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History of mathematics as a tool to educate for anti-racism

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History and Epistemology in Mathematics Education

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HISTORY OF MATHEMATICS AS A TOOL TO EDUCATE FOR ANTI-RACISM

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ABSTRACT

In the present paper, we analyze how the history of mathematics – and that of STEM disciplines in general – can become an effective tool to educate and heighten awareness about the Memory of the Shoah, with the aim of promoting an antiracist attitude. Several actions to achieve this goal have always been conducted in schools within the humanities. However, we would claim the important role assumed by the history of science in this perspective, starting from the analysis of some educational activities proposed in schools within the project ‘*Mathematics knows no races or frontiers*’. *Ideas for historical-mathematical reflection for an antiracist education*.

1 Towards an antiracist education

While the awareness towards the Shoah had always been prerogative of humanities in educational context, only in recent years it emerged as unspoken potential of scientific disciplines. Hence, we would discuss the opportunity and the impact of introducing these issues in schools starting from a different perspective, the one of the history of mathematics and the history of science. The need to promote antiracist attitudes among the youngest stems also from the increasing episodes of intolerance. The growth of migration flows and the persistent economic difficulties have risen concerns at all levels regarding the return of racist and xenophobic attitudes among the Italian population. In the last few years, the national monitoring centre denounced the spread of acts of intolerance against those who are regarded as ‘different’ for reasons of ethnicity, race, religion, gender, sexual orientation, physical or mental disability. A substantial percentage of these acts takes place in schools and, having the younger generation as protagonist, is amplified by social networks. The task of educating to inclusion and antiracism, also included in the goals of the United Nations’ action program “Agenda 2030” in connection with the development of global citizenship, is one of the most compelling challenges of our educational system.

Although there are several initiatives in this direction, they are often piecemeal and fragmented and there is the risk of ‘losing’ their true meaning.

The best practices of didactics of the Shoah stress that racist preconceptions essentially spring from ignorance, and only thanks to the knowledge of what happened in the past it is possible to acquire the tools needed to identify and combat the symptoms of intolerance. The national guidelines *Per una didattica della Shoah a scuola*⁸⁸ identify the interdisciplinary and transcultural perspective as the root cause that makes the anti-racism education one of the most complex educational challenges, since it involves different skills and specializations. Conversely, the cross-cultural dimension invoked is typical of the multi-disciplinary approach of the history of science and mathematics. Because of their universal language, which transcends every race and frontier, STEM disciplines lend themselves well to develop these themes. A good teaching of the history of scientific disciplines can contribute to counter the cognitive-essentialist bias which underlies the racist views of new generations (Corbellini, 2020; Rutherford, 2020). Historical-scientific research may become an educational tool and a carrier of public engagement aimed at ‘disarming’ false arguments, stereotypes and slogans which are often puerile but simple, and therefore effective (Castelnuovo, 1997; Segre, 2018). The plots of the internal history of STEM, can make the future generations aware of the instrumental use that was made of such disciplines, which are usually considered – by their very nature – impervious to ideological conditioning. In this regard, it is hardly necessary to recall that state racism in the German Reich and in the Fascist Italy, justified by the false myth of ‘racial purity’, broadly benefitted from the scientific support and the complicity of some men of science to promote forms of discrimination and persecution, besides to plan and perpetrate eugenic and/or extermination programs. However, the horizons of historical research regarding the correlation between mathematical, statistical, biological, medical, psychiatric sciences and racisms did not end with the Short Century and have been progressively extended to the short and long term, in a perspective of connected history, taking into account social Darwinism, the relationships racism-colonialism, science-apartheid, etc.

