RELATIVITY THEORY AS A THEORY OF PRINCIPLES: A READING OF CASSIRER'S *ZUR EINSTEIN'SCHEN RELATIVITÄTSTHEORIE*

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In his *Zur Einstein'schen Relativitätstheorie*, Ernst Cassirer presents relativity theory as the last manifestation of the tradition of the "physics of principles" that, starting from the nineteenth century, has progressively prevailed over that of the "physics of models." In particular, according to Cassirer, the relativity principle plays a role similar to the energy principle in previous physics. In this article, I argue that this comparison represents the core of Cassirer's neo-Kantian interpretation of relativity. Cassirer pointed out that before and after Kant, the history of physics presents significant instances in which the search for formal conditions that the laws of nature must satisfy preceded and made possible the direct search for such laws. In his earlier years, Cassirer seems to have regarded principles like the energy principle, the relativity principle, and the principle of least action as a constitutive but provisional form of a priori, imposing specific limitations on the form of the allowable laws of nature. Only in his later years, by attributing an autonomous status to these statements of principle, did Cassirer attribute a definitive but merely regulative meaning to the a priori. This does not impose specific requirements on natural laws but only a motivation to search for them.

1. Introduction

Shortly after the Eddington-Dyson eclipse expeditions "confirmed" general relativity, Einstein became an international celebrity (Kennefick 2009), hailed, especially by the British and American press, as the physicist who had "dethroned Newton." In the context of continued national tensions after the First World

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War, in November 1919, Einstein attempted to present relativity theory in plain terms to the English-speaking public with a brief article for the *Times*, the prestigious London daily newspaper. He famously characterized relativity theory as a "principle theory" rather than a "constructive theory" (Giovanelli 2020). Constructive theories, like the kinetic theory of gases, try to "construct a model [*ein Bild*] of the phenomena" (Einstein 1919a) that behaves according to given laws of nature (a mechanical model of a gas). Principle theories, like thermodynamics, starting from universally recognized empirical facts (there is perpetuum mobile of the second kind), search for mathematically formulated criteria that any possible laws of nature must satisfy if those facts are to hold (the second principle of thermodynamics) (Einstein (1919a).

In the past decade, this distinction has acquired a life of its own and attracted considerable attention among philosophers of physics (Brown 2005; Janssen 2009). However, Einstein's principle/constructive theory opposition, surprisingly, played no role in the debate about relativity that engaged the philosophical community in the 1920s, especially in Germany (Hentschel 1990a, 1990b). Neither Moritz Schlick nor Hans Reichenbach—Einstein's two primary philosophical interlocutors—mentioned the article in the *Times* or referred to relativity theory as a "theory of principles." As I show in this article, it has seldom been noticed that Ernst Cassirer was, to a certain extent, an exception to the rule. There is no evidence that Cassirer ever read Einstein's short article for the *Times* either. However, in his 1921 book on relativity (Cassirer 1921b), he arrived at a strikingly similar conclusion. In Cassirer's view, relativity theory was nothing but an example of the prevalence of the "physics of principles" over the "physics of models" (*Bilder* or *Modelle*)—a trend that had started to emerge at least from the second half of the nineteenth century. The convergence between Einstein and Cassirer on this issue is less surprising than one might think. At that time, the opposition between two styles of doing physics, one based on general principles and the other on detailed models, was a common trope (see, e.g., Sommerfeld 1915). Most of all, most early relativists agreed that special relativity did not introduce a new theory in the usual sense but a new principle constraining first-level theories (Laue 1911/1961b, 185f.; Minkowski 1908, 55).

However, Cassirer was possibly the only philosopher who not only appreciated and emphasized this point but made it the core of his neo-Kantian interpretation of the theory of relativity. In Cassirer's eyes, the "physics of principles" had unmistakable Kantian overtones, at least in light of the "Marburg" interpretation of Kant, which was part of his philosophical training.[1] Throughout his

1. Cassirer often emphasized that it was one of Cohen's (1885) fundamental merits to have shown that the "barycenter" of Kant's first *Critique* was not the intuitions of space and time or the categories

career, Cassirer repeatedly insisted that certain universal principles can be singled out in the structure of all physical theories (e.g., the virtual work principle, the "energy principle," the least action principle) and remain unaltered despite the changes undergone by physical laws. These principles, unlike the usual physical laws, do not directly say anything about the properties of any specific physical system; rather, they impose general constraints on lawlike statements concerning them. The latter do not qualify as proper physical laws unless they satisfy these conditions. In insisting on the importance of "principles" in physics, Cassirer did not make any claim to originality. He simply meant to report on a "practice of principles" (Seth 2010) that was widespread in nineteenth- and early twentieth-century physics.[2] The compatibility of candidate physical laws (laws of mechanics, electrodynamics, etc.) with abstract principles had progressively become more important than constructing detailed intuitive pictures (*Bilder*) according to such laws.[3] This strategy allowed for empirical predictions but avoided speculations about the nitty-gritty of the physical systems at stake (Wilson 2017).

In Cassirer's view, despite its revolutionary nature, relativity theory was ultimately nothing but the last manifestation of the well-established tradition of the physics of principles that has progressively prevailed over that of the physics of models. Although Cassirer's interpretation of relativity has attracted considerable

but the "synthetic principles" (*Grundsätze*; Cassirer 1907, 624n2; 1912; 1918a, 187). From this perspective, according to Cassirer (1920a, 11ff.), the dualisms that plague Kant's philosophy—between subjective and objective, form and content, intellect and sensibility, and so on—are ultimately resolved in the correlation between the universal and the particular, which is dealt with in the *Kritik der Urheilskraft*. Special empirical laws are all special cases of the synthetic principles because the latter are the conditions that *any* possible laws of nature must satisfy to be recognized as such. We know a priori that all empirical laws satisfy such conditions because otherwise they would have been rejected from the outset. The synthetic principles (*Grundsätze*) are said to be *constitutive* in that they impose specific formal constraints on the possible candidate laws of nature. The search for the actual empirical laws of nature is guided by the expectation that the latter are organized in a progressively more coherent system—that is, the principle (*Prinzip*) of the "formal finality of nature." The latter is merely a regulative principle because it provides only a guide for the search of the empirical laws without imposing any specific constraints on them (Stadler 1874). Thus, I disagree with the widespread interpretation that the Marburg School defended a regulative meaning of the a priori.

2. The importance of the role of principles in physics goes back at least to the work of Hermann von Helmholtz (Bevilacqua 1993). At the turn of the century, Hendrik H. Lorentz (1900, 1905/1934; see Frisch 2005) and Henry Poincaré (1904, 1905; see Darrigol 1995; Giedymin 1982, 42–89) presented the opposition between the physics of principles and the physics of models as commonplace (Bordoni 2017) for other examples.

3. Macroscopic thermodynamic properties of a system, for example, might be reduced to the motion of its microscopic constituents according to the laws of mechanics (mechanical models of gases). Mechanical properties, such as inertial mass, might be reducible to electromagnetic effects implied by Maxwell's equations (electromagnetic models of the electron).

scholarly attention in recent decades (Ferrari 1994, 1996, ch. 4; Ryckman 2003, 2005, ch. 2), Cassirer's characterization of relativity theory as a "theory of principles" has not been fully appreciated. Indeed, it is undeniable that this motif is not always easy to disentangle from other lines of argument that run through Cassirer's book. As Hermann Weyl (1954) perceptively pointed out, although Cassirer was a "mind of rare universality," his writings resemble "more a suite of bourrées, sarabands, minuets, and gigues than variations on a single theme" (624). If one pushes Weyl's comparison further, one might claim that Cassirer's prose resembles more the work of a post-Romantic composer rather than works of the classical age. Cassirer seemed to focus, so to speak, on texture over structure, often proceeding by local associative transitions rather than global motivic development. For this reason, in reading Cassirer's book on relativity, one might be persuaded by this or that line of argument; however, one is often left struggling to identify the overall message that he is attempting to convey.

By situating Cassirer's work on relativity in the context of his early philosophy of physics, I hope to show that the characterization of relativity theory as a "theory of principles" can be considered the book's central message. In particular, Cassirer attributed to the relativity principle the same role he had attributed to the energy principle in previous works, most prominently in his first monograph on Leibniz (Cassirer 1902): The relativity principle, like the energy principle, is not one law of nature among others; it is a second-order constraint that we impose on the laws of nature. To allow the reader to appreciate this point, this article divides the development of Cassirer's philosophy of physics up to the 1920s into two phases, in each of which Cassirer focuses mainly, although nonexclusively, on a single "principle":

- Energy principle (sec. 2): In his oft-neglected first monograph on Leibniz, *Leibniz' System in seinen wissenschaftlichen Grundlagen* (Cassirer 1902), the young Cassirer contends that Leibniz treated the principle of conservation of mechanical work not as a consequence of the laws of mechanics but as a general principle, a requirement that *any* law of nature has to satisfy. By taking the form of a "condition" on first-order laws (e.g., the collision rules), the principle does not postulate the existence of mechanical energy as a separate 'thing'. In the immediately following years, Cassirer generalized this conclusion in two directions: in his historically oriented work, he presented conservation principles as an instantiation of those "invariants of experience" that serve as possible candidates for a priori conditions that any good physical theory in general must meet (Cassirer 1906a), whereas in his theoretical work, the energy principle was presented as a significant example of the transition from "substance-concepts" to "function-concepts" (Cassirer 1910; see also Giovanelli 2023).

