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A COURNOT EQUILIBRIUM BETWEEN DARK NET MARKET AND STREET MARKET

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A Cournot equilibrium between Dark Net Market and Street market

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Abstract

This article contributes to the economic analysis of the illegal drug trade on either the Street or the Dark Net Market (DNM). For the sake of simplicity, it is assumed that there is a continuum of consumers with unitary demand for one drug. Their demand price varies from one market to the other according to the risks they bear in accessing it. The lower risk of violence in the DNM implies that, *ceteris paribus*, the good delivered there is deemed higher quality. Vendors compete à la Cournot in quantity in their “home” market, selling homogeneous goods. However, the other market exerts a vertical competitive threat. The two markets are intertwined, and we model the case in which both are simultaneously in equilibrium.

Keywords: Dark Net Market, Street Market, Drug Market, Cournot Equilibrium, Vertical competition, Constant Elasticity Inverse Demand

JEL classification: D04, D43, L13, K42, O17

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1 Introduction

This paper aims to study drug vendors' and consumers' behavior in a framework in which they can meet either on the web, in the so-called Dark Net Market (DNM from now on), or the Street Market. By Street Market we mean a representative local market in which drugs are traded, characterized by the fact that consumers can reach it on foot or at negligible transport costs.

As for the DNM, we resort to the Cournot oligopoly model, and for simplicity of treatment, we assume that identical vendors compete in quantity and sell a homogenous given drug (like, e.g., cocaine) online. The role of platforms is not explicitly modeled: it is assumed that the costs of services of platforms contribute to determining the costs borne by vendors in this market. While this is a limitation of our approach, the presence of many competing platforms operating in this field and the frequency of multihoming severely limit the market power of platforms.

We also model the Street market for the same drug type as a Cournot oligopoly since some competition also arises in the Street, at least at the borders of catchment areas. Moreover, the Cournot approach also allows for the possibility of having a unique vendor, thus encompassing the case of a monopolistic market as well. The risk of violence on the Street is considered in our model as a feature that negatively affects the quality of the goods traded in the Street market. The drug sold in the DNM is thus perceived as higher quality since the risk incurred during the trade is lower, i.e., the two markets are vertically differentiated.

The whole drug market, consisting of DNM and Street, is described as a Cournot-type market, in which, however, competition between the two markets occurs in quality instead of quantity (Gabszewicz and Thisse, 1979). We adopt a short-term perspective and assume that suppliers in each market are identical, specialize in one market, cannot move to the other or operate in both, and cannot modify the riskiness of accessing their market.

As for consumers, it is assumed that there is a continuum of them, each demanding one unit of a given drug, with no substitute or complement. Their demand price varies from the DNM to the Street according to the riskiness of the environment.

Vendors in both markets compete in quantity within their home market and also face the vertical threat coming from the other market. We present the conditions characterizing an equilibrium in which both markets are active.

The recent literature in the field of drug markets has focussed on the moral hazard problems raised by the fact that drugs sold in the streets are experience goods whose quality can be assessed only after buying them (Galenianos and Gavazza, 2017). Other papers (Galenianos et al., 2012) concentrated on the search process that arises in the Street market. As for the DNM, it has been pointed out that customers' ratings signal the high quality of products delivered there. At the same time, the DNM continuing expansion shows that it is resilient to disruptive events or specific risks, such as exit scams (Bhaskar et al., 2019).

Our contribution aims at building a simple model describing the current scenario, characterized by the fact that information asymmetry is now less relevant in the DNM. At the same time, there is a vertical differentiation between the DNM and the Street since the latter is perceived as being riskier than the DNM. We aim to describe both the internal functioning of each market and the interaction between them, and for the latter sake, we exploit the classical approach to vertical differentiation à la Gabszewicz and Thisse (Gabszewicz and Thisse, 1979). Notwithstanding the strong simplifying assumptions needed to render this approach manageable, our model provides a rich basic framework for analyzing policy interventions, which might target the relative riskiness of the two markets and/or the other parameters affecting the internal functioning of each of them.

The rest of the article is organized as follows. In Section 2, we introduce back-

ground information about the DNM drug trade. Section 3 presents the standard Cournot model in which suppliers compete in quantity, and the equilibrium which would arise if each market was the only active one. We introduce and study the global equilibrium in the two markets in section 4. Section 5 discusses illustrative example and policy implications, while the final section concludes.

2 Background

The typical DNM functions as a traditional e-commerce business with many sellers competing for the consumer's attention. One of the most significant advantages of the DNM platform is a new and convenient way to order drugs. A person can browse drug listings from anywhere. The drug ordering process is temporally detached from its delivery process, creating an additional security buffer for users. There is no need anymore to meet with dealers. Third parties, such as postal services or delivery companies, handle drug shipments without even knowing it.

In contrast to the DNM, vendors and buyers are exposed on the Street. The violence and apprehensions are two major downsides of this exposure. While using DNM, only dealers are susceptible to violence and arrests during meetings with wholesalers. End consumers never meet with sellers from the DNMs.

DNMs offer sellers a unique opportunity to enlarge their distribution channels worldwide. DNM vendors are not physically attached to any specific place like the Street dealers. On the Street market, the territory for drug sales is usually divided between criminal groups, which intermittently resort to violence to deter competitors.

In DNM trading, the decisive risk occurs at the level of delivery. To reduce the shipping and delivery risks, buyers and sellers share information about the best practices (Aldridge and Askew, 2017).

Aside from collecting information about “safe delivery” techniques, the DNM helps construct private and anonymous drug communities. There is no need to build an interpersonal relationship on the online platform like on the Street market, as the former offers a review system for both vendors and consumers. Moreover, through escrow system utilization, market administrators enforce contracts. Platform owners keep consumers’ money in escrow until the delivery is confirmed. Even though the escrow system is designed to protect buyers and sellers, it tempts a platform owner to perform an exit scam with all the escrowed money in their possession.

To mitigate the platform’s moral hazard problem, the usage of multi-signature¹ escrow schemes is advised. Still, this precaution measure is rarely used, possibly because of the additional fees to be paid (Bhaskar et al., 2019). Therefore, DNM drug sellers and consumers are exposed to the risk of losing money due to the platform’s exit scam. However, the market proved to be quite resilient after episodes of scams (Bhaskar et al., 2019).

Accessing DNM for the first time could be challenging for new buyers. “Knowing the right people” may be restated to knowing the right way on the internet (Leukfeldt, 2017). Still, the DNMs drug market is an open market, and everyone with a computer and an internet connection is welcomed. The “DarkNet Bible” exists to answer the most common questions and doubts.² Additionally, on ClearNet,³ there are forums such as DarkNet on Reddit,⁴ where DNM drug users can ask for advice or share information.

On the DNMs, consumers are not tied to specific dealers, and users can freely

¹Multi-signature escrow means that money is released only when the seller, buyer, and platform put their signatures.

²See <https://archive.is/yo4oF>, accessed 10.12.2021.

³ClearNet (Surface Web) is a everyday part of the Internet accessible by search engines as Google (Weimann, 2016).

