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Variability and substantiality. Kurd Lasswitz, the Marburg school and the neo-Kantian historiography of science

Marco Giovanelli

Università degli Studi di Torino, Department of Philosophy and Educational Sciences, Via S. Ottavio, 20 10124 Torino, Italy

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ABSTRACT

A trained physicist, Kurd Lasswitz (1848–1910) is best known as a novelist, the father of modern German science fiction, and as a historian of science, the initiator of the modern historiography of atomism. In the late 19th century, Lasswitz engaged in an intense dialogue with the emerging Marburg school of neo-Kantianism, contributing to shaping most of its defining tenets. By the end of the decade, this research had grown into a two-volume *Geschichte der Atomistik* (1890), which remains the most successful example of neo-Kantian historiography of science. Lasswitz combined attention to historical detail with the search for the intellectual tools (*Denkmittel*) without which the 'fact of science' would be impossible. In particular, Lasswitz regarded Huygens' kinetic atomism as a historical model of a successful scientific theory, shaped by the interplay of two conceptual tools: (a) substantiality, the requirement for identity of the subject of motion through time, which found its scientific expression in the extensive atom; (b) variability, the intensive tendency to continue in an instant, which found its conceptual fixation in the notion of 'differential'. By raising the problem of individuality in physics, Lasswitz offers a unique perspective on the utilization of the history of science in 19th-century neo-Kantian thought.

0. Introduction

Kurd Lasswitz (1848-1910) studied mathematics and physics in Berlin and Breslau, where he had the opportunity to attend lectures by the philosopher Wilhelm Dilthey, who would become one of his major academic sponsors (Azzouni, 2009). He completed his physics dissertation on the formation of water droplets in 1873 (Lasswitz, 1873, 1874b). The thesis was dedicated to his advisor, Oskar Emil Meyer (1877), one of the leading proponents of the kinetic theory of gases.[1] In the 1870s, the latter had proven successful in explaining observable macroscopic properties of gases, such as viscosity or diffusion, by resorting solely to the motions and collisions of atoms. However, the theory was far from achieving consensus. A widespread positivistic denial of or at least agnosticism about the existence of atoms persisted. Meanwhile, others explored alternative theories of matter aimed at avoiding the introduction of atoms as 'unexplained explainers'. In German philosophical circles, dynamical theories remained popular, positing point-like atoms interacting at a distance through repulsive and attractive forces (see Lange, 1873–75, 2:192f.). British physicists were suspicious of these obscure non-local interactions and favored a kinetic continuum theory of matter, in which atoms were merely stable 'vortices' in an ethereal fluid (see Kragh, 2002). Lasswitz appeared to have been convinced early on that all these attempts were ultimately flawed on epistemological, rather than empirical, grounds (Lasswitz, 1873, 78).

After securing a position as a high school teacher in Gotha, Lasswitz engaged in tireless philosophical advocacy for kinetic atomism (Lasswitz, 1878a, 1879b) from a broadly Kantian point of view (Lasswitz, 1883). Lasswitz argued that continuous theories of matter were all incapable of ensuring the identifiability of matter parcels over time without surreptitiously introducing indestructible atoms (Lasswitz, 1879b, 286). He supported this theoretical defense of modern kinetic atomism by embarking on a systematic investigation of its early history and subsequent decline in the 18th century (Lasswitz, 1874a, 1879a, 1882, 1884a). Moreover, he carved out time to develop his literary pursuits, writing poems and short stories (see, e.g., Lasswitz, 1878b).

From 1883 to 1895, Lasswitz also began acting as a reviewer of the latest scientific and philosophical works for several journals.[2] Through this activity, Lasswitz (1884b, 1884c, 1885a, 1887b, 1887c) established contact with a group of scholars centered around Hermann

E-mail address: marco.giovanelli@unito.it.

[1] See Wolff (1994).
[2] For a comprehensive bibliography, see Roob (1981).
[3] See Beiser (2018)
[4] See Giovanelli (2016)

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Cohen,[3] a Marburg professor of philosophy who had just published his controversial[4] work *Das Princip der Infinitesimal-Methode* (Cohen, 1883). Lasswitz (1885a, 1885b) realized that the interpretation of the infinitesimal calculus proposed in the book aligned with his previous work on the foundation of physics (Lasswitz, 1878a). This unexpected ally was welcomed with open arms in Marburg. Cohen's right-hand man, Paul Natorp, recently appointed as the editor of the *Philosophische Monatshefte* requested Lasswitz's regular collaboration with the journal. Alongside the *Vierteljahrsschrift für wissenschaftliche Philosophie*, edited by Richard Avenarius, it became Lasswitz's primary platform for publication.

Building on Cohen's ideas, Lasswitz began to forge his own philosophical perspective (Lasswitz, 1888a). The task of philosophy is to delve into the past of physics, searching for the intellectual tools or *Denkmittel*[5] that are indispensable for distinguishing an objective nature from the flux of our subjective experiences. If ancient metaphysics was dominated by the thought-instrument of substantiality, modern science, as Cohen intuited, introduced a new thought-instrument of variability with the discovery of the infinitesimal method (Lasswitz, 1888a). Lasswitz wove this somewhat half-baked philosophy into a series of papers on early modern theories of matter (Lasswitz, 1888b, 1889a, 1889b). Lasswitz's decade-long historiographical research culminated in the comprehensive two-volume *Geschichte der Atomistik* (Lasswitz, 1890), spanning nearly a thousand pages, published in 1890. The book heralded Christiaan Huygens as the pivotal figure in modern science. In Huygens' kinetic atomism, the two *Denkmittel* work together exemplarily. The atom serves as the scientific expression of thought-instrument of 'substantiality', ensuring the individual identity of the subject of motion over time; the thought-instrument of *variability* enables the determination of continuous changes in the distribution of motion among invariable atoms as regulated by the principle of mechanics, namely the conservation of kinetic energy and momentum.

The *Geschichte der Atomistik* was well-received. In the 1890s, Dilthey recommended Lasswitz for philosophy chairs in Breslau, Bonn, and Würzburg (see Azzouni, 2009, 66). He also made it to the final round for a professor position in Marburg, which Natorp eventually filled in 1893 (Cohen to Bosse, Jul. 22, 1893; Holzhey, 1986, Vol. 2, Doc. 498). However, Lasswitz ultimately never managed to secure an academic post. As the general editor of Kant's *Akademie Ausgabe*, Dilthey selected Lasswitz as the editor for the pre-critical writings. Yet, the editorial work conflicted with Lasswitz's ambitions as an author of future-themed novels (Lasswitz, 1897) and philosophical essays (Lasswitz, 1900). Lasswitz's efforts to combine Fechner's panpsychism with Kantianism progressively distanced him from the Marburg school. He passed away in 1910, when Cassirer's (1910) and Natorp's (1910) major monographs appeared in print. Today, Lasswitz is remembered as a pioneering historian of atomism (Lüthy, 2003) and, above all, as the father of German science fiction (Willmann, 2018), with a standing akin to that of Jules Verne in France and H.G. Wells in the English-speaking world. However, his role as a 'non-resident member' of the so-called Marburg school of neo-Kantianism has been largely forgotten. Lasswitz's name is barely mentioned in the otherwise well-informed historical literature on neo-Kantianism (Ferrari, 1997; Köhnke, 1986), or more specifically, on its Marburg variant (Dussort, 1963; Ferrari, 1988). By exploring the years from 1885 to 1895 in Lasswitz's career, this paper aims to show that this neglect is unwarranted. Lasswitz was one of the few scholars who embraced Cohen's controversial interpretation of the infinitesimal method and incorporated it into his own historical work. In doing so, Lasswitz produced the most successful example of neo-Kantian historiography of science. *Geschichte der Atomistik* remained the benchmark in atomism studies for decades (see Weyl, 1927, 161, Cassirer, 1929, 547ff., Bachelard, 1933, 9f.), in contrast to Cohen's *Das Princip der Infinitesimal-Methode*, which never acquired a similar status in the historiography of mathematics.

In this sense, Lasswitz should be recognized as one of the most important forerunners of what we now call 'history and philosophy of science' (see, *e.g.*, Schickore, 2011). Lasswitz's neo-Kantian approach to the history of science might appear too 'on the nose'; however, the goal of this paper is to show that, precisely for this reason, Lasswitz's work unwittingly exposes the difficulties in which the neo-Kantian use of the history of science remains entangled. As Cassirer (1929) pointed out, celebrating his legacy in the late 1920s, Lasswitz not only provided "an excellent exposition" of Huygens' kinetic atomism, but attempted a "critical justification, or 'transcendental deduction' of it" (Cassirer, 1929, 547; tr. 1957, 469). Only by introducing impenetrable and indestructible atoms can one ensure the most fundamental characteristic of matter, the individuality and trans-temporal identifiability of its parts. Without the latter, Lasswitz argued, no sensible theory of matter is possible. However, as Cassirer repeatedly pointed out, the history of 20th-century physics questioned Lasswitz's conclusion at his core (Cassirer, 1929, 547-562). In particular, modern quantum mechanics had to renounce to the very idea of individual distinguishable particles (Cassirer, 1936, sec. V.2). Paradoxically, the neo-Kantian historiography of science succeeds when it fails, as an alleged 'necessary condition for the possibility of science' is ultimately revealed to be dispensable.