- Relativity principle (sec. 3): In his monograph on relativity, *Zur Einstein'schen Relativitätstheorie* (Cassirer 1921b), Cassirer returned, to a certain extent, more explicitly to the approach of his first book. The special and general relativity principles are compared explicitly with the energy principle. Just as the energy principle does not imply the reduction of physics to mechanics, the relativity principle does not imply its reduction to electrodynamics. Both principles impose constraints on laws of nature rather than being mere accidental by-products of those laws. The relativity theory, like the energy principle, can be considered an instance of the general tendency of modern physics to move from the physics of models to the physics of principles. Cassirer soon integrated this line of argument into his growing body of work on the role of 'symbols' in physics, showing how, in the history of physics, the 'symbolism' of principles has prevailed over the 'schematism' of pictures and models (Cassirer 1927a, 1929).

In this article, I conclude that Cassirer's book on relativity has the merit of grasping quite clearly what Eugen Paul Wigner (1949), two decades later on the occasion of Einstein's seventieth birthday (a few years after Cassirer's death), called the "reversal of the trend" (5). Instead of deriving the principles from what are believed to be the laws of nature, physicists test the acceptability of the laws of nature through certain general principles. Historically, Wigner pointed out, the power of these principles was established "so firmly that we have to be reminded that they are based only on experience" (5). Einstein had certainly made extensive use of this heuristic technique, but Cassirer could argue with good reason that this 'principle strategy' has always been practiced in the history of physics from Leibniz to Poincaré.

In Cassirer's assessment, it was Kant's merit to have articulated the philosophical importance of the search for criteria, separating the wheat from the chaff, the lawlike statements that can be taken as 'laws of physics' (see n. 1) from those that cannot. However, as Cassirer conceded, Kant mistakenly believed that a set of selection criteria could be fixed once and for all. Cassirer's historical-critical analysis of the role of 'principles' in physics can ultimately be considered his life-long attempt to avoid the shortcomings of Kant's original program by preserving its key insight. In doing so, Cassirer was forced to unravel a more complex dialectic between a priori and a posteriori, constitutive and regulative. This dialectic marks the main stages of Cassirer's work in the philosophy of physics and in particular of his conception of the a priori. It is often claimed that, whereas the young Reichenbach (1920) promoted a constitutive but relativized a priori (Friedman 2001), Cassirer and the Marburg School defended a regulative but absolute form of the a priori as the limit of a process of progressive discovery (Ryckman 2005, 245ff.; Friedman 2005, 2008). However, I hope to offer evidence that shows that the shift toward a regulative conception of the a priori happened only in Cassirer's (1936) later work (for more details, see Giovanelli 2022).

Up to the 1920s, Cassirer seems to have considered the surprising "resilience" of some principles, despite the chaotic rise and fall of individual theories, as a sign that they could be considered, at least provisionally, as candidates for constitutive, a priori conditions of what counts as a law of nature in general. Cassirer conceded that the development of science can always force us to search for "better" constitutive principles in an infinite convergent process. After relativity theory, the refutation of principles previously held as a priori principles had become more than a theoretical possibility. By the 1920s, Cassirer, nearly imperceptibly, started to suggest that only the overall process of searching for progressively more adequate constitutive principles should be considered a priori. As has been shown, Cassirer did not attempt to resolve this ambiguity until the 1930s: (1) he attributed explicitly to the "statements of principle" an autonomous role in the structure of physical theories (as opposed to the statements of law and statements of measurement) while denying them, even provisionally, a priori status (Cassirer 1936, 66; 1956, 52–53); (2) he shifted the position of the *statements* a priori—for example, the causality principle—to a deeper level by attributing to them an absolute but regulative meaning (Cassirer 1936, 75; 1956, 60). The a priori motivates and guides the search for the laws of nature without providing any particular insight into their structure.

2. The Principle of Conservation of Energy in Cassirer's Early Marburg Phase

After having studied in Berlin under the guidance of Georg Simmel, Cassirer moved to the small university of Marburg in 1896. At that time, the so-called Marburg School of neo-Kantianism was perhaps at the beginning of its golden age (Ferrari 1988; Sieg 1994). Over the years, Hermann Cohen and Paul Natorp had used a series of philosophical prizes (*Preisaufgaben*) to support Marburg's doctoral students and develop some of the core insights of a "little school" that had started to gather in Marburg (Cohen to Natorp, April 19, 1897; Holzhey 1986, vol. 2, document [doc.] 42). For the years 1898–99, the argument proposed by Natorp required an examination of Leibniz's philosophy of the foundation of mathematics and mechanics (Holzhey 1986, 1:382). The winner of the competition was the young Cassirer, who soon emerged as the "rising star" of the Marburg group (Natorp to Görland, November 21, 1898; Holzhey 1986, vol. 2, doc. 45). Cassirer further worked on the manuscript, using part of it as the basis for his dissertation on Descartes (Cassirer 1899), which became the first chapter of a book on Leibniz by the end of 1901 (Cassirer 1902). When sending Natorp the first drafts of his *Leibniz' System*, Cassirer (1902) invited him to consider it as part of a larger "study on the prehistory of criticism" (Cassirer to Natorp, November 26, 1901; Cassirer 1995–, vol. 18, doc. 43) that he was working on.[4]

The "whiggish" project of a "prehistory of criticism" had been introduced by Natorp (1882a, 1882b, 1882c) himself two decades earlier, and it became one of the tenets of Marburg historiography (Cohen 1883, 1885). In particular, the second part of Cassirer's (1902, pt. 2) book dedicated to the "fundamental concepts of mechanics" can be considered as a prototypical example of this sort of 'typological' interpretation of the history of science and philosophy.[5] On the one hand, Cassirer emphasized that Leibniz treated principles such as vis viva conservation, the continuity principle, and so forth, not as individual laws of nature among others—such as specific rules about impact and collisions among bodies—but as second-order conditions that we impose on any dynamic law. On the other hand, Cassirer argued that by acknowledging the role of such principles in physics, Leibniz "anticipated" the Kantian concept of the a priori, even if he could not free himself from its metaphysical underpinnings. By regarding Leibniz's scientific work as part of the prehistory of Kant's a priori, Cassirer, ultimately following Cohen (1885), could clarify the latter concept. Certain statements are not considered a priori because of their *origin* (because they are part of the structure of the mind or derived from the table of categories); they are a priori because of their role (*Leistung*) as a condition of possibility of the mathematical science of nature. In this form, Cassirer argued, more or less explicitly, that Kant's combination of the a priori and the transcendental maintains its validity for the philosophical assessment of modern science.

2.1. Leibniz and the Conservation of Mechanical Work: Cassirer's *Leibniz' System*

Some of the details of Cassirer's reconstruction of Leibniz's role in the history of the discovery of the energy principle can be used to make Cassirer's interpretative

4. The study was the first outline of *Das Erkenntnisproblem*, which would be concluded only a few years later (Cassirer 1906a).

5. *Typology* is a method of biblical interpretation in which events, persons, and so on in the Old Testament are seen as "types" that prefigure the corresponding antitypes of Christ found in the New Testament.

strategy clear. As Cassirer (1902, 308ff.) pointed out, in the historical literature of his time (Dühring 1873; Helm 1887; Mach 1872; Planck 1887), Leibniz's contribution to the discovery of energy conservation was usually downplayed and limited to the establishment of the conservation of kinetic energy mv^2 in the collision of elastic bodies. Nevertheless, according to Cassirer, the vis viva controversy was, for Leibniz, only a polemical opportunity. Leibniz's contribution to the history of energy conservation appears minor if one only considers his role in the discovery of the energy principle (for more detail, see Cassirer 1904b, 267 n. 205, 315 n. 256). However, in Cassirer's view, Leibniz's contribution is of fundamental importance if one considers Leibniz's justification of his conservation principle. According to Cassirer (1904b), Leibniz realized that the principle of conservation of vis viva was not a single empirical law among others describing a certain class of phenomena but a "general principle," a constraint that *all* particular laws of nature have to satisfy to be recognized as such: "From here Leibniz first acquires the general version of the principle of *conservation of energy*, which he conceives not only as an *individual theorem* of analytical mechanics but as a *fundamental rule* of all physics" (117; last emphasis added).

