare $V = (\overline{1M, .89}; 1M, .11)$ and $V = (\overline{1M, .89}; B, .11)$ - where the bar indicates the risk which has been already borne; that is, he will face the prospects a_1 and a_2. In this way the agent will choose a_1 at the choice node - consistently with his choice at the beginning of the tree;

2. by inserting the continuation of each tree branch in the conditional preference function $V_{1M,.89}=(\bullet)$, so that he will compare $V_{1M,.89}=(1M, .11)$ and $V_{1M,.89} = (B, .11)$, where it is defined that $V_{x1,p1;x2,p2}(Z) = V(x_1,p_1;\ x_2,\ p_2;\ Z,\ (1-p_1-p_2)$. As above, the agent will face prospects a_1 and a_2 and choose a_1, being consistent with his previous choice.

Both procedures have the same effect on the agent's behaviour: they allow the agent to implement at the choice node the plan adopted at the beginning of the decision problem, and therefore to be dynamically consistent.

This model of choice is substantially equivalent, even if differently formalised, to McClennen's model of Resolute Choice[7], which will be discussed extensively below, so that I shall imply that a MNEU and a Resolute Chooser (RC) are the same kind of agent.

There are still two kinds of agent which are to be considered. One is the expected utility (**EU**) agent ('α -people' for Machina), to whom the problem of dynamically inconsistent choice does not apply. The other agent is the Sophisticated Chooser (**SC**) ('δ -people' for Machina).

As discussed previously, this agent determines the optimal strategy in the tree by a process of 'backward induction': he starts from the last decision nodes in the tree, considers the set of prospects following from those nodes and determines the optimal choice at those nodes by using his original preferences. The process is repeated for all previous choice nodes, given the path so determined out of each terminal node, till the agent determines the path from each choice node to the beginning of the tree. The SC agent will be consistent, as he will follow at each node the optimal choices so determined when moving along the tree.

In the example in Figure 4, the agent who prefers the lottery to the sure outcome, will forecast that he would go Up when at the decision node in the left hand-side tree, and will then pre-commit to go Up instead of Down when at the beginning of the tree (a_2), therefore avoiding inconsistent behaviour in his future choice. In the right hand-side tree he will choose Up for a_3. The agent who prefers the sure outcome to lottery B, will forecast that he would go Down in the right hand-side

[7] According to Machina, Resolute Choice represents one of the "antecedents of the formal approach" presented in his paper.

tree, and will therefore choose Down at the beginning of the tree (a_4), avoiding inconsistency. In the left hand-side tree he will go Down for a_1. Therefore, in this dynamic choice situation the SC agent will behave as an EU agent, while exhibiting non-expected utility preferences in the static Allais problem of Figure 2.

In the context of the simple dynamic choice problem considered, four different kinds of agents - or different approaches to choice available to the agent in a dynamic choice situation - can be outlined. An EU agent who will not be inconsistent. A NEU agent who is liable to inconsistency. A MNEU and a SC agent who are NEU agents but avoid inconsistent behaviour, even though the strategies they adopt and their choices differ.

4. Models of dynamic choice and dynamic choice conditions

After having considered Raiffa's and Machina's formulations of the dynamic inconsistency problem, it may be useful to summarise the arguments by considering the decision trees and the different axioms implied by the arguments and choice strategies discussed above.

The Allais pairs of prospects considered in Figure 2 can be represented by the following two decision trees, with choice of A being equivalent to a_1=(1 chance of $1million), choice of B to a_2=(.10 chance of $5million; .89 chance of $1million; .01 chance of $0), choice of C to a_3=(.10 chance of $5million; .90 chance of $0) and choice of D to a_4=(.11 chance of $1million; .89 chance of $0).

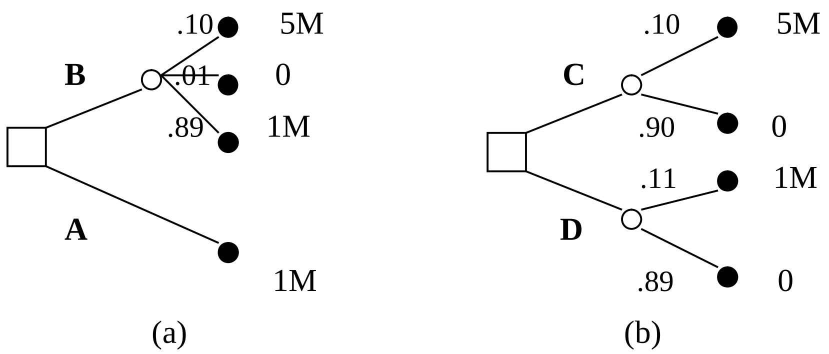
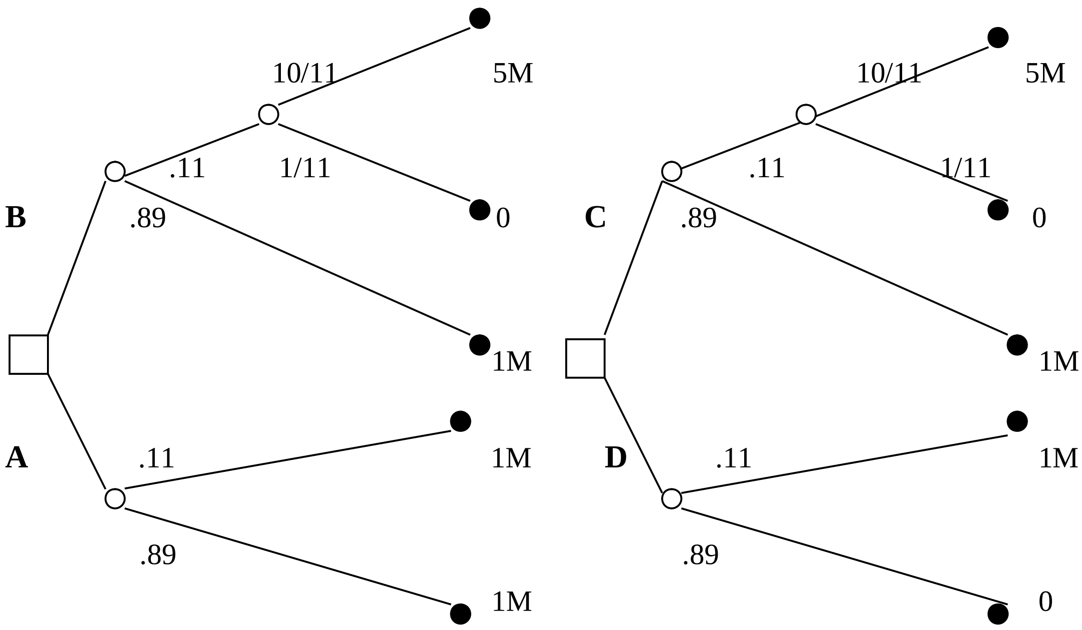


Figure 5 – Decision trees representing the Allais paradox prospects

As considered above, given the RCLA **reduction** condition, these prospects are probabilistically equivalent to, respectively:



(a) (b)

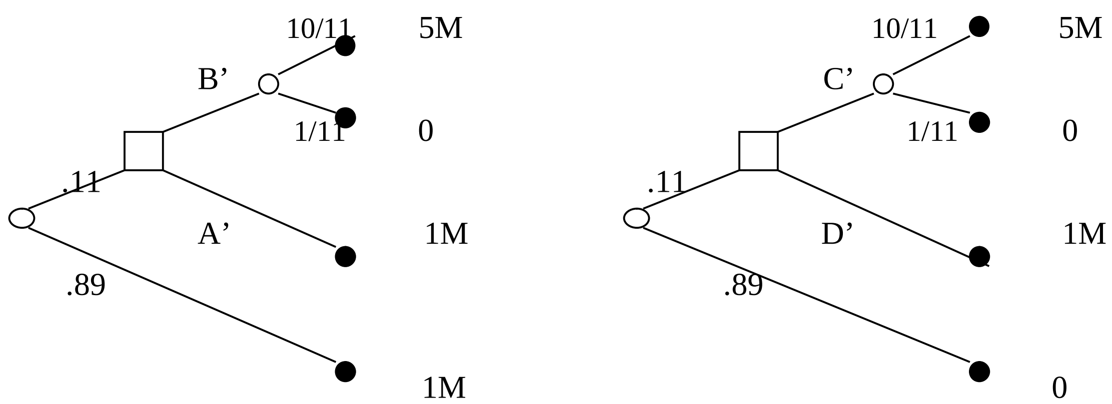
Figure 6 – Decision trees representing the Allais paradox prospects with reduction

In Figure 6, the **independence** condition requires that

$A \succ B \Leftrightarrow (1M, 1) \succ (5M, 10/11; 0, 1/11) \Leftrightarrow D \succ C$.

