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BETWEEN COMMITMENT AND FLEXIBILITY: REVEALING ANTICIPATED REGRET AND ELATION

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Between commitment and flexibility: revealing anticipated regret and elation

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Abstract

This paper introduces and characterizes behaviorally a model of choice over menus of actions in which the individual experiences regret or elation if, after uncertainty resolves, the choice from the menu is inferior or superior to available alternatives. The revealed preference characterization of the model combines two contrasting forces: a preference for having fewer options in order to reduce ex post regret, and a preference for having more options in order to increase ex post elation. An application of the model to information acquisition shows that instrumental information is always valuable. Anticipated elation drives an apparently irrational aversion to delegate choices to an informed agent. Lastly, anticipated elation also generates a desire to include options that will not be selected from the menu, a behavior that is often ascribed to naive time-inconsistency.

KEYWORDS: REGRET, ELATION, FLEXIBILITY, COMMITMENT, INFORMATION
JEL CLASSIFICATION: D01, D91

1 Introduction

Individuals experience regret if after observing the outcome of a decision they realize that a better course of action was possible. Symmetrically, they experience elation when they realize that things could have been worst (Bell, 1982; Loomes and Sugden, 1982). Both regret and elation arise from the ex post comparison between the choice and the available alternatives, therefore they affect the individual's desire to include or exclude options. On one side, an individual anticipating regret can have a desire to have fewer options so as to reduce the possibility that her choice is ex post inferior to the alternatives. On the other side, anticipating elation generates a preference for having more options so as to increase the possibility that her choice is ex post superior to the alternatives. Being opposite forces, it seems that only the relative strength of regret and elation is identifiable by observing revealed preferences, while their absolute identification would require additional

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data. For example, by observing a preference for having a larger menu, we can infer that the value of elation was stronger than the cost of regret, but it is not clear how to separately "measure" the two (see the example in Section 2.2).[1]

In this paper, we solve this identification problem and we show that revealed preferences over menus of (Anscombe-Aumann) acts are sufficient to separately identify regret and elation. We introduce a two-period model of choice in which the value of a menu of acts is determined as if the individual selects an act from the menu before uncertainty resolves, and experiences regret or elation if her choice is ex post inferior or superior to other acts in the menu. We show that anticipated regret is captured by a limited form of a preference for commitment according to which the individual wants to eliminate options that won't be selected from the menu and are unable to generate elation. Symmetrically, anticipated elation is captured by a limited form of preference for flexibility according to which the individual wishes to include options that cannot induce ex post regret, even if these won't be selected from the menu. Strengthening these two conditions characterizes particular cases of the model in which regret, elation, or both disappear. Imposing the full preference for flexibility characterizes the absence of anticipated regret. Assuming a preference for commitment, à la Sarver (2008), characterizes the absence of anticipated elation. Lastly, imposing the strategic rationality of Kreps (1979) characterizes the case in which neither regret nor elation plays a role.

We apply the model to two choice situations in which regret is known to affect behavior: information acquisition and delegation. Differently from standing models of regret that generalize expected utility, in our model instrumental information is always weakly valuable. This happens because information helps making better choices from the menu (it is instrumental), but it does not affect what is compared with the choice to determine ex post regret or ex post elation, namely the ex post best and worst payoffs in the menu. Under particular conditions on the choice problem, the value of information is proportional to well-known measures of uncertainty: the average reduction in entropy or variance between the prior and the posteriors. Concerning delegation, we show that anticipated elation leads to the apparently irrational behavior of avoiding delegation to a perfectly informed agent. Suppose that the individual is facing a choice between a menu and the "optimal selection" from the menu, namely the action that delivers the best payoff from the menu in each state of the world. We interpret choosing the optimal selection as delegating to a perfectly

[1] For a numerical parallelism, it is like identifying two numbers by only knowing their difference.

informed agent. According to our model, the individual can turn down delegation because commitment eliminates costly ex post regret, but it also eliminates the "thrill" of ex post elation. If the anticipated value of elation is larger than the anticipated cost of regret plus the value of optimizing the second-period choice, delegation is suboptimal.

We then study conditions under which the second-period choice is consistent with the interpretation of the model. A new axiom called Sophistication relates first- and second-period choices. It posits that an act is strictly valuable from the first-period point of view if either it will be selected in the second period or it adds "valuable" elation. Thus, the model predicts an elation-driven preference for having unchosen options, acts that are strictly valuable in the first period but are not selected in the second. A preference for unchosen options can rationalize, for example, the choice to buy an expensive health club membership (a monthly pass) even if the subsequent attendance is low (DellaVigna and Malmendier, 2006). Such a behavior is often ascribed to naive dynamic inconsistency, but the present model offers an alternative rationalization. Exercising even once in a month generates more ex post elation after buying the monthly pass rather than after buying the alternative pay-per-visit pass. If the anticipated value of elation in the monthly pass compensates its higher per-visit cost, choosing the monthly pass becomes optimal even if the individual knows that she will not exercise much.

Lastly, we provide a comparative notion of proneness to regret and elation and we characterize it in terms of revealed preferences among menus. Parametrically, an individual j is less regret and elation prone than i, if they share the same prior and the parameters measuring regret and elation for j are equal to that of i but scaled by a common factor smaller than one. Behaviorally, whenever the choices of individual i are aligned with her anticipated second-period choice, hence "not affected by regret and elation", so are the choices of j.

The paper is structured as follows: Section 2 introduces the framework and a motivating example. The model is introduced and characterized behaviorally in Section 3. This section also contains the comparative static analysis. Section 4 discusses applications of the model to information acquisition and delegation. Section 5 introduces the axioms relating first- and second-period choices. Lastly, Section 6 contains a review of the relevant literature, and Appendix A the proofs of the results in the main text.

2 Framework and a motivating example

2.1 Framework and notation

Given a topological space M, we denote by $\Delta(M)$ the set of probabilities defined on the Borel σ-algebra of M and we endow ΔM with the weak* topology. Uncertainty is modeled with a finite set Ω of states of the world. An act is a function from Ω to consequences $f:\Omega\to X$, where X is a compact, convex subset of a topological vector space. A constant acts, i.e. $f(\omega)=x$ for all $\omega\in\Omega$, is identified with the element $x\in X$ that it delivers. We denote by $\mathscr{F}$ the set of all acts, and by $\mathscr{A}$ the set of all nonempty and compact subsets of $\mathscr{F}$ (the menus). For simplicity, the singleton $\{f\}$ is denoted by f, the doubletons $\{f,g\}$ by $f\cup g$, and the menus $\{f,g,h\}$ by $f\cup g\cup h$. As usual, the mixture + of two acts, $\alpha f+(1-\alpha)g$ for all $\alpha\in[0,1]$ is performed state-wise, $(\alpha f+(1-\alpha)g)(\omega)=\alpha f(\omega)+(1-\alpha)g(\omega)$, and the mixture of two menus $\alpha F+(1-\alpha)G$ for any $\alpha\in[0,1]$ is a menu that contains all the mixtures of the elements in F and G, $\alpha F+(1-\alpha)G=\{\alpha f+(1-\alpha)g : f\in F,\ g\in G\}$. The primitive of our approach is a binary relation $\succcurlyeq$ representing the individual's preference over $\mathscr{A}$. The interpretation is that the individual selects a menu expecting to choose an action from the menu before uncertainty resolves. The binary relations $\sim$ and $\succ$ represent indifference and strict preference and are defined from $\succcurlyeq$ in the usual way.

2.2 A motivating example

Consider three different vaccines a,b and c against a disease. There are two states of the world representing uncertainty about the payoffs of each vaccine (e.g. efficacy in preventing the severe form of the disease) and each state occurs with probability $\frac{1}{2}$. The payoffs (in utils) are the following:

	ω_1	ω_2
a	10	0
b	0	8
c	$-\epsilon_1$	$8+\epsilon_2$

Table 1: Vaccine and payoffs

for some $0\le\epsilon_1,\epsilon_2$ with $\epsilon_2-\epsilon_1\le 2$.

Suppose that an individual can decide to get the vaccine a or the vaccine b, while c is currently not recommended for her age group. From the ex ante point of view, the vaccine a gives expected utility 5 and b expected utility 4, so the individual will get a. If the true state of the world turns out to be ω_2, she can suffer regret, since getting b would have been better. If the true state turns out to be ω_1, she can enjoy elation (or rejoicing), "the extra pleasure associated with knowing that, as matters have turned out, (s)he has taken the best decision" (p. 808 in Loomes and Sugden, 1982). It is clear that both preferences $a \succcurlyeq a \cup b$ and $a \cup b \succcurlyeq a$ are rationalizable by a model of anticipated regret and anticipated elation.

Suppose now that the third vaccine c becomes available. Accepting a is still the best action from the ex ante point of view (the expected utility of c is $4 + \frac{1}{2}(\epsilon_2 - \epsilon_1) \leq 5$ by the condition $\epsilon_2 - \epsilon_1 \leq 2$). However, if $\epsilon_2 > 0$ the vaccine c is superior to both a and b in state ω_2. Thus in that state, selecting a in the presence of c generates a larger ex post regret with respect to the case in which the unique alternative was b. If $\epsilon_1 > 0$ and the realized state is ω_1, ex post elation is also larger in the presence of c. Indeed, c would have been the worst choice and selecting a generates more elation with respect to the case in which the unique alternative was b. Hence, from the first-period point of view, having more options has an ambiguous effect on the individual's welfare. The third vaccine c increases costly regret if the state is ω_2, and increases valuable elation if the state is ω_1. If the anticipated larger cost of regret of having c on top of a and b overcomes the additional anticipated elation deriving from having c, she will prefer having only the two vaccines a and b. Otherwise, she will prefer having the three vaccines $a \cup b \cup c$. As a consequence, anticipated regret and elation are consistent with both preferences, $a \cup b \cup c \succcurlyeq a \cup b$ and $a \cup b \succcurlyeq a \cup b \cup c$ and observing one of the two preferences can only suggest that anticipated regret is "stronger" than elation or vice versa. Even if we can observe the preference between a and $a \cup b$, the identification issue remains.

Despite these difficulties, the identification of anticipated regret and elation from revealed preferences is still possible in our setting. Consider the case $\epsilon_1 > 0$ and $\epsilon_2 = 0$. The ex post regret in the two menus $a \cup b$ and $a \cup b \cup c$ is now the same. Indeed, choosing a does not generate regret if the realized state is ω_1, and generates the same regret in $a \cup b$ and $a \cup b \cup c$ if the realized state is ω_2 (since the maximum payoff attainable would be 8, that is equal for b and c). However, if the state ω_2 realizes, choosing a in the presence of c generates more elation than choosing a when b

represents the only available alternative. Therefore, anticipating elation makes the value of $a \cup b \cup c$ larger than that of $a \cup b$, so that we will observe $a \cup b \cup c \succcurlyeq a \cup b$. More importantly, the difference in utility between the value of $a \cup b \cup c$ and $a \cup b$ is completely determined by anticipated elation, thus we can "measure" it directly from preferences.