According to Leibniz, for qualitatively different phenomena (e.g., elasticity, gravitational free fall, hydrostatic pressure) to be quantitatively compared, the "general definition of an abstract unity" was needed (Cassirer 1902, 304). In principle, the choice of measuring standard is arbitrary. However, it must be assumed that the measurement yields identical results in the chosen units (304). In this assumption, Cassirer argues, "the essential content of the principle of conservation is already implicit" (306). For any quantity that arises ex nihilo and disappears *ad nihilum* without being compensated for, the invariability of the chosen unit would not be granted. In this sense, "Leibniz's conservation principle is regarded as the *necessary condition for the application of pure mathematics to reality*" (306; emphasis added)—that is, for establishing the quantitative equivalence of qualitatively different phenomena.

According to Cassirer, Leibniz's contribution to the history of the energy principle was the choice of "mechanical work" as a common denominator.[6] Leibniz's choice of mechanical work as a unit of measure did not imply the reduction of all phenomena to mechanics. It was grounded on the fact that mechanical effects are more familiar and easily measurable than other effects. 'Causes' are to be called equal when they produce equal 'effects'—that is, if they are able to perform an equal amount of mechanical work as measured in work

6. Cassirer (1904a, 1:249f., 249n184) provides some additional details in his commentaries on the German translation of some of Leibniz's *Hauptschriften*, which he was working on at that time.

units—if they produce an equal degree of tension in an equal number of elastic springs, raise an equal weight to the same height, communicate to an equal number of bodies the same amount of velocity, and so forth (Cassirer 1902, 305). The postulation of such one-to-one coordination exhausts the essential content of the energy principle without any need to introduce mechanical energy as a separate reality.[7]

On the one hand, Leibniz, like many of his contemporaries, took for granted that "all happening can be traced back to mechanical processes and can only be fully explained by reference to them" (Cassirer 1902, 318). On the other hand, according to Cassirer, Leibniz recognized that "the value and validity of the concept of conservation" does not depend "on special ideas about the nature of physical forces" (319); on the contrary, the forces of nature of whatever kind must satisfy the conservation of mechanical work. Thus, Leibniz did not regard his conservation principle as the *consequence* of the "mechanical interpretation of the phenomena" (319); the latter is only an *instance* of a worldview that satisfies that principle: "Such a reversal [*Umkehrung*] does not change the outlined worldview, but it does change the doctrine of principles of scientific knowledge" (319). Cassirer could then project more or less explicitly onto Leibniz a debate on the use of models or *Bilder* that was fashionable among physicists at the turn of the century (Deltete 1999). In general, according to Cassirer (1902), Leibniz's physics can avoid the construction of "hypothetical models, in which one tries to grasp the essence of the phenomena" (319) and relies only on general principles that any such models, of whatever nature, have to satisfy.

According to Cassirer (1902), this showed how the conservation of mechanical work "relates to *experience*"—that is, "in which proportion a priori and empirical elements contribute to its justification" (320). In Leibniz's work, Cassirer pointed out that the "equivalence of cause and effect and the principle of the impossibility of the perpetuum mobile resulted in two different basic motifs of the conservation law" (319–20). If the equality of cause and effect were not satisfied, and if it were possible to create work out of nothing, then the absurdity of a perpetuum mobile would ensue. On the one hand, Leibniz's conservation principle appears to be a posteriori, being based on the repeated experience of failure in constructing a perpetual motion machine. On the other hand, it is a priori because the equivalence of cause and effect serves as "a criterion for assessing and differentiating the value of given experiences" (321). Cassirer explains his stance clearly in the following passage by referring to his derivation of rules for the collision of bodies:

7. On Cassirer's notion of coordination, see Ryckman (1991).

> The relationship between the conservation law and experience cannot be described more clearly than in this case. The value of the law lies in its fertility as a *principle* for the exact investigation of the phenomena. For this very reason, it is independent of "experience" in the trivial sense of the latter word, which only denotes a disordered set of random observations. Compared to this indefinite chaos of perceptual content, the equivalence "of cause and effect" contains a rule of judgment, through which only scientific experience can emerge from the lawless combination of the immediate content of the mind. In this sense, we can call the energy law an a priori law—provided that we use the expression of the a priori as the fundamental tool for gaining knowledge from the mere descriptions of existing facts. The attribution of such *status* to the principle could still be considered a daring philosophical stance in Leibniz's time; however, it appears to modern development as a sober expression of a historical fact: . . . [Leibniz's approach] has become a maxim of scientific research. (321)

Leibniz expressly conceded that the exceptions to vis viva conservation appear at first sight to be overwhelming. Unlike momentum, which is always conserved, the quantity of vis viva appears to be conserved only in elastic collisions. In inelastic collisions, such as when a ball of soft clay strikes another ball of soft clay, momentum mv is conserved but vis viva mv^2 is lost. Because macroscopic collisions are at least partially inelastic, "the entire material of observations therefore forms a single major contradiction to the principle" (Cassirer 1902, 321). However, rather than abandoning the universality of his principle in the face of the empirical evidence, Leibniz considered it to be more fundamental than the latter. It is not that vis viva is conserved because bodies are elastic but rather the other way around: because vis viva is conserved, bodies *must* always have some degree of elasticity (308).

In this sense, the a posteriori–a priori opposition does not fully grasp Leibniz's contribution to the justification of the energy principle (Cassirer 1902, 318). According to Cassirer, the apparent contradiction can be resolved if one does not look at the "origin" (*Ursprung*) of the principle but at its "function" (*Leistung*) in the overall system of physics. If mechanical work were not conserved, then causes would produce different effects depending on the unit of measure chosen, and nature would be without laws; the whole science of dynamics would become something indeterminate and contradictory, *quiddam vagum et absonum* (Leibniz 1850, 3:210). "As Leibniz says against Johann Bernoulli, this requirement means nothing less than a *condition of the possibility of dynamics as a science*" (Cassirer 1902, 402; emphasis added). In attributing to

Leibniz this sort of proto-transcendental argument, Cassirer aimed to emphasize what he saw as Leibniz's fundamental contribution to the history of the energy principle. For Leibniz, the conservation of mechanical work was not some mathematical *equation* among others that describes a certain group of phenomena. Something much more essential was at stake—namely, the very possibility of dynamics as a science. Thus, Leibniz treated the conservation of mechanical work as a fundamental principle that lurks behind all equations of dynamics.

In Cassirer's judgment, Leibniz's role in the discovery of the energy principle in its generality was indeed minor; the modern concept of "energy" was acquired only much later in the nineteenth century by taking into account not only mechanical energy but other forms of energy (e.g., thermal, electromagnetic). Nevertheless, Leibniz's role in defining the strategy for the justification of the principle has been historically relevant: "In the *proof* of the fundamental law, Leibniz can now clearly distinguish between the two lines of thought, which have also been expressed separately in the development of modern theory" (Cassirer 1902, 317). Indeed, in justifying the energy principle, its discoverers in the nineteenth century, Robert Julius von Mayer (1842) and Hermann von Helmholtz (1847), were confronted with a choice between the a posteriori or bottom-up approach and the a priori or bottom-down approach. "While Robert Mayer started from the equality of cause and effect" as a metaphysical principle, Cassirer points out that "Helmholtz based his investigation on the principle of the excluded perpetual motion machine," which he considered as an empirical generalization (Cassirer 1902, 317).

In mechanics, the impossibility of perpetual motion can easily be demonstrated. Thus, Helmholtz started from the assumption that all processes in nature were mechanical—for example, that heat was nothing but motion.[8] The energy principle was "identical to the assumption that all effects in nature can be traced back to attractive and repulsive forces, the intensity of which depends only on the distance between the points acting on one another" (Cassirer 1902, 318). On the contrary, Mayer thought of the energy principle in terms of the correlation of numerical values—a given number of work units always corresponds to a fixed number of heat units. Thus, Mayer could ignore the question of the nature of heat for the purpose of relating it to a mechanical equivalent. Following Mayer, "the law of conservation is obtained and carried out independently of every special conception of nature, in particular every special mechanical interpretation of individual physical processes" (Cassirer 1902, 318). However, in Cassirer's view, the alternative between Mayer's top-down approach, which builds on a self-evident principle, and Helmholtz's bottom-up approach, which relies on

8. For the importance of Helmholtz in neo-Kantianism, see Biagioli (2016).

a broad empirical generalization, is not exhaustive. As Leibniz had already sensed, the issue is ultimately irrelevant. The justification of the conservation principle resides entirely in its capacity to serve as an effective selection criterion for the acceptability of individual laws.

