



2–7 February 2022, Bozen-Bolzano, Italy

Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education

Editors: Jeremy Hodgen, Eirini Geraniou, Giorgio Bolondi, Federica Ferretti

Organised by: Free University of Bozen-Bolzano

Year: 2022

Editors

Jeremy Hodgen and Eirini Geraniou

IOE, UCL's Faculty of Education and Society, University College London, UK

Giorgio Bolondi and Federica Ferretti

Faculty of Education, Free University of Bozen-Bolzano, Italy

Editorial Board

Mario Sánchez Aguilar; Linda Marie Ahl; Chiara Andrà; Jonas Bergman Ärlebäck; Fatma Aslan-Tutak; Michal Ayalon; Anna Baccaglioni-Frank; Anette Bagger; Berta Barquero; Angelika Bikner-Ahsbabs; Irene Biza; Laura Black; Nina Bohlmann; Hilda Borko; Gülay Bozkurt; Lina Brunheira; Orly Buchbinder; Gözde Kaplan Can; Martin Carlsen; Susana Carreira; Renata Carvalho; Clelia Cascella; Aurélie Chesnais; Maria Chimoni; Renaud Chorlay; Anna Chronaki; Alison Clark-Wilson; Alf Coles; Jenny Christine Cramer; Annalisa Cusi; Nelleke Den Braber; Javier Díez-Palomar; Paul Drijvers; Viviane Durand-Guerrier; Andreas Ebbelind; Kirstin Erath; Nataly Essonnier; Eleonora Faggiano; Fiona Faulkner; Janne Fauskanger; Carla Finesilver; Ignasi Florensa; Birte Friedrich-Pöhler; Marita Eva Friesen; Daniel Frischmeier; Michael Gaidoschik; Ingólfur Gíslason; Inés M^a Gómez-Chacón; Orlando Rafael Gonzalez; Alejandro González-Martín; Gilbert Greefrath; Rikke Maagaard Gregersen; Helena Grundén; Ghislaine Gueudet; Tanja Hamann; Çiğdem Haser; Tracy Helliwell; Dave Hewitt; Kees Hoogland; Mark Hoover; Veronika Hubeňáková; Paola Iannone; Jenni Ingram; Britta Eyrih Jessen; Heather Johnson; Seçil Yemen Karpuzcu; Ronnie Karsenty; Sibel Kazak; Beth Kelly; Cecilia Kilhamn; Boris Koichu; Ulrich Kortenkamp; Jenneke Krüger; Macarena Larrain; Aisling Leavy; Nicolas Leon; Esther Levenson; Peter Liljedahl; Andrea Maffia; Božena Maj-Tatsis; Francesca Martignone; Michela Maschietto; Janka Medova; Siún Nic Mhuiri; Morten Misfeldt; Simon Modeste; Miguel Montes; Hana Moraova; Francesca Morselli; Reidar Mosvold; Andreas Moutsios-Rentzos; Edyta Nowinska; Kate O'Brien; Antonio M. Oller-Marcén; Shai Olsher; Alik Palatnik; Chrysi Papadaki; Caterina Primi; Luis Radford; Maryna Rafalska; Chris Rasmussen; Elisabeth Rathgeb-Schnierer; David A. Reid; Jorunn Reinhardtsen; Sebastian Rezat; Miguel Ribeiro; Ornella Robutti; Helena Roos; Bettina Rösken-Winter; Sabrina Bobsin Salazar; Libuse Samkova; Piers Saunders; Judy Sayers; Florian Schacht; Petra Scherer; Stanislaw Schukajilow; Abdel Seidouvy; Anna Shvarts; Nathalie Sinclair; Michele Stephan; Pernille Bødtker Sunde; Osama Swidan; Michal Tabach; Athina Thoma; Daniel Thurm; Melih Turgut; Marianna Tzekaki; Behiye Ubuz; Beatrice Vargas; Michiel Veldhuis; Olov Viirman; Hanna Viitala; Katrin Vorhölter; Christof Weber; Mei Yang; Stefan Zehetmeier.