1. Lasswitz and the early Cohen's group

1.1. Lasswitz and Cohen's historical work on the infinitesimal method

At the close of 1883, Georg Cantor (the father of modern set theory) began a correspondence with Lasswitz regarding a paper by the latter on Giordano Bruno's atomism (Lasswitz, 1884a). In passing, Cantor revealed that he had just submitted a critical review of Cohen's (1883) book on the history of the infinitesimal method to the *Deutsche Literaturzeitung* (Cantor, 1884). Cantor's criticisms unintentionally drew Lasswitz's attention to Cohen's booklet. Although Lasswitz's response has not been preserved, based on Cantor's impatient reactions, it can be inferred that Lasswitz tried to defend Cohen's stance (Cantor to Lasswitz, Mar. 9, 1884; Eccarius, 1985, Doc. 8). Lasswitz had already reviewed the work of other members of the Marburg group (Lasswitz, 1884b, 1884c). After his correspondence with Cantor, he quickly wrote a review of *Das Princip der Infinitesimal-Methode* and Lasswitz (1885a). Lasswitz did express frustration about "the difficulties of the subject matter and the obscure writing style" (Lasswitz, 1885a, 494). However, unlike Cantor, Lasswitz appreciated Cohen's effort to link the discovery of the concept of the differential dx to Kant's rather obscure claim that 'reality has an intensive magnitude' (B202-207).

Causality and substantiality establish a relation between something that is already 'given' (something that is conserved, that changes, etc.). In order to define this 'something' itself – as Lasswitz summarized Cohen's point of view – one needs a different *Denkmittel*: "This thought-instrument is called *reality*" (Lasswitz, 1885a, 498). In Kantian parlance, the category of 'reality' (*Realität*) does not refer to 'existence' (*Wirklichkeit*); rather, it indicates the quality that defines 'something' as opposed to something else. If space and time are extensive magnitudes, the physical 'reality' extended in space and time is characterized by its intensive magnitude. In Cohen's reading, the 'differential' was invented to provide a mathematical expression for this 'intensive reality'. The 'differential' expresses the intensive tendency to generate extension. Applying this reasoning to the problem of the constitution of matter, Cohen claimed that the "atomic hypothesis" has become unnecessary (Cohen, 1883, §93). Generally, Cohen argued, bulk matter should be 'produced' from the infinitely small as a definite integral (Cohen, 1883, §94), rather than being composed of 'given' extended atoms as an aggregate: "the differential takes the place of the atom" (Lasswitz, 1885a, 500).

[5] For the history of this expression, see Klein (2021); following Klein, I translate '*Denkmittel*' with 'thought-instrument'

As we shall see, Lasswitz found this last point contentious (Section 1.2). However, he was broadly sympathetic to Cohen's attempt to present "the differential (dx) as an example of intensive magnitude" (Lasswitz, 1885a, 501). Still, Lasswitz expressed some concerns about Cohen's use of the notion of 'differential': "One may recognize the infinitesimal reality as a most fruitful thought-instrument, and yet be in doubt as to its relation to that of the mathematician" (Lasswitz, 1885a, 502). Lasswitz, in fact, wondered whether the intensive reality "can really be identified with the mathematical differential or rather with the functional connection with the differential of a second variable," that is, in the relation between dx and dy in the differential quotient (Lasswitz, 1885a, 502). This is particularly evident in the case of the differentials of higher orders (Lasswitz, 1885a, 502f.). The 'quality' or 'reality' "only appears in the differential equation" (Lasswitz, 1885a, 503) not in the isolated 'differential'.

Cohen felt gratified by Lasswitz's positive review, one of the few he had received (Cohen to Lasswitz, Apr. 7, 1886; Holzhey, 1986, Vol. 2, Doc. 8). However, the review did not address a more controversial issue. Lasswitz could not agree with Cohen's arguments against atomism. By reading Cohen's book, one could be led to the conclusion that "atomism could no longer be justified as soon as one agrees to a theory of knowledge that rests on the ground of Kant's critique" (Lasswitz, 1885b, 137f.). Thus, Lasswitz wrote an extensive rebuttal (Lasswitz, 1885b, 139), drawing upon his previous work, in which he insisted on the compatibility between atomism and criticism (Lasswitz, 1878a) and supported kinetic atomism against the vortex theory of atoms (Lasswitz, 1879b, 207).

1.2. Lasswitz's 'justification' of kinetic atomism

Lasswitz's philosophical outlook is quite conventional. The task of physics involves reducing any change of the different types of sensation to the single type of change that allows for mathematical representation, that is, 'motion' or change of position in time. Motion is empirically characterized by visual and muscular sensations, which correspond to two different aspects of motion. (a) the sensation of sight leads to the 'phoronomic' motion, *i.e.*, the displacement of individualized parts against a background (*Ortsveränderung*)[6] (b) the capacity of the body to overcome obstacles in virtue of its motion gives rise to the sensation of impetus (*Andrangsempfindug*), which distinguishes physical motion from purely geometrical change of position.

The task of science is to conceptually fix (*begrifflich fixiren*) these 'sensible' facts, transforming them into measurable quantities and entering into a functional relationship with other measurable quantities. Phoronomy (or kinematics) defines 'motion' as the relation among *extensive quantities*, distances traveled in a time interval as measured from a particular reference frame. However, phoronomic motion has no physical reality; past motion does not exist anymore, future motion does not exist yet, and in an instant, there is no change in position and, therefore, no motion. Thus, Lasswitz argued, "[t]he fact of dynamics requires a new unity; here, to the extensive quantities of phoronomy, an intensive quantity is added" (Lasswitz, 1885b, 140f.). In an instant, where there is no change of position, one can still define the intensive tendency to continue with the same velocity. This tendency represents the physical 'reality' of motion, which manifests subjectively in the sensation of impetus:

> The 'sensation of impetus' to which I sought to trace the formation of the concept of the atom is, in fact, the starting point for the solution of the basic mechanical problem. However, the solution lies in its conceptual formulation as an intensive quantity that signifies a tendency toward motion. Cohen has convincingly demonstrated that the category of reality can express this tendency in connection with the principle of intensive magnitude. I welcome this insight. This intensive magnitude, which is objectified by the thought-instrument [*Denkmittel*] of reality, is what I understood with the sensation of impetus [*Andrangsempfindung*] as 'the real of the motion' [*dem Realen der Bewegung*]. I recognize Cohen's expression as the more appropriate term for what I imperfectly called the 'sensation of impetus'. (Lasswitz, 1885b, 142)

This excerpt outlines the reasons Lasswitz was drawn to Cohen's approach. In his first monograph, Lasswitz (1878a) gave a *psychological* interpretation of the physical content of motion, using the concept of *Andrangsempfindung*. Cohen provided a *conceptual* account of the same issue by resorting to Kant's category of reality and the related notion of 'intensive magnitude'. The notion of 'intensive reality' expresses the problem that Galileo was trying to grasp when he resorted to expressions such as 'the impetus, the moment of descent, the tendency to motion, etc.' (see Cohen, 1883, 51). Leibniz and Newton provided the mathematical solution to the problem with the discovery of the 'differential'.

According to Lasswitz, however, Cohen made the mistake of applying the same reasoning not only to motion, but also to matter itself. As a consequence, Cohen and more generally neo-Kantian scholars rejected atomism and expressed some sympathy for dynamic theories of matter *à la* Boscovich (Cohen, 1883, 94) or *à la* Kant (Stadler, 1883), in which extended bulk matter is grounded in unextended points that have an intensive tendency to expand. However, according to Lasswitz, dynamical theories of matter are doomed to fail. The category of 'reality' and intensive magnitude is necessary to define physical motion; however, it is not the appropriate conceptual tool to define the 'subject' of motion, the 'something' that moves. According to Lasswitz, this problem "cannot be solved through the concept of intensive reality, but only through the concept of substance" (Lasswitz, 1885b, 144).

Kant's theory of matter shares the same shortcomings of all 'plethoric' theories of matter, which conceive matter as a continuum (see Lasswitz, 1879b). Such theories are incapable "of separating out from the general matter a certain part as a body, which acts as a finite quantum 'as a whole'" (Lasswitz, 1885b, 143). The notion of the material 'particle' (that is, a physically individuated parcel of space) was motivated by the necessity of being able to identify the trajectory of the parts of a homogeneous medium. Particles are supposed to be impenetrable since the individuality of identical particles would be lost if they overlap. If they are impenetrable, they must be extended, since inextended particles cannot touch without overlapping. The parts of such extended particles must cohere so that they can be transferred as a whole; otherwise, the problem of individuation would be shifted from the whole to the parts.