⁸⁸ i.e. *For a didactics of the Shoah in schools.*

2 *A response to local needs*

Our project fits in a well-defined context, that of Piedmont, whose chronicle has recently recorded many serious acts of racism and anti-Semitism: writings on walls, drawings of swastikas, insults of despicable violence, which bring back in vogue the most forbidden Nazi-fascist stereotypes. These are intolerable facts for a society that wants to be inclusive, free, and democratic, and even more for a city like Turin, Gold Medal of Resistance. The perception that these are not isolated episodes and the consequent concern about the surfacing of forms of hatred that make the past dramatically close, are reflected in the statistical data: from January 1, 2018, to the end of February 2019, 118 acts of racism were reported in Piedmont, 36% of which in Turin and its metropolitan area. In 8% of cases, they occurred in a school context. On the other hand, according to *IRE Piemonte* annual report for 2020, only 13.67% of the population considers racism and other forms of discrimination to be of concern and, due to Covid-19 emergency, this percentage dropped to 9.5% in 2021. In the same year, however, there was a 13% increase in acts of intolerance compared to the previous year. In our region – but also in Italy in general – it is regrettable to note the scarcity of proposals and materials concerning STEM disciplines, whose ancient and recent history can make a valid contribution to a deeper understanding of this issue. This is even more regrettable considering that the Piedmontese scientific community was among those most affected by the racial laws of 1938 (Capristo, 2014; Luciano, 2018). Hence, came about the idea of the Research Group in History of Mathematics at the University of Turin (coord. by prof. Luciano) to design educational activities and training courses to sensitize teachers and students to Memory and awareness, starting from the critical re-reading of some aspects and moments of research and teaching of mathematics and science during the fascist dictatorship and motivating the direct involvement in research of sources and investigation of facts. Delving into the historical past, studying and analysing it with the typical lucidity of scientific thought and logical-deductive argumentation is a significant operation also in relation to our present and to the multi-identity and multi-ethnic society in which we live.

3 *Educational activities*

In the school years 2020-21 and 2021-22 three schools were involved in the project. During the first year, students (8th degree) of the lower secondary school “U. Foscolo” in Turin delved into the personal and professional trajectories of scientists who were victims of racial persecution and experienced the effects of state anti-Semitism, reading and commenting on the correspondence of that ‘dark’ period. In the following year, they focused on the impact of racial laws in Italy on the lives of female mathematicians and scientists. The fates of women examined were divided into three macro-scenarios: emigration, permanence in Italy in hiding, and deportation to concentration camps. In both cases, to make their research accessible to a wide public, students created a freely navigable multimedia content which was published on the school website on the Holocaust Memorial Day.⁸⁹

Two different paths were taken by the two high schools that took part in the project. At the IIS “Santorre di Santarosa” in Turin, three classes (11th-12th degree) with biochemistry study address opted to focus on the eighteenth-century debate on polygenism and racism and on the instrumental use of mathematics (statistics, demography, ...) and science (biology, anthropology, medicine, ...) to justify colonialism, racism and anti-Semitism, in modern and contemporary times.⁹⁰ In addition, some students were involved in deepening the figure of three scientists who lived during the Nazi-fascist period, chosen in relation to their address of study: J. Mengele, E. Segrè and R. Levi Montalcini. Another class began with the history of the school: its building is one of the finest examples of Fascist-era architecture in the city of Turin, with a Littoria Tower identical to that of the city’s central square. Starting from the local context, they analysed the effects of the racial laws on the city of Turin with special attention to scientific faculties.

At the Liceo “G. Peano” of Cuneo, the Mission Memory path was pursued: teachers and students rediscovered personal and professional trajectories of their colleagues who were persecuted for racial reasons, with particular focus on the partisans struggle in the Langhe. After having listened to the testimony

⁸⁹ The final products are available online at the links below.

https://www.icfoscolo.org/wp-content/uploads/2021/01/MOOC_Giornata-della-Memoria.pdf

https://www.icfoscolo.org/wp-content/uploads/2022/01/Scienziate-ebree_MOOC.pdf

⁹⁰ To consult their work, see <https://www.dropbox.com/sh/17xeqssf4dfgvse/AABPWWTa-HqxMl63gyUkGzuHa?dl=0>.

of the Montanari family, one of the oldest Jewish families in Cuneo, and having discussed with the scholar of Judaism A. Cavaglioni, they privileged the historical events linked to the territory. Older students (13th degree) wanted to pay tribute to two of the many scientists of Jewish origin who brought fundamental developments to their disciplines and were affected by the consequences of racial laws: V. Volterra and T. Levi-Civita. Starting with archival research, younger students (11th degree) focused on the reconstruction of the biography of Ugo Levi, who taught mathematics and physics in Saluzzo in the historical period considered. They also interviewed some of his former students (B. Segre, an engineer, and A. Bosi, in turn then professor of history and philosophy at the same school) to learn more about his personality as a teacher and how he was affected by the anti-Semitic wave that swept through Fascist Italy. At the end of this path, a web page was created, containing the research and reflections of the students, with the aim of disseminating the results and to give back to the territory a piece of its history (<https://liceocuneo.it/progetto-pls/>). This project was likewise an opportunity for a rapprochement between retired and current teachers and students and for a rediscovery of their roots.

