Augmented reality for conceptualizing covariation through connecting virtual and real worlds

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The Augmented Reality learning environment connects both real-world and virtual representations simultaneously. This contribution investigates how students coordinate conceptual facets belonging to the two worlds when conceptualizing the time-distance relationship of a cube sliding down along an inclined plane.

Keywords: Augmented reality, covariational reasoning, conceptual facets, conceptual understanding, functional thinking.

Introduction

One of the goals of teaching and learning mathematics is to allow students to quantify, interpret, and understand the world. To achieve this goal, curricula worldwide integrate modeling activities to engage students in mathematizing real-world phenomena. Since the mathematical representation and the real world are not explicitly connected, and due to the lack of tools that juxtapose both worlds, the link between the real phenomenon and the mathematical representations is made when both worlds remain separate. Recently, by using dedicated augmented reality (AR) technology, the juxtaposition of the two worlds has become possible.

So far, the assumption is that juxtaposing the two worlds may help students mathematize the real-world phenomena and endow them with meanings (Swidan et al., 2019). In this contribution, we want to shed light on the specific role of AR in how students interpret a real phenomenon. So, this study aims to understand how students use mathematical representations to analyze a real phenomenon and what mathematical knowledge they use for this purpose. To achieve this goal, we used Prediger and Zindel's (2017) model of conceptual facets to understand the transition between the two worlds and the covariational reasoning framework (Thompson & Carlson, 2017) to understand what knowledge students use. Understanding the ways in which students transit from one world to another carries out theoretical and methodological implementation. Theoretically, this study may shed light on the role of AR technology in mathematizing real-world phenomena. Methodologically, we modified Prediger and Zindel's (2017) model to fit the analysis of modeling activities; this modification may allow the use of the model by other researchers to analyze the learning resulting from the transition between the two worlds.

Theoretical framework

Conceptual understanding can be intended as a dense network of pieces of knowledge called conceptual facets (Hiebert & Carpenter, 1992). This network of facets can be built by compacting facets into denser concepts (Aebli, 1981). Prediger and Zindel (2017) propose a model of conceptual facets of understanding functional relationships (Figure 1) in tune with these theoretical ideas. The model is based on a definition of the conceptual understanding of functional relationships as “the ability to adopt different perspectives in different representations and to coordinate them by addressing the facets” (Prediger & Zindel, 2017, p. 4166). Upwards and downwards movements in the facet model reveal the processes of compacting and unfolding the conceptual facets. The following example outlines the functioning of the model. Thinking of the law of a certain motion of a car, a student claiming that a certain function tells for which time you have a certain distance traversed by the car is identifying correctly which are the ||dependent|| and ||independent variable|| and is unfolding the ||functional dependency|| on the medium level of the facet model. The words between || || are the corresponding facets that will be marked in the analytical model.

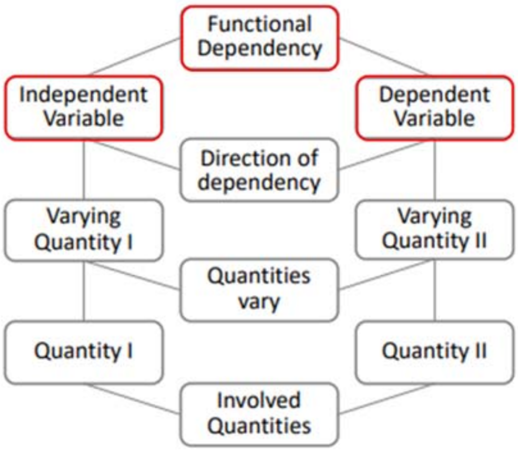

Figure 1: Facet model

Our second theoretical perspective is covariational reasoning, which considers the ability to hold a sustained image, in the mind, of two quantities' values (magnitudes) that change simultaneously (Thompson & Carlson, 2017). The covariation concept is deeply inherent in several dynamic phenomena, such as filling a bottle with water (volume-height) or a rolling ball (time-distance). Arzarello (2019) and Bagossi (2022) further elaborated the idea of covariational reasoning to include what they call “second-order covariation”, which considers covarying between quantities and mathematical objects. For example, considering the changes between a car's velocity (quantity) and the graph of motion (mathematical object) is a form of second-order covariational reasoning.

