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TOP EARNERS: A LABOR PRODUCTIVITY PROCESS

CLAUDIO CAMPANALE

ROCÍO FERNÁNDEZ-BASTIDAS

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Top Earners: a Labor Productivity Process

Claudio Campanale* and Rocío Fernández-Bastidas†

Università di Torino Universidad de Alicante

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Abstract

In the present paper we confront standard wage processes used in the quantitative literature on the optimal tax progressivity and a process with heterogeneous life-cycle profiles that we propose against the data. We find that the former fail to capture several features of the earnings dynamics at the very top of the distribution while our proposed model improves along some of these dimensions.

JEL classification: D31, E24, J24, J31

Keywords: top earners, earnings dynamics, heterogeneous income growth

*Departement of Economics and Statistics (ESOMAS), Università di Torino.

†Departamento de Fundamentos del Análisis Económico (FAE), Universidad de Alicante

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1 Introduction

Under the influence of the large increase in concentration of economic means occurred in the last 30 years, recent research has tried to analyze the level of tax progressivity that maximizes revenues for the government and or welfare. A key pre-requisite for the quantitative models that are used to explore these issues is a process for wages that enables the model to match the observed concentration of earnings especially at the very top of the distribution. Two approaches have been used so far to achieve this goal. The first approach used by Guner et al. (2016) consists of adding to a standard autoregressive process an ex-ante distribution of fixed earnings skills with a small fraction of individuals possessing a very high realization, sometimes termed "awesome state". The second approach proposed by Kindermann and Krueger (2020) adds to the standard autoregressive process stochastic awsome states with limited persistence. In both cases the processes are calibrated to match the cross-sectional distribution of household earnings. However their implications for the dynamics of those earnings at the very top of the distribution is not confronted with the data. This dynamics, and in particular the persistence of superstar earnings is key in determining the effects of tax progressivity and indeed these works deliver substantially different results in terms of the revenue maximizing and optimal income tax rates.

In this short essay we exploit recent empirical evidence that accurately describes the properties of earnings at the very top of the distribution in Guvenen et al. (2021a) to better shed light on the processes for top earnings that are the input to taxation models. We do this along two lines. First we examine the properties of the processes that follow the approaches of Guner et al. (2016) and Kindermann and Krueger (2020). Second we propose an alternative process for wages whose key feature is the presence of ex-ante heterogeneous growth rates. The distribution of growth rates of earnings skills features both ordinary states and awesome states, the latter being key to generate the top tail of the distribution. We show that traditional approaches fail along several of the dimensions explored in the data by Guvenen et al. (2021a), our approach on the contrary while still retaining the ability to match the cross section of the earnings distribution also improves the model performance along these extra dimensions. We thus see the wage process proposed here as an important building block of models aimed at studying the problem of the revenue and welfare maximizing tax progressivity.

2 Model Economy

We consider an overlapping generations model with idiosyncratic earnings risk, incomplete markets and elastic labor supply. Our model special feature is the addition of an Heterogeneous Income Profile (HIP), where we allow for a small fraction of agents to experience extremely high earnings growth.

The model period is one year. Agents are born and work until their retirement age J_R. Agents of age $j-1$ survive to the next period with probability s_j. They live up to a maximum age J. Population grows at rate n.

Households differ in their productivity $e(z,\theta,j)$, due to stochastic shocks z_j, a common age-productivity profile, $\bar{e}_j$, and ex-ante heterogeneity, summarized by the vector $\theta^i = (\alpha^i, \beta^i)$ where α^i denotes the level of earnings skills and β^i is their growth rate. Insurance markets are incomplete and households can accumulate a risk-free asset a_j. Borrowing is not allowed.

Agents maximize

$$\mathbb{E}\Big[\sum_{j=1}^{N} \delta^{j-1}\Big(\prod_{i=1}^{j} s_i\Big)\frac{c_j^{1-\sigma}}{1-\sigma} - \nu\frac{l_j^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}}\Big] \tag{1}$$

where $\chi \geq 0$ is the parameter that governs the Frisch elasticity of labor supply, ν measures the disutility of labor and σ is the coefficient of relative risk aversion. The discount factor is given by δ.