Publisher

Free University of Bozen-Bolzano, Italy and ERME

ISBN 9791221025378

© Copyright 2022 left to the authors

Recommended citation for the proceedings

Hodgen J., Geraniou E., Bolondi G., & Ferretti, F. (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*. Free University of Bozen-Bolzano, Italy and ERME.

Recommended citation for single entries in the proceedings

[Authors](#). (2022). [Title of paper/poster](#). In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*. (pp. xxxx-yyyy). Free University of Bozen-Bolzano, Italy and ERME.

Teachers' professional development: a cultural matter. How to describe cultural contexts?

Riccardo Minisola¹ and Carola Manolino¹

¹University of Turin, Italy; riccardo.minisola@unito.it, carola.manolino@unito.it

Teachers' professional development in collaborative contexts is a growing trend in Mathematics Education research. Particularly, Japanese Lesson Study has seen a great focus on its dissemination around the world. Research shows that Japanese culture is one of the main reasons that makes Lesson Study effective: understanding Lesson Study means understanding the cultural context in which it originated. We attempt to describe the Japanese and the Italian cultural contexts. Since there exists no consensus on what is essential to analyse in order to understand a cultural context, we present two approaches to this description, and consider some advantages and shortcomings. We hope to sprout discussion on the possibility to create guidelines for describing cultural contexts, shared by the community of researchers in Mathematics Education: awareness of beliefs, identity and practice is a sensitive element for successful mathematics teacher professional development.

Keywords: Cultural context, culture, Japan, Lesson Study, Italy.

Introduction

Over the last 20 years, teachers' professional development (TPD) in collaborative contexts has received ever-growing attention from the community of researchers in Mathematics Education (Robutti *et al.*, 2016), and a recent survey by Bakker and colleagues (2021) confirmed the trend. Among the many different collaborative methodologies for TPD in Mathematics and Mathematics Education, Lesson Study has seen a great deal of research focused on its dissemination (i.e.: Huang *et al.*, 2019; Huang & Shimizu, 2016; Quaresma *et al.*, 2018). Lesson Study (LS) is a collaborative TPD methodology, part of the Japanese paradigmatic infrastructure (Winsløw, 2011) since the 1880s, focused on the co-responsibility in the lesson-planning process of the involved teachers and knowledgeable others (Huang *et al.*, 2019).

LS is also the focus of the authors' doctoral dissertations (in progress), which also aim at introducing LS in the context of Italian TPD. During YESS11, TWG1 dedicated to *teacher education and professional development* saw four out of eleven papers focused on LS in different contexts (Italy, Mozambique, Portugal, and Switzerland). In the discussions around the four papers, one question resulted relevant: what is the *cultural context* in which the research takes place? Indeed, being aware of their cultural context is one of the essential competences of mathematics teachers (and researchers) to gain awareness of their beliefs, their identity and their professional practice and to develop their teaching knowledge: a sensitive element for successful mathematics teacher professional development, and a demand evermore necessary and therefore not negligible (Andrews, 2010). Yet, issues arise when we try to address such demand: in the following, we will attempt to describe the Japanese and Italian cultural and institutional context and discuss such issues.

Literature review

Unsuccessful attempts at translating LS outside of its cultural context (Demir *et al.*, 2012; Fernandez *et al.*, 2003) suggest that, if LS is not introduced in a cultural context with proper consideration to the differences with the Japanese cultural context, it might be rejected by the institutions. Ebaegu & Stephens (2014) suggest to address the cultural compatibility of LS. A number of scholars proposed different theoretical lenses to analyse why LS is so widespread in Japan (i.e.: Krainer, 2011; Lewis,

2016). We can suppose that LS exists because of the Japanese culture, and Japanese culture is one of the main reasons why LS is effective: LS is a cultural activity (Stigler & Hiebert, 2016). The question arises if maintaining the efficacy of LS across different cultural contexts is feasible.

Despite the rising awareness on the importance of studying cultural contexts and identities to contextualize global trends in Mathematics Education (Bakker *et al.*, 2021), the majority of reports on LS around the world seems to depict LS as an isolated practice in the Japanese panorama of TPD practices (Miyakawa & Winsløw, 2019) and seemingly ignores that the Japanese definition of LS is not as clear cut as the American one (Miyakawa & Winsløw, 2013). This suggests that “to develop a deeper understanding of Lesson Study in a post-modern global world, there is a need to seek views beyond those presented from an American perspective” (White & Lim, 2008, p. 915).