AR is an innovative technology that combines layers of virtual objects and information about physical objects from the real world, such as texts, images, graphs, etc. This creates an environment in which virtual and real objects coexist (Azuma, 1997). In addition, AR allows uncovering invisible mathematical details embedded in dynamic phenomena and presenting them simultaneously. This suggests opportunities for real-world modeling phenomena, where the covariation concept is inherent and creates meanings through combining both real and virtual worlds.

The research question guiding this study is: How do students coordinate conceptual facets of the real and virtual world representations as they learn covariation in an AR environment?

Method

The learning experiment here analyzed was conducted with a group of three 11th graders, Sagi, Noam, and Alex, from Israel. The experiment explores the time-distance relationship as a cube slides down along an inclined plane, the so-called Galileo experiment. The graph representation, as well as the table of values with numerical measurements of time and distance (virtual world in the facets model), were layered over the real inclined plane with the sliding cube (real-world in the facets model). The group worked on a corresponding task sheet. Data were collected through video recordings documenting all actions and interactions in the learning environment. Since data provided by AR and observed by the students through their AR headset is not available or seen by the researchers, it was mirrored on a screen to allow us better understand students' observations and explanations (Figure 2).

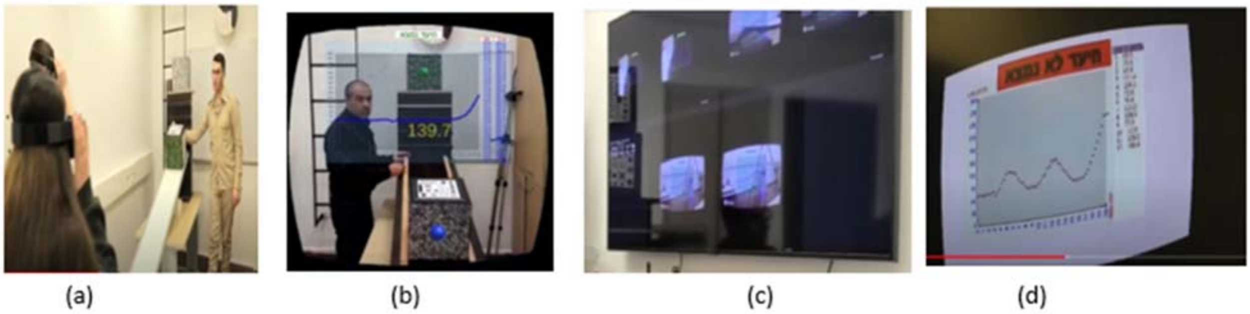

Figure 2: **(a) Galileo experiment, (b) Galileo experiment as seen through AR headset, (c) Screen mirroring students' AR headsets, (d) Mirrored data**

To analyze the data, we identified episodes revealing forms of covariational reasoning and documenting students while combining both real and virtual worlds. Eventually, we used the Prediger and Zindel (2017) multi-facet model to analyze students' conceptualization of covariational reasoning. The analysis of students' reasoning is visualized by the facet model in Figures 3, 4, and 5. Facets referred to the real-world phenomenon are framed in blue, while the ones referred to the virtual world are framed in red. Connectors denote connections between the two worlds. In the analysis, the addressed facets in the model are remarked by using || ||.

Results

This episode illustrates how Alex connects the real-world phenomenon with the virtual representation.

1	Alex:	As the height (of the inclined plane) is greater, then the faster the cube speed is, and then it passes the distance in a shorter time than a lower height.
2	Sagi:	From second to second, the distance simply increases.
3	Alex:	Yeah, like, it (the cube) takes less time to pass it (distance) because the inclination is... more drastic.
4	Sagi:	If I'm not wrong, the distance... the difference between the distances from point to point is greater at the top (of the graph), right?
5	Alex:	Yes, it sounds correct.
6	Noam:	At a specific time, the cube traveled a certain distance, ... mm... while the plane's inclination... brought to… as if... it (cube) had an acceleration that was growing... dependent on time... which have been created...
7	Alex:	The acceleration affects the graph that is created…