⁴See <https://www.reddit.com/r/darknet/>, accessed 10.12.2021.

choose their suppliers from any part of the world. The entry barriers for suppliers on the DNMs are low. Vendors only need to register on the DNM and write product descriptions. The setup costs are modest, consisting of a vendor bond, which is about \$200. Then, the vendor pays a fee of around 2-4% of each transaction (Aldridge et al., 2018). This leads to a simultaneous presence of vendors on multiple DNMs. A growing number of vendors and review systems increase the competition between the sellers and make vendors improve the product quality. On DNMs, while presenting their products, vendors should write the drug's composition and purity. If the description does not coincide with the actual product, it will mirror the reviews. The high quality of the drugs is one of the main reasons why consumers prefer DNMs (Caudevilla et al., 2016).

DNM is a relatively new phenomenon as the first one was created in 2011. Technologies used on DNM are constantly evolving. Nowadays, the biggest challenge for law enforcement agencies is the usage of untraceable cryptocurrency instead of Bitcoin. Therefore, the probability of apprehension is perceived to be lower on the DNM than on the Street: only 4.1% of drug users are concerned about being caught by law enforcement while buying drugs from DNM (Barratt et al., 2016).

3 The model

Let us first of all model the functioning of each market in isolation, assuming that it is the only one active. Then we will consider the case in which both markets are active.

3.1 Demand functions

For simplicity, let us assume that all consumers are indexed over the interval $[0, 1]$; that is, any real number $x \in [0, 1]$ denotes a single customer belonging to a

(continuous) infinite population of consumers. Moreover, again for simplicity, we assume that each consumer x is willing to purchase *one unit* of a homogeneous illicit good (e.g., cocaine), while only one market, i.e., either the DNM or the Street, is active. The same model, in which vendors compete in quantity, is used to describe both markets: of course, the relevant parameters for a calibration (such as the demand price, the number of firms, etc.) would differ from one market to the other.

In line with empirical evidence (Gallet, 2014), we assume that the inverse market demands are characterized by a constant elasticity in each market. They are denoted by $R_D(x)$ and $R_S(x)$ for the DNM and the Street respectively; for each $x \in [0, 1]$, $R_D(x)$ represents the inverse market demand in the DNM, while $R_S(x)$ represents the inverse market demand in the Street.

We characterize both markets with constant elasticity inverse demand functions:

$$R_D(x) = \theta_D x_D^{-\varepsilon_D} \quad \text{and} \quad R_S(x) = \theta_S x^{-\varepsilon_S}, \quad (1)$$

where θ_D and θ_S are positive constants and $-\varepsilon_D$, $-\varepsilon_S$ denote the (negative) constant elasticities of the DNM and Street markets respectively, i.e.,

$$\frac{R'_D(x)x}{R_D(x)} \equiv -\varepsilon_D \quad \text{and} \quad \frac{R'_S(x)x}{R_S(x)} \equiv -\varepsilon_S \quad \text{for all } x \in [0, 1].$$

We assume that $\theta_D = \theta_S = 1$ and $|\varepsilon_D| > |\varepsilon_S|$, which is equivalent of having (the absolute values of) the *elasticities of the direct demand curves*, $\frac{1}{|\varepsilon_D|}$ and $\frac{1}{|\varepsilon_S|}$, satisfying $\frac{1}{|\varepsilon_D|} < \frac{1}{|\varepsilon_S|}$ and having the inverse demand curves in the two markets such that $R_S(x) < R_D(x)$ ⁵ for all $x \in [0, 1)$ and $R_D(1) = R_S(1) = 1$.

⁵Our assumption of $R_S(x) < R_D(x)$ is consistent with empirical evidence (Rhumorbarbe et al., 2016). This assumption could be explained as follows. Drug users are willing to pay higher prices on the DNM in exchange to safer transactions, greater choice and more convenient process of buying drugs compared to the Street market.

3.2 Oligopoly with either the DNM or the Street market active

We study the two markets in parallel since the same standard Cournot competition model is used to describe the respective functioning.

There are n_D vendors in the DNM and n_S vendors in the Street market. The n_D vendors face the same marginal cost $c'_D(y)$ to provide any amount of the homogeneous drug $y \geq 0$ in the DNM, while the n_S vendors face the same marginal cost $c'_S(y)$ to provide any amount of the homogeneous drug $y \geq 0$ in the Street market. Vendors are all equal in each market and compete in quantity within their market, that is, in equilibrium, each vendor in the DNM sells $y_D = \frac{Y_D}{n_D}$ units of the drug, while each vendor in the Street market, if the latter is the only one active, sells $y_S = \frac{Y_S}{n_S}$ units of the drug, where Y_D and Y_S denote the total quantities offered in equilibrium in each market. Specifically, according to formula (16.4) on p. 290 in (Varian, 1992), the equilibrium conditions in each market, as long as it is the only one active, are:⁶

$$R_D(Y_D) \left(1 - \frac{\varepsilon_D}{n_D}\right) = c'_D(y_D) = c'_D\left(\frac{Y_D}{n_D}\right) \quad (2)$$

$$R_S(Y_S) \left(1 - \frac{\varepsilon_S}{n_S}\right) = c'_S(y_S) = c'_S\left(\frac{Y_S}{n_S}\right). \quad (3)$$

Clearly, as both inverse demands $R_D(Y_D)$ and $R_S(Y_S)$ denote prices, the terms $R_D(Y_D)$ and $R_S(Y_S)$ are strictly positive; similarly, we assume that the cost structure in both markets increase in the quantity sold, i.e., $c'_D(y_D)$ and $c'_S(y_S)$ are strictly positive as well⁷. Hence, in order to be meaningful, conditions $\left(1 - \frac{\varepsilon_D}{n_D}\right) >$

⁶Note that in equations (16.3) and (16.4) on p. 290 (Varian, 1992) the term ε denotes the (negative) elasticity of *direct demand*, while here we use ε_D and ε_S to denote the absolute value of the elasticity of the *inverse demands*; therefore, the expressions inside the brackets in the LHS contain the terms $-\frac{\varepsilon_D}{n_D}$ and $-\frac{\varepsilon_S}{n_S}$ instead of $\frac{1}{n_D \varepsilon_D}$ and $\frac{1}{n_S \varepsilon_S}$ as in Varian (1992).

⁷Indeed, on the DNM, cost increases due to shipping costs, and on the Street, dealers' cost

0 and $\left(1 - \frac{\varepsilon_S}{n_S}\right) > 0$ in equations (2) and (3) must hold. As we do not want a priori rule out monopolistic markets, in line with the other research (Payne et al., 2020), we shall adopt the following assumption.

A. 1 *The elasticity parameters of the isoelastic inverse demand functions in (1) satisfy $|\varepsilon_S| < |\varepsilon_D| < 1$.*

Assumption A.1, by requiring both inverse demands to be inelastic in the relevant domain $[0, 1]$, allows for a single monopolistic seller in both markets to operate on the elastic part of the direct demand.

3.3 The inverse supply function

Conditions (2) and (3) imply that prices will settle in each market according respectively to (4) and (5):

$$R_D(Y_D) = \frac{c'_D\left(\frac{Y_D}{n_D}\right)}{1 - \frac{\varepsilon_D}{n_D}} \quad (4)$$

$$R_S(Y_S) = \frac{c'_S\left(\frac{Y_S}{n_S}\right)}{1 - \frac{\varepsilon_S}{n_S}}, \quad (5)$$

For different inverse demand functions the equilibrium quantities will differ; therefore, by considering all possible (constant elasticity) inverse demand functions it is possible to build the whole *inverse supply functions* L^8 for each market by taking

 increases due to payments for protection.