4 Conclusive remarks

Despite the limits of a temporally limited intervention, positive results in terms of appreciation and involvement of schools indicate the usefulness of continuing the path of Shoah education also from the perspective of history of STEM. According to the feedbacks of teachers and students, some aspects are particularly significant. Firstly, this kind of awareness-raising to the Memory of the Shoah can ensure a more inclusive and interdisciplinary didactic approach and can be declined in many ways and at different school levels. At the same time, it contributes to promote social integration and cultural inclusion of students from fragile backgrounds: indeed, reflecting on past forms of discrimination can help to avoid repeating the same mistakes in the present.

Secondly, these experiences reveal the effectiveness of dealing with historical sources: beyond their intrinsic importance, they captivate students, thus facilitating a major level of commitment. The analysis of historical documents was appreciated by the students as they 'got closer' and empathized with those who experienced first-hand the effects of state anti-Semitism and, from these readings, developed some reflections on today's context, actualizing what they

learned.⁹¹ Moreover, such critical reading helps to make mathematics and science living subjects in students' eyes.

Lastly, educating for tolerance is notably significant relating to civic education: learning to appreciate other people and their differences is one of the key skills of the rising generations of citizens. To this end, history of STEM becomes a resource, a vehicle for improving students' consciousness: "history is something that can make us aware of who we are, and how we have come to be the individuals that we are" (Radford, 2014, p. 89).

The history of mathematics and STEM disciplines, hitherto little involved in the field of antiracist education, can therefore provide an important means of combating ignorance, with the goal of defeating racial discrimination and intolerance, and can contribute to identify appropriate antiracist resources to incorporate into school curricula. We hope that the ideas, insights and experiences illustrated can form a basis for new educational actions in this direction, which are especially needed in our society, crossed by currents of racial hatred and contempt for diversity.

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⁹¹ In this regard, some student comments are given here: "In the letters you can see that Jewish scientists tried to find work even in a place far away from where they worked, taking their family with them, because they were thinking about their children and their future."; "It is important to understand that it doesn't matter if you follow a different religion or speak a different language from others because we are all equal."; "What happened to the Jews certainly must not happen again because we have to think about what they had to suffer and that there were too many victims who died unjustly.".

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“MODERN MATHEMATICS” IN ITALY

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ABSTRACT

This contribution deals with the history of the attempts to introduce the so-called "Modern Mathematics" into teaching in the 1960s. After a short presentation of the global situation, we focus on the case of Italy. We will present the development of the phenomenon and focus on the opinion of teachers and mathematicians who were involved in the debate. We will mention some of them and we will focus on why "Modern Mathematics" in Italy has had a limited diffusion, except for some aspects.

Introduction

The term "Modern Mathematics" is intended to indicate the new approach to mathematics based on set theory and algebraic, order and topological structures. In the intentions of the proponents, mathematics, even at school, had to be transformed from a study on entities to a study of the properties common to these entities, according to the proposals coming from the "Bourbaki Group", a collective of mainly French mathematicians who, starting from 1930s, proposed to found the whole mathematics on algebraic, order and topological structures. Their main work was *Éléments de mathématique* [Elements of mathematics] (in eleven volumes, 1939-2016, about 7000 pgs.). In other countries, primarily the United States, similar, albeit different, movements are labelled "New Math", see (Furinghetti & Menghini, 2023a).

1. “Modern mathematics”

The OECD (Organisation for Economic Cooperation and Development) Roy-aumont (France) Seminar, November 1959, is a milestone of the reform of European “Modern Mathematics”. As reported in (Furinghetti & Menghini, 2023a, p. 59) in short, “the theme of the Seminar was not only the need for new thinking in both mathematics and mathematics education – including changes

in curricula and teacher training – but also the development of appropriate follow-up action (OEEC, 1961). Conviction of some participants was that introduction of new topics should facilitate the study of abstract algebra, analysis, applications of physics and other sciences in university courses.