A government finances public spending G with individual taxes and accidental bequests. Agents pay taxes on labor and capital income, and the tax schedule T_j has three components: a flat-rate tax τ_I on total income I meant to capture state and local taxes, a flat-rate capital income tax τ_k, and a non-linear income tax scheme, T_f, representing the federal tax system. Hence, given total income $I = we(z,\theta,j)l + ra$, the total amount of individual taxes paid by the household is:

$$T_j = T_f(I) + \tau_I I + \tau_k ra \tag{2}$$

The nonlinear tax function[1] is given by $t(\tilde{I}) = 1 - \lambda \tilde{I}^{-\tau}$, where $t(\tilde{I})$ is the average tax rate at the relative income level $\tilde{I}$. Hence, federal taxes paid amount to $T_f = It(\tilde{I})$. The parameter λ defines the level of the average tax rate, while $\tau \geq 0$ controls the progressivity of the tax function.

Households pay a payroll tax τ_p to finance a pay-as-you-go pension system, and receive the pension benefit, b_j, when retired.

[1] The tax system is modelled following Guner et al. (2016).

Let $x = \{a, z, \theta, j\}$ be the vector of state variables. The household's problem, described in recursive way, is given by

$$V(a, z, \theta, j) = \max_{c,l,a'} \left[\frac{c^{1-\sigma}}{1-\sigma} - \nu \frac{l^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} \right] + \delta s_{j+1} \mathbb{E}_{z',\theta'|z,\theta} V(a', z', \theta', j+1) \quad (3)$$

subject to the budget constraint

$$c + a' = (1+r)a + \lambda_w (1-\tau_p) we(z, \theta, j) l + (1-\lambda_w) b_j - T_j \quad (4)$$

and inequality constraints

$$c \geq 0, \quad a' \geq 0, \quad 0 \leq l \leq 1 \quad (5)$$

where r denotes the rental rate of capital and w is the wage rate per unit of effective labor. The indicator function λ_w equals 1 when the agent is working and zero otherwise.

Output is produced according to $Y = AF(K, L) = AK^{\omega} L^{1-\omega}$. Capital depreciates at rate Δ. The representative firm hires capital K and labor L in perfectly competitive markets.

The equilibrium for this economy can be defined in the standard way and is omitted for brevity.

3 Calibration

Agents are born at age 25, retire at age 65, living up to a maximum of 100 years. The population growth rate is $n = 0.011$ (1.1%). The discount factor δ equals 0.982 to match a capital-output ratio of 2.95. The disutility of labor ν is set to 8.4 for an average of hours worked equal to one third. The Frisch elasticity χ is set to a value of 1 as in Guner et al. (2016), and the risk aversion coefficient σ equals 1. The parameter A is normalized to have a wage rate equal to 1. We set the capital share ω to 0.35 and the depreciation rate Δ equal to 0.06.

Regarding the tax system, we also follow Guner et al. (2016) and set $\lambda = 0.911$ and $\tau = 0.053$. The state tax τ_I equals 0.05, the corporate tax rate is $\tau_k = 0.074$, and the payroll tax τ_p equals 0.122.

The key element in the model is the labor process. We specify the process for the log-hourly wage of an agent by way of the following equation:

$$\log(e(z, \theta, j)) = \alpha^i + \beta^i (j/10) + \bar{e}_j + z_j, \qquad z_j = \rho z_{j-1} + \varepsilon_j, \qquad z_0 = 0 \quad (6)$$

where $\varepsilon_j \sim N(0, \sigma_\varepsilon^2)$. Agents draw at birth α^i, distributed as $\mathcal{N}(0, \sigma_\alpha^2)$, and their individual-specific growth rate of earnings β^i, that follows $\mathcal{N}(\mu_\beta, \sigma_\beta^2)$.

With respect to the calibration of the parameters of the given process we follow the procedure in Guner et al. (2016) and Kindermann and Krueger (2020) of taking parameters for the bottom part of the distribution from standard estimates while for the top part of the distribution where such estimates do not exist we pick parameters so that the model output matches certain available moments in the data.