⁸More properly these functions represent the *Locus of quantities offered in equilibrium in each market* (for a discussion of this topic, see (Klemperer and Meyer, 1989).

the RHS terms in (4) and in (5) as functions of all the possible quantities Y :

$$L_D(n_D, \varepsilon_D, c'_D, Y) = \frac{c'_D \left(\frac{Y}{n_D} \right)}{1 - \frac{\varepsilon_D}{n_D}} \quad (6)$$

$$L_S(n_S, \varepsilon_S, c'_S, Y) = \frac{c'_S \left(\frac{Y}{n_S} \right)}{1 - \frac{\varepsilon_S}{n_S}}. \quad (7)$$

4 Towards a definition of global equilibrium

When both markets are active, consumers will choose the market which provides the best deal. This implies considering the *consumer surplus (rent)* $E_D : [0, 1] \rightarrow \mathbb{R}_+$ and $E_S : [0, 1] \rightarrow \mathbb{R}_+$ that consumers would get in each market, defined over the whole population $[0, 1]$ of consumers as:

$$E_D(x) = R_D(x) - p_D^* \quad \text{for the DNM,}$$

$$E_S(x) = R_S(x) - p_S^* \quad \text{for the Street,}$$

where E_D and E_S denote the consumer surplus as functions of the consumers' index x in DNM and in the Street respectively, while $R_D(x)$ and $R_S(x)$ have the form in (1), and p_D^* and p_S^* denote the *prices that are actually being paid in equilibrium* in the DNM and in the Street market respectively. Prices p_D^* and p_S^* turn out to be complex objects that must be carefully discussed; for the moment let us consider them as abstract equilibrium prices in the two markets. Our key assumption is the following.

A. 2 *Whenever either $E_D(x) \geq 0$ or $E_S(x) \geq 0$ (that is, consumer x purchases at least in one of the two markets), the following hold:*

i) *consumer $x \in [0, 1]$ purchases on the DNM if $E_D(x) > E_S(x)$,*

- ii) consumer $x \in [0, 1]$ purchases on the Street if $E_D(x) < E_S(x)$,
- iii) consumer $x_m \in [0, 1]$ will be labelled as the marginal consumer (i.e., she is indifferent between DNM and the Street) if $E_D(x_m) = E_S(x_m) \geq 0$.

Assuming that the inverse demand functions in the two markets have constant elasticity as in (1) which satisfy $\theta_D = \theta_S = 1$ and, according to Assumption A.1, $\varepsilon_D > \varepsilon_S$, consumers x close to the left endpoint 0 are eager to pay higher prices on the DNM than on the Street for one unit of the same drug. Independently of any assumption on the oligopolistic inverse supply functions provided by (6) and (7), we assume that consumers x close to the left endpoint 0 always prefer to purchase on the DNM⁹, that is $E_D(x) > E_S(x)$ for small values of x . As $\theta_D = \theta_S = 1$, $\varepsilon_D > \varepsilon_S$ implies that $R_D(x) > R_S(x)$ for all $x \in [0, 1)$ and $R_D(1) = R_S(1) = 1$, necessarily R_D decreases faster than R_S . If we also assume that the inverse supply functions L_D and L_S have values sufficiently close in $Y = 0$, $L_D(0) \sim L_S(0)$, and that the former increases faster than the latter, one expects that there exists a (possibly unique) *marginal consumer* $x_m \in (0, 1)$ which is indifferent between going to the DNM or to the Street market, i.e., such that $E_S(x_m) = E_D(x_m)$, and that all consumers $x \in (x_m, x_S^*] \subseteq (x_m, 1]$, where x_S^* is the consumer having reserve price equal to the equilibrium price p_S^* in the Street market, will go to the Street market; that is, $E_D(x) < E_S(x)$ for all $x \in (x_m, x_S^*]$. For simplicity let us assume that the marginal consumer x_m is unique.

⁹The intuition behind this assumption relies on the empirical evidence (Bancroft and Reid, 2016),(Moeller et al., 2021) of some customers ready to buy drugs only on DNM and to pay higher prices for the perceived lower risk and higher quality.

4.1 The equilibrium in the Street market

Let $0 < x_m < x_S^*$; then in equilibrium each consumer indexed by $x \in [0, x_m)$ purchases one unit of drug, so that a total quantity given by

$$Y_D = x_m - 0 = x_m > 0 \quad (8)$$

is being sold by all the n_D vendors in the DNM. Conversely, each consumer indexed by $x \in (x_m, x_S^*]$ purchases one unit of drug, so that a total quantity given by

$$Y_S = x_S^* - x_m > 0 \quad (9)$$

is being sold by all the n_S vendors in the Street market. Specifically, (9) shows that the marginal consumer x_m is the first consumer entering the argument of the supply function $L_S(Y)$ in the Street, i.e., consumer x_m from the demand perspective corresponds to consumer $Y = 0$ from the supply perspective in the Street market, as clearly vendors cannot distinguish among consumers and are interested only in the quantity to be sold. Such an observation leads to the conclusion that, while in the DNM vendors in principle face the actual inverse demand function $R_D(x)$ (in the following we shall see that this is not exactly true), in the Street market vendors face the portion of the $R_S(x)$ demand starting from $x = x_m$. In other words, the actual inverse demand function faced by vendors in the Street is a new function obtained through a parallel shift of the original inverse demand function $R_S(x)$ towards the left by x_m ; i.e., for any given x_m , the new inverse demand function is defined by

$$\hat{R}_S(x_m, x) = R_S(x_m + x) \quad \text{for } (x_m, x) \in [0, 1] \times [0, 1 - x_m], \quad (10)$$

Notice that $\hat{R}_S(x_m, \cdot)$ ceases to have constant elasticity, as its absolute value is given by

$$\hat{\varepsilon}_S(x_m, x) = \left| \frac{\frac{\partial}{\partial x} \hat{R}_S(x_m, x) x}{\hat{R}_S(x_m, x)} \right| = \left| \frac{R'_S(x_m + x) x}{R_S(x_m + x)} \right|.$$

Under our assumption in (1), it holds:

$$\hat{\varepsilon}_S(x_m, x) = \left| \frac{-\varepsilon_S(x_m + x)^{-\varepsilon_S - 1} x}{(x_m + x)^{-\varepsilon_S}} \right| = \varepsilon_S \frac{x}{x_m + x}, \quad (11)$$

which clearly depends on both x_m and x .

In order to define a global equilibrium across both markets we must consider that in the Street market the equilibrium must be determined as the intersection point between the “*horizontally shifted*” inverse demand function $\hat{R}_S(x_m, x)$ defined in (10) and, according to the Cournot equilibria discussed in Subsection 3.3, the non-constant elasticity inverse supply function defined as

$$\hat{L}_S(x_m, x) = \frac{c'_S \left(\frac{x}{n_S} \right)}{1 - \frac{\hat{\varepsilon}_S(x_m, x)}{n_S}}, \quad (12)$$

where $\hat{\varepsilon}_S(x_m, x)$ is the elasticity defined in (11). As $\hat{R}_S(x_m, x)$ defined in (10) is the actual inverse demand function faced by vendors in the Street market, in order to guarantee a Cournot equilibrium their inverse supply function must be adapted as well according to (12).