Therefore, reduction and independence together require that $A \succ B \Leftrightarrow D \succ C$ in Figure 5.

Consider now the dynamic interpretation of the independence condition in the manner of Raiffa and Machina. Compare Figure 6 with the following dynamic decision problems which reverse the temporal order of the first decision node and the chance node and generate the same opportunity sets as the Allais paradox:



(a) (b)

Figure 7 – Dynamic decision trees which generate the same opportunity sets as the Allais paradox

The condition of **dynamic consistency** requires that agent's choices will be the same in the two sets of problems: the agent who prefers A to B in 6a will prefer A' to B' in 7a, and the agent who prefers D to C in 6b will prefer D' to C' in 7b.

A ≻ B ⇔ A' ≻ B' and D ≻ C ⇔ D' ≻ C'.

Besides, in Figure 7 the condition of **consequentialism** according to Machina (1989) requires that A' ≻ B' ⇔ D' ≻ C', as the consequences of these prospects are identical in the two pairs of decision trees. That is, according to consequentialism, in Figure 7 the payoff at the end of the lowest branch is not relevant at the decision node.

Therefore, dynamic consistency and consequentialism together require that

A ≻ B ⇔ D ≻ C in Figure 6. So reduction, dynamic consistency and consequentialism together require that A ≻ B ⇔ D ≻ C in Figure 5.

Therefore, it results that whoever offers an Allais-type response in Figure 5 must violate at least one of reduction, dynamic consistency and consequentialism.

Recalling the different strategies available to the agent in this situation, it is possible to see that the straight NEU myopic agent will violate independence and therefore dynamic consistency but respect consequentialism and reduction, so that in Figures 5 and 6, A ≻ B and C ≻ D, while in Figure 7 A' ≻ B' and D' ≻ C'.

A dynamically consistent MNEU agent will respect reduction and dynamic consistency, while violating consequentialism, so that A ≻ B and C ≻ D in Figures 5 and 6, while A' ≻ B' and C' ≻ D' in 7.

A SC sophisticated agent will obey consequentialism and dynamic consistency, so that A ≻ B and D ≻ C in Figures 6 and A' ≻ B' and D' ≻ C' in 7, but violate reduction, as for this agent A ≻ B and C ≻ D in Figure 5.

So far I have considered the argument on the dynamic inconsistency of non-expected utility agents through the examples of Raiffa and Machina, and the different models of dynamic choice discussed by Machina.

It is important to note that other authors have discussed the issue of the linearity of the expected utility function and dynamic inconsistency. Karni and Safra (1989a) show that linearity of expected utility is a necessary and sufficient condition for dynamic consistency in ascending bid auctions. Karni and Safra (1990) argue also that the same is true in the more general case where the set of choices is not restricted to lotteries induced by auctions.

In the next paragraph I will consider McClennen's approach to the problem of the dynamic inconsistency of individuals with non-expected utility preference patterns. The discussion will illustrate the conditions of dynamic choice and the different models of behaviour in a dynamic context through some examples, leaving the formal part of McClennen's argument in the Appendix. This will allow to examine further the myopic and sophisticated models of choice, and illustrate the model proposed by McClennen as an alternative solution to the problem of dynamic inconsistency - Resolute Choice.

Before introducing McClennen's argument, it is useful to illustrate briefly the general theoretical debate in which the dynamic inconsistency argument finds its place and McClennen's argument develops.

The problem of the dynamic inconsistency of non-linear preference orderings is part of the general debate on the justification of expected utility as a normative theory. On the basis of a growing empirical evidence of systematic violations of expected utility maximisation, many non-expected utility models have been formulated (for references see Machina, 1989). Most of these models maintain the ordering and continuity conditions and relax or generalise the independence axiom, which accounts for the linearity in the probabilities. Many different arguments have been formulated against non-expected utility models, in the context of static as well as dynamic choice (Machina, 1989; McClennen, 1990). According to Machina, the problem of dynamic inconsistency is one of the most "formidable" arguments against the validity of these models as normative for choice under risk and uncertainty and in support of the normative superiority of expected utility.

Hammond's consequentialist argument (1988a,b;1989) is the most formal normative justification of expected utility. According to Hammond's argument, violations of one of the two expected utility axioms of ordering and independence make the agent's preferences subject to dynamically inconsistent changes. If this occurs, the agent may find himself in a dynamic choice situation in which what at the present moment he prefers to choose at a later point in the decision tree is not what he will prefer to choose when he actually arrives at that decision point.

The problem with dynamically inconsistent shifts in preferences is that they are not compatible with the agent always maximising with respect to his preferences for consequences. The agent may act according to a plan that is strictly dominated by another available plan with respect to preferences for outcomes.

Hammond argues that the consequentialist principle– that acts have to be valued only by their consequences - suffices for the derivation of the axioms of ordering and independence, which are implied by consequentialism, given a condition of consistency. In particular, Hammond shows that ordering and independence can be recovered as theorems under an axiomatic formalisation of the principle of consequences.

5. The problem of dynamic inconsistency in McClennen

McClennen (1986,1990) develops his contribution to the analysis of the dynamic inconsistency problem in the context of the debate on the normative validity of expected utility theory and two of its axiomatic presuppositions, the principles of weak ordering and independence. The discussion concentrates on the justification from a pragmatic perspective of a normative interpretation of the two axioms, that is, the justification of "the claim that a rational decision maker ought to avoid violating these two principles" (McClennen 1990, page 3).

McClennen's contribution to the debate focuses on the discussion of what he considers the most formal and complex of the pragmatic arguments in defence of the two axioms of ordering and independence, Hammond's consequentialist argument. In common with the other pragmatic arguments, this includes reference to a dynamic choice framework. McClennen constructs his argument by introducing a set of conditions for rational dynamic choice, which have the purpose of factoring Hammond's consequentialist principle. The conditions provide a model for the consequentialist argument constructed in favour of the two axioms: McClennen shows that the same two axioms can be recovered as theorems from the conjunction of the dynamic choice conditions. Moreover, the conditions play an important role in a more general pragmatic approach.

It is of particular interest to consider how McClennen uses the rational dynamic choice conditions to model agent's behaviour in a dynamic choice context. For this purpose, it is sufficient to give here only a description of the conditions necessary to follow McClennen's construction. A more detailed notation and formal definitions of the conditions and models of choice are given in the Appendix.

5.1 Some notation and definitions

A *dynamic choice problem*, as defined in section 1 above, is represented by a *decision tree T*, composed by initial chance or choice nodes, a set of following *choice* or *chance nodes*, and a set of *terminal nodes* or *outcomes* $O(T) = \{o_1, ..., o_n\}$ associated with *T*. In line with the assumption of the decision problem to be bounded, it is assumed that an outcome is not terminal, insofar as chance events and probabilities enter explicitly into its description.

Define *S* as the set of *plans* available to the agent in *T*, and *s*, *r*, ... as elements of *S*. For any decision tree *T*, it is possible to define $T(n_i)$ as the truncated part of the tree from node n_i to all terminal nodes that can be reached from n_i. $s(n_i)$ will be a representative truncated plan available to the agent at point n_i and $S(n_i)$ the set of truncated plans available at n_i.