On the other side, consider the case $\epsilon_1 = 0$ and $\epsilon_2 > 0$. Again $a \succcurlyeq c$, but now c does not add anticipated valuable elation to a menu containing a and b. However, if the realized state is ω_2, choosing a in the presence of c generates more regret with respect to the situation in which b represents the only available alternative. As a consequence, c becomes costly and committing to $a \cup b$ is optimal, hence we will observe $a \cup b \succ a \cup b \cup c$. As before, the difference in utility between the value of $a \cup b$ and $a \cup b \cup c$ is completely determined by anticipated regret, thus we can "measure" it directly from preferences.

We note that allowing for the positive effect of elation generates a violation of the preference for "commitment" of Sarver (2008) that characterizes anticipated regret. His main axiom (adapted to the present setting) posits that, when $f \succcurlyeq g$ and $f \in A$, then $A \succcurlyeq A \cup g$. The preference $f \succcurlyeq g$ implies that g will not be selected from a menu containing f. However, if after uncertainty resolves g turns out to be the optimal choice, the individual will suffer regret. Therefore, committing to A is weakly preferred to $A \cup g$. In the vaccine example $a \succcurlyeq c$ but it is possible that $a \cup b \cup c \succ a \cup b$. Such an elation-driven preference for larger menus is distinguishable from alternative rationales leading to same preference, for example costly learning (Hyogo, 2007; Dillenberger, Lleras, Sadowski, and Takeoka, 2014; Pennesi, 2015; Oliveira, Denti, Mihm, and Ozbek, 2017). In these models, the individual acquires information before choosing an action from the menu. Facing larger menus allows to better tailor the second-period choice to the information. However, in these models, a state-by-state dominated action is never strictly valuable. This is because, regardless of the acquired information, a state-by-state dominated action will never be selected in the second period. For example, if $\epsilon_2 = 0$, the vaccine c is dominated in both states by the vaccine b, therefore, c cannot add value to a menu containing b in a model of costly learning. To the contrary, c can add value in our model of anticipated elation.

3 The model

3.1 Representation

In this section, we introduce the Regret and Elation representation of a binary relation $\succcurlyeq$.

Definition 1. *A Regret and Elation representation (R&E) of* $\succcurlyeq$ *is a tuple* (u, p, γ, θ) *where* $u : X \to \mathbb{R}$ *is non-constant and affine,* $p \in \Delta\Omega$ *and* $\gamma, \theta \geq 0$*, such that* $\succcurlyeq$ *is represented by* $V : \mathscr{A} \to \mathbb{R}$*:*

$$V(A) = \max_{f \in A} \mathbb{E}_p \left[u(f(\omega)) - \gamma \left(\max_{g \in A} u(g(\omega)) - u(f(\omega)) \right) + \theta \left(u(f(\omega)) - \min_{g \in A} u(g(\omega)) \right) \right]. \tag{R\&E}$$

In our interpretation, the individual evaluates a menu A as if her choice from the menu A is made before uncertainty resolves, and she experiences regret or elation if her choice is ex post inferior or superior to other acts in A. Therefore, she considers the maximum expected utility she can get from A subject to the anticipated cost of regret and the anticipated value of elation. Ex post regret in state ω arises from the comparison between the selected act and the best act that could have been selected from A in state ω, $\max_{g \in A} u(g(\omega)) - u(f(\omega))$. Symmetrically, ex post elation in state ω arises from the comparison between the selected act and the worst act that could have been selected from A in state ω, $u(f(\omega)) - \min_{g \in A} u(g(\omega))$. From the first-period point of view, the values of elation, regret and the second-period choice are weighted by the prior p. The parameters γ and θ measure the marginal cost of regret and marginal value of elation, respectively. The R&E representation can be decomposed in three parts:

$$V(A) = \underbrace{\max_{f \in A} \mathbb{E}_p[u(f)]}_{\text{Material value}} \underbrace{- \gamma \left(\mathbb{E}_p \left[\max_{g \in A} u(g) \right] - \max_{f \in A} \mathbb{E}_p[u(f)] \right)}_{\text{Anticipated Regret}} \underbrace{+ \theta \left(\max_{f \in A} \mathbb{E}_p[u(f)] - \mathbb{E}_p \left[\min_{g \in A} u(g) \right] \right)}_{\text{Anticipated Elation}}.$$

The "material value" $\max_{f \in A} \mathbb{E}_p[u(f)]$ represents the maximum expected utility that can be obtained from A by choosing an action based on the available information. The term $\mathbb{E}_p \left[\max_{g \in A} u(g) \right]$ represents the ex ante expected utility of learning the true state of the world before choosing an act from A. Symmetrically, $\mathbb{E}_p \left[\min_{g \in A} u(g) \right]$ represents the expected "worst-case choice" from A. Therefore, the term $R(A, p) = \mathbb{E}_p \left[\max_{g \in A} u(g) \right] - \max_{f \in A} \mathbb{E}_p[u(f)]$ measures the anticipated cost of regret, the average difference between the maximum achievable payoffs and the actual choice. Symmetrically, the term $E(A, p) = \max_{f \in A} \mathbb{E}_p[u(f)] - \mathbb{E}_p[\min_{g \in A} u(g)]$ measures the anticipated value of elation, the average difference between the actual choice and the worst choices.

Remark 1 (Choosing a vaccine). Consider the payoffs in Table 1. According to the R&E model, the value of having two vaccines is $V(a \cup b) = \frac{1}{2}10 + \frac{1}{2}0 - \gamma\left(\frac{1}{2}10 + \frac{1}{2}8 - 5\right) + \theta\,(5-0) = 5 - \gamma(4) + \theta(5)$. By adding the third vaccine c, we obtain $V(a \cup b \cup c) = 5 - \gamma\left(\frac{1}{2}10 + \frac{1}{2}(8+\epsilon_2) - 5\right) + \theta\left(5 - \frac{1}{2}0 - \frac{1}{2}(-\epsilon_1)\right) = 5 - \gamma\left(4 + \frac{1}{2}\epsilon_2\right) + \theta\left(5 + \frac{1}{2}\epsilon_1\right)$. Therefore, $V(a \cup b \cup c) \geq V(a \cup b)$ if and only if $\theta\epsilon_1 \geq \gamma\epsilon_2$. As hinted in section 2.2, if $\epsilon_1 = 0$ and $\epsilon_2 > 0$, $V(a \cup b) \geq V(a \cup b \cup c)$, whereas if $\epsilon_1 > 0$ and $\epsilon_2 = 0$, $V(a \cup b \cup c) \geq V(a \cup b)$.

3.2 Behavioral characterization

This section contains the behavioral restrictions characterizing the R&E model. The first four axioms are standard:

Axiom (Weak Order)**.** *The binary relation $\succsim$ is a weak order and there exist $f, g \in \mathscr{F}$ such that $f \succ g$.*

Axiom (Continuity)**.** *If $F \succ G \succ H$, then there are $\gamma, \gamma' \in (0,1)$ such that $\gamma F + (1-\gamma)H \succ G \succ \gamma' F + (1-\gamma')H$.*

The third axiom is the classical independence axiom: preferences are not reversed after mixing with a common menu:

Axiom (Independence)**.** *For all menus $F, G, H \in \mathscr{A}$ and $\alpha \in (0,1)$, $F \succsim G \iff \alpha F + (1-\alpha)H \succsim \alpha G + (1-\alpha)H$.*

The next axiom restricts the preference $\succsim$ only over singleton menus and corresponds to the Anscombe-Aumann monotonicity axiom:

Axiom (Singleton Monotonicity)**.** *If $f(\omega) \succsim g(\omega)$ for all $\omega \in \Omega$, then $f \succsim g$.*

According to the Anscombe-Aumann Theorem (Anscombe and Aumann, 1963), Weak Order, Continuity, Independence and Singleton Monotonicity are necessary and sufficient for the restriction of $\succsim$ to $\mathscr{F}$ to have a subjective expected utility representation:

Lemma 1. *The binary relation $\succsim$ satisfies Weak Order, Continuity, Independence and Singleton Monotonicity if and only if there are $p \in \Delta\Omega$ and a non-constant $u : X \to \mathbb{R}$ such that $f \succsim g$ if and only if $\mathbb{E}_p[u(f)] \geq \mathbb{E}_p[u(g)]$.*

We now define three binary relations on $\mathscr{A}$ that can be derived from the restriction of $\succcurlyeq$ to singletons and have intuitive interpretations. The first is denoted by $\succcurlyeq_0$ and defined as:

$$A \succcurlyeq_0 B \iff \exists f \in A: \; f \succcurlyeq g, \;\; \forall g \in B.$$

By Lemma 1,

$$A \succcurlyeq_0 B \iff \max_{f\in A} \mathbb{E}_p[u(f)] \geq \max_{g\in B} \mathbb{E}_p[u(g)].$$

The relation $\succcurlyeq_0$ represents the material value of a menu A because it reflects the value of each menu in terms of the anticipated second-period choice, in the spirit of Kreps (1979). In our interpretation, the second-period choice occurs before uncertainty resolves, hence it is based on the prior and the utility u. The preference $f \succcurlyeq g$ then means that g won't be selected from a menu containing f. Therefore, $A \succcurlyeq_0 B$ whenever the second-period choice from A is, in expectation, superior to the second-period choice from B. The relation $\succcurlyeq_0$ reflects the preference of a individual who does not anticipate ex post regret or ex post elation.