4.2 The equilibrium in the DNM

Under the assumption that $x_m < x_S^*$ and because the (actual) excess demand function in the Street market, $E_S(x) = R_S(x) - p_S^*$, is strictly decreasing in x , necessarily $E_S(x_m) = R_S(x_m) - p_S^* > 0$ must hold, which, by definition of marginal consumer, in turn implies that $E_D(x_m) = R_D(x_m) - p_D^* > 0$ must hold as well. In

other words, any definition of global equilibrium across the two markets must incorporate the property that the DNM market actually is in *disequilibrium*, at least according to the standard notion of equilibrium stating that supply must equal demand. However, no exception actually emerges if one considers that suppliers in the DNM are aware that consumers, beginning from the marginal one, would shift from the DNM to the Street as long as, notwithstanding the lower quality of the product delivered there, thanks to a lower enough price they would get a larger consumer rent. Under the threat posed by the other market, vendors in the DNM revise their profit maximization problem. The Cournot equilibrium in the DNM is reached when their revised assumption about the quantity demanded in their market is compatible with the equilibrium in both markets, so that the following condition is satisfied:

$$E_D(x_m) = R_D(x_m) - p_D^* = E_S(x_m) = R_S(x_m) - p_S^* > 0. \quad (13)$$

$E_D(x_m)$ is the minimum consumer rent that customers must obtain in the DNM in order to prevent them from shifting to the other market. To discuss the equilibrium when both markets are active we thus modify the inverse demand function $R_S(x)$ in the Street as shown in Subsection 4.1 and we change the original inverse demand function $R_D(x)$ by shifting it downward as in (13). Hence, the actual inverse demand function faced by vendors in the DNM is the new inverse demand function obtained through a rigid downward shift of the original inverse demand function $R_D(x)$ by a magnitude corresponding to the minimum consumer rent $E_D(x_m)$ in (13). Under the latter assumption, again, the new inverse demand function in the DNM ceases to have constant elasticity, as the new inverse demand function

$$\hat{R}_D(x_m, x) = R_D(x) - E_D(x_m), \quad (14)$$

which must be interpreted as a function of the only variable x , while $E_D(x_m)$ is a *constant* (it is the minimum consumer rent value in equilibrium), has elasticity given, in absolute value, by

$$\hat{\varepsilon}_D(x_m, x) = \left| \frac{\frac{\partial}{\partial x} \hat{R}_D(x_m, x) x}{\hat{R}_D(x_m, x)} \right| = \left| \frac{R'_D(x) x}{R_D(x) - E_D(x_m)} \right|.$$

As, according to the definition of Cournot equilibrium discussed in Subsection 3.2, we only need to consider the elasticity value on the marginal consumer x_m , we can define $\hat{\varepsilon}_D(x_m, x)|_{x=x_m} = \hat{\varepsilon}_D(x_m)$ as

$$\hat{\varepsilon}_D(x_m) = \left| \frac{R'_D(x_m) x_m}{R_D(x_m) - E_D(x_m)} \right|,$$

which, under the functional form in (1), becomes

$$\hat{\varepsilon}_D(x_m) = \left| \frac{-\varepsilon_D (x_m)^{-\varepsilon_D - 1} x_m}{(x_m)^{-\varepsilon_D} - E_D(x_m)} \right| = \varepsilon_D \frac{(x_m)^{-\varepsilon_D}}{(x_m)^{-\varepsilon_D} - E_D(x_m)}. \quad (15)$$

Therefore, once again the inverse supply in the DNM must be adapted because the oligopolistic vendors face an actual inverse demand function characterized by the elasticity defined in (15). According to the Cournot equilibria discussed in Subsection 3.3, such a non-constant elasticity inverse supply is pointwise defined on $x = x_m$ as

$$\hat{L}_D(x_m) = \frac{c'_D\left(\frac{x_m}{n_D}\right)}{1 - \frac{\hat{\varepsilon}_D(x_m)}{n_D}}, \quad (16)$$

where $\hat{\varepsilon}_D(x_m)$ is the elasticity defined in (15). It remains to determine the value of the minimum consumer rent $E_D(x_m)$ in (13). As it depends on everything at the same time¹⁰ $E_D(x_m)$ is a key element in the following definition of global

¹⁰Specifically, the downward shifted inverse demand function in the DNM, $\hat{R}_D(x_m, x)$ in (14), the inverse supply in the DNM, $\hat{L}_D(x_m)$ in (16), the value of the inverse demand function in the

equilibrium across the markets, which is itself based on the definition of marginal consumer x_m as in Assumption A.2 (iii).

4.3 A definition of equilibrium across the two markets

The pivotal element on which the whole definition of a global equilibrium rests is the minimum consumer rent $E_D(x_m)$ [defined in (13)] enjoyed by the *marginal consumer* x_m [as specified in Assumption A.2(iii)], i.e.,

$$\hat{R}_D(x_m) = \hat{R}_D(x_m, x) \Big|_{x=x_m} = R_D(x_m) - E_D(x_m), \quad (17)$$

must hold in equilibrium.¹¹

To clarify ideas we introduce an abstract definition of equilibrium.

Definition 1 *Consider a population of consumers indexed by $x \in [0, 1]$ who have the opportunity to choose on whether to purchase one unit of a homogeneous illicit good either on the DNM or on the Street market. Each consumer x has a reservation price $R_D(x)$ if she purchases in the DNM and a reservation price $R_S(x)$ if she purchases in the Street market, and all consumers are ordered in the interval $[0, 1]$ so that they have decreasing inverse reservation price functions $R_D(x)$ and $R_S(x)$. Let $E_D(x, p_D) = R_D(x) - p_D$ and $E_S(x, p_S) = R_S(x) - p_S$ be the consumer rent functions in the DNM and in the Street market respectively; p_D and p_S denote the prices that are actually being paid in each market and $E_D(x, p_D) > E_S(x, p_S)$ for values of x close to zero.*

Street market corresponding to the marginal consumer x_m , $R_S(x_m)$, and the equilibrium price in the Street market, p_S^* , itself depending on the modified inverse demand and supply functions in the Street market, $\hat{R}_S(x_m, x)$ in (10) and $\hat{L}_S(x_m, x)$ in (12).

¹¹In fact, the whole definition of the downward shift $\hat{R}_D(x_m, x) = R_D(x) - E_D(x_m)$ of $R_D(x)$ in the DNM defined in (14) as a function of x is not required in our definition, only its value at the marginal consumer x_m , i.e., $\hat{R}_D(x_m)$ according to (17), suffices, as the whole equilibrium rests on the excess demand function $E_D(x_m)$ defined in (13).

We say that the two markets, the DNM and the Street, are in equilibrium if quantities $x_D^* > 0$ and $x_S^* > 0$ and prices $p_D^* > 0$ and $p_S^* > 0$ exist such that the following conditions are satisfied:

- i) $E_D(x_D^*, p_D^*) = E_S(x_D^*, p_S^*) > 0$ and
- ii) $E_S(x_S^*, p_S^*) = 0$.