Before introducing the rational choice conditions, it is useful to consider (more formally in the Appendix) a crucial problem in McClennen's approach - the evaluation of plans and the different perspectives from which plans can be evaluated. The difference in the perspectives of evaluation are the basis for the differences in the conditions and models of dynamic choice.

i. The evaluation of plans

D(S) is defined as the set of acceptable plans, that is, the subset of S consisting of those plans judged by the agent to be acceptable, by whatever criteria is employed.

ii. The evaluation of plans at subsequent nodes

According to McClennen there are three different perspectives from which the agent who finds himself at a certain (chance or choice) node n_i in a tree *T* can evaluate the plans available at that point. These give rise to three different kinds of evaluation:

- Evaluation of truncated plans. This concerns the evaluation of the alternatives available to the agent at node n_i *from the perspective of node* n_i *itself*. Given that the agent finds himself at some node n_i, he will face the truncated tree $T(n_i)$, and will have to evaluate the set of plans $S(n_i)$ available to him at n_i.
- Evaluation of plan continuations. This concerns the evaluation of all the possible plans available to the agent *before any move - by nature or by the agent - has taken place.* For each truncated plan $s(n_i)$ available at n_i, there must exist some plan *s*, such that $s(n_i)$ is the continuation of *s* from the point n_i on. At n_i the agent can perceive he faces not a set of *truncated* plans, but a set of plan *continuations*. From this perspective, the agent will have to consider which of all the plan continuations he is actually facing were considered acceptable by him initially. Then, it is possible to define for any node n_i the set of all acceptable plans that the agent can still implement from n_i.
- Evaluation of truncated plans as de novo trees. This concerns the evaluation of the agent in the case he is given a 'de novo' decision problem identical to the one he is considering. The perspectives of evaluation considered above concern the general problem of how the agent should *choose at a certain point within a decision tree*. Choice so intended has a background of past choices and events, of paths that are "collateral but now counterfactual" (page 106). However, it is possible to consider the agent as facing, at some node n_i, a *completely new* decision tree; a full tree that starts de novo at node n_i and is like $T(n_i)$ except for "having no history and collateral paths".

<u>Prospects</u>. For any tree *T* and plan *s* in *S*, a prospect g_s - associated with plan *s* - is assumed to consist of a well-defined probability distribution over a set of outcomes, which can be reached by implementing plan *s*. The notion of prospects presupposes that the agent's preference ordering over gambles satisfies the condition of reduction RD (defined in 2.1). This means that the agent is indifferent to gambles that can be reduced to the same probability distribution over the same set of outcomes.

A more formal definition of a prospect is to be found in the Appendix.

5.2 Dynamic choice conditions

McClennen considers four *conditions* or *principles of rational dynamic choice,* together with other conditions which enter in their definition: Simple Reduction, Normal and Extensive form Coincidence, Dynamic Consistency, Separability.

As noted above, definition and discussion of the dynamic choice conditions is central in McClennen's analysis, as all the dynamic models of choice which McClennen develops are defined through the conditions that they violate or observe. Therefore, in what follows I will try to consider the part of the discussion which is necessary to understand McClennen's argument, while leaving the more formal part of the definitions and discussion in the Appendix.

5.2.1 Conditions of reduction

<u>Simple Reduction (SR)</u>

SR (as defined in the Appendix) is the extension to dynamic choice of the simple reduction condition RD previously defined. According to SR, the decision tree problem is equivalent to a simple non-sequential choice among different gambles, where n_0 is the only choice node in the tree, and all other nodes are chance nodes. This condition requires that the ranking of plans in this kind of dynamic choice problem depends on the ranking of the outcomes or prospects associated with those plans. Thus, it requires that the evaluation that the agent makes of the prospects, and his subsequent choice, rules the evaluation and choice of strategies. The condition is plausible, if the RD version of the condition - that only outcomes and probability distributions are important in evaluation of gambles - holds in static choice. SR not only implies that RD holds, but is the extension of RD to a dynamic context.

<u>Plan Reduction (PR)</u>

This condition extends the previous SR, by eliminating the requirement that the plan be implemented by a single initial choice (see the Appendix for a formal definition). This is in line with the consideration that a plan has no value in itself, but that its value depends *only* on the value of the prospects attached to the plan and their probabilities. Then, if two plans lead to prospects which are considered equivalent, the two plans are to be judged equivalent. The condition is stated in terms of acceptable plans. It requires a coincidence between acceptable sets over prospects and acceptable sets over plans that map into those prospects.

The SR condition of reduction is more limited than PR, and therefore directly implied by it. Besides, by "factoring" PR (as shown in the Appendix), it is possible to see that PR adds to SR the requirement that, insofar as evaluation is concerned, no distinction exists between plans in normal and extensive form.

In order to explain the difference between normal and extensive form of a plan, one can consider a plan *s* in *S(T),* where *T* is modified so that the agent can implement any plan in *T* by means of a single choice, for example, by the existence of a mechanism which automatically performs from the start all the choices required by plan *s*, and consider this as the normal form version of the plan. Given the normal form of *s* and the normal version of *T*, then, it is possible to formulate the condition of:

Normal-form/extensive form coincidence (NEC).

NEC (formally defined in the Appendix) implies that there is no difference to choice whether the agent is presented with the extensive or the normal form version of any plan[8].

While PR ensures the coincidence between the set of acceptable plans and the set of the corresponding acceptable prospects, NEC ensures the coincidence between the set of acceptable plans associated with a tree and the set of acceptable normal form plans associated with the same tree.

There exists a relation between the conditions SR, NEC and PR. McClennen establishes the relation formally through two propositions, which prove that PR is equivalent to the conjunction of NEC and SR. The formal demonstration of the propositions will not be given here. However, the link connecting the different conditions of reduction will be better understood below, when the principles of rational dynamic choice are used to define the different models of choice in the context of a dynamic decision tree.

5.2.2 Dynamic consistency (DC)

As mentioned in paragraph 1, dynamic consistency is defined in terms of the "disparity between the plan originally adopted and subsequent choice" (McClennen 1990, page 118). As it is assumed that the plan adopted by the agent is a plan that he judges acceptable at the initial decision point, and that the plan chosen at any subsequent point is among the truncated plan judged acceptable from that standpoint, dynamic consistency requires coincidence between what the agent chooses at a certain node by the strategies judged acceptable *at the outset*, and the set of the strategy continuations from that point on, which are judged acceptable *at that node*.

More specifically, the condition of dynamic consistency requires a relation between the set of plans judged acceptable by the agent (in particular what it implies about the set of plans that the agent can still implement from a particular choice node), and the set of truncated plans judged acceptable which are available to the agent at the *subsequent* choice node under consideration.

Formally, the DC condition of dynamic consistency is obtained by the combination of two different conditions – of Inclusion (DC-INC) and Exclusion (DC-EXC). The formal definition of the two conditions and of the resulting dynamic consistency is given in the Appendix.

5.2.3 Separability (SEP)

Separability requires that the evaluation of the options still available to the agent at some point within the decision tree be independent of the context of chance events and choices already occurred. That is, how the agent would choose if he were to confront choices *de novo* determines how he chooses at any given point in the tree. Therefore, the condition requires coincidence between the set of strategy continuations judged acceptable at a particular decision point and the set of strategies which would be judged acceptable by the agent if he were to confront the sub-tree starting at that decision node as a new decision tree.

This condition of separability requires that how an agent will choose among different alternatives at a node in the decision tree is determined by the way he would choose among those alternatives *de novo*. That is, the choice of the agent at a node n_i in the decision tree does not depend on "what might have happened, or what alternatives might have been available, under conditions that do not in fact obtain at choice point n_i", that is on "alternative but counterfactual choices or moves by nature" (page 122).

The role of the SEP condition can be better understood by considering a condition of *reduction on truncated plans*, TR, which generalises the plan reduction condition PR to the case where $n_i = n_0$, and therefore directly implies it.

TR (defined in the Appendix) requires the ordering of the set of plan truncations to depend only on the ordering of the prospects associated to them - which the agent can still have at one node in the tree - so that once the ordering of prospects of a truncated tree is known, the ordering of plan truncations is also known, independently of the rest of the tree.

An important implication of this is that the ordering of the truncated plans at a node n_i in the tree is independent of the prospects that might have been open to the agent earlier in the tree, had the agent chosen differently, or had nature moved differently at choice or chance nodes before n_i.

[8] A thorough explanation of what is meant by normal and extensive form is not given in McClennen, as noted by Cubitt (1996).

In particular, it can be shown (for this see the Appendix) that by factoring TR, it is possible to derive TR by PR by adding the condition of separability SEP to it. Therefore, the SEP condition is what has to be added to PR in order to get TR.

5.3 Different models of dynamic choice

As discussed previously, two different approaches to choice have been traditionally considered in the literature on dynamic choice and changing preferences: myopic and sophisticated choice. McClennen evaluates and compares these dynamic choice models in terms of the conditions on dynamic choice considered above and formulates the new approach of Resolute Choice[9].

The way of framing the choice strategies in dynamic choice situations with a potential for inconsistency follows Strotz's analysis. However, despite the definition of the strategies differs, the decision context of McClennen's analysis is the same considered by Hammond - of dynamic inconsistency caused by non-expected utility non-linear preferences.

Following McClennen's analysis, I will use the same simple dynamic decision problem to evaluate the three different decision models in terms of the dynamic conditions.