As one can see, the conditions that the parts of matter must satisfy to serve as the individual subject of motion seem in fact to describe what is traditionally called an 'atom': "as soon as substance appears as a principle or means of individuating matter, we have atomism" (Lasswitz, 1885b, 144). The impenetrability, extension, cohesion, etc., of atoms should not be confused with the homonymous sensible properties of macroscopic bodies; they are conceptual conditions necessary for the individuation of matter. Lasswitz concluded that, contrary to Cohen's claim, "the concept of the differential does not exhaust the thinking tools of natural science" (Lasswitz, 1885b, 146); the concept of the atom is equally essential: "The differential serves to describe motions, but the moving object, as soon as it appears as an independent whole, requires the concept of the atom"(Lasswitz, 1885b, 177f.).

In this manner, Lasswitz believed to have offered a 'justification' of the kinetic theory of matter, wherein all phenomena are explained by the collision between strictly impenetrable and rigid atoms. In this context, 'collision' should not be confused with the collision of macroscopic bodies; rather, it simply indicates that the subsequent motion of two atoms after their encounter is determined by their motion prior to it.

[6] E.g., the motion of a blue region on a blue background cannot be perceived

When two atoms approach each other within a defined distance (known as the radius of the 'sphere of action'), their direction and velocity change instead of proceeding in a straight line. The collision rules are the conditions of the univocal determinability of the velocities after the collision, if the velocities before the collision are known. Relying on a paper by Lübeck (1877), Lasswitz could show that conservation of momentum $\sum mv = 0$ and kinetic energy $\sum \frac{1}{2}mv^2 =$ const. are necessary and sufficient for this task (see also Meyer, 1874, 1877, 239f.). These are, of course, the laws of elastic collisions; however, this is only an accident; atoms satisfy those laws not because they are elastic, but because their velocities are supposed to be fixed unambiguously.

2. The development of Lasswitz's philosophy

Lasswitz's positive review of Cohen's book was the exception rather than the rule. The young Edmund Husserl was baffled by Lasswitz's "enthusiastic reception" of Cohen's "nonsensical profundity or profound nonsense" (Husserl to Brentano, Dec. 29, 1886; Husserl, 1994, 5). Inevitably, Lasswitz's work attracted the attention of Cohen's circle, as Adolf Elsas, a physicist close to Cohen (1895), confirmed to Lasswitz in private correspondence (Elsas to Lasswitz, Ja 7, 1887; Holzhey, 1986, Vol. 2, Doc. 11). Indeed, at about the same time, Natorp, who had just become the editor of the *Philosophische Monatshefte* asked Lasswitz to start collaborating with the journal as a book reviewer (Natorp to Lasswitz, Sep. 24, 1886; Holzhey, 1986, Vol. 2, Doc. 10). As a first assignment, Natorp asked Lasswitz to review a monograph by Ferdinand August Müller (1886), a former doctoral student of Cohen's who was critical of his approach to the 'infinitesimal method'. In 1888, the first volume of *Philosophische Monatshefte* edited by Natorp was published. In the introductory editorial (Natorp, 1888), Natorp emphasized the importance of maintaining a lively relationship between philosophy and the sciences. It was probably not a coincidence that the issue opened with Lasswitz's review, which he had expanded into a comprehensive article, *Das Problem der Continuität* (Lasswitz, 1888a), which detailed his philosophical views.

2.1. Substantiality and variability

According to Lasswitz (as one might expect from a 19th-century neo-Kantian), the task of philosophy is to discover the conditions necessary to transform subjective sensations into an objective 'nature' (Lasswitz, 1888a, 8f.). Lasswitz preferred to call these conditions *Denkmittel* (or thought-instrument), indicating that they are intellectual tools used in the process of 'constructing' nature. Analyzing the logical structure of scientific theories as the 'result' of this process, one can refer to these tools as *Grundsätze* (or principles), as was more common in the neo-Kantian literature (Lasswitz, 1888b, 460). The *Denkmittel* of substantiality dominated ancient metaphysics; attempts to implement the *Denkmittel* of causality, however, failed: "Until the seventeenth century, it was not possible to combine substantiality and causality in a manner that would allow for the explanation of nature [*Naturerklärung*]; there was no means of representing causal events *mathematically*" (Lasswitz, 1888a, 16). The thought-instrument of causality presupposes the changeability of things. However, the thought-instrument of substantiality cannot understand the 'becoming' as Eleatic philosophy clearly shows: "The thing either remains unchanged, or it is no longer the thing" (Lasswitz, 1888a, 17). The 'transition' itself is not comprehensible. Thus, there is no mediation between substantiality and causality, between what stays the same and what becomes different. A new thought-instrument is necessary to understand the moment of passage. Lasswitz called it the *Denkmittel* of variability.

The *Denkmittel* of variability was discovered in an attempt to overcome the Eleatic objection against the possibility of motion, which claims that the arrow is at rest during each instant. As we have seen, the intensive notion of the infinitesimal came to the rescue. In the instant where there is no motion, one can still define a tendency to continue with the same velocity. Nevertheless, "[t]he formulation of the principle of intensive magnitude and its connection with the problem of continuity and the infinitesimal method contains, despite Cohen's great merit, still something problematic" (Lasswitz, 1888a, 29). Cohen should not have identified the dx with the 'intensive reality'; it would have been preferable to identify the intensive reality with the dy, that is, with the derivative function $dy = f'(x)dx$. In Lasswitz's view, the connection of the intensive magnitude with the category of reality lies in the 'concept of function' (*Functionalbegriff*). Functional concepts presuppose 'variables'. Thus, Lasswitz preferred to speak of the thought-instrument of variability rather than of the category of 'reality':

> The category of reality is thus contained in what we have called the thought-instrument of variability, something that is a unitary element in itself but has a tendency to change. [...] Without the thought-instrument of variability, the flying arrow would rest at every point of its trajectory. This thought-instrument permits the abstraction of extension without eliminating the tendency. [...] The latter is denoted mathematically by a differential, and the sign dy should be suitable for this, because according to mathematical school usage, dx means the differential of the independent variable, while dy represents that of the function. The connection between the principle of intensive magnitude and the category of reality with the infinitesimal method only becomes clear through the reference to the concept of function [...] In order to maintain this distinction between dy and dx [...] we have chosen the neutral expression '*Denkmittel* of variability' [...] for the principle of intensive magnitude. (Lasswitz, 1888a, 29)

Lasswitz was confident that the choice of the derivative function dy instead of the differential dx did not contradict Cohen's claim that reality designates the thought-instrument that takes something as given independent of everything else (without relation) (Lasswitz, 1888a, 30). The function expresses the law of development for something, thereby capturing its quality or reality (Lasswitz, 1888a). With this clarification, Lasswitz claimed to have addressed the issue that mathematicians had with Cohen's interpretation of dx (Lasswitz, 1888a, 30).

The *Denkmittel* of variability provides a means of reconciling substantiality and causality, which ancient philosophy had been unable to achieve. The *Denkmittel* of substantiality requires the identity of the subject of motion over time, and this was expressed in the scientific concept of the 'atom'; on the other hand, the *Denkmittel* of variability required the possibility of defining motion in the instant, and this was achieved by the concept of the 'differential'. By combining these two *Denkmittel*, science was ultimately able to account for the causal action of one atom to another. The importance of continuity lies not in matter distribution in space but in the distribution of motion over time: "the world is no less continuous because it consists of atoms, as long as world events are continuous" (Lasswitz, 1888a, 21).

2.2. Galilei and the Denkmittel of variability

Natorp was delighted by Lasswitz's paper and was eager to read his "research on the genesis of modern science" (Natorp to Lasswitz, Oct. 8, 1888; Holzhey, 1986, Vol. 2, Doc. 15). In fact, Lasswitz's philosophical pronouncements were not made for their own sake. On the one hand, Lasswitz claimed to have 'found' the thought-instrument of substantiality of variability, causality, etc. by investigating the history of science. On the other hand, Lasswitz used those *Denkmittel* as interpretive tools in his work as a historian. Not without some clumsiness, Lasswitz tended to present figures in the history of science for their contribution to the definition of a particular *Denkmittel*. A two-part paper on Galilei's theory of matter (Lasswitz, 1888b) published in 1888 is a typical example of Lasswitz's historiographical style.

According to Lasswitz, Galileo's outstanding achievement was the scientific expression of the *Denkmittel* of variability in the history of modern science. Galileo is often credited for having formulated the 'principle of virtual velocities'. When bodies are in equilibrium, "there is already the moment as a tendency to fall" (Lasswitz, 1888b, 470). Bodies are trying to descend, but are mutually hindered and hence have not achieved any actual motion yet. Two bodies are in equilibrium with each other if they are in an inverse ratio to the velocity that the weights would acquire simultaneously by motion compatible with the constraints. The product mv is called 'moment'. The general cause of equilibrium is the equality of *moments*, not the equality of absolute weights. Galileo discovered that "the concept of velocity is already present here, even if only in a virtual sense" (Lasswitz, 1888b, 470).