Before introducing the second relation, for each menu A and an $\omega \in \Omega$, we denote by $A(\omega) \subseteq X$ the set $A(\omega) = \{g(\omega) \in X : g \in A\}$. The *optimal selection* from A is the act $f_A^* \in \mathscr{F}$ defined as follows $f_A^*(\omega) \in \operatorname{argmax}_{x\in A(\omega)} u(x)$. The act f_A^* is unique (up to indifference) and represents the best action that can be selected from A when knowing the true state of the world. For example, with the payoffs of Table 1, $(a \cup b \cup c)(\omega_1) = \{10, 0, -\epsilon_1\}$, whereas $(a \cup b \cup c)(\omega_2) = \{0, 0, 8+\epsilon_2\}$ and $f^*_{a\cup b\cup c}$ is defined as $f^*_{a\cup b\cup c}(\omega_1) = 10$, $f^*_{a\cup b\cup c}(\omega_2) = 8+\epsilon_2$. The second relation, denoted by $\succcurlyeq^*$, is defined as:

$$A \succcurlyeq^* B \iff f_A^* \succcurlyeq f_B^*$$

It ranks two menus according to the ex ante value of their optimal selection. Indeed, by Lemma 1,

$$A \succcurlyeq^* B \iff \mathbb{E}_p\left[\max_{f\in A} u(f)\right] \geq \mathbb{E}_p\left[\max_{g\in B} u(g)\right].$$

The last relation derived from $\succcurlyeq$ is symmetrical to $\succcurlyeq^*$ and depends on the *worst selection* from A which we denote by $f_*^A \in \mathscr{F}$. This act is defined as $f_*^A(\omega) \in \operatorname{argmin}_{x\in A(\omega)} u(x)$. It represents the worst payoffs that can be achieved in A when knowing the true state of the world. For example, with the payoffs of Table 1, $f_*^{a\cup b\cup c}$ is $f_*^{a\cup b\cup c}(\omega_1) = -\epsilon_1$, $f_*^{a\cup b\cup c}(\omega_2) = 0$. The worst selection defines a binary relation over menus, denoted by $\succsim_*$, in the following way:

$$A \succsim_* B \iff f_*^B \succsim f_*^A$$

A menu A is preferred to a menu B according to $\succsim_*$ if the ex ante value of the worst selection from A is lower than the ex ante value of the worst selection from B. By Lemma 1, $\succsim_*$ is represented by

$$A \succsim_* B \iff \mathbb{E}_p\left[\min_{g\in B} u(g)\right] \geq \mathbb{E}_p\left[\min_{f\in A} u(f)\right].$$

The next axiom implies that the value of a menu is completerly determined by three components: its material value, its optimal and its worst selections.

Axiom (Consistency)**.** *For all $A, B \in \mathscr{A}$, if $A \sim_0 B$, $A \sim^* B$ and $A \sim_* B$, then $A \sim B$.*

The last two axioms contain the behavioral restrictions that capture anticipated regret and anticipated elation, as informally discussed in Section 2.2. The first axiom is a weaker version of the *Dominance* axiom of Sarver (2008):

Axiom (Conditional Dominance)**.** *If $f \succsim g$ and for each $\omega \in \Omega$ there is $h_\omega \in A$ such that $g(\omega) \succsim h_\omega(\omega)$, then $f \in A$ implies $A \succsim A \cup g$.*

Note that by Singleton Monotonicity, an act $h \in \mathscr{F}$ such that $g(\omega) \succsim h(\omega)$ always exists and it is equal to constant act $x \in X$ that pays the worst payoff of g in all states. However, the axiom is much weaker, as each ω could be associated to a payoff h_ω such that $g(\omega) \succsim h_\omega(\omega)$. Since $f \succsim g$, adding g to a menu containing f does not increase its material value, because g is not going to be selected from $A \cup g$. However, g can either increase anticipated elation or anticipated regret. But the latter possibility is excluded because in each state, there is a payoff in A that is "worse" than the payoff of g in that state. Therefore, g cannot increase ex post elation, but only ex post regret. Thus committing to A is weakly preferred to $A \cup g$.

The next axiom has a symmetric interpretation:

Axiom (Conditional Flexibility)**.** *If $f \succsim g$ and for each $\omega \in \Omega$ there is $h_\omega \in A$ such that $h_\omega(\omega) \succsim g(\omega)$, then $f \in A$ implies $A \cup g \succsim A$.*

Again by Singleton Monotonicity, an act h such that $h(\omega) \succsim g(\omega)$ always exists and it is equal to the constant act $x \in X$ that pays the best payoff of g in all states. However, the axiom is much weaker, as each ω could be associated to a h_ω such that $h_\omega(\omega) \succcurlyeq g(\omega)$. Since $f \succcurlyeq g$, adding g to a menu containing f does not increase its material value, because g is not going to be selected from $A \cup g$. As before, g can still increase anticipated elation or anticipated regret. But the former possibility is excluded because in each state, there is a payoff in A that is "better" than the payoff of g in that state. Therefore, g cannot add ex post regret, but only ex post elation. Thus the extra-flexibility of having g in A is valuable. As hinted in Section 2.2, Conditional Flexibility distinguishes our model from models of costly learning (Hyogo, 2007; Dillenberger et al., 2014; Pennesi, 2015; Oliveira et al., 2017). In these models, if $h(\omega) \succcurlyeq g(\omega)$ for all $\omega \in \Omega$, $h \in A$ implies $A \sim A \cup g$. A state-by-state dominated action is never strictly valuable, since regardless of the acquired information, g cannot be strictly superior to h. Conditional Flexibility however, allows for a strict preference $A \cup g \succ A$, if g generates anticipated elation.

The next theorem is the main result of this section and shows that the previous axioms are necessary and sufficient to obtain a R&E representation of $\succcurlyeq$:

Theorem 1. *A binary relation $\succcurlyeq$ satisfies Weak Order, Continuity, Independence, Singleton Monotonicity, Consistency, Conditional Dominance and Conditional Elation if and only if $\succcurlyeq$ has a R&E representation* (u, p, γ, θ).

The sketch of the proof is the following: the axioms Weak Order, Continuity, Monotonicity and Independence are necessary and sufficient to obtain a representation V of $\succcurlyeq$ that is affine with respect to mixture of menus. By construction, the three derived relations $\succcurlyeq_0$, $\succcurlyeq^*$ and $\succcurlyeq_*$ also have affine representations on $\mathscr{A}$. Moreover, Weak Order and Independence ensure that each menu is indifferent to its convex hull, so that we can restrict our attention to convex menus. Since convex menus form a mixture space and V, V_0, V^* and V_* are affine, the Consistency axiom allows us to write V as $V(A) = \alpha_1 \max_{f \in F} \mathbb{E}_p[u(f)] + \alpha_2 \mathbb{E}_p[u(f_A^*)] + \alpha_3 \mathbb{E}_p[u(f_*^A)] + \alpha_4$ for some $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbb{R}$. Because the restriction of V to singletons is represented by the expected utility $\mathbb{E}_p[u]$, the function V can be rewritten as $V(A) = (1+\gamma+\theta) \max_{f \in F} \mathbb{E}_p[u(f)] - \gamma \mathbb{E}_p[u(f_A^*)] - \theta \mathbb{E}_p[u(f_*^A)]$ for some $\gamma, \theta \in \mathbb{R}$. The remaining axioms, Conditional Dominance and Conditional Flexibility, ensure that $\gamma, \theta \geq 0$ as required by the R&E representation.

The last result of this section illustrates the uniqueness properties of the representation in Theorem 1. We say that $\succcurlyeq$ has a *non-degenerate* R&E representation (u, p, γ, θ) if there is a measurable $E \subseteq \Omega$ such that $0 < p(E) < 1$.

Proposition 1 (Uniqueness). *If $(u', p', \gamma', \theta')$ is another R&E representation of $\succsim$, then $u = au' + b$ for some $a > 0$ and $b \in \mathbb{R}$, $p = p'$, $\theta = \theta'$ and, if $\succsim$ is regular, $\gamma = \gamma'$.*

We conclude this section by showing how to identify the parameters γ and θ from revealed preferences. Suppose that $f \succsim g$ and for each $\omega \in \Omega$ there is $h_\omega \in A$ such that $h_\omega(\omega) \succsim g(\omega)$, then $f \in A$ implies $A \cup g \succsim A$. It can be seen that

$$V(A \cup g) - V(A) = \theta \left(\mathbb{E}_p \left[\min_{f \in A} u(f) \right] - \mathbb{E}_p \left[\min_{f \in A \cup g} u(f) \right] \right)$$

Thus, the identification of θ follows from finding an act g and a menu A for which the difference $\mathbb{E}_p \left[\min_{f \in A} u(f) \right] - \mathbb{E}_p \left[\min_{f \in A \cup g} u(f) \right]$ is strictly positive. A similar argument, conditional on $\succsim$ having a non-degenerate representation, allows us to identify γ.

3.3 Special cases

In this section, we strengthen the Conditional Dominance and Conditional Flexibility axioms to characterize three particular cases of the R&E model. The first axiom is the standard Preference for Flexibility and it is a strengthening of Conditional Flexibility:

Axiom (Preference for Flexibility). *If $B \subseteq A$, then $A \succsim B$.*

A preference for having larger menus is intuitively inconsistent with the anticipation of ex post regret. Indeed, having more options increases the probability that the actual choice is worse than the available alternatives. Suppose that $f \succsim g$ and $f \in A$, then Preference for Flexibility implies that $A \cup g \succsim A$. If g does not add material value to A because it will not be selected in the second period, it is possible that it adds valuable ex post elation or costly ex post regret. However, the menu $A \cup g$ is weakly preferred to A regardless of the act g, so also for those g that can generate ex post regret, meaning that regret does not really affect preferences.

Corollary 1 (Elation Only). *If the binary relation $\succsim$ has a R&E representation with $\gamma = 0$ it satisfies Preference for Flexibility. The converse holds if $\succsim$ has a non-degenerate R&E representation.*

The next axiom strengthens Conditional Dominance and it is an adaptation to the present setting of the Sarver (2008)'s main axiom:

Axiom (Dominance). *If $f \succsim g$ and $f \in A$, then $A \succsim A \cup g$.*

If g does not add material value to A because it will not be selected in the second period, it is possible that it adds valuable ex post elation or costly ex post regret. However, the menu A is always weakly preferred to $A \cup g$ regardless of the act g, so also for those g that can generate ex post elation, meaning that elation does not really affect preferences:

Corollary 2 (Regret only)**.** *If binary relation $\succcurlyeq$ has a R&E representation, it satisfies Dominance if and only if $\theta = 0$.*

The last axiom is the Strategic Rationality of Kreps (1979) and it is implied by assuming both the Preference for Flexibility and the Dominance axioms.