Understanding LS means understanding the context in which it originated. At the same time, to introduce LS in a new context, it is essential to know the TPD practices already in existence (Miyakawa & Winsløw, 2013). Yet, there exist no consensus on what is essential to analyse in order to understand a cultural context. What is *culture*? This paper has two aims: to provide arguments to the importance of understanding the cultural contexts involved in the research, and to provide a currently-missing description of the Italian TPD context in the English language. We provide a tentative analysis of the Japanese and the Italian cultural and institutional contexts to guide future studies on LS in Italy, and we also hope to sprout discussion on the possibility to create guidelines for describing cultural contexts that might be shared by the community of researchers in Mathematics Education.

Theoretical Framework

Culture may be described as “any aspect of the ideas, communications, or behaviours of a group of people which give them a distinctive identity and which is used to organise their internal sense of cohesion and membership” (Scollon & Scollon, 1995, p. 127) or as “[t]he system of shared beliefs, values, customs, behaviours, and artefacts that the members of society use to cope with their world and with one another, and that are transmitted [...] through learning” (Bates & Plog, in Freimuth, 2006, p. 2). Anthropologists have not reached a shared definition (Spencer-Oatey, 2012), and proposing one would be outside of our expertise. In fact, our aim is not to propose our own definition but to observe how existing approaches and definitions may interact with learning and teaching processes in Mathematics, particularly in TPD. It is a facet of our doctoral research, especially within a semiotic context (Manolino, 2021). Here we rely on a *popular* understanding of what *culture* is, as the definition is not central to this paper. In the following, we propose two different approaches to the definition and description of *culture* and *cultural contexts*: the first one is synthetic, the second one is descriptive. We hope to show some advantages and shortcomings of each of them, which should provide a mean to engage in this discussion.

Hofstede’s Dimensions of Culture

Hofstede defines culture as “the collective mental programming of the people in an environment. Culture is not a characteristic of individuals; it encompasses a number of people who were conditioned by the same education and life experience” (in de Mooij, 2010, p. 48). Hofstede identifies basic value orientations of a certain national culture. These values “are broad preferences for a certain state of affairs (e.g., preferring equality over hierarchy) [...] transmitted by the environment [...] shaped by the time we hit 10-12 years of age” (<https://news.hofstede-insights.com/news/what-do-we-mean-by-culture>) described in five dimensions, scored over 100 points, and represent:

- *Power Distance* Index: “the extent to which less powerful members of a society accept and expect that power is distributed unequally” (de Mooij, 2010, p. 75). The higher the score, the more hierarchical a society is.
- *Individualism* vs Collectivism: “[...] people looking after themselves and their immediate family only, versus people belonging to in-groups that look after them in exchange for loyalty” (de Mooij, 2010, p. 77). Higher scores indicate individualistic values.
- *Masculinity* vs Femininity: “The dominant values in a masculine society are achievement and success; the dominant values in a feminine society are caring for others and quality of life” (de Mooij, 2010, p. 79). Lower score indicates a feminine society. Please note that this label is problematic as it reinforces harmful gender stereotypes, and in the following we will use the alternative “*Tough* vs *Tender*”.
- *Uncertainty Avoidance* Index: “[...] the extent to which people feel threatened by uncertainty and ambiguity and try to avoid these situations” (de Mooij, 2010, p. 82). The higher the score, the less open to changes is a society.
- *Long-Term* vs Short-Term Orientation: “[...] the extent to which a society exhibits pragmatic future-oriented perspective rather than a conventional historic or short-term point of view” (de Mooij, 2010, p. 85). Lower scores point to a society that prefers short-term planning.

One peculiarity of Hofstede’s dimensions is that “[t]he country scores on the dimensions are relative, in that we are all human and simultaneously we are all unique. In other words, culture can only be used meaningfully by comparison” (<https://hi.hofstede-insights.com/national-culture>).