The persistence ρ of the shock z equals 0.958, and the variance σ_ε^2 equals 0.017, following Kaplan (2012). The age-productivity profile $\bar{e}_j$ comes from Guner et al. (2016). We assume that α^i and β^i are uncorrelated. We set $\sigma_\alpha^2 = 0.25$ in line with evidence in Kaplan (2012). From estimates in Guvenen et al. (2021b), we obtain $\sigma_\beta^2 = 0.0384$.

We discretize the support of β^i with a 5-point grid. To allow for the extremely high earnings growth that are fundamental in trying to capture the dynamics at the very top of the distribution, we consider two extra values for the β^i, β^{6*} and β^{7*}, with $\beta^{6*} < \beta^{7*}$. Agents are born in these states with probabilities $\{\pi_{6*}, \pi_{7*}\}$. Ordinary growth states are fixed, the superstar growth states though are very persistent but not fully permanent. The following transition matrix summarizes the mobility across earnings growth states

$$
\mathrm{P}_{\beta,j} = \left[\begin{array}{c|ccccccc}
 & \beta^1 & \beta^2 & \beta^3 & \beta^4 & \beta^5 & \beta^{6*} & \beta^{7*} \\
\hline
\beta^1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta^2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\beta^3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\beta^4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\beta^5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\beta^{6*} & p_{61}(j) & p_{62}(j) & p_{63}(j) & p_{64}(j) & p_{65}(j) & (1-p_{67})(1-p_6^e(j)) & p_{67} \\
\beta^{7*} & 0 & 0 & 0 & 0 & 0 & p_7^e(j) & 1-p_7^e(j)
\end{array}\right] \tag{7}
$$

where the $p_{6i}(j)$ for $i = 1, \ldots, 5$ are given by $p_{6i}(j) = w_i(1 - p_{67})p_6^e(j)$. The w_i term denotes the probability mass of each $\{\beta^i\}_{i=1}^5$ in the stationary distribution of β^i absent the superstar growth states.[2] The probability p_{67} of moving from β^{6*} to β^{7*} is constant during the life cycle. The probability of moving from a given superstar state to lower growth states $p_i^e(j)$ is given by the following logistic

[2] Hence, $\sum_{i=1}^5 w_i = 1$. The sixth row sums up to 1 since $\sum_{i=1}^5 p_{6i}(j) = p_6^e(1 - p_{67})$ and $\sum_{i=1}^6 p_{6i}(j) = 1 - p_{67}$. This specification implies that an agent falling from state 6 down to ordinary states ends up in each of the ordinary states with the same probability an ordinary agent is born in them.

function

$$p_i^e(j) = \frac{L_i}{1 + p_1 \exp(-p_2(j - j_0))} \tag{8}$$

for $i = \{6, 7\}$. This specification delivers an age-dependent probability of exit that increases as agents advance in their working career. This allows us to match the earnings growth of top earners during the life cycle. We set exogenously the initial probabilities $\{\pi_{6*}, \pi_{7*}\}$ to 0.025 and 0.005, respectively. The other parameters are endogenously chosen to match several features of top earners growth, and are summarized in Table 1.

Table 1: Endogenously calibrated parameters for HIP process

Parameter	Value	Targets
μ_β	0.1	Earnings growth bottom 99%
j_0	30.0	Change in slope earnings growth at age 45
p_1	10^{-5}	Prob. exiting superstar shocks zero at age 25
p_2	1.1	Earnings growth for second 0.9% after age 45
p_{67}	0.012	Prob. top 0.1% moving to bottom 99% in 5 years
L_6	0.13	Earnings growth top 0.1% after age 45
L_7	0.17	Share earnings top 1%
β^{6*}	0.78	Mean earnings growth second 0.9% until age 45
β^{7*}	1.31	Mean earnings growth top 0.1% until age 45

Some parameter values mostly impact only one moment while others have stronger interactions. Among the former group we set β^{6*} and β^{7*} to match the growth rate of earnings up to age 45 for agents in the top 0.1% and next 0.9% of the lifetime earnings distribution and p_1 is set so that superstar states are initially virtually permanent. The parameter j_0 controls the age at which the probability of exiting superstar states start to change rapidly. This by a composition effect reduces the growth rate of earnings of the top groups to ordinary values so j_0 is set to match the data in this dimension. The values of p_2, L_6, L_7 determine how fast and by how much exit probabilities increase once they start to do so and again by a composition effect determine the growth rate of earnings of top lifetime earners late in the working life and the share of earnings in the top 1% of the cross sectional distribution. Finally, adding a small probability of moving from state 6 upwards to state 7 helps aligning the probability of moving from the top 0.1% to the bottom 99% of the current earnings distribution in the model and in the data, although we could not match this target perfectly.