Axiom (Strategic Rationality)**.** *If $f \succcurlyeq g$ and $f \in A$, then $A \sim A \cup g$.*

If g does not add material value to A because it will not be selected in the second period, it is possible that it adds valuable ex post elation or costly ex post regret. However, if the menu A is indifferent to $A \cup g$ regardless of the act g, it means that neither ex post regret nor ex post elation can play a role:

Corollary 3. *If the binary relation $\succcurlyeq$ has a R&E representation with $\gamma = \theta = 0$ then it satisfies Strategic Rationality. If the binary relation $\succcurlyeq$ has a R&E representation and satisfies Strategic Rationality, then $\theta = 0$ and, if $\succcurlyeq$ is non-degenerate, $\gamma = 0$.*

3.4 Comparative analysis

In this section, we perform a comparative static analysis aimed at capturing a comparative notion of proneness to regret and elation. We consider two individuals i and j each of which has a preferences represented by a R&E model. The following definition behaviorally describes the notion of being comparatively "less regret and elation prone":

Definition 2. *Suppose that $\succcurlyeq^i$ and $\succcurlyeq^j$ are represented by two R&E $(u_h, p_h, \gamma_h, \theta_h)$ for $h = i, j$. We say that $\succcurlyeq^j$ is* less regret and elation prone *(LREP) than $\succcurlyeq^i$ if and only if for all $A, B \in \mathscr{A}$,*

$$A \succcurlyeq^i_0 B, \text{ and } A \succcurlyeq^i B \implies A \succcurlyeq^j B.$$

An individual j is less regret and elation prone than an individual i if, whenever the preference of the former is aligned with her material preference, so is the preference of the latter. Intuitively, if anticipated regret and elation do not affect the preference of i, then the same is true for j. The next theorem characterizes the parametric restrictions entailed by definition of LREP:

Theorem 2. *Given binary relations $\succsim_h$, $h = i, j$ with R&E representations $(u_h, p_h, \gamma_h, \theta_h)$. Then $\succsim^j$ is LREP than $\succsim^i$ if and only if $u_j = au_i + b$ for some $a > 0$ and $b \in \mathbb{R}$, $p_j = p_i$, and there exists $\kappa \in [0,1]$ such that $\gamma_j = \kappa\gamma_i$ and $\theta_j = \kappa\theta_i$.*

An individual j is less regret and elation prone than i if they share the same prior p and utility u, but the parameters measuring the anticipated marginal cost of regret and marginal value of elation or j are uniformly smaller that those of i.

4 Applications

This section contains two applications of the R&E model to choice situations in which regret often plays a role.

4.1 Regret, elation and the value of information

Information is not always valuable for a regret-averse individual. For example, not knowing the payoffs of alternative unchosen actions reduces if not eliminates ex post regret (e.g. Golman, Hagmann, and Loewenstein, 2017). In this section, we show that in the R&E model information is actually always valuable when it is *instrumental*, namely when it arrives before the second-period choice.

Given the prior $p \in \Delta\Omega$, we denote by $\Gamma(p)$ the family of all experiments that are Bayesian-consistent with respect to p:

$$\Gamma(p) = \left\{ \mu \in \Delta\Delta\Omega : p = \int_{\Delta\Omega} q\mu(q) \right\}.$$

Each element μ of $\Gamma(p)$ represents an "experiment" or "source" of information that induces a posterior $q \in \Delta$ with probability $\mu(q)$, by Bayesian updating the prior p.[2] The condition $p = \int_{\Delta\Omega} q\mu(q)$ derives from the consistency property of Bayesian updating: the average probability of a state

[2] An equivalent approach that is more common in the game theory literature assumes the existence of a set of signals S that are correlated with the true state of the world, together with a function that assigns to each signal a posterior.

according to the posteriors is equal to the prior probability of that state. The next definition introduces the classical Blackwell (1953)'s informativeness order on $\Gamma(p)$:

Definition 3. *Given $\mu, \nu \in \Gamma(p)$ for some $p \in \Delta\Omega$, ν is Blackwell more informative than μ, written $\nu \trianglerighteq \mu$, if*

$$\int_{\Delta\Omega} \phi(q)\,d\nu(q) \geq \int_{\Delta\Omega} \phi(q)\,d\mu(q)$$

for all convex and continuous functions $\phi : \Delta\Omega \to \mathbb{R}$.

An experiment ν is Blackwell more informative than μ if any convex payoff function ϕ prefers ν to μ. To highlight the dependence of V on a belief $q \in \Delta\Omega$, in this section we write $V(A, q)$ in place of $V(A)$. Consider the value of acquiring information through an experiment $\mu \in \Gamma(p)$ *before* selecting an action from the menu A, and denote such a value by $\mathbb{V}(\mu, A)$, then

$$\mathbb{V}(\mu, A) = \int_{\Delta\Omega} V(A, q)\,d\mu(q).$$

It is the average value of A conditional on the posteriors q. The next proposition shows that more informative experiments in the sense of Blackwell are weakly valuable.

Proposition 2. *If $\nu \trianglerighteq \mu$ then $\mathbb{V}(\nu, A) \geq \mathbb{V}(\mu, A)$ for all $A \in \mathscr{A}$.*

The result follows from noting that the material value of a menu increases with the Blackwell's order, because better information makes the second-period choice more likely to pick the best action in A. However, information does not affect what is compared with the actual choice to determine ex post regret or ex post elation, namely the best and worst selections from A. Since both information sources μ and ν are Bayesian-consistent with p, the values of the optimal and worst selections from A are, on average, equal to their values under prior. Therefore, the value of information is completely determined by the additional material value of tailoring the second-period choice to the realized posterior. To formalize this point, for a belief $q \in \Delta\Omega$, we can rewrite the R&E model as $V(A, q) = (1+\gamma+\theta)\max_{f\in A}\mathbb{E}_q[u(f)] - \gamma\mathbb{E}_q[\max_{g\in A} u(g)] - \theta\mathbb{E}_q[\min_{g\in A} u(g)]$, then

$$\mathbb{V}(\nu, A) = (1+\gamma+\theta)\int_{\Delta\Omega} \max_{f\in A}\mathbb{E}_q[u(f)]\,d\nu(q) - \gamma\int_{\Delta\Omega} \mathbb{E}_q[\max_{g\in A} u(g)]\,d\nu(q) - \theta\int_{\Delta\Omega} \mathbb{E}_q[\min_{g\in A} u(g)]\,d\nu(q).$$

Since ν is Blackwell more informative than μ, the material value of A is higher under ν than under μ, that is $\int_{\Delta\Omega} \max_{f\in A}\mathbb{E}_q[u(f)]\,d\nu(q) \geq \int_{\Delta\Omega} \max_{f\in A}\mathbb{E}_q[u(f)]\,d\mu(q)$. On the other side, the two remaining terms are averages of expected utilities of the optimal and worst selections from A, respectively. By the Bayesian consistency property of ν and μ with respect to p,

$$\int_{\Delta\Omega} \mathbb{E}_q[\max_{g\in A} u(g)]\, d\nu(q) = \mathbb{E}_p[\max_{g\in A} u(g)] = \int_{\Delta\Omega} \mathbb{E}_q[\max_{g\in A} u(g)]\, d\mu(q)$$

and

$$\int_{\Delta\Omega} \mathbb{E}_q[\min_{g\in A} u(g)]\, d\nu(q) = \mathbb{E}_p[\min_{g\in A} u(g)] = \int_{\Delta\Omega} \mathbb{E}_q[\min_{g\in A} u(g)]\, d\mu(q).$$

Thus, from the point of view of the information acquisition stage, the values of the optimal and worst selections are identical under ν and μ. It is also interesting to consider the difference $\mathbb{V}(\nu, A) - \mathbb{V}(\mu, A)$ which measures the "value of better information" given a menu A. It follows from the previous argument that

$$\mathbb{V}(\nu, A) - \mathbb{V}(\mu, A) = (1+\gamma+\theta)\left(\int_{\Delta\Omega} \max_{f\in A} \mathbb{E}_q[u(f)]\, d\nu(q) - \int_{\Delta\Omega} \max_{f\in A} \mathbb{E}_q[u(f)]\, d\mu(q)\right) \tag{1}$$

for all $A \in \mathscr{A}$. The term $1+\gamma+\theta$ multiplying the parenthesis in equation (1) shows that better information, in the sense of Blackwell, has a triple effect: it increases the material value of a menu, it reduces the anticipated cost of regret and, it increases the anticipated value of elation. The next examples relates equation (1) with measures of uncertainty that are well-known in the literature (e.g. Frankel and Kamenica, 2019).

Example 1 (Entropy)**.** *Suppose that the space $X \subseteq \mathbb{R}_{++}$ and consider the menu:*

$$A = \left\{ c \in X^{\Omega} : a \leq c(\omega), \sum_{\omega\in\Omega} \pi_\omega c(\omega) = W \right\}, \;\; \textit{for some } W > 0 \tag{2}$$

where $0 < a$. The $c \in X^{\Omega}$ are state-contingent consumption bundles, π_ω is the "state price" and W is the total wealth. Then, if $u(x) = \ln(x)$ and for any $q \in \Delta\Omega$ with $q(\omega) > 0$ for all $\omega \in \Omega$

$$\mathbb{E}_q\left[\max_{c\in A} u(c(\omega))\right] = -H(q) - \mathbb{E}_q[\ln(\pi_\omega)] + \ln(W)$$

where $H(q) = -\sum_{\omega\in\Omega} q(\omega)\ln q(\omega)$ is the entropy of $q \in \Delta\Omega$. It follows that (proof in Appendix A)

$$\mathbb{V}(\nu, A) - \mathbb{V}(\mu, A) = (1+\gamma+\theta)\left(\int_{\Delta\Omega} H(q)\, d\mu(q) - \int_{\Delta\Omega} H(q)\, d\nu(q)\right).$$

If $\mu = \delta_p$, the uninformative experiment, $\mathbb{V}(A, \nu) - V(A, p) = (1+\gamma+\theta) \int_{\Delta\Omega} H(p) - H(q) d\nu(q)$ and the value of information is proportional to the average entropy reduction between the prior and the posteriors.

Example 2 (Variance)**.** *A different measure of uncertainty is represented by the variance of the prior (see Frankel and Kamenica, 2019). It arises in the R&E model when the choice problem requires matching the state under a quadratic utility. Consider $\Omega \subset \mathbb{R}$ and assume that X is the convex closure of Ω, i.e. $X = \overline{co}\,\Omega \subset \mathbb{R}$. Hence $X = [a, b]$ for some $a < b$ and we define the following menu:*

$$A = \left\{ f_x \in \mathscr{F} : f_x(\omega) = x - \omega, \text{ for some } x \in X \right\}. \tag{3}$$

The action f_x maps a state of the world ω to its distance from x. If the utility is quadratic $u(f_x(\omega)) = -(x-\omega)^2$, the optimal action matches the state. Therefore, for any $q \in \Delta\Omega$ (proof in Appendix A):

$$\mathbb{E}_q \left[\max_{f_x \in A} u(f_x(\omega)) \right] = -Var(q)$$

where $Var(q) = \mathbb{E}_q[(\mathbb{E}_q(\omega) - \omega)^2]$ and

$$\mathbb{V}(\nu, A) - \mathbb{V}(\mu, A) = (1+\gamma+\theta) \left(\int_{\Delta\Omega} Var(q) d\mu(q) - \int_{\Delta\Omega} Var(q) d\nu(q) \right).$$

If $\mu = \delta_p$, the uninformative experiment, $\mathbb{V}(A, \nu) - V(A, p) = (1+\gamma+\theta) \int_{\Delta\Omega} Var(p) - Var(q) d\nu(q)$ and the value of information is proportional to the average variance reduction of variance between the prior and the posteriors.