Levels of Co-Determination

We start from “the notion of [teaching] “practice” as a link between culture [...] and the larger cultural contexts” (Hatano & Inagaki, 1998, p. 80). In the Anthropological Theory of the Didactic, practices are described in terms of praxeologies: the know-how (*praxis*) and the know-why (*logos* - the discourses that justify the know-how) related to a task. Chevallard (1985) suggests that teachers’ praxeologies are shaped by a plurality of agents (politicians, scholars...) and historical or institutional conditions that defines the boundaries of what teachers can or cannot do, their *noosphere* (the sphere of those who thinks). Chevallard (2002) pictures the complex relations of the factors influencing teachers’ praxeologies, which are influenced not only by the teachers’ decision, but at a higher level by the society in which the teachers and students are immersed, as shown in Figure 1.

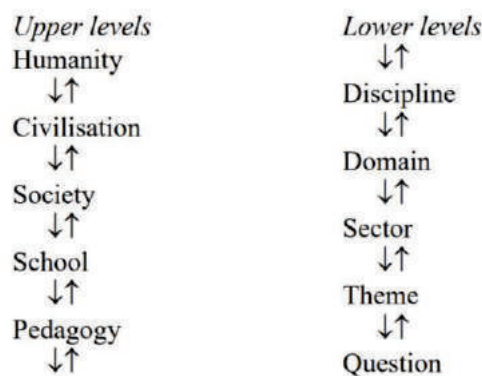


Figure 1: Scale of levels of co-determination (Florensa *et al.*, 2018, p. 5)

Similarly, we suggest that professional development practices are influenced by the context in which teachers and their educators are immersed. It is important to notice that Chevallard’s framework does not use the term *culture*.

Context Analysis

Hofstede’s Cultural Dimensions

Ebaeguin and Stephens (2014) suggest that comparing Hofstede’s scores might be a starting point for studying the introduction of LS in Australia, as they connect the efficacy of LS in Japan to the Japanese scores. The scores for Japan and Italy according to Hofstede’s cultural dimensions can be freely collected from the website <https://www.hofstede-insights.com/product/compare-countries/> and are shown in Table 1. The labels are simplified because of space constraints.

Table 1: Hofstede’s score for cultural dimension for Italy and Japan

| | Power Distance | Individualism | Tough | Uncertainty Avoidance | Long-Term |
|-------|----------------|---------------|-------|-----------------------|-----------|
| Italy | 50 | 76 | 70 | 75 | 61 |
| Japan | 54 | 46 | 95 | 92 | 88 |

Within Hofstede’s description, the two cultures appear different in almost all categories: according to these descriptors, Japanese culture appear less individualistic, more prone to success (and therefore more competitive), less open to “thinking outside of the box”, and keener to long-term planning. There are some similarities, as Japanese and Italian cultures seem to have a shared approach to hierarchy.

Levels of Co-Determination

Using Chevallard’s didactic transposition lens and moving within the co-determination levels, we attempt to provide a description of the Japanese and Italian institutional contexts. Since many sources are available on the Japanese context, the description will be briefer. The description of the Italian context, on the contrary, will be as detailed as the format allows, inspired to that proposed for Japan by Miyakawa and Winsløw (2019).

Japan is an East-Asian country, influenced by countries of “Confucian Heritage Culture” (Mason, 2014) such as China and Korea. These countries generally share some cultural values that are reflected in the school system, and can be considered part of the Japanese system of school-related beliefs:

a high regard for education [...]; [...] the cultivation of the self; a strong work ethic [...]; a belief [...] that success depends more on effort than on innate capacity [...]; respect for teachers [...] (Mason, 2014, p. 2).

In Japan, the national curriculum is detailed and rigid. Textbooks are essential for lesson planning. Long-term planning is centralized at the prefectural or school level, so teachers’ attention is focused on learning units and lessons. Classes are homogeneous by level: strict entrance tests are usually required for accessing high schools, while students with disabilities attend special schools. Japanese teachers spend the working day at school, where they have their personal workspace in a room shared with the whole teaching staff: in this space, they prepare lessons and discuss with their colleagues. In-service professional development is compulsory and takes place during working hours (Bartolini Bussi & Ramploud, 2018). LS is only part of Japanese TPD activities, which have many common

features: in particular, the practice of open classes with observers is common (Miyakawa & Winslow, 2013). Participating in optional TPD activities increases teachers' chances of career advancement (Miyakawa & Winslow, 2019).