As an example of the application of Strotz's perspective to Hammond's problem, consider the following decision problem described by McClennen, and originally formulated by Kahneman and Tversky (1979).

Suppose an individual is facing the following prospects:

g_1 = ($2400, 1)

g_2 = ($2500, 33/34; $0, 1/34)

g_3 = ($2400, 34/100; $0, 66/100)

g_4 = ($2500, 33/100; $0, 67/100)

where g_1 means that the agent will get $2400 with certainty; g_2 means that he will get $2500 with probability 33/34, and $0 with probability 1/34, and so on.

In what follows it is going to be assumed that the agent prefers g_1 to g_2, and g_4 to g_3. This preference pattern corresponds to a class of violation of expected utility theory and the independence axiom, known as the "common ratio effect".

According to expected utility, the preference for $g_1 \succ g_2$ implies that

$u(\$2400) \succ 33/34u(\$2500) + 1/34u(\$0)$, which can be rewritten as

$34/100u(\$2400)+66/100u(\$0) \succ 33/100u(\$2500) + 1/100u(0)+66/100u(\$0)$ or

$34/100u(\$2400)+66/100u(\$0) \succ 33/100u(\$2500) + 67/100u(\$0)$.

9 An experimental analysis of the three model of dynamic choice is attempted in Hey and Lotito (2009).

Therefore, expected utility implies $g_3 \succ g_4$ and not $g_4 \succ g_3$.

It is also possible to show how the pattern of choice considered violates the independence axiom. Given the RCLA, it is possible to rewrite the prospects as

g_1 = (A)

g_2 = (B)

g_3 = (A, 34/100; $0, 66/100)

g_4 = (B, 34/100; $0, 66/100),

where A ≡ ($2400, 1) and B ≡ ($2500,33/34; $0,1/34).

As argued above, the condition implied by independence is:

$A \succ B \;\Leftrightarrow\; [A, p; Q, 1\text{-}p] \succ [B, p; Q, 1\text{-}p] \qquad \forall Q, p.$

Having defined A ≡ ($2400, 1) and B ≡ ($2500,33/34; $0,1/34), it results that the condition above implies

$g_1 \succ g_2 \;\Leftrightarrow\; A \succ B \;\Leftrightarrow\; g_3 \succ g_4.$

Following Machina's analysis, the preferences in the common ratio effect problem can be shown to violate the independence axiom through violation of mixture separability over sublotteries.

Recalling the representation of the prospects when RCLA holds, it can be seen that the agent who prefers g_1 to g_2 and g_4 to g_3 prefers the lottery ($2400, 1) to the lottery ($2500,33/34; $0,1/34) in a direct choice between the two, but not when they are mixed with the other lottery ($0, 66/100). That is, for this agent

($2400, 1)≻ ($2500,33/34; $0,1/34), but not

($2400, 34/100; $0, 66/100) ≻ (($2500,33/34; $0,1/34)), 34/100; $0, 66/100),

in violation of mixture separability and of the independence axiom.

Suppose now that this other prospect is also available

g_{3+} = ($2401, 34/100; $1, 66/100),

where the agent can have slightly higher payoffs with the same probability than in g_3. If g_4 is strictly preferred to g_3 it is always possible to construct g_{3+} so that very small increments in g_3 will still result in a prospect less preferred than g_4, so that the agent will prefer g_4 to g_{3+} and g_{3+} to g_3.

The choice of the agent among these prospects can be represented by the following decision tree, once the notation and definitions considered above are assumed.

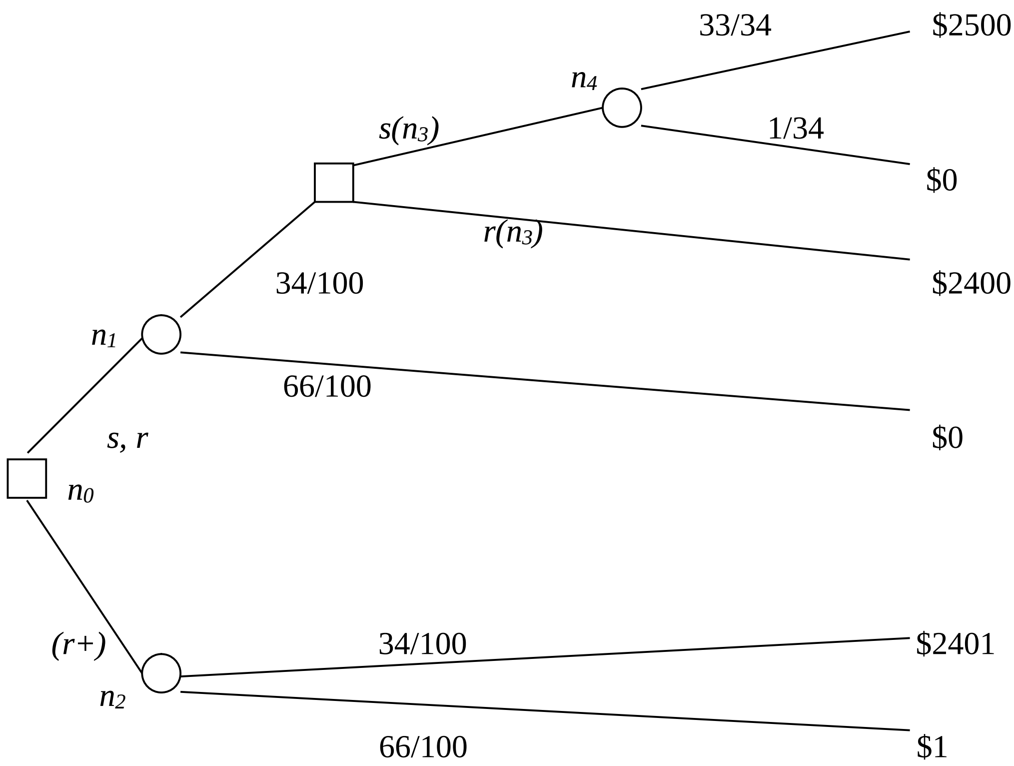


Figure 8 – Decision tree representing choice of a variation of the "common ratio effect" prospects

Consider the options available to the agent in this situation.

At the first choice node n_0 the agent is facing the possibility of choosing plan *s* or plan *r*, by going Up at choice node, or going Down by choosing *(r+)* and getting prospect g_{3+}. In case he chooses to go Up at n_0, he is in fact facing the two prospects g_3 and g_4. In case at n_3 he chooses $s(n_3)$, he will get (($2500, 33/34; $0, 1/34) with probability 34/100; and $0 with probability 66/100), which is just g_4 = ($2500, 33/100; $0, 67/100). In case at n_3 he chooses $r(n_3)$, he will get (($2400, with probability 34/100; and $0 with probability 66/100), which is just g_3 = ($2400, 33/100; $0, 66/100).

According to McClennen there are three different models of behaviour which the agent can adopt in this dynamic decision problem – Myopic Choice, Sophisticated Choice and Resolute Choice.

5.3.1 Myopic Choice (MC)

When acting according to a myopic approach, the agent selects at each point the strategies, or strategy continuations, which he judges acceptable from the perspective of that point.

Remind that the agent prefers g_1 to g_2 and g_4 to g_3 (and to g_{3+}). Therefore. he will choose plan *s* at n_0 in order to choose $s(n_3)$ at node n_3. However, in case he actually finds himself at node n_3, the agent will in fact decide to go for $r(n_3)$ and drop $s(n_3)$, as he is now facing prospects g_1 and g_2 and prefers the first to the latter.

Therefore, when faced with this decision problem, at n_0 the agent will choose plan *s*, while at n_3 he will choose the plan continuation $r(n_3)$ instead of the continuation of the plan originally adopted $s(n_3)$, then behaving in a dynamically inconsistent manner - adopting a plan and leaving it at a later node.

McClennen uses the conditions of dynamic choice defined above to offer a model of the agent's behaviour. A detailed description of how this is done can be found in the Appendix. Here it is sufficient to conclude that the agent who has the preferences considered above – that is, who ranks g_1 over g_2 and g_4 over g_3 - and follows all three conditions SR, NEC (which together imply PR) and SEP, will select plan *s* at n_0 and $r(n_3)$ at node n_3. In this way, he will violate the condition of dynamic consistency DC.