In his talk delivered at the Royaumont Seminar, Jean Dieudonné - a leading exponent of the “Bourbaki Group” – launched his famous motto "*À bas Euclide! Mort aux triangles!*". Dieudonné's provocation is emblematic, meaning the outdatedness of traditional teaching and in particular of Euclidean geometry.

Some members of the CIEAEM - Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques [International Commission for the Study and Improvement of Mathematics Teaching] took part in this conference. After his Royaumont lecture, in 1964 Gustave Choquet published *L'enseignement de la géométrie* (Italian translation in 1967).

At Royaumont only the general lines of a clear separation from tradition were laid. The following year (August 1960) in Dubrovnik (now Croatia) the concrete indications for the teaching of mathematics in lower and upper secondary schools were drawn up. In October 1961, the OECD published the "Dubrovnik Program" with the title *Un programme moderne de mathématiques pour l'enseignement secondaire* [A modern mathematics curriculum for secondary education].

In the same years, Jean Piaget's theories on the close correlations between the construction of the child's mental structures and mathematical structures were disclosed. This very concise reference to Piaget's thought is to remind his participation in the debate stated, for instance, by his vice-presidency of CIEAEM.

The new programs of maths were applied in the secondary schools, first in France and Belgium. Almost immediately, the movement of modern mathematics was criticized for the excessive formalism and the claim to replace "traditional" mathematics, especially Euclidean geometry, with "modern" mathematics since the earliest grades of elementary school. Famous mathematicians, such as René Thom (1923-2002) and Hans Freudenthal (1905-1990), have expressed criticisms. Thom (1970) argued that we shouldn't privilege the formal rigor, but the meaning of the mathematical objects involved, because children do not base their thinking on abstract structures that pre-exist in their mind, but rather on concrete experiences.

2 The teaching of mathematics at the beginning of the 1960s in Italy

At the beginning of the 1960s, universities in Italy began to update their courses according with the new proposal from ‘beyond the Alps’ introducing, for instance, abstract Algebra in the first year of the degree course in mathematics. On the contrary, primary and secondary school education remained rather traditional, despite the various proposals for renewal after institution of Unified Middle School (for pupils aged 11-14) in 1962; see (Linati, 2016).

Several Italian mathematicians and teachers participated in international seminars. Luigi Campedelli (Florence University) and Emma Castelnuovo (Middle School "Tasso", Rome) were the Italian delegates at the OECD Roy-aumont Conference (1959). Mario Villa (Bologna University) and Ugo Morin (Padua University) were present at the OECD Dubrovnik meeting, 1960.

The International Commission on Mathematical Instruction (ICMI) and the Commissione Italiana per l’Insegnamento Matematico (CIIM) [Italian commission for mathematical teaching] organized an international conference in Bologna from 4 to 8 October 1961 on the teaching of modern mathematics in secondary schools. Following the proposals that emerged at the Bologna Conference, a course for teachers was organized, and 40 "pilot classes" were set up in 1962-63. It was an experimentation of a "Bourbakist" type of teaching: see (Villa, 1962). For this experimentation, a group of mathematicians wrote the school textbooks: Mario Baldassarri, Ugo Morin, Mario Villa, Luigi Campedelli, Tullio Viola among others. We would like to highlight (Morin & Busulini, 1962). The 40 "pilot classes" took place in upper secondary schools (lyceums and institutes for prospective primary teachers). The experimentation generally involved the penultimate grades of secondary school, even if up to that point the students had already been trained on completely different topics and methods (Ciarrapico & Berni, 2017).

Vita (1986) notes that the outcomes of the "pilot classes" were never collected and publicized. He claims that the failure of that experience was due to the lack of a global plan.

3 “Modern mathematics“ and renovation of Italian teaching

Also Italian mathematicians and teachers criticized the pedagogical proposal of the "modernists", assuming a position that was in line with the most part of opinions at international level (Fabris, 2021; Furinghetti & Menghini, 2023b). Some mathematicians and teachers elaborated their alternative proposals.