The Italian school system is centralized. Recent reforms (2010 and 2012 respectively for secondary and primary school) stressed the importance of inclusiveness (law 133 and 169/2008). Italian school is structured around the concept of equity, and special schools do not exist: all students are given the same opportunities to reach the same goal, plus aids if needed. The Ministry of Education provides the *Indicazioni Nazionali* (National guidelines), which contain contents and aims for each subject, and its number of hours in a year. These contents are not prescriptive, but at the end of the 8th and 13th grades there are two national exams. Each teacher has the responsibility of the didactical plan for their classes, also according to the *Piano Triennale dell'Offerta Formativa* (Three-year Educational Plan – describing the cultural-pedagogical inspiration and the curricular, extracurricular, didactic and organisational design of the proposed activities). The contents of this document are specific of each school and decided by the collegiality of teachers and school staff.

Freedom of teaching, understood as professional autonomy in carrying out teaching activities and free cultural expression of the teacher, is guaranteed as a constitutional right: Article 33 of the Constitution states “Art and science are free and free is their teaching”. Institutionally, the duration of the lesson is 60 minutes. The teacher can have up to three consecutive lessons in the same class, without interruptions. During the lesson, the teacher is usually the only adult figure in the class. The Italian teacher works at school from one to six hours a day, dedicated to classroom lessons. The planning of individual lessons is not part of the working hours, nor there are places in the school dedicated to this activity: the teacher's paradidactic activity takes place in personal and private time and space. There are no compulsory contents or practices for TPD, they are chosen by teachers according to their own needs. In-service TPD is compulsory (law 107/2015), there is no minimum number of hours per year, and must be carried out outside working hours. Teachers' career advancement is based exclusively on seniority, although some economic incentives are given to those that take relevant roles in the school organization (Blandino, 2008; Capperucci, 2008).

On paper, teachers have numerous occasions for improving their professionalism. The Ministry¹ attests more than 500 agencies offering TPD opportunities. Universities, academic associations, teachers' associations, and educational companies which fulfil quality standards defined by the Ministry, are registered in a national database and can publish their TPD proposals on a digital platform (S.O.F.I.A.). The in-service professional development “system” is conceived as a “lifelong learning environment” for teachers and is intended as a “network of opportunities for professional growth and development for teachers” (law 107/2015). At national level, proposals come from the national education centre, academic associations, teachers' associations, educational companies. At regional level, regional school offices intervene by supporting, managing, and publicising the proposals. At local level, experienced teachers also offer courses in their school, sometime opens to teachers in the surrounding area. No official account is given on how many teachers participate in TPD. Yet, the impression is that this vastity of opportunities does not correspond to a high-quality offer: the Ministry states that the quality of TPD programmes is compromised by the general “low quality of models and methodologies” (law 107/2015) suggesting that teachers might be easily lost

¹ <https://www.miur.gov.it/accreditamento-enti-e-qualificazione-associazioni>

and caught in low quality programmes. The Ministry does not provide guidance to orientate in this labyrinth.

Some Reflections

At the end of this section, we can ask ourselves: is this description complete? Did we miss any essential point? Did we provide too much information, and made our description useless? Is this description reliable? Hofstede's synthetic data, for example, provides a quick overlook on the differences between two cultures and invite to carefully consider LS introduction in Italy. Yet, this description is problematic as it eliminates complexity, and the risk of overgeneralizing is high. Furthermore, these scores are open to interpretation. The similar scores for PDI might suggest that Italy and Japan have a shared view on hierarchy, but we propose a different interpretation: while Japanese invites consciousness of one's hierarchical position in any social setting and act accordingly (Ebaegu & Stephens, 2014), Italian culture dislike control and formal supervision. An analysis based on Hofstede's cultural dimensions, in support of our assertions, can be found in Giordanengo's Master Degree dissertation (2020)².