4.2 "Irrational" delegation aversion

A second application of the R&E model concerns an apparently "irrational behavior" driven by anticipated elation. Suppose that the individual can choose between a menu A and the optimal selection from A, namely the act $f_A^* \in \mathscr{F}$ defined above. An interpretation is that f_A^* represents delegating to a a perfectly informed agent. The following proposition, which proof is immediate, shows that the R&E model is consistent with a strict preference for A over f_A^*:

Proposition 3. *For any $A \in \mathscr{A}$, $V(A) > \mathbb{E}_p[\max_{f\in A} u(f)]$ if and only if*

$$\theta\left(\max_{f\in A}\mathbb{E}_p[u(f)] - \mathbb{E}_p[\min_{f\in A} u(f)]\right) > (1+\gamma)\left(\mathbb{E}_p\left[\max_{f\in A} u(f)\right] - \max_{f\in A}\mathbb{E}_p[u(f)]\right).$$

Aversion to delegation cannot occur in the absence of elation $\theta = 0$. This is consistent with the intuition that costly regret prompts individuals to delegate decisions (Steffel and Williams, 2017). However, anticipated elation potentially reverses this result. The simple rationale of Proposition 3 is that, although commitment to the optimal action eliminates the possibility to experience ex post regret, it also eliminates the "thrill" of ex post elation. If the ex ante value of elation in A is larger than the material value of committing to f_A^* plus the reduction in the cost of regret, rejecting delegation becomes optimal.

5 Second-period choice

The interpretation of the R&E model is that second-period choices are made before uncertainty resolves. Therefore, the choice from a menu A consistent with such an interpretation selects the act(s) that maximizes the expected utility $f \mapsto \mathbb{E}_p[u(f)]$ in A:

$$C(A) = \operatorname*{argmax}_{f\in A}\mathbb{E}_p[u(f)].$$

To verify the consistency of first- and second-period choices however, we need to observe both the preferences over menus $\succcurlyeq$ and the choice from the menu. In this section, we introduce a new primitive, a choice correspondence $C : \mathscr{A} \to \mathscr{A}$ such that $C(A) \subseteq A$ representing the observed second-period choices. The following axioms combines $\succcurlyeq$ and C so as to make C consistent with the interpretation of the R&E model.

Axiom (WARP)**.** *For all $A, B \in \mathscr{A}$, if $f, g \in A \cap B$, $f \in C(A)$ and $g \in C(B)$, then $g \in C(B)$.*

WARP is a classic rationality requirement (e.g. Arrow and Fisher, 1974).

Axiom (Closed Graph)**.** *The correspondence C is upper hemicontinuous.*[3]

[3] A correspondence $C : M \to Z$ between topological spaces is upper hemicontinuous at x if, for every neighborhood U of $C(x)$, there is a neighborhood V of x such that $z \in V$ implies $C(z) \subset U$. We say C is upper hemicontinuous on M if it is upper hemicontinuous at every point of M. Upper hemicontinuity and the fact that $A \in \mathscr{A}$ is compact (and closed) implies that the graph of C is closed (see Aliprantis and Border, 2005, Theorem 17.11).

Closed Graph is technical property and implies that the graph of C is closed. The last axiom, called Sophistication, is the substantive requirement as it relates first- and second-period choices. Sophistication reflects a potential "preference for unchosen options" arising from anticipated elation. A preference for unchosen options means that

$$A \cup f \succ A \quad \text{but} \quad f \notin C(A \cup f).$$

The individual has a strict preference for having f available in the first period, even if f is not selected from $A \cup f$ in the second period. Thus, the value of including f in A is purely non-instrumental. Evidence of a preference for unchosen options abounds. For instance, individuals buy expensive gym memberships even if they rarely exercise later on (DellaVigna and Malmendier, 2006). Similarly, individuals often buy products that never use after. In the R&E model, a preference for unchosen options derives from anticipated elation. Even if it will not be selected from the menu, an option that generates anticipated elation is valuable in the first period. The following axiom formalizes this intuition:

Axiom (Sophistication)**.** *For all $f \in \mathscr{F}$ and $A \in \mathscr{A}$,*

$$A \cup f \succ A \implies C(A) = f \;\; \textit{or} \;\; \exists \omega \in \Omega : g(\omega) \succ f(\omega), \; \forall g \in A.$$

Either f will be selected in the second period, or there must be a state ω in which f is the worst action, so that f increases the anticipated elation with respect to A. The next theorem is the main result of this section:

Theorem 3. *Let $\succcurlyeq$ be a preference with a R&E representation (u, p, γ, θ) with u unbounded above. Then a choice correspondence C satisfies WARP, Closed Graph and Sophistication if and only if for any $A \in \mathscr{A}$:*

$$C(A) = \underset{f \in A}{\operatorname{argmax}} \mathbb{E}_p[u(f)].$$

Consider the "match-the-state" decision problem of Example 2. The value of the menu A is given by

$$V(A) = -Var(p) - \gamma(\mathbb{E}_p[\max_{f_x \in A} -(x-\omega)^2] + Var(p)) + \theta(-Var(p) - \mathbb{E}_p[\min_{f_x \in A} -(x-\omega)^2]).$$

It can be seen that $E_p[\max_{f_x \in A} -(x-\omega)^2] = 0$ since knowing the state allows to perfectly match it, and $\mathbb{E}_p[\min_{f_x \in A} -(x-\omega)^2] = -\sum_{\omega \geq \frac{b-a}{2}} (a-\omega)^2 p(\omega) - \sum_{\omega < \frac{b-a}{2}} (b-\omega)^2 p(\omega)$. Therefore,

$$V(A) = -Var(p) - \gamma Var(p) - \theta \left(Var(p) + \sum_{\omega \geq \frac{b-a}{2}} (a-\omega)^2 p(\omega) + \sum_{\omega < \frac{b-a}{2}} (b-\omega)^2 p(\omega) \right).$$

The second-period choice from the menu is $C(A) = f^*_{\mathbb{E}_p[\omega]}$ where $f^*_{\mathbb{E}_p[\omega]}(\omega') = \mathbb{E}_p[\omega] - \omega'$. A preference for unchosen options implies that $V(A) > V(f^*) = -Var(p)$, a condition that holds if

$$\theta \left(\sum_{\omega \geq \frac{b-a}{2}} (a-\omega)^2 p(\omega) + \sum_{\omega < \frac{b-a}{2}} (b-\omega)^2 p(\omega) \right) > (\gamma + \theta) Var(p).$$

Although the second-period choice coincides with the prior's mean, the individual strictly prefers the flexibility of the whole set A if the value of anticipated elation is large enough. Clearly, if $\theta = 0$ (no elation), the previous inequality cannot hold and $V(f^*) \geq V(A)$.

Example 3 (Paying not to go to the gym)**.** *DellaVigna and Malmendier (2006) found that individuals often buy expensive gym memberships (monthly passes with an average price of \$75) while they would save up to \$300 per year by switching to pay-per-visit passes (average price \$10 per visit). Anticipated elation is able to rationalize why people "pay not to go to the gym". In each menu, the monthly pass and the pay-per-visit pass, actions represent how many times the individual will visit the gym in the following month. We denote by $u(n)$ the utility of visiting the gym n times in a month and we normalize $u(0) = 0$. The net utility of choosing n visits under the monthly pass is $u(n) - P$, where P is the price of the pass (e.g. \$75). The net utility of choosing n visits from the pay-per-visit pass is $u(n) - p \cdot n$, where $p < P$ is the price of a single visit (e.g. \$10). For simplicity, we assume $\gamma = 0$ (no regret) and that uncertainty is irrelevant (e.g. there are two states of the world and in one state all actions give zero utility). The individual anticipates that she will visit the gym n^* times during the following month. Choosing n^* visits from the monthly pass menu gives utility $u(n^*) - P$. The worst choice from the monthly pass menu is to make 0 visits, a choice that has utility $u(0) - P = -P$. Thus, the value of a monthly pass is $u(n^*) - P + \theta(u(n^*) - P - (-P)) = (1+\theta)u(n^*) - P$. Choosing n^* visits from the pay-per-visit menu has utility $u(n^*) - p \cdot n^*$ and we assume that 0 visits is also the worst choice in the pay-per-visit menu. The latter choice has net utility $u(0) - p \cdot 0 = 0$. Thus, the value of the pay-per-visit pass menu is $u(n^*) - p \cdot n^* + \theta(u(n^*) - p \cdot n^* - 0) = (1+\theta)(u(n^*) - p \cdot n^*)$.*

A strict preference for the monthly pass holds if

$$P < (1+\theta)p \cdot n^*.$$

Note that, without elation ($\theta = 0$), the monthly pass is preferred to the pay-per-visit pass if $P < p \cdot n^*$, which is the rational choice. With the data of DellaVigna and Malmendier (2006), $75 < 10 \cdot n^*$ holds if $n^* > 7.5$. Thus, in absence of elation, only those who anticipates to visit the gym at least 8 times will buy the monthly pass. The data in DellaVigna and Malmendier (2006) show that individuals buying the monthly pass visit the gym an average of $n^* = 4$ times during the following month. Allowing for anticipated elation ($\theta > 0$), the R&E model can rationalize the data of DellaVigna and Malmendier (2006). Indeed, the inequality $75 < (1+\theta)10 \cdot 4$ holds if $\theta > \frac{7}{8}$ and a strict preference for the monthly pass holds even if the individual knows that she will visit the gym only 4 times. The reason is that the larger cost of the monthly pass makes the worst action from the associated menu (0 visits at cost P) worse than the worst action from the pay-per-visit menu (0 visits at zero cost). Therefore, if the larger anticipated elation in the monthly pass menu compensates the lower net utility of making n^* visits ($u(n^*) - P$ is smaller than $u(n^*) - p \cdot n^*$ for $n^* = 4$), the monthly pass is preferred to the pay-per-visit pass.

The literature proposed different rationalizations of a preference for having unchosen options. In the context of temptation (without objective uncertainty), Ahn, Iijima, Le Yaouanq, and Sarver (2019) consider the preference $A \succcurlyeq C(A)$ as representing naivete about the strength of future temptations. Individuals desire to include normative options because they underestimate the future cost of resisting temptation. In the same context, Kopylov (2012) generalizes Gul and Pesendorfer (2001) and provides a rationalization of a preference for unchosen options due to a rational perfectionism striving. A perfectionist values including normatively optimal actions even if she knows that they will not be selected in the second period. Differently from perfectionism, elation makes an action f valuable if, in at least one state, f is worse than all the actions in A.

We conclude by noting that, as in Sarver (2008), by only observing second-period choices is not possible to determine if the individual anticipates regret and/or elation. Indeed, the choice correspondence C is generated by maximization of an expected utility, whereas models of regret defined over Anscombe-Aumann acts generalize expected utility (e.g. Hayashi, 2008; Stoye, 2011). This observation extends the Sarver's intuition that identifying regret and elation could require choices over future opportunities (i.e. menus).