Of the two Italian participants in Royaumont Seminar, Luigi Campedelli (1903-1978), president of CIIM in 1964-1972, showed a certain detachment, or at least caution, regarding modern mathematics (Furinghetti, 2019).

Emma Castelnuovo (1913-2014) tried to follow the new guidelines on teaching modern mathematics, adapting the contents to the skills and competences of middle school students. She believed it necessary, however, to introduce the concepts of modern mathematics in a natural way as soon as the opportunity arose. She opted for a dynamic and active didactic, based on movement, on borderline cases, on intuitive infinitesimal reasoning, on simple models and on the presentation of counterexamples. She was author of school textbooks, promoter of mathematics exhibitions, and author of publications for teachers' training.

Bruno de Finetti (1906-1986) proposed a genetic method, antithetical to the axiomatic one of modern mathematics. He agreed about a radical simplification and revision of the mathematical tools and advocated a unified vision of different mathematical topics. He considered "Euclid's geometry so unnatural and heavy because of the lack of distinction between affine and metric properties" (de Finetti, 1965, p. 124).

The attempt to introduce modern mathematics into primary and secondary schools, although essentially unsuccessful, has had the positive effect over time of stimulating a rethinking of mathematics teaching and the subsequent development of projects. We would like to mention, in particular, the "Prodi Project" (proposed and coordinated by Giovanni Prodi, 1925-2010) and his volumes of *Matematica come scoperta* [Mathematics as a discovery] (published between 1975 and 1982). Other important textbooks: Lucio Lombardo Radice and Lina Mancini Proia, *Il metodo matematico* [The mathematical method], 1977; Francesco Speranza and Alba Rossi Dell'Acqua, *Il linguaggio della matematica* [The language of mathematics], 1979; Walter Maraschini and Mauro Palma, *Problemi e modelli della matematica* [Problems and models of mathematics], 1981; Bruno Spotorino and Vinicio Villani, *Matematica: idee e metodi* [Mathematics: ideas and methods], 1982. About these textbooks, see (Tomasi, 2012).

We recall the theme on "Correspondences and structural analogies" in the ministerial programs for the middle school of 1979. It was introduced with the following words: "[It] will not give rise to a separate discussion. During the three years, whenever the opportunity arises, similarities and differences between different situations will be recognized" (our translation). This statement

well summarizes the orientation in Italian mathematics education in the seventies and eighties.

In the primary school, it became usual to start by introducing logical concepts and operations on sets (including intersection, union, complement, ...) and only later to introduce the concept of natural number. A project coordinated by the University of Pavia included activities linked to logic and set theory for the introduction of connectives, or activities on relations between sets. Some teachers attempted integration between different approaches, also by using materials such as Dienes logic blocks (Furinghetti & Menghini, 2023b).

4 Conclusion: "Modern Mathematics", what remains?

What is left of "Modern Mathematics" in the 1970s and in subsequent years, in Italy? It can be observed that today there is a large gap between mathematics presented in secondary school and that proposed in university teaching. Provocatively, we can say that mathematics taught in secondary school is still stuck, for some aspects, to a century ago! Instead, in the universities the new topics find a lot of space.

The most widespread Italian textbooks report specific parts dedicated to geometric transformations. There is also a partial reorganization of the proposal of traditional themes: e.g., the tracing of curves obtained with translations and symmetries of elementary curves with respect to the Cartesian axes. Some schools promote supplementary courses on the topics such as algebraic structures and topology. We recall the experience of the "Mathematical Lyceums" in which the teaching of mathematics is enhanced also through the contents proposed by the modern mathematics.

Main merit of the debate on the modern mathematics is to have questioned the tradition. In the programs for Primary School (1985), it is stated that "logic and theory of sets are no longer the foundation of mathematics, but a means to analyse the mathematical discourse and guide its development" (Pellerey, 1989, p. 176). Giovanni Prodi recognises that "Undoubtedly today there would not be such a widespread awareness of the unity of mathematics if Bourbakism had not existed" (Prodi, 1995, p. 416). Summing up, the phenomenon of "Modern Mathematics" appears to be a booster of the change that has taken place in the Italian school and a promoter of international openings.

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