Discussion

It emerges that the Japanese and the Italian cultural and institutional contexts share some similarities and come with a number of differences. Hofstede's cultural dimensions show divergent basic values. However, to consider Hofstede's analysis *complete enough* to understand the similarities and differences between Japan and Italy would be preposterous, and similar numbers can still be interpreted with very different founding values. A detailed look suggests similarities between the two educational contexts, yet striking differences in the institutional and paradidactic organization. We believe this is sufficient to justify the importance of a cultural approach when practices from a cultural context are brought in different contexts. A tentative description of the Italian institutional (school) context was provided as a reference for future studies. Italian researchers are invited to amend this description, which is certainly lacking details.

This paper answers no research questions but considering our attempt to respond to the need to provide a description of the Italian institutional context in which students, teachers, and researchers work every day, leaves us with some questions. Are we satisfied with the result? No, we are not: the description misses many details, and we are not sure that what we provided is enough to really understand the context. How is Mathematics as a school subject considered at a cultural level? How is the teacher role considered in each society? Many questions are left unanswered, yet we often hear from reviewers that we should focus on describing just some aspects. How detailed can these descriptions be? Too little or too many information will lead to the same result: little understanding of the cultural context. Can we really achieve this *correct* kind of detail? Again, the answer, in our opinion, is no. The gap is embedded in the notion of cultural context and in any possible analysis of it. A number of scholars (e.g., Lotman, 1990) have declared the impossibility of a full knowledge of culture, as we are embodied in it and in what François Jullien - sinologist - calls the *unthoughts* (for a broader understanding, see Mellone *et al.*, 2019). To be aware of these unthoughts may not be

²https://sia.unito.it/studenti/intesi/Ricerca_tesi_libera/ricerca_tesi_dettaglio.asp?id_upload=192959&cdl_tesi=&cdl=&matricola=781420

enough anymore. What is incumbent on us is to frame our research accordingly, as to provide careful attention to their influence on teaching and learning processes in mathematics.

References

- Andrews, P. (2010). The importance of acknowledging the cultural dimension in mathematics teaching and learning research. *Acta Didactica Napocensia*, 3(2), 3–16.
- Bakker, A., Cai, J., & Zenger, L. (2021). Future themes of mathematics education research: an international survey before and during the pandemic. *Educational Studies in Mathematics*, 107(1), 1–24. <https://doi.org/10.1007/s10649-021-10049-w>
- Bartolini Bussi, M. G., & Ramploud, A. (2018). *Il lesson study per la formazione degli insegnanti*. Carocci.
- Blandino, G. (2008). *Quando insegnare non è più un piacere : la scuola difficile, proposte per insegnanti e formatori*. Cortina.
- Capperucci, D. (2008). *Dalla programmazione educativa e didattica alla progettazione curricolare : modelli teorici e proposte operative per la scuola delle competenze*. FrancoAngeli
- Chevallard, Y. (1985). *La Transposition didactique: du savoir savant au savoir enseigné*. La Pensée Sauvage.
- Chevallard, Y. (2002). Organiser l'étude. 3. Écologie & regulation. *Act Es de La 11e École d'Été de Didactique Des Mathématiques*, 3–22.
- de Mooij, M. (2010). *Global marketing and advertising: Understanding cultural paradoxes*. SAGE.
- Demir, K., Sutton-Brown, C., & Czerniak, C. (2012). Constraints to Changing Pedagogical Practices in Higher Education: An example from Japanese lesson study. *International Journal of Science Education*, 34(11), 1709–1739. <https://doi.org/10.1080/09500693.2011.645514>
- Ebaegu, M., & Stephens, M. (2014). Why Lesson Study Works in Japan: A Cultural Perspective. *Mathematics Education Research Group of Australasia*, 199–206.
- Fernandez, C., Cannon, J., & Chokshi, S. (2003). A US–Japan lesson study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education*, 19(2), 171–185. [https://doi.org/10.1016/S0742-051X\(02\)00102-6](https://doi.org/10.1016/S0742-051X(02)00102-6)
- Florensa, I., Bosch, M., Cuadros, J., & Gascón, J. (2018). Helping lecturers address and formulate teaching challenges: an exploratory study. In V. Durand-Guerrier, R. Hochmuth, S. Goodchild, & N. M. Hogstad (Eds.), *Proceedings of the INDRUM 2018* (pp. 373–382). University of Agder and INDRUM.
- Freimuth, H. (2006). Language and culture. *Ugru Journal*, 2. UAE University
- Hatano, G., & Inagaki, K. (1998). Cultural contexts of schooling revisited: A review of “The Learning Gap” from a cultural psychology perspective. In *Global prospects for education: Development, culture, and schooling*. (pp. 79–104). American Psychological Association. <https://doi.org/10.1037/10294-003>
- Huang, R., & Shimizu, Y. (2016). Improving teaching, developing teachers and teacher educators, and linking theory and practice through lesson study in mathematics: an international perspective. *ZDM*, 48(4), 393–409. <https://doi.org/10.1007/s11858-016-0795-7>
- Huang, R., Takahashi, A., & Ponte, J. P. (Eds.). (2019a). *Theory and Practice of Lesson Study in*