Condition i) establishes the existence of a marginal consumer x_D^* who is indifferent between going to the DNM or to the Street market, as in both markets she earns the same (strictly positive) consumer surplus; condition ii) states that all consumers indexed to the right of x_D^* , i.e., $x \in (x_D^*, x_D^* + x_S^*]$, go to the Street market, where the price p_S^* satisfies the standard definition of equilibrium (supply equals demand), i.e., $R_S(x_S^*) = p_S^*$. Moreover, the DNM is in disequilibrium according to the standard definition, as $R_D(x_D^*) > p_D^*$, where p_D^* is the price at which the illicit good is being sold in the DNM.¹²

The next proposition provides a characterization of the equilibrium introduced in Definition 1 when the supply structures in both markets are oligopolistic in the sense of Cournot according to the discussion in Subsection 3.2. We still make no assumptions on the demand structures other than the basic properties recalled in Definition 1, however we will assume that there is one *unique* equilibrium.

Proposition 1 *Suppose that there are n_D identical vendors in the DNM, each facing the same marginal cost $c'_D\left(\frac{Y}{n_D}\right)$, and n_S identical vendors in the Street market, each facing the same marginal cost $c'_S\left(\frac{Y}{n_S}\right)$, where Y is the total quantity sold in each market. All vendors behave oligopolistically and they are eager to sell*

¹²Clearly a similar approach could be applied to the opposite case, in which $E_D(x, p_D) < E_S(x, p_S)$ for values of x close to zero. In this alternative scenario, a suitable minimum consumer rent would be given to consumers in the Street.

at Cournot-type equilibrium prices satisfying (6) and (7), that is,

$$p_D = R_D(x) = L_D(x) = \frac{c'_D\left(\frac{x}{n_D}\right)}{1 - \frac{\varepsilon_D(x)}{n_D}}$$

$$p_S = R_S(x) = L_S(x) = \frac{c'_S\left(\frac{x}{n_S}\right)}{1 - \frac{\varepsilon_S(x)}{n_S}},$$

where $\varepsilon_D(x)$ and $\varepsilon_S(x)$ denote the absolute values of elasticities of the inverse demand functions $R_D(x)$ and $R_S(x)$ in the DNM and in the Street market respectively satisfying Assumption A.1.

Then, conditions i) and ii) of Definition 1 are equivalent to the following system of two equations in the unknowns x_m and x :

$$\begin{cases} R_D(x_m) - \hat{L}_D(x_m) = R_S(x_m) - \hat{L}_S(x_m, x) \\ \hat{R}_S(x_m, x) = \hat{L}_S(x_m, x), \end{cases} \quad (18)$$

where $\hat{L}_D(x_m)$ is defined by (16) together with the elasticity $\hat{\varepsilon}_D(x_m)$ defined in (15), $\hat{R}_S(x_m, x)$ is defined by (10) and $\hat{L}_S(x_m, x)$ is defined by (12) together with the elasticity $\hat{\varepsilon}_S(x_m, x)$ defined in (11). The solution (x_m^*, x_S^*) of system (18) represents the total quantity sold in the DNM, corresponding to the marginal consumer $x_m^* = x_D^*$, and the total quantity sold in the Street, x_S^* . The sum $x = x_m^* + x_S^*$ corresponds to the total quantity sold in both markets. The equilibrium prices are $p_D^* = \hat{L}_D(x_m^*)$ in the DNM and $p_S^* = \hat{L}_S(x_m^*, x_S^*) = \hat{R}_S(x_m^*, x_S^*)$ in the Street.

For a proof see the Appendix.

For inverse demand functions having constant elasticities, i.e., given by (1), and affine marginal costs¹³ faced by vendors, condition (18) in Proposition 1 can be further specified so to obtain a numerically computable equilibrium.

¹³Affine marginal costs of the form $c'\left(\frac{x}{n}\right) = a + b\frac{x}{n}$ correspond to quadratic total costs of the form $c\left(\frac{x}{n}\right) = a\frac{x}{n} + \frac{b}{2}\left(\frac{x}{n}\right)^2 + d$, which are increasing and convex whenever $a, b > 0$.

Corollary 1 *Assume that the inverse demand functions have constant elasticity and in both markets vendors face affine marginal costs; specifically, the inverse demand curves have the form $R_D(x) = \theta_D x^{-\varepsilon_D}$ and $R_S(x) = \theta_S x^{-\varepsilon_S}$ where $\theta_D = \theta_S = 1$ and $\varepsilon_D, \varepsilon_S$ satisfy Assumption A.1, while marginal costs are given by $c'_D\left(\frac{x}{n_D}\right) = a_D + b_D \frac{x}{n_D}$ and $c'_S\left(\frac{x}{n_S}\right) = a_S + b_S \frac{x}{n_S}$, with non-negative parameters a_D, a_S, b_D, b_S . Then, there exists one unique equilibrium characterized by the following specification of system (18):*

$$\begin{cases} \left(1 - \frac{\varepsilon_D}{n_D}\right) (x_m)^{-\varepsilon_D} - \frac{b_D}{n_D} x_m - a_D = (x_m)^{-\varepsilon_S} - \frac{a_S n_S + b_S x}{n_S - \varepsilon_S \frac{x}{x_m + x}} \\ (x_m + x)^{-\varepsilon_S} = \frac{a_S n_S + b_S x}{n_S - \varepsilon_S \frac{x}{x_m + x}} \end{cases} \quad (19)$$

For a proof see the Appendix.

Remark 1 *Note that the LHS in the first equation of system (19) contains exactly the two sides that would define the Cournot oligopolistic equilibrium for the marginal consumer x_m in the DNM according to equation (2): the inverse demand $R_D(x_m) = (x_m)^{-\varepsilon_D}$, multiplied by the term $\left(1 - \frac{\varepsilon_D}{n_D}\right)$,¹⁴ and the vendors' marginal cost $c'_D\left(\frac{x_m}{n_D}\right) = a_D + b_D \frac{x_m}{n_D}$. However, such a Cournot equilibrium in the DNM requires these two terms to be equal; here, instead, the equilibrium in the DNM is characterized by a strictly positive minimum consumer rent $E_D(x_m) = \left(1 - \frac{\varepsilon_D}{n_D}\right) R_D(x_m) - c'_D\left(\frac{x_m}{n_D}\right) > 0$, so that $\left(1 - \frac{\varepsilon_D}{n_D}\right) R_D(x_m) \neq c'_D\left(\frac{x_m}{n_D}\right)$ and this market turns out to be in disequilibrium according to the standard concept.*

¹⁴Note that the term $\left(1 - \frac{\varepsilon_D}{n_D}\right)$ is the reciprocal of the constant markup typical of an oligopolistic market with a constant elasticity inverse demand.

5 Illustrative example and policy implications

This chapter will provide just one of many possible scenarios to illustrate the mechanics of the model. Since our model shows two markets in equilibrium, we would like to study how policymakers can influence the markets to decrease the equilibrium number of sales. In line with other research, we believe that the arrival of DNM increases the number of drug users.¹⁵ Therefore, shifting online drug dealers back to the streets or making them reconsider their online career path could make a difference in the war on drugs.