Alex in [1] shifts from the real world to the virtual one while demonstrating covariational reasoning. His words "as…then…" in [1] suggest that he describes a ||functional dependency|| within the real world. It seems that Alex is aware of the ||independent variable|| (height), and the ||dependent variable|| (speed). He also outlines the ||direction of dependency||: "As the height is greater, then the faster the cube speed is". After that, Alex focuses on the virtual representations, and similarly, he describes a ||functional dependency|| between time and distance. In the same utterance, he connects them with the height of the inclined plane. This connection suggests that Alex considers the height as a quantity, which varies in the real world: "It (cube) passes the distance in a shorter time compared with a lower height".

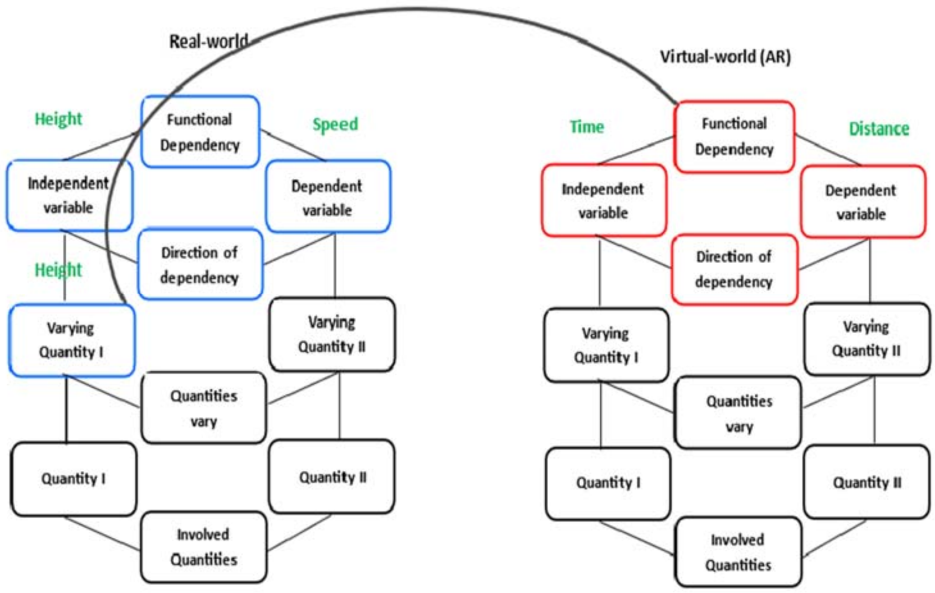

Figure 3: Facet model for utterance [1]

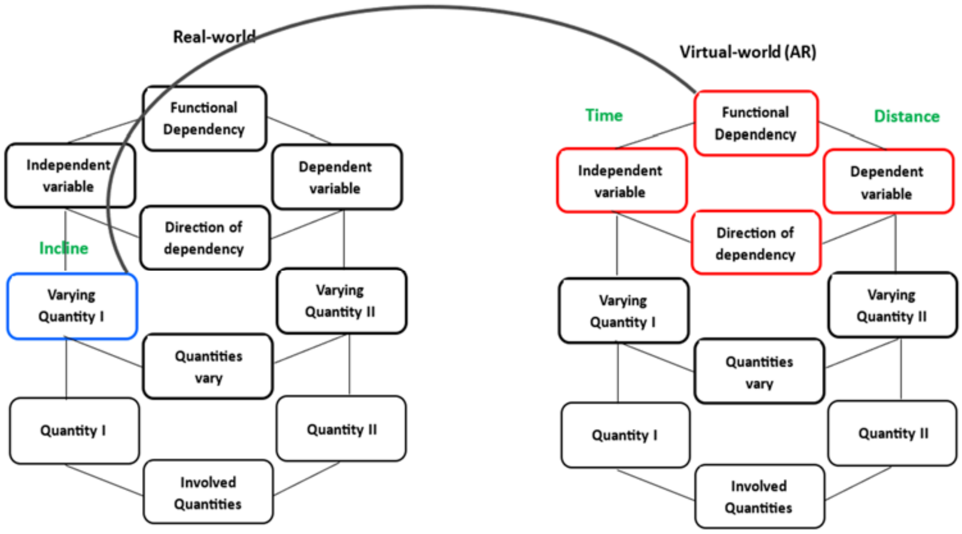

Figure 4: Facet model for utterance [3]