Mathematics. Cham, Switzerland: Springer International Publishing.
<https://doi.org/10.1007/978-3-030-04031-4>

- Krainer, K. (2011). Teachers as stakeholders in mathematics education research. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 47–62. Ankara, Turkey: PME.
- Lewis, C. (2016). How does lesson study improve mathematics instruction? *ZDM*, 48(4), 571–580. <https://doi.org/10.1007/s11858-016-0792-x>
- Lotman, J. M. (1990). *Universe of the mind. A semiotic theory of culture*. IB Taurus.
- Manolino, C. (2021). The Semiosphere lens to look at Lesson Study practices in their cultural context: a case study. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 263–272. Khon Kaen, Thailand: PME.
- Mason, M. (2014). Culture and educational outcomes in “Confucian heritage” societies in Asia. *Revue Internationale d’éducation de Sèvres*. <http://journals.openedition.org/ries/3812>
- Mellone, M., Ramploud, A., Di Paola, B., & Martignone, F. (2019). Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic. *ZDM*, 51(1), 199–212. <https://doi.org/10.1007/s11858-018-0992-7>
- Miyakawa, T., & Winsløw, C. (2013). Developing mathematics teacher knowledge: the paradidactic infrastructure of “open lesson” in Japan. *Journal of Mathematics Teacher Education*, 16(3), 185–209. <https://doi.org/10.1007/s10857-013-9236-5>
- Miyakawa, T., & Winsløw, C. (2019). Paradidactic infrastructure for sharing and documenting mathematics teacher knowledge: a case study of “practice research” in Japan. *Journal of Mathematics Teacher Education*, 22(3), 281–303. <https://doi.org/10.1007/s10857-017-9394-y>
- Quaresma, M., Winsløw, C., Clivaz, S., da Ponte, J. P., Ní Shúilleabháin, A., & Takahashi, A. (Eds.). (2018). *Mathematics Lesson Study Around the World*. Springer International Publishing. <https://doi.org/10.1007/978-3-319-75696-7>
- Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., Goos, M., Isoda, M., & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration: June 2016. *ZDM*, 48(5), 651–690. <https://doi.org/10.1007/s11858-016-0797-5>
- Scollon, R., & Scollon, S. W. (1995). *Intercultural Communication: A Discourse Approach*. Basil Blackwell.
- Spencer-Oatey, H. (2012). What is culture? A compilation of quotations. *GlobalPAD Core Concepts*, 1–22.
- Stigler, J., & Hiebert, J. (2016). Lesson study, improvement, and the importing of cultural routines. *ZDM*, 48(4), 581–587. <https://doi.org/10.1007/s11858-016-0787-7>
- White, A. L., & Lim, C. S. (2008). Lesson study in Asia Pacific classrooms: local responses to a global movement. *ZDM*, 40(6), 915–925. <https://doi.org/10.1007/s11858-008-0138-4>
- Winsløw, C. (2011). A Comparative Perspective on Teacher Collaboration: The Cases of Lesson Study in Japan and of Multidisciplinary Teaching in Denmark. In *From Text to “Lived” Resources* (pp. 291–304). Springer Netherlands. https://doi.org/10.1007/978-94-007-1966-8_15