Galileo was then able to transform the 'static' concept of 'moment' into its 'dynamic' counterpart by postulating that velocity can be defined in every arbitrarily (*beliebig klein*) small part of time. In this way, a pure rational element is substituted for a sensible and intuitive one. Galileo assumed that the "quality of velocity is not eliminated with the quantity of time" but remains as what "characterizes the process of motion" as such (Lasswitz, 1888b, 473). In this way, "through the conceptual characterization of the intensity of motion" Galileo was able to explain how "in the unit of time the tendency [to motion] still remains" even if there is no change of position, that is, no motion (Lasswitz, 1888b, 473). The Eleatic objection to motion was overcome. By modeling his dynamical concept of 'moments of velocity' upon the static concept of 'moments of weight', Galileo was able to understand acceleration; the motion of a free falling body can be regarded as an aggregate of infinitely many momenta.

2.3. Gassendi and the Denkmittel of substantiality

As Cohen himself conceded (Cohen to Lasswitz, Dec. 6, 1888; Holzhey, 1986, Vol. 2, Doc. 16), Lasswitz successfully integrated his insights into his historiographical work. However, in the second part of the paper, published a year later, Lasswitz (1889a) articulated his objection against Cohen's support of dynamical theories of matter. The thought-instrument of variability, which was applied successfully to *Zeiterfüllung*, 'the filling of time', cannot be expected to apply with equal success to the *Raumerfüllung*, the 'filling of space' (Lasswitz, 1889a). Galileo attempted to solve the problem of the rarefaction and condensation of matter by using the analogy of the *rota Aristotelis*: a body could be composed of an infinite number of unquantifiable atoms, or *non-quanta*, just as the total speed of a body is the sum of an infinite number of indivisibles of speed. The transition from the solid to the liquid state is comparable to the transition from rest to motion (Palmerino, 2001). According to Lasswitz, endeavors of this nature are doomed to failure.

If Galileo's approach were pursued systematically, it would lead to the "theory of intensive points" formulated by Boscovich in a Newtonian setting or to its Kantian plethoric version (Lasswitz, 1889a, 45). However, dynamical theories of this kind fail in the "individualization of matter into closed bodies" (Lasswitz, 1889a, 143). Therefore, the sole thought-instrument of variability is not sufficient for developing a proper theory of matter. "The predicate of *common* motion of the parts of a space quantum can only be attributed to it by thought-instrument of substantiality" (Lasswitz, 1889b, 46). As we have seen, the latter leads to the corpuscle, and ultimately to the atom.

In a paper published the same year, Lasswitz credited Pierre Gassendi, Galileo's contemporary, with grasping the issue at stake. Gassendi introduced the property of 'solidity', which includes impenetrability and rigidity, as the basic features of atoms. According to Lasswitz, "one would not grasp the concept of solidity adequately if one wanted to understand it as the idealization of a sensual property, hardness" (Lasswitz, 1889b, 460). Solidity is the term used by Gassendi to describe the property of the parts of space through which they exist as "space-asserting individuals [*raumbehauptende Individuen*]" (Lasswitz, 1889b, 461). The solidity is the instantiation of the requirement of the *Denkmittel* of substantiality; it is the conditions without which the individualization of matter would not be possible.

Gassendi understood that solid atoms were necessary as the proper 'subjects' of motion; however, he failed to grasp how they exchange motion. According to Gassendi, atoms possess a *vis motrix* as an intrinsic and essential property, which they can never gain or lose (see, *e.g.*, LoLordo, 2008). An atom may be temporarily impeded by an obstacle, but as soon as it is released, it resumes its natural velocity. The different velocities observed in nature are due to the alteration of rest and motion, much like the way varying densities occur from the combination of emptiness and fullness. In Lasswitz's parlance, Gassendi did not have the *Denkmittel* of variability and the concept of infinitesimal. As a consequence, he could not conceive motion as a continuum and grasp the idea of *quantitative* distribution of *vis motrix* among atoms (Lasswitz, 1889b, 468). To account for the variety of phenomena, he had to multiply the *qualitative* differences among atoms. However, the more hypotheses about the shapes of atoms one introduces (such as endowing them with corners, protrusions, hooks, etc.), the less suitable atomism becomes for mathematical treatment: "In the conceptual foundation of physics, therefore, Gassendi did not advance beyond ancient atomism" (Lasswitz, 1889b, 468).

3. Lasswitz on the history of kinetic atomism

The contours of Lasswitz's historical scheme were beginning to become apparent. Galileo successfully applied the *Denkmittel* of variability to motion, but he failed to appreciate the importance of *Denkmittel* of substantiality in defining the subject of motion; Gassendi was guilty of the opposite mistake: "the cross-pollination could only take place in the future; Huygens made it possible by exhibiting the principles of mechanics" (Lasswitz, 1889b, 470). By that time, Lasswitz had already completed collecting the results of these and previous historical investigations (Dilthey to Lasswitz, Dec. 1, 1888; Dilthey, 2011–22, Vol. 2, Doc. 718) in a monumental two-volume work titled *Geschichte der Atomistik* (Lasswitz, 1890). The *Vorwort* is dated October 1889 (Lasswitz, 1890). A few weeks later, Natorp wrote to Lasswitz that he had seen the first volume on Cohen's desk and was already planning to write a review (Natorp to Lasswitz, Dec. 22, 1889; Holzhey, 1986, Vol. 2, Doc. 18).

Geschichte der Atomistik represents the culmination of Lasswitz's decades-long studies on atomism. The present paper cannot adequately convey the wealth of information contained in a thousand-page account of the history of atomism, stretching from the church fathers to Newton. However, the book has a recognizable protagonist. Between Galileo and Descartes at the beginning of the 1600s, on the one hand, and Leibniz and Newton at the turn of the 1700s, Lasswitz indicated Huygens as the central figure of modern science. Lasswitz's choice is far from obvious. Even today, Huygens remains surprisingly under-researched. To Lasswitz, Huygens appeared as the last stage of a three-step historical–philosophical scheme: "The objectification of sensation to the law-like moving atomic world takes place in the development, which is designated by the three names: Gassendi, Galileo, Huygens" (Lasswitz, 1890, 2:376).

Huygens' great achievement was to show how "[t]he *Denkmittel* of variability does not apply only to the change of the velocity of a single body", as in the case of Galileo, but to the "distribution of velocities" (Lasswitz, 1890, 2:378) among invariable atoms. In particular, Huygens must be credited with establishing laws of motion for the atoms. Atoms act on each other by 'collision' (*Stoß*). According to Lasswitz, for Huygens, 'collision' meant nothing but the fact that the motion of two atoms after their encounter is determined 'univocally' (*eindeutig*) by their motion before their encounter. Huygens was able to prove that the principle of conservation of the algebraic sum of momenta $\sum mv = 0$ and of *vis viva* $\sum mv^2 =$ const. are the necessary conditions for determining the motion uniquely. The rigid atom provides the subject of motion, and the laws of mechanics regulate the continuous exchange of velocities among atoms. Here, for the first time, the essential conceptual tools necessary to separate objective reality from the ever-changing flux nature of our experiences are brought together in a coherent form.

3.1. Huygens and the peak of kinetic atomism

It is well-known that in his 1692-94 correspondence with Huygens, Leibniz pointed out the elephant in the room of Huygens' atomism (see Lange, 1873, 2:202). If one assumes that Huygens' collision laws apply to atoms, one is faced with a dilemma (Lasswitz, 1890, 2:367): (a) the atoms must either themselves be elastic or (b) *vis viva* must be at least partly lost in their collision. However, (a) is not possible since the atoms per definition do not have movable parts; (b) contradicts the principle of the conservation of *vis viva* from which Huygens derived his collision rules: "Consequentely, says Leibniz and it is said in general, absolutely unchangeable atoms are an absurdity" (Lasswitz, 1890, 2:367). However, Huygens denied that the alternative between (a) and (b) is exhaustive. He insisted that atoms must be absolutely hard and still not lose any of their motion in their impact: Huygens promises to explain his view *un jour*. "Has that day appeared? Or have these reasons remained hidden from us forever?" (Lasswitz, 1890, 2:367).