PR requires coincidence between acceptable sets over prospects and acceptable sets over plans that map into those prospects. Given the agent's preferences for g_4 over g_3, g_4 is in the set of acceptable prospects, while g_3 is not. Thus, according to PR plan *s* is in the set of acceptable plans, while plan *r* is not.

According to SEP, the agent will choose at n_3 as if facing a new tree. Then, for SR at the new tree starting at n_3, the prospect associated with plan *s* is g_2 and the prospect associated with plan *r* is g_1. Then, *r* will be in the set of acceptable plans, *s* will not. Then, for PR (SR+NEC) and SEP, the agent will select *s* at n_0 and *r* at n_3, violating DC.

This kind of behaviour corresponds to the myopic choice model of behaviour introduced by Strotz.

5.3.2 Sophisticated Choice (SC)

When acting according to a sophisticated approach, the agent always makes his ex post choice according to his ex ante plans, and avoids violating dynamic consistency. The SC agent will avoid choosing inconsistently with the plan adopted at the start, by restricting the set of feasible plans. Besides, the sophisticated agent is committed to SEP, as his preferences over options at a future time are only shaped by considerations for those consequences that remain realisable at that future time, and abstract from earlier evaluations. He violates NEC, allowing for a disparity between how he evaluates outcomes abstractly considered and strategies which access those outcomes.

Consider the behaviour predicted by this strategy in the above example.

When at the starting choice node n_0, the SC agent will anticipate that by moving Up towards n_1 he will choose $r(n_3)$ over $s(n_3)$ in case chance takes him at node n_3. Then, he will consider plan *s* as not feasible. Therefore, having dropped *s*, which constitutes a choice of g_4, he is now confronting the two plans which lead to prospects g_3 and g_{3+}, that is, r and $(r+)$ respectively. As, by assumption, g_{3+} is preferred to g_3, the agent will decide to adopt plan $(r+)$ at the start.

In order to see how the conditions of rational dynamic choice can offer a model of the agent's behaviour in the SC case, it is necessary to consider a refinement of the reduction conditions, and their relation with SC. This will be done in detail in the Appendix, and applied to the decision problem in the example.

From the refinement of the reduction conditions it will result that the SC agent makes choice as required by the SR, SEP and DC conditions; but - when he faces a potential violation of DC - he commits only to a restricted modified form of PR. This modification of PR is characterised by a rejection of the NEC condition as applicable to all possible plans available to the agent at n_0.

PR requires that the set of acceptable plans coincides with the set of acceptable prospects.

NEC requires that the acceptable set of plans associated with any tree coincides with the acceptable set of normal form (NF) plans associated with the same tree. Taking any tree T – like the one in Figure 8 – and modifying it so that at the outset the agent can implement any plan in T with a single choice, one gets a tree T^n – like the one in Figure 8a - which is the normal version of T. A plan s^n in T^n is the normal version of plan s in T.

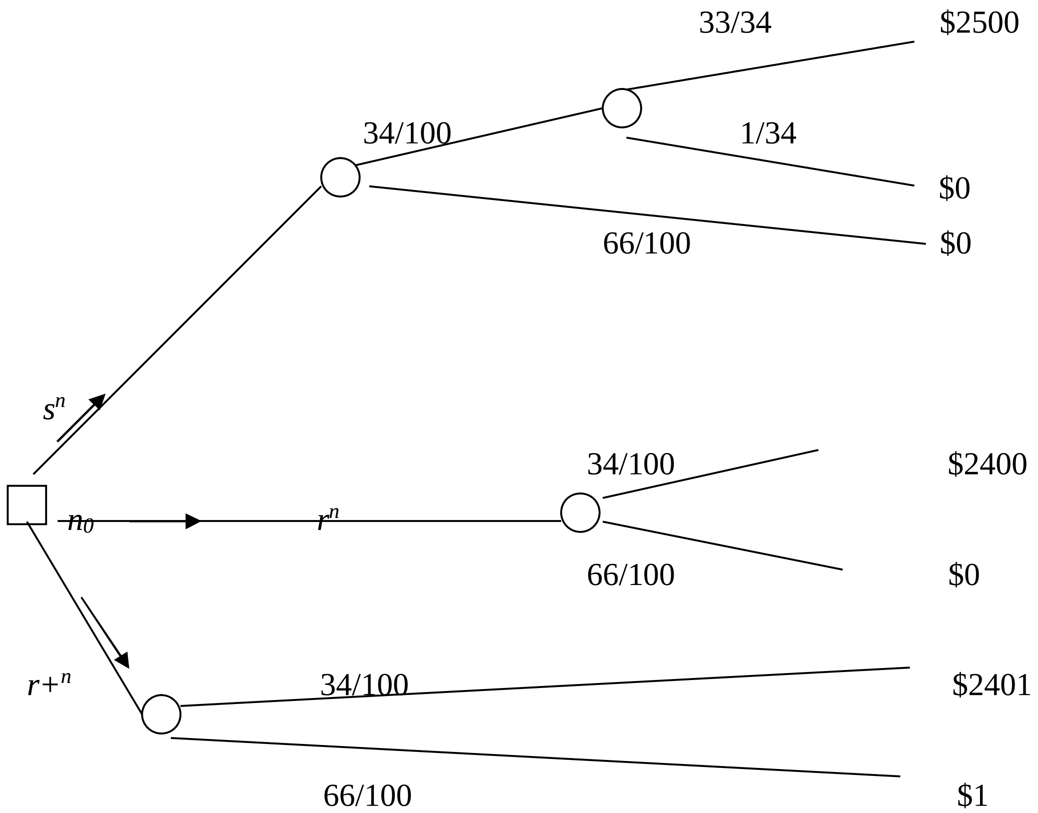


Figure 8a – Decision tree T^n representing the normal form version of the tree in Figure 8

According to NEC, the acceptable set of plans in T coincides with the acceptable set of plans in T^n.

SR requires that the set of acceptable plans coincides with the set of acceptable prospects if the decision tree is equivalent to a simple non-sequential choice among lotteries, where n_0 is the only choice node in the tree, and all other nodes are chance nodes. According to SR, the acceptable set of NF plans in T^n coincides with the acceptable set of corresponding prospects.

But plans in T^n and in T map into the same prospects, so that s^n and s map into g_4, r^n and r map into g_3, and $r+^n$ and $r+$ map into g_{3+}.

The SC agent violates NEC, as the acceptable set of plans in T do not coincide with the acceptable set of plans in T^n. In T^n plan s^n - leading to prospect g_4 - is in the acceptable set, while in T plan s is not in the acceptable set.

However, the SC agent maintains SR, which is a rather slack condition: the acceptable sets of plans in T^n coincides with the acceptable sets of corresponding prospects: if g_4 is acceptable, s^n is acceptable.

By violating NEC, the SC agent violates PR, as the set of acceptable plans in T does not coincide with the set of acceptable prospects. g_4 is acceptable, but s is not in the set of acceptable plans, as acceptable plans in T^n and in T do not coincide.

An agent who accepts the conditions SR, SEP and DC and rejects NEC provides a model for Hammond's sophisticated choice and Strotz's consistent planning.

5.3.3 Resolute Choice (RC)

The characteristic feature of the sophisticated choice strategy is that of avoiding dynamic inconsistency. According to McClennen, in the literature concerned with dynamic inconsistency, sophisticated choice is considered the only strategy which offers a solution to the problem, with the exception of Johnsen and Donaldson (1985)[10]. Resolute choice is an alternative to sophisticated choice in solving the problem of dynamic inconsistency. As seen above, the resolute agent resolves to act according to a plan judged best from an ex ante perspective, and intentionally acts on that resolve when the plan imposes on him ex post to make a choice he disprefers. By so choosing he avoids to act in a dynamically inconsistent manner.

However, the resolute agent violates the separability condition: he does not evaluate outcomes at tree continuation points as he would evaluate them *de novo*. A disparity exists between how the resolute agent evaluates options *ex post* and how he evaluates the same options *de novo*. Violation of separability means that choice at decision nodes within the tree cannot be explained only with reference to the options that the agent confronts at those points: his preferences over options at a future time do *not* abstract from earlier evaluations.

Consider this in more detail. A more formal description is given in the Appendix.

Sophisticated choice avoids dynamic inconsistency by making ex post choice behaviour shape ex ante choice of a plan. However, the condition of dynamic consistency only requires consistency between present and future choice, but does not specify *how* this choice is found, on which basis to determine what alternative is best at a future given point.