Similarly, in [3], Alex refers to the inclination as a ||varying quantity|| in the real world. Then, he shifts his focus of attention to the virtual world. There he identifies ||independent|| and ||dependent variables||, and describes explicitly the ||functional dependency|| and the ||direction of dependency|| between time (independent variable) and distance (dependent variable) when the cube rolled down "it takes the (cube) less time to pass it (distance) because the inclination is ... more drastic".

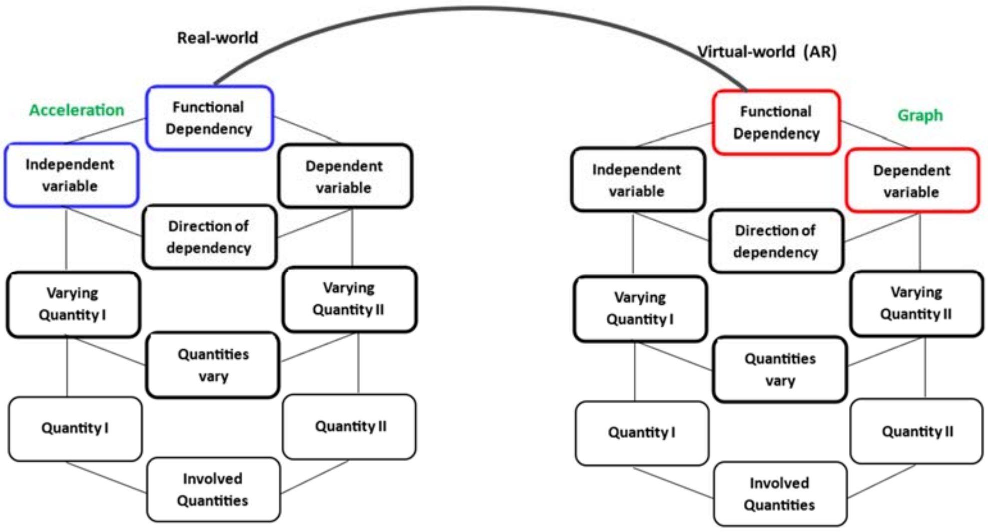

Figure 5: Facet model for utterance [7]

In [7], Alex refers to acceleration as an ||independent variable|| in the real world. Then, he shifts to the virtual world and refers to the graph as a ||dependent variable||: "The acceleration affects the graph that is created". Alex identifies a ||functional dependency|| between cube acceleration and the graph that represents the real phenomenon. The verb 'affect' used by Alex suggests that he expresses the functional dependency as second-order covariational reasoning between the acceleration and the graph.

Final remarks

As visualized through the facets model, Alex's thinking process indicates the frequent transition between real-world and virtual-world representation facets. He demonstrates the ability to describe functional dependency between dependent and independent variables that he explicitly addresses. In addition, he also refers to the direction of dependency when he describes the relations between the variables. Such description also relates to the changes in the varying quantities of the variables. This path of translations among several conceptual facets and unfolding relationships on lower levels of the facet model are indicators of a developed conceptual understanding. Coordinating conceptual facets of the virtual and real-world representations is attributed to the potential of AR technology which brings both worlds to coexist (Azuma, 1997). Juxtaposing virtual representations with the real-world environment seemingly afford the meanings making of covariation concept in learning processes as conjectured by Swidan et al. (2019). The analysis presented in this contribution aims at being a preliminary attempt to adapt the conceptual facets model to a learning environment offering a coexistence of two worlds, the real and the virtual one. Indeed, instead of addressing the specific representations involved, we focused on the facets belonging to the two worlds. The use of this model revealed two main issues: first, the difficulty of analyzing rich covariational reasoning involving more than two quantities [1-3]; second, the inadequacy of the model to describe forms of second-order covariational reasoning in which not only quantities are involved but also mathematical objects [7]. Both these issues will be object of our future research.

Acknowledgment

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