6 Related literature

The seminal papers of Bell (1982) and Loomes and Sugden (1982) introduced regret and elation in models of choice over lotteries that generalize the expected utility model. Regret aversion can explain the Allais' Paradox as well as other failures of the expected utility, such as the coexistence and gambling and insurance. Axiomatic characterizations of regret models have been proposed by Sugden (1993); Diecidue and Somasundaram (2017); Fishburn (1989); Quiggin (1994), and recently by Lanzani (2019), using revealed preferences over lotteries and by Hayashi (2008); Stoye (2011) using preferences over Anscombe-Aumann's acts. The present work takes a different approach and studies choices over menus of acts in the spirit of Sarver (2008), but in a different framework. In Sarver (2008)'s model, the primitive is a revealed preference over menus of lotteries and the decision maker is uncertain about her future tastes (as in Kreps, 1979; Dekel, Lipman, and Rustichini, 2001). Similarly to our interpretation, the choice from the menu occurs before the resolution of the subjective uncertainty and the individual experiences regret if she realizes that a better choice was available. In the Sarver (2008)'s approach uncertainty is endogenous and not part of the description of the problem, this makes the separate identification of regret and elation difficult. Our primitive is a preference over menus of Anscombe-Aumann's acts, hence uncertainty is exogenous even if the prior is subjective. Our richer setting allows us to identify both elation and regret from revealed preferences. In particular, in a setting with endogenous uncertainty axioms such as Conditional Dominance and Conditional Flexibility cannot be defined. An interesting open question is how to identify elation (and regret) in the context of choice over menus à la Dekel et al. (2001), hence extending the work of Sarver (2008). The present paper shares the primitive, choices over menus of acts, with models of information acquisition (Dillenberger et al., 2014; Oliveira et al., 2017). In these models, flexibility is always valuable because information arrives before the second-period choice. Moreover, the second-period choice is random as it depends on the realized posterior. In our model, the second-period choice is deterministic as it maximizes a subjective expected utility. In a slightly different context, Epstein (2006) provided a model of choice over menus of acts that features a preference for commitment. In particular, his model satisfies Set-Betweenness (if $A \succcurlyeq B$, then $A \succcurlyeq A \cup B \succcurlyeq B$). In the Epstein (2006) model, commitment is valuable to a sophisticated subject who anticipates her non-Bayesian reaction to information. Our model violates Set-Betweenness, for example, with the payoffs of Table 1, $a \succcurlyeq b$ but is possible that $a \succcurlyeq b \succ a \cup b$ (if $1 < \gamma 4 - \theta 5$). Thus, the two models are behaviorally distinguishable

A Proofs

Proof of Theorem 1. The restriction to $\mathscr{F}$ of $\succcurlyeq$ satisfies Weak Order, Continuity, Singleton Monotonicity and Independence. Since $\mathscr{F}$ is a convex subset of a vector space, by the Anscombe-Aumann theorem, this is equivalen to the existence of an affine and cardinally unique function $u : X \to \mathbb{R}$ and a probability $p \in \Delta\Omega$ such that $f \mapsto \mathbb{E}_p[u(f)]$ represents the restriction of $\succcurlyeq$ to singleton menus, i.e. $f \succcurlyeq g$ if and only i f $\mathbb{E}_p[u(f)] \geq \mathbb{E}_p[u(g)]$. Since $\succcurlyeq$ satisfies Weak Order and Independence (and the state-space is finite), a proof similar to that of Lemma S6 part two in the Supplementary Appendix of Dekel, Lipman, Rustichini, and Sarver (2007), shows that $\succcurlyeq$ satisfies Indifference to Randomization, i.e. for all $A \in \mathscr{A}$, $A \sim coA$ where coA is the convex hull of A. Indifference to Randomization allows us to restrict our attention to the family of convex menus in $\mathscr{A}$, that we denote by $\mathscr{A}^c$. The latter family is a mixture space. Therefore, the axioms Weak Order, Continuity and Independence and an application of the Mixture Space Theorem of Herstein and Milnor (1953) are equivalent to the existence of an affine representation of $\succcurlyeq$, that is a function $V : \mathscr{A} \to \mathbb{R}$ such that $V(A) \geq V(B)$ if and only if $A \succcurlyeq B$ and such that $V(\alpha A + (1-\alpha)B) = \alpha V(A) + (1-\alpha)V(B)$ for all $\alpha \in [0,1]$ and all $A, B \in \mathscr{A}^c$. By the uniqueness of the subjective expected utility representation, the restriction to $\mathscr{F}$ of V, must be an affine transformation of $\mathbb{E}_p[u(f)]$, so we can renormalize, if necessary, u such that $V(f) = \mathbb{E}_p[u(f)]$. W.l.o.g. let assume that $[0,1] \subseteq u(X)$. Now define $V_0, V^*, V_* : \mathscr{A}^c \to \mathbb{R}$ as follows: $V_0(A) = \max_{f\in A} \mathbb{E}_p[u(f)]$, $V^*(A) = \mathbb{E}_p[\max_{f\in A} u(f(\omega))]$ and $V_*(A) = \mathbb{E}_p[\min_{f\in A} u(f(\omega))] = -\mathbb{E}_p[\max_{f\in A} -u(f(\omega))]$. The three functionals V_0, V^*, V_* are affine on $\mathscr{A}^c$ and non-constant. Suppose now that for $A, B \in \mathscr{A}$, $V_0(A) = V_0(B)$, $V^*(A) = V^*(B)$ and $V_*(A) = V_*(B)$, the Consistency axiom implies $V(A) = V(B)$. Therefore, Theorem 2 in Fishburn (1984) implies the existence of $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbb{R}$ such that $V(A) = \alpha_1 V_0(A) + \alpha_2 V^*(A) + \alpha_3 V_*(A) + \alpha_4$ for all $A \in \mathscr{A}^c$. By definition, $V(x) = u(x) = \alpha_1 u(x) + \alpha_2 u(x) + \alpha_3 u(x) + \alpha_4$ for all $x \in X$, that implies $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $\alpha_4 = 0$. Therefore, $V(A) = (1-\alpha_2-\alpha_3)V_0(A) + \alpha_2 V^*(A) + \alpha_3 V_*(A)$ for all $A \in \mathscr{A}^c$. Suppose that $f \succcurlyeq g$, $f \in A$ and for each $\omega \in \Omega$, there is $h_\omega \in A$ with $g(\omega) \succcurlyeq h_\omega(\omega)$. Axiom Conditional Dominance implies $V(A) \geq V(A \cup g)$. Since $f \in A$, $V_0(A) = V_0(A \cup g)$. Moreover, $V_*(A) = V_*(A \cup g)$ since for each $\omega \in \Omega$, there is $h_\omega \in A$ with $g(\omega) \succcurlyeq h_\omega(\omega)$. Therefore, $V(A) = (1-\alpha_2-\alpha_3)V_*(A) + \alpha_2 V^*(A) + \alpha_3 V_*(A) \geq V(A \cup g) = (1-\alpha_2-\alpha_3)V_0(A\cup g) + \alpha_2 V^*(A \cup g) + \alpha_3 V_*(A \cup g) = (1-\alpha_2-\alpha_3)V_0(A) + \alpha_2 V^*(A \cup g) + \alpha_3 V_*(A)$ implies $\alpha_2 V^*(A) \geq \alpha_2 V^*(A \cup g)$. Since V^* is monotone with respect to set inclusion $V^*(A \cup g) \geq V^*(A)$, the inequality $\gamma V^*(A) \geq \gamma V^*(A \cup g)$ holds only if $\alpha_2 \leq 0$. Suppose that $f \succsim g$, $f \in A$ and for each $\omega \in \Omega$, there is $h_\omega \in A$ with $h_\omega(\omega) \succsim g(\omega)$. Axiom Conditional Flexibility implies $V(A \cup g) \geq V(A)$. Since $f \in A$, $V_0(A) = V_0(A \cup g)$. Moreover, $V^*(A) = V^*(A \cup g)$ since, for each $\omega \in \Omega$, there is $h_\omega \in A$ with $h_\omega(\omega) \succsim g(\omega)$. Therefore, $V(A) = (1-\alpha_2-\alpha_3)V_*(A)+\alpha_2 V^*(A)+\alpha_3 V_*(A) \leq V(A\cup g) = (1-\alpha_2-\alpha_3)V_0(A\cup g)+\alpha_2 V^*(A\cup g)+\alpha_3 V_*(A\cup g) = (1-\alpha_2-\alpha_3)V_0(A)+\alpha_2 V^*(A)+\alpha_3 V_*(A)$ implies $\alpha_3 V_*(A) \geq \alpha_3 V_*(A\cup g)$. Since V_* is antimonotone with respect to set inclusion $V_*(A\cup g) \leq V_*(A)$, the inequality $\alpha_3 V^*(A) \leq \alpha_3 V_*(A\cup g)$ holds only if $\alpha_3 \leq 0$. To extend $V : \mathscr{A}^c \to \mathbb{R}$ to the whole $\mathscr{A}$, we exploit indifference to randomization. Indeed, $A \sim coA$ for all $A \in \mathscr{A}$, hence we can define $V(A) = V(coA)$ for any $A \in \mathscr{A}$. The extension is well-defined since $A \sim B$ and indifference to randomization imply $coA \sim A \sim B \sim coB$, hence $V(A) = V(B)$. Lastly, defining $\gamma = -\alpha_2$ and $\theta = -\alpha_3$ concludes the proof. □

Proof of Proposition 1. Since $V : \mathscr{A}^c \to \mathbb{R}$ is affine with respect to mixtures of menus, the uniqueness part of the Mixture Space Theorem of Herstein and Milnor (1953) ensures that $V' = \alpha V + \beta$ for some $\alpha > 0$ and $\beta = 0$. This also implies $u' = \alpha u + \beta$. By the uniqueness property of the Subjective expected utility representation $p = p'$. Consider $A = x \cup y$ with $x \succ y$ (they exist by non-triviality of $\succsim$). Then $\theta' = \frac{V'(x\cup y)-u'(x)}{u'(x)-u'(y)} = \frac{\alpha V(x\cup y)-\alpha u(x)}{\alpha u(x)-\alpha u(y)} = \frac{V(x\cup y)-u(x)}{u(x)-u(y)} = \theta$. Since $\succsim$ is non-degenerate, there is a measurable $E \subseteq \Omega$ such that $p(E) \in (0,1)$. Take $x \succ y$, and consider $A = xEy \cup yEx$. Then $f_A^* = x \succ xEy$ and $x \succ yEx$, therefore, $V^*(A) > V_0(A)$. Then $\gamma' = \frac{V_0'(A)-V'(A)-\theta'(V_0'(A)-V_*'(A))}{V'^*(A)-V_0'(A)} = \frac{\alpha V_0(A)-\alpha V(A)-\alpha\theta(\alpha V_0(A)-\alpha V_*(A))}{\alpha V^*(A)-\alpha V_0(A)} = \gamma$.