Sophisticated choice gives an answer to the problem of dynamic inconsistency by specifying that choice at the initial node is constrained by expected choices at each following node**.**

Resolute Choice adopts the opposite perspective: it makes choice of a plan judged best ex ante shape ex post behaviour.

Compare the behaviour prescription of SC and RC in the decision problem given in the example of Figure 8. SC makes the agent choose plan *(r+)*, even if the agent would prefer *s* if it were feasible. RC makes the agent choose *s* and then "intentionally choose" $s(n_3)$ when he gets to node n_3.

The RC agent resolves to implement the plan he originally adopted - despite this implies at some future node choice(s) that the agent would not have liked to make - and therefore behaves consistently. Besides, by moving through the tree for implementing his initial plan, the RC agent shows his commitment to the NEC condition, as he does not consider the difference between the normal and extensive form of plans. However, observing the DC condition in this way, forces the agent to introduce a difference in the way he values the same alternatives at a given node in the tree, and the way he would value them if he faced a new decision tree starting at that choice node.

In the previous example, the RC agent ranks plan *s* - which requires choice of $s(n_3)$ and allows to get prospect g_4 - over plan *r* - which requires choice of $r(n_3)$ and allows to get prospect g_3. In case the agent reaches node n_3, he implements his decision, and chooses $s(n_3)$, despite the fact that he would have chosen $r(n_3)$ over $s(n_3)$ in an outright choice between the two, if these two alternatives had been the only two plans of a *de novo* decision tree starting at n_3. Therefore, in avoiding dynamic inconsistency, the RC agent violates the separability condition SEP.

Violation of SEP means that the agent does not choose at a node in the decision tree by abstracting from choices or moves by nature which could have been available under conditions that do not occur at that choice node - "alternative but counterfactual choices or moves by nature". In the example, in order to observe SEP, the agent should have considered as irrelevant at node n_3 what might have happened at node n_1 (the probability of 66/100 of getting $0), but did not occur,

[10] As previously discussed, Machina (1989) formulates a model of dynamic choice behaviour alternative to SC, and substantially equivalent to RC. In his paper are also references of other formal antecedents of his formulation: Anand (1987), Donaldson and

since he is now at n_3. If the agent considered that event as relevant at n_3, then he would cut off that branch from the tree, and face only a tree starting at n_3, that is, prospects g_1 and g_2. By violating SEP, the agent considers that branch as relevant, and therefore faces prospects g_3 and g_4 instead.

For this agent, as Machina (1989) phrases it, “risk which is borne but not realised is gone in the sense of having been *consumed*, rather than gone in the sense of *irrelevant*” (page 1647).

6. Other models for Sophisticated Choice

The strategy of sophisticated choice has been previously considered in the context of dynamic inconsistency with changing preferences, both in the consumption model of Strotz, and in the more general case of Hammond’s (1976) potential addict. In that context, the sophisticated chooser has been described as an agent who anticipates his future choice and chooses the best option among those that he is ready to follow. In the context of dynamic choice under risk and uncertainty, where inconsistency is a problem in case preferences are non-expected utility, sophisticated choice can be characterised similarly. As discussed above, McClennen has offered a model of sophisticated behaviour in this context by modelling the strategy through dynamic choice conditions. Machina has discussed a strategy which represents the equivalent of sophisticated choice in the same dynamic context. In what follows I discuss other models of sophisticated behaviour in dynamic choice under risk.

Consider the following dynamic choice decision problem, introduced by Karni and Safra (1989b) and discussed by Dardanoni (1990). This example shows how violation of the independence axiom on the part of the agent’s preferences generates a problem of dynamically inconsistent choice and allows also to outline the different predictions of the myopic and sophisticated choice models.

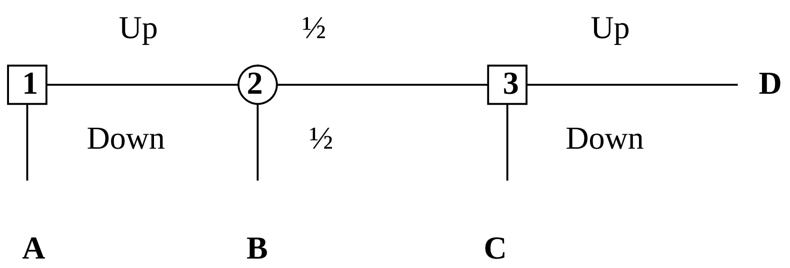


Figure 9 – Decision tree showing the potential dynamic inconsistency of violations of the Independence Axiom

Selden (1981), Machina (1981), Loomes and Sugden (1986), Yaari (1985), McClennen, (1988a,1988b,1990), which is RC itself.

The agent faces the outcomes A, B, C and D, which can be considered to be lotteries. Suppose that his preferences are such that D $\succ$ C but (½ B + ½ C) $\succ$ (½ B + ½ D), in violation of the independence axiom, which would imply that since D $\succ$ C, then (½ B + ½ D) $\succ$ (½ B + ½ C).

In addition, assume also that

A $\succ$ (½ B + ½ D) but (½ B + ½ C) $\succ$ A.

At each decision node, the agent has to decide whether to go Up or Down.

At the beginning of the tree, the agent has three possible courses of action to take:

- Up at the first node, and Up at the second, which implies lottery (½ B + ½ D)
- Up at the first node, and Down at the second, which implies lottery (½ B + ½ C)
- Down at the first node, which implies lottery A.

At the second decision node, the agent faces two courses of action: going Up, which implies lottery D, and going Down, which implies lottery C.

According to the agent's preferences, at the beginning of the tree, the best action to take would be to go Up at the first decision node, and Down at the second. However, at the second decision node, the best action is Up. Therefore, the optimal continuation at the second decision node does not coincide with the continuation of the optimal course of action at the initial decision node. There is a potential for dynamic inconsistent choice, which is a consequence of the violation of independence.

An agent choosing according to myopic choice, will incur in dynamically inconsistent behaviour. He will follow at each decision node the strategy that is most preferred at that node, on the basis of the options which result from the continuation of the strategy, ignoring any inconsistency which may arise from this behaviour. The myopic agent is described by Machina (1989) as a " β -type" decision maker. In the above example he will choose to go Up at the first decision node and will plan to go Down at the second decision node, following his preferred lottery, (½ B + ½ C). At the second choice node, however, he will be left with the options resulting from the continuation of the strategy, C and D, and will choose D. The myopic agent will act differently from his original plan and obtain an outcome which is dispreferred.

An agent choosing according to the equivalent of the sophisticated choice model in dynamic choice under certainty, will anticipate his future choice, and choose at each decision node the strategy which he will be able to follow, eliminating those plans which he anticipates he would not be able to implement. In the example, the agent "folds backwards"; he anticipates that at the second decision node he would choose the dispreferred lottery D, and rules out the possibility of making such a choice, by going for lottery A at the first decision node.

Karni and Safra's model of "behavioural consistency" is a way of implementing the sophisticated choice approach (Machina, 1989), and represents a solution to the problem of dynamic inconsistency with non-linear preferences, "equivalent (...) to the 'sophisticated' approach to dynamic consistency in choices under certainty" (Dardanoni, 1990). The decision maker is represented by a "collection" of agents, one at each node of the decision tree, and behaviour is modelled so to represent the subgame perfect equilibrium (Selten, 1975) of the game among the agents representing the decision maker at the different decision nodes.

In the above example, Karni and Safra consider "unreasonable" to suppose that the agent would choose the course of action (Up, Down) at the beginning of the tree, being aware of his preferences at the second decision node, and suggest as more "plausible" that he will reject that plan as "self-deceiving". The reasonable choice remains Down at the first decision node. This is achieved by backward induction.

The course of action of going Down at the first decision node is the unique subgame perfect equilibrium of the game between two players situated at the first and the second decision nodes. The behaviour which represents the subgame perfect equilibrium of the game among the agents at the different nodes, is *consistent*, and the course of action it generates is *behaviourally consistent*.

The notion of Nash equilibrium as a solution to the choice of an optimal consumption plan when tastes are endogenously changing had been already proposed by Peleg and Yaari (1973). As discussed previously in the chapter, references of similar approaches and their applications to the problem of inconsistent dynamic choice under risk are found in Machina (1989). He defines the dynamically consistent non-expected utility agents, who adopt a backward induction for determining the optimal strategy in a dynamic decision problem, as ∂-type decision makers[11].