2.2. Beyond Leibniz: From *Das Erkenntnisproblem* to *Substanzbegriff und Funktionsbegriff*

It is not my goal in this article to systematically assess the historical accuracy of Cassirer's reconstruction, which certainly raises the eyebrows of many of today's scholars. What is relevant for this article is the extent to which Cassirer's historical work reveals his philosophical agenda. In Cassirer's first book, one can recognize the emergence of themes that would soon become trademarks of his philosophy of physics. It is not by chance that when the book was published, Cassirer, then back in Berlin, was already working on the manuscripts of some of his later major works (Cassirer to Natorp, December 13, 1902; Cassirer 1995–, vol. 18, doc. 55). When he sent a copy of the book to Natorp, Cassirer wrote that in addition to preparing a German anthology of Leibniz's writings with Arthur Buchenau (Leibniz 1904), he was still dealing with his "work on the prehistory of the critique of reason" (Cassirer 1995–, vol. 18, doc. 55). Indeed, Cassirer had apparently already "finished the first part concerning the predecessors and the philosophical-mathematical problems of the seventeenth century" (doc. 55). However, he soon realized that the sections on Kant required further work (doc. 55).

After moving back to Berlin, Cassirer (1906b) maintained strong ties with the Marburg group. This relationship is shown by his strenuous defense of Cohen (1902) against the attacks of Leonard Nelson (1905). By 1905, the first volume of *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit* was finished, and Cassirer already planned to add a second volume. However, he was not fully satisfied with the title. In reality, Cassirer wanted not only to describe a series of theories of knowledge but also to capture the logic of its development.[9] For this reason, already at that time, he realized that a third systematic volume was required (Cassirer to Natorp, July 31, 1905; Cassirer 1995–, vol. 18, doc. 70). *Das Erkenntnisproblem* (Cassirer 1906a) was presented as a habilitation thesis at the University of Berlin in April. In July 1906, Cassirer held his *Probevorlesung* and obtained the *venia legendi* for philosophy. The title of the

9. As Cassirer (1906a) pointed in the introduction of the first volume of *Das Erkenntnisproblem*, the "Apriori" is not a fixed "psychological or physiological 'disposition'" (6). It must be discovered in the history of science as "the conservation of a general logical structure in all consecutive conceptual systems" (17f.).

lecture, "Substanzbegriff und Funktionsbegriff" (Cassirer 1906c), reveals that the core idea of what would become the theoretical counterpart of *Das Erkenntnisproblem* was already fully formed (Cassirer 1910). In particular, in the physics part of the *Probevorlesung*, Cassirer used the historical development of the concept of 'energy' as a prime example of how a substance-concept could be transformed into a function-concept. In this way, Cassirer developed an idea that was already present in his book on Leibniz but without indulging in the somewhat cryptic jargon of the Marburg School.

As Cassirer (1906c) pointed out, it was initially natural to assume that energy is "a constant thing 'behind' the phenomena" (11). However, it was progressively realized that the function of the concept of energy is only to establish quantitative equivalences among different phenomena (e.g., heat, motion, or electricity): "All reality of energy resolves itself, from the point of view of knowledge, in the assessment of equivalence relations" (11). For this purpose, a common numerical scale is constructed, the unit of which as the unit of energy serves as a common denominator for comparison. It is at this point that energy as a concept of substance has been replaced by a concept of function (11ff.). The "substantiality" that is ascribed to energy means "nothing other than a *constancy of pure numerical relationships*" (12). A constant numerical coefficient is assigned to each form of energy, called the "mechanical equivalent" of that form of energy. This coefficient serves to convert any quantity of that energy into a corresponding amount of mechanical work, just as one can convert feet into meters: "The 'essense' of the individual types of energy is sufficiently known and clarified if one specifies the fixed *equivalence values* that univocally regulate the connection and the transition between the different areas can be specified" (Cassirer 1909, 93)

At the time the lecture was held, Cassirer was probably already working on the corresponding chapters of his monumental monograph of the same title, which he finished in July 1910. In the physics sections of the book, the relational conception of energy (Hiebert 1962) that, not without some arbitrariness, Cassirer attributed to Leibniz in his historical work was now developed systematically, relying on a tradition that, from Mayer (1842) and William John Macquorn Rankine (1855), leads to Georg Ferdinand Helm's (1898) and Wilhelm Ostwald's (1902) energetics. It suffices to quote a significant passage:

> The law of energy directs us to coordinate every member of a manifold with one and only one member of any other manifold, in so far as to any *quantum* of motion there corresponds one *quantum* of heat, to any *quantum* of electricity, one quantum of chemical attraction, and so on. In the concept of work, all these determinations of magnitude are related

> to a common denominator. If such a connection is once established, then every numerical difference that we find within one series can be completely expressed and reproduced in the appropriate values of any other series. The unit of comparison, which we take as a basis, can arbitrarily vary without the result being affected. If two elements of any field are equal when the same amount of work corresponds to them in any series of physical qualities, then this equality must be maintained, even when we go over to any other series for the purpose of their numerical comparison. *In this postulate, the essential content of the principle of conservation is already exhausted; for any quantity of work, which arose 'from nothing' would violate the principle of the mutual one to one coordination of all series*. . . . In any case, it appears that energy in this form of deduction is never a new *thing*, but is a unitary *system of reference* on which we base measurement. All that can be said of it on scientific grounds is exhausted in the quantitative relations of equivalence, that prevail between the different fields of physics. (Cassirer 1910, 253f.; 1923b, 190f.)

According to Cassirer (1910), the energy principle opened up the possibility of turning "nature into a *system*, without our being obliged to require representation of this system in a unitary, intuitive *picture* [*Bilde*], like the one offered by mechanism" (266; 1923b, 200). "Mechanism" was the attempt to unify the phenomena by reducing everything to one class, mechanical motions, by providing mechanical models of thermodynamic or electromagnetic phenomena. Modern energetics, if properly understood, indicates the possibility of unifying different classes of phenomena under a common principle: the assumption that energy, whether it is derived from motion, heat, or electricity, is always equivalent to a proportional amount of mechanical work. Against this conception, Cassirer pointed out, the objection has been raised that the qualitative difference between separate classes of phenomena has not been abolished but glorified into a sort of neo-Aristotelian qualitative physics (see Duhem 1903, 197ff.). However, Cassirer replied that this criticism is unjustified. The quantitative equivalence between different phenomena "gives a no less definite logical connection than reduction to a common mechanical model" (Cassirer 1910, 202; 1923b, 269).

Whereas all attempts to reduce all phenomena to mechanics have repeatedly failed, the principle of conservation of energy has survived the rise and fall of the most disparate theories and models. This surprising resilience cannot be a coincidence. According to Cassirer, the provisional hypothesis could be made that principles of this kind are constitutive conditions a priori for the very acceptability of particular physical theories in general (see Cassirer 1936, 207;

1956, 117). A theory that does not comply with such principles would probably be rejected from the outset. Using a geometrical metaphor, Substanzbegriff und Funktionsbegriff famously defined transcendental philosophy as a "theory of the invariants of experience." It singles out the stable elements in the synchronic and diachronic succession of different if not rival theories as possible candidates for a priori principles.[10] By using this analogy, Cassirer attempted to spread Marburg's brand of historicized Kantianism beyond the Marburg inner circle. Despite Cohen's lukewarm reaction (Cohen to Cassirer, August 10, 1910; Cassirer 1995–, vol. 18, doc. 45), Cassirer's endeavor turned out to be very successful.

Substanzbegriff und Funktionsbegriff (Cassirer 1910) rapidly became the most respected and widely read work outside of Marburg, followed by Natorp's (1910) *Die logischen Grundlagen der exakten Wissenschaften*, which was published at around the same time. The Marburg School had reached its peak. However, when the 70-year-old Cohen retired in 1912, the school (Cassirer 1912; Natorp 1912) was already starting to dissolve (Natorp to Görland, June 6, 1912; Holzhey 1986, doc. 131). Cassirer's (1918b, 1920b, 1926, 1943) intense personal and philosophical relationship with Cohen remained unaffected. In a letter to Natorp from the beginning of 1914, written on the occasion of the latter's sixtieth birthday, Cassirer continued to emphasize his gratitude for his "Marburg apprenticeship" (Cassirer to Natorp, January 23, 1914; Cassirer 1995–, vol. 18, doc. 208). However, the publication of Cassirer's "studies on the history of the German spirit," collected in 1916 in *Freiheit und Form*, could be seen as the transition from an early Cassirer, the neo-Kantian "philosopher of science," to a mature Cassirer, the well-rounded "philosopher of culture." As Cassirer conceded in a letter to Natorp, "the actual external connection to the school [*Schulzusammenhang*] is loosening more and more" (Cassirer to Natorp, January 1, 1917; Cassirer 1995–, vol. 18, doc. 226). Nevertheless, Cassirer also pointed out that "the more each of us tries to continue on our own path, the closer we ultimately get to the same problems and tasks. And this is basically

10. *Substanzbegriff und Funktionsbegriff* expresses the same conception of the a priori as *Das Erkenntnisproblem* (see n. 9) by using an analogy with Klein's (1872) Erlangen program. The historical development of science, Cassirer (1910) claims, "must leave a certain body of principles [*Prinzipien*] unaffected" (355; 1923b, 268). It is surmised that this invariance is not a coincidence: "those moments, which persist in the advance from theory to theory *because* they are the conditions of *any* theory" (Cassirer 1910, 357; 1923b, 269; first emphasis added). At no given stage can the goal of finding these conditions be fully achieved; "nevertheless it remains as a *demand*" to search for them. Strictly speaking, only "the ultimate *logical invariants*" (Cassirer 1910, 357; 1923b, 269) should be considered a priori. However, we can never be sure to have them. In my view, at this point, Cassirer seems to consider each invariant as constitutive, although provisional a priori. Thus, I do not agree with those who attribute to Cassirer a "regulative" conception of the a priori already in the 1910s (see, e.g., Ryckman 2005; Friedman 2008; Heis 2014a).

the best and safest proof that we can hope for our intimate relationship" (Cassirer to Natorp, January 1, 1917; Cassirer 1995–, vol. 18, doc. 226).