□

Proof of Corollary 1. A preference $\succsim$ represented by a R&E model with $\gamma = 0$ clearly satisfies Preference for Flexibility. For sufficiency, take $x \succ y$ (they exist by non-triviality). Since $\succsim$ is non-degenerate, there is $E \subseteq \Omega$ such that $0 < p(E) < 1$. Let define an act $f = xEy$, where xEy denotes an act that pays x if $\omega \in E$ and y otherwise. Consider now $f \sim x_f$ (it exists by Lemma 1) and $x \in X$ with $f(\omega) \succsim x$ for all $\omega \in \Omega$. Let define $\Omega_f = \{\omega \in \Omega : f(\omega) \succ x_f\}$ (it is nonempty, since f is non-constant). Then, Preference for Flexibility implies $V(f \cup x \cup x_f) = u(x_f) - \gamma\left(\sum_{\omega\in\Omega_f} p(\omega)u(f(\omega)) - u(x_f)\right) + \theta(u(x_f)-u(x)) \geq u(x)+\theta(u(x_f)-u(x)) = V(x\cup x_f)$ that implies $\gamma \leq 0$, hence $\gamma = 0$. □

Proof of Corollary 2. Necessity is straightforward. For sufficiency, take $x \succ y$ (they exist by non-triviality). Suppose that Dominance holds and $\theta > 0$. Then $V(x) > V(y)$ and Dominance implies $V(x) \geq V(x\cup y) = u(x)+\theta(u(x)-u(y)) > V(x)$ a contradiction. Hence, $\theta = 0$. □

Proof of Corollary 3. Necessity is straightforward. For sufficiency, take $x \succ y$ (they exist by non-triviality). Suppose that Strategic Rationality holds and $\theta > 0$. Then $V(x) > V(y)$ and Strategic Rationality imply $V(x) = V(x \cup y) = u(x) + \theta(u(x) - u(y)) > V(x)$ a contradiction. Hence, $\theta = 0$. Since $\succcurlyeq$ is non-degenerate, there is $E \subseteq \Omega$ such that $0 < p(E) < 1$. Let define $f = xEy$ and consider $f \sim x_f$ (it exists by Lemma 1) and $x \in X$ with $f(\omega) \succcurlyeq x$ for all $\omega \in \Omega$. Let define $\Omega_f = \{\omega \in \Omega : f(\omega) \succ x_f\}$ (it is nonempty, since f is non-constant). Then, Strategic rationality implies $V(f \cup x \cup x_f) = u(x_f) - \gamma\left(\sum_{\omega \in \Omega_f} p(\omega) u(f(\omega)) - u(x_f)\right) + \theta(u(x_f) - u(x)) = u(x) + \theta(u(x_f) - u(x)) = V(x \cup x_f)$ that implies $\gamma = 0$. □

Proof of Theorem 2. For Necessity, we can renormalize u_j to be equal to u_i. Since $p_j = p_i$, the functionals V_0^i, V^{*i} and V_*^i are equal to the functionals V_0^j, V^{*j} and V_*^j, respectively, and we can denote them by V_0, V^*, V_*. The conditions $\gamma_j = \kappa\gamma_i$ and $\theta_j = \kappa\theta_i$ for some $\kappa \in [0,1]$ allow us to rewrite $V^j(A) = \kappa V_0(A) + (1-\kappa)V^i(A)$. Suppose that $A \succcurlyeq_0^i B$ and $A \succcurlyeq^i$, then it follows that $V^j(A) = \kappa V_0(A) + (1-\kappa)V^i(A) \geq \kappa V_0(B) + (1-\kappa)V^i(B) = V^j(B)$, hence $\succcurlyeq^j$ is less regret and elation prone than $\succcurlyeq^i$. For sufficiency, given $f, g \in \mathscr{F}$, $f \succcurlyeq_0^i g$ implies $f \succcurlyeq^i g$, and since $\succcurlyeq^j$ is LREP than $\succcurlyeq^i$, then $f \succcurlyeq^j g$. This means that the restrictions of $\succcurlyeq^i$ and $\succcurlyeq^j$ to singletons are equivalent. By the uniqueness properties of the subjective expected utility model, u_i and u_j are cardinally equivalent and $p_i = p_j$. Then, $(\succcurlyeq_0^i, \succcurlyeq^{*i}, \succcurlyeq_*^i)$ and $(\succcurlyeq_0^j, \succcurlyeq^{*j}, \succcurlyeq_*^j)$ are represented (after renormalization if necessary) by the same functionals that we denote by V_0, V_*, V^*, respectively. By the LREP condition and Theorem 2 in Fishburn (1984). $V^j(A) = \alpha V_0(A) + \beta V^i(A) + \delta$ for some $\alpha, \beta \geq 0$ and $\delta \in \mathbb{R}$. Since $V^j(f) = \mathbb{E}_p[u(f)] = \alpha V_0(f) + \beta V^i(f) + \delta = \alpha \mathbb{E}_p[u(f)] + \beta \mathbb{E}_p[u(f)] + \delta$, it follows that $\alpha + \beta = 1$ and $\delta = 0$. Rearranging yields $V^j(A) = V_0(A) - (1-\alpha)\gamma_i(V^*(A) - V_0(A)) + (1-\alpha)\theta_i(V_0(A) - V_*(A))$, hence the conclusion follows by defining $\kappa = 1 - \alpha$.

□

Proof of Propositon 2. The proof follows from rewriting, for a given posterior $q \in \Delta\Omega$, the R&E model as

$$V(A, q) = (1 + \gamma + \theta)\max_{f \in A} \mathbb{E}_q[u(f)] - \gamma \mathbb{E}_q[\max_{f \in A} u(f(\omega))] - \theta \mathbb{E}_q[\min_{f \in A} u(f(\omega))].$$

Then, after defining $\kappa = 1+\gamma+\theta$:

$$\begin{aligned}
\mathbb{V}(\nu, A) = &\kappa \int_{\Delta\Omega} \max_{f\in A} \mathbb{E}_q[u(f)]\,d\nu(q) - \gamma \int_{\Delta\Omega} \mathbb{E}_q[\max_{f\in A} u(f(\omega))]\,d\nu(q) - \theta \int_{\Delta\Omega} \mathbb{E}_q[\min_{g\in A} u(g(\omega))]\,d\nu(q) \\
=&\kappa \int_{\Delta\Omega} \max_{f\in A} \mathbb{E}_q[u(f)]\,d\nu(q) - \gamma \mathbb{E}_p[\max_{f\in A} u(f(\omega))] - \theta \mathbb{E}_p[\min_{g\in A} u(g(\omega))] \\
\geq&\kappa \int_{\Delta\Omega} \max_{f\in A} \mathbb{E}_q[u(f)]\,d\mu(q) - \gamma \mathbb{E}_p[\max_{f\in A} u(f(\omega))] - \theta \mathbb{E}_p[\min_{g\in A} u(g(\omega))] \\
=&\kappa \int_{\Delta\Omega} \max_{f\in A} \mathbb{E}_q[u(f)]\,d\mu(q) - \gamma \int_{\Delta\Omega} \mathbb{E}_q[\max_{f\in A} u(f(\omega))]\,d\mu(q) - \theta \int_{\Delta\Omega} \mathbb{E}_q[\min_{g\in A} u(g(\omega))]\,d\mu(q) = \mathbb{V}(\mu, A)
\end{aligned}$$

where the second equality follows from the Bayesian consistency condition of ν (i.e. $\nu \in \Gamma(p)$), the inequality follows from $\nu \trianglerighteq \mu$ and the convexity of $q \mapsto \max_{f\in A} \mathbb{E}_q[u(f)]$, the third equality from the Bayesian consistency of μ. □

Proof of the results in Example 1. For a belief q in the interior of $\Delta\Omega \subset \mathbb{R}_+^{|\Omega|}$, the individual solves

$$\max_{c\in A} \sum_{\omega\in\Omega} q(\omega)\ln(c(\omega)) - \lambda\left(\sum_{\omega\in\Omega} \pi_\omega c(\omega) - W\right).$$

The first order conditions are $q(\omega)\frac{1}{c(\omega)} = \lambda\pi_\omega$, clearly $\lambda > 0$ hence the constraint is binding. Therefore, $q(\omega) = \pi_\omega \lambda c(\omega)$. Summing over Ω, $1 = \sum_{\omega\in\Omega} q(\omega) = \lambda \sum_{\omega\in\Omega} \pi_\omega c(\omega) = \lambda W$. Then, $\lambda = W^{-1}$. By the FOC, $c^*_\omega = W\frac{q(\omega)}{\pi_\omega}$. Substituting gives $\sum_{\omega\in\Omega} q(\omega)\ln\left(\frac{q(\omega)}{\pi_\omega}\right) + \ln(W)$. Therefore, $\max_{c\in A} \mathbb{E}_q[\ln(c)] = -H(q) - \mathbb{E}_q[\ln(\pi_\omega)] + \ln(W)$. For the second part, let denote by $\kappa = 1+\gamma+\theta$, then

$$\begin{aligned}
\mathbb{V}(\nu, A) - \mathbb{V}(\mu, A) =&\kappa \int_{\Delta\Omega} \max_{c\in A} \mathbb{E}_q[\ln(c)]\,d\nu(q) - \gamma \int_{\Delta\Omega} \mathbb{E}_q[\max_{c\in A} \ln(c)]\,d\nu(q) - \theta \int_{\Delta\Omega} \mathbb{E}_q[\min_{c\in A} \ln(c)]\,d\nu(q) \\
&-\kappa \int_{\Delta\Omega} \max_{c\in A} \mathbb{E}_q[\ln(c)]\,d\mu(q) - \gamma \int_{\Delta\Omega} \mathbb{E}_q[\max_{c\in A} \ln(c)]\,d\mu(q) - \theta \int_{\Delta\Omega} \mathbb{E}_q[\min_{c\in A} \ln(c)]\,d\mu(q) \\
=&\kappa \left(\int_{\Delta\Omega} \max_{c\in A} \mathbb{E}_q[u(c)]\,d\nu(q) - \int_{\Delta\Omega} \max_{c\in A} \mathbb{E}_q[u(c)]\,d\mu(q)\right) \\
=&\kappa \left(\int_{\Delta\Omega} -H(q) - \mathbb{E}_q[\ln(\pi_\omega)] + \ln(W)\,d\nu(q) - \int_{\Delta\Omega} -H(q) - \mathbb{E}_q[\ln(\pi_\omega)] + \ln(W)\,d\mu(q)\right) \\
=&\kappa \left(\int_{\Delta\Omega} H(q)\,d\mu(q) - \int_{\Delta\Omega} H(q)\,d\nu(q)\right)
\end{aligned}$$

where the first equality follows by definition, the second equality by the Bayesian consistency of μ and ν with p (i.e. $\mu, \nu \in \Gamma(p)$), the third equality by the result above and the last equality again by the Bayesian consistency of μ and ν that allows to cancel the terms $-\mathbb{E}_p[\ln(\pi_\omega)]$ in the two integrals.