Karni and Safra (1989b) analyse ascending-bid auctions with non-expected utility preferences, and replace dynamic consistency with the model of behavioural consistency, maintaining consequentialism and reduction of compound lotteries. Behavioural consistency is considered also in Karni and Safra (1990) as a necessary assumption when preferences are non-expected utility to avoid dynamic inconsistent choices. In Karni and Safra (1990) behavioural consistency is applied to the analysis of optimal stopping rules.

The model of dynamic choice considered, which will be referred to as sophisticated choice, resolves the problem of inconsistency. However, it may give rise to "undesirable" behaviour. Machina (1989) argues that the sophisticated approach may generate different choices in decision

[11] Machina does not use directly the term sophisticated choice for this kind of approach.

trees which are strategically equivalent, with a consequent non-indifference between these trees; and may exhibit aversion to costless information (pages 1654-5)[12].

Dardanoni (1990) gives an example of how the sophisticated approach can lead to a choice of 'dominated' strategies. In the previous example, sophisticated consistent behaviour was leading to an outcome dominant with respect to the myopic choice outcome according to the preferences of the agent at the beginning of the tree. Different results can be obtained in case a perspective of Pareto dominance is adopted as a criterion for strategy evaluation. Dardanoni (1990) shows that it is possible to construct examples, where Pareto dominance is not implied by dominance according to the first agent's preferences. Furthermore, he gives an example of a game where behavioural consistency leads to a Pareto dominated outcome with respect to myopic choice.

These results resemble the ones in Grout (1982), where it is shown that myopic choice can be dominant over sophisticated choice in a dynamic choice situation in conditions of certainty with changing preferences.

Dardanoni (1990) may be considered as an extension of Grout's result to the case of unchanging preferences under conditions of risk.

[12] Aversion to information is one of the dynamic arguments against non-expected utility. In Machina (1989) the argument concerns an agent who behaves according to sophisticated choice. Karni and Safra have pointed out that in game theory terms, aversion to information is a "pre-commitment strategy" on the side of the fist agent against moves on the part of the later stage agents. Strategic use of pre-commitment allows Cubitt, Starmer and Sugden (1998a) to interpret some results they obtain in their experiment as sophisticated choice behaviour.

7. Conclusion

In this paper the main contributions to the discussion of the problem of inconsistency in a dynamic decision problem have been reviewed.

In a context of choice under certainty the attention has been focused on the initial contribution by Hammond (1976, 1977), which contains many elements on which the following literature will develop.

In a context of choice under risk and uncertainty the problem occurs when preference orderings violate Expected Utility Theory. Here the case of violation of Expected Utility through violation of the Independence Axiom has been considered. The problem is introduced through the example by Raiffa (1968), which uses the Allais paradox to show the link between violation of independence and dynamic inconsistency. A review of Machina (1989) follows. Particular attention has been given to the general example of how any violation of independence generates dynamically inconsistent choices, and to the application of this example to the prospects of the Allais paradox. This application allows to introduce the specific example of the dynamic inconsistency of non-expected utility agents with Allais kind of preferences. In the context of these decision problems, the different models of choice available to the agent have been discussed, with particular attention to Machina's solution to the problem of inconsistent choice and his discussion of *consequentialism*.

The different arguments, conditions and strategies of choice have then been summarized and related through some simple decision trees.

McClennen's (1990) contribution to the debate has then been reviewed. First, the general theoretical frame of the dynamic inconsistency problem has been mentioned, as McClennen's and Machina's arguments are contributions to that debate. Then, the elements of McClennen's analysis have been considered. A description of the rational dynamic choice conditions has been introduced, and the models of choice available to the agent - myopic, sophisticated, and resolute choice - discussed in the context of a dynamic decision problem, when the agent's preferences violate independence. Different models of choice maintain and violate different conditions. Resolute Choice maintains dynamic consistency and is McClenenn's solution to the inconsistency problem.

To conclude, other contributions have been considered which discuss the model of Sophisticated Choice.

8. Appendix - Notation and formal definitions of McClennen's dynamic choice conditions and models of choice

8.1 Some notation and definitions

1) The evaluation of plans

The minimal requirement on the coherence of the ranking is made, that for any *T* for which a finite set of plans is defined, there is at least one plan that the agent does not rank below the others. Therefore, D*(S)* is not empty for any *S*.

2) The evaluation of plans at subsequent nodes

• Evaluation of truncated plans

$D(S(n_i))$ is defined as the set of truncated plans that the agent judges acceptable from the new point n_i.

• Evaluation of plan continuations

From this perspective, the agent will have to consider which of the present plan continuations are continuations of members of *D(S)* defined at any point n_i. Then, it is possible to define for any node n_i, $D(S)(n_i)$ as the set of all plans in *D(S)* that the agent can still implement from n_i, so that $D(S)(n_i)$ is the restriction of *D(S)* to the continuations from n_i onward of plans in *D(S)*.

• Evaluation of truncated plans as de novo trees

For any *T* define $T(n_i)^d$ as the truncated tree $T(n_i)$ conceived as a full tree that starts de novo at node n_i, and is like $T(n_i)$, except for "having no history or collateral paths". Then, $S(n_i)^d$ is the set of plans available to the agent in $\mathrm{T}(n_i)^d$ and $D(S(n_i)^d)$ is the set of plans for $\mathrm{T}(n_i)^d$ acceptable to the agent.

The different perspectives from which the agent who finds himself at a certain node n_i in a tree *T* can evaluate the plans from that point are the following three.

- $D(S)(n_i)$ concerns the evaluation made by the agent of all the possible plans available to him before any move by nature or by the agent himself has taken place.
- $D(S(n_i))$ concerns the evaluation of the alternatives available to the agent at node n_i, from the perspective of node n_i itself.
- $D(S(n_i)^d)$ concerns the evaluation of the agent in the case he is given a 'de novo' decision problem identical to the one he is considering.

Prospects

For any tree *T* and plan *s* in *S*, g_s is defined as the prospect associated with *s*, and G_S the set of prospects associated with *S.*

For any truncated plan $s(n_i)$ at n_i, $g_{s(ni)}$ is defined to be the prospect associated with $s(n_i)$, and $G_{S(ni)}$ the set of prospects associated with $S(n_i)$.

8.2 Dynamic choice conditions

8.2.1 Conditions of reduction

Simple Reduction (SR). *Given T with associated S, such that each s in S requires for its implementation a single choice "up front" by the agent, and the set* G_S *of prospects associated with those plans, then for any s in S and associated* g_s *in* G_S*, s is in D(S) iff* g_s *is in the set of acceptable prospects* $D(G_S)$*.*

Plan Reduction (PR). *Given T with associated S, and the set of prospects associated with those plans* G_S*, then for any s in S and associated* g_s *in* G_S*, s is in D(S) iff* g_s *is in* $D(G_S)$*.*

By "factoring" PR it is possible to show that PR adds to SR the condition that, insofar as evaluation is concerned, no distinction exists between plans in normal and extensive form. Given s^n the normal form of *s* and T^n the normal version of *T*, it is possible to formulate the following condition:

Normal-form/extensive form coincidence (NEC). *Let T be any decision tree with associated S, and* T^n *the decision problem resulting by converting each s in S into its normal form, so that each s in S is mapped into* s^n *in* S^n*. Then, for any s in S, s is in D(S) iff* s^n *is in* $D(S^n)$*.*

SR ensures the coincidence between the set of acceptable plans *D(S)* and the set $D(G_S)$ of the corresponding acceptable prospects. NEC ensures the coincidence between the set of acceptable plans *D(S)* associated with a tree and the set of acceptable normal form plans $D(S^n)$ associated to the same tree.

8.2.2 Dynamic consistency (DC)

For any choice point n_i *in a decision tree T, if* $D(S)(n_i)$ *is non-empty and* $s(n_i)$ *is in* $D(S(n_i))$*, then* $s(n_i)$ *is in* $D(S)(n_i)$*; and if* $s(n_i)$ *is in* $D(S)(n_i)$*, then* $s(n_i)$ *is in* $D(S(n_i))$*.*

In order to reach the formulation of DC given above, two relations have to be established.