3. From the Physics of Models to the Physics of Principles: Cassirer and the Principle of Relativity

After more than a decade as a *Privatdozent* in Berlin, in 1919, Cassirer became an ordinary professor at the recently founded University of Hamburg. At about the same time, the success of the Eddington-Dyson eclipse rushed Cassirer back to the philosophy of the natural sciences.[11] Toward the end of the year, the *New York Times* announced the confirmation of general relativity, and Einstein (1919b) himself wrote a celebrated article in the London *Times* on November 28, 1919. In it, he famously declared relativity theory to be a theory of principle (e.g., classical thermodynamics) rather than a constructive theory (e.g., the kinetic theory of gases), a theory based on empirically based abstract principles rather than on the construction of detailed models, or *Bilder* (Einstein 1919b). On December 14, 1919, the front page of the *Berliner Illustrierte Zeitung* (vol. 28, no. 50) printed a profile of Einstein in which he was described as a "new great figure in world history," comparable to Copernicus, Kepler, and Newton.

It is not clear whether Cassirer read Einstein's article for the London *Times*. As far as I can see, it is never mentioned in his writings. However, Cassirer reached a surprisingly similar conclusion, probably relying on common sources. Relativity theory appeared to Cassirer as an example of the progressive prevailing of the physics of principles over the physics of models, which was still dominant in the nineteenth century. In the following months, he was able to rapidly finish a short but insightful philosophical book on relativity (Cassirer 1921b). He sent the first draft of the manuscript to Einstein for feedback in May 1920 (Cassirer to Einstein, May 10, 1920; Einstein 1987–, vol. 10, doc. 11).[12] Einstein's reaction

11. The Marburg School had developed a philosophical interest in relativity theory early on. Natorp wrote an often-criticized but, after all, well-informed section on relativity in his 1910 monograph (392ff.). Just afterward, Cassirer wrote to Natorp that a student of Cohen's, Otto Buek (1904, 1912), planned to "work on the relativity principle" (Cassirer to Natorp, May 16, 1911; Cassirer 1995–, vol. 18, doc. 168). Some years later, in 1914, Cohen wrote to Natorp that Buek was attending Einstein's lectures in Berlin (Cohen to Natorp, November 28, 1914; Holzhey 1986, vol. 2, doc. 145). Curiously, Buek became a good friend of Einstein's and signed a pacifist manifesto with him in 1918 (Einstein 1987–, vol. 6, doc. 8).

12. Cassirer to Einstein, July 15, 1920, 3–393, Albert Einstein Archives at the Hebrew University of Jerusalem (EA). Einstein read the book while traveling to the Netherlands to meet his friend Paul Ehrenfest (Einstein to Elsa Einstein, May 19, 1920; Einstein 1987–, vol. 10, doc. 19; see also Einstein to Vahinger, June 3, 1920; Einstein 1987–, vol. 9, doc. 41).

was respectful but somewhat lackadaisical (Einstein to Cassirer, June 5, 1920; Einstein 1987–, vol. 9, doc. 44), reflecting Einstein's mild hostility toward Kant's philosophy at that time (Beller 2000).[13]

Nevertheless, their relationship remained friendly.[14] In August 1920, Cassirer announced to Einstein that the revised manuscript (Cassirer 1921b) was ready for publication (Cassirer to Einstein, August 28, 1920; Einstein 1987–, vol. 10, doc. 112). Amid numerous political and antisemitic attacks against Einstein (1920b), Cassirer hoped the book would help to dispel "the spiritual confusion that still persists in so many minds about these things and which seems to be deliberately exploited from some quarters" (Cassirer to Einstein, August 28, 1920; Einstein 1987–, vol. 10, doc. 112). Indeed, the book was also written to rebuke the charges of "relativism," "abstractness," and "unintuitiveness" made against "Einsteinery" in some philosophical circles (see Cassirer 1920c). Faithful to his continuist approach to the history of science, Cassirer meant to show that the allegedly unsettling features of the new theory simply brought to light tendencies that were all clearly recognizable in previous theories.

3.1. Cassirer's *Zur Einstein'schen Relativitätstheorie* and his 1920–1921 Lectures on Relativity

Cassirer's book was published at the beginning of 1921 (Ryckman, 2005). In the 1920–21 winter term, from early October to the end of January, Cassirer (1920–21) also gave a series of 13 lectures on relativity, the notes for which constitute a neglected source for understanding the core argument of Cassirer's book. In particular, the lectures seem to offer an additional glimpse into

13. As usual, Einstein conceded to the Kantians that "one must approach experiences with some sort of conceptual functions in order for science to be possible." However, he insisted that he did not believe "that our choice of these tools is constrained *by virtue of the nature of our intellect*" (Einstein to Cassirer, June 5, 1920; Einstein 1987–, vol. 9, doc. 44). Although Mach (1872) failed to appreciate the nonempirical origin of some physicists' conceptual tools, he was right in recognizing that the latter must be "replaceable." Einstein praised Cassirer for having grasped the spirit of the theory. Nevertheless, he complained that the manuscript did not sufficiently emphasize the empirical origins of both the special and the general principle of relativity (Einstein to Cassirer, June 5, 1920; Einstein 1987–, vol. 9, doc. 44). This issue was of great concern to Einstein at that time. He feared that both of his theories were perceived as overly speculative and abstract (see Einstein 1920a, 1). Cassirer promised a substantial revision of the manuscript in light of Einstein's commentaries (Cassirer to Einstein, June 16, 1920; Einstein 1987–, vol. 9, doc. 58).

14. Cassirer invited Einstein to be his personal guest and stay at his house during his visit to Hamburg to deliver a lecture (Cassirer to Einstein, July 15, 1920, 3-393, EA), which was held on July 17, 1920 (Reich, 2000). After dinner, a discussion session was organized at Cassirer's home at 26 Blumenstrasse. The encounter is vividly recollected by Cassirer's wife, Toni Bondy Cassirer (1981, 134–35), in her biography of Cassirer, giving the erroneous date of January 1921 (Reich 2000).

Cassirer's sources. Besides Einstein's (1917) popular work on relativity, Cassirer had clearly been influenced by Max Planck's (1909, 1910, 1915b) lectures from the 1910s and, most of all, by the writings of Planck's student Max von Laue. Laue (1911, 1913, 1921) not only authored the first textbook on relativity (in 1911) but also published some papers with philosophical content on the topic. In particular, I argue that Laue's reading of relativity offered Cassirer a *trait d'union* between his old writings on the prehistory of energy conservation and his philosophical analysis of Einstein's new theories. Cassirer (1902) returned more directly to the argumentative structure of his book on Leibniz by giving the relativity principle the same status as the principle of conservation of energy. Both of these principles are constraints that we impose on the laws of nature rather than a simple accidental by-product of these laws. This comparison is indeed prominent in the "conclusion" of Laue's (1911/1961b, 184ff.; 1913) textbook and in his more popular writings.

Possibly following Einstein's suggestion, these lectures emphasize the empirical origin of the theory. On this point, Cassirer (1920–21) refers to Laue: "For the justification of the principle of relativity—says Laue (1911, 104)—this series of experiments plays the same role as the attempts to build a perpetuum mobile played in the justification of the energy principle" (101). The continuing series of failures to construct a perpetuum mobile finally cemented the belief that they could not be an accident. The question was then turned upside down. It has been repeatedly verified that it is in no way possible—by mechanical, thermal, chemical, or other processes—to obtain a perpetuum mobile of the first kind. What must the laws of nature governing those processes be like if the perpetuum mobile is to be impossible? The energy principle was the answer to this question. This principle plays the role of a constraint that all dynamic laws governing those processes have to satisfy if the perpetuum mobile of the first kind is to be impossible.