Table 2: Earnings Distribution

	Quintiles					Top				
	1st	2nd	3rd	4th	5th	10%	5%	1%	0.1%	Gini
Data[a]	-0.1	4.2	11.7	20.8	63.5	47.0	35.3	18.7	6.6	0.636
HIP Model	0.0	4.6	11.0	18.1	66.3	50.7	37.8	18.4	6.7	0.649
Guner et al.	0.0	4.7	11.0	19.9	64.4	47.9	35.6	18.6	4.2	0.636
K & K	0.0	5.1	9.4	19.9	65.5	53.3	41.0	18.7	5.7	0.654

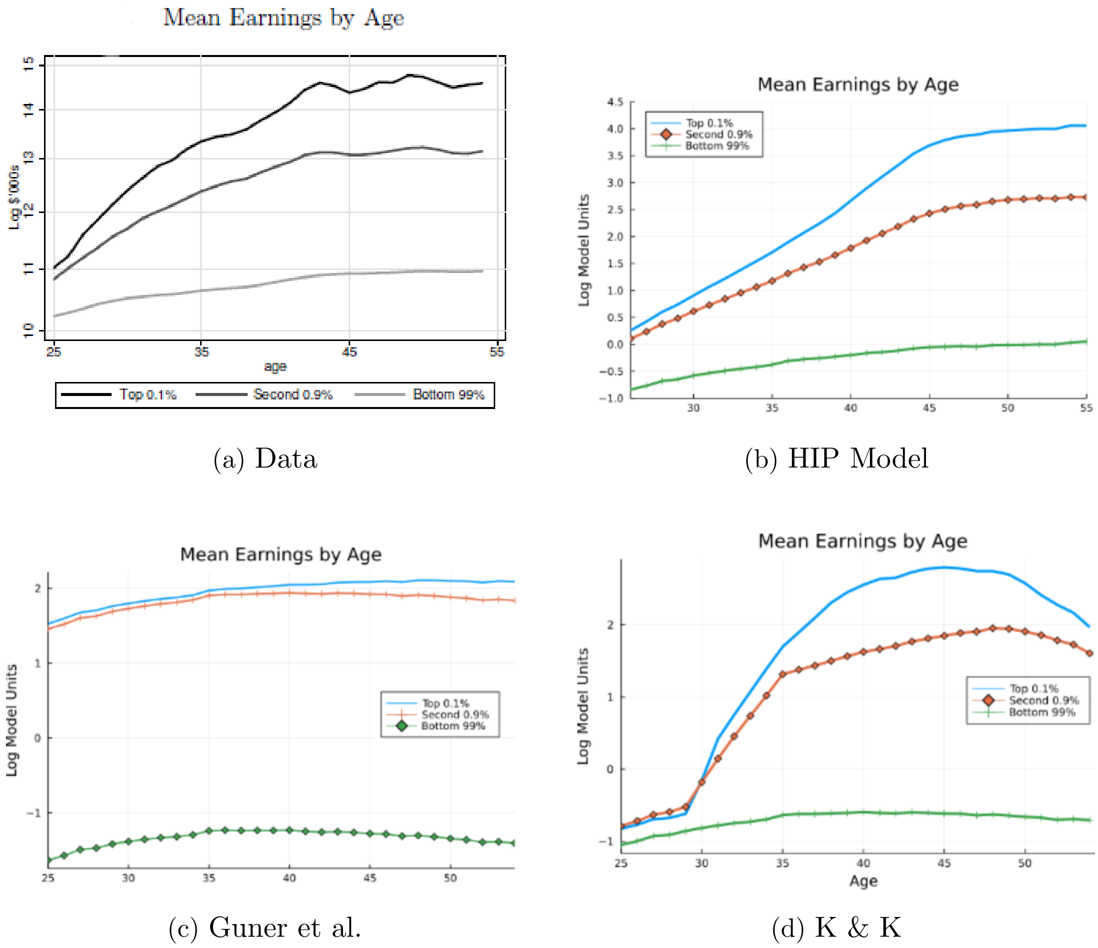
[a]Data from the 2007 Survey of Consumer Finances.