(1) Inclusion (DC-INC). *For any choice point* n_i *in T, if* $D(S)(n_i)$ *is non-empty and* $s(n_i)$ *is in* $D(S(n_i))$*, then there is some plan s* in D(S) such that* $s(n_i) = s^*(n_i)$ *is the plan continuation of s* at* n_i*, and hence such that* $s(n_i) = s^*(n_i)$ *is in* $D(S(n_i))$*.*

Dynamic consistency can be formulated in terms of a relation between what is an acceptable choice to make at a node n_i from the point of view of the beginning of the tree, and what is an acceptable choice to make at n_i from the point of view of n_i itself, that is, a relation between $D(S)(n_i)$ and $D(S(n_i))$. First of all, it is necessary to assume that at a node n_i at which the set $D(S)(n_i)$ is non-empty, the intersection between the two sets $D(S)(n_i)$ and $D(S(n_i))$ is itself non-empty. If this does not hold, then it is possible that the agent will choose inconsistently with the plan adopted, as that plan is a member of *D(S)*, and what the agent chooses at n_i is supposed to be a member of $D(S(n_i))$. Besides, given that this holds, it is possible that there exists at n_i a plan continuation $r(n_i)$, which is an element of $D(S(n_i))$, but that no plan *r* exists in *D(S)* such that $r(n_i)$ is its continuation. Then, by choosing $r(n_i)$ at n_i, the agent will choose inconsistently with the original plan. The DC-INC condition avoids this by assuming that there must exist some plan *r* in *D(S)* whose continuation at node n_i is $r(n_i)$.

(2) Exclusion (DC-EXC). For any choice point n_i in T, if $s(n_i)$ is defined and is not in $D(S(n_i))$, then s is not in D(S).

So far it has been required that, if a plan continuation at point n_0 is in the set of acceptable plans from the point of view of that node, but in the set of acceptable plans there is not a plan whose continuation is in the set of plans acceptable at the node from the point of view of the beginning of the tree, then there is a possibility for inconsistent choice. That is, plan *s* must be included in *D(S)* if there is a possibility for its continuation $s(n_i)$ to be chosen.

The DC-EXC condition requires in addition that a plan *s* should not be in the set of acceptable plans *D(S)*, if there are conditions for which its continuation $s(n_i)$ will not be chosen, because not in the set $D(S(n_i))$ of plans acceptable at a node in the tree from the point of view of the node itself. In case this does not hold, it is possible for the agent to choose inconsistently with his original plan. That is, plan *s* must be excluded from *D(S)* if there is a possibility for its continuation $s(n_i)$ not to be chosen.

The condition DC of dynamic consistency defined above is obtained by combining the above conditions of inclusion and exclusion.

8.2.3 Separability (SEP)

SEP requires the agent to choose in the truncated tree $T(n_i)$ in the same way he would choose in the separate tree $T(n_i)^d$ corresponding to $T(n_i)$, therefore stating a connection between $D(S(n_i))$ and $D(S(n_i)^d)$. SEP is defined as follows:

For any tree T and node n_i in T, let $T(n_i)^d$ be a separate tree that begins at node n_i and coincides with $T(n_i)$, and let $S(n_i)^d$ be the set of plans available in $T(n_i)^d$ that correspond *one to one with the set of truncated plans $S(n_i)$ available in $T(n_i)$. Then, $s(n_i)$ is in $D(S(n_i))$ iff $s(n_i)^d$ is in $D(S(n_i)^d)$.*

The role of SEP can be better understood by considering the following condition:

Truncated plan reduction (TR). *Let n_i be any node in a decision tree T, and $S(n_i)$ be the set of truncated plans that can be associated with $T(n_i)$. Then $s(n_i)$ is in $D(S(n_i))$ iff $g_{s(ni)}$ is in $D(G_{S(ni)})$.*

By factoring TR, it is possible to see that TR can be derived by PR by adding the condition of separability SEP to it.

Consider the distinction between the truncated tree $T(n_i)$ and the truncated tree $T(n_i)^d$, identical to $T(n_i)$ but conceived as a new tree starting at node n_i. The condition PR will apply to $T(n_i)^d$ as this is a whole tree. Then, $s(n_i)^d$ is in $D(S(n_i)^d)$ iff $g_{s(ni)}{}^d$ is in $D(G_{S(ni)}{}^d)$. This means that $s(n_i)^d$ is in $D(S(n_i)^d)$ iff $g_{s(ni)}$ is in $D(G_{S(ni)})$, as $G_{S(ni)}{}^d = G_{S(ni)}$. So, TR follows if one assumes in addition that $s(n_i)$ is in $D(S(n_i))$ iff $s(n_i)^d$ is in $D(S(n_i)^d)$, which is just SEP.

8.3 Different models of dynamic choice

8.3.1 Myopic Choice (MC)

According to MC, the agent selects at each point the strategies or strategy continuations, which he judges acceptable from the perspective of that point.

In the decision tree of Figure 8 the agent who ranks g_1 over g_2 and g_4 over g_3, and follows all three conditions SR, NEC and SEP, will select plan *s* at n_0 and $r(n_3)$ at node n_3 and violate DC.

According to the PR condition, which is equivalent to the conjunction of SR and NEC, plan *s* is in $D(S)$, while plan *r* is not. As the prospects associated with plans *s* and *r* are respectively g_4 and g_3, and the first is preferred to the latter, g_4 is in the set of acceptable prospects, while g_3 is not, and according to PR, its associated plan *s* is in the set of acceptable plans, while *r* is not.

Conditions SR and SEP require that $r(n_3)$ is in the set $D(S(n_3))$, while $s(n_3)$ is not. According to SEP the agent will choose at decision node n_3 as if he were facing a new tree starting from n_3. For SR the plan *s* in the new tree is associated with prospect g_2, while plan *r* with the preferred prospect g_1, so that plan *r* will, while plan *s* will not, be in the set of acceptable plans, and will be chosen. Then, for SEP $r(n_3)$ and not $s(n_3)$ will be in the set of acceptable plans at n_3.

8.3.2 Sophisticated Choice (SC)

Consider the following definition of sophisticated choice:

A SC agent regards a plan *s* as not feasible, and then as not in $D(S)$, if he projects that at some point *n* for which $s(n_i)$ is defined, $s(n_i)$ is not in $D(S(n_i))$.

In terms of the dynamic inconsistency problem, the implication of this definition is that the SC agent will avoid violation of DC by restricting the set of feasible plans.

In order to see how the rational dynamic choice conditions can offer a model of SC consider the following refinement of the reduction conditions and their relation with SC.

Sophisticated choice and reduction - weakening of the PR condition

Recall the condition of plan reduction PR which has been defined above. In the following, PR is going to be weakened. The restricted version of plan reduction which will be considered, corresponds to the following restricted conception of feasibility:

Separable feasibility (SF). A plan is feasible iff $s(n_i)^d$ *is in* $D(S(n_i)^d)$ *for every choice point* n_i, $n_i \neq 0$, *for which s is defined.*

Restricted plan reduction (RPR). For any plan s, such that s satisfies SF, s is in D(S) iff g_s *is in* $D(G_S)$.

RPR requires that the mapping of acceptable plans in the decision tree *T* into acceptable plans in its normal form T^n, does not apply to all possible plans in *T*, but only to the separably feasible plans. That is, in its restricted version, PR holds only with respect to the evaluation of the plans which are feasible according to SF. But then, what characterises the change in a commitment to PR here is a rejection of NEC as applicable to all possible plans at n_0, that is, the emergence of a disparity between the evaluation of plans at the moment of decision, and the evaluation of plans in a once-and-for-all choice, so that whether plans are in extensive or normal form makes a difference. Given SF, NEC can be understood to apply only to sets of plans that are separably feasible, not to all plans at n_0.

Therefore, an SC agent makes choice as required by SR, SEP and DC; but when he faces a potential violation of DC, he commits to PR only in its restricted modified form of RPR. This modification of PR is characterised by a rejection of NEC as applicable to all possible plans available to the agent at n_0. The SC agent accepts the conditions SR, SEP and RPR.

8.3.3 Resolute Choice (RC)

In formal terms, dynamic consistency only requires that the (non-empty) restriction of $D(S)$ to n_i, that is, $D(S)(n_i)$, coincides with $D(S(n_i))$.

SC requires that $D(S)(n_i)$ has to be constrained to $D(S(n_i))$ for any n_i at which $D(S)(n_i)$ is non-empty, and that $D(S(n_i))$ has to be constrained to $D(S(n_i)^d)$. That is, what the agent expects to choose is conditioned by SR, SEP, and his preferences with respect to the prospects associated at each new tree corresponding to each node n_i.

RC requires to apply PR to n_0 in order to determine $D(S)$, and then to constrain $D(S(n_i))$ to $D(S)(n_i)$ for any n_i for which $D(S)(n_i)$ is non-empty.

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