As previously stated, the drug market is expanding to a different domain, now present on the Street and online Dark Net market. This expansion has changed the nature of drug dealers and drug users. To succeed on the DNM, the platform’s participants should have a specific level of digital literacy. This fact implies acquiring new knowledge by Street dealers or forming a new drug vendor type: the technology-educated dealer. According to an online survey of about 4000 individuals, 38% had completed a university degree. For example, the founder of the DNM “Silk Road” had a master’s degree in material science and engineering, and the founder of the DNM “Silk Road-2” did his internship in SpaceX (Ladegaard, 2019). In our paper, we do not consider the knowledge path of drug dealers since it is a necessary condition to start online trading on DNM.

The goal of this chapter is not to analyze a drug dealership per se but consider the determinants of the illegal drug trade. Moreover, we analyze the ways of shifting DNM dealers back to the streets since it is more difficult to apprehend online criminals (Bahamazava and Nanda, 2022) than offline counterparts.

Let us consider two types of cocaine markets: Street market and DNM. The population of vendors is characterized by an affine marginal cost function in

¹⁵A curious reader can turn to (Aldridge et al., 2018) to examine this issue more carefully.

each market:

$$c' \left(\frac{x}{n} \right) = a + b \frac{x}{n}, \quad (20)$$

where, in our case, a represents delivery¹⁶ costs that a vendor bears while selling drugs, b decomposes to b_i - insurance cost to avoid violence and b_s - scam cost¹⁷ to avoid deception from other participants in the drug selling business, that is $b = b_i + b_s$, and n is a number of vendors in each market.

Let us specify affine marginal costs in the two markets as:

$$c'_S \left(\frac{x}{1} \right) = 0.6 + 20 \left(\frac{x}{1} \right) \quad \text{and} \quad c'_D \left(\frac{x}{15} \right) = 1 + 16 \left(\frac{x}{15} \right). \quad (21)$$

We set n_S to 1 and n_D to 15 to stress the more competitive nature of the cocaine trade on the DNM. We assume that the delivery costs a are higher on the DNM than on the Street market since the DNM vendors send drugs to each end user while on the Street market delivery occurs only within drug cartel network. As for the b costs, the insurance costs b_i are less on the DNM since vendors do not meet with the buyers. The scam costs are more substantial on the DNM market than on the Street market because of the involvement of potentially anonymous intermediaries (platform's administrators). We set $b_{S_i} = 18$ and $b_{S_s} = 2$ on the Street market, while on the DNM, $b_{D_i} = 2$ and $b_{D_s} = 14$ ¹⁸.

Using (21), the equilibrium condition (19) becomes:

$$\left\{ \begin{array}{l} \left(1 - \frac{0.90}{15} \right) (x_m)^{-0.90} - \frac{16}{15} x_m - 1 = (x_m)^{-0.84} - \frac{0.6 \cdot 1 + 20 \cdot x}{1 - 0.84 \frac{x}{x_m + x}} \\ (x_m + x)^{-0.84} = \frac{0.6 \cdot 1 + 20 \cdot x}{1 - 0.84 \frac{x}{x_m + x}} \end{array} \right. \quad (22)$$

¹⁶Delivery was the most common risk identified by vendors (Aldridge and Askew, 2017).

¹⁷The increased occurrence of real or potential scams could disrupt the trust in the DNM ecosystem based on profit, ideology, and blockchain.

¹⁸Our example takes hypothetical values to illustrate the mechanics of the model.

```

1 function [x, res, niter]=newtons(F,J,x0,toll,imax)
2 niter=0;
3 err=toll +1;
4 x=x0;
5 while err ≥ toll & niter < imax
6 JF=J(x);
7 FF=F(x);
8 Δ = -JF\FF;
9 x= x + Δ;
10 err =norm(Δ,inf);
11 niter=niter +1;
12 end
13 res=norm(F(x),inf);
14 if (niter==imax & err > toll)
15 fprintf(['\nll method doesnt converge in max',...
16 'number of iterations.The last iteration\n',...
17 'calculated has residual equal to %e.\n'],res);
18 else
19 fprintf(['\nll method converges in %i iterations ', ...
20 'with a residual equl to %e.\n'], niter, res)
21 end

```

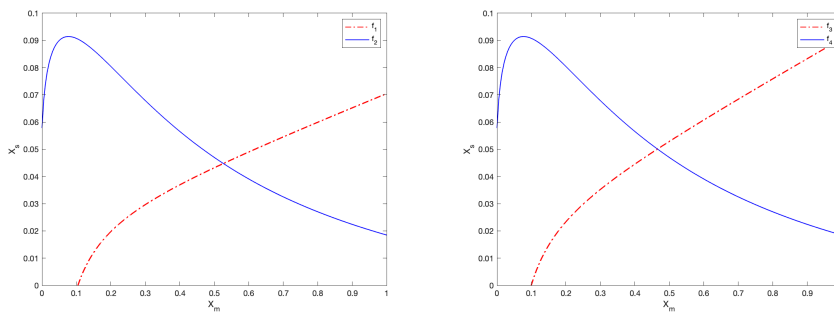
FIGURE 1: Matlab software code for Newton-Raphson method

where ε_S is equal to 0.84 and ε_D is equal to 0.90.¹⁹ To solve the system of equations (22) numerically in Matlab software, we utilized the Newton-Raphson method (Figure 1). We decided to follow this approach since this numerical method is the best-known iteration approach to find a real or complex root of a differentiable function (Denis and Rose, 2006). In the code (Figure 1), F is the function in which we defined the system of equations (22), J is the function in which we defined the Jacobian matrix of the system (22), x_0 is our initial guess for x_m and x , toll is tolerance, and imax is the maximum number of iterations. In the following examples, functions f_1 to f_8 present contour plots of each of the two equations in the system (22) for different values of the parameters a_D and b_D .

We set the tolerance to 10^{-6} , and the max number of iterations - to

¹⁹Consistent with empirical evidence (Payne et al., 2020) and our intuition discussed in Section 3.2 in the Footnote 5.

1000. Solving (22) for the interval $(0, 1)$, we obtain the solution (x_m, x) , where x_m represents the equilibrium quantity sold on the DNM, while x denotes the equilibrium quantity sold on the Street market. In our example, $(0.5263, 0.0448)$ is a unique solution on the interval $(0, 1)$ (Figure 2a). The total number of cocaine sold on both markets is $x_m + x$, which is equal to 0.5711.