Finally, we consider two alternative model economies. The first one, labeled *Guner et al.*, features their earnings process with a permanent superstar state. The second one, labeled $K\&K$, takes the process from Kindermann and Krueger (2020). We modify the value of the highest shock in $K\&K$, and the superstar state in *Guner et al.* to obtain a share of earnings for the top 1% of 18.7% as in the SCF 2007. Other parameters, like the discount factor and disutility of labor, are recalibrated in each version to match the same targets as in the HIP model.

4 Results: Top Earners Statistics

Results are reported in Figure 1 and tables 2, 3 and 4. We report the data and the corresponding values from our HIP model and for comparison our version of the models in Guner et al. (2016) and Kindermann and Krueger (2020). We focus on the statistics for the three groups of ordinary households that make the bottom 99 percent of the distribution and on the top 1% and 0.1% of the distribution of lifetime earnings. Since comparable literature on taxation of the top of the distribution focuses on matching the shares at different percentiles and in particular at the top 1% we check first that all the three models satisfy this criterion. This is done in Table 2 where we see that all the three models can match quite well the distribution of earnings at different percentiles beyond the top 1% that was used as a target. They also do a good job at the 0.1% percentile except perhaps the model in Guner et al. (2016) which is only two thirds of the data value

Figure 1 shows the results in terms of the lifetime earnings growth for the very top percentiles and for the ordinary households in the economy. Since our HIP process is calibrated to match this target it reproduces quite well both the overall earnings growth and its age profile for the three groups. The model in



(a) Data (b) HIP Model

(c) Guner et al. (d) K & K

Figure 1: Age Profile by lifetime Top Earnings Groups

Guner et al. (2016) fails in this dimension since its fixed skill levels implies that lifetime profiles are just scaled version of each other. The model in Kindermann and Krueger (2020) can generate earnings growth for the two top groups early in life that is reasonably close to the data, although later in life there is a more substantial fall for the top 0.1% group as a consequence of the low persistence of the superstar shocks.

This is confirmed in Table 3 which reports the 5-year transition matrix across the three groups. Panel A in the table reports the data. Results from our HIP model, reported in panel B show a very close match especially in the persistence within each of the two top groups. The probability of moving out of the two top groups is also well matched except for some more notable deviations between model and data for the probability of moving from the top 0.1% to the bottom 99% of the distribution. Panel C shows that the model in Guner et al. (2016) generates too high persistence within the two top groups and too low a probability of moving from these two groups to the bottom 99% of the distribution. Panel D

Table 3: Transition Probabilities across Top Earnings Groups.
Five-Year Earnings, Five-Year Transitions

Panel A: Data				
	Top 0.1%	Second 0.9%	Bottom 99%	Exit Sample
Top 0.1%	0.40 (0.59)	0.22 (0.32)	0.06 (0.09)	0.32
Second 0.9%	0.05 (0.07)	0.46 (0.61)	0.24 (0.32)	0.25
Bottom 99%	<0.01 (<0.01)	<0.01 (<0.01)	0.72 (0.96)	0.27

Panel B: HIP Model				
	Top 0.1%	Second 0.9%	Bottom 99%	Exit Sample
Top 0.1%	0.37 (0.59)	0.16 (0.25)	0.11 (0.17)	0.37
Second 0.9%	0.06 (0.09)	0.45 (0.64)	0.19 (0.27)	0.30
Bottom 99%	0.00 (0.00)	0.00 (0.01)	0.84 (0.99)	0.15