Huygens never explained the rebound of hard atoms in any letter to Leibniz or other published works during his lifetime. However, Lasswitz claimed that Huygens did provide an 'explanation' in his posthumous book on impact. Huygens' 'explanation', however, was nothing but the 'proof' (*Beweis*) of his rules of collision. According to Lasswitz, Leibniz saw that Huygens' collision rules applied to the behavior of perfectly elastic macroscopic bodies; thus, he inferred that atoms must be elastic (see Blackwell, 1977). On the contrary, Huygens derived the rules of collision from the principles of mechanics; thus, he inferred that atoms must satisfy those rules: "These principles of mechanics are the conditions for the possibility of atomism and that the laws of collision are derived from them, not the other way round, that the movement of matter presupposes the laws of collision" (Lasswitz, 1890, 2:368). The two principles of mechanics are justified by the fact that they are necessary conditions that allow the unambiguous determination of the velocity of atoms after the impact if now their velocities before the impact:

> Therefore, Huygens' assumptions are equivalent to these two principles of mechanics: the law of conservation of the center of gravity and that of the conservation of energy. Even if they initially appear here in the form of theorems [*Lehrsätze*], this is only an incidental aspect of the formulation. What is essential and decisive in Huygens is that he does not start from sensible intuitions or anthropomorphic representations, but from mechanical facts. The principles of mechanics are fundamental because they are necessary and sufficient to unambiguously [*eindeutig*] determine the motions of bodies, *i.e.*, to calculate their velocities and directions, if those before the collision are given. It is not because bodies are elastic that their *vis viva* is conserved after the impact; but because *vis viva* must be conserved, the impact occurs in the way observed in bodies which we call elastic. The elastic displaceability of the parts, this sensuous fact, is not a condition of the laws of impact. Huygens does not refer to the bodies he is dealing with as elastic but hard; and this does not imply a sensuous property, but rather the same concept of solidity, the property of the substance to assert its space unchangeably. The space-assertion [*Raumbehauptung*] of individual substances and the principles of mechanics are therefore made by Huygens the basis of the theory of matter; from them, he derived the laws for the modification of the motion of atoms. The fact that we find the same laws in the sensuous impact of elastic bodies is entirely irrelevant

Huygens did not simply apply to the atoms empirical laws of collision valid for elastic bodies; rather, he established the condition that any empirical law of collision among atoms must satisfy if their velocity after the impact is to be uniquely determined. Indeed, "Huygens does not refer to the bodies he deals with as elastic, but hard; this reflects the Gassendian concept of solidity" (Lasswitz, 1890, 2:370). Thus, Huygens does not view 'hardness' as a sensory property of macroscopic bodies; nor does he attempt to derive 'hardness' from an analysis of the mutual forces maintaining the shape and volume of individual atoms in equilibrium. According to Lasswitz, the meaning of the term 'hard' is *implicitly defined* by the two conservation principles (see, e.g. Mormino, 1996):

> However, the exchange of velocities is mathematically defined by the principles of mechanics, since $\sum mv$ and $\sum mv^2$ are constant quantities. Speculating about what happens to the atoms when they collide should be dismissed as entirely impermissible, because it presupposes a sensory intuition of the atoms. Atoms are often envisioned as small, hard bodies colliding, similar to what we observe in the case of tangible bodies. However, this is precisely where Huygens made progress in transforming corpuscular theory into a science, as he transcended this sensory conception and replaced it with rational and mathematically formulated concepts. The absolute atom and the totality of atoms in motion are theoretical constructs [*begriffliche Gebilde*], and their encounter in space no longer implies anthropomorphic collision but rather geometric determination of their position at a given time. Moreover, their behavior after the so-called collision is not inferred from the analogy of the rebounding of macroscopic bodies but rather determined by the mathematical formula governing the distribution of velocities. Engaging in speculation about what must occur when immutable bodies collide in a manner analogous to tangible bodies leads only to fruitless conjecture. (Lasswitz, 1890, 2:374f.)

In Huygens' view, the laws of collisions determine the values of the atoms' velocities *just before and after* the collision. Any attempt to investigate what happens *during* the collision of atoms itself is based on confusion with the collision of atoms and that between small macroscopic bodies. Atoms are not the ultimate parts of macroscopic matter but are necessary conditions for any mechanical explanation of the behavior of macroscopic matter. Gassendi's 'atoms' were introduced as an expression of the *Denkmittel* of substantiality, to ensure the individuality of the moving parcels of space (Lasswitz, 1890, 2:379). Galileo's 'moment' was introduced to satisfy the requirement of the *Denkmittel* of variability so that the velocity of a falling body is defined at each instant (Lasswitz, 1890, 2:379). Huygens' merit was to have combined these two *Denkmittel* by formulating the laws of the distribution of velocities among atoms. In this way, for the first time, he allowed for the mathematical representation of the causality and reciprocal action among atoms, that is, the laws of the continuous distribution of kinetic energy among them (Lasswitz, 1890, 2:380).

3.2. The decline of kinetic atomism. Leibniz and Newton

Lasswitz credited Huygens with transforming ancient philosophical atomism into a modern scientific theory. However, contemporaries fundamentally misunderstood Huygens' result. If atoms are perfectly hard, it was argued, then all changes in velocity would have to be discontinuous, since two atoms colliding would have to instantly alter their velocities if they did not experience any deformation from the collision. Thus, the velocity would not be defined univocally at the instant of impact, contrary to Huygens' claim. With some simplification, one can argue that, according to Lasswitz, two distinct approaches have been taken to address this issue. The first approach, as championed by Leibniz, relied on contact action. The second, drawing from Newton's work, relied on distant action.

Leibnizian program Leibniz famously argued that the notion of perfectly hard atoms colliding is inconsistent with the principle of continuity. The latter requires elasticity, which in turn presupposes movable parts. Leibniz concluded that matter must be an elastic plenum, infinitely divided by the different motions of its parts. As a result, however small, no part of matter remains the same for no longer than a moment. Thefore, it becomes impossible to determine the identity and individuality of parcels of matter: "What distinguishes one particle from another in motion, what gives unity to the moving part of space?" (Lasswitz, 1890, 2:239). Since he could not define the identity of the subject of motion by applying the thought-instrument of substantiality to space elements, Leibniz was compelled to refer to the time element. Leibniz was left with the sole thought-instrument of variability and identified the substance with the 'force', the 'constant tendency' to continue from state to state (Willmann, 2012).

Newtonian program After the success of his theory of gravitation based on the notion of attraction at a distance, Newton attributed to atoms forces of attraction and repulsion (Lasswitz, 1890, 2:480). Boscovich carried out the full implications of Newton's program. Like Leibniz, he recognized the conflict between continuity and atomism. However, he avoided the idea of an infinite regression of elastic parts by conjecturing that the basic elements of matter must be just simple points (Lasswitz, 1890, 2:563) endowed with repulsive forces. In this way, the possibility of applying the *Denkmittel* of substantiality is lost, and one is left with the sole variability. For this reason, as Lasswitz had repeatedly argued, dynamic theories of matter *à la* Boscovich cannot explain how "a unitary mass particle is formed [...] how a sum of such points could achieve unitary movement" (Lasswitz, 1890, 2:52).

Lasswitz concluded that "the great physicist and mathematician Newton arrived at the same result as the great philosopher and mathematician Leibniz" (Lasswitz, 1890, 2:580). Whereas the Leibnizian program inevitably leads to a *metaphysical* monadology, the Newtonian program leads to a *physical* monadology.

Both programs appeared to Lasswitz as an epistemological regress with respect to Huygens's kinetic atomism. Both were the consequence of an attempt to reply to an apparently cogent objection against atomism. If atoms were hard, their velocity at the moment of impact would be undefined. However, Lasswitz argued, "this objection rests only on the old mistake of bringing sensible intuition back into the motion of those theoretical entities (*rationalen Gebilde*) that we call atoms" (Lasswitz, 1890, 2:380). The interaction between substances in space "is not at all sensually representable; it is a transcendental principle of experience" (Lasswitz, 1890, 2:380). If one tries to describe two atoms at the moment of their collision, one proceeds in the same way as if one tries to imagine the flying arrow at a point on its path. The discontinuity lies in the subject of the motion, not in the motion itself (Lasswitz, 1890, 2:379).

3.3. Condition and ideal. Critical philosophy and the history of science

Lasswitz made no secret that his work was a textbook application of what Cohen (1885) had called the 'transcendental method'. Critical philosophy starts with the "fact of science [*Faktum der Wissenschaft*]" (Lasswitz, 1890, 2:383) and searches for the conditions of its possibility (see also Lasswitz, 1887a). One might even say that Lasswitz was the first to attempt to apply this method in a systematic way as a practicing historian of science. According to Lasswitz, the history of atomism offers the "suitable material for studying the things-in-themselves on which mechanical natural science is based and for learning about the transcendental conditions for the possibility of an objective nature" (Lasswitz, 1890, 2:384).

According to Lasswitz, the transcendental method relates to the mathematical science of nature in the same way as the latter relate to empirical reality; the transcendental method is an analogue of the empirical method. The things-in-themselves are not derived *a priori* from some higher instance; they are found *a posteriori* in the history of science: "The discovery of the principles of mechanics is the *empirical fact* on which critical idealism could be based" (Lasswitz, 1890, 2:384). As a consequence, critical philosophers must always be aware that their conclusions are provisional:

> Critical philosophy cannot define the conditions of experience and the principles of physics *a priori*. Instead, it can do so only through the historical process. Just as physical knowledge changes, the theory of transcendental conditions of experience will also change over time. The essential difference between the transcendental principles and the changing theories is not in how the principles of scientific knowledge are formulated in the consciousness of a given epoch, but in the fact that they must be formulated. There is an eternal determination for the direction of consciousness, a supreme law of objectivization. It is an insoluble problem to predict which intellectual instruments will be discovered and which ones will vanish from human consciousness. However, each cultural epoch becomes aware of its own intellectual instruments as the synthetic unities that guarantee the possibility of scientific experience amid the vacillations and gropings of special investigations and hypotheses. This is achieved by showing the shifting theoretical content to be dependent not only on empirical accident, but also on an enduring trend of consciousness. (Lasswitz, 1890, 2:393)

Lasswitz was aware that his 'transcendental deduction' crucially depended on his hypothesis that "the epistemological foundations of physics are complete with Huygens" (Lasswitz, 1890, 2:384). Lasswitz broke the circularity of this reasoning by transforming kinetic atomism from a fact into a program. Huygens' kinetic atomism plays a double role in his system: (1) it is a *condition* (*Bedingung*) of physics, encapsulating the conditions of *possible* physics, the two *Denkmittel* of substantiality and variability, in their uncontaminated form; (2) it represents the *ideal* (*Ideal*) toward which *actual* physics must ultimately converge.