The special theory of relativity is the result of the very same strategy. As Cassirer (1920–21) put it, using one of his favorite quotes from Goethe, "The highest art in intellectual life and worldly life—Goethe once wrote to Zelter—consists *in turning the problem into a postulate* that enables us to get through" (101; Goethe to Zelter, August 9, 1828; Goethe and Zelter 1832, 61). Like the discoverers of the energy principle, Einstein "made a virtue out of necessity" (101). Experience has increasingly shown that a privileged ether system cannot be found by experiments that rely on either mechanical, optical, or electromagnetic processes. The theory of relativity surmised that this failure could not be an accident, and it raised this conjecture about the status of a postulate (Einstein 1905). It introduced as a "heuristic maxim the most general assumption that such a system *cannot* and *must* not exist" (Cassirer 1920–21, 121–22). The apparently coincidental failure of all ether drift experiments was raised to the level of the *principle* [*Prinzip*] that for the physical description of natural processes, no particular inertial reference body should be singled out (121–22). In this way, the true philosophical import of relativity theory was to have transformed "a mere negative expression into a positive expression," "a *limitation* of physical knowledge into a *principle* of such knowledge" (Cassirer 1921b, 74; 1923b, 408). As Cassirer (1920–21) put it in his lecture notes:

> The principle of the constancy of the speed of light combined with the principle of relativity proves to be suitable to give a complete account not only of all mechanical but also of all optical-electrical phenomena. In their connection, the two principles no longer represent *a unified thing that exists in nature*; but they formulate *a uniform basic relation which all special laws of nature obey*: a most general moment of our knowledge of nature. *They create the framework for the form of natural law in general*, and this form is truly uniform, although it is precisely through it that it is required that the particular space and time values that are determined within different systems do not agree, but only stay in a determinate functional relationship, in mutual coordination [*Zurordung*]. This methodical state of affairs *methodische Sachverhalt* is expressed in the proposition that forms the actual core of the theory of relativity: . . . "the general laws of nature are covariant with respect to Lorentz transformations." (81)

Thus, the content of special relativity takes (so to speak) the form of an imperative, a demand that all laws of nature have to satisfy if the relativity and light postulates are to hold. Every fundamental law of nature must be so constituted that it is transformed into a law of exactly the same form when, instead of the space-time variables x, y, z, t of the coordinate system K, we introduce new spacetime variables x', y', z', t' of a coordinate system K'. The relationship between the unprimed and primed variables is given by the Lorentz transformation. The latter are not laws of nature but serve as criteria for selecting the allowable law of nature. If we express a lawlike statement mathematically in a system K by applying the Lorentz transformation, then we obtain its expression with respect to K'. If the two expressions are identical, then the candidate law is well formulated; if not, then it has to be rejected or modified so that it complies with the relativity principle.

To further support this interpretation, Cassirer (1921b) could quote Einstein's repeated characterization of the relativity principle as "a general maxim" that should serve as a "heuristic aid when searching for the general laws of nature" (38; 1923b, 377; see Einstein 1917, 29). Thus, special relativity is not a theory in the proper sense of the word (e.g., mechanics or electrodynamics) that entails the individual laws of nature (Maxwell's equations, Newton's particle dynamics, hydrodynamics, elasticity theory, etc.). It is rather the expression of a "law of lawlikeliness [*Gesetz der Gesetzlichkeit*]" (Cassirer 1920–21, 88), "a criterion of their validity and admissibility for all special physical areas and for all special physical theories" (Cassirer 1921b, 33; 1923b, 359). In this "formal limitation, which is placed on natural laws by this maxim, lies—as Einstein himself urged—the characteristic 'sagacity' [*Spürkraft*] of the principle of relativity" (Cassirer 1921b, 38; 1923b, 377f).[15] Indeed, the theory does not simply assert that the actual laws of nature that we know happen to be Lorentz invariant—the theory makes the bolder claim that all possible laws of nature must be Lorentz invariant. In this form, a merely analytic principle describing the formal properties of certain equations is transformed into a synthetic one that allows for new results. Maxwell's equations turned out to already be Lorentz invariant, whereas, on the contrary, Newton's equations of motion were not. Thus, they had to be modified in conformity with the new kinematics. From this modification, Einstein obtained testable consequences—the velocity dependence of mass that could be observed in fast-moving electrons.[16]

The status that Cassirer (1902) attributes to the relativity principle is thus similar to the status that he attributed to the energy principle in his book on Leibniz. Both principles, like any other physical statement, are expressed in the form of functional equations. However, they are not used as equations expressing this or that individual law of nature but rather as principles that impose a formal requirement on the analytical formulation of all possible laws. As

15. The expression is borrowed from Einstein (1917, 67).

16. As Cassirer rightly noted, this result was not new; Lorentz (1903–4) had obtained the same law of the velocity dependence of the mass of the electron: "In the modern electron theory, it follows from the well-known investigation of Kaufmann (1906) that the 'mass' of an electron is not unchangeable, but that it rapidly increases with the velocity of the electron as soon as the latter approaches the velocity of light" (Cassirer 1921b, 66; 1923b, 400–401). By assuming that the particles constituting the electron experience a contraction in the direction of motion, Lorentz arrived at the same law of motion as Einstein, which was later confirmed (Bucherer 1908). As Cassirer (1921b) points out, the theory of relativity leads to the same result, "but it reveals in this too its peculiar nuance and character" (66; 1923b, 400–401). Lorentz (1903–4) maintained Newton's second law and shifted the explanatory burden to electrodynamics. The mass, which is supposed to be constant according to the old mechanics, varies because the electron self-field (i.e., the magnetic field generated by the electron) resists its motion. On the contrary, Einstein (1905, sec. 10) modified Newton's point dynamics so that it satisfies the relativity principle. Thereby, he obtained the velocity dependency of masses of any kind. No particular model of the electron was required. Electrons served only to test the theory because electrons in β-rays move with a velocity close to that of light.

Cassirer (1921b) points out, explicitly referencing his earlier monograph, it was Leibniz himself who "referred to this logical moment in it" (45; 1923b, 384). As we have seen, the young Cassirer (1921b) had already pointed out that, in the nineteenth century, "the same process of thought" introduced by Leibniz was "repeated on broader physical lines in the discovery and grounding of the modern principle of energy" (46; 1923b, 384).

Helmholtz "tried to express the principle of conservation of energy in the language of classical mechanics and tried to prove it with the formulas of this mechanics" (Cassirer 1921a, 288). For the young Helmholtz, the energy principle was the consequence of the fact that all forces in nature are mechanical—that is, central forces whose intensity depends only on distance (Cassirer 1920–21, 64). On the contrary, Cassirer (1921a) argues, "Mayer's idea took a different turn from the start" (288). Whether, for example, heat is "in its essence nothing other than molecular motion" remains open; "it is sufficient that the constant exchange ratio between the two is established and brought to an exact expression" (288). Instead of investigating the peculiar qualitative nature of heat or mechanical motion, Mayer only postulated the constant quantitative proportion between work and heat units. The latter imposes a constraint on the fundamental forces that there could be, without addressing the question of the metaphysical nature of the individual forces that happen to be (e.g., electromagnetic, mechanical).

We are confronted with the same methodological approach in the foundation of the special relativity principle. Einstein's formulation of the relativity principle did not imply the reduction of all physics to electrodynamics, just as Mayer's formulation of the energy principle does not imply the reduction of all physics to mechanics. The initial contradiction between mechanics and electrodynamics that is revealed by the negative result of ether drift experiments was overcome not "by using the electrodynamic processes as a key to the mechanical" but by establishing "*a far more perfect and deeper unity between the two than previously* existed" (Cassirer 1921b, 33; 1923b, 373; emphasis added). The unification of the two separate fields of theoretical physics—electrodynamics and mechanics—is not obtained through a process of horizontal integration, a reduction of the one to the other, but through a vertical integration, a subsumption of both theories under a higher principle. Rather than the reduction of mechanics to electrodynamics and of electrodynamics to mechanics, in relativity theory, "a *truly universal principle*, a heuristic maxim of investigation in general, is established, which claims to contain a *criterion* of the validity and permissibility of *all* particular physical fields and theories" (Cassirer 1921b, 32f; 1923b, 373; emphasis added). This result was reached not simply by heaping up experimental data but rather "rests on a critical transformation of the system of fundamental physical concepts" (Cassirer 1921b, 33; 1923b, 373).[17]

Cassirer (1921b) could then point out that, despite its apparent radical novelty, that the relativity principle, like the energy principle, is ultimately an instance of "that general direction of physical thought, which has been called the 'physics of principles' in contrast to the physics of pictures and mechanical models" (16; 1923b, 359). In Cassirer's (1920–21) view, the history of the energy principle shows that "physics in the nineteenth century more and more ceased to be a physics of *models* [*Bilder*] in order to transform itself instead into a physics of *principles*" (63). Physics previously devised "a separate model for each of the different areas, and it was often satisfied with simply placing these often very different models side by side and lining them up" (123). It is true that when we observe the evolution of physics, at first we are facing an ever-changing "chaos of pictures and opinions." We might therefore ask, "Which is the constant component [*Bestandteil*] of the physical theories and hypotheses, if their model-like [*bildlicher*] component does not play this role? What established some sort of objective connection among such models?" (64). The energy principle, the least action principle, and so forth have survived despite the demise of the particular models that attempted to describe a certain class of natural processes directly. The reason for this stability is that the formulation of "a 'principle' . . . never refers directly to things," but it "sets up a general rule for complex functional dependencies and their mutual connection" (Cassirer 1921b, 17; 1923b, 359). Principles show an astounding resilience, despite the rapid succession of triumphs and collapses of the individual theories because they are the very criteria for selecting what can be considered to be an acceptable physical theory.