Panel C: Guner et al.				
	Top 0.1%	Second 0.9%	Bottom 99%	Exit Sample
Top 0.1%	0.61 (0.72)	0.23 (0.28)	0.00 (0.00)	0.16
Second 0.9%	0.04 (0.05)	0.80 (0.94)	0.01 (0.01)	0.16
Bottom 99%	0.00 (0.00)	0.00 (0.00)	0.84 (1.00)	0.16

Panel D: K & K				
	Top 0.1%	Second 0.9%	Bottom 99%	Exit Sample
Top 0.1%	0.23 (0.31)	0.17 (0.23)	0.33 (0.46)	0.27
Second 0.9%	0.02 (0.03)	0.26 (0.32)	0.53 (0.65)	0.19
Bottom 99%	0.00 (0.00)	0.00 (0.00)	0.84 (1.00)	0.16

Note: numbers in parenthesis report the transition rates conditional on remaining in the sample (normalized by one minus the exit rate).

shows that the reverse is true for the model in Kindermann and Krueger (2020).

An alternative way of looking at the dynamics of very top earnings is provided in Table 4. This table presents the fraction of years that agents in the three groups of lifetime earnings spend in the same age specific earnings groups. Panel A reports the data. Looking at the first column, the data tell us that an agent that is in the top 0.1% of the lifetime earnings distribution spends about a third of her working years in each of the three age specific earnings groups, while someone in the next 0.9% lifetime earnings group spends 38 percent of her working life in the same age specific earnings group and 58 percent in the bottom group.

Table 4: Mean Fraction of Working Years in Age-Specific Group

Panel A: Data			
	Lifetime Earnings Group		
Age-Specific	Top 0.1%	Second 0.9%	Bottom 99%
Top 0.1%	0.33	0.04	0.00
Second 0.9%	0.36	0.38	0.00
Bottom 99%	0.31	0.58	0.99

Panel B: HIP Model			
	Lifetime Earnings Group		
Age-Specific	Top 0.1%	Second 0.9%	Bottom 99%
Top 0.1%	0.41	0.04	0.00
Second 0.9%	0.32	0.43	0.00
Bottom 99%	0.27	0.53	0.99

Panel C: Guner et al.			
	Lifetime Earnings Group		
Age-Specific	Top 0.1%	Second 0.9%	Bottom 99%
Top 0.1%	0.66	0.04	0.00
Second 0.9%	0.34	0.95	0.00
Bottom 99%	0.00	0.01	1.00

Panel D: K & K			
	Lifetime Earnings Group		
Age-Specific	Top 0.1%	Second 0.9%	Bottom 99%
Top 0.1%	0.27	0.08	0.01
Second 0.9%	0.12	0.18	0.01
Bottom 99%	0.61	0.74	0.98

Our HIP model matches these figures remarkably well: An agent in the top 0.1% of the lifetime earnings distribution spends 41 percent of her working life in the same age specific earnings group, 32 percent in the next 0.9% and 27 percent in the bottom group. An agent in the next 0.9% of the lifetime earnings distribution in the model spends 43 percent of her working life in the same age specific earnings group, and 53 percent in the bottom group. Panel C and D report the same statistics for the papers in Guner et al. (2016) and Kindermann and Krueger (2020). Consistent with the findings in Table 3 we find excessive persistence in the model of Guner et al. (2016) where the top 0.1% and next 0.9%

of the lifetime earnings distribution spend respectively 66 and 95 percent of their time in the same age specific earnings group. We also find too little persistence in the model of Kindermann and Krueger (2020) where the top 0.1% and next 0.9% of the lifetime earnings distribution spend respectively 27 and 18 percent of their time in the same age specific earnings group and 61 and 74 percent in the bottom group of the age specific distribution respectively.

5 Conclusion

Models of optimal tax progressivity need to generate the correct amount of earnings concentration especially at the top of the distribution. In this paper we have shown that processes for wages used so far while correctly matching top earnings shares fail in other dimensions of their dynamics that are critical to the taxation result. We have also proposed an alternative approach to modeling the wage process featuring heterogeneous growth profiles with superstar states. This approach improves along those dimensions and with further refinements in the calibration could become a candidate for further work in this area.

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