Lasswitz conceded that the 'condition' and the 'ideal' never fully coincide. Kinetic atomism aims to describe all physical processes without assuming either force or potential energy. However, this form of kinetic atomism was at most able to account for the behavior of ideal monatomic gases. For more complex cases, 19th-century physics could not avoid falling back on the hybrid 'Laplacian–Newtonian' atomism, which introduces potential energy and central forces acting at a distance between material particles. Indeed, Lasswitz conceded that modern physics was still "far away" from the ideal of a purely kinetic atomism (Lasswitz, 1890, 2:394). At the same time, Lasswitz saw some encouraging signs that physics was "was edging closer" to this goal (Lasswitz, 1890, 2:394). The development of modern energetics (Helm, 1887)[7] showed the tendency of replacing central forces with different types of spatial energy (*e.g.*, distance energy, surface energy) (Lasswitz, 1893). Lasswitz hoped that this approach might act as an intermediary step toward ultimately reducing all forms of energy to the kinetic energy of moving atoms. This conclusion is somewhat surprising given the anti-atomistic bent of energetics (Ostwald, 1895). As far as I can see, Hertz's (1894) forceless mechanics comes close to the ideal of a physical theory that Lasswitz seemed to have in mind.

Like Hertz (1894, §605), Lasswitz argued that the 'potential energy' of a visible system must be reduced to the kinetic energy of a hidden system. As an example, Lasswitz considered the law that the ascent height of the pendulum is equal to its fall height (Lasswitz, 1890, 2:373). In a Newtonian setting, when a pendulum is at its highest

[7] See, Lasswitz to Ostwald, Apr. 9, 1892; Ostwald, 1961, Doc. 117.

point, its kinetic energy is fully transformed into gravitational potential energy. However, in Lasswitz's Huygensian approach, all energy is supposed to be kinetic. Hence, one has to assume that atoms of a 'gravitational fluid' are set into motion and take over the kinetic energy of the visible pendulum (Lasswitz, 1890, 2:373). According to Lasswitz, this peculiar form of mechanical explanation represented the 'ideal' form of a proper explanation since it perfectly satisfies the 'conditions' of any physical explanation: "All reality of natural events can only be based on a lawful change in the distribution of intensive magnitudes in space, whose possibility is linked to the substantial existence of spatial individuals" (Lasswitz, 1890, 2:394).

Regrettably, the physics of the 19th century not only fell short of meeting the 'conditions' Lasswitz deemed necessary for the possibility of science but also did not appear to advance toward Lasswitz's crypto-Hertzian 'ideal'. Toward the end of the century, even a champion of the kinetic theory of gases, such as Ludwig Boltzmann, after attempting to pursue Hertz's program, reverted to the "old distinction between potential and kinetic energy" (Boltzmann, 1896–98, 1:3). Lasswitz would probably have forgiven this lapse, confident that physics would sooner or later return to the righteous path toward a purely kinetic-atomic theory. However, such a stance became progressively more untenable as the disparity between the *condition* and the *ideal* showed no signs of diminishing. By the turn of the century, the dominance of the 'mechanical worldview', which sought to reduce all physics to mechanics, was seriously challenged by a burgeoning 'electromagnetic worldview', which sought to reduce all physics to electrodynamics. The program was launched in 1900 by Wilhelm Wien, who explicitly described it as "diametrically opposed to Hertz's foundation of mechanics" (Wien, 1900, 512).

4. Conclusion. Lasswitz and the Marburg school

The Marburg reception of Lasswitz's *Geschichte der Atomistik* was overwhelmingly positive. Elsas, in his review, credited Lasswitz for providing the most convincing attempt "to further explain Cohen's pathbreaking book about the principle of the infinitesimal method" (Elsas, 1891, 301). At the same time, the opposition between 'variability' and 'substantiality' was considered philosophically problematic. Both Natorp (1891) and Cohen (1896, XLV) took the trouble to reply to Lasswitz,[8] but their more articulated response was left to their doctoral students. In particular, the Russian *émigré* Otto Buek (1905), following Cohen's suggestion, proposed Michael Faraday's field theory of matter as an alternative to Huygens's atomism defended by Lasswitz. On Faraday's view, particles are nothing but high-intensity field regions, propagating through empty space like water waves across the water surface. Whereas Lasswitz deemed atoms necessary to ensure the identity of the 'subject of motion' over time, Buek (1905, 161f.) argued that modern physics opened the possibility for a 'motion without a subject' (see Giovanelli, 2024, for more detail).

As far as I can see, Lasswitz never responded to these objections in published writings. At the turn of the century, Lasswitz (1900) appeared to be more interested in articulating a synthesis between Kant's idealism and Fechner's animistic worldview, centered on the notion of 'lived experience' as the mediator between subjective and objective, science and poetry (Fechner, 1906; Lasswitz, 1896, see Heidelberger, 2004, 12f., Azzouni, 2009, 70). Such a philosophical program did not resonate with his Marburg interlocutors. However, there were no signs of ill feelings. As late as 1910, Natorp wrote a friendly letter to Lasswitz, recognizing his success as a novelist and announcing the publication of his *Die logischen Grundlagen der exakten Wissenschaften* (Natorp, 1910). The Marburg school, he recounted, was gaining a progressively more prominent role in the German philosophical scene (Natorp to Lasswitz, Feb. 2, 1910; Holzhey, 1986, Vol. 2, Doc. 121). Unfortunately, Lasswitz did not live to see the golden age of Marburg Neo-Kantianism. He passed away in October 1910, shortly after the publication of Cassirer's celebrated *Substanzbegriff und Funktionsbegriff* (Cassirer, 1910). In this book, Cassirer, with his customary conciliatory approach, can be said to have settled the Buek–Lasswitz debate. The history of the concept of 'atom' and that of the concept of 'field' were both presented as instances of the progressive transition from substance-concept to function-concept (Cassirer, 1910, 207–216).

However, nearly two decades later, Cassirer (1929) seems to have realized that Lasswitz, by relating the notion of 'substance' to that of 'individuality', had brought up a more subtle issue that could no longer be ignored. The success of Einstein's field theory of gravitation convinced many that the full reduction of matter to the field was at hand (Cassirer, 1921, 16f.). However, if material particles ar nothing but changing portions of the field, there "is no longer any meaning in speaking of one and the same matter at different times" (Cassirer, 1929, 552; tr. 1957, 473). Within a field-theoretical approach, the property of identifiability, traditionally considered one of the essential features of ordinary matter, becomes dispensable. In this respect, Cassirer continues, if we compare "modern relativistic physics" with "Lasswitz's picture of kinetic atomics", one can appreciate the conceptual shift physics has undergone in recent decades (Cassirer, 1929, 550; tr. 1957, 471).

Lasswitz, Cassirer continued, aimed at a "critical [*erkenntniskritische*] justification, or 'transcendental deduction'" of kinetic atomism (Cassirer, 1929, 547; tr. 1957, 469). The latter is the only theory that can account for the interaction between parcels of matter while preserving their individual identity. Thus, in Lasswitz's view, Huygens' atomism is not a theory among others; it is "the norm and prototype [*Vorbild*] of an exact natural science in general" (Cassirer, 1929, 547; tr. 1957, 469). In Huygens' work, the *Denkmittel* of variability and substantiality, the continuous distribution of motion, and the identity of what is moving are placed in a "perfect balance" (Cassirer, 1929, 547; tr. 1957, 469). Still, as Cassirer pointed out, Lasswitz was fully aware that "the critical [*erkenntniskritische*] deduction" is always a "hypothetical deduction" (Cassirer, 1929, 547; tr. 1957, 470). It always "relates to a definite historical stage, which it treats as an underlying 'fact of science' [*Faktum der Wissenschaft*]". As a "strictly critical thinker", Lasswitz could not regard the 'fact' of kinetic atomism as "immutable and definitive" (Cassirer, 1929, 547; tr. 1957, 470). In this way, Lasswitz's work presents in a stylized form the dilemma that any neo-Kantian approach to the history of science seems to face.