□

Proof of the results in Example 2. Given $\mathbb{E}_q\left[\max_{f_x\in A} -(x-\omega)^2\right]$, the FOCs imply that the maximum is attained at $x=\mathbb{E}_q[\omega]$, it follows that

$$\max_{f_x\in A}\mathbb{E}_q[-(x-\omega)^2]=\mathbb{E}_q[-(\mathbb{E}_q[\omega]-\omega)^2]=-Var(q)$$

For the second part, let denote by $\kappa=1+\gamma+\theta$, then

$$\begin{aligned}
\mathbb{V}(\nu,A)-\mathbb{V}(\mu,A)=&\kappa\int_{\Delta\Omega}\max_{f_x\in A}\mathbb{E}_q[u(f_x)]d\nu(q)-\gamma\int_{\Delta\Omega}\mathbb{E}_q[\max_{f_x\in A}u(f_x)]d\nu(q)-\theta\int_{\Delta\Omega}\mathbb{E}_q[\min_{f_x\in A}u(f_x)]d\nu(q)\\
&-\kappa\int_{\Delta\Omega}\max_{f_x\in A}\mathbb{E}_q[u(f_x)]d\mu(q)-\gamma\int_{\Delta\Omega}\mathbb{E}_q[\max_{f_x\in A}u(f_x)]d\mu(q)-\theta\int_{\Delta\Omega}\mathbb{E}_q[\min_{f_x\in A}u(f_x)]d\mu(q)\\
=&\kappa\left(\int_{\Delta\Omega}\max_{f_x\in A}\mathbb{E}_q[u(f_x)]d\nu(q)-\int_{\Delta\Omega}\max_{f_x\in A}\mathbb{E}_q[u(f_x)]d\mu(q)\right)\\
=&\kappa\left(\int_{\Delta\Omega}-Var(q)d\nu(q)-\int_{\Delta\Omega}-Var(q)d\mu(q)\right)\\
=&\kappa\left(\int_{\Delta\Omega}Var(q)d\mu(q)-\int_{\Delta\Omega}Var(q)d\nu(q)\right)
\end{aligned}$$

where the first equality follows by definition, the second equality by the Bayesian consistency of μ and ν with p (i.e. $\mu,\nu\in\Gamma(p)$), the third equality by the result above and the last equality by rearranging terms.

□

Proof of Theorem 3. Necessity is straightforward. For sufficiency, suppose that $f\in A$ is such that $\mathbb{E}_p[u(f)]\geq\mathbb{E}_p[u(g)]$ for all $g\in A$. Take the lowest payoff of f on a non-null state, namely $x=f(\omega)$ such that $p(\omega)>0$ and $f(\omega')\succ f(\omega)$ for all $\omega'\in\Omega\setminus\omega$ with $p(\omega')>0$. Now define $f'_\epsilon\in\mathscr{F}$ to be $f'_\epsilon(\omega')=f(\omega')$ for all $\omega'\in\Omega\setminus\omega$ and $f'_\epsilon(\omega)$ is such that $u(f'_\epsilon(\omega))=u(f(\omega))+\frac{\epsilon}{p(\omega)}$ for $\epsilon>0$ small enough. By continuity of u the act f'_ϵ is well-defined (if $f=x\in X$, the fact that u is unbounded above implies the existence of x' such that $u(x')=u(x)+\epsilon$). Clearly, $\mathbb{E}_p[u(f'_\epsilon)]>\mathbb{E}_p[(f)]$. Moreover, $A\cup f'_\epsilon\succ A$, indeed, $V(A\cup f'_\epsilon)=\mathbb{E}_p[u(f'_\epsilon)]-\gamma[\mathbb{E}_p[\max_{h\in A\cup f'_\epsilon}u(h(\omega))]-\mathbb{E}_p[u(f'_\epsilon)]]+\theta[\mathbb{E}_p[u(f'_\epsilon)]-\mathbb{E}_p[\min_{h\in A\cup f'_\epsilon}u(h(\omega))]]$. By definition, $R(A\cup f'_\epsilon,p)\leq R(A,p)$, because either $h(\omega)\succcurlyeq f'_\epsilon(\omega)$ for some

$h \in A$ or $f'_\epsilon(\omega) \succ h(\omega)$ for all $h \in A$. The former case implies $R(A \cup f'_\epsilon, p) \leq R(A,p)$. In the latter case:

$$\begin{aligned}
R(A \cup f'_\epsilon, p) = & \sum_{\omega' \in \Omega \setminus \omega} p(\omega') \max_{h \in A \cup f'_\epsilon} u(h(\omega)) + p(\omega)[u(f(\omega))] + \epsilon - \mathbb{E}_p[u(f'_\epsilon)\\
= & \sum_{\omega' \in \Omega \setminus \omega} p(\omega') \max_{h \in A} u(h(\omega)) + p(\omega)[u(f(\omega))] + \epsilon - \mathbb{E}_p[u(f)] - \epsilon\\
= & R(A,p).
\end{aligned}$$

Therefore, $V(A \cup f'_\epsilon) > V(A)$, moreover for no $\omega \in \Omega$ and no $g \in A$, $g(\omega) \succ f'_\epsilon(\omega)$. By Sophistication $f'_\epsilon = C(A \cup f'_\epsilon)$. Letting $\epsilon \to 0$, Closed Graph implies $f \in C(A)$. Now take $\mathbb{E}_p[u(g)] < \mathbb{E}_p[u(f)]$ and consider $h \in \mathscr{F}$ such that $g \prec h \prec f$. Let define g' in a similar way we defined f' given f and such that $f \succcurlyeq g'$ and consider $B = \{g, g'\}$. Then $V(B) > V(g)$ and for no $\omega \in \Omega$, $g(\omega) \succ g'(\omega)$, by Sophistication $C(B) = g'$. Now define $A' = A \cup g'$ and note that $f \in \text{argmax}_{h \in A'} \mathbb{E}_p[u(h)]$, that implies $f \in C(A')$. Suppose that $g \in C(A)$, by WARP, $g \in C(A')$, since $f \in C(A') \cap A$ and also $g \in C(B)$ since $g' \in C(A') \cap B$, a contradiction to the fact that $C(A') = g'$. Therefore, $g \notin C(A)$. □

References

Ahn, D. S., R. Iijima, Y. Le Yaouanq, and T. Sarver (2019, 01). Behavioural Characterizations of Naivete for Time-Inconsistent Preferences. *The Review of Economic Studies 86*(6), 2319–2355.

Aliprantis, C. D. and K. C. Border (2005). *Infinite Dimensional Analysis.* Springer.

Anscombe, F. J. and R. J. Aumann (1963). A definition of subjective probability. *Annals of Mathematical Statistics 34*(1), 199–205.

Arrow, K. J. and A. C. Fisher (1974). Environmental preservation, uncertainty, and irreversibility. *The Quarterly Journal of Economics 88*(2), 312–319.

Bell, D. E. (1982). Regret in decision making under uncertainty. *Operations Research 30*(5), 961–981.

Blackwell, D. (1953, 06). Equivalent comparisons of experiments. *Ann. Math. Statist. 24*(2), 265–272.

Dekel, E., B. L. Lipman, and A. Rustichini (2001, July). Representing Preferences with a Unique Subjective State Space. *Econometrica 69*(4), 891–934.

Dekel, E., B. L. Lipman, A. Rustichini, and T. Sarver (2007). Representing preferences with a unique subjective state space: A corrigendum. *Econometrica 75*(2), 591–600.

DellaVigna, S. and U. Malmendier (2006, June). Paying not to go to the gym. *American Economic Review 96*(3), 694–719.

Diecidue, E. and J. Somasundaram (2017). Regret theory: A new foundation. *Journal of Economic Theory 172*, 88–119.

Dillenberger, D., J. S. Lleras, P. Sadowski, and N. Takeoka (2014). A theory of subjective learning. *Journal of Economic Theory 153*(0), 287 – 312.

Epstein, L. G. (2006). An axiomatic model of non-Bayesian updating. *The Review of Economic Studies 73*(2), 413–436.

Fishburn, P. C. (1984). On Harsanyi's utilitarian cardinal welfare theorem. *Theory and Decision 17*(1), 21.

Fishburn, P. C. (1989). Non-transitive measurable utility for decision under uncertainty. *Journal of Mathematical Economics 18*(2), 187–207.

Frankel, A. and E. Kamenica (2019, October). Quantifying information and uncertainty. *American Economic Review 109*(10), 3650–80.

Golman, R., D. Hagmann, and G. Loewenstein (2017). Information avoidance. *Journal of Economic Literature 55*(1), 96–135.

Gul, F. and W. Pesendorfer (2001). Temptation and self-control. *Econometrica 69*(6), 1403–1435.

Hayashi, T. (2008). Regret aversion and opportunity dependence. *Journal of economic theory 139*(1), 242–268.

Herstein, I. N. and J. Milnor (1953). An axiomatic approach to measurable utility. *Econometrica, Journal of the Econometric Society*, 291–297.

Hyogo, K. (2007). A subjective model of experimentation. *Journal of Economic Theory 133*(1), 316–330.

Kopylov, I. (2012). Perfectionism and choice. *Econometrica 80*(5), 1819–1843.

Kreps, D. M. (1979, May). A Representation Theorem for "Preference for Flexibility". *Econometrica 47*(3), 565–577.

Lanzani, G. (2019). Correlation made simple: Applications to salience and regret theory. Technical report.

Loomes, G. and R. Sugden (1982). Regret theory: An alternative theory of rational choice under uncertainty. *The Economic Journal 92*(368), 805–824.

Oliveira, H., T. Denti, M. Mihm, and K. Ozbek (2017). Rationally inattentive preferences and hidden information costs. *Theoretical Economics 12*(2), 621–654.

Pennesi, D. (2015). Costly information acquisition and the temporal resolution of uncertainty. *Journal of Mathematical Economics 60*, 115–122.

Quiggin, J. (1994). Regret theory with general choice sets. *Journal of Risk and Uncertainty 8*(2), 153–165.

Sarver, T. (2008). Anticipating regret: Why fewer options may be better. *Econometrica 76*(2), 263–305.

Steffel, M. and E. F. Williams (2017, 07). Delegating Decisions: Recruiting Others to Make Choices We Might Regret. *Journal of Consumer Research 44*(5), 1015–1032.

Stoye, J. (2011). Axioms for minimax regret choice correspondences. *Journal of Economic Theory 146*(6), 2226–2251.

Sugden, R. (1993). An axiomatic foundation for regret theory. *Journal of Economic Theory 60*(1), 159–180.

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