17. As Natorp (1910, 392ff.) had already realized a decade earlier, it was in Minkowski's (1909) work that this radical change in the concept of space and time found its most fruitful expression. However, Cassirer, in contrast to Natorp, did not attempt to "immunize" but to "revise" Kant, or at least to locate the spirit of Kantian philosophy at a deeper level (Hentschel 1990b). In Cassirer's (1920–21) view, the main result of Kant's philosophy of space and time was the recognition that "the reality assigned to space and time is not that of things [*Dingen*], but of conditions [*Bedingungen*]" (112). They are not "things" that exist in *natura rerum* but rather indispensable "conditions" of all our empirical knowledge. From the point of view of critical idealism, their empirical reality resolves in their objective validity (Cassirer 1920–21, 112). The passage from classic to Minkowski "spacetime" does not change this fundamental result. Minkowski's spacetime is nothing but a symbolic representation of certain analytical relations between variables that appears in the laws of nature. "The 'postulate of the absolute world,'" as Minkowski defines this requirement, is ultimately "a postulate of absolute method" (Cassirer 1921b, 117; 1923b, 445f). The opposition of the electromagnetic worldview and the mechanistic worldview was overcome not ontologically, by reducing one set of phenomena to the other, but by methodologically conforming both sets of laws to a common principle (see Corry 2010).

According to Cassirer, the passage from special to general relativity shows the same epistemological tendency. At the same time, it testifies to the fact that physicists must be ready to abandon some previously selected principles in favor of more general ones. Einstein's initial goal was to formulate a special relativity theory of gravitation starting from Newton's action-at-a-distance theory. As in any other field of physics (e.g., classical point dynamics, elasticity, hydrodynamics), Einstein attempted to modify Newton's gravitational laws so that they conformed to the new relativistic kinematics. However, he realized early on that the modification did not proceed successfully, which revealed the limitations of special relativity. Meanwhile, the method of special relativity pointed beyond itself, leading to an extension of the relativity principle to accelerated motions. Einstein understood that the notion of a class of privileged nonaccelerated frames that still hold in the special theory could not be empirically singled out because of the identity of inertial and gravitational mass. Again, the problem was transformed into a postulate. The general principle of relativity, according to which every coordinate system is just as good as any other, was elevated to a condition that we impose on the formulation of the laws of nature a powerful selection principle that restricts the range of possible laws. Cassirer (1920–21) makes this point very clearly in his lecture note:

> We can only call those lawlike statements laws of nature (i.e., give them objective universality) if their form is independent of the peculiarity of our empirical measurements, of the special choice of the variables x_1, x_2, x_3, x_4, which express the space and time parameters. In this sense, the principle of general relativity—that the general laws of nature do not change form in any transformation of space-time variables—could be considered an *analytical* assertion: a definition of a "general" law of nature. However, one can call *synthetic* the requirement that there must be such last invariants. To justify this demand, physicists can ultimately only rely on a "*transcendental principle*," on a principle of the "possibility of experience." One cannot prove the invariance that they claim as a metaphysical absolute; one can only show that physics as a science is and remains dependent on such an assumption. (103)

The reference to the notion of 'transcendental' was probably too "on the nose," and it did not appear in the corresponding passage of the book (Cassirer 1921b, 45). However, it is revealing of Cassirer's agenda. Like the special relativity principle, the general relativity principle ultimately serves as a "formal constraint that is placed on the laws of nature" (38; 1923b, 377–78). Cassirer could not avoid emphasizing the Kantian overtones of Einstein's insistence on the general relativity principle as a requirement, a constraint, a demand that one imposes on the laws of nature.[18] "In fact, it can be shown that the general doctrine of the invariance and univocality of certain values, which is given in the first place by the theory of relativity, must recur in some form in *any* theory of nature" (Cassirer 1921b, 45; 1923b, 384). At first sight, the general principle of relativity seems merely to be an empty analytic description of what a law of nature happens to be. Indeed, all laws of nature can trivially be written in a generally covariant form (Kretschmann 1917). However, as Einstein (1918b) argued, they acquire a synthetic meaning once they are combined with the principle of simplicity. One needs to search for the laws of nature, which cannot be simplified through the choice of a particular coordinate system. In this way, the principle of general relativity acquires its characteristic *Spürkraft*, a strong restricting power. Together with the assumption that the field equations are of the second order, this requirement was sufficient to limit the number of possible generalizations of the Poisson equation for gravity to one possibility, which led to Einstein's 1915 theory of gravitation.

3.2. Principles and Symbols

Many other themes and lines of argument are intertwined in Cassirer's book; however, most of them pertain, so to speak, to its *pars destruens*. Cassirer aims to show that the new relativity theory is opposed to both naive empiricism and naive realism (Cassirer 1921b, 47; 1923b, 386); it is better framed within Cassirer's own "critical idealism." However, as far as I can see, most interpreters have missed the main line of the argument, which constitutes the *pars construens* of the book: the characterization of relativity theory as an example of the physics of principles as opposed to the physics of models. The relativity principle, like the energy principle, is an abstract requirement that we impose on the formulation of the laws of nature rather than an intuitive model-like description of the properties of some physical system. In this way, Cassirer arrived at a conclusion that is surprisingly similar to Einstein's (1919b) article for the London *Times* in which special and general relativity are classified as "principle theories" (e.g., thermodynamics), as opposed to "constructive theories" based on *Bilder*

18. I disagree with the otherwise excellent Ryckman (2005), who claims that Cassirer regarded the principle of general covariance as a "regulative" principle but still 'constitutive' principle. On my reading, the principle of general covariance must be regarded as constitutive because it imposes a rather strict condition on the formulation of the laws of nature. Cassirer would embrace a 'regulative' conception of the a priori only in the 1930s. For a systematic overview of Cassirer's conception of the a priori, see Heis (2014b).

(e.g., the kinetic theory of gases). As we have surmised, no evidence shows that Cassirer read that article. Nevertheless, the convergence of Einstein and Cassirer on this issue is less surprising than one might judge at first sight. On the contrary, what is surprising is that no other contemporary philosopher emphasized this point.

The opposition of a physics of principles and a physics of models was widespread in the scientific and philosophical debate to the point of having become platitudinous (Lorentz 1900, 1905/1934; Poincaré 1904; Sommerfeld 1915). Cassirer (1909, 1910) referred to it more or less explicitly in his previous writings. Moreover, the idea that relativity theory was ultimately a second-order theory that imposes constraints on other, first-level dynamical laws was shared by virtually all early relativists (although with different nuances)—not only by Einstein but also by Minkowski (1908), Planck (1915b), and most of all Laue (1911/1961b), who was one of Cassirer's main sources.[19] The temptation to give a Kantian (or better, neo-Kantian) reading of the role of those 'principles' with which all laws of nature seem to comply was hard to resist. At this point, Cassirer seems somewhat ambivalent in identifying each of these principles (at least provisionally) as constitutive a priori, relative to the historically given fact of science.[20] He also seems reluctant to consider a priori only the general tendency to search for progressively more general principles.[21]

This vacillation was not fully resolved, which left Cassirer open to attack by his critics, who viewed his conception of the a priori as either to narrow or too

19. Among the many possible sources, see esp. Einstein (1914), in which the relativity principle is compared with the energy principle, both being requirements imposed on first-order laws. Minkowski (1915) explicitly compares the relativity principle to the energy principle; both are not customary laws of nature but demands that we impose on possible laws of nature (see also Minkowski 1908). Planck used the principle of least action and the relativity principle to construct "relativistic general dynamics," encompassing mechanics and electrodynamics as special cases; see Liu (1997). Laue (1911/1961b, 185f.) also resorted to a comparison between the relativity principle and the energy principle, emphasizing that both are "criteria for the admissibility" of possible theories.

20. In my view, contrary to what is usually claimed, in this period, Cassirer's stance does not seem to differ significantly from the young Reichenbach's (1920) "method of successive approximations" (66). In other words, both claim that it is always possible to find "better" constitutive principles in an infinite convergent process. The difference between Cassirer and Reichenbach lies in the nature of such principles: for Cassirer, they are principles that constrain the form of the laws of nature; for Reichenbach, they are principles that coordinate the form of the laws of nature with their empirical content (see Padovani 2009).