When a new theory comes along, the 'fact' (*Faktum*) can come into conflict with the 'norm' (*Vorbild*). Critical philosophers could react by rejecting the 'fact' or by changing the 'norm'. As we have seen, Lasswitz ultimately chose the first option and dismissed much of post-Huygensian physics as epistemologically unsound. Cassirer took the second alternative, but at the price of having to progressively weaken the ambitions of the neo-Kantian program (Giovanelli, 2022). "Modern physics," he wrote, "cannot dispense with Lasswitz's two basic intellectual instruments [*Denkmittel*], 'substantiality' and 'variability'. But in making use of these instruments, it moves them into a new systematic relationship" (Cassirer, 1929, 550; tr. 1957, 471). By the time the third volume of the *Philosophie der symbolischen Formen* was published in 1929, the vision of a field theory of matter was superseded by the new quantum theory. However, the problem of the 'numerical identity' of particles did not disappear but became even more pressing (Cassirer, 1936, sec. V.2). In *Determinismus und Indeterminismus* Cassirer once again credited "Lasswitz, in his excellent presentation of Huyghens' atomism" (Cassirer, 1936, 182; tr. 1956, 146) for having shown that the classical 'atom' was not an indivisible 'thing', but rather the expression of a conceptual 'condition', namely the requirement of the identifiability of matter parcels over time. However, *pace* Lasswitz, this condition turned out not to be necessary for the possibility of a theory of matter.

[8] See also Natorp to Lasswitz, Jun. 3, 1891; Holzhey, 1986, Vol. 2, Doc. 19 and Cohen to Lasswitz, Jul. 22, 1891; Holzhey, 1986, Vol. 2, Doc. 22.

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Marco Giovanelli: Writing – review & editing, Writing – original draft, Conceptualization.

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References

Azzouni, Safia (2009). How Wilhelm Dilthey Influenced Popular Science Writing: Kurd Laßwitz's "Homchen. Ein Tiermärchen aus der oberen Kreide". In Uliana Feest (Ed.), *Historical perspectives on Erklären and Verstehen* (pp. 61–79). Dordrecht Heidelberg London New York: Springer.

Bachelard, Gaston (1933). *Les intuitions atomistiques (essai de classification)*. Paris: Boivin et Cie., Éditeurs.

Beiser, Frederick C. (2018). *Hermann Cohen: An intellectual biography*. Oxford University Press.

Blackwell, Richard J. (1977). Christiaan Huygens' The motion of colliding bodies. *Isis*, *68*(4), 574–597.

Boltzmann, Ludwig (1896–98). *Vorlesungen über Gastheorie*. 2 Vols., Leipzig: Barth; Brush, Stephen G. (1964). *Lectures on Gas Theory*. Berkeley: University of California Press.

Buek, Otto (1905). *Die Atomistik und Faradays Begriff der Materie*. Berlin: Reimer.

Cantor, Georg (1884). Review of Cohen, *Das Princip der Infinitesimal-Methode* [Cohen 1883]. *Deutsche Literaturzeitung*, *5*, 266–268.

Cassirer, Ernst (1910). *Substanzbegriff und Funktionsbegriff: Untersuchungen über die Grundfragen der Erkenntniskritik*. Berlin: B. Cassirer, repr. in Cassirer, 1998-2007, Vol. 6.

Cassirer, Ernst (1921). *Zur Einstein'schen Relativitätstheorie: Erkenntnistheoretische Betrachtungen*. Berlin: B. Cassirer, repr. in Cassirer, 1998-2007, Vol. 10.

Cassirer, Ernst (1929). Philosophie der symbolischen Formen. In *Dritter Teil: Phänomenologie der Erkenntnis*. Berlin: B. Cassirer, repr. in Cassirer, 1998-2007, Vol. 13.

Cassirer, Ernst (1936). Determinismus und Indeterminismus in der modernen Physik: Historische und systematische Studien zum Kausalproblem. *Göteborgs Högskolas Årsskrift*, *42*(3), repr. in Cassirer (1998–2007), Vol. 19.

Cassirer, Ernst (1956). *Determinism and indeterminism in modern physics: Historical and systematic studies of the problem of causality*. New Haven: Yale University Press.

Cassirer, Ernst (1957). *The philosophy of symbolic forms: Volume three: The phenomenology of knowledge. With an introduction by Charles W. Hendel*. New York: Yale University Press.

Cassirer, Ernst (1998–2007). In Birgit Recki (Ed.), *Gesammelte Werke*. 25 Vols., ed. by Birgit Recki, Hamburg: Meiner.

Cohen, Hermann (1883). *Das Princip der Infinitesimal-Methode und seine Geschichte: Ein Kapitel zur Grundlegung der Erkenntnisskritik*. Berlin: Dümmler, repr. in Cohen 1977-, Vol. 5/I.

Cohen, Hermann (1885). *Kants Theorie der Erfahrung* (2nd ed.). Berlin: Dümmler, repr. in Cohen 1977-, Vol. I/1.

Cohen, Hermann (1895). "Worte an der Bahre von Adolf Elsas, Professor der Physik an der Universität Marburg." Gesprochen im Trauerhause am 16. Mai 1895. In Albert Görland, & Ernst Cassirer (Eds.), *Schriften zur Philosophie und Zeitgeschichte* (pp. 396–397). Berlin: B. Cassirer (in German).

Cohen, Hermann (1896). Einleitung mit kritischem Nachtrag. In Friedrich Albert Lange (Ed.), *Geschichte des Materialismus und Kritik seiner Bedeutung in der Gegenwart* (5th ed.). Leipzig: Baedeker, repr. in Cohen (1977-)-, Vol. 5/2.

Cohen, Hermann (1977-). *Werke*. 19 Vols., ed. by Helmut Holzhey, Hildesheim: Olms.

Dilthey, Wilhelm (2011–22). *Briefwechsel*. Vol. 4, Ed. by Gudrun Kühne-Bertram, Göttingen at al: Vandenhoeck & Ruprecht.

Dussort, Henri (1963). *L'École de Marbourg*. Paris: Ed. by Jules Vuillemin, Presses Universitaires de France.

Eccarius, Wolfgang (1985). Georg Cantor und Kurd Laßwitz: Briefe zur Philosophie des Unendlichen. *NTM -Schriftenreihe für Geschichte der Naturwissenschaften, Technik und Medizin*, *22*(1), 29–52.

Elsas, Adolf (1886). *Über die Psychophysik: Physikalische und erkenntnisstheoretische Betrachtungen*. Marburg: Elwert.

Elsas, Adolf (1891). Review of Lasswitz, *Geschichte der Atomistik* [Lasswitz 1890]. *Zeitschrift für Philosophie und Philosophische Kritik*, *99*, 290–303.

Fechner, Gustav Theodor (1906). In Kurd Lasswitz (Ed.), *Zend-Avesta oder über die Dinge des Himmels und des Jenseits: Vom Standpunkt der Naturbetrachtung*, Hamburg and Leipzig: Voss.

Ferrari, Massimo (1988). *Il giovane Cassirer e la scuola di Marburgo*. Milan: Angeli.

Ferrari, Massimo (1997). *Il Neokantismo*. Rome and Bari: Laterza.

Giovanelli, Marco (2016). Hermann Cohen's *Das Princip der Infinitesimal-Methode*: The history of an unsuccessful book. *Studies in History and Philosophy of Science*, *58*, 9–23.

Giovanelli, Marco (2022). Motivational Kantianism: Cassirer's late shift towards a regulative conception of the a priori. *Studies in History and Philosophy of Science*, *95*, 118–125.

Giovanelli, Marco (2024). Kurd Lasswitz, the Marburg School, and the problem of individuality in physics. In Helmut Pulte, & et al. (Eds.), *New Perspectives on Neo-Kantianism and the Sciences* (pp. 107–125). New York and London: Routledge.

Heidelberger, Michael (2004). *Nature from within: Gustav Theodor Fechner and his psychophysical worldview*. Pittsburgh: University of Pittsburgh Press, 1993.

Helm, Georg Ferdinand (1887). Lehre von der Energie: Historisch-kritisch entwickelt. *Nebst beiträgen zu einer allgemeinen energetik*. Leipzig: Felix.

Hertz, Heinrich (1894). Die Prinzipien der Mechanik in neuem Zusammenhange dargestellt: Mit einem Vorworte von H. von Helmholtz. In *Gesammelte Werke*. Vol. 3, Leipzig: Barth.

Holzhey, Helmut (1986). *Cohen und Natorp*. Basel: 2. Vols., Schwabe.

Husserl, Edmund (1994). *Briefwechsel*. Ed. by Samuel Ijsseling, Dordrecht: Kluwer.

Klein, Alexander (2021). How American is pragmatism? *Philosophy of Science*, *88*(5), 849–859.

Köhnke, Klaus Christian (1986). *Entstehung und Aufstieg des Neukantianismus: Die deutsche Universitätsphilosophie zwischen Idealismus und Positivismus*. Frankfurt am Main: Suhrkamp.

Kragh, Helge (2002). The vortex atom: A Victorian theory of everything. *Centaurus*, *44*(1–2), 32–114.

Lange, Friedricha Albert (1873). *Geschichte des Materialismus bis auf Kant*: *vol. 1*, (2nd ed.). Leipzig: Baedeker.

Lange, Friedrich Albert (1873–75). *Geschichte des Materialismus bis auf Kant*, (2nd ed.). Leipzig: 2 Vols., Baedeker.

Lasswitz, Kurd (1873). *Ueber Tropfen, welche an festen Körpern hängen und der Schwerkraft unterworfen sind* (PhD diss.), Universität Breslau.