(A) $f_1 = 0$ and $f_2 = 0$ for initial equilibrium

(B) $f_3 = 0$ and $f_4 = 0$ after increasing scam costs

FIGURE 2

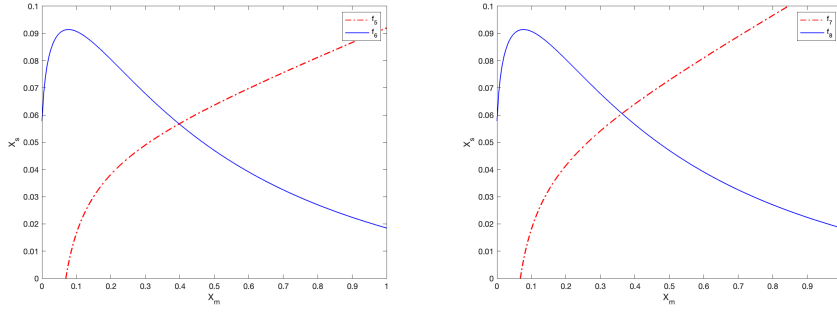
Increasing the scam cost b_{Ds} would deter occasional consumers from buying cocaine on the DNM and “shift” some of the usual buyers towards the Street market. Increasing scam costs by 50% (from 14 to 21) on DNM gives us a unique equilibrium solution (x_m, x) on the interval $(0, 1)$ which is equal to $(0.4656, 0.0501)$, respectively (Figure 2b). The new total number of cocaine sold on both markets is 0.5157. The 50% increase of scam costs would diminish the quantity of cocaine sold by 11.53% on DNM and decrease the total number by 9.7%. In the long term perspective (not considered in the model presented in the paper), increasing the scam cost b_{Ds} would increase the cost of doing drug business for DNM vendors shifting some sellers to the Street market or discouraging some of them to participate in the drug business at all.

The announcements related to DNM activities affected DNM’s participants’ behavior (Ladegaard, 2018); therefore, it is possible to influence the illegal

drug trade through these announcements. Application of the proposed policy of increased scam cost would “shift” some vendors toward the Street market and deter occasional traders. Considering the starting presumption of the cocaine Street market being easier to control than DNM by the law enforcement agencies, our example showed one of the possible ways of how to influence the illegal drug trade. The DNM sellers react to announcements related to DarkNet activity. More announcements could be made to create panic around the exit scam’s scheme. The speculations announcing the “insider’s information” about prepared exit scams schemes could increase costs in the cost and benefit analysis of doing business on DNMs. Vendors having the information about the possible exit scams on the specific DNMs would feel reluctant to continue their drug business as they risk losing not only cryptocurrencies linked to the trades but all the information about their customers and ongoing deals. Therefore, some of the sellers would cease their online drug business, perhaps shifting to the Street market. If it is easier to arrest criminals on the Street than on the DNM, our approach could be used to diminish the illegal drug trade.

Another proposed policy implication is to increase delivery costs a_D to “shift” DNM’s vendors to the Street market or make them refrain from the drug business. Increasing delivery costs on DNM by 50% (from 1 to 1.50) would give us a unique solution (0.3979, 0.0568) on the interval (0,1) for equilibrium quantity sold on DNM and Street, respectively. Due to increased delivery costs by 50% on DNM, the quantity of cocaine sold on DNM would diminish by 24.4%, and the total cocaine quantity would decrease by 20.38% (Figure 3a).

For example, this policy could be introduced by announcing the possibility to trace DNM orders. Since ordinary postal services handle all the DNM purchases, it is feasible to trace them. With the possibility of being traced back, vendors would be forced to use different techniques while packaging to diminish



(A) Contour plot for $f_5 = 0$ and $f_6 = 0$ after rising delivery costs (B) Contour plot for $f_7 = 0$ and $f_8 = 0$ after combined policy

FIGURE 3

the probability of orders being intercepted. This new packaging policy, in turn, would increase the cost of the DNM drug business. Due to increased costs of doing business and increased probability of being apprehended, some vendors would be reluctant to continue their online drug business. Some vendors, and according to the new equilibrium yielded by the new vendors' strategy, some consumers would return to the Street market, while occasional users would cease their online business. Note that this policy is more effective than the “increasing scam costs” policy in “shifting” drugs vendors from DNM to the Street market.

In the case of using both policies together, the unique solution would be $(0.3628, 0.0606)$ for DNM and Street market, respectively (Figure 3b). The DNM cocaine sale would diminish by 31.07%, and the total quantity sold on both markets would decrease by 25.86%.

Our numerical example, consistent with other research (Martin et al., 2020), shows that some DNM vendors are reluctant to become Street dealers. Therefore, it is essential to research and explore the ways to influence DNM drug dealers. In this example, we consider delivery, insurance, and scam costs as factors affecting drug dealers' marginal cost function. We show that impacting these factors through media sources may shift or discourage selling drugs through DNM.

We particularly stress the necessity of utilizing the Internet and social media for these announcements since we believe DNM drug dealers are more susceptible to these kinds of announcements than Street dealers.

6 Conclusion

In this paper, we modeled the internal functioning of two drug markets, namely, the Street and the DNM, and considered their interactions. Taking into account the relationship between the two markets seems of growing relevance in a framework that in recent years was characterized by the resilience and expansion of the DNM. Still, the Street market remains well active. Policy interventions in the field have been traditionally designed concerning the Street market, which, being active for a longer time, has been more thoroughly studied and has become more familiar to the police and the responsible authorities. The DNM has only recently attracted researchers' and public authorities' attention.

We stress the lower risk that buyers face in the DNM as a factor that translates into a higher perceived quality of supply therein. The Street market, however, can compete in terms of price, thus attracting the demand of consumers with a lower willingness to pay and inducing as a response a less than full exploitation of the quality advantage by vendors in the DNM. Our model provides a simple basic framework for describing the equilibrium in the two markets and for discussing policy interventions in the new scenario, in which both the old and the new forms of drug commerce are present.

DNM requires a different approach from Law Enforcement Agencies than the Street market for numerous reasons, including the diverse nature of drug dealers. Underlining the distinction between DNM and Street, we present possible determinants of the drug trade and ways to influence these determinants.

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Appendix

Proof of Proposition 1. Note that in the first equation of (18) the whole LHS and the first term in the RHS depend only on the marginal consumer x_m ; only the second term in the RHS, in addition to depending on x_m , also depends on x . This is because the equilibrium point x_m is sufficient to define the Cournot equilibrium in the DNM as equality by the vendors' inverse supply $\hat{L}_D(x_m)$ defined in (16) and the downward shifted inverse demand curve $\hat{R}_D(x_m)$ defined in (17); otherwise, the RHS depends on the equilibrium price in the Street market, which, according to the second equation in (18), is defined as the intersection point between the *whole* leftward shifted inverse demand curve $\hat{R}_S(x_m, x)$ defined in (10) (by “whole” we mean that, in addition to depending on x_m , it also depends on the variable x over the whole interval $[0, 1]$) and the *whole* inverse supply $\hat{L}_S(x_m, x)$ defined in (12) (again, by “whole” we mean that, in addition to depending on x_m , it also depends on the variable x over the whole interval $[0, 1]$).