21. Cassirer seems to come close to defending this position in a letter to Schlick (Cassirer to Schlick, October 23, 1920; Cassirer 1995–, vol. 18, doc. 88). *Stricto sensu*, he claimed, only the unity or legality of nature in general is a priori. However, the requirement of the unity of nature, as Schlick (1921, 102) pointed out, does not impose any specific constraint on physical laws. In other words, the a priori thus conceived is not constitutive anymore. However, in my view, only in *Determinismus und Indeterminismus* did Cassirer (1936) explicitly concede that the a priori has only a regulative meaning.

vague (Schlick 1921, 102).[22] Objections of this kind might have been one of the reasons why Einstein's reaction to Cassirer's manuscript was less positive than one might expect (Einstein to Schlick, August 10, 1921; Einstein 1987–, vol. 12, doc. 202).[23] In Einstein's view, "principles" were based on empirical generalization and were certainly not a priori (see Einstein to Besso, August 28, 1918 [Einstein 1987–, vol. 8, doc. 607]; cf. Einstein to Besso, September 8, 1918 [doc. 612]). On the contrary, they were obtained by breaking the alleged a priori validity of certain previously accepted concepts (see, e.g., Einstein 1916, 120). Moreover, the goal of physics, according to Einstein, ultimately remained that of constructing "models" of natural phenomena (we want to know whether matter is made of particles or fields, which is the mathematical structure of the latter, etc.; the search for "principles" is ultimately a provisional strategy that helps physicists to restrict the range of available models when they are confronted with an *embarras de richesses* [Einstein 1918a, 701]).

On the contrary, Cassirer (1922b) saw the history of physics as a one-directional progression from the physics of models to the physics of principles, a progressive liberation from particular images of the world toward a general abstract unity: "Latest conception: physics of principles. Planck: detachment from the peculiarity of the individual images. Removal of everything that is only a sensual and descriptive element. Search for unity. He wants to embrace the whole of unity. Sens[itive] and [I]tuit[ive] are only accidents, anthropomorphic features. . . . Such a development of relativity theory can be characterized in this manner. Level of pure symbolic expression" (fragment from June 13, 1922). The use of the word "symbol" in this unpublished fragment shows how the opposition between the physics of principles and the physics of models became entangled with the "symbolic turn" in Cassirer's (1921b) philosophy that was announced in the last pages of *Zur Einstein'schen Relativitätstheorie* (126ff.; 1923b, 454ff.), in which the notion of symbolic form is introduced. In a talk that was given in Hamburg in July 1921, titled *Die Begriffsform im mythischen*

22. A few years later, Einstein (1924) made this point quite clearly in his review of Elsbach (1924). Referring explicitly to Cassirer, Einstein (1924, 1688f.) points out that when confronted with scientific theories that do not satisfy the previously recognized a priori principles, transcendental philosophers are at a crossroads: they can (*a*) concede that they were mistaken (what they thought to be an "invariant" of experience turned out not to be) or (*b*) consider the proper constitutive a priori only as the never reachable limit of an approximation process. Ultimately, as Einstein put it, neo-Kantians are neither with Mohamed nor with the Prophet (1688). In both cases, nothing in their system is ever really a a priori (i.e., a condition of the possibility of physical sciences *in general*). Indeed, according to Einstein (1928, 162), philosophy should concede that it should at most describe how science happens to work, not dictate how it must work.

23. Cassirer's approach is more akin to the "Kantian" stance of Hilbert (1921, 1923); see Ryckman (2008) for more details.

Denken, and published in an expanded form the following year, Cassirer (1922a) started to apply the "transcendental method" to myth, language, and the entire foundation of the philosophy of culture. Whereas the emerging logical empiricism aimed to proceed "analytically" by investigating the details of single sciences or even physical theories, Cassirer proceeded "synthetically" by considering natural sciences as cultural "facts" among others. This approach required an enormous undertaking that culminated in the first two volumes of *Philosophie der symbolischen Formen*, on language and myth (Cassirer 1923a, 1925).

4. Conclusion

Such a symbolic turn soon made its way into Cassirer's (1927b, 35) philosophy of natural science. In the sections dedicated to physics in the third "epistemological" volume of the *Philosophie der symbolischen Formen*, which was finished in 1927, Cassirer (1929) incorporated the opposition between the physics of principles and the physics of models into his reflections about the symbolic nature of physical knowledge. The history of nineteenth-century physics is described as "the progress from the 'model' to the 'principle'" (538; 1957, 461). Cassirer returned in some detail to the case of the discovery and justification of the energy principle and the Helmholtz-Meyer controversy. He located in Planck's (1910) talk in Königsberg a "decisive methodological conclusion," in which "the *primacy of principles over models* is recognized and carried out in every particular" (Cassirer 1929, 540; 1957, 463). The key methodological issue was not one of images but of principles, an attempt to encompass different and even conflicting natural laws in one supreme, all-embracing rule. In this respect, Cassirer (1929) saw that "a definite and unmistakable line runs from the principle of the conservation of energy to the general principle of relativity" (537; tr. 1957, 460). In modern physics (Cassirer 1927a), the tendency toward unification has triumphed over the tendency toward representation: "*The schematism of images has given way to the symbolism of principles*" (Cassirer 1929, 547; 1957, 467; emphasis added).

In a series of lectures titled "Die Einheit der Wissenschaft," given for the new cultural program *Hochschulfunk* on the national radio station *Deutsche Welle* in 1931, Cassirer (1931) described this tendency by relying again on Planck's authority[24]:

24. The *Deutsche Welle* was intended to serve as the central German radio station, as opposed to the regional networks. The program *Hochschulfunk*, or radio university, was introduced in 1930 as a cultural program.

> According to Planck's account, the entire history of physics appears to be nothing other than the constant, more or less conscious, struggle for this goal; as a single coherent process of progressive unification . . . physicists are not satisfied with the immediate sensory experience, but instead design their own model, an ideal schema of the knowledge of reality. . . . *Older physics devised a separate model for each of the different areas*, and it was often satisfied with simply placing these often very different models side by side and lining them up. However, the further science progressed, the more this mere juxtaposition of images had to be abandoned. Instead of a physics of models, modern natural science became a physics of principles. Moreover, even these latter could not stand next to each other as an unrelated multiplicity. An attempt had to be made to understand them *as emanations from a basic principle* and to interpret them as its applications. . . . The content of this highest physical principle of unity has, of course, been understood and determined differently in different epochs of natural science, depending on the state of empirical research. (125f.; emphasis added)

Cassirer appreciated the fact that, in Planck's view, the unity of the physical worldview was not to be understood as a reduction of different branches of physics to one another, as their integration under common "general principles." The unifying principle has changed over time, but the general tendency of searching for progressively more general principles has remained constant (Cassirer 1931, 125). Although the energy principle dominated nineteenth-century physics, by 1915, Planck (1915a) had indicated the least action principle as a fundamental unifying principle that contained the energy principle as a special case. The least action principle would become Cassirer's go-to example of a "principle" in the next decade. However, by that time, Cassirer's attitude toward the role of "principles" in physics had changed.

Up to the 1920s, Cassirer considered each of these unifying principles (e.g., the energy principle) as a provisional candidate for a constitutive a priori, imposing a specific constraint on the possible laws of nature. By the end of the 1920s, Cassirer suggested—mostly in private correspondence—that only the quest for the unity of nature in general (rather than for a specific unifying principle) should be appropriately considered a priori (Cassirer to Schlick, October 23, 1920; Cassirer 1995–, vol. 18, doc. 88). However, he did not fully embrace this position. As I have shown in a separate article (Giovanelli 2023), Cassirer (1936) attempted to solve this ambiguity in the 1930s in his last major book on the philosophy of science, which was dedicated to the new quantum theory. On the one hand, Cassirer introduced "statements of principle" as a separate class of statements but deprived them of their a priori status (60; 1956, 52f.). On the other hand, Cassirer transformed the a priori into a regulative principle that motivates the search for the laws of nature without imposing any specific constraints on them (75; 1956, 60).

In this way, Cassirer seems to have settled for a sort of motivational Kantianism, which does not purport to search for the conditions without which physics would be impossible but only for the conditions without which physics would not be worth pursuing. One might rightly wonder whether Cassirer's mature stance still deserves to be called a form of "Kantianism" (Ferrari 2009). However, infamous "isms" aside, Cassirer (quite isolated among twentieth-century professional philosophers) has the merit of having perceived the importance of the "metacharacter" of some statements in physics—an issue that has recently gained new momentum in contemporary philosophy of science (Lange 2009, 2016). In particular, as I have tried to show in this article, Cassirer was the only philosopher who realized that relativity theory was not a theory in the usual sense of the word, entailing individual laws of nature; it was a second-order theory that provided general constraints on all possible laws as long as space-time variables entered into their formulation.

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