Lasswitz, Kurd (1874a). Der Verfall der 'kinetischen Atomistik' im siebzehnten Jahrhundert. *Annalen der Physik und Chemie*, *153*, 373–386.

Lasswitz, Kurd (1874b). Ueber Tropfen an festen Körpern insbesondere an Cylindern. *Annalen der Physik und Chemie*, *Ergänzungsband 6*, 441–477.

Lasswitz, Kurd (1878a). *Atomistik und Kriticismus: Ein Beitrag zur erkenntnistheoretischen Grundlegung der Physik*. Vieweg: Braunschweig.

Lasswitz, Kurd (1878b). *Bilder aus der Zukunft: Zwei Erzählungen aus dem vierundzwanzigsten und neununddreißigsten Jahrhundert*. Breslau: Schottlaender.

Lasswitz, Kurd (1879a). Die Erneuerung der Atomistik in Deutschland durch Daniel Sennert und sein Zusammenhang mit Asklepiades von Bithynien. *Vierteljahrsschrift für wissenschaftliche Philosophie*, *3*(4), 408–434.

Lasswitz, Kurd (1879b). Über Wirbelatome und stetige Raumerfüllung. *Vierteljahrschrift für Wissenschaftlicher Philosophie*, *3*, 206–215; 275–294.

Lasswitz, Kurd (1882). *Die Lehre von den Elementen während des Überganges von der scholastischen Physik zur Corpusoulartheorie*. Gotha: Gymnasium Ernestinum.

Lasswitz, Kurd (1883). *Die Lehre Kants von der Idealität des Raumes und der Zeit: Im Zusammenhange mit Seiner Kritik des Erkennens Allgemeinverständlich Dargestellt*. Berlin: Weidmannsche Buchhandlung.

Lasswitz, Kurd (1884a). Giordano Bruno und die Atomistik. *Vierteljahrsschrift für wissenschaftliche Philosophie*, *8*, 18–55.

Lasswitz, Kurd (1884b). Review of Natorp, *Descartes' Erkenntnisstheorie* (Natorp, 1882). *Göttingische Gelehrte Anzeigen*, *18*, 758–765.

Lasswitz, Kurd (1884c). Review of Stadler, *Kants Theorie der Materie* [Stadler 1883]. *Göttingische Gelehrte Anzeigen*, *13*, 532–536.

Lasswitz, Kurd (1885a). Review of Cohen, *Das Princip der Infinitesimal-Methode* [Cohen 1883]. *Vierteljahrsschrift für wissenschaftliche Philosophie und Soziologie*, *9*, 494–503.

Lasswitz, Kurd (1885b). Zur Rechtfertigung der kinetischen Atomistik. *Vierteljahrsschrift für wissenschaftliche Philosophie*, *9*, 137–161.

Lasswitz, Kurd (1887a). Review of Cohen, *Kants Theorie der Erfahrung 2* [Cohen 1885]. *Zeitschrift für Mathematik und Physik*, *32*, 137–138.

Lasswitz, Kurd Review of Elsas, *Über die Psychophysik* (Elsas, 1886). *Deutsche Literaturzeitung*, *8*, 3–4,

Lasswitz, Kurd (1887c). Review of Müller, *Das Problem der Continuität im Mathematik und Mechanik* [Müller 1886]. *Deutsche Literaturzeitung*, *28*, 1003–1004.

Lasswitz, Kurd (1888a). Das Problem der Continuität. *Philosophische Monatshefte*, *24*, 9–36.

Lasswitz, Kurd (1888b). Galileis Theorie der Materie, I. *Vierteljahrsschrift für wissenschaftliche Philosophie*, *4*, 458–476.

Lasswitz, Kurd (1889a). Galileis Theorie der Materie, II. *Vierteljahrsschrift für wissenschaftliche Philosophie*, *13*, 32–50.

Lasswitz, Kurd (1889b). Über Gassendi's Atomistik. *Archiv für Geschichte der Philosophie*, *2*(3), 459–470.

Lasswitz, Kurd (1890). *Geschichte der Atomistik*, 2 Vols., Hamburg and Leipzig: Voss.

Lasswitz, Kurd (1893). Die moderne Energetik in ihrer Bedeutung für die Erkenntniskritik. *Philosophische Monatshefte*, *29*(29), 1–30; 177–197.

Lasswitz, Kurd (1896). *Gustav Theodor Fechner*. Stuttgart: Frommann.

Lasswitz, Kurd (1897). *Auf zwei Planeten: Roman in 2 Büchern*. Weimer: Selber.

Lasswitz, Kurd (1900). *Wirklichkeiten: Beiträge zum Weltverständnis*. Berlin: Felber.

LoLordo, Antonia (2008). Epicurean and Galilean motion in Gassendi's physics. *Philosophy Compass*, *3*(2), 301–314.

Lübeck, Gustav (1877). Ableitung des elastischen Stosses zweier Atome. *Zeitschrift für Mathematik und Physik*, *22*, 126–132.

Lüthy, Christoph (2003). Atoms & corpuscles: Kurd Lasswitz and the historiography of atomism. *Intellectual News*, *11*(1), 7–16.

Meyer, Oskar Emil (1874). Berichte über Sections-Sitzungen. Physikalische Section. 22. September. *Tageblatt der 47. Versammlung Deutscher Naturforscher und Ärzte in Breslau*, *7*, 173–174.

Meyer, Oskar Emil (1877). *Die kinetische Theorie der Gase in elementarer Darstellung mit mathematischen Zusätzen*. Breslau: Maruschke & Berendt.

Mormino, Gianfranco (1996). Atomismo e meccanicismo nel pensiero di Christiaan Huygens. *Rivista di storia della filosofia*, *51*(4), 829–863.

Müller, Ferdinand August (1886). *Das Problem der Continuität im Mathematik und Mechanik: Historische und systematische Beiträge*. Elwert: Marburg.

Natorp, Paul (1882). *Descartes' Erkenntnisstheorie: Eine Studie zur Vorgeschichte des Kriticismus*. Marburg: Elwert.

Natorp, Paul (1888). Zum Eingang. *Philosophische Monatshefte*, *24*, 1–8.

Natorp, Paul (1891). Review of Lasswitz, *Geschichte der Atomistik* [Lasswitz 1890]. *Philosophische Monatshefte*, *27*, 334.

Natorp, Paul (1910). *Die logischen Grundlagen der exakten Wissenschaften*. Leipzig Berlin: Teubner.

Ostwald, Wilhelm (1895). *Die Überwindung des wissenschaftlichen Materialismus. Vortrag, gehalten in der 3. allgemeinen Sitzung der Versammlung der Gesellschaft Deutscher Naturforscher und Ärzte zu Lübeck am 20. 1895*. Leipzig: Veit.

Ostwald, Wilhelm (1961). *Aus dem wissenschaftlichen Briefwechsel Wilhelm Ostwalds. Vol. I: Briefwechsel mit Ludwig Boltzmann, Max Planck, Georg Helm und Josiah Willard Gibbs*. Ed. by Hans-Günther Körber and Grete Ostwald, Berlin: Akademie Verlag.

Palmerino, Carla Rita (2001). Galileo's and Gassendi's solutions to the *rota Aristotelis* paradox: A bridge between matter and motion theories. In Christoph Lüthy, William R. Newman, & John E. Murdoch (Eds.), *Late medieval and early modern corpuscular matter theories* (pp. 381–422). Leiden. Brill.

Roob, Helmut (1981). *Kurd Laßwitz. Handschriftlicher Nachlaß und Bibliographie seiner Werke*. Gotha: Veröffentlichungen der Forschungsbibliothek.

Schickore, Jutta (2011). More thoughts on HPS: another 20 years later. *Perspectives on Science*, *19*(4), 453–481.

Stadler, August (1883). *Kants Theorie der Materie*. Leipzig: Hirzel.

Weyl, Hermann (1927). Philosophie der Mathematik und Naturwissenschaft. In Alfred Bäumler, & Manfred Schröter (Eds.), *Handbuch der Philosophie*. München Berlin: Oldenbourg.

Wien, Wilhelm (1900). Ueber die Möglichkeit einer elektromagnetischen Begründung der Mechanik. In Johannes Bosscha (Ed.), *Recueil de travaux offerts par les auteurs à H.A. Lorentz: À l'occasion du 25me anniversaire de son doctorat le, 11 Décembre 1900* (pp. 96–107). Le Haye: Nijhoff.

Willmann, Françoise (2012). Leibniz's metaphysics as an epistemological obstacle to the mathematization of nature: the view of a late 19th century neo-Kantian, Kurd Lasswitz. In Ralf Krömer, & Yannick Chin-Drian (Eds.), *New essays on Leibniz reception in science and philosophy of science 1800-2000* (pp. 25–39).

Willmann, Françoise (2018). *Science – Philosophie – Fiction: L'œuvre de Kurd Lasswitz (1848-1910)*. Nancy: Presses Universitaires de Nancy Éditions universitaires de Lorraine.

Wolff, Stefan L. (1994). Von der Hydrodynamik zur kinetischen Gastheorie: Oskar Emil Meyer. *Centaurus*, *37*, 321–348.