The first equation in (18) characterizes the marginal consumer x_m by equating the strictly positive minimum consumer rent $E_D(x_m)$ (the consumer sur-

plus in the LHS) required to keep the marginal consumer in the DNM to the largest possible consumer surplus $E_S(x_m, x)$ (the RHS) earned in the Street market by the same marginal consumer. Both $E_D(x_m) = R_D(x_m) - \hat{L}_D(x_m)$ and $E_S(x_m, x) = R_S(x_m) - \hat{L}_S(x_m, x) = \hat{R}_S(x_m, x) - \hat{L}_S(x_m, x)$ depend on the inverse supply value $\hat{L}_D(x_m)$ defined in (16) and the inverse supply function $\hat{L}_S(x_m, x)$ defined in (12), themselves depending on the elasticities of the inverse demand functions actually faced by the oligopolistic vendors in both markets, i.e., the value of the downward shifted inverse demand curve $\hat{R}_D(x_m)$ defined in (17) at x_m and the leftward shifted inverse demand curve $\hat{R}_S(x_m, x)$ defined in (10) respectively. For a given value of x_m , the second equation in (18) establishes the standard equilibrium in the Street market corresponding to the quantity x_S^* that equates the inverse demand function [the LHS, $\hat{R}_S(x_m, x)$] to the inverse supply [the RHS, $\hat{L}_S(x_m, x)$]. ■

Proof of Corollary 1. Clearly, $R_S(x_m) = (x_m)^{-\varepsilon_S}$ and $\hat{R}_S(x_m, x) = (x_m + x)^{-\varepsilon_S}$ according to definition (10). Under the assumption of affine marginal cost in the Street market, $c'_S\left(\frac{x}{n_S}\right) = a_S + b_S \frac{x}{n_S}$ and noting that, according to (11), the elasticity of the (leftward shifted) inverse demand curve in the Street market is $\hat{\varepsilon}_S(x_m, x) = \varepsilon_S \frac{x}{x_m + x}$, according to definition (12) $\hat{L}_S(x_m, x) = \frac{a_S n_S + b_S x}{n_S - \varepsilon_S \frac{x}{x_m + x}}$. Therefore, the RHS of the first equation and both sides in the second equation of (19) are equivalent to the corresponding sides in system (18).

The explicit form of the LHS in the first equation, corresponding to $E_D(x_m) = R_D(x_m) - \hat{L}_D(x_m) = (x_m)^{-\varepsilon_D} - \hat{L}_D(x_m)$ (the minimum consumer rent for consumers in the DNM) in the first equation in (18) is a bit trickier to obtain because of the expression of $\hat{L}_D(x_m)$ according to (16). Recall that the elasticity of the inverse demand function in the DNM is given by (15), so that, at the value

$x = x_m$, it holds

$$\hat{\varepsilon}_D(x_m) = \varepsilon_D \frac{(x_m)^{-\varepsilon_D}}{(x_m)^{-\varepsilon_D} - E_D(x_m)}, \quad (23)$$

where, under the assumptions of constant elasticity inverse demand function, $R_D(x_m) = (x_m)^{-\varepsilon_D}$, and affine marginal cost in the DNM, $c'_D\left(\frac{x}{n_D}\right) = a_D + b_D \frac{x}{n_D}$, the minimum consumer rent $E_D(x_m)$ defined in (13) after some algebra becomes:

$$\begin{aligned} E_D(x_m) &= R_D(x_m) - \hat{L}_D(x_m) = (x_m)^{-\varepsilon_D} - \frac{c'_D\left(\frac{x_m}{n_D}\right)}{1 - \frac{\hat{\varepsilon}_D(x_m)}{n_D}} = (x_m)^{-\varepsilon_D} - \frac{a_D + b_D \frac{x_m}{n_D}}{1 - \frac{\hat{\varepsilon}_D(x_m)}{n_D}} \\ &= (x_m)^{-\varepsilon_D} - \frac{a_D n_D + b_D x_m}{n_D - \hat{\varepsilon}_D(x_m)}. \end{aligned}$$

Substituting the last expression into (23) yields

$$\begin{aligned} \hat{\varepsilon}_D(x_m) &= \varepsilon_D \frac{(x_m)^{-\varepsilon_D}}{(x_m)^{-\varepsilon_D} - E_D(x_m)} = \varepsilon_D \frac{(x_m)^{-\varepsilon_D}}{(x_m)^{-\varepsilon_D} - (x_m)^{-\varepsilon_D} + \frac{a_D n_D + b_D x_m}{n_D - \hat{\varepsilon}_D(x_m)}} \\ &= \varepsilon_D \frac{(x_m)^{-\varepsilon_D}}{a_D n_D + b_D x_m} [n_D - \hat{\varepsilon}_D(x_m)], \end{aligned}$$

which is equivalent to

$$\left[1 + \frac{\varepsilon_D (x_m)^{-\varepsilon_D}}{a_D n_D + b_D x_m} \right] \hat{\varepsilon}_D(x_m) = \frac{\varepsilon_D n_D (x_m)^{-\varepsilon_D}}{a_D n_D + b_D x_m} \iff \hat{\varepsilon}_D(x_m) = \frac{\frac{\varepsilon_D n_D (x_m)^{-\varepsilon_D}}{a_D n_D + b_D x_m}}{1 + \frac{\varepsilon_D (x_m)^{-\varepsilon_D}}{a_D n_D + b_D x_m}},$$

that is,

$$\hat{\varepsilon}_D(x_m) = \frac{\varepsilon_D n_D (x_m)^{-\varepsilon_D}}{a_D n_D + b_D x_m + \varepsilon_D (x_m)^{-\varepsilon_D}}. \quad (24)$$

By replacing (24) into the definition of $\hat{L}_D(x_m)$ according to (16) we get

the second term in the LHS of (19):

$$\begin{aligned}
\hat{L}_D(x_m) &= \frac{c'_D \left(\frac{x_m}{n_D} \right)}{1 - \frac{\hat{\varepsilon}_D(x_m)}{n_D}} = \frac{a_D n_D + b_D x_m}{n_D - \hat{\varepsilon}_D(x_m)} = \frac{a_D n_D + b_D x_m}{n_D - \frac{\varepsilon_D n_D (x_m)^{-\varepsilon_D}}{a_D n_D + b_D x_m + \varepsilon_D (x_m)^{-\varepsilon_D}}} \\
&= \frac{(a_D n_D + b_D x_m) [a_D n_D + b_D x_m + \varepsilon_D (x_m)^{-\varepsilon_D}]}{n_D (a_D n_D + b_D x_m) + \varepsilon_D n_D (x_m)^{-\varepsilon_D} - \varepsilon_D n_D (x_m)^{-\varepsilon_D}} \\
&= \frac{(a_D n_D + b_D x_m) [a_D n_D + b_D x_m + \varepsilon_D (x_m)^{-\varepsilon_D}]}{n_D (a_D n_D + b_D x_m)} \\
&= \frac{a_D n_D + b_D x_m + \varepsilon_D (x_m)^{-\varepsilon_D}}{n_D} \\
&= a_D + \frac{b_D}{n_D} x_m + \frac{\varepsilon_D}{n_D} (x_m)^{-\varepsilon_D}. \tag{25}
\end{aligned}$$

Using the expression of $\hat{L}_D(x_m)$ in (25) in the LHS of the first equation in (19) we get

$$\begin{aligned}
E_D(x_m) &= R_D(x_m) - \hat{L}_D(x_m) = (x_m)^{-\varepsilon_D} - a_D - \frac{b_D}{n_D} x_m - \frac{\varepsilon_D}{n_D} (x_m)^{-\varepsilon_D} \\
&= \left(1 - \frac{\varepsilon_D}{n_D} \right) (x_m)^{-\varepsilon_D} - \frac{b_D}{n_D} x_m - a_D,
\end{aligned}$$

and the